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Page 1404, Table 11, Column R, line 3(*b*), insert "+ 0.510"; and change "*CN* (in 0.1 ft)" to "*CN* (in 0.01 ft)."

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Page 990, change Eq. 50*a* to read:

$$\Delta L = \frac{8}{3 \sec^2 \alpha} \left[\frac{f^2}{L} - \frac{(f + \Delta f)^2}{(L + \Delta L)} \right] + \Delta L_d \sec \alpha \text{ (very nearly)} \ldots (50a)$$

Page 991, lines 12 to 15, delete the last sentence of this paragraph.

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EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

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WITH DISCUSSION BY MESSRS. F. C. ROBERTS, JR., ROLF ELIASSEN, FREDERIC BASS, DARWIN W. TOWNSEND, LLOYD H. FLICKINGER, ISADOR W. MENDELSOHN, RALPH E. FUHRMAN, R. W. ROWEN, AND GEORGE J. SCHROEPFER.

SYNOPSIS

The Minneapolis-St. Paul (Minn.) sewage treatment plant began operating on June 1, 1938. In reporting the experiences of the first two years of operation, data are included on the performance of the plant and the cost of operation and maintenance, as well as on the improvement of the Mississippi River which has resulted from its operation. Some of the problems that arose in early operation, and the methods used to overcome them, are discussed, as well as other improvement changes which have been incorporated to simplify operation or effect economies. It is hoped that such statements will encourage the presentation of data on similar experiences at other plants, as well as make available for the benefit of others the experiences at this plant.

DEGREE OF TREATMENT

Because of its importance to a proper consideration of the results of operation and the statements concerning experiences made herein, as well as because it is the most important factor in the consideration of any pollution abatement project, some space is devoted to the degree of treatment provided in this project.

The Mississippi River at St. Paul fluctuates widely, both as to discharge (which varies from a minimum of 864 cu ft per sec to a maximum estimated flow of 107,000 cu ft per sec with an average annual flow of 8,800 cu ft per sec) and in the physical, chemical, and biological factors that affect its capacity to receive and assimilate pollution. This condition, and the fact that the Twin City sewage formerly discharged, and now the effluent of the treatment plant discharges, into pools created by the canalization project, affects to a greater

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or lesser extent the type of treatment selected, the degree, and the duration for which various degrees of treatment must be maintained. In the studies of treatment necessary, the information collected on river conditions by various authorities over the relatively long period of practically continuous record from June, 1926, to October, 1934, when the decision on type of treatment was made, was of prime importance in adjusting proposed sewage treatment to river requirements. This record of analytical data has been continued, so that at the present time (1941) it covers a period of fifteen consecutive years.

The pollution load on the river is predicted to reach a 1,300,000 population-equivalent for the Twin City metropolitan area by 1945, exclusive of South St. Paul and Newport, Minn., from a tributary population of 910,000. Computations using the 42-yr period of river record, from 1892 to 1933, inclusive, indicated, for the foregoing pollution load, (1) that sedimentation alone (36% B.O.D. reduction—that is, 5-day biochemical oxygen demand) would suffice to meet river standards as interpreted herein, except for 12.5% of the time; (2) that sedimentation plus effluent filtration (45% B.O.D. reduction) would be sufficient except for about 8% of the time; and (3) that sedimentation and effluent filtration, plus chemical treatment (to 70% B.O.D. removal), would maintain standards except for 1.5% of the time.

A 5-day B.O.D. removal of 70% was adopted as the degree of treatment necessary to meet all reasonable river requirements satisfactorily. A variety of treatment processes were investigated progressively in the period from 1927 to 1934 to provide this removal. As a result, sedimentation plus effluent filtration, augmented at times of critical river flow by chemical treatment, was adopted as the process which would meet the river requirements most satisfactorily and economically.

A clearer conception of the variation in treatment requirements can be had from the following: Considering river flows in the period from 1892 to 1933, and the expected 1945 pollution load, chemical treatment would be required during only 40 months of the total of 504 months, and the average B.O.D. removal required during the 40 months that chemical treatment was necessary was approximately 58%. Expressed in another way, sedimentation plus effluent filtration would be sufficient treatment based on past river flows for a continuous period of fourteen years; and then, after a 2-yr period when chemical treatment would be required, preliminary treatment again would be sufficient for another 12-yr period. Sedimentation plus effluent filtration would be sufficient, and no chemical treatment would be required for thirty years out of the 42-yr period of record.

General Plant Description

The treatment plant[2] was designed for an average daily flow of 134 mgd expected from a tributary population of 910,000 in 1945. The intercepting sewer leading to the plant has a capacity of 610 mgd, which flow will be reached on infrequent occasions during storms.

Described briefly, the sewage treatment plant consists of screen and grit chambers, flocculating and settling tanks, and effluent filters, with provision

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[2] For a more detailed description of the treatment process see "Layout and Plan of Operation of the Minneapolis-St. Paul Sewage Treatment Plant," by George J. Schroepfer, *Sewage Works Journal*, Vol. 9, 1937, p. 913.

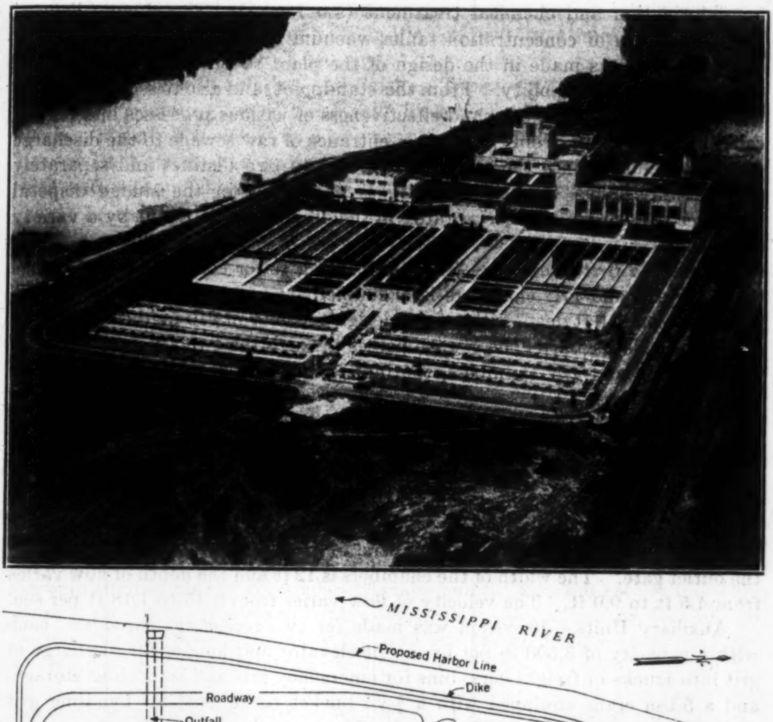
MISSISSIPPI RIVER
Proposed Harbor Line
Dike
Roadway
Outfall
Downward Flow Filters
Settling Tanks
Administration Building
Pump and Blower Building
By Pass
Combined Flocculation and Settling Tanks
Chemical Tower
Screen and Grit Chambers
Combined Flocculation and Settling Tanks
Filter Service Building
Downward Flow Filters
Settling Tanks
Filtration and Incineration Building
Sludge Concentration Tanks
Fuel Oil Storage Tanks
Property Line
ST. PAUL BRIDGE & TERMINAL RY. CO. LEASED TO CHICAGO GREAT WESTERN RY. CO.

FIG. 1.—CHEMICAL TREATMENT PLANT, MINNEAPOLIS-ST. PAUL SANITARY DISTRICT

for chlorination and chemical treatment (see Fig. 1). The sludge disposal process consists of concentration tanks, vacuum filters, and incinerators. A distinct effort was made in the design of the plant to provide a maximum of flexibility and dependability. From this standpoint, and also to secure information on the relative economy and effectiveness of various processes and chemicals, the entire plant, from the point of entrance of raw sewage to the discharge of treated effluent into the river, is divided into two distinct and separately operated divisions. Similar flexibility is obtainable in the sludge disposal processes. Features are incorporated in the design for treatment by a variety of processes.

Screen and Grit Chambers.—The intercepting sewer conducting the sewage to the screen and grit chambers is a double-barrel section, each of which is 9.5 ft wide and 10 ft deep, with a total design capacity of 610 mgd. From each intercepting sewer barrel the sewage passes through the two separate batteries of the screen and grit chambers, the individual units of which are described briefly, as follows:

Coarse Bar Racks.—Two hand-raked racks in channels 10 ft wide with a maximum flow depth of 10 ft and with clear openings of 6 in.

Bar Screens.—Four mechanically and automatically cleaned bar screens with one-inch clear openings, each 10 ft wide, with a maximum sewage depth of 8.7 ft.

Grit Chambers.—Eight grit chambers, 62.75 ft long, from the inlet gate to the outlet gate. The width of the chambers is 12 ft and the depth of flow varies from 4.5 ft to 9.0 ft. The velocity of flow varies from 0.45 to 1.18 ft per sec.

Auxiliary Units.—Provision was made for two screenings shredders, each with a capacity of 3,500 lb per hr; a grit elevator and hopper for discharge of grit into trucks or freight cars; bins for emergency grit and screenings storage; and a 5-ton crane equipped with a ¾-yd bucket to be used for handling grit and screenings from these bins, as well as for several other purposes.

Chemical Control Building.—The chemical control building is a part of the superstructure over the screen and grit chamber. It houses the dry chemical feeders, chlorinators, pneumatic conveying equipment, and storage bins, described in more detail as follows:

Chemical Feeders.—Three dry chemical feeders of the gravimetric type, each having a feeding rate infinitely variable between 400 and 2,500 lb per hr.

Lime Feeders.—Two lime feeders, each of which has a capacity range from 400 to 2,500 lb per hr, equipped with lime slakers.

Chlorinators.—Five solution feed chlorinators, each with a capacity range from 850 to 6,000 lb of chlorine per twenty-four hours.

Pneumatic Conveying Equipment.—Pneumatic conveying equipment with a capacity of 10 tons per hr for unloading and elevating dry chemicals.

Chemical Storage Bins.—A total of ten concrete storage bins, each having a capacity of approximately 2,550 cu ft.

Flocculating and Settling Tanks.—Included under this heading are the venturi meters, combination flocculation and settling tanks, and the settling tanks.

Venturi Meters.—Each of the two venturi meters has an inlet diameter of 9 ft and a throat diameter of 5 ft, and is designed for a minimum flow of 43 mgd and a maximum flow of 350 mgd.

Combination Flocculating and Settling Tanks.—Although they are designated as flocculating tanks, these units have been arranged for multiple functioning, including their use as a by-pass channel, settling tanks, post-chlorination contact tanks, and grease separation tanks, in addition to flocculation with and without chemicals. The tanks proper consist of two units, one adjacent to each battery of settling tanks, each unit consisting of two passes which are each 17.75 ft wide and 290 ft long with an average water depth of 15.5 ft. The tanks provide a detention period of 24.5 min at the average flow of 134 mgd. When used as settling tanks, they therefore add 24.0 min to the sedimentation period.

Settling Tanks.—The two batteries of settling tanks consist of three units, each 56 ft wide, 290 ft long, and with an average water depth of 15.5 ft. Each tank has three passes with a width of 18.0 ft. The detention period in the settling tanks at the average flow of 134 mgd is two hours. The tanks are provided with sludge removal and skimming mechanisms. A rather comprehensive system of effluent weirs has been provided, having a total length of 480 ft for each tank and providing 0.047 million gal per ft of weir length at average flow.

Pump and Blower Building.—In this building are located various services used in connection with flocculation and sedimentation, including two 300-gal-per-min sump-drainage and under-drainage pumps; two 200-gal-per-min scum pumps; eight 210-gal-per-min triplex displacement sludge pumps; two 500-gal-per-min centrifugal sludge pumps for peak and emergency purposes; one 2,500-gal-per-min tank dewatering pump; and three blowers, one 1,200 cu ft per min, one 1,800 cu ft per min, and one 2,400 cu ft per min, along with the switchboard for control of the various equipment, a small office, and various minor miscellaneous services.

Effluent Filters.—The effluent filters are of the downward-flow magnetite sand type, are eight in number, and are divided into two batteries. The total filter area is 31,200 sq ft, which provides a filter rate of 3 gal per sq ft per min at average flow of 134 mgd. Each filter bed is 16 ft wide and 244 ft long.

Filter Service Building.—In this building are located three 1,500-gal-per-min wash-water pumps, two 300-gal-per-min sump pumps, one pump of 300-gal-per-min capacity, and two pumps of 450-gal-per-min capacity each to pump plant effluent for use in sluicing ash or dissolving and transporting chemicals; and three motor generator sets of 50-hp capacity each.

SLUDGE DISPOSAL

General.—The sludge removed from the settling tanks is pumped to the sludge concentration tanks. After concentration the sludge is conducted to the conditioning tanks, thence to vacuum filters, and finally to the incinerators.

Concentration Tanks.—Two such tanks, each 95 ft long, 18 ft wide, and with a total water depth of 24 ft (effective capacity of 41,000 cu ft each), are provided, with sludge collector mechanisms and means for removing scum.

The number of tanks in use, the depth of sludge, and therefore the period of storage are varied with the sludge load.

Filtration and Incineration Building.—In this building, in addition to the vacuum filters, incinerators and their appurtenances, the machine shop, boiler room, and electrical control equipment are located.

Vacuum Filters.—The sludge from the concentration tanks is fed to either of two bucket elevators which discharge into three air-agitated conditioning tanks. From these tanks the sludge discharges by gravity into six vacuum filters arranged in two batteries. Each filter is 11.5 ft in diameter and 14.0 ft long, and has an effective filter area of 500 sq ft. Among the main appurtenances of the vacuum filters may be mentioned three vacuum pumps, each with a capacity of 1,850 cu ft per min; three blowers, each with a capacity of 800 cu ft per min; three filtrate pumps, each with a capacity of 250 gal per min; and two belt conveyers with weighing scales, each having a capacity of from 2 to 30 tons per hr.

Incinerators.—Three incinerators of the multiple-hearth type have an outside diameter of 22.25 ft, with eight hearths, each incinerator having a nominal capacity of 60 tons of dry solids. The incinerator ash is discharged into hoppers, from which it is sluiced to ash pumps, and thence is discharged to the dump. The main appurtenances of the incinerators are three plate-type preheaters, three hot-air fans each with a capacity of about 11,000 cu ft per min, and three hot-gas fans each with a capacity of about 36,000 cu ft per min.

Chemical Handling and Feeding.—A pneumatic conveying system with a capacity of 6 tons per hr has been provided for unloading dry chemicals and discharging them into storage bins.

Chemical feed equipment was installed for feeding lime with either chlorinated copperas, ferric sulfate, or ferric chloride, the use of which will depend on the efficiency and market prices of the various chemicals from time to time. Two gravimetric lime feeders and slakers with a capacity range from 200 to 1,200 lb per hr each, as well as two copperas or ferric sulfate feeders with a capacity range from 100 to 600 lb per hr each, have been installed. In addition to the aforementioned feeders, two solution feeders for ferric chloride have been provided, each with a capacity range from 12 to 120 gal per hr.

PLANT OPERATION

Operation of Plant Started.—Shortly after the dedication of the treatment plant on May 16, 1938, preliminary operation and testing of the equipment were begun. On June 1 the first sewage was turned into the intercepting sewers and into the plant, and by July 15, 1938, practically all of the sewage of the Twin Cities was conducted to the plant.

The plant was placed in operation under a planned and definite schedule which called for a gradual starting of the various units. As already stated, the plant is divided into two separately operated halves or batteries that can be operated independently of each other. Advantage was taken of this fact in starting the plant. The various plant units were placed in operation one battery at a time. The schedule of placing the plant in operation (which was carefully adhered to) was as follows:

June 1—One battery of screen and grit chambers placed in operation.
June 7—Operation of second battery begun.
June 9—One battery of settling tanks started.
July 2—Vacuum filtration of sludge started and filter cake hauled to dump.
July 6—Settled effluent turned into magnetite filters.
July 7—Second battery of settling tanks placed in service.
July 21—Incineration of sludge begun.

Operation of the various units was begun without unusual difficulty except as might be expected with mechanical equipment of the type required in a large plant of this type. The manufacturers of the magnetite effluent filters and the incinerators experienced the greatest difficulties in placing their equipment in satisfactory operation. Some of these difficulties are discussed in more detail subsequently.

Continuous 24-hr operation of the plant was begun on July 20. From then on all the equipment that had been accepted by the Minneapolis-St. Paul Sanitary District performed in an acceptable manner. However, because of the manufacturer's difficulties with the incinerators, it was necessary to by-pass varying quantities of sewage from time to time, depending on the quantity of sludge that could be incinerated. After October 3 no sewage was by-passed at the plant with the exception of excess storm flows, the excess sludge that could not be handled by the incinerators being pumped directly to the river. It was not until the latter part of November, 1938, that the incinerators were capable of incinerating all the sludge produced at the plant.

With the large volume of equipment in a modern sewage treatment plant, some of which has been used in this field only a few years, the process of placing a large plant in operation is rather involved, especially from a mechanical standpoint. The writer wishes to record with satisfaction the fine cooperation accorded by manufacturers of various equipment. It is gratifying to note that, although extensive guarantee and maintenance bonds of various types were provided for in the mechanical contracts, it has not been necessary to invoke any of their compliance provisions.

General.—The sewage flow arriving at the plant was less than the estimated flow of 120 mgd for the year 1938, averaging 98.8 mgd for the period from July 1, 1938, to May 31, 1940, inclusive. The strength of the incoming sewage, however, was somewhat greater than was anticipated, averaging 275 ppm on a suspended-solids basis, and 210 ppm on a 5-day B.O.D. basis. This compares with expected strengths of 185 ppm and 200 ppm, respectively. The increased strength on a suspended-solids basis places a greater load on the treatment plant, especially on the sludge disposal processes, than was anticipated.

Up to May 31, 1940, a total of 66,864.1 million gal of sewage was treated by the plant, from which a total of 79,944 cu ft of screenings and 501,278 cu ft of grit was removed. In addition, solids equivalent to a total of 183,497.6 tons of filter cake were removed and incinerated. More detailed operating data are given in Table 1.

As already stated, the plant was designed so that treatment by a variety of processes can be provided to meet varying river requirements. Operation

TABLE 1.—Analytical and Operating

Month	Average Daily Sewage Flow in Millions of Gal			Screenings, in cu ft per million gal	Grit, in cu ft per million gal	B.O.D.				
						PPM		Percentage of Removal		
	Total	Minneapolis	St. Paul (by difference)			Raw sewage	Settled effluent	Settling tanks	Effluent filters	Total
(a) Six Months in 1938										
July[a]	93.6	70.4	23.2	....	....	170	105	35.9	....	....
August[b]	71.357	65.776	....	1.15	6.3	180	115	35.8	....	....
September	87.397	65.966	21.4	1.05	6.3	175	120	29.3	....	....
October	89.819	64.322	25.5	0.98	7.2	210	130	38.6	....	....
November[c]	92.835	....	....	1.15	6.9	230	160	31.3	....	....
December	98.59	67.133	31.5	1.00	6.2	245	135	44.7	....	....
Average	88.9	66.6	22.3	1.07	6.6	200	130	36.0	....	....
(b) Twelve Months in 1939										
January	99.266	73.814	25.4	1.12	6.0	230	130	42.5	....	42.5
February	96.481	69.481	26.9	1.17	4.8	235	155	34.3	....	34.3
March	114.543	81.288	33.2	1.24	7.5	200	135	31.9	....	31.9
April	97.532	69.568	28.8	1.61	8.75	210	130	38.5	....	38.5
May	100.383	73.788	26.6	1.76	10.65	205	130	35.0	....	35.0
June[d]	110.836	78.629	33.0	1.28	13.9	170	91	45.7	....	45.7
July[e]	107.105	74.926	32.2	1.03	12.4	165	98	40.2	....	40.2
August[f]	108.150	75.365	32.8	1.04	9.39	170	105	39.6	....	39.6
September	108.389	74.122	36.8	1.17	10.1	190	115	39.7	....	39.7
October[g]	106.157	....	....	1.52	9.3	220	130	40.2	....	40.2
November[g]	94.112	....	....	1.25	7.8	250	145	40.0	2.8	42.8
December	92.286	71.009	21.3	1.11	4.7	240	145	39.6	2.6	42.2
Average	102.937	74.199	29.7	1.28	8.77	205	125	38.9	....	39.4
(c) Five Months in 1940										
January	91.321	....	....	0.88	3.1	240	140	41.8	....	....
February	90.164	....	....	0.96	3.4	235	140	39.4	0.7	40.1
March	109.314	77.051	32.3	1.08	6.5	200	130	33.8	1.6	35.4
April	108.212	67.490	38.1	1.57	7.1	220	140	35.8	0.1	35.9
May	104.898	66.031	38.9	1.08	5.4	220	135	39.0	0.9	39.9
Average	100.782	70.191	36.4	1.11	5.1	225	135	38.0	0.8	37.8

[a] Plant placed in partial operation, 24-hr basis, July 20. [b] Considerable by-passing during August. [c] Minneapolis
[f] Trial chemical treatment using ferric sulfate. [g] 27 days of operation. [h] Gross removals, 70.5% and 77.5%, were
was used in the average for the year. [i] Gross removals, 70.0% and 71.2%, were actually reduced to 59.5% and 60.7%,

to date (1941) indicates that with all units in operation the following removals can reasonably be expected:

Treatment process	5-day B.O.D.	Suspended solids
Sedimentation only	40 to 45%	70 to 75%
Flocculation plus sedimentation	45 to 50%	75 to 80%
Flocculation and sedimentation, plus effluent filtration	50 to 55%	80 to 85%
Flocculation with chemicals, sedimentation, and effluent filtration (dependent upon quantities of chemicals)	55 to 70%	85 to 95%

DATA ON SEWAGE TREATMENT

Suspended Solids					Settleable Solids					Alkalinity				pH		Temperature, in Degrees F		Detention period in tanks, in hr
PPM		Percentage of Removal			Ml per Liter		Percentage of Removal			Raw Sewage		Settled Effluent						
Raw sewage	Settled effluent	Settling tanks	Effluent filters	Totals including screenings and grit	Raw sewage	Settled effluent	Settling tanks	Effluent filters	Total	Methyl orange	Phenolphthalein	Methyl orange	Phenolphthalein	Raw sewage	Settled sewage	Raw sewage	Effluent	
(a) Six Months in 1938																		
235	62	72.9	..	..	7.0	0.3	92.4	..	..	270	0	265	0	7.3	7.3	68	68	2.4
235	75	66.5	..	..	5.9	0.5	91.2	..	..	280	0	270	0	7.2	7.3	70	70	1.5
225	81	62.1	..	..	6.0	1.2	80.0	..	..	290	0	285	0	7.1	7.2	66	..	1.2
255	67	73.0	..	..	6.9	0.6	91.9	..	..	320	0	310	0	7.2	7.3	65	..	1.85
250	83	65.9	..	..	6.7	0.6	90.9	..	..	330	0	325	0	7.2	7.3	61	..	2.45
255	67	73.0	..	..	7.2	0.4	94.3	..	..	335	0	325	0	7.1	7.3	58	57	3.5
240	72	68.8	..	..	6.6	0.6	90.0	..	..	305	0	295	0	7.2	7.3	65	..	2.15
(b) Twelve Months in 1939																		
245	74	70.0	..	73.6	7.0	0.7	88.4	..	88.4	325	0	315	0	7.3	7.5	56	56	2.1
280	105	61.2	..	67.0	7.8	1.7	77.3	..	77.3	330	0	320	0	7.4	7.5	56	55	0.9
360	125	63.3	..	70.8	6.7	1.9	68.4	..	68.4	260	0	255	0	7.4	7.5	54	52	1.2
330	97	69.0	..	75.9	7.2	1.2	83.6	..	83.6	220	0	210	0	7.4	7.5	55	54	1.3
275	85	63.5	..	75.6	6.1	1.3	77.6	..	77.6	270	0	260	0	7.4	7.5	61	60	1.0
310	63	77.9	..	84.6	6.5	0.8	87.5	..	87.5	270	0	260	0	7.5	7.5	65	64	1.5
285	69	74.4	..	82.5	6.3	0.9	86.5	..	86.5	240	0	235	0	7.5	7.5	69	67	1.2
245	75	67.7	..	78.0	6.7	0.9	87.1	..	87.1	265	0	255	0	7.4	7.5	69	68	1.0
250	77	68.4	..	77.9	6.9	0.9	88.0	..	88.0	285	0	275	0	7.5	7.5	67	66	1.0
280	77	72.3	..	78.1	7.8	0.8	89.8	..	89.8	290	0	285	0	7.5	7.5	65	64	1.1
300	88	70.3	2.5	76.2	8.8	0.9	90.4	0.7	91.1	300	0	300	0	7.4	7.5	61	59	1.5
280	81	60.6[h]	4.3	68.6[h]	7.5	0.5	93.7	1.3	95.0	300	0	300	0	7.5	7.5	59	57	1.5
285	85	68.2	..	75.7	7.1	1.0	84.9	..	85.0	280	0	275	0	7.4	7.5	61	60	1.3
(c) Five Months in 1940																		
265	78	59.5[e]	60.7[e]	64.0	7.7	0.5	94.3	0.2	94.5	305	0	300	0	7.5	7.5	56	55	1.5
310	75	74.7	75.7	77.8	7.9	0.3	96.2	0.3	96.5	290	0	285	0	7.5	7.5	56	54	2.4
310	100	65.0	66.5	72.0	6.1	0.8	85.5	0.6	86.1	220	0	220	0	7.4	7.5	52	50	1.2
310	100	66.9	66.9	73.4	6.3	1.1	83.6	0.5	84.1	245	0	240	0	7.4	7.5	53	53	1.1
300	76	73.8	75.4	78.5	7.0	0.4	94.4	0.7	95.1	250	0	245	0	7.5	7.5	58	58	1.3
300	86	68.0	69.0	73.1	7.0	0.6	90.8	0.5	91.3	260	0	260	0	7.5	7.5	55	54	1.5

venturi meters out of adjustment part of November. [d] Trial chlorination and trial flocculation. [e] Trial chlorination.
actually reduced to 60.6% and 68.6%, respectively, because of lighter solids returned to the river; this reduced percentage
respectively, because of lighter solids returned to the river

Screen and Grit Removal.—Sewage was first taken through the screen and grit chambers on June 1, 1938. From that time this unit of the plant functioned in a satisfactory manner. Until May 31, 1940, a total of 79,944 cu ft of screenings and 501,278 cu ft of grit was removed. Related to the 66,864.1 million gal of sewage treated, this amounts to an average of 1.2 cu ft per million gal of screenings and 7.5 cu ft per million gal of grit. Based on a study of 117 screen installations throughout the United States, the average quantity of screenings removed by screens with one-inch openings was 3.1 cu ft per million gal. The quantity of screenings produced by the Twin City plant fortunately was only about one third of the quantity that might have been expected. The quantity of grit removed compares with an average in a number of plants in the United States of 4.1 cu ft per million gal. A relatively clean grit containing an average of 11.7% of volatile solids was produced.

Velocities in the grit chambers were originally maintained at 0.75 ft per sec. In 1940 velocities during dry weather flows were gradually increased to 1.20 ft per sec for the definite purpose of securing a greater percentage of inert material in the sludge, thereby increasing the solids content of the sludge and reducing the quantity of conditioning chemicals required.

Because of the small quantity of screenings in proportion to grit, it was possible to bury the screenings with an appropriate depth of the relatively clean grit. Shredders were used only to test out the equipment and periodically to keep the equipment in satisfactory operating condition. A variety of material was removed on the hand-raked coarse bar racks (6-in. openings) at the entrance to the plant. Material removed at this point has included both vegetable and animal matter and has ranged from logs and timbers, a section of boiler plate 8 ft by 4 ft, and whole trees to dead dogs and quantities of cattle paunches.

A number of improvement changes were made in this part of the plant which resulted in better and more simplified operation. Some difficulty with wear because of sand entering the lower and intermediate bearings on the screw conveyer grit washers was experienced; but this was partly corrected during 1939 by the manufacturers at no expense to the Sanitary District. However, more positive methods of reducing wear still further have been installed by the Sanitary District, consisting essentially of sealed water-washed and lubricated bearings. After almost one year of operation several bearings of this type installed for observation showed little, if any, wear, and as a result of this experience all bearings have been modified to the new type. From a very troublesome installation requiring replacement of the former type bearings after only one month of operation, this equipment has been converted into a very smooth and satisfactory operating installation. Another item of equipment with which considerable difficulty was experienced in the early part of 1939 was the grit bucket elevator. With grit of certain characteristics, principally that containing high moisture content and a large percentage of "fines," the elevator would not unload satisfactorily, and material would enter the bearings and accumulate on the pulleys and otherwise cause difficulty. By the installation of an air jet to sweep the material out of the buckets, and by installing sealed bearings of a special type, the difficulties with this equipment have been largely overcome.

Sedimentation.—From the time the settling tanks were placed in operation on June 9, 1938, they have operated continuously and satisfactorily. The removals accomplished by the settling tanks were unusually high, from the standpoint of the removals expected, and compared with the removals by most of the other primary settling tanks throughout the United States. The fact that the tanks operated satisfactorily, both from the viewpoint of reliability of equipment and efficiency of removals, is gratifying, considering the fact that the long tanks represent, to some degree, departure from precedent, both for the purpose of effecting construction economies and of accomplishing the higher removals expected from a consideration of a number of theoretical factors.

An effort was made to curtail the removals by the settling tanks, until the South St. Paul and Newport plants were in operation, to an average of approximately 60% removal of suspended solids, by reducing the detention period. With an average detention period for the two years of only 1.5 hr, as compared with a possible detention period of somewhat more than three hours at present flows, the average removal of suspended solids was 69% and of 5-day B.O.D. 38%. Considering the four months of 1939 and 1940, when detention periods of more than 2 hr were maintained (averaging 2.4 hr), the removal of suspended solids averaged 74.4% and of B.O.D. 42.9%. By using all of the settling tanks, a detention period of 3.2 hr can be obtained at present flows, which should result in still higher removals.

Because of the long tanks, the settling tank equipment is operated continuously. Sludge is pumped once each shift, and a determined effort is made to secure as concentrated a sludge as possible. That such efforts were successful is shown by the fact that for the 2-yr period the solids concentration in the raw sludge pumped from the settling tanks averaged 7.7% (92.3% moisture). The skimmings removed from the settling tanks are ejected to an area south of the plant where they are covered with incinerator ash without nuisance, by the simple device of discharging the two materials at the same point.

The aforementioned removals may be compared with expected removals (for a detention period of approximately 2.5 hr) of 56% on a suspended-solids basis and 36% on a 5-day B.O.D. basis. These removals were based on a study of thirty sedimentation plants throughout the United States, reported[3] in 1933. It has been suggested that the relatively high removals were due to a relatively low percentage of volatile matter in the raw sewage, but this opinion was determined to be without foundation. As an indication of the fact that the volatiles in the raw sewage are in line with what is experienced elsewhere, the average for the year 1939 was 71.1%.

A number of improvements were made in the appurtenances to the settling tanks to increase dependability, to reduce costs, and to simplify operation. Perhaps the most important is the method of handling scum after its removal from the settling tanks. Automatic skimming mechanisms remove the material from the settling tanks and deposit it in scum troughs from which, with some flushing, it is transported to scum manholes. In the original installation, scum was discharged from this manhole to a sump from which the suction of the scum pumps was drawn. Difficulties were encountered early because of segregation of materials in this sump, and in an effort to remedy this condition the suction was corrected directly to the pumps and various methods of agitation were installed. However, suction and discharge lines, as well as the pumps themselves, frequently became plugged, and in addition to using large quantities of transporting water it was necessary to screen out all material retained on ¾-in. racks. Nevertheless, after four months of operation, the cross-sectional area of the scum discharge line had been reduced to one tenth of its original area by grease incrustations. A number of different methods of handling were considered, from which an ejection method was selected, chiefly because of its possibilities of handling relatively dry scum, which is an important consideration in connection with final disposal. This system was installed with the result that dry material could be handled without the necessity of previously removing large pieces of material, and without attendant stoppage, all of which greatly simplified disposal problems. Although this problem may appear unimportant from the standpoint of general plant performance, it has greatly reduced the labor required in this part of the treatment process.

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[3] "Factors Affecting the Efficiency of Sewage Sedimentation," by George J. Schroepfer, *Sewage Works Journal*, Vol. 5, 1933, No. 2, pp. 209–232.

No difficulties were experienced with freezing of sewage in the settling tanks of effluent filters during the winter months. Even with temperatures as low as 25° F below zero, sewage temperatures dropped only about 4° F through the plant. Some difficulty was experienced, however, with the automatic skimming mechanisms during cold weather. Heavy frost gathered on moving parts of the equipment during the period that the mechanism was stationary, with the result that some attachment links were broken. It was found much more economical under these conditions to skim the material into the scum troughs by hand than to attempt to keep the equipment operating.

It was expected at the time of design that sludge as removed from the settling tanks usually would contain from 3% to 6% solids. By careful control of sludge pumping, it has been possible to remove sludge from the tanks averaging between 7% and 8%. In order to handle heavy sludge, containing at times as high as 20% solids, without frequent stoppages, it was found necessary to make provisions for blowing back suction lines with plant effluent or compressed air, on a routine schedule. In this manner, difficulties with sludge suction lines as long as 300 ft were eliminated. Careful control of pumping is required to secure a concentrated sludge. The tendency generally is to pump for too long a period. Except under conditions of storm flow, the usual practice is to operate the sludge pumps three times daily, continuing pumping as long as the sludge remains heavy. As a means of maintaining as concentrated a sludge as possible, specific gravities are determined by weighing calibrated containers of sludge periodically during the pumping period.

One of the problems in the early operation of the plant was the difficulty created by mud and silt that were carried into settling tanks during thaws and storms. This material, which was too fine to be removed by the grit chambers, collected in large quantities in the settling tanks and on several occasions stalled the collector mechanisms, with the result that the tanks had to be pumped down and flushed out with a hose. The difficulties resulted partly from the inability to move the mud and silt through the sludge suction pipes, due to stoppage in the lines, which, even after they were cleaned with compressed air, removed only water. Climatic conditions require the use of large quantities of sand and cinders on the streets during the winter months, amounting to a total of approximately 20,000 cu yd during the winter of 1938–1939. Thaws and early rains carry in a fair percentage of this material. Samples taken at half-hour intervals before, during, and after rains in the early summer indicated suspended-solids strength up to 6,000 ppm, compared with averages of approximately 275 ppm. The percentage of volatile material drops very low during these times. Depending on the intensity and interval between rains, present indications are that flows of more than 250% of the average usually result in settling tank and sludge pumping difficulties. It was found that, when a heavy blanket of normal sewage sludge is carried in the tanks previous to storm flows, more inert material can be removed effectively from the tanks because of the evident loosening of the mud and silt which ordinarily packs and results in difficult removal problems. As an indication of practice and experience elsewhere, a study of twenty five of the larger plants in the United States showed that the average of the peak flows treated for these plants was 180% of the average flow treated.

It was found desirable to replace the pistons and the collars on the sludge pumps with heavier parts made in the shop of the Sanitary District at a fraction of the cost of such parts from the manufacturers. Although no unusual difficulty was experienced with the lighter pistons, it was felt that it would be economical to install heavier pistons that would permit resurfacing from time to time, thereby prolonging the life. The only difficulty of any type with the sludge collection equipment in the settling tanks, and that of a minor nature, was the breakage of some of the links of the drive chains, especially during the winter months. A completely satisfactory explanation of the cause of this difficulty has not been found as yet, but after experimenting with different materials and types of chains the conclusion has been reached that the installation of chain tighteners will largely eliminate the previous difficulties. Data on removals by sedimentation for the various periods are presented in Table 2.

TABLE 2.—DATA ON REMOVALS BY SEDIMENTATION

Period of observation	Detention period, in hours	RAW SEWAGE STRENGTH (PPM)		PERCENTAGE REMOVAL			Percentage of total solids in raw sludge
		5-Day B.O.D.	Suspended solids	5-Day B.O.D.	Suspended solids	Settleable solids	
July, 1938, to December, 1938, inclusive	2.15	200	240	36.0	68.8	90.0	7.15
1939	1.3	205	285	38.9	68.9	84.9	7.79
January, 1940, to May, 1940, inclusive	1.5	225	300	38.0	70.1	90.8	8.09

Because of early difficulties in placing the plant in operation, the period from July to December, 1938, is not truly representative of normal operation. Considering the latter two periods, the removals of suspended solids averaged 69.5% and of B.O.D. 38.4%, with a detention period of 1.4 hr (less than one half of the possible period with all tanks in service). The increase in the solids content of the raw sludge with experience is shown in Table 2.

Effluent Filters.—The downward-flow magnetite filters were first placed in operation on July 6, 1938. This equipment was purchased under guarantees providing for removals of suspended solids, as follows:

Influent strength (ppm)	Reduction through filters (ppm)
30	2
50	9
70	18
90	29
110	40
130	50

The filters were designed for an average rate of 3 gal per sq ft per min and a short-time maximum rate of 6 gal. This compares with average rates at other plants generally between 2 and 2.5 gal per sq ft per min.

Early operation indicated that the filters would meet the guaranteed removals, but difficulties with sand erosion and displacement occurred at flows greater than the average. The earlier difficulties were felt to be associated with the hydraulics of the structure, considerable erosion near the inlet ports of the beds being experienced. As a means of overcoming this difficulty, the contractor enlarged the influent ports, which apparently solved the erosion problem. However, with this solved, another problem appeared: The sand was displaced, laterally and longitudinally, by the action of the cleaner carriages in passing over the beds. It is a pleasure to report that, as a result of continued efforts over a long period of time, the manufacturer has apparently corrected these difficulties.

The removals of suspended solids by the effluent filters during thirty-four days of intermittent operation, between May 1 and December 28, 1939, have been as follows:

Suspended Solids (ppm):	
Raw sewage	280
Filter influent	78
Filter effluent	56
Percentage of Removal by:	
Settling tanks	71.1
Effluent filters	8.2
Total	79.3
Detention period in settling tanks, in hr	1.6
Filter rate, in gal per sq ft per min	2.5

During the foregoing period, settling only was used prior to filtration (that is, there was no flocculation or chemical treatment). The sedimentation period prior to filtration was only about one half of the normal detention period in the tanks. Somewhat higher removals prior to filtration can be effected, especially if flocculation (without chemicals) is practiced prior to sedimentation. Partly because of these difficulties, as well as the fact that there operation was not required in any event, the filters were operated only a relatively few days during the 2-yr period covered by this paper.

SHORT-TIME TESTS OF VARIOUS OPERATIONS

Flocculation Without Chemicals.—During the first two years of operation, treatment consisted essentially of plain sedimentation, but flocculation without chemicals, effluent filtration, chemical treatment, and chlorination were practiced for varying periods of time for test purposes. During June, 1939, flocculation without chemicals was practiced in one battery, and plain sedimentation was continued in the other battery for comparison. An average of approximately 0.035 cu ft of air per gallon was used for flocculation. The average removals in suspended solids and 5-day B.O.D. were as follows:

Description	Flocculated and settled	Settled only
Suspended solids	80.1	75.5
5-day B.O.D.	49.8	44.2
Period of flocculation, in min	32.0	...
Period of sedimentation, in hr	1.70	1.64

Chemical Treatment Trials.—During 1939, chemical treatment was not practiced; nor was it considered necessary except to test the various equipment for four days. During this short test period it was impossible to adjust flocculating conditions to secure optimum removals. Ferric sulfate was used as a coagulant and no attempt at *pH*-control was tried. For this test, again, the plant was divided into two halves, in one of which sedimentation alone was practiced; in the other, chemical treatment was used. The results of this short test are as follows:

Description	Plain sedimentation	Chemical treatment
Detention Period:		
Flocculation tanks, in min	...	34.3
Sedimentation tanks, in hr	1.4	1.85
Air, in cu ft per gal	...	0.025
pH-value, raw sewage	7.4	7.4
Chemicals added (in ppm of iron)	...	12
Percentage of Removal:		
Suspended solids	73.0	83.4
5-day B.O.D.	35.3	55.4

Neither of the foregoing tests include effluent filtration, the inclusion of which would increase the removals accomplished.

Disinfection Trials.—Sewage was disinfected with chlorine in half the plant for eleven days in June and July for the purpose of running operating tests on the equipment and obtaining some information on effective dosages. One 16-ton tank car of chlorine was used, and chlorine dosages of 4, 6, and 8 ppm were tried. Pre-chlorination post-chlorination, and split chlorination were used—that is, trials were made applying the chlorine (*a*) to the raw sewage only (pre-chlorination), (*b*) to the settled effluent only (post-chlorination), and (*c*) dividing the dosage between the raw sewage and the effluent (split chlorination). The flocculating tanks were used to provide approximately 30 min contact time for the chlorine added to the settling tank effluent. To avoid

TABLE 3.—DISINFECTION OF SEWAGE (AVERAGE VALUES)

Chlorine dosage (ppm)	BACTERIAL COUNT IN RAW SEWAGE (COLONIES PER ML, 48 HR; AGAR, 37° C)		COLIFORM ORGANISMS IN RAW SEWAGE (*Coli-Aerogenes* PER ML; MOST PROBABLE NUMBER)	
	Total	Percentage reduction	Total	Percentage reduction
4	9,200,000	67.1	170,000	56.1
6	7,600,000	97.9	230,000	97.4
8	6,200,000	99.64	200,000	99.95

any possible changes in bacterial counts in stored samples, only grab samples collected during the morning and afternoon were used to measure the bacterial kill or disinfection. A summary of the results obtained is given in Table 3.

The results of pre-chlorination, post-chlorination, and split chlorination are not shown separately in Table 3. Although they were quite uniform, the tests covered too short a time to justify any definite conclusion regarding the relative effectiveness of the various points of chlorine application. The results indicate that a satisfactory kill of bacteria can be obtained with a comparatively small chlorine dosage of 6 to 8 ppm.

Recirculation of a Percentage of Incinerator Ash.—During the latter part of September, 1940, the return of a percentage of incinerator ash to the raw sewage ahead of the screen and grit chambers was begun. This experiment was conducted as a result of laboratory experiments made approximately two years previously in an effort to improve vacuum filtration and reduce the quantity of conditioning chemicals required. Because this test was begun only recently, it is too early to present conclusive data, but present indications are that very satisfactory reductions in chemicals required for sludge conditioning have been effected, in addition to which appreciable increase in the efficiency of sedimentation resulted. It is the expectation that flocculation will further increase the removals accomplished.

SLUDGE DISPOSAL

Sludge Dewatering; Character of Sludge.—The raw sludge delivered to the concentration tanks varies over quite a wide range in quantity and in the characteristics that affect filtration and incineration. Referring to Table 4, the raw sludge as removed from the settling tanks in the period from September 1, 1938, to May 31, 1940, averaged as follows: *pH*, 6.0; moisture content, 92.26%; and volatile solids, 64.8%. After concentration for a period which generally varies from one to two days, the *pH*-value of the sludge had changed to 5.9, the moisture content to 90.88%, and the volatile solids to 63.6%.

Sludge Conditioning.—The chemicals used for conditioning, except for short periods when ferric chloride alone was used as an experiment, have been lime and ferric chloride. During the period covered by this paper, the quantity of ferric chloride has averaged 2.18% of the weight of dry sewage solids, and the lime 4.87%. However, during the last six months of this period, the quantity of chemicals averaged somewhat less, or 1.89% ferric chloride and 4.37% lime, as compared with the requirements of 2.68% and 9.74%, respectively, during the first six months of operation. Control of the quantity of chemicals used for sludge conditioning has been by a physical examination of the cake and the sludge by the operators, checked by relatively frequent laboratory tests. Early in 1940 continuous control of filtration by the filter operators was begun by means of Bueckner funnel tests. It is of interest to mention that the period of sludge conditioning has been gradually shortened so that in 1940 it averages about 6 min.

Proper mixing and agitation of conditioned sludge prior to filtration is of utmost importance, especially with heavy sludge. The use of large quantities of low-pressure air administered at a number of points resulted in improvement in filtration and a consequent reduction in chemicals required. As originally installed, high-pressure air was supplied through porous tubes. Upon failure of this arrangement to perform in a completely satisfactory manner, various methods were tried, culminating in one in which a total of eighteen small pipes was added to each tank and excess low-pressure air from the blowers, provided for removing cake from the filters, was utilized for agitation. Means for distributing the chemicals more uniformly through the tank were provided, and experiments were made to determine the proper sequence of chemical additions. Studies are under way to effect additional improvements in sludge conditioning.

TABLE 4.—SUMMARY OF DATA ON SLUDGE CHARACTERISTICS AND VACUUM FILTRATION

(September, 1938, to May, 1940, Inclusive)

Month	*pH*-Value		Total Solids (%)			Volatile Solids (%)			Average Tons of Material Daily			Conditioning Chemicals, % of Sewage Solids		Average rate, in lb per sq ft per hr
	Raw sludge	Thickened sludge	Raw sludge	Thickened sludge	Filter cake	Raw sludge	Thickened sludge	Filter cake	Dry sewage solids filtered	Wet cake produced	Incinerator ash produced	Ca O	$FeCl_3$	
1938:														
Sept.	6.4	5.9	7.80	9.10	34.5	59.3	59.5	51.4	44.5	136.7	23.0	10.30	3.30	7.31
Oct.	6.3	6.3	6.29	8.20	34.3	65.0	64.2	52.6	60.8	194.0	31.5	11.70	2.69	5.68
Nov.	5.9	6.1	7.40	9.16	35.1	63.5	62.7	52.6	75.0	216.0	36.0	10.70	2.58	5.02
Dec.	5.8	5.8	7.10	8.65	34.2	67.9	66.2	54.7	97.6	317.0	49.0	11.20	2.80	5.05
1939:														
Jan.	6.2	5.9	7.70	9.52	36.1	64.6	63.9	55.9	101.6	326.6	52.1	8.78	2.36	6.21
Feb.	6.1	6.0	7.18	8.87	34.8	67.8	66.8	62.3	104.0	309.4	41.0	5.76	2.34	4.47
Mar.	6.1	6.0	10.25	11.18	39.1	56.6	55.7	54.3	121.1	285.5	50.0	3.24	1.76	4.90
Apr.	6.0	5.9	8.04	9.61	35.0	65.2	63.2	59.1	102.2	313.3	46.7	4.80	2.06	4.84
May	5.8	5.7	9.08	10.06	35.3	64.8	64.1	60.1	107.7	320.4	48.7	4.72	2.02	5.29
June	5.8	5.7	8.53	10.74	38.3	59.2	56.5	53.9	124.2	317.6	57.2	4.61	2.07	5.29
July	5.7	5.6	7.58	9.05	35.3	62.9	60.8	56.6	90.9	273.5	42.6	5.83	2.44	4.97
Aug.	5.7	5.6	7.22	8.42	33.9	64.7	63.1	58.2	83.7	269.3	39.0	5.97	2.14	4.71
Sept.	5.8	5.7	6.63	8.22	33.0	66.4	64.7	60.6	97.7	307.5	41.4	5.63	2.11	4.53
Oct.	5.8	5.8	6.64	7.85	32.7	69.3	67.3	62.9	99.9	320.2	38.9	5.82	1.89	4.32
Nov.	5.7	5.7	7.29	7.82	31.6	71.9	71.1	65.5	97.9	305.4	33.4	6.71	1.92	3.73
Dec.	5.7	5.8	7.33	8.18	31.7	72.4	71.8	66.8	89.1	290.4	30.5	6.26	2.06	3.99
1940:														
Jan.	5.8	5.8	7.21	8.05	31.9	72.3	72.2	68.6	86.1	264.7	26.5	4.65	2.17	3.80
Feb.	5.9	5.8	7.87	8.81	33.7	67.7	67.3	64.4	110.1	337.0	40.4	4.31	1.90	4.35
Mar.	6.2	6.0	8.77	10.3	38.1	57.0	56.4	54.6	131.8	340.2	56.7	3.79	1.63	4.88
Apr.	6.2	6.1	8.69	10.4	37.8	58.6	56.6	54.6	119.9	300.1	51.3	3.53	1.70	4.61
May	6.1	6.0	7.93	9.43	36.2	63.1	61.4	58.6	121.7	324.3	47.4	3.70	1.88	4.27
Average	6.0	5.9	7.74	9.12	34.9	64.8	63.6	58.5	98.5	289.0	42.1	6.29	2.18	4.87

It has been found that the quantity of chemicals required for sludge is less with heavy sludge, which condition is usually associated with rains carrying inert material into the sewage. With the particular design of sludge piping installed in this plant, sludge containing more than 10% to 12% solids usually requires thinning with plant effluent to permit its movement through the piping system.

Vacuum Filtration.—Dewatering of sludge was begun on July 2, 1938. The sludge cake produced averaged 34.9% solids (65.1% moisture) in the 2-yr period. As a monthly average, during March, 1939, the moisture content was as low as 60.9%. Daily averages have been lower than 50%, these low moisture contents being associated with rains and periods when the volatile solids were exceptionally low. The volatile solids in the filter cake vary widely. During the period from September, 1939, to May, 1940, the volatile solids in the filter cake averaged 58.5% with a range in monthly averages from 51.4% to 68.6%. Daily variations are still greater. In the period since operation started to May 31, 1940, 183,497.6 tons of filter cake were produced by the plant. For the period from September 1, 1938, to May 31, 1940, the quantity of dry sewage solids has averaged 98.5 tons daily, equivalent to 289.0 tons of filter cake daily.

As in sludge conditioning, a number of improvement changes were made in this part of the plant which have resulted in simplified operation and marked reductions in operating cost. As an example of the effect of these changes, including those in sludge conditioning, during early operation approximately 3% ferric chloride and 10% lime were required, which has been reduced by various means to somewhat less than 2% and 5%, respectively. On the basis of 100 tons of dry sewage solids daily, and chemical prices current at this time, this amounted to a saving of $100 per day in chemical costs.

The reduction in the use of lime resulted in other operating advantages. In early operation, difficulty was experienced with lime-blinding of the cloths, and after a period of several months' operation the openings on the screen supporting the cloth had been reduced to less than one quarter of their original area by calcium carbonate deposits. A number of methods of removing this material were tried. At one time it appeared desirable to purchase new screens at a cost of $193 per filter. Finally the screens were removed and sandblasted at a cost of only $15 per filter, with entirely satisfactory results. With the reduction in the quantity of lime, the necessity of cleaning screens has been practically eliminated.

To remove calcium carbonate from the screens, conditioning with ferric chloride alone was tried. It was determined, after a short time of operation on two of the filters, that this method, although effective, was not economical. However, the experiment permitted a determination of the effectiveness of this chemical alone for conditioning. The rate of filtration as compared with lime and ferric chloride was reduced, ranging from 3 to 4 lb per sq ft per hr for dosages of 3% to 4% of ferric chloride.

For the entire period the average filter rate was 4.87 lb per sq ft per hr. During the early period when higher chemical dosages were used, the rates were considerably higher, averaging as much as 10 lb for various periods of time. In addition to the higher operating costs to secure greater rates, the production of a thick cake presented certain undesirable operating problems in the conveying of the cake. In view of the fact that the increased power cost by using more filters was insignificant, and that the extra cost of cloths was small in comparison with the savings in chemicals which could be effected, the desirability of operating with lower filter yields was apparent. In emergencies, or under extreme peak conditions, it is necessary to use higher chemical dosages to increase the filter rate.

The problem of increasing filter cloth life has been given due consideration, brushing and washing with steam and air being tried progressively. In the early period of operation, cloths generally had to be removed after about 150 hours of use. The need for removal was dictated by the fact that lime-blinding necessitated higher chemical dosages and more frequent washing, and ripping or rotting at seams required the use of a large number of tin patches (as many as 30 to 40 on a filter) even with this short life. With reduced lime dosage the economical cloth life has been increased to somewhat more than 300 hours (periods of usage of 500 to 600 hours were common for several months). Beyond this time increased chemical feed is required, incommensurate with the cost of cloth replacement, especially in view of the reduction in cost of cloth which has been effected. The ability to purchase identical cloths locally at a fraction of their original cost has made it possible to change filter cloths more frequently than was formerly the case. In addition, the new cloths are sewed with a different type of thread and with a lock stitch so that difficulties at the seams, requiring patching, have been eliminated.

Changes have been made in the location and angle of incidence of water sprays used in washing filters as compared with the type ordinarily manufactured. Various types of sprays arranged at different angles relative to the drum surface were tried. The original sprays were reversed, and located so that water would impinge almost at right angles to the cloth on the drum. The new arrangement permitted better washing in one third of the time previously required. Less water is used, and the necessity of brushing the cloth has been eliminated, thus insuring a longer cloth life.

The problem of conveying filter cake, containing varying quantities of water, on belts presents many difficulties that are sometimes not recognized. Stoppages are quite frequent with relatively minor restrictions in openings around and over belts and at transfer points. These problems are aggravated with thick cake (heavier than ⅜ in.). It was found early that large blocks of cake, as long as 8 ft and as wide as the filter drum, were broken off as the drum revolved. Various methods of breaking up this cake were considered, resulting in the selection of the simple and effective expedient of installing sloping ¼-in. rods on 12-in. centers in the filter take-off plates. This device cuts the cake into ribbons one foot wide, which break off irregularly and eliminate difficulties from this cause.

Incinerators.—Incineration of sludge was begun on July 21, 1938, and 24-hr operation on July 29. On August 4, 1938, the first incinerator placed in operation became heated to the point at which the manufacturers' representatives considered that it was necessary to shut down in order to insulate the rabble arms on certain of the hearths. Incinerator temperatures up to 2,200° F were recorded as compared with expected maximum temperatures of 1,600° F. The high temperatures required the replacement of several of the rabble arms and a section of the central shaft on this incinerator.

The explanation for this condition is simple. Instead of requiring approximately $2,500 worth of fuel oil a month, as estimated, it was found early that under practically all conditions of operation an excess of heat existed, and no fuel was required for actual incineration. It was the manufacturers' expectation that the location of burning, and the temperatures of combustion, would be controlled by the use of varying quantities of oil introduced at different points. Instead, it was necessary to dissipate heat to the extent of more than 6,000,000 Btu per hr per incinerator. In the beginning the manufacturers attempted to accomplish this by introducing water into the furnace through sprays. Although this was effective, it was considered unsatisfactory by the Sanitary District to use the large quantity of water required. After experimenting with different methods for solving their difficulties, the manufacturers installed a by-pass to conduct preheated air direct to the stack. In this way air at room temperature could be used for combustion, and the water would be reduced to reasonable quantities and its addition required on less frequent occasions.

Continuous operation of all the incinerators was possible beginning November 17, 1938, and in accordance with the specifications the acceptance tests were begun thirty days later, on December 17, 1938, and completed satisfactorily on March 17, 1939.

The method of correcting some of the early difficulties with overheating, etc., was a matter of thorough investigation over a period of five months, and presented some rather interesting problems. Progressively, a number of solutions were considered, and the methods finally adopted were those selected after careful consideration of all factors by the Sanitary District and the contractor. Some of the difficulties appeared insuperable at times, but it is a pleasure to report that they were successfully overcome by the contractor, at no expense to the Sanitary District.

From July 21, 1938, to May 31, 1940, 182,097.6 tons of filter cake were incinerated. The average Btu per pound of combustible solids in the sludge during this period was 10,800, and the average volatile solids in the sludge cake during the period covered herein amounted to 58.5%. The sludge cake was reduced by incineration to an ash containing less than 2% volatile matter (actual average 1.6% during guarantee period). As shown in Table 4, the moisture content in the sludge cake as produced by the filters has averaged 65.1% during the period from December, 1938, to May, 1940, inclusive.

The comparison between the quantity of material incinerated and the guaranteed capacity of the incinerators is interesting. Over a 24-hr period, a maximum of 523.8 tons of filter cake or a total of approximately 225 tons of dry solids have been incinerated. The capacity of each of the three incinerators is 60 tons of dry solids per twenty-four hours. As much as 78.5 tons of dry solids have been incinerated in one incinerator in a 24-hr period. Hourly rates considerably in excess of the foregoing have been handled successfully. For the period from December 1, 1938, to May 31, 1940, the number of tons of dry solids incinerated per day in each incinerator has averaged 55.1 tons as compared with the maximum guaranteed capacity of 60 tons daily.

During the period from December, 1938, to May, 1940, inclusive (see Table 5), 123,911 gal of fuel oil were required by the incinerators. Practically all of this was required in bringing furnaces up to temperatures from a cold condition or in cooling them down, and in holding temperatures in furnaces out of service. The aforementioned quantity of oil amounts to an average of 2.3 gal per ton of dry solids incinerated. Power required for incineration amounted to an average of 17.3 kw per hr per ton of dry solids.

Miscellaneous Sludge Disposal.—Investigations are in progress in connection with the utilization of sludge and ash for various purposes. A research fellowship has been established with the Agricultural College of the University of Minnesota, at Minneapolis, to explore the fertilizing or soil conditioning possibilities of sludge cake as produced by the filters and its use after storing for periods of time. Experiments have been made on the production of partly dried material by removal of a portion of the sludge from the top hearths of the incinerater, using the heat of the remainder of the sludge incinerated for drying purposes. An unusual degree of interest has been shown in the use of the material by farmers, nurseries, golf courses, parks, etc., but to 1940 no material was distributed except to those working in cooperation with the Agricultural College. This has been done with the express purpose of determining the advantages and disadvantages of the material, as well as the proper conditions of use, before general distribution and sale. From the interest and requests to date, it appears likely that a relatively large proportion of the material may be disposed of in this manner during a part of the year.

TABLE 5.—DATA ON SLUDGE INCINERATION, DECEMBER, 1938, TO MAY, 1940

Month	TONS OF SLUDGE INCINERATED		Dry tons per incin-erator day	Oil used (gal)	Power (kw-hr)
	Cake basis	Dry basis			
(1)	(2)	(3)	(4)	(5)	(6)
1938:					
Dec. .	9,826.4	3,346.5	53.8	7,573	58,603
1939:					
Jan. . .	10,119.0	3,654.5	60.4	4,195	54,782
Feb. .	8,663.2	3,022.3	56.6	6,069	50,943
Mar. .	8,851.9	3,349.5	61.3	6,793	55,829
Apr. .	9,398.7	3,321.6	61.4	5,634	54,205
May .	9,932.2	3,579.9	61.5	10,470	61,569
June .	9,528.9	3,649.1	64.0	6,188	55,572
July. .	8,479.3	3,007.7	57.5	10,763	52,061
Aug. .	8,348.5	2,847.6	55.0	14,064	52,069
Sept. .	9,223.5	3,071.4	55.0	5,977	52,503
1939:					
Oct. . .	9,284.5	3,036.4	51.2	6,629	54,761
Nov. .	7,023.4	2,222.3	45.0	6,836	45,257
Dec. .	9,002.6	2,850.9	46.5	3,557	53,822
1940:					
Jan. . .	8,206.1	2,618.0	43.5	2,767	52,509
Feb. . .	9,773.0	3,295.2	51.4	9,776	61,504
Mar. .	10,177.2	3,874.7	58.5	10,938	66,098
Apr. . .	8,964.5	3,391.4	55.7	6,604	56,604
May. .	9,844.0	3,564.6	53.3	8,078	59,278
Total . .	164,646.9	57,793.6	. . . .	132,911	997,969
Average	9,147.0	3,211.0	55.1	7,384	55,443

The use of ash has been investigated. Some possible uses that have been suggested are: A binder or filler for various products, a soil conditioner, an ingredient for certain kinds of cement, etc. About 400 tons of the material have been disposed of as filler, in lieu of pulverized limestone, for a commercial fertilizer. A typical analysis of the ash from this plant is as follows:

Silica as $Si\ O_2$	43.6%
Calcium as $Ca\ O$	26.3%
Aluminum as Al_2O_3	9.9%
Iron as Fe_2O_3	6.9%
Potassium as K_2O	2.1%
Phosphorus as P_2O_5	0.7%
Miscellaneous	10.5%

Another interesting possibility relates to the utilization of the heat resulting from incineration. Various possibilities have been developed which may result in the generation of sufficient steam for all heating purposes and electric power requirements.

Everything considered, the methods of sludge disposal used at this plant have proved quite satisfactory, and a number of possibilities for still further improvement are under consideration. The successful and economical operation has been very gratifying, and at times such operation was an absolute necessity, especially when it is considered that the quantities of material to be handled were considerably greater than originally expected (monthly averages as high as 131.8 tons daily, as compared with expected average quantities of 76.5 tons daily).

Cost Features

Construction Cost.—The total construction cost of the plant was approximately $3,750,000 or $28,000 per million gal of capacity. The distribution of this cost was about as follows: Screen and grit chambers with chemical tower, $540,000; settling tanks, effluent filters, and appurtenances, $1,630,000; filtration and incineration building, $1,240,000; laboratory and administration building, $175,000; and miscellaneous, $165,000. Equipment represents about $1,500,000 of the total cost of $3,750,000.

Operation and Maintenance Costs.—The total operation and maintenance costs for the period from June 1, 1938, when preliminary treatment was first begun, to May 31, 1940, was $587,136.03 (see Table 6(*a*)). During this period 66,864.1 million gal of sewage was treated, bringing the average cost per million gallons to $8.80. The entire flow was not received until about July 15, 1938, and furthermore operation during the early months was curtailed because of the manufacturer's difficulties with incinerator operation. For these and other reasons the costs during the early period of operation in 1938 are not typical. During 1938, for example, the average cost per million gallons was $10.65, which compares with $8.20 for 1939, and $8.35 for the first five months of 1940. Of the total cost during the first two years of operation, maintenance costs totaled $45,884.23, or 7.8% of the total.

Table 6(*b*) shows the distribution of operation and maintenance costs by departments for the three periods. In order of descending cost importance the following are the most important items: Vacuum filtration, incineration, supervision and engineering, administrative expense, laboratory expense, sedimentation, etc. This order will be varied, of course, whenever chemical treatment and chlorination are practiced.

Although what follows concerning the comparison between actual expenditures and funds available (see budget provisions, Table 6(*b*)) may not be of general interest, it is nevertheless a matter of local gratification that it has been possible to hold expenditures within budgeted funds. During the first two years of operation, expenditures totaled $587,136.03 as compared with budget provisions of $917,164.81. It is a fact that, although the deliberate reduction in treatment provided (until communities and industries immediately downstream provided comparable treatment) resulted in certain savings, a large part of the reduction in cost was possible by the operation economies effected.

It is interesting to note the large proportion of the total cost directly chargeable to sludge disposal. During 1939, for example, $136,815.43 was expended for vacuum filtration and incineration, which represents $3.70 per million gal, or approximately 45% of the entire cost of operation and maintenance. In 1938 the total cost of sludge disposal was $69,840.65, during which period a total of 10,248 tons of dry sewage solids were processed. The cost of dry sewage solids was approximately $6.80 per ton. For the more representative period of 1939, during which time reductions in sludge conditioning chemicals were effected, the total cost per ton was $3.63, which was further

TABLE 6.—COSTS OF SEWAGE TREATMENT AND SLUDGE DISPOSAL IN THE TWO YEARS ENDING MAY 31, 1940

Dept. No.	Description	June 1, 1938 Dec. 31, 1938	Jan. 1, 1939 Dec. 31, 1939	Jan. 1, 1940 May 31, 1940	June 1, 1938 May 31, 1940
	(a) TOTAL AND UNIT COSTS[a]				
	Total operation and maintenance[b]	157,868.13	301,538.98	127,728.92	587,136.03
	Maintenance only	9,564.26	24,012.46	12,307.51	45,884.23
	Sewage Treatment:				
	Mgd	(14,826.1)	(36,705.3)	(15,332.7)	(66,864.1)
	Dollars per mgd	10.65	8.20	8.35	8.80
	Disposal:				
	Tons of dry sludge solids	(10,248.0)	(37,703.0)	(17,100.0)	(65,051.00)
	Dollars per ton of dry solids	15.40	7.98	7.45	9.05
	Cost of sludge disposal only	69,840.65	136,815.43	54,082.66	260,738.74
	Cost of sludge disposal, per ton of dry solids	6.80	3.63	3.15	4.01
	(b) DISTRIBUTION BY DEPARTMENTS				
1	Administrative expense	13,180.40	27,822.37	11,712.34	52,715.11
2	Supervision and engineering	13,044.72	30,268.52	12,306.69	55,619.93
3	Laboratory expense	10,908.35	19,715.30	7,759.84	38,383.49
4	Grit and screenings removal	6,658.31	11,978.33	3,910.26	22,546.90
5	Grit and screenings disposal	7,024.42	10,764.19	3,950.09	21,738.70
6	Chemical handling and feeding (sewage treatment)	789.76	895.12	76.65	1,761.53
7	Disinfection	227.42	1,132.38	3.11	1,362.91
8	Flocculation	490.99	1,460.15	177.80	2,128.94
9	Sedimentation	12,207.31	18,482.73	6,803.05	37,493.09
10	Scum disposal	4,483.56	3,285.05	855.66	8,624.27
11	Effluent filtration	1,783.46	2,369.11	1,031.68	5,184.25
12	Effluent water supply	1,371.37	3,498.06	905.83	5,775.26
13	Vacuum filtration	50,419.48	99,448.39	33,590.09	183,457.96
14	Sludge incineration	21,042.50	37,365.04	13,785.79	72,193.33
15	Heating	....[c]	9,815.25	5,175.50	14,990.75
16	Shop	....[c]	2,862.27	1,098.18	3,960.45
17	Roads, walks, grounds	1,848.95	2,485.67	420.85	4,755.47
18	Odor control	....	10.00	....	10.00
19	Interceptor sewers	670.37	1,234.32	100.25	2,004.94
20	Miscellaneous and unassigned	....[c]	8,292.39	7,786.02	16,078.41
	Total	146,151.37	293,184.64	111,449.68	550,785.69
	Miscellaneous income	....	713.40	222.01	935.41
	Cost of operation and maintenance	146,151.37	292,471.24	111,227.67	549,850.28
	Increases in assets, materials and supplies inventory, etc.	11,736.76	9,065.74	16,501.25[d]	37,303.75
	Grand total	157,888.13	301,536.98	127,728.92	587,154.03
	Budget provisions	240,097.93	478,066.88	199,000.00	917,164.81

[a] All units are dollars except values in parentheses. [b] Including capitalized improvement changes. [c] Distributed in other departments. [d] Includes $12,307.51 undistributed maintenance costs.

reduced in the first five months of 1940 to $3.15 per dry ton. Monthly data on the cost of sludge disposal is shown in Table 7.

Of the total cost of disposal for the period from January, 1939, to May, 1940, inclusive ($191,311.19), filtration cost was $137,310.48, or 72%, and incineration $54,000.71, or 28%, of the total cost. As further shown in Tables 8 and 9, the principal item of cost in connection with sludge dewatering is material and supplies, whereas salaries and wages constitute the principal cost factor in sludge incineration. Maintenance (including wages) amounts to

TABLE 7.—OPERATION, MAINTENANCE, AND UNIT COSTS OF SLUDGE FILTRATION AND INCINERATION
(December, 1938, to May, 1940, Inclusive)

Month	OPERATION AND MAINTENANCE COST			TONS SLUDGE		OPERATION AND MAINTENANCE COST PER DRY TON		
	Vacuum filtration	Inciner-ation	Total	Dry sewage solids basis	Cake basis	Vacuum filtration	Inciner-ation	Total
1938:								
Dec.	$13,737.75	$5,680.77	$19,418.52	3,067	9,826.4	$4.48	$1.85	$6.33
1939:								
Jan.	10,594.05	2,716.65	13,310.70	3,150	10,119.0	3.37	0.86	4.23
Feb.	9,334.35	2,977.01	12,311.36	2,920	8,663.2	3.19	1.02	4.21
Mar.	8,364.06	2,800.75	11,164.81	3,755	8,851.9	2.22	0.75	2.97
Apr.	8,095.15	2,444.49	10,539.64	3,065	9,398.7	2.64	0.80	3.44
May	8,021.87	2,960.57	10,982.44	3,339	9,932.2	2.40	0.89	3.29
June	9,178.08	3,179.92	12,358.00	3,727	9,528.9	2.46	0.86	3.32
July	7,239.88	3,630.36	10,870.24	2,818	8,479.3	2.57	1.29	3.86
Aug.	6,481.29	3,329.77	9,811.06	2,593	8,348.5	2.50	1.29	3.79
Sept.	7,704.60	3,651.00	11,355.60	2,929	9,223.5	2.63	1.25	3.88
Oct.	8,180.68	3,798.52	11,979.20	2,896	9,284.5	2.82	1.31	4.13
Nov.	7,844.92	3,257.00	11,101.92	2,251	7,023.4	3.48	1.45	4.93
Dec.	8,212.56	3,639.68	11,852.24	2,762	9,002.6	2.97	1.32	4.29
1940:								
Jan.	7,079.14	3,049.43	10,128.57	2,668	8,206.1	2.65	1.14	3.79
Feb.	6,590.02	2,976.13	9,566.15	3,192	9,773.0	2.07	0.93	3.00
Mar.	7,521.09	3,760.62	11,281.71	4,087	10,177.2	1.84	0.92	2.76
Apr.	8,883.73	3,642.30	12,526.03	3,596	8,964.5	2.47	1.01	3.48
May	7,344.90	3,235.30	10,580.20	3,772	9,844.0	1.96	0.86	2.82
							Average:	
Total	$150,408.12	$60,730.27	$211,138.39	56,587	164,647	$2.67	$1.06	$3.73

TABLE 8.—DISTRIBUTION OF OPERATION AND MAINTENANCE COSTS FOR THE PERIOD JANUARY 1, 1939, TO MAY 31, 1940

Description	FILTRATION		INCINERATION		TOTAL	
	Cost	%	Cost	%	Cost	%
Operation:						
Salaries and wages	33,198.51	24.2	27,217.36	50.4+	60,415.87	31.5
Material and supplies	75,603.40	55.0	5,606.26	10.4	81,209.66	42.5
Power and light	12,652.80	9.2	13,248.20	24.5	25,901.00	13.6
Miscellaneous	4,651.42	3.4	389.08	0.7	5,040.50	2.6
Maintenance	11,204.35	8.2	7,539.81	14.0+	18,744.16	9.8
Total	137,310.48	100.0	54,000.71	100.0	191,311.19	100.00

about 10% of the entire cost of disposal. Although it is true that maintenance costs in the first two years of operation of a plant are expectedly lower than they will be after, say, ten years (and that therefore it might be held to follow that costs will increase after a period of years), operation of this plant to 1940 indicates that such apparent increase will be offset by economies effected in other cost items of greater magnitude.

TABLE 9.—SUMMARY OF UNIT COSTS

Treatment	DRY SEWAGE SOLIDS		FILTER CAKE		Average cost (cost per month)
	Tons	Cost per ton	Tons	Cost per ton	
Filtration	....	$2.56	....	$0.89	$8,080
Incineration	....	1.01	....	0.35	3,180
Total	53,520	$3.57	154,820.6	$1.24	$11,260

The total operating and maintenence personnel, including administration, supervision, laboratory, etc., numbers 74 employees, involving a monthly payroll of approximately $13,500. Operation and maintenance costs are financed in the two cities by sewer rental charges which in one city are based on the volume of water consumed, and in the other on the basis of the size of water meter installed on the premises.

MISSISSIPPI RIVER CONDITIONS

General Data.—A very marked improvement in the physical, biochemical, and bacteriological condition of the Mississippi River became apparent shortly after the Twin Cities' sewage treatment plant was placed in operation. With the passage of time and the gradual elimination of "loose ends" of pollution, the river condition through Minneapolis and St. Paul has gradually approached that entering Minneapolis at Camden, Minn. (see Fig. 2).

Through the Twin Cities the floating scum, sleek and large masses of sludge which in former summers covered as much as 50% of the water surface in some areas, has practically disappeared. The foul odors of past years are no longer in evidence. Game fish are returning to this section of the river which previously did not provide satisfactory environment for even the rougher forms of fish life. Boating is being enjoyed by an increasing number of people.

Occasionally some debris is seen floating on certain sections of the river following heavy storms. The Twin Cities' sewer systems are combined and therefore both storm water and sewage must overflow to the river during heavy storms. The sewer departments of both cities strive to hold such overflow to a minimum by the use of automatic regulators and by inspecting the sewer outlets following storms. The Sanitary District also makes a monthly inspection of all former sewer outlets and reports its findings to the sewer departments of both cities.

The river below the Twin Cities has shown an improvement although it is still seriously affected by the untreated sewage and packing plant wastes from South St. Paul and Newport. Some idea of the relative effect of the sewage and

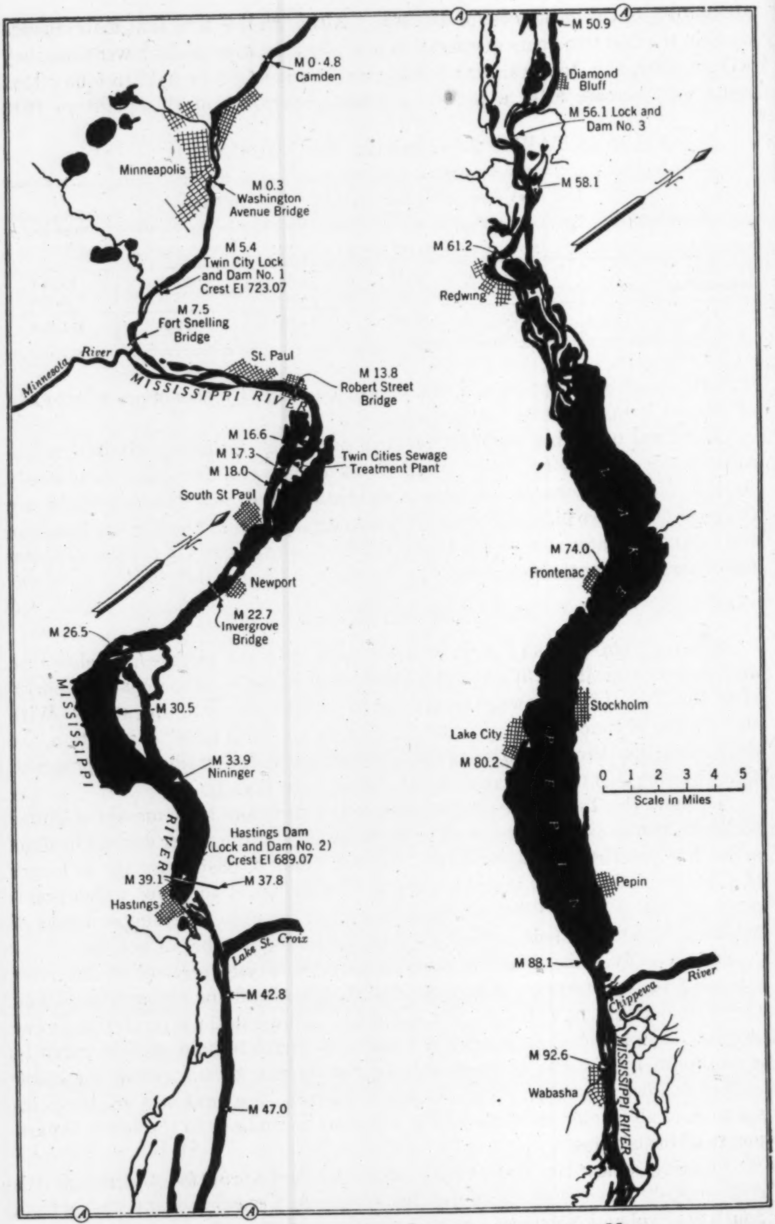
M 0-4.8
Camden
Minneapolis
M 0.3
Washington
Avenue Bridge
M 5.4
Twin City Lock
and Dam No. 1
Crest El 723.07
M 7.5
Fort Snelling
Bridge
Minnesota River
St. Paul
MISSISSIPPI RIVER
M 13.8
Robert Street
Bridge
M 16.6
M 17.3
M 18.0
Twin Cities Sewage
Treatment Plant
South St Paul
Newport
M 22.7
Invergrove
Bridge
M 26.5
MISSISSIPPI
M 30.5
M 33.9
Nininger
RIVER
Hastings Dam
(Lock and Dam No. 2)
Crest El 689.07
M 39.1
M 37.8
Hastings
Lake St. Croix
M 42.8
M 47.0
A
A
A
A
M 50.9
Diamond
Bluff
M 56.1 Lock and
Dam No. 3
M 58.1
M 61.2
Redwing
LAKE
M 74.0
Frontenac
Stockholm
Lake City
M 80.2
PEPIN
0 1 2 3 4 5
Scale in Miles
Pepin
M 88.1
Chippewa River
M 92.6
Wabasha
MISSISSIPPI RIVER

FIG. 2.—LOCATION MAP, MISSISSIPPI RIVER AT MINNEAPOLIS, MINN.

wastes from South St. Paul and Newport can be had from the fact that the population equivalent of the untreated sewage and wastes from these communities varies from 200,000 to 400,000, depending upon the animal kill, as compared with a total population equivalent of untreated sewage from the Twin Cities of from 1,000,000 to 1,200,000. It is unfortunate that at this writing (1940) all sewages and wastes from the metropolitan area are not receiving treatment so as to permit a more definite comparison of river conditions farther downstream "before and after."

Each winter commercial fishermen seine rough fish from the river at Grey Cloud Island, 14 miles below St. Paul. They report the presence of crappies and sunfish for the first time during the winter of 1939, indicating that some of the better forms of fish life are returning even to this section of the river. At other points on the river, both upstream and downstream from the treatment plant, catches of game fish are reported by fishermen, where formerly only the more hardy varieties of "rough fish" were able to survive.

Analytical Data.—The general evidence of improved river conditions discussed herein is substantiated by the analytical data on the river water samples collected regularly for analysis. The continuous record of analytical data on the Mississippi River and its tributaries in the period from 1926 to 1940 is valuable in a comparison of river conditions.

Data contained in Table 10 show the improved condition of the water at the Twin City Lock and Dam during the summers of 1938 and 1939 compared with previous summers before the discharge of sewage into the pool had been stopped. The years 1929, 1930, 1935, and 1937 were selected for comparison because the river flows for those summers most nearly correspond with the flows for the summers of 1938 and 1939. As indicated in Table 10, the dissolved oxygen in the river passing the Twin City Lock and Dam has materially increased, whereas the biochemical oxygen demand and bacterial counts have greatly decreased since the Twin Cities' sewage disposal system was placed in operation in the summer of 1938. This improvement is shown further in Fig. 3, containing a plot of the river data at this station since 1926.

TABLE 10.—AVERAGE ANALYTICAL DATA AT STATION M 5.4, TWIN CITY LOCK AND DAM, FOR JULY, AUGUST, AND SEPTEMBER

Year	1929	1930	1935	1937	1938	1939
Dissolved oxygen, ppm	3.60	3.85	1.70	1.75	6.60	6.60
B.O.D., ppm	4.40	4.65	3.85	4.25	1.80[a]	1.85[a]
Total bacterial count per ml	480,000	237,000	400,000	375,000	17,000[a]	11,000[a]
Coliform organisms per ml	3,900	8,900	3,100	2,800	325[a]	110[a]
River flow at Twin City Lock and Dam, cu ft per sec	4,190	3,680	2,885	2,845	4,525	3,274

[a] At station M 7.5, Fort Snelling Bridge.

That the river condition through the Twin Cities is gradually approaching that entering Minneapolis is shown by the data contained in Table 11 (see also Table 10). Some added pollution is indicated, however, probably made up from the Minnesota River pollution, from storm water overflow through the

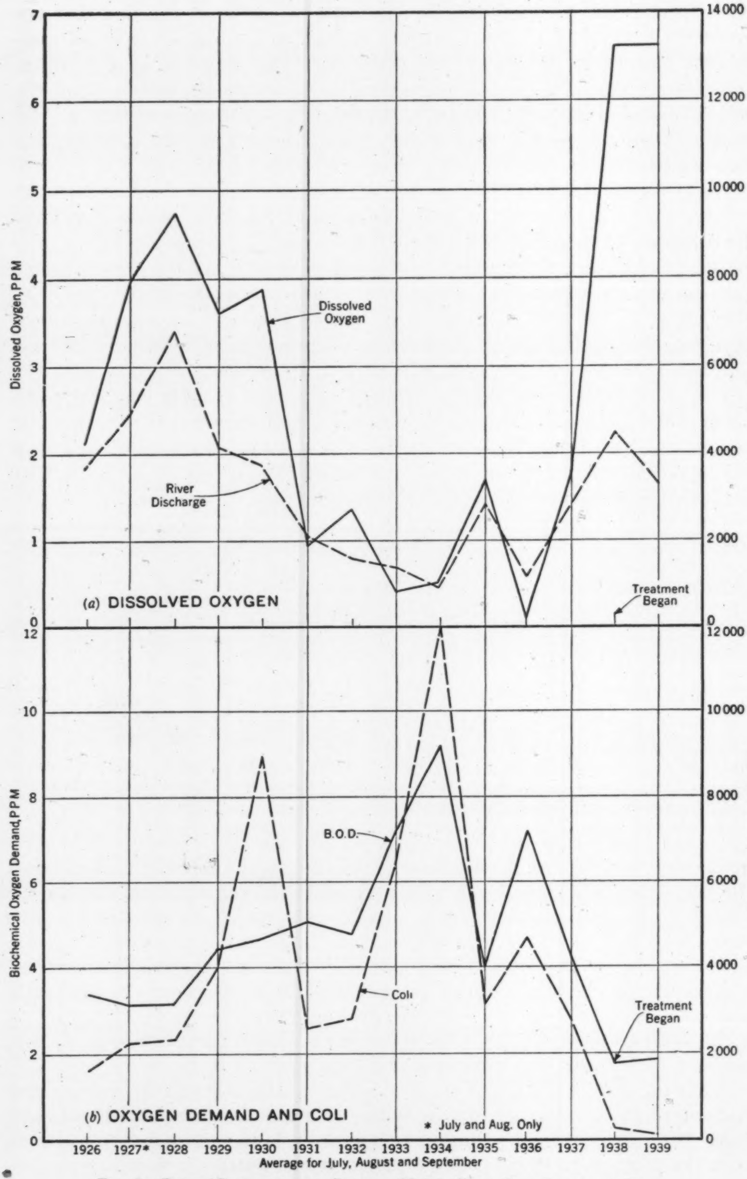

FIG. 3.—RIVER CONDITIONS AT STATION M 5.4, TWIN CITY LOCK AND DAM, BEFORE AND AFTER SEWAGE TREATMENT

old sewer outlets, occasional leakage from the latter, unaccounted-for sources of pollution, and some residual effect from the old sludge deposits in this area.

TABLE 11.—AVERAGE ANALYTICAL DATA FOR JULY, AUGUST, AND SEPTEMBER, 1937 AND 1939, MISSISSIPPI RIVER, THROUGH THE TWIN CITIES

Station (see Fig. 2)	1937				1939			
	PPM		COUNT PER ML		PPM		COUNT PER ML	
	Dissolved oxygen	B.O.D.	Total bacteria	Coliform organisms	Dissolved oxygen	B.O.D.	Total bacteria	Coliform organisms
M 0–4.8	7.85	1.55	1,700	2.0	7.40	1.45	3,000	4.0
M 0.3	6.10	4.25	340,000	1,100	7.65	1.65	5,000	42
M 7.5	0.80	4.20	340,000	2,000	6.65	1.85	11,000	110
M 13.8	....	....	....	....	6.65	2.10	12,800	150
M 16.6	....	....	....	....	6.60	2.20	9,200	110
M 17.3	1.25	3.50	210,000	2,100	....	....	....	....

A more complete picture of the effect of the operation of the Twin Cities' plant on river conditions is shown in Table 12, which contains data on dissolved oxygen concentration at various sampling stations in a 100-mile stretch of the Mississippi River. The location of the sampling stations in relation to the Twin Cities and the treatment plant is shown in Fig. 2. Referring to Table 12, it will be seen that oxygen concentrations through the Twin Cities (stations

TABLE 12.—A COMPARISON OF MISSISSIPPI RIVER FLOWS (CU FT PER SEC) AND DISSOLVED OXYGEN CONTENT (PPM)

(Average for July, August, and September 1931–1939, Inclusive)

Station[a]	DISSOLVED OXYGEN (PPM)								
	1931	1932	1933	1934	1935	1936	1937	1938[b]	1939
St. Paul[c]	2,430	2,140	1,715	1,090	3,382	1,476	4,000	7,740	4,000[d]
M 0–4.8	6.65	7.55	7.50	8.35	7.55	9.05	7.85	8.40	7.40
M 0.3	5.05	4.90	4.60	1.75	6.15	2.70	6.10	7.50	7.65
M 5.4	0.95	1.35	0.40	0.50	1.70	0.10	1.75	6.60	6.60
M 7.5	....	....	....	0.40	1.65	0.15	0.80	6.85	6.65
M 13.8	....	....	....	....	....	....	....	6.85	6.65
M 16.6	....	....	....	....	....	....	....	6.00	6.60
M 17.3–18.0	0.70	1.40	0.25	0.35	1.30	0.80	1.25	5.55	5.05
M 22.7	0.60	0.80	0.30	0.20	1.05	0.75	1.25	3.35	2.00
M 26.5	....	....	....	....	1.80	2.65	1.35	2.90	1.25
M 30.5	....	....	....	....	2.95	3.35	2.55	2.60	1.95
M 33.9	....	....	....	....	....	....	....	3.10	3.25
M 37.8	1.45	3.90	2.95	5.25	4.85	5.95	5.30	4.60	6.00
M 39.1	5.65	7.40	7.25	7.10	7.30	7.15	7.20	6.15	6.50
M 42.8	....	....	....	....	....	....	6.70	6.40	6.00
M 47.0	....	....	....	....	....	....	6.30	6.45	6.00
M 50.9	....	....	....	....	....	....	6.65	6.35	6.50
M 56.1	....	....	....	....	....	....	6.60	6.30	6.60
M 58.1	....	....	....	....	....	....	....	6.05	6.65
M 61.2	5.10	6.60	6.15	6.75	6.05	5.90	6.60	6.05	6.85
M 74.0	6.35	8.50	7.00	7.35	7.20	7.90	7.00	6.70	6.80
M 80.2	6.55	7.50	8.00	8.10	9.45	6.60	7.10	6.30	6.25
M 88.1	6.80	7.15	6.30	7.55	8.15	7.05	6.35	6.85	5.80
M 92.6	6.65	7.20	6.35	7.35	7.30	6.70	5.80	6.40	6.15

[a] Indicates mileage from the head of navigation at the lower Northern Pacific Railway bridge in Minneapolis—see Fig. 2. [b] Twin Cities' sewage treatment plant was placed in operation during the summer of 1938. [c] River flow at St. Paul. [d] Estimated from flow at Lock and Dam No. 1.

M 0–4.8 to M 16.6) are, as would be expected, appreciably higher than before the sewage was intercepted and treated. Immediately below the plant the dissolved oxygen dropped to 5.05 ppm, and at a point 10 to 14 miles below the Minneapolis-St. Paul plant (9 miles below South St. Paul and Newport) dropped to its low average value of 2.60 ppm in 1938 and 1.25 ppm in 1939. The standard tentatively fixed for this section of the river (pool of the Hastings Dam) is 2 ppm of dissolved oxygen during the summer months. Below this point the river quickly recovered, so that at Hastings, approximately 22 miles below the Twin Cities' treatment plant, the water contained more than 6 ppm of oxygen.

In the winter periods, critical conditions formerly prevailed during January, February, and March. This was from the standpoint of fish life below the confluence with the St. Croix River, for which a tentative standard of 4 ppm of dissolved oxygen has been set. Table 13 contains comparative data for February, 1935, and February, 1939 and 1940. February, 1935, was selected because during that month the river flow of 1,613 cu ft per sec at St. Paul was extremely critical. Instead of 4 ppm below the St. Croix River, as tentatively set, individual samples collected in Lake Pepin have been practically devoid of oxygen at various times. In February, 1940, with flows of 1,400 cu ft per sec, the lowest individual sample of dissolved oxygen below the St. Croix River was 4.5 ppm, and the average minimum for the month was 5.9 ppm. During this period the Minnesota River entered the Mississippi River with an average of only 1.5 ppm (lowest 0.6 ppm) of oxygen, and the packing plants at South St. Paul and Newport were enjoying an unusually heavy volume of business with a consequent heavy pollution load. The river flow at St. Paul in February, 1940, was the lowest winter flow, with the exception of one month, than at any time in the 53-yr period of record.

TABLE 13.—DISSOLVED OXYGEN CONTENT OF MISSISSIPPI RIVER IN FEBRUARY OF VARIOUS YEARS (PPM)

Station	1935[a]	1939[b]	1940[c]
M 0–4.8 River entering Minneapolis	6.1	6.3	7.5
M 0.3	6.9	7.6	8.7
M 5.4	4.5	7.9	8.8
M 13.8	3.3	6.7	8.0
M 16.6 Twin City sewage treatment plant	2.7	6.9	7.3[d]
M 20.0 South St. Paul	1.8	6.6	....
M 22.7	1.2	6.3	5.3
M 30.5	0.7	4.5	1.0
M 37.8 Hastings Dam	0.4	3.2	0.3
M 39.1 Hastings Bridge	7.2	....	....
M 42.8 Below junction with St. Croix River	8.0	6.9	....
M 47.0	7.8	6.3	....
M 50.9	7.6	5.8	5.9
M 56.1 Red Wing Dam	7.3	6.2	6.0
M 61.2	7.0	7.8	6.5
M 74.0 Frontenac	4.5	6.9	6.2
M 80.2 Lake City	3.3	6.9	9.0
M 88.1 Outlet of Lake Pepin	3.5	7.0	14.0
M 92.6 Wabasha	....	7.9	....

[a] Flow at St. Paul, February, 1935, 1,613 cu ft per sec. [b] Flow at St. Paul, February, 1939, 3,730 cu ft per sec. [c] Flow at St. Paul, February, 1940, 1,400 cu ft per sec. [d] Station M 18.0.

River Bottom Conditions.—In October, 1939, a study was made of the river bottom conditions in that part of the Hastings pool below the Twin Cities' sewage treatment plant, a distance of 21 miles. Cross-section soundings were made at 22 stations and composite bottom samples collected at 14 stations. It was found that this stretch of river divided itself into three sections, on which some of the pertinent data are summarized in Table 14.

TABLE 14.—River Bottom Study (Treatment Plant to Hastings Dam)

Section	Cross-Sectional Water Area, in Sq Ft		Relative River Velocity		5-Day B.O.D. of Bottom Material, in PPM		Physical character of bottom material
	Average	Maximum	Average	Minimum	Average	Maximum	
(*A*) M 16.6–18.5	5,800	7,900(M 16.8)	1.0	1.0	630	690(M 16.8+380 ft)	Dirty sand and gravel
(*B*) M 18.5–26.5	9,600	15,100(M 20.5)	0.6	0.5	1,050	1,650(M 19.5)	Considerable sludge and silt
(*C*) M 26.5–37.5	15,000	28,000(M 35.5)	0.4	0.3	820	895(M 28.5)	Some sludge and silt deposits

Section (*A*) is a 2-mile stretch of river extending from a point immediately upstream from the Twin Cities' sewage plant to a point just upstream from the South St. Paul packing plants. The data showed that the river bottom through this section was of uniform character, being mostly sand and gravel with just enough organic matter to give the comparatively low average B.O.D. of 630 ppm. The bottom was quite clean, with practically no sludge deposits. Such material as existed may originate from storm runoff and overflow from sewers during storms, by-passing of excess storm flows at the sewage treatment plant, general river silt (of which the Minnesota River contributes a large proportion), and any suspended solids remaining in the treatment plant effluent.

Section (*B*) is an 8-mile stretch of river extending from a point just upstream from the South St. Paul packing plants to a point 4 miles downstream from the Invergrove Bridge. This is the section most affected by the packing plant wastes in respect to bottom deposits. There are considerable deposits in this section which exert a large oxygen demand on the overlying water. The oxygen demand of the bottom samples from this section was about twice as great as in section (*A*) and does not appear to have decreased much, if any, in comparison with recent years. The untreated sewage and packing wastes of South St. Paul and Newport discharging into the upper part of this section no doubt have contributed heavily to these deposits. In general, the river bottom in section (*B*) may be described as sand and gravel covered to a considerable extent with deposits of sludge, silt, and straw.

Section (*C*) is an 11-mile stretch of river extending from a point 4 miles downstream from the Invergrove Bridge to the Hastings Dam. This section contains many large but shallow backwater areas more or less cut off from the main river and not believed to affect the channel velocity appreciably. In general, the river bottom in this section may be described as mostly sand, gravel, and rock, covered to some extent with deposits of sludge and silt, which exert a somewhat smaller oxygen demand on the overlying waters than in section (*B*) but greater than in section (*A*). The oxygen demand of the bottom material in this section appears to be about the same as in 1937, the last previous year of complete cross-sectional sampling in this area. The oxygen demand exerted by the sludge deposits in sections (*B*) and (*C*) tends to reduce the dissolved oxygen in the overlying waters very appreciably and correspondingly affects the maintenance of the desired minimum dissolved oxygen in this part of the river.

General.—It is unfortunate that it is impossible to make a complete evaluation of the recovery of the Mississippi River through, and downstream from, the Twin Cities from its former heavy pollution load because of the fact that comparable treatment is not being provided by downstream communities. The writer has looked forward anxiously to the time when the original river calculations, on which the degree of treatment decided upon was based, could be checked by actual observations and a determination made of several factors affecting deoxygenation and re-aeration after the system was in operation. Such preliminary calculations as have been made, substantiated by analytical data and observation, indicate that the original assumptions, if anything, were conservative, and that the type of treatment selected will meet maximum requirements and at the same time will permit variations of treatment provided with river dilution conditions and requirements.

ACKNOWLEDGMENT

The Minneapolis-St. Paul Sanitary District is under the direction of a Board of Trustees of seven members. In charge of plant operation are Clifford R. Raiter, John C. Sager, Warren H. Sleeger, Assoc. M. Am. Soc. C. E., and Benjamin M. Storey, plant operators. Kerwin L. Mick is chief chemist in charge of the laboratory, and Stanley J. Larsen is mechanical engineer in charge of maintenance. The writer wishes to acknowledge the fine cooperation of all the members of his staff, as well as of superintendents of other plants and of other engineers engaged in this field, the effect of whose assistance and free interchange of information and experience has been a constant source of encouragement in the operation of this plant as it has in many others.

CONCLUSION

The foregoing review of experiences in the first two years of operation of the Twin Cities' plant should be considered as in the nature of a progress report. So many interesting possibilities of producing economies and effecting simpler and even better operation have presented themselves that operations here cannot be considered to be on a standardized basis. As a matter of fact, it is the writer's hope and expectation that operation will not be considered to have reached that point for some time, but rather that the possibilities of new developments will be continually explored. As many plant operators will agree, it is these possibilities that make operation of a sewage treatment plant, over a period of years, a succession of interesting problems, rather than a repetition of routine and standardized tasks.

DISCUSSION

F. C. Roberts, Jr.,[4] Assoc. M. Am. Soc. C. E. (by letter).—In this very complete paper, Mr. Schroepfer has outlined the various steps involved in the new plant for the Minneapolis-St. Paul Sanitary District. When all units of this plant are in service, the sewage passes through the following steps: A coarse bar screen with 6-in. openings and a bar screen with 1½-in. openings; then from these screens the sewage flows to the grit chambers; and thence to the flocculating tanks, the settling tanks, and finally the magnetite filter units. Before flocculation, the sewage entering these basins may be dosed with chemicals to aid in sedimentation and clarification.

One of the features of this plant is the vacuum filtration and incineration of solids removed from the sewage. It is to be noted from the information presented by Mr. Schroepfer that the cost of this disposal represents approximately one half of the plant operation cost. A question arises as to the relative cost of this process over the digestion tank processes, or its necessity, in so far as construction and operation costs are concerned.

Throughout this paper it is apparent that the objective was the improvement of the pollution of the Mississippi River. The paper is presented in terms of the tonnage load of pollution delivered to the river. In most public health work engineers are not concerned with the percentage of removal of solids by a particular unit of a disposal plant. They are not interested in the percentage efficiency of the settling basin, or the secondary or tertiary processes; but they are interested in the final effluent that reaches the river. If the plant effluent is consistently of the same degree of purity, so that the river is neither a nuisance nor a public health menace, then they are satisfied. With the most complicated disposal process, the final results may not be satisfactory to the river, and a river pollution condition results. It is of no little interest to note that this paper refers to the end product that must be met, and the plant is built back from this criterion.

This plant was not designed unit by unit for maximum flow or average flow conditions, but rather to meet the variable flow conditions that might be expected at the plant. As Mr. Schroepfer states, the period of time that chemical treatment will be needed will be about 29 days in one year. One may imagine the necessity for chemical treatment during this relatively short period. The availability of chemical treatment will furnish the operator with a tool with which to meet adequately a condition that will arise at intervals, and a uniform result may be expected at the point of plant outfall. Flexibility appears to be the cardinal virtue of this plant.

The writer has had no experience with that part of the country around Minneapolis and St. Paul. In discussing chemical treatment, it is necessary that his remarks be confined to the six or seven chemical treatment plants in Arizona in order to furnish some basis of comparison with the plant presented in the most thorough and excellent paper by Mr. Schroepfer. It was thought that it might be of value to discuss the Scott-Darcy process as it has been developed in Arizona. The development of this process has been the one major contribution that Arizona has made to the field of chemical treatment to date.

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[4] Tucson, Ariz.

The needs within Arizona are different from those encountered elsewhere in the United States. Because of its freight rates, high temperatures, and wide variations in sewage flow due to a high rate of water usage, it was necessary to use some chemical treatment process that was flexible, and that would be available when needed and stable until needed. A chemical that might be delivered to any point with a minimum of cost and inconvenience was also a requisite.

There are four general types of chemical treatment: First, chemical treatment is used to aid in coagulation and flocculation, as provided in the Minneapolis-St. Paul plant; second, sewage is treated with certain chemicals to remove objectionable characteristics, such as the sulfur compounds that cause the most objectionable odors and the deterioration of concrete sewers; third, chemicals are used to maintain the sewage in a fresh condition until it reaches the plant; and fourth, sewage is sterilized to remove the bacteria.

The first classification of this purely arbitrary grouping is accomplished with any of the metal precipitants, such as ferric iron, alum, etc. The second process is the application of ferrous iron or iron in such form that it combines readily with sulfur. The third and fourth types are differentiated only in the degree of treatment that is used. These types of treatment make use of chlorine and its compounds of hypochlorite. The latter type satisfies the entire demand of the sewage and then leaves a "residual," or excess, for the complete sterilization of the flow.

In Arizona the problems have involved all of these types. The Scott-Darcy process has been used for the manufacture of iron in such form that it may be used for the first and second uses outlined previously. By modifying the equipment, the plant may be used for the other two purposes, although this has not been the general practice to the present time (1941). Many installations in Arizona have been made on collection and outfall sewer systems for the purpose of removing the sulfides from the field of action, to reduce odors, and to retard the corrosive action of the sulfides on concrete sewers.

It will be recalled that chlorine and some of its compounds have been used for a number of years for the aforementioned purpose. As long as the sewage is sterilized in its initial or very fresh stage, the use of chlorine has been effective. After decomposition has proceeded to a certain point, the use of chlorine is expensive and does little to remove sulfides already formed.

In line with the pioneer work of the Scott-Darcy process as established in Oklahoma, the City of Tucson, Ariz., installed the first Scott-Darcy plant in Arizona. This was for the purpose of removing sulfides already formed in the sewage, and for the addition of sufficient iron to the sewage to react with the sulfide delivered to the outfall from the many interceptors that discharge into it below the point of application. Also, a major purpose of this installation was to control odors at the disposal plant. An installation similar to the original Scott-Darcy was installed. This installation allows superchlorinated water to enter one end of a contact or reaction tank that is filled with scrap iron. After a period of contact therein, the iron and chlorine solution flows out of the far end of the tank and enters the sewage.

This installation has resulted in a lower "free sulfide content" of the sewage to a point 2 miles below the point of iron-chloride application, where the sulfides again begin to increase until the plant is reached. It is apparent that either a larger dosage of iron-chloride at the present plant or another plant is necessary at the point where the sulfides again begin to increase.

This same type of installation was made at Chandler, Glendale, Phoenix, and Wickenburg, Ariz. In each instance the control of odors was of extreme importance, and the plants have achieved this purpose. It has been expensive to operate them, yet in each instance any chemical or other treatment would have been expensive. The installation of these plants has forestalled some expensive litigation, which in at least two instances had been instituted against the cities or towns.

Phoenix has stopped the deterioration of concrete sewers with the process by the installation of some four plants throughout the system. This fact has been determined by visual inspection only. Prior to the installations, the wall thickness of some concrete sections had deteriorated 50% in four years. It is hoped that this process will extend the life of these sewers to last out the length of the bond issue, or until 1962. Chandler has stopped the odors from a septic tank. Lawsuits were pending against the city. By the addition of Scott-Darcy, these lawsuits have been dropped at least temporarily. Glendale has corrected an odor condition at this plant, and is using the iron as a precipitant after the addition of lime to form the iron hydrate. This installation has worked satisfactorily for the manufacture of deodorant and precipitant. Wickenburg has installed such a plant, but it needs further investigation to determine how effectual it will be.

During this period of development, the more modern Scott-Darcy process was developed at the Tovrea Packing Plant. It was the purpose of this installation to provide odor control and a precipitant, to have free chlorine available for the coagulation of blood in the slaughterhouse waste, and to have available a sterilizing solution if it was found to be of use. The use of the Scott-Darcy plant for the latter two purposes has not been found to be necessary to the present time (1941). The raw waste entering the plant ranges in strength from 1,000 to 5,000 ppm of suspended solids. After flocculation and sedimentation with the use of iron chloride from the Scott-Darcy unit, the final effluent has a suspended solid range from 65 to 442 ppm. The effluent has been reduced in B.O.D. strength, with a range of from 750 to 2,200 ppm down to 113 to 300 ppm.

The difference between this Scott-Darcy plant and the other plants is that the iron-chlorine solution is recirculated through special pumps so that the iron solution may be chlorinated to that particular point desired by the operator, and the required type of iron chloride produced. The flexibility of the plant to meet changing hourly demands has proved an excellent tool in the solution of this problem. The packing plant officials, health agencies, and the neighbors are all satisfied with this plant.

The conditioning of this sewage has made available an irrigation water supply that is used on 160 acres of land for the production of forage crops to be fed to the beef cattle held at the plant. Prior to the installation of this plant, packing house waste was discharged into the Salt River to the embarrassment of the various health departments and to the despair of those living in that vicinity.

From the detail of information offered in Mr. Schroepfer's analysis of the Minneapolis-St. Paul plant, and from these casual observations, it might be concluded that chemical treatment is vitally necessary when needed. It furnishes a flexibility of operation that is difficult to attain otherwise. It may be for only a short period of time during the day, week, or month that chemical treatment might be needed. When available, it levels off a peak effectively, and improves operation.

In conclusion, the writer desires to express appreciation to J. A. Carollo and Dario Travaini, Associate Members, Am. Soc. C. E., Phil J. Martin, Jr., of Tucson, and W. J. O'Connell of San Francisco, Calif., for the generous assistance that they have provided in the preparation of this discussion.

Rolf Eliassen,[5] Assoc. M. Am. Soc. C. E. (by letter).—Complete operation and cost data on the vacuum filtration and incineration of the raw sludge at the Minneapolis-St. Paul Sewage Treatment Plant are presented in this paper. Data of this nature are essential in order to obtain a better economic evaluation of the various methods of sludge treatment and disposal.

Most of the larger cities in the United States have complex problems of sludge disposal. The sludge is usually digested, followed by drying on sand beds or vacuum filters and ultimate disposal on land as fertilizer or fill. Some cities prefer to subject raw or digested sludge to vacuum filter and either dry it mechanically or incinerate it before ultimate disposal. As the author has stated, in the Minneapolis-St. Paul plant the raw sludge is conditioned with chemicals, partly dewatered on vacuum filters, then dried and incinerated in multiple-hearth furnaces.

The City of New York, N. Y., is more fortunate in that it is located on the Atlantic Ocean and can utilize that body of water for disposal of its sludge. In 1940, Wellington Donaldson,[6] M. Am. Soc. C. E., described New York's method of sludge disposal. The sludge from three treatment plants is transported in sludge vessels designed for that purpose. They are three sea-going vessels of the latest design and have sludge storage capacities of 55,000 cu ft each. The sludge dumping grounds are 34 miles out to sea from the Ward's Island Sewage Treatment Plant. These grounds are far enough removed from the shore to preclude any danger of harbor or coastal pollution.

Utilizing data presented by the author and Mr. Donaldson,[6] an interesting comparison may be made between these two distinctly different methods of sludge disposal.

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[5] Assoc. Prof., San Eng., Coll. of Eng., New York Univ., New York, N. Y.

[6] *Waterworks & Sewerage*, December, 1940, p. 583.

These comparative data for the year 1939 are as follows:

	Minneapolis-St. Paul	New York City
Initial cost, in dollars	1,240,000	1,500,000
Total sewage flow treated, in million gal	36,705.3	62,696.1
Total sludge, dry tons	37,703	54,088
Operating and Maintenance Costs (in Dollars):		
Total	136,815.43	137,931.50
Per million gal of sewage treated	3.74	2.20
Per ton of dry sludge	3.63	2.55

In the case of Minneapolis-St. Paul, the initial cost includes the cost of constructing the filtration and incineration building and appurtenances. For New York, the cost is that of constructing the three sludge vessels. As far as one may ascertain from an analysis of the supporting data and discussions, these cost analyses are on a comparable basis.

Further economies are possible in both cases. This is evidenced by the fact that data for the first five months of 1940 at Minneapolis-St. Paul show costs of sludge disposal at $3.15 per ton of dry solids, a reduction of 13.2%. This has resulted from more efficient use of chemicals, filters, and incinerators. Even greater economies are possible in New York as means are devised for the thickening of the sludge before loading on the vessels. The 55,000 cu ft of storage capacity should be filled with the heaviest possible sludge in order that the tonnage of dry solids may be at a maximum for each trip to the dumping grounds.

FREDERIC BASS,[7] M. AM. SOC. C. E. (by letter).—The Minneapolis-St. Paul sewage disposal plant was located and designed after a preliminary study extending over a period of seven years, from 1927 to 1934. During this period the science and art of sewage disposal was rapidly advancing, and the design ultimately adopted embodied a number of features not previously available. Perhaps the most notable characteristic of the design is its flexibility, which enables it to function with economy under the varying conditions of flow in the Mississippi River. Among other features which, at the time of design, were considered by some engineers as innovations, but which were included after careful observation and study, were the unusual length of the sedimentation tanks, the elimination of sludge digestion, and the incineration of filtered sludge. The performances and costs of operation, presented in the paper, appear to justify the many studies made by the author.

Although conclusions from the limited period of operation covered by the author must be regarded as tentative, they unmistakably hold special interest for all sanitary engineers. For the purposes of this discussion the author has made available to the writer certain operating data obtained later than April 30, 1939. The records previous to January 1, 1939, because of the usual process of adjustment in a new plant, probably do not represent normal averages of performance and cost as closely as do the subsequent records. Table 15 contains statistics of operation for the year 1940.

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[7] Cons. Engr.; Head, Dept. of Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

The quantity of sewage was somewhat less than expected, the strength expressed in B.O.D. was approximately equal to that expected, but the suspended solids (perhaps because of changing industrial conditions and effect of surface runoff) were more than 50% greater than the expected amounts. This heavy load fortunately was offset by the unusually high removal of suspended solids in the sedimentation tanks. In these tanks (290 ft long with unusually long overflow weirs) the average removal of 71.1% (detention period 1.5 hr) of suspended solids, which itself is greater than the performance of most plants, has been raised under average conditions over a period of several weeks to approximately 80% by the use of air flocculation at the trifling cost of 6 cents per million gal treated. The expected 56% removal of 185 ppm would have left 81 ppm in the effluent, whereas the actual performance of 80% removal of 300 ppm has left only 60 ppm in the effluent.

TABLE 15.—OPERATING DATA FROM JANUARY 1, 1940, TO DECEMBER 31, 1940

Item	Description	Monthly			Expected at time of design
		Maximum	Minimum	Average	
1	Flow (mgd)	113.87	90.16	104.17	122
	Sewage Strength (ppm):				
2	Suspended solids	345	245	300	185
3	B.O.D.	240	160	210	200
4	Screenings (cu ft per million gal)	1.96	0.78	1.06	3.1
5	Grit (cu ft per million gal)	9.3	3.1	6.2	4.1
	Total Removal in Plant (%):				
6	Suspended solids	83.5	64.0	77.2	56
7	Settleable solids	96.5	84.1	91.5	....
8	B.O.D.	53.0	35.4	43.2	36
9	Filter cake produced (tons), average daily	340.2	264.7	308.3	235
10	Cake moisture (%)	68.1	61.1	64.8	68
11	Cake volatiles (%)	68.6	51.4	59.8	55
12	Ash from sludge (tons)	1,760	820	1,275	....
	Sludge Conditioning Chemicals (%):				
13	Ferric chloride	2.32	1.59	1.83	3.0
14	Lime	6.53	3.53	4.51	10.0

The B.O.D. removal was boosted by flocculation as well as by removal of suspended solids, the author's figures showing 49.8% with flocculation as against 44.2% without flocculation. The total capacity for removals of suspended solids and B.O.D. with the use of chemical precipitation will be awaited with much interest. From the performance of the plant already given, the maximum removals as expected by the author, in the tabulation preceding the heading "Plant Operation: Screen and Grit Removal," appear to be readily obtainable. The bacteriological results from the use of chlorine, although of short duration, are interesting.

The conclusions from this phase of the operation seem to extend the usefulness of this type of flexible plant design to many situations where a variable river flow exists. For such conditions, a careful study of quantity and quality of both river and sewage may result in economy of both first cost and operation.

The sludge disposal department of the plant appears to have been as successful as the sewage treatment department. Drawing sludge frequently from the sedimentation tanks to the concentration tanks and stopping withdrawal with thinning sludge have resulted in a heavier sludge. A quantity of sludge 35% higher than expected, offset by higher heat value, less moisture, higher volatiles, and unusually close control of sludge conditioning and filter operation, has led to a lower over-all cost of sludge disposal than was estimated. It is remarkable that practically no oil fuel is necessary. The costs given by the author compel attention.

The improvement of the river above the plant is very satisfactory in appearance, as the analytical results indicate. Improvement of conditions in the river below the plant can be appraised only after the installations of treatment plants for the large packing plants located there.

Supplementary data on cost supplied by the author permit the summary in Table 16 for the 12-month period ending December 31, 1940:

TABLE 16.—COST OF OPERATION AND MAINTENANCE, JANUARY 1, 1940, TO DECEMBER 31, 1940

Item	Description	Operation	Maintenance	Total	Percentage of total
1	Administration, engineering laboratory....	$74,637.24	$1,326.01	$75,963.25	25.6
2	Sewage treatment........................	43,843.65	8,917.08	52,760.73	17.8
3	Sludge disposal..........................	112,580.62	18,218.60	130,799.22	44.1
4	Miscellaneous............................	33,508.90	3,376.68	36,885.58	12.5
	Total..................................	$264,570.41	$31,838.37	$296,408.78	100.0

In this period a total of 38,038 million gal of sewage was treated at a cost of $7.80 per million gal for all operating and maintenance items. During this period the magnetite filters have not been in full operation and chemical precipitation has not been used. Consequently, over a sequence of years, which includes a considerable proportion of drought when all facilities of treatment are used, the costs may be somewhat higher. It might be concluded that the cost of administration, engineering, and laboratory, at about 25% of the total cost of operation and maintenance, is unduly high, but it is probable that every dollar thus spent is reflected in much greater savings in the other items.

It has been a pleasure to examine the detailed records of performance and accompanying cost data available in the office of the author. It will be a great public benefit when all sewage works are as carefully and economically managed as the one in Minneapolis and St. Paul; plant operators will be stimulated to greater interest when comparative figures are made available. These cities are indeed fortunate in having conceived and built a plant so well fitted to the existing conditions and so effectively and economically operated.

The emphasis in this paper upon performance, and particularly upon costs, deserves more than passing notice at this time. A sewage treatment plant is but one of the agencies of the promotion of health and cleanliness of a community; there are others of perhaps even greater importance, which need consideration and call for the expenditure of public funds. Since public financial resources are limited, the analysis of costs and relative value of various public works is an important duty resting upon engineers who are becoming increasingly responsible for the conservation of national resources.

Darwin W. Townsend,[8] M. Am. Soc. C. E. (by letter).—The practice of pollution control by cities on the upper Mississippi River has been marked during the past decade (1930–1940). Modern sewage treatment works are now (1941) in operation in the cities of Minneapolis-St. Paul, and South St. Paul; Rock Island, Ill.; and Davenport, Iowa, as well as in other populated centers of lesser magnitude. The Minneapolis-St. Paul plant, which Mr. Schroepfer's excellent paper treats, and the South St. Paul plant, designed by the writer's firm, are the two most northerly plants of magnitude on the Mississippi River in the foregoing group. Upon the completion of the Newport, Minn., treatment works, the pollution control in the populous and highly industrial Minneapolis-St. Paul metropolitan area will have been effectively accomplished. Of unusual note, perhaps, is the fact that three separate treatment plants will serve this area, these plants being within sight of each other.

It may be of interest, by way of contrast, to comment briefly upon sewage B.O.D. concentrations obtaining in each of the aforementioned instances. Sewage delivered into the Minneapolis-St. Paul treatment plant has a 5-day B.O.D. concentration—approximating 200 ppm, as stated by Mr. Schroepfer. At this concentration, a sewage of average domestic character is indicated, in which case little, if any, difference might be anticipated between human and equivalent populations.

Available dilutions and dissolved oxygen (D.O.) supply afforded by the waters of the Mississippi River for the assimilation of organic pollution indicated that B.O.D. removals from Minneapolis-St. Paul sewage, approximating 70% for ten months and 33% for two months, would prove adequate to maintain in the receiving waters a 2.0 ppm D.O. residual at all except infrequent times of unusually low river stage. It is evident from the data presented by Mr. Schroepfer that, when contrasted with the volume of river flows shown, plant removals thus far obtained have been quite adequate to accommodate the D.O. standard established.

The requirements as to degree of treatment for the South St. Paul-Newport area differ vastly from those of the Minneapolis-St. Paul area and call for a much higher degree of treatment. In South St. Paul, as well as in Newport, the meat-packing industry predominates, and as a result the sewage treatment problem is essentially that of treating packing-house and stockyard wastes containing an average B.O.D. concentration approximating 1,000 ppm, the peak concentrations reaching as high as 3,000 ppm. The human population of South St. Paul approximates 15,000, whereas, at times, the equivalent population may exceed 300,000.

To comply with the requirement of the Minnesota Board of Health (namely, that the effluent from the South St. Paul treatment plant contain a B.O.D. concentration that would not be greater per unit of volume than the Minneapolis-St. Paul effluent), it was necessary to design treatment works of biologic character to obtain 90% to 95% B.O.D. removal, these works to include three stages of sedimentation, flocculation, grease removal, and trickling filters. Effluent chlorination is to be practiced during the summer months.

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[8] Cons. Engr. (Consoer, Townsend & Quinlan), Milwaukee, Wis.

The lead taken by the Minneapolis-St. Paul Sanitary District in pollution control of Mississippi River waters provided the necessary stimulus to downstream cities for subsequent participation in work of similar character, some of which have been referred to herein.

Although Mr. Schroepfer has performed a yeoman's task in preparing and presenting a well-conceived and well-written paper, supplemented with interesting data, the writer takes the liberty of suggesting that, in the closing discussion of his paper, something additional be written regarding the extensive river-water studies made by himself and the late J. A. Childs, M. Am. Soc. C. E., former sanitary engineer for the District. The writer had occasion to study this work while serving the District as consultant, and it is most certain that the results of these water reoxygenation studies would appeal to many sanitary engineers.

Of particular interest to the writer is the subject of sludge disposal as practiced at the Minneapolis-St. Paul plant. This installation is believed to be the most extensive of its particular kind in existence, and may be considered unusual, perhaps, in that raw, rather than digested or otherwise treated, sewage solids are filtered and incinerated.

The partial dehydration of sewage solids by means of vacuum filters has become standard practice during comparatively recent years, the first sizable installation having been placed in operation at Milwaukee, Wis., about 1925. This installation dehydrates activated sludge, which subsequently is heat-dried for use as a fertilizer. The solids filtered at this plant may be classified as raw activated sewage solids, as differentiated from digested solids, and raw untreated solids. They possess radically different filtering characteristics from those of the two last-named materials. Sludge filtering at Milwaukee, therefore, is not comparable with generally similar work done at the Minneapolis-St. Paul plant.

Many treatment plants throughout the United States now use vacuum filters and appropriately designed incinerators as a means of sludge disposal, and this method appears to be becoming increasingly popular. However, general practice recognizes certain economies indicated and attainable through the use of an intermediate step in sludge-disposal procedure—namely, that of reducing sludge volume through the process of digestion.

Instances may be cited wherein the available power contained in the gas resulting from controlled sludge digestion equals or exceeds the entire power and heat requirements of a treatment plant. Since power is a substantial item of expense in any sewage treatment plant (more so in some than in others, depending upon plant type and prevailing rates), savings that may accrue to a city through the use of gas-engine equipment, motivated by gas produced from the sewage within the plant, have in recent years met with almost universal favor.

Perhaps the author may make some reference in his closing discussion to vacuum filtration of raw solids, and incineration versus digestion and gas utilization as studied in connection with the Minneapolis-St. Paul project. Such references would be most interesting.

LLOYD H. FLICKINGER,[9] ASSOC. M. AM. SOC. C. E. (by letter).—The subject of sewage treatment has received a valuable contribution by Mr. Schroepfer. Designers and operators of many treatment plants, whether of the chemical-mechanical type or other types, can learn valuable lessons from this paper. When this plant was designed, it was more than twenty times as large as any other one of its type then in operation. Many new ideas were incorporated in the plant and it is gratifying to find that the results have been very satisfactory, with a few minor exceptions.

The writer was employed on the Minneapolis-St. Paul project throughout the design and construction period of the treatment plant, and for a short time after the plant was placed in operation. The purpose of this discussion is to call attention to certain features of the design and operation that merit close attention by future designers of similar plants.

The kind of treatment selected for this project was of a type having great flexibility to provide a degree of treatment in direct proportion to the demands of the Mississippi River. This procedure is not subscribed to by many state boards of health. Too often they require "complete treatment" with little regard for the ability of the stream to absorb a pollution load. It is economically unsound to provide a continued high-grade treatment where a large part of the pollution load can be absorbed by the stream without nuisance or danger to the community. This procedure would be comparable to operating a furnace in a home at full capacity regardless of the temperature outdoors, merely because of the available capacity of the heating plant.

The type of plant selected for this project is economically sound and provides a very high degree of treatment only when it is required. This is sound economics and should be observed.

The plant was designed on a basis of 185 ppm suspended solids in the raw sewage. The actual amount found by operation was about 275 ppm. Contrasted to this the estimated B.O.D. was 200 ppm and the actual was 210 ppm. The unusual amount of suspended solids has never been explained fully. The quantity of sewage in gallons was about 20% less than estimated for the years 1938–1940. This accounts only partially for the high suspended solids content.

In 1933 principal sewers were weired and samples collected for about two weeks. The principal outlets were attended by a sample collector 24 hr per day throughout the test period, and the quality of the analytical work has never been questioned. The only deduction is that something caused a great change in the sewage in the 5-yr interval from 1933 to 1938. However, no substantial industrial change took place except in the brewing and malting plants.

The extra-long settling tanks, almost 300 ft, were selected because they fit the site very well and were lower in first cost than the shorter ones. The estimated removal of suspended solids was 56% and the actual was about 70%. Designers should note that these tanks have a very high weir ratio. The amount of weir length per million gallons of sewage was about halfway between that used for circular tanks and that usually used for rectangular tanks. These tanks are 56 ft by 290 ft and have 480 lin ft of weir per tank. The unusual results from the extra long tanks with high weir ratios should be carefully noted by all designers of settling tanks.

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[9] Capt., Corps of Engrs., Quartermaster Corps, U. S. Army, Operations Officer, Kansas Ordnance Plant, Parsons, Kans.

The scum handling methods originally installed proved unsatisfactory in operation. Pneumatic ejectors definitely proved their worth. Line stoppages can be eliminated with this method, if the scum influent lines to the ejector are short and have an appreciable slope.

The plant is arranged so that several kinds of chemical can be used. Engineers have long suspected that the price of many chemicals does not depend on the cost of production or the supply. It is suspected that the price is controlled, perhaps on the basis of all the traffic will bear. Engineers can meet this practice by designing for several chemicals. All the added cost can be saved easily in purchases of the most economical chemical. In other words, while the chemical producers are using chemical prices as so many poker chips in an industrial game of chance, the sanitary engineer can buy the chemical that is in the "bear" market.

The sludge in the concentration tanks actually became too concentrated to be handled at times. The need for concentration can be questioned, but the storage facilities providing a reservoir on the line proved the worth of the tanks. Future concentration tanks can properly include facilities for diluting the sludge when necessary.

Sludge containing 10% to 12% solids needs thinning for proper handling through a piping system of any length. Thick sludge frequently disobeys some of the fundamental laws of hydraulics. Sludge pipe design demands that a little "salt" be taken with theoretical results assumed using hydraulic computations for water.

It is gratifying to note the low chemical dosage required to condition the sludge. The writer recommends no change from the original dosage of 3% ferric chloride and 10% lime, as a basis of design. The chemical demand for the conditioning of sludge usually cannot be determined before the plant is put n operation, with the exception of additions to existing plants and plants other than the first in a city.

The favorable results of vacuum filtration of raw sewage solids at this plant definitely put an end to the argument that digestion is necessary to satisfactory and economical vacuum filtration. Mr. Schroepfer's experiments with different kinds of filter cloth are very enlightening, and similar tests should be made at other plants to confirm the results.

Incineration of sewage solids was in its infancy when the Minneapolis-St. Paul plant designers decided to use this method of sludge disposal in 1936. The use of extensive and delicate heat-recovery units does not appear economically sound in the light of present knowledge. They are very costly and expert operation is necessary to prevent serious damage to the units. The problem at the Minneaplis-St. Paul plant seemed to be one of disposing of excess heat instead of recovering it.

The writer recommends that screenings and grit be disposed of by incineration, omitting the extensive grit washing operation and screening grinders.

This waste can be carried direct to the incinerators by one belt conveyer. The belt can be operated manually as required, dumping the unwashed grit and unground screenings on a horizontal belt conveying the sludge cake to the incinerators. Close control of the grit chamber velocities becomes unnecessary because high putrescible contents of grit to be incinerated are not objectionable. Due to existing arrangements at the Minneapolis-St. Paul plant, it is probably not practical to do this, but designers of new plants can well consider this possibility.

The profession is indebted to Mr. Schroepfer and the operating staff of the Minneapolis-St. Paul Sanitary District for the extensive and valuable data obtained from the first two years of operation. The plant is so designed that one half can be used as a control and the other half for experimental work. At the present rate more valuable contributions to the science of sewage treatment can be expected from this plant.

The writer is very pleased to learn of the vast improvement in the condition of the Mississippi River and is particularly pleased to have been a part of the design staff that made this improvement possible.

Isador W. Mendelsohn,[10] M. Am. Soc. C. E. (by letter).—As expected, Mr. Schroepfer has presented a fitting complement to his notable contribution on "Economics of Sewage Treatment" published in 1938.[11] This paper is more than a repository of interesting experiences for the first two years of operation of the sewage treatment plant serving the Minneapolis-St. Paul Sanitary District. It should prove a source of unending stimulation to operators and designers of sewage treatment plants, large and small, for it is replete with ever-promising, economy-producing experiments and studies in mechanical equipment and plant operation. Throughout all of these studies, the engineers never lost sight of the purpose of producing as economically as possible an effluent that would merge with the varying receiving body of water in a manner not prejudicial to public health and comfort.

The lengthy and remarkable list of tests on plant equipment and operation performed by Mr. Schroepfer and his organization in this period includes:

For screen and grit removal—

(1) Increasing the velocities of flow from 0.75 to 1.20 ft per sec to secure a greater percentage of inert material in the sludge, thereby increasing the solids content of the sludge and reducing the quantity of conditioning chemicals required;

(2) Burial of the proportionately small quantity of screenings with an appropriate depth of relatively clean grit;

(3) Reducing sand wear on the lower and intermediate bearings on the screw conveyer grit washers by installing sealed water-washed and lubricated bearings;

(4) Overcoming difficulty with the grit bucket elevator by installing an air jet to sweep high moisture content, large "fines" percentage grit out of the buckets, and installing sealed bearings of a special type.

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[10] Capt., Corps of Engrs., U. S. Army, Utilities Officer, Fort Worth, Tex.

[11] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1210.

For sedimentation—

(5) Careful pumping control of raw sludge from the settling tanks once during each of the three shifts so as to average 7.7% solids (92.3% moisture);

(6) Providing an ejection method of handling scum after its removal from the settling tanks, thereby reducing labor required;

(7) Providing for blowing back sludge suction lines as long as 300 ft with plant effluent or compressed air to handle heavy sludge containing as much as 20% solids;

(8) Carrying a heavy blanket of normal sewage sludge in the tanks previous to storm flows to remove the inert material more effectively because of evident loosening of the mud and silt which ordinarily packs and produces difficult removal problems;

(9) Replacing pistons and collars on sludge pumps with heavier parts made in the Sanitary District shop at a fraction of the cost of such parts from manufacturers to permit resurfacing from time to time, thereby prolonging their life;

(10) Installing chain tighteners on the sludge collection equipment to reduce breakage of links of drive chains.

For magnetite effluent filters—

(11) Overcoming sand erosion and displacement difficulties at greater than average flows by enlarging influent ports and other improvements.

For sludge conditioning—

(12) Shortening the period of sludge conditioning to an average of 6 min with continuous control of filtration by means of funnel tests;

(13) Improving sludge for filtration and at the same time reducing the quantity of chemicals used by agitation with excess low-pressure air from the blowers, provided for removing cake from the filters;

(14) Providing means for distributing the chemicals more uniformly through the conditioning tanks;

(15) Determining the proper sequence of chemical (lime and ferric chloride) additions.

For vacuum filtration—

(16) Recirculation of a percentage of incinerator ash to the raw sewage ahead of the screen and grit chambers to improve vacuum filtration, reduce the quantity of conditioning chemicals, and increase sedimentation efficiency;

(17) Reducing the quantities of chemicals (3% ferric chloride and 10% lime) to less than 2% and 5%, respectively, amounting to a saving of approximately $100 a day in chemical costs alone;

(18) Increasing filter cloth life from about 150 hr of use to more than 300 hr;

(19) Purchasing identical filter cloths locally at a fraction of their original cost;

(20) Sewing filter cloths with a different type of thread and with a lock stitch to eliminate seam difficulties;

(21) Improving the washing of filters by impinging the water at right angles to the cloth on the drum, thereby reducing the washing time to one third, using less water, and eliminating brushing of the cloth;

(22) Installing sloping ¼-in. rods on 12-in. centers in the filter take-off plates to break up the cake and prevent stoppages.

For incinerators—

(23) By by-passing preheated air direct to the stack to save rabble arms and other incinerator parts.

For sludge disposal—

(24) Exploring the fertilizer or soil conditioning possibilities of sludge cake as produced by the filters and its use after storing for periods of time;

(25) Experimenting with the production of partly dried material by removing portions of the incinerator top-hearth sludge, using heat from the remainder of the sludge incinerated for drying purposes;

(26) Investigating the use of ash for binder or filler for various products, such as a soil conditioner or an ingredient for certain cements;

(27) Testing the utilization of heat from incineration in the generation of steam for all heating purposes and electric-power requirements.

For flocculation without chemicals—

(28) Testing for a brief period to determine the degree of improvement that may be obtained in sedimentation.

For chemical treatment—

(29) Testing for four days to determine possible sedimentation-removal improvements.

For chlorination—

(30) Experimenting on bacterial removal with chlorine dosages of 6 to 8 ppm with favorable results. A large part of the great reduction (approximately $330,000) in operation costs of the first 2 yr is attributed to the operation economies effected by the tests mentioned.

The paper is notable also for the detailed operation and maintenance cost data presented. For comparative purposes with other sewage treatment plants, it would be desirable, when more complete operation data are available, to show for each plant unit the construction cost, the operation and maintenance costs, the specific parts-per-million removals in suspended solids and B.O.D., and all three costs per 1% reduction in suspended solids and B.O.D.

The fact that vacuum filtration and incineration of sludge cost $3.70 per million gal, or approximately 45% of the entire plant operation and maintenance costs, is noteworthy. The relative cost seems high for sludge disposal alone. It presents a great opportunity for improvement. Whether the cost of sludge disposal would have been lower with some other process or combination of processes cannot be determined from the data presented. As bearing on this question, it would be of interest to note what studies were made regarding sludge disposal in the design of the plant, what construction, operation, and maintenance costs were estimated for each method considered, and why vacuum filtration and incineration were adopted.

As between vacuum filtration and sludge incineration operation and maintenance costs, it would be desirable, for comparison with similar data in other plants, to learn the basis of allocating labor and supervision expenditures to each.

With reference to the incinerator, it is surprising to learn that "It was the manufacturers' expectation that the location of burning, and the temperatures of combustion, would be controlled by the use of varying quantities of oil introduced at different points" (see heading, "Sludge Disposal: Incinerators"). Acceptance tests on multiple-hearth sludge incinerators at Auburn and Elmira, N.Y., sewage treatment plants in May, 1937, disclosed excess heat, requiring by-passing of preheated air direct to the stack to prevent damage to incinerator parts, and use of oil only for warming up and maintenance of temperature in off-burning periods.

Mr. Schroepfer refers to extensive guarantee and maintenance bonds of various types included in mechanical equipment contracts for the plant. In view of his experience with such provisions, it would be of value to learn his views as to what kind of guarantees and maintenance bonds should be included in specifications and for what kind of mechanical equipment, what tests should be made to determine whether or not the guarantees are complied with, how the tests should be conducted, and for what time periods.

Before concluding these remarks, it is most important to note that the worthy results obtained in the operation of this sewage treatment plant are the outgrowth of a capable and energetic operator with an experienced organization, adequate laboratory and plant recording equipment, and above all a body of intelligent, sewage-minded Sanitary District officials, representing a community that desires, and is willing to back, disposal of its sanitary sewage in a manner which will not prove harmful. Similar fully trained, equipped, and cooperating groups are a vital necessity in all communities, however large or small, for sanitary disposal of their sewage.

RALPH E. FUHRMAN,[12] Assoc. M. AM. SOC. C. E. (by letter).—In a comprehensive manner, Mr. Schroepfer has described the development of the sewage treatment scheme for the Twin Cities. From the inception of the project, the primary purpose of the plant (that of eliminating pollution from the Mississippi River) has received first consideration.

The design of the plant has, as its fundamental basis, the varying requirements of the receiving stream. This rational basis has provided, through the author's analysis, the degree, and probable duration and frequency of varying degrees of treatment imposed by river conditions. The result, a chemical-mechanical sewage treatment plant, provides the range of treatment and flexibility to relieve the river of pollutants economically.

The District of Columbia sewage treatment plant and receiving stream present some interesting parallels with the Twin City plant and its receiving stream, namely:

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[12] Asst. Supt., Sewage Treatment Plant, District of Columbia, Washington, D. C.

Description	Twin City	District of Columbia
Design flow (mgd)	134	130
Full-time operation begun	July 20, 1938	August 1, 1938
Original construction cost	$3,750,000	$3,400,000
Construction cost per design (mgd)	$28,000	$26,100
Minimum flow of receiving stream (cu ft per sec)	864	700
Maximum estimated flow (cu ft per sec)	107,000	480,000
Annual average flow (cu ft per sec)	8,800	9,480

Similarity ends, however, when the plant processes themselves are considered. At the Twin City plant, sewage is screened, grit is removed, coagulating chemicals may be added prior to metering, flocculation, and sedimentation. When desired, more solids may be removed by use of the effluent filters. At the District of Columbia plant, sewage is pumped, grit is removed, the sewage is metered and aerated for grease separation before sedimentation, the last step in the sewage treatment process. At the Minneapolis-St. Paul plant, raw solids are dewatered on vacuum filters and incinerated. At Washington, D. C., raw solids are digested, digested solids are elutriated and dewatered by vacuum filters. Most of the sludge cake is shipped to the District of Columbia penal institutions for agricultural uses. Between the two plants, practically all current sewage and sludge treatment methods except activated sludge are used.

Since the dates of design and initial operation of the two plants are very close, some operating experiences have been common to both plants. For instance, the failure of the underwater and grit thrust bearings of the grit conveyers described by Mr. Schroepfer was also experienced at Washington, D. C. At the District of Columbia plant this failure occurred at an early date and the difficulty was corrected by installing standard ball thrust bearings at the shaft coupling above the sewage level. Completely satisfactory operation of the conveyers has been gained by this alteration.

Although the sedimentation tanks of the District of Columbia plant are circular, experiences in tank skimming have been similar to those at the Minneapolis-St. Paul plant, except for difficulties caused by climatic conditions. At the Washington, D. C., plant, completely automatic skimming was not provided originally. Collection of the scum from the surface of the sedimentation tank was mechanical and continuous, but removal of the scum was manual and intermittent. New skimmers have been installed which give continuous skimming and removal of scum. Minor stoppages have occurred in the lines conveying scum to the hoppers from which the sludge pumps take suction to pump the scum to the sludge digestion tanks; but as these lines are short, clogging is easily relieved mechanically.

Because of the completely different nature of sludge treatment by the two plants, an operating cost comparison of sludge treatment is interesting. From Mr. Schroepfer's paper, Twin City costs for the 1939 calendar year for vacuum filtration and incineration were $136,815.43, giving unit costs of $3.70 per million gallons or $3.63 per ton of dry solids treated. At Washington, D. C., for the 1939 fiscal year (July 1, 1938, to June 30, 1939), the operating costs for sludge digestion, sludge elutriation, sludge dewatering (vacuum filtration) and sludge cake disposal were $54,486, giving unit costs of $1.59 per million gallons or $4.41 per ton of dry solids removed. These data should not be taken in direct comparison, as they are based on early operating periods of both plants. They do indicate, however, that ultimate operating costs of both methods will be reasonable and of a comparable nature. The final cost analysis must include fixed charges, and energy recoveries by both methods. In the Minneapolis-St. Paul layout this will mean heat from the incinerators and ash value, whereas at Washington, D. C., it will mean sludge gas, both as heat and power and the fertilizer value of the sludge produced.

R. W. ROWEN,[13] ESQ. (by letter).—This was the first large plant to adopt filtration and incineration of undigested sludge. After a careful investigation of the various types of sludge incinerators, it was decided to install three multiple-hearth, mechanically rabbled incinerator units. When they were placed in operation it was found that the filter cake contained less moisture and a higher percentage of combustible matter than anticipated. As a result, no auxiliary fuel was required and, in fact, there was an excess of heat that had to be dissipated. Therefore, hot air from the air preheater units was by-passed directly to the stack, and air at ordinary temperature was used for combustion. The center shaft and rabble arms on the hottest hearths were insulated, and this has effectively overcome the danger of overheating.

Grit and screenings are not incinerated at the Minneapolis-St. Paul plant, although they are now being incinerated at Detroit, Mich., and at several other smaller plants using multiple-hearth incinerators.

The volume of waste, deodorized gases leaving the incinerators at about 1,000° F would produce enough steam for plant heating and all electric power requirements of the plant and still have an excess of steam with only one incinerator unit in operation. The raw sludge filter cake, containing an average of about 65% moisture, 20% combustible material, and 15% ash on the wet basis, has sufficient heat content not only to be self-combustible, but also to furnish all heat and energy required for the entire plant. The economics of heat and power generation are being investigated, and it may prove advantageous to install waste heat boilers and turbo generators.

Another means of utilizing this waste heat is being studied at the Minneapolis-St. Paul plant, which consists of feeding a part of the thickened sludge, containing about 90% water, directly to the incinerator without mechanically dewatering. As the author has pointed out, filtration cost is 72% and incineration is only 28% of the total cost of sludge disposal. By feeding a part of the liquid sludge directly to the incinerator and utilizing the excess heat from the part that is filtered, a further substantial saving in operation costs may be obtained through the elimination of filtration costs on part of the sludge. This is along the line of developments made during 1940 at the Piqua, Ohio, and Ashland, Ohio, sewage treatment plants where all of the undigested, unfiltered sludge is incinerated directly as a liquid in multiple-hearth incinerators, eliminating entirely the cost, mechanical difficulties, and skill formerly required for filtration at these two plants.

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[13] Vice-Pres., Nichols Eng. & Research Corp., New York, N. Y.

The type of sewage treatment and means for sludge disposal at the Minneapolis-St. Paul plant were decided only after long and intensive study of the requirements. A program of investigation, studies, and tests during operation has resulted in economies well beyond those anticipated when the plant was constructed. Frequent meetings of the operating staff for discussion of minor mechanical matters, as well as major developments or theorizing on some idea, have borne fruit.

George J. Schroepfer,[14] Assoc. M. Am. Soc. C. E. (by letter).—The writer has enjoyed reading the discussions of his paper and appreciates the time spent by the various writers in the preparation of their comments. The discussions developed no controversial issues, but rather were replete with suggestions that will serve as a source of encouragement in improvement of the operation of the Minneapolis-St. Paul sewage treatment plant.

Professor Eliassen makes an interesting comparison between the costs of sludge disposal by dumping at sea, as practised in New York City, and by filtration and incineration, as used at Minneapolis and St. Paul. It is evident that even the seemingly simple expedient of hauling to sea is a comparatively expensive process, involving costs approximating two thirds to three fourths of those at Minneapolis and St. Paul. As Professor Eliassen has stated, the initial cost of $1,500,000 includes only that of the three sludge vessels. To be comparable it would be desirable to include also the costs of the rather extensive sludge storage building and storage tanks, as well as of dockage facilities and other appurtenances required at the Ward's Island plant in connection with sludge barging.

The performance of the unusually long sedimentation tanks, as discussed by Professor Bass, has been a constant source of gratification to the writer. It appears likely that even greater removals can be effected with graded adjustment of the long effluent weirs—which are now set at a uniform elevation—in relation to the varying detention periods provided at points along the length of the tanks.

Mr. Townsend suggests the desirability of further statements by the writer concerning the question of digestion versus no digestion, prior to vacuum filtration. Throughout the paper the writer attempted to confine himself to factual information concerning the Minneapolis-St. Paul project. Realizing that every sludge disposal problem is (or at least should be) a special subject of investigation for the peculiar conditions of a particular situation, and remembering the sagacious proverb to the effect that "the most consistent thing about sewage is its inconsistency," the writer would prefer not to generalize concerning the merits or demerits of various combinations of processes of sludge disposal. Referring only to the Minneapolis-St. Paul conditions, however, the writer can state that the decision concerning the filtration and incineration of raw sewage sludge was well justified, as is indicated by the relatively low total costs of sludge disposal at this plant.

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[14] Chf. Engr. and Supt., Minneapolis-St. Paul San. Dist., St. Paul, Minn.

Mr. Mendelsohn has requested the writer's views concerning guarantees and maintenance bonds that should be included in specifications. This is an important point. The only equipment on which guarantees and appropriate bonds insuring compliance with these guarantees were required, in connection with Sanitary District contracts, related to the screenings centrifuge, magnetite filters, and incinerators. At the time this equipment was designed in 1935 and 1936, these items of equipment were relatively new, at least in their applications in this particular plant, and in some cases competition was precluded by reason of their special and patentable nature. Operating experience over a period of time has now furnished the designing engineer with what information he requires concerning the performance of such equipment before he makes a selection. Venturing an opinion in a field in which views are certain to differ, the writer would state that qualified engineers, having prepared satisfactory specifications, need not require guarantee bonds on performance of proved equipment in the sanitary engineering field, since the direct and hidden costs resulting therefrom might exceed by several times the risks involved.

Maintenance bonds in the amount of 50% of the contract price for a period of one or two years were required on all equipment. Such bonds were written in connection with the general performance bond without increase in premium. They act as a protection to the qualified manufacturer and contractor, and the city alike, since they serve notice in a forceful way that what is required is well-designed and well-constructed equipment, which will perform successfully and satisfactorily without unusual maintenance costs for a specified period of time.

Mr. Fuhrman presents rather interesting comparative values on the Washington, D. C., and the Minneapolis-St. Paul, plants. As stated, these plants are of comparable size and were constructed and placed in operation about the same time. Enlarging somewhat on the information presented in Mr. Fuhrman's comparison, an interesting point concerning comparisons between various plants can be made:

Description	Minneapolis-St. Paul	Washington, D. C.
Total annual cost of sludge disposal	$136,815.43	$54,486
Million gallons treated	36,705.3	34,200
Tons of dry solids removed	37,703.0	12,350
Cost per million gallons treated	$3.70	$1.59
Cost per dry tons of solids removed	$3.63	$4.41

It will be observed that the quantity of sewage treated is approximately the same in the two plants. The quantity of dry solids removed from the sewage in the two plants differs markedly, however, being only about one third in the case of the Washington, D. C., plant. The difference is probably due to difference in strength of the raw sewage and in removals accomplished. It is for these reasons that the commonly used comparison on a million-gallon basis is inapplicable in many cases, even for plants using similar processes. Strength of raw sewage may easily vary from 150 to 300 ppm of suspended solids in two different plants, and removals by sedimentation may differ in the range of 40% to 75%. Using these illustrative examples, it is manifestly improper to compare on a million-gallon basis the costs at a plant having a dilute sewage, on which a low removal is effected, with another plant treating a strong sewage containing quantities of industrial wastes and which removes high percentages of material. For the illustrative range chosen, which is by no means uncommon in practise, one plant might remove and have to dispose of four times the quantity of material that another plant at the other extreme of the range would. It is this fact that in general makes comparisons on a million-gallon basis unsatisfactory and indicates the greater propriety of a comparison on a tonnage basis. Especially is this true of comparisons of sludge disposal costs in which many of the major cost elements vary in direct proportion to the quantity of material handled. The foregoing statements are of a general nature, and are not presented solely because of the specific comparison Mr. Fuhrman has made. It is simply an expression of an opinion that the writer has had for some time.

Returning to Mr. Fuhrman's discussion, it should be stated that in the case of the Minneapolis-St. Paul plant it is possible that at certain times incineration (which is not employed in the Washington, D. C., plant) may be eliminated, at a saving of approximately $1 per ton. This could be accomplished by farmer hauling of sludge cake for fertilizer purposes, at no expense to the District. Rather, it appears quite likely from present indications that a certain income may be realized.

Mr. Rowen points out possible additional savings in connection with incinerator operations that are under consideration. One such possibility that was thoroughly investigated in the period since the paper was presented (January, 1941) and has since become a reality, is the following:

> As a result of the elimination of the preheater on one furnace, marked savings in power have been effected. As mentioned under the heading "Sludge Disposal: Incinerators" in the paper, the average power consumption in the two years of operation reported on therein has been 17.3 kw per hr per dry ton. The average power consumption for two months in the furnace from which the preheater has been removed has been approximately 5.5 kw per hr per dry ton. Similar changes will be made in the other two units at an estimated total annual saving in power, replacement, and maintenance costs of $12,000 annually.

Several discussions made mention of flexibility of the plant in regard to the degree of treatment capable of being provided to meet varying river dilution conditions. Although such possibilities might not be desirable in the very small plants, the writer agrees that flexibility in the degree of treatment is extremely advantageous, especially for a large project in which continuous examination of the requirements of the receiving body is made. After all, the aim is to meet required standards determined upon in relation to stream use, and therefore a certain end product is required, varying with the capability of the stream to assimilate pollution, and with the use to be made of the water.

The writer would like to have answered each point of information raised, but inasmuch as several of them involved matters of a specific rather than a general nature, and since many such questions have already been discussed in other publications, they are not specifically replied to herein.

At the time the paper was presented, the guarantee tests had not as yet been completed on the magnetite effluent filters, because of certain difficulties, principally those occasioned by displacement of sand in the beds. It is a pleasure to be able to report that as a result of the efforts of the manufacturer the filters have been placed in satisfactory operation and the performance tests relating to removals have been acceptably accomplished. The average removals in the four guarantee tests covering different seasons of the year were as follows:

Suspended solids	ppm
Filter influent	78
Filter effluent	52
Removal by filters	26

The writer wishes to emphasize a matter relative to type of personnel, which is sometimes not sufficiently well recognized, especially on the part of elective public officials. This relates to the desirability of having a sufficient number of qualified professional engineers and chemists on the staff of a sewage treatment plant of any magnitude. Especially is this important when a plant is looked upon as a necessary and considerable investment to be improved and perfected with the rapidly changing status of the art. What might appear on the surface to be a disproportionate expense in "overhead" pays ample dividends in present and future savings in operation and maintenance costs. The Minneapolis-St. Paul Sanitary District is fortunate in having on its staff thirteen engineers and chemists, in addition to a number of others with subprofessional experience, out of a total staff of seventy-nine employees. Lest an impression be created that all such men need be supervisors and "straw bosses," the fact is that many are far from such in the case of the Minneapolis-St. Paul plant, but rather carry their proportionate share of actual work.

The writer appreciates the complimentary remarks concerning the operation of this plant by several of the discussers, and passes on to those who have earned and deserve it whatever credit is due. It is generally realized that to effect important improvement changes, and to increase efficiency and economy, a conscientious, cooperative, and loyal staff is essential.

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Paper No. 2129

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

BY THOMAS E. STANTON,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. W. CARLSON, BAILEY TREMPER, HUBERT WOODS, N. T. STADTFELD, W. C. HANNA, J. C. WITT, R. F. BLANKS, J. MACNEIL TURNBULL, ROBERT A. KINZIE, JR., AND THOMAS E. STANTON.

SYNOPSIS

Tests have demonstrated that excessive expansion of concrete may occur through chemical reactions between cements of relatively high alkali content and certain mineral constituents in some aggregates, such as certain types of shales, cherts, and impure limestones found along the coast of California between Monterey Bay on the north and Los Angeles County on the south. A new test procedure is described in this paper through which it is possible, in a comparatively short time, to develop the deleterious characteristics of cement-aggregate combinations similar to those reported in the California study. The procedure consists of curing the specimens in sealed containers at normal temperatures.

INTRODUCTION

Numerous tests conducted during recent years have established the value of low-heat cements in minimizing the shrinkage of concrete, and the value of the sulfate-resistant standard and puzzolanic-type cements in resisting attack by the sodium and magnesium sulfates and other deleterious constituents in alkali soils and sea water. Designers must now consider a third important factor in cement composition which may so affect the characteristics of a portland cement concrete as to produce excessive expansion and possible ultimate failure of a structure even if it is subjected to only normal curing and weathering conditions.

Given a properly designed cement-aggregate combination, such a condition can occur even when the cement is perfectly sound (in the generally accepted interpretation of the word) and of such a chemical composition as to be highly

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NOTE.—Published in December, 1940, *Proceedings*.

[1] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

resistant to exterior attack by deleterious agents. This is fully and conclusively demonstrated through the tests described in this paper. A new test procedure is also described without which it would probably have been impossible to make the laboratory tests.

Continual wetting or alternate wetting and drying of test specimens failed to develop (in any reasonable period of time, at least) any of the abnormal expansion characteristics experienced on many projects in certain districts of the state. It is now known that at least some of the field conditions of expansion can be duplicated in the laboratory when the specimens are cured in sealed containers, thereby retaining the original mixing water with few, if any, additions or subtractions through immersion or drying. It was not until this fact was discovered that invariable measurable expansion was obtained with certain cement-aggregate combinations, and that negligible, if any, expansion occurred with other combinations.

The complete documentary evidence sustaining the conclusions of the tests are presented for the first time in this paper, although a very brief preliminary report has been presented elsewhere.[2]

Salinas Valley Pavement Failure

Early in 1938, a section of concrete pavement north of the town of Bradley in the Salinas Valley, Monterey County, Calif., failed because of excessive expansion. An intensive field and laboratory investigation followed to determine the causes of the failure and to develop safeguards to prevent a recurrence of the condition in future work. The failed pavement was built in the fall, winter, and spring of 1936–1937. Most of it had passed through two winters when portions began to show distress in the spring of 1938. The distress became manifest in the form of excessive expansion which caused buckling at the expansion joints and a severe cracking throughout the length of certain slabs (Fig. 1(*a*)).

A coarse aggregate from a local commercial plant, located on a tributary to the Salinas River, was used throughout. The fine aggregate came from two sources, one the local deposit (herein referred to as Oro Fino), from which all of the coarse aggregate was secured, and the other (Coyote) imported from a commercially operated deposit near San Jose, Calif., in the San Francisco Bay area. The sands from these two deposits were used during alternate periods so that it was easy to identify the source of the fine aggregate in each section or day's run. A close inspection of all sections readily disclosed that excessive expansion had occurred only in those sections in which the local (Oro Fino) fine aggregate had been used, and it was obvious, therefore, that aggregates from this area contained at least one of the sources of trouble.

Earlier Failures

The Bradley pavement failure culminated a series of concrete failures along the coast area from Monterey County in the north to the northern part of Los Angeles County in the south, throughout which aggregates of the same typical characteristics are encountered.

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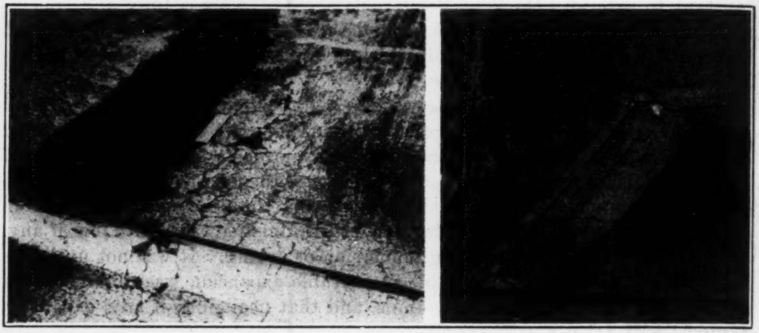
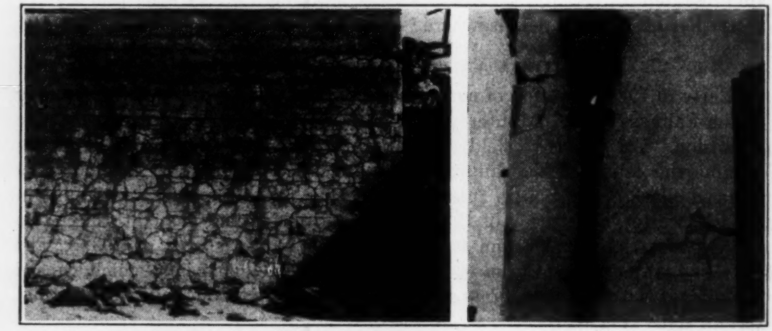
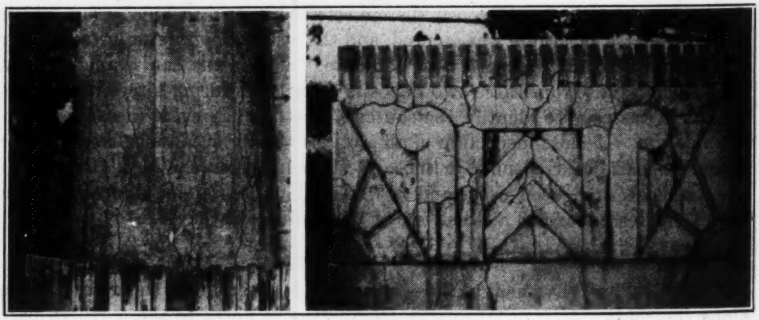
[2] "Influence of Cement and Aggregate on Concrete Expansion," by Thomas E. Stanton, *Engineering News-Record*, February 1, 1940, p. 59.



(*a*) Pavement (*b*) Bridge Girder



(*c*) Sea Wall (*d*) School Building



(*e*) Bridge Pier (*f*) Bridge Pylon

FIG. 1.—TYPICAL CONCRETE FAILURES RESULTING FROM COMBINATIONS OF CERTAIN AGGREGATES AND HIGH-ALKALI CEMENTS

The first structure in which distress developed to such an extent as to attract attention was the King City Bridge (Fig. 1(*e*)) built in 1919–1920 across the Salinas River at King City, Monterey County. This bridge consists of an all-concrete trestle approach to a series of concrete piers and steel trusses over the main channel of the river. The channel structure consists of steel trusses resting on concrete piers of two circular sections with battered sides connected by a 12-in. diaphragm wall.

The coarse aggregate was imported from an unquestioned source, but the fine aggregate was taken from the bed of the Salinas River, or a small tributary (Pine Creek) emptying into the river a short distance upstream from the bridge.

Within three years after construction, well-defined cracks were in evidence in the caps of the piers. Later, these cracks extended into the columns of the piers. By July, 1924, all of the pier caps were more or less affected.

The concrete in the pile and trestle construction has shown little, if any, distress, the severe distress being confined to the piers. Although it is possible that the cement was not of a uniform quality, so far as is known the two concretes differed only in the sands used. Pine Creek sand was used at first; then sand from pits in the stream bed of the Salinas River and later surface sand from the bed of the same river, deposited during the winter rains, was used.

The condition of the King City Bridge has been investigated by a number of agencies, including representatives of the Bureau of Public Roads, the Bureau of Standards, the Portland Cement Association, and the cement companies, as well as engineers and chemists of the California Division of Highways. In general, the number of opinions expressed equaled the number of reports made. The causes of failure advanced by the different investigators were the sand; unsound cement; excess water; extreme variations in temperature; faulty curing; faulty concrete design; variable coefficient of expansion between different parts (including reinforcing steel); electrolysis; overstressing or under design; and rusting of anchor bolts, reinforcing steel, iron pipes, washers, etc. All of the assumed causes, however, were purely speculative without any substantiating foundation and (in 1940) have remained in the category of interesting theories.

The King City Bridge has been enphasized because it was the first partial failure of a major structure to attract attention and because the definite cause of the difficulty has been an unsolved mystery. Subsequent to the King City Bridge trouble, several concrete trestles, built afterward in Monterey County, in which fine aggregate from the Salinas River or its tributaries had been used, developed serious distress (Fig. 1(*b*)).

Similar trouble had been noted in Ventura County where the concrete piers of the Southern Pacific Main Line Bridge over the Santa Clara River near Montalvo, built in 1914–1915, had developed map cracking similar to the Salinas Valley structures. On June 5, 1933, George W. Rear, M. Am. Soc. C. E., bridge engineer for the Southern Pacific Railroad Company, wrote that the piers showing distress had been built in 1914, using local sand from the bed of the stream, whereas imported sand had been used the following year in constructing the piers which were still in good condition after eighteen years.

Other pavement and structure failures were noted in Los Angeles, Ventura, Santa Barbara, San Luis Obispo, and Monterey counties, where the aggregate was all of the same general type, containing from 4% to 15% total of shale, cherty shale, and chert. A number of the more important failures were some sections of pavement and sea walls in Ventura and Santa Barbara counties (Fig. 1(*c*)).

Although the sulfates in sea water undoubtedly contributed to the serious deterioration of the sea walls, it was obvious that this deterioration was accelerated by the infiltration of sea water through cracks resulting from excessive expansion occasioned by other causes. The concrete walls of a school building in Santa Barbara, built in 1931, had cracked to such an extent by 1934 as to require at least partial reconstruction (Fig. 1(*d*)). All of the failures noted followed the same general pattern and were apparently due to the same underlying cause.

The solution of the problem was complicated by the fact that, although without exception cracking of the nature described was noted only when local fine aggregate had been used, there were numerous instances where the apparently same local aggregate performed in an entirely normal and satisfactory manner.

Immediately north of the pavement failure in Montery County is a section of pavement, constructed at least six years earlier, in which fine aggregate from the Salinas River was used, but which is still (1940) in excellent condition and shows no evidence of excessive expansion or cracking.

Development of Test Procedure

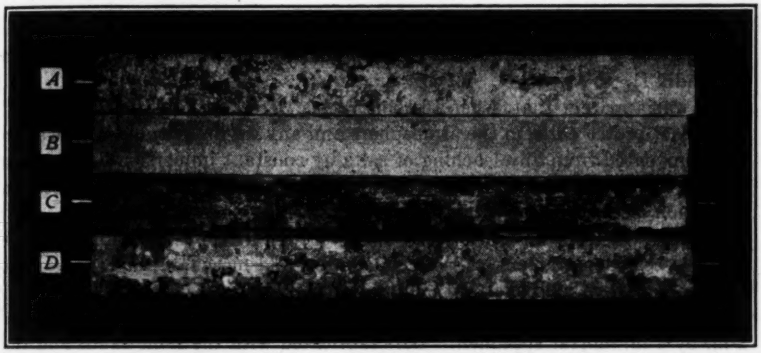
Upon conclusive evidence that the local sand used in the Bradley pavement, constructed in 1936–1937, had played an active part in the failure, a laboratory study was undertaken to determine, if possible, the cause or causes of the trouble and the reason for the inconsistencies that had been noted not only in the locality under immediate investigation, but likewise in all of the coast counties from Monterey to and including the northwesterly part of Los Angeles County.

The first step was to make up mortar and concrete bars of the local (Oro Fino) and the imported (Coyote) fine and coarse aggregates, and then subject some of these bars to continual wetting, some to continual dry exposure, under normal laboratory temperature and humidity conditions, and some to alternate wetting and drying. In one series, after initial curing for seven days, the specimens were dried for three days in an oven at 150° F and then soaked in water at 70° F for four days, the complete cycle taking one week. Another set was dried for two days in the oven at 150° F and soaked seven days in water at 160° F, the complete cycle requiring nine days.

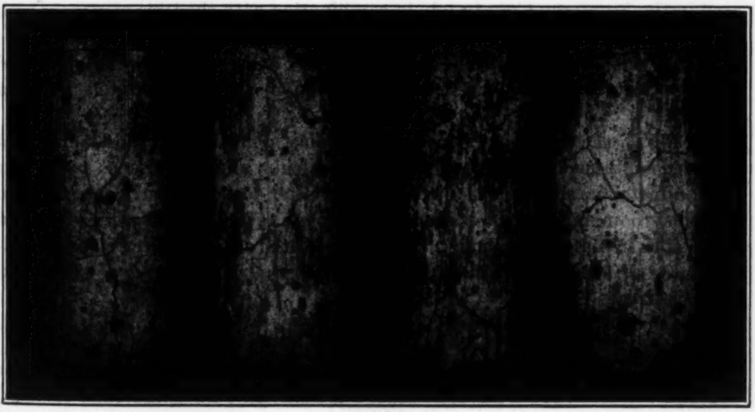
Under none of these curing methods was any excessive expansion observable, even up to more than one year, regardless of the source or nature of the sand, except in the case of a coarse aggregate comprising 100% shale. In that case the expansion was obviously due to a physical expansion of the shale which, when present in large or concentrated quantities, had the effect, ultimately, of rupturing the concrete, but, when present in the normal percentages found in the aggregate under suspicion, had no measurable deleterious effects for the period under observation. In the case of the coarse aggregate with 100% shale, failure occurred on the fifty-first cycle of wetting and drying, at which time the expansion was 0.193%.

AN ACCIDENTAL DISCOVERY

In the meantime, however, upon removing the cover from a 2-in. by 4-in. mortar cylinder which had been cast and retained in a tin container for one year, the specimen was observed to be covered with blotches fringed with a white efflorescence (subsequently analyzed as sodium carbonate), and in a short time the entire specimen became covered with cracks similar to those shown in Fig. 2(*b*) and also to the crazing noted in the pavement and other structures in the area under investigation. A similar condition will be noted in



(*a*) Relative Length of 1-in. by 1-in. by 10-in. Mortar Bars (Mix 1 : 3) Cured for Four Months Under Various Conditions



(*b*) Mortar Specimens (Mix 1 : 2) After Curing Seven Months in Sealed Containers

FIG. 2.—CONDITION OF LABORATORY SPECIMENS AFTER CURING IN AIR, WATER, AND SEALED CONTAINERS

the case of the mortar bars in Fig. 2(*a*). These specimens are identified, by letters, as follows:

Specimen (see Fig. 2(*a*))	Curing method	Expansion after four months, in millionths of an in. per in.
A	Sealed container	+7,310
B	Air	−1,350
C	Water	+ 40
D	Sealed container	+7,230

All bars contain the same quantity of high-alkali cement (1.14%) and a neutral aggregate to which was added 10% of the deleterious mineral No. 28039 (see Appendix I).

It was apparent that, when test specimens were kept in sealed containers, or at least protected from the atmosphere and any drying out, but at the same time prevented from any leaching of salts by constant immersion or alternate wetting and drying, a chemical reaction was going on which caused an excessive expansion of the mortar, with ultimate failure.

A series of tests was then started on 1-in. by 1-in. by 10-in. mortar bars cured in sealed containers, as well as in water, and measured for expansion at different ages. Experience with hundreds of specimens, subsequently fabricated, has invariably checked the results accidently noted in the case of the 2-in. by 4-in. mortar specimen. Bars fabricated with the same sands and the same cement have invariably followed the same expansion pattern.

The local Oro Fino fine aggregate (sand No. 4, Appendix II, and Fig. 3) used in the Monterey County job, in combination with the type of cement used in the work, always shows excessive expansion in a few months (considerable even at twenty-eight days); but the imported (Coyote) sand (No. 20) with the same cement has shown negligible expansion in periods exceeding eighteen months.

In Fig. 3, the specimens were of a 1 : 3 plastic mortar mix, and were cured in sealed containers. The C_3A-content (tricalcium aluminate) does not appear to be a factor in the expansion reactions. Later tests on specimens stored in the moist room, but protected from excess moisture, indicate the same expansion reaction as in sealed containers. However, for the sake of uniformity and convenience, all specimens, other than those constantly wet or dry, were cured in airtight cans.

CURING PROCEDURE

Immediately following fabrication, the test bars are cured twenty-four hours in the moist room under standard conditions. They are then removed from the molds, identified by marking with an indelible pencil, measured, and immediately placed in the containers.

The containers used are regular 6-in. by 12-in. tin cylinder molds with the joints soldered to prevent loss of moisture. The slip cover is probably sufficiently tight to seal the specimens without any other precautions, but if there is any doubt a strip of adhesive tape is wrapped around the joint.

Only enough free water (30 cc to 50 cc) is placed in the bottom of the container to insure keeping the air humid. The specimen rests on the ½-in. monel metal gage point, and therefore the mortar of the bar is not in direct contact with the water.

Eighteen 1-in. by 1-in. by 10-in. specimens (a day's run) are usually stored in one container. The containers are stored under ordinary room temperature, no special effort being made to keep the temperature constant. The specimens are removed from time to time for measurement and then immediately returned to the containers for further curing and later tests. If rusted badly, the containers are replaced.

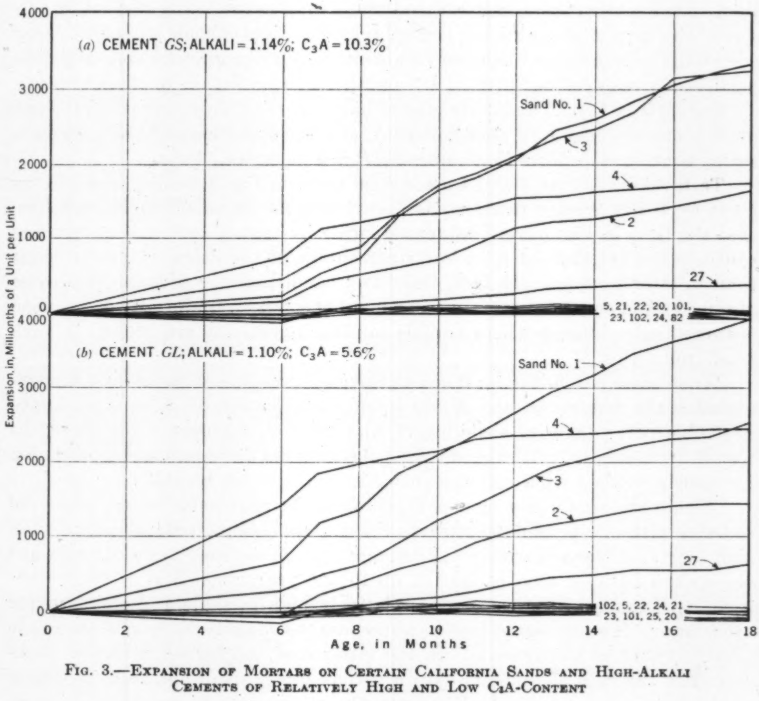


FIG. 3.—EXPANSION OF MORTARS ON CERTAIN CALIFORNIA SANDS AND HIGH-ALKALI CEMENTS OF RELATIVELY HIGH AND LOW C_3A-CONTENT

CEMENT

Other concrete structures in this area, in which the same type of local sand but a different cement had been used, developed no distress up to five or six years at least. This fact gave rise to the suspicion that the expansion resulted from a chemical reaction between certain ingredients in the aggregate and some ingredients in the cement. Alkali in the cement in the form of the sodium and potassium oxides was suspected and, as the cement used on the Bradley job contained approximately 1.14% total alkalis, the tests were repeated with another cement low in alkali (0.45%), with definite results, in that there was little, if any, expansion with the low-alkali cement in combination with the local (Oro Fino) sand which had shown so much expansion with the 1.14% alkali cement. The tests have been repeated many times, not only with the two aforementioned cements, but with other California cements of intermediate alkali content and sands from within and without the coast area.

The resultant expansion has almost invariably been proportional to the alkali content of the cement, whenever used in combination with aggregates containing minerals of the deleterious type found in the area under invesitgation. On the other hand, mortars containing commercial sands originating outside of this area, and containing negligible percentages of shale and chert, being essentially of quartz and feldspar fragments with some granitic and sandstone particles, have developed little, if any expansion, even with high alkali cements.

The data in Figs. 3 and 4 are offered in support of this observation, the mix in each case being a 1 : 3 plastic mortar, and the specimens all being cured in sealed containers. Note that cement *HP* is a puzzolanic type.

Tests on sands Nos. 2 and 4 made after those in Fig. 3 showed expansion as great as, if not greater than, sands Nos. 1 and 3. It would seem, therefore, that the tests in Fig. 3 were influenced either by lack of sufficient moisture to continue the reaction, or by a lower percentage of the deleterious ingredients than in later samples. In fact, these two conditions are probably the cause of some lack of consistency in the tests on other sands, as uniformity of sampling is exceedingly difficult where trouble-causing ingredients are present in such small quantities.

When sand No. 5, Fig. 3, failed to develop excessive expansion, a mineralogical study was made, and it was found that this sample of sand contained negligible percentages of shale, chert, and impure limestone. Therefore, the results are consistent with the nature of the material and confirm the conclusion previously reached regarding the probable source of the trouble.

Within the area under investigation, all of the aggregates from commercial sources, with one possible exception, contain appreciable percentages of shale and chert. All sands with small percentages, or entirely free, of shale and chert show little, if any, expansion up to one year, regardless of the cement used. All sands in the coast area relatively high in shale and chert, on the other hand, develop expansions in a few months roughly in proportion to the alkali content of the cement. In the latter case, negligible expansion is observed in the case of cements with less than 0.5% alkali and high expansion in the case of the 1.14% alkali cement, the other cements ranging in between roughly in proportion to the alkali content (see Fig. 4). Some inconsistencies are noted, but not of such magnitude that they cannot be accounted for by variations in the aggregate.

It is obvious, therefore, that in the presence of certain minerals an otherwise sound but high-alkali cement is a contributing factor to the excessive expansion and subsequent failure of a concrete structure. Under certain conditions this expansion may not be sufficient to disrupt the structure and may cease as soon as the reaction has been completed in any particular cement-

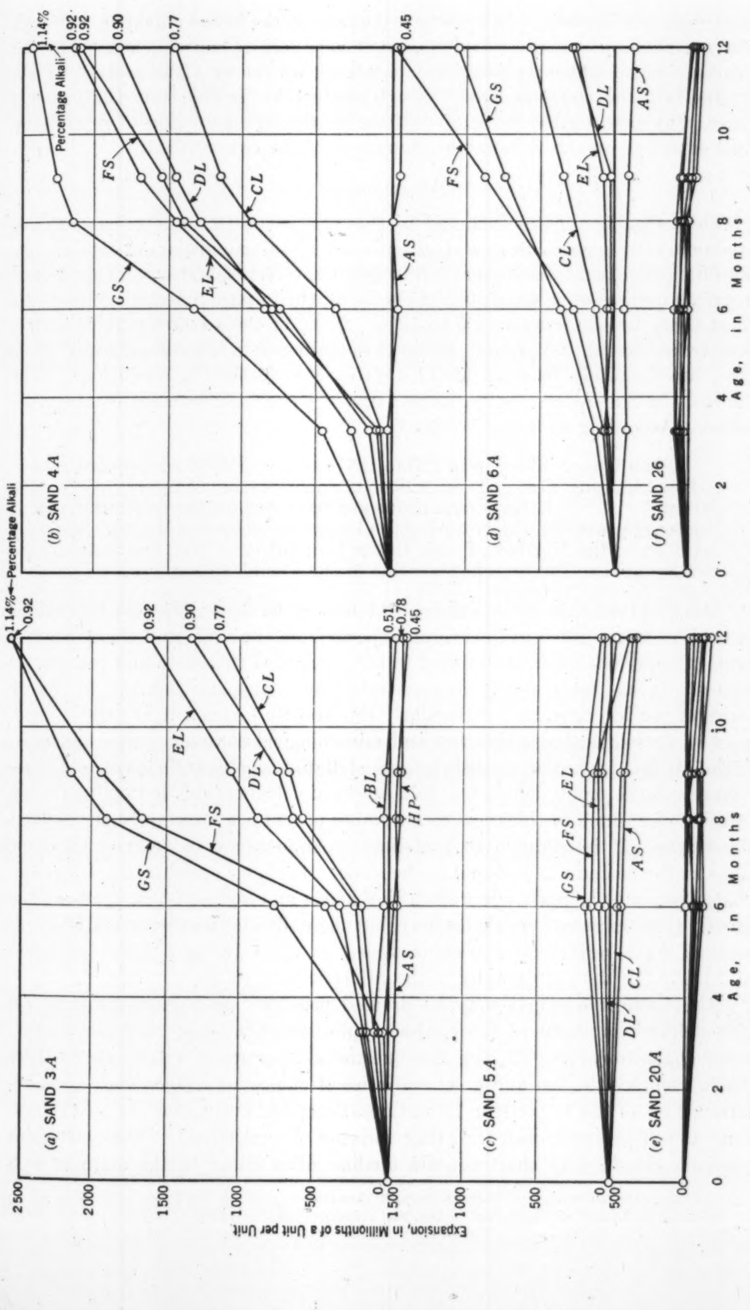


FIG. 4.—EXPANSION OF SAND-CEMENT MORTARS AS INFLUENCED BY THE TYPE OF SAND AND THE ALKALI CONTENT OF THE CEMENT

aggregate combination. In other cases, however, the initial cracking, although above normal but not excessive, may have very serious consequences when the structure is subsequently subjected to attack by sea or alkali waters which, seeping into the concrete mass through fine cracks formed from the primary cause, may take up the work of destruction through corrosion of reinforcing steel or attack on the vulnerable compounds of the cement itself.

AGGREGATES

The coast aggregates included in this study consist of mineral particles predominantly granitic in character with some quartz-feldspar, sandstone, impure limestone, shale, and chert. The granite, quartz-feldspar, and sandstone usually comprise more than 90% of the total; the impure limestone, shale, and chert comprise the remaining 5 to 10%. None of the adverse reactions have been traced to the first group. Some of the minerals in the second group, however, are definitely reactive under appropriate weathering conditions. The minerals in this latter group originate in the Upper Miocene sedimentary deposits. According to Ralph D. Reed,[3]

> "The Upper Miocene in California was a period of widespread seas in which deposition of organic siliceous shale of the Monterey type took place * * *. Whatever conditions are necessary for the deposition of this material prevailed widely during the Upper Miocene from Lower California at least to the Northern Coast Range province and from the San Joaquin Valley area to the present shoreline of the Pacific Ocean."

Many of the types of shale and chert found in the Upper Miocene formations of California are different in several respects from the shale and chert of other geological periods commonly found in other parts of this state and the United States. It has been clearly demonstrated by geologists that the Monterey series of the Miocene, and particularly the Modelo formation, is unique in regard to petrological classification and mineralogical content. Although most of the Modelo formation consists largely of diatomaceous, tuffaceous, and other siliceous shales, many lenses and thin layers of calcareous shale ranging from a fraction of an inch in thickness to a number of feet are intermixed throughout the strata. It is common to find lime, opaline silica, or some chalcedonic silica as the cementing material in the shale. In many cases opaline silica and chalcedony are the main minerals in thin lenses and layers. The opaline silica greatly predominates over chalcedony, but these silicified bands, layers, and concretions have nevertheless been classified as chert by geologists and other authorities on the petrography of the Miocene formations.

The California cherts may be divided into two main classifications: (1) The deleterious cherts of the opaline types, which may or may not contain some chalcedony; and (2) the less questionable group of quartzose particles which includes the red and green varieties of chalcedonic silica such as radiolarian chert of the Franciscan formations, described and named by A. C. Lawson,[4] as well as jasper and the other varieties of chalcedony. The deleterious particles classified as chert contain opaline silica either in the main or as a

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[3] "Geology of California," by Ralph D. Reed, 1933, p. 188.

[4] Geological Atlas, U. S. Geological Survey, San Francisco, Folio 193, 1914.

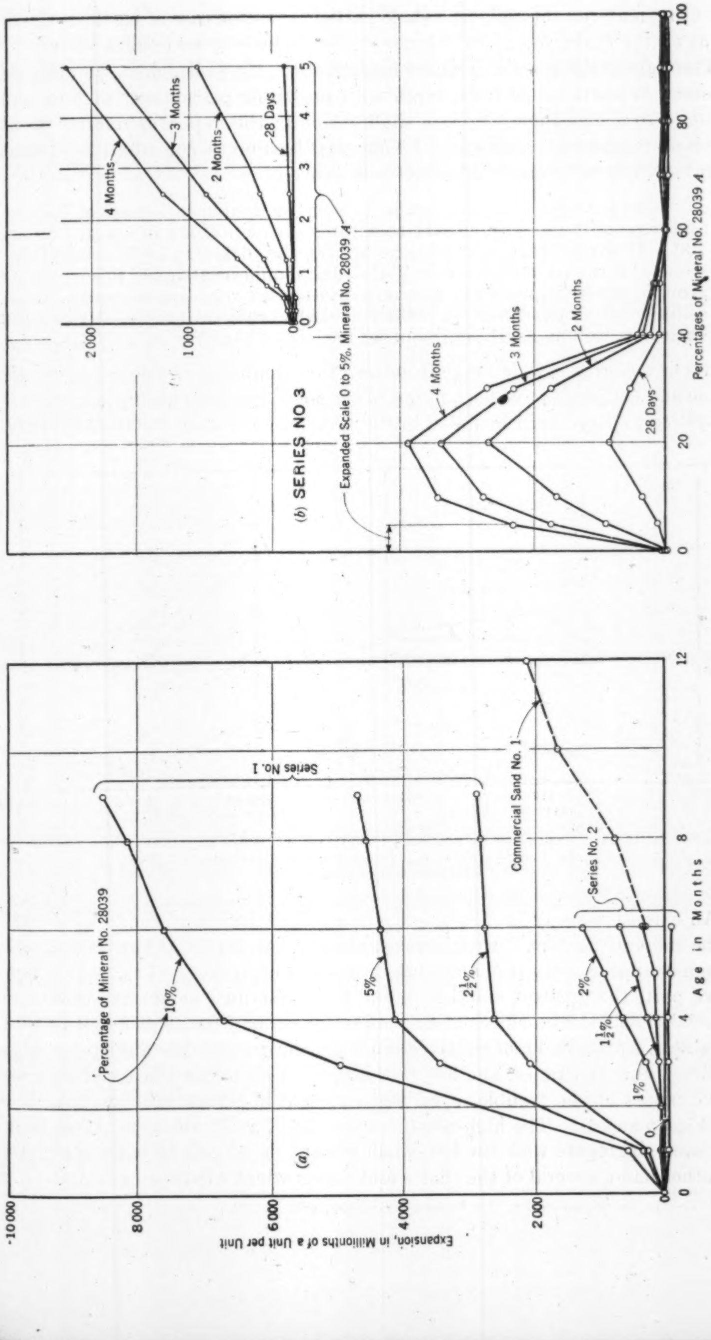

FIG. 5.—COMPARATIVE EXPANSION OF MORTARS CONTAINING SAND NO. 21, IN RELATION TO AGE AND PERCENTAGE OF MINERAL NOS. 28039 AND 28039 A

cementing material throughout a shale particle. In the case of particles classified as cherty shale, some of the laminæ contain little or no opaline silica.

Throughout the coast area under investigation, the shale, chert, and impure limestone deposits range from types with negligible percentages of lime and magnesia to a high-lime magnesia shale which in this report is referred to as "siliceous magnesian limestone." This classification is consistent with that given by William Twenhoffel,[5] who states that:

> "The per cent of calcium carbonate or the double carbonate of calcium and magnesium in sedimentary rock ranges from nothing to approximately 100. If the per cent equals or exceeds 50, the rock may be termed a limestone. If the per cent is below 50 the rock should be assigned to some other group. There is, however, no sharp division between limestones and other sedimentary rocks; they grade without sharp break into the sandstones and shales, as well as nearly every other variety of rock."

Rocks covering a wide range from zero to a comparatively high percentage of lime and magnesia have been found in the coast area, and a sufficient number of these rocks have been included in the present study to cover the field fully.

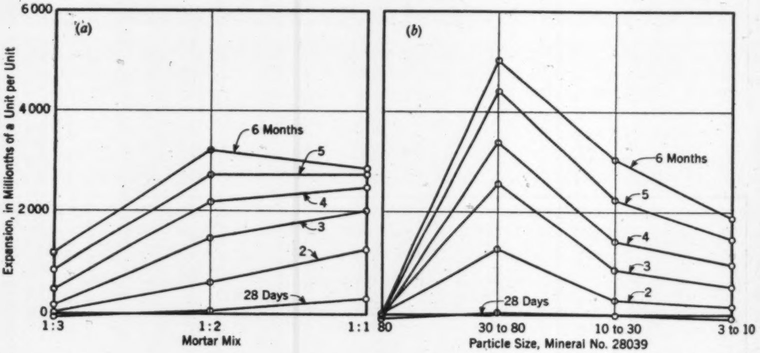

FIG. 6.—COMPARATIVE EXPANSION OF MORTARS CONTAINING SAND NO. 21, IN RELATION TO MORTAR MIX AND PARTICLE SIZE, THE PERCENTAGE OF MINERAL NO. 28039 REMAINING CONSTANT AT 5%

An extensive study has been made in an effort to isolate the "bad actors" in the mineral content. Considerable success has attended this effort. Although it appears very probable that some form of hydrous silica plays a very active part, the greatest reaction in the tests described herein has been with material classified as a siliceous magnesian limestone (Nos. 28039 and 28039*A* in Table 4, Appendix I), of relatively high specific gravity, low absorption, high sodium sulfate resistance, and low rattler loss. This mineral is one of the most active causes of the trouble, even when present in percentages less than 1.00 (see Figs. 5 and 6). The high-alkali cement *GS* (Fig. 3) was used throughout. The same aggregate with the low-alkali cement developed no expansion. On the other hand, several of the shales and cherts which expanded moderately in water showed negligible expansion in sealed containers, even with a high-alkali cement (see Table 1), as contrasted with the siliceous magnesian limestone (No. 28039), which developed no expansion when cured in water but high expansion in sealed containers. It has not been determined if the reaction in the case of this impure limestone is due entirely to some form of silica or in part to the magnesium carbonate. If it is due in any part to the magnesium carbonate, the probable chemical reactions in this respect are as follows:

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[5] "Treatise on Sedimentation," by William Twenhoffel, 1932, p. 238.

TABLE 1.—COMPARATIVE EXPANSION OF MORTARS OF SAND NO. 21 WITH 10% OF DIFFERENT TYPES OF CRUSHED SHALE CHERT, AND SILICEOUS MAGNESIAN LIMESTONE AND A HIGH-ALKALI CEMENT

(In Millionths of a Unit per Unit)

Age	Mix[a]							
	1 : 3					1 : 2		
	No. 19374	No. 28037	No. 28038	No. 28039	No. 28042	No. 28044	No. 28045	No. 28046
28 days	+90	....	....	....	....	+55	+ 3	+ 25
2 months	....	....	....	....	....	+90	+42	+190
3 months	+238	+18	+47	+1,910	+62	+90	+370	+1,030
6 months	+1,220	− 5	+45	+4,140	+80	....	....	....
8 months	+2,065	− 3	+35	+5,175	+70	....	....	....
9 months	....	−38	− 3	+5,557	+72	....	....	....
1 year	....	−90	−37	+6,195	+50	....	....	....

[a] See Appendix I; all these specimens were cured in sealed containers.

The sodium oxide in the cement hydrates to form sodium hydroxide. The sodium hydroxide, in turn, probably reacts with at least some form of the magnesium carbonate in the siliceous magnesian limestone to form magnesium hydroxide and sodium carbonate; thus:

$$\mathrm{Mg\,CO_3} + 2\,\mathrm{Na\,OH} \rightarrow \mathrm{Mg(OH)_2} + \mathrm{Na_2CO_3} \ldots\ldots\ldots\ldots (1)$$

The sodium carbonate ultimately crystallizes with seven to ten molecules of water, with an increase in volume, and thus causes expansion of considerable magnitude, resulting in stresses sufficient to rupture the concrete. The approximate volume change may be computed theoretically from Eq. 1, assuming the following specific gravities:

Description	Specific gravity
Magnesium carbonate in the rock	3.037
Sodium hydroxide	2.130
Magnesian hydroxide	2.300
Sodium carbonate including 10 molecules of H_2O	1.460

If the molecular weight of the reacting components and the end products are divided by the specific gravities, the following respective volumes are obtained:

$$\left.\begin{array}{l} \mathrm{Mg\ CO_3} = \dfrac{\text{Molecular weight}}{\text{Specific gravity}} = \dfrac{84.33}{3.037} = 27.7 \\ \quad + \\ 2\ \mathrm{Na\ OH} = \dfrac{80.016}{2.13} = 37.6 \end{array}\right\} 65.3 \text{ (Combined volume of reacting components)}$$

reacts to form:

$$\left.\begin{array}{l} \mathrm{Mg(OH)_2} = \dfrac{58.336}{2.3} = 25.36 \\ \quad + \\ \mathrm{Na_2CO_3,\ 10\ H_2O} = \dfrac{286.17}{1.46} = 196.00 \end{array}\right\} 221.36 \text{ (Combined volume of reaction products)}$$

or an increase in volume of 239%.

To check the accuracy of this assumption, tests were made to determine the action of sodium hydroxide upon various materials that might be present in the aggregate, such as magnesium carbonate, calcite, dolomite, etc., as well as on the siliceous magnesian limestone.

The method used was to subject the substance under observation to the action of a normal sodium hydroxide solution for seventy-two hours, excluding the possibility of contamination by atmospheric carbon dioxide. A portion of the clear liquid was titrated with normal hydrochloric acid using methyl orange as an indicator. A second volume equal to that used in the methyl orange titration was treated with barium chloride solution to precipitate the carbonates and then titrated with normal hydrochloric acid with phenolphthalein as the indicator. The difference between the volumes of acid used in these determinations was computed to sodium carbonate.

Treated in this manner a sample of:

(*a*) Standard calcite yielded only a trace of sodium carbonate as Na_2CO_3;
(*b*) Bakers calcium carbonate likewise yielded only a trace;
(*c*) Dolomite with 7.1% Mg O yielded 1.06% Na_2CO_3;
(*d*) Technical hydrated magnesium carbonate yielded 51.00% Na_2CO_3; and
(*e*) The siliceous magnesian limestone with 15.5% Mg O yielded 8.65% Na_2CO_3.

All material was crushed to pass an 80-mesh screen before test.

These results indicated that sodium hydroxide reacts with certain forms of magnesium carbonate, but not with carbonate of lime. It appears that some similar reaction occurs in the case of at least some of the hydrous silicates. Although some early expansion due to physical causes may be expected from some of the shales, and therefore the percentage of such shales should be held to a reasonable minimum (controlled by the sodium or magnesium sulfate test), nevertheless, when held to comparatively low limits, the expansive force may not be sufficient to cause rupture except in the nature of popouts when particles are close to the surface. This conclusion is confirmed by observation of numerous structures which, except for surface popouts, show no distress over a period of years even if the aggregate contains a relatively high percentage of many of the types of shale found in the area under investigation. With some cherts and shales, however, in addition to the siliceous magnesian limestone, a chemical reaction of a decidedly destructive nature always takes place in the presence of a high-alkali cement.

An astonishing development was the effect of a very small percentage of the high lime-magnesia rock. As will be seen by reference to Fig. 5(*a*), as low as 1.00% of this mineral caused considerable expansion in six months, the expansion in that period being almost identical with the expansion mortars of sand No. 1 in the same period of time, indicating that sand No. 1 contains not more than 1.00% of deleterious particles but that even this low percentage is objectionable.

The particle size of this mineral also has an important bearing on the result. It was thought that, by crushing the deleterious particles to pass 200-mesh completely, an accelerated reaction might be had. The reverse was the case, as specimens fabricated with −80-mesh particles developed no expansion in sealed containers, whereas the −30-mesh to +80-mesh particle specimens showed the greatest expansion (see Fig. 6(*b*)). Therefore, it appears that the reaction between the reactive ingredient in the aggregate and the alkali in the cement, when the aggregate is in a finely divided state, is either dissipated throughout the mass in such a way as to cause no high expansive forces or the reaction is largely over before the concrete attains permanent set. One theory is that the expansion is taken up in the interstices and by the gel, thus reducing the measurable manifestation of the reaction to negligible proportions.

MERRIMAN ALKALINITY AND FREE ALKALI TEST

The Merriman test[6] for alkalinity and free alkali was applied to the cements, but it was found that this test was not always consistent with the results. One cement, relatively low in the Merriman test, but with high alkali as determined by chemical analysis, developed high expansion; and its action, therefore, was consistent with the total alkali as determined from the chemical analysis rather than with the percentage of soluble alkali as determined by the Merriman test. Another cement with a high Merriman test, but low in alkali as determined by the chemical analysis, gave low expansion results. The reason for the discrepancy appears to be that the soluble portions of some cements go into solution much more slowly than others. The conclusion has been reached, therefore, that the Merriman test procedure, in its present form at least, is not suitable for a proper classification of cements in the order of the reactions encountered in the California tests.

EXTENT OF TESTS

Tests have been made on fine aggregate from all of the commercial sources within the region affected and from several sources outside of this area for comparison purposes; they have been made also on most of the commercial cements manufactured in California ranging in alkali content from less than 0.50% to approximately 1.14% (Table 5, Appendix III).

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[6] Specifications for Cement, Board of Water Supply, City of New York.

As previously stated, negligible expansion developed with sands from outside the affected area. With one exception, all of the local sands showed high expansion in proportion to the alkali content of the cement. With the exception of the one sand, all of the samples contained from 5% to 10% of shale, chert, and the siliceous magnesian limestone. The sample of sand (sand No. 5) from the one deposit which failed to react was subsequently analyzed and found to contain only a very small percentage of the deleterious particles.

Numerous combinations have been made of aggregate from all commercial deposits and cement from most of the California mills. For check purposes, the study has been expanded to include, also, several cements from other parts of the United States.

Following a visit to the laboratory, when he was in California during the summer of 1939, the late Thaddeus Merriman, M. Am. Soc. C. E., who was much interested in the study, forwarded a sample of low-alkali cement (0.47%) from a Lehigh Valley plant (cement *N* in Table 5, Appendix III); and Ira Paul of the New York State Highway Department furnished a sample of 0.64% alkali cement from a New York plant (cement *R*). Through F. V. Reagel of the Missouri State Highway Department and W. E. Gibson, Assoc. M. Am. Soc. C. E., of Kansas, cements were secured similar to those used in pavements in Missouri and Kansas (cements *M* and *O*). These out-of-state cements have all been tested in combination with the Oro Fino and Coyote sands.

The expansion reactions have paralleled the reactions of the California brands. Cement *M* with a high-alkali content (0.83%) has developed excessive expansion with the Oro Fino sand (4,010 millionths of a unit per unit in fourteen months) and negligible expansion with the Coyote sand, whereas cements *N*, *O*, and *R*, comparatively low in alkali, have developed little if any expansion with either the Oro Fino or Coyote sands.

More than 3,000 1-in. by 1-in. by 10-in. mortar specimens of all combinations were fabricated and tested for expansion under different curing methods. The present status of knowledge of the subject was only arrived at through following numerous leads, some of which led into blind alleys and others of which were productive of results pointing the way and suggesting additional tests or lines of investigation.

Series I.—This series was started in May, 1938, and included only the two sands used on the Bradley job. Several coarse aggregates were used, including chert, shale, granite, and an altered dacite.

The cements in this first series were of the brand used in the Bradley job (a high-alkali cement) and in addition a low-alkali cement from another mill. The test specimens consisted of 2-in. by 2-in. by 11¼-in. bars of six-sack concrete, using 50% sand and 50% coarse aggregate, and the maximum size was ¾ in. One set of bars was cured in water, one set in air, and other sets alternately soaked and dried at air and water temperatures ranging from 70° F to 160° F. These tests produced no results of value, even up to one year, and at no time duplicated field experience with the local fine aggregate and job cement.

All specimens cured in water showed a slight expansion in the earlier period, but by 200 days all showed shrinkage from the peak length. At one year all specimens were shorter than the original length with the exception of the shale coarse aggregate specimens (ledge shale from Oro Fino Canyon), which showed a slight expansion (0.009%).

All specimens cured in air at laboratory temperature showed shrinkages ranging from 0.1% to 0.3%, but no cracking. In the alternately wet and dry group, there was shrinkage or negligible expansion with the exception of the coarse aggregate specimen of 100% shale which, by the fifty-first cycle, had expanded 0.193% and failed by cracking.

Series IA.—Shortly after Series I was started, supplementary specimens of the same size were placed under similar test, using aggregates from various spots in the Oro Fino deposit, and also from other commercial sources throughout the district between Monterey and Los Angeles counties. Again the results were negative.

Series II.—After Series I and IA had been under way approximately three months, the condition of the 2-in. by 4-in. mortar specimens cured in a sealed container was observed, and a new series was immediately started, the specimens in this series being fabricated in September and October, 1938. Because of the expense and labor involved in fabricating and measuring concrete specimens, and because it was apparent from an inspection of the mortar specimen cured in the sealed container that the fine aggregate was considerably more reactive, at early stages at least, than the coarse aggregate, Series II consisted of fine-aggregate mortar specimens only, and the specimens in all subsequent series, except one, were mortar specimens. At the start, six 1-in. by 1-in. by 10-in. 1 : 3 mortar bars were fabricated from each mix for curing under different conditions, and the results reported are the average of several specimens. As the 1 : 2 mortars were found much more reactive, all later specimens were fabricated from this mix.

Sands from fourteen sources were included in Series II—five from commercial sources throughout the affected area; seven from outside sources, free of shale and chert; and in addition a crushed diorite and a crushed quartz. With little information at that time relative to the most probable source of the trouble, and to ascertain if the situation could be corrected by the use of a low C_3A cement, two types of the brand of cement used on the Bradley job were included: (1) The job cement with 10.3% C_3A and 1.14% alkali, and (2) a modified cement from the same mill containing 5.6% C_3A and 1.10% alkali. To hold the amount of testing to a minimum, no other cements were included in Series II.

The mixes were put up in duplicate, one set with the high-alkali and comparatively high C_3A, and the other with the high-alkali and comparatively low C_3A cement. A uniform water-cement ratio (0.94) was used, the slump varying $1\frac{3}{4}$ in. to $2\frac{1}{8}$ in. with the natural sands, 1 in. to $1\frac{1}{4}$ in. for sands having rock chips in the coarser sizes, and $\frac{5}{8}$ in. to $1\frac{1}{4}$ in. for the crushed quartz and diorite. Some of the specimens were cured continuously in water, some in air, some alternately wet and dry, and some in sealed containers in the presence of a slight amount of free water.

As no material reaction was expected short of six months, the sealed containers were not opened or the specimens measured until six months had elapsed. The reaction was found to be so extensive by this time (Fig. 3) that in all subsequent series the measurements were started at earlier periods. After measuring, the specimens were returned to the sealed containers for further curing and measurement at subsequent test periods, usually monthly. At six months all of the sands from the shale-chert infected area, except one, showed considerable expansion. The exception was the sand which was subsequently found to be low in shale and chert.

All of the sands from outside the infected area, except one (all of which have a good construction history), showed either only a slight expansion or a shrinkage at six months. This relative condition has persisted throughout the entire test period to date. The expansion of the one exception (No. 27 in Fig. 3) was not especially high, but, as it was decidedly out of line with the others, a thorough study is being made (1940) to ascertain the cause.

Mortar cylinders (2 in. by 4 in.) for compression tests, fabricated from the same mixes as the bars and cured in water, developed equal strengths up to one year for the poor and the good sands. Later specimens cured in sealed containers showed a gradual falling off in strength with age whenever the infected sands were used.

Series III.—As soon as Series II was well under way and time permitted, a third series was started similar to Series II, but including a number of other California brands of cement ranging from low to high alkali. In addition to the bars, 2-in. by 4-in. mortar cylinders were cast and cured in sealed containers for breaking at twenty-eight days, six months, and one, two, and five years.

As in the previous series, all bars cured continuously in water expanded slightly at first and then began to shrink. The first expansion measurements of the specimens cured in sealed containers were made at one hundred days. Even at that period, considerable expansion was noted in the case of the infected sands.

Series IV.—Having determined definitely from Series II and III that certain sands were reactive, the next step was to determine, if possible, which mineral ingredient or ingredients in the sands were the source of the trouble. Since all minerals except the shale and chert had been removed from suspicion by previous tests on selected sands, four suspected minerals were selected from the infected area, including a diatomaceous shale (No. 28037); a chert (No. 28038); and a calcareous shale (No. 28039), later classified as a siliceous magnesian limestone (Table 4, Appendix I). These shales and cherts were crushed to pass a No. 3 screen and were combined in various percentages with a neutral sand (No. 21) from the Russian River, north of San Francisco Bay.

This series was later expanded to include a serpentine found in the coast area, a calcareous shale (No. 19374) with approximately 6% Ca O but low magnesia, a siliceous chert (No. 19375), and two opaline cherty shales (Nos. 28045 and 28046). An opal and an opalized chert were likewise included to determine the effect of an opaline silica. Crushed limestone was used in other specimens.

The conclusion from this test was that the principal ingredient in the sand contributing to the abnormal volume change was the "siliceous magnesian limestone" and that probably the magnesia in the form of $Mg\ CO_3$ in this rock was one of the active reagents with the sodium hydroxide from the sodium oxide in the cement. However, excessive expansion was not confined to the specimens of No. 28039 but was likewise noted, although to a lesser degree, with the laminated calcareous shale No. 19374, and the opaline cherty shales Nos. 28045 and 28046 (Table 4, Appendix I).

The early tests in this series were made with 2.5%, 5%, and 10% additions to the Russian River sand. As evidence accumulated that excessive expansion occurred with even as low as 2½% of No. 28039, the series was expanded to include specimens with lower percentages ranging down to as low as 0.10% (see Fig. 5(*a*)).

OTHER TESTS

Later test series have included observations to determine the effect of:

(1) Age of cement;

(2) Washing soluble salts out of the cement, treating the aggregate with this resultant liquor, and then testing for expansion;

(3) Admixtures such as pumicite, celite, ground Ottawa sand, calcium chloride, and Vinsol resin;

(4) Treating the sand with sodium hydroxide and dilute hydrochloric acid;

(5) Varying the quantity of cement, using 1 : 3, 1 : 2, and 1 : 1 mortar mixes; and

(6) Methods of accelerating reaction by heating so as to reduce the time of test.

Most of these later tests have not been conducted long enough to produce conclusive data of value. Apparently, however, the test cannot be accelerated by heat, by the autoclave, or by pulverizing the deleterious materials, but can be accelerated by using 1 : 2 mortar mixes instead of 1 : 3 and adding from 5% to 20% of the reactive minerals crushed and screened to the size range, a −30-mesh to +80-mesh size.

The addition of a puzzolanic material seems to be effective, as 25% pumicite substituted for an equivalent weight of high-alkali cement reduced the expansion to a negligible amount at early periods. Part of the reduction may have been due to the lower percentage of total alkali in the combined mix. Ground Ottawa sand, however, was much less reactive and calcium chloride had little effect (see Fig. 7). The reaction with the puzzolanic-type cement included in Series II (*HP* in Fig. 4) was likewise considerably less than with other standard cements of the same or less alkali content. Vinsol resin appears effective to a certain extent.

Treating the Oro Fino sand with water mixed with, and then leached from, a high-alkali cement, or with a solution of sodium hydroxide, apparently stops any subsequent reaction; but treating with a hydrochloric acid solution is ineffective. This would indicate that it may be some form of silicate rather than carbonate which causes the trouble.

CONCRETE TESTS

With the exception of Series I and IA, all of the tests described were made on mortar specimens. A later series consisted of 6-in. by 6-in. by 30-in. specimens of six-sack concrete, using different combinations of good and bad coarse and fine aggregates. Specimens of each mix were cured, sealed in water, and were kept continuously dry at laboratory humidity and temperature. This test has been in progress too short a time, however, to produce information of value.

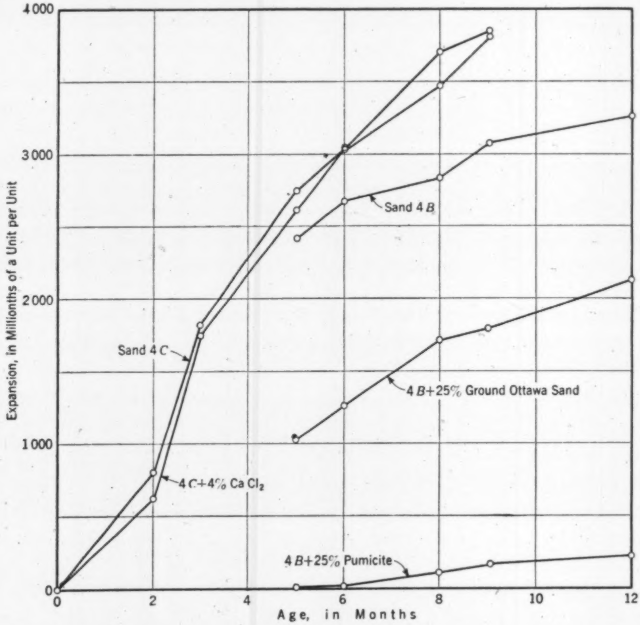


Fig. 7.—Effect of Various Admixtures on the Expansion of a High-Alkali Cement and Reactive Sand Mortar (1 : 3 Mix)

Shale and Chert Popouts

Although the laboratory investigations described herein conclusively demonstrate a very destructive chemical reaction between a high-alkali cement and some ingredient in the siliceous magnesian limestones, nevertheless one cannot overlook the fact that some chemical action is also taking place in the presence of some types of the low lime-magnesian shales and cherts, as evidenced by the data shown in Table 1, as well as by a study of the extensive popouts observed in numerous structures, including pavements, bridges, and buildings where shale or chert particles are invariably found at the base of the popouts, frequently accompanied by a soft viscous substance analyzed as a silica gel with some form of sodium silicate. Figs. 8(*a*) and 8(*b*) are views of large popouts from the basement of the old Times Building in Los Angeles. Fig. 8(*a*) is a part of a 5-in. popout showing at: (*a*) Sodium silicate gel; (*b*) an intermediate zone of partly altered shale; and (*c*) apparently unaltered rock. The chemical analyses of these samples, given in Table 2, were necessarily made on very small specimens and may therefore be inaccurate in some details. Fig. 8(*b*) shows altered and unaltered parts of the mineral that caused the popout.

The aforementioned silica gel frequently has been observed exuding from cracks or pores in the face of concrete structures in which aggregates and cements similar to those under investigation had been used. This type of gel has also been found throughout the cores and pieces of concrete examined from all of the failed pavements and structures. In some cases a number of aggregate particles had entirely altered to gel, so that it often swelled out above a freshly cut surface of a concrete specimen. In many cases the reaction has only partly progressed into particles identified as siliceous shales or cherts of a type relatively free from lime and magnesium carbonates. Although the gel has usually been found in a plastic state in fresh popouts, and in specimens cut from beyond the exposed surfaces of the concrete pavements and structures, it hardens readily and shrinks when dried on exposure to the air.

TABLE 2.—CHEMICAL COMPOSITION OF ROCK AND GEL FOUND IN THE TWO TYPICAL POPOUTS SHOWN IN FIGS. 8(*a*) AND 8(*b*)

Description	Formula	Fig. 8(*a*)				Fig. 8(*b*)	
		Gel	Intermediate altered shale	Apparently Unaltered Rock		Gel	Apparently unaltered rock
				Upper layer	Lower layer		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Moisture	....	12.63	8.92	3.12	3.23	9.69	3.38
Combined H_2O and organic matter	....	9.81	5.84	2.80	3.41	6.96	5.00
Iron	Fe_2O_3	2.40	1.92	3.83	3.83	3.19	2.55
Aluminum	Al_2O_3	4.38	5.84	1.09	6.37	4.61	4.69
Lime	$Ca\,O$	2.90	2.36	0.92	2.50	2.62	1.36
Magnesia	$Mg\,O$	0.58	0.72	0.54	1.09	0.85	0.79
Alkalis	Na_2O	12.93	9.62	2.04	4.90	12.77	5.00
Carbon dioxide	CO_2	....	2.50	0.71	1.43	3.93	1.79
Chlorides	Cl	....	0.72	0.40	0.24	0.48	0.16
Silica	$Si\,O_2$	53.86	60.16	83.30	71.22	53.40	74.00
Undetermined	....	0.51	1.40	1.25	1.78	1.50	1.28
Total	....	100.00	100.00	100.00	100.00	100.00	100.00

A detailed examination was made of approximately 200 popout and core specimens removed from the affected buildings, sea walls, bridges, and pavements over an area extending from Los Angeles on the south to the northern part of the Salinas Valley. As defined by Allan H. Nicol, mineral technologist for the State Division of Highways, each popout particle had associated with it a considerable amount of optically isotropic material of very low index of refraction. This material was identified as opaline silica with complex sodium silicates and was found to have indexes ranging from 1.41 to 1.54. This substance, which was slightly plastic before drying, is hereafter referred to as "gel." It was found that the index of refraction of the dried gel increased with the color. The clear, glassy variety had indexes ranging from 1.41 to 1.47.

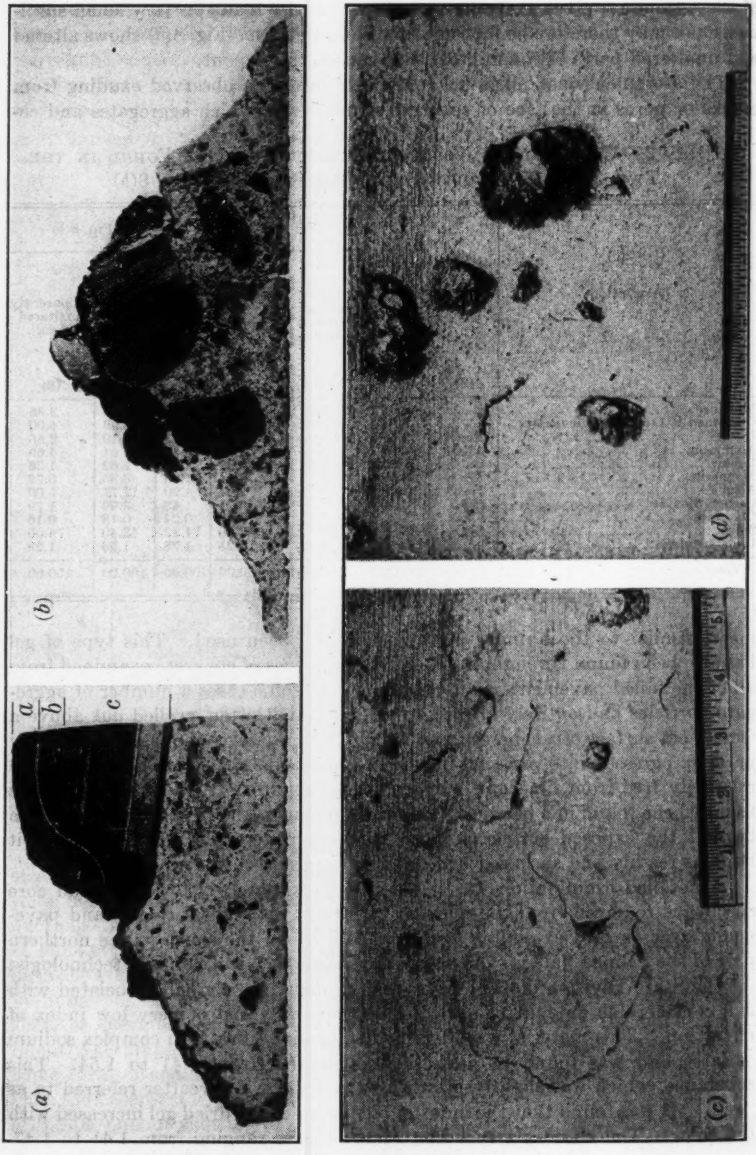

(a)
a
b
c
(b)
(c)
1
2
3
4
5
(d)

FIG. 8.—TYPICAL SHALE POPOUTS

The yellow and pale brown types showed values from 1.46 to 1.49, whereas with the brown to dark brown types the values ranged from 1.47 to 1.54.

Microchemical tests showed that the type of gel with a low refractive index was a nearly pure opaline silica, relatively free from sodium, magnesium, aluminum, calcium, and iron salts, whereas the yellow, amber, and brown varieties, which predominated in many specimens, contained increasing percentages of silicates and other compounds of these and other elements. The presence of these compounds, together with a variable content of adsorbed water, undoubtedly could account for the variable refractive indexes.

Most of this gel is fluorescent under ultraviolet light. A strong fluorescence was observed in the expansion bars and mortar cylinders which carried various percentages of No. 28039 (siliceous magnesian limestone; see Appendix I). These specimens had been subjected to the sealed curing test. The fluorescence in this case is due to gel derived from the particles of No. 28039 in the aggregate, as the rock in its original unaltered condition does not fluoresce.

The gel-like material associated with these specimens contains opaline silica, with varying amounts of sodium, magnesium, aluminum, calcium, and iron compounds. All of the gel examined on popouts, and in the cores and broken concrete from the various sources, shows similarity of optical properties and chemical composition.

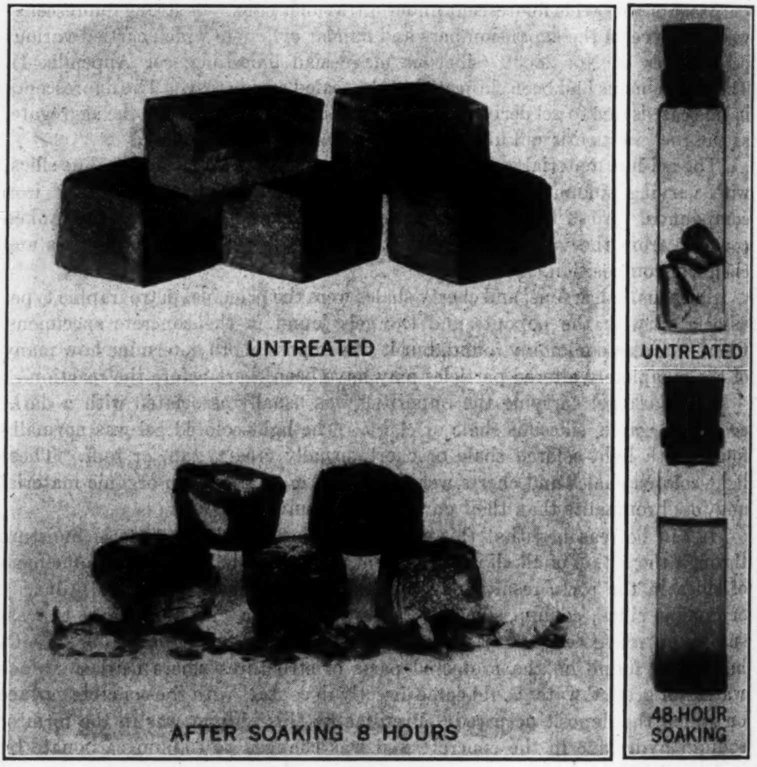
Siliceous, calcareous, and cherty shales were the principal petrographic types associated with the popouts and the gels found in the concrete specimens. Chert was also commonly found, but it was impossible to determine how many of the completely altered particles may have been chert before the reaction.

The dark gel carrying the impurities was usually associated with a dark-colored, organic, siliceous shale or chert. The light-colored gel was normally found with light-colored shale or chert, usually cream, tan, or buff. These light-colored shales and cherts were generally more free from organic material and fossil remnants than their dark-colored equivalents.

It can be reasoned that this substance forms in the presence of moisture through the action of alkali in the cement or possibly in the rock on some form of silica in the rock, resulting in a gel-like end product consisting primarily of opaline silica, sodium silicates, and water. As a partial proof of this assumption, white deposits or incrustations containing more than 50% Na_2CO_3 have been found on the protected parts of structures where neither surface water nor ground water could come directly in contact with the concrete surface on which the deposit occurred. Presumably this sodium was in the form of sodium hydroxide in the concrete and was changed to sodium carbonate by carbon dioxide derived from the air.

Following this reasoning, mineralogy has well established the fact that hydrous forms of silica, such as opaline, are readily dissolved in either sodium or potassium hydroxide solutions, and also that these forms of silica are found in abundance throughout some of the Miocene formations in the area under discussion. All siliceous shales and cherts, therefore, are subject to question, in connection with the chemical reaction resulting in the gel substance. Extensive tests show that many types of shale and chert found in the gravel deposits are readily dissolved and disintegrated by soaking in a 10% solution of sodium hydroxide. In Fig. 9 the effects of soaking are shown for two materials—specimens cut from ledge rock, and ½-in. shale and chert particles from stream-bed gravel deposits.

Although these tests show only that the gel type of chemical action accompanying popouts and expansion failures can be approximately duplicated in the laboratory by the use of a sodium hydroxide (Na OH) solution, additional tests indicate that a similar action occurs when these types of aggregate are soaked in the mixing water from high-alkali cements. The chemical composition of water containing alkali, leached from a low-alkali (0.45%) and a high-alkali (1.14%) cement, is shown in Table 3, together with the change in each solution, after digesting for 120 hr at 200° F with Oro Fino sand containing about 9% of chert and shale particles. It is significant that the water from the



(*a*) Specimens Cut from Ledge Rock

(*b*) Stream-Bed Gravel

FIG. 9.—EFFECT OF A 10% SODIUM HYDROXIDE SOLUTION (AT 70° F) ON SOME TYPES OF SILICEOUS (CHERTY) SHALE AND CHERT FROM THE CALIFORNIA MIOCENE FORMATIONS

high-alkali cement dissolved approximately ten times more silica during the treatment than the water from the low-alkali cement.

Figs. 8(*c*) and 8(*d*) show two typical popouts of the type accompanied by relatively large deposits of gel. It will be noted from Table 2 that these shale particles have chemical compositions somewhat similar to No. 28042 (Table 4, Appendix I), particularly in regard to lime, magnesia, and alkali, although the shale in Figs. 8(*a*) and 8(*b*) is very much harder than that in Figs. 8(*c*) and 8(*d*), or the soft shale represented by No. 28042. Even if the gel type of reaction accompanies many popouts and other expansion failures observed in the field and also can be accounted for, at least theoretically, when a high-alkali cement is used, the volume change and the exact source of the stress causing the popouts and accompanying the reaction require further investigation.

TABLE 3.—CHEMICAL COMPOSITION OF WASH WATERS FROM A LOW-ALKALI AND A HIGH-ALKALI CEMENT AND THE SAME SOLUTIONS AFTER DIGESTING FOR 120 HR AT 200° F WITH ORO FINO SAND CONTAINING ABOUT 9% OF SHALE AND CHERT PARTICLES

(In Parts per Million)

Description	Cl	CO_3	H CO_3	OH	SO_3	Si O_2	Al_2O_3 (Fe_2O_3)	Ca O	Mg O	Na_2O[a]
WASH WATER FROM:										
Low-alkali cement........	20	75	....	1,400	10	7	....	450	....	2,200
High-alkali cement........	60	240	....	3,200	36	39	....	110	....	6,000
SOLUTIONS AFTER DIGESTING WITH SAND:[b]										
Low-alkali cement........	4	250	....	870	23	590	42	....	....	1,800
High-alkali cement........	50	660	....	2,800	45	6,100	....	....	....	5,600

[a] Alkalies (Na and K) reported as Na_2O. [b] 3-mesh to 10-mesh sand particles.

CONCLUSION

The tests described herein justify the following conclusions:

1. Certain mineral constituents in concrete aggregates contribute to expansion of concrete and sometimes develop stresses of such magnitude as to cause failure.

2. Some shales expand excessively when saturated with water or when they are alternately wet and dry and, therefore, the percentage of such material should be kept to a practical minimum. This action, however, appears to be physical and of much less intensity than a chemical reaction with other minerals.

3. Excessive expansion, sufficient to rupture a concrete mass, may occur when certain minerals are present. The reaction in this case is chemical, and evidence indicates that it always takes place with the siliceous magnesian lime rocks found in the aggregates from the Upper Miocene sedimentary deposits of the state and frequently in the presence of some of the low-magnesia, low-lime shales and cherts.

4. The chemical reaction producing excessive expansion apparently occurs only when the portland cement component contains an appreciable percentage of alkali in the form of sodium and potassium oxides. It is of an intensity proportional to the percentage of such oxides, apparently being of such low order as to be negligible when the alkali content is less than 0.6%.

ACKNOWLEDGMENTS

The investigations and tests described in this paper were conducted as a research project of the California Division of Highways, of which State Highway Engineer Charles H. Purcell, Assoc. M. Am. Soc. C. E., is chief. The writer is indebted to the following members of his staff, all of whom have contributed valuable suggestions: Lester Meder, assistant physical testing engineer, for having first observed the accelerated expansion of specimens cured in sealed containers and for subsequent direction of the fabrication and measurement of test specimens and intelligent analysis of results; G. H. P. Lichthardt, chief chemist, for the direction of all chemical analyses and the development of the theory of the probable reactions; O. J. Porter, senior physical testing engineer and head of the Aggregate and Soils Department, and his assistant, Mr. Nicol, for their contribution to the geological and aggregate classification and discussion, identification of the mineral constituents, and suggestions as to probable reactions. In the field District Engineers L. H. Gibson, District V, San Luis Obispo, and S. V. Cortelyou, M. Am. Soc. C. E., District VII, Los Angeles, afforded wholehearted cooperation.

APPENDIX I

MINERAL IDENTIFICATION

The chemical analysis and physical properties of a few of the Miocene shales, cherts, and impure limestones, found in the coast area between Monterey and Los Angeles, Calif., are given in Table 4. Each sample is identified by a laboratory test number that identifies the following detailed description of each one:

No. 19374—Laminated Calcareous Shale:

Groundmass an impalpable mixture of clay, opaline silica, and sericite, carrying abundant fragments of quartz, orthoclase, chalcedony, and calcite, with some marcasite. Organic matter and fossil relics, including foraminifera and diatoms, are found occasionally as stringy accumulations parallel with the laminæ. The chalcedony occurs chiefly as fragmental grains, with an occasional veinlet. A few small clay lenses were observed without any appreciable embedded mineral grains of sufficient size to be identified. The laminations in this rock are quite high in clay and opaline silica, and carry the thickest layers of organic matter and fossils. This could explain the ease with which this material splits apart.

No. 19375—(Chalcedonic) Cherty Shale:

Predominant evidence of chalcedonic replacement of the shaly portions of this rock. The groundmass is predominantly chalcedony of the flamboyant type, with some areas not entirely replaced. Sizable fragments of recognizable minerals are lacking. Organic matter, low in content, occurs in layers that tend to parallel the bedding. The entire specimen is pierced throughout by chalcedony veinlets. Quartz and feldspar fragments are lacking. Opaline silica not observed in this material. Fossils absent.

TABLE 4.—CHEMICAL ANALYSIS AND PHYSICAL PROPERTIES OF SHALES, CHERTS, AND LIMESTONES
(All Values Are Percentages)

No.	Description	Formula	LABORATORY TEST NOS.:			
			19374	19375	28037	28038
1	Moisture	....	0.75	0.22	3.88	2.14
2	Organic matter[a]	....	1.42	0.98	2.21	2.63
3	Iron	Fe_2O_3	3.38	2.64	2.59	2.59
4	Alumina	Al_2O_3	1.40	3.15	13.11	0.79
5	Lime	Ca O	6.01	0.65	2.71	Trace
6	Magnesia	Mg O	0.60	0.51	1.69	Trace
7	Alkali	Na_2O	0.58	0.78	2.11	1.43
8	Manganese	Mn_2O_3	0.01	0.03	0.36	....
9	Carbonates	CO_2	9.46	0.68	2.31	....
10	Undetermined	....	0.69	0.14	0.61	0.48
11	Silica	Si O_2	75.70	90.22	68.42	89.94
12	Totals	....	100.00	100.00	100.00	100.00
13	Average loss, Na_2SO_4 (percentages)	....	22.0	....	36.4	28.7
	Rattler Test:[b]					
14	100 revolutions	....	5.2	....	8.0	9.4
15	500 revolutions	....	23.0	....	31.2	34.6
	Specific Gravity:					
16	Le Chatlier	....	....	....	2.20	2.00
17	Saturated surface dry	....	1.82	....	1.80	1.89
	Absorption:					
18	Percentages	....	8.0	....	10.2	4.3

TABLE 4—(*Continued*)

No.	28039	28039A	28042	28045	28046	30704
1	0.42	0.92	3.92	4.74	5.09	3.8
2	4.15	0.58	3.12	Trace	2.19	8.3
3	0.83	0.99	3.24	3.19	3.52	1.1
4	0.27	1.22	12.40	5.09	3.90	3.5
5	26.24	30.85	1.80	5.18	3.58	0.4
6	15.55	8.75	1.33	0.52	0.52	Trace
7	0.20	0.53	4.21	2.09	1.87	....
8	0.02	....	....	....	....	....
9	33.92	37.03	0.44	2.50	2.30	....
10	0.58	0.39	0.84	0.44	0.66	1.6
11	17.82	18.74	68.70	76.25	76.37	85.1
12	100.00	100.00	100.00	100.00	100.00	100.00
13	2.8	....	72.7	56.3	48.2	....
14	4.8	....	11.2	10.2	....	....
15	20.0	....	41.8	36.0	....	....
16	2.62	....	2.50	2.28	2.16	....
17	2.43	....	1.90	1.94	1.94	....
18	1.7	....	7.1	8.4	7.7	....

[a] Combined water and organic matter. [b] Los Angeles Rattler Test.

No. 28037—Diatomaceous Shale (Soft Siliceous Type):

Carries a few fragments of plagioclase and quartz. Groundmass consists chiefly of diatoms and other opaline material, together with clay, some mica (probably sericite), and a variable amount of organic matter. Groundmass is quite uniform.

No. 28038—Chert:

Opaline material, with some organic matter, and a few fine chalcedony fragments constitute the groundmass of this speciman. The groundmass is pierced throughout the entire length of the slide by a series of fine chalcedony veinlets that average about 0.15 mm in width. These veinlets are all more or less parallel with each other and are apparently conformable with the bedding. They also parallel the heavy layers of organic matter. Abundant evidence of chalcedonic replacement throughout groundmass. "Canals" or veinlets are in portions partly opaline. No fossils or diatoms were noticed. Chalcedony of veinlets is of flamboyant type and carries minute opaque inclusions, possibly marcasite. Many canals or veinlets not entirely sealed from wall to wall by opaline cement, leaving innumerable fissures and openings.

No. 28039—Siliceous Magnesian Limestone:

A groundmass composed principally of chalcedony, opal, dolomite, and calcite. The dolomite occurs as rhombohedrons; calcite shows excellent twinning lamellæ; the chalcedony is flamboyant and shows aggregate polarization under *X* nicols. A few diatoms present in the groundmass, many showing replacement by calcite, dolomite, or chalcedony. Irregular patches of organic matter, concentrated near the chalcedony.

No. 28042—Soft Shale (Santa Margarita Formation):

Quartz and other mineral grains are very abundant in this material (over 25% of groundmass). Groundmass carries abundant organic matter and finely disseminated marcasite. No evidence of siliceous replacement. No diatoms or other micro-fossils observed. Cracks and fissures numerous. Cement of groundmass chiefly clay. No evidence of recrystallization—hence, material is probably quite soft and friable. Average size of grains approximately 0.039 mm. Consist principally of quartz, with some plagioclase and actinolite. Quartz has inclusions, possibly rutile. Organic matter strung out parallel to laminæ. This sample also carries a considerable amount of an anisotropic, pleochroic mineral (index greater than balsam) found in popouts.

No. 28045—Opaline Cherty Shale:

This rock shows strong similarity to No. 19374 and to No. 28046, under low-power magnification. It is of the same geological age as No. 19374 and No. 28046, as evidenced by the foraminifera and other fossils, which show partial chalcedonic replacement. The groundmass appears to be an intimate mixture of chiefly opaline silica with a small amount of diagenetic glauconite and clayey portions, with traces of organic matter. The latter is of fairly uniform distribution throughout the slide. Embedded in this groundmass are abundant coarse detrital grains of quartz, orthoclase, and plagioclase. Numerous feldspars appear partly glauconitized. Chalcedony is not very abundant in the groundmass, except as a replacement of the fossils. No free calcite or dolomite as fragments or veins was observed. This material differs from No. 19374 by a lower calcite-dolomite content, and by a higher amount of detrital quartz and feldspars, as well as a more uniform distribution of the organic matter.

No. 28046—Opaline Cherty Shale:

This rock is of the same geological age as No. 19374 and No. 28045 and strongly resembles them. The following differences, however, are noteworthy: In No. 28046 the detrital quartz and feldspar content is lower, and these grains are smaller; spherules of chalcedony are fairly abundant in the groundmass; glauconite and organic matter are more plentiful than in No. 28045 and occur in concentrated form parallel with the laminæ; although the groundmass is composed chiefly of opaline silica, it nevertheless appears partly pleochroic under high power and indicates the presence of intermixed diagenetic minerals (formed after deposition). No free dolomite or calcite was observed in this material, and what lime content is found on chemical analysis will be due in large part to the fossils and the detrital plagioclase grains. Due to concentration of organic matter and clay in definite layers, this rock is more laminated than No. 28045, and the evident lack of opaline replacement along these laminæ suggests the easy cleavability of this material.

No. 30704—(Dark) Opaline Chert:

Dark brown stringy accumulations of organic matter parallel the bedding, which is frequently distorted. An occasional quartz or orthoclase fragment is enclosed by opaline groundmass. Sample carries a few channels of brecciated material, cemented by opaline veinlets and clay. The quartz and orthoclase are the only recognizable anisotropic minerals observed in this material. Matrix predominantly opaline silica with some undistinguishable clay minerals. Material shows no evidence of chalcedonic replacements, and no fossils.

APPENDIX II

SAND IDENTIFICATION

Group I.—In this group rock fragments consist chiefly of granitic rock and sandstone with less than 15% quartzite, rhyolite, andesite, siltstone, and limestone, and from 4% to 15% shale and chert. The mineral grains consist chiefly of quartz and feldspar, with less than 10% of pyroxene, amphibole, mica, magnetite, hematite, limonite, serpentine, and calcite. Sands Nos. 1 to 6 in this group were secured from the following places:

Sand No.	Source
1	Santa Clara River at Saticoy, Ventura County
2	Sisquoc River at Sisquoc, Santa Barbara County
3	Piru Creek at Santa Clara River, Ventura County
4	Oro Fino, tributary to Salinas River, Monterey County
5	Bank adjacent to Salinas River at Atascadero, San Luis Obispo County
6	Salinas River at Templeton, San Luis Obispo County.

Group II.—Rock fragments in Group II consist chiefly of granitic rock and gneiss in Southern California, and of graywacke (sandstone) in Northern California, with less than 10% of basalt, andesite, quartzite, and jasper and usually less than 1% of shale and chert. Mineral grains in Group II are similar to Group I. Sands Nos. 20 to 24 in this group were found in:

Sand No.	Source
20	Coyote Creek, Santa Clara County
21	Russian River, Healdsburg, Sonoma County
22	Livermore Valley, Alameda County
23	Near Olympia, Santa Cruz County
24	Roscoe, San Fernando Valley, Los Angeles County
25	Irwindale, San Gabriel Valley, Los Angeles County
26	Monrovia, San Gabriel Valley, Los Angeles County
27	San Joaquin River at Friant, Fresno County

Group III.—The sands in Group III were made by crushing ledge rock to the desired gradation. Sands Nos. 101 and 102 in this group were found in:

Sand No.	Source
101	Crushed hornblende diorite, Logan, San Benito County
102	Crushed quartz, Jackson, Calaveras County.

APPENDIX III

CEMENT CLASSIFICATION

The cements used in the investigation were analyzed as shown in Table 5.

TABLE 5.—CEMENT ANALYSES

Cement identification	OXIDE ANALYSIS (PERCENTAGES)							COMPOUND COMPOSITION (PERCENTAGES)				MERRIMAN ALKALI TEST		Autoclave test (percentage expansion)
	SiO_2	Fe_2O_3	Al_2O_3	CaO	MgO	SO_3	Alkali as Na_2O	C_4AF	C_3A	C_3S	C_2S	Alkalinity	Free alkali	
AS	22.76	2.00	4.02	66.04	1.80	1.54	0.45	6.1	7.2	62.3	17.9	3.3	1.0	0.05
BL	23.76	2.48	4.34	64.58	1.61	1.61	0.51	7.6	7.2	41.1	37.2	4.1	4.1	0.12
CL	20.42	5.68	5.92	62.86	1.57	1.74	0.77	17.3	5.8	52.5	17.6	3.6	2.5	0.09
DL	22.26	3.60	4.73	63.18	2.93	1.38	0.90	10.9	6.4	48.5	26.8	4.1	5.1	0.21
EL	21.57	3.52	4.49	62.44	4.08	1.50	0.92	10.6	6.1	50.8	23.3	3.8	3.7	0.25
FS	21.00	3.12	5.39	64.80	1.84	1.51	0.92	9.4	9.0	61.0	13.5	3.8	3.6	0.10
FL	23.34	3.96	4.28	64.32	1.09	0.87	0.50	12.1	4.6	49.5	28.8	3.5	2.6	0.05
FA	25.35	3.87	3.42	63.52	1.62	1.22	0.56	11.8	2.6	34.6	46.2	3.6	2.0	0.01
GS	21.48	2.72	5.64	64.50	1.72	1.71	1.14	8.2	10.3	54.1	20.3	5.1	6.3	0.09
GL	20.02	5.35	5.49	62.80	1.65	1.81	1.10	16.4	5.6	58.2	12.0	5.0	6.2	0.05
HS	21.14	2.48	6.43	64.16	2.23	1.78	0.69	7.5	12.8	49.5	23.0	3.3	1.5	0.18
HL	21.59	4.80	5.17	62.61	1.72	1.89	0.53	14.6	5.6	45.2	27.3	3.5	1.4	0.05
HP	31.10	2.40	6.77	53.30	1.86	1.57	0.78	Portland puzzolanic type				....	....	0.01
IS	22.56	2.40	5.52	65.33	1.48	1.49	0.44	7.3	10.6	50.1	26.7	3.2	2.1	0.17
M	21.30	2.90	5.70	63.44	2.78	1.97	0.83	8.8	10.3	49.4	23.4	4.2	4.4	0.12
N	22.34	3.78	4.78	63.32	3.44	1.51	0.47	11.5	6.3	46.1	29.3	3.1	1.6	0.10
O	21.65	3.55	5.65	63.35	1.45	1.85	0.51	10.8	9.0	46.0	27.0	3.7	3.9	0.01
R	22.37	4.00	4.35	64.65	1.36	1.45	0.64	12.2	4.8	54.5	22.8	3.8	2.7	0.01

In the identifying letters given in the first column, the first letter refers to the brand and the second letter to the type of cement, as follows:

S = Standard cement; in some cases the standard product contains less than 8% of C_3A and therefore might also be classified as a low or moderately low C_3A cement.
L = Cement having less than 8% C_3A and complying with American Association of State Highway Officials Specification M-60-38 as distinguished from the standard product of the same manufacturer with higher C_3A content.
A = Low-heat type "*A*."
P = Portland-puzzolan; insoluble, 15.44; all other cements less than 0.85% insoluble; cements *M*, *N*, *O*, and *R* are from mills outside California.

DISCUSSION

R. W. Carlson,[7] Assoc. M. Am. Soc. C. E. (by letter).—The disclosure of excessive expansion of concrete due to chemical reaction between cement and aggregate has opened a vast new field of concrete research. The importance may be tremendous, depending upon just how much bad concrete is attributable to this cause. A scattering of aggregates that are susceptible to reaction with alkalis doubtless will be found in all parts of the country.

If recurrence of cement-aggregate expansion can be prevented, the average quality of concrete structures should be raised appreciably. Recurrence of this type of concrete expansion probably can be prevented with least hardship to cement and aggregate producers when the following questions are answered:

1. Is the trouble due only to the formation of an alkali carbonate, only to the formation of an alkali silicate, or both?
2. Are the alkalis of sodium and potassium, which are usually lumped together in analysis, equally harmful?
3. Is it possible that the combination of moderate-alkali cements with the susceptible aggregates will produce expansion in correspondingly longer time?
4. Conversely, is it possible that the combination of high-alkali cements with moderately good aggregates will ultimately produce trouble?
5. Is the alkali carbonate likely to be formed by carbonation of the alkalis in aggregates, especially those like crushed feldspar containing a considerable amount of alkali?
6. Is it possible that concretes containing good aggregates but cements of high-alkali content are likely to develop alkali carbonates, and subsequently shrink excessively due to drying?
7. Are the alkalis in the cement equally harmful whether free or combined, either as sulfates, aluminates, or otherwise?

The great need, of course, is for an accelerated test to which questionable combinations of cement and aggregate can be subjected. Due to the slowness of the reaction, doubtful aggregates or cements cannot be proved soon enough merely by making up common specimens and observing their expansion. In some cases, the potential expansion might not develop for a year or more. Therefore, some sort of reliable accelerated test needs to be developed that will give the result in a shorter time. At present, it is indicated that certain sizes of aggregate are more harmful than others, also that certain proportions of cement to aggregate are more sensitive. Undoubtedly, temperature is also a factor. Some combination of conditions may be found that will give an indication of the long-time expansion in a few weeks. It is hardly to be expected that any type of autoclave test applied for only a few hours will be reliable.

Bailey Tremper,[8] Esq. (by letter).—The physical consequences that may result from chemical reactions between high-alkali cements and certain mineral constituents of aggregates have been presented convincingly in this paper. The author's disclosures serve to focus attention on concrete in other localities where the outward appearance of internal expansion is similar to that illustrated in Fig. 1.

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[7] Associate Prof., Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[8] Materials Engr., Dept. of Highways, State of Washington, Olympia, Wash.

In 1935, H. S. Mattimore and G. A. Rahn, Jr.,[9] Assoc. Members, Am. Soc. C. E., reported on the condition of some experimental concrete test walls that had been exposed to weather in Pennsylvania and were then 4½ years old. Two cements and two fine aggregates were used in the concrete. The alkali content of the cements was as follows:

Chemical	Brand A	Brand B
Na_2O	0.43	0.24
K_2O	0.57	0.37
	1.00	0.61

There was not much difference in the composition of the two cements in other respects.

According to the scheme of rating used, all of the specimens made with brand B cement were rated as 70% intact or better. The ratings for specimens made with brand A cement were from 0 to 80%, averaging 51% and 75%, respectively, for the two sands.

In the State of Washington there are a few examples of concrete exhibiting patterns of open cracks while other concrete made with aggregates from the same sources is in excellent condition. The defects are connected definitely with the source, and probably with the alkali content, of the cements. In one instance the aggregate used in the defective concrete was almost entirely igneous in origin.

These occurrences give rise to the uncomfortable suspicion that deleterious expansion may result from the use of high-alkali cement with a variety of aggregates that is very much more extensive than the sedimentary group described by the author.

Mr. Stanton's physical test, consisting of the measurement of length changes of 1 by 10-in. mortar bars, seems to have served very well in distinguishing between good and bad combinations of cement and aggregate in the locality in which he has worked. It is not known, however, that the test will serve equally well with other types of aggregate. It is unfortunate, moreover, that several months at least are required to obtain significant results.

These considerations point to the desirability of knowing more concerning the chemical nature of the reactions involved. When this information is available it should be possible to devise a more rapid method of eliminating suspicion from satisfactory aggregates.

Mr. Stanton points to two types of chemical reactions as possible causes of expansion. The first is a reaction between sodium hydroxide and the magnesium carbonate occurring in the siliceous magnesian limestone. The second is a reaction between the alkalis and silica in the aggregate.

The first reaction does not appear to the writer to be the basic cause of expansion for the following reasons:

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[9] "Research on Concrete Disintegration," by H. S. Mattimore and G. A. Rahn, Jr., *Proceedings*, A. S. T. M., Vol. 35, 1935, p. 410.

(1) Expansions were observed with aggregates containing magnesia in relatively small amounts.

(2) Although the paper does not report the separate percentages of sodium oxide and potassium oxide in the cements involved, it is logical to assume that they were present in varying ratios. The observed expansions were in good agreement with the total alkali content of the cements. This indicates that sodium and potassium compounds were equally active. The latter does not give the theoretical increase in volume indicated for the sodium-magnesium reaction.

(3) Calcium hydroxide, always present in relatively large amounts in hardened concrete, should prevent the formation of sodium or potassium carbonates.

The reaction involving silica in the aggregate, as illustrated in Table 3, seems to be a more valid explanation of the expansion. Rocks that show puzzolanic action when finely ground are generally soluble in part in alkali solutions. Alumina as well as silica may be dissolved. If such a reaction is the cause of expansion in concrete, then one might expect rocks such as rhyolite and andesite, particularly when formed by explosive volcanic action, to be troublesome with high-alkali cements.

In the consideration of such a reaction, the effect of dissolved calcium hydroxide cannot be ignored. Nearly all calcium salts are less soluble than the corresponding sodium or potassium salts. Therefore, the final products would be expected to be calcium silicates or aluminates with the alkalis converted back into the hydroxide form and available for additional attack upon the minerals. If the alkalis play a catalytic rôle of this nature, their potency in relatively small percentages may be more readily comprehended.

Reactions of this kind would continue in the presence of moisture until stopped by exhaustion of the active minerals, exhaustion of the calcium hydroxide, the accumulation of high pressures within the concrete, or the formation of reaction products in amounts sufficient to form a barrier against the migration of the reacting constituents.

If reactions of this kind are the true cause of expansion, further studies must take into consideration the change in solubility of calcium hydroxide in varying concentrations of sodium and potassium hydroxides, the effect of temperature on solubilities, the density and permeability of the concrete, and its capability to resist the pressure built up by the reaction products. It is hoped that Mr. Stanton's studies will progress to the point that the chemical phases can be discussed more fully.

It is worthy of emphasis that it was only with aggregates from a relatively small area in the State of California that deleterious effects were found through the use of high-alkali cements. D. G. Miller and P. W. Manson[10] have determined the alkali contents of 106 commercial cements from 74 mills. Their analyses, therefore, should represent a typical cross section of cement as produced in the United States. Of the 106 cements, more than one half contained alkalis (computed as Na_2O) of more than 0.6%, the maximum set by Mr. Stanton for freedom from deleterious expansion. If an aggregate has been used previously with cements from several mills, it is probable that in at least one instance the cement was high in alkali. Examination of concrete in service, then, should form a basis for estimating the probable effects on proposed construction of that aggregate with high-alkali cement even though the alkali content of previous cements is not known.

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[10] "Tests of 106 Commercial Cements for Sulfate Resistance," by D. G. Miller and P. W. Manson, *Proceedings*, A. S. T. M., Vol. 40 (1940).

HUBERT WOODS,[11] ESQ. (by letter).—Unquestionably, this paper provides, for the first time, an adequate explanation of a number of baffling cases of concrete deterioration that have occurred in the coastal region of California. It also provides new incentive for a more comprehensive study of aggregates in their relation to cement composition. There is great need for further studies on the chemical nature of the expansive reaction between certain natural minerals and the alkalis in cements. Only when this reaction is well understood can intelligent steps be taken to forestall or prevent it.

It would be as unreasonable to conclude that all aggregates will react expansively with alkalis in cement as to conclude that only those aggregates peculiar to the coastal area of California will do so. The widespread satisfactory behavior of concrete, in general, attests the fact that such troubles are not omnipresent, and where local aggregates and cements have had a long record of satisfactory performance there is no reason for alarm. On the other hand, with aggregate from a new source, without adequate history of performance, a thorough study is justified, and there is great need of a rapid test by which such aggregates may be judged with respect to possible expansive reaction with alkali.

In cases where the only aggregates economically available are likely to give trouble through reaction with alkalis in cement, it might be concluded that a simple solution is to put a limit on the alkali content of cement. As a matter of fact, at present that is about all that can be done; but it is not a satisfactory final solution. It is not economically desirable. All cements made in the United States contain more or less alkali, not by intent but by reason of the fact that nature has put alkali-containing minerals in practically all clays and shales used in cement making. The alkalis are partly, but not wholly, volatilized in the rotary kilns. In order to reduce the alkalis in his cement, the manufacturer may have to use other raw materials containing less of the alkali minerals, and these may be very difficult to find and very expensive to use. It would be highly desirable to find some inexpensive means, perhaps by additions to the cement, whereby the expansive reaction would be circumvented, and a diligent search for such means would seem to be essential.

N. T. STADTFELD,[12] M. AM. SOC. C. E. (by letter).—In this paper Mr. Stanton has made a valuable contribution, based on a painstaking investigation. He eliminated all other possible factors to tie in definitely the disintegration of concretes containing certain aggregates with the alkali content of the portland cement used.

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[11] Chf. Chemist, Riverside Cement Co., Riverside, Calif.

[12] Div. Engr., Board of Water Supply, New York, N. Y.

Although they are present in small quantity, the alkalis, nevertheless, have been under suspicion for a number of years. In 1929 the late Thaddeus Merriman,[13] M. Am. Soc. C. E., gave the results of an extensive investigation, which had led him to conclude that the rate of formation of the hydration products obtained when water is added to cement is dependent on the alkalinity of the solution. Within a certain zone of alkalinity, hydration is slow and orderly; but alkalinities either above or below this range were found to increase the rate of hydration. Thus the addition of calcium chloride to the mixing water, which lowers the alkalinity, will accelerate the hardening process. Similarly, the addition of sodium hydroxide, which increases the alkalinity, not only speeds up the hardening process but increases the maximum hydration temperature, as measured by a calorimeter in which a cement paste is hardening. After several years of investigation and numerous tests on the sulfate-resistant cement used in the tunnels and spillway of the Fort Peck (Mont.) Dam, Mr. Merriman finally embodied a restriction on the soluble alkali content in the specifications for portland cement of the Board of Water Supply (BWS) of the City of New York.[6] Cement to the extent of three million barrels made at 10 plants, complying with these specifications, has now been delivered to the various sections of the Delaware aqueduct. The principle involved in restricting the alkali content to obtain a uniform rate of hydration and to protect men against alkali burns when they work the wet concrete is different from the principle involved in restricting alkali content because of the deleterious effect on the concrete when used in conjunction with certain aggregates. Nevertheless, the fact remains that a well-defined beginning has been made to produce, consistently, a portland cement of a low-alkali type; and therefore, experience gained during four years of procedure may be of interest.

It is true that, whereas the BWS specifications confine their restrictions to water-soluble alkalis, the author uses the total alkali content, computed as sodium oxide (Na_2O), as the criterion of fitness. In his investigation these two did not always keep pace with one another, but records from plants producing BWS cement do show a close correlation. In addition, some records from plants not making BWS cement, which were made available to the writer, also show substantial correlation. In every case, cement passing BWS specifications would have been under the 0.5% Na_2O limitation, and cement failing in one would have failed in the other.

Table 6 gives the results of a series of tests run at one of the plants which consistently meet the BWS specifications. The raw mix was kept constant and, as the temperature was lowered, samples were removed and alkali determinations made. Although no accurate record of the temperatures was made, the samples were arranged in accordance with descending temperature.

It is apparent that potassium can be more readily eliminated by temperature increases than sodium; this was to be expected as potassium compounds volatilize at lower temperatures. Note the decrease in potassium content from 0.79% to 0.26% and the decrease in sodium content from 0.29% to 0.16%, a ratio of more than 3 to 1 for the potassium to a ratio of less than 2 to 1 for the sodium. Also note that, as the author's limit of 0.5% (total alkalis) is approached, the titration value of 3.5 for the BWS specifications is similarly reached. The method of determination is described in the specifications, as follows:

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[13] "Cement," by Thaddeus Merriman, Paper No. 308, *Proceedings of the World Engineering Congress*, Tokyo, 1929, Vol. XXXI, p. 345.

TABLE 6.—COMPARISON OF TOTAL ALKALIS WITH FREE ALKALIS (WATER SOLUBLE)

Method	Sample Numbers:									
	1	2	3[a]	4[a]	5[a]	6[a]	7	8	9	10
Free alkali[b] (cc)	1.20	1.70	1.80	1.80	3.95	3.95	6.40	6.80	11.30	11.40
Smith[c] (%):										
Na_2O	0.16	0.20	0.17	0.18	0.23	0.19	0.32	0.19	0.31	0.29
K_2O	0.26	0.27	0.29	0.28	0.36	0.36	0.53	0.55	0.72	0.79
Computed as Na_2O	0.38	0.42	0.41	0.41	0.53	0.49	0.76	0.65	0.91	0.95

[a] Sample 4 is a duplicate of Sample 3; and Sample 6 is a duplicate of Sample 5. [b] Merriman method.
[c] J. Lawrence Smith method (see Scott's "Standard Methods of Chemical Analysis," 5th Ed., 1939, p. 1612).

> "The alkalinity of the cement shall not be greater than 3.8 and its content of free alkali shall not exceed 3.5. These characteristics shall be determined as follows: Weigh out 800 grams of cement and put into an enameled saucepan with 500 c.c. of distilled water. Stir frequently for two hours, then filter through large folded filter paper for 10 minutes. If filtrate is not clear, refilter. Titrate 25 c.c. of the filtrate with N/2 HCl, using methyl orange as the indicator. The number of c.c. of acid required to neutralize the filtrate to the methyl orange end point is the measure of the alkalinity.
>
> "The free alkali content of the cement shall be determined as follows: Measure out 100 c.c. of the filtrate obtained in the alkalinity test of the preceding paragraph into a small beaker, and add 30 to 35 c.c. of a saturated filtered solution of $Ba(OH)_2$. Let stand, filter and wash with H_2O. Pass CO_2 into the filtrate for five minutes. Let stand, filter and wash the precipitate with H_2O. Heat to boiling and, if a precipitate forms, filter it out. Then boil the total filtrate plus wash water down to about 50 c.c., filter it and make it up to 100 c.c. with distilled water. Now take 25 c.c. of the solution and titrate it with N/2 HCl in the presence of methyl orange. The number of c.c. of the acid required to neutralize the 25 c.c. to the end point of the methyl orange is the measure of the free alkali content. Both of these tests are to be made at room temperature."

Close correlation holds true at other plants meeting the specifications. The opinion may be ventured that this is due to the fact that in all cases the so-called "flue dust" is discarded. "Flue dust" is the fine material blown from the rear end of a rotary kiln and caught under waste-heat boilers, in precipitators, or by other means. In many cases it is fed back to the raw mix and sometimes to the finished cement. In the former instance it builds up the alkali content, occasionally to such an extent that even with high temperatures it becomes impossible to clinker the mix properly, thus causing waste in fuel and lowering of quality. In the latter case it tends to increase the alkali content of the cement and mainly the non-water soluble alkali content.

Had the paper included such items in the history of the clinker as burning-zone temperature, speed of kiln rotation, subsequent treatment of the clinker, and a notation as to the disposition of the "flue dust," additional and valuable deductions might have been made.

A plea is made here for placing restrictions on water-soluble alkalis rather than on the total alkalis, because the determination of the former is extremely simple and excellent checks are obtained between laboratories, whereas determination of the latter is laborious (3½ days for an experienced chemist) and checking between laboratories is difficult.

Regardless of the form which the restrictions on alkalis may take, however, some additional burden is likely to be placed on producers, as disintegration of parts of the Parker Dam[14] in southern Arizona has been traced recently to the destructive action of the alkalis in the portland cement on a hydrous silica occurring in the aggregate, an aggregate which had passed all known tests for durability. A trend may well be expected toward the incorporation of low alkali content in future specifications for cement as a safeguard against possible disintegration. With this in mind a review of some of the difficulties encountered in manufacturing such cement may be of value.

All BWS cement is made from well-burned clinker assured by:

(1) Insistence on continuous high temperature (about 2,700° F) in the burning zone of the kiln, as recorded by pyrometers placed outside and in front of the kilns;

(2) Insistence on regularity of kiln rotations coordinated with the occasional rise and fall of the temperature; and

(3) Insistence on satisfactory performance in the "sugar test" which indicates the degree of completion with which the various constituents in the clinker have been combined.

The BWS test for sugar solubility is described in the specifications as follows:

> "The sugar solubility of the cement shall not be greater than 8.0 to the phenolphthalein end point, nor greater than 10.0 to the final clear point. These values shall be determined as follows: A sample of about 100 grams of the cement is passed through the 200-mesh screen and put into a glass bottle closed with a rubber stopper. From this bottle 15 grams are then weighed out and placed into a Nessler tube containing 100 c.c. of a 15-percent. solution of cane sugar in distilled water (commercial granulated sugar such as 'Jack Frost'); this solution shall not be more than three days old. The tube and its contents are then quickly shaken by hand and placed on a wheel revolving about 60 times per minute. At the end of about 1 hour and 50 minutes the mixture is poured into a filter paper and funnel and allowed to filter for ten minutes when the beaker containing the filtrate is removed. (In case the filtration time of 10 minutes is too short to produce a volume of filtrate of 30 c.c., it may be lengthened by shortening the time of shaking, but the total time from the putting of the sample into the solution to the end of the filtration must be exactly 2 hours.) Twenty-five c.c. of the filtrate is now titrated with N/2 HCl in the presence of phenolphthalein and the number of c.c. of

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[14] See minutes of December 6, 1940, meeting of Committee C-1 on cement, A.S.T.M., p. 5.

> acid to the end point is the first measure of the sugar solubility. At this stage of the test, in the case of a thoroughly burned cement which has been kept dry, the solution will be practically clear and only traces of ferric oxide and alumina will be in suspension. In the case of an underburned cement, or one which has been exposed to moisture, the solution will be heavily clouded and the end point must be approached slowly and with caution. When so performed, the phenolphthalein end point can be definitely determined, as the color changes from light pink to yellow. The titration is then continued until the solution is crystal clear and nothing remains in suspension. The total number of c.c. of acid from the beginning of the titration to this final clear point is the second measure of the sugar solubility. This test shall be made at room temperature. In addition to disclosing the quality of the cement as above stated, this test further indicates the character of the hydration products which will be realized in the completed concrete."

In most cases adherence to principles (1) to (3) resulted in a cement of satisfactory alkali content. However, it was found in a few of the plants that with proper burning and low sugar-test results the alkali content was higher than allowed by the specifications. After some experimentation it developed that, where potassium was present, shortening of the burning flame and consequently impinging it on the hot clinker bed would drive off this element. This could be noted through blue glasses, which showed up the unmistakable purple hue of potassium vapors.

Where even this technique was insufficiently effective, probably due to the presence of larger proportions of sodium (which, as previously mentioned, does not respond as well as potassium), addition up to 0.5% by weight of calcium chloride was made to the raw mix, which changed some of the alkalis, in whatever form present, to alkali chlorides. These volatilize easily and appear to be driven off completely, as none of the resultant cement showed any trace of chlorides, but did show a large reduction in alkali content.

To summarize, therefore, with present knowledge based on four years of experience, the following is suggested for the consistent production of cement low in alkali content:

(*a*) Discard all "flue dust" and incidentally save on fuel;

(*b*) Install pyrometers and kiln-rotation recorders and note the alkali content with varying burning temperatures;

(*c*) Experiment with length of flame in the burning zone; and

(*d*) As a last resort, add calcium chloride up to 0.5% by weight to the raw mix.

In conclusion it may be stated that, although the disintegration of concrete, as found by the author and in the Parker Dam, may well be due to the action of alkalis in cement on localized aggregates not generally encountered, the limitation on the alkali content should be encouraged in any case for additional safeguard and the production of a decidedly better product, as outlined in the first part of this discussion.

W. C. HANNA,[15] ESQ. (by letter).—That very reactive aggregates exist in certain districts of California has been known for many years. The occasional defective concrete constructions made with these aggregates have probably contained cements of all brands used within the state, and this includes cements manufactured in foreign countries.

As early as 1934, the company with which the writer is connected investigated the case given by Mr. Stanton in Fig. 1(*d*) (a building completed about 1931), recognized the abnormal condition, and decided to determine the cause. The cement used in the construction was of most excellent quality, sound in every respect, and with both the manufacturer's tests and the independent inspection of a well-known commercial laboratory to prove the quality. A study was started at once and the company assigned Ira C. Bechtold (later, Portland Cement Association Fellowship, National Bureau of Standards) to devote considerable time to the problem.

Inspection of concrete cores from this structure revealed the gel formation which evidently had caused the cracks. From the study of the concrete it was possible to trace the bad aggregate to its source and to collect specimens of it for laboratory study. Photographs of the concrete showing the gel formation, together with Mr. Bechtold's findings, were exhibited to a number of organizations and individuals who were interested in the matter. This discussion is based on the resultant report.

Examination of the concrete disclosed that it contained many particles that were evidently foreign to the remainder of the aggregate material. These particles ranged in color from a dark brown to a light tan or buff. They were quite hard, but brittle, when dry and intact. They were sometimes cracked and frequently disintegrated and contracted in the center. Practically all such particles showed a hard, shiny shell, and in some cases small spheres were noted near them which appeared to be hard and white but brittle when dry. These particles varied in size from very small grains to pieces about $\frac{1}{4}$ in. by $\frac{1}{4}$ in.

To this point all specimens were kept dry. Then they were immersed in water for a period of three days, after which they were removed, allowed to dry at room temperature for three days, and then again immersed in water. After twenty-four hours in water the foreign particles became very soft and began to disintegrate; and from many of them a definite formation of gel occurred, in some cases in a quantity almost equal to the volume of the original particle. The envelope around many particles became wet and soft but did not expand appreciably. The aforementioned spheres also softened. Some of the particles which showed marked disintegration before wetting did not form large amounts of gel. In one case a specimen from a core developed a large amount of gel from an area that became cracked as the piece remained wet; and finally a small piece of concrete was forced away by the gel.

Some cracks in concrete specimens from cores were found to pass entirely through the specimen. In several such cases it was found that gel was being squeezed out of the crack, sometimes in several spots along the crack. Where there was a crack it was always found that there was at least one particle developing gel within it.

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[15] Chf. Chemist and Chemical Engr., California Portland Cement Co., Colton, Calif.

An attempt was made to determine the chemical nature of the gelatinous material being formed from the particles under consideration. It was almost impossible to secure the pure gel material for test purposes because, in the process of formation, it always became contaminated with cement and small fragments of aggregate. Furthermore, it is to be expected that this type of gel would absorb calcium hydroxide and other material from the water around it. However, the following approximate analysis was obtained on a small portion of relatively clean gel scraped from the specimen: SiO_2, 41%; Al_2O_3, 11%; CaO, 33%; and H_2O, 17%.

Mr. Bechtold expressed an opinion that reaction of the calcium hydroxide freed from the cement by water is involved in the formation of this gel. The gel formation appears to be irreversible. Evidently, the gel progressively swells after it is first formed, but fortunately, after once being dried and contracted, it will not swell again when brought into contact with water. This point on the irreversibility of this reaction is important. It indicates that during the life history of a given piece of concrete construction there is a gradual slowing up of the reaction between calcium hydroxide and the reactive aggregate. In the early stages this reaction may be severe enough to cause cracking and "pop outs," but after a period of several years, depending on how reactive the aggregate happens to be, the disruptive reactions will have spent themselves.

Further research reported in 1936 indicated that volume changes and the extent of cracking evidenced in the ordinary laboratory specimens were not indicative of the intensive destructive forces that appeared to be responsible for the failures in question. Laboratory specimens will not indicate the same extent of change observed in the field. The expansive force of the gel can derange the particles of concrete so much that readjustment to the former position is impossible. When such readjustment is forced by shrinkage under drying conditions, the deranged particles may act as wedges to force the cracks open.

The shales vary tremendously in activity. They have not been found to be active to water alone to the extent of forming gel, although disintegration has been observed. In saturated limewater many of them produce large quantities of gel. In some cases the gel produced has been estimated to have a volume of 6 to 8 times that of the shale producing it. Other specimens of apparently the same material produce little gel and some show no sign of activity. Concentrations of even less than 10% of shale could produce stresses that would exceed the tensile strength of the strongest concrete.

At least a part of the bad aggregates in the aforementioned structure produced gel with limewater which, of course, is found in all normal concrete. There was a sufficient amount of such aggregate in the concrete to account for the damage. The gel formed in the concrete did not indicate that the alkalis of sodium and potassium were necessary for the reaction.

Mr. Stanton's bad aggregate is not the same as those detected in the aforementioned structure. It seems to resist the limewater but is readily attacked by a 10% solution of either K_2SO_4 or K OH. It was of interest to note that the reaction with these potassium compounds was more severe than with the corresponding sodium compounds.

The Stanton test, using sealed containers, is a valuable one for estimating the relative activity of certain California aggregates. For the present, the test should be classed as specific, because the writer has records of aggregates which have proved to be unsatisfactory in the field but which failed to show any excessive expansion when subjected to a Stanton test lasting several months. Mr. Stanton was admittedly dealing with a bad aggregate as revealed both in field work and by his unique laboratory test. All users of concrete should recognize his contribution in developing a test that is easily made and that certainly reveals doubtful quality of the aggregate he was testing.

Mr. Stanton has indicated that expansions he has encountered may be due in part to a reaction between $MgCO_3$ and Na OH. It is indicated that $Na_2CO_3 \cdot 10\,H_2O$ is a product of this reaction and that, due to the addition of ten molecules of water to the Na_2CO_3, an enormous increase in volume results. This in turn has a disruptive effect on the concrete. The writer is convinced that this possible reaction does not explain the expansion effects encountered by Mr. Stanton. From a practical standpoint, it is known that dolomitic limestone or magnesian limestone, when free of reactive siliceous material, is classed as one of the best of aggregates. Many widely distributed concrete structures, thirty or forty years old and containing dolomitic limestone as aggregate, are in excellent condition. It seems logical to conclude that the magnesian limestone part of this bad aggregate is not at fault. Some instability of the siliceous phase of this aggregate may be the cause of the failures reported by Mr. Stanton.

A close study of the paper will reveal a sincere attempt to establish the limitations of concrete made from portland cement, aggregate, and water. A clear conception of the function and limitation of each ingredient entering into concrete is essential.

Portland cement is very reactive chemically, and must be so to be useful. The compound composition of cement is complex, and the reactions taking place upon hydration are involved. The products of reaction are basic in character. The basic compounds may react within the cement composition itself or with some added material such as reactive aggregate. Aggregates that are reactive to portland cement compounds enter the field of puzzolanic materials. Some materials that may be definitely classed as poor and unsound aggregates, upon being interground with portland cement, will produce a sound and high-quality puzzolanic cement. In some respects there is a relation between the reaction products of portland cement and the reaction products from unsound aggregates. Both are active chemically and yield complex calcium silicates. Placing a relatively large particle of reactive aggregate within a concrete structure is always dangerous. The reactivity of the piece of aggregate may persist for an indefinite period, and there is always danger of failure of concrete, if the destructive forces set up are sufficient to overcome the inherent strength of the concrete structure. Construction engineers should attempt to approach the ideal in formulating concrete; that is, they should use a high quality portland cement plus a properly graded inert aggregate. This thought, of course, is not a new one; but it is worth repeating.

The record of failure as reported by Mr. Stanton is conclusive and indicates that constructive work toward improvement must be done by all concerned. The cement manufacturer should be required to produce the best quality of cement consistent with available raw materials. Great care should be taken as to the quality of aggregate, and the governing factors are relative inertness, high inherent strength, proper size distribution, and reasonable availability to the construction site. Further constructive work must be done in the field of physical and chemical testing of cements and aggregates. It is difficult to devise practical tests of this kind, but the work should be continued until satisfactory engineering tests are developed.

The field of better design for concrete structures should not be overlooked. There is still plenty of room for improvement in the design of concrete structures. Another study of a more fundamental nature involves a better knowledge of the chemistry of cement compounds. Considerable is known of the function of the recognized oxides in cement, such as silica, iron oxide, alumina, lime, magnesia, and sulfuric anhydride. Small amounts of other materials such as titanium dioxide, phosphorus compounds, and alkalis are present in all commercial cements. The effect of these incidental materials should be the subject of further study.

The chemical form of the sodium and potassium compounds present in cement is not known. There is evidence that potassium in cement often occurs as K_2SO_4. The possibility of complex calcium-sodium silicates should not be overlooked; nor should the alkalis in the aggregates be forgotten. The writer has made many Stanton tests in which 1% sodium or potassium hydroxide was deliberately added to the normal portland cement used in the test. In general, the addition of active alkali appeared to lower, or had little effect on, the degree of expansion.

The time has not come to establish a definite specification limit for alkali in portland cement. Considerable difficulty has been encountered in obtaining check results on alkali determinations. The writer has seen a great discrepancy in results on check samples submitted to representative laboratories. Considerable cooperative work is being done on standardizing the methods for estimating sodium and potassium oxides in portland cement. In the future, chemists probably will be able to estimate these compounds with reasonable certainty, but for the present they are inclined to allow considerable tolerance in any estimate of alkalis.

Out of the maze of discussion that has resulted from the commendable work of Mr. Stanton and others on alkali in portland cement, much good will result. Every one seems to be "alkali conscious," and the writer considers the current agitation as a "one track" proposition. All factors that can lead to unsound concrete should be investigated carefully. Cement manufacturers will do their utmost to cooperate with every one interested in the fabrication of better concrete structures.

J. C. Witt,[16] M. Am. Soc. C. E. (by letter).—In his very interesting paper Mr. Stanton presents some field and laboratory data on the expansion of concrete. On the basis of fact and assumption he points out relations that may exist between the two groups of phenomena. However, some statements are made with a "q.e.d.-ness," or finality, which the data do not justify. Reference to a few points will be sufficient to make this clear.

Under the heading "Earlier Failures" this statement appears:

> "Although the sulfates in sea water undoubtedly contributed to the serious deterioration of the sea walls, it was obvious that this deterioration was accelerated by the infiltration of sea water through cracks resulting from excessive expansion occasioned by other causes."

The deterioration of concrete exposed to sulfate solutions has been observed all over the world. It has not been restricted to cements or aggregates of any particular composition, nor to any combination of the two. The reason is not apparent why the deterioration of the sea walls mentioned in the paper is due primarily to the nature of the cement and aggregate present—the sea water simply being a contributing agent.

In the laboratory, specimens containing cements relatively high in alkali compounds and certain fine aggregates were subjected to storage conditions involving a wide range of moisture and temperature. After more than a year no excessive expansion was observed. Accidentally it was found that expansion could be obtained easily by placing specimens in sealed containers. This procedure does not represent the type of exposure to which most concrete is subjected.

The following statement is quoted from the author's discussion of Table 1:

> "It has not been determined if the reaction in the case of this impure limestone is due entirely to some form of silica or in part to the magnesium carbonate. If it is due in any part to the magnesium carbonate, the probable chemical reactions in this respect are * * *."

This is followed by a computation (Eq. 1) indicating that, when magnesium carbonate and sodium hydroxide react, the products are magnesium hydroxide and sodium carbonate (with ten molecules of water of crystallization), accompanied by an increase in volume of 239%. Apparently this is the basis for such statements in the "Conclusion" as: "Certain mineral constituents in concrete aggregates contribute to expansion of concrete and sometimes develop stresses of such magnitude as to cause failure," and "The chemical reaction producing excessive expansion apparently occurs only when the portland cement component contains an appreciable percentage of alkali in the form of sodium and potassium oxides."

Here are a few of the comments that could be made:

(*a*) The volume of the water of crystallization is not included in the "combined volume of reacting compounds." In conformity with the author's nomenclature, this volume is 180.16. It is equivalent to 276% of the combined volume of the listed reacting compounds and to 80% of the reaction products.

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[16] Technical Director, Marquette Cement Mfg. Co., Chicago, Ill.

(*b*) There is a series of hydrous sodium carbonates. There is no basis for assuming that $Na_2CO_3 \cdot 10\,H_2O$ alone would crystallize from solution—the only manner in which any member of the series is formed. A crystal containing a given number of molecules of water does not develop directly into a crystal containing more water. Re-solution and recrystallization are necessary. The compound $Na_2CO_3 \cdot 10\,H_2O$ is unstable—losing water when in contact with air.

(*c*) If crystallization occurs when the cement paste is plastic, no cracking of a specimen would be expected. After the cement has hardened, very little water is available in the interior of a specimen stored in a sealed container. At the surface some water is available, but crystallization here would not cause cracks of any importance.

Following Eq. 1 this statement appears with reference to the volume increase computations: "To check the accuracy of this assumption, tests were made to determine the action of sodium hydroxide upon the various materials that might be present in the aggregate, * * *." Several substances were treated with a large excess of normal sodium hydroxide solution. The conditions differed greatly from the other tests described in the paper, and the results do not serve to check any of the assumptions.

R. F. Blanks,[17] M. Am. Soc. C. E. (by letter).—The far-reaching discovery, reported by Mr. Stanton, that reactions between alkalis in cement and some mineral constituents of aggregates explain certain cases of concrete expansion, cracking, and disintegration is not surprising. Since about 1930, there has been a growing suspicion that the minor compounds in cement (including alkalis) might have a greater influence on the quality of cement and the resulting concrete than was generally realized. The incredible part, in the light of developments since Mr. Stanton's discovery, is that it has taken engineers and investigators so long to arrive at this explanation.

The Constituent Materials in Concrete.—During recent years the quality of portland cement has been improved notably, and this has been reflected in specification requirements. Limitations on various compounds that have been found to be objectionable in portland cement have been imposed, and in some cases manufacturing methods and procedure have been limited by specification requirements. Free lime, magnesia, alumina, and, in special cases, iron, have been subjected to varying degrees of restriction. During the past few months (1941) the alkalis of sodium and potassium have likewise been limited in some special cases. However, far too little is yet known regarding the true constitution of portland cement, in what forms the various compounds really exist, or what reactions are actually involved in the hydration processes.

It cannot be said that even comparable progress has been made in the specification and use of aggregate materials. The popular conception that aggregates are merely inert filler materials and that most all rocks, with a few exceptions, will make good concrete still persists. However, as P. H. Bates has indicated,[18] all aggregate materials are subject to chemical reaction in concrete and the real problem is whether or not they are adversely reactive. Mr. Bates also pointed out that the compounds of calcium, as well as of sodium and potassium, should not be overlooked in searching for the solutions to the alkali-aggregate reaction problems. Present criteria by which the suitability of aggregates for use in concrete is judged are woefully inadequate, and basic information for intelligent guidance is almost entirely lacking.

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[17] Senior Engr. (Civ.), Bureau of Reclamation, Denver, Colo.

[18] *Proceedings*, Conferences on Problems Related to Alkalies in Cement and Their Effect on Aggregates and Concretes, Bureau of Reclamation, February, 1941, p. 20.

In the light of the aforementioned conditions, it is again not surprising that, since Mr. Stanton's discovery, many cases of concrete disintegration similar to those that he described have been revealed. Concrete structures either known to be or strongly suspected of suffering from alkali-aggregate reactions are now under observation in other parts of California, and in Arizona, New Mexico, Washington, Idaho, Oregon, Nebraska, Canada, Iowa, Virginia, and Pennsylvania. The list of aggregates known to be reactive in the presence of alkalis includes materials having such descriptive terms as andesitic, rhyolitic, felsitic, granitic, slaty, schistose, impure limestones, and others as yet unidentified, in addition to the shale, chert, and siliceous limestone reported by Mr. Stanton.

Preventive Measures.—The predominating question in the minds of all who are concerned with the alkali-aggregate reaction problems, of course, is "How can future cases of such concrete disintegration be avoided?" From the cement manufacturer's viewpoint, alkali-reactive aggregates should be excluded from use. He substantiates his position by stating that most of the concrete in existence is rendering satisfactory service even though high-alkali cement was used in much of that concrete. Concrete investigators emphasize that there is, as yet, no certain method of determining what combination of cement and aggregate might be expected to cause trouble under various service conditions. Furthermore, the occurrence of alkali-reactive aggregates is not a condition peculiar to any one locality but is apparently widespread.

The engineer and builder, in some cases, finds himself in a quandary. For most ordinary concrete work, particularly where aggregates with long service records in combination with a relatively high-alkali cement are available, the problems probably need to occasion little concern, except an uneasy realization that alkalis in cement are not desirable constituents. For large, important, and costly works there is, for the moment, only one available expedient—the imposition of the lowest practicable limit on the alkali content of the cement. Whether this expedient affords adequate protection in all cases, whether some aggregates should be condemned for use in concrete, or whether there are other more effective corrective measures whereby the destructive chemical reactions can be neutralized yet remains to be determined.

Comparison of Mr. Stanton's Findings with Those of the Bureau of Reclamation at Parker Dam.—The Parker Dam case has been discussed in detail in the technical press.[19,20] It is sufficient to state here that Parker Dam was completed in 1938, and, after less than two years of service, random-pattern cracking (Fig. 10) and internal expansion of the concrete developed on an extensive

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[19] "Concrete Deterioration at Parker Dam," by R. F. Blanks, *Engineering News-Record*, March 27 1941, p. 46.

[20] "Cracking in Concrete Due to Expansive Reaction Between Aggregate and High-Alkali Cement As Evidenced in Parker Dam," by H. S. Meissner, Assoc. M. Am. Soc. C. E., *Journal*, A. C. I., April, 1941.

scale. As high-alkali cement was used in the construction of Parker Dam, the possibility of alkali-aggregate reactions, similar to those described by Mr. Stanton, was immediately investigated by means of his 1 : 3 mortar-bar expansion test at 70° F. The results obtained from such tests at six months (Fig. 11(*a*)) show about the same expansion for the Parker sand, Grand Coulee sand, and the local Denver sand from the Platte River (approximately 300 millionths) with a cement containing 1.13% total alkalis. In comparison with Fig. 4, this might be interpreted as meaning that all the aforementioned sands are mildly reactive in combination with a high-alkali cement. On the other hand, the foregoing results might be interpreted as meaning that none of these sands are reactive, the recorded uniform expansions being due simply to increased moisture content of the mortar bars.

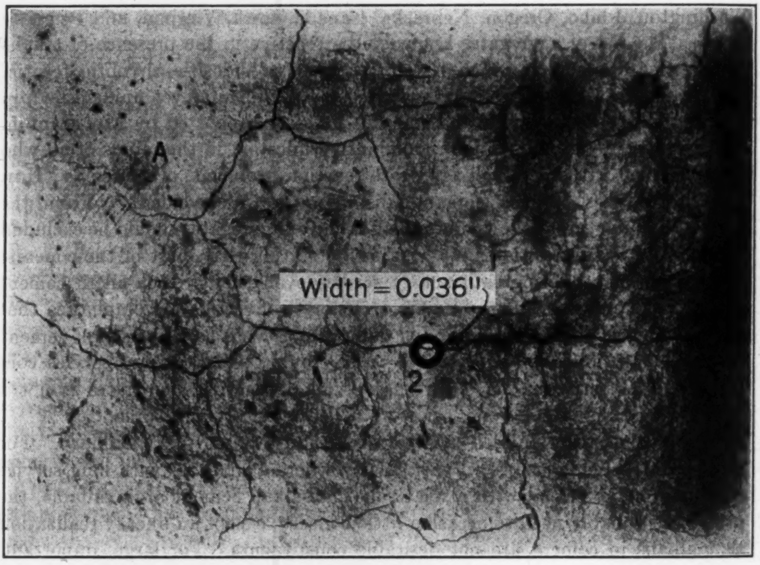

FIG. 10.—TYPICAL RANDOM-PATTERN CRACKING OF THE PARKER DAM SURFACE CONCRETE AS A RESULT OF INTERNAL EXPANSION

Conclusive evidence to explain the condition of the Parker Dam concrete was not obtained until cores were drilled. When the Parker concrete cores were stored in the presence of moisture, a gel-like ooze appeared on their surfaces and also was found in the small cavities and voids within the concrete mass (see Fig. 12). Upon drying, this gel became a hard whitish deposit similar to that described by Mr. Stanton. Chemical analysis of the gel revealed it to be essentially sodium silicate. Further investigation showed that the gel-like substance was apparently a reaction product between the alkalis in the cement and as yet unidentified mineral constituents carried by small percentages of andesitic and rhyolitic aggregate particles in the Parker gravel (see Fig. 13).

Whereas it is inferred from Mr. Stanton's paper that only the sand in the California aggregates carried alkali-reactive materials, 6-in. cobbles have been found undergoing chemical alteration in the Parker concrete.

After the reactive aggregates in the Parker gravel were identified by means of the concrete-core studies, samples of the andesite rocks were hand picked and the material subjected to tests corresponding to those shown in Fig. 6(*b*) for siliceous limestone. The comparative responses of a 10% crushed Parker andesite and the 5% siliceous magnesian California limestone (as reported by Mr. Stanton) to the mortar-bar expansion test at 70° F are shown in Figs. 14(*b*) and 14(*a*), respectively.

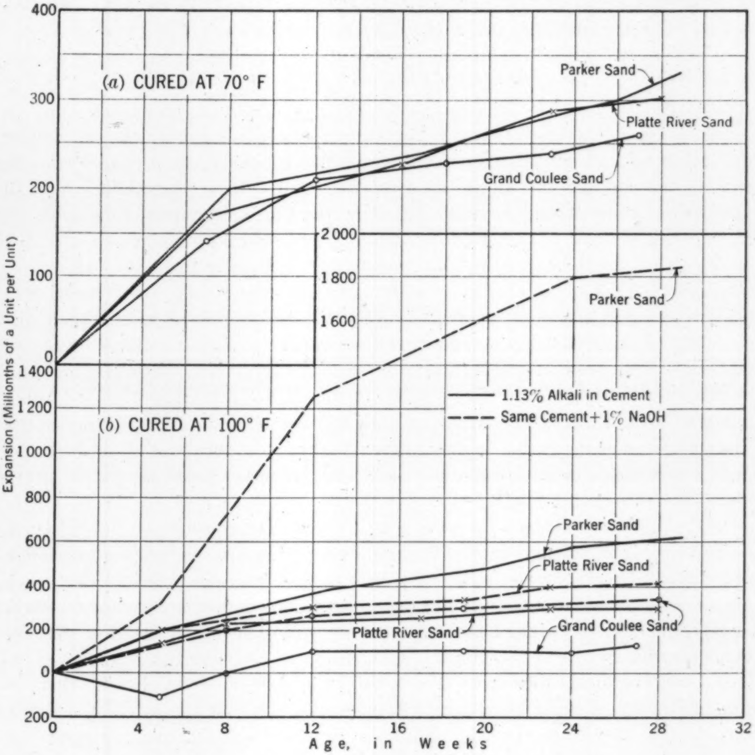

FIG. 11.—1 : 3 MORTAR BARS CURED IN SEALED CONTAINERS

In an effort to develop an accelerated test for detecting alkali-reactive aggregates, the mortar-bar test was conducted at 100° F with a cement containing 1.13% alkalis and also with the same cement to which had been added 1% of Na OH. The results from these tests are shown in Fig. 11(*b*). Again, the correct interpretation of these results is not very clear. However, the differences in the expansions obtained with and without the addition of Na OH are believed to be significant. A close comparison of the data shown in Fig. 11(*b*) with those represented in Fig. 11(*a*) also indicates some inconsistencies.

The most significant fact developed by Mr. Stanton's investigations, corroborated by those conducted by the Bureau of Reclamation, is that "bad" aggregates apparently make "good" concrete in combination with cements containing less than 0.60% total alkalis. The question immediately arises: "Is an aggregate that is reactive in the presence of a high-alkali cement 'bad' or is the high-alkali cement 'bad'?" This is an important question that has not as yet been answered satisfactorily.

For concrete work in those areas where alkali-reactive aggregates are known to occur, the California Highway Department has limited the alkali content of cement to 0.60%. The Bureau of Reclamation has imposed such a limit on cements being used in current large concrete works. Practically all of the natural sand and gravel aggregates available in the Western States contain varying percentages of andesitic, rhyolitic, and felsitic materials. Not all particles of andesite and rhyolite found in the Parker concrete show evidence of chemical alteration and, likewise, examples of felsite in concrete containing high-alkali cement have been observed to be chemically inactive. However, methods for predetermining when such materials may or may not be reactive in the presence of excessive quantities of alkalis in cement have not been discovered. It is reasonably certain that the reactive minerals in these aggregate materials are products of alteration, and the intensive petrographic and microscopic investigations of aggregates, cements, and concretes that are being initiated should clarify the situation. In this connection, a comparison of the findings from the present investigations with the experiences described[21] by J. C. Pearson and G. F. Loughlin in 1923 with feldspar aggregates should prove extremely interesting.

Effect of Environmental Conditions on Concrete Deterioration at Parker Dam.—The drying conditions in the Parker Dam region are very severe, although observations on concrete structures in the arid regions indicate that such exposure is not particularly trying to concrete or conducive to unusual crack development. At Parker Dam, the cracking is most severe in the more massive concrete members and the abutment blocks at or near the roadway elevation. The cracking diminishes from the top of the dam downward, and there is practically no cracking at or near water levels. Also, at the top of the superstructure in the relatively thin wall, floor, and roof slabs of the gate-operating house, the cracking is insignificant.

For varying depths, to 30 ft, 10-in. by 20-in. cores have been drilled from the top of the spillway crest. The strength and Young's modulus values obtained with these cores are much higher than those previously reported[19] for cores taken at higher elevations. The average strength of the more recent 10-in. cores is 4,750 lb per sq in.; modulus, 3,375,000 lb per sq in.; and Poisson's ratio, 0.16. However, the recent cores from all depths profusely develop the characteristic exuding deposit of sodium silicate when stored in the presence of moisture. The gel-like deposit is found in the small pores within the concrete

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[21] "An Interesting Case of Dangerous Aggregate," by J. C. Pearson and G. F. Loughlin, *Proceedings*, A. C. I., Vol. XIX, 1923, p. 142.

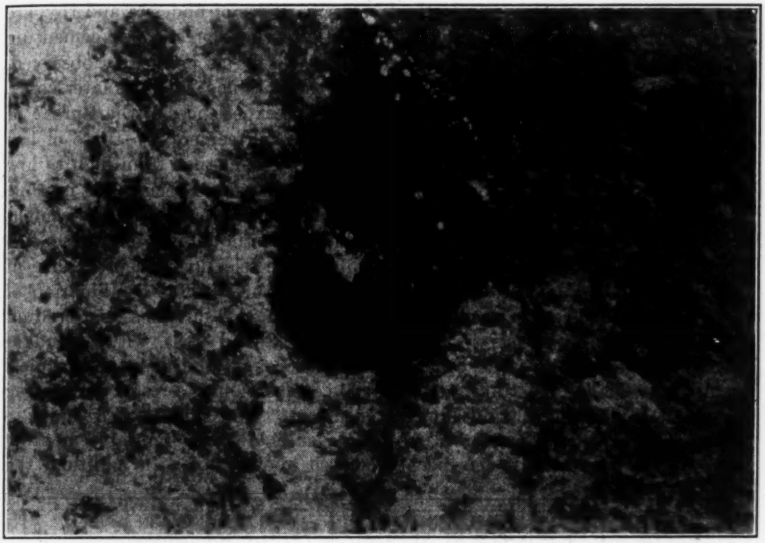

FIG. 12.—SODIUM SILICATE GEL IS FOUND IN THE PORE SPACES OF PARKER DAM CONCRETE

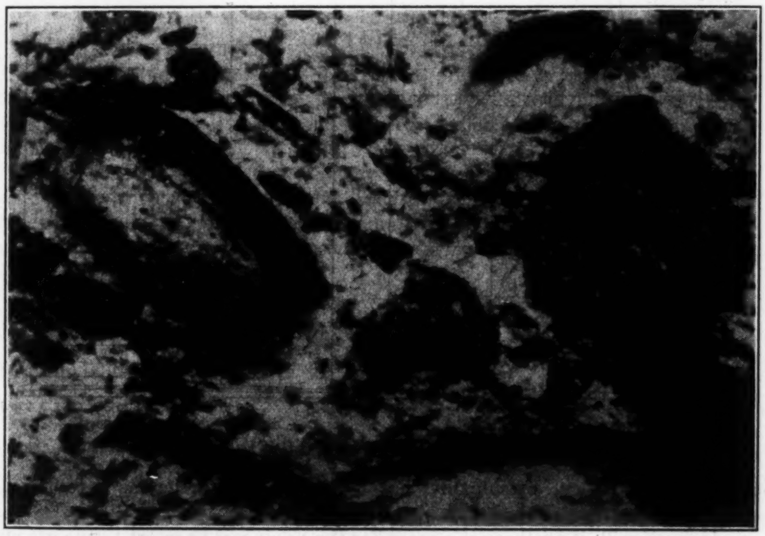

FIG. 13.—ANDESITIC AGGREGATE PARTICLES IN PARKER DAM CONCRETE ALTERED BY CHEMICAL REACTION WITH ALKALIES IN THE CEMENT

mass, and evidence of chemical attack upon particles of andesite and rhyolite aggregate can be seen—all as revealed by the earlier cores. The cementing matrix does not show the same degree of dissociation as previously observed, as might be expected from the higher strengths obtained with the later cores.

Improved Quality of Low-Alkali Cement.—The experiences of the Bureau of Reclamation in changing from relatively high-alkali cements to those of low alkali have developed interesting comparisons. The change from high to low alkalis with attendant improvement in other qualities has been made possible for certain Reclamation work through the splendid cooperation of the cement manufacturers in developing practical production methods. In most cases the alkali content has been reduced by controlling the selection and processing of raw materials and by increased burning temperatures. The combination of raw materials required for producing low-alkali cement results in a final product that is considerably higher in total silicates (C_2S and C_3S), with attendant lower C_4AF and C_3A, and the cement is more thoroughly burned. The typical examples in Table 7 illustrate the improved composition as a result of reducing the alkali content of certain low-heat cements. The sum of C_2S and C_3S (which is sometimes considered as one measure of the potential cementing value of a given cement) has been increased about 10%. In addition, the cementing efficiency, presumably, has been increased further as a result of higher burning temperatures.

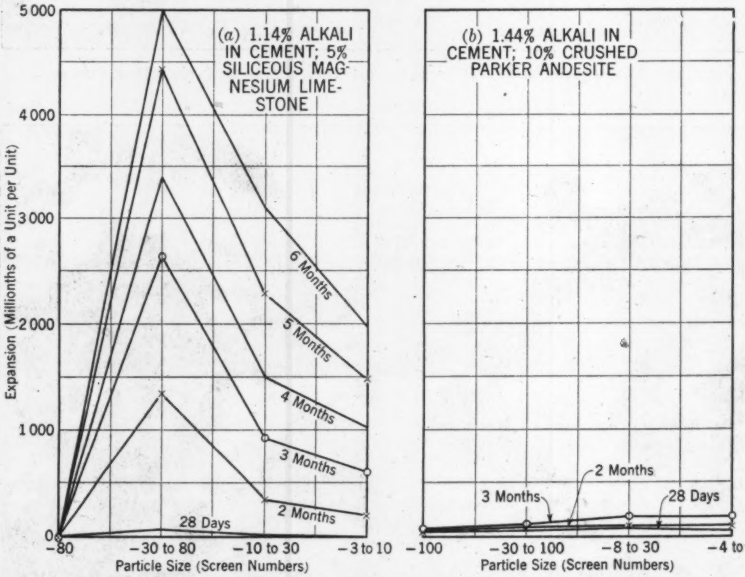

Fig. 14.—Reaction of Aggregates to the Mortar-Bar Expansion Test at 70° F

In some mills it has been necessary for the producers to resort to the use of calcium chloride or other expedients in addition to raw-material selection in reducing the alkali content of their cement below the 0.60% limit. Calcium chloride apparently will effect a reduction in alkalis of at least 0.3% to 0.4%, depending upon manufacturing conditions. When it has been used, no evidence of chlorides has been found in the finished cement. In one case a heat-treating process of the clinker efficiently reduces the alkali content about 0.3%. In addition, the 7-day strength of this cement is increased 25%, for the same theoretical composition and fineness, by the heat treatment.

Experience to date indicates that the increased cost of producing cement that meets the 0.60% alkali limit may vary from practically nothing to 20 cents per bbl, depending upon the individual mill conditions.

TABLE 7.—EFFECT OF REDUCING THE ALKALI CONTENT OF CEMENT (PERCENTAGES)

Constituent	MILL A		MILL B		MILL C	
	Normal alkali	Low alkali	Normal alkali	Low alkali	Normal alkali	Low alkali
C_4AF	15	9	12	7	16	14
C_3A	6	5	6	6	6	4
C_2S	43	49	48	52	46	45
C_3S	29	32	29	30	25	32
Alkalis	1.2	0.4	0.9	0.4	1.3	0.4

Conclusion.—The engineering profession, the cement and aggregate industries, and the users of concrete are all indebted to Mr. Stanton. Without question, his discovery will ultimately lead to far-reaching improvement in concrete and better understanding of its ingredient materials.

J. MacNeil Turnbull,[22] Assoc. M. Am. Soc. C. E. (by letter).—Investigations and tests of great interest are presented in this paper and, by demonstrating one effect of the presence of the alkalis in portland cement, the importance of these minor and hitherto neglected constituents has been emphasized.

F. M. Lea and C. H. Desch[23] have given the percentage of the total alkali content of various types of portland cement that are dissolved on shaking the cement in water for forty-eight hours. The amounts of the alkalis that can go into solution under these conditions exhibit a wide range of values—20% to 85%.

R. H. Bogue[24] has given some of the possible compounds that the alkalis may form in portland cement.

The writer believes that the percentage of soluble alkalis depends upon the proportion of the alkalis which are combined with the cement in stable compounds. Any excess greater than this will be present in a free, or easily liberated, state, and available for extraction by water or reaction with any chemically active substance in the aggregate.

During 1936 the writer investigated the increase in the caustic alkalinity of water in contact with hardened concrete. He had hoped to be able to remove a considerable proportion of the alkalis from the cement by washing it in water before addition to the aggregate. This was done by adding the cement to three and one half times its weight of water and keeping the mixture well stirred for a period of 30 min or longer. The cement was allowed to settle and the free water decanted, the surplus, greater than the water-cement ratio desired, being removed by centrifuging. The wash water was filtered and submitted to chemical analysis. Table 8 gives the composition of this wash water after washing 1,400 g of cement with 4,900 ml of tap water for 30 min; 4,275 ml of water were recovered, leaving a water-cement ratio of 0.45 by weight in the cement paste.

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[22] Asst. Engr., Hume Pipe Co. (Australia) Ltd., Melbourne, Victoria, Australia.

[23] "The Chemistry of Cement and Concrete," by F. M. Lea and C. H. Desch, Edward Arnold and Co., London, 1935, p. 330.

[24] "Constitution of Portland Cement Clinker," by R. H. Bogue, *Proceedings of the Symposium on the Chemistry of Cements*, Stockholm, 1938, pp. 69, 70, and 88.

TABLE 8.—CHEMICAL COMPOSITION OF WASH WATER FROM A MEDIUM-ALKALI CEMENT

In Parts per Million

Cl	CO_3	OH	SO_4	$Si\,O_2$	Ca	Mg	Na	K	NO_3
21	15	984	1,325	2.0	1,444	0.4	19	492	trace

The cement used was cement *C* of Table 9, from which it will be seen that this cement contains considerably more potash than soda. The principal compounds dissolved out of the cement appear to be calcium sulfate, calcium hydroxide, and potassium hydroxide.

TABLE 9.—PERCENTAGE OF ALKALI IN PORTLAND CEMENT

Cement	Na_2O	K_2O	Total alkali[a]
A (low-heat)	0.11	0.23	0.26
B (low-heat)	0.64	1.22	1.44
C (standard)	0.14	0.78	0.65
D (standard)	0.70	0.40	0.96
E (standard)	0.98	0.55	1.34

[a] Calculated as Na_2O.

Tests of hardened cement mortar, 1 : 2.2 by weight, in the form of a pipe lining ⅜ in. thick, 3¼ in. internal diameter, 12 in. long, made of cement *C*, with and without washing, confirm the analysis given in Table 8. After hardening, the pipes were filled with tap water and left standing for a period of seven days. The water was then removed and replaced with fresh water, and left for a further period of seven days. The composition of this water is given in Table 10.

The washing of cements *A* and *D*, Table 10, had no effect in reducing the caustic alkalinity, but cement *C* was considerably improved. Cements *B* and *E* were not tested but, due to their very high alkali content, are included in Table 9 as a matter of interest.

Although the soda cannot be washed out of the cement itself, it can be leached out of the hardened cement mortar or removed by washing the cement for longer than the normal period of final set. L. Forsén[25] mentions that the alkali appears as free hydroxide as soon as the gypsum or any other calcium salt is consumed by the aluminates.

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[25] "The Chemistry of Retarders and Accelerators," by L. Forsén, *Proceedings of the Symposium on the Chemistry of Cements*, Stockholm, 1938, p. 301.

The writer suggests that these differences between the behavior of the sodium and potassium hydroxides in unhardened and hardened cement may explain the lack of coordination between the author's results and the Merriman test.

TABLE 10.—CHEMICAL COMPOSITION OF TAP WATER AFTER CONTACT WITH CEMENT MORTAR FOR SEVEN DAYS (AVERAGE OF TWO SUCCESSIVE PERIODS)

In Parts per Million

Cement	Prior treatment of cement	Age, in weeks[a]	Cl	CO_3	OH	SO_4	SiO_2	Ca	Mg	Na	K	NaOH[b]	Remarks
A	None	11	9	24	33	51	58	55	Trace	11	33	84	Palatable water[c]
A	Washed	11	9	25	44	16	82	53	Trace	11	33	93	Palatable water[c]
C	None	4	8	39	86	35	105	37	Trace	26	166	223	Objectionable taste[d]
C	Washed	4	8	21	47	62	78	42	Trace	26	70	109	Palatable water[e]
D[f]	None	4	..	..	..	..	...	..	....	..	..	329	Unpalatable water
D[f]	Washed[g]	4	..	..	..	..	...	..	....	..	..	340	Unpalatable water[h]
Tap water			13	10[i]	..	..	...	3	2	7	2	....	....

[a] Age of lining at test. [b] Caustic alkalinity computed as sodium hydroxide. [c] Caustic alkalinity due largely to calcium hydroxide. [d] Caustic alkalinity due largely to potassium hydroxide. [e] Potassium hydroxide reduced 58%. [f] Chemical composition not determined. [g] Washed for 1.5 hr. [h] No reduction of caustic alkalinity. [i] As HCO_3.

The results of the author's tests with finely ground siliceous magnesian limestone No. 28039 suggest that the use of this material as an admixture (apparently by inducing a rapid reaction with the alkalis in the cement) may prevent any subsequent reaction with the chemically active material in the aggregate.

Acknowledgment is made to D. Avery and V. G. Anderson, industrial, research and analytical chemists, for the chemical analyses.

ROBERT A. KINZIE, JR.,[26] ESQ. (by letter).—The results of Mr. Stanton's work, and his interpretation of the results, were both interesting. Since the test method was new, the writer was mainly interested in the problems of reproducing the results, and to this end he secured aggregates from the same source.

The work was arranged so that the effect of the sand, method of curing, and the alkali in the cement could be determined. Mr. Stanton's tests showed that a portland-puzzolan cement which he tested had a much smaller expansion than would be predicted from its alkali content. This cement was included in the writer's series.

The bars in this series of tests were made according to the method given by Mr. Stanton; that is, they were cured in four ways:

(1) Wet............Immersed in water.
(2) Moist...........Cured in sealed cans, over but not touching a layer of water on the bottom of the can.

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[26] Supt., Santa Cruz Portland Cement Co., Davenport, Calif.

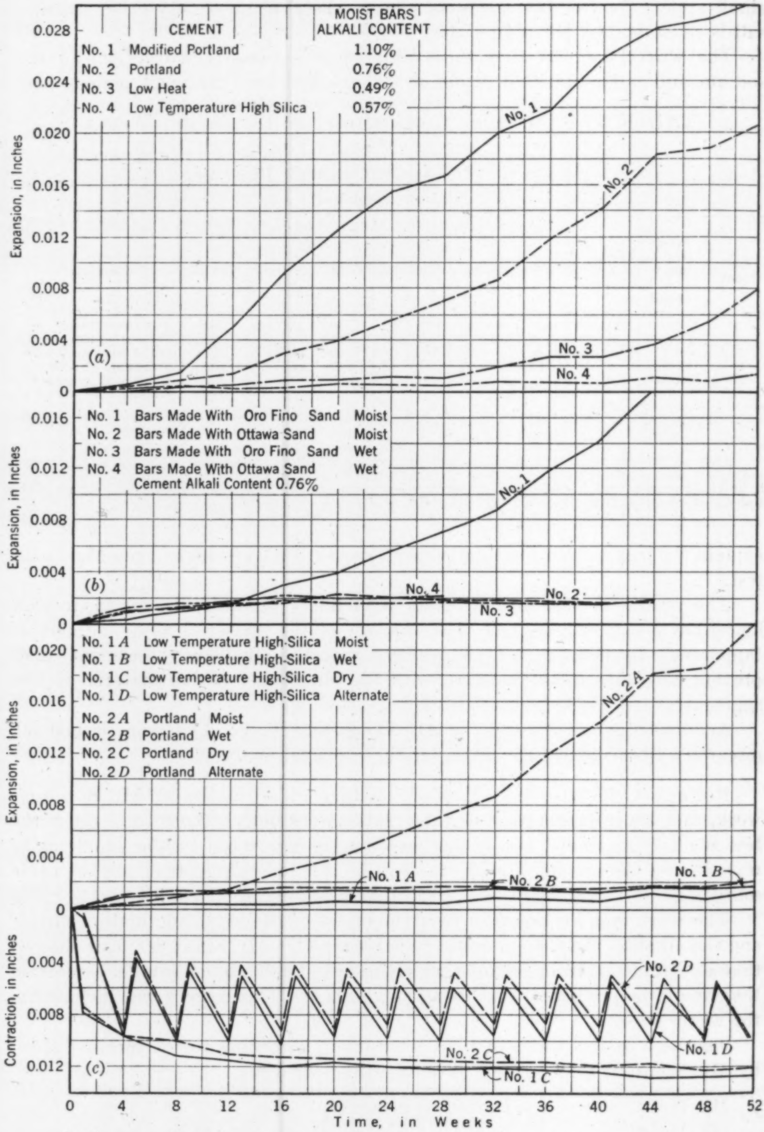

FIG. 15.—REPRODUCTION OF THE STANTON TESTS (ORO FINO SAND, GROUP I, NO. 4)

(3) Alternate........One week immersed in water and then dried in air at 70° F for three weeks. The relative humidity was not closely controlled, but was about 60%.

(4) Dry............Kept in air at 70° F and a relative humidity of about 60%.

Portland cements, made to Government specifications, were used, with the exception of the low-temperature high-silica cement, which was a portland-puzzolan cement. The results of the writer's tests may be summarized as follows:

A test on bars made with Oro Fino sand (Group I, No. 4) was designed to show the effect of alkali in the cement in relation to this sand. Therefore, three cements of different alkali content were selected.

The portland-puzzolan cement was included to check Mr. Stanton's results. The curves in Fig. 15(*a*) show that with "moist curing" the expansion of the mortar made with Oro Fino sand increased with increased alkali content of the cement. The portland-puzzolan cement had a much lower expansion than would be expected from its alkali content.

The curves in Fig. 15(*b*) were drawn to show that the expansion is caused by a combination of "moist curing" and Oro Fino sand. The cement is the same as cement No. 2 in Fig. 15(*a*). With "wet curing" there was an initial expansion of some magnitude in all cases and then a much slower change. For both types of curing the Ottawa sand bars behaved normally.

The curves in Fig. 15(*c*) were included as a further check on the results of Mr. Stanton. The cements were low-temperature high-silica, the same as cement No. 4, Fig. 15(*a*), and the portland cement was the same as cement No. 2, Fig. 15(*a*). The bars were cured in four ways to determine whether the method of curing was the main factor in the expansion and to check further the behavior of the low-temperature high-silica cement. All of the bars were made with Oro Fino sand.

This series of tests showed that the method of testing developed by Mr. Stanton's laboratory will give results that can be checked by other laboratories.

THOMAS E. STANTON,[27] M. AM. SOC. C. E. (by letter).—Considerable "water has gone over the dam" since this paper was first published in December, 1940. The presentation and data as of that date were based on laboratory tests and field studies conducted during a period of approximately eighteen months, an entirely too short time for the development of much of the data that have subsequently come to light (as of August, 1941). The subsequent developments, however, do not change the original conclusions in any material respect, but have served, rather, to confirm them and, in addition, indicate the possible answer to several as yet unsolved or at least only partly answered problems.

In closing the discussion on this important topic, the writer desires to express his appreciation to those who have so kindly participated, and likewise to repeat his grateful acknowledgment of the assistance of the members of his staff, without which most of the valuable data presented would have been unavailable. Many valuable suggestions have been made and it is hoped that the information presented in the paper and in the subsequent discussion will be of value in either solving some important problems relative to Portland cement concrete or pointing the way to further studies.

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[27] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

Parker Dam.—Interest in the subject has been accentuated by the findings of the U. S. Bureau of Reclamation that the severe cracking that developed in 1940 in parts of the Parker Dam on the Colorado River can be traced to a reaction between the cement and aggregate. The story of the Parker Dam is treated fully in the discussion by Mr. Blanks.[19,20] The Bureau has since started a very extensive and thorough nationwide investigation in cooperation with cement manufacturers.

A. S. T. M. Activities.—P. H. Bates, chairman of Committee C-1 of the American Society for Testing Materials (A. S. T. M.), has appointed a working committee on the "Effects of Alkalies in Portland Cement on the Durability of Concrete," of which Mr. Blanks is chairman and the writer a member.

Further Bradley Pavement Developments.—It will be recalled that the early (1.5 years) failure of the pavement north of Bradley, Monterey County, Calif., was confined to the sections in which the local (Oro Fino) sand had been used, and that the other sections in which Oro Fino coarse aggregate and imported (Coyote) sand had been used had, to the date of the original report, failed to show appreciable distress.

Two years later (spring, 1941) the Oro Fino coarse aggregate-Coyote fine aggregate sections had begun to show some distress, thereby indicating that under conditions similar to the California experience the reactive particles in the coarse aggregate may be expected to cause trouble ultimately, although apparently at a somewhat slower rate, and possibly to an ultimate less degree, than fine aggregate of a similar composition. This conclusion is checked by subsequent laboratory tests on coarse and fine aggregate containing reactive particles.

TABLE 11.—MINERAL COMPOSITION AND REACTION CHARACTERISTICS OF DIFFERENT MINERALS SIMILAR TO THE CONSTITUENTS OF ROCK NO. 28039

Mineral	Approximate percentage in rock No. 28039	Percentage in bar	Age in months	Expansion units
Chalcedony	15	10	12	60
Opal Type 1	12	2.5	6	9,920
Opal Type 2	1	10.0	10	11,490
Limonite	1	5	10	30
Glauconite	1	5	6	10
Dolomite	10	10	6	10
Calcite	60	10	12	50
Combined[a]	100	10	12	7,470

[a] Sample including all minerals.

Reactive Properties of Opal.—The probable rôle of the hydrous silicates was fully discussed in the paper. It was pointed out that a sodium silicate gel was at the base of most, if not all, of the popouts and was also found throughout broken sections of concrete as well as exuding on the surface through pores and cracks.

However, in the early tests, specimens containing 10% of either opal or the highly opalized chert No. 28038 had failed to develop excessive expansion, whereas excess expansion was invariably encountered with the same percentage of the siliceous magnesian limestone No. 28039. This experience led to the effort to develop a formula based on a possible reaction between magnesium carbonate and sodium hydroxide.

The next step was to make chemical and petrographic analyses of the siliceous magnesian limestone No. 28039 to determine its mineralogical composition and then to make up expansion bars of each mineral component singly and in combination in the relative percentages found in nature. Table 11 shows the results of this study.

Although the combined synthetic sample was made up of the relative percentages found in No. 28039 and 10% of this combination added to a neutral sand, the tests on each mineral separately were not at the relative percentages found in the original rock but in some cases in higher percentages, the main purpose of the tests being to reveal the probable reaction characteristics of each mineral when present in a sufficiently large percentage.

At the same time, however, excessive expansion has been observed even with much smaller percentages: 1% of Type 2 opal expanded 1,310 units in eight months and 5% of the same mineral expanded 10,250 units, or 1.025%, in the same period of time. From this study it was determined that in the California experience the opal constituent was at the base of most, if not all, of the trouble, and, therefore, the original study of the rôle of soluble silica in the form of opal was resumed.

It was then found that the reason little, if any, reaction was originally developed with the highly opalized material was that the mixture had been overdosed. If the entire coarse or fine aggregate is composed of opaline particles, there is no undue expansion, but when present in what might be called a "pessimum" amount, which appears to be substantially less than 10% or 15% of the total aggregate, a high expansion may be expected in the presence of a high-alkali cement. Table 12 shows the expansion results with a number of rocks, all of which contain opal in varying amounts.

TABLE 12.—Relative Degree of Expansion Induced by 2½%, 5%, and 10% of Different Reactive Rocks Added to a Neutral Sand-High-Alkali Cement Mortar

(1-In. by 1-In. by 10-In. Mortar Specimens; Mix 1 : 2; Age at Time of Measurement, Six Months)

Percentage added to a neutral sand	Rock No.									
	19374	22100[a]	23806	28038	28039-A	28039-B	28045	28056	28057	Opal[b]
	Expansion in Millionths of an Inch per Inch									
2.5	+220	+12,580	+6,310	+7,470	+3,850	+2,910	+320	+11,040	+4,850	+9,920
5.0	+1,150	....	+5,520	+5,460	+5,390	+4,160	+730	+8,490	+4,460	+8,820
10.0	+180	+110	+3,650	+210	+8,420	+7,470	+260	+130	+1,910	+1,740

[a] Only a small quantity of this rock available at time of fabrication of specimens. Analysis shows it to be similar to No. 28038. [b] Opal from dredger dumps on American River near Auburn, Calif.

In the case of the California aggregates the quantity of deleterious particles required for maximum expansion appears to be related both to the opal content and to the type of opal or hydrous silicate. There is definite evidence that opal with high water content is more soluble and reacts quicker than opal with low water content. Much work needs to be done along this line, however.

Further attention is called to the low percentage of opal required for maximum results, as well as the remarkably low percentage (in some cases less than 0.5%) which produces excessive expansion in a relatively short time (Table 13). This would appear to be conclusive evidence that, when expansion does not occur in the field until after a year or more, the percentage of deleterious material may be as low as 0.75% and possibly even less.

Test results available by July, 1940, led to the conclusion that, since little, if any, measurable expansion was observed with deleterious particles below 80-mesh or 100-mesh size, the reason for the falling off in expansion as the percentage of such particles was increased beyond a certain percentage might be an accelerated reaction between the cement and finely dispersed mineral particles.

TABLE 13.—SHOWING EXPANSION EFFECT OF DIFFERENT PERCENTAGES OF TWO HIGHLY REACTIVE ROCKS[a]

(High-Alkali Cement *GS* Used Throughout; 1-In. by 1-In. by 10-In. Mortar Specimens; Mix 1 : 2; Age, Four Months)

Material	PERCENTAGE OF SILICEOUS MATERIAL ADDED TO A NEUTRAL SAND									
	0.25%	0.5%	0.75%	1%	2%	3%	4%	5%	10%	15%
	Expansion in Millionths of an Inch per Inch									
Opal[b]	+70	+1,010	+3,400	+5,660	+9,400	+9,950	+7,750	+4,230	+390	+10
No. 28038[c]	+70	+ 110	+1,790	+2,380	+2,880	+5,080	+3,080	+ 240	+210	....

[a] Rock particles substituted in percentages shown for an equivalent percentage of neutral Russian River sand. [b] Opal secured from dredger dump on the American River near Sacramento, Calif. [c] Rock No. 28038 has been identified as one of the constituents of Oro Fino sand and was secured from ledge deposits found in Monterey County.

Although the underlying cause may be that originally suspected, later tests show that neutralization also occurs in the presence of excess percentages of coarser particles from which all finer particles have been removed. From this it would appear that, for maximum reaction, there is a relation between the amount of reactive mineral in any unit area and the alkali in the cement, and that, when the area of such a mineral available for reaction with a given amount of alkali exceeds a certain amount, the magnitude of the reaction products at the locus of each particle may be insufficient to cause excessive expansion and rupture.

The power of the disruptive action is well illustrated in Fig. 16. In this case a mortar bar, 1 in. by 1 in. by 10 in., was fabricated from a high-alkali cement and an all limestone fine aggregate to determine the reaction, if any, between limestone and the cement. All specimens in this series showed no expansion except one which developed the cracks shown in Fig. 16. Upon breaking this specimen along the line of the transverse crack, it was found that a small piece of reactive mineral had inadvertently been included, and that, although this foreign mineral particle was only about 1.0% of the cross section of the 1-in. by 1-in. bar, the reaction had sufficient force to cause rupture.

Type of Mineral Aggregate.—In so far as the early reactions are concerned, the source of the California troubles was definitely defined as resulting from some mineral in the 5% to 15% shale and chert portion of the aggregate, the bulk of the remaining 85% to 95% consisting of unreactive granitic rock, sandstone, quartzite, rhyolite, andesite, siltstone, and limestone.

The subsequent identification of the trouble with certain chert aggregates led to a suspicion in some quarters against all types of chert. It now appears probable that only those cherts which contain soluble silica of the opaline type are definitely reactive and that other cherts may be absolved of any suspicion in so far as any present evidence based on California experience is concerned. At the same time it has not yet been proved that all opaline cherts are equally reactive. It may be that only the highly soluble types are to be feared and then only when the percentage of solubility exceeds a certain amount. There is ample field for further investigation along this line.

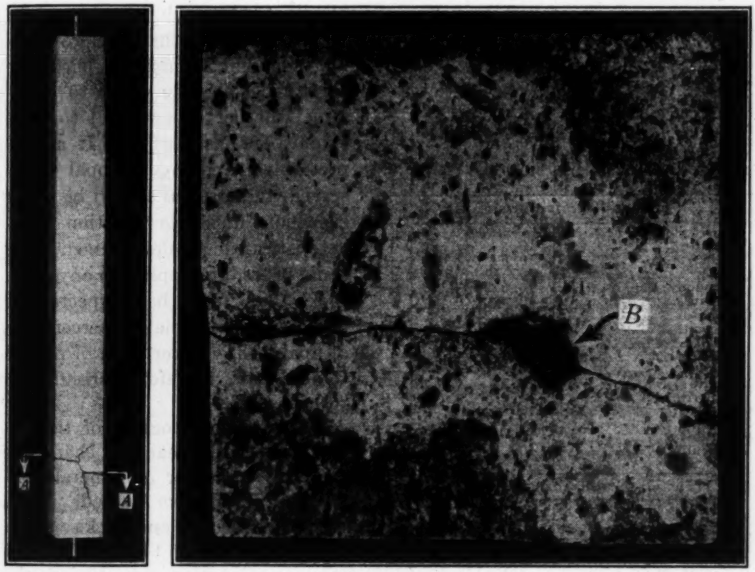

FIG. 16.—RUPTURING EFFECT OF A SMALL PARTICLE OF REACTIVE MINERAL ACCIDENTALLY INCLUDED IN A NEUTRAL LIMESTONE FINE AGGREGATE, HIGH-ALKALI CEMENT MORTAR (SECTION *A–A* SHOWS REACTIVE PARTICLE INCLUSION AT *B*)

Mr. Twenhoffel states[28] that "Opaline or amorphous silica is wanting or extremely rare in Mesozoic and older flints and cherts, but may be found in those of the Tertiary and later.”

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[28] "Treatise on Sedimentation," by William Twenhoffel, 1932, p. 521.

TABLE 14.—REACTIVE AND NON-REACTIVE MATERIALS

Identification	Percentages [a]	Fluorescence
(*a*) REACTIVE MINERALS		
Opal	25 to ¼, 2½ [b]	–
Selenite [c] (gypsum)	10	+
(*b*) REACTIVE ROCKS		
No. 19374 (Table 4)	10, 5, 2½ [b]	–
No. 28038 (Table 4)	10, 5, 2½ [b]	–
No. 28039 (Table 4)	100 to 0.1, 20 [b]	–
No. 28045 (Table 4)	10,[b] 5, 2½	–
No. 28046 (Table 4)	10,[b] 5, 2½	–
No. 30704 (Table 4)	25	–
No. 22100 (opaline chert)	10, 2½ [b]	+
No. 23806 (opaline chert)	10, 5,[b] 2½	–
No. 28056 (opaline chert)	10, 5, 2½ [b]	–
(*c*) NON-REACTIVE MINERALS		
Aragonite	10	+
Biotite	10, 5	–
Calcite	100, 10	+
Chalcedony	10, 5, 2½	+
Dolomite	10, 5, 2½	–
Gehlenite	10, 5, 2½	–
Glauconite	10, 5, 2½	–
Limonite	5, 2½	–
Magnesite	5	+
Natrolite	10	–
Pectolite	10	+
Prochlorite	10	–
Quartz	100	–
Sericite	10	–
Stellerite	100	–
Vermiculite	10	–
(*d*) NON-REACTIVE ROCKS		
Limestone	100	–
Quartz monzonite	100	–
Hornblende diorite	100	–
Amphibole schist	100	–
No. 19375, Table 4	10, 5, 2½	+
No. 22001, soft shale	10	–
No. 28042, soft shale	10, 5, 2½	–
No. 28044, soft shale	10, 5, 2½	–
No. 28053, sandstone	10	–
No. 28054, sandstone	10	–
No. 28055, sandstone	10	–
Serpentine	10, 5, 2½, 1¼	–

[a] Percentage substituted for an equivalent percentage of neutral sand.

[b] This percentage was found to be the most reactive.

[c] Expansion not believed to be due to cement-alkali reaction.

Any type of aggregate that contains opal may be reactive. However, due to the prevalence of opaline cherts and cherty shales in the Tertiary formations west of the Rocky Mountains, the West is more likely to experience this difficulty than the East, so far as cherts in general are concerned.

It is evident, therefore, that all cherts, as such, should not be blacklisted, but that, before passing judgment against any particular chert, it is first in order to determine whether or not, and to what extent, opal is present. If an appreciable percentage of opal is observed, the mineral should be under suspicion until its reaction characteristics can be determined. The same procedure applies to any other rock that may be suspected of containing appreciable percentages of opal or any other form of readily soluble and, therefore, presumably reactive silica.

During the progress of this investigation the reaction characteristics of a number of minerals and rocks at 70° F were studied. A list of these minerals and rocks will be found in Table 14. Tests were made on mortar bars using the high-alkali cement, *GS*, combined with the percentages of mineral indicated in figures after the name. The material under investigation was added in the indicated percentages to a neutral sand.

The reactivity was determined by measuring the expansion characteristics of mortar bars of high-alkali cement, non-reactive sand, and variable percentages of materials under investigation. Test specimens were stored and tested at 70° F. As will be noted, chalcedony, as well as a number of other minerals of the type tested, is not reactive, at least in a short period of time.

All fluorescence observations on minerals in Table 14 apply strictly to samples tested with equipment in the laboratory of the California Division of Highways. Other specimens of the same mineral species from different localities may show different fluorescence results. In all of the California series, storage and tests were at a normal laboratory temperature of approximately 70° F. Prof. Roy F. Carlson and others have been able to get expansion at 110° F and 150° F not obtainable at 70° F.

TABLE 15.—EXPANSION OR SHRINKAGE OF CONCRETE SPECIMENS OF ORO FINO AND COYOTE AGGREGATES CURED IN WATER, IN SEALED CONTAINERS, AND IN AIR

(Age, Twenty-one Months; Expansion in Millionths of an Inch per Inch)

Description	CEMENT							
	GS; 1.14% Alkali				*AS*; 0.45% Alkali			
	1	2	3	4	1	2	3	4
AGGREGATE:								
Fine	Coyote	Oro Fino	Coyote	Oro Fino	Coyote	Oro Fino	Coyote	Oro Fino
Coarse	Coyote	Oro Fino	Oro Fino	Coyote	Coyote	Oro Fino	Oro Fino	Coyote
6-IN. BY 6-IN. BY 34-IN. CONCRETE BEAMS; MAXIMUM SIZE OF COARSE AGGREGATE, 1½ IN.; 1 : 2 : 3.8 MIX								
STORED:								
In water	+100	+4,220	+67	+83	−8	0	−42	−37
Sealed	+80	+1,027	+72	+82	−63	−63	−92	−83
In air	−1,120	−1,027	−803	−923	−1,150	−927	−755	−877
2-IN. BY 2-IN. BY 11-IN. CONCRETE BARS; MAXIMUM SIZE OF COARSE AGGREGATE, ¾ IN.; 1 : 2.5 : 2.5 MIX								
STORED:								
In water	+100	+90	+120	+90	+10	+30	+20	+40
Sealed	+10	+2,320	+730	+2,280	−100	−50	−40	−60
In air	−1,570	−950	−1,130	−1,170	−1,340	−750	−1,100	−1,080

Autoclave Tests on Feldspars and Other Materials.—Inasmuch as several investigators have questioned feldspar as a mineral constituent, some studies were made to determine the effect on various unaltered feldspars and other petrographic types of aggregates when immersed in normal Na_2CO_3 and Na OH solutions after a method similar to that used by E. A. Stevenson.[29] The material tested included: (1) Three different types of opal, (2) Parker Dam andesite, (3) siliceous magnesian limestone No. 28039, (4) orthoclase, (5) microcline, (6) oligoclase, (7) andesine, and (8) labradorite. (Feldspars 4, 5, 6, 7, and 8 were obtained from Ward's Natural Science Establishment, Rochester, N. Y.)

The specimens submerged in the test solution were sealed in an autoclave at a pressure of approximately 300 lb per sq in., temperature 430° F, for six days. Results indicated the ready solubility of the opal and of parts of the siliceous magnesian limestone No. 28039, but the feldspars and the Parker andesite were practically unaffected by this treatment.

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[29] *Journal of Geology*, 1916, Vol. 24, p. 180.

Curing in Sealed Containers Not Necessary with Large Concrete Specimens.—The discovery that excessive expansion of mortar specimens occurred only when cured in sealed containers defined the test procedure requisite to a determination of the reactions between the fine aggregate and the cement. Hundreds of mortar-bar tests conducted during this study have confirmed the necessity for the sealed container or some equivalent method of storage for this size specimen.

Recent developments, however, demonstrate just as conclusively that, in the case of concrete specimens of 6-in. cross section, it is not necessary to cure in sealed containers, and that, in fact, the greatest reaction may be developed when such specimens are constantly immersed in water.

Tables 15 and 16 are illustrative of this development. It will be noted in Table 16 that some of the 6-in. by 12-in. cylinders and 6-in. by 6-in. by 34-in. beams composed of Oro Fino fine aggregate, stored continuously in water, developed excessive expansion and lowered strength in eighteen months, whereas the companion 2-in. by 2-in. by 10-in. concrete specimens, using aggregates from $\frac{3}{4}$-in. and smaller failed to develop expansion when stored in water twenty-one months, but have developed considerable expansion in sealed containers (Table 15). There would appear, therefore, to be a critical size of specimen below which the reaction will not take place unless the specimen is stored in a sealed container or in some other way in which the moisture can be controlled properly.

Concrete Tests.—Under the heading "Concrete Tests," reference was made in the paper to the 6-in. by 6-in. by 34-in. specimens of six-sack concrete, using different combinations of non-reactive and reactive coarse and fine aggregates, the specimens of each mix being cured under three conditions—(*a*) sealed, (*b*) in water, (*c*) exposed to air,—at laboratory temperature (approximately 70° F) and humidity. Similarly, 6-in. by 12-in. cylinder specimens were treated for compression strength tests.

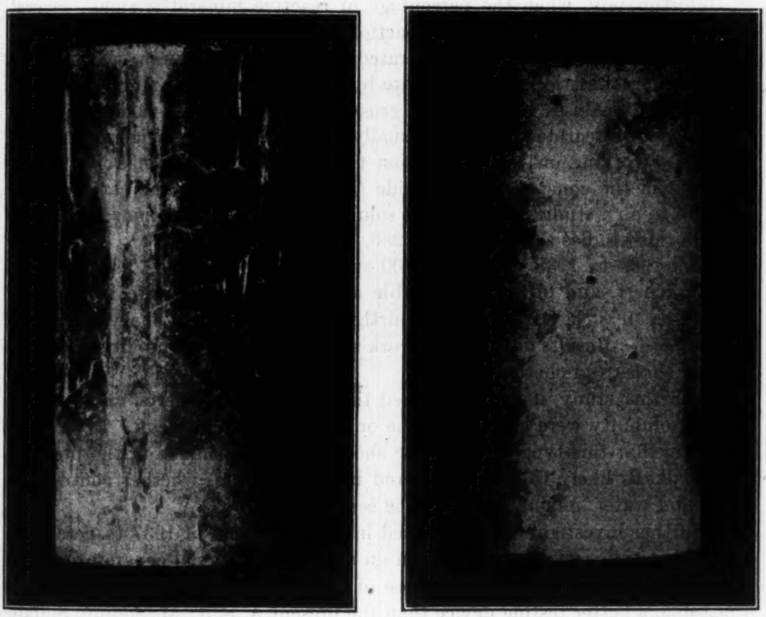
The following mixes were used: (1) Oro Fino fine and coarse aggregate; (2) Oro Fino fine aggregate, Coyote coarse; (3) Coyote fine aggregate, Oro Fino coarse; and (4) Coyote fine and coarse aggregate.

Two sets of specimens were made—one with the high-alkali cement *GS*, and the other with the low-alkali cement *AS*. The test had been in progress too short a time when the report was closed to produce information of value. The writer now reports the situation as of one year later (1941).

The specimens were fabricated in September, 1939. At one year, no appreciable expansion had taken place in any of the specimens, regardless of the method of cure. When next inspected, at eighteen months, a very marked expansion had taken place in the Oro Fino fine and coarse aggregate specimens stored in water, and some expansion in similar mixes covered with a watertight wrapping. Table 15 shows the expansion at twenty-one months for all concrete specimens in this series.

TABLE 16.—Condition of Concrete Beam and Cylinder Specimens After Storage in Water for Eighteen Months

(*a*) 6-In. by 6-In. by 34-In. Beam



(*b*) (*c*)

Observation Period	6-In. by 6-In. by 34-In. Beam		6-In. by 12-In. Cylinders	
	(*a*)		(*b*)	(*c*)
Cement	*GS*	*AS*	*GS*	*AS*
21 months[a]	4,220	0	....	....
28 days[b]	....	....	4,030	4,590
18 months[b]	....	....	2,440	5,800

[a] Expansion in millionths of an inch per inch. [b] Compressive strength, in pounds per square inch.

Attention is called particularly to the fact that little, if any, expansion and no cracking were noted at twelve months, whereas very marked expansion and deterioration took place in some specimens between twelve months and eighteen months, thus paralleling the Bradley job experience in which the same aggregates and cement were used, and where excessive expansion became apparent between twelve and eighteen months after construction.

Tests to Determine Aggregate Characteristics.—The only known suitable tests to determine the reactive characteristics of aggregates conclusively have been long-time expansion tests of mortar and concrete specimens, although petrographic analyses are of valuable assistance in furnishing rapid advance information relative to the presence of mineral inclusions that may be reactive.

Unfortunately, when the percentage of reactive mineral is small, several years may be required before the reaction becomes apparent. Attention has turned, therefore, to a study of accelerated methods of determining the probable reaction characteristics of an aggregate by chemical analysis and other means.

Chemical analyses of the white crust and efflorescences taken from the surface of disintegrated concrete usually indicate a rather high content of Na_2CO_3. Working on the assumption that the sodium is probably present originally in the concrete as hydroxide (later converted to the carbonate on exposure to air), studies of aggregate solubility in Na OH solutions were made in California studies as far back as 1936.

From 1936 to 1938 more than 500 specimens, representing various types of shale, chert, and other questionable aggregate particles were individually soaked in a 10% Na OH solution, in airtight containers, at ordinary laboratory temperature. Results of this early work showed the ready solubility of certain types of shales and cherts.

On further study, it was discovered that these shales, cherts, and particles of each solubility were high in opaline or hydrous silica content. It was felt, therefore, that this type of aggregate should be objectionable in the presence of any alkalis likely to be encountered in the concrete, whether contributed by ground water, mixing water, or the cement.

A further investigation was started in the early part of 1937 to determine the solvent action of Na OH solution on certain commercial concrete sands of the State of California. The purpose of this later investigation was to find, if possible, a better testing device than the present A. S. T. M. Sodium Sulfate Soundness Test, for those sands known to contain shale and chert of a deleterious nature.

After much experimentation, the method finally selected as showing the most promise consisted of grinding 50 g of each sand to pass the 100-mesh sieve, soaking in a 10% Na OH solution for fifteen hours at a temperature of 210° F, then determining the amount of dissolved carbonate by titration, and assuming the remainder as silicate. Results, however, showed generally as much solvent action on certain sands of apparent durability as on sands known to contain deleterious particles. Therefore, as the results were inconsistent and inconclusive, no further investigation along this line was made until June, 1939.

In June, 1939, after the Bradley pavement failure investigation was started, chemical analyses were made on solutions from treating Oro Fino and Coyote sand, respectively, in (*a*) tap water, (*b*) low-alkali cement leaching water, and (*c*) high-alkali cement leaching water. Again it was noted that the non-reactive Coyote sand was nearly as soluble as the reactive Oro Fino.

When the more recent developments demonstrated that in all probability soluble silica of an opaline nature was "at the root" of the California trouble, a new series of solubility tests was started, using a weaker sodium hydroxide solution and a less severe procedure, on the theory that, as opal may be more soluble than other silica minerals, a procedure might be developed under which the opal could be dissolved more rapidly than other minerals, thereby enabling comparative determinations to be made.

The results of this later study have, to date (September, 1941) been disappointing and it is somewhat doubtful if the sodium hydroxide solubility test is sufficiently selective to be dependable in the case of aggregates containing quite small but at the same time objectionable percentages of reactive minerals. The study is being continued, however, in the hope that a satisfactory and dependable procedure can be developed.

Fluorescence.—Studies have been continued to ascertain, if possible, the nature and significance of the fluorescence observed and described in the paper, in the hope that an effective and reliable aid might be developed for detecting reactive aggregates. The results of this study have been none too encouraging, as the observations are too frequently inconsistent.

Efforts have been made to compare the amount of observed fluorescence of both reactive and non-reactive rock particles in concrete. Results indicate that, although the gel is fluorescent in most cases where expansion occurs through cement-aggregate reactions, there is likewise a noticeable fluorescence in samples where no reaction has taken place. In many cases fluorescent rims are observed surrounding durable aggregates. Many non-reactive mineral grains that fluoresced in the sand sample continue to fluoresce after the sand has been incorporated in concrete cylinders, so that it is difficult to distinguish between the non-reactive and the reactive particles. In addition, not all reactive particles show fluorescence after several months in the sealed curing test.

The observations in connection with this work were made with a standard commercial type of mercury-vapor lamp with built-in filter. Since the ultraviolet radiation characteristics of the many commercial lamps now available undoubtedly vary with the type and manufacturer, and since no other type of lamp was available for use in the experiments, it is not possible to say, at this time, what results would have been obtained from these same samples if different ultraviolet equipment, having different wave-length ranges and different filters, had been used.

It is possible that, for certain gels and reactive rims, as well as for the reactive aggregates themselves, different lamps may show decidedly different fluorescence effects. It is likewise very probable that many types of aggregates now known to be reactive, but which have not shown fluorescence under the mercury-vapor lamp used in the test, may fluoresce under a higher or lower wave length. Additional work needs to be done in this field (see Table 14 for fluorescent reactive and non-reactive materials).

Modulus of Elasticity by Means of Sonic Vibrations.—Considerable progress has been made during recent years in the development of equipment and procedure for determining modulus of elasticity by means of sonic vibrations.[30] Suggestions were made in several quarters that it might be possible to determine the effects of the expansive reaction between cements and aggregates more readily and at an earlier date by means of sonic vibrations than through expansion measurements.

A number of investigators are now studying this phase, including the Research Department of the Portland Cement Association, the Bureau of Reclamation, the Washington State Highway Department, the Riverside Cement Company, and undoubtedly many others, as well as the California Division of Highways.

TABLE 17.—SONIC MODULUS OF ELASTICITY

(1 : 2 Mortar; Specimens, 1-In. by 1-In. by 10-In.; *GS* Cement with Russian River Sand; and Varying Amounts of 30-Mesh to 80-Mesh Opal[a])

Material	Age of Specimen in Weeks[b]									
	1[b]	7[b]	2	3	4	5	6	7	8	10
	Modulus of Elasticity × 10^{-6}									
Russian River sand.	1.48	3.31	3.34	3.36	3.48	3.54	3.63	3.67	3.68	3.69
+2½% opal........	1.59	3.67	3.85	3.79	3.55	3.18	2.39	1.57	1.40	1.25
+5% opal.........	1.47	3.70	3.86	3.75	3.73	3.73	3.74	3.70	3.68	3.12
+10% opal........	1.39	3.36	3.55	3.46	3.43	3.52	3.52	3.52	3.53	3.48
	Expansion in Millionths of an Inch per Inch									
Russian River sand.	....	....	....	....	−60	....	−120	....	−120	−110
+2½% opal........	....	....	....	....	+60	....	+720	....	+1,650	+2,910
+5% opal.........	....	....	....	....	+50	....	+270	....	+520	+1,090
+10% opal........	....	....	....	....	+80	....	+150	....	+190	+220

[a] Opal from dredger dumps near Roseville, Calif.

[b] The first two columns (1 and 7) are in days.

The results of some of the California studies are shown in Table 17. The measurements shown were made first on the specimens immediately upon removal from the molds and thereafter at weekly intervals. Each value is the average of measurements made on four sides of three bars in each set, or a total of twelve determinations.

All of the tests were made on specimens without end gage plugs. A duplicate set of specimens was made with end plugs for expansion determinations. Modulus tests on the bars with end plugs were somewhat erratic and therefore are not shown in this table.

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[30] "Measuring Young's Modulus of Elasticity by Means of Sonic Vibrations," by T. C. Powers, *Proceedings*, A. S. T. M., Vol. 38, Pt. II, 1938, p. 460.

The control specimens were made from a neutral sand and high-alkali cement. These control specimens have shown a steady increase in modulus, whereas all specimens containing variable added percentages of reactive mineral in the form of opal reached a peak modulus in not more than three weeks and then a retrogression in direct relation to the expansion characteristics. However, although the modulus decreases as expansion develops, the decrease at early ages is not appreciable except in the case of a high expansion. The expansion measurements are more marked, and it appears doubtful, therefore, if the sonic method will be found more satisfactory than direct expansion measurements.

Also, as most commercial fine aggregates contain substantially less deleterious particles than the synthetic sands reported in Table 17, a longer time before the reaction becomes detectable by the sonic method may be anticipated with reactive commercial sands. Commercial sands have not been tested long enough to produce information of any value.

Correctives.—The study of correctives has been continued with little practical success to date. At first it was thought that if the sodium hydrate could be converted to a chloride the sodium (or potassium) chloride might be harmless. However, such apparently is not the case. Calcium, ferric, and aluminum chloride have all been added to the mixing water, but none have proved effective in reducing the expansion. In fact, in most cases the expansion has been considerably increased.

Other possible correctives have included diatomaceous earth, resins, and sodium alginate but none of these materials has proved sufficiently effective in reducing expansion to warrant their use. Likewise, magnesium carbonate was added to react with the sodium hydroxide, but without success.

The writer's attention was called to work done in which sodium sulfate was added to cement to increase its alkali (sulfate) resistance.[31] However, a 4% addition of anhydrous sodium sulfate had the effect of increasing instead of decreasing the expansion, even with the low-alkali cement *AS*, to such an extent that, although this cement is non-reactive under normal conditions, the expansion in twenty-eight days in the presence of the sodium sulfate was even greater than in the case of the high-alkali cement *GS*.

In this short period the *AS* cement specimens expanded 3,800 units and the *GS* cement specimens 2,640 units per unit. The specimens were of a 1 : 2 mix made with Russian River sand to which had been added 2.5% opal.

Aluminum powder has proved effective in stopping the expansion. When 0.05% by weight of the cement was added, the high-alkali cement bars have shown no expansion up to four months, but the mortar was so fluffed by the expulsion of gas that this method is not considered suitable for construction purposes.

Silica or Puzzolanic Admixtures.—As reported in the paper, the high-silica or puzzolanic type cement HP (Fig. 4) had developed very little expansion in twelve months although the cement contained 0.78% alkali. At the end of twenty-eight months the same specimens show even a slight shrinkage, whereas specimens containing normal cement CL with 0.77% alkali have continued to expand, reaching 3,600 units or 0.36% at twenty-eight months.

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[31] *Industrial and Engineering Chemistry*, Industrial Ed., Vol. 33, No. 5, May, 1941, p. 692.

This fact gave encouragement to the enlargement of this series to include all California commercial puzzolanic type cements and likewise to include a series of blends of the high-alkali cement *GS* to which had been added, or for a part of which had been substituted, a percentage of certain silica admixtures.

These studies have not progressed sufficiently to be considered conclusive, but they have developed one interesting fact: When the siliceous material is added without reducing the cement content, little benefit is noticed. The expansion is reduced effectively only when the siliceous material is substituted for an equivalent amount of cement. This is revealed in Table 18.

TABLE 18.—EFFECT ON CEMENT-AGGREGATE EXPANSION REACTION, OF GROUND[a] MINERAL ADDITIONS TO, AND SUBSTITUTIONS FOR, EQUIVALENT PERCENTAGES OF CEMENT; IN MILLIONTHS OF AN INCH PER INCH

(Sand, 90% Russian River Plus 10% No. 28039-B; 1 : 2 Mortar Specimens)

Type of admixture[a]	Age, in months	PERCENTAGE OF SILICEOUS MATERIAL:							
		0	5	10	15	20	30	40	50
(*a*) SILICEOUS MATERIAL ADDED TO CEMENT									
Monterey shale	4	+5,370	+6,670	+6,070	+4,980	+4,050	+1,240	....	....
Pumicite	4	+4,900	+5,380	+5,930	+4,920	+4,580	+3,540	....	....
Opalite[b]	4	+5,640	+5,940	+6,250	+5,970	+6,290	+6,430	....	....
No. 28039-A	4	+3,430	+3,980	+3,800	+3,630	+3,360	+2,410	....	....
(*b*) SILICEOUS MATERIAL SUBSTITUTED FOR CEMENT									
Monterey shale	4	+3,860	....	+1,140	....	+30	−10	−40	−120
Pumicite	4	+4,650	....	+3,250	....	+1,180	+140	+100	+40
No. 28039-A	4	+5,070	....	+4,740	....	+2,610	+140	+50	+30
Opalite[b]	4	+4,560	....	+4,150	....	....	+2,710	....	....
Ground Ottawa sand	4	+4,550	....	+4,660	....	+4,280	+2,340	+1,030	+150
Opal[c]	4	+4,550	....	+410	....	+20	−50	....	....
No. 28038	4	+4,560	....	+180	....	+60	+10	....	....

[a] All mineral additions ground to pass 325-mesh. [b] From Arkansas reported to be a disintegrated chert of nearly pure opaline silica. [c] From dredger dump near Roseville, Calif.

Comments on Discussions.—Having brought all test data up to date, it is possible to comment on the discussions of the original paper.

Professor Carlson propounded several questions. The answers to some of these are at least partly known and some are definitely in the process of determination. The solution of others may be long deferred but offer a definite goal for the interested researcher.

The trouble is not due entirely, if at all, to the formation of an alkali carbonate but undoubtedly also in major part, if not entirely, to the formation of an alkali silicate. No further evidence has been developed either to sustain or to disprove the original suggestion that an alkali carbonate might be one of the contributing factors.

Such tests as are available indicate that the alkalis of sodium and potassium are equally objectionable, but this phase is to be explored thoroughly as a part of the program instigated and being conducted by the Reclamation Bureau.

Although it may be that a combination of moderate-alkali cements with susceptible aggregates may produce expansion in correspondingly longer times, the results of at least twenty-four-month tests with high percentages of No. 28039, and shorter periods in the case of other reactive minerals, indicate that this is highly improbable, at least for any commercial aggregates studied where the percentage of offending mineral is as low as that present in the California aggregates.

It was found, however, that, if a high percentage of a relatively low-alkali cement is used with a highly reactive aggregate, expansion may occur, probably due to the presence of a high total alkali content. In illustration, cement *AS* with 0.45% alkali, which showed no expansion in eighteen months in the 1 : 3 and 1 : 2 mortars, shows a definite expansion (2,210 units) in eighteen months in the 1 : 1 mix with no expansion of the same mix at eight months.

Tests up to thirty-three months do not indicate that certain combinations of high-alkali cement and non-reactive aggregate will ever cause trouble, although it is admitted that longer tests may upset this conclusion.

There is no answer at present to the inquiry as to the possibility of the formation of alkali carbonate through the carbonation of the alkalis in aggregates, especially those like crushed feldspar containing a considerable amount of alkali; nor to the last two questions raised by Mr. Carlson. This is a fertile field for study.

Mr. Woods has well stated the problems facing the cement manufacturers in reducing the alkalis in cement. It would undoubtedly be an ideal solution if some inexpensive addition to the cement to circumvent the expansive reaction could be found. Such a solution may be found in the nature of a silica admixture that will not only nullify the expansive reaction, but likewise result in a less expensive product. In this respect the data shown in Table 18 are of interest.

Mr. Stadtfeld makes a plea for placing restrictions on water-soluble alkalis rather than on total alkalis and a further plea for the Merriman free alkali test and a limit of 3.5% free alkali under this test. As stated in the paper, under the heading "Merriman Alkalinity and Free Alkali Test," it was found that in the case of the California cements this test is not always consistent with results and, therefore, is not a suitable test as a substitute for total alkalis as determined by chemical analyses. This phase of the subject was discussed with Mr. Merriman when he visited the Laboratory of the California Division of Highways in July, 1939, just prior to his untimely death. After leaving Sacramento, Mr. Merriman wrote the writer as follows:

> "Just a word to say how much I enjoyed my visit of Saturday and how instructive it was. You are on the track and close to the goal of demonstrating that all cements do not always act alike. To me it was inspiring

to see what you have done. Far too few ever look beyond the obvious because they are blind even before they start.

"The literature is full of results attained by the use of unknown materials which are assumed to act alike under all conditions and in all locations. Their history and origin are never known! Is that engineering?"

It may be that, as Mr. Stadtfeld states, the Merriman alkalinity and free alkali tests are suitable quality tests under average conditions, but it is evident that they do not fully satisfy the situation covered by this paper and, on the other hand, may be too severe for universal application.

It is possible that, under certain circumstances, the destructive action taking place in concrete may be one of base exchange involving the calcium in solution, the alkali, alumina, and the silica, somewhat in the manner suggested by Mr. Tremper. If this should be the case, the alkali would be replaced and liberated by the calcium and then could combine with the silica.

Search of available literature yielded no information regarding the expansion, if any, which occurs when the reaction $Na_2OH + Si\,O_2$ takes place. Probably the lack of data is due to the large number of possible combinations of alkali and silica, the majority of which are noncrystalline or colloidal.

Although it is true that large amounts of calcium hydroxide result from the hydration of cement, only a small part of it is in solution and available for chemical reaction.

The solubility of calcium hydroxide at several temperatures is as follows:

1 g of calcium hydroxide is soluble in—

(cc of water)	(Degrees centigrade)
540	0
630	25
1,300	100

As there is not enough water present in concrete to dissolve all of the calcium hydroxide formed, it is not necessary that sufficient sodium carbonate be present to combine with all of the lime hydrate to form the carbonate before the sodium carbonate can crystallize as stated in Mr. Tremper's discussion.

It must be understood that the possibility of the reactions—magnesium carbonate plus sodium hydroxide yielding magnesium hydroxide plus sodium carbonate ($Mg\,CO_3 + 2\,Na\,OH \rightarrow Mg(OH)_2 + Na_2CO_3$); sodium hydroxide plus silica ($Na_2O + Si\,O_2$); or any other chemical activity—is visualized as taking place only at the points of contact of the reactive aggregate and the sodium hydroxide from the cement and not throughout the entire mass of the concrete. This localizes the reaction and further reduces the amount of calcium hydroxide available for the calcium carbonate reaction to which reference has been made.

Mr. Tremper's suggestion that the Na OH acts in a roving commission rather than becoming fixed as a silicate is interesting and a possible situation, but would be more probable if the $Ca(OH)_2$ were more soluble or if there were more free water available.

Mr. Hanna, in his comments, differentiates between the sodium and potassium hydroxide. It was found, in a limited series of tests in which Na OH and K OH were added to the mixing water, that the addition of either accelerated and increased the expansion of the specimens up to a certain percentage addition. Additions above this amount retarded and decreased the action. The writer's work was done on a synthetic sand, made by adding No. 28039 to a neutral sand. Data published by the Bureau of Reclamation show that an increase of Na OH beyond the amount added by the writer accelerates the action with Parker andesite. It is probable that there is a different "pessimum" amount of alkali for various minerals, just as there is a different "pessimum" amount of different minerals for a given alkali content.

The same study indicated that there is very little difference in the effect of the K OH and the Na OH when they are used in equimolar amounts. The molecular weight of sodium hydroxide is 40, whereas that of potassium hydroxide is 56. If the bases are used in that proportion, or equimolar, they react very similarly.

Mr. Blanks lists many materials which he states are known to be reactive. It should not be concluded that all materials of the types listed, such as andesite, rhyolite, felsite, and granite are, of necessity, reactive in all cases, or even at any time reactive just because they are of the types noted, but rather that when reactive it may be because they contain a sufficient percentage of reactive soluble silica of the opaline type.

It is unfortunate if the paper was so worded as to justify the inference of Mr. Blanks that only the sand in the California aggregates carried alkali-reactive materials, as such, of course, is not the case. The rôle of the coarse aggregates has been fully covered in the earlier part of this discussion and need not be repeated. Mr. Blanks has submitted a very valuable contribution to the discussion on this subject.

It is gratifying to note the extent to which Mr. Kinzie, reproduces the results secured in the California studies described in this paper. As previously stated, it is apparent that one highly potential corrective procedure is through the use of suitable puzzolanic admixtures, and this phase merits thorough exploration.

Summary.—In summarizing developments since the paper was published in *Proceedings*, it is interesting to note the extent to which different investigators are now tracing some of their hitherto unexplained concrete failures to a reaction between alkali in the cement and the aggregate.

There is still considerable to be learned relative to the mineral ingredient or ingredients at the base of the trouble, but in the California case, at least, the evidence points conclusively to the opaline types of rocks as contributing at least a major part. Experiences of others, however, indicate rather definitely that opal may not be the only reactive mineral. It is possible that any form of silica that is readily soluble in Na OH may in time contribute to undue expansion, although the writer's tests to date, at normal temperature, do not indicate any excessive expansion in the case of non-opaline rocks.

The maximum amount of alkali that can be tolerated is still a matter for future determination. The California tests indicate that with the California aggregates and California standard cements little concern need be felt if the alkali content is kept below 0.6%, but whether this is universally true, or even locally over a greater period of years than covered by the present series of tests, can only be determined by future developments. At the same time, however, there is considerable ground for hope that some corrective will be found that will afford ample protection in any doubtful cases.

The nationwide investigations now being made by the Bureau of Reclamation will be a most valuable contribution to the study and it is hoped that the Bureau will make the data collected available to the profession at large.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2130

AN INVESTIGATION OF STEEL RIGID FRAMES

BY INGE LYSE,[1] M. AM. SOC. C. E., AND W. E. BLACK,[2]
JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. J. POSEY, W. J. ENEY, LAMOTTE GROVER, WILLIAM R. OSGOOD, M. HIRSCHTHAL, JAROSLAV J. POLIVKA, A. C. BARROW, AND W. E. BLACK.

SYNOPSIS

Tests on two riveted-steel, rigid-frame models (scale, 4 to 1) are described in this paper. In one frame, the knee sections were approximately square, having a sharp reentrant angle at the inner corner. The other frame had a large circular fillet at the inside corner of the knee. The frames were tested chiefly as two-hinged structures under working loads.

In general, the structural behavior of the two rigid frames was in accordance with conventional theory. At the knees of both frames, however, the normal stress distribution departed markedly from the usual straight-line relationship. In the square knee, a concentration of stress existed at the inner corner but was found to be of minor importance. In the curved knee, compressive stresses in the flange of the curved fillet were considerably greater than those computed by either the straight-beam or the curved-beam theories. Furthermore, a transverse variation of the stress in the outstanding legs of the curved flange angles increased the high compressive stresses in the curved knee. On the basis of the test results, recommendations for the analysis and design of each type of rigid frame have been made and are presented herewith.

INTRODUCTION

During the decade 1925–1935 there was a growing appreciation of the many structural and esthetic advantages of the rigid-frame type of construction, particularly as applied to short-span bridges. However, due to the lack of available information regarding the stress distribution in this type of structure, especially at the knee section, the steel rigid frame has been viewed with some concern by many engineers.

NOTE.—Published in November, 1940, *Proceedings*.

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In order to remedy this situation, the American Institute of Steel Construction (A. I. S. C.) initiated two experimental investigations of the structural behavior of steel rigid frames. One was conducted on rigid-frame knee sections by the National Bureau of Standards at Washington, D. C.;[3,4,5] and the second investigation, which is presented for discussion in this paper, was conducted on complete one-quarter size models, and was made possible by the establishment of a cooperative research fellowship by the A. I. S. C. and Lehigh University at the Fritz Engineering Laboratory, Bethlehem, Pa.

In the investigation conducted at the National Bureau of Standards, the primary purpose was to determine the stress distribution at the knee section. At Lehigh University, two complete rigid-frame models were tested to secure a check upon the stress distribution obtained by the testing of knee specimens only, and also to permit the observation of other important data regarding the frame as a whole. The focal point of interest was the knee section, although such subjects as movement of foundations and accuracy of conventional methods of analysis and design were also studied.

TEST PROGRAM

The investigation was planned with the view of studying the behavior of the frames when hinged at the supports. The test program was laid out with the following objectives in mind:

A. Determination of stress distribution in the knee section (1—principal stresses and maximum shears, 2—normal stresses and shears on arbitrary sections);

B. Effect of the stress distribution in the knee upon the behavior of the frames as a whole (1—on horizontal reaction, 2—on stresses away from the knee, and 3—on deflections);

C. Effect of simulated foundation slippage (1—on horizontal reaction, 2—on stresses away from the knee, and 3—on deflections);

D. Determination of restraint provided by flat-plate base;

E. Comparison between experimental data and calculated results.

TEST SPECIMENS

In order to compare the results of this investigation with similar tests at the National Bureau of Standards,[3,4,5] the shapes of the knee sections of the two model frames were made respectively similar to two of the specimens tested at the Bureau.

The chief point of difference between the two models was the shape of the knee. One, referred to as the square-knee frame, had a sharp reentrant angle of slightly more than 90° at the inside corner of the knee; whereas the other, designated as the curved-knee frame, had a large circular fillet at the inside of the knee. The details of the two frames are shown in Fig. 1. The method

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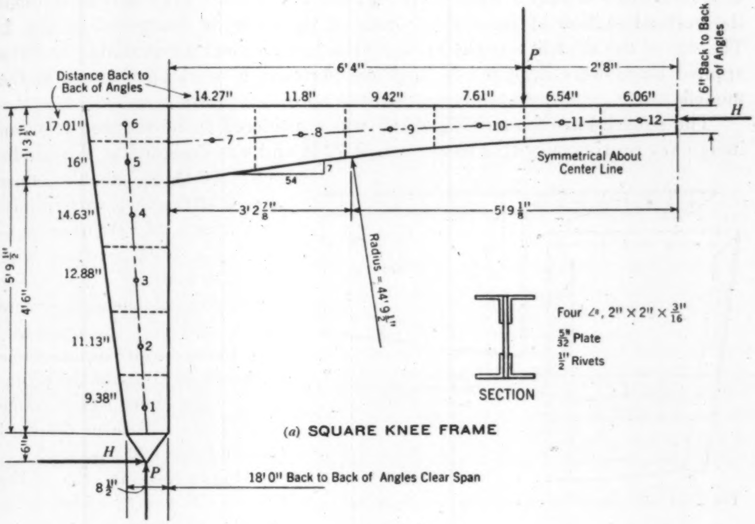
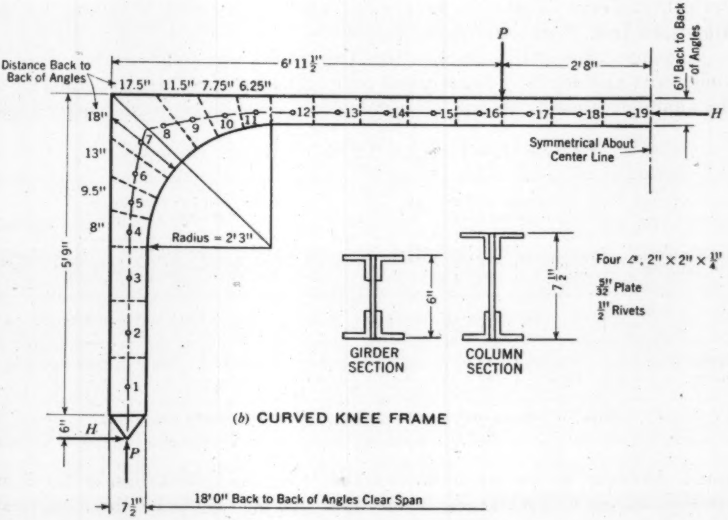
[3] "Strength of a Riveted Steel Rigid Frame Having Straight Flanges," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, M. Am. Soc. C. E., *Research Paper No. 1130*, *Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 269.

[4] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, *Research Paper No. 1161*, *loc. cit.*, p. 853.

[5] "Strength of a Welded Steel Rigid Frame," by Ambrose H. Stang and Martin Greenspan, *Research Paper No. 1224*, *loc. cit.*, Vol. 23, 1939, p. 145.


Distance Back to
Back of Angles
17.01"
16"
14.63"
12.88"
11.13"
9.38"
1' 3 1/2"
5' 9 1/2"
4' 6"
6"
H
P
8 1/2"
14.27"
11.8"
9.42"
7.61"
6.54"
6.06"
6' 4"
2' 8"
P
6" Back to Back of Angles
H
1
2
3
4
5
6
7
8
9
10
11
12
7
54
3' 2 7/8"
5' 9 1/8"
Symmetrical About Center Line
Radius = 44' 9 1/2"
Four ∠s, 2" × 2" × 3/16"
5/32" Plate
1/2" Rivets
SECTION
18' 0" Back to Back of Angles Clear Span

(a) SQUARE KNEE FRAME


Distance Back to
Back of Angles
17.5"
11.5"
7.75"
6.25"
18"
13"
9.5"
8"
5' 9"
6"
H
P
7 1/2"
6' 11 1/2"
2' 8"
P
6" Back to Back of Angles
H
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Symmetrical About Center Line
Radius = 2' 3"
6"
7 1/2"
GIRDER SECTION
COLUMN SECTION
Four ∠s, 2" × 2" × 1/4"
5/32" Plate
1/2" Rivets
18' 0" Back to Back of Angles Clear Span

(b) CURVED KNEE FRAME

FIG. 1.—DETAILS OF MODEL FRAMES (SCALE RATIO, 4 : 1)

of fabricating the square knee, showing that the web does not extend through the vertical section at the inside corner of the knee, is illustrated in Fig. 2. The size of the models was chosen to fit the testing machine available, and the applied loads were designed to produce permissible working stresses in the models.

The square-knee frame (Fig. 1(*a*)) was considered to be reduced from an imaginary prototype with a clear span of 72 ft and was designed as the middle one of three such frames spaced 15 ft apart, with framed floor beams and stringers supporting a 36-ft roadway with an H-20 loading. The linear dimensions of the model were one fourth and the cross-sectional areas approximately one sixteenth of those of the prototype. The curved-knee frame (Fig. 1(*b*)) was later designed for the same conditions.

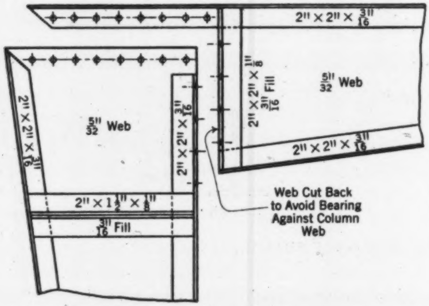


FIG. 2.—FABRICATION DETAILS OF A SQUARE-KNEE FRAME

Due to the small size of the frames, it was considered advisable to have them fabricated at a plant equipped for small-scale iron work rather than at an ordinary structural shop. The square-knee frame was fabricated by the Bethlehem Fence Works, in Bethlehem, and the curved-knee frame by the Allentown Iron Works, in Allentown, Pa.

In general, the workmanship of the two frames was satisfactory. Over-all dimensions and depths of the sections were sufficiently accurate to permit use of the nominal values in all but a few instances. One objectionable feature was found, however, in the square-knee frame. At the intersection of the compression flanges at both knees, where tight bearing should be obtained, small gaps existed. It was considered advisable to fill these gaps with shims that were tackwelded in place, but which produced tight bearing only along the

TABLE 1.—TENSILE TESTS OF STEEL COUPONS

Coupons cut from	STRESSES IN KIPS[a] PER SQUARE INCH			Elongation in 8 in.	STRESSES IN KIPS[a] PER SQUARE INCH			Elongation in 8 in.
	Modulus of elasticity	Yield point	Tensile strength		Modulus of elasticity	Yield point	Tensile strength	
	(*a*) SQUARE-KNEE FRAME				(*b*) CURVED-KNEE FRAME			
Angles..........	28,400	43.1	62.7	24.1	29,370	39.5	59.3	30.0
Plates..........	28,570	45.8	54.4	22.0	29,400	....[b]	63.6	15.7

[a] Kips = "kilo-pounds" = thousand pounds. [b] No definite yield point.

outstanding legs of the girder flange angles. Thus, a loose fit was eliminated at the expense of a concentration of stress at the bearing. Tensile properties of the material in the two frames are given in Table 1.

TEST METHODS AND OBSERVED DATA

Loading Apparatus.—The frames were tested in a 300,000-lb machine having a 21-ft beam which provided an excellent base on which to set the 18-ft models. The load was transferred from the movable head of the testing machine to two fixed load points on the frames by a system of bars and loading beams, as illustrated in Fig. 3, on the square-knee frame.

The horizontal reaction was resisted by a $\frac{3}{4}$-in. round bar extending between the two column bases in all tests. To allow adjustment of the reaction and the span length of the frame, the ends of the tie bar were threaded and fitted with nuts. Rollers under one of the column bases insured that only a negligible amount of friction might affect the horizontal reaction. The tie-bar attachment and rollers are shown in Figs. 3 and 4.

In order to prevent lateral buckling or twisting of the flexible horizontal girder, trussed frames were built up from the testing machine at midspan, and just inside the inner face of each column, as shown in Fig. 3. The frame had a tendency to bear against all of these lateral supports under load; but only at the

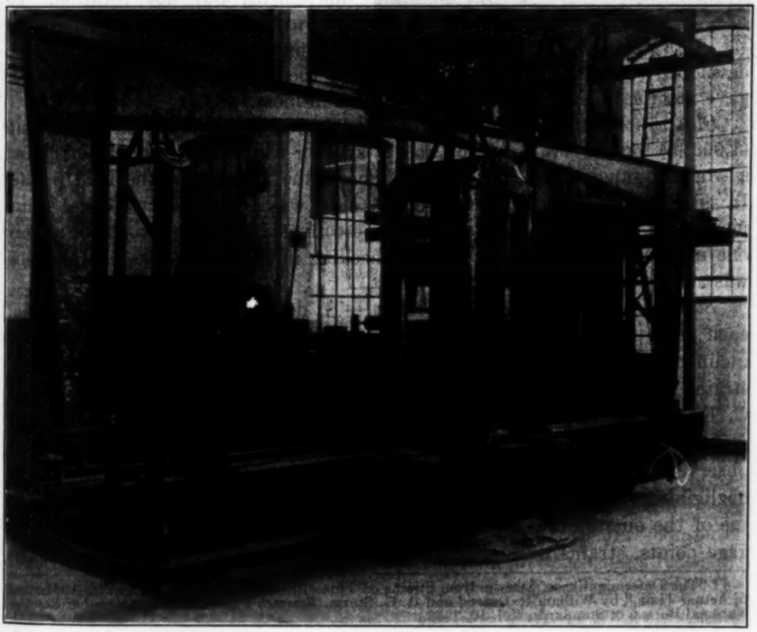


FIG. 3.—SQUARE-KNEE FRAME IN A TESTING MACHINE

center was there any appreciable deflection where frictional resistance to vertical movement might be developed. Comparative tests, with and without the center support, gave practically identical results, so the frictional restraint was considered negligible.

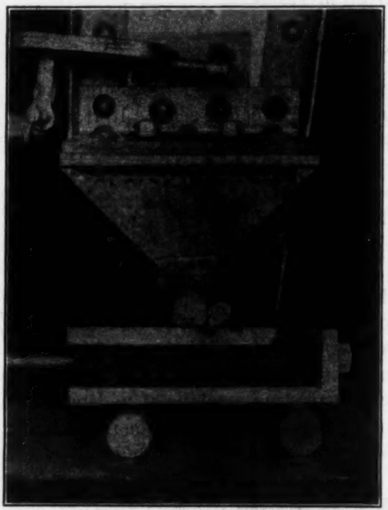


FIG. 4.—MOVABLE HINGED BASE

For the most part the rigid frame specimens were tested as two-hinged frames. Therefore, the apparatus shown in Fig. 4 was developed to provide a point bearing for the frame supports and at the same time transmit the horizontal reaction to the tie bar. The bearing area of the pin was always less than 1 in. wide, which was sufficiently small for the assumption of a point support.

It was also considered worth while to investigate how much restraining moment was produced by the base when the frame rested upon flat bases which restrained horizontal movement, but not rotation. To provide this condition, the two sets of apparatus shown in Fig. 5 were developed (the frame proper is indicated by the shaded areas). Since the two gave identical results, Base No. 2 was used in most tests.

Observations and Test Data; Loading.—In all tests the total load was one kip (1,000 lb) for the zero condition and 13 kips for the working condition, giving a working range of 12 kips. Fig. 1 shows the location of the load points. In some of the early tests, the working load was applied in two equal increments, but for the most part it was found convenient to apply the load in only one increment. For the purpose of checking in the latter case, each loading was always repeated.

Stress Distribution at Knee Sections.—To determine the state of stress at each gage point on the web, three strain readings (horizontal, vertical, and inclined at 45°) were observed. Stresses were obtained from the observed strains by the graphical method developed by W. R. Osgood, M. Am. Soc. C. E., and R. G. Sturm,[6] Assoc. M. Am. Soc. C. E. At each flange gage point, only the strains parallel to the longitudinal dimension of the flange were observed, as the transverse stresses in the flange may generally be considered negligible. Wherever possible, flange strains were observed at both heel and toe of the outstanding legs of the angles and on the edge of the web. At all gage points, strains were observed simultaneously on both sides of the frame in order to eliminate the effect of lateral bending. These strain observations were made with tensometers having 1-in. gage lengths. With these instruments, stresses could be obtained within an expected accuracy of 300 lb per sq in. The instruments were held in position on the web plate by means of $\frac{1}{8}$-in. bolts and holes tapped in the web. As it was desirable to keep the number of holes at a minimum, only three strain readings were observed at each point. Instead of the check usually obtained by the fourth reading, a repetition was made of each loading and observation. The tensometer attachments for web and flange strain measurements are shown in Figs. 3 and 6.

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[6] "The Determination of Stresses from Strains on Three Intersecting Gage Lines and Its Application to Actual Tests," by William R. Osgood and R. G. Sturm, *Research Paper No. 559, Journal of Research*, National Bureau of Standards, Vol. 10, 1933.

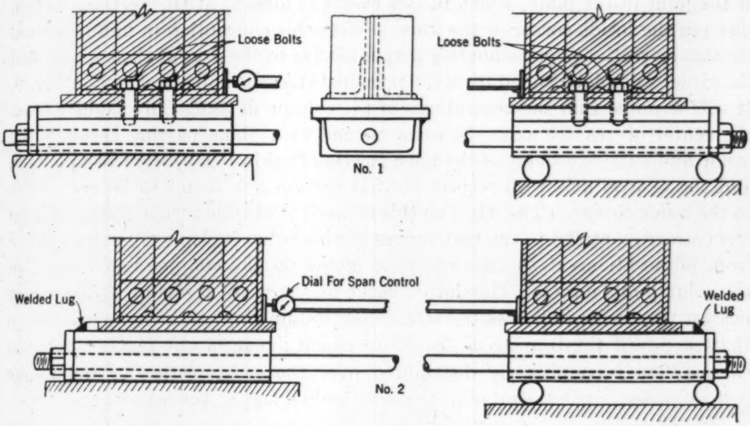


FIG. 5.—FLAT BASE DETAILS

Three complete sets of stress-distribution data were obtained for the square-knee frame, one set for each knee of the frame at normal span length, and one for the east knee with the supports allowed to move outward $\frac{1}{4}$ in. under load. In general, the results for the two knees were so similar that the data from only one are presented herein. The variation of distribution due to movement of the supports is discussed in another section.

For the curved-knee frame (see Fig. 7), two complete sets of stress-distribution data were observed, of which only one is presented herein, for the reason given in the preceding paragraph. The bases were maintained at the normal distance apart for these tests.

From the strains observed at each gage point on the web of each rigid-frame knee, principal stresses and maximum shearing stresses were determined graphically by the method mentioned previously. On the backs of the flanges longitudinal strains only were observed, from which stresses were computed directly. The values of principal stresses and maximum shearing stresses are indicated by lines of equal stress (contour lines) for the square knee in Fig. 8(*a*), and for the curved knee in Fig. 8(*b*). The directions and approximate magnitudes of the principal stresses at each of the gage points are also shown in the diagrams.

In the square-knee frame the knee joint may be considered as a rigid beam-and-column connection in which the column extends to the top of the frame. Whether the girder or column extends through the knee depends on the location of the joint in the plate, which in this case was located at the vertical section through the inside corner of the knee. With this conception in mind, normal stresses on sections approximately perpendicular to the axes of the column and the girder were determined from the principal stresses, and are shown in Fig. 9. It will be noted that the neutral axis of the column deviates only slightly from the center of gravity axis. Stresses normal to a plane passing through the inside and outside corners of the knee (section *O–O* in Fig. 9) were also plotted, and the neutral axis with respect to that section was found to be very close to the inside corner. The stress on this plane at the inside corner was computed from stresses parallel to the two flanges at this point, which were extrapolated from observations at the nearest gage points on the compression flanges of the column and girder. The dotted curve on the vertical section in the girder nearest the corner indicates the stress distribution that might have existed on that section if the bearing of the compression flange of the girder upon the column had been properly distributed over the cross section of the flange instead of being concentrated in the outstanding legs of the angles.

In the testing of the curved-knee frame it was observed that the stresses at the toes of the outstanding legs of the curved flange angles were only 40 to 70% of the stresses at corresponding points at the heels of the angles or on

FIG. 6.—TEST SET-UP SHOWING ARRANGEMENT OF TENSOMETERS ON THE SQUARE-KNEE FRAME

the edge of the web. The stresses at the heels of the angles and on the edge of the web agreed very closely. Fig. 10 shows flange stresses observed on three sets of gage lines: On the edge of the web, at the heels of the angles, and at the toes of the angles. This transverse variation of stress in the outstanding legs of flange angles did not exist to any extent on straight flanges whether subjected to tension or compression. The explanation of this phenomenon was found to be a kind of buckling action in which the outstanding legs of the curved flange angles bent away from their center of curvature, thus elongating relative to the edge of the web and relieving some of the compression in the outstanding legs. For the observed differential in stress between the edge of the web and the toes of the angles to have taken place, the toes of the outstanding legs would have had to deflect about 0.01-in. relative to the edge of the web. The slight transverse bending observed, as measured by a transverse bending stress of about one kip per sq in., was not sufficient, however, to produce this deflection, indicating that some rotation was probably taking place about the rivet line. The stresses on all three sets of gage lines increased approximately in proportion to the load.

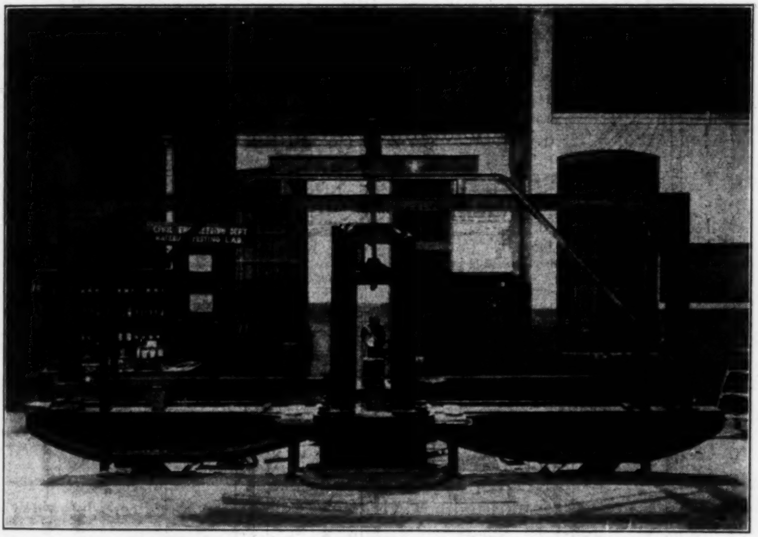
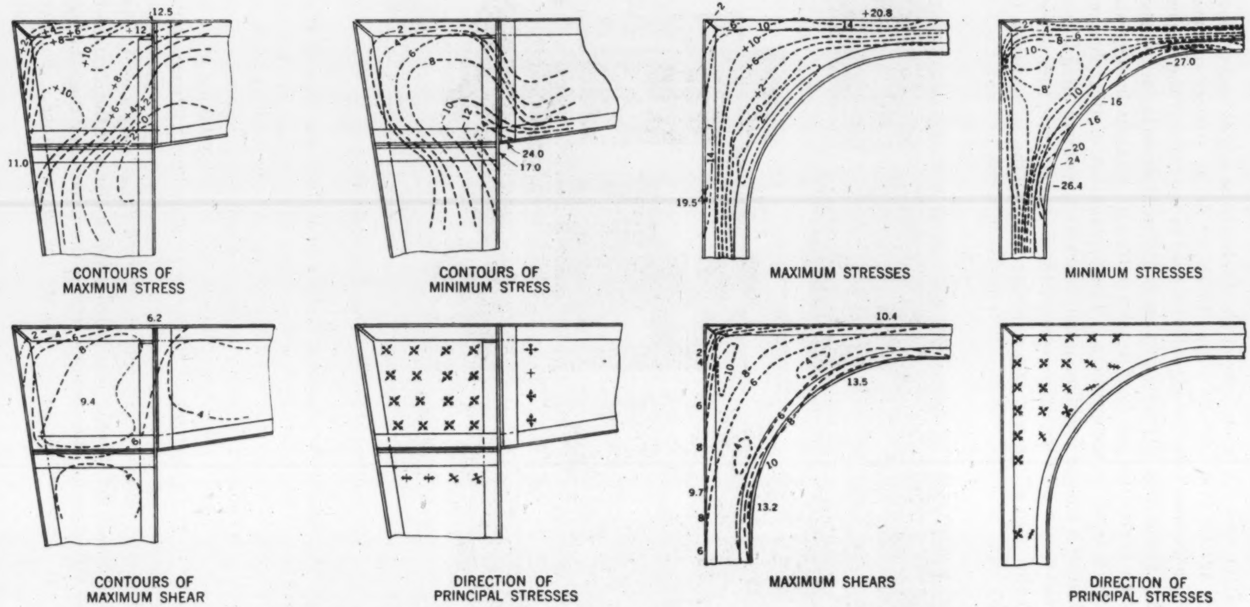


FIG. 7.—CURVED-KNEE FRAME AND DEFLECTION APPARATUS

Normal stresses on planes radial to the curved flange are shown in Fig. 11. It will be noted that the straight-line distribution of stress of the straight-beam theory does not exist on the sections within the knee, and that the neutral axis is between the center-of-gravity axis and the curved flange. The average



12.5
+12
+10
+8
+6
+4
+2
11.0
2
4
6
8
10
12
24.0
17.0
+20.8
14
10
6
2
+10
+6
+2
0
19.5
-27.0
-16
-20
-24
-26.4
6.2
9.4
4
10.4
13.5
10
13.2
9.7
8

CONTOURS OF MAXIMUM STRESS

CONTOURS OF MINIMUM STRESS

MAXIMUM STRESSES

MINIMUM STRESSES

CONTOURS OF MAXIMUM SHEAR

DIRECTION OF PRINCIPAL STRESSES

MAXIMUM SHEARS

DIRECTION OF PRINCIPAL STRESSES

(*a*) SQUARE KNEE FRAME

(*b*) CURVED KNEE

FIG. 8.—OBSERVED PRINCIPAL STRESSES, IN KIPS PER SQUARE INCH

stress values at the back of the curved flange indicated in Fig. 11 were obtained by assuming that the stress was constant across the backs of the angles at each section, that the neutral axis would remain where observed, and that the moment of the compression area about the neutral axis would be the same as observed. From the observed stresses and the position of the neutral axis, the average stress was then computed. The difference between the maximum and average stresses was as much as 25% at some gage points.

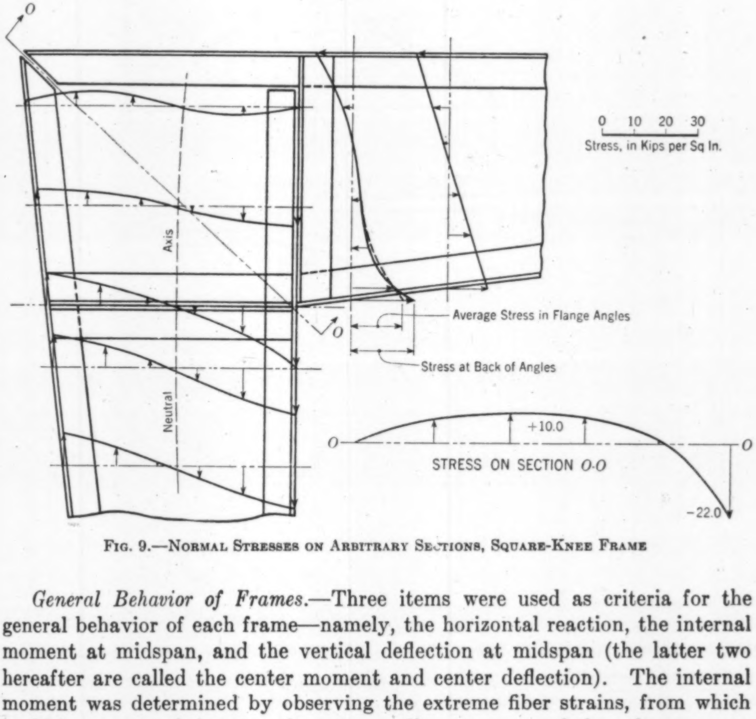


FIG. 9.—NORMAL STRESSES ON ARBITRARY SECTIONS, SQUARE-KNEE FRAME

General Behavior of Frames.—Three items were used as criteria for the general behavior of each frame—namely, the horizontal reaction, the internal moment at midspan, and the vertical deflection at midspan (the latter two hereafter are called the center moment and center deflection). The internal moment was determined by observing the extreme fiber strains, from which could be computed, in turn, the extreme fiber stresses and then the moment, assuming that the conventional theory of flexure held true at this location. The horizontal reaction was determined by observing the strain in the tie bar with a 10-in. strain gage and computing the stress and load therefrom. The vertical deflection of the frame at midspan was measured by a 0.001-in. Ames dial between the top flange of the frame and a framework built up from the bases of the frame as shown in Fig. 7.

Load-reaction curves and load-deflection curves for each frame, shown in Fig. 12, illustrate that the frame as a whole behaved as an elastic structure; that is, the observations varied in direct proportion to the load. Individual stress observations were generally made in only one increment; but at several random points load-stress data were observed, and a straight-line relationship was obtained.

Since only six 1-in. extensometers were available for use (all of these being stationary by nature) many repetitions of loading were necessary to obtain all the desired data. Strain observations on the tie bar were made at regular intervals throughout the course of the testing, and no appreciable change in reaction was noticed. Evidently such phenomena as permanent set due to high localized stresses and slippage of rivets, if such existed, had a negligible effect on the frame as a whole. However, a decrease of about 5% in the high localized stresses at the inside corner of the square knee occurred during the course of the testing.

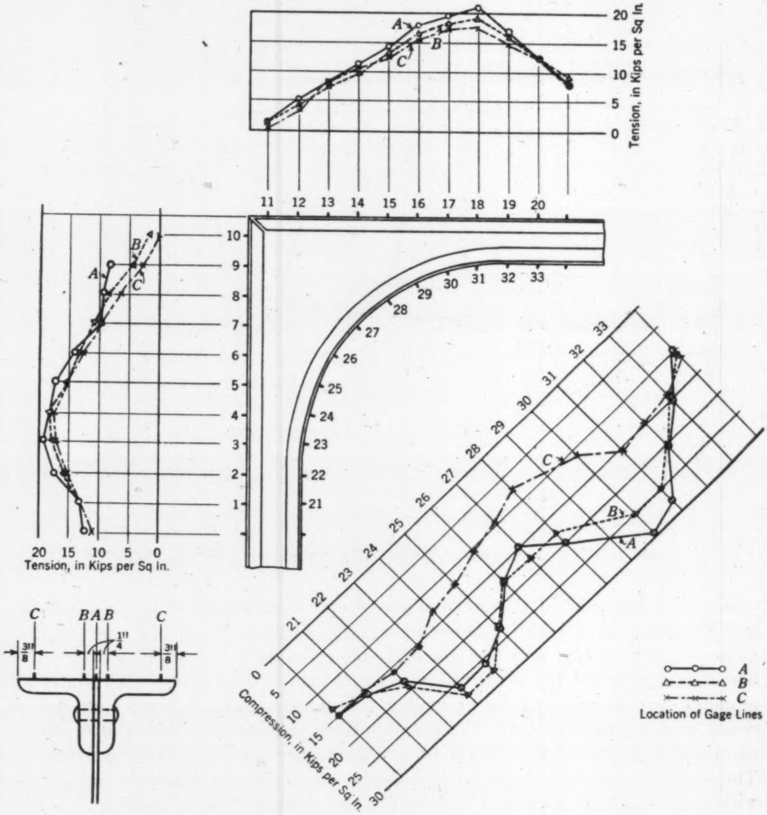


Fig. 10.—Flange Stresses on Various Gage Lines, Curved-Knee Frame

Horizontal Movement of Supports.—In order to simulate one of the most important problems in rigid-frame construction—that of horizontal foundation movement—the bases of the frames were moved inward or outward as the load was being applied. Each frame was studied under five such conditions, the span length being varied from the no-load state by the following amounts: $-\frac{1}{2}$ in., $-\frac{1}{4}$ in., 0 in., $+\frac{1}{4}$ in., and $+\frac{1}{2}$ in. A $\frac{1}{4}$-in. movement in the model corresponded to a 1-in. movement in the prototype.

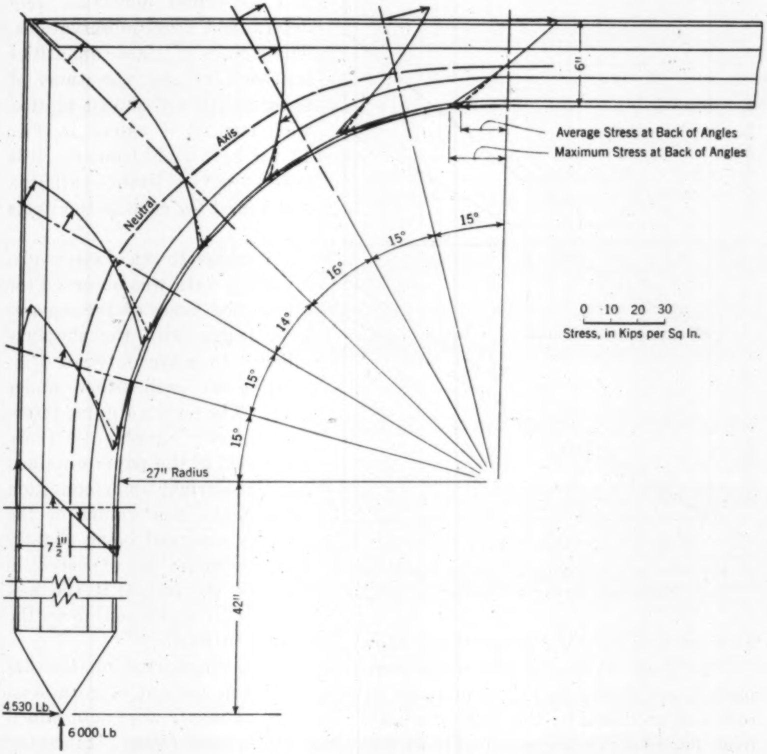

6"
Average Stress at Back of Angles
Maximum Stress at Back of Angles
Neutral Axis
15°
15°
16°
14°
15°
15°
0 10 20 30
Stress, in Kips per Sq In.
27" Radius
$7\frac{1}{2}$"
42"
4 530 Lb
6 000 Lb

FIG. 11.—NORMAL STRESSES ON RADIAL SECTIONS, CURVED-KNEE FRAME

The adjustment of the span length was controlled by a 0.001-in. Ames dial bearing against one column base and fastened to a long light angle clamped at its opposite end to the other column base. Rollers supported the angle along its length. The two ends of this device are shown in Fig. 5. Readings at zero load were always taken at normal span length; that is, the span length (distance between supports) under no load. As the load was being applied, the span was adjusted by the nuts on the tie bar, so that, at maximum load, the span length was as desired, either normal, or plus or minus a given quantity.

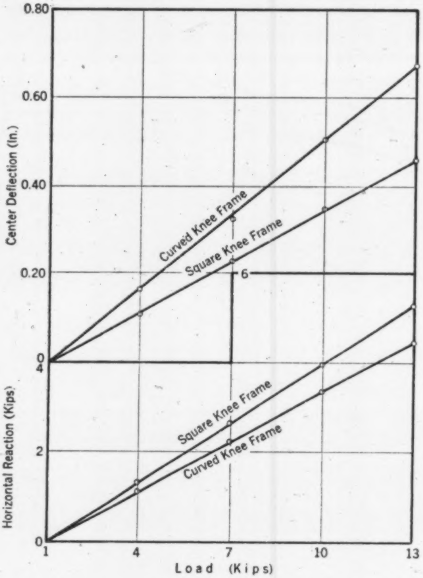


FIG. 12.—OBSERVED LOAD-DEFLECTION AND LOAD-REACTION CURVES FOR NORMAL SPAN

For each test, the horizontal reaction, the center moment, and the center deflection were determined from observations. The values of these quantities for each of the variations of span length are shown plotted with computed values in Fig. 13 for both rigid frames. It is noted that a linear variation was found for each of the items observed.

A complete set of stress-distribution data was observed for one of the knees of the square-knee frame with the supports allowed to move outward $\frac{1}{4}$ in. relative to each other under load. The results obtained were qualitatively similar to those presented in the paper obtained for the normal span condition (Fig. 8(*a*)), the values of the stresses observed being slightly less, as would be expected in view of the fact that the moments and shears acting on the knee are smaller if the supports are allowed to move outward.

Flat Base Tests.—Both frames were tested while supported on the flat plate bases shown in Fig. 5 in order to determine the amount of rotational restraint produced by this type of a base. Internal moments were determined from flange strain measurements at two locations in each frame. Since the horizontal reaction was also observed, it was then possible to compute the location of the point of inflection in the column, if it existed. In both frames, the point of inflection was found to be so close to the base of the column that this type of base was considered to produce, in effect, a hinged support.

Further evidence that the frames resting on the flat plate bases acted as hinged frames is presented in Table 2, which gives ratios of observed to computed horizontal reactions, center moments, and center deflections for both hinged and flat-base tests. All computations were based on the assumption that a hinge existed at the base. It is noted that the two base conditions gave very similar results for both frames.

RELATION OF TEST DATA TO ANALYSIS

Moment of Inertia.—A test was made to determine the correct moment of inertia for use in computations. With the curved-knee frame inverted in the testing machine, the girder that was of uniform section was tested as a simple beam with two equal and symmetrically situated loads. At a working load for the girder, strains were observed on the top and bottom of the girder. Stresses were computed from the observed strains, and the effective moment of inertia was computed from the extreme fiber stresses, applied loads, and dimensions of the girder.

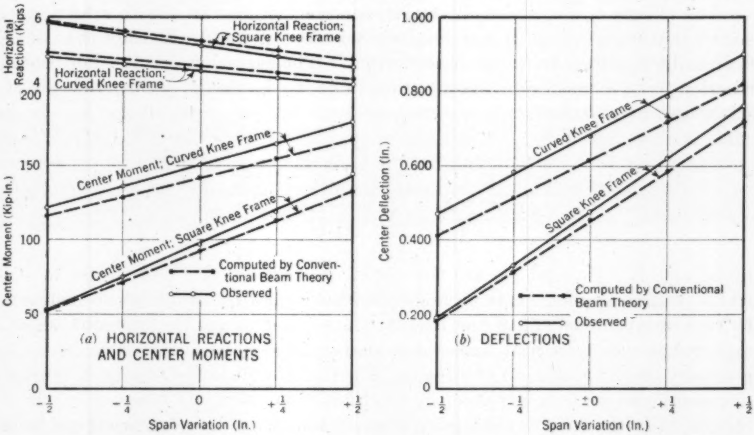


FIG. 13.—COMPARISON OF OBSERVED AND COMPUTED REACTIONS, MOMENTS, AND DEFLECTIONS

The effective moment of inertia was slightly less than the gross moment of inertia, and almost exactly equal to that obtained by assuming the total volume of the rivet holes to be uniformly distributed along the length of the girder. This average moment of inertia was used in all computations. If the gross moment of inertia had been used, the error would have been about 3% for this girder.

TABLE 2.—COMPARISON BETWEEN HINGED-BASE AND FLAT-BASE RESULTS

$$\left(\text{Ratio} = \frac{\text{Observed}}{\text{Computed}} \right)$$

Type of base	(a) SQUARE-KNEE FRAME			(b) CURVED-KNEE FRAME		
	Horizontal reaction	Center moment	Center deflection	Horizontal reaction	Center moment	Center deflection
Hinged	0.985	1.06	1.02	0.965	1.07	1.10
Flat	0.975	1.08	1.04	0.955	1.09	1.11

Horizontal Reaction.—In order to compare the test results with computed values, analyses were made for both frames by the Maxwell-Mohr (unit dummy load) method as applied to a two-hinged arch. Since both frames were of non-uniform cross section, the members of each frame were arbitrarily divided into short sections and the necessary integrations were made as algebraic summations. The arbitrary sections are shown in Fig. 1.

The effects of shear and direct stress were included in the computations, since for comparison with test results a greater degree of accuracy than that ordinarily required in design was desired. The equation for the horizontal reaction (H), considering deformations due to bending, shear, and direct stress, can be written:

$$H = \frac{\sum \frac{M_1\, m_h\, \Delta s}{E\, I} + \sum \frac{V_1\, v_h\, \Delta s}{A_w\, G} + \sum \frac{N_1\, n_h\, \Delta s}{A\, E}}{\sum \frac{(m_h)^2\, \Delta s}{E\, I} + \sum \frac{(v_h)^2\, \Delta s}{A_w\, G} + \sum \frac{(n_h)^2\, \Delta s}{A\, E}} \qquad \text{(1)}$$

in which: M_1, V_1, and N_1 are the actual moment, shear, and thrust in the frame, due to applied loads on a simple frame (that is, assuming no restraint to horizontal movement of the bases); m_h, v_h, and n_h are the moment, shear, and thrust due to a unit horizontal load applied at the supports; Δs is an arbitrary length of section of a frame; E is the modulus of elasticity in tension and compression; G is the shearing modulus of elasticity; I is the moment of inertia; A is the total cross-sectional area; and A_w is the cross-sectional area of the web. Actual evaluation of the different quantities indicated that, for rigid frames of types similar to the two models, the last two terms of the numerator and the last term of the denominator may be neglected for all practical purposes, giving:

$$H = \frac{\sum \frac{M_1\, m_h\, \Delta s}{E\, I}}{\sum \frac{(m_h)^2\, \Delta s}{E\, I} + \sum \frac{(v_h)^2\, \Delta s}{A_w\, G}} \qquad \text{(2)}$$

Neglecting the remaining shear term would introduce an error of about 3% for the square-knee frame and about 1% for the curved-knee frame.

The application of Eq. 2 for H is a routine matter provided the structure behaves in accordance with the assumptions on which the theory is based. Figs. 9 and 11 show, however, that at the knees of both rigid frames straight-line distribution of normal stresses does not exist, and, in all probability, sections that are plane before bending do not necessarily remain plane after bending. As a result, the actual bending deformations $\frac{M\, \Delta s}{E\, I}$ and the approximate shearing deformations $\frac{V\, \Delta s}{A_w\, G}$ in the sections at the knee will differ from those computed by the Maxwell-Mohr theory[7] for straight beams. Therefore, a study of the stress distribution at the knee of each frame was made to determine the treatment of the corner sections that would most nearly approach actual conditions as interpreted from observed data.

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[7] "Elementary Treatise on Statically Indeterminate Stresses," by J. I. Parcel and G. A. Maney, Members, Am. Soc. C. E., John Wiley & Sons, Inc., New York, 2d Ed., 1936, Chap. 1.

The changes in theoretical horizontal reaction due to variations in span length were determined directly from Eq. 2. The numerator of Eq. 2 represents the horizontal deflection of the supports due to the applied loads when the supports are free to move horizontally; that is, the variation in span length for a 100% change in reaction. Since the change in reaction is proportional to the variation in span length for a given loading, the horizontal reactions for the various span lengths were obtained by direct proportion.

Deflections.—Vertical deflections at the center of each frame were computed by the Maxwell-Mohr method, involving the application of a vertical unit load at the point at which the deflection was desired. For convenience, it is recommended that the moments due to the unit load be computed for the determinate frame (supports free to move horizontally), that is:

$$\delta = \sum \frac{M\, m\, \Delta s}{E\, I} \qquad (3)$$

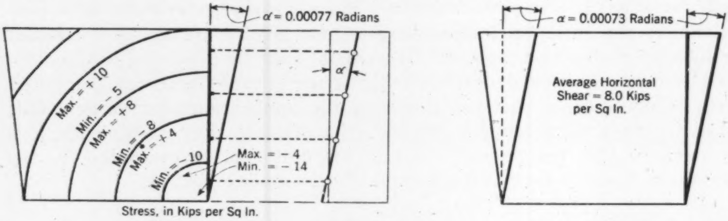
in which: δ = deflection at any point; M = actual moments in the structure due to applied load; and m = moments due to a unit load applied to the determinate structure at the location of, and in the direction of, the deflection desired.

Corner Section, Square-Knee Frame.—Of primary importance is the degree to which the sections within the knee may influence the determination of the horizontal reaction. Total neglect of knee sections in the computations for horizontal reaction (that is, considering the knee to be infinitely stiff) increased the computed value by about 3%.

An approximation to the actual deformation of the square knee was obtained from the information in Fig. 8(*a*). Since the principal stresses within the knee were approximately circumferential and radial, the knee was divided into circumferential bands, as shown in Fig. 14(*a*). Average radial and circumferential stresses were assigned to the bands in accordance with the observed stress distribution. From the average principal stresses the elongation of each band was computed, which, when plotted together, gave the total angular change between the two internal faces of the square knee, also shown in Fig. 14(*a*). This angular change was practically the same as that computed from the observed shears in the knee (Fig. 14(*b*)), indicating that bending, as ordinarily considered, contributed very little to the deformation of the knee.

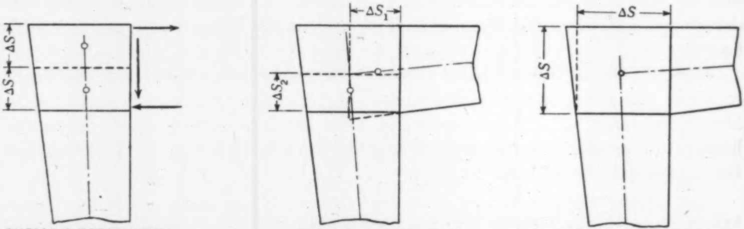
The effect of shear in the knee is represented by the analysis presented in Fig. 14(*c*), in which $\sum \frac{V\, \Delta s}{A_w G} = 0.00072$. This practice of treating the column as a free body acted upon by the reactions of the frame and by the flange forces and end shear of the girder (neglecting the moment and thrust carried by the web) is used now by some designers. When the knee is considered to be loaded in this manner, bending about the neutral axis of the column will contribute little to the effective deformation of the knee which was already indicated by test results in Fig. 14(*a*) and Fig. 14(*b*).

Two conventional methods of treating the corner of a square rigid-frame knee for mechanical integration are shown in Fig. 14(*d*), in which $\sum \frac{M\,\Delta s}{E\,I} = 0.0018$; and Fig. 14(*e*), in which $\frac{M\,\Delta s}{E\,I} = 0.0017$. The computed bending deformations at the knee for the two cases were approximately equal and considerably greater than the deformation indicated by the analysis shown in Figs. 14(*a*) and 14(*b*). If, however, the effect of shear is neglected throughout the frame (as would be done in design), the large bending deformation assigned to the knee tends to offset the absence of shear deformation in the frame as a whole. The results thus obtained differed only slightly from those in which shear was considered. A comparison between the observed horizontal reaction and the values obtained by the several treatments of the knee section is as follows:


α = 0.00077 Radians
Max. = +10
Min. = −5
Max. = +8
Min. = −8
Max. = +4
Min. = −10
Max. = −4
Min. = −14
α
Stress, in Kips per Sq In.
α = 0.00073 Radians
Average Horizontal Shear = 8.0 Kips per Sq In.

FROM OBSERVED PRINCIPAL STRESSES
(*a*)

FROM OBSERVED HORIZONTAL SHEARS
(*b*)


ΔS
ΔS
ΔS₁
ΔS₂
ΔS
ΔS

SHEARING DEFORMATION
(*c*)

BENDING DEFORMATION
(*d*)

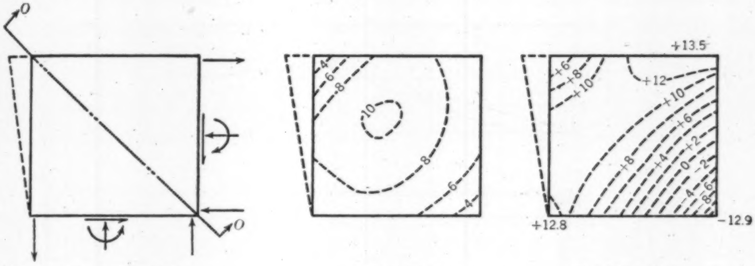
BENDING DEFORMATION
(*e*)

FIG. 14.—DISTORTION OF THE SQUARE KNEE AND POSSIBLE CHOICES OF SECTIONS FOR MECHANICAL INTEGRATION

Description	Horizontal reaction (lb)
Observed	5,280
Computed by Analysis in Fig. 14(*c*):	
Shear included	5,340
Shear neglected	5,480
Computed by Analysis in Figs. 14(*d*) or 14(*e*):	
Shear included	5,320
Shear neglected	5,360

Computations of all theoretical values used for comparison with observed values were based upon the treatment shown in Fig. 14(*c*), including the effect of shear.

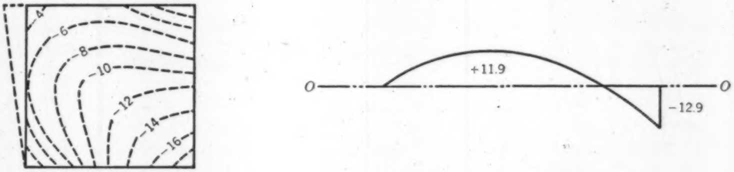
In connection with the rigid-frame investigation conducted at the National Bureau of Standards, a theoretical analysis[3] of a rectangular rigid-frame knee was developed. The analysis involved an application of the theory of elasticity to a rectangular plate loaded at the edges with shears, thrusts, and moments in its own plane as shown in Fig. 15. The resulting contours of principal stresses and maximum shears, which were computed by H. D. Hussey, M. Am. Soc. C. E., for a rigid-frame knee of practically identical proportions as the square knee are also given in Fig. 15. By comparison with Fig. 8(*a*) it is noted that the general agreement between theoretical and observed results is very satisfactory.



ASSUMED LOADING OF PLATE

CONTOURS OF MAXIMUM SHEARS

CONTOURS OF MAXIMUM STRESS



CONTOURS OF MINIMUM STRESS

NORMAL STRESS ON 45° PLANE, STRESSES IN KIPS PER SQ IN.

Fig. 15.—Analysis of Rectangular Knee by the Theory of Elasticity

A comparison of observed and computed extreme fiber stresses in the vicinity of the knee is presented in Fig. 16. Computed stresses are based on the conventional formulas for flexure and direct stress. In plotting the computed tension values it was assumed that the stresses reach a maximum at the section that passes through the inside corner of the knee, and decrease uniformly along the flanges of the knee to zero at the outer corner. The stresses represented by triangular dots on the vertical section passing through the inside corner of the knee were computed on the assumption that the flange angles of the girder transmitted all of the moment and thrust in the girder to the column. This assumption is reasonable if it is noted that the web splice is located at this section.

A simple method for computing normal stresses on the 45° plane through the inside corner of the knee was developed from the test data shown in Fig. 9. In accordance with the stress distribution on the diagonal plane, the neutral axis for this section was assumed to be at one fifth the length of the diagonal from the inside corner, the tension stress area was assumed to be a second-degree parabola, and the compression-stress area a triangle. With these assumptions and the equilibrium equations for direct stress and bending on the section, the maximum values of tension and compression stresses were determined. Considering the compression flange (but not the tension flange of the girder nor the horizontal stiffening angles) to be acting with the web, the maximum tension and compression stresses computed by the foregoing method fell within 5% of the observed values.

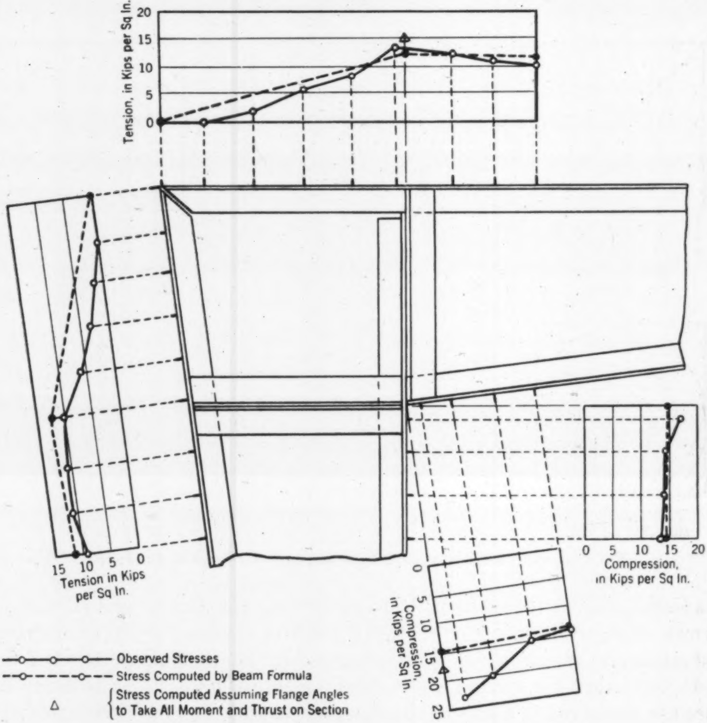
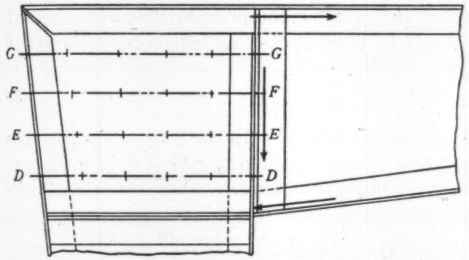


FIG. 16.—COMPARISON OF OBSERVED AND COMPUTED FLANGE STRESS

Fig. 8(*a*) shows that the maximum shearing stresses are very nearly constant over most of the web in the knee and are all approximately horizontal and vertical, being inclined at 45° to the direction of the principal stresses. Computations show that the average of the horizontal shearing stresses over the entire knee is 98.4% of the average of the maximum shearing stresses. Thus, a correct determination of the horizontal shearing stresses would be adequate for design purposes. Fig. 17 presents a comparison between the horizontal shearing stresses determined from observations and those computed on the assumption that the web resists a horizontal shear equal to the tension in the top flange of the girder. The girder flanges have been assumed to take all the moment and thrust on the section at the column face as shown in Fig. 17.



SECTION	COMPUTED	OBSERVED
D	8 240	8 630
E	8 040	8 120
F	7 860	7 990
G	7 670	7 580

Computed on the Basis of Indicated Loading

FIG. 17.—AVERAGE HORIZONTAL SHEARING STRESSES IN THE SQUARE KNEE, IN POUNDS PER SQUARE INCH

Corner Section, Curved-Knee Frame.—In the curved-knee frame the horizontal reaction was 3.5% below the computed value. Average compression-flange stresses at the knee varied from 15% to 50% in excess of stresses computed by the conventional formulas for flexure and direct stress as shown in Fig. 18. Therefore, the bending deformations in sections within the knee were probably greater than those computed by conventional methods—that is, the knee was not as stiff as assumed. To remedy this condition, a simple and arbitrary method for reducing the moments of inertia and section moduli of sections within the knee was developed from the test results. Using these modified values, computations gave a horizontal reaction and compressive stresses within the knee in close agreement with the observed data.

For knees of this type radial sections are the most convenient to use and were employed in all analyses involving mechanical integration. To obtain what will be called the "effective" section modulus (S') for each section, the moment at a given section was divided by the average observed compressive stress at the extreme inner fibers. The ratio of "effective" section modulus to actual section modulus $\left(\frac{S'}{S}\right)$ was determined for each section and plotted against a central angle C, as shown in Fig. 19. The approximately linear relationship between the ratio $\left(\frac{S'}{S}\right)$ and the angle C can be expressed by the linear equation,

$$\frac{S'}{S} = 1 - \tfrac{1}{2}\left(\frac{C}{45}\right) \quad \ldots\ldots\ldots\ldots (4a)$$

that is,

$$S' = S\left(1 - \frac{C}{90}\right) \quad \ldots\ldots\ldots\ldots (4b)$$

Due to the origin of Eq. 4, the compressive stresses based on the "effective" section moduli agreed closely with the average observed compressive stresses. Furthermore, the same principle used in the modification of the section modulus may be applied to the moment of inertia in order to correct for the discrepancy in observed and computed horizontal reactions. The procedure in an actual design, assuming that the stress in the extreme compressive fibers is constant across the width of the flange, would be to compute the "effective" moment of inertia, I', from the relationship:

$$I' = S'c \ldots\ldots\ldots\ldots (5)$$

in which c is one half the actual depth of section. In the case of the curved-knee frame, however, the deformation of the knee depended not on the average flange stresses, but on the maximum compressive stresses which occurred at the

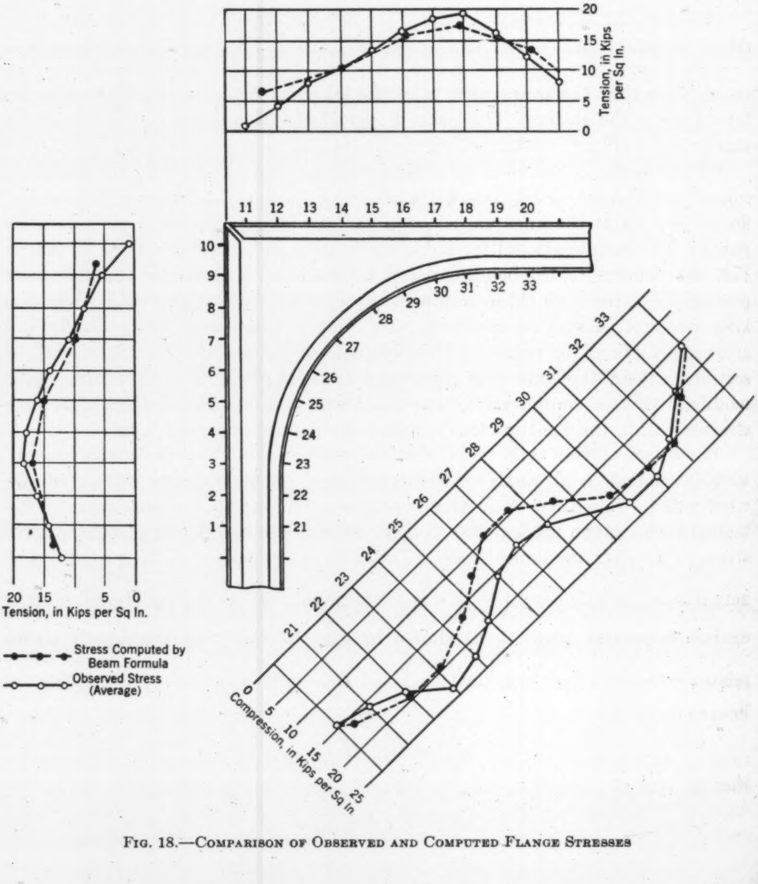


Fig. 18.—Comparison of Observed and Computed Flange Stresses

edge of the web. By using the "effective" moments of inertia based on the maximum compressive stresses in the web, the value of the theoretical horizontal reaction (neglecting shear) was reduced from 4,690 lb to 4,480 lb, as compared with the observed value of 4,530 lb.

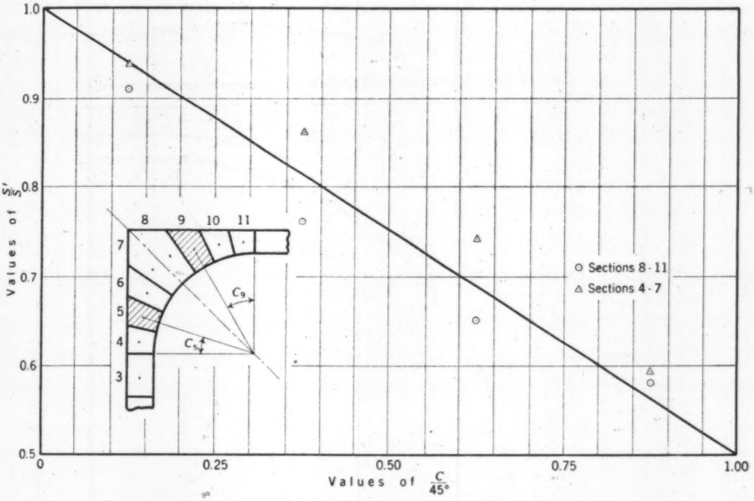


FIG. 19.—GRAPH FOR THE DETERMINATION OF A MODIFIED SECTION MODULUS

Maximum tensile stresses at the knee occurred at the outer extreme fibers of sections near the points of tangency of the curved flange and were in fairly close agreement with values computed by the conventional formulas for flexure and direct stress (Fig. 18). On the radial section through the exterior corner of the knee, however, the maximum tensile stress cannot be computed in this manner. A simple solution for this section was derived from the test results in Fig. 11, identical in principle with the method applied to a corresponding section in the square-knee frame. The assumptions made were that the neutral axis was one quarter the distance from the compression flange to the exterior corner, and that the stress distribution followed a second-degree parabola on the tension side and a triangle on the compression side. Assuming that the tension flange angles did not act with the web, and applying the equations of equilibrium, the maximum observed tension and compression stresses on the section were checked within about 10%. Since this is not likely to be a critical section, the check was considered satisfactory.

In connection with the tests on rigid-frame knees made at the Bureau of Standards, an analysis[4,8] was developed by Mr. Osgood for the knee frame with the curved inner flange. This analysis determines normal and shearing stresses on arbitrary circular-cylindrical sections for members having nonparallel

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[8] "A Theory of Flexure for Beams with Nonparallel Extreme Fibers," by W. R. Osgood, *Journal of Applied Mechanics*, September, 1939.

extreme fibers. No consideration is taken of the possibility that the edges of the member may be curved. A comparison of normal stresses computed by this method and observed normal stresses is presented in Fig. 20 for the curved-knee frame. In general, the observed stresses at the extreme fibers are somewhat higher than the computed stresses, although the agreement is considerably better than that obtained by comparison with the theory of flexure for straight beams with parallel edges. The neutral axis, as located in Fig. 20, is the axis of zero stress rather than the true bending neutral axis.

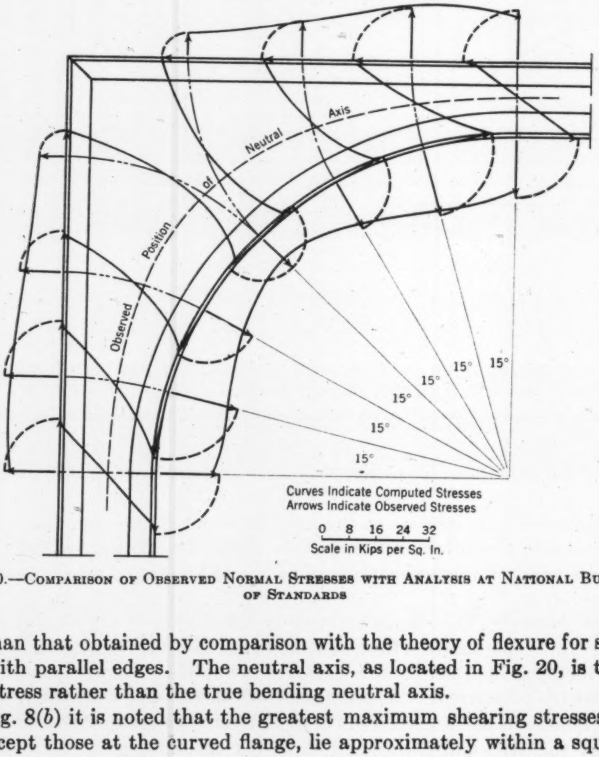
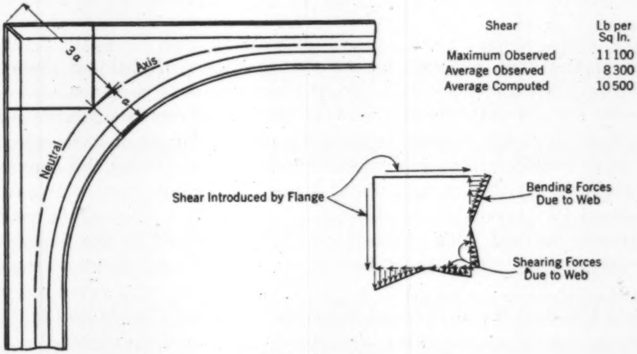

Observed Position of Neutral Axis
15°
15°
15°
15°
15°
15°
Curves Indicate Computed Stresses
Arrows Indicate Observed Stresses
0 8 16 24 32
Scale in Kips per Sq. In.

FIG. 20.—COMPARISON OF OBSERVED NORMAL STRESSES WITH ANALYSIS AT NATIONAL BUREAU OF STANDARDS

In Fig. 8(*b*) it is noted that the greatest maximum shearing stresses in the knee, except those at the curved flange, lie approximately within a square, at the exterior corner of the knee, with two of its boundaries coinciding with the straight flanges of the knee and with its interior corner on the observed neutral axis (see Fig. 11). This square is shown in detail in Fig. 21. From Fig. 8(*b*) it is also noted that the maximum shearing stresses within the square are nearly all horizontal and vertical. The forces acting on this section of the web are illustrated in Fig. 21, the shears introduced by the flange tensions being very great in comparison to the bending loads due to the web. Therefore, the horizontal and vertical shears in the square were considered to be approximately equal to the shear introduced by the flanges. The total stress in the angles that must be unloaded into the web through the rivets along the two exterior sides of the square can be computed by the conventional formulas for flexure and direct stress. Assuming the square to be isolated from the remainder of the knee and loaded with horizontal and vertical shears equal to the average of the tensions in the two flanges, which were nearly equal, the resulting average shearing unit stress agreed closely with the observed maximum shearing stress. The comparison between observed and computed results is shown in Fig. 21. The high shearing stresses along the curved flange are a direct function of the normal stresses there, and they will not become excessive if the normal stresses do not exceed working limits.


3a
Axis
a
Neutral
Shear Introduced by Flange
Bending Forces Due to Web
Shearing Forces Due to Web

Shear	Lb per Sq In.
Maximum Observed	11 100
Average Observed	8 300
Average Computed	10 500

FIG. 21.—SHEARING STRESSES IN THE CURVED KNEE

General Behavior of Frames.—The extent to which the frames, each considered as a whole, behaved in accordance with currently accepted theory is shown in Fig. 13. For both frames the observed horizontal reactions were slightly less than the computed values, varying from 0 to 4% for the square-knee frame, and being constant at about 4% for the curved-knee frame. The observed center moments and deflections were greater than the computed values, varying from 0 to 8% for the square-knee frame, and from 5 to 13% for the curved-knee frame. According to the laws of the statics, observed reactions and center moments should always check each other. In general they agreed very well. Any excessive deformation at the knee sections would be reflected

TABLE 3.—COMPARISON BETWEEN TEST DATA AND COMPUTED VALUES FOR TESTS ON NORMAL SPANS

Description	(a) SQUARE-KNEE FRAME			(b) CURVED-KNEE FRAME		
	Horizontal reaction (lb)	Center moment (kip-in.)	Center deflection (in.)	Horizontal reaction (lb)	Center moment (kip-in.)	Center deflection (in.)
Observed values	5,280	98	0.465	4,530	153	0.678
Computed values	5,340	93	0.455	4,690	143	0.615
Computed from observed horizontal reaction	....	98	0.485	....	154	0.700

in the value of the horizontal reaction obtained, but not in the relation between the observed reaction and observed moments.

By comparing the observed values of horizontal reaction, center moment, and center deflection with theoretical values and with semi-theoretical values obtained by using the observed horizontal reaction, a good indication of the efficiency of the knee joint as well as of the frame as a whole was obtained. Such a comparison is given for each frame in Table 3. All values refer to the normal span condition.

DISCUSSION OF RESULTS

Stress Distribution, Square-Knee Frame.—In general, the parts of the frame outside of the knee behaved in accordance with conventional theory. On sections only a few inches away from the boundaries of the knee, straight-line distribution of normal stresses was obtained. However, immediately adjacent to the knee, observed stresses in the compression flanges of both column and girder were considerably greater than stresses computed by the conventional beam formula for flexure and direct stress. Probably two principal causes contributed to this condition. First, the web of the frame was discontinuous through the vertical joint at the knee, which resulted in the flanges of the girder carrying more than their share of the moment and thrust on the section at the column face. This premise is upheld by the normal stress distribution shown in Fig. 9 on the girder section next to the knee. It is noted that on this section the web is understressed and the flange angles overstressed. Second, the presence of shims between the compression flange of the girder and the column caused practically all of the compression in the girder to be transmitted to the column through the outstanding legs of the flange angles. At a point on the girder 2.5 in. from the column face, the outstanding legs were carrying about three fourths of the total load in the compression-flange angles. The average stress in the compression flange agreed closely with the stress computed on the assumption that the girder flanges carry all the moment and thrust in the girder at the knee joint. The high stress noted in the compression flange of the column at the inside corner of the knee was probably caused, in part, by the extreme concentration of bearing at that point. It seems probable that any additional concentration of stress in the column or girder at the knee because of the sharp reentrant angle was of small magnitude.

Whatever concentration of stress does exist is directly dependent upon the bearing condition at the corner. An accurate fit along the entire joint will produce lower stresses in the flanges but higher stresses in the web. Local variations of this nature are of general occurrence in steel structures, where these high localized stresses are usually disregarded as unimportant. Therefore, with regard to concentrations of stress at sections adjacent to the knee, the rigid frame may be treated as any other steel structure. However, a loose fit is to be avoided because of the possible effect on the horizontal reaction and therefore on the stresses at midspan.

Within the knee the stress distribution cannot be determined by any simple theoretical analysis. The application of the theory of elasticity is too complicated and tedious for practical use by designers. Moreover, Figs. 9 and 16 indicate that the critical sections for normal stresses at the knee are the horizontal and vertical sections through the inside corner of the knee, which were discussed in the preceding paragraph.

Shearing stress is apparently quite important in a rigid-frame knee of this type. The unit shearing stress in the web within the knee was about twice the stress just outside the knee, which is ordinarily considered to be the critical section with respect to shear. A study of the maximum shears (Fig. 17) showed that designing for horizontal and vertical shearing stress in the knee, on the basis of an external shear equal to the total tension in the top flange of the girder, was both adequate and correct.

Stress Distribution, Curved-Knee Frame.—As in the case of the square-knee frame, the parts of the frame outside the knee gave results in accord with conventional theory. Within the knee, the stress distribution differed markedly from that obtained by the ordinary beam theory. Throughout the entire curved part of the compression flange at the knee, the observed stresses were much greater than the computed values. Two factors probably contributed to the observed difference. First, the neutral axis has a pronounced curvature at the knee. However, an application of the curved-beam theory could account for only a small part of the differences between observed and computed stresses, particularly on the compression flange. The second and possibly the most important factor is the rapid change of section that occurs at the knee. This latter problem introduces complexities that may be solved reasonably only by a highly theoretical analysis, or by an arbitrary procedure based on test results. The arbitrary procedure presented in the section on analysis gave good agreement with the observed results, but it made no differentiation between the separate effects of the two aforementioned factors. Such a differentiation would be necessary if the method of analysis were to be extended to knees having decidedly different degrees of curvature or different proportions of flange area to web area.

The stress distribution was complicated by a stress relief in the outstanding legs of the curved flange angles. The radial component of compression due to the curvature of the flange caused the outstanding legs to deflect away from the center of curvature and thus elongate relative to the edge of the web. However, if there had been a cover plate on the backs of the curved flange angles, it is possible that no appreciable radial displacement of the outstanding legs could have occurred. Not only would the outstanding legs have been reinforced by an additional thickness of metal, but any rotation of the individual angles about the rivet line would have been effectively reduced. In the section on analysis it was assumed that cover plates would be present. Consequently the effect of stress relief in the outstanding legs was eliminated by using an average extreme fiber stress, although this phenomenon is too important to be overlooked in design.

The analysis developed by Mr. Osgood was applied to the curved-knee frame, and it gave fairly good agreement with the observed results, although the observed extreme fiber stresses along the curved flange were somewhat higher than those computed by this method. In the Bureau of Standards' tests on a rigid-frame knee of similar shape and proportions, the observed extreme fiber stresses along the curved flange were slightly lower than the computed stresses. This apparent discrepancy might have been caused, in part, by the presence of the transverse variation of stress across the outstanding legs of the curved flange angles. In the tests described herein, strains were measured on several gage lines (Fig. 10) and an attempt was made to determine a properly weighted average extreme fiber stress. To the best knowledge of the writers, in the Bureau of Standards' tests, strains were measured on two symmetrically situated gage lines, each about midway out on the outstanding leg of a flange angle. The difference in the manner of observation would cause the Lehigh results to be higher relatively than the Bureau of Standards' results, and may afford the explanation for the apparent disagreement cited. Since this method of analysis did not take into consideration the curvature of the frame at the knee, it is not without reason that the observed normal stresses in the curved flange might be slightly greater than the computed values. If the effect of the curvature of the neutral axis were taken into account, it is probable that an even better agreement with the observed compressive stresses could be obtained.

General Behavior.—In both frames the observed horizontal reactions were slightly lower than the computed values, 1.5% for the square-knee frame and 3.5% for the curved-knee frame. These relations refer to the normal span condition. Although this degree of accuracy appears to be satisfactory, the discrepancy is on the dangerous side. A decrease in the horizontal reaction will produce an increase of much greater magnitude in the center moment. A greater degree of accuracy in the computations was obtained for the curved-knee frame by arbitrarily reducing the moments of inertia within the knee in accordance with observed stresses.

In general, observed center moments and deflections agreed very well with corresponding values computed from the observed horizontal reactions. Therefore, the accuracy with which stresses and deflections can be computed is merely a reflection of the accuracy of the computations for the horizontal reaction. In other words, the knee of a two-hinged rigid frame affects the frame as a whole only in so far as it affects the horizontal reaction.

The observed normal span deflections of 0.46 in. and 0.67 in. for the square-knee and curved-knee frames, respectively, seem large when it is considered that the deflections in the prototypes are four times as great. In this connection, reference is made to deflection tests,[9] reported by R. M. Hodges, M. Am. Soc. C. E., on three structural-steel rigid-frame bridges in Westchester County, New York. In these tests, the floor systems and cutoff walls, which were not included in the deflection calculations, evidently acted with the frame girders to a great extent. The observed deflections in some cases were only one eighth of the computed values.

Foundation Slippage.—The importance of preventing horizontal movement of the supports of rigid frames is clearly illustrated by Fig. 13. For an increase in span length of $\frac{1}{4}$ in. (that is, each support deflecting outward $\frac{1}{8}$ in.) the horizontal reaction for the square-knee frame decreased 7% and the center moment increased 20%. Corresponding variations were not quite as great for the curved-knee frame because of its greater flexibility.

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[9] "Deflection Tests Show Rigidity of Steel Rigid-Frame Bridges," by R. M. Hodges, *Engineering News-Record*, September 3, 1931.

The accuracy of computations for the effect of horizontal movement of the supports upon the frame as a whole depends chiefly on the accuracy with which the horizontal reaction is determined. Changes in horizontal reaction for known variations of span length did not check the computed values exactly (the computed and observed curves in Fig. 13 are not exactly parallel), but the agreement was satisfactory for all practical purposes.

Flat Base Tests.—It is of interest to note that practically no rotational restraint was developed at the supports by allowing the base plates of the rigid-frame models to rest upon flat plates which resisted only horizontal movement. In fact, the horizontal reactions were less than the values computed, assuming the supports to be hinged, whereas the presence of any reaction moment should have the effect of increasing the reaction. This might have been due to a slight misalinement of the frames. If the bases were not truly horizontal, the reaction might easily have been located inside the centroid of the base. Such a condition would have produced a reduced horizontal reaction. However, the test results shown in Table 2 definitely indicate that, unless specific provisions are made to prevent rotation at the base of a rigid frame, the frame will act as a two-hinged structure.

Recommendations for Design

For most practical purposes, horizontal reactions in two-hinged rigid frames of the types tested in this investigation may be computed satisfactorily by any theoretically sound method of analysis. Fig. 13 presents a criterion of the accuracy that can be expected.

If greater accuracy is desired for the curved-knee frame, the following recommendations are made: Arbitrarily reduce the moments of inertia of sections within the knee by means of the graph in Fig. 19. Shear is of secondary importance and need not be considered when radial sections at the knee are used.

Design.—Concerning the design of the knee in each frame, the following recommendations are made:

For a square knee,

1. The horizontal and vertical sections through the inside corner of the knee are critical sections with respect to normal stresses. Apply the usual formula for flexure and direct stress to the horizontal section. On the vertical section, assume that the flange angles carry all the moment and thrust in the girder, and apply the same formula.
2. Design the web to take a total horizontal shear equal to the tension in the top flange of the girder computed in step 1.
3. If the web plate is thin in comparison to that in the test specimen, it may be well to investigate the tension in the web on the 45° plane through the inside corner. For this section, the neutral axis will be located at about one fifth of the length of the diagonal from the inside corner, and the stress distribution may be considered to take the form of a second-degree parabola on the tension side, and of a triangle on the compression side. Maximum stresses

may then be computed from the equations of equilibrium for bending and direct stress on the section.

For a curved knee,

1. Critical sections for direct stress at the knee occur within 15° from the points of tangency of the curved flange. Maximum compression stresses may be determined by use of conventional formulas for flexure and direct stress in straight beams if the section moduli are reduced in accordance with Fig. 19. Maximum tension stresses may be determined in the same manner, but using nominal section moduli. Flange stresses may also be computed satisfactorily by the Osgood theory.[8]

2. The web should be designed for shear on the basis of the arbitrary method illustrated in Fig. 21. Shears outside the square will be smaller than those computed for the square.

3. If the web plate is thin in comparison to that in the test specimen, the 45° plane through the outside corner should be investigated, using the same procedure that was recommended for a corresponding section in the square knee. However, in the curved knee the neutral axis is located at a distance from the compression flange equal to about one fourth the entire diagonal distance.

SUMMARY

The important findings in this investigation may be summarized as follows:

(*a*) The square knee did not act as a continuous homogeneous corner. It may be analyzed and designed as a rigid girder and column connection, in which the reactions of the girder upon the column consist chiefly of a top-flange tension, a bottom-flange compression, and a vertical shear, neglecting the moment and thrust carried by the web at the splice.

(*b*) Stress concentrations at the sharp reentrant angle of the square knee were due principally to imperfect bearing rather than to any inherent property of rigid-frame knees.

(*c*) Normal stresses upon radial sections of the curved knee did not exhibit a linear relationship. The neutral axis lay between the centroidal axis and the compression flange. On the radial section through the exterior corner the neutral axis was about one fourth the distance from the compression flange to the exterior corner.

(*d*) Maximum stresses in the curved knee occurred on sections just inside the points of tangency of the curved flange. Average extreme fiber stresses in the curved flange were from 15 to 50% higher than stresses computed by the conventional formulas for flexure and direct stress in straight beams.

(*e*) A simple method of reducing section moduli and moments of inertia within the curved knee produced close agreement between observed and computed values for compressive stresses in the curved flange and also for the horizontal reaction.

(*f*) The outstanding legs of the curved-flange angles deflected away from the center of curvature under the radial component of compression introduced by the curvature. As a result, the stresses observed near the toes of the outstanding legs at various points along the curved flange were only from 40 to 70% of the stresses at corresponding points on the heels of the angles and on the edge of the web. Maximum stresses on the edge of the web were as much as 25% greater than the average values used for comparing test results with computations.

(*g*) Conventional methods of rigid-frame analysis gave horizontal reactions slightly greater than those observed; 1½% for the square-knee frame and 3½% for the curved-knee frame at normal span.

(*h*) Conventional methods gave center moments and deflections as much as 10% less than observed values for both frames.

(*i*) Center moments and deflections generally agreed closely with values computed from observed reactions and applied loads.

(*j*) Changes in horizontal reaction, center moment, and center deflection due to horizontal movement of supports also checked theoretical values fairly closely.

(*k*) No appreciable rotational restraint was developed by setting the base plates of the rigid frames on flat plates which resisted horizontal movement of the supports, but not rotation. Rigid frames in which no special precaution for preventing base rotation is taken, therefore, will act as two-hinged frames.

(*l*) The results of this investigation agreed very well in general with the results obtained at the National Bureau of Standards and with the English tests reported by J. J. Leeming and S. C. Redshaw[10] in 1939.

ACKNOWLEDGMENTS

The program for this investigation was prepared by the Technical Research Committee of the Institute, consisting of Jonathan Jones, M. Am. Soc. C. E., chairman; O. E. Hovey, Hon. M. Am. Soc. C. E.; the late H. G. Balcom, the late Aubrey Weymouth, and F. H. Frankland, H. D. Hussey, and J. R. Lambert, Members, Am. Soc. C. E. Acknowledgment is due to the members of the committee for their active interest in the work and their advice and guidance; to E. L. Durkee, M. Am. Soc. C. E., G. L. Gray, Jun. Am. Soc. C. E., and W. B. McLean Assoc. M. Am. Soc. C. E., of Fabricated Steel Construction, Bethlehem Steel Company, for the design of the square-knee model and for their valuable assistance in the theoretical analysis of the frames; to Mr. Hussey for the design of the curved-knee model; and to the members of the laboratory staff for their assistance in testing the models and the preparation of the report.

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[10] "The Testing of Two Portal Frame Girders," by J. J. Leeming and S. C. Redshaw, *Structural Engineer*, No. 17, 1939, pp. 124–133 and 156.

DISCUSSION

C. J. Posey,[11] Assoc. M. Am. Soc. C. E. (by letter).—When continuous corners of structural steel first came into use not many years ago, their design usually was based upon the methods that had proved satisfactory for the design of beams and girders. Structurally, however, a corner is very different from a beam, although it may be built with the same cross-sectional shape. The fact that serious consequences may result from designing a corner by the methods used for designing a beam is gradually becoming known. The authors' tests and the companion investigation at the National Bureau of Standards reveal some of the basic differences in structural action of the two types of members.

The most important structural characteristic of a steel corner of I-shaped cross section having a curved flange is the tendency of the web to buckle.[12] When the inside of the corner is in compression (as was the case with the frames tested by the authors) the curved flange puts a large radial compressive loading on the web, and if the web is not adequately reinforced with stiffeners, it will buckle. The tendency to buckle appears at comparatively low loads.

The existence of such an effect is recognized in the report on the curved inner flange riveted steel rigid frame tested at the Bureau of Standards. Messrs. Stang, Greenspan, and Osgood include the following statement among their conclusions:

> "The design was such that the frame failed by elastic instability. The inner-flange angles at failure were stressed to less than half their yield point. This emphasizes the necessity of providing adequate bracing for the inner flange at the knees of rigid frames of this type."[4]

What is vaguely referred to as "elastic instability" is nothing other than the tendency of the web to buckle due to the radial compressive forces. Bracing the inner flange, except by means of stiffeners, would not decrease the stress in the web.

The authors do not mention this important phenomenon at any point in their paper. Fig. 7 seems to show that sidewise deflection of the inner flange was prevented to some extent by the frame holding the specimen. Such support is seldom, if ever, provided when curved knees are used in construction. Even with this type of bracing, which decreases the effective slenderness ratio of the web and thus increases its resistance to buckling, the tendency toward "elastic instability" should become evident. The data included with the present paper do not seem to permit any comparison that would reveal this, although it may be noted that the span with curved flanges was more flexible, in comparison with computed values, than was the span with a sharp corner, although the fabrication of the latter was admittedly inferior.

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[11] Associate Engr., Iowa Inst. of Hydr. Research; Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa, Iowa City, Iowa.

[12] "Handling Corners in Rigid Frames," by B. J. Lambert, M. Am. Soc. C. E., and C. J. Posey, *Engineering News-Record*, August 4, 1938, p. 147.

W. J. ENEY,[13] ASSOC. M. AM. SOC. C. E. (by letter).—In 1938 the writer completed a study of a rigid frame celluloid model to determine the effect of the shape of the knee and footing on the reactions, moments, and deflections. The data obtained are presented herein with the belief that they will prove useful in appraising the investigation reported by Messrs. Lyse and Black. Unfortunately the study was well under way before the American Institute of Steel Construction (A.I.S.C.) tests were started; otherwise the same prototype could have been used. The prototype (shown in Fig. 22) was one of the steel frames in a 96.5-ft span highway bridge erected over the Pennsylvania Railroad on City Line, Philadelphia, Pa. The general dimensions of the frame are given in Table 4 and Fig. 23. The model was constructed of sheet celluloid 0.125 in. thick to a geometric scale of 1 in. = 30 in. At corresponding sections, the width of the model, in inches, was made numerically equal to the cube root of the moment of inertia of the structure in foot units. The initial square, filleted, knee model (designated as Model L 2) was progressively altered at the knee and footing to obtain successively Models L 3, L 4, L 5, and L 6. The general nature of each of these changes is given in Fig. 24, together with the detailed dimensions. Influence lines for horizontal reaction, reaction moment, moment at and deflection of load point 10 L, were determined for both the hinged-base and fixed-base condition.

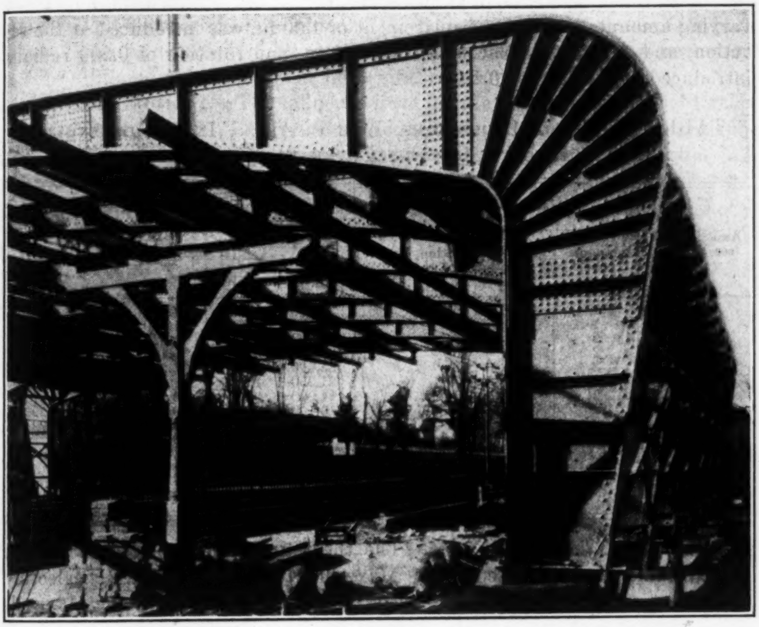

FIG. 22.—PROTOTYPE OF CELLULOID MODELS

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[13] Asst. Prof., Civ. Eng., Lehigh Univ., Bethlehem, Pa.

The deformeter apparatus used in the study is shown in Fig. 25 attached to Model L 3. The displacements of the load points were measured with a scale oriented in the direction of loads acting on the prototype. This scale, graduated to one hundred divisions per inch, was read easily with a watchmaker's loupe to an accuracy of plus or minus 0.002 in. The scale slides along a reference bar permitting the measurement of the displacement of all load points at one setting of a gage.

Two types—a reaction gage and an internal deformeter—were used. They permitted the introduction of known shear, thrust, or rotation displacements of varying amounts. A shear displacement of 1.00 in. was introduced at the reaction, and a thrust displacement of 0.500 in. and rotation of 0.200 radians introduced at load point 10 L.

TABLE 4.—FRAME DIMENSIONS: WEB PLATES, ½ IN.; FOUR ANGLES, 6 IN. BY 6 IN. BY ¾ IN.; AND COVER PLATE, 14 IN. BY ¾ IN.

Axis division (see Fig. 23)	SECTION DETAILS		MOMENT OF INERTIA OF STRUCTURE, I_s		Width of model, I_s (in.)	COORDINATES[a] (FT)	
	No. of cover plates	Depth of section, back to back of angles (in.)	In.^{4}	Ft^4		*X*	*Y*
1	1	46.5	31,600	1.52	1.150	−0.30	3.70
2	1	59	53,400	2.57	1.370	−0.85	11.15
3	2	76	125,800	6.05	1.823	−0.35	18.20
4	2	56	65,200	3.14	1.465	6.10	20.00
5	2	42	35,600	1.72	1.200	12.50	20.85
6	2	33	21,350	1.03	1.010	19.00	21.40
7	2	27	14,000	0.675	0.877	25.50	21.80
8	2	23	10,100	0.486	0.786	32.00	22.10
9	2	20	7,500	0.361	0.712	38.50	22.40
10	2	18.5	6,450	0.311	0.673	45.00	22.50
Center line	..	....	....	....	....	48.25	22.40

[a] Coordinates for the division of center structure (see Fig. 23).

Computed Influence Ordinates.—The theoretical values presented for the hinged frame were computed by the Maxwell–Mohr method (Eq. 2, neglecting the shear term); and those for the fixed frame were computed by the Maxwell-Mohr method and checked by the method of Elastic Weights. The theoretical prototype had a round knee as in Model L 2, and a base corresponding to that of Model L 6. The frame was divided into section lengths varying from 6.30 ft to 7.42 ft for the algebraic summations in Eq. 2.

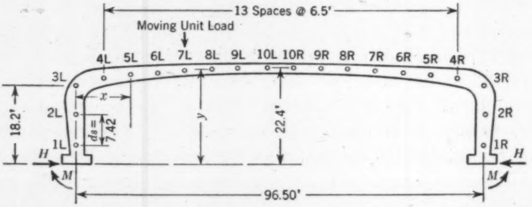

FIG. 23

Horizontal Reaction.—The influence lines for horizontal reaction are shown in Fig. 26. All of the observed values for the various modifications of the hinged end models lie along one curve. The ratio of the observed ordinates to the theoretical ordinates was 1.00 for the square-knee model L 2 and 0.975 for the round-knee model L 6. The corresponding ratios for the steel model were 0.985 and 0.965.

The effect of the modifications of the knee on the fixed frame horizontal reactions are negligible as the values for Models L 2, L 3, and L 4 are almost identical and one curve suffices for the three models. The observed ordinates are 92% of the theoretical ordinates. The reduction in the stiffness of the base of the frame as represented by converting Model L 4 to Model L 5 caused an 8.5% additional reduction in the horizontal reactions.

Reaction Moments.—The influence lines for the moment at the right reaction are shown in Fig. 27. The computed maximum moment at the right support is approximately 11% greater than the observed value for Model L 3. The modifications of the knee, changing Model L 2 to L 3 and then to L 4 and from L 5 to L 6, produced an increase in the moments of approximately 5.5%. Notching the footing, changing Model L 4 to L 5, caused a decrease of approximately 20% in the moments.

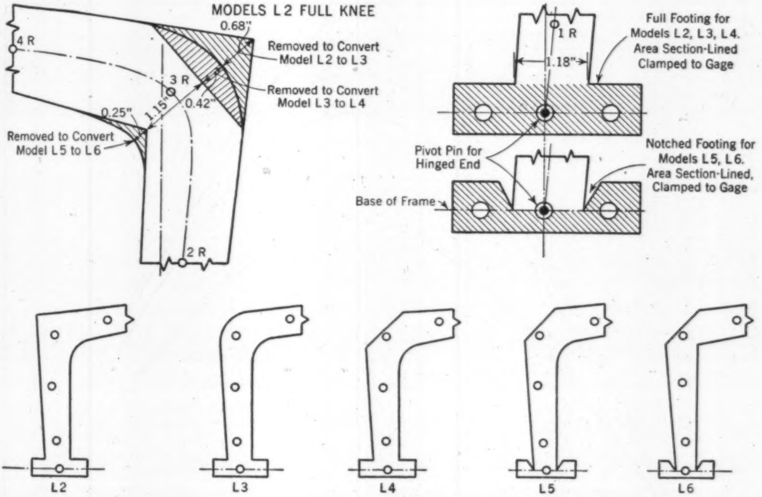
MODELS L2 FULL KNEE
0.68"
4 R
3 R
1.15"
0.42"
0.25"
2 R
Removed to Convert Model L2 to L3
Removed to Convert Model L3 to L4
Removed to Convert Model L5 to L6
1 R
1.18"
Full Footing for Models L2, L3, L4. Area Section-Lined Clamped to Gage
Pivot Pin for Hinged End
Notched Footing for Models L5, L6. Area Section-Lined, Clamped to Gage
Base of Frame
L2
L3
L4
L5
L6

FIG. 24.—DETAILS OF CELLULOID MODELS 12 TO 16, INCLUSIVE

This 20% decrease in reaction moments is accompanied by a corresponding 8.5% decrease in the horizontal reaction, as stated. Since these produce opposing moments in the frame, the reduction of the stiffness in the footing has little effect on moments at the knee or center of the frame. The moments

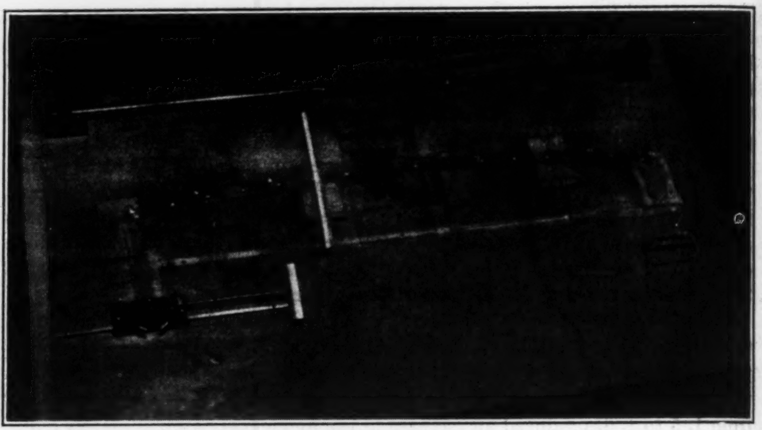

FIG. 25.—DEFORMETER APPARATUS AND MODEL L 3

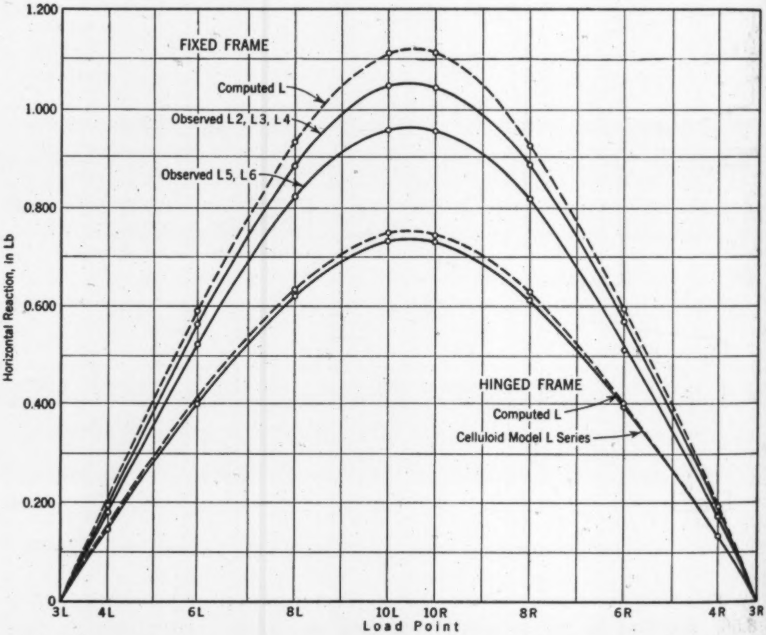

FIG. 26.—INFLUENCE LINES—HORIZONTAL REACTION, FIXED FRAME AND HINGED FRAME: CELLULOID MODELS

at the knee, computed from the observed reactions, disagreed by approximately 5% with the theoretical moments.

Influence Lines for Thrust at Load Point 10 L.—The influence lines for thrust at load point 10 L were observed for Models L 3, L 4, L 5, and L 6 for

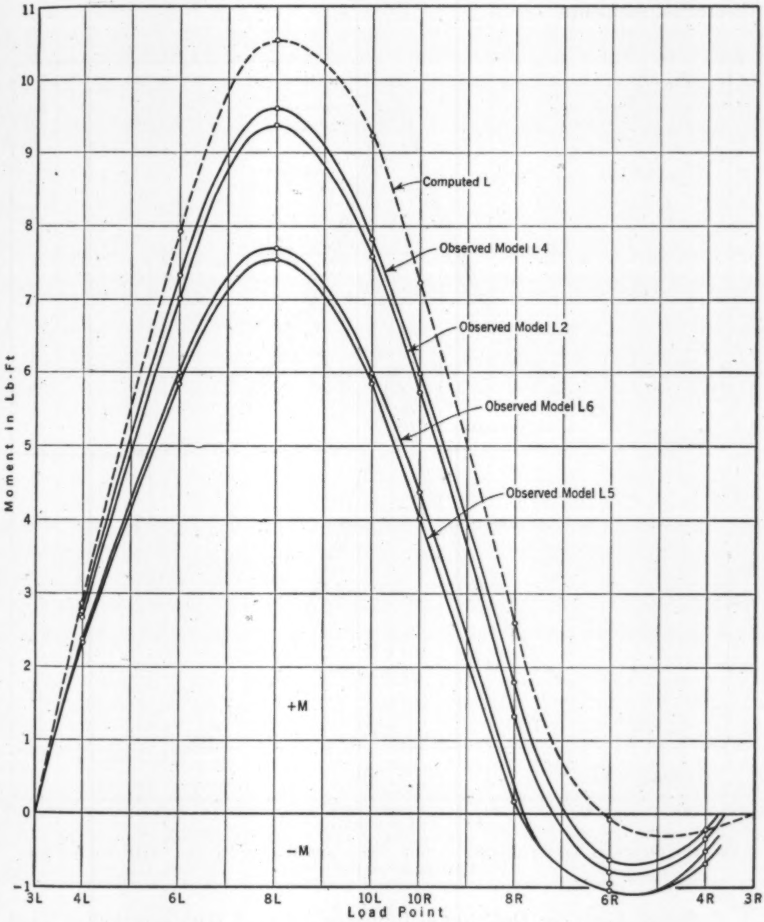

FIG. 27.—INFLUENCE LINE, MOMENT AT RIGHT FOOTING, FIXED FRAME

both hinged and fixed frame conditions. The values were in agreement with the corresponding horizontal reactions and are therefore represented by Fig. 26.

Moment Near Center Frame.—The influence lines for moment at 10 L are given in Fig. 28. The effect of the modifications of the knee and the footing on the moment at 10 L in the case of the hinged end model was negligible—

one curve fits the values for all models. In the case of the fixed frame the modification of Model L 2 progressively to L 6 caused an increase in the moment at 10 L of approximately 5%. The comparison between the theoretical and observed ordinates reveals the same relationship as was found with the steel model—the observed moments were from 6% to 11% larger than the theoretical moments.

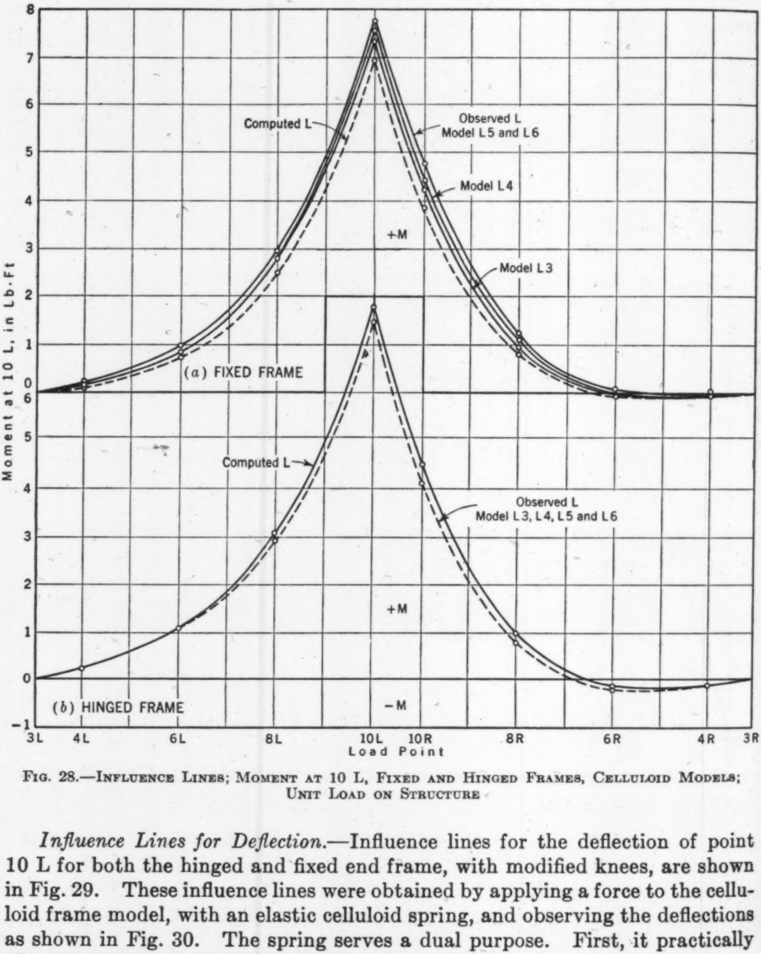

FIG. 28.—INFLUENCE LINES; MOMENT AT 10 L, FIXED AND HINGED FRAMES, CELLULOID MODELS; UNIT LOAD ON STRUCTURE

Influence Lines for Deflection.—Influence lines for the deflection of point 10 L for both the hinged and fixed end frame, with modified knees, are shown in Fig. 29. These influence lines were obtained by applying a force to the celluloid frame model, with an elastic celluloid spring, and observing the deflections as shown in Fig. 30. The spring serves a dual purpose. First, it practically eliminates the annoying creep of the plastic model; and second, it serves as a convenient means of applying an accurate force to the model, eliminating the pulleys necessary where weights are used. An identical force can be applied to different models by means of a graduated scale on the spring.

The relation between the deflection of corresponding points on the model and structure can be expressed by the equation:

$$\delta_s = \delta_m N^3 \left(\frac{I_m}{I_s}\right)\left(\frac{E_m}{E_s}\right) \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

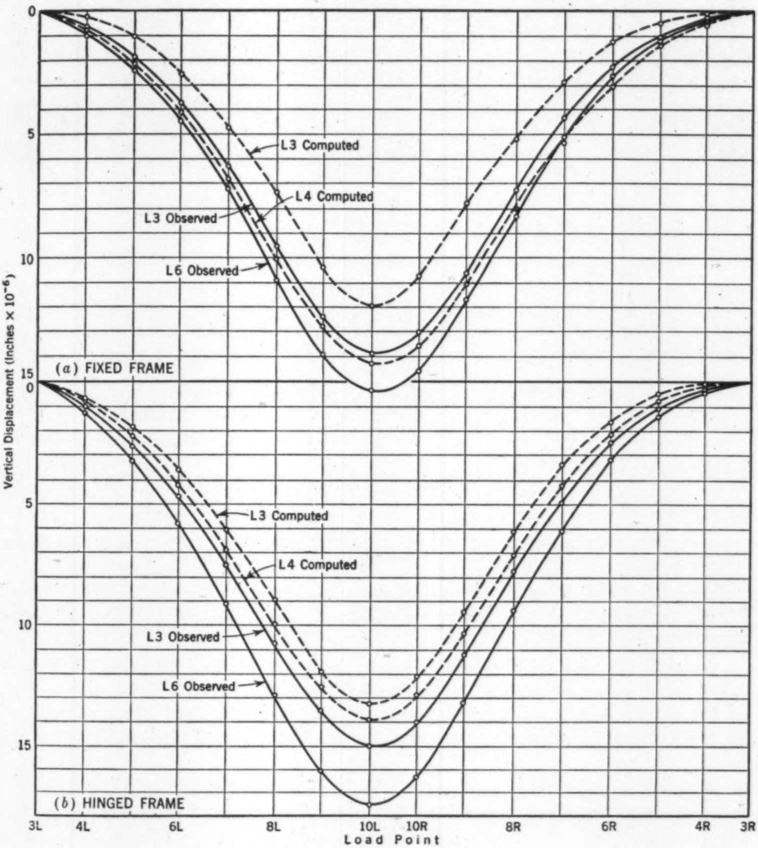

FIG. 29.—INFLUENCE LINES, DEFLECTION OF POINT 10 L, UNIT LOAD ON STRUCTURE

in which: δ_s = deflection of structure; δ_m = deflection of corresponding point on model; $\frac{I_m}{I_s}$ = ratio moment of inertias of model and structure at corresponding sections = a constant for the model, and $\frac{E_m}{E_s}$ = ratio modulus of elasticity of model and structure; and N = units of length in the structure represented by one unit of length in model.

In proportioning the model a section of the structure having a moment of inertia of 1 ft^4 was represented by a section of model 0.125 in. thick and 1.000 in. wide. The modulus of elasticity was determined to be 380,000 lb per sq in. by applying a force to a cantilever beam cut from the same sheet of celluloid as the frame model. Therefore:

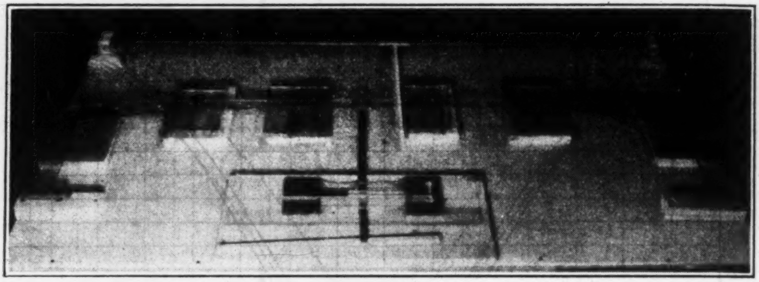

Fig. 30.—Measuring Deflections of Celluloid Model

$$\delta_s = \delta_m N^3 \left(\frac{I_m}{I_s}\right)\left(\frac{E_m}{E_s}\right) = \delta_m 30^3 \left(\frac{0.125 \times 1^3 \times \frac{1}{12}}{1 \times 12^4}\right)\left(\frac{380{,}000}{30 \times 10^6}\right) = \frac{\delta_m}{5{,}820}.$$

The frame model, therefore, deflects 5,830 times the deflection of the structure for equal loads.

The data presented in Fig. 29 show that the shape of the knee has a small effect on the frame deflections. Chamfering the outside of the knee and removing the fillet, converting Model L 3 to L 6, increased the hinged frame deflection at point 10 L approximately 15% and the fixed frame deflections approximately 10%.

The computed influence lines marked "L 3 computed" were determined by using the method of dummy unit loading and theoretical moments computed by the elastic theory, whereas the one marked "L 4 computed" was determined by the tangent offset method in which a conjugate beam was loaded with the $\frac{M}{I}$-diagram and the moments obtained by the mechanical analysis of Frame L 4. These computed deflections did not include the effect of shear or rib shortening. The deflection would be increased 0.6% due to rib shortening and approximately 2.0% due to shear. The observed deflections are from 6% to 17% greater than the computed deflections. Messrs. Lyse and Black found the observed deflections for the hinged, round knee steel model to be 11% greater than the computed deflections.

Comparing the observed deflection of Model L 3 one finds the fixed frame deflects about 90% as much as the hinged frame. This slight increase in rigidity is easily destroyed by a very slight rotation of the base. The largest observed rotation of the footing of the hinged model amounted to 0.00206 radians for a unit load of 10 L. As Messrs. Lyse and Black show, unless the footing is firmly restrained against rotation the flat base frame acts as a hinged frame.

Conclusions.—Although it is true that there is probably a vast difference between the action of a steel frame with a knee stiffened radially in Fig. 22, and the round celluloid knee of the model, the results obtained with the celluloid model nevertheless are indicative of the effect of the shape of the knee on frame action. These results are in agreement with those of Messrs. Lyse and Black and, in addition, indicate the probable effect of the knee and footing on the fixed frame.

LAMOTTE GROVER,[14] M. AM. SOC. C. E. (by letter).—The tests described have made an excellent start in the investigation of stresses in rigid-frame knees. The authors are to be commended for their thorough interpretation of results and especially for giving structural designers practical rational methods of stress analysis that will yield sufficiently accurate results at least for knees of shapes and proportions similar to those tested, when they are subjected to static loading. These methods have the virtue of focusing the attention of the designer upon the structural behavior of such knees and the stress distribtion in them, thus providing at least some orientation in the matter of designing for dynamic loadings.

It is important to recognize that this investigation, together with its companion investigation at the National Bureau of Standards, covers too small a range of knee shapes to warrant conclusions as to the best general type, such as quite a number of structural engineers seem to have assumed after reading the reports; although no such conclusions are represented by the investigators. None of the tests has included a welded curved knee. The welded knee tested by the Bureau of Standards had straight sloping inner flanges.

It was perhaps inopportune that the Lehigh riveted curved knee was proportioned similar to the one tested by the Bureau, which had confirmed the belief that serious elastic instability is inherent in such widely expanded and flexible knees as have been used for some building frames, usually to conform to architectural plans. It might have been better to strive for proportions that would provide the best distribution of stresses possible with a curved inner flange of smaller radius, decreasing the large flexible web area and increasing the compression flange stiffness. In the design tested, flange angles were used alone without cover plates.

In comparing the general behavior of the square-knee frame with that of the curved-knee frame, it must be borne in mind that the members themselves in the square-knee frame were considerably deeper and stiffer.

The riveted square knee, as tested, was designed principally with the viewpoint of facilitating fabrication, which is of paramount importance. High concentrations of stress were found at the sharp inside corner but it was concluded that these were due largely to extreme concentration of bearing at that point, resulting from shims driven in between the face of the vertical leg and the outstanding legs of the compression flange angles of the girder, where it is very difficult to obtain reliable bearing in riveted construction. Little significance was attributed by the authors to the general effect of the sharp reentrant angle. This conclusion seems highly questionable in view of the drastic effects upon stress distribution that have been observed for such sharp angles or "notch effects" in other investigations. Even higher local concentrations of stress might have been found if the corner areas had been explored more fully by differential tensometer readings.

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[14] Structural Welding Engr., Applied Eng. Dept., Air Reduction Sales Co., New York, N. Y.

It is questionable whether the normal stresses on the sections perpendicular to the axes of the leg and the girder (Fig. 9) carry much significance within the square knee and immediately adjacent to it. Normal stresses on diagonal or radial sections passing through the inner corner would seem more significant in view of the orientation of the maximum principal stresses in this vicinity, a fact recognized by the authors in discussing Fig. 14(*a*).

One of the conclusions was that any effect of plastic flow or permanent set and slippage of rivets upon the behavior of the frame as a whole was negligible. However, Table 3 shows an appreciable decrease in observed horizontal reaction and an approximately corresponding increase in observed center moments and deflections as compared with computed values, for both types of frames. Although Fig. 12 indicates elastic behavior throughout the range of loading, it must be remembered that the loading was repeated many times in gathering all the data for the investigation (because of the limited number of only six tensometers available for use). Initial loadings, prior to taking the readings for Fig. 12, could have produced permanent relative rotations between the axes of the intersecting members at the knees, which would result in initial no-load stresses for the next following tests. Under these circumstances, elastic behavior would be expected unless the initial load were exceeded. The effect of rivet slippage, and permanent set or plastic flow, might be more pronounced in a multiple-span frame with three-branch knees at the intermediate supports.

As inferred at the beginning of this discussion, the tests conducted thus far cannot be considered much more than a good start in an adequate investigation of rigid-frame knee design. To gain an impression of the ground that remains to be studied, as well as an indication of the nature of further investigations that are needed, it should be helpful to review briefly what has been done by other investigators in this field, and to point out wherein their conclusions seem to be reinforced by the results of the Lehigh tests and also where there appears to be disagreement.

In Europe, development as well as research in rigid-frame construction has been based for the most part upon welded details because of their inherently greater effectiveness in providing rigid connections with a minimum amount of material.

In Belgium and Germany extensive studies of stress distribution in rigid-frame joints of various types have been made along with many tests under static and fatigue loadings, of large and small scale steel models of built-up flanged specimens as well as plane models. Meticulous determinations of stresses and their distribution have been made throughout the webs and flanges, and transversely across the latter, by means of tensometer readings on short gage lengths—some of them differential readings, where stresses vary rapidly. These quantitative readings have been supplemented by photoelastic studies and also by the determination of stress patterns as revealed by the cracking under tensile stress of a brittle, tightly adhering coating of varnish that is applied to specimens before loading, for investigation of tensile stress, or applied while the specimen is under load, for investigation of compression stress. Some of these investigations have included three-branch joints simulating the rigid connection of a flexural member to a column or chord carrying considerable direct stress.

Investigators agree that in knees of all types, as indicated by mathematical analyses, the neutral axis lies between the centroidal axis and the inner flange, and the distribution of normal stresses departs from a straight-line distribution within the knee. Both tangential and radial stresses are concentrated at and near the inner flange, increasing considerably as the curvature or haunch angles of this flange become sharper.

There is general agreement that under usual conditions of design loading the critical section for tensile stresses as well as compression lies between the diagonal section through the center of the knee and the section where the knee joins one of the members, usually close to the latter if not directly at it. This suggests the possibility of fatigue failure at these critical sections under repeated stress, even though the frame would fail by buckling of the compression flange well within the knee under a static loading of considerably greater magnitude.

Quite a number of investigators have concluded that abrupt angles or sharply curved inner flanges must be avoided to eliminate high, local stress concentrations which are especially objectionable under dynamic loading. As in the Lehigh investigation, they have found that the radial component of the stress in a curved compression flange, which becomes greater as the curvature becomes sharper, causes transverse bending or cupping of that flange with consequent concentration of flange stresses near the junction with the web, and reduction of the effective flange area. Because of unavoidable lack of perfect symmetry of the cross section, due to slight inaccuracies in fabrication, an eccentricity develops which accentuates the effect of the radial compression force. However, when quite large radius haunches are used, the knee area is so large and the web so flexible, even though stiffened, that it will buckle under comparatively small loads and permit lateral deflection and buckling of the compression flange. It is necessary, therefore, to balance a design between consideration of stress distribution on the one hand and elastic stability on the other.

Conclusions reached from the Lehigh curved-knee test attribute major importance to the rapid change of section occurring at the junctions of the members with the knee. Belgian and German investigators have concentrated much of their attention upon this same feature. Their objective has been to reduce the rate of change in curvature at the beginning of curvature, and to avoid discontinuities at these critical sections, thereby eliminating stress concentrations or rapidly increasing principal stresses, which are distinctly revealed in the stress-line patterns by sharply curved lines. Stress lines, especially those along the inside edges of the knees, have been studied in many models involving rectilinear inner edges or flanges and geometric curves of various shapes and sizes, parabolic, hyperbolic, elliptical and sinusoidal (symmetrical and unsymmetrical or deformed, depending upon the relation between the direct stresses and bending stresses in the members joined).

An assumption commonly made by proponents of knees with rectilinear borders is that bending stresses are largely channeled in the flanges, and that the logical arrangement is that of straight flange elements combined with stiffeners in such a way as to form triangular force systems in equilibrium somewhat similar to those in trusses. Some of those technicians who favor curved inner flanges oppose the use of stiffeners or other special appendages for the transmission of stresses, because they have been held partly responsible for rather sharp transverse deviations in stress paths in the flanges to which they have been joined. It is apparent from test results, however, that curved compression flanges and adjacent web areas must be stiffened to reduce transverse bending of the flanges and to prevent lateral flange deflection and buckling in the web, unless it is practical to make the proportions of the width, thickness, and curvature of the flange and the thickness of the adjacent web such that elastic stability and good transverse stress distribution are assured. L. C. Maugh, Assoc. M. Am. Soc. C. E., has concluded from model tests at the University of Michigan, Ann Arbor, Mich., that stiffeners are required when the quantity $\frac{b^2}{4\,t\,r}$ exceeds 0.3 (b and t being the width and thickness of flange and r its mean radius of curvature). This expression does not evaluate the effect of web thickness. Professor Maugh feels that stiffeners should extend the full depth of the web so that the compression flange will receive lateral support from the tension flange through the stiffened web. Other investigators consider it sufficient to provide only small triangular transverse corner diaphragms that stiffen the compression flanges and adjacent web areas. A combination of the two types of stiffeners seems expedient, with all of them welded to the web and to the compression flange.

Unless a very decided advantage can be shown for the curved knee in the way of eliminating high stress concentrations that would contribute to real structural damage under repeated stress at the critical sections, and unless that advantage would be reflected in a substantial saving in weight of material, the economic preference will undoubtedly be in favor of the square or rectilinear knee for riveted rigid-frame bridges as well as buildings. Any such economic preference for the rectilinear knee is much less pronounced in the case of welded fabrication. One must also consider whether there is a general architectural preference for either the curved knee or the sharp-corner knee, one over the other.

The several rational methods of design that have been formulated by various investigators provide a much better insight into the behavior of rigid-frame knees than that afforded by conventional methods applied to many other commonly used structural details. Stresses in the members at sections only a short distance away from the knee have been found to conform well to computations by conventional theory. Designers should not hesitate, therefore, to use rigid frames.

Nevertheless, further tests of a comprehensive nature are needed in the United States to confirm or refute the conclusions of foreign investigations and perhaps to correct misinterpretations that have been made of meager tests in this country. This is the more important now that the completion of European tests has been interrupted by war activities. More information is needed about the influence of knees upon the behavior of a frame as a whole, especially the effects of plastic deformation or permanent set in cases of overloading, small settlements and horizontal movements of supports; also information to enable a better evaluation of the significance and importance of comparatively high shearing stresses that are known to exist within rigid-frame knees.

It is not merely the matter of a possibility of saving a comparatively small amount of material in the knee itself, as opposed to a generous proportioning of that element. The shape and type of knee and its rigidity exert a marked influence upon the stresses at the critical sections, adjacent to the members of the frame, which members, therefore, are likewise influenced at least locally. It is very desirable to evaluate these influences under repeated stress.

It may be that foreign investigators, as well as some students of stress analysis in the United States, have very greatly exaggerated the importance of sharp deviations in stress paths in the knees of rigid-frame bridges. Practical minded American engineers would surely not be willing to add much cost to their structures merely for the sake of an elegant solution to a stress problem. However, if there is really no practical value in refinements in the proportioning of rigid-frame knees, this should be proved definitely once and for all.

In many other structural details it has been proved quite important to avoid forms of details that cause abrupt disturbances in stress paths at critical points, even in statically loaded structures. It is evident from test results that there is a distinct correlation between fatigue strength and stress distribution. Therefore, it is logical to pursue further studies of rigid frames largely through fatigue tests. The most favorable proportions for each type should be determined for comparison to evaluate relative advantages. This can be done more quickly and at less expense if the fatigue tests are supplemented by static tests of models coated with brittle varnish to reveal the stress patterns and thereby guide the investigators more rapidly to a determination of the best details for resisting fatigue stresses, as well as static load stresses.

The program of further tests should be arranged objectively and without prejudice. In too much of the research work that has been done already, one type or another has been virtually disqualified at the start, as a result of subjective or premature conclusions.

WILLIAM R. OSGOOD,[15] M. AM. SOC. C. E. (by letter).—It is very much worth while to have tests checked in different laboratories. Such checking or duplication is common in many scientific laboratories such as laboratories of physics, chemistry, medicine, etc. It is not so common in engineering laboratories, possibly because of the frequently greater costs involved. When it can be done, however, it is to be welcomed. The authors have confirmed test results obtained at the National Bureau of Standards and have introduced ideas of their own. The paper contains much that can be a source of only satisfaction to those interested in rigid frames. Some points will bear discussion and comment.

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[15] Materials Engr., National Bureau of Standards, Washington, D. C.

The authors state (see "Synopsis") "In the square knee, a concentration of stress existed at the inner corner but was found to be of minor importance." In view of the fact that a comparable square-knee frame tested at the National Bureau of Standards[3] failed by buckling of the outstanding flanges of the angles at the inner corner, it would be interesting to know on what grounds the authors base their statement.

The results shown in Fig. 20 confirm the validity of the theory developed at the Bureau of Standards[4, 8] even better than the tests which led to that development, at least so far as the stresses in the flanges are concerned. Before making too much of disagreement between theory and test, it is suggested that equilibrium of the part of the frame on one side of a circular section be considered. The theoretical stresses on such a section are in equilibrium with the applied loads on either side of it. Are the observed stresses in equilibrium with these loads? If not, the observed stresses are manifestly in error; and if the necessary adjustment of the observed stresses to bring about equilibrium is of the order of the difference between observed stress and theoretical stress at any point, then it may be impossible to draw any conclusions as to how imperfect the theory is.

The "Observed Position of Neutral Axis" in Fig. 20 may be misleading. If this axis is the axis of zero normal stress on radial sections, it bears no relation to the points of zero normal stress on the circular sections shown in the figure.

The authors make the statement that for a curved knee "critical sections for the direct stress at the knee occur within 15° from the points of tangency of the curved flange." The implication is that these sections will always occur there, regardless of the positions of the applied load. The writer does not think that this will be the case. If the approximate rational method of determining the critical sections is used, as suggested in reference 4, the critical curved-flange sections for the loading shown in Fig. 11 occur at 9°.8 from the upper point of tangency and 7°.4 from the lower point of tangency. These sections are close to the sections of maximum stress shown in Fig. 18. If the ratio of P to H in Fig. 1(*b*) had been considerably larger, for example, it is probable that the upper critical section would have been found at an angle greater than 15° from the point of tangency of the curved flange.

The theory presented by the writer[8] in 1939 is not of the simplest, as the authors intimate, but on the other hand, it is not at all difficult to use. Three sets of curves are given[8] in which the ratio of stress computed by the ordinary beam theory to stress computed by the tapered beam theory is plotted against the ratio of flange area to total circular sectional area. It is only necessary then to compute the stress by the ordinary beam theory, enter the chart at the proper ratio of flange area to total area, and read the ratio of the two stresses at the curve drawn for the proper angle measured from the point of tangency of the curved flange.

It is perhaps significant that the authors do not even mention radial stiffeners and buckling of the web of the frame having the curved inner flange. Evidently there was no evidence of buckling due to radial compressive stresses, even though the average compressive fiber stress in the curved flange was as high as 24,000 lb per sq in. and the highest nearly 30,000 lb per sq in. (Figs. 18 and 10). This finding is of importance because it has not infrequently been the practice to stiffen the web radially, and "in massive doses." The authors' test was not carried to destruction, it is true; but the Bureau of Standards' test on a similar frame was, and there was no evidence that the web buckled. The radial compressive stresses were low, with probable maximum values not exceeding 12,000 or 13,000 lb per sq in. at spots. "Failure occurred suddenly, by lateral deflection of the curved inner portion of the frame," [4] that is, as a sidewise displacement of the inner flange as a whole, not as a curling over of the flange. This kind of failure can only be prevented by lateral bracing—radial stiffeners are useless to prevent it.

M. HIRSCHTHAL,[16] M. AM. SOC. C. E. (by letter).—Tests of large-scale models of indeterminate structures are of the utmost value to the engineering profession as a verification of theory (or its disproof), as well as for the clarification or decision of controversial questions in connection therewith. However, to accomplish such results, meticulous care must be exercised so that neither a predetermined theory nor methods pursued in similar investigations exercise any influence on the researcher in his tests. It is to be feared that the authors permitted themselves to be somewhat influenced in both ways. There is no question of the great advantage to be gained in planning tests that will either confirm or deny results previously obtained by the selection of similar forms for tests, but no attempt should be made to read into the results identity with the prototypes (tests) or to resort to "modified sections" to confirm theoretical analysis in which assumptions must necessarily be made as a basis for logical results. The tests were well planned, yet it is questionable whether the degree of accuracy was sufficiently great when the variations between assumed theory and stresses obtained from strain readings are of approximately the same amount at times as the expected accuracy of 300 lb per sq in. in the instruments used. The authors state that the number of readings or observations was governed by the desirability of limiting the number of holes made in the angles. Could not the remedy have been found in welding and thus this difficulty been eliminated? Gage lines along the gravity lines of each of the angles of the curved inside section might have furnished valuable information. In selecting the configuration for the curved-knee frame (Fig. 1(*b*)), the authors made the crown thickness 6 in., carrying the soffit of the beam parallel to the extrados, whereas the leg is $7\frac{1}{2}$ in. thick at the base. The result is that the sections at the diagonal through the knee are not equal—the vertical section through the beam is $13\frac{7}{8}$ in. deep and the horizontal section through the leg is $15\frac{3}{8}$ in. deep (Fig. 31). In the case of the square-knee frames, these sections are of the same depth, which is preferable.

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[16] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

An examination of Fig. 18 discloses the fact (and it is pointed out by the authors) that, from the tangent point 22 (opposite 2) around the curve to the tangent point 32 (opposite 19), the observed stresses (computed from the strains) are considerably higher than those computed by the "beam formula." Beyond these points, both in the horizontal beam and in the vertical legs, the stresses are lower than those computed theoretically. The writer differs from the authors on two points as to the reasons and the results of these conditions.

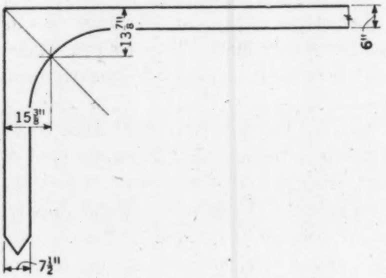

FIG. 31

First: Since points 32 and 22 and the points opposite them are subjected respectively to the maximum compressive and maximum tensile stresses, it is evident that (*a*) the sections through these points (practically the points of tangency) are at the points of maximum negative moment of the respective horizontal and vertical spans; (*b*) that they are the "fixed ends," and (*c*) that the section between, or the knee section, furnishes the restraint for such fixedness. Under such conditions (Fig. 11) the stress distribution along a diagonal across the knee would not follow the "straight-beam" theory, but rather, more nearly, that resulting from a thrust against this diagonal section. The resistance to this thrust is offered by the remainder of the frame (the other half of the knee and the leg) down to the hinged support (Fig. 32). The bending moment at the end of the beam span, resulting in tension in the upper plane and compression in the lower, together with the vertical shear, may be considered replaced by a thrust acting eccentrically on the section, the eccentricity being equivalent to that resulting in the aforementioned bending moment. From the observed location of the zero strain (therefore, zero stress), the resultant thrust is evidently applied very close to the inside edge of the diagonal. The point of zero stress under this assumption would be located at three times the distance from the inside edge to the point of application of the resultant, similar to the condition of a

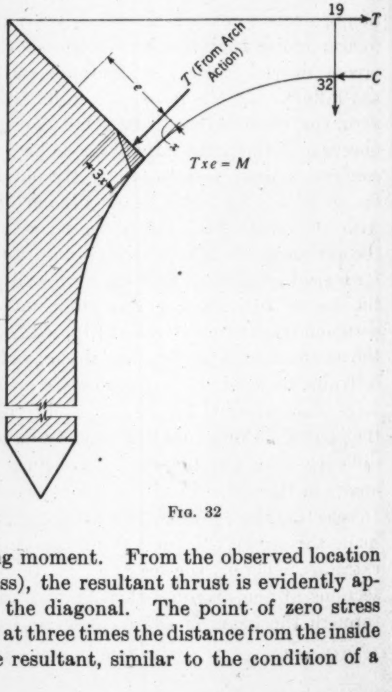

FIG. 32

footing on rock or other unyielding material, in which the resultant of the pressure falls outside of the middle third of the base. As long as there is no yielding under the point of maximum compression (in other words, if the surface resists distortion), there is no possibility of a tensile stress at the outer edge.

Second: From the fact that the extreme fiber stresses between the tangent points of the horizontal and vertical members, as computed from the observed strains, were considerably greater than those calculated from the theoretical assumptions, the authors conclude that the knee was "not stiff enough." The writer differs with this conclusion, and feels that on the contrary the knee, because of its very stiffness, was capable of taking greater stresses than assumed. Thus it relieved the sections of the horizontal beam, and the vertical legs beyond the knee, of part of the stresses or moments, resulting in the lower actual observed stresses noted in those sections. The situation is similar to that occurring at the junction of two members, such as a column and a beam, subjected to a bending moment, in which case the stiffer member takes the larger share of the moment. Likewise, within a member itself the section having the greater stiffness will take the greater part of the moment. Perhaps this is the explanation of the fact that a reinforced-concrete beam never yields a deflection anywhere near that calculated for the particular condition of loading. A well-proportioned beam has about one third of its tensile reinforcement bent up for shear and diagonal tension, and this reinforcement is carried into the opposite plane for proper anchorage, thus providing a cross section at the supports far stiffer than that assumed for the theoretical consideration of a reinforced-concrete beam design from which the deflection is ordinarily computed.

The condition at the knee is not unlike that posed when a deep girder is connected with a relatively light column and the column selected is found to be overstressed. An increase in the column section will result only in an increase in the share of the bending moment it will be required to resist. Thus, it is evident that, to relieve the column of its overstress, it will be necessary to increase the girder section at its junction with the column so as to take a greater share of the moment because of its increased stiffness. This, however, does not end the problem, which cannot be treated herein.

Square Knee.—In Fig. 16, which shows the distribution of stress for the square-knee section, it is to be noted that the compressive stress in the corner (the junction of the soffit of the horizontal beam with the vertical leg), from gage observation, is much higher than that which had been computed—the theoretical stress. F. E. Richart, M. Am. Soc. C. E., in his tests of reinforced-concrete knees at the University of Illinois, found a similar concentration of stress at the inside corner of the specimen. At that time, the writer had suggested that this might have resulted from the fact that the horizontal section through the leg and the vertical section through the beam, at their junction, have a common extreme fiber in compression and such a concentration of stress was to be expected. Where these sections are at right angles to each other, the resultant compressive stress would be the square root of the sum of the individual squares of the individual extreme fiber compressive stresses; and, where the depth of section and reinforcement are the same in both sections, this resultant stress would be 1.41 times the compressive stress of either. This would also apply to the structural steel section. This assumption appears to be practically verified in Fig. 16 where the observed stress in the horizontal member is about 22 kips, whereas the theoretical stress is only 15 kips per sq in., although the section for the vertical member does not quite corroborate this value. For the foregoing reason, the writer would suggest including this consideration in the recommendation for design that the authors include in recommendation 1.

In the opinion of the writer the results of the tests are not quite conclusive and do not justify the other definite conclusions embraced in the recommendations.

JAROSLAV J. POLIVKA,[17] M. AM. SOC. C. E. (by letter).—The investigation is a valuable contribution to the analysis of framed structures with variable moments of inertia. In the analysis of such types of structures, certain assumptions are made, and it is important to know how nearly these assumptions conform with the actual behavior of the structure, as proved by reliable tests.

The authors' comparison between the results of the tests and conventional methods of analysis shows that a fair conformity may be obtained. The observed horizontal reactions differ slightly from the computed values, considering the influence of shear on deformation; but the computed center moments and deflections are as much as 10% less than observed values—a fact affecting, seriously, the design of the structure and its safety.

The writer has analyzed, completely, both types of hinged frames subjected to investigation, using the "ellipse-of-elasticity" method, and has found a very good conformity with the results of the tests. The method is essentially graphical. Better accuracy may be obtained by expressing the constants of the ellipse of elasticity by algebraic values. Generally, the method requires some knowledge of geometry (especially projective geometry), but the analysis is greatly simplified in cases where the change of moment of inertia can be expressed by algebraic terms permitting the use of tabular values. Three of the most common of such cases are demonstrated in Fig. 33.

Eqs. 1 and 2 are correct as long as the sections Δs are very small so that the elastic centroid of the section may be properly assumed as coinciding with the geometrical center of gravity. For sections in Fig. 33 (relatively long with respect to the change of the depth of the structural part) the elastic centroid approaches considerably nearer to the smaller cross section. It affects the elastic weight $\frac{\Delta s}{E\,I}$ and other elastic constants of the section considered, such as semi-axes of ellipse of elasticity, angular rotations, and respective centers of elastic rotations.

In Fig. 33, let: s = distance between elastic centroid S and face B; G = the elastic weight of the element $A\ B$, concentrated at elastic centroid S; and d = the distance between center of rotation (produced by a force F acting along the face B) and face B. Then the following equations express the elastic

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[17] Research Associate, Univ. of California; Cons. Engr., Berkeley, Calif.

deformations of the face B with respect to the face A due to pure bending:[18]
Angular rotation—

$$\Delta\delta = F\,G\,s \quad \ldots\ldots\ldots\ldots (7)$$

Displacement along the line of action of the force F—

$$\Delta = d\,\Delta\delta = F\,G\,s\,d \quad \ldots\ldots\ldots\ldots (8)$$

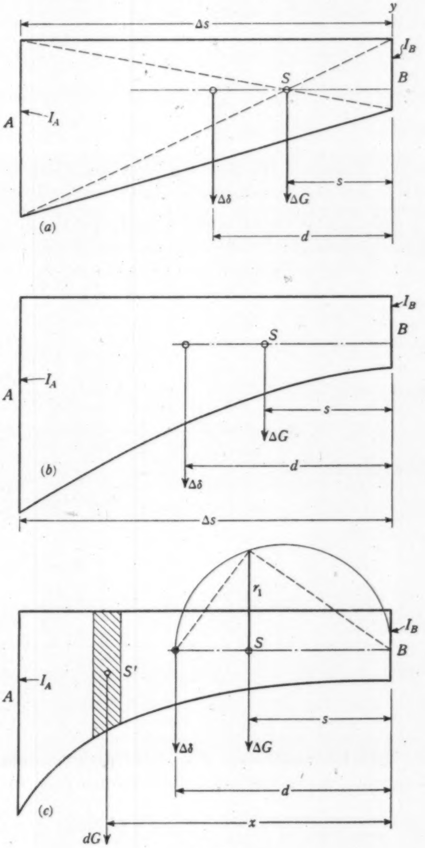

FIG. 33

Moment of inertia of elastic weight with respect to the face B—

$$I_G = G\,s\,d \quad \ldots\ldots (9)$$

Square of longitudinal semi-axis of ellipse of elasticity—

$$r_1^2 = (d - s)\,s \quad \ldots. (10)$$

Considering the effect of shear on deformation, Eq. 9 becomes (see Fig. 33)—

$$I_G' = \int_0^{\Delta s} \left[x^2 + (r_2')^2\,k\,\frac{E}{S} \right] dG \,. \quad (11)$$

in which: S is the modulus of elasticity in shear; r_2' is the radius of gyration of an element ds; and k is a numerical constant depending on the cross-sectional shape of the member (section). For prismatic members the value of k is $\frac{6}{5}$; and for **I**-shaped members it is approximately $\frac{A}{A'}$, in which A is the full cross-sectional area and A' is the cross section of the web.

The elastic constants of sections having the shapes shown in Fig. 33 are computed as follows:[19]
Straight line—

$$G = G_B\,\frac{2 + C}{2\,(1 + C)^2} \quad \ldots\ldots\ldots\ldots (12a)$$

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[18] "Graphical Analysis of Framed Structures on the Basis of Ellipse of Elasticity," by Jaroslav Polivka, Soc. of Eng. Students, Institute of Technology, Prague, 1919; also, "Kontinuierlicher Bogentraeger auf elastisch nachgiebigen Stuetzen," by Jaroslav Polivka, *Der Brueckenbau*, 1920, Vols. 4, 5, and 6.

[19] "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by Jaroslav Polivka, Berkeley, Calif., 1940 and 1941.

in which I_B is the moment of inertia of cross section at B; and,

$$G_B = \frac{\Delta s}{E\,I_B} \ldots\ldots\ldots\ldots (12b)$$

$$C = \sqrt[3]{\frac{I_A}{I_B}} - 1 \ldots\ldots\ldots\ldots (12c)$$

$$s = \Delta s \frac{1}{2 + C} \ldots\ldots\ldots\ldots (12d)$$

$$d = \frac{\Delta s}{C^3} [2 \log_e (1 + C)(1 + C)^2 - C (2 + 3\,C)] \ldots\ldots (12e)$$

and

$$G\,s\,d = G_B \frac{\Delta s^2}{C^3} \left[\log_e (1 + C) - C \frac{2 + 3\,C}{2\,(1 + C)^2} \right] \ldots\ldots (12f)$$

Eq. 12*d* is determined exactly by the intersection of diagonals.

Parabolic—

$$G = \frac{G_B}{8} \left[\frac{5 + 3\,C}{(1 + C)^2} + \frac{3}{\sqrt{C}} \text{arc tan} \sqrt{C} \right] \ldots\ldots (13a)$$

$$G\,s = G_B\,\Delta s \frac{2 + C}{4\,(1 + C)^2} \ldots\ldots\ldots\ldots (13b)$$

and

$$G\,s\,d = \frac{G_B\,\Delta s^2}{8} \left[\frac{C - 1}{C\,(1 + C)^2} + \frac{1}{C\,\sqrt{C}} \text{arc tan} \sqrt{C} \right] \ldots\ldots (13c)$$

Sharply curved—

$$G = G_B \frac{1 + b}{2} \ldots\ldots\ldots\ldots (14a)$$

in which

$$b = \frac{I_B}{I_A} \ldots\ldots\ldots\ldots (14b)$$

$$s = \Delta s \frac{1 + 2\,b}{3\,(1 + b)} \ldots\ldots\ldots\ldots (14c)$$

and

$$d = \Delta s \frac{1 + 3\,b}{2\,(1 + 2\,b)} \ldots\ldots\ldots\ldots (14d)$$

Elastic constants for these three shapes are compiled in tables.[20] For example, using the tabulated values p, q, and u:[20]

$$G = (p + q)\,G_B \ldots\ldots\ldots\ldots (15)$$

$$s = \frac{p}{p + q} \Delta s \ldots\ldots\ldots\ldots (16)$$

and

$$d = (1 - u)\,\Delta s \ldots\ldots\ldots\ldots (17)$$

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[20] Discussion by Walter Ruppel, Assoc. M. Am. Soc. C. E., of "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

These formulas are valid for each shape shown in Fig. 33. Only the values of d are affected by shear. Considering the shear, the moments of inertia of the elastic weight with respect to face B are $I_G' = I_G + I_G''$, in which the values of I_G'' (due to shear) are given by the following equations (in which $(r_2')^2 = \frac{I_B}{A_B}$):

Straight line—

$$I_G'' = k\,\frac{E}{S} \times (r_2')^2 \times \log_e \frac{1 + C}{C}\,G_B \ldots\ldots\ldots\ldots\ldots (18a)$$

Parabolic—

$$I_G'' = k\,\frac{E}{S} \times (r_2')^2 \times \frac{1}{\sqrt{C}} \text{arc tan } \sqrt{C}\,G_B \ldots\ldots\ldots\ldots (18b)$$

Sharply curved—

$$I_G'' = k\,\frac{E}{S} \times (r_2')^2 \times \frac{3}{4} \times \frac{1}{1 - b}\,(1 - \sqrt[3]{b^4})\,G_B \ldots\ldots\ldots (18c)$$

The framed structure is divided into sections allowing the application of the foregoing formulas (Fig. 34). The redundant horizontal reaction is computed by that one of the following methods of analysis, based on the principles of the ellipse of elasticity, which appears most suitable for a given case:

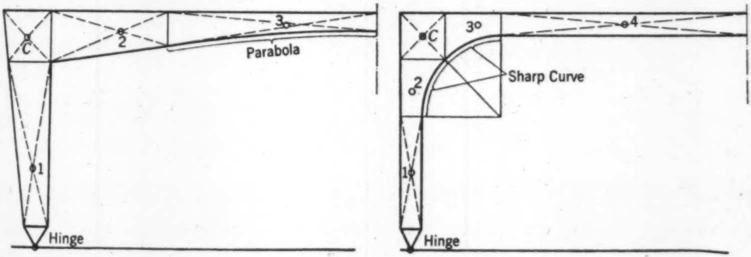

FIG. 34

(1) Compute the angular rotation $(\delta)_M$ for a simply supported beam $A\,B$ due to the given loading, and its center D_M (Fig. 35(a)). The horizontal reaction is then

$$H = \frac{(\delta)_M}{G\,s_H} \times \frac{d_M}{d_H} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which G is the total elastic weight of the framed structure concentrated at the elastic centroid S; s_H is the lever arm of the elastic weight G with respect to the line of action of the force H (passing through hinges C and D); d_H is the perpendicular distance between the center of rotation D_H of the force H and the line $C\,D$, D_H being the antipole of $C\,D$ with respect to the ellipse of elasticity of the whole structure; and d_M is the perpendicular distance between the center of rotation D_M of a simply supported beam $A\,B$ due to the given loading.

(2) Consider $A\ B$ as fixed at A and B (Fig. 35(b)), and compute the angular rotations of the columns at A and B due to unit moment. These angular rotations define and determine the elastic weights G_A and G_B of supports A and B. Combine the elastic weights of the supports with the elastic weight G_0 of the beam $A\ B$ and determine the end moments M_A and M_B of the restrained beam.

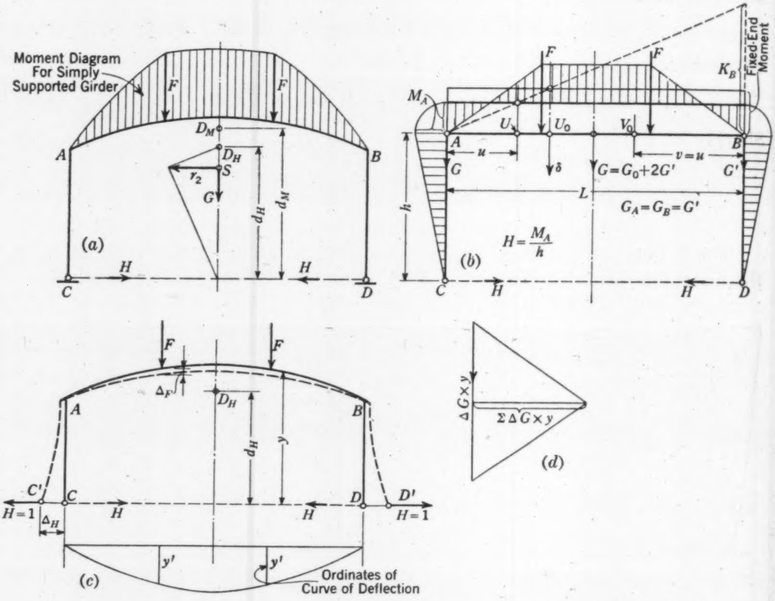
Moment Diagram For Simply Supported Girder
F
F
D_M
D_H
S
r_2
G
d_H
d_M
A
B
(a)
H
H
C
D
F
F
M_A
K_B
Fixed-End Moment
U
U_0
V_0
A
B
u
$v=u$
G
δ
$G=G_0+2G'$
G'
L
$G_A=G_B=G'$
h
$H=\frac{M_A}{h}$
(b)
C
H
H
D
F
F
Δ_F
D_H
A
B
d_H
y
C'
C
D
D'
$H=1$
Δ_H
H
H
$H=1$
y'
y'
(c)
Ordinates of Curve of Deflection
$\Delta G\times y$
$\Sigma\Delta G\times y$
(d)

FIG. 35

(3) Determine the deflection Δ_F at the point of application of the force F and the displacement Δ_H along $C\ D$ due to a force $H = 1$. From the basic principle of reciprocity of deformations, the magnitude of the horizontal reaction is found to be:

$$H = -\frac{F\,\Delta_F}{\Delta_H} \qquad (20)$$

(4) When the loading consists of a group of concentrated forces, or of uniformly distributed weights, it is preferable to draw the deflection curve due to $H = 1$, acting along $C\ D$, with the pole distance $\Sigma\Delta G\ y$, which is the influence line of the redundant force H (Fig. 35(c)):

$$H = -\frac{F\,y'}{d_H} \qquad (21)$$

in which d_H has the same significance as in Fig. 35(*a*). As far as the stress distribution within the knee of the investigated frames is concerned, the results reported by the authors were compared with photoelastic tests, at the University of California,[21] relating to stress concentrations at sharp reentrant angles and curved fillets.

Conclusion.—The tests reported in this paper are checked very closely by the analysis outlined herein.

A. C. Barrow,[22] Assoc. M. Am. Soc. C. E. (by letter).—The data presented in this paper can be tested by applying the formula:

$$f = \frac{P}{A} \pm \frac{P\,e\,c}{I} \qquad (22)$$

in which: f represents a unit stress at either edge of the section, depending on whether the plus or minus sign is used; P represents the force acting on a section, symmetrical about at least one axis through the center of gravity of the section; A represents the area of section under consideration; e equals the eccentricity of the line of action of the force F; c is the distance from the center of gravity of the section to the edge of the section where the value of f is to be determined; and I is the moment of inertia of the section about an axis through the center of gravity and at right angles to the axis through its center of gravity and the point of application of the force P. The flange stresses throughout the rigid frame can be computed by Eq. 22, both for square knees and curved knees.

The Curved Knee.—The knee in these frames (see Fig. 18) may be considered to be loaded or stressed by a moment produced by an eccentric loading, consisting of the horizontal reaction acting through the hinge and the vertical reaction acting through the same point and decreasing the resulting moment of the horizontal reaction.

The writer made a drawing of this frame to a scale of 6 in. equals 1 ft, and the lever arms of these forces were scaled from it. First, a vertical section was passed through point 27, Fig. 18. The moment of the horizontal reaction about the center of this section was obtained. This was decreased by the moment of the vertical reaction of 6,000 lb about the same point. The result was multiplied by the distance of the outermost fiber from the center of the section, and this result was divided by the moment of inertia of the section about its center and at an angle of 90° with the vertical axis of the plate. The horizontal reaction of 4,350 lb was divided by the area of the vertical section. This latter value was subtracted from the result already obtained to find extreme fiber stress in the outer flanges and added to find the stress in the inner flanges. The gross moment of inertia was used. The author states (see heading "Relation of Test Data to Analysis: Moment of Inertia") that "The effective moment of inertia was slightly less than the gross moment of inertia * * *," and that "If the gross moment of inertia had been used, the error would have been about 3%, for this girder." Therefore, 3% was added to the foregoing results. The inner edge of the knee was at an angle of 45° at point 27 and the value found for the inner flanges was multiplied by the secant of 45°.

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[21] "Some Stress Relationships in Photoelasticity," by J. J. Polivka and H. D. Eberhart, Assoc. M. Am. Soc. C. E., Proceedings of the Tenth Semi-Annual Eastern Photoelasticity Conference, Cambridge, 1939; also "A New Method of Determination of Separate Stresses from Photoelastic Results on the Basis of First Derivatives of Airy's Function," paper presented by J. J. Polivka before California-Stanford Seminar, March 6, 1940.

[22] Cons. Engr., Mount Sterling, Ky.

TABLE 5.—COMPARISON OF OBSERVED AND COMPUTED STRESSES
(In Kips per Square Inch)

Description	POINT (SEE FIG. 18):														
	3	4	5	6	7	14	15	16	17	18	27	28	29	30	31
Eq. 22[a]	17.5	18.9	16.4	12.5	10.5	8.2	11.4	15.2	17.6	16.2	13.8	15.1	18.4	19.9	18.2
Fig. 18	18.5	18.0	16.0	14.0	11.0	10.0	13.0	16.0	18.5	19.0	14.0	15.5	18.5	22.0	22.5

[a] Computed values rounded out to nearest 100 lb per sq in.

Table 5 shows the results of the values thus determined compared with the observed values as shown in Fig. 18.

A different procedure is required for finding the flange stress in the vertical leg of this frame. It is obvious that the flange stress at point 26 is the same as that at point 28, except that it carries its proportion of uniformly distributed force, producing the vertical reaction of 6 kips, over a horizontal section through point 26. Fig. 18 shows the unit stress at point 26 to be slightly more than at point 28. From this consideration there should be a difference of about 1 kip. The difference in the stress at these points increases down the vertical leg, and this is as it should be because the total stress is the same (6 kips) and the area of the section decreases to and including point 23.

For point 27, the agreement is close (Table 5), but for other points on the inner edge of the knee the difference between the computed values and the observed values increases. Eq. 22 is valid only for short increments of length, and it is the elasticity of the frame at these points that causes a greater stress and increased differences. They are given to show how far Eq. 22 may be used to check the observed values.

The Square Knee.—Applying Eq. 22 to the square knee and using a horizontal reaction of 5,280 lb, the stress at the inner corner is 16,571 lb per sq in. and the observed stress is about 17,000 lb per sq in. Just above this point, there is a computed stress of 14,473 lb per sq in. and an observed stress of about 14,000 lb per sq in. (Fig. 16).

In the paper it is stated (see heading "Test Specimens") that

> "At the intersection of the compression flanges at both knees, where tight bearing should be obtained, small gaps existed. It was considered advisable to fill these gaps with shims that were tackwelded in place, but which produced tight bearing only along the outstanding legs of the girder flange angles."

This connection produces another case of eccentric loading, and Eq. 22 determines a stress of 23,055 lb per sq in. at this point. Fig. 16 shows an observed stress of 23,000 lb per sq in. in the horizontal flanges some distance from this inner corner. The stress decreases rapidly from the inner corner and, if this line is produced to a point in the graph directly opposite the inner corner, a stress of 25,000 lb per sq in. would be indicated.

The observed and computed stresses both show clearly the danger of imperfect bearings at the inner corner of the rigid frame with a square knee.

W. E. BLACK,[23] JUN. AM. SOC. C. E. (by letter).—Those who submitted discussions have contributed many interesting comments and constructive criticisms. It was gratifying to note that the results of the tests made at the National Bureau of Standards were brought into the discussion frequently. The two investigations represent the only experimental work on steel rigid frames reported in the United States to date (1941), and any discussion of the analysis or design of steel rigid frames should rightly refer to both. Much of the discussion centered about two controversial points—the concentration of stress at the sharp inner corner of the square knee, and the stability of the curved knee. Therefore, each will be discussed individually.

Concentration of Stress in Square Knee.—Several of the discussers have taken exception to the writers' conclusion that the concentration of stress at the inner corner of the square knee was of relatively minor importance. In a continuous and elastic knee of this type, it is generally recognized that the compression stress at the inner corner of the knee is extremely high under the usual manner of loading. Photoelastic tests have demonstrated this fact incontrovertibly. However, it is the writers' contention that, due to the method of fabrication, the type of knee tested does not act as a continuous member. The reactions of the horizontal girder upon the inner face of the column may be assumed to consist of the tension in the top flange, the compression in the bottom flange (both acting approximately at the flange centers of gravity), and a vertical shear. This assumes that the web transmits very little, if any, of the moment or thrust across the connection, a contention that is supported by Fig. 9.

If the action of the knee is represented accurately by the foregoing assumptions, the intensity of stress at the inner corner will be directly dependent upon the type of bearing obtained. In the Lehigh tests, an extreme concentration of bearing was encountered, and a high stress resulted. However, the average stress in the compression flange was not excessive. In the Bureau of Standards tests, tight bearing was not obtained, and the maximum stress in the compression flanges at the inner corner of the knee was comparatively low. Mr. Osgood points out that failure in the Bureau of Standards' square knee occurred by local buckling of the compression flanges adjacent to the inner corner. Is it strange that such would be the case when the failure load was two and one half times the "design load," and when any accepted method of analysis will locate the point of maximum stress in the knee at the inner corner? Furthermore, in the usual rigid-frame knee, the compression flange will include cover plates, and local buckling will be less likely to occur, and probably not before the yield point of almost the entire compression flange is reached. However, the possibility of this type of failure should be recognized, and steps should be taken in design to prevent its occurrence.

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[23] Instr., Dept. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

Mr. Grover warns of the possibility of fatigue failure at the knee. The writer believes that repeated stress will not be an important factor in the design of steel rigid frames of this type since a reversal of stress is not likely to occur. The endurance limit of structural steels, tested from zero to a maximum stress in either tension or compression, is well above the yield point. A fatigue failure at the point of greatest stress concentration, the inner corner, is practically impossible since it is almost always subjected to a compression stress. A tensile failure at the opposite side of the member is equally improbable since the greatest stress there will seldom be as high as the allowable working stress.

One of the worst mishaps that may befall a square type rigid-frame knee is the occurrence of a loose fit at the inner corner. This condition would be equivalent to a decrease in stiffness of the knee. Although complete failure could not result from this cause, there would result excessive stresses and deflection at midspan. On the other hand, an extreme concentration of stress at the inner corner, if not accompanied by local buckling of the compression flanges, would be relieved by the ductility of the metal. In general, even though the stress at the inner corner should reach the yield point, nothing serious would happen until practically the entire compression flange was so stressed, a condition which could not be obtained until the design load was greatly exceeded under the method of design recommended by the writers.

In Mr. Hirschthal's discussion of the maximum compression stress at the reentrant corner of a rectangular rigid-frame knee, he is perhaps underestimating the magnitude of the theoretical maximum compression stress. The determination of this stress is scarcely as simple as Mr. Hirschthal suggests. T. J. Dolan[24] found by photoelastic tests that the maximum compression stress at the reentrant corner of a similarly-shaped bakelite model was about 2.5 times the stress computed by the flexural theory. This stress, however, is extremely local, and in the case of a riveted steel rigid frame, it may be much lower, depending upon the condition of bearing at the corner, the degree of continuity in the knee, and the angle of the reentrant corner.

Stability of the Curved Knee.—Attention was focused on the instability of certain types of rigid-frame knees by the results of tests on rigid-frame knees at the Bureau of Standards.[3, 4, 5] A knee with a large circular fillet (very similar to the Lehigh curved knee frame) failed by lateral buckling of the entire curved flange at a load but little greater than the design load. Similar knees having sharp reentrant corners at the inside of the knee supported more than twice their design loads and then failed by local buckling of the compression flange near the reentrant corner. Each of the latter specimens had at least one pair of web stiffeners at the knee. Whether or not web stiffeners would have prevented buckling of the curved knee is problematical. Professor Posey is inclined to think they would, whereas Mr. Osgood expressed the belief that lateral bracing would be necessary to properly stiffen such a knee frame.

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[24] "An Investigation of Rigid Frames, Part I, Tests of Reinforced Concrete Knee Frames and Bakelite Models," by F. E. Richard, T. J. Dolan, and T. A. Olson, *Bulletin No. 307*, Univ. of Illinois Eng. Experiment Station, November, 1938.

Mr. Grover suggests that web stiffeners may be used to reduce lateral deflection of the compression flange as a whole, and, in addition, stiffen the outstanding legs of the compression flange against transverse bending of the type found in the Lehigh tests. The writers are not at all certain that web stiffeners would prevent, completely, lateral buckling of the curved flange of a knee of this type. However, the compression flange should be stiffened to prevent transverse bending of the outstanding legs, and a web stiffener welded to or bearing tightly against the compression flange, as suggested by Mr. Grover, might help to solve both problems. Further tests should be made to determine the effectiveness of web stiffeners in preventing lateral buckling of knees of this type.

It is to be regretted that more information regarding buckling was not obtained in the writers' tests, but since the primary purpose of the investigation was the determination of stress distribution, other related phenomena had to be neglected because of time limitations. However, as Mr. Osgood states, with the curved flange supported laterally at one point, no web or flange buckling was noticed in the Lehigh curved-knee frame, even though the flange stresses developed were as high as 29,000 lb per sq in. Further information regarding the ultimate strength of rigid frames may be forthcoming, as the American Institute of Steel Construction has made arrangements for testing to destruction the two rigid frames which were tested under design loads by the writers.

General Discussion.—Professor Eney has described a method of investigation of structural frame models that is particularly well adapted to studying the effects of varying the relative proportions of different parts of the frame. Thus he has investigated the effects of varying the stiffness of both the knee and the footing of a fixed frame, a procedure which could not be followed for larger steel models without prohibitive expense. It is gratifying to note that where his results can be compared with those obtained by the writers, a qualitative agreement was obtained.

The writers are indebted to Mr. Grover for his excellent résumé of the work done in this field. With his recommendations regarding further tests the writers are in whole-hearted agreement.

Mr. Osgood's comment regarding the position of the neutral axis in Fig. 20 is quite correct. The observed neutral axis for stresses on the circular sections would probably fall quite close to the points of zero stress shown in the figure.

Mr. Hirschthal points out the fact that the observed stresses near the tangent points of the curved knee were greater than computed stresses, and that away from the knee, observed stresses were slightly the lower (Fig. 18). It might be well to add that the observed stresses at midspan were slightly greater than corresponding computed stresses. The reason for these differences between computed and observed stresses away from the knee is that the computed stresses were determined from the theoretical external reactions, and there was about 3.5% difference between the theoretical and observed horizontal reactions for the curved knee frame. Using the observed external reactions for the computation of flange stresses away from the knee, the agreement with observed stresses was almost exact.

Mr. Hirschthal's discussion of the curved knee includes one misleading item. If, in Fig. 32, the resultant thrust T is meant to be the resultant of the three forces shown at the section through points 19 and 32, it is somewhat misplaced. Fig. 1(*b*) may be used to clarify the matter. The force T must be equal, collinear, and opposite to the resultant of the pin reactions at the base of the vertical leg ($P = 6{,}000$, $H = 4{,}500$ approximately), and therefore must intersect the diagonal section of Fig. 32 at a point almost coincident with the center of curvature of the curved flange, and not within the section at all. To resist such a loading, considerable tension, as well as compression, must be developed within the section. A consideration of the forces acting upon a differential rectangle of the web plate at the extreme outside corner of the knee will show that there can be no tensile stress on the diagonal section at that point, although between that point and the neutral axis, considerable tension must exist (Fig. 11).

Mr. Polivka has presented a method of analysis which appears to be particularly well adapted for application to frames of varying moment of inertia, and in which some of the approximations usually made are eliminated. The agreement obtained between Mr. Polivka's computations and the writers' test results was gratifying.

Mr. Barrow has presented a simple and familiar method of computation, which, with a few modifications, gives surprisingly close agreement with the observed test results. The use of such simple methods of analysis is to be encouraged wherever applicable. That Mr. Barrow's computed stresses for the curved flange of the curved knee were somewhat less than the observed stresses demonstrates further that straight-beam formulas are not quite adequate for this type of member.

Even now, more tests on steel rigid frames are being made in the United States, and definite answers may soon be provided for some of the still debatable questions. It would be well to know, among other items, the most economical and efficient shapes for rigid-frame knees, how to stiffen the more flexible types of knees most effectively, and the effect of cyclic or repeated loads on rigid frames. Nevertheless, the writers are of the opinion that sufficient information concerning the stress distribution in rigid frames and the accuracy of current methods of analysis is now available so that engineers may design rigid frames with confidence and perhaps more economically than has been done in the past.

It is a source of regret to the writer that the preparation of these closing comments could not have been shared by Professor Lyse, now resident in Trondheim, Norway. Distance and disturbed communication conditions, however, have made this virtually impossible.

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TRANSACTIONS

Paper No. 2131

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

By DONALD M. BURMISTER,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Gordon E. Thomas and M. N. Sinacori, E. J. Kilcawley, Dimitri P. Krynine, B. K. Hough, Jr., T. B. Rights, and Donald M. Burmister.

SYNOPSIS

Laboratory investigations made to determine the general suitability of the Flushing Meadow in New York, N. Y., for development as a permanent park and for supporting the temporary and permanent structures of the World's Fair, are described in this paper. The problems encountered in the reclamation and development of the site arise from the peculiar nature of the soils of the meadow, which are quite typical of such tidal marsh deposits. The tests included routine studies to determine the general physical character, and consolidation and shear tests to determine the behavior characteristics of the materials encountered. They were made primarily to furnish information on what could be done and how it should be done. The tests and their interpretation are discussed in relation to these problems, and certain suggestions are made as to the correlation of test results which permit the use of data obtained from the simpler tests as a basis for predicting the more general behavior of soils.

INTRODUCTION

The laboratory investigations for the development of the Flushing Meadow Park site for the New York World's Fair were interesting in many respects, not only because of the practical use made of the results of soil tests, but also because of the interesting nature of the problems encountered. The first series of tests was made during the latter part of 1935 for a preliminary report, by the consulting foundation engineers, on the suitability of the Flushing Meadow for development as a permanent park and for supporting the permanent and temporary structures of the World's Fair,[2] and for a later report by the same engineers during 1936 for certain permanent structures.

Note.—Published in January, 1941, *Proceedings*.

[1] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[2] "The Application of Soil Mechanics in Building the New York World's Fair," by George L. Freeman, M. Am. Soc. C. E., Hamilton Gray, Jun. Am. Soc. C. E., and George W. Glick, *Civil Engineering*, October, 1940, p. 649; see also "Soil Survey of the Flushing Meadow Park Site, Long Island, New York," by George L. Freeman, *Paper No. C-2*, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. I, p. 25.

The Flushing Meadow was a low-lying tidal marsh, only about 2 ft above mean tide, consisting of deposits of very soft silty clays extending to a depth of 60 ft or more in some places, and underlaid by sands and gravels. The materials of the deposit were sampled by forty-eight undisturbed samples from twelve borings, 3 in. in diameter, which were tested (see Table 1) primarily to furnish information on the nature of the problems to be encountered, and to serve as a guide for making recommendations as to:

TABLE 1.—SCHEDULE OF SOIL TESTS, FLUSHING MEADOW PARK

Series	Borings	Samples	Routine tests	Consolidation tests	Shear tests
Preliminary report.....	12	48	48	20	19
Permanent structures..	25	70	70	19	48

(1) Methods of filling and grading the site, stable slopes, etc., to prevent lateral subsurface soil movements;

(2) The quantity of fill material required and finished grade elevations to compensate for the expected settlement due to the weight of the fill; and

(3) The allowable foundation pressures in different parts of the area and the settlement to be expected when the structures were erected.

In addition, seventy undisturbed samples from twenty-five borings, 3 in. in diameter, taken at the proposed sites of permanent structures, were tested to furnish detailed information as to the foundation conditions and for use in selecting and designing the most satisfactory and economical types of foundations for the different types of structures. A more detailed study and analysis of the results of the soil tests than was possible at the time, together with the results of subsequent researches on the Flushing Meadow soils, are herewith offered to complete the information on the development of the Flushing Meadow Park site.

PHYSICAL CHARACTERISTICS

Routine tests were made on all samples for the purpose of classifying the soils. These tests included: Natural moisture content, grain-size analysis, liquid, plastic and shrinkage limits, specific gravity, loss on ignition, and a routine squeeze test.

These routine tests gave information on the general character of the soils encountered and their uniformity or variability. Furthermore, they showed whether the Flushing Meadow soils had regular characteristics[3] usually associated with materials of their grain-size distribution, or whether they had unusual or peculiar characteristics that would be expected to influence their behavior greatly. For similar grain-size distributions, soils having regular characteristics have been found to possess similar properties. The peculiar nature of the Flushing Meadow soils is shown by the extremely high values of the liquid limit and plasticity index in relation to those for regular soils. These characteristics are due inherently to the mineralogical and chemical nature of the true clay fraction of the soil.

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[3] "The Physical Characteristics of Soils with Special Reference to Earth Structures," by Donald M. Burmister, *Bulletin No. 6*, Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

Microscopic studies and X-ray diffraction patterns show that the clay minerals consist chiefly of kaolinite and hydromica. The hydromica clay minerals, which are an intermediate alteration product of a scale-like crystalline structure formed by the weathering of feldspathic rocks to kaolinite, and the high organic content are responsible for the unusual character of the Flushing Meadow soil.

The natural moisture content is characteristically high for these materials, as shown in Fig. 1, being about 100% by dry weight, or more. A striking feature shown by these tests is the fact that the moisture content is practically uniform below the meadow mat down to the underlying sand and gravel layers, and about equal to the liquid limit, a condition which seems to be characteristic of such deposits. These facts explain why this material is so extremely soft in character, and indicate that any serious disturbance would tend to reduce it to consistencies approaching the liquid-limit condition. Although the meadow mat had still higher moisture contents of 200% or more, it had considerable strength because of the strong root system, a fact that made possible, and greatly facilitated, the grading operations. A marked peculiarity of samples from certain areas of the Flushing Meadow was the swelling properties they exhibited, due to the release of dissolved gases (probably marsh gas).

A most important property of the silty clay deposit is its natural structure, which gives the undisturbed material a much higher strength than when remolded or disturbed. It is important, therefore, to obtain preliminary information on the structure and consistency of all samples. The small cylinder test (or cube test) has been used for such purposes, on medium to stiff clays, by Karl Terzaghi,[4] M. Am. Soc. C. E., and by others. A simplified small squeeze test device[5,6] was developed, for the very soft materials encountered in the Flushing Meadow, to obtain preliminary information on the structural capacity of these materials. A small sampling tube is used to cut a cylindrical specimen 2 sq cm in area by 1 cm high. The average final diameter, in centimeters, of the squeeze specimen, after applying a 2-kg load, is taken as the squeeze test value.

The structure of the clay is indicated in the soil profile of Fig. 1 by plotting the undisturbed squeeze test diameter, which is determined primarily by structure, and the remolded value, which is considerably greater and is influenced largely by moisture content.

It is interesting to note that the consistency of the material, as indicated by the undisturbed squeeze test value, does not increase appreciably with depth for boring No. 17, whereas for boring No. 2 the consistency of the upper layers

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[4] "Determination of Consistency of Soils by Means of Penetration Tests," by Karl Terzaghi, *Public Roads*, February, 1927, Vol. 7, p. 240.

[5] "Squeeze Test for Integrity of Soil Samples," by Donald M. Burmister, *Engineering News-Record*, April 22, 1937, p. 588.

[6] "A New Method for Determining the Relative Consistency of Soils," by Donald M. Burmister, *Paper No. A-13*, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. II, p. 43.

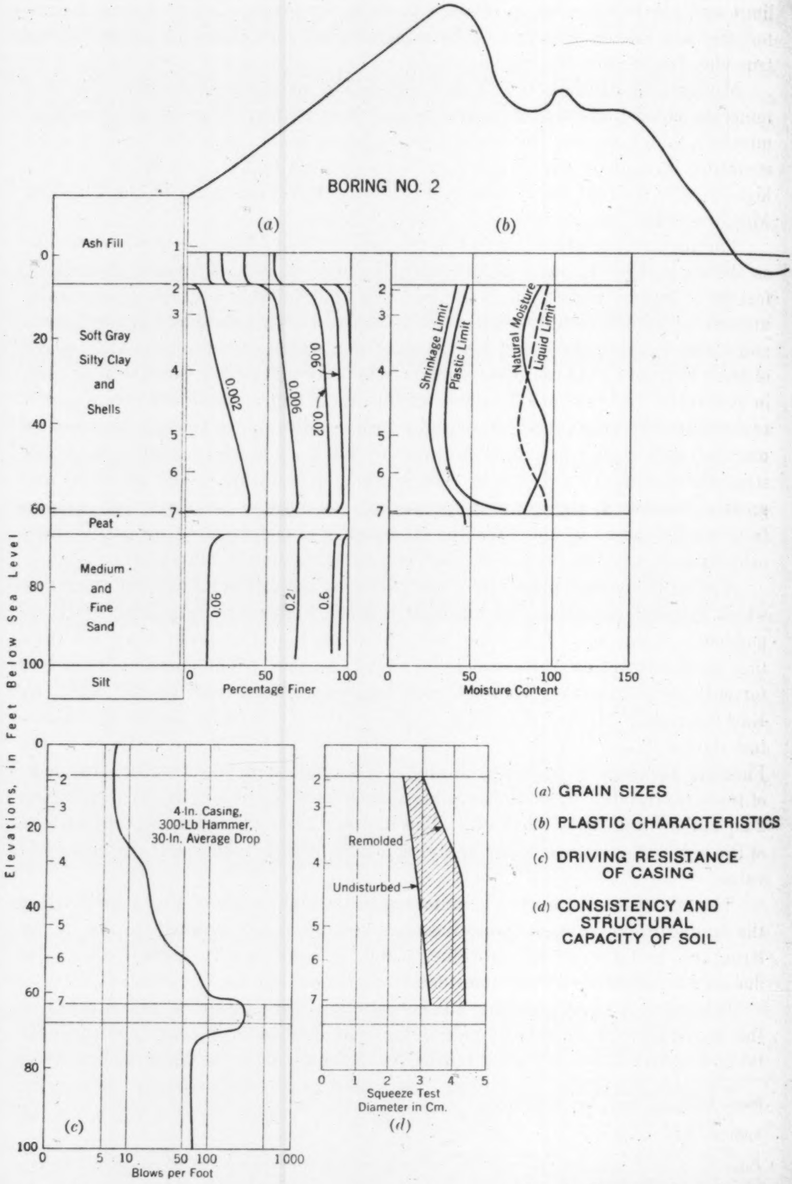
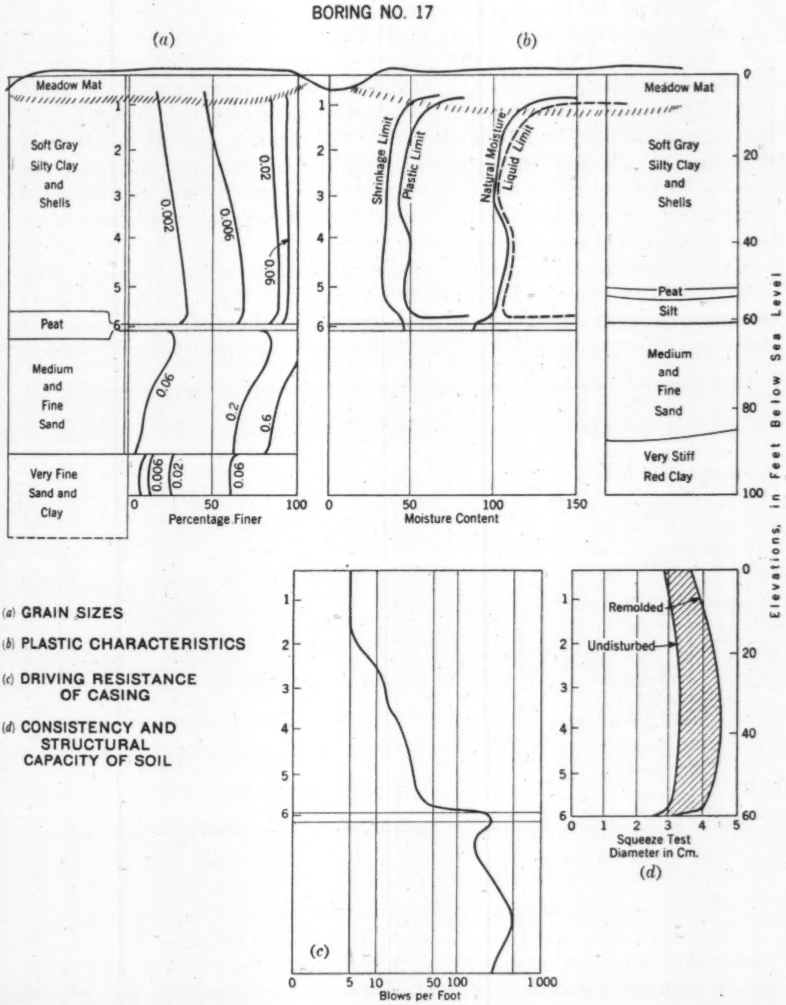


FIG. 1.—PHYSICAL CHARACTERISTICS



OF THE FLUSHING MEADOW SOILS

directly beneath the ash dump is much stiffer as a result of the consolidation of the material under the weight of the ash fill.

The structural capacity of the soil is indicated by the numerical difference or spread between the squeeze test diameters for the undisturbed and remolded states, about one centimeter indicating a fairly undisturbed and representative sample for these soft Flushing Meadow soils. Correlations with shearing tests on these materials showed that a spread of one centimeter represented a ratio of 6 : 1 between the shearing strength in the undisturbed and remolded states. A comparison of the squeeze test diameters in the remolded state in Fig. 1 with the values obtained for the liquid limit and plastic limit conditions of remolded materials, respectively 3.70 and 1.60 cm, indicates how serious would be the effect of any disturbance to the natural structure of these silty clays with the consequent loss of shearing strength. The shearing strength at the liquid limit is approximately 0.02 kg per sq cm. The routine squeeze test may be used in the laboratory for selecting the more representative and least disturbed samples for the consolidation and shear tests, and also has possibilities in the field for control of sampling operations.

The extremely soft consistency of the materials encountered in each boring is also clearly indicated by typical driving records in Fig. 1. The presence of the underlying sand and gravel layers, together with the relative compactness of these deposits, is shown by the marked increase in driving resistance of the casing.

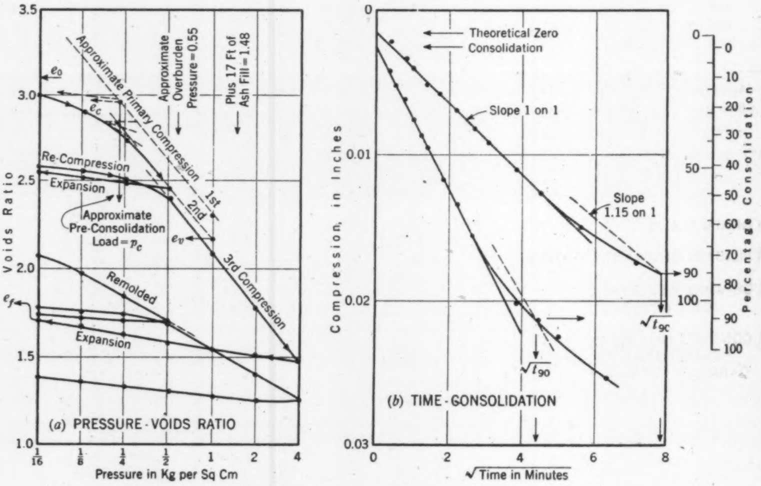


FIG. 2.—TYPICAL CONSOLIDATION CHARACTERISTICS OF THE FLUSHING MEADOW SOILS

CONSOLIDATION CHARACTERISTICS

The consolidation characteristics of the Flushing Meadow deposits were obtained from consolidation tests, which indicated that settlements of considerable magnitude were to be expected from the consolidation of the soft material under the weight of the fill, and of the Fair structures.

Typical pressure-voids ratio and time-consolidation curves are shown in Fig. 2. The subscript v denotes compression or volume-change phenomena in general. It is applicable to all materials, whether simple compression takes place, as in permeable, cohesionless soil; or whether consolidation takes place, as in saturated plastic clays. The subscript c denotes phenomena specifically related to consolidation. The consolidation characteristics required for settlement analyses are as follows:

(1) The voids ratio e_v at a pressure of p' equal to 1 kg per sq cm;

(2) The compression index c_v (slope of the pressure-voids ratio curve on a semi-log plot);

(3) The pre-consolidation load p_c, as defined by Arthur Casagrande,[7] Assoc. M. Am. Soc. C. E.;

(4) The coefficient of consolidation c_c, as determined either by the Casagrande[8] or the Taylor method.[9]

According to the Taylor diagram (Fig. 2) these coefficients are:

Item	Undisturbed	Remolded
$2\,h_o$, in cm	0.405	0.497
$\sqrt{t_{90}}$, in min	4.4	7.8
t_{90}, in min	19.4	60.9
c_c, in sq cm per min	0.0018	0.00078
Load increment, in kg per sq cm	$\frac{1}{4}$ to $\frac{1}{2}$	$\frac{1}{4}$ to $\frac{1}{2}$

A comparison of consolidation coefficients is afforded by reference to Table 2.

TABLE 2.—CHARACTERISTICS OF MATERIAL IN BORING NO. 9, SAMPLE 5 (DEPTH = 52 FT), AT BRIDGE C ACROSS HORACE HARDING BOULEVARD

CONSOLIDATION CHARACTERISTICS				PHYSICAL CHARACTERISTICS	
Item	Symbol	Undisturbed	Remolded	Definition	Value
Voids ratio at pre-consolidation load	e_c	2.96	....	Liquid limit	93.0
Voids ratio at $p' = 1$ kg per sq cm	e_v	2.17	1.55	Plastic limit	42.2
Compression index	c_v	0.361	0.216	Plasticity index	50.8
Coefficient of compressibility ($\frac{1}{4}$ to $\frac{1}{2}$ kg)	c_a	1.25	0.56	Specific gravity	2.73
Coefficient of consolidation in sq cm per min	c_c	0.0018	0.00078	Squeeze Test:	
				Undisturbed	2.9[a]
				Remolded	4.2[a]

[a] Squeeze test units are centimeters.

DEFINING EQUATIONS

Two defining equations used in this paper are:

$$e = e_v - c_v \log_e \frac{p}{p'} \qquad (1)$$

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[7] "The Determination of the Pre-Consolidation Load and Its Practical Significance," by Arthur Casagrande, *Paper No. D-34, Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. III, p. 60.

[8] "New Facts in Soil Mechanics from the Research Laboratories," by Arthur Casagrande, *Engineering News-Record*, September 5, 1935, p. 320.

[9] "Improved Soil Testing Methods," by Glennon Gilboy, Assoc. M. Am. Soc. C. E., *loc. cit.*, May 21, 1936, p. 732.

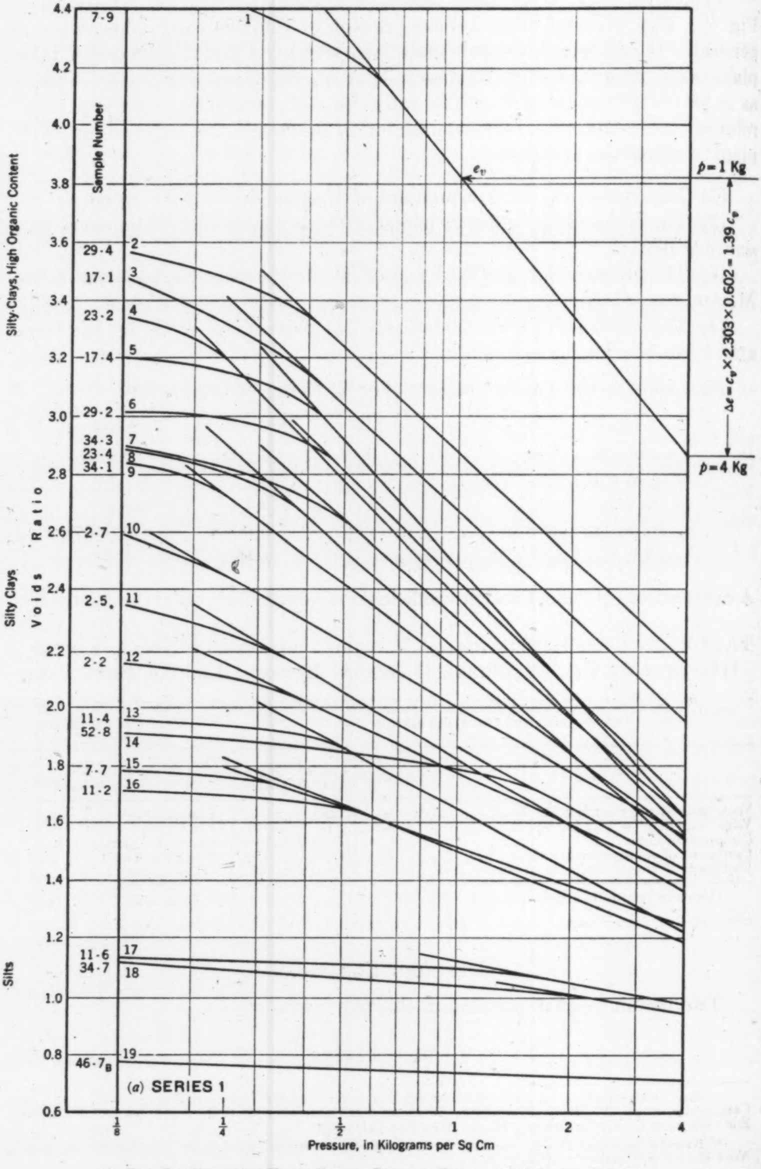

FIG. 3.—PRESSURE-VOIDS RATIO CURVES, FLUSHING MEADOW SOILS

and, total settlement by the summation method,

$$S = \sum \Delta h \frac{e_1 - e_2}{1 + e_1} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which $(e_1 - e_2)$ corresponds to the initial and final pressures, p_1 and p_2, respectively, at the center of the soil layer, the thickness being Δh.

The organic silty clays of the Flushing Meadow deposits are characterized by steep pressure-voids ratio curves, high initial voids ratios, low pre-consolidation loads, and very flat expansion curves, as shown in Fig. 2, which are consistent with the unusual physical characteristics shown in Fig. 1. An unloading and reloading cycle was made in the region just beyond the estimated pre-consolidation load in order to observe and study the characteristics of the consolidation curve in a loading cycle similar to that resulting from the sampling operation.

The marked effect of remolding or disturbance is shown by the fact that a compression of about 25% occurs under the pre-consolidation load as a result of complete remolding before equilibrium under this same pressure is again reached. The spread between the two curves is a measure of the natural structural capacity of the material. It is interesting to note in Fig. 2 that the breakdown of the natural structure and the dispersion of the soil grains by remolding, but without change in moisture content, also tend to decrease or slow up the rate of consolidation appreciably. The coefficient of consolidation is determined by the Taylor method,[9] plotting compression against the square root of time. The parabolic nature of the initial stages of the time-consolidation curve of Fig. 2(*b*) is shown by the linear relationship to about 60% consolidation, which facilitates making preliminary forecasts of settlement.

The pressure-voids ratio curves of series 1 are plotted in Fig. 3 to show the range and characteristics of the materials of the deposits. A striking feature of these curves is the consistent tendency for steep curves of compressible materials to be associated with high values of the voids ratio at the 1 kg pressure, and for flat curves of relatively incompressible materials to be associated with low values. The practical significance of these interrelationships was first presented and discussed orally by Arthur Casagrande and Philip C. Rutledge, Assoc. Members, Am. Soc. C. E. (Fall Meeting of the Society in October, 1937), who suggested that a simple characteristic soil test such as the liquid limit could be used practically for obtaining preliminary estimates of the consolidation characteristics of undisturbed materials.

In a natural state of consolidation, a material passes from a suspension by the simple process of settling or sedimentation to a point where a real structure begins to form, which is capable of carrying stress as the overburden accumulates. A certain amount of pressure (although very small) is then required either to deform or to further compress the material. This moisture content is characteristic and may be considered as the starting point of true consolidation, and the transition from the suspended liquid state to the plastic state. The liquid limit, a characteristic moisture content of a remolded material, represents a moisture condition somewhat below this transition point. However, the same inherent physical characteristics of the material,

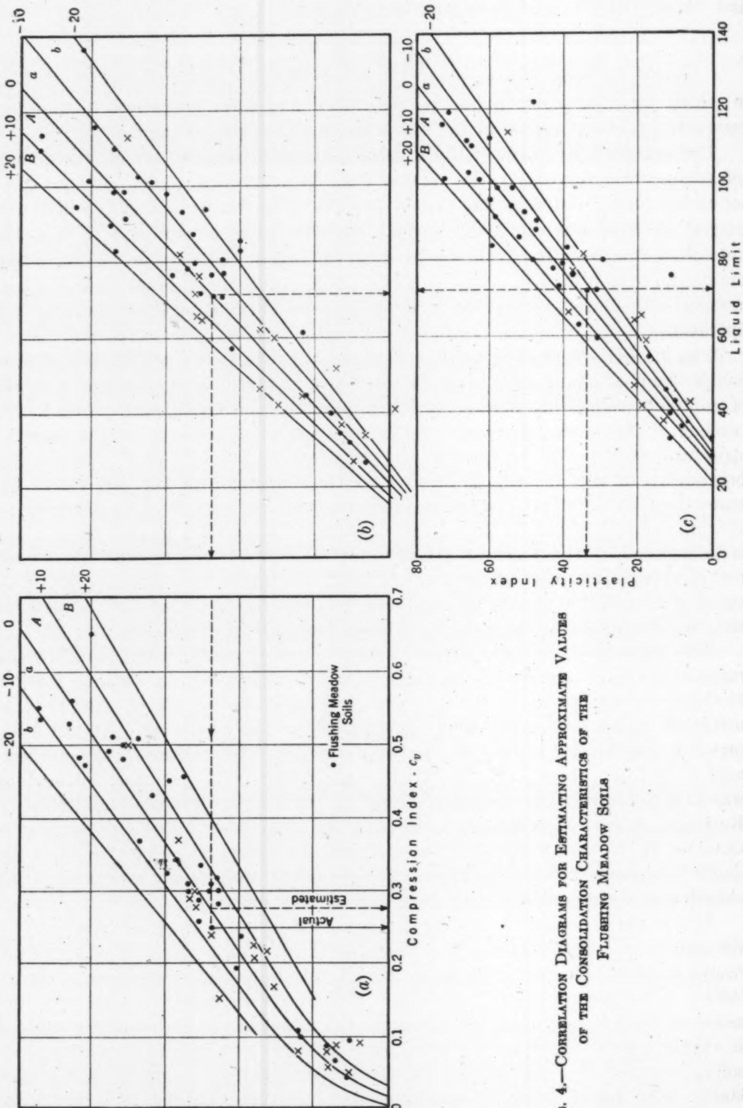

FIG. 4.—CORRELATION DIAGRAMS FOR ESTIMATING APPROXIMATE VALUES OF THE CONSOLIDATION CHARACTERISTICS OF THE FLUSHING MEADOW SOILS

which determine the liquid-limit condition, determine not only this characteristic moisture content at the starting point of consolidation before a real soil structure forms, but also the form of this structure, which is a determining factor in the compressibility of the undisturbed material in a natural state of consolidation.

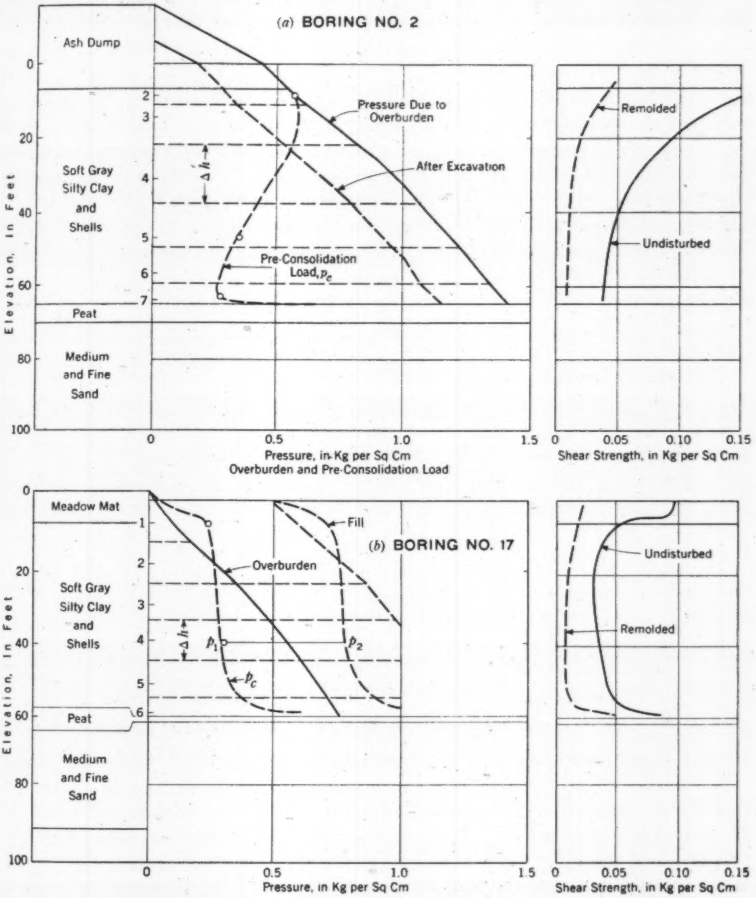

FIG. 5.—STRESS CONDITIONS AND SHEARING STRENGTH, FLUSHING MEADOW SOILS

A subsequent study of these interrelationships for the Flushing Meadow soils and for soils from other localities by C. R. Horne, Jr.,[10] Jun. Am. Soc.

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[10] "Correlation of Compressibility and Atterberg Limits," by C. R. Horne, Jr., *Thesis No. 494*, Dept. of Civ. Eng., Columbia Univ., New York, N. Y., June, 1938.

C. E., is sufficiently interesting and of practical value to be included here in the diagrams of Fig. 4. About 60% of the values of the consolidation characteristics fall within a band of ± 10%.

Although the relations are not as consistent as desirable, studies seem to indicate that a material of low plasticity, falling in a zone (*a*) of Fig. 4(*c*), might be expected likewise to fall in corresponding parts of zone (*a*) of Figs. 4(*b*) and 4(*a*), tending as a consequence to have slightly lower values of both e_v and c_v. On the other hand, a material of more plastic nature than usual might be expected to lie in zone (*A*) of all diagrams of Fig. 4. These relations show the fundamental nature of the liquid and plastic limits as a characteristic moisture content in relation to consolidation phenomena, and indicate the need for fundamental research on the liquid and plastic limit tests to establish these relations more definitely for all types of materials. The effect of partial disturbance, inaccurate determinations of the liquid and plastic limits, and uncertainties associated with the consolidation test itself accounts for many of the discrepancies. An important fact to remember in all soil testing is variability of material which is always to be expected either in specimens taken from closely spaced samples in a boring or in different borings at the same elevation.

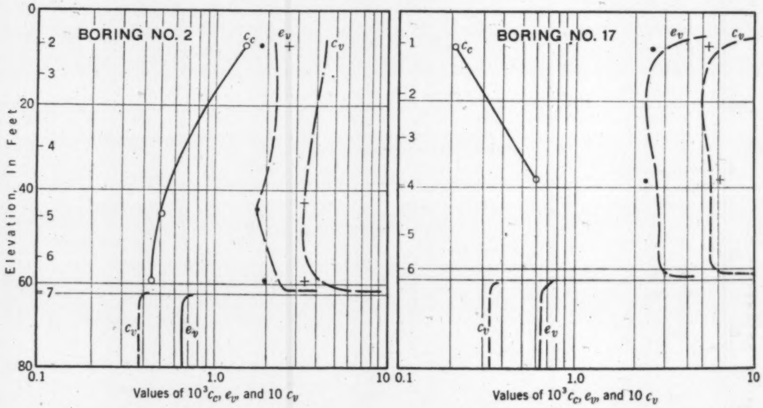

FIG. 6.—CONSOLIDATION CHARACTERISTICS, FLUSHING MEADOW SOILS

The possibility of making quite satisfactory preliminary estimates of the consolidation characteristics from the simpler routine soil tests, and of thereby extending and supplementing the information available for settlement analysis, is illustrated in Figs. 5 and 6. The profiles in Fig. 5 indicate the uniformity of the properties of the Flushing Meadow soils, the extremely compressible character of the meadow mat and the peat layer, and the relatively incompressible nature of the underlying silt and sand layers, the latter values being interpolated from other borings for comparison.

SHEARING CHARACTERISTICS

The major consideration in the development of the Flushing Meadow was to prevent and control subsurface movements in the soft material during grading operations and in the construction of foundations, so as to preserve the natural structure of the material. The shearing characteristics of the Flushing Meadow deposits were obtained from results of direct shear tests of the "quick" and "delayed" shear types.[11, 12, 13, 14] The first group of tests was made in a ring type of shear apparatus with shearing load applied at a constant rate (water loading), and the later tests were made in a Harvard type of shear

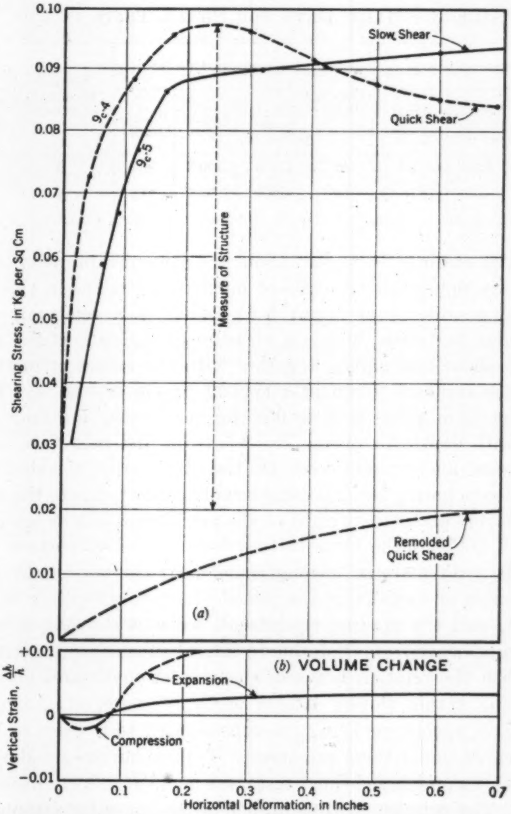

FIG. 7.—SHEARING STRESS-DEFORMATION AND VOLUME-CHANGE CURVES OF FLUSHING MEADOW SOILS AS DETERMINED BY DIRECT SHEAR TESTS OF THE STRAIN CONTROL TYPE

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[11] "The Application of Theories of Elasticity and Plasticity to Foundation Problems," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 206.

[12] "The Shearing Resistance of Saturated Soils and the Angle Between the Planes of Shear," by Karl Terzaghi, *Paper No. D-7, Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. I, p. 54.

[13] "Die Coulombsche Gleichung für den Scherwiderstand biundiger Böden," von Karl Terzaghi, *Die Bautechnik*, 1938, Heft 26, p. 337.

[14] "The Shearing Resistance of Remolded Cohesive Soils," by M. J. Hvorslev, Assoc. M. Am. Soc. C. E., *Paper No. E-1, Proceedings*, Soils and Foundation Conference, U. S. Eng. Dept., Boston, Mass., June 17-21, 1938.

box with a strain-control loading at a rate of about 0.05 in. per min. The shearing stress for the undisturbed material in a rapid test in Fig. 7 increases characteristically to a maximum value and thereafter drops off to about 85% of the maximum. In the slow shear test (see Table 3), the stress increases less rapidly with strain. It reaches about the same maximum value but shows only a slight dropping off. A ratio of maximum stresses in the undisturbed and remolded conditions of about 5 : 1 indicates a relatively undisturbed sample for these materials, and is a measure of their structural capacity.

TABLE 3.—BASIC DATA FOR SHEAR TESTS IN FIG. 7

Sample No.	SHEARING RATE			Depth, in ft	MOISTURE CONTENT			Normal pressure, in kg per cm	SQUEEZE TEST	
	Description	Speed, in in. per min	Total time, in hr		Sample as received (w)	Liquid limit	Plastic limit		Undisturbed	Remolded
9_c-4	Quick	0.05	....	42	116.5	100.8	56.7	0	3.0	4.6
9_c-5	Slow	....	18 to 24	52	106.1	93.0	50.8	$\frac{1}{16}$	2.9	4.2

The quick shear test values, together with the values estimated from the routine squeeze test, are given in a typical profile of Fig. 5. The shearing strength varies from about 0.03 to 0.08 kg per sq cm, and does not increase appreciably with depth for boring No. 17, from just below the meadow mat down to the sand and gravel layers. On the other hand, the shear strength of the upper layers in boring No. 2 is considerably greater, due to the consolidation of the upper layers under the weight of the ash dump; but at lower depths the shear strength is about the same as that found in other parts of the deposit. However, this weak material, strengthened near the surface by the meadow mat, was capable of supporting the initial blanketing (with a fill to a height of about 4 ft) and the grading equipment, without showing appreciable disturbance or mud-wave formation due to lateral subsurface movements.

To establish the relation between shearing strength and normal pressure for determining stable slopes, lateral earth pressure, etc., delayed[11] shear tests were made, specimens being pre-consolidated to a series of normal pressures and sheared under these pressures. Typical curves are shown in Fig. 8. The original series of delayed shear tests was made at a fairly rapid rate of 0.05 in. per min. The primary loading branch of the curve for sample 9_c–4 (depth 42 ft) passes through the origin, yielding a value of 20° 35′ for the apparent angle of friction. Because insufficient time was allowed in the rapid test for the volume-change adjustments to take place during shearing, excess hydrostatic or neutral stresses[12, 13, 14] were induced in the pore water, which could not be dissipated by the escape of water. This means that only part of the total applied normal stresses (possibly only 60% to 70%) were effective in mobilizing shearing strength.

Subsequently, slow shear tests were made on sample 9_c–5 from the same boring (depth 52 ft) at a very slow rate (about a 24-hr test), so that the hydrostatic stresses were at all times equal to zero. Therefore, the total applied stresses

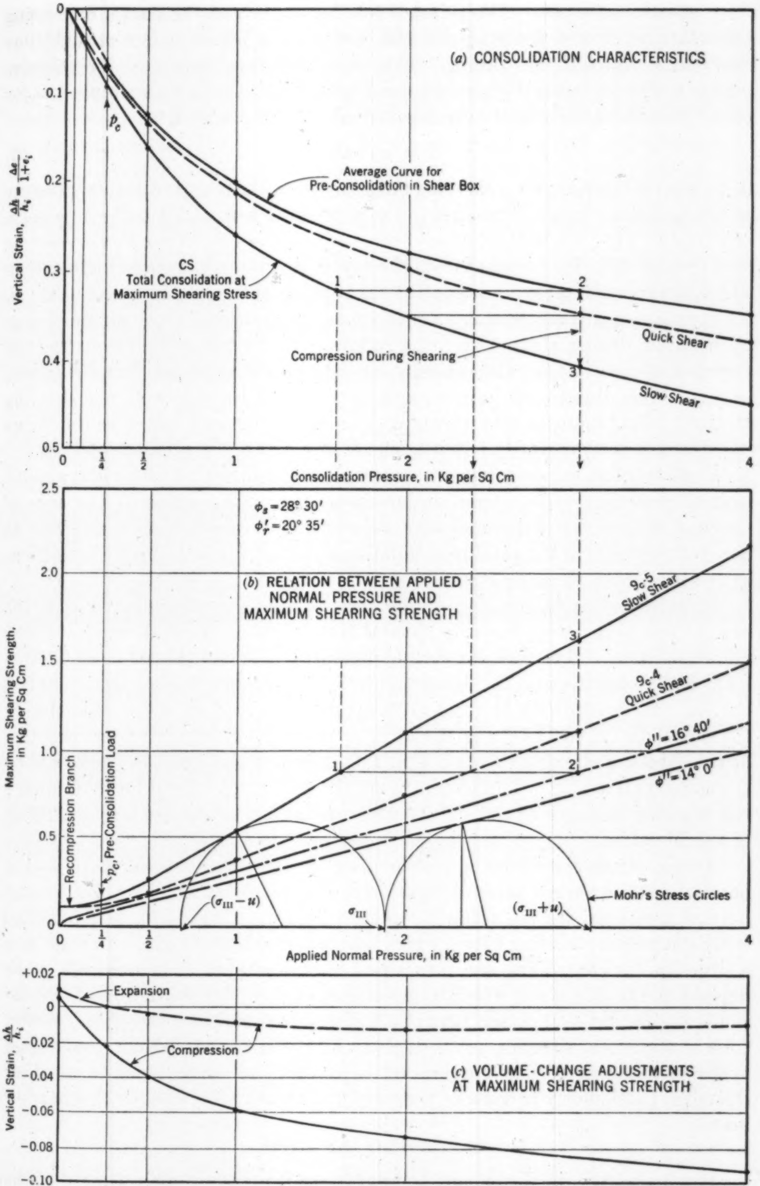

FIG. 8.—SHEARING CHARACTERISTICS OF THE FLUSHING MEADOW SILTY CLAYS AS DETERMINED BY DIRECT SHEAR TESTS OF THE STRAIN CONTROL TYPE

were effective stresses. The primary loading branch of the curve, expressing the relation between shearing strength and normal pressure, is a straight line that passes through the origin. This direct relation is typical for cohesive plastic soils in a natural state of consolidation, and Coulomb's simple law for cohesionless materials applies to the primary loading branch:

$$S = p_N \tan \phi \qquad (3)$$

in which S is the maximum shearing strength, p_N is the effective normal pressure which equals the applied pressure in the slow shear test, and ϕ is the true angle of friction. The angle of friction obtained from the slow shear test on sample 9_c–5 was 28° 3′. It is evident from the curve of volume-change adjustments in Fig. 8 that considerable consolidation takes place during a slow shear test. In a large mass of this material it is conceivable that practically no volume-change could occur during a rapid shearing failure. Under these conditions the apparent angle of friction, theoretically, may be half that of the slow shear test, or less. Also, due to the impermeable character of the material, the shearing strength would increase very slowly with consolidation and therefore the shearing strength is temporarily independent of any externally applied loadings.

Consolidation curve *A* represents complete consolidation under the applied normal pressures. The consolidation-shear curve *CS*, which is the sum of curve *A* and of the volume-change adjustment at maximum shear stress in curve *B*, represents the total consolidation required for equilibrium conditions at the maximum shearing stress.

In interpreting the results of the test, it must be remembered that the sample has been taken from its natural state with consequent relief of stress and expansion. Therefore, for normal pressures, less than the pre-consolidation load, one is operating on the reloading cycle of the consolidation and shear test curves. However, with the typical flat unloading and reloading branches of the consolidation curves of Fig. 2, the quick shear test gives a fair approximation of the shearing strength of the soil in the natural state, and the values are on the conservative side. Therefore, the quick shear test is useful for determining the shearing strength of the soil for such operations as the grading and filling of the Meadow.

Actual stress conditions approximating those in the natural state do not obtain until the normal pressure equals the pre-consolidation load. For greater normal pressures, the reloading curve then becomes tangent to, and coincides with, the primary loading curve, which passes through the origin. In analyzing a situation for controlling and preventing lateral subsurface movements in the deposit due to filling and grading operations, or due to construction of foundations, the profile of shearing strength in Fig. 5 is useful in locating the critical zone of maximum shearing stress as given by the stress conditions for different types of loadings of Leo Jürgenson[11] in relation to critical values of shear strength, particularly in view of the very low values just beneath the meadow mat.

SETTLEMENT ANALYSIS

It is necessary, in settlement computations, to make some estimate of the pre-consolidation load; that is, the probable state of stress in the soil in the natural condition as shown in Fig. 2. Where three or more consolidation tests could be made per boring, a good idea could be obtained of this state of stress; but where fewer samples were tested, it is necessary to obtain an estimate of this stress condition for settlement computations. Because of the very flat expansion curve in consolidation and the fairly consistent relation between shear strength and pre-consolidation load, a correlation curve was obtained for these materials for estimating the approximate value of the pre-consolidation load from the results of the quick shear and routine squeeze tests. The curve was also very useful for estimating the place to make a loading cycle in the region of the pre-consolidation load.

The values of the pre-consolidation load determined from consolidation tests, and obtained from estimates, are plotted in the soil profiles of Fig. 5. It is interesting to note that below a depth of 10 to 20 ft the pre-consolidation load is approximately constant and considerably less than the weight of the overburden. There are two possibilities: Either the material is incompletely consolidated and stress is temporarily carried by hydrostatic pressure in the pore water; or there is permanent artesian water pressure in the sand and gravel layers extending up on either side to the flanks of the adjacent hills, a pressure gradient being established by upward flow through the silty clay deposit. These represent two limiting conditions for settlement analyses.

In the computation of settlement under a given external loading, it is convenient to divide the depth of the deposit into a number of increment sections of thickness Δh, as shown in Fig. 5, and take the appropriate values of pressure, p_1 and p_2, at the center of each section from the pre-consolidation and external loading curves. The summation method can be made to take adequate account of the changes in the character of a deposit. Where consolidation tests have been made, the corresponding values of $(e_1 - e_2)$ are obtained directly from the pressure-voids ratio curves. Eq. 2 is then used directly in the settlement computations. Where the consolidation characteristics are estimated from Fig. 4, it is simpler to reconstruct the primary pressure-voids ratio curves, from the voids ratio at 1 kg of pressure and the voids ratio at 4 kg of pressure, which is equal to $e_v - c_v \log_e \frac{4}{1}$ or $e_v - 1.385\, c_v$ (see Eq. 1).

CONCLUSIONS

The investigations have shown quite clearly that, if a limited amount of time is available for testing, it is very important to make a few carefully conducted consolidation and shear tests to obtain very complete information on the behavior characteristics of a few representative samples as control tests in the given area. The Flushing Meadow soils, although possessing unusual characteristics, exhibit physical properties consistent with their character. The simpler routine soil tests, carefully made on all soil samples, will not only give information on the nature and on the variability or uniformity in the character of the materials, but also will provide (after the correlation curves have been established for different soil properties by the control tests) a practical means for obtaining preliminary estimates on the consolidation characteristics and shearing strength of all samples, and for extending and supplementing the information for settlement and stability analyses.

ACKNOWLEDGMENTS

The writer wishes to express his thanks and appreciation to Moran, Proctor, Freeman and Mueser, Consulting Foundation Engineers, for the opportunity of making the soil investigations in connection with the development of the Flushing Meadow Park site of the New York World's Fair, 1939. The testing was done in the Soils Laboratory of the Department of Civil Engineering at Columbia University, New York City.

DISCUSSION

Gordon E. Thomas,[15] Assoc. M. Am. Soc. C. E., and **M. N. Sinacori,**[16] Jun. Am. Soc. C. E. (by letter).—In presenting a practical application of soil mechanics to foundation engineering the author is to be congratulated. The correlations between Atterberg limits and both shear and consolidation characteristics are indicative of more efficient methods to be used in soil investigations, especially from the preliminary and time-saving standpoints. In order to make these correlations of value, it is quite apparent that proper methods should be used in field sampling as well as in the simplest of laboratory tests.

With reference to the Atterberg limits, the writers wish to show, in passing, what correlations were found to exist in connection with a similar foundation investigation in another part of the United States. They particularly wish to show a further use of these Atterberg limits in borrow-pit studies for suitable impervious rolled-fill material for earth dams in which the material is composed of clay and silt mixtures.

The aforementioned studies were conducted on the foundation and borrow-pit areas of an earth dam situated in the Lower Tennessee River Valley about twenty-three miles from the river mouth. The earth structure is principally on the river flood plains, which are covered with a brown silty clay to a depth of from 10 to 30 ft. At a few locations layers of gray, carbonaceous, silty clay occur beneath the brown clay. In general, however, a 15-ft thickness of sand and clay underlies the top clay blanket, and this in turn overlies the fluvial valley fill consisting of sand and gravel. Samples of the foundation materials were taken with a drill rig using an "undisturbed" clay sampler, and also, where possible, by hand in cubic-foot boxes. Samples from the borrow areas were obtained by using hand augers of the bucket type.

The writers have found that for clays in the Lower Tennessee River Valley the relationship between liquid limit and plasticity index is very close with only small variations from the mean curve. This fact made it possible to use only one of the Atterberg tests in the relations with the other characteristics. For example, there appears to be a relationship between plastic limit and the consolidation characteristics (namely, compression index and voids ratio), taken at 10,000 lb per sq ft. Because of the uniformity of the material and the narrow spread of characteristics, the number of samples tested was not as great as the number presented by Professor Burmister. The samples taken at the start of the investigation were 2 in. in diameter. The diameter was later changed to 3 in. The samples of larger diameter tend to give a more definite correlation. This is probably due to less disturbance in the material. It was also found that the accuracy of the correlation was influenced to a considerable degree by the presence of lenses and pockets of fine sand in some of the samples. The pressure-voids ratio curves obtained from these tests had initial voids ratios of from 0.7 to 0.9, thus placing the material in the range designated by Professor Burmister as silt in Fig. 3. In the case of shear characteristics, the writers found that the natural moisture content of the material had to be introduced as a third variable before a relationship of definite practical value could be realized.

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[15] Capt., Corps of Engrs., U. S. A.; Forest Glen, Md.

[16] Associate Materials Engr., TVA, Gilbertsville, Ky.

These simple routine tests can be used to great advantage in obtaining information on the type of borrow material available for a rolled-fill section.

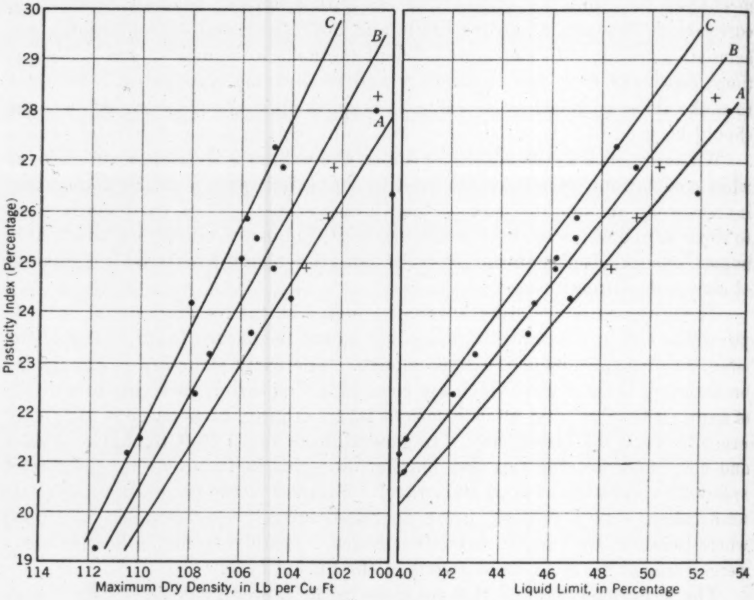

FIG. 9.—RELATION BETWEEN ATTERBERG LIMITS AND MAXIMUM DENSITY

The borrow areas for this particular earth dam are flat, long, and relatively shallow. The intrusion of sand layers, and high moisture contents at shallow depths, together with the necessity of leaving a prescribed thickness of blanket to minimize under-seepage of the dam, resulted in a small depth of borrow. All these factors made it necessary to investigate large areas, one of which was one-half square mile. Due to the stratification and variability of the soil, as well as the great area, a considerable number of compaction tests would have been required to cover this area completely. To save time and work, enough compaction tests were run to cover the various types of materials, as indicated by the pint jar samples from the auger borings. Then curves were drawn correlating liquid and plastic limits with compacted densities. From these original auger samples enough liquid and plastic limit tests were run to establish, more thoroughly, the compaction characteristics of the entire borrow area. Other general tests were run on typical borrow material to determine permeability, shear, and consolidation.

The liquid limit and plasticity index can be correlated very closely with the maximum density of the compaction curve by using the zone method. Fig. 9

shows the plots for a borrow pit on Twenty-Seven Mile Island, Kentucky. However, the plastic limit results, when plotted directly against the compacted maximum densities, show small variations from the mean curve, and are sufficiently accurate for correlating results. Consequently, these plastic limit tests alone were used as a rapid means of estimating maximum densities (Fig. 10). From the foregoing relationships all borrow-pit materials were classified very closely.

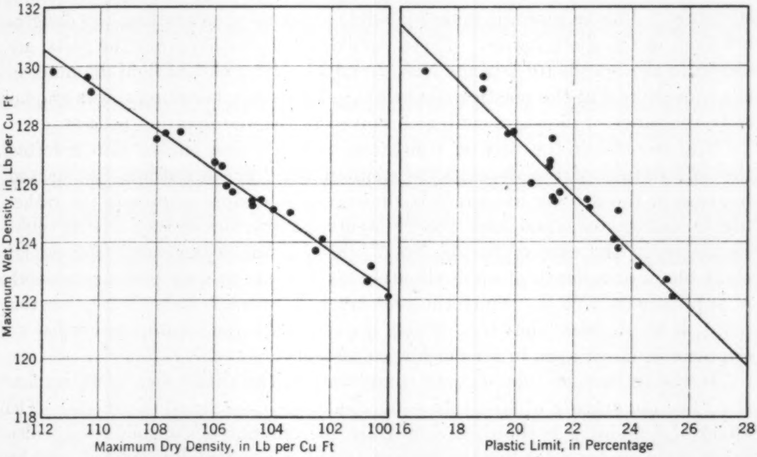

Fig. 10.—Relation Between Plastic Limits and Maximum Density

In dealing with borrow materials, the soil is thoroughly mixed when prepared for compaction, and any local variation in the sample is eliminated. In general it may be stated that Atterberg limits can be more closely correlated with borrow materials than with foundation soils taken in the natural state.

E. J. Kilcawley,[17] Esq. (by letter).—An unusual opportunity is afforded, by this paper, for a comparison of results and conclusions reached under rush conditions and those determined by retesting and study in a more leisurely manner. It represents a comparison and suggested correlations of physical and mechanical test results.

In this later detailed analysis, a method of presenting results of routine tests has been developed and applied. These methods seem to aid materially in visualizing actual field conditions which, in turn, make quick and reasonable conclusions possible. The necessary extent and method of sampling, and further detailed testing to determine and evaluate mechanical characteristics of the deposits, can likewise be done on the basis of the fundamental facts established by simple and quickly performed soil tests.

An excellent example is the work reported in the use of the squeeze test.

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[17] Prof. of Soil Mechanics and San. Eng., Rensselaer Polytechnic Inst., Troy, N. Y.

The modification of the three-dimensional test proposed by Mr. Jürgenson[18] in 1934 offers an efficient and handy tool for selecting samples, and for showing changes in the structure of the deposit. The method of plotting the final average diameter of the specimen measured in centimeters as abscissa for both undisturbed and remolded samples against the depth at which the sample was taken as an ordinate offers a quick and fairly reliable method of evaluating structure. Furthermore, it makes possible a means of estimating the dangers involved in the disturbance. These results, as concluded from the tests, are shown in the graphs for borings Nos. 2 and 17. The plot of final diameter in the squeeze test at the plastic and shrinkage limits, furthermore, shows graphically the range of moisture content involved in the stabilization process.

The records of the driving resistance of the casing shown also reflected the soft nature of the deposits, as pointed out by the author. A marked increase in the driving resistance is shown as the casings approach the rather highly compacted sand and gravel layers underlying the soft, silty clay. However, in the case of boring No. 2, this resistance increased from about eight blows at a depth of approximately 20 ft to about fifty blows at a depth of approximately 53 ft. Since the peat layer is approximately 10 ft below this point, it would seem that this reflects the fact that quick compression has an appreciable effect even in soils of low consistency.

In the further consideration of consolidation, the author has again emphasized the importance of structure as it affects the consolidation process. The effect of remolding is shown in Fig. 2 by the amount of consolidation which takes place under a loading equivalent to the pre-consolidation load, and by the spread between the consolidation curves for the undisturbed and remolded samples. This spread again offers a measure of the natural structure of the material. The results of repeated compression tests (also plotted in Fig. 2) show graphically the resulting change in structure and internal stress conditions. This part of the paper might well be expanded. Such a method of analysis might be applied to a study of stress history of deposits and of effective stress conditions.

The comparison of pressure-voids ratio curves in Fig. 3 shows both the range and the characteristics of the materials, and confirms the prediction of consolidation characteristics as indicated by the simple soil tests. The data included from the thesis of Mr. Horne[10] show the relation between consolidation characteristics and the liquid limit determination. They further point out the possibility of establishing a reliable guide in making good estimates of mechanical behavior and a means of establishing a maximum range of variation in these values of soil type characteristics. More extended study on this phase of behavior would be valuable.

In the analysis of quick, slow and delayed shear tests, the author has definitely shown the importance of hydrostatic equilibrium and effective stresses in the determination of shearing resistance. The significance of hydrostatic equilibrium as first pointed out by Professor Terzaghi is clearly shown. Volume change and structural adjustment are likewise emphasized and related to the important time factor in shear testing. An important consideration of an evaluation of stress conditions in undisturbed deposits, and in the test before any attempt is made to analyze results, is appropriately pointed out by the author.

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[18] "The Shearing Resistance of Soils," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 242.

In presenting the results of the tests and the correlations with the fundamental theories of Messrs. Terzaghi,[12] Hvorslev,[14] Jürgenson[11] and others, Professor Burmister has made a definite contribution. Certain parts of the paper could have been expanded. More information on the effect of recompression might do much toward explaining varying results obtained on soils which are presumably the same. The underlying lesson taught in this work is the possibility of establishing a maximum range of variation for soil type groups and for the evaluation of physical and mechanical characteristics. Such a possibility deserves the best consideration of all soil physicists.

DIMITRI P. KRYNINE,[19] M. AM. SOC. C. E. (by letter).—An interesting and comprehensive laboratory approach to the solution of a very important engineering problem is given in this paper.

Importance of the Problem.—Material termed "silt," "organic silt," "clayey silt," "silty clay," and "mud," is very common along the Atlantic coast, at least close to the New York metropolitan area. This is organic silt mixed with more or less clay, and sometimes mixed with shells. The thickness of a layer of this material is from a few feet to 100 ft or more. The writer's observations showed this material to be mostly underlain with sand of variable coarseness. At Flushing Meadow this material served as a foundation for the World's Fair; tunnels under the Hudson River pierce horizontally thick silt layers; H-piles supporting the bridges over the Potomac River in Maryland and over the Thames River in eastern Connecticut pierce it vertically to reach the ledge rock or reliable sand. Cutoffs from old roads along the coast often pass through similar materials; and it is very probable that some muddy areas on the shore may sometime become airports, as in Baltimore, Md.

Hence it is very important to correlate different studies of this material in order to draw some general conclusions. From this point of view, Professor Burmister's paper, containing abundant data on the properties of the given soil, is a welcome beginning.

Field and Laboratory Solutions.—No civil engineering problem can be solved definitely in the laboratory. Structures stand in the field and conditions of their stability (particularly properties of the soil) should be studied both in the field and in the laboratory. Laboratory data alone are only indicative—not conclusive. From this point of view, it is highly desirable to take advantage of a situation such as that under discussion. There can be no better large-scale consolidation tests and no better large-scale shearing tests than those made by the City of New York in dumping ashes on Flushing Meadow. It is true that the sequence of operations in dumping is not well known and hence the influence of the time-factor cannot be defined very clearly. Some limiting assumptions, however, can be made in similar cases. For instance, the presence of a mud wave means that the shearing strength of the material at a given place is overcome, and vice versa; the material should be considered safe so far as shear action is concerned, if there is no mud wave at a given place. Professor Burmister has made interesting observations on the penetration of the casing (Fig. 1), and it was possible for the construction engineer to correlate his data with the actual pile-driving records since many structures of the World's Fair were on piles driven through the given material to reach resistance.

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[19] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

Driving of the Casing.—A 4-in. casing was driven at Flushing Meadow starting with some 5 or 7 blows per ft (Fig. 1). This number gradually increased to about 100 blows per ft in approaching the firm base. Apparently this fact should be ascribed to the increase in the point resistance of the casing due to the support from below. The number of blows in the sand layer was more or less constant although not the same in the case of borings Nos. 2 and 17. A similar picture was observed by the writer at the various times that he had an opportunity to be present.

A variation of the casing-driving diagram (Fig. 1) would correspond to the situation in which the material at the top is so soft that the casing sinks down by its own weight. The writer has observed this phenomenon on the Thames River, at New London, Conn., when a 4-in. casing, similar to that used in the Flushing Meadow investigation, penetrated in this manner from several feet to 39 ft down through the clayey silt. Actual H-piles sank at an average of 25 ft through such material, the range of fluctuations being perhaps 5 ft. The weight of the casing used was 15 lb per ft, and that of the piles 89 lb per ft. In addition to the possibility of computing the coefficient of friction, such studies perhaps may offer as a by-product an opportunity to obtain a general idea of what percentage of the cross section of an H-pile driven into any soil takes active part in the friction against that soil and what percentage remains neutral.

Shearing Resistance.—According to Fig. 5 the shearing strength of undisturbed samples was greater close to the top and to the bottom of the layer, the average shearing strength at a certain depth being of the magnitude of 0.04 kg per sq cm which equals about $0.04 \times 2{,}000 = 80$ lb per sq ft. A very high shearing strength at the top of boring No. 2 is undoubtedly due to the compressing action of the ash pile. The meadow mat and the deep peat layer apparently acted as reinforcing agents.

Besides the direct laboratory shear test in the silty clay in the Thames River, the writer participated in the following simple field tests in Connecticut: (*a*) Brass cylinders containing undisturbed samples extracted from a depth were carefully cut with a hack saw along two parallel planes normal to the length of the cylinder, without disturbing the sample. A force necessary to pull out the ring thus formed was then measured. This test furnished a shearing strength of the order of 100 lb per sq ft; (*b*) when dredging for a cofferdam, huge undisturbed samples were taken directly from the bucket and tested immediately in a field, direct-shear, apparatus (size of the box 8 in. by 12 in.) designed and constructed by Philip Keene, Assoc. M. Am. Soc. C. E., soil mechanics engineer for the Connecticut State Highway Department. This test also furnished a shearing strength of about 100 lb per sq ft, or more. The sample tested was taken 12 ft below the bottom of the river, the depth of water being 44 ft.

Slopes.—Slopes of fills in this material should be designed rather flat using results of the quick shearing tests, as Professor Burmister advises. The shearing strength of the remolded material is perhaps $\frac{1}{4}$ or $\frac{1}{5}$ of that of undisturbed samples (Fig. 5). So far as cuts are concerned, this material is able to maintain astonishingly steep slopes. Reference is made to an article[20] that describes the construction of the present highway bridge at New London, Conn. The dredging operations are thus described in that article: "The dredging was therefore performed as an independent preliminary operation, with a clamshell dredge, and carried down 18 to 23 ft. below the river bottom, the material standing on a slope one to one."

Natural Moisture Content.—The writer agrees with Professor Burmister's statement that the natural moisture content is characteristically high for these materials, being about 100% by dry weight, or more. Smaller percentages may be found also.

The writer wishes to call attention to the fact that the moisture content as determined from an undisturbed sample of such clayey silt, extracted by using a sampler, and the moisture content of the same material in a bucket during the dredging may be different, the latter moisture content being somewhat smaller. The writer suspects that undisturbed samples may be oversaturated.

Odor.—According to the writer's observations, this material has the odor of methane (marsh gas) in the upper layers only, perhaps to a depth of 40 or 50 ft. Again, if an odorous sample is left alone for a few weeks, it does not smell any longer.

Pre-Consolidation Load.—Assuming that the procedure used for determining the pre-consolidation load is correct (and the writer believes that it is), Professor Burmister states that either the Flushing Meadow deposit was not fully consolidated at the time of investigations or that there was artesian water in the underlying sand layer. Assuming the former alternative, it should be concluded that, in the case of boring No. 2, the settlement of unloaded cuts should still have proceeded during the existence of the World's Fair notwithstanding the relief in load due to the excavation of the ash dump.

In this connection, it would be very interesting to study some values of settlement at the Flushing Meadow as computed and as actually observed.

Conclusion.—Professor Burmister is to be commended for his sound views on the value of the soil laboratory work. He calls the attention of the reader to the variability of material either in samples or in different borings at the same elevation, which means that absolutely conclusive results cannot be obtained in the laboratory. In making his settlement analysis he proceeds with extreme care and checks the values of the pre-consolidation load by a special shear-test correlation curve. Finally, he tends to simplify the laboratory procedure as much as possible by trying to correlate consolidation characteristics of a soil with such simple tests as the determination of the liquid limit and of the plasticity index.

B. K. HOUGH, JR.,[21] ASSOC. M. AM. SOC. C. E. (by letter).—In his "Synopsis," the author conveys the impression that the tests described in his paper were conducted for certain very practical purposes, such as determination of the general suitability of the Flushing Meadow for the intended development. It is stated (see "Synopsis") that "They [the tests] were made primarily to furnish information on what could be done and how it should be done." The paper itself is disappointing in that the description of the application of test data to practical problems is very meager.

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[20] *Engineering and Building Record*, October 11, 1890, p. 296

[21] Area Engr., U. S. Engr. Dept., Elmira, N. Y.

Not only is it a matter of general interest to learn how test data are applied in practice, but, in the writer's opinion, the validity of the data cannot be judged apart from its application. In numerous instances the writer has encountered the popular belief that a given soil has certain characteristic constants which are readily determinable by laboratory tests and which can be applied under any conditions. Opposed to this belief is the theory that testing must simulate (at least in some respect) the anticipated construction procedure, and hence both testing procedure and the type of test data to be secured must be fitted to the proposed work. If space permits in the author's closure, further discussion on application of data might be undertaken.

Other factors bearing directly on the reliability of test results are the equipment and technique used in obtaining samples. Through first-hand knowledge of the Flushing soil formations, obtained during subsurface exploration at various times in Flushing Bay and at the La Guardia Airport, the writer believes that there is some doubt as to the reliability of samples obtained as described by the author, in 3-in. borings. The author's statement that the ratio of maximum stresses in the undisturbed and remolded shear tests was as great as six to one is probably a favorable indication however (see "Physical Characteristics"). Another point in connection with sampling is that observations on the number of blows per foot to drive the casing are not as valuable as the number of blows required to drive the sample spoon at each sampling point. The former reflects a highly complex condition of side friction on the casing which is almost certain to increase with depth since the length of casing steadily increases; whereas the latter indicates with some reliability the resistance encountered at each elevation.

The main thought in the paper appears to be that relatively complicated and expensive characteristics tests need not be run on all samples if some correlation can be established between such data and the results of relatively simple routine tests. This thesis is one that merits further development since the objective of soils engineering is to reduce, not to increase, the over-all cost of earthworks construction. For his contribution to this subject, the author is to be commended.

T. B. RIGHTS,[22] ASSOC. M. AM. SOC. C. E. (by letter).—Professor Burmister is to be congratulated on "lifting the veil" on soil mechanics and the mechanics of making tests. However, the average structural engineer (one who has been out of college ten years or more) is still almost as much "in the dark" as he has always been in regard to soil analysis. The writer realizes that this subject is in the transition period. In fact, as far as the writer knows it is only in the last few years that a few books in the English language have been published on this subject. A few years ago engineers were forced to depend upon mimeographed notes or isolated articles in the technical journals. Investigations made by the committee headed by Robert A. Cummings, M. Am. Soc. C. E., laid the groundwork for the researches that have revealed what is known at present.

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[22] Roselle, N. J.

When soil mechanics started getting publicity about 1935 or 1936, structural men heaved a sigh and said, "Fine, give us the formulas." When no formulas arrived and the structural men discovered that they would have to make expensive and extensive subsurface explorations, they went back to their old method, which is still good if done by an experienced engineer. The old method, according to the writer, consisted in going out and looking at the site, maybe making one loading test of a 1-ft square area, holding a consultation, and stating "Our company has always used a value of 2 tons per sq ft." After spending most of their time on the design of the superstructure, when the foundation settled extensively or tilted, they consoled themselves with the fact that foundations are tricky things and hoped for better luck next time.

More and more subjects are being pushed into the realm of specialization; but structural engineers should have a basic knowledge of the subject of soil mechanics. The design of large structures should be referred to consulting engineers, but the small structures will still have to be built without their services. This is where the average structural engineer should have a basic knowledge of the subject. Where will he obtain this knowledge? Few colleges are giving this subject in extension courses. There is no place for the man residing outside of very large cities to study this subject, except through extensive reading. Many libraries, even in the larger cities, do not have reference material in soil mechanics. It is not surprising that many engineers and designers do not have enough knowledge to design foundations properly. An example of this was recently reported in the metropolitan papers. A building within 5 miles of Flushing Meadow Park is settling badly. Near the beginning of the news article, the engineers are reputed to have stated that the end of the settling process would occur in two weeks. Near the end of the same article was another statement that no one can tell when the settlement will stop.

These news statements should be contrasted with what was done at Flushing Meadow.[23] After the settlement analysis for the Flushing Meadow grounds had been computed, settlement contours were plotted. Additional fill was placed so that, at the end of the 2-yr settling period, the ground would be properly graded. The same investigation could have been made for the aforementioned building as was made for the New York World's Fair. Although this building is on piles, a complete soil investigation would have determined proper pile lengths to support it without extensive settlements.

A paper like that by Professor Burmister does much to bring before the engineering profession a scientific method of making a correct and complete settlement analysis. The undisturbed samples taken in these soil investigations were 3 in. in diameter. The writer understands that in Boston, Mass., 5-in. samples are used; 6-in. diameter samples would produce more accurate results in soil investigations. The possibility of the disturbance in 3-in. samples is very great. If Professor Burmister and others interested in soil mechanics can persuade clients and those paying for undisturbed samples to use 6-in. sizes, they will do much to establish foundation practice on a scientific basis.

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[23] "The Application of Soil Mechanics in Building the New York World's Fair," by George L. Freeman, George W. Glick, and Hamilton Gray, *Civil Engineering*, October, 1940, p. 649.

The writer believes that the correlation diagrams in Fig. 4 will prove a "milestone" in soil investigations. If another diagram showing voids ratio versus shear strength is added, it will act as a partial check on the other tests. If a sample falls in the same shear-strength range as the other samples, but the compression index falls outside the same range, it might be presumed that the running of the compression test might be in error. If the sample falls outside the shear-strength range, investigation could be made to see whether a real undisturbed sample was used or whether the sample was disturbed or remolded in either the sampling operation or at some later period.

Donald M. Burmister,[24] Assoc. M. Am. Soc. C. E. (by letter).—It is very gratifying to find that Messrs. Thomas and Sinacori share many of the writer's views and have made practical use of correlation methods on a large project. In Figs. 9 and 10 they have shown how more effective methods were developed in borrow-pit studies, especially from the preliminary, and time and expense saving, standpoint, for obtaining information on the suitability of impervious rolled-fill materials for earth dams. The writer has made similar use of correlation methods in earth-dam investigations.[3] Although generalizations for all soil types are not possible, satisfactory regional correlations can be made for localized areas, where similar soil types are encountered; that is, soils formed essentially by the same geological processes and having essentially the same mineralogical character and grain-size distribution.

An important task of the engineer is to develop more effective methods of soil investigation for specific problems, especially from the time and expense saving standpoint. This involves the application of "Control Test Methods" by making a few carefully conducted and complete soil tests (consolidation, shearing, routine classification tests, etc.) on typical representative soils to define the correlation band or bands for the soil types encountered on the given project. The methods may be extended to the cohesionless soil types by using the "density limits," that is, the loose limit *L*, and the dense limit *D*, which serve the same purpose and reflect the physical character of cohesionless soils in a manner similar to the consistency limits for cohesive-plastic soils. Once a satisfactory correlation has been established, the simple routine tests can be made quickly and at a relatively small expense on a large number of samples in order to obtain estimates of the consolidation, shearing, permeability, and other properties of soils, in as great a detail over an area as is considered necessary for an adequate investigation and solution of the problem. It is much more important to obtain information on the possible range of the estimates of these properties, which have a direct bearing on the analysis of the problem, as given by the width of the correlation band, than it is to know the exact values for a few isolated samples in a given area. Such an analysis would not only permit the engineer to visualize and analyze the situation more completely, but would also serve a very useful purpose for design and construction. Experience will be the best guide in teaching engineers how to make correct practical applications of this information.

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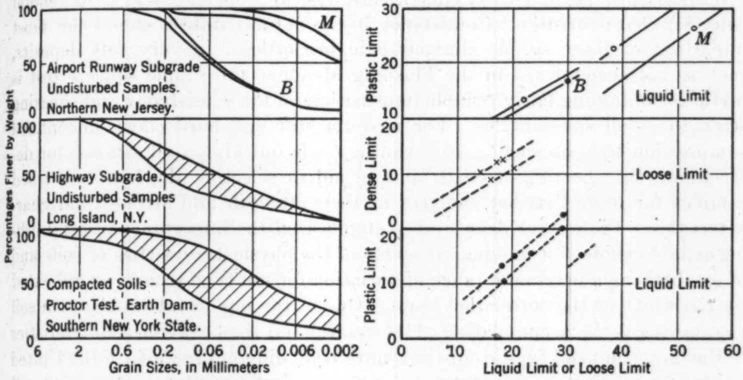
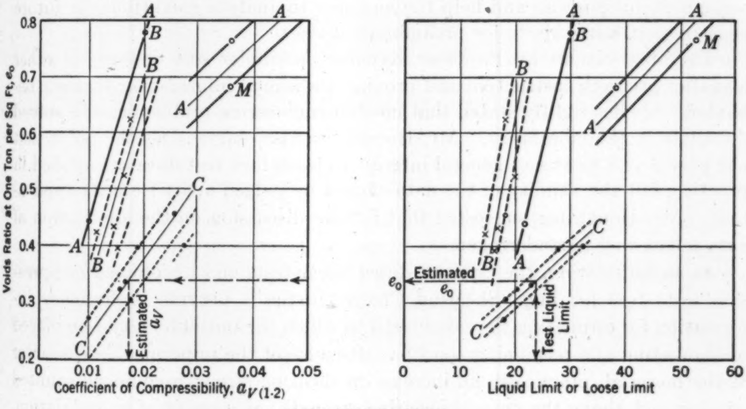
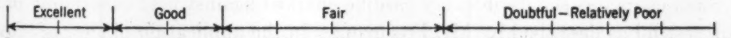
[24] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.



(*a*) GRAIN SIZE DISTRIBUTION (*b*) CONSISTENCY AND DENSITY LIMITS



(*c*) CORRELATION DIAGRAMS



(*d*) CLASSIFICATION OF SETTLEMENT AND BEARING PROPERTIES

FIG. 11.—CONSOLIDATION PROPERTIES

To illustrate further the possibilities of such methods, data for three cases have been analyzed in Fig. 11 for the consolidation properties of the soils. This permits a more satisfactory classification either of artificially placed material or of natural deposits as to their relative settlement and bearing properties for highway subgrades, airport runways, and the like.

Professor Kilcawley states that an important part of foundation investigations is to present the field data as well as the results of laboratory tests so that subsurface conditions can be visualized clearly and interpreted correctly. This requires an accurate knowledge of the kinds of soils encountered—their extent, uniformity, and variability, their physical properties, and their general behavior characteristics. Consistency in the natural state is one of the most important of these simple characteristic properties. In very soft deposits, such as are encountered in the Flushing Meadow, the simple squeeze test is useful for obtaining fairly reliable information on the consistency and shearing strength of all soil samples. For medium stiff and hard clays, unconfined compression tests on small prisms can be made quickly on all samples for determining the shearing strength and to aid in selecting the least disturbed samples for more detailed analyses of their strength and stress-strain characteristics. Professor Kilcawley has emphasized the importance of establishing reliable guides for making estimates of the physical properties of soils and of establishing a maximum range of variation for soil type groups, as indicated by the width of the correlation band. One of the important problems in soil engineering is the accumulation of information on the behavior characteristics of the important soil type groups encountered in different sections of the United States, for example, such as those presented in Figs. 4, 9, 10, and 11, which will serve a useful purpose and help the engineer to analyze conditions for future construction, particularly for preliminary studies.

The observations by Professor Krynine on similar soft deposits in other localities are very instructive and provide additional information on their behavior. He has rightly stated that no civil engineering problem can be solved definitely in the laboratory. Mr. Hough, likewise, emphasizes the point that not only is it a matter of general interest to learn how test data are applied in practice, but the validity of the data cannot be judged apart from its application. Mr. Hough has suggested that further discussion on the application of data might well be undertaken.

As an illustration, the "quick" direct shear tests and unconfined compression tests (see heading "Shearing Characteristics") provide preliminary information for estimating the safe height to which the initial fill could be placed in blanketing and reclaiming the site. Because of the impermeable character of the material, practically no increase in shearing strength could be counted on over and above the natural shearing strength, as a result of consolidation. Such estimates should in every case be checked against field experience, or by a test fill as described by Mr. Freeman.[2] In the application of the theories of elasticity and plasticity to foundation problems, Mr. Jürgenson[11] has given the critical shearing stress conditions beneath the toe of an embankment approximately as follows: Plastic state—

$$S = \frac{p}{\pi} \quad \ldots\ldots \quad (4a)$$

and, failure—

$$S = \frac{p}{\pi + 2} \quad \ldots\ldots \quad (4b)$$

The shearing strength varies with depth as shown in Fig. 5. The situation would be more serious if it were not for the fact that the meadow mat had a very important spreading effect on stresses and strengthened the surface of the underlying soft material so that is was capable of supporting the initial blanketing to a height of about 4 ft and the grading equipment. The critical zone is deeper and is confined to a considerable extent by this mat layer; thus:

Depth	Plastic condition in meadow mat layer	Failure in underlying soft deposit
$p = w\,h =$	$2{,}000\,\pi\,S$	$2{,}000\,(\pi + 2)\,S$
h	$\frac{2{,}000}{100}\,\pi \times 0.08$	$\frac{2{,}000}{100}\,(\pi + 2)\,0.04$
h (in feet)	5.1	4.1

Experience in reclaiming and filling the Flushing Meadow showed that only in a few instances were any appreciable subsurface movements started, if the height of fill was kept within recommended safe limits.

To illustrate the application of the consolidation test results in a settlement analysis, the situation at boring No. 17, Figs. 1, 5, and 6, is considered. The following steps are taken in estimating the settlement to be expected as a result of a blanket fill, for example, of 10 ft:

1. The profiles of the overburden pressure, pre-consolidation pressure, and the pressure due to the fill, are estimated in Fig. 5 for boring No. 17, using the data on the approximate moisture content in Fig. 1 and the consolidation test data for two samples from boring No. 17 in Fig. 3 (also Fig. 2). The profile is subdivided, as shown, into a convenient number of increments of depth (located halfway between each sample, except at changes in the character of the soil).

2. The settlement property of the soils, which is defined by the compression index c_v, is estimated from the results of two consolidation tests in Fig. 3 and from the correlation diagrams of Fig. 4 for the soils of the Flushing Meadow for the remainder of the samples. The liquid limits of the samples of boring No. 17, which are the correlation arguments, are taken from Fig. 1. For settlement analyses it is more important to obtain an estimate of the possible range of the settlement properties of the soils in a deposit at the point where settlement estimates are desired than it is to know an exact value for an individual sample at a certain depth only. Accordingly, for the range of liquid limit for these samples, the approximate maximum range of the estimates of the compression index c_v for the inner correlation band of Fig. 4 is found to be about $\pm 15\%$ from the mean curve of the correlation, which, it is believed, should be considered a fairly satisfactory accuracy considering all of the uncertainties involved in such analyses.

3. The natural stress conditions in the soil (which were discussed in the paragraphs preceding "Conclusions") are assumed for Case 1 to be represented by the dotted curve of Fig. 5, of pre-consolidation pressure (and complete consolidation). The approximate corresponding voids-ratio in the natural

state (e_n) at the elevation of each sample is either taken directly from the pressure-voids ratio curves of Fig. 3 or is computed from Eq. 1. The stresses due to the 10-ft fill are computed on the basis of a unit weight (compacted) of 100 lb per cu ft.

4. In making the settlement estimates by the Increment Depth Method, Eq. 2 is solved by substituting the values of the voids ratios e_1 and e_2, corresponding to pressures p_1 and p_2 from Fig. 5 as follows:

$$\text{Settlement} = h \times \left(\frac{c_v}{1 + e_n}\right)\left(\log_e \frac{p_2}{p_1}\right) \ldots\ldots\ldots\ldots\ldots (5)$$

The approximate estimates of settlement are made as shown in Table 4, which contains the values obtained for each of the foregoing items.

TABLE 4.—APPROXIMATE SETTLEMENT ESTIMATES FOR BORING NO. 17, CASE 1, DUE TO A BLANKET FILL OF 10 FT

Sample No.	Depth, in ft	H, in ft	w_L	c_v	e_v	e_n	$\frac{c_v}{1 + e_n}$	p_1	p_2	$\log_e \frac{p_2}{p_1}$	S, in ft
..	0	6	(151.2)	0.815	3.52	4.77	0.141	0.10	0.60	1.38	1.17
1	6.0	5	151.2	0.517[a]	2.67[a]	3.39[a]	0.118[a]	0.23	0.73	1.154	0.69
..	11.0	..	....	....	....	....	....	....	....	....	....
2	17.0	11	104.5	0.500	2.45	3.04	0.123	0.25	0.75	1.095	1.47
3	27.5	10	105.3	0.506	2.45	3.10	0.123	0.27	0.77	1.045	1.29
4	38.0	11	116.2	0.680[a]	2.57[a]	3.41[a]	0.154[a]	0.29	0.79	1.000	1.70
5	49.5	14	106.0	0.511	2.48	3.06	0.126	0.32	0.82	0.940	1.65
..	57.0	..	....	....	....	....	....	....	....	....	....
6	59.0	7	(122.0)	0.635	2.90	3.30	0.146	0.50	1.00	0.460	0.47
..	64.0	..	....	....	....	....	....	....	....	....	....
										$S = 8.44$[b]	

[a] Consolidation test values. [b] Possible range of estimates ±15%, or 7.3 to 9.7 ft.

To obtain some idea of the effect of differences between the possible stress conditions in the deposit and those assumed, the overburden pressure (full line) is assumed to represent the actual stress conditions for Case 2. The corresponding settlement estimate is equal to 7.7 ft. The corresponding settlement estimates for a 5-ft fill are equal to 5.3 and 4.9 ft, respectively. The settlement in the meadow mat layer at the top and the peat layer at the bottom probably occurred largely during the filling operation because of their more permeable character, as evidenced by the continuous flow of water from the toe of the advancing fill as it was constructed. Hence this settlement would be compensated for by additional fill in the operation of filling and grading to the required surface elevation, as suggested by Mr. Rights. Deducting these amounts, the estimates of subsequent settlement to be expected for a 10-ft fill (total height), for Cases 1 and 2, are 6.1 and 5.5 ft, respectively, and for a 5-ft fill, 3.9 and 2.7 ft, respectively.

The rate of settlement is estimated roughly on the assumption of an average value of the coefficient of consolidation, C_c, of 0.0012 cm² per min for the deposit, and of the natural voids ratio, e_n, of 3.32, and a deposit 47 ft thick, exclusive of the meadow mat and peat layers. The estimated times t in years, corresponding to different percentages of the ultimate settlement and time factors T_v, are estimated by the equation:[25]

$$t = \left(\frac{h}{1 + e_n}\right)^2 \frac{T_v}{1{,}400\, C_c} = 70.8\, T_v \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and are given in Table 5.

In regard to the driving records of the casing and sampler, the writer agrees with Mr. Hough's statement that the number of blows per foot to drive the sampler indicates more reliably the resistance encountered at each elevation. In the Flushing Meadow deposits, only about one or two blows were required to drive the sampler because of its soft consistency. Since, for all important structures, pile foundations were required, the driving resistance of the casing was considered to be a better indication of total resistance, in this case.

TABLE 5.—ESTIMATES OF THE RATE OF SETTLEMENT

Time[a]	PERCENTAGE OF SETTLEMENT				
	10	20	30	40	50
T_v	0.02	0.08	0.17	0.31	0.50
t	1.4	5.6	12.0	22.0	35.4

[a] T_v = time factor; and t = time, in years.

In foundation studies for buildings and earth dams, the writer has devised and used the tentative values listed in Table 6, which has proved to be very useful for investigating and analyzing subsurface conditions with respect to their foundation properties.

TABLE 6.—TENTATIVE VALUES OF APPROXIMATE DEGREES OF COMPACTION

No.	Granular cohesionless soils	No. of blows[a]	Density[b] %	Cohesive plastic soils
1	Very loose silt..................................	<5	.....	Very soft clay
2	Loose sand or silt..................................	5–15	<40	Medium soft clay
3	Loose sand and gravel; medium compact sand or silt...	15–35	40–70	Medium stiff clay
4	Medium compact sand and gravel; compact sand or silt.	35–70	70–90	Stiff or medium hard clay
5	Compact sand and gravel; very compact sand or silt...	>70	90+	Very hard clay

[a] Blows per foot; 3-in. sampler; driving energy, 7,200 in-lb; weight of hammer, 300 lb; average drop, 24 in.
[b] Relative density, expressed as a percentage.

The liquid limit of cohesive plastic soils and the loose limit of cohesionless soils, as laboratory references, are assumed to be 40% relative density based on experience with the measured loosest states found in natural deposits. The plastic limit and the dense limit, respectively, as laboratory references, are assumed to be 100% relative density. On this basis very few natural deposits have a relative density greater than 100%.

The relative densities D_d are determined by the relations: Cohesive plastic soils,

$$D_d = 40 + 60\, \frac{w_L - w_N}{w_L - w_p} \ldots\ldots\ldots\ldots\ldots\ldots (7a)$$

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[25] Progress Report of Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 777.

and cohesionless soil,

$$D_d = 40 + 60 \frac{w_l - w_N}{w_l - w_d} \dots\dots\dots\dots\dots\dots\dots (7b)$$

in which:[26] w_L = liquid limit; w_p = plastic limit; w_N = approximate natural moisture content; w_l = loose limit, as a moisture content; and w_d = dense limit, as a moisture content.

A distinction is made in the driving resistance of sand-gravel moistures and sand or silt because the presence of gravel increases the resistance quite as much as an increase in the degree of compactness. There is no simple direct relation for different sizes of samplers and casings, and therefore the numerical values must be determined from field experience. However, for different weights of hammers and drops, fairly consistent values are obtained, if the blows per foot are made inversely proportional to the driving energy (weight of hammer times drop).

The tentative values in Table 6 have not only made it possible to visualize subsurface conditions better as to their foundation properties, bearing capacity, length of piles, etc., but also have served a useful purpose in providing numerical values by which to compare different situations, particularly in cases where settlements are known, where loading tests have been made, etc.; or, in cases where the allowable bearing capacity is to be estimated—for example, from the Boston Building Code, which differentiates between the values for loose and compact conditions of the soils.

In conclusion, the writer wishes to express his gratitute to the discussers for their stimulating comments.

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[26] See "Soil Mechanics Nomenclature," *Manual of Engineering Practice No. 22*, Am. Soc. C. E.

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HYDRAULICS OF SPRINKLING SYSTEMS FOR IRRIGATION

By J. E. Christiansen,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Arthur F. Pillsbury, Tom A. Bither, Ralph W. Powell, Harry F. Blaney, and J. E. Christiansen.

Synopsis

Portable systems for sprinkling agricultural crops were first used in the Sacramento Valley of California in 1931, although in 1930 there were a few scattered installations in the southern part of the state. Sprinkling with stationary systems had been confined largely to citrus orchards and truck crops because of the relatively high investment required. Low-cost portable systems, however, made sprinkling a feasible method of irrigating large acreages of field crops such as beans, peas, onions, and sugar beets.

In 1932 the Division of Irrigation of the University of California began a study of sprinkling, principally to determine (1) the hydraulic characteristics of rotating sprinklers, (2) the loss of water by evaporation, (3) the hydraulic characteristics of sprinkler lines, (4) the cost of applying water by sprinkling, and (5) the general success of sprinkling as a method of irrigation. The scope of this paper is defined by the first three of these items.

Portable Sprinkler Systems for Field Crops

In brief, a portable sprinkler system is a line of light-weight pipe with quick-acting couplings (on which rotating sprinklers are mounted) together with a pumping plant. Sprinkler pipe comes in standard lengths of 20 ft; other lengths are furnished on special order. Sprinklers are usually spaced 40 ft apart; but sometimes the spacing is 20 ft (or 30 ft when 30-ft pipe is used). The pipe is moved across the field, usually by two men who carry one length at a time. Several different makes of systems are now available, each with its own kind of coupling.[2] Formerly, only 4-in. pipe was available; but now one can obtain some of the systems in $1\frac{1}{2}$-in., 2-in., 3-in., 4-in., 5-in., and 6-in. sizes and in any

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Note.—Published in January, 1941, *Proceedings*.

[1] Irrig. and Drainage Engr., U. S. Regional Salinity Laboratory, U. S. Dept. of Agriculture, Riverside, Calif.

[2] "Irrigation by Sprinkling," by J. E. Christiansen, *Journal*, Am. Soc. of Agri. Engrs., December, 1937, p. 533, Fig. 1.

desired pipe length. Special quick-acting couplings are also made for use on smaller pipe for orchard systems.

As a rule, the systems operate with a portable pumping plant that pumps from a field ditch along one side of the field, or through the center. The plant shown in Fig. 1 is operating on two quarter-mile lines of 4-in. and 5-in. pipe,

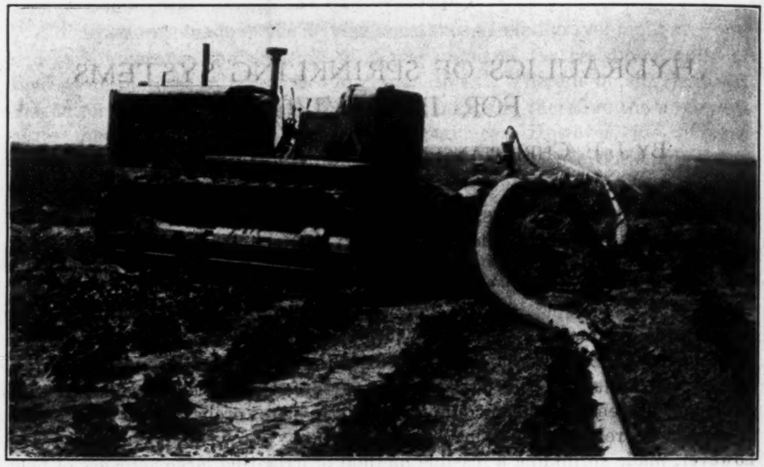

FIG. 1.—TYPICAL PORTABLE PUMPING PLANT

FIG. 2.—MOVABLE SPRINKLER MACHINE PUMPING 500 GAL PER MIN FROM FIELD DITCH

with 61 sprinklers. Its capacity is approximately 1,100 gal per min. The most economical arrangement is to have the supply ditch through the center of the field with a sprinkler line extending in both directions. This minimizes the friction loss in the pipe line and necessitates shutting down only one line at a time when moving pipe, thereby appreciably increasing the operating efficiency of the system.

If the water supply is obtained from wells or if open ditches through the field are not feasible, a stationary pump and a pressure supply line are used. The latter may be either a portable pipe similar to the sprinkler line, or a stationary pipe with valve outlets. On stationary lines the valves are usually spaced about three times the distance the pipe is moved, so that three settings are made from each valve.

Another type of low-pressure system, consisting of light-weight perforated pipe, has recently come into use for field crops and pasture. Water is distributed over a strip 25 to 50 ft wide. These systems operate satisfactorily on pressures as low as 5 or 6 lb per sq in., but the effective width covered increases with the pressure up to about 25 lb per sq in. These systems apply the water at a relatively high rate, which limits their use to pervious soils and necessitates moving the pipe frequently.

Rotating nozzle lines are also used to some extent for irrigating truck crops and nurseries. They are probably more common in Eastern United States than in California.

One innovation is a movable sprinkler machine (Fig. 2) that pumps water from a field ditch and distributes it through two large part-circle sprinklers while traveling very slowly but continuously along the ditch. Several such machines[3] were constructed in 1938. They have capacities up to 600 gal per min and effectively cover a strip about 250 ft wide when operating at a pressure of about 60 lb per sq in.

ORCHARD SPRINKLING SYSTEMS

The several types of orchard sprinkling systems may be divided into two general groups: The overhead systems with sprinklers mounted above the trees; and the under-tree systems with sprinklers mounted on short risers so that they distribute the water under the branches of the trees.

There are three types of overhead sprinkler systems: (1) Those with stationary sprinklers mounted above the trees; (2) those with portable sprinklers and risers that are moved from place to place on a stationary pipe system; and (3) those using portable pipe and sprinklers. There are also different types of under-tree systems, some using stationary sprinklers, others using portable sprinklers attached to hose or mounted on portable pipe. Stationary supply lines with valve outlets are generally used for portable systems.

The under-tree systems are gaining in popularity; some stationary overhead systems are even being converted into them. Under-tree sprinklers are usually spaced the same distance apart as the trees, so that each sprinkler covers an area

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[3] "Our New Movable Sprinkler Machines," by J. E. Christiansen, Pacific Rural Press, October 15, 1938.

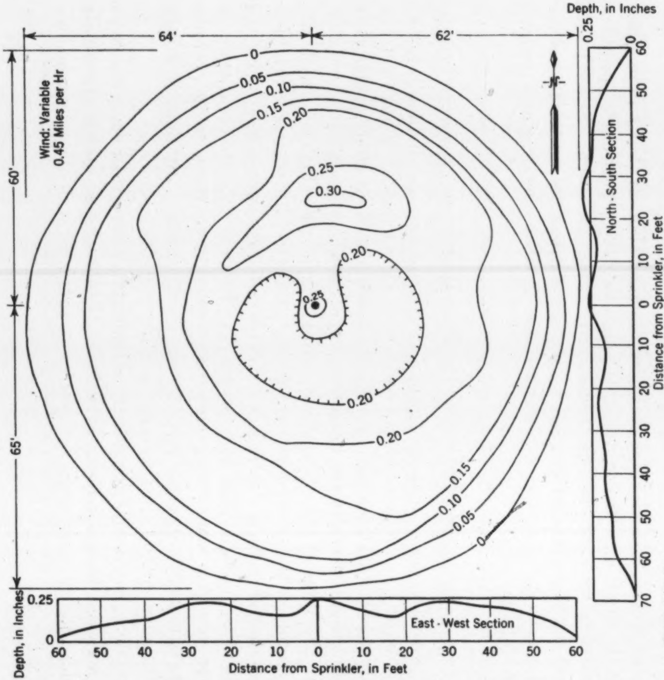


FIG. 3.—TEST 16. DISTRIBUTION OF WATER FROM SPRINKLER WITH $\frac{9}{32}$-IN. AND $\frac{3}{16}$-IN. NOZZLES, OPERATING AT 45-LB PRESSURE; SPRINKLER ROTATION, 0.7 RPM

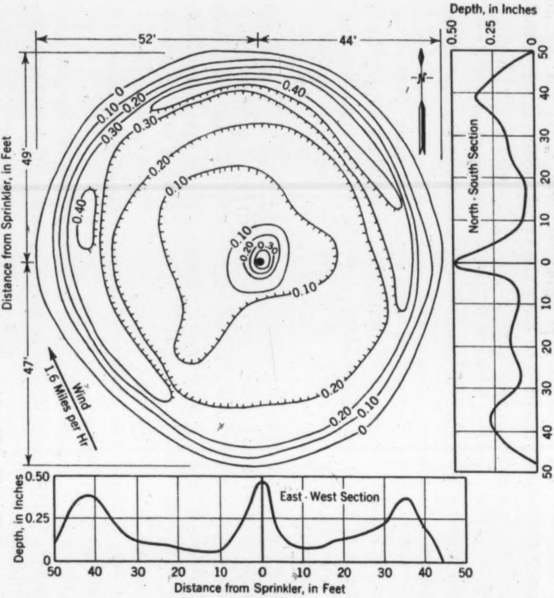


FIG. 4.—TEST 11. EFFECT OF LOW PRESSURE (20 LB PER SQ IN.) ON DISTRIBUTION OF WATER FROM SPRINKLER WITH $\frac{5}{16}$-IN. AND $\frac{5}{32}$-IN. NOZZLES; SPRINKLER ROTATION, 0.6 RPM

equal to that occupied by one tree, whereas stationary overhead sprinklers are generally spaced three to four tree rows apart each way. Consequently the distribution of water is much better for the under-tree systems, since each tree is supplied with essentially the same quantity of water, although the distribution over the area occupied by each tree may not be uniform. Very little is gained by wetting the leaves, and often this practice is detrimental in that it spreads disease. Small under-tree sprinklers operate satisfactorily on pressures of 10 to 20 lb per sq in., whereas the larger sprinklers on overhead systems require 30 to 50 lb per sq in. The low pressures can sometimes be supplied by gravity, whereas pumping plants would be required for the higher pressures.

Distribution of Water from Sprinklers

In studying the hydraulic characteristics of sprinklers, more than 300 tests were made at Davis, Calif., to determine the distribution of water from sprinklers. Most of these tests were for periods of one hour, some for ½ hour. For most of the experiments, cans were spaced 10 ft apart (or closer) in both directions over the entire area covered by the sprinkler; and, for certain tests, as many as 280 cans were used. The water was measured to the nearest cubic centimeter (equivalent to 0.005 in. in depth) in a graduated cylinder. During most of the tests, wet-and-dry bulb temperatures were taken for use in evaporation studies. To measure the wind velocities, a four-cup anemometer was mounted in an exposed position about 10 ft above the ground. By means of a return-pressure line from the base of the sprinkler, with a pressure gage mounted near the control valve, a nearly constant pressure was maintained. The plotting of the tests is illustrated in Fig. 3, which shows the distribution obtained with one sprinkler under favorable conditions (sufficient pressure, no wind, and a fairly slow and uniform rotation).

The effect of low pressure is shown in Fig. 4. With a pressure of 20 lb per sq in. most of the water was thrown to the outside edges of the area covered, with very little falling between 10 and 20 ft from the sprinkler. Fig. 5 shows the effect of wind. The area covered is offset about 2 ft for each mile per hour of wind, with a high concentration near the sprinkler, especially on the windward side.

A cause of uneven distribution is often a variable rate of rotation of the sprinklers. Every sprinkler tested varied somewhat in this respect, some much more than others. In one case (Fig. 6), the angular velocity was eight times as great when the main nozzle was discharging to the east as when it was discharging to the southwest. For a study of rotation, an anemometer recorder was converted into a rotation recorder by gearing it so that the drum rotated at a speed of one inch in 50 sec. Attached to the sprinkler was a commutator arrangement that closed an electrical circuit every 30° or 60°, as desired. In this way the actual rate of rotation for a large number of revolutions was recorded automatically. Uneven rotation is not caused by wind, as is commonly supposed.

The rate of rotation greatly affects the distribution of water. For maximum coverage, a sprinkler should rotate less than 1 rpm. Often it may rotate as

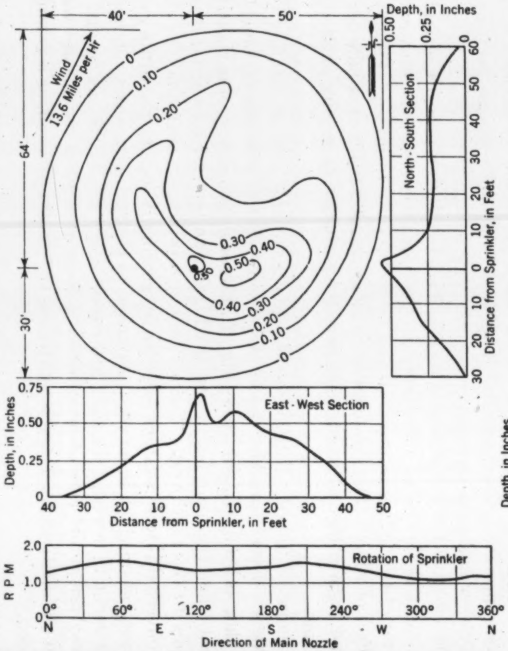


Fig. 5.—Test 112. Effect of Wind (13.6 Miles per Hr) on Distribution of Water from Sprinkler with $\frac{4}{16}$-In. Main Nozzle. Pressure, 40 Lb; Rotation, 1.3 rpm

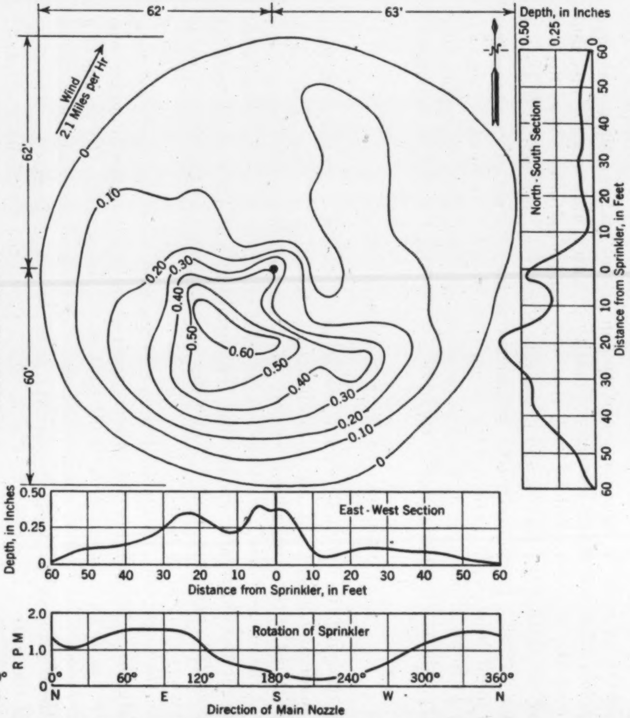


Fig. 6.—Test 103. Distribution of Water from Sprinkler with Variable Rate of Rotation; Pressure, 40 Lb per Sq In.

slowly as $\frac{1}{3}$ rpm. The slower the rotation, the larger the area covered. When improperly adjusted and allowed to rotate at 20 rpm or more, sprinklers cover only about half the area they would cover at a slow rate under the same pressure. In one instance, a sprinkler was observed to be rotating very rapidly, and a count showed the speed to be about 90 rpm. The effect of rapid rotation is shown in Fig. 7, in which case a speed of 13.6 rpm reduced the area covered to about 55% of what it would have been at 1 rpm. Unless such sprinklers are mounted closer together, the uniformity of application will be poor. In Fig. 7 the nozzles were $\frac{9}{32}$ in. and $\frac{5}{32}$ in., and the pressure was 40 lb per sq in. Under similar conditions, but with rotation less than 1 rpm, the diameter covered would have been about 115 to 120 ft.

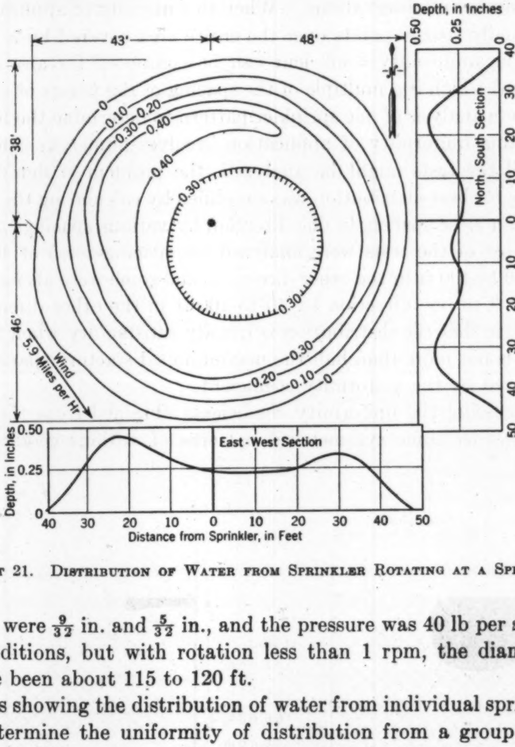


FIG. 7.—TEST 21. DISTRIBUTION OF WATER FROM SPRINKLER ROTATING AT A SPEED OF 13.6 RPM

Patterns showing the distribution of water from individual sprinklers can be used to determine the uniformity of distribution from a group of sprinklers spaced any distance apart. The resulting distribution from sprinklers with patterns corresponding to Fig. 3, for the customary spacing of 40 ft by 50 ft, is shown in Fig. 8(*a*). The variation is from about 0.7 in. to slightly more than one inch. Fig. 8(*b*) shows the resulting distribution for patterns corresponding to Fig. 4, illustrating the effect of low pressure. Here the variation is from about 0.2 in. to more than one inch. Fig. 8(*a*) illustrates the uniformity obtained under favorable conditions and Fig. 8(*b*) illustrates what occurs when pressures are inadequate.

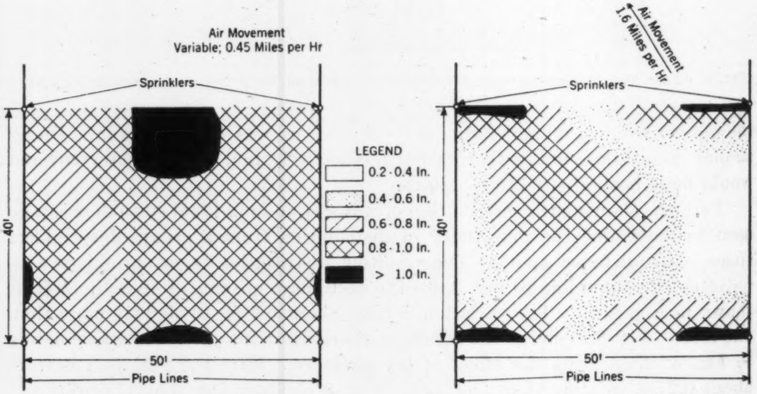
To compare sprinkler patterns and to determine how various spacings affect the resulting distribution of water, one needs a numerical expression that can serve as an index of the uniformity secured. For this purpose an expression called the uniformity coefficient was adopted. The uniformity coefficient (C_u) expressed as a percentage is defined by the equation

$$C_u = 100\left(1.0 - \frac{\Sigma x}{M\,n}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which x is the deviation of individual observations from the mean value m, and n is the number of observations. When the intensity of application at any number of equally spaced points over the entire area covered by a sprinkler is determined, the uniformity coefficient can be computed for any spacing (in either direction) which is a multiple of the spacing of the points of observation. Thus a complete analysis of one sprinkler pattern to determine the best spacing and the resulting uniformity of application involves numerous computations.

So that all the tests might be analyzed, the problem of determining the proper spacing for best distribution was simplified by calculating the uniformity coefficients for a close spacing in one direction by various spacings in the other direction. Most of the tests were analyzed for spacings of 5 or 10 ft in one direction by 40 to 100 ft in the other direction, and some were also analyzed for spacings of 40 ft in one direction by 40 to 100 ft in the other direction. The analyses indicate that the distribution is usually satisfactory when the spacing along the line is not more than half the maximum satisfactory spacing between lines as indicated by the uniformity coefficient.

Before discussing the uniformity coefficients obtained from the tests, one might well consider some symmetrical patterns of various geometric shapes



(*a*) PATTERN IN FIG. 3; 45 LB PER SQ IN. (*b*) PATTERN IN FIG. 4; 20 LB PER SQ IN.

FIG. 8.—DISTRIBUTION RESULTING FROM SPRINKLERS HAVING VARIOUS PATTERNS WHEN SPACED 40 FT BY 50 FT APART

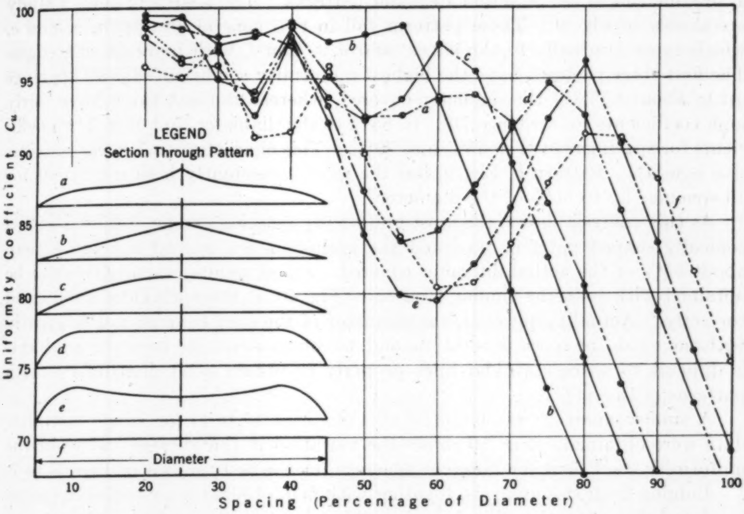


FIG. 9.—COEFFICIENTS OF UNIFORMITY FOR DIFFERENT SPACINGS OF GEOMETRICAL PATTERNS

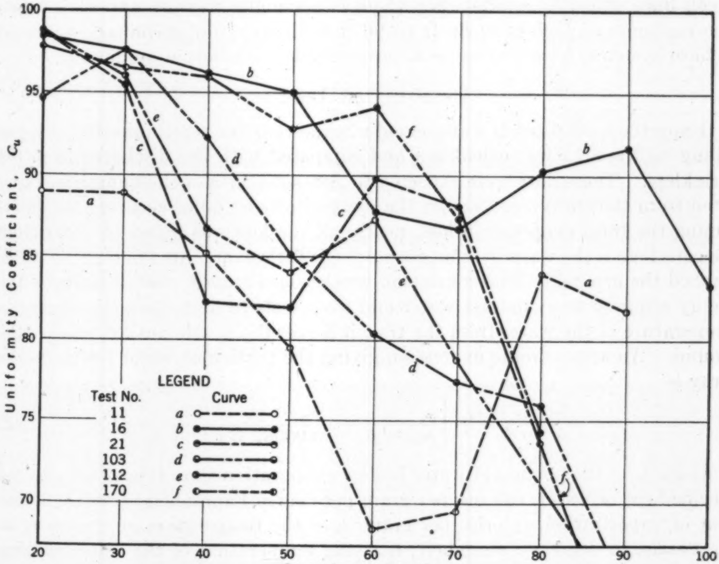


FIG. 10.—UNIFORMITY COEFFICIENTS FOR THE SPRINKLER PATTERNS SHOWN IN FIGS 3 TO 7

approximating those of actual sprinkler patterns. The results of such a study are shown in Fig. 9. These patterns fall in two general groups: *a*, *b*, and *c*, which taper gradually to the edges; and *d*, *e*, and *f*, with more abrupt edges. The first three patterns have the highest uniformity coefficients for all spacings up to about 65% of the diameter covered, whereas the last three have fairly high coefficients for spacings 70% to 85% of the diameter, but very low coefficients for spacings between 45% and 70%. This characteristic indicates excessive overlap. Pattern *b*, Fig. 9, has the most consistently high coefficient for all spacings up to 55% of the diameter.

As this analysis shows, the most uniform application can be obtained with a conically shaped pattern, provided the sprinklers are spaced not more than about 55% of the actual diameter covered. Good results, apparently, can be obtained with patterns similar to *d* or *e* (Fig. 9), if the sprinklers are spaced correctly. Actually, however, the diameter of the area covered varies greatly with the wind, the speed of rotation, and, to some extent, the pressure, so that it is difficult to space sprinkler lines properly to obtain good distribution with patterns of this type.

A similar analysis was made of all the sprinkler tests for which complete data were obtained. Fig. 10 shows the variation in the average values of the uniformity coefficient for different spacings for the tests shown in Figs. 3 to 7.

Judging from this analysis, together with field observations, sprinklers must be placed closer together than is indicated by the study of symmetrical patterns. The usual spacings of 20 ft, 30 ft, and 40 ft along the line and 50 ft to 70 ft between lines give fair results when there is adequate pressure and little wind. The customary spacings of 60 ft to 90 ft both ways for stationary overhead orchard systems, however, cause an appreciable variation in application.

EVAPORATION LOSSES

From the quantities of water caught in the cans the total quantity of water falling on the area was calculated and compared with the discharge from the sprinklers. These data were expected to give some idea of the quantity evaporated from the spray. Although the method proved quite accurate for determining the total evaporation loss, nearly all the loss was found to occur from the cans before the water was measured; very little evaporated before the water reached the ground. This conclusion was reached from a consideration of the energy required to evaporate water and from tests to determine the change in temperature of the water from the time it leaves the nozzle until it reaches the ground. An approximate expression giving the portion of water lost from the spray is

$$E = \frac{c\,(t_1 - t_2)}{L}\left[\frac{p_w - p_a}{p_w - p_a - 0.00037\,B\,(t_a - t_w)}\right]\ldots\ldots\ldots(2)$$

in which E = the portion of water loss by evaporation from the spray; c = the specific heat of water, calories per gram per degree Fahrenheit; L = the latent heat of vaporization, calories per gram; t_1 = the temperature of the water at the nozzle, in degrees Fahrenheit; t_2 = the temperature of the water striking the ground, in degrees Fahrenheit; t_w = the mean water temperature, in degrees

Fahrenheit; t_a = the air temperature, in degrees Fahrenheit; p_w = the vapor pressure of water at temperature t_w, in inches of mercury; p_a = the pressure of the water vapor in the air, inches of mercury; and B = the barometric pressure, in inches of mercury.

Evaluating the constants, and letting $c = 0.555$, $L = 585$, and $B = 30$, Eq. 2 becomes

$$E = 0.00095\,(t_1 - t_2)\left[\frac{p_w - p_a}{p_w - p_a - 0.011\,(t_a - t_w)}\right]\ldots\ldots\ldots(3)$$

One can derive a similar expression from the general evaporation formula,[4]

$$E = \frac{I - H_s - k - R_b}{L\,(1 + r)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

by assuming that the insolation I, the conduction k, and the back radiation R_b are negligible and can be omitted. The sensible heat, H_s, is indicated by the change in the temperature of the water. The other terms are as follows: L, the heat of vaporization; and r, the ratio of energy transfer by convection to that by evaporation (called Bowen's ratio), which is a function of the vapor-pressure deficit and of the difference in air and water temperature.

Several determinations of the temperature drop in the water between the sprinkler and the ground were made on field installations and at Davis. The evaporation losses for these tests, computed from Eq. 4, did not exceed one per cent. The drop in temperature, and therefore the evaporation loss, depends largely upon the size of drops, which in turn depends upon pressure, wind, and type of sprinkler. Apparently, however, the loss from the spray is very little, and the principal loss by evaporation is from the wet surface of soil and plants during and following the application of water.

The rate of this loss probably exceeds that from a free water surface because of the relatively larger area exposed. The rate decreases rapidly, however, as the surfaces dry, and becomes negligible after about a week, during which time the total loss may have exceeded an inch of water. In the interior valleys of California, rates of evaporation from standard Weather Bureau pans often exceed 0.40 in. per day; hourly rates frequently exceed 0.05 in., and average about 0.04 in. during the afternoon. The total evaporation loss frequently amounts to a large part of the application, especially when frequent light applications are made.

HYDRAULICS OF SPRINKLER LINES

One interesting problem in connection with sprinkler systems concerns the hydraulic losses at branch outlets along sprinkler lines. All the water does not flow through the entire line; it is distributed through side outlets equally spaced along the line. At each of these outlets there is a decrease in mean velocity in the pipe (assuming no change in pipe size) accompanied by an increase in pressure due to conversion of velocity head to pressure. There is also an hydraulic loss between the main and the sprinkler outlets. Consider a typical outlet along the line, sections 1, 2, and 3, representing, respectively, sections upstream and downstream from the outlet and in the side outlet. From Bernoulli's theorem, assuming no energy loss, one can show that

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[4] "Evaporation as a Function of Insolation," by Burt Richardson, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), pp. 996–1019.

$$p_2 - p_1 = \frac{w\,v_1^2}{2\,g}(2\,r_q - r_q^2) \qquad (5)$$

In Eq. 5, p_2 and p_1 are the pressure intensities on sections 2 and 1, respectively, v_1 = the mean velocity at section 1; g = acceleration due to gravity; r_q = ratio of $\frac{Q_3}{Q_1}$, or the proportion of water leaving the side outlet; and w = the weight of a cubic foot of water. John A. Oakey,[5] Assoc. M. Am. Soc. C. E., making a different assumption, derives an equation that can be written

$$p_2 - p_1 = \frac{w\,v_1^2}{2\,g}(3\,r_q - r_q^2) \qquad (6)$$

His experimental data, however, show that for values of r_q less than 0.7 the actual increase in pressure was approximately that given by Eq. 5. The energy loss between sections 1 and 3 can be determined only by experiment. Professor Oakey[5] gives values of K_3 from which the loss can be determined from the expression

$$e_1 - e_3 = \left(1 - r_q + \frac{K_3\,r_q^2}{r_p^2}\right)\frac{v_1^2}{2\,g} = \frac{v_1^2}{2\,g}(1 - r_q) + \frac{K_3\,v_3^2}{2\,g} \qquad (7)$$

in which e = energy; r_p is the ratio of the pipe area at section 3 to the pipe area at section 1; and K_3 is an empirical coefficient having values dependent both upon r_p and r_q. For small values of r_p, apparently, the value of the coefficient K_3 would be more nearly constant if the expression were written

$$e_1 - e_3 = \frac{v_1^2}{2\,g} + \frac{K_3\,v_3^2}{2\,g} \qquad (8)$$

Professor Oakey's data show that for $r_p = 0.0557$, K_3 varied from 0.53 for $r_q = 0.1$ to 0.36 for $r_q = 1.0$. Since values of v_3 are fairly constant along a sprinkler line and seldom exceed 8 ft per sec, the last term in Eq. 8 will seldom exceed 0.5 ft head. The conclusion reached is that the velocity in the sprinkler line has little effect on the flow in the outlets, and that the energy loss between sections 1 and 3 is of small consequence because, in comparison with the pressure required, it is appreciable only near the upper end of the line where the pressure is excessive.

Field tests were made to determine the friction factors or coefficients for the special sprinkler pipe and couplings used for portable sprinkler systems. These tests presented some difficulty because pipe-line velocities were sometimes as high as 15 ft per sec with pressures of 60 lb or more per sq in. A transverse pitot tube for measuring the flow did not prove satisfactory. The method finally adopted for these tests consisted of calibrating the sprinklers to determine the relation between pressure and discharge for all nozzle combinations and then to determine the pressure along the line by means of a pressure gage and a special fitting, slipped over the main nozzle of the sprinkler. It was necessary to correct for the increase in pressure caused by shutting off the flow of the nozzle being tested. As tests show, this correction was approximately constant along a line; one could determine it readily by noting the increase in pressure on another gage installed on the line or simply by momentarily stopping the flow of another nozzle while the gage was being used on one of the sprinklers. A pressure reading was taken on each sprinkler on a line from which the sprinkler discharge and the total discharge were determined from the calibration. The pressures were plotted on a large sheet of coordinate paper, and a smooth curve was drawn through the points. The pressure drop for each space between sprinklers was then taken from the curve and plotted against the discharge on logarithmic paper. The curve was drawn through these points, showing the relation between flow and pressure drop along the line. An interesting fact about these curves was that the best-fitting straight line showed the pressure drop to be a function of the square of the velocity in the pipe, instead of some lower power as indicated by most of the empirical formulas. For this reason the Weisbach formula,

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[5] "Hydraulic Losses in Short Tubes Determined by Experiments," by John A. Oakey, *Engineering News-Record*, June 1, 1933, pp. 717–718.

$$h_f = \frac{f\,l\,v^2}{2\,g\,D} \qquad (9)$$

was adopted, and the average value of f for 4-in. pipe was found to be about 0.018, there being no appreciable difference between the different makes of pipe tested. (In Eq. 9, h_f = head loss due to friction; f = coefficient of friction; l = length of pipe; and D = diameter of pipe.)

However, when the values of f were computed from the pressure drops in each section of pipe between sprinklers, corrected by adding the pressure rise due to the decrease in velocity at the sprinklers to obtain the true friction loss, the values of f obtained were not constant but showed the usual form of curve when plotted against Reynolds' number.

In sprinkler problems one usually thinks of discharge in terms of gallons per minute and friction losses in pounds per square inch. For convenience the Weisbach formula can be written

$$p_f = \frac{0.0134\,f\,l\,Q_g^2}{D_i^5} \qquad (10)$$

in which p_f = the friction loss in pounds per square inch, l = the length of the line in feet, Q_g = the flow in gallons per minute, and D_i = the inside diameter of the pipe in inches.

A difficult task in the design of sprinkler systems is to determine pressure losses along a line with a large number of sprinklers. Although one can compute the loss between each two adjacent sprinklers, for long lines with twenty or more sprinklers this procedure becomes wearisome. Furthermore, in such computations one must begin at the distal end of the line and work back toward the source, beginning with an assumed pressure. A precise algebraic expression for the pressure loss in a long line of pipe with multiple outlets is very involved and therefore is of little practical value. Approximate expressions, however, from which the pressure loss can be determined quite accurately for limited conditions are very useful. If one assumes that the sprinklers are spaced *s* feet apart and that each discharges the same quantity of water (*q*),

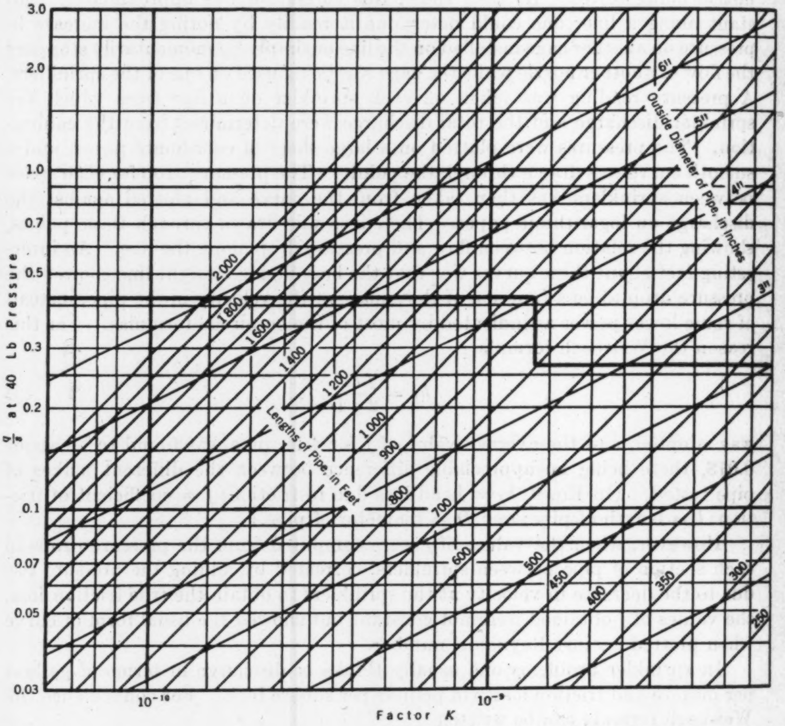


FIG. 11.—LOGARITHMIC DIAGRAM FOR DETERMINING

the total pressure loss for a line with N sprinklers and N spaces would be

$$p_f = \frac{0.0134\, f\, s\, q^2\, \Sigma N^2}{D_i^5} \qquad (11)$$

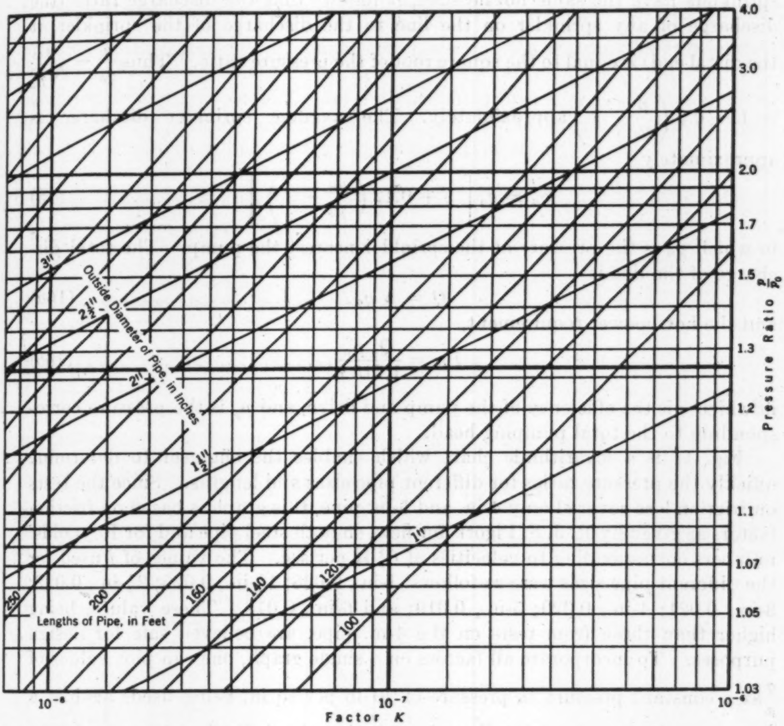
Because of the friction loss along a line, however, the sprinklers do not discharge the same quantity of water, even if the same nozzle sizes are used throughout. If the total pressure loss is relatively small, as compared with the pressure on the line, and if the computations are based on the average discharge of the sprinklers, the results will check very closely with those determined by detailed computations. The value of ΣN^2 can be determined from the expression

$$\Sigma N^2 = \frac{N^3}{3} + \frac{N^2}{2} + \frac{N}{6} \qquad (12)$$

Eq. 11 can be written

$$p_f = \frac{0.0134\, f\, L\, Q_g^2}{D_i^5}\,\frac{(\Sigma N^2)}{N^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which the first part of Eq. 13 would be the friction loss if all the water flowed through to the end of the line. From Eq. 12,



PRESSURE LOSSES IN PORTABLE SPRINKLER LINES

$$\frac{\Sigma N^2}{N^3} = \frac{1}{3} + \frac{1}{2\,N} + \frac{1}{6\,N^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The simplest method of obtaining friction loss along a sprinkler line, therefore, is first to compute the loss, assuming that all the water flows through the entire line, and then to multiply this by a factor that depends only on the number of sprinklers. For this purpose any friction-loss table or graph applying to the pipe can be used.

The friction loss along a sprinkler line is directly proportional to the pressure at any point on the line. It has been found convenient to think in terms of pressure ratio $\frac{p}{p_o}$, the ratio of the pressure at any point on the line to the pressure at the distal end. This ratio is constant at a given point on a line regardless of the operating pressure at the pump. Assuming that all sprinklers have the same nozzle sizes, it follows that the discharge ratio (the discharge of any sprinkler on the line to the discharge of the sprinkler at the distal end) is equal to the square root of the pressure ratio. Thus $\frac{q}{q_o} = \sqrt{\frac{p}{p_o}} = 1 + 0.5\left(\frac{p}{p_o} - 1\right)$ approximately. The average sprinkler discharge is approximately

$$q_a = q_o\left[1 + 0.12\left(\frac{p_n}{p_o} - 1\right)\right] \quad \ldots\ldots\ldots\ldots\ldots (15)$$

in which p_n is the pressure at the sprinkler nearest the pump. The total discharge of the line is

$$Q = N\,q_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

and the horsepower requirement,

$$P_H = \frac{Q_g\,p_p}{1{,}715\,\eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

in which η is the efficiency of the pump and drive, and p_p is the pressure corresponding to the total pumping head.

Fig. 11 is a logarithmic chart which enables the engineer to determine quickly the pressure ratios for different pipe sizes and lengths. Since the tests on friction loss covered only 4-in. and 5-in. pipe, this graph is based on friction factors as given by R. J. S. Pigott[6] for new, smooth steel pipe and for Reynolds' numbers corresponding to velocities of 6½ ft per sec. The values of f used for the different pipe sizes were as follows: 1-in., 0.028; 1½-in., 0.025; 2½-in., 0.022; 3-in., 0.021; 4-in., 0.020; 5-in., 0.019; and 6-in., 0.018. These values, being higher than those from tests on the 4-in. pipe, are believed safe for design purposes. To incorporate all factors on a single graph, one can plot values of $\frac{q}{s}$ at a constant pressure (a pressure of 40 lb per sq in. being used) against a parameter K, and then plot K against the length of the line, l, to obtain the pressure ratio $\frac{p}{p_o}$, in which p is a pressure at any distance, l, from the distal end of the line.

To use the graph one must first determine the discharge of the sprinkler at a pressure of 40 lb per sq in. If the discharge of the sprinkler at any pressure is known, the discharge at 40 lb per sq in. can be determined readily, since the discharge of the sprinkler is proportional to the square root of the pressure. For example, to find the friction loss in 800 ft of 4-in. pipe with sprinklers discharging 19 gal per min at 50 lb of pressure, spaced 40 ft apart, one must first determine the discharge at 40 lb of pressure. Thus, $q_{40} = 19\sqrt{\frac{40}{50}}$

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[6] "The Flow of Fluids in Closed Conduits," by R. J. S. Pigott, *Mechanical Engineering*, August, 1933, pp. 497–501, and 515.

$= 17$. With sprinklers spaced 40 ft apart, $\frac{q_{40}}{s} = \frac{17}{40} = 0.425$. Entering the left side of the diagram with this value, follow to the right to the intersection with 4-in. pipe. Then drop vertically downward to the intersection with the 800-ft line, and read the pressure ratio $\frac{p}{p_o} = 1.27$ on the right margin. If the pressure at the distal end, p_o, equals 30 lb per sq in., the pressure 800 ft from the end would be $1.27\ p_o = 38.1$ lb; the pressure loss would be 8.1 lb. This diagram has proved very convenient for determining the proper pipe size under a given set of conditions or for determining the permissible length of pipe of each size in a combination line having two or more pipe sizes.

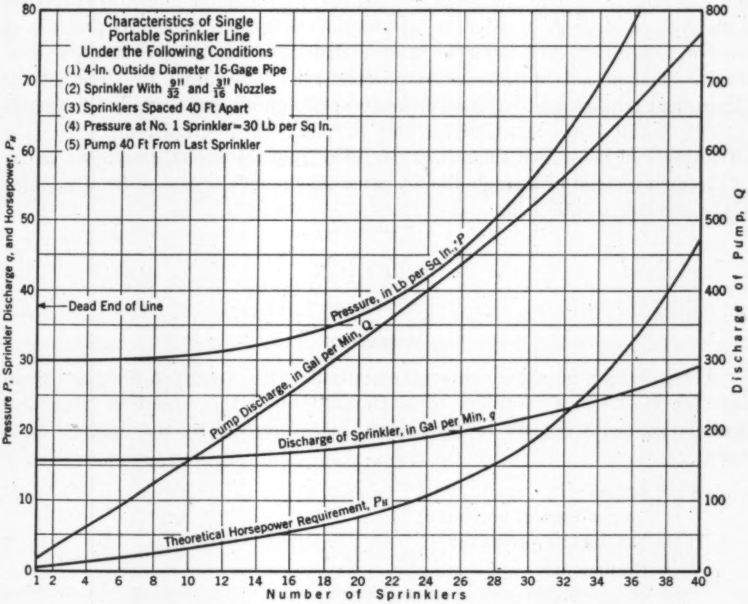

FIG. 12.—HYDRAULIC CHARACTERISTICS OF A PORTABLE SPRINKLER LINE

In designing a sprinkler system it is customary to start with an assumed value of p_o equal to the minimum pressure for which satisfactory sprinkler performance can be obtained. The dimensions of the field govern the length of the line; and since farm tractors required for other farm work are generally used as a source of power, the available power is generally known in advance. One must then ascertain the sizes of the sprinkler nozzles, the pipe, and the pump required.

When no provision is made for regulating the discharge of sprinklers along the line, it is a good practice to proportion the pipe sizes and nozzle sizes so that the over-all pressure ratio does not exceed 1.2. This arrangement limits the discharge ratio to approximately 1.1, permitting a 10% variation in sprinkler discharge along the line. On large systems it may not be advisable to limit the ratio to this value because pipe larger than 4 in. in diameter is heavy and difficult to carry. Many systems in use have pressure ratios in excess of 2; one tested had a ratio of approximately 3. Needless to say, such systems are very inefficient, and the pressure at the distal end of the line is usually far too low for satisfactory sprinkler performance. Lately, on one make of sprinkler system, the risers are provided with a shut-off cock and a pressure-valve arrangement so that the pressure on all sprinklers on the line can be determined readily and equalized.

Fig. 12 shows the hydraulic characteristics of a portable sprinkler line under the following conditions: (*a*) 16-gage pipe, outside diameter 4 in.; (*b*) sprinklers with $\frac{9}{32}$-in. and $\frac{3}{16}$-in. nozzles; (*c*) spacing of sprinklers, 40 ft; (*d*) pressure at the No. 1 sprinkler, 30 lb per sq in.; and (*e*) distance from pump to last sprinkler, 40 ft. The actual horsepower requirement is determined by dividing the theoretical requirement by the efficiency of the pump and drive. Efficiencies of 50% to 70% are assumed for this purpose. This graph emphasizes the advantages of using two short lines supplied from a source through the center of the field instead of a single line of twice that length, supplied from a source along one side of the field.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Hydraulics compiled by a Committee of the American Standards Association[7] with Society representation and approved by the Association in 1929:

A = area:
  A_1 = area of section 1;
B = barometric pressure;
C = coefficient:
  C_u = uniformity coefficient;
c = specific heat of water;
D = diameter:
  D_i = inside diameter of pipe in inches;
E = evaporation; the portion of water loss by evaporation;
e = energy;
f = friction coefficient;
g = gravity constant;
H = heat:
  H_s = sensible heat;
h = hydraulic head:
  h_f = head loss due to friction;

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[7] A. S. A.—*Z* 10*b*—1929.

I = insolation;
K = an emperical coefficient:
  K_3 = an emperical coefficient having values dependent on both r_p and r_q;
k = thermal conductivity; conduction;
L = latent heat of vaporization;
l = length;
M = mean value;
m = mass;
N = the total number of;
n = any number of observations;
P = power:
  P_H = horsepower requirement;
p = pressure intensity; pressure loss:
  p_a = pressure of water vapor in the air;
  p_f = pressure loss due to the pipe friction;
  p_n = pressure intensity at the sprinkler nearest the pump;
  p_o = pressure intensity at the distal end of a pipe;
  p_p = the pressure corresponding to the total pumping head;
  p_w = vapor pressure of water at t_w;
  p_1 = pressure intensity at section 1;
Q = quantity, or rate of flow:
  Q_g = flow in gallons per minute;
q = rate of flow, in gallons per minute, through a single sprinkler:
  q_a = average discharge of the sprinklers on a line;
  q_o = flow through a sprinkler at the distal end of pipe;
R = radiation:
  R_b = back radiation;
r = ratio of energy transfer by convection to energy transfer by evaporation (called Bowen's ratio):
  r_p = ratio of the pipe area at Point 3 to the pipe area at Point 1;
  $r_q = \frac{Q_3}{Q_1}$, or the proportion of water leaving the side outlet;
s = spacing of sprinklers;
t = temperature:
  t_a = air temperature;
  t_w = the mean water temperature;
  t_1 = temperature of water at the nozzle;
  t_2 = temperature of water as it strikes the ground;
v = velocity:
  v_1 = velocity at section 1;
w = weight of a cubic foot of water;
x = deviation from the mean;
η = efficiency.

DISCUSSION

Arthur F. Pillsbury,[8] Assoc. M. Am. Soc. C. E. (by letter).—In California, sprinkling systems for irrigation often have failed to perform satisfactorily because of faulty design, failure to obtain good distribution characteristics for the sprinklers with the spacings provided, and failure to interpret, properly, the limitations of the soil in absorbing the precipitation. Therefore, the excellent dissertation by Mr. Christiansen, based on a wealth of experimental evidence, is timely.

Sprinkling systems are useful where: (1) A high water level exists, and less uniform surface applications would cause that level to rise; (2) the topsoil is shallow and pervious, and less uniform surface applications would create a temporary water table above a subsoil of different texture; (3) the soil is extremely pervious resulting in waste of water in parts of a planting; (4) irrigation is infrequent, the good topsoil is shallow, and for other reasons it is not practical to establish satisfactory grades for surface irrigation; and (5) extremely light and frequent applications are desired, as with some pastures, lawns, and gardens.

In short, sprinkler irrigation is useful where surface methods cannot be economically and efficiently utilized. Its use with field crops is limited to areas of high water table, or where extremely light frequent or infrequent supplemental irrigations are desired, and with orchards to areas of shallow soil, extremely pervious soil, or unfavorable topography.

In orchard irrigation, under-tree systems are unquestionably preferable to the overhead type. The essential sprinkler objective is to improve distribution of water. With the under-tree system, the distribution is less affected by wind and by the trees, it is better adapted to operation by the usual farm laborer, and it should be less expensive.

Mr. Christiansen's data on distribution of water from sprinklers are of greatest importance with the overhead type used for field crops. With under-tree orchard irrigation, sprinklers are generally spaced only 18 to 24 ft apart, and the greatest essential is to insure approximately equal discharge of each sprinkler, rather than absolutely uniform distribution in each space bounded by four trees, which is seldom attained. A good pattern is nevertheless desirable.

For under-tree work the slow revolving sprinklers and perforated pipe systems generally are used for low-pressure (4 to 15 lb per sq in.) operation with medium to high rates of discharge (2 to 8 gal per min, or equivalent for perforated lines). Whirling sprinklers are used for intermediate pressures (10 to 20 lb per sq in.) with all usual rates of discharge (1 to 8 gal per min) and for higher pressures with low to medium rates of discharge (1 to 4 gal per min). Fixed heads are used for high pressure (20 lb per sq in. and up) with high rates of discharge (4 to 8 gal per min) and for lower rates of discharge when spacing is closer than usual or when exceptionally high pressure is available. Intermediate and high-pressure systems usually result in economies in material where sufficient pressure is available without pumping. On the other hand, low-pressure systems may eliminate the necessity of pumping, or may minimize the pumping cost.

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[8] Asst. Prof. of Irrig., Univ. of California, Los Angeles, Calif.

The rate of application depends upon general farm practices or upon the maximum infiltration rates permissible without appreciable runoff and erosion. Infiltration rates decrease somewhat with increase in the proportion of the finer particles of a soil. More important is the physical condition of the soil. Generally, infiltration rates increase with decrease in original moisture content, but powdery dry soils may not take water readily. Organic matter, such as cover crops and manure, increases infiltration rates. Chemically "soft" waters (relatively low in calcium or with a high percentage of sodium in relation to other cations), and some inorganic fertilizers, may tend to leave a top sodium-saturated crust on the surface which is relatively impervious. Similarly, impervious crusts may be formed by physical compaction. Cultivation will improve infiltration by breaking up and mixing such crust with other soil.

Of frequent practical importance in sprinkler design is the fact that the size of sprinkler drops (with equivalent spray trajectories) is closely correlated with physical compaction ("puddling") of the surface crust. For instance, on a bare soil classified[9] as Yolo loam, with sprinkler precipitation at a constant rate of 0.59 in. per hr, a 5.4-in. depth of fine spray was applied without runoff, while with a somewhat coarser spray only 2.5 in. could be so applied. Relative size of drops was roughly checked with the Bentley technique.[10] Detailed investigations are now (1941) being conducted by the U. S. Soil Conservation Service on the effect of drop size on erosion and infiltration.[11]

In general, size of drop with a given sprinkler increases with distance from the sprinkler, and, comparing different sprinklers producing equivalent patterns, drop size decreases as pressure increases. This point is stressed because many sprinkler systems have failed due to serious runoff and erosion that developed before sufficient water could be applied. Later investigations have indicated: (1) That the soil was too heavy for satisfactory sprinkler use under the cultural practices followed, and (2) that the condition might have been prevented had sprinklers with a finer spray been used. It must be recognized, then, that low-pressure sprinklers (with a coarse spray) are sometimes not suitable for irrigation of bare soils, whereas higher pressure sprinklers would be suitable.

Mr. Christiansen's data on evaporation losses indicate that loss from the sprinkler spray is insignificant. Some increase in evaporation, as compared with furrow irrigation, may result, however, from the greater ground surface wetted and from water intercepted by plant foliage. His data on the hydraulics of sprinkler lines are quite complete. Comments on the hydraulics, having in mind the small under-tree units commonly used in Southern California, follow:

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[9] "Soil Survey of the Los Angeles Area, California," Bureau of Soils, U. S. Dept. of Agriculture in cooperation with the Univ. of California Agricultural Experiment Station, 1916.

[10] "Studies in Raindrops and Raindrop Phenomena," by W. A. Bentley, *Monthly Weather Review*, Weather Bureau, U. S. Dept. of Agriculture, October, 1904, pp. 450–456.

[11] "Recent Studies in Raindrops and Erosion," by J. Otis Laws, *Agricultural Engineering*, Am. Soc. of Agri. Engrs., Vol. 21, No. 11, November, 1940, pp. 431–433.

TABLE 1.—For Selecting Sizes of Portable Under-Tree Sprinkler Units

Spacing (ft)	Maximum Recommended Discharge per Sprinkler, at 20 Lb per Sq In., in Gal per Min, for Units with the Following No. of Sprinklers:													
	3	4	5	6	7	8	9	10	11	12	14	16	18	20	22
(a) ¾-In., Type M, Copper Pipe—Solder Fittings															
16	7.6	4.5	3.1	2.3	1.8	1.4	1.2	1.0	...	...	...	...	...	...	...
20	6.8	4.0	2.7	2.0	1.6	1.2	1.0	0.9	...	...	...	...	...	...	...
24	6.0	3.6	2.5	1.8	1.4	1.2	0.9	0.8	...	...	...	...	...	...	...
(b) 1-In., Type M, Copper Pipe—Solder Fittings															
16	...	9.1	6.2	4.5	3.5	2.8	2.3	2.0	1.7	...	...	...	...	...	...
20	...	8.0	5.4	4.0	3.1	2.5	2.1	1.7	1.5	...	...	...	...	...	...
24	...	7.2	4.9	3.6	2.8	2.2	1.9	1.6	1.3	...	...	...	...	...	...
30	...	6.4	4.3	3.2	2.5	2.0	1.6	1.4	1.2	...	...	...	...	...	...
(c) 1¼-In., Type M, Copper Pipe—Solder Fittings															
16	...	...	10.9	7.9	6.1	4.9	4.1	3.4	...	2.5	2.0	...	...	...	...
20	...	...	9.4	7.0	5.4	4.4	3.6	3.0	...	2.2	1.7	...	...	...	...
24	...	...	8.6	6.3	4.9	3.9	3.2	2.7	...	2.0	1.6	...	...	...	...
30	...	...	7.6	5.6	4.3	3.4	2.9	2.4	...	1.8	1.4	...	...	...	...
(d) 1-In. Electrical Conduit ($1\frac{3}{64}$-In. Inside Diameter), Galvanized, with Constricting Airhose-Type Couplings															
16	9.1	5.5	3.8	2.8	2.2	...	...	...	...	...	...	...	...	...	...
20	8.1	4.9	3.4	2.5	2.0	...	...	...	...	...	...	...	...	...	...
24	7.4	4.5	3.1	2.3	1.8	...	...	...	...	...	...	...	...	...	...
(e) 1-In. Electrical Conduit ($1\frac{3}{64}$-In. Inside Diameter), Galvanized, with No Constricting Couplings															
16	12.7	7.6	5.2	3.9	3.0	2.5	...	1.7	...	1.3	...	...	...	...	...
20	11.2	6.8	4.6	3.4	2.7	2.2	...	1.5	...	1.1	...	...	...	...	...
24	10.1	6.1	4.2	3.1	2.4	1.9	...	1.4	...	1.0	...	...	...	...	...
(f) 1¼-In. Electrical Conduit (1⅜-In. Inside Diameter), Galvanized, with No Constricting Couplings															
16	...	...	10.6	7.9	6.2	5.0	4.1	3.5	...	2.6	2.1	...	...	...	...
20	...	...	9.5	7.1	5.5	4.4	3.7	3.1	...	2.3	1.8	...	...	...	...
24	...	...	8.6	6.4	5.0	4.0	3.3	2.8	...	2.1	1.6	...	...	...	...
30	...	...	7.7	5.7	4.4	3.6	2.9	2.5	...	1.9	1.5	...	...	...	...
(g) 1½-In. Galvanized Sprinkler Pipe (1.40-In. Inside Diameter) with Quick Couplings															
16	...	...	...	8.6	6.7	5.4	4.5	3.8	...	2.9	2.2	1.8	...	...	...
20	...	...	...	7.7	6.0	4.8	4.0	3.4	...	2.5	2.0	1.6	...	...	...
24	...	...	...	7.0	5.5	4.4	3.6	3.0	...	2.3	1.8	1.5	...	...	...
30	...	...	...	6.2	4.8	3.9	3.2	2.8	...	2.1	1.6	1.3	...	...	...
(h) 2-In. Galvanized Sprinkler Pipe (1.90-In. Inside Diameter) with Quick Couplings															
16	...	...	...	...	...	12.1	...	8.4	...	6.4	4.9	4.0	3.4	2.8	2.4
20	...	...	...	...	...	10.7	...	7.5	...	5.6	4.4	3.6	3.0	2.5	2.2
24	...	...	...	...	...	9.7	...	6.8	...	5.1	4.0	3.2	2.7	2.3	2.0
30	...	...	...	...	...	8.6	...	6.0	...	4.5	3.5	2.9	2.4	2.0	1.7

(1) The recommendations of Mr. Pigott[6] for friction loss in new steel and copper pipes are very satisfactory for use in design of $\frac{3}{4}$-in. to $1\frac{1}{2}$-in. units, as indicated by recent tests of friction losses in such pipes.

(2) Generally, the under-tree units have sprinklers at each end of the line, and are uniformly spaced. Sufficient accuracy is attained if it is conservatively assumed that friction losses will be one third the loss that would occur if the full flow of all sprinklers was carried to the end of the line.[12]

(3) Table 1 shows the maximum recommended sprinkler discharge in gallons per minute at a pressure of 20 lb per sq in. and an over-all pressure ratio of 1.2. The sprinklers are assumed to be all similar, equally spaced, and placed at each end of a unit. Spacings that are not included can be interpolated. In Table 1, approximate lengths of pipe that can be dragged readily by one man, under favorable conditions, are underlined. Assuming a maximum allowable over-all pressure ratio of 1.2, Table 1 is satisfactory for use in designing the portable units. Included are all the various types of pipe now used for such portable units. The usual steps in designing a unit are: (*a*) Select the application rate desired (generally 0.2 to 1 in. per hr); (*b*) compute the desired sprinkler discharge by the formula

$$\text{Sprinkler discharge in gal per min} = \frac{\text{Area (sq ft) per sprinkler} \times \text{rate of application (in 1 hr)}}{96.3} \quad \ldots\ldots(17)$$

(*c*) select sprinkler and operating pressure; (*d*) determine what the discharge per sprinkler would be at 20 lb per sq in. (discharge proportional to square root of pressure); and (*e*) select pipe and length of unit from Table 1. Usually sprinklers with devices for manually adjusting pressure at each sprinkler are not satisfactory for under-tree systems, but whirling sprinklers are being developed which automatically regulate discharge if pressure is greater than 8 lb per sq in. If these prove satisfactory, it will not be necessary to limit the pressure ratio when using them but simply to insure adequate pressure at the distal end of each unit.

TOM A. BITHER,[13] ASSOC. M. AM. SOC. C. E. (by letter).—This excellent paper deals with a type of irrigation that is becoming more and more extensively adopted for the irrigation of row crops, pastures, and orchards, resulting in a saving in water and more uniform distribution, as well as increased yields. It has been the good fortune of the writer to be located near Davis and the Sacramento Valley where most of Mr. Christiansen's experimental work and field tests have been conducted. The paper does not adequately indicate to the reader the vast amount of experimental work that Mr. Christiansen has done during the eight years since 1933; nor is the author able, in the space available, to cover in full the many tests on sprinklers and sprinkler lines that he has conducted during that period.

Some sprinkler lines have been sold without any consideration of the fundamental principles of design. In some cases, for example, claims have been made that a higher pressure would be obtained at the outer (distal) end of the line than at the inlet end, even though the line was to operate on level ground. It is to be hoped that with the publication of Mr. Christiansen's data and design formulas this practice can be stopped.

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[12] "The Design of Overhead Irrigation Systems," by E. S. West and A. Howard, *Pamphlet No. 50*, Council for Scientific and Industrial Research, Commonwealth of Australia, Melbourne, 1934.

[13] With California Corrugated Culvert Co., Berkeley, Calif.

In regions where natural rainfall alone is relied upon for crop production, droughts frequently occur. Supplementary irrigation during dry periods will greatly increase crop yields and often will save valuable crops and prevent serious injury to orchards. The average seasonal rainfall is usually more than enough to grow good crops, but the rains do not always come at the right times, with the result that crops suffer if not protected by some type of supplemental irrigation. Any farmer should be able to increase production with the aid of irrigation since he can then assure himself of always having sufficient moisture available when needed. The overhead or sprinkler system of supplementary irrigation has been found to be the most suitable method for many crops and conditions. By this method, water may be applied efficiently and economically since it enables the irrigator to apply a uniform depth of water over his land regardless of topography, and without waste. This method requires less water than surface irrigation methods. Canals and ditches are eliminated, thereby doing away with seepage losses.

On uneven or rolling ground, surface methods are unsatisfactory since they result in heavy applications and water logging in the low spots, and light applications or no water at all on the high spots. A considerable quantity of water is also lost by deep percolation at the upper ends of checks and furrows where surface methods are used. The cost of preparing land for satisfactory surface irrigation is an important consideration. Extensive leveling and grading not only cost money but also have the serious objection of removing the surface topsoil from the high spots and leaving a less fertile subsoil. Where the soil is shallow, the high spots may be completely ruined by deep grading.

Expensive leveling of land is entirely eliminated where sprinkler irrigation is used to apply water. This method applies water in the form of rain and is applicable to all types of soil and to any condition of topography. It permits the utilization of a small head of water that would be entirely inadequate for surface irrigation methods. It insures a uniform distribution of water; permits keeping the soil in the best condition at all times; and, by applying water at the proper rate, prevents serious soil erosion.

Mr. Christiansen should have explained that the sprinkler distribution tests (Figs. 3 to 7) were made using sprinklers that are now considered obsolete and inadequate for agricultural work. These sprinklers were of the reaction-drive type and have been almost entirely supplanted by the impulse-drive sprinkler, a type which rotates at a uniform speed when properly adjusted. These sprinklers do not produce the unfavorable patterns shown in Figs. 3 to 7. Also, as a result of Mr. Christiansen's investigations, these later types of sprinklers produce a cross-sectional pattern approximating that shown in Fig. 9, pattern *b*. Sprinkler manufacturers have also developed agricultural sprinklers operating at lower pressures than those he describes, some of them, for under-tree work, operating at pressures as low as 2 lb per sq in. with satisfactory patterns for this work.

As a result of personal observations, the writer agrees with Mr. Christiansen that the loss due to evaporation in the spray between the nozzle and the ground is negligible. Also, the total losses incurred from the sprinkler method of irrigation are less than that from any surface method of application since water is lost neither by deep percolation at the upper ends of furrows and checks, nor from seepage from ditches.

The writer is of the opinion, as the result of field tests on operating lines, that the pressure losses determined from the logarithmic chart (Fig. 11) and from Eq. 11 are too high. This is undoubtedly due to the assumption that all sprinklers are discharging the same quantity of water, which is not the case, as indicated by Fig. 12. Although a logarithmic chart of the form of Fig. 11 is undoubtedly convenient and useful for determining pressure losses, it has been found, in the preparation of handbooks and manuals for field men, that tables are more convenient than charts. As a consequence, the writer has computed and compiled tables for various sizes of pipe and of sprinklers placed at several spacings on the line. These tables which give pressures, total line discharge, and equivalent rainfall enable the field man to determine quickly the size of pipe as well as the size and spacing of sprinklers for any particular installation, depending upon the size of the field to be irrigated, the size of the power unit or pump, the amount of water available, the type of soil, or other governing factor.

Ralph W. Powell,[14] M. Am. Soc. C. E. (by letter).—The writer wishes to comment on only one section of this interesting paper—namely, that on the hydraulics of sprinkler lines, and principally on only one phase of it, which is the recovery of pressure head in the main line of pipe when some of the flow is drawn off through a side outlet. This has been discussed in the technical press, but as Wallace M. Lansford,[15] Assoc. M. Am. Soc. C. E., has pointed out, no "complete experimental and analytical solution * * * has yet been published." Probably the most thorough experimental study of the question was made by the Bureau of Reclamation at the Fort Collins (Colo.) laboratory in 1932; but their report[16] deals primarily with the energy loss, and the only data explicitly given regarding pressure recovery seem to be those applying to points opposite the exact center of the junctions, where the recovery might be expected to be only half completed, or perhaps less than that.

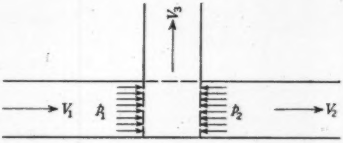

Fig. 13

For many years the writer has been puzzled by the result arising from a momentum treatment of this problem. In Fig. 13, if the unbalanced force acting in the liquid within the dashed lines is made equal to the change in momentum per second in the usual manner,

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[14] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[15] *Engineering News-Record*, April 10, 1941, p. 522. For previous discussions of the same subject, see *Engineering News-Record*, November 21, 1940, p. 697, and January 16, 1941, p. 78.

[16] "Model Studies of Penstocks and Outlet Works," *Bulletin No. 2*, Boulder Project Final Repts., U. S. Dept. of the Interior, 1938, especially pp. 49–64. (The reference, on p. 62, to a paper by the writer is unfortunate, as the paper was never published.)

$$p_2\,A - p_1\,A = \frac{w\,Q_1\,v_1}{g} - \frac{w\,Q_2\,v_2}{g} \qquad (18)$$

which reduces to

$$p_2 - p_1 = \frac{w\,v_1^2}{g}\,(2\,r_q - r_q^2) \qquad (19)$$

In Eq. 19, $p_2 - p_1$ is just twice the value given by Eq. 5. The frictional drag around the periphery of the free body has been neglected, but an approximate estimate shows that it would be negligible, and at any rate it is in the wrong direction to explain the discrepancy. One may first be tempted to state that there is a question as to whether Bernoulli's theorem applies between two points at which the quantity of flow is different, and that, since momentum considerations give an exact solution without the loss term which is required in the energy equation, Eq. 19 is more likely to be right than Eq. 5. However, when he investigates the reported experiments, he finds that the actual recovery is less than that given by Eq. 5, rather than more.

The explanation seems to lie in the unequal velocity distribution in the main pipe just downstream from the junction. Following the usual notation,[17] the momentum equation should have been written

$$p_2\,A - p_1\,A = \frac{w\,Q_1\,\beta_1\,v_1}{g} - \frac{w\,Q_2\,\beta_2\,v_2}{g} \qquad (20)$$

which leads to

$$p_2 - p_1 = \frac{w\,v_1^2}{g}\,[\beta_1 - \beta_2\,(1 - r_q)^2] \qquad (21)$$

Then, calling the ratio of recovered pressure head to original velocity head K_2:

$$K_2 = \frac{(p_2 - p_1)\,2\,g}{w\,v_1^2} = 2\,\beta_1 - 2\,\beta_2\,(1 - r_q)^2 \qquad (22)$$

Table 2 gives the average values of K_2 as reported by Professor Oakey.[1] As the author states, these agree with Eq. 5 better than with Eq. 6. Table 2 also gives the values of K_2 deduced from material published by the Reclamation Service.[18] The agreement between the two sources is very satisfactory. The measurements of the Reclamation Bureau were on a much larger scale and are

TABLE 2.—VALUES OF K_2 AND β_2

Description	PROPORTION OF WATER LEAVING SIDE OUTLET $r_q = Q_3/Q_1$										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Empirical Coefficient K_2:											
John A. Oakey	0.00	0.21	0.38	0.53	0.66	0.74	0.80	0.83	0.82	0.78	0.69
Reclamation Service	0.02	0.21	0.38	0.53	0.66	0.76	0.84	0.85	0.80	0.70	0.56
Coefficient β_2	1.01	1.13	1.30	1.54	1.92	2.56	3.75	6.61	15.5	67.0	∞

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[17] "Mechanics of Liquids," by Ralph W. Powell, The Macmillan Co., New York, N. Y., 1940, p. 125. (One w is printed erroneously there as W.)

[18] "Model Studies of Penstocks and Outlet Works," *Bulletin No. 2*, Boulder Project Final Repts., U. S. Dept. of the Interior, 1938, Fig. 30 and the first equation on p. 39 (after supplying a missing parenthesis around the last three terms).

probably more accurate. The last line gives the values of β_2 necessary to make the Reclamation Bureau values of K_2 satisfy Eq. 22, β_1 being taken as 1.02.

The values of β_2 in Table 2 may seem impossibly high, but it must be realized that, when a large part of the flow is diverted to a side outlet, the remaining flow will be distributed very unequally across the pipe, possibly even to the extent of a reversal of flow on the side toward the outlet. If the velocity averaged zero over half the cross section and $2\,v$ over the other half, β would exceed 2.

It would seem to the writer that in this problem there is no hope of deducing how much head will be recovered and how much lost, but that one must depend on experiments.

Harry F. Blaney,[19] M. Am. Soc. C. E. (by letter).—Sprinkling irrigation has been used extensively in the United States for many years. The paper on this subject by Mr. Christiansen presents some excellent data but may give some readers the impression that sprinkling systems for irrigation are comparatively new. Information obtained from the 1920 Census of Agriculture, and from other sources, indicates that more than 12,000 acres were thus irrigated in 1919. As early as 1916 the U. S. Department of Agriculture made some general studies on spray irrigation and the results were published.[20] In 1924 the University of California made a study of irrigation by overhead sprinkling[21] in Southern California. However a comparison of these reports with the author's paper will indicate the change that sprinkler equipment has undergone. Tests of typical German and American sprinkler equipment in 1928 by the U. S. Department of Agriculture at the Arlington Experiment Farm under eastern conditions are also interesting.[22]

Many agricultural crops are now being successfully irrigated by portable sprinkling systems at reasonable cost, which was not deemed feasible years ago, and Mr. Christiansen's paper is a valuable contribution to engineering literature on this subject.

The writer agrees with the author's conclusions regarding evaporation losses. In the case of citrus fruits, losses by both surface evaporation and deep penetration below the root zone are unavoidable when water is applied so that the maximum tree growth is obtained. Tests made under the direction of the writer indicate that the efficiency of irrigation under the sprinkler method may not be greater than under surface irrigation. Conditions under which irrigation by sprinkling has an economic advantage over surface irrigation are those of steep or uneven topography. A higher efficiency

TABLE 3.—Irrigation Efficiencies

Grove	Water applied, acre-in. per acre	Water placed in root zone, acre-in. per acre	Percentage efficiency
A	0.62	0.18	29
B	1.48	0.62	42
B	2.36	1.47	62
C	7.50	3.50	47

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[19] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.

[20] "Spray Irrigation," by Milo B. Williams, *Bulletin No. 495*, U. S. Dept. of Agriculture, 1917.

[21] "Irrigation by Overhead Sprinkling," by H. A. Wadsworth, *Circular No. 4*, California Agri. Extension Service, 1926.

[22] "Tests of Spray Irrigation Equipment," by F. E. Staebner, *Circular No. 195*, U. S. Dept. of Agriculture, 1931.

of irrigation can be maintained under such conditions than is possible with surface irrigation. Under sprinkler irrigation, surface runoff usually is a negligible factor if the water is properly applied.

The variations of efficiencies of irrigation under overhead sprinkling with light, medium, and heavy applications of water as determined from intensive soil moisture studies,[23] San Diego County, California, 1926, are shown in Table 3.

During the season of 1939 the Division of Irrigation, in cooperation with the Los Angeles Bureau of Water Works and Supply, conducted experiments on applying irrigation water with low-head sprinklers and by furrows. The average efficiency of irrigation under the sprinkler system was 49% and with the furrow method 47%. Two types of sprinklers were used with pressures varying from 8 to 18 lb per sq in.

J. E. CHRISTIANSEN,[24] ASSOC. M. AM. SOC. C. E. (by letter).—The interest shown in this paper is appreciated sincerely. Professor Pillsbury has contributed greatly to the subject, especially with respect to undertree sprinkler systems for orchards. Although the writer fully agrees with Professor Pillsbury's statements in regard to the conditions under which sprinkling is useful, he also believes that sprinkling will be satisfactory on most crops, and under most conditions, provided sufficient water is applied, at the proper rates. Exceptions are found in crops adversely affected by fungi or bacterial diseases; sprinkling may aggravate such trouble or make control more difficult. Grapes, a typical example, are not sprinkled because of mildew. Sprinkling is simply a method of irrigation in which the operator has good control over the water. If he applies the water at such a rate that the soil can absorb it, he can apply as little or as much as he wishes and can secure any depth of penetration desired.

The primary limitation is one of economics—that is, the cost of sprinkling as compared with other methods of irrigation. In general, sprinkling costs more; and unless the results, as measured by increased returns, are better, it cannot be considered economical. Since only 1% or 2% of the total irrigated area in California is sprinkled, evidently most farmers are either satisfied with their present methods, or do not believe that increased returns will compensate for higher costs.

Mr. Bither lists many advantages of sprinkling. An increased return resulting from one or another of these advantages will sometimes more than offset higher costs, but not always. One crop grown extensively in California that thrives under sprinkling is Ladino clover. Being shallow rooted, it requires rather frequent irrigation—that is, every week or 10 days for best results. Most of it is grown on rather poor, shallow, undulating soil, where extensive leveling is objectionable. It is generally irrigated by the border-strip flooding method, and considerable water is wasted. Nevertheless, thousands of acres of clover are grown profitably, mainly for fattening lambs, where the gross return averages about $25 per acre. This total gross return, however, would scarcely pay the cost of applying water with sprinklers; and so, although the production might be appreciably increased, sprinkling would apparently not be feasible. The economic phases of this subject do not seem pertinent in this paper and are covered in another report.[25]

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[23] "Irrigation Water Requirement Studies of Citrus and Avocado Trees in San Diego County, California," by S. H. Beckett, Harry F. Blaney, and Colin A. Taylor, Assoc. M. Am. Soc. C. E., *Bulletin No. 489*, Univ. of California, Coll. of Agriculture, 1930.

[24] Irrig. and Drainage Engr., U. S. Regional Laboratory, U. S. Dept. of Agriculture, Riverside, Calif.

All the tests illustrated in Figs. 3 to 7 were made on sprinklers that have been replaced by later models. However, many tests on types now generally used indicate that the uniformity of application usually is no better than that produced by the pattern shown in Fig. 3. Furthermore, the new sprinklers are similarly affected by wind and to some extent by low pressure. They do rotate at a fairly steady, uniform rate and cannot be thrown out of adjustment so that they rotate rapidly. The uniformity of application of water by sprinklers is the subject of a paper published early in 1941.[26]

The logarithmic diagram (Fig. 11) for determining pressure losses in portable sprinkler lines is based upon coefficients slightly more conservative than those obtained from the field tests on sprinkler pipe. This diagram was prepared as a guide for designing sprinkler lines with all types of couplings. Although tests on several makes of lines indicate rather small differences in friction factors, some allowance was made for deterioration in carrying capacity caused by increasing roughness of the pipe with age. This allowance was very small as compared with standard engineering practices in designing pipe lines. The assumption that all sprinklers discharge equal quantities of water does not result in higher losses, provided proper friction factors are used and provided the total friction loss is not excessive. The writer recognizes the advantage of tables for the purpose mentioned. In this paper, however, tables would be impracticable because of the space required to cover the many combinations of pipe and sprinkler sizes in use.

In deriving Eq. 19 by the momentum treatment, Professor Powell does not consider the pressure difference on the upstream and downstream sides of the side outlet near the main pipe. Undoubtedly there is a zone of high pressure on the downstream side of the outlet pipe, and a zone of low pressure on the upstream side. Whether or not the net downstream thrust on the outlet pipe is sufficient to harmonize Eqs. 5 and 19 is not known. As Professor Powell states in private correspondence with the writer, the effect of this unequal pressure on the sides of the outlet pipe is to reduce the values of β_2 given in Table 2. It is surprising that such an intriguing problem in hydraulics has not been investigated more completely. The writer was concerned principally with the effect of the pressure recovery on the friction loss in the pipe. The friction factor f, computed on the basis of pressure and elevation difference at two points on a sprinkler line, with differences in velocity neglected, was found to remain rather constant for large differences in Reynolds' number. When corrections were made for the pressure recovery at each outlet, however, the customary relation between f and R was obtained. In other words, the pressure recovery at a side outlet increases the exponent of V to approximately 2 in an empirical formula for flow of water in pipes, the magnitude of the effect depending upon the spacing and number of outlets on the pipe. For this reason the Weisbach formula, with constant value for *f* of each pipe size, was used as a basis for the friction-loss diagram.

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[25] "Irrigation by Sprinkling," by J. E. Christiansen, College of Agriculture, Univ. of California, Berkeley, Calif. (bulletin in press).

[26] "The Uniformity of Application of Water by Sprinkler Systems," by J. E. Christiansen, *Agricultural Engineering*, Am. Soc. of Agri. Engrs., Vol. 22, No. 3, March, 1941, pp. 89–92.

The writer thanks Mr. Blaney for emphasizing that sprinkling is not a new practice. Portable systems of the types now common, however, are relatively new; and their use has greatly extended the acreage irrigated by this method. As Mr. Blaney also states, sprinkling does not always increase irrigation efficiencies; many installations are very inefficient, mainly because of poor equipment, poor design of the system, and improper operation. In the writer's opinion, however, it is possible to obtain higher irrigation efficiencies by sprinkling than are generally possible by surface methods, although this is not always accomplished. Farmers seldom understand the hydraulics of sprinkler systems; and often some minor change in equipment or in methods of operation will improve the efficiency of the system materially.

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Paper No. 2133

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

By Charles S. Whitney,[1] M. Am. Soc. C. E.

With Discussion by Messrs. L. E. Grinter, Basil Sourochnikoff, R. W. Stewart, George C. Ernst, Homer M. Hadley, Robert W. Abbett, Paul Andersen, R. A. Caughey, Roberto Contini, A. A. Eremin, Jaroslav J. Polivka, Paul W. Abeles, O. G. Julian, and Charles S. Whitney.

Synopsis

It is the purpose of this paper to present a realistic method for the design of reinforced concrete members that should result in more efficient use of the materials. The theory of elasticity has proved satisfactory for the determination of the forces acting on the various sections of continuous frames or arches and much attention has been devoted to various methods of its application. The result has been a general extension of its use and a marked improvement in the understanding, by designers, of the action of reinforced concrete structures.

Corresponding progress has not been made in proportioning the members of a structure after the external forces have been determined. The theory of elasticity, as applied to design of sections, is too inflexible and inaccurate to be entirely satisfactory. The writer proposes the adoption of the plastic theory which recognizes the true characteristics of the material determined by research since the standard "straight-line theory" was adopted about a generation ago. The plastic theory takes into account the plasticity of the material and is in itself sufficiently "plastic" to be adjusted empirically to actual conditions. It has already been given recognition in the general adoption of the standard column formula, and the formulas proposed herein by the writer extend the same theory to beams, eccentrically loaded columns, and arch ribs. The equations are much simpler than the standard formulas and agree better with test results. They are based on the cylinder strength of the concrete and the yield strength of the steel without the use of modular ratio n. A number of

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Note.—Published in December, 1940, *Proceedings*.

[1] Cons. Engr., Milwaukee, Wis.

attempts[2,3,4] have been made since 1936 to eliminate the modular ratio, n, but none of them is as practical or as well adapted to American practice as that given herein. The writer has previously published parts of this theory,[5,6] but has now improved and extended the equations so that he hopes they may be recognized as forming a satisfactory basis for practical design and further research.

INTRODUCTION

There are three general steps in the design of an efficient structure:

(1) The determination of service conditions depending on the purpose and location of the structure, such as the nature and intensity of loading, the foundation conditions, and the exposure to deteriorating influences;

(2) The computation of the external forces to which the various sections of all the members will probably be exposed during the life of the structure; and

(3) The proportioning of the different parts of the structure so that the limits of their useful strength may be reached, but not exceeded, under the loading specified.

Step 1 leads an experienced designer to the selection of materials and a preliminary layout of the structure. It has been demonstrated by many investigations that, in the case of frames or arches of reinforced concrete, the external forces can be computed satisfactorily (Step 2) by means of the theory of elasticity, based on the assumption of direct proportionality of stress to strain. In spite of the plasticity of concrete this is true within the limits of the useful strength of the structure, except in some special cases where cracking at certain sections sufficient to cause redistribution of moments might not be detrimental. In such special cases, advantage might be taken of the increased strength due to redistribution of moments, but this must be done with caution.

A number of tests since about 1936[7] have shown that the maximum strength of a redundant structure is not determined by the first section to reach its maximum strength but by the combined strength of the redundant and primary members or sections, as explained in 1939 by J. A. Van den Broek,[8] M. Am. Soc. C. E.

Ordinarily, in reinforced concrete structures, the useful load limit of the structure occurs when the first section reaches its allowable limit, as determined by the permissible cracking or yielding of the concrete while the steel stress is still considerably below its yield point. For this reason, it is best to design the

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[2] "Bruchzustand und Sicherheit im Eisenbetonbalken," by Rudolf Saliger, *Beton und Eisen*, October, 1936, p. 317.

[3] "The Modular Ratio—A New Method of Design Omitting m," by K. Hajnal-Konyi (and discussion by various others), *Concrete and Constructional Engineering*, January, February, March, May, June, July, August, September, October, 1937.

[4] "Considérations sur le Calcul et la Sécurité des Pièces Fléchies. Moments de Rupture," by R. Coppée, *Publications*, International Assoc. for Bridge and Structural Eng., Vol. 3, 1935, p. 19.

[5] "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March-April, 1937, p. 483.

[6] "Eccentrically-Loaded Reinforced Concrete Columns," by Charles S. Whitney, *Concrete and Constructional Engineering*, London, November, 1938.

[7] "The Redistribution of Moments in Reinforced Concrete Beams and Frames," by W. H. Glanville and F. G. Thomas, *Journal*, Inst. of Civ. Engrs., June, 1936, p. 291.

[8] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105, 1940, p. 638.

structure so that, as nearly as practicable, all the sections involved may reach their allowable limits without exceeding them under the service loading.

Under such a condition, the theory of elasticity is the best guide to the forces acting on the different sections.[7] The probable redistribution of moments caused by yielding under higher loads than the design loads will effect the factor of safety of the structure against collapse and aid in the selection of the proper allowable limits of the strengths of the different sections.

The third step in the design is the proportioning of the different parts of the structure so that the limits of their useful strength may be reached but not exceeded under the loading specified. In dealing with a plastic material such as concrete, this limit can best be determined by a theory that recognizes its true characteristics. The plastic theory is based as closely as practicable on the actual behavior of concrete in combination with steel and makes possible the determination of the useful strength of concrete members with more accuracy than the present standard straight-line stress theory. The strength and reliability of the present-day concrete, and the detailed knowledge gained by practical research certainly justify a complete re-examination of the methods of design.

THE STRESS-STRAIN CHARACTERISTICS OF CONCRETE

The behavior of concrete under stress is extremely complex because it is influenced by combinations of so many variables that even under well-controlled conditions the prediction of true unit stress is not possible.

The classic conception of the stress-strain curve is that of a parabola with its lower part approximately straight and ending at the top tangent to the horizontal. The lower third of the curve being nearly straight, it formed the basis of the straight-line formula in which working stresses are used. The entire curve was used in the parabolic formula based on ultimate strength. Both of these formulas involve Young's modulus of elasticity of the concrete, which has been proved inaccurate by exhaustive research because the stress-strain relation is an indeterminate variable depending on the concrete, the manner and intensity of loading, and on time. Change in strain with time is due to plastic flow and shrinkage.

After years of extensive researches, R. E. Davis, M. Am. Soc. C. E., H. E. Davis, Assoc. M. Am. Soc. C. E., and Elwood H. Brown state that[9]

> "* * * it is not yet possible quantitatively to state with any degree of certainty what is likely to be the magnitude of either plastic flow or shrinkage under the conditions which surround any given concrete structure; nor is it possible quantitatively to state with accuracy what is the effect of plastic flow and shrinkage upon the magnitude and distribution of stresses."

It has been shown that deformations of stressed concrete change with time, and that stresses in a restrained member are greatly affected by plastic flow.[10] Even short-time measurements are affected by plasticity. On account of the variability of this stress-strain relation, it is not possible to determine the unit stresses at any particular point in the concrete member by measuring strains, except to a very limited degree.

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[9] "Plastic Flow and Volume Changes of Concrete," by R. E. Davis, H. E. Davis, and Elwood H. Brown, *Proceedings*, A. S. T. M., Pt. II, Vol. 37, 1937, p. 318.

[10] "Plain and Reinforced Concrete Arches," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March, 1932.

The modulus of elasticity of concrete used in design formulas is an empirical value assumed because it appeared to make them give satisfactory results. However, the results cannot be made entirely satisfactory, even by this method of choosing n.

An almost universal misconception of the behavior of concrete in flexure is the assumption that at maximum load the highest unit stress is at the outer surface. The descending branch of the compression stress-strain curve has been generally disregarded because it is difficult to measure and has been considered of no particular interest.

Fig. 1 shows some typical stress-strain curves for concrete cylinders selected from those measured in 1938 by O. G. Kiendl and J. A. Maldari.[11] It will be noted that the maximum stress occurs at a strain of approximately 0.002 in. per in., and that the failure does not occur suddenly but that the stress is reduced gradually as the strain continues to increase.

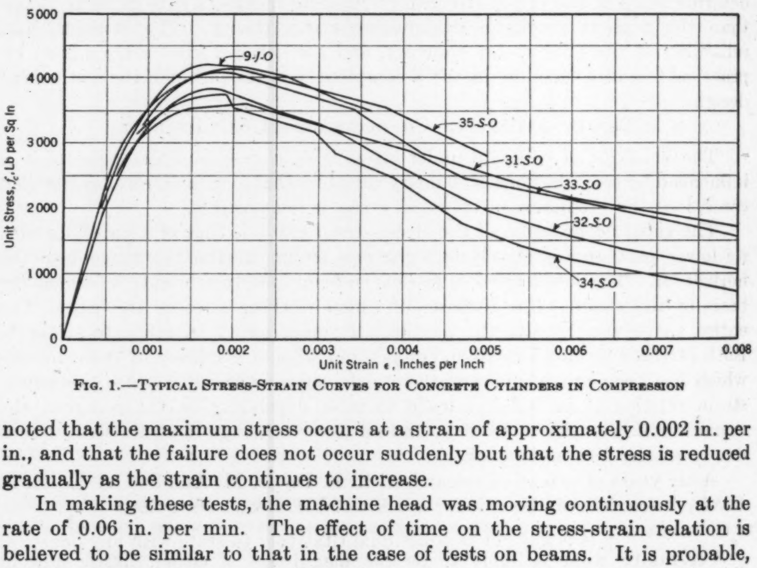

FIG. 1.—TYPICAL STRESS-STRAIN CURVES FOR CONCRETE CYLINDERS IN COMPRESSION

In making these tests, the machine head was moving continuously at the rate of 0.06 in. per min. The effect of time on the stress-strain relation is believed to be similar to that in the case of tests on beams. It is probable, therefore, that these curves indicate the form of compressive stress distribution in a beam at ultimate load in the concrete in the zone where the unit strain exceeds about 0.002 in. per in. In the case of a beam under partial load for a long time, plastic flow will modify the stress-strain relation so that the descending portion of the curve is perhaps of no significance. It is the writer's purpose in this paper to present a method of predicting ultimate strengths. No attempt will be made to determine actual stresses.

In considering the effect of the large strains occurring in a beam as the maximum load is approached it should be remembered that, under favorable circumstances, concrete strained beyond its point of maximum resistance will regain its strength after a rest.[12]

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[11] "A Comparison of Physical Properties of Concrete Made of Three Varieties of Coarse Aggregate," a thesis by Oscar G. Kiendl and Joe A. Maldari, presented to the University of Wisconsin, Madison, Wis., in 1938, in partial fulfilment of the requirement for the degree of Bachelor of Science in Civil Engineering.

Because the straight-line formula applied to test results indicates that the concrete stress at maximum load in an over-reinforced beam is much higher (about 50%) than the cylinder strength, and because the maximum strain in the beam is much higher than that in the cylinder at maximum load, it has been concluded quite generally that the concrete does not behave the same in the beam and in the cylinders. Although there is undoubtedly some difference due to the shape of the unit and the distribution of stress, it will be shown that this difference may not be substantial and that the behavior in the compression side of the beam may be fully explained by the stress-strain characteristics of the cylinder.

It is obvious from a comparison of the different curves of Fig. 1 that even in controlled tests there are such great differences in the strains that no very refined theory can be justified. There are general characteristics, however, that form the basis of the plastic theory and indicate the form of the equations, leaving certain empirical constants to be determined by actual tests on beams. In this manner, simple formulas can be derived that have a more satisfactory theoretical basis than the present standard formulas and that give more accurate results.

The ideal stress-strain curve in Fig. 1 was derived from cylinder tests of concrete with a strength of about 4,000 lb per sq in. It has been proved that the strain in the compression side of a beam increases practically in direct proportion to the distance from the neutral axis. Therefore, the ideal curve to the left of any particular ordinate corresponding to the strain at the outer surface of the beam will represent the variation of unit compression stress in the concrete. If the strain in the steel is known, the location of the neutral axis and the resisting moment of the beam can be computed. The tension in the concrete will be neglected because the effect is small at high loads and the error is on the side of safety.

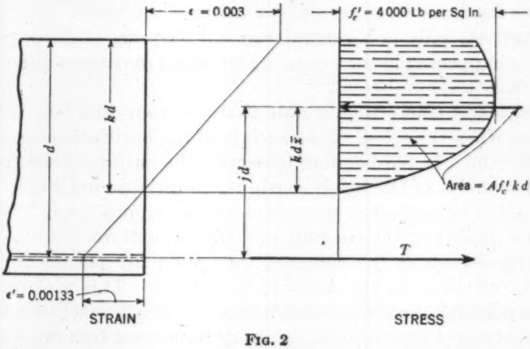

Fig. 2

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[12] "Autogenous Healing of Concrete," by M. W. Loving, Assoc. M. Am. Soc. C. E., *Bulletin No. 13*, Am. Concrete Pipe Assoc., 1936; and "Autogenous Healing of Cement and Concrete," by Leslie Turner, London, International Assoc. for Testing Materials, April, 1937.

As an example, Fig. 2 shows the distribution of stress in the beam assuming a unit strain of $\epsilon = 0.003$ at the face of the concrete and $\epsilon' = 0.00133$ at the steel corresponding to a steel stress of 40,000 lb per sq in.

The computed value of k is $\frac{\epsilon}{\epsilon + \epsilon'} = 0.692$. By integrating the stress-strain curve from zero to the ordinate where the strain ϵ is 0.003, it is found that the area is $0.803\,\epsilon f_c'$ in which f_c' is the maximum concrete stress and ϵ is the maximum strain. Furthermore, the center of gravity of the area is $\bar{x}\,\epsilon = 0.571\,\epsilon$ from the origin. Applying this area to the depth of the beam above the neutral axis, substituting $k\,d$ for ϵ, the area represents the total compressive force in the beam; and it equals $C = 0.803 f_c'\,k\,d = 0.556 f_c'\,d$. Then the moment arm from the steel to the center of compression is $j\,d = d - k\,d\,(1 - 0.571) = 0.703\,d$. The bending moment for a unit width of beam is then:

$$M = 0.556 f_c'\,d \times 0.703\,d = 0.391 f_c'\,d^2 \ldots\ldots\ldots\ldots (1)$$

Corresponding values of moment have been computed for other concrete and steel strains (see Fig. 3(*a*)), using the formula

$$\frac{M}{f_c'\,b\,d^2} = k\,A\,(1 - k + k\,\bar{x}) \ldots\ldots\ldots\ldots (2)$$

in which

$$A = \frac{\int f_c\,d\epsilon}{f_c'\,\epsilon} \ldots\ldots\ldots\ldots (3a)$$

and

$$\bar{x} = \frac{\int \epsilon\,dA}{A\,f_c'\,\epsilon^2} \ldots\ldots\ldots\ldots (3b)$$

In these computations, the values of distance $\bar{x}$ and area A given in Fig. 3(*b*) were used, based on the ideal stress-strain curve. The maximum ordinates in Fig. 3(*a*) give the maximum resisting moments of the beam. They are significant quantitatively only in a general way but they are of great value in indicating what is happening in the beam. The actual maximum value can best be determined from tests on beams.

It is particularly interesting to note that the maximum bending moment is not developed when the stress at the surface of the beam reaches the maximum cylinder strength. As the bending increases, the surface stress decreases but the resisting moment of the beam continues to increase until the strain at the surface is roughly twice that of the cylinder at maximum load. This ratio will vary with the quality of the concrete and the steel strain.

These large strains in the concrete are principally plastic and result in a redistribution of stress as the moment is applied. This yielding brings the center of the compressive stress much nearer the tensile steel than indicated by the standard straight-line formula. A very important feature of this redistribution of stress is that it apparently does not affect the fatigue limit of the beam adversely. Tests made by J. L. Van Ornum,[13] M. Am. Soc. C. E., on heavily reinforced beams that failed in compression showed that the fatigue limit at which beams failed under repeated loading was between 50% and 60% of the static strength and practically the same percentage as the fatigue limit of cylinders under repeated compression. The strength of the beams under 1,000 repetitions was about 70% of the static strength.

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[13] "The Fatigue of Concrete," by J. L. Van Ornum, *Transactions*, Am. Soc. C. E., Vol. LVIII, June, 1907, p. 294.

Referring to the ideal curve in Fig. 3, it will be seen that a moment of 50% of the ultimate may produce initially a stress greater than 80% of the cylinder strength. It might be expected that a repetition of such load would cause failure, but the plasticity of the concrete at such a high stress causes a readjustment before failure takes place. This is important because it indicates that although the stress in the outer portion of the beam may be higher than the fatigue limit as shown by cylinders, there is a redistribution of stress in the beam so that the fatigue limit of the beam is more than 50% of the ultimate static strength of the beam.

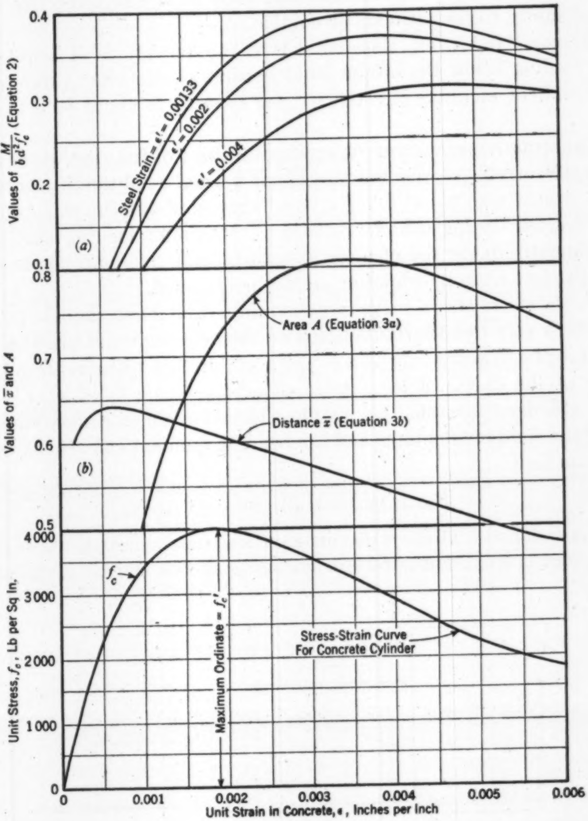

Fig. 3

It has been proved that plastic flow is approximately proportional to stress at low stresses, but for higher stresses the plastic strain increases much more rapidly than the stress.[14] This is to be expected from the shape of the ordinary stress-strain curve.

It is obvious that if the fatigue limit is a reasonably definite percentage of the ultimate strength, the ultimate strength is itself a better measure of the useful strength of the beam than is the unit stress calculated by the standard straight-line formula based on an arbitrary value of the modulus of elasticity. The useful strength can then be determined most effectively by applying a factor of safety to the ultimate strength.

Professor Van Ornum also found that the deflections of beams under repeated loading below the fatigue limit became elastic after a number of repetitions, and the modulus of elasticity was reduced to about 80% of the initial value.

No information is at hand regarding the effect of full reversal of loading on heavily reinforced sections, but this is not a usual condition in concrete structures. Beams with about 0.3% of steel showed about the same fatigue limit under reversed loading as under repeated. Maximum loading conditions occur so infrequently in the life of most structures that this may not be important ordinarily; but more information should be obtained.

Since the stress-strain curves are so variable and indeterminate (as indicated by Fig. 1), a very exact formula based on this relation is not practical. Fortunately it is practicable to derive a simple formula that will give the approximate location of the center of the compression force more satisfactorily than the present standard formula. A simple expression for the maximum limiting value of the compression force and the bending moment can also be determined from beam tests.

RECTANGULAR BEAMS AND SLABS

If it is assumed that the stress distribution in a concrete beam at failure has the shape of the cylinder stress-strain curve shown in Fig. 4, the total compression, C, is the area bounded by the curve. The line of action of C lies through the center of gravity of this area. From an examination of the ideal stress-strain curve in Fig. 3, it is found that, if the actual area is replaced by an equivalent rectangular area of width equal to $0.85 f_c'$ and depth equal to a (as shown in Fig. 4), the location of the center of gravity of this rectangle corresponds closely with that of the actual area.

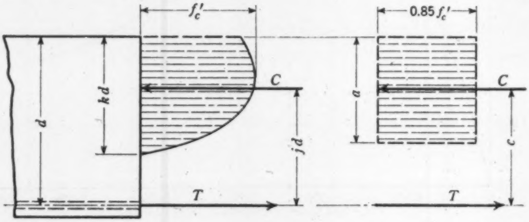

FIG. 4

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[14] "Short-Time Creep Tests of Concrete in Compression," by R. S. Jensen and F. E. Richart, M. Am. Soc. C. E., *Proceedings*, A. S. T. M., Pt. II, Vol. 38, 1938, p. 417.

In other words, the actual irregular stress block is replaced for simplicity with a rectangular stress block of equal size that has an average stress intensity equal to $0.85 f_c'$. This does not mean that this method is based on the assumption of rectangular stress distribution. The use of the rectangle is merely a mathematical device to approximate the effect of the true distribution. It would be possible to use any curved shape that would give the same area and center of gravity, but the rectangular area appears to be entirely satisfactory and gives the simplest possible mathematical solution. This conception forms the basis of all the following formulas, and they give good agreement with tests. The assumption of a concrete stress equal to $0.85 f_c'$ corresponds to the actual strength in columns and permits a consistent treatment through the full range from pure flexure (eccentricity $e = \infty$) to axial load ($e = 0$).

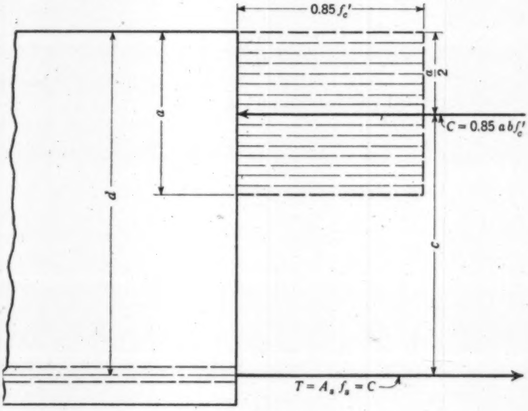

FIG. 5

It should be noted that the depth a does not correspond with the depth to the neutral axis, $k d$, and it bears no definite relation to it. The value of $k d$ is determined by the strains at the top and bottom of the beam but a is determined by the strengths of the materials.

For practical purposes, the action of the beam may now be described as follows (see Fig. 5):

If the beam is under-reinforced so that the primary failure will occur in the tensile steel, the concrete will crack as the steel stretches, and the equivalent depth of the beam in compression, a, will decrease until the average effective concrete stress reaches the maximum. The concrete will then fail progressively, reducing the lever arm of the steel and causing failure of the beam.

Therefore[14b]:

$$a = \frac{A_s f_s}{0.85\, b f_c'} = \frac{A_s m}{b} \qquad (4a)$$

or

$$\frac{a}{d} = p\, m \qquad (4b^*)$$

[14b] Equation Nos. marked with an asterisk (*) indicate essential design formulas.

in which: A_s = area of tensile steel; f_s = yield point stress of steel (or in the case of steel without a definite yield point, the stress producing a total unit strain equal to 0.004 in. per in.); b = width of beam; f_c' = standard concrete cylinder strength; the steel percentage is

$$p = \frac{A_s}{b\,d} \quad \text{(5a)}$$

and

$$m = \frac{f_s}{0.85\,f_c'} \quad \text{(5b)}$$

The lever arm of the steel reinforcement is then

$$c = d - \frac{a}{2} = d - \frac{A_s\,m}{2\,b} \quad \text{(6a)}$$

or

$$\frac{c}{d} = 1 - \frac{p\,m}{2} \quad \text{(6b*)}$$

The ultimate resisting moment of the beam as controlled by steel failure can be written:

$$M = c\,A_s f_s = A_s f_s \left(d - \frac{A_s\,m}{2\,b} \right) \quad \text{(7a)}$$

or

$$\frac{M}{b\,d^2} = p\,f_s \left(1 - \frac{p\,m}{2} \right) \quad \text{(7b*)}$$

Also, for under-reinforced beams,

$$M = 0.85\,f_c'\,a\,b \left(d - \frac{a}{2} \right) \quad \text{(8)}$$

from which

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35}{f_c'} \frac{M}{b\,d^2}} \quad \text{(9a*)}$$

and

$$\frac{c}{d} = \frac{1}{2} \left(1 + \sqrt{1 - \frac{2.35}{f_c'} \frac{M}{b\,d^2}} \right) \quad \text{(9b*)}$$

The foregoing expressions are extremely simple and are independent of n. It should be noted that all of the equations in this paper apply only to ultimate load conditions and are not intended to predict stresses under working loads.

The required steel area is

$$A_s = \frac{M}{c\,f_s} \quad \text{(10*)}$$

In case the moment is less than that required to cause a primary compression failure in the concrete, from Eq. 7b

$$p = \frac{1}{m} - \sqrt{\frac{1}{m^2} - \frac{2}{f_s\,m} \frac{M}{b\,d^2}} \quad \text{(11*)}$$

The limiting value of the depth of equivalent compression, a, for equal concrete and steel strengths in flexure can best be determined experimentally. If there is at least sufficient steel to fully develop the strength of the concrete, additional steel does not materially increase the strength of the beam.

The limiting value of $\frac{a}{d}$ computed from tests reported by Inge Lyse, and the late W. A. Slater,[15] Members, Am. Soc. C. E., is given in Table 1. Eliminating four groups of beams that appear erratic, the mean value of $\frac{a}{d}$ is 0.537 and $\frac{c}{d}$ is 0.732. The corresponding ultimate resisting moment is given by the expression

TABLE 1.—BEAM TESTS REPORTED BY SLATER AND LYSE

Group No.	d (in.)	b (in.)	Ratio p	Cylinder strength (lb per sq in.)	Actual maximum moment (lb-in.)	Eq. 9a $\frac{a}{d}$	Eq. 9b $\frac{c}{d}$	Computed maximum moment from Eq. 12 (lb-in.)	ACTUAL MAXIMUM MOMENT DIVIDED BY MAXIMUM MOMENT AS COMPUTED BY		
									Eq. 12	A.C.I. formula	Parabolic formula
1	10.2	8.2	0.021	1,390	597,000	>1.0[a]	....	396,000	1.508[a]	2.06[a]	1.50[a]
2	10.3	8.2	0.028	2,790	818,000	0.544	0.728	811,000	1.007	1.51	1.03
3	10.3	8.2	0.037	4,070	1,152,000	0.515	0.742	1,183,000	0.973	1.50	0.98
4	10.1	8.2	0.047	4,800	1,360,000	0.554	0.723	1,342,000	1.013	1.54	1.00
5	10.2	8.3	0.056	5,740	1,584,000	0.481	0.759	1,660,000	0.955	1.46	0.95
6	14.2	8.2	0.030	2,590	1,805,000	0.910[a]	0.545[a]	1,430,000	1.262[a]	1.85[a]	1.27[a]
6A	14.1	8.2	0.039	4,130	2,196,000	0.518	0.741	2,250,000	0.976	1.50	0.97
7	12.2	8.3	0.028	2,950	1,240,000	0.553	0.723	1,220,000	1.016	1.56	1.03
8	8.0	8.1	0.031	2,760	543,000	0.670[a]	0.665[a]	478,000	1.135[a]	1.66[a]	1.13[a]
9	5.9	7.9	0.032	2,900	310,000	0.708[a]	0.646[a]	266,000	1.166[a]	1.69[a]	1.14[a]
10	4.1	8.0	0.030	2,820	129,000	0.552	0.724	127,000	1.016	1.48	1.00
10A	4.1	8.0	0.040	3,810	179,000	0.576	0.712	171,000	1.046	1.55	1.03
Average						0.537	0.732	....	1.00	1.52	1.00
Standard Deviation						±0.024	±0.012	....	±0.024	±0.028	±0.024

[a] These values not included in average.

$$\frac{M}{b\,d^2} = 0.85\,f_c'\,\frac{a}{d}\left(1 - \frac{a}{2\,d}\right) = \frac{f_c'}{3} \quad \text{(12*)}$$

For comparison, Table 1 gives the maximum moment for the test beams as computed from the American Concrete Institute (A.C.I.) formula and from the parabolic formula[16] using the measured value of n. The standard deviation is practically the same in the three cases indicating that there is no advantage in using a formula involving n.

Fig. 6 shows the value of $\frac{M}{b\,d^2\,f_c'}$ given by 36 beams tested by Professors Slater and Lyse,[17] and 33 beams tested by the late Richard L. Humphrey, M. Am. Soc. C. E., and Louis H. Losse[18] which had sufficient steel to cause a primary compression failure. The Humphrey beams, with stronger concrete, were not included because there was not sufficient reinforcement to develop such high concrete strength fully although they may have been reported as compression failures.

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[15] "Compressive Strength of Concrete in Flexure," by W. A. Slater and Inge Lyse, *Journal*, Am. Concrete Inst., June, 1930, p. 831.

[16] *Loc. cit.*, p. 861.

[17] *Loc. cit.*, p. 850.

[18] "The Strength of Reinforced Concrete Beams," by Richard L. Humphrey and Louis H. Losse, *Technological Paper No. 2*, National Bureau of Standards, 1912.

Confusion no doubt exists because it is difficult to differentiate between primary steel or concrete failures unless the steel percentage is considerably less than the critical required to develop full concrete strength. The difference cannot be detected by measuring the maximum strain in the concrete as it may be as large in either case. A slight stretching of the steel will reduce the concrete compression area and will cause, in an under-reinforced beam, what appears like a concrete compression failure.

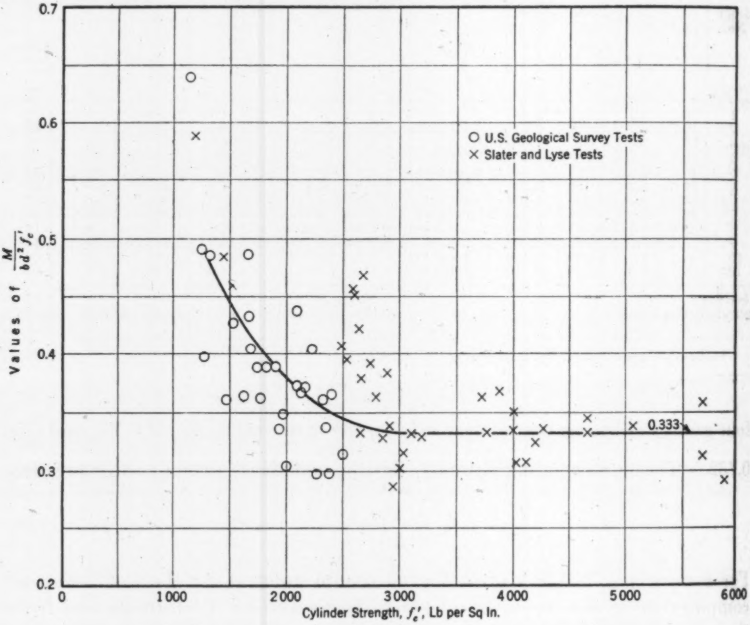

FIG. 6.—VALUE OF $\frac{M}{b\,d^2 f_c'}$ FROM TESTS OF BEAMS

The value of $\frac{M}{b\,d^2 f_c'}$ shown in Fig. 6 is unaffected by the concrete strength between 3,000 and 6,000 lb per sq in., although it rises about 50% for weaker concretes from 3,000 to 1,000 lb per sq in. With present methods and materials it is not probable that structural concrete will have a strength less than 2,500 or 3,000 lb per sq in., and if a weaker concrete is used, a greater factor of safety is appropriate. Therefore, it appears that a value of 0.333 for $\frac{M}{b\,d^2 f_c'}$ can be used generally.

The critical percentage of steel required to develop the full compressive strength of the concrete is (see Eq. 4):

$$p_0 = \frac{0.537}{m} = 0.456\frac{f_c'}{f_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13^*)$$

For beams with less steel, the allowable bending moment is given by Eq. 7. Table 2 shows the results of the application of Eq. 7 to 72 beams of the series

TABLE 2.—COMPARISON OF COMPUTED AND ACTUAL ULTIMATE STRENGTH OF BEAMS

Item No.[a]	Concrete		Steel			Actual maximum moment (lb-in.)	$\frac{c}{d}$ from Eq. 9b	Computed maximum moment from Eq. 7 (lb-in.)	Actual maximum moment divided by computed
	Age, in weeks	Cylinder strength (lb per sq in.)	d (in.)	100 p	Yield point (lb per sq in.)				
				(a) GRANITE AGGREGATE					
1	4	3,241	10	0.49	42,490	177,000	0.9621	160,500	1.103
2	4	2,807	10	0.98	43,043	321,000	0.9115	307,500	1.043
3	4	3,158	9½	1.96	41,857	492,000	0.8350	507,000	0.970
4	52	5,589	10	0.49	42,573	193,600	0.9780	163,500	1.183
5	52	5,086	10	0.98	42,320	340,000	0.952	319,000	1.066
6	52	4,957	9½	1.96	41,877	543,000	0.895	545,000	0.996
				(b) LIMESTONE AGGREGATE					
7	4	2,087	10	0.49	41,653	164,000	0.9425	154,000	1.064
8	4	2,600	10	0.98	40,797	291,000	0.9095	289,000	1.006
9	4	2,725	9½	1.96	38,037	471,000	0.815	451,000	1.044
10	52	4,069	10	0.49	41,003	183,000	0.9708	157,000	1.165
11	52	4,360	10	0.98	41,413	317,000	0.9451	307,000	1.033
12	52	4,230	9½	1.96	37,673	532,000	0.890	487,000	1.092
				(c) GRAVEL AGGREGATE					
13	4	3,441	10	0.49	43,447	169,000	0.9635	164,000	1.030
14	4	3,440	10	0.98	42,600	321,000	0.9285	310,000	1.035
15	4	3,376	9½	1.96	38,633	504,000	0.857	482,000	1.045
16	52	5,186	10	0.49	44,140	173,000	0.9754	169,000	1.022
17	52	5,242	10	0.98	40,107	356,000	0.9559	300,000	1.186
18	52	5,467	9½	1.96	38,643	533,000	0.9120	512,000	1.042
				(d) CINDERS AGGREGATE					
19	4	1,629	10	0.49	37,357	163,200	0.9337	138,000	1.182
20	4	1,555	10	0.98	41,170	281,000	0.8475	273,000	1.030
21	4	1,643	9½	1.96	37,887	403,000	0.7315	376,000[b]	1.071
22	52	2,944	10	0.49	38,343	172,600	0.9624	145,000	1.188
23	52	2,619	10	0.98	41,103	306,000	0.9090	293,000	1.044
24	52	2,763	9½	1.96	38,047	488,000	0.827	458,000	1.064
						Average			1.071
						Standard deviation			±0.05

[a] Three beams each item.

[b] Determined by concrete strength, Eq. 12.

tested by Messrs. Humphrey and Losse[18] except Item 21 for which Eq. 12 controls because of low-concrete strength. The average ratio of actual maximum moment to that given by Eq. 7 is 1.071 to 1. Eliminating the low-strength cinder concrete beams (Table 2(*d*)), the ratio is 1.062 to 1. Classified according to age, the ratio is 1.052 to 1 at 4 weeks, and 1.09 to 1 at 52 weeks.

According to percentage of steel, it is 1.117 for $p = 0.0049$, 1.054 for $p = 0.0098$, and 1.042 for $p = 0.0196$, showing the increasing influence of the tension in the concrete as the percentage of steel is reduced. There appears to be no correlation between the modulus of elasticity of the concrete and the accuracy of Eq. 7.

The value of the steel lever arm, $\frac{c}{d}$, from Eq. 6 shows a substantial improvement over the j-value given by the standard formula. This is clearly shown by the aforementioned tests.[17, 18] The general trend of the comparative values is shown in Table 3 computed for different percentages of steel in a beam with 3,000 lb per sq in. concrete and 50,000 lb per sq in. yield point steel. It will be noted that, for the smaller percentages of steel, $\frac{c}{d}$ is larger than j and this increase is fully justified by the Humphrey tests. As the percentage of steel increases to more than that commonly used, $\frac{c}{d}$ becomes subtantially smaller than j. The importance of this reduction is shown in the tests by Messrs. Slater and Lyse[15] who reported that the steel stresses at failure of the beams were considerably higher than the theoretical values. Table 4 gives a comparison for four series of beams for which strains were measured close to the ultimate load. The values of j at failure have been estimated from the observed values. It is seen that the actual j was much lower than the theoretical and corresponds very well with $\frac{c}{d}$ as given by Eq. 6.

TABLE 3.—COMPARISON OF j AND $\frac{c}{d}$

($f_c' = 3{,}000$ Lb per Sq In., $f_s = 50{,}000$ Lb per Sq In., $n = 10$, and $m = 19.6$)

p	STANDARD METHOD		PLASTIC THEORY
	$k = \sqrt{2pn + (pn)^2} - pn$	j	$\left(\frac{c}{d} = 1 - \frac{pm}{2}\right)$
0.005	0.270	0.910	0.951
0.010	0.3585	0.880	0.902
0.0113	0.376	0.875	0.889
0.020	0.400	0.867	0.804
0.027	0.516	0.828	0.732

TABLE 4.—COMPARISON OF j AND $\frac{c}{d}$ AT FAILURE

Description	SERIES NO.			
	2	3	4	5
Values of j Theoretical	0.812	0.801	0.808	0.798
Actual	0.718	0.740	0.730	0.735
Plastic theory; values of $\frac{c}{d}$ by Eq. 6	0.728	0.743	0.723	0.759

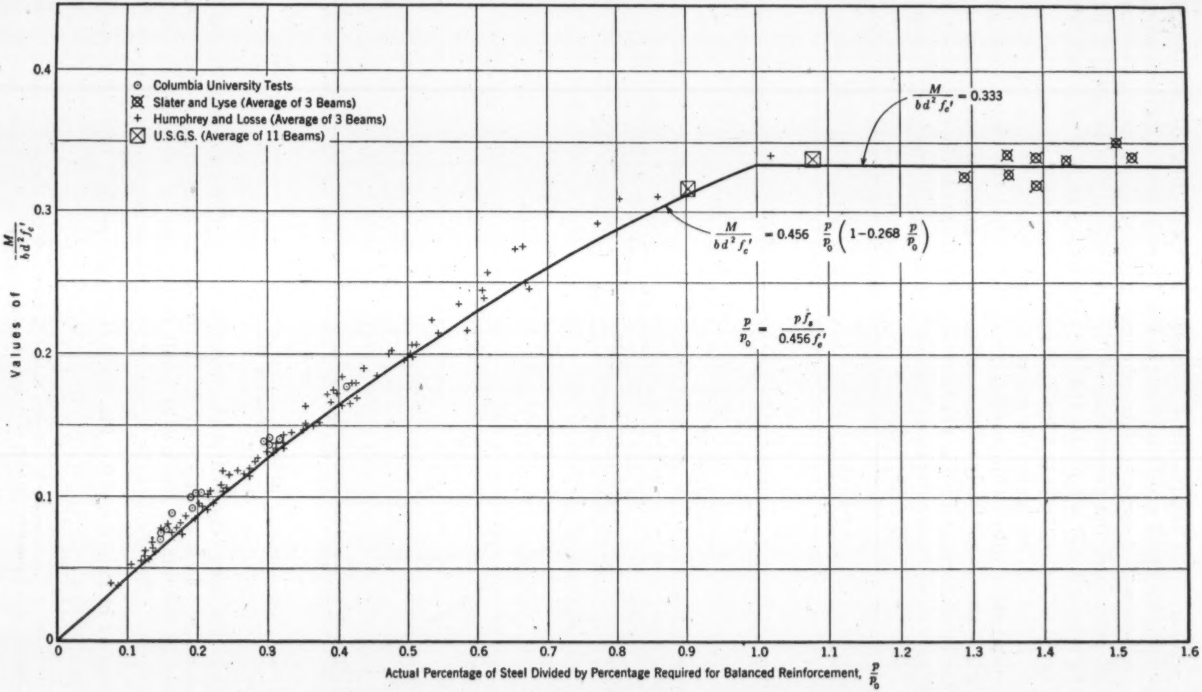

FIG. 7.—COMPUTED ULTIMATE STRENGTH OF CONCRETE BEAMS COMPARED WITH THE RESULTS OF 309 TESTS

A comparison of the results of 309 beam tests with the ultimate strength values given by Eqs. 7 and 12 is shown in Fig. 7 (see Appendix for description). Only well-controlled series of tests have been included and European tests are omitted because of the uncertainty of the translation of cube strength into cylinder strength. It is probable that the cylinder is a more satisfactory index of strength of the concrete in a beam or column than is the cube.

Eq. 7 does not recognize the excess strengths that have been reported in beams and slabs with small percentages of steel,[19] but the use of small percentages of steel is not likely to be as satisfactory in practical construction as it is in the laboratory, and it has not been demonstrated that the excess strength will not disappear under repeated loading. Considerable cracking of slabs in buildings has been observed in the past few years. This is evidently due to shrinkage and lack of plasticity in the relatively high-strength and rapid-hardening concrete, so the use of small percentages of steel should not be encouraged. The effect of the foregoing formulas on rectangular beam design is indicated in Figs. 8 and 9. Fig. 8 shows the allowable $\frac{M}{b\,d^2}$ given by Eqs. 7 and 12 for different values of p with a factor of safety of $2\frac{1}{2}$ as compared with

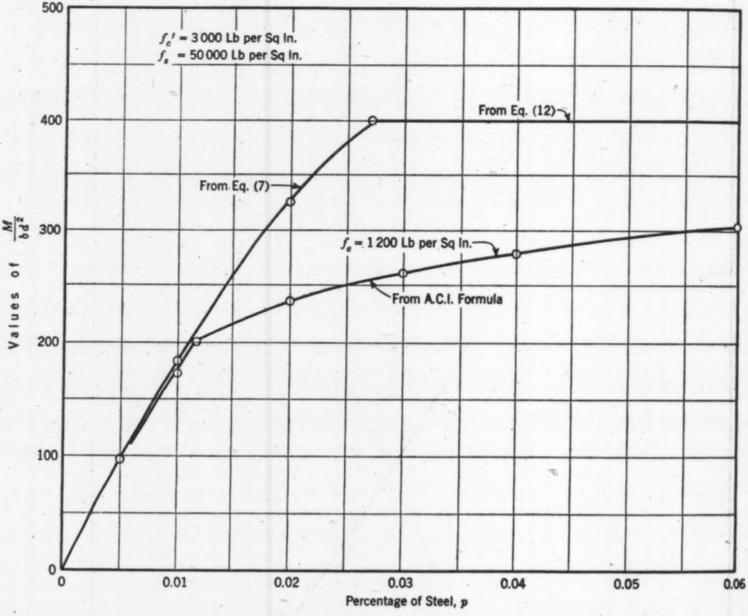

FIG. 8.—COMPARISON OF RESISTING MOMENT OF RECTANGULAR BEAMS COMPUTED FROM VARIOUS FORMULAS

the standard formula of the American Concrete Institute when $f_c' = 3{,}000$ lb per sq in. and $f_s = 50{,}000$ lb per sq in. The new formulas raise the critical percentage of steel from 0.0113 to 0.0273, and the allowable $\frac{M}{b\,d^2}$ from 197.5 to 400.

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[19] "A Study of Reinforcement in Concrete Slabs," by Inge Lyse and George R. Wernisch, *Journal*, Am. Concrete Inst., September–October, 1936, p. 1.

Fig. 9 is a universal design diagram giving values of $\frac{M}{b\,d^2}$ and p for all grades of concrete and steel. The advantage of the simplicity of the basic formulas is obvious, and since the equations conform so well to tests covering the full range of steel and concrete strengths used in practice, there appears to be no reason that they should not be used for design.

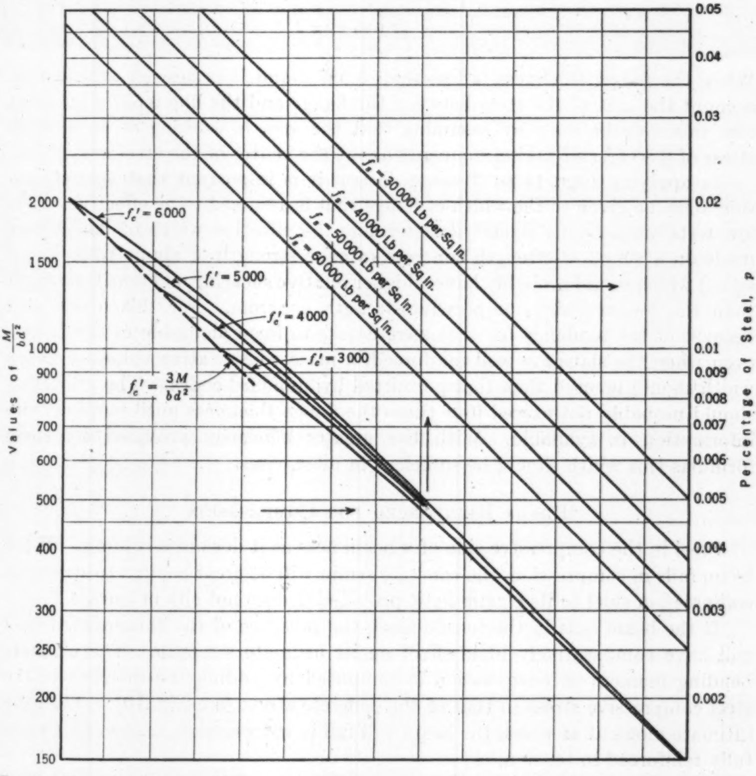

FIG. 9.—ULTIMATE BENDING MOMENT AND PERCENTAGE OF STEEL FOR RECTANGULAR BEAMS, BASED ON EQS. 7 AND 12

DESIGN OF T-BEAMS

The foregoing formulas for rectangular beams (Eqs. 7 and 12) can be directly applied to T-beams when the depth of compression, a, does not exceed the flange thickness t. For larger moments, when t is less than 0.537 d, it can be assumed that the entire flange area is working at an average stress of 0.85 f_c' with the resultant at the center depth of the flange. Then the ultimate moment is given by

$$\frac{M}{b\,d^2} = 0.85\,f_c'\left(1 - \frac{t}{2\,d}\right)\frac{t}{d} \tag{14a*}$$

and

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} \tag{14b*}$$

When the flange thickness is less than 0.537 d and it is desired to take into account the area of the stem between the flange and the line 0.537 d from the top, this may be done by assuming that the area is working at an average stress of 0.85 f_c' and taking moments about the center of the steel area.

In applying Eqs. 14 to T-beam design, it is important that special consideration be given to the width of flange that is assumed to be effective. The few tests made by C. Bach[20] to determine the effective width of flange were made on a very low-strength concrete (a cube strength of about 1,700 lb per sq in.) which would probably have a higher relative shearing and tensile strength than the modern high-compressive-strength concrete. For this reason and because of the tendency for shrinkage cracks to form alongside of the beams, even when the slabs are well reinforced, a more conservative value for flange width should be used than that permitted by standard codes. The projection should probably not exceed four times the flange thickness until more definite information is available. With the greater efficiency provided by these formulas this width should be sufficient in most cases.

Beams Reinforced for Compression

Steel in the compressive side of a beam acts as it does in a column. If the beam fails in compression, the concrete strain will be great enough to stress the compression steel to its elastic limit provided it does not slip or buckle.

If the beam fails in the tensile steel, the presence of the compression steel will have comparatively little effect on its ultimate strength. The ultimate bending moment in compression is computed by adding the moment of the steel compressive stress to that of the concrete stress (see Fig. 10). Then the ultimate moment at which the beam will fail in compression, assuming it to be fully reinforced in tension, is

$$M = \tfrac{1}{3}\,b\,d^2 f_c' + d'\,A_s'\,f_s \tag{15a*}$$

or

$$\frac{M}{b\,d^2} = \frac{f_c'}{3} + \frac{d'}{d}\,p'\,f_s \tag{15b*}$$

in which A_s' is the area of the compression steel, and d' is its lever arm. If the yield point of the compressive steel has a value of f_s' different from f_s, it can be taken into account in the equation by changing the value of A_s' used in Eq. 15a to the value of $\frac{f_s'}{f_s} A_s'$ without changing the equation. This applies to all equations involving compressive steel.

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[20] "Versuche mit Eisenbetonbalken," von C. Bach and O. Graf, *Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Heft 90 and 91, Berlin, 1910.

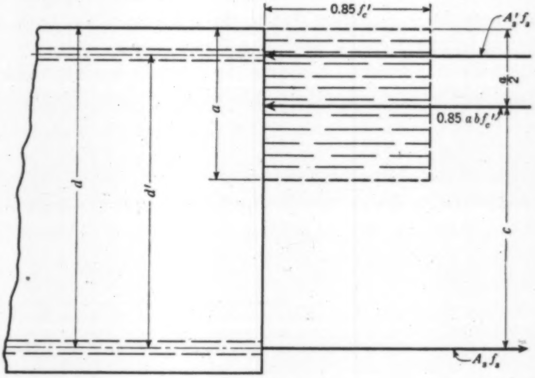

FIG. 10

When the beam is under-reinforced in tension, the lever arm of the tensile steel can be computed as if the compressive steel is stressed to its elastic limit and the remainder of the compression is on the concrete. The calculation is made as if the tensile steel were in two parts, one to balance the compressive steel and the other to balance the stress on the concrete. The ultimate tensile moment is then

$$M = d' A_s' f_s + (A_s - A_s') \left[d - \frac{(A_s - A_s') m}{2 b} \right] f_s \ldots\ldots (16^*)$$

or

$$\frac{M}{b d^2} = \frac{d'}{d} p' f_s + (p - p') \left[1 - \frac{(p - p') m}{2} \right] f_s \ldots\ldots\ldots (17^*)$$

and the tensile steel required is

$$p = p' + \frac{1}{m} - \sqrt{\frac{1}{m^2} - \frac{2}{f_s m} \frac{M - d' A_s'}{b d^2}} \ldots\ldots\ldots\ldots (18^*)$$

Eqs. 16 and 17 assume that A_s is greater than A_s'. If A_s' is the greater, the lever arm of the tensile steel can be taken as d' and Eq. 16 becomes:

$$M = d' A_s f_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Eqs. 15 to 18 have been checked against the results of tests with satisfactory agreement provided that failure is not caused by shear or bond. The results of tests of beams with compressive steel made by C. Bach and O. Graf at Stuttgart, Germany, and reported by Fritz von Emperger[21] are of interest because they show the danger of bond failure. The full yield point stress of 35,000 lb per sq in. was developed in compression, but with bars of 60,000 lb per sq in. yield point the failure was in bond before the full strength was developed.

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[21] "Handbuch für Eisenbetonbau," by Fritz von Emperger, Vol. 1, 3d Ed., p. 225.

Tests made with twin twisted bars in the compression side of beams[22] indicate that, because of buckling due to the twisting of the bars, they are not as fully effective in compression as they are in tension.

It is extremely important, of course, to provide against failure by bond, shear, and buckling in highly reinforced beams.

Flexure and Direct Load on Rectangular Sections

The strength of the compression side of the section will be the same when the member is subjected to bending and direct load as it is under flexure alone. Therefore, Eq. 15*a* can be used to predict the resistance to compression failure of an eccentrically loaded rectangular section.

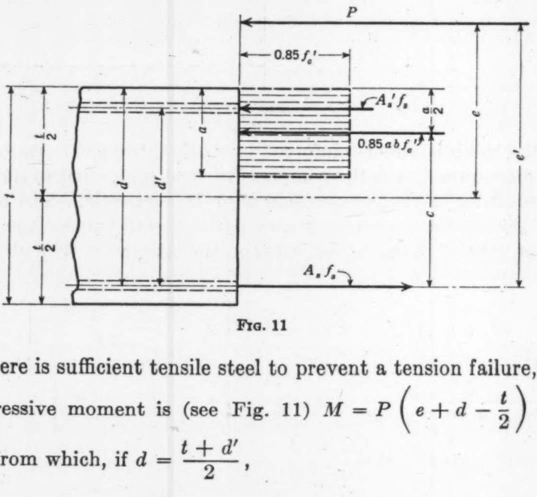

Fig. 11

When there is sufficient tensile steel to prevent a tension failure,[23] the ultimate compressive moment is (see Fig. 11) $M = P\left(e + d - \frac{t}{2}\right) = \frac{1}{3}bd^2f_c' + d'A_s'f_s$ from which, if $d = \frac{t + d'}{2}$,

$$P = \frac{2A_s'f_s}{\frac{2e}{d'} + 1} + \frac{btf_c'}{\frac{3te}{d^2} + \frac{6dt - 3t^2}{2d^2}} \quad \ldots\ldots\ldots\ldots\ldots (20^*)$$

Eq. 20 gives the theoretical value when the eccentricity of the load is greater than the eccentricity of the resisting forces in the compression side of the section. Because of its derivation it has no theoretical meaning for smaller eccentricities but it can be adjusted to the smaller range by making *P* approach the proper value for an axially loaded column as *e* approaches zero. The first term giving the steel strength needs no adjustment because it equals $2\,A_s'f_s$ when e equals zero. It is known that the concrete strength is $0.85\,b\,t\,f_c'$ for axial load so the second term becomes $\dfrac{b\,t\,f_c'}{\dfrac{3\,t\,e}{d^2} + 1.178}$, and with this change, Eq. 20 becomes:

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[22] "Versuche über Zielsichere Betonbildung und an druckbewerten Balken," by Rudolf Saliger, *Beton und Eisen*, January 20, 1935.

[23] "Plain and Reinforced Concrete Arches," Report of Committee 312, by Charles S. Whitney, Chairman, *Proceedings*, Am. Concrete Inst., September, 1940.

$$P = \frac{2\,A_s'f_s}{\dfrac{2\,e}{d'} + 1} + \frac{b\,t\,f_c'}{\dfrac{3\,t\,e}{d^2} + 1.178} \ldots\ldots\ldots\ldots\ldots\ldots (21^*)$$

which gives the strength of the rectangular member as controlled by compression strength. Eq. 21 gives values which agree satisfactorily with tests, as will be shown subsequently.

The formula for the strength of the member as controlled by ultimate tensile strength of the steel for the larger eccentricities will now be derived. If the member is over-reinforced on the compression side so that the compression steel is sufficient to take the total compression force without help from the concrete, the value of the ultimate load can be expressed by taking moments, and

$$P = A_s f_s \frac{2\,d'}{2\,e - d'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22^*)$$

When there is not sufficient compression steel to take all the compression force, it may be assumed for practical purposes that both the tension and compression steel will be stressed to the yield point at ultimate load, and the remainder of the compression load on the section acts on the compression side of the concrete over an equivalent depth equal to a. This total compression on the concrete is then $0.85\,a\,b\,f_c'$ and it follows that $P + A_s f_s = A_s' f_s + 0.85\,a\,b\,f_c'$ or, $0.85\,a\,b\,f_c' = P + (A_s - A_s')\,f_s$ and $a = \dfrac{P}{0.85\,b\,f_c'} + (A_s - A_s')\,\dfrac{m}{b}$, in which $m = \dfrac{f_s}{0.85\,f_c'}$.

Then taking moments about the tension steel,

$$M = P\left(e + \frac{d'}{2}\right) = d'A_s'f_s + 0.85\,f_c'\,a\,b\left(\frac{d'}{2} + \frac{t}{2} + \frac{a}{2}\right) \ldots\ldots (23)$$

Substituting the previous values and solving for P, we find: $P = 0.85\,t\,b\,f_c' \times$

$$\times\left\{\sqrt{\left[\frac{e}{t} - 0.5 + (p - p')\,m\right]^2 + \frac{2\,d'}{t}\,p'\,m + (p - p')\,m\left[\frac{d'}{t} + 1 - (p - p')\,m\right]}\right.$$

$$\left. - \left[\frac{e}{t} - 0.5 + (p - p')\,m\right]\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24a^*)$$

For the symmetrical reinforcement, when $p = p'$, this reduces to

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.5\right)^2 + \frac{d'}{t}\,p_t\,m} - \left(\frac{e}{t} - 0.5\right)\right\} \ldots (24b^*)$$

in which, $p_t = p + p' = \dfrac{A_s + A_s'}{b\,t}$. When there is no compressive steel, $p' = 0$ and the ultimate load equals

$$P = 0.85\,t\,b\,f_c' \left\{ \sqrt{\left(\frac{e}{t} - 0.5 + p\,m\right)^2 + \left(\frac{d'}{t} + 1 - p\,m\right) p\,m} - \left(\frac{e}{t} - 0.5 + p\,m\right) \right\} \ldots\ldots (24c^*)$$

On account of the assumption that the steel on the side of the column away from the load is in tension, Eqs. 24 have no significance when e is very small. That case is covered by Eq. 21.

When the value of $\frac{e}{t}$ is large, it is difficult to obtain satisfactory accuracy in solving Eqs. 24 with a slide-rule. Formulas can be derived in another form to cover the same case in the same manner as Eqs. 16 and 17. This can be done most conveniently in terms of e', the eccentricity measured from the tensile steel.

Assuming the compressive steel to be working at yield-point stress, the percentage of tensile steel required to balance it will be $A_s'\left(1 - \frac{d'}{e'}\right)$. Subtracting this quantity from A_s, the percentage of tensile steel that is dependent on the development of compression in the concrete for its fulcrum to work against is determined. The value of P that can be resisted by these two portions of the tensile steel can be written and combined as follows:

$$P = f_s \left[A_s - A_s' \left(1 - \frac{d'}{e'}\right) \right] \frac{\beta}{e' - \beta} + \frac{d'}{e'} A_s' f_s \ldots\ldots (25^*)$$

in which

$$\beta = \frac{1}{2} \left\{ d + e' - \sqrt{(e' - d)^2 + \frac{2\,e'\,m}{b} \left[A_s - A_s' \left(1 - \frac{d'}{e'}\right) \right]} \right\}$$

When the reinforcement is symmetrical, $A_s' = A_s$ and Eq. 25 reduces to

$$P = \frac{2\,d'\,A_s\,f_s}{e' - d + \sqrt{(e' - d)^2 + \dfrac{2\,d'\,A_s\,m}{b}}} \ldots\ldots (26a^*)$$

When there is no compression steel $A_s' = 0$ and this becomes

$$P = A_s\,f_s \frac{e' + d - \sqrt{(e' - d)^2 + \dfrac{2\,e'\,A_s\,m}{b}}}{e' - d + \sqrt{(e' - d)^2 + \dfrac{2\,e'\,A_s\,m}{b}}} \ldots\ldots (26b)$$

Tables 5 and 6 compare the values of ultimate load given by Eqs. 21 to 26 with the actual loads found in several series of tests by Messrs. Bach and Graf,[24] F. E. Richart and T. A. Olson,[25] and A. G. Hayden,[26] M. Am. Soc. C. E. In the case of the Bach and Graf tests (Table 5), it was necessary to translate the cube strength into cylinder strength, and it is assumed the strength of the 30 cm cube per sq in. is 1.13 times that of the standard 6-in. by 12-in. cylinder.[27]

TABLE 5.—COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS FOR COLUMNS TESTED UNDER ECCENTRIC LOADS

Eccentricity, e/t	Equation No.	ACTUAL LOAD (LB)		Computed ultimate load (lb)	$\frac{\text{Col. 3}}{\text{Col. 5}}$	ACTUAL LOAD (LB)		Computed ultimate load (lb)	$\frac{\text{Col. 3}}{\text{Col. 5}}$
		Ultimate	First crack			Ultimate	First crack		
(1)	(2)	(3)	(4)	(5)	(6)	(3)	(4)	(5)	(6)
	(*a*) SERIES I; $p = 0.005$ AND $p' = 0$					(*b*) SERIES IV; $p = 0$ AND $p' = 0$			
0	21	618,000	....	596,000	1.04	609,000	....	597,000	1.02
0.25	24*c*	....	....	....	....	300,000	208,000	299,000	1.00
0.375	24*c*	....	....	....	....	180,500	91,900	149,000	1.21
0.50	24*c*	204,700	75,700	203,000	1.01	52,800	52,500	0	....
0.75	24*c*	132,700	44,800	123,300	1.07	....	....	....	....
1.25	24*c*	66,000	23,500	62,500	1.07	....	....	....	....
	(*c*) SERIES II; $p = 0.005$ AND $p' = 0.005$					(*d*) SERIES III; $p = 0.0095$ AND $p' = 0.0095$			
0	21	735,000	....	729,000	1.01	892,000	....	835,000	1.07
0.25	21	447,000	293,000	423,000	1.05[a]	496,000	358,000	486,000	1.02[a]
0.50	24*b*	273,000	77,800	257,000	1.06	347,000	99,000	343,000	1.01
0.75	24*b*	153,400	45,200	148,000	1.03[a]	231,500	58,900	219,000	1.05[a]
1.25	24*b*	71,300	22,300	68,500	1.04[a]	118,000	27,700	116,400	1.01

[a] Two tests each; all other determinations based on three tests each.

The Bach and Graf tests (Table 5) show very close agreement with the formulas for the full range, with the actual strengths slightly larger in all cases. They were made on 49 columns about 16 in. square with an equivalent cylinder strength of 2,830 lb per sq in. The steel had yield strengths of 53,700 lb per sq in. and 50,900 lb per sq in. with p equal to 0.005 and 0.0095, and p' equal to zero, 0.005, and 0.0095. The tests therefore covered a very significant range. The details of the sections, and the agreement between the actual and theoretical strengths, have been presented elsewhere.[28]

The Richart and Olson tests (Table 6(*a*)) on knee frames[25] show excellent agreement with the exception of No. 9 which had about 3% of tensile steel and failed at about 85% of the load given by Eq. 26*b*. Cracks formed before failure parallel to the compression face at the knee, indicating that it may have been caused by high shearing stresses due to the sharp curvature of the line of compression stress around the angle of the knee. The other frames with about 1% of steel failed primarily in tension and indicated no weakness in compression due to the concentration of stress at the knee when the ordinary percentage of reinforcement is used. More information is needed regarding the knee effect with higher steel percentages.

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[24] "Tests of Reinforced and Unreinforced Concrete Columns Under Axial and Eccentric Load," by C. Bach and O. Graf, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Vols. 166 to 169.

[25] "Rapid and Long Time Tests on Reinforced Concrete Knee Frames," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1937, p. 459.

[26] "Tests of Knees for Continuous Frame Concrete Bridges," by Arthur G. Hayden, *Engineering News-Record*, January 18, 1923, Vol. 90, p. 108.

[27] "Effect of Size and Shape of Test Specimen on Compressive Strength of Concrete," by H. F. Gonnerman, M. Am. Soc. C. E., *Bulletin No. 16*, Structural Materials Research Laboratory, Lewis Inst., Chicago Ill., 1925.

[28] *Journal*, Am. Concrete Inst., Vol. 12, No. 1, September, 1940, p. 1 (compare Figs. 5, 6, 7, and 8).

TABLE 6.—TESTS OF REINFORCED CONCRETE KNEE FRAMES

Frame No.	STEEL AREA (SQ IN.)		f_c' (lb per sq in.)	DIMENSIONS (IN.)[a]			Actual ultimate load[b] (lb)	COMPUTED ULTIMATE LOAD		$\frac{\text{Col. 7}}{\text{Col. 8}}$
	A_s	A_s'		d	d'	e'		(Lb)	Eq. No.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) RAPID AND LONG-TIME TESTS ($b = 12$ IN. AND $f_s = 48{,}300$ LB PER SQ IN.)[e]										
1	1.77	0	3,545	14.50	....	62.5	31,900	31,800[c]	26*b*	1.003
2	1.77	0	3,460	13.64	....	54.07	35,700	34,800[c]	26*b*	1.025
3	1.77	0	3,310	12.78	....	45.64	42,200	38,800[c]	26*b*	1.09
4	1.77	0	3,305	13.64	....	54.07	35,000	34,500[c]	26*b*	1.015
5	1.77	0	3,350	13.30	....	45.64	41,650	41,300[c]	26*b*	1.010
6	1.77	0	3,330	14.50	....	62.50	32,450	31,400[c]	26*b*	1.033
7	1.77	0	2,920	13.30	....	45.64	42,500	40,000[c]	26*b*	1.062
8	1.77	1.77	2,985	14.50	13.0	62.5	32,650	32,400[c]	26*a*	1.008
9	1.77	3.56	3,145	14.50	13.0	62.5	34,350	31,700[c]	22	1.082
10	1.77	1.77	3,035	13.30	11.25	45.64	42,500	41,300[c]	26*a*	1.030
11	1.77	3.56	3,055	13.30	11.25	45.64	43,300	39,500[c]	22	1.096
19	4.84	0	4,185	13.75	....	61.75	57,125	67,700[c]	26*b*	0.845[d]
(b) KNEES FOR CONTINUOUS-FRAME CONCRETE BRIDGES ($b = 18$ IN. AND $f_s = 45{,}000$ LB PER SQ IN.)[e]										
A	1.25	0	3,640	9.0	....	37.65	19,300	16,000	26*b*	1.206
B	1.18	0	3,360	9.0	....	37.65	16,000	15,100	26*b*	1.060
C	1.18	0	2,790	9.75	....	39.6	23,330	21,900[c]	26*b*	1.064
D	1.18	0	2,640	9.75	....	39.6	25,600	21,800[c]	26*b*	1.174
E	1.18	0	3,390	9.75	....	39.6	25,800	22,300[c]	26*b*	1.158
F	1.18	0	1,810	9.0	....	37.65	15,900	13,700	26*b*	1.15

[a] In Col. 6, e' is measured perpendicular to the axis of the leg of the member. [b] Average of two frames. [c] $P\sqrt{2}$. [d] Possible shear failure. [e] Part (*a*) refers to footnote 25; and part (*b*) refers to footnote 26.

Eqs. 21 to 26 give the ultimate strengths of rectangular members for the full range of conditions from axially loaded columns to beams under flexure alone. Since it is customary to provide a greater factor of safety for axially loaded columns than for beams, it is necessary to modify some of these formulas for use in design. Without changing their form they can easily be adjusted to any desired combination of beam and column factors of safety.

For instance, in the case of the compression control Eq. 21, it would be reasonable to assume that the factor of safety should vary from that now provided by the present standard formula for axially loaded columns to $2\frac{1}{2}$ for the case of pure bending. When plotted with P as ordinate and $P\,e$, as abscissa, Eq. 21 is a straight line, and it can be adjusted by lowering the y-axis intercept to any desired point.

The standard formula for the axially loaded column is

$$P = 0.4\, A_{st}\, f_s + 0.225\, A_c\, f_c' \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which A_{st} is the total area of longitudinal steel; and A_c = area of concrete in compression. Eq. 21 can be adjusted so that P will be $2\frac{1}{2}$ times this value when e is zero and it then becomes

$$P = \frac{2\, A_s'\, f_s}{\frac{2\, e}{d'} + 1} + \frac{A_c\, f_c'}{\frac{3\, t\, e}{d^2} + 1.775} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28^*)$$

Eq. 28 can be used as an ultimate strength formula and will provide the desired variation in factor of safety. The relation between Eqs. 21 and 28 is

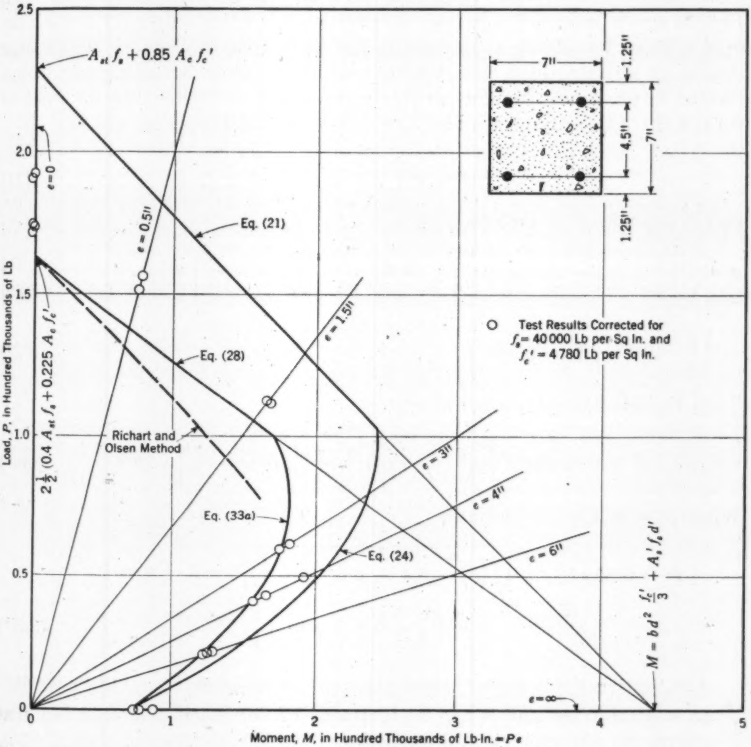

FIG. 12.—TEST RESULTS, SERIES F E, CORRECTED FOR f_s = 40,000 LB PER SQ IN. AND f_c' = 4,780 LB PER SQ IN.

shown by Fig. 12, assuming: d = 5.75 in.; d' = 4.5 in.; $A_s = A_s'$ = 0.392 sq in.; f_s = 40,000 lb per sq in.; f_{cu} = 6,250 lb per sq in.; and f_c' = 4,780 lb per sq in.

The allowable axial load for rectangular tied columns, according to standard practice, is

$$P = 0.8\,(0.4\,A_{st}\,f_s + 0.225\,A_c\,f_c') \ldots\ldots\ldots\ldots\ldots (29)$$

and the allowable eccentric load when the reinforcement is symmetrical may be obtained by multiplying Eq. 28 by $\frac{0.8}{2.5}$ which gives

$$P = \frac{0.32\,A_{st}\,f_s}{\dfrac{2\,e}{d'} + 1} + \frac{0.18\,A_c\,f_c'}{\dfrac{1.69\,t\,e}{d^2} + 1} \ldots\ldots\ldots\ldots\ldots (30^*)$$

Eq. 30 covers the case of the rectangular tied column when compression controls. When the eccentricity is so large that the tension in the steel controls the load, Eqs. 24 can be modified to provide an additional factor of safety for the smaller eccentricities by adding a constant to the eccentricity. Because of the large strains at ultimate load, the deflection of the column (assuming the length limited to ten times t) might possibly increase the eccentricity by as much as $0.1\,t$, and if this amount is added to e used in the derivation of Eq. 24*a*,

$$M = P\left(e + \frac{t}{10} + \frac{d'}{2}\right) \ldots\ldots\ldots\ldots\ldots (31)$$

and it becomes: $P = 0.85\,t\,b\,f_c' \times$

$$\times\left\{\sqrt{\left[\frac{e}{t} - 0.4 + (p - p')\,m\right]^2 + \frac{2\,d'}{t}\,p'\,m + (p - p')\,m\left[\frac{d'}{t} + 1 - (p - p')\,m\right]} - \left[\frac{e}{t} - 0.4 + (p - p')\,m\right]\right\} \ldots\ldots (32^*)$$

Then Eq. 24*b* becomes, when $p' = p$—

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.4\right)^2 + \frac{d'}{t}\,p_t\,m} - \left(\frac{e}{t} - 0.4\right)\right\} \ldots (33a^*)$$

When $p' = 0$, Eq. 24*c* becomes

$$P = 0.85\,t\,b\,f_c'\left\{\sqrt{\left(\frac{e}{t} - 0.4 + p\,m\right)^2 + \left(\frac{d'}{t} + 1 - p\,m\right)p\,m} - \left(\frac{e}{t} - 0.4 + p\,m\right)\right\} \ldots\ldots\ldots (33b^*)$$

The comparison of Eqs. 33*a* and 24*b* are shown by Fig. 12. The addition of $\frac{t}{10}$ to e is arbitrary and it can be increased or decreased if desired but that amount appears to give good results as compared with tests. Eqs. 25 and 26 are only used for large eccentricities and need no reduction.

The writer has compared[6] the ultimate loads computed from Eqs. 28 and 33*a* with the actual loads found in tests made by Messrs. Richart and Olson[29] and F. G. Thomas.[30] The test results by Mr. Thomas are generally lower than the values computed from Eqs. 21 and 24*b* but higher than Eqs. 28 and 33*a*. This is consistent with his tests on axially loaded columns in the same series and the low strength appears to be due to the design of his test specimens. The reinforcing steel did not extend to the bearing plates but was embedded in enlarged ends which were smaller than those that proved unsatisfactory in the Illinois and Lehigh tests.[31] The test results indicated a slipping of the compression steel. The results of one series of tests are plotted in Fig. 12.

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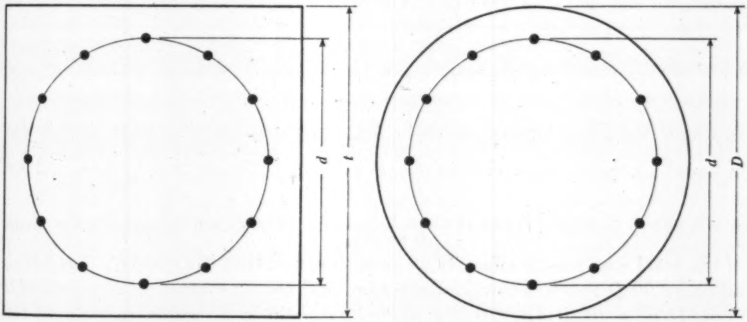
[29] "The Resistance of Reinforced Concrete Columns to Eccentric Loads," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1938, p. 401 (see especially Tables 2 and 4).

The results of the Richart and Olson tests[29] on 10-in. square columns also lie between the two sets of equations, but for some reason the axially loaded columns also failed to develop their expected strength. Messrs. Richart and Olson stated:[32]

> "Comparing the axially loaded columns of this series with previous tests in which 'flat-ended' columns were used, it appears that these 'hinged-ended' columns are weaker by roughly 15 per cent. It seems reasonable that the eccentrically loaded columns of this series are similarly weaker than columns rigidly framed into other members of a building frame."

COLUMNS WITH ROUND CORES

The writer has derived formulas for the strength of round cored columns and they will be given herein, although further tests may show that the empirical constants can be improved.



(*a*) SQUARE COLUMN (*b*) ROUND COLUMN

FIG. 13.—COLUMNS WITH ROUND CORES

Square Columns.—When, in a square column (Fig. 13(*a*)), the longitudinal reinforcement is distributed around the circumference of a circle with a diameter *d*, Eqs. 28 and 33 can be adapted by substituting for the value of d' the equivalent effective steel distance in terms of d. The equations seem to agree quite well with tests when $0.67\,d$ is used, and assuming half the steel effective on each side of the section, Eq. 28 becomes, for ultimate load producing compression failure:

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[30] "The Strength and Deformation of Reinforced Concrete Columns Under Combined Direct Stress and Bending," by F. G. Thomas, *Studies in Reinforced Concrete No. VI*, Building Research Station, London; see also, *Concrete and Constructional Engineering*, March, 1938, p. 165.

[31] "First Progress Report on Column Tests at Lehigh University," by W. A. Slater and Inge Lyse; and "Second Progress Report on Column Tests at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 27, 1931.

[32] "The Resistance of Reinforced Concrete Columns to Eccentric Loads," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1938, p. 406.

$$P = \frac{A_{st} f_s}{\dfrac{3\,e}{d} + 1} + \frac{0.85\,A_c f_c'}{\dfrac{10.2\,t\,e}{(t + 0.67\,d)^2} + 1.51} \quad \text{(34}a^*\text{)}$$

With a factor of safety of 2½ Eq. 34*a* approaches $P = 0.4\,A_{st} f_s + 0.225\,A_c f_c'$ as e approaches zero. Making the same substitution in Eq. 33, it becomes, for ultimate load producing tension failure:

$$P = 0.85\,t^2 f_c' \left[\sqrt{\left(\frac{e}{t} - 0.4\right)^2 + 0.67\,\frac{d}{t}\,p_t\,m} - \left(\frac{e}{t} - 0.4\right) \right] \quad \text{(34}b^*\text{)}$$

Round Columns.—To adapt Eq. 34*a* for compression failure to the case of the round column with round core, the effective depth of the concrete section, t, can be changed to $0.8\,D$ and Eq. 34*a* becomes

$$P = \frac{A_{st} f_s}{\dfrac{3\,e}{d} + 1} + \frac{0.85\,A_c f_c'}{\dfrac{8.16\,D\,e}{(0.8\,D + 0.67\,d)^2} + 1.51} \quad (35)$$

For the case of tension failure, a new equation can be derived in the same manner as Eq. 24*b*. For this purpose the distance from the center line of the section to the center of gravity of the area of concrete in compression can be expressed with sufficient accuracy as

$$\bar{x} = 0.211\,D + 0.293\left(0.785\,D - \frac{2\,A}{D}\right) \quad (36)$$

in which A is the compression area. Assuming that the total steel compression is equal to the total steel tension: $P = 0.85\,A\,f_c'$; or, $A = \dfrac{P}{0.85\,f_c'}$. Then $\bar{x} = 0.211\,D + 0.293\left(0.785\,D - \dfrac{2\,P}{0.85\,D\,f_c'}\right)$. Then assuming that four tenths of the total steel area is effective on each side, and that the effective d' is $0.75\,d$, adding $0.09\,D$ to e to allow for deflection, and taking moments: $P\,(e + 0.09\,D + 0.375\,d) = (0.40\,A_{st} \times 0.75\,f_s\,d) + P\,(\bar{x} + 0.375\,d)$ which reduces to the following equation for the ultimate load causing tension failure in a round column with a round core:

$$P = 0.85\,D^2 f_c' \left[\sqrt{\left(\frac{0.85\,e}{D} - 0.3\right)^2 + \frac{d\,p_t\,m}{2.5\,D}} - \left(\frac{0.85\,e}{D} - 0.3\right) \right] \quad (37^*)$$

Although some of the assumptions used in deriving these equations may be questioned as being contrary to present conceptions based on the elastic theory, they may nevertheless be sufficiently close to the true conditions at failure so that the simple and convenient form of the equations will prove satisfactory in practice. A comparison[6] with the tests reported by Messrs. Richart and

Olson[29] indicates that this is true, although the writer feels that sufficient tests have not been made to show whether or not the empirical constants in the equations can be improved. The equations appear to be more convenient than any others that have yet been proposed.

METHOD OF DESIGN AND FACTOR OF SAFETY

The foregoing equations provide a simple and consistent treatment for the full range of conditions from axially loaded columns to beams under flexure alone. Designers who follow current methods are inconsistent in applying the plastic theory to axially loaded columns, and at the same time attempting to combine it with the theory of elasticity as soon as the eccentricity is introduced. Messrs. Richart and Olson[29] suggest that design of eccentrically loaded columns for the range of eccentricities with e less than $0.25\,t$ should be based on the transformed section modulus of the uncracked section, involving the modular ratio. They propose the use of the standard straight-line stress formulas for large eccentricities. For the allowable unit stress, they propose an empirical variation from $0.225\,f_c'$ for a concentric load to $0.4\,f_c'$ for larger eccentricities. Fig. 12 shows the relative strength computed by their method.

The method suggested by Thomas[30] using the present formulas based on the straight-line no-tension theory is similar to the method proposed by Messrs. Richart and Olson. Thomas introduces an empirically varying allowable concrete stress only in the range between $e = 0$ and $0.5\,d$. As applied to the full range of eccentricities such a method merely forms an empirical transition between the standard plastic treatment of the axially loaded column and the standard straight-line working stress treatment for beams, which is not as realistic nor as simple as the method proposed by the writer.

The equations of this paper, except Eq. 30, are ultimate strength equations. They are not intended to be used to predict stresses under working loads.

Actual stresses under working loads are affected by so many conditions that they are quite indeterminate. It has been proved[33] that shrinkage and plastic flow entirely upset stress computations based on the elastic theory but they have little effect on ultimate strength because before failure the strains are great enough to cause redistribution of stress.

This is also true of pre-stressed reinforcement which has been shown to have no appreciable effect on ultimate strength although it does increase the load at which tension cracks appear in the concrete.[34]

The factor of safety should obviously depend on the nature of the structure and its loading. It may be desirable to provide different factors for different kinds of loads in some cases. Reasonably well-controlled concrete is such a reliable material at present that the use of any factor of safety to provide for sub-standard concrete can be eliminated by using f_c' as the minimum cylinder strength instead of the mean. With care, sufficient concrete strength can readily be provided and the concrete can be placed on the same design basis as steel.

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[33] "Effect of Time Yield in Concrete Upon Deformation Stresses in a Reinforced Concrete Arch Bridge," by W. M. Wilson, M. Am. Soc. C. E., and R. W. Kluge, Assoc. M. Am. Soc. C. E., *Bulletin No. 275*, Eng. Experiment Station, Univ. of Illinois, 1935.

[34] "Cracking in Reinforced Concrete," by F. G. Thomas, *The Structural Engineer*, July, 1936, p. 298.

If the minimum fatigue strength of reinforced concrete members, under load repeated thousands of times, is more than 50% of the ultimate static strength as indicated by tests, it will be still higher under loads repeated as infrequently as the maximum loading conditions in an ordinary structure. As a general proposition it appears that allowable dead plus live loads could be four tenths of the ultimate loads; and, for extreme combinations of dead and live loads, and wind, temperature, and shrinkage effects, one half of the ultimate might not be excessive. This is premised on the use of the elastic theory (and not the principle of moment redistribution based on yielding) to determine moments and thrusts so that the actual forces will not exceed the useful limit of the strength of the member. Each case should be given careful consideration and it is obvious that the designer should have as correct an understanding as possible of the true action of structures.

There is one important peculiarity of members subjected to both bending and direct load which has not been properly recognized. As can be seen from Fig. 12, when the tensile strength controls (Eq. 24*b*), the bending moment that can be resisted increases with the direct load. Therefore, it is possible for a reduction of direct load without change in moment to cause failure. It may not be safe, therefore, to consider that in case of overload, both the direct thrust and the moment will increase proportionately. It is obvious that in the case of a bridge arch, where the thrust is due largely to dead load and the bending moment due to live load, an increase in bending is much more probable than an increase in thrust. The use of ultimate strength formulas makes it possible to design a member for any combination of multiples of the thrust and moment. Instead of using a simple factor of safety as for an axially loaded column or a beam, in the case of a member subjected to both direct load and bending, consideration should be given to probable combinations to which the member might be subjected. A further analysis of the desirable factor of safety of arch ribs has been made by the writer.[23]

COMPARISON OF PLASTIC THEORY WITH THE STANDARD THEORY

The plastic theory proposed by the writer for the design of concrete members is based on three assumptions: (1) No tension in the concrete; (2) linear variation of strains; and (3) a simple equivalent stress block which closely approximates the effect of the real distribution at maximum load. It differs from the ordinary straight-line stress distribution theory only in that it recognizes the plastic action of concrete at failure instead of attempting to predict the stresses at working loads.

The objections that may be raised to the adoption of the plastic theory are about as follows: (*a*) It is unfamiliar and untried; (*b*) it is an ultimate strength method instead of a working stress method; (*c*) the present method has given satisfactory service for years and could be modified to permit higher stresses if they are justified; and (*d*) it may lead to the use of beams and slabs that are too shallow and cause excessive deflections.

Each of these ideas will be discussed in order, as follows:

(*a*) The application of the plastic theory is much simpler than the present method, and it is as well supported by tests as the latter. In fact, its closer agreement with tests is one of its greatest advantages. It can be taught as easily as the present method and will lead designers to a better understanding of the actual structural action.

(*b*) In spite of the designer's familiarity with the working-stress method of design and his preference for it, the ultimate-strength method is superior if it is a better guide to the useful strength of members and provides a more consistent factor of safety against damaging overload. Tests indicate that the useful strength bears a more definite relation to the ultimate strength than to theoretical working stresses given by the standard formulas.

(*c*) The present method has been useful during a time when knowledge and experience with reinforced concrete were being accumulated. It provides, in some cases, what is now an excessively high factor of safety against concrete failure. It leads to uneconomical design and inconsistent factor of safety. It is actually based on an empirical value of $\frac{E_s}{E_c}$ which could be modified together with the allowable stresses to give practically the same results as the new theory as far as the resisting moment of the concrete in compression is concerned. This would lead to the use of nominal working stresses which would appear unreasonably high; but, more serious than that, as previously stated, the method does not give as satisfactory values of the steel lever arm as the new method. This would lead to the use of more steel in lightly reinforced beams and a reduction in factor of safety against steel failure in heavily reinforced beams as compared with the new method. Table 7 shows the average unit concrete stress, at failure, of the test beams reported by Messrs. Slater and Lyse,[15] as computed by seven different methods. The last column indicates the consistency with which the formula might predict the results of the tests if the correct unit stresses were used in the formula.

TABLE 7.—COMPARISON OF CYLINDER STRENGTH WITH MAXIMUM UNIT STRESS IN OVER-REINFORCED BEAMS

Item No.	Value of *n* in the formula	Ratio[a] $\frac{f}{f_c'}$	Mean variation from the average
	By the Straight-Line Formula:		
1	American Concrete Institute, 1928[b]	1.57	±0.045
2	Joint Committee, 1924[c]	1.50	±0.059
3	Actual	1.62	±0.075
	By the Parabolic Formula:		
4	American Concrete Institute, 1928[b]	1.10	±0.034
5	Joint Committee, 1924[c]	1.07	±0.041
6	Actual	1.05	±0.035
7	By the plastic theory, Eq. 12[d]	1.05	±0.037

[a] Computed unit stress divided by the cylinder strength.

[b] "Reinforced-Concrete Building Regulations and Specifications," *Proceedings*, Am. Concrete Inst., Vol. XXIV, 1928, Section 601.

[c] "Standard Specifications for Concrete and Reinforced Concrete," Report of Joint Committee, *Proceedings*, Am. Soc. C. E., October, 1924, Section 103.

[d] Ratio for Item 7 is the computed unit stress divided by 0.85 f_c'.

It is interesting to note that the straight-line formula with the actual value of *n* (Item 3, Table 7) gave the poorest results, and that the parabolic formula

(Item 6) and the plastic formula (Item 7) gave the best results. The plastic theory appears more desirable than a modified straight-line theory.

(*d*) The new formulas permit the use of more slender members, which is an advantage in structures for which dead weight and temperature or shrinkage moments are important. It will also be beneficial at the haunches of T-beams. It is always important to guard against excessive deflections, no matter what method is used, and it is always necessary to design carefully for shear and bond. It will not generally be found economical to use maximum percentages of steel in beams and slabs; but the designer will be given more freedom in design, and, having simpler tools with which to work, he can pay more attention to other important considerations.

APPENDIX

DESCRIPTION OF TESTS PLOTTED IN FIG. 7

The tests by Messrs. Humphrey and Losse[18] were made on 333 8-in. by 11-in. beams of gravel, limestone, granite, and cinder concrete varying in strength from 1,500 to 5,700 lb per sq in. Reinforcement varied from 0.49% to 1.96% of steel with a yield point averaging about 40,000 lb per sq in. Of the total, 84 beams are not plotted in Fig. 7 because the concrete cylinder strength exceeds the testing machine capacity and, therefore, f_c' was not determined. The cylinders tested are 8 in. by 16 in. and f_c', the strength of 6-in. by 12-in. cylinders, is calculated as $\frac{100}{95}$ times the reported value. Each point plotted represents the average of three beams.

The U. S. Geological Survey Tests[35] were made at St. Louis, Mo., in 1908 on a series of 8-in. by 11-in. beams of gravel concrete with 3.02% of steel with an average yield point strength of 38,210 lb per sq in. Only beams with concrete strengths greater than 2,000 lb per sq in. are included, and because individual steel strengths are not reported, only the averages are plotted for two groups of eleven beams each, one under-reinforced and one over-reinforced. The concrete cylinders were 8 in. by 16 in. and f_c' is calculated to be $\frac{100}{95}$ times the reported strengths.

The Columbia University tests[36] were made on thirteen rectangular 12-in. by 13-in. beams reinforced with ordinary steel and twin twisted bars with yield points from 34,800 lb per sq in. to 58,250 lb per sq in. and areas varying from 0.451% to 1.245%. The concrete strengths varied from 2,510 to 3,443 lb per sq in. The yield point of the twin twisted bars was taken as the stress producing a unit strain of 0.004.

The tests by Messrs. Slater and Lyse[37] are those previously quoted. The four sets excluded from the averages in Table 1 are not plotted. Each point represents the average of a group of three beams.

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[35] *Proceedings*, Am. Concrete Inst., Vol. 16, 1920, p. 127.

[36] *Concrete and Constructional Engineering*, March, 1937, p. 200; *Journal*, Am. Concrete Inst., November–December, 1936; and *Proceedings*, Am. Concrete Inst., Vol. 33, p. 183.

[37] *Journal*, Am. Concrete Inst., Vol. 33, p. 488.

DISCUSSION

L. E. Grinter,[38] M. Am. Soc. C. E. (by letter).—The procedure for the structural design of reinforced concrete that is proposed in this paper is certain to receive repeated and probably favorable consideration in the coming years. There is reason to question if the knowledge of the action of reinforced concrete frames under ultimate loads is adequate today to answer the question as to whether the author's procedure is logical in all cases; and, of course, the proposed method of allowing for plasticity is but one of many possible procedures. It is true, however, that designers have already taken certain steps in this direction—notably, the adoption of a column formula that omits consideration of relative moduli of elasticity by basing load capacity upon the yield strength of the steel. On the other hand, this step was a much more obviously acceptable one in that high steel stresses in columns are known to exist as a normal condition even under ordinary working loads. Elimination of the term n in present formulas for the design of reinforced concrete beams presupposes flow beyond conditions ordinarily normal in concrete structures.

The author's castigation of the use of the ratio n in reinforced concrete design nevertheless seems justified by many recent findings. It is true that measured strains in concrete cannot be transferred into stresses of great significance, and yet one cannot question that the stress corresponding to a given strain will bear a relation, however complex, to the modulus of elasticity. The directness of this relationship may be expected to reduce as the load is increased beyond working conditions. As shown clearly by Fig. 1, the straight-line relationship between stress and strain may be expected to exist up to one half of the ultimate strength, and engineers do not plan to design structures for working stresses of even this magnitude. On the other hand, Fig. 1 does not express the stress-strain relationship as it is influenced by time yielding since this influence seems largely restricted to dead-load conditions.

Evidently, then, the acceptability of any method of design by means of a theory of plasticity should be studied in the light of one's belief in design for failure. After most careful study, the writer has concluded that, in many instances, a steel structure can be designed far more logically if failure is accepted as the basis. By failure is meant a condition of deformation dependent upon the passing of the elastic limit, not necessarily a condition of exaggerated distortion. Similarly, in reinforced concrete design, it would seem logical to design the structure like the famous "shay" so that it would fail simultaneously in all of its parts. A criterion, then, is whether the proposed plastic theory will achieve this condition more closely than use of the ordinary straight-line formulas for the design of reinforced concrete beams.

In order that a continuous frame may fail simultaneously at many places (and nearly all concrete structures are continuous frames of some type or other), the designer must not only consider plasticity in the design of the individual sections but also plasticity for the distribution of moments. This is precluded in the author's procedure by his belief that "* * * the theory of elasticity is the best guide to the forces acting on the different sections." It is precluded in the writer's opinion by the simple fact that engineers do not know enough about the influence of plasticity upon structural action to predict properly the influence of time yield or of plastic flow upon moments.

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[38] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

The other point that deserves consideration is the fact that the parts of a structural frame include columns, beams, slabs, and their joints. To be better than the present methods, a new theory must predict the failure of each of these parts more accurately than standard procedures, and this must be true even if concrete of different strengths (and different moduli of elasticity) are used in the several parts of the structure. In early attempts at analysis, the factor *n* was introduced consistently for columns, beams, and slabs, and perhaps within reasonable limitations designers achieved uniformity of action under light loads for which elastic conditions obtained. The realization, however, that column rods picked up additional stresses through time flow in the concrete spoiled this apparent consistency and has led the author to attempt a readjustment of beam design to parallel the now accepted method of column design. Some revision of beam design seems desirable, one reason being that the factor of safety is probably greater in beam design than in column design by present methods. A problem for consideration is that present column formulas are justified by time yielding, whereas the proposed beam formulas depend upon plastic flow near failure. Evidently these are quite different conditions.

As was mentioned earlier, there are many ways that plasticity can be introduced into design. One procedure would be to accept the parabolic stress distribution curve seriously proposed in the early days of reinforced concrete. The author's rectangular stress pattern, being non-physical in its aspects, can be justified only by simplicity, and, actually, it does not seem to simplify, greatly, the common methods of design. Hence, although the writer agrees with the probable desirability of considering failure as a criterion for design, he finds it difficult to justify a procedure resulting in an entirely artificial stress-distribution relationship for the concrete above the neutral axis. It seems unfortunate that one dimension of the stress diagram does not represent the maximum concrete stress and that the other dimension is not associated with the depth of the beam in a physically significant manner.

It must not be forgotten, when theories are proposed for introduction into standard codes, that these theories must be presented to undergraduate students. It is always feasible to teach facts that may be substantiated in any reasonable way. It is also possible to present hypotheses that have some reasonable connection with observed phenomena and to develop theories from such hypotheses. It is simply not reasonable, however, to ask students to accept theories that do not follow physically observable phenomena or even the accepted hypothesis as to physical action. An example of this is the possible use of a stress diagram for compression in the concrete that is inconsistent with the hypothesis of a straight-line variation of strain throughout the depth of the beam. Such an inconsistency would militate against the logical use of any formulas that might be developed, irrespective of their simplicity or other desirable features.

The author's paper will serve a most valuable purpose in pointing out the present inconsistencies in the design of reinforced concrete structures. His proposal will call forcefully to the attention of structural engineers the desirability of basing design procedures for all members of a structure upon a single hypothesis. The importance of plasticity both as an influence upon the stress distribution at a section and upon the moment distribution throughout the structure will be reweighed as to its importance. Time yielding at low loads will be reconsidered and compared in its significance with plastic flow beyond normal working stresses. The many possibilities of stress distribution over the beam cross section may be reconsidered again to advantage. Finally, the purpose to be achieved by any theory of stress analysis that is known to be far from exact in its representation of conditions in a material that is neither homogeneous nor even continuous must be re-evaluated. The author's excellent paper will bring each of these matters again to the attention of structural engineers.

Basil Sourochnikoff,[39] Esq. (by letter).—In order to simplify computations, Mr. Whitney replaces the curvilinear stress distribution with a rectangular stress block. This appears to be quite satisfactory, as the results of formulas obtained by using this approximation check very closely with experimental data. However, it may be of interest to establish formulas based directly on the curvilinear stress distribution without using any simplifying device. Of course, these formulas cannot be expected to be more exact than the stress-strain curve on which they are based. A comparison of results thus concluded with the experimental data may show just how closely the idealized stress-strain curve represents actual conditions.

Under-Reinforced Beams.—From the conditions of equilibrium of any cross section it follows that

$$\kappa = \frac{f_s'}{f_c'} \times \frac{p}{A} \ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

and

$$\frac{M}{f_c'\,b\,d^2} = \kappa\,A\,[1 - \kappa\,(1 - \bar{x})] = \frac{f_s}{f_c'}\,p\left[\,1 - \frac{f_s'}{f_c'}\,\frac{p}{A}\,(1 - \bar{x})\,\right] \ldots . (38b)$$

in which, in addition to the author's notation, f_s' is the yield stress in steel. When the steel is strained above the yield point, f_s' remains practically constant and independent of ϵ'. Then, for each particular beam, the resisting moment will vary with $\frac{1 - \bar{x}}{A}$ alone. The maximum resisting moment will correspond to the minimum of $\frac{1 - \bar{x}}{A}$. This last quantity, for each given grade of concrete, depends on the stress curve only. For curves of Fig. 3:

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[39] Registered Structural Engr., St. Paul, Minn.

$1{,}000\,\epsilon$	$\frac{1 - \bar{x}}{A}$
1.0	0.710
1.5	0.585
2.0	0.540
2.5	0.538
3.0	0.538
3.5	0.543
4.0	0.575
4.5	0.602
5.0	0.638

Therefore, if the steel is overstressed, the resisting moment increases until the strain in the concrete reaches a value of about 0.00275. Then it would decrease if the strain exceeds this value. The ultimate strain in the concrete is therefore about 0.00275 and does not depend on the percentage of steel. The ultimate resisting moment is determined from the expression:

$$\frac{M}{f_c'\, b\, d^2} = \frac{f_s'}{f_c'}\, p \left[1 - 0.538 \frac{f_s'}{f_c'}\, p \right] \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

which is of the same form as Eq. 7*b*. The coefficient $\frac{m}{2}$ in Eq. 7*b* is $\frac{f_s'}{2 \times 0.85\, f_c'} = \frac{0.59\, f_s'}{f_c'}$. For $f_c' = 4.000$, $f_s' = 6.000$ and $p = 3\%$. This means a difference of only 3.2% on the ultimate moment. Assume that, at the yield point, the steel is strained to 0.002. Since, in the present case, the steel is overstressed, $\epsilon' > 0.002$. Then, if the concrete fails after the steel is overstressed, failure occurs at $\frac{\epsilon'}{\epsilon} > \frac{0.002}{0.00275}$ or 0.73. From Eq. 38*a* this is possible only if $p < \frac{0.46\, f_c'}{f_s'}$. Therefore, in the present case, it seems that $\frac{0.46 \times 4{,}000}{60{,}000}$ or 3% is the limiting ratio of reinforcement. The beam would be under-reinforced if $p < 3\%$, and over-reinforced if $p > 3\%$. This again checks with results obtained by Mr. Whitney.

Over-Reinforced Beams.—From the conditions of equilibrium it follows again that:

$$f_c'\, A \frac{\epsilon}{\epsilon + \epsilon'} = E\, \epsilon'\, p \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Therefore,

$$\frac{\epsilon'}{\epsilon} = \tfrac{1}{2} \left(\sqrt{1 + \frac{4\, f_c'\, A}{E\, p\, \epsilon}} - 1 \right) \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

and

$$C = \frac{M}{f_c'\, b\, d^2} = A\, \kappa \left[1 - \kappa \left(1 - \bar{x} \right) \right] \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Using again the curves of Fig. 3, the constants for different steel ratios are given in Table 8. This shows that the ultimate resistance occurs for $\epsilon = 0.003$, approximately. The quantity $C = \dfrac{M}{f_c'\, b\, d^2}$ depends on p and increases asymptotically to 0.455. This result is at variance with that obtained by Mr. Whitney, as it seems to show that for over-reinforced beams the ultimate resistance does increase with the steel ratio, although the rate of increase is slow.

TABLE 8.—CONSTANTS FOR VARIOUS STEEL RATIOS

$1{,}000\,\epsilon$	STEEL REINFORCEMENT (FOR $p = \infty$—$\epsilon' = 0$; $\kappa = 1$; AND $C = A\,\bar{x}$)									
	3%			4%			5%			∞
	$1{,}000\,\epsilon'$	κ	C	$1{,}000\,\epsilon'$	κ	C	$1{,}000\,\epsilon'$	κ	C	C
1	1.105	0.474	0.203	0.905	0.525	0.220	0.78	0.563	0.233	0.330
2	1.75	0.535	0.310	1.44	0.582	0.330	1.21	0.624	0.346	0.442
3	2.10	0.590	0.352	1.71	0.638	0.370	1.44	0.678	0.384	0.455
4	2.28	0.640	0.362	1.84	0.686	0.375	1.56	0.720	0.385	0.431
5	2.35	0.681	0.348	1.86	0.730	0.362	1.60	0.760	0.368	0.392

It should be reemphasized that the numerical results cannot be more exact than the stress-strain curve from which they are derived.

R. W. STEWART,[40] M. AM. SOC. C. E. (by letter).—Many tests cited by the author, and other published tests not cited,[41] show that the concrete stress in a reinforced concrete beam is substantially lower at rupture than the value computed by the straight-line stress distribution assumption that is used almost universally for design purposes. This proves that the tensile steel is the weak partner and that by increasing the steel percentage a beam of given thickness can be made suitable for carrying substantially heavier loads. The author makes this situation very clear and proposes a method of analysis which he demonstrates to give accurate results. This method eliminates the modular ratio n that is recognized as an arbitrary factor if the stress-strain ratio for concrete does not plot as a straight line or some known mathematical curve. However, factor n can be eliminated only by the introduction of some other arbitrary device, and the author has selected the rectangular stress block, which he describes.

Flexure formulas for ultimate loads have been published heretofore on the assumption that the stress deformation curve for the concrete is a parabola.[42] The author's investigation shows, however, that a parabola of higher order than the second degree must be used to represent the stress distribution in the concrete in order to compute with accuracy the ultimate resisting moment of a beam. Table 9(a) indicates a sequence of beam sections, each of which has the proper amount of tensile steel to fix its neutral axis at the same percentage of its depth as the beam analyzed by the author and illustrated by Fig. 2. Item 4 shows the relative moment-resisting capacity of each of the beam sections. It is to be noted that the ideal curve for concrete stress distribution, presented by the author, will permit a beam, if sufficiently reinforced, to carry 47% more load than if designed by the usual straight-line formula. It may be noted also that using the fourth-degree parabola will give very nearly the same

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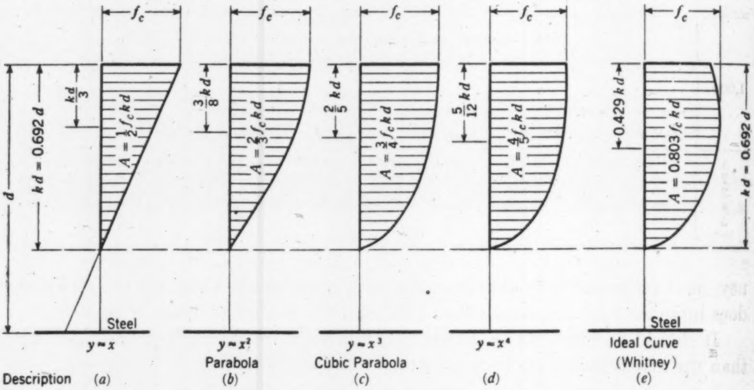
[40] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[41] "Flexural Resistance of Shallow Concrete Beams," by Conde B. McCullough, M. Am. Soc. C. E., *Engineering News-Record*, September 19, 1935, p. 406.

[42] "Concrete Engineers' Handbook," by George A. Hool and Nathan C. Johnson, McGraw-Hill Book Co., Inc., New York, N. Y., 13th printing, p. 279.

TABLE 9.—COMPARISON OF SLAB DESIGNS
(f_s = 18,000; f_c = 1,000; Slab Thickness = d + 2 In.; d = 10 In.)


f_c
f_c
f_c
f_c
f_c
d
$kd = 0.692\,d$
$\frac{kd}{3}$
$A = \frac{1}{2} f_c kd$
$\frac{3}{8} kd$
$A = \frac{2}{3} f_c kd$
$\frac{2}{5} kd$
$A = \frac{3}{4} f_c kd$
$\frac{5}{12} kd$
$A = \frac{4}{5} f_c kd$
$0.429\,kd$
$A = 0.803\,f_c kd$
$kd = 0.692\,d$
Steel
Steel
$y = x$
$y = x^2$
Parabola
$y = x^3$
Cubic Parabola
$y = x^4$
Ideal Curve
(Whitney)
Description
(a)
(b)
(c)
(d)
(e)

No.	Description	Straight line (a)	Parabola (b)	Cubic parabola (c)	Fourth-degree parabola (d)	Ideal curve (Whitney) (e)
	(a) PHYSICAL CHARACTERISTICS; UNIFORM VALUE, k = 0.692					
1	$C/(f_c\,k\,d)$	0.500	0.667	0.750	0.800	0.803
2	$j\,d$, in inches	7.69	7.40	7.23	7.11	7.03
3	$M/(f_c\,d^2)$	0.266	0.341	0.375	0.394	0.391
4	Ratio	1.0	1.28	1.41	1.48	1.47
5	d, in inches	10.0	10.0	10.0	10.0	10.0
6	k	0.692	0.692	0.692	0.692	0.692
	(b) COST COMPARISONS; MOMENT, IN ALL CASES, 26,600 FT-LB PER FT OF WIDTH; d VARIABLE					
7	Slab thickness, in inches	12.0	10.83	10.42	10.22	10.25
8	$j\,d$, in inches	7.69	6.54	6.09	5.85	5.80
9	Area of steel, A_s, in sq inches	2.31	2.72	2.92	3.04	3.06
	Steel Reinforcement:					
10	Ratio	0.0192	0.0227	0.0243	0.0253	0.0255
11	Weight of steel in lb per ft of beam	7.85	9.25	9.93	10.33	10.40
	Dollar Costs:					
12	Steel (at 5 cents per lb) per ft of beam	0.392	0.462	0.496	0.516	0.520
13	Concrete (at $16 per cu yd) per ft of beam	0.593	0.535	0.515	0.505	0.506
14	Totals, per ft (concrete at $16 per cu yd)	0.985	0.997	1.011	1.021	1.026
12	Steel (at 5 cents per lb) per ft of beam	0.392	0.462	0.496	0.516	0.520
15	Concrete (at $24 per cu yd) per ft of beam	0.889	0.802	0.772	0.757	0.759
16	Totals, per ft (concrete at $24 per cu yd)	1.281	1.264	1.268	1.273	1.279

result. The cubic parabola, with a small error on the side of safety, appears to the writer to be a suitable curve to use. It is pictorially superior to a rectangular stress block in representing the actual condition of fiber stress and involves less departure from established procedure.

A comparison of costs of slabs designed by the proposed method and by the straight-line formula was not included in the paper. Such data are presented in Table 9 (*b*), which shows the make-up of a series of slabs, each one designed to resist the same bending moment using the same fiber stresses. The differences in slab thicknesses and percentage of reinforcing steel are due to the different assumptions as to stress distribution as shown by Table 9 (*a*). It is noted that, although the author's proposed concrete stress distribution and the higher order of parabolic distributions show saving in concrete, the increased cost of the additional steel required will more than offset this saving so that some special inducement to use thin slabs is required in order to justify, economically, the abandoning of the straight-line theory of concrete stress distribution. The cost of forms is properly excluded from the cost of concrete in this comparison, so it is seen that, unless the unit cost of the concrete is considerably higher than the usual bid prices, the straight-line stress distribution formula will give the most economical result.

It may be noted also that Fig. 1 indicates that the stress-strain graph for the tests of concrete reported appears as a straight line up to a unit stress of 2,000 lb per sq in.

GEORGE C. ERNST,[43] ASSOC. M. AM. SOC. C. E. (by letter).—The rational approach to the design of reinforced concrete members, as presented in this paper, has interested the writer since its introduction in 1937.[5] The straightforwardness shown in the concept of the action of the two combined materials is refreshing and should be welcomed by engineers engaged in the structural field.

It is important to note in Fig. 12 that the results of the series of tests by F. G. Thomas,[30] in which high-strength mixtures were used, do not conform to the proposed formulas although the tests of Bach and Graf[24] with a medium-strength mixture show good agreement. The Thomas tests on specimens of low-strength concrete also agree well with the formulas as shown in Table 10, but the author has suggested that the lack of agreement shown in Fig. 12 for the higher strength concrete appeared to be due to the use of enlarged ends on the specimens and slipping of the compression steel. (The use of the minimum concrete strength in Table 10 is consistent with the author's suggestion under "Method of Design and Factor of Safety." Manipulations with average, minimum, nominal, or maximum concrete or steel strengths, however, do not affect the relative positions of the groups.) The design of the ends of the specimens in the low-strength group, however, was the same as for the high-strength group. It is apparent also that the lack of agreement is just as evident for the larger eccentricities where the effect of the enlarged ends and slipping of the compression steel are virtually negligible. It is inconceivable to the writer that the design of the eccentrically loaded specimens affected the results to any noticeable degree unless it is due to the smallness of the cross section (about one sixth the area of the Bach and Graf test columns) when used with the higher strength mixtures. In this connection it is to be noted that the 10-in. square test columns of Richart and Olson had an area of only about one third of the Bach and Graf specimens. Although Messrs. Richart and Olson suggest the possibility that about a 15% reduction in the capacity of "hinged-ended" columns exists as compared to "flat-ended" columns, nevertheless the tests of Thomas fail to confirm this belief when enlarged ends are used. Since the effect of the enlarged ends should be essentially the same for "hinged" and "fixed" ended specimens, the tests by Professor Thomas are definitely inconclusive in this matter (see Table 10). With regard to the five points plotted in Fig. 12 on the ordinate $e = 0$, the lowest value and the next to the highest value are for "fixed-ended" conditions, whereas the remaining three are "hinged-ended." Two other "hinged-ended" columns, not shown would fall above the entire diagram although the concrete strengths of these

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TABLE 10.—COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS FOR COLUMNS TESTED UNDER AXIAL AND ECCENTRIC LOADS

Item No.	Ratio, $\frac{e}{t}$ (1)	Tests averaged (2)	ULTIMATE LOADS, IN POUNDS		Eq. No. (5)	Ratio, $\frac{\text{Col. 3}}{\text{Col. 4}}$ (6)
			Actual (3)	Computed[a] (4)		
SERIES I: $p = 0.0245$; $p' = 0.0045$; f_c' (AVERAGE) = 1,480; f_c' (MINIMUM) = 1,370; AND f_s (AVERAGE) = 45,600						
1	0	2	85,300	77,200	21	1.10
2	0[b]	4	105,600	77,200	21	1.37
3	0.143	2	54,450	51,100	21	1.06
4	0.357	2	41,700	34,100	21	1.22
5	0.715	2	22,900	21,900	21	1.05
SERIES II: $p = 0.0080$; $p' = 0.0080$; f_c' (AVERAGE) = 1,960; f_c' (MINIMUM) = 1,630; AND f_s (AVERAGE) = 41,900						
6	0	4	113,800	100,900	21	1.13
7	0[b]	2	113,900	100,900	21	1.13
8	0.071	2	86,500	81,400	21	1.06
9	0.214	2	66,100	59,000	21	1.12
10	0.571	2	30,300	33,400	24*b*	0.91
11	0.857	2	18,500	20,700	24*b*	0.90
SERIES III: $p = 0.0080$; $p' = 0.0080$; f_c' (AVERAGE) = 4,640; f_c' (MINIMUM) = 4,370; AND f_s (AVERAGE) = 43,700						
12	0	3	183,200	216,400	21	0.85
13	0	2	280,500	216,400	21	1.31[c]
14	0[b]	2	174,200	216,400	21	0.81
15	0.071	2	157,800	172,300	21	0.92
16	0.214	2	112,400	123,100	21	0.91
17	0.429	2	58,600	77,700	24*b*	0.75
18	0.571	3	43,700	51,900	24*b*	0.84
19	0.857	3	23,100	25,800	24*b*	0.90

[a] Minimum f_c' and average f_s used in each group. [b] Tested with fixed ends. All others tested with knife-edge end bearings. [c] The cylinder strength (converted) exceeded the average by 17.5%.

two were only 17.5% above the average of the group. Furthermore, it is difficult to attribute the lack of strength in the tests of Messrs. Richart, Olson, and Thomas to hinged ends if the Bach and Graf tests are considered as hinged-ended for the eccentrically loaded conditions.

All evidence points to a reduction in relative capacity due to the use of the higher strength mixtures, although it must be recognized also that the size of the specimens of Bach and Graf are comparable to actual construction, whereas those of Thomas are somewhat small. It would seem best to the writer to place a limit on the concrete strength at approximately 3,000 lb per sq in. for the proposed formulas until more conclusive tests have been conducted using the higher strength mixtures under combined bending and direct stress.

Although Table 6(*a*) is entitled "Rapid and Long-Time Tests," the frames subjected to long-time loading by Messrs. Richart and Olson are not tabulated and it would prove of value if the author would supply the missing information for the purpose of showing that the effect of sustained loading is negligible. The writer believes, also, that much of the value of Table 6(*a*) is lost unless the frame details are described briefly to show the effect of various sizes of fillets and chamfers. Such details raise a question as to the position of the critical section, and in so far as the writer has been able to determine it occurs, for these frames, just inside the fillets, whether circular or straight. Chamfering the back corner may bring the critical section into the corner if the effective section at the corner is reduced sharply, although this does not appear to be the case for frames 6 or 7.

TABLE 11.—COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS FOR RECTANGULAR FRAME TESTS

(Clear Opening, 4 Ft by 2 Ft; Third-Point Loading; Low-Strength Concrete Group; $f_c' = 1{,}510$; $f_s = 56{,}800$ at the Yield Point; Steel at Midspan = 1.47%; $p' = 0$)

No.[a]	Ratio, $\frac{e}{t}$	ULTIMATE LOADS (LB)		Ratio, $\frac{\text{Col. 2}}{\text{Col. 3}}$
		Actual	Computed	
	(1)	(2)	(3)	(4)
1[b,c]	4.50	10,200	9,100	1.12
2[c,d]	4.50	8,800	9,100	0.97
3[c,d,e]	2.67	4,800	4,960	0.97
4[d,f]	8.90	6,000	6,200	0.97
5[d,e,f,g]	2.67	4,800	4,960	0.97

[a] One test for each item. [b] No hinges. [c] Constant 6-in. depth and 3.5-in. width. [d] Bases hinged. [e] Hinged at crown. [f] Verticals varied from 6 in. at the knee to 3 in. at each base hinge. [g] Top horizontal varied from 6 in. at the knee to 3 in. at the hinges.

Tests on rectangular frames made at the University of Maryland in 1940 (Table 11) show good agreement with the proposed formulas, but it must be noted that the concrete mixture was of low strength. Tests upon frames very closely approximate actual conditions of restraint in the application of the eccentric load.

The important feature revealed by the comparison of the author's formulas with test results seems to be the need for additional information on rectangular and circular sections subjected to eccentricities from zero to pure bending and covering a wide range of concrete strengths. Until such additional tests are made it would be exceedingly difficult for the writer to justify the use of the proposed formulas without an upper limit of 3,000 lb per sq in. placed upon the concrete cylinder strength.

HOMER M. HADLEY,[44] ASSOC. M. AM. SOC. C. E. (by letter).—There are several matters concerned with the fundamentals of Mr. Whitney's proposed theory of design on which it is hoped additional information will be furnished.

(1) Fig. 1, from which the proposed theory is developed, shows six curves, one of which terminates at a unit strain of 0.0036 in. per in., a second at a unit strain of 0.005 in. per in,, whereas the remaining four are shown extending unbroken to the right-hand edge of the diagram. Standard 6-in. by 12-in. cylinders being used, the unit strain 0.005 in. per in. gives a total deformation of 0.06 in. The text states (see heading "The Stress-Strain Characteristics of Concrete") "the machine head was moving continuously at the rate of 0.06 in. per min." On this basis the range of stresses shown in the diagram occurred in slightly more than 1½ min. It is most immediate, practically instant, plasticity that is disclosed.

These are highly unusual stress-strain curves for concrete cylinders. Ordinarily the attainment of maximum stress is closely followed by failure. For a few seconds while it holds together the load drops off rapidly; then the cylinder shatters and breaks into fragments. In Fig. 1, however, the cylinders do nothing of the kind. After maximum stress has been attained (that is, after maximum load has been applied) they do not fracture but simply deform with reducing unit stress. It might almost be said that they are made to deform by taking load off of them, which, of course, is an improper statement. The truth is that they are made to deform uniformly and they offer sustained but decreasing resistance. Complete details regarding these tests and such anomalous behavior are most desirable.

(2) Under the foregoing heading is the statement that

> "The effect of time on the stress-strain relation is believed to be similar to that in the case of tests on beams. It is probable, therefore, that these curves indicate the form of compressive stress distribution in a beam at ultimate load in the concrete in the zone where the unit strain exceeds about 0.002 in. per in."

Considering that the cylinder test is one of direct stress over a full section, whereas the beam is a case of flexure with fibers at varying distances from the neutral axis; and considering the highly unusual character of the curves in Fig. 1, the writer hopes there will be further exposition of these statements. Has it definitely been "proved" or has it definitely been assumed "that the strain in the compression side of a beam increases practically in direct proportion to the distance from the neutral axis"?

(3) When the stress distribution in a beam has been explicitly stated to be "probably" like those of the cylinder curves of Fig. 1, why should the direct

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declaratory form of statement subsequently be used regarding this stress distribution—for example, in the second paragraph following Eq. 3*b*? The validity of an assumption is not established simply by its being postulated and formulated.

(4) Why was the particular form of stress curve shown in Fig. 4 selected? This curve appears to be identical with the curve of Fig. 2 which is definitely stated to be based on an ϵ value of 0.003. If it is assumed that the four curves of Fig. 1 do indeed terminate at the margin of the diagram at a unit deformation of 0.008 in. per in., then the average value of ϵ of the six curves would be 0.0068—say, 0.007. If this average value of the anomalous evidence were used, as apparently it might well be, then a different stress distribution in the beam would result with a different position of the center of gravity, etc. Of all the "fibers" (so-called) in a beam, the extreme topmost one is most advantageously placed with respect to the centroid of tensile stresses for developing resistance to external forces. In view of this fact is there not an inherent improbability in the assumption that a less advantageously placed fiber with a substantially lesser lever arm carries a higher stress than the topmost fiber? In the general economy of nature things do not work in this fashion.

This matter of so-called "plastic flow" in concrete is an annoying one. There is a nucleus of truth in the term. A certain slight amount of increased deformation does occur at early ages under sustained load; but how this is to be separated and distinguished from the deformation that occurs when concrete dries out and shrinks in plain, common every-day fashion is difficult to determine. The concrete knee-frame tests under sustained load, reported by Messrs. Richart and Olson,[25] had the specimens continuously wet until placed under load and it was while under load that they dried out. So likewise continuously wet until placed under load were the specimens of the A. C. I. column tests at Lehigh University, Bethlehem, Pa., and the University of Illinois, Urbana, Ill. Plastic flow is such an easy explanation of other things. Some one forgets to put camber in a bridge span and it settles slightly. "Plastic flow" is the immediate whispered explanation. Embarrassing deflections from some other cause—insufficient reinforcement, for example—develop and—"Oh, that's plastic flow in the concrete. All the research laboratories are studying it these days"—and so on. It is worthy of note that most concrete structures, if they are any the worse for it, do not reveal that fact to any appreciable extent. In a publication of Northwestern Technological Institute of Northwestern University dated February 1, 1941, Prof. G. A. Maney,[45] M. Am. Soc. C. E., reaches the conclusion that "Concrete is not an appreciably plastic material under working stresses and the time changes, in volume and dimension, of concrete under working loads are not due to plastic flow but to shrinkage." Mere change of nomenclature, of course, does not eliminate the volume change but it does simplify matters; it does eliminate obscurantism.

The standard concrete compression-test specimen, a cylinder 6 in. in diameter and 12 in. high, is an interesting object to consider. It is necessary to have some means of comparing various mixtures of concrete made at different places and at different times; and it has come to pass that the standard cylinder is the generally accepted means of making such comparisons; but, out of the complete widespread acceptance of this cylinder and of its unit compressive strength as a basis of comparison, has developed the attribution to it of wholly unwarranted powers. Of these none is more lacking in substantial basis than the assumption that it measures, directly, the flexural strength of its concrete when that concrete is put in a beam or slab and its ultimate fiber stress is computed by the straight-line theory with some one of the various values of n. Just why it should be a cylinder 6 in. in diameter and 12 in. high that should do this rather than some other cylinder 6 in. in diameter and of a different height is not, on earnest consideration, apparent. In Europe concrete is tested in 6-in. cubes and has some other unit strength than it has in America. Let no one think that shape of test specimen and conditions of testing do not have a pronounced effect upon the nominal unit strength of concrete. Some years ago S. H. Woodard, M. Am. Soc. C. E., tested numerous cylinders and prisms with sheets of blotting paper, impregnated with heavy grease, introduced between the ends of the specimens and the heads of the testing machine. By so doing he assuredly did not change the inherent properties of the concrete but he eliminated conical and pyrimidal breaks completely and caused their strengths to drop markedly below those of companion specimens tested without the blotting paper.[46] The paper by H. F. Gonnerman,[47] M. Am. Soc. C. E., indicates that the height of the 6-in. diameter cylinder should be about 4 in. to have it measure, directly, the flexural strength of the Slater-Lyse and other test beams when computed by the straight-line theory.

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[45] "'Warping' or Non-Uniform Shrinkage Over Cross-Section Found to Be Source of All Time Yield in Loaded Concrete Members," *Studies in Engineering—No. 1*, Northwestern Technological Inst. of Northwestern Univ., Evanston, Ill.

However, when it is possible to multiply the unit strength of the standard 6 by 12 cylinder by 1.5 and thereby obtain a value that reasonably approximates the flexural unit strength of the concrete in test beams, what could be simpler than to do that? The writer is not especially "enamored" of the straight-line theory, but on the other hand it is to be recognized that it has the virtue of great simplicity. Furthermore, it is the established and practically world-wide basis for designing reinforced concrete and has been taught continuously in all engineering schools and colleges ever since there has been a standard method of design. It is not to be lightly abandoned and put aside.

Therefore, as a constructive proposal it is suggested that the ordinary cylinder values be multiplied by 1.5 to obtain flexural values conservatively less than the ultimate flexural strength, that the straight-line theory be retained, and that working stresses be based on the flexural, not on the cylinder, strength. Certainly there is need for a revision of concrete design practice to effect a more reasonable and more economical use of the material than at present obtains. Fundamentally such a revision will make it permissible for concrete structures to carry greater loads than now allowed. That is what concrete structures are for—to carry loads. To make them do that effectively, economically, and in reasonable conformity with their potential abilities means the revision of various "books of rules" and a marked increase in present maximum permissible stress values.

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[46] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1091.

[47] *Proceedings*, A. S. T. M., 1925, Pt. 2, p. 237.

ROBERT W. ABBETT,[48] M. AM. SOC. C. E. (by letter).—The theory proposed in this paper, for proportioning structural members of reinforced concrete, is based on the assumptions that: The material is plastic; stress is not proportional to strain; and, in the case of gradually applied loads, the material will fail at a stress intensity less than maximum. These conditions are known to exist when the order of magnitude of compressive stress in the concrete is near the ultimate. Therefore, the author's theory should give an accurate indication of the behavior of a reinforced concrete member at the point of impending failure, provided that the strength of the concrete is critical and that the applied loads are of long duration.

In the case of loads suddenly applied and quickly removed, the stress-strain curve for concrete is not very similar to the ideal curve in Fig. 3 in that failure usually occurs near the point of maximum stress. This is true even when the ratė of load application is that usually adopted as standard for the testing of cylinders in the laboratory. Frequently, in laboratory tests of cylinders, the failure is in the nature of a sudden explosion at or near the point of maximum stress.

If the reinforcement of a beam is under-designed, presumably ultimate failure will occur because of yielding of the steel when the concrete stress is at some point below the maximum for the material. When this occurs the concrete stress is likely to fall in a region of the stress-strain curve which is a closer approximation of the properties of an ideal elastic material than those of an ideal plastic material. In this connection it should be noted that, even when the design is perfectly balanced, failure will usually occur because of yielding of the reinforcement, and that under any circumstances a compression failure of the concrete is very rare. Furthermore, when a factor of safety of 2 or more is used in the design, the working stresses in the concrete are in the elastic range of the stress-strain curve as in the case of the under-reinforced beam at the point of failure.

In view of the foregoing conditions, the writer has drawn the following conclusions with regard to the plastic theory:

(1) In the case of structural members for which the greater part of the load to be carried is dead load or live load permanently fixed in position, the plastic theory gives a better indication of the ultimate strength of the member than the present standard theory based on the theory of elasticity. Typical of such members are the column, footing, arch rib, and floor girder.

(2) When movable live load comprises the greater part of the total load to be carried, the plastic theory may give an erroneous indication of the strength and behavior of the member, and the elastic theory is to be preferred. Included in this class of members are most building floors, bridge decks, beams, and stringers.

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[48] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

(3) The plastic theory gives very little indication as to what is occurring in a member working at the average, or design, stresses when the factor of safety is 2 or more. On the other hand, the elastic theory gives results that are relatively close approximations for a large class of structures.

(4) The prediction of deflections under working conditions, although very uncertain with the elastic theory, is virtually impossible with the plastic theory.

(5) There are uncertainties in the behavior of reinforced concrete members that are ignored by both the plastic and elastic theories. These are probably the cause of greater variations than those due to the two theories mentioned. Typical of these uncertainties are the effect of tension resistance in the concrete, variation in effective moment of inertia, and stress conditions in both the concrete and steel in the vicinity of the small cracks which are always present in a stressed beam.

(6) Apparently the balance is about even when it comes to a choice of the two theories with a slight advantage in favor of the elastic theory simply because it is in agreement with the procedure standardized for other materials. However, the entire idea of "working stress" as a basis for design is questionable. There is a growing tendency to think of design in terms of useful limiting load but until this becomes universal there is no reason why the plastic theory should not be adopted for the class of structures for which it is most suitable. This has been done already in the case of columns.

Finally, the writer would like to pay his respects to the author, whose paper has contributed so much to the theory of reinforced concrete structures.

Paul Andersen,[49] Assoc. M. Am. Soc. C. E. (by letter).—The conventional method of designing reinforced concrete is unsatisfactory. Attempts have been made to eliminate from the analysis the modular ratio, *n*. At present, columns concentrically loaded are analyzed by means of formulas that carefully avoid this quantity. If, however, the load is moved slightly off center so as to produce combined flexure and compression, the analysis is immediately changed to one that makes extensive use of the modular ratio. Therefore, the author should be congratulated upon his efforts in correcting these contradicting procedures and presenting methods of analysis consistent for all types of members.

The plastic theory encourages the use of much higher steel percentages than the usual straight-line theory. According to the former theory, the critical percentage of steel required to develop the full compressive strength of the concrete is expressed by Eq. 13.

According to the straight-line theory this percentage can be found by equating total tension and total compression; thus: $\frac{1}{2} k\, j\, b\, d^2 f_c' = A_s f_s\, j\, d$, and

$$p_0 = \frac{A_s}{b\,d} = \frac{k}{2} \times \frac{f_c'}{f_s} = \frac{n f_c'}{2(n f_c' + f_s)} \qquad (43)$$

To compare Eqs. 13 and 43 to actual tests, six beams with varying steel percentages were made up and tested at the age of 28 days in the Materials

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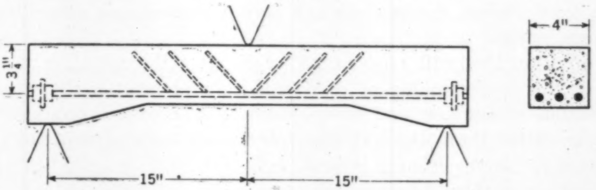
[49] Associate Prof., Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

TABLE 12.—DETAILS AND SCHEDULE OF REINFORCEMENT, TEST DATA PLOTTED IN FIG. 14



Description	Beam No. 1	Beam No. 2	Beam No. 3	Beam No. 4	Beam No. 5	Beam No. 6
Number of bars	2	3	2	3	2	3
Diameter of bars, in inches	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$

Testing Laboratory of the University of Minnesota, Minneapolis, Minn. In each beam the bottom reinforcement differed, as shown in Table 12. The

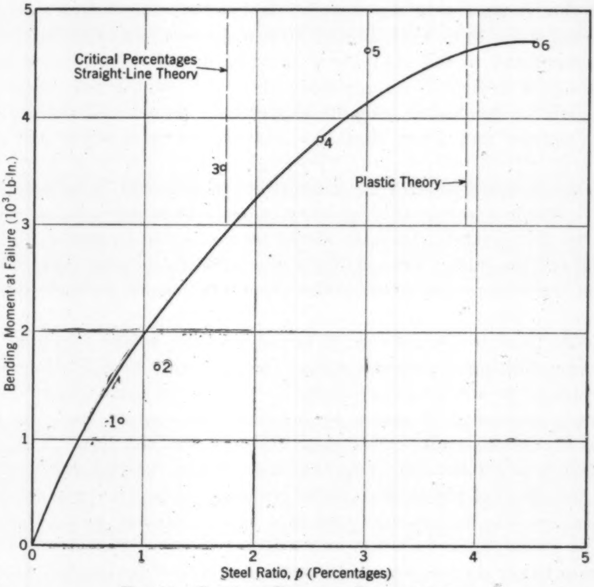


FIG. 14.—TESTS ON BALANCED REINFORCEMENT

compressive strength of the concrete (average of two cylinders) was 4,060 lb per sq in.; the secant modulus of elasticity at 75% of ultimate strength was 3.67×10^6 lb per sq in.; and the yield point of the reinforcement was 47,000 lb per sq in. Bending moments at failure plotted against steel ratios are shown in Fig. 14. Critical steel percentages according to Eqs. 13 and 43 are indicated. These tests, together with those cited by the author, support the contention that the conventional theory does not permit percentages of steel sufficient for balanced design.

The question that will arise immediately after the adoption of a plastic theory for the proportioning of reinforced concrete sections is whether the determination of moments and shears should not be by a procedure based on limit design rather than elastic theory. If the different parts of the structure are designed for forces existing prior to failure it appears logical to determine the distribution of these forces on the same basis.

R. A. CAUGHEY,[50] M. AM. SOC. C. E. (by letter).—The attention of the engineering profession has been directed to a problem that should receive serious attention and that should call forth various suggestions for the improvement of certain phases of reinforced concrete design. The writer, however, feels that the problem is not finally solved and that certain details should, and will, receive attention before the proposed method is considered for general use.

From the viewpoint of an engineering teacher, the writer feels that, in beam design, it would be very difficult to substitute a rectangle for the present stress diagram and to tell students that "The use of the rectangle is merely a mathematical device to approximate the effect of the true distribution." Most students in reinforced concrete classes have tested cylinders in concrete materials courses and know that the relation between stress and strain in concrete is a difficult problem; but they might reasonably expect a stress-strain curve closer to the results of the laboratory than would be afforded by this "device." Table 1 seems to suggest that, outside of the method suggested, a return to the parabolic formula would be a solution in the case of beams; but designers generally, recognizing the variations in the material, uncertainties in the flexural behavior of the beam, etc., would scarcely desire such a change.

It should be noted in the case of beams that the author's comparisons are made between ultimate-strength values (see Tables 1 and 2) when what the engineer desires to consider in concrete design is what the concrete does under working stresses. Fig. 1 seems to indicate that, up to half the ultimate strength, the stress-strain curve approaches a straight line. True enough, plastic flow is not taken into account, but neither does the author's method consider the effect of plastic flow under working loads. It seems to the writer that designers have had enough test data and experience with design, by the method of straight-line variation and working stresses, to warrant confidence in design but they do need some information on the effect of plastic flow under working loads. Mr. Whitney attempts to make allowance for the plastic flow by use of the rectangle but confines his attention to consideration of ultimate loads.

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[50] Prof. of Structural Eng., Dept. of Civ. Eng., Iowa State Coll., Ames, Iowa.

It will be noted that all of the formulas of the paper are based on ultimate strengths. At first this would seem to be desirable in these days of rigid frames but the designer still has the moments of inertia of the members of frames to deal with in determining bending moments; and it would seem to offset, to some extent, the advantages of an ultimate-strength method to use moments of inertia which, in turn, infer straight-line variation in concrete stresses.

Dropping the use of working stresses would also be objectionable because members of all other structural materials are designed on that basis. Most designers have regarded the abandonment of the ultimate-strength method of design as a very desirable step and would scarcely welcome a return to that method. Concrete design at present, with available curves, tables, etc., compares well in simplicity with design of other materials; and this simplicity with similarity in design methods to methods used in other materials is very desirable.

What the writer has stated thus far applies principally to beams. It is a generally recognized fact that structural engineers have gone far in considering plastic flow in formulas for columns[51] carrying concentric loads. Stress distribution over the section of a concentrically loaded column, however, is a problem that differs from the case of a beam. The unit stress in the concrete is practically the same over the entire section whereas, in the case of the beam, it presents a major problem. The writer feels that more tests of columns under combined direct stress and moment should be made, with the idea of determining strain distribution under working loads. With the distribution of strains determined, it might be possible to extend the work of J. R. Shank,[52] M. Am. Soc. C. E., and others to the determination of stresses with plastic flow taken into account.

The aforementioned method might be used in the case of beams, possibly by arriving at some method of adjusting the value of n so that the present convenient method of design might be more satisfactory.

Roberto Contini,[53] Jun. Am. Soc. C. E. (by letter).—Although the method of ultimate strength for purposes of design appears very logical and satisfactory, the developments for the determination of the strength of structural members seem to be rather uncertain. All the computations are based on the consideration that the equivalent rectangular stress diagram at failure must have a height equal to 0.85 f_c' in order for the center of gravity of that rectangle to coincide with the center of gravity of the actual stress diagram.

This develops, it seems, from the shape of the stress-strain diagram for concrete and from the fact that the maximum bending moment is developed when the maximum concrete strain is about twice the one causing the maximum stress. However, as the author himself states (see paragraphs following Eq. 3),

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[51] Final Report of Committee 105, Reinforced Column Investigation, *Proceedings*, Am. Concrete Inst., Vol. 29, p. 275.

[52] "The Mechanics of Plastic Flow of Concrete," by J. R. Shank, *Loc. cit.*, Vol. 32, pp. 149–180.

[53] Graduate Student, Columbia Univ., New York, N. Y.

"This ratio will vary with the quality of the concrete and the steel strain." As the height of the rectangular equivalent diagram is certainly correlated with that ratio, the value of 0.85 f_c' appears questionable.

Moreover, even if that value is satisfactory for concrete at failure, it could be erroneous, to some extent, in case of under-reinforced beams. In this case, in fact, the full concrete strength is not yet developed when the steel yields. The concrete stress diagram has a different shape than the one at maximum concrete strength, and its center of gravity is in a different position, so that the lever arm of the steel is different.

The case of under-reinforced beams is to be given consideration, because very often the size of a beam is controlled by a large bending moment confined to only a part of it, or by conditions other than the maximum bending moment. When this occurs, it would be wasteful to provide the full percentage of reinforcement required to balance the section, at least for some parts of the beam.

The writer thinks, however, that it is possible to avoid the use of uncertain values such as 0.85 f_c', and to proceed, instead, with the analysis of reinforced concrete members by simple considerations very similar in principle to the ones used in the usual theory of reinforced concrete. Such analysis, outlined hereafter, leads to the plotting of diagrams that are very easy to use.

The study is limited to simple bending of rectangular and T-beams, with tensile reinforcement only; but, if the system should prove satisfactory, it should not be difficult to extend it to any type of load and reinforcement. The following assumptions are made: (*a*) The strain distribution is linear through the entire depth of the beam; (*b*) the tensile action of concrete is negligible; (*c*) the stress-strain characteristics for concrete are well known and can be represented by means of a curve such as Fig. 3; and (*d*) the steel follows Hooke's law to the yield point—that is, to failure.

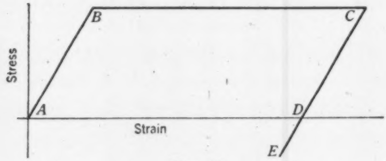

FIG. 15

The first assumptions are made also by the author. In order to justify assumption (*d*) the writer refers to a paper by J. A. Van den Broek, M. Am. Soc. C. E., in which a stress-strain diagram[54] for structural steel is plotted as shown in Fig. 15. Professor Van den Broek states:

> "Although the curve [Fig. 15] may appear idealized, this is not necessarily the case to any great degree. To be sure, stress-strain curves frequently are recorded with a bulge instead of the sharp angle shown at Point *B* * * *. It must be noted that the things tested are specimens and not filaments. For example, an ordinary steel bar suffers a differential stress between material in the outside and the center of the bar due to the cooling of the core (on the cooling bed) after the outside of the bar is already black and hardened. Furthermore, the rate of running the testing machine affects the curve. In other words, variations from stress-strain curves of mild steel, as shown in the figure, may frequently be assigned to the test specimen rather than to the material. In several

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[54] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Vol. 105 (1940), Fig. 2, p. 640.

hundred tests reported in 1918[55] the writer found the horizontal part of the curve very pronounced and coincident with the proportional limit."

It must be noted that Professor Van den Broek refers to tests made by strain control method, which is the one used for determining the stress-strain curve for concrete given by the author.

Rectangular Beams.—On these assumptions the following relations can be written (using the author's notation): From the linear strain relation—

$$\epsilon' = \frac{1 - k}{k} \epsilon \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

Because of Hooke's law for steel—

$$T = A_s f_s = A_s \epsilon' E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44b)$$

in which E = Young's modulus up to yield point. Since $T = C = A f_c' k b d$:

$$A_s \epsilon' E = A f_c' k b d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44c)$$

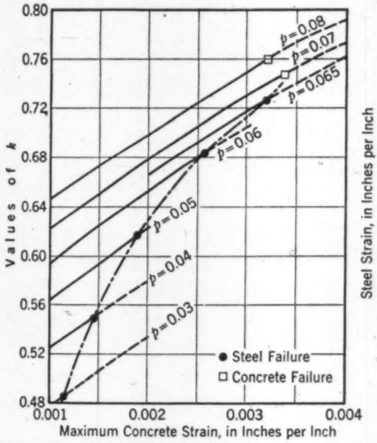

FIG. 16

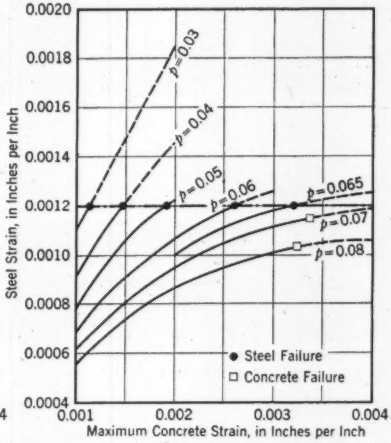

FIG. 17

Eliminating ϵ' from Eq. 44c by means of Eq. 44a:

$$A_s \frac{1 - k}{k} \epsilon E = A f_c' k b d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45a)$$

Therefore,

$$k^2 + k \frac{A_s \epsilon E}{A f_c' b d} - \frac{A_s \epsilon E}{A f_c' b d} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

and

$$k^2 + k \frac{p \epsilon E}{A f_c'} - \frac{p \epsilon E}{A f_c'} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45c)$$

from which, by solving for k and taking the positive root:

$$k = -\frac{p \epsilon E}{2 A f_c'} + \sqrt{\left(\frac{p \epsilon E}{2 A f_c'}\right)^2 + \frac{p \epsilon E}{A f_c'}} \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

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[55] "The Effect of Cold Working on the Elastic Properties of Steel," Carnegie Scholarship *Memoirs*, Iron and Steel Inst., Vol. IX, 1918; see, also, *Engineering*, July, 1918.

For any value of ϵ, the corresponding value of A can be selected from Fig. 3(*b*). Therefore, everything being known concerning the right side of Eq. 46, it will be possible to plot a series of curves k against ϵ for different values of p (see Fig. 16) and, by using Eq. 44*a*, to plot ϵ' against ϵ. All the elements are now available to plot $\frac{M}{f_c'\,b\,d^2}$ against ϵ by using Eq. 2. The curves given in Fig. 17 were plotted by using the following characteristics of the materials: (*a*) For concrete, the curves given by the author, in Figs. 3(*b*) and 3(*c*) with $f_c' = 4{,}000$ lb per sq in. (The limit of x when ϵ approaches zero should be 0.667.) (*b*) For steel, $E = 30{,}000{,}000$ lb per sq in. and f_s' = yield point stress = 36,000 lb per sq in.; consequently, ϵ_y' = strain at which yield begins = 0.0012.

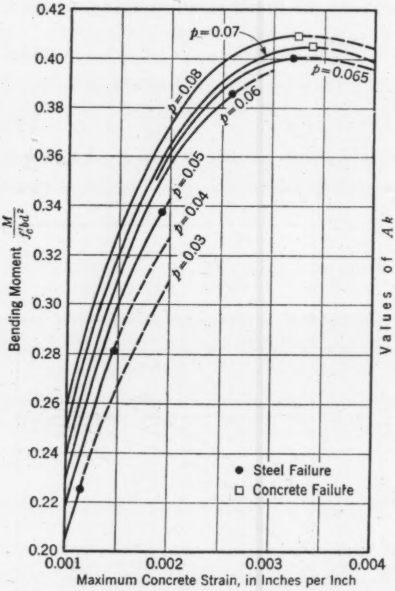

FIG. 18

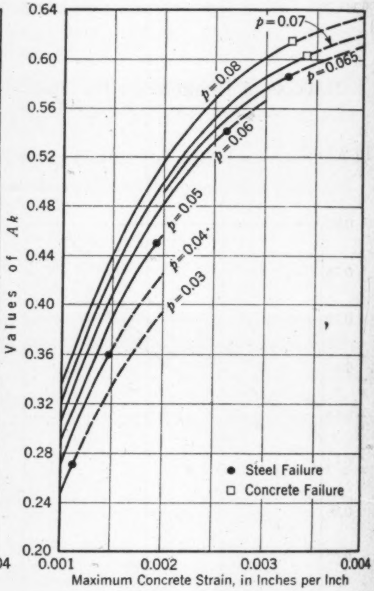

FIG. 19

The curves were plotted as if the steel followed Hooke's law indefinitely; therefore they are of no significance beyond the point at which the steel strain reaches the value $\epsilon_y' = 0.0012$ (dotted parts of curves). As shown by Figs. 17 and 18, for small values of p the beam fails by tension in the steel, because the steel strain reaches the value ϵ_y' before the full strength of the concrete is developed. If p is high, the maximum resisting moment, as controlled by concrete, is reached when the steel strain is still less than ϵ_y' and the beam fails by compression in concrete.

A "balanced" design is obtained when the maximum resisting moment, as controlled by concrete, is reached at the same time that the steel strain reaches the value ϵ_y'. At that point the beam is on the verge of failure both by compression in concrete and by tension in steel.

For the selected characteristics of the materials, the value of p for balanced design happens to be a little more than 0.065. For under-reinforced beams the value of ϵ, corresponding to ϵ_y', can be found also (in addition to the foregoing) by plotting the curve $A\,k$ (Fig. 19) and noting that $T_{max} = \epsilon_y'\,E\,A_s$. This quantity is known; and, as $T = C$, one can write: $\epsilon_y'\,E\,A_s = A\,f_c'\,k\,b\,d$; or,

$$\frac{\epsilon_y'\,E\,p}{f_c'} = A\,k \quad \text{(47)}$$

in which the left term is known. The value of ϵ, corresponding to the value of $A\,k$ found by means of Eq. 47, is read from the curves.

In Fig. 20 the value of the maximum resisting moment is plotted against p. This, therefore, is the fundamental diagram for design purposes. Also the value of k at failure is plotted.

T-*Beams.*—For T-beams with tensile reinforcement (using the notation of Fig. 21), if $k < t$, the formulas found for rectangular beams apply as well.

The author states that, in his theory, formulas for rectangular beams apply directly to T-beams if the compression depth a is less than the thickness of the flange. It seems, however, that in any case the length $k\,d$ should be considered instead of a; otherwise it becomes implicitly assumed that the stress distribution is rectangular, which, on the contrary, is sharply denied by the author.

If $k > t$, the total compressive force will be the same as for a rectangular beam of width b, less the part corresponding to the shaded area in Fig. 21(a). If: $\frac{b - b'}{b} = s$; C_1 = compressive force for rectangular beam of width b; C_2 = compressive force corresponding to the shaded area, to be subtracted; and $C = C_1 - C_2$ = effective compressive force; then:

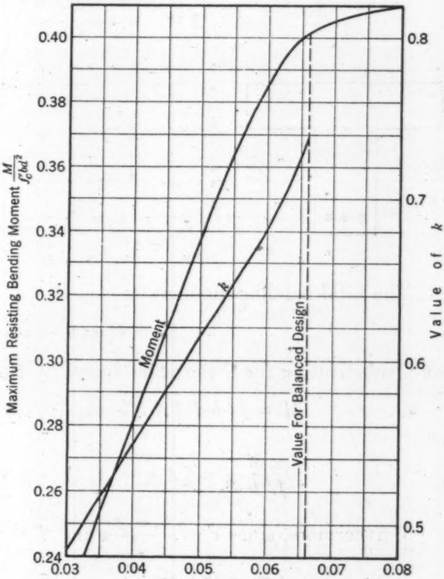

FIG. 20

$$C_1 = A\,f_c'\,k\,b\,d \quad \text{(48a)}$$

$$C_2 = A_2\,f_c'\,(k - t)\,b\,d\,s \quad \text{(48b)}$$

and

$$C = C_1 - C_2 = f_c' \, b \, d \, [A \, k - A_2 \, (k - t) \, s] \quad \text{..........} (48c)$$

Area A is read from Fig. 3(b) in relation to ϵ, maximum concrete strain, and A_2 is read in relation to ϵ_2, the concrete strain at the lower face of the flange. The linear strain distribution yields the relation:

$$\epsilon_2 = \frac{k - t}{k} \, \epsilon \quad \text{........................} (49)$$

Assume that: $\bar{x}_1 \, k \, d$ = distance of the line of action of C_1 from the neutral axis; and $\bar{x}_2 \, k \, d$ = distance of the line of action of C_2 from the neutral axis. The moment of $C = C_1 - C_2$ about the neutral axis will then be:

$$(C_1 - C_2) \, \bar{x} \, k \, d = C_1 \, \bar{x}_1 \, k \, d - C_2 \, x_2 \, (k - t) \, d \quad \text{..........} (50)$$

in which $\bar{x} \, k \, d$ is the distance of the line of action of C from the neutral axis. Solving for $\bar{x}$:

$$\bar{x} = \frac{C_1 \, \bar{x}_1 - C_2 \, \bar{x}_2 \left(1 - \frac{t}{k}\right)}{C_1 - C_2} \quad \text{.................} (51)$$

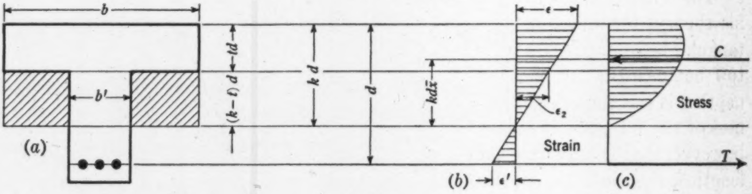

FIG. 21

The total bending moment is:

$$M = C \, d \, (1 - k + k \, \bar{x}) \quad \text{...................} (52)$$

and, substituting for C the value found by Eq. 48c:

$$M = f_c' \, b \, d^2 \, [A \, k - A_2 \, (k - t) \, s] \, (1 - k + k \, \bar{x});$$

or,

$$\frac{M}{f_c' \, b \, d^2} = [A \, k - A_2 \, (k - t) \, s] \, (1 - k + k \, \bar{x}) \quad \text{..........} (53)$$

Furthermore, since $\epsilon' = \frac{1 - k}{k} \, \epsilon$ and $T = A_s \, f_s = A_s \, \epsilon' \, E = C$:

$$A_s \, \epsilon' \, E = f_c' \, b \, d \, [A \, k - A_2 \, (k - t) \, s] = A_s \, \epsilon \, E \, \frac{1 - k}{k} \quad \text{......} (54)$$

That is:

$$\frac{p \, \epsilon \, E \, (1 - k)}{f_c' \, k} = A \, k - A_2 \, (k - t) \, s \quad \text{.............} (55a)$$

or

$$k^2 \, (A - A_2 \, s) + k \left(\frac{p \, \epsilon \, E}{f_c'} + A_2 \, s \, t\right) - \frac{p \, \epsilon \, E}{f_c'} = 0 \quad \text{......} (55b)$$

If $s = 0$, Eq. 55*b* becomes the formula for rectangular beams; if $s = 1$ the action of the compressed part of the web is neglected.

Eq. 55*b* is not as simple as the one for rectangular beams, and actually is not solvable for k by simply assigning values to ϵ, as in the case of rectangular beams, because A_2 is unknown. This quantity, in effect, is read from Fig. 3(*b*), the determination of which, by means of Eq. 49, requires a knowledge of k. Therefore Eq. 55*b* can be solved either by trial or graphically. In this latter case, for each value of ϵ, A_2 should be plotted against k both by means of Eq. 55*b* and by means of Eq. 49 and Fig. 3(*b*). The intersection of the two curves gives the value of k corresponding to the inherent value of ϵ.

Once k has been plotted against ϵ for given values of t and s and for a series of values of p, all the elements are available for plotting the following curves (all with ϵ as abscissas): ϵ_2, by means of Eq. 49; A_2, from Fig. 3(*b*); $\bar{x}$, from Eq. 51 after having determined $\bar{x}_1$ and $\bar{x}_2$ from Fig. 3(*b*); and finally, for determining the maximum resisting moment as controlled by concrete $\frac{M}{f_c' \, b \, d^2}$ by means of Eq. 53.

For steel failure ϵ' will be plotted against ϵ, by means of Eq. 44*a*, and the designer can proceed in the determination of the resisting bending moment as he did for rectangular beams.

All these curves are plotted for a series of values of p. Finally, the resisting moment can be plotted against the steel ratio p; and, here again, this will be the fundamental curve for purposes of design.

In conclusion, sets of diagrams should be plotted for each combination of steel and concrete; in the case of T-beams, a set of diagrams would be necessary for each combination of t and s. As a matter of course, the suggested method should be tested with experimental results. The writer has attempted to show that, for purposes of design, it is not necessary to rely on formulas based on more or less empirical and uncertain data, when it is possible to devise diagrams for easy application, based on simple and fundamental considerations. In this sense, the discussion is an attempt to support and improve the arguments presented by Mr. Whitney.

A. A. EREMIN,[56] ASSOC. M. AM. SOC. C. E. (by letter).—It is well known that the modulus of elasticity for compression in concrete varies with its stress, age, and physical qualities. Attempts were made in this paper to develop the equations for stresses, considering variation of the modulus of elasticity.

Interesting equations were developed by Fritz J. von Emperger in 1936 for a beam with a rectangular section.[57] He considered the variation of the modulus of elasticity in terms of the ratio "n," which he computed for the maximum stresses in concrete and steel, and the percentage of reinforcement in the section. Mr. von Emperger expressed variation of stresses in the rectangular section by a trapezoid with the maximum stresses at the top of beam section continued to a depth of $\frac{2}{3}$ a and varied according to a straight line from the maximum stresses to zero stresses at the neutral axis, a being a distance from the top of beam to the neutral axis (Fig. 22). Mr. von Emperger has shown that, in a reinforced concrete beam with light reinforcement, a variation of the ratio "n" has a slight effect on the stresses. Furthermore, like the author, he reached the conclusion that varying the ratio "n" and the percentage of reinforcement in a beam section made it possible to increase the maximum stresses in concrete and steel.

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[56] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[57] "Der Beiwert '*n*,'" by Fritz J. von Emperger, *Beton und Eisen*, October, 1936, p. 324.

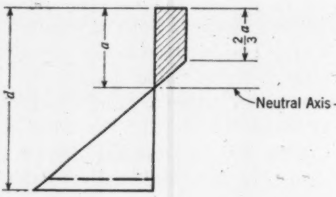

FIG. 22.—DISTRIBUTION OF STRESSES

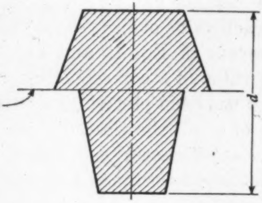

FIG. 23.—EQUIVALENT SECTION

In his formula for stresses (Eq. 3), instead of varying the ratio "n," Mr. Whitney assumed the maximum strains in concrete and steel. Furthermore he assumed that the stresses in the rectangular section vary as a rectangular prism with the total resulting stresses concentrated in center. Evidently the resulting equations may produce an economic design of reinforced concrete beams with small ratio of depth to span length. However, deep girders designed according to the author's formulas will require an excessive percentage of reinforcement.

To insure further accuracy in the equations for stresses in reinforced concrete beams, the influence of the section width should be taken into account. J. Melan has shown that the distribution of stresses in deep sections is not the same as in wide sections.[58]

Interesting equations for computation of stresses in concrete beams were developed by Rudolf Kern.[59] Like the author, Mr. Kern based his equation on the stress and strain curve obtained from a cylinder test. However, instead of assuming a rectangular prism distribution of stresses, Mr. Kern computed an equivalent trapezoidal section for the rectangular section of a beam, Fig. 23. Stresses in the equivalent trapezoidal section of a beam may be computed with standard formulas based on a straight-line variation. It is interesting to note that the equivalent beam sections determined by Mr. Kern may be used also for the computation of the elastic properties in rigid frames. Mr. Kern applied his equations in the computation of stresses in reinforced concrete arch ribs.

The equations of this paper should be modified to make them strictly applicable to the analysis of stresses in rigid frames or arches.

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[58] *Beton und Eisen*, 1936, p. 315.

[59] *Loc. cit.*, January, 1930, p. 29.

JAROSLAV J. POLIVKA,[60] **M. AM. SOC. C. E.** (by letter).—The characteristics of the plastic flow of concrete just before the failure of a structural member are different from those at working loads,[61] and more research is needed to prove whether, and how far, the stage preceding the failure can be considered as a reliable basis for reinforced concrete design. The adoption of the plastic theory is very attractive for its simplicity of analysis and design, compared with the generally accepted theory using the modular ratio n. A further simplifying step is suggested that may be extended for analysis and design of any irregular shape of the concrete section having unsymmetrical reinforcement and subject to any kind of loading.

Eq. 12 may be derived, assuming the value a equal to the distance between the centroidal axis of the prismatic cross section and the compression face:

$$a = \frac{t}{2} \qquad (56)$$

When the value a is substituted in the equation of equilibrium,

$$M = C_f \times f_c' \times a\, b \times c \qquad (57)$$

the ultimate resisting moment of the beam as controlled by concrete failure (Eq. 12) becomes equal to $\frac{f_c'}{3}$, maintaining the coefficient $C_f = 0.85$ and setting $d = 0.93\,t$. Then, $a = 0.538\,d$ and $c = 0.731\,d$, which values check very well with those computed from tests reported by Professors Slater and Lyse[15] (0.537 d and 0.732 d).

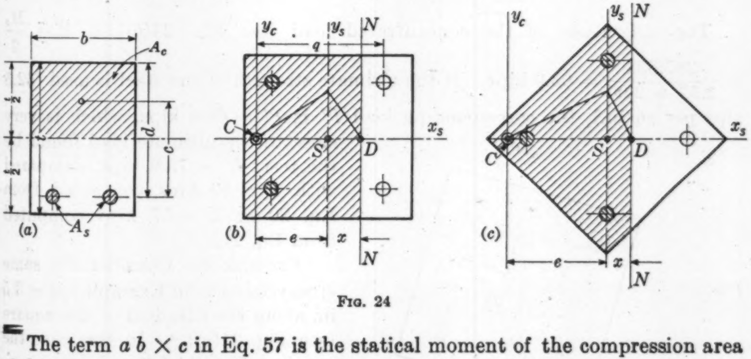

FIG. 24

The term $a\, b \times c$ in Eq. 57 is the statical moment of the compression area of the cross section with respect to the centroidal axis of the reinforcement:

$$M_c = A_c\, c = a\, b\, c \qquad (58)$$

This principle may be adopted for any shape of the cross section, considering the area of compression as acted upon by a uniformly distributed imaginary stress $C_f f_c'$. The compression area is determined by the neutral axis of the full homogeneous cross section (section of concrete). For pure bending the neutral axis is defined as a centroidal axis of the cross section, conjugate to the plane of bending. For eccentric loading the neutral axis is the antipolar of the point of application of the concentrated load with respect to the ellipse of inertia of the homogeneous cross section.

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[60] Research Associate, Univ. of California,; Cons. Engr., Berkeley, Calif.

[61] "Design of Concrete Columns Reinforced by Steel of High Strength," by J. J. Polivka, *Stavitelské Listy* (*Structural Journal*), Vol. 29, 1933, Prague, pp. 175–178 and 188–191; see also "Le Béton Translucide," by J. J. Polivka—published by Les Études des Composés Siliceux, Bruxelles, 1934.

The coefficient C_f is equal to 0.85 when the compression face is parallel with the neutral axis and must be modified for other positions of the compression face according to the varying distance between the points of the boundary in compression and the neutral axis.

Example 1.—Consider an eccentrically loaded column 6 in. by 6 in., with symmetrical reinforcement of four $\frac{3}{8}$-in. round bars, and an eccentricity of the concentrated load $e = 2.50$ in. (see Fig. 24(*b*)).[62] The radius of gyration of the cross-sectional area is given by $r^2 = \frac{t^2}{12} = 3$ in.^{2} and $r = 0.289\,t$; and the distance between the neutral axis and the centroidal axis is $x = \frac{r^2}{e} = 1.2$ in.

For $f_c' = 4.02$ kips per sq in. ("kips" equal "kilo-pounds"; that is, 1,000 lb), $f_s = 43$ kips per sq in. (at the yield point), and $C_f = 0.85$, the statical moment of compressive stresses with respect to the centroidal axis of the tensile bars (in kip-in.) is

$$\begin{aligned} 25.2 \times 0.85 \times 4.02 \times (2.1 + 0.8) &= 249.714 \\ 2 \times 0.11 \times 43 \times 4.0 &= \underline{37.840} \\ \text{Moment } M_c &= 287.554 \end{aligned}$$

The magnitude of the concentrated load (see Fig. 24(*b*)) is $P = \frac{M_c}{q} = \frac{287.554}{2.50 + 2.00} = 63.9$ kips. If the ultimate strength of the steel is used (62.5 kips per sq in.), the corresponding load P' is $P' = 68.6$ kips, which is very close to the ultimate load found by the test: $P' = 73.9$ kips, compared with $P = 53$ kips computed from Eq. 20 or $P = 57$ kips computed from Eq. 21.

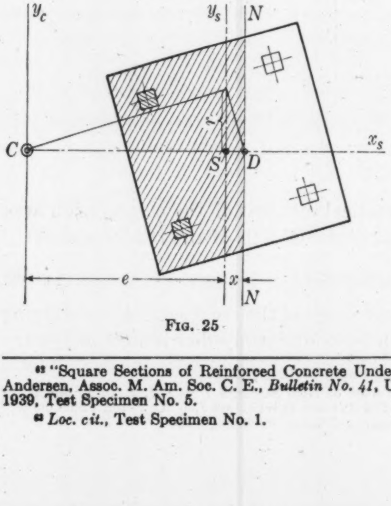

FIG. 25

Example 2.—Consider the same cross section as in Example 1, $e = 3.5$ in. along the diagonal of the square (see Fig. 24(*c*)).[63] In this case the constant C_f must be modified as previously stated. One method would consist in computing the stress volume for which the base is the area of compression, the stress remaining constant in planes perpendicular to the neutral axis and decreasing toward the corners. Then C_f becomes $\frac{2}{3} \times 0.85 = 0.567$.

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[62] "Square Sections of Reinforced Concrete Under Thrust and Nonsymmetrical Bending," by Paul Andersen, Assoc. M. Am. Soc. C. E., *Bulletin No. 41*, Univ. of Minnesota, Minneapolis, Minn., Vol. XLII, 1939, Test Specimen No. 5.

[63] *Loc. cit.*, Test Specimen No. 1.

Considering the fact that f_s is decreasing toward the neutral axis, the statical moment of compressive forces with respect to the centroidal axis of the tensile reinforcement (in kip-in.) is:

(concrete) $0.567 \times 3.98\,(18 \times 4.242 + 6.979 \times 2.387) = 209.907$

(extreme bar) $0.11 \times 43 \times 5.656 = 26.753$

(two bars in the compression zone) $0.22 \times 43 \times \dfrac{0.867}{3.695} \times 2.828 = 6.278$

Moment $M_e = 242.938$

The intensity of the concentrated load is $P = \dfrac{242.938}{3.50 + 2.828} = 38.4$ kips.

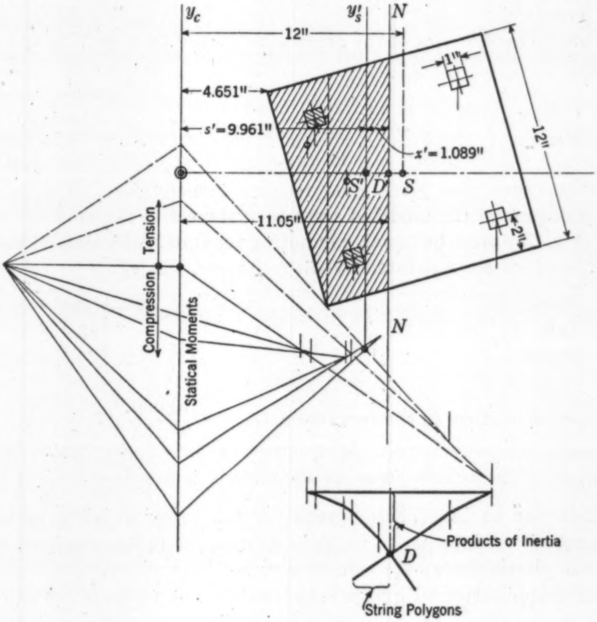

FIG. 26

If the ultimate strength of the steel is used, the corresponding load is $P' = 40.7$ kips. Both values are close to the ultimate load, 36.3 kips, of the test.

Example 3.—Comparison of the suggested method with the conventional analysis of eccentrically loaded concrete sections (Fig. 25) shows a square 12-in. by 12-in. section reinforced with four 1-in. square bars and subjected to a concentrated load $P = 10$ kips acting at a point C, which is 12 in. from the centroid S of the section; the plane of bending makes an angle of 15° with one of the principal axes.[64] The following definition of the neutral axis has been formulated by the writer, relating to the conventional elastic theory:

> The neutral axis N–N is a line parallel to a diameter of the ellipse of inertia that is conjugate to the plane of eccentric bending. It is determined by the condition that the products of inertia of the areas in compression and tension with respect to the neutral axis and to a parallel y_C through the point of application are equal. The products of inertia are determined by two string polygons (Fig. 26), applying the statical moments of cross-sectional areas with respect to the line y_C as forces acting parallel to y_C.

Both string polygons intercept at a point D, which locates the neutral axis N–N. The distance of the neutral axis N–N from the point of application C is found graphically: $s' + x' = 11.05$ in. (compared with 10.99 in. found by Mr. Andersen).[64]

The products of inertia of the areas with respect to axes N–N and y_C check very well:

Zone	Product of inertia
Compression	1,552 in.^{4}
Tension	1,551 in.^{4}

Another check also gives good results—namely, the condition that the statical moments of the modified areas with respect to the modified centroidal axis y_s' (Fig. 26) must be equal, computing separately the statical moments in both zones which are separated by the centroidal axis y_s':

Zone	Sum of statical moments
Left	125.455 in.^{3}
Right	125.454 in.^{3}

The compressive stress in the concrete is $f_c = -\dfrac{10{,}000}{98.193}\left(1 + \dfrac{5.31}{1.09}\right) = -598$ lb per sq in. The tensile stress in the steel is $f_s = \dfrac{10{,}000}{98.193}\left(1 - \dfrac{6.94}{1.09}\right) \times 10 = 5{,}472$ lb per sq in. These stresses would mean a safety factor of 7.9, assuming $f_c' = 4.7$ kips per sq in., $f_s = 43$ kips per sq in., and $n = 10$.

For the plastic theory the compression zone is confined by the antipolar of the point of application with respect to the ellipse of inertia of the homogeneous cross section. The radius of gyration is given by $r^2 = \dfrac{t^2}{12} = 12$ in.,^{2} and the distance of the antipolar from the centroid of the homogeneous cross section is $x = \dfrac{12}{12} = 1$ in. The coefficient C_f is computed in a manner similar to that of Example 2: $C_f = 0.335 \times 0.85 = 0.285$. Then the ultimate load of the column is $P = 51.6$ kips and, since the working load considered in the conventional theory was 10.0 kips, the safety factor is only 5.2.

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[64] "A Graphical Method of Analyzing Eccentrically Loaded Concrete Sections," by Paul Andersen, *Civil Engineering*, January, 1940, p. 37.

Paul W. Abeles,[65] **Esq.** (by letter).—As explained by Mr. Whitney, the formulas of this paper do not take into account the excess strength that has been reported in beams and slabs with small percentages of steel. The reason for this neglect is that, in Mr. Whitney's opinion, the use of small percentages of steel is not likely to be as satisfactory in practical construction as in the laboratory; and, furthermore, it has not been demonstrated that the excess strength will remain after repeated loading. Against this statement it may be emphasized that small percentages of steel have become more and more common owing to the use of medium and high-grade steel. Before the use of new steel grades of unusually high yield point was permitted, many tests had to be made in various countries. In this connection (for example, on structures reinforced with twin twisted steel bars), tests with repeated loads were conducted which have proved that the behavior with such a loading is similar to that of structures reinforced with mild steel. It may be stated that a repeated loading in cases of higher percentages of steel also causes less favorable results than tests on beams, whose loading is gradually increased until failure.

The excess of strength in cases of small percentages of steel is especially great when high-grade materials are used. The excess of strength can be shown best by the theoretical lever arm of test specimens at failure c_t calculated for the breaking moment:

$$c_t = \frac{M}{A_s f_s} \qquad (59)$$

This value c_t may be compared with the value c according to Eq. 6

$$c = \left(1 - \frac{p\,m}{2}\right) d = \left(1 - \frac{p f_s}{2 \times 0.85 \times f_c'}\right) d = \left(1 - \frac{p}{1.7} k\right) d \qquad (60)$$

in which $k = \frac{f_s}{f_c'}$ represents the ratio of the steel yield point stress and the concrete cylinder strength. Although the ratios $\frac{c}{d}$ in Mr. Whitney's theory represent an improvement over the values j determined by usual designing method, they differ materially from the values $\frac{c_t}{d}$, calculated on the basis of test results, which may be shown in the following. In Table 1 only tests are dealt with whose percentages of steel are greater than 2%. Table 2 contains tests, conducted in 1912, of small percentages of steel. In this case, the ratio $\frac{c_t}{c}$ (which demonstrates the difference between real behavior and theory according to Mr. Whitney) varies between 1.006 and 1.188 for percentages of 0.49 and 0.98. As long as no greater differences are in question there would be no reason for any improvement; but newer tests, especially in cases of high-grade

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[65] Cons. Engr., London, England.

TABLE 13.—COMPARISON OF THEORETICAL STEEL LEVER ARM WITH TEST RESULTS

Beam No.	100 p	Stresses (Lb per Sq In.)		k	$\frac{c_t}{d}$	$\frac{c}{d}$	j	$\frac{c_t}{c}$	$\frac{c_t}{j}$
		Yield point, f_s	Cylinder strength, f_c						
(a) Slabs (Lyse and Wernisch); $f_c = f_c'$; and $k = \frac{f_s}{f_c'}$									
9	0.35	46,000	2,890	15.92	1.45	0.976	.....	1.50	.....
10	0.303	55,300	2,795	19.66	1.47	0.965	.....	1.52	.....
6	0.755	47,500	2,784	17.06	1.31	0.924	.....	1.42	.....
7	0.754	50,000	2,640	18.95	1.32	0.916	.....	1.44	.....
32	0.725	50,000	5,750	8.76	1.30	0.963	.....	1.35	.....
22	1.32	48,500	3,140	15.45	1.02	0.880	.....	1.16	.....
24	1.328	50,300	3,140	16.20	0.985	0.874	.....	1.13	.....
12^a	0.343	95,500	3,390	28.20	1.20	0.943	.....	1.275	.....
29^a	0.77	93,000	5,230	17.70	1.04	0.920	.....	1.130	.....
(b) Rectangular Beams and T-Beams (Columbia Tests); $f_c = f_c'$ and $k = \frac{f_s}{f_c'}$									
C_4	0.451	56,330	3,376	16.62	1.135	0.956	.....	1.185	.....
C_{14}	0.456	56,330	2,837	19.85	1.13	0.947	.....	1.18	.....
C_7	0.798	55,730	3,304	16.87	1.03	0.921	.....	1.13	.....
C_{17}	0.820	55,730	3,340	16.78	1.02	0.919	.....	1.11	.....
$D_3{}^b$	0.843	50,470	3,247	15.52	1.03	0.923	.....	1.115	.....
$D_{13}{}^b$	0.812	50,470	2,960	17.04	1.04	0.918	.....	1.13	.....
C_1	0.620	35,600	2,510	14.18	1.125	0.948	.....	1.188	.....
C_{11}	0.623	35,600	3,320	10.7	1.141	0.961	.....	1.18	.....
C_5	1.245	35,800	3,243	11.04	1.0	0.919	.....	1.09	.....
C_{15}	1.230	35,800	3,028	11.82	0.97	0.914	.....	1.06	.....
(c) Rectangular Beams (Abeles); $k = \frac{4}{3}\frac{f_s}{f_c}$; and $n = 15$ for j and $\frac{c_t}{j}$									
Concrete 1:									
1	0.38	37,600	$2{,}060^c$	24.32	1.025	0.946	0.905	1.085	1.13
2	0.85	37,100		24.02	0.955	0.880	0.868	1.085	1.10
4	0.38	95,200	$2{,}060^c$	61.75	1.04	0.862	0.905	1.195	1.15
5	0.85	91,000		58 90	0.72	0.705	0.868	1.022	0 83
Concrete 4:									
18	0.38	37,600	$8{,}400^c$	5.96	1.167	0.987	0 905	1.183	1.29
19	0.85	37,100		5.94	1.110	0.970	0.868	1.145	1.28
20	1.50	37,100		5.94	1.041	0.948	0.839	1.098	1.24
22	0.38	95,200	$8{,}400^c$	15.10	1.385	0.966	0.905	1.425	1.53
23	0.85	91,000		14.45	1.232	0.928	0.868	1.327	1.42
24	1.50	87,000		13.80	1.007	0.878	0.839	1.146	1.20
(d) Spun Tubular Beams[d] (Král); $j = j_0$									
$\mathrm{I}_c{}^e$	0.19	47,025	.....	.....	1.995	.....	0.936	.....	2.13
$\mathrm{II}_a{}^e$	1.51	42,902	.....	.....	1.206	.....	0.928	.....	1.30
$\mathrm{III}_a{}^e$	3.32	41,992	.....	.....	1.145	.....	0.902	.....	1.27
$9_b{}^f$	0.37	57,307	.....	.....	1.806	.....	0.927	.....	1.95
$7_a{}^f$	1.33	56,311	.....	.....	1.365	.....	0.928	.....	1.47
$10_b{}^f$	2.01	55,316	.....	.....	1.280	.....	0.909	.....	1.41
$16_b{}^g$	0.30	47,025	.....	.....	2.775	.....	0.907	.....	3.07

[a] The corresponding beams with the same steel percentages do not agree regarding loading and arrangement, respectively. [b] T-beams. [c] Cube strength. [d] Beams with the smallest breaking load. [e] Octagonal tube, 11 in. high, mild steel. [f] Octagonal tube, 11 in. high (upper width, $b = 4.8$ in.), twin twisted steel bars. [g] Spun beam 8.7 in. total height with lower flat base, mild steel.

materials, give much greater differences. This may be seen by reference to Table 13, which shows the relation between the ratio $\frac{c}{d}$ of beams or slabs and their reinforcement (between 0 and 2%). The values of $\frac{c_t}{d}$ calculated for several test results are tabulated. The ratio $\frac{c_t}{d} = 1$ represents a limit, the lever arm being identical with the distance of the reinforcement from the upper fiber, which case actually would be impossible should Eq. 59 be correct. This limit is materially surmounted, as shown in Table 13. The following test results have been taken into account: (1) Tests on slabs, conducted at Lehigh University,[19] Bethlehem, Pa.; (2) tests on rectangular and T-beams at Columbia University,[66] New York; (3) tests on rectangular beams by the writer[67] in Vienna, Austria; and (4) tests on tubular spun beams by Prof. C. Král[68] at Ljubljana, Yugoslavia.

Table 13(*a*) contains a selection of the Lehigh tests.[19] Two steel types were taken into account, one type with a yield point stress between 46,000 and 55,300 lb per sq in. (three different percentages of steel, two or three separate results for each percentage, respectively), and the other type between 93,000 and 95,000 lb per sq in. (two different percentages and only one test result each). Table 13(*b*) contains a selection of the Columbia University tests with two different steel types (yield point stresses 35,600 to 35,800 and 50,470 to 56,330 lb per sq in.), two different percentages in each case, no difference resulting between rectangular beams C_7 and C_{17} and T-beams D_3 and D_{13}. In Tables 13(*a*) and 13(*b*), also, the concrete cylinder strengths are given. The rectangular beams reported in Table 13(*c*), with a cross section 8 in. by 9 in., had a span of 8 ft and were loaded with two single loads in a distance of 3.3 ft; four different concrete mixes were tested, each with two different grades of steel (yield point stresses 37,100 to 37,600 and 87,000 to 95,200 lb per sq in.). From Table 13(*c*) (which contains average values of two specimens each), it can be seen that only concretes 1 and 4 have been taken into account, concrete 1 being of very low strength, much different from the high-grade concrete 4. For comparison the values $\frac{c}{d}$ have been computed on the assumption that $\frac{3}{4}$ of the cube strength given in the Table 13(*c*) (obtained from 8-in. cubes) is identical with the cylindrical concrete strength. The values j for $n = 15$ and ratios $\frac{c_t}{j}$ have been added in Table 13(*c*) for purposes of comparison. The tubular spun beams, according to Table 13(*d*), do not actually agree with this comparison, since they represent a combination of tubes and beams, which works very favorably, the automatic pre-stressing of the transverse reinforcement (owing to the centrifugal force during spinning) being of great advantage in regard to reduction of the principal stresses. Nevertheless, their results have been included to show the steel saving that is possible if consideration is given to the actual behavior of reinforced concrete structures with small reinforcement. In Table 13(*d*) only the results with the smallest breaking moments have been considered. In this case the percentages have been computed for the upper breadths of the tubular beams, and the ratios $\frac{c_t}{d}$ have been calculated with the assumption of an existing secondary reinforcement of one round bar (area = 0.22 in.) in either lower outside corner of the octagon, the values j_0 representing the ratios of the lever arms of T-beams $d - \frac{t}{2}$ and their depths d (t being the thickness of the slab). The tests were conducted on beams of 3.6-ft span and 7.4-ft span, subjected to two single loads in a distance of 1 ft and 1.5 ft, respectively.

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[66] "The Modular Ratio," by K. Hajnal-Kónyi, *Concrete and Constructional Engineering*, Vol. 32, 1937, pp. 129, 189, 521.

[67] "Versuche mit Rechteckbalken, bewehrt mit besonders hochwertigem Stahl," by Paul Abeles, *Beton und Eisen*, Vol. 36, 1937, p. 295; also "Elastizität des Betons," by Paul Abeles, *Zement*, Vol. 26, 1937. p. 619.

[68] "Spun Tubular Beam," by P. Abeles, *Concrete and Constructional Engineering*, Vol. 35, 1940, p. 233.

From Table 13(*b*) it is seen that the results of tests made at Columbia University agree very well with those reported in Table 2. The ratios $\frac{c_t}{c}$ vary between 1.06 and 1.185; but the difference is much greater according to tests on slabs (Table 13(*a*)) and on rectangular beams (Table 13(*c*)). There is a discrepancy of 50% with a small percentage of medium-grade steel tests (Table 13(*a*)), whereas with a high-grade steel only a smaller excess of strength of 27.5% occurs. The Viennese tests, conducted by the writer, agree very well with those of Lehigh University, but in the first case slabs have been tested, and in the second case rectangular beams. Concrete 1, of very low strength, gives only an excess of strength with a small percentage by use of high-grade steel, whereas with a greater percentage no use can be made of the reinforcement, since the concrete strength is too low. In this case mild steel causes better results than high-grade steel similar to the test results (Table 13(*a*)). In neither case was the concrete strength very great. With concrete 4, of great strength, the excess of strength is greater when high-grade steel is used. (Similar results have been obtained with concretes 2 and 3.) The test results of spun tubular beams surpass those of rectangular beams and slabs materially. Attention is drawn to values $\frac{c_t}{j_0} = 2.13$ and 1.95 in the case of octagonal beams and of 3.07 (!) in case of beams with flat lower base, the concrete tensile zone being thus increased. Tests on these beams have proved that the concrete tensile zone, even at breaking stage, still resists to a large extent, in spite of the discontinuity caused by the cracks.

Even if the results of tests on tubular spun beams are eliminated, because of their different behavior, those of the tests in Tables 13(*a*) and 13(*c*) show an excess of strength of up to 50%, which fact is reason enough to study the matter more closely in order to make use of it in design. First, the fact itself may be investigated as to how these results can accord with the assumption that the yield point stress, or one connected with a definite elongation, is the reason for failure, the concrete tensile zone not cooperating due to cracks. The latter will be the case if the bond between concrete and steel is destroyed and the concrete strength is low. Actually, with increased concrete strength, the concrete area in the tensile zone cooperates to some extent, which causes the tensile stress of the reinforcement, between the cracks (especially in cases of small percentages), to be much lower than in the cracks. In the latter it ought to increase to a maximum stress calculated from the breaking moment, the tension in the concrete being neglected. Such a straining would cause the steel stresses between the cracks to increase gradually on both sides of the cracks, owing to the decreased bond, and thus obtain in the points of the cracks their greatest values. Such an irregular straining with maximum points at intervals seems to be unlikely to agree with the reality. It may be expected that a redistribution of stresses takes place and the greatest steel stresses are smaller than those computed for the crack points. This hypothesis was offered by the writer in 1936.[69] Besides, as long as the cracks are not too wide, it can be assumed that the actual steel stresses are materially greater than the yield point stress or its equivalent. Even if elongations of 5% and more are taken into account (the equivalent for yield point straining is only 0.2% and 0.4%, respectively, for permanent and total elongations), no great harm can be caused if the elongation extends only to a small length. The latter depends on concrete tensile and compressive strength, on the distribution of the reinforcement, its diameter and behavior (strain-stress diagram), etc., to avoid wide cracks and to insure a good bonding between the cracks. From this point of view, at failure, even stresses as high as the full strength may act in the steel at the cracks, as long as the latter have not surpassed a limit width. Besides, there is another possible explanation of the excess. The strain-stress diagrams of steel have been obtained on specimens of a certain length. It may be expected that the behavior of the material is quite different for a gage corresponding with the width of a crack, say, 0.2 in. From all these considerations it will be realized that further investigations would be of the greatest importance in finding the bases necessary for design. These particulars must be dependent on test results. K. Hajnal-Kónyi[70] has examined such a proposition which agreed very well with the tests reported in Table 13(*a*); but in this method only the compressive strength of the concrete is taken into account, and the results can therefore only correspond with the reality as long as a definite ratio exists between compressive and tensile strength, the respective bonding also being of importance. That method, for example, disagrees with the tests in Table 13(*c*). Concretes 2, 3, and 4, of different tensile and compressive strengths, differ little in their carrying capacities.

From this point of view it seems of greatest importance to make further investigations on beams and slabs reinforced with small percentages of steel to find the proper basis for an improvement in design, since small percentages become more and more common, owing to the use of high-grade material and of steel pre-stressing. For a definite concrete grade the basis can be found by a number of tests and can be applied to the design if definite equality of the concrete is guaranteed.

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[69] *Monatsnachrichten Oester. Beton Ver.*, April/Mai 1936.

[70] Discussion by K. Hajnal-Kónyi of "A Study of Reinforcement in Concrete Slabs," by Inge Lyse and George R. Wernisch, *Journal*, Am. Concrete Inst., Vol. 33, 1937, p. 16–1.

O. G. JULIAN,[71] M. AM. SOC. C. E. (by letter).—In 1904 Arthur Newell Talbot, Past-President, Am. Soc. C. E., stated:[72]

> "The principles governing the strength and action of reinforced concrete construction have not been fully established and opinions and theories presented by engineers are somewhat conflicting."

That statement is as true today as it was 37 years ago. This is evinced by the discussion by sixteen authorities from eight different countries[3] regarding Prof. Rudolf Saliger's method published in 1937, by numerous articles regarding the effect of plastic flow published within the past fifteen years,[73] and by the discussion on The Joint Committee Report[74] published in 1940.

Concrete is not an elastic material but has characteristics not unlike those of a viscous fluid as defined by James Clerk Maxwell:[75]

> "If the stress when it is maintained constant causes a strain or a displacement in the body which increases continually with the time, the substance is said to be viscous. When this continuous alteration of form is only produced by stresses exceeding a certain value, the substance is called a solid, however soft it may be. When the very smallest stress if continued long enough will cause a constantly increasing change of form the body must be regarded as a viscous fluid, however hard it may be."

Messrs. W. H. Glanville and F. G. Thomas state:[76]

> "From the tests at the Building Research Station, it appears that if there is such a yield stress for concrete below which no inelastic deformation occurs as a result of loading, the value of this stress is so small as to be negligible."

General practice for reinforced concrete design is based on the Bernoulli-Eulerian theory that plane sections remain plane and that stresses follow Hooke's law. Experimental evidence indicates that, neglecting the effect of shearing detrusion, plane sections remain plane but that Hooke's law does not apply. If, as was the advocated practice prior to 1924, working stresses are limited to about one third of the ultimate unit strength of test cylinders, and if the variation of the equivalent modulus of elasticity with the duration of the loading is taken into account, the order of magnitude of the actual stresses and deflections may be computed on the basis that Hooke's law is valid. On account of the number of factors involved, the evaluation of the equivalent modulus of elasticity of concrete, even for low stresses, is highly speculative. As allowable stresses are raised, a point is reached where the action of the concrete is not even approximately elastic. At this point the application of Hooke's law and the use of the concept, equivalent modulus of elasticity, should be abandoned. It is at least questionable if this point has not been passed when working stresses equal $0.45\,f'_c$ for ordinary loads ($0.60\,f'_c$ with the effect of wind loads included) as is advocated at present. A proved method of design that does not involve the highly speculative quantity E_c should be accepted as a boon to the profession. Such a method, which is also simple, easily understood, and frankly recognizes concrete for what it is—a viscous or plastic material rather than an elastic material—is presented in the author's paper. For those cases in which static loads only are involved the method appears rational and amply proved empirically. The paper is comprehensive in as far as members subjected to unilateral flexure and such flexure combined with direct stress are concerned. However, it does not explicitly cover the important case of bilateral flexure and direct stress. Practically all columns of reinforced concrete frames and especially corner columns are subjected to such moments and direct stresses. It is hoped that the author will cover explicitly this important case in his closing discussion.

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[71] Chf. Structural Engr., Jackson & Moreland, Boston, Mass.

[72] *Bulletin No. 1*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1904.

[73] "Flow of Concrete Under Sustained Compressive Stress," by Raymond E. Davis, *Proceedings*, Am. Concrete Inst., 1928, p. 303 *et seq.*; "Plain and Reinforced Concrete Arches," by Charles S. Whitney, *Journal*, Am. Concrete Inst., Vol. 3, 1931–1932, p. 485 *et seq.*; and "Studies in Reinforced Concrete, Further Investigations on the Creep or Flow of Concrete Under Load," by W. H. Glanville and F. G. Thomas, *Technical Paper No. 21*, Building Research Dept. of Scientific and Industrial Research, London, England, October, 1939.

[74] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2; discussion in September, 1940, p. 1351; November, 1940, p. 1710; December, 1940, p. 1846; February, 1941, p. 247; March, 1941, p. 457; April, 1941, p. 727; May, 1941, p. 879; and June, 1941, p. 1087.

[75] "Theory of Heat," by James Clerk Maxwell, London, Longmans, Green, 1899, p. 295; see also discussion by J. H. Griffith of "Plastic Flow in Concrete Arches," by Lorenz G. Straub, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 688.

[76] "Studies in Reinforced Concrete, Further Investigations on the Creep or Flow of Concrete Under Load," by W. H. Glanville and F. G. Thomas, *Technical Paper No. 21*, Building Research Dept. of Scientific and Industrial Research, London, England, October, 1939.

It has been proved conclusively by independent tests[77,78] that, in the case of reinforced concrete columns subjected to axial forces only, the ultimate strength is independent of the elastic moduli, plastic flow, shrinkage, and temperature effects. For such columns, provided they are adequately reinforced laterally, the ultimate strength is a function of the yield point strength of the steel and the ultimate strength of the concrete only. In such a member the concrete, being viscous, gradually flows away from the load and in so doing transfers stress to the steel. When the load becomes equal to the yield strength of the steel, the latter becomes plastic, yields, and transfers stress to the concrete. Upon application of further increments of load the steel continues to carry load equal to its yield point strength, the remainder, of necessity, being carried by the concrete. When the concrete becomes stressed to its ultimate strength, it and the column fail. The ultimate load, P, for the column is given by the well-known formula,

$$P = R\,f'_c\,A_c + f_s\,A_s \ldots\ldots\ldots\ldots\ldots\ldots (61)$$

in which R is the ratio of the ultimate strength of the concrete in the column to f'_c of comparable concrete in test cylinders. For unreinforced columns

$$R = \frac{P/A}{f'_c} \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

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[77] "Fourth Progress Report on the Column Tests Made at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 28, 1932, p. 279 *et seq.*

[78] "Fourth Progress Report on Column Tests at Lehigh University," by Inge Lyse and C. L. Kreidler, *loc. cit.*, Vol. 3, 1931–1932, p. 317 *et seq.*

Tests[77] indicate variation in the value of R from 0.73 to 1.00 for members in air storage and from 0.46 to 0.80 for comparable members stored moist. It is somewhat questionable as to whether the value 0.85 used by the author is warranted, unless a high factor of safety is used.

It appears reasonable that much the same process, as described herein for an axially loaded column, takes place in a member subjected to flexure. Eq. 22, which is derived on the basis that there is sufficient compression steel to take all the compression without exceeding the yield point, implies that the concrete flows viscously and in time transfers all its load to the steel. By taking moments about the tension steel the yield point strength of the compression steel necessary to make Eq. 22 valid is found to be

$$A'_s f'_s = -P\left(\frac{2e + d'}{2d'}\right) \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

in which the negative sign is used to signify compression. The sum of $A'_s f'_s$ and $A_s f_s$, of course, equals P. In this case the concrete contributes nothing toward resisting the load. A basic principle of the theory is that for loads of protracted duration the concrete in time relieves itself of all stress, provided the contiguous steel is not stressed beyond its yield point, and then, provided it is sufficiently strong, the concrete resists that part of the load that is not taken by the steel working at yield point stress. It should be emphasized that, in order to act in this manner, the concrete must have time to flow. This being the case, it appears that the theory may not be applicable to cases involving suddenly applied loads of transitory or cyclic nature such as impact or those resulting from the vibration of high-speed machines. It may be that the stress-strain relation for concrete subjected to very rapidly applied loads is practically linear up to the point of failure and that this point is materially different from that for static loads. Data on these points are not readily available.

The possible exclusion of the theory from the realm of impact loads leaves it a wide field of application, and there appears to be no good reason why it should not be widely used. It is interesting to note that Hardy Cross, M. Am. Soc. C. E., in 1931 stated:[79]

> "The method which we always use in America of computing the stresses in such [compressive] reinforcement on the basis of an assumed value of n is quite naive. * * * It may not be proper to go so far in this case as it is proposed to do in column design and depend upon the steel to carry load up to its yield point, but the tendency is evidently in that direction."

Since, as pointed out by the author, a balanced ultimate strength design requires several times the quantity of steel needed for a balanced design by the straight-line elastic theory, high yield point steel must be used in small beams if the design approaches balance for ultimate strength and the beams are not unduly crowded with steel.

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[79] Discussion by Hardy Cross of "Flow of Concrete Under the Action of Sustained Loads," by Raymond E. Davis and Harmer E. Davis, *Journal*, Am. Concrete Inst., Vol. 3, 1931–1932, p. 265 *et seq.*

Differentiating M with respect to f'_c in the equation given in Fig. 7, the following is obtained,

$$\frac{\partial M}{\partial f'_c} = 0.456 \left[\frac{p}{p_0} - 0.256 \left(\frac{p}{p_0} \right)^2 \right] b\, d^2 \ldots\ldots\ldots\ldots\ldots (64)$$

This quantity is small and practically equal to $0.456 \frac{p}{p_0} b\, d^2$ when $\frac{p}{p_0}$ is small but approaches $\frac{1}{3} b\, d^2$ as $\frac{p}{p_0}$ approaches unity. For a beam of given cross section there is little increase in the resistance moment for a given increase in f'_c in case the steel ratio is small; but there is considerable increase when balanced design is approached. It appears, therefore, that there is little advantage in using high-strength concrete with a low steel ratio but that there is considerable advantage in using such concrete with balanced or nearly balanced design.

It is believed that the curved stress diagrams in Figs. 2 and 4 should indicate the maximum stress at the upper extremity of the beam. The diagrams as drawn imply that, in the process of loading the member, the laminæ near the upper edge of the beam were stressed to their ultimate strength and then, instead of failing, were relieved somehow of some stress while the load was still increasing. It is understood that, as the ultimate strength of the member is approached, the laminæ nearest the top of the beam transfer stress to those nearer the neutral plane, and that this process may continue until the stress for a substantial depth of the beam is practically constant—that is, rectangular stress distribution is approached. However, that laminæ stressed to their ultimate strength (in other words, broken) transfer a part of, but not all, the stress they were carrying to adjacent laminæ appears impossible. It is difficult to imagine the stress caused by a given strain being less than that caused by a lesser strain. These stress diagrams appear to require further justification. However, the point is largely academic and in no way invalidates the author's main argument. The exact shape of the stress diagram is of secondary interest. Other authorities have assumed the stress diagram to be represented by various curves, including parabolas up to the fifth degree.[80] These may all be replaced by the hypothetical rectangular diagram used by the author and others[80,81] as a mathematical tool. This concept, although it does not indicate the location of the neutral axis, is the simplest and hence the one best adapted for use in practical design work.

It is customary, in design, to refer the moment of the external loads to the centroidal axis of the member. However, in the equation immediately following Fig. 11 and in Eqs. 23 and 31 the quantity M is referred to the axis of the tension steel rather than to the axis customarily used. It would appear preferable to omit M from these equations, as it may lead to confusion.

Regarding factor of safety the author states (see heading "Flexure and Direct Load on Rectangular Sections"), "* * * it is customary to provide a greater factor of safety for axially loaded columns than for beams * * *." Factors of safety for columns, according to current codes,[82] vary from 4.5 for lightly reinforced tied columns to 2.75 for heavily reinforced spiral columns. The actual factor of safety of over-reinforced beams is not $\frac{f'_c}{f_c}$—which, according to current codes, is 2.22—but, as has been shown by the author (Table 7), 1.62 times that amount or 3.60. Furthermore, if full cognizance is taken of the eccentric loading to which practically all columns are subjected, and the author's method of design is used, there appears to be no good reason why comparable columns and beams should not be proportioned for identical safety factors. As stated by the author the first pronounced crack in a beam may occur at about 25% of the ultimate load; the corresponding percentage for a column increases as e decreases. If the appearance of the first pronounced crack is to be taken as a criterion, there appears to be considerable justification for using a higher factor of safety for members subjected to loadings with large eccentricities than for comparable members subjected to loadings with lesser eccentricities. This is the reverse of the author's statement as to customary practice. The writer cannot concur with the author's statement (see heading "Method of Design and Factor of Safety"),

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[80] Discussion by Inge Lyse of "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., Vol. 8, 1936–1937, p. 498–21 *et seq.*

[81] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, Chap. 24, p. 168; and "Strength of Materials," by S. Timoshenko, D. Van Nostrand, 1930, Chap. 7, p. 237 (for materials with well-defined yield points).

> "Reasonably well-controlled concrete is such a reliable material at present that the use of any factor of safety to provide for sub-standard concrete can be eliminated by using f'_c as the minimum cylinder strength instead of the mean."

On jobs where considerably better than average supervision was exercised, the concrete strength was found to vary 30% either way from the mean. With sympathetic, omnipresent, and omnipotent supervision it has been found that the mean strength may be raised about 36% and the variation from this mean reduced to less than 10%. However, it is doubtful if such supervision will be available for all jobs. Miles N. Clair, Assoc. M. Am. Soc. C. E., reports[83] (under the heading "Variations of Compressive Strength of Concrete on Fourteen Projects Representing Approximately 100,000 Yards of Concrete") deviations from average 28-day compressive strength for any one project as follows:

	Plus (%)	Minus (%)
Maximum	63.0	65.6
Average	40.5	34.4
Minimum	18.5	10.1

These tests were made on concrete proportioned on the basis of 1 : 6 dry and loose by volume. The strongest cylinder for any job tested 4,980 lb per sq in., whereas the weakest tested 777 lb per sq in. The designer cannot know the minimum cylinder strength positively until 28 days after the last concrete is installed. Considering the possibility of loads being increased from the design loads, the variations that may occur in the available materials, and the difficulty of consistently obtaining proper supervision, a factor of safety of the order of 4.5 may not be excessive. This is in line with taking $f_c = 0.33\,f'_c$ in designing a beam according to the straight-line elastic theory and somewhat higher than that required by current codes. In case the loads are known definitely, satisfactory material is known to be available, and good supervision can be obtained, this factor may be reduced as found expedient.

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[82] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2, p. 65.

[83] "Concrete Technology," by Miles N. Clair, *Journal*, Boston Soc. of Civ. Engrs., July, 1940, p. 195.

CHARLES S. WHITNEY,[84] M. AM. SOC. C. E. (by letter).—The questions raised in the discussion have followed quite closely the pattern anticipated by the writer. They have for the most part been covered in the paper, but it is probable that additional consideration will help to clarify the subject.

The position that the present methods are best because they are familiar and have been used many years is not tenable unless they can be brought up to date and adjusted to new developments. Structural design methods have not advanced, in the twenty years since 1921, as much as the design of machinery and mechanical products, which have been practically revolutionized by the results of research. This has been due in part to the fact that the average structural designer is so circumscribed by codes that he thinks in terms of standard methods instead of underlying principles. A large proportion of structural research has been directed at the determination of factors involved in standard formulas without much examination of the meaning of the formulas themselves. The existence of the standard method has a tendency to direct the thinking of both the designer and the teacher and produce great inertia against the introduction of new ideas in practice. This inertia is of great value in so far as it protects against the adoption of unsound ideas, but it should not be permitted to discourage progress.

The plastic theory of reinforced concrete design has not been presented as a perfected standard but as a basis for research leading toward modernization of standard practice. It is the result of an attempt to build a practical design theory consistent with all pertinent facts that have been discovered regarding the nature of the materials, not with a few incomplete characteristics selected for convenience. There is some resistance to the conception of the rectangular stress block to represent the compressive stress in the concrete. Dean Grinter and Professor Caughey suggest that it is so contrary to physically observable phenomena that it is not sufficiently reasonable to be taught to students. The writer was interested in this reaction and has discussed it with a number of students and teachers who expressed themselves as being entirely satisfied with the use of the rectangle. It may well be that it would rouse the students' interest and stimulate them to pay more attention to the reasons for it, whereas the triangular distribution is so easily accepted that it leads to superficial consideration and a lack of understanding of the real conditions.

As a matter of fact, if the proper phenomena are observed, the rectangular block is truly consistent. There is no evidence that a better shape can be devised nor that any amount of research will ever lead to an accurate predetermination of the actual stress distribution under varying conditions.

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[84] Cons. Engr., Milwaukee, Wis.

Fortunately, the determination of the actual stress distribution is unimportant if the total stress and its line of action are known. The stress-strain curves of Fig. 1 were given merely to indicate why the assumption of an equivalent rectangle is perfectly rational. Whether or not the maximum compressive stress at failure of the beam occurs at the face or in the interior is also unimportant.

The writer believes that there is a reduction in the stress toward the compressive face but that it may be small in amount and may occur very close to the ultimate load, especially in the case of high-strength concrete. He has observed the failure of concrete cylinders of strengths ranging up to 9,000 lb per sq in. and appreciates the reason for Mr. Hadley's skepticism regarding the descending portion of the stress-strain curve. With low-strength concrete, it is a simple matter to observe that beyond the maximum load the stress drops off gradually as the strain increases. With high-strength concrete, however, the failure appears to be sudden, and it is only at very slow testing speeds that a reduction in load just before failure can be noted. It should be realized that there is a sudden increase in strain that accompanies the sudden decrease in stress, due to the elasticity of the testing machine, and the stress-strain curve does not drop vertically.

In a beam, the concrete at the face is supported by the concrete in the interior, and compression failure does not occur until a sufficient area of the interior concrete is highly stressed. While the interior is being stressed, plastic flow effects a reduction of the stress at the face.

The writer does not feel that the stress-strain relation is sufficiently determinate to justify its direct application as suggested by Mr. Contini.

Since this paper was submitted for publication, the writer has received from Anton Brandtzaeg, M. Am. Soc. C. E., a copy of the report[85] on his tests to determine the strain and stress distribution in concrete members under bending or eccentric loading. The tests were made on heavily reinforced beams and struts about 6 in. deep, of concrete with cylinder strengths ranging from about 1,250 to 5,000 lb per sq in. Mr. Brandtzaeg assumed a stress distribution in the concrete as shown in Fig. 27.

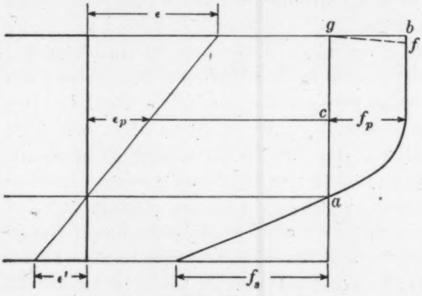

FIG. 27

The location of the neutral axis, a, is determined from the maximum strains in the concrete and steel, ϵ and ϵ', assuming the sections to remain plane. At c, where the strain is equal to the maximum strain given by prism tests, the stress is assumed to be equal to the prism strength, f_P, and the variation from a to c follows the stress-strain curve for the prism or an equivalent parabola.

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[85] "Der Bruchspannungzustand und der Sicherheitsgrad von rechteckigen Eisenbetonquerschnitten unter Biegung oder aussermittigem Druck," by Anton Brandtzaeg, Norwegian Technical High School, Trondheim, 1935.

From c to g at the compression face, the stress is uniform and equal to the prism strength. Mr. Brandtzaeg suggests that there may be a falling off of the stress along such a line as $f\,g$ as the maximum load is approached but he does not consider it important. Rudolph Saliger reports[86] indications of a falling off similar to that in Fig. 4. Mr. Brandtzaeg reports that the loads and lever arms calculated according to Fig. 27 agreed satisfactorily with the actual measurements.

The value of $\frac{\epsilon}{\epsilon_P}$ for his strongest concrete (equivalent $f'_c = 5{,}000$ lb per sq in.) was 2.13, and for the weakest (equivalent $f'_c = 1{,}250$ lb per sq in.) it was 6.87. The value of prism strength f_P was 0.75 to 0.80 times the strength of 30-cm cubes. If the cube strength is assumed at 1.13 times the cylinder strength, this would be 85% to 90% of the cylinder strength, which agrees very well with the writer's theory. Mr. Brandtzaeg also suggests the use of an equivalent rectangle, but his solution involves the ratio $\frac{\epsilon}{\epsilon_P}$, which is rather erratic although it depends in a general way on the strength of the concrete.

TABLE 14.—COMPARISON OF ACTUAL AND THEORETICAL LOADS FOR THOMAS TESTS

SERIES II; f'_c (AVERAGE) = 1,960 LB PER SQ IN.		SERIES III; f'_c (AVERAGE) = 4,640 LB PER SQ IN.	
Item No.	Ratio[a]	Item No.	Ratio[a]
6	0.919	12	0.778
7[b]	0.993	13[c]	1.059
8	0.922	14[b]	0.787
9	1.007	15	0.845
10	0.827	16	0.875
11	0.905	17	0.754
....	....	18	0.860
....	....	19	0.860
Average	0.930	Average	0.853

[a] Actual load divided by computed load (see Col. 6, Table 10).

[b] Fixed ends.

[c] $f'_c = 5{,}400$ lb per sq in.

Although the writer believes with Professor Ernst that more tests are needed, he does not agree that the tests already made indicate that the equations are not correct for concrete strengths up to 5,000 lb per sq in. In his analysis of the Thomas tests, Professor Ernst used in the formulas the minimum concrete strength and the average steel strength for each series. Therefore, he did not get as true a check on the accuracy of the formula as he would have if correct values were used in each case. If the individual strengths are used for each case, the comparison of actual to computed strengths of the columns is given in Table 14 following the form of Professor Ernst's Table 10.

TABLE 15.—RELATIONS BETWEEN COLUMN AND CYLINDER STRENGTHS

Description	Air storage				Moist storage			
6-in. by 12-in. cylinder strength, in lb per sq in.	2,515	4,565	5,705	Average	2,685	4,445	6,510	Average
Column strength divided by cylinder strength	0.92	0.79	0.88	0.86	0.71	0.68	0.75	0.71

The writer believes that these results are probably caused by two conditions:

First, inadequate anchorage of the reinforcing steel caused slipping of the bars and splitting of the concrete ends. Contrary to Professor Ernst's statement, the writer believes this would affect the tests with large eccentricity, as well as the others, and Table 14 indicates no consistent trend. There is no doubt but that the efficiency of the specimens was reduced by the inadequate ends.

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[86] "Versuche über Zielsichere Betonbildung und an druckbewerten Balken," by Rudolph Saliger, *Beton und Eisen*, Nos. 1 and 2, 1935.

Second, the control tests were made on 6-in. cubes and the tests on plain concrete columns made at the same time showed the following relations between column and cube strengths:

6-in. cube strength, in lb per sq in.	Column strength divided by cube strength
2,460	0.67
5,780	0.61

This reduction of the ratio of column-to-cube strength for the stronger concrete might be expected to account for a reduction in the strength of the reinforced columns. This may indicate that the standard 6-in. by 12-in. cylinder is a better control specimen than the 6-in. cube because the column tests made at the University of Illinois showed no such trend. The latter are reported in Table 15.[87]

Even more conclusive evidence that there is no reduction in the relative capacity of members of concrete with strengths up to at least 5,000 lb per sq in. is furnished by the high-strength specimens of the Slater and Lyse tests (Table 1) and the Brandtzaeg tests. In the range above 5,000 lb per sq in., evidence is lacking. There is no definite explanation for the lack of strength of the Richart and Olson specimens, but it is significant that those under axial load were also deficient.

The writer does not agree with Professor Abbett that the plastic theory is appropriate only for cases of sustained loading. The equations are based on tests made at ordinary testing speeds and should be suitable for general use. They are intended to indicate the strength of the member accurately so that the proper factor of safety can be provided. If they accomplish that, the calculation of theoretical working stresses is of no particular value. The writer believes that the theoretical stresses calculated by the straight-line formula are inaccurate and not satisfactorily indicative of the true factor of safety, especially for the higher percentages of steel. In that case, the straight-line formula indicates steel stresses lower and concrete stresses higher than the actual stresses to such a variable extent that proper interpretation of the results is difficult or impossible.

The significance of the comparison made by Mr. Stewart is questionable because there appears to be no justification for the assumption of a fixed value of $k\,d$. The sections do not have equal strengths, either actually or theoretically, and therefore are not comparable. Using the same bending moment (26,600 in-lb) and unit costs (steel at 5 cents per lb and concrete at $16 per cu yd), the writer has made a comparison of costs for different percentages of steel according to the plastic theory and for balanced reinforcement according to the straight-line formula. This comparison is shown in Table 16.

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[87] "Fourth Progress Report on the Column Tests Made at the University of Illinois," by F. E. Richart and G. C. Staehle, *Journal*, Am. Concrete Inst., Vol. 28, 1932, p. 296.

This indicates very little difference in cost of beams of low percentages of reinforcement designed by the two methods. The plastic-theory formula requires a little less steel than the straight-line formula when p is less than 0.01 in. It also indicates a minimum cost at about 2% of steel for the conditions assumed, but several factors may operate to raise the most economical percentage above that value. They are the cost of forms, variation in dead load, and the fact that only a part of the beam requires maximum reinforcement. It may be cheaper to use a higher percentage of steel in that part than to increase the concrete section for the full length. This is especially true at the ends of continuous beams where haunches or deep stems may be avoided. The general use of the maximum percentage of steel will probably not be economical, but it will undoubtedly be desirable in special cases. Mr. Eremin may have had this in mind when he said the formulas require an excessive percentage of steel in deep girders, but the writer does not understand that there is any difference in their application to deep or shallow sections. The designer is left free to determine the most economical percentage of steel in any case, and need not use the maximum.

TABLE 16.—COMPARISON OF COST OF BEAMS

d (inches)	p	A_s, in sq in.	$\frac{c}{d}$	Cost per lin ft
PLASTIC THEORY (f'_c = 3,000 LB PER SQ IN.; f_s = 50,000 LB PER SQ IN.; FACTOR OF SAFETY = 2.5; AND b = 12 IN.)				
12.5	0.010	1.46	0.89	$0.964
10.6	0.0136	1.74	0.865	0.920
9.1	0.020	2.19	0.802	0.904
8.4	0.025	2.52	0.755	0.943
8.2	0.0274	2.70	0.731	0.959
STRAIGHT-LINE FORMULA (f_c = 1,350 LB PER SQ IN.; f_s = 20,000 LB PER SQ IN.; n = 10; AND b = 12 IN.)				
10.6	0.136	1.74	0.866	$0.920

A treatment of irregular cases such as is suggested by Mr. Polivka should be possible, but the writer believes it is a mistake to base calculations on the full section of the concrete. When the steel on one side is in tension, the concrete between the tensile steel and the outside of the concrete has no effect on the strength of the member and could not influence the location of the neutral axis nor the compression area.

Mr. Abeles is correct in his statement that further investigation is necessary to determine the proper basis of design with small percentages of high-strength steel. The apparent excess strength as compared with mild steel is no doubt due to the lack of a definite yield point. The writer has adopted what seems to be a conservative definition of yield point (the stress corresponding to a total unit strain of 0.004), and, if it can be proved that the actual useful strength is greater with high-strength steel than the equations indicate, an allowance might be made for it by changing the definition of the yield point, using a greater strain. There is not sufficient data to prove whether or not 0.004 is the best value.

Mr. Julian's discussion of the suggestion that "* * * any factor of safety to provide for sub-standard concrete can be eliminated by using f'_c as the minimum cylinder strength instead of the mean" indicates that the writer has not made his meaning clear. The writer wishes to propose that the value of f'_c be selected so as to provide in itself the proper factor of safety for substandard concrete, taking into account the job conditions. The concrete will then be designed for a strength higher than f'_c by an amount depending on the accuracy of the job control that is to be expected. This will have the effect of focusing attention on the concrete strength and simplifying the design by eliminating consideration of that factor from the design formulas. It will help to avoid the application of a standard factor of safety to jobs where it should not be used because of a variation in the concrete control methods.

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DYNAMIC STRESS ANALYSIS OF RAILWAY BRIDGES

By R. K. Bernhard,[1] M. Am. Soc. C. E.

With Discussion by Messrs. R. S. Chew, and R. K. Bernhard.

Synopsis

Since it is a known fact that stresses due to dynamic impact often become an important factor in the design of a railway bridge, it is the purpose of this paper to indicate a simple method by which these stresses may be determined. In order that this method may be presented in the shortest manner, it will be restricted first to the main trusses or girders of single-track spans with known static loads. It is assumed that, as a steam locomotive crosses such a span at the "critical speed," the greatest vibrations are set up by the action of the free and unbalanced centrifugal forces of the moving parts. The various formulas deduced herein are set up in the form of nomographic charts. Seven of these are located side by side for ease of manipulation. If the damping capacity of any vibrating structure is known, the true value of the stresses due to impact can be read from the chart. One problem is worked through, showing the advantages, and also the limitations, of the method. Finally, a comparison is given with the impact percentage of standard specifications.

Introduction

One of the members of a steel railway bridge in Europe failed recently while a train of comparatively light weight was passing over it at a relatively low speed. The result has been a revival of the discussion of dynamic overloads. This question is often "sidetracked," particularly by practical field engineers, since purely mathematical methods[2–10] of developing the analysis may become extremely involved.

Note.—Published in January, 1941, *Proceedings*.

[1] Prof. and Head, Dept. of Eng. Mechanics, The Pennsylvania State College, State College, Pa.

[2] "Discussion of Differential Equations Relating to the Failure of Railroad Bridges," by G. G. Stokes, *Transactions*, Cambridge Philosophical Soc., Vol. VIII (1849), p. 707.

[3] "Vibration of a Girder Under Moving Loads," by H. Zimmerman, W. Ernst and Sohn, Berlin, 1896.

[4] "Forced Vibrations of Prismatic Bars," by S. Timoshenko, *Journal für Mathematik und Physik*, Vol. 59 (1911), p. 163.

[5] "Impact in Steel Railway Bridges of Simple Span," by J. B. Hunley, A.R.E.A., Vol. 37, No. 380, October, 1935.

(footnotes continued on next page)

Most impact formulas of standard specifications, which make an allowance for the dynamic increment in the design of bridges, are derived more or less by experiment. Experimental methods for finding the impact value by field tests, using various types of measuring apparatus, are usually rather time-consuming. Even if the matter is approached from a strictly mathematical point of view, only approximate solutions can be obtained. For some time past many investigators have attempted to find a method of computing this impact value which would combine sufficient accuracy with ease of manipulation. One of the most comprehensive papers on this subject was published by J. B. Hunley in 1935.[5]

Following are nomographic charts which have been developed leading to an approximate solution of the dynamic problem by combining theoretical investigations with the results of numerous field tests that have been conducted both in the United States and in Europe.

There are various methods of including the impact factor in the design, but the simplest seems to be to make the computation in two steps: First, the structure should be designed for dead load and moving load in the usual static manner; and second, a supplementary computation must be made of the stresses caused by those locomotives which produce the maximum dynamic effect. This second computation is often the determining factor because, although the static investigation is based on the heaviest moving loads, larger dynamic stresses due to resonance effects may be set up by light or intermediate loads.

The computation as given herein does not provide a general solution for all possible cases. For the sake of simplicity, the method is made applicable only to such conditions as often occur in designs. Among these may be mentioned single-track, simple spans from 60 to 300 ft in length.

ASSUMPTIONS

The following assumptions are made:

(1) For main bridge girders the largest dynamic effect will occur at the critical speed. This critical speed is that at which the period of the unbalanced masses of the steam locomotive drivers, causing alternating forces of a sine form, coincides approximately with the natural frequency of the bridge. Hence, the fundamental consideration of dynamic design is based on this resonance case.

(2) Supplementary weights to equalize, partly, the reciprocating masses of steam locomotives passing at their highest allowable speed may cause centrifugal forces attaining, as a maximum, 15% of the vertical loads of drivers.[6]

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[6] "Vibration During Acceleration through a Critical Speed," by F. M. Lewis, *Transactions*, A.S.M.E., 1932, APM-54-24-253.

[7] "Report of the Bridge Stress Committee," by C. E. Inglis; published under the authority of His Majesty's Stationery Office, London, 1928, p. 201.

[8] "Vibration of Locomotive Driving Wheels Caused by Unbalance," by O. J. Horger and C. W. Nelson, *Journal of Applied Mechanics*, Vol. 6, No. 4, December, 1939.

[9] "Impact Stresses and Vibrations of Main Girders for Simple Span Railroad Bridges," by W. Hort, *Bautechnik*, 1928, No. 3, p. 37, and No. 4, p. 50.

[10] "Mathematical Studies of Vibrations of Girders in Particular Cases," by R. Desprets; publication of the International Railroad Assoc., 1930 (Madrid) and the International Bridge Assoc., 1930 (Luettich).

(3) The damping capacities of bridges are taken from experimental values of published tests.[11,12,13]

(4) Furthermore, experiments have shown that in the resonance case periodic forces may cause an amplifying factor of 16, with the loads acting in the middle of the span only. This decreases to 13, with the same loads moving.[9] The reduction factor becomes, therefore, 13 : 16 = 0.81. Hence, to simplify the design, the effect of the motion of the loads (the increasing influence in the center of the bridge and the decreasing influence at the ends) may be taken into consideration by this reduction factor.

(5) The main girders in the resonance case may oscillate in their fundamental vertical mode; for trusses, therefore, one impact value for all members is justified. Other periodic vibrations, such as horizontal or torsional vibrations, or even nonperiodic vibrations, are negligible in general. Theoretical derivations have proved that they do not exceed 7%, assuming the dynamic stresses due to the wheel effect at resonance to be 100%.[9] Old locomotives of British railways produced a strong "hammer-blow" effect.[7] The lifting of the driving wheels from the rails causes heavy shocks,[8] which, however, are of primary importance for the rails and track girders only. Hence, nonperiodic vibrations, excited by direct impact on the track girders (rail joints or flat spots on wheels), which in certain cases might increase the impact value for single members of the main girders (overtones), are neglected.

(6) The dynamic design is developed only for statically determinate main girders and trusses of railroad bridges, with span lengths from 60 ft to 300 ft. The wheel effect does not come into consideration for span lengths less than 60 ft, since the critical speed is usually higher than the highest allowable train speed. The nomographic charts herein take into consideration all possible maximum and minimum values of the dynamic properties of loaded and unloaded bridges within the aforementioned span range.

COMPUTATION

Determination of the Natural Frequency of the Bridge.—The natural frequency ω_n of the bridge in cycles per second may be computed by the formula:[13]

$$\omega_n = \frac{\pi}{2}\sqrt{\frac{5}{384}\frac{g}{y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which (see Appendix) g = acceleration due to gravity in feet per second per second and y = deflection of the bridge in feet.

In order to produce the deflection for the equivalent load, two extreme values must be taken into consideration: The minimum, being the dead load alone, and the maximum, being the dead load combined with the load of all vehicles on the bridge. For the moving loads, a uniformly distributed load

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[11] "Dynamic Tests by Means of Induced Vibrations," by R. K. Bernhard, *Proceedings*, A.S.T.M., Vol. 37, Pt II, pp. 634–649.

[12] "Mechanical Vibrations of Bridges," by E. Homann and R. K. Bernhard, *Verkehrstechnische Lehrmittelgesellschaft*, Berlin, 1933.

[13] "Dynamic Relations Between Moving Loads and Structures," by R. K. Bernhard, *Mechanical Engineering*, Vol. 60, No. 9, p. 697 et seq.; and "Dynamic Method of Investigating Stresses in Buildings," by R. K. Bernhard and W. Spaeth, *Der Stahlbau*, No. 6, 1926.

may be assumed. In most cases, the natural period with dead plus moving load is critical.

Determination of Critical Speed, Caused by Wheel Effect on the Bridge.—The critical speed in miles per hour may be determined by the equation:

$$V_c = 0.68\,\pi\,D\,\omega_n \qquad (2)$$

in which D = diameter in feet of the drivers of the passing locomotive.

The maximum and minimum values of V_c are determined by assuming the largest and smallest diameters of the drivers of the passing locomotives. A special case is the critical speed for wheels with flat spots on heavy freight cars with small wheel diameters. It is obvious that if the critical speed is higher than the highest allowable train speed, no further dynamic investigation is necessary.

Determination of the Number of Vertical Impulses Caused by Unbalanced Masses of the Passing Steam Locomotives.—The number of impulses, N_i, alternating with a sine form caused by unbalanced masses of a single driving wheel, may be determined by the formula:

$$N_i = \frac{L}{\pi D} \qquad (3)$$

in which L = length of span in feet.

Determination of the Number of Vertical Impulses Necessary to Build up Oscillation.—The logarithmic damping decrement (θ) of bridges tested in recent years by the writer varies between 0.05 and 0.20.[11] It may be determined by recording power-input frequency diagrams (resonance curves) by means of mechanical oscillators. The width at the half point of the diagram, divided by the resonance frequency and multiplied by π, is equal to the damping decrement.[12] An average value of $\theta = 0.075$ may be suggested until more results are available. The nomographic charts, however, permit the use of maximum and minimum values.

The amplifying factor, F, for the resonance case (stable stage) and the number of periods, N_t, necessary to build up the oscillation (transient stage) may be determined with sufficient accuracy from the decrement, θ, by the following equations:[13]

$$F = \frac{\pi}{\theta} \qquad (4a)$$

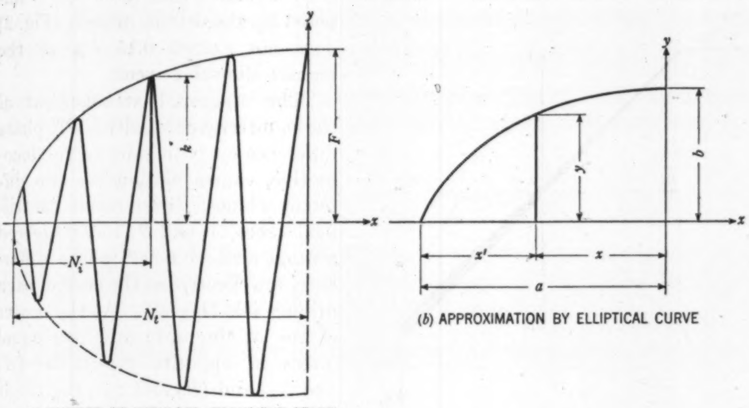
and

$$N_t = \frac{2.3}{\theta} \qquad (4b)$$

In other words, F is the multiplication factor for obtaining the maximum influence for the dynamic forces in the resonance case.

The time to build up the oscillation in the resonance case is determined by the number of cycles that are necessary to reach the maximum excursion (stable stage). To attain this stable equilibrium (that is, constant amplitudes) an infinite time is necessary; to reach a finite time value, however, it may be assumed that the equilibrium is reached as soon as the amplitudes have increased up to nine tenths of their final theoretical value.[12]

The computation is still accurate enough if, for the sake of simplicity, the amplitudes are assumed to increase from 0 to N_t cycles on an elliptic envelope (Fig. 1(*a*)). The following simple formula can be developed from Fig. 1(*b*):



(*a*) NUMBER OF IMPULSES (TRANSIENT STAGE)

(*b*) APPROXIMATION BY ELLIPTICAL CURVE

FIG. 1.—BUILDING UP VIBRATIONS

$$y = \frac{b}{a} \sqrt{a^2 - x^2} \qquad (5a)$$

or, for $x' = a - x$:

$$y = \frac{b}{a} \sqrt{2\,a\,x' - (x')^2} \qquad (5b)$$

Substitute $y = k$; $b = F$; $a = N_t$; and $x' = N_i$.

$$k = \frac{F}{N_t} \sqrt{2\,N_t\,N_i - N_i^2} \qquad (6)$$

The value k represents that part of the amplifying factor which, after N_t cycles, is still effective.

Substituting for F and N_t the values of Eq. 4,

$$k = 1.365 \sqrt{2\,N_t\,N_i - N_i^2} \qquad (7)$$

For most practical cases, the first part of the curve with small, and the last part with large, values of k do not occur; hence, the assumption of the elliptical envelope is accurate enough.

For further simplification, the increasing influence caused by the motion of the live loads near the center of the span, as well as the decreasing effect at the ends, may be taken into consideration by a reduction factor (experimental value) of 0.81:

$$k_w = 0.81 \times 1.365 \sqrt{2\,N_t\,N_i - N_i^2} = 1.11 \sqrt{2\,N_t\,N_i - N_i^2} \qquad (8)$$

Hence, k_w is equal to that part of the amplifying factor (F) caused by vertical forces of unbalanced locomotive drivers which is effective after N_i impulses. The symbol k_w may be called the amplifying number.

Determination of the Additional Vertical Forces.—The maximum value of the additional vertical forces can be determined by assuming that the load P_A imposed by the driving wheels (Fig. 2) does not exceed 0.15 P_A at the highest allowable speed.

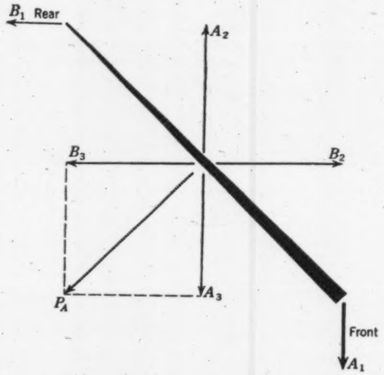


FIG. 2.—DISTRIBUTION OF UNBALANCED FORCES OF STEAM LOCOMOTIVE DRIVER AXLES

The staggered arrangement of the counterweights, with a 90° phase difference on both sides of the locomotive, causes motions in two different planes. Two equal centrifugal forces, A_1 and B_1, in a staggered arrangement of a 90° phase difference, are effective at the ends of each driving axle (Fig. 2). At the center of the driving axle add two equal forces in opposite directions—(A_2 and A_3) and (B_2 and B_3); the result is two staggered couples, composed of forces (A_1 and A_2) and (B_1 and B_2) with a 90° phase difference.

The remainder may be replaced by two staggered single forces, A_3 and B_3, with a 90° phase difference, and a resultant force $P_A = A\sqrt{2} = 1.4\,A$, assuming that $A_3 = B_3 = A$.

The first couple (forces A_1 and A_2) will excite the bridge to torsional vibrations; the second couple of forces (B_1 and B_2) will excite it to horizontal vibrations. Both couples may be neglected in the foregoing case. Only the vertical component $P_A = A\sqrt{2}$, alternating in a sine form and effective along the center line of the bridge, is to be considered. The numerical example has been computed, however, with the wheel load P_a, and not with $\frac{1}{2}$ times the vertical component P_A. This is to simplify any comparison with the static calculation, where often wheel loads of the theoretical train (standard specifications) are used. Finally, the free forces of the different driving wheels of one locomotive may be combined into one average force Z.

Knowing that the centrifugal forces increase with the square of the speed (angular velocity), the following equation is valid:

$$Z = 0.15 \times N_w \times P_A \left(\frac{V_c}{V_m}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which V_m = highest allowable speed of the locomotive, and N_w = number of driving wheels of one locomotive.

The assumption is made that the case of two or even more locomotives on the bridge, with their counterweights in phase at the same time, is a rare exception which need not be taken into consideration; furthermore, the damping influence of adjacent cars, especially on longer bridges, may be neglected.

The effective vertical additional forces to build up the vibration are:

$$Z_w = k_w \times Z \quad \text{(10)}$$

Hence, Z_w is the maximum dynamic load on the bridge in tons, which becomes effective during the passage of the locomotive. In other words, it is the load that builds up the maximum oscillation under most unfavorable dynamic conditions.

Determination of Dynamic Impact.—The total stress is a combination of a static pre-stress and a superimposed dynamic stress (Fig. 3). The maximum value of the static pre-stress may be taken from the routine static computation, but without impact, or from a design based upon the actual static loads of the previously chosen locomotive, whichever causes the most unfavorable conditions.

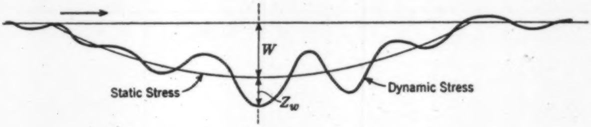


FIG. 3.—STATIC AND DYNAMIC STRESS DIAGRAM OF BRIDGE

This static load may be equal to W tons. The dynamic load has already been determined by Eq. 10 to be Z_w tons. Eq. 11 gives the final dynamic impact number:

$$\phi = \frac{W + Z_w}{W} \quad \text{(11)}$$

Hence, to obtain the maximum static and dynamic stresses (total stress) for a certain vehicle (locomotive), the static loads must be multiplied by ϕ. Thus, for special cases, ϕ may replace the impact value in standard specifications.

NOMOGRAPHIC CHARTS

The nomographic charts in Figs. 4 and 5 make it convenient to determine the dynamic effect for an existing bridge, as well as for a new bridge design. Normally, space permitting, these charts should all be on the same sheet. In many cases it is sufficiently accurate to determine the final value with seven adjacent nomographic charts and one double scale. Networks of curves have the advantage of being independent of the quality of the drawing paper (no changing scale through humidity); however, they may cause confusion among different curves.

A special advantage of the nomographic chart is the easy determination of maximum and minimum values simultaneously. All scales and indicators, corresponding to Eqs. 1 to 11, could be drawn in logarithmic scales.

The seven nomographic charts in Figs. 4 and 5 may be constructed as a simple mechanism. The connecting lines can be drawn on celluloid strips, mounted with flexible links on the scales. After having determined the fundamental values on the proper scales, the result will be indicated automatically.

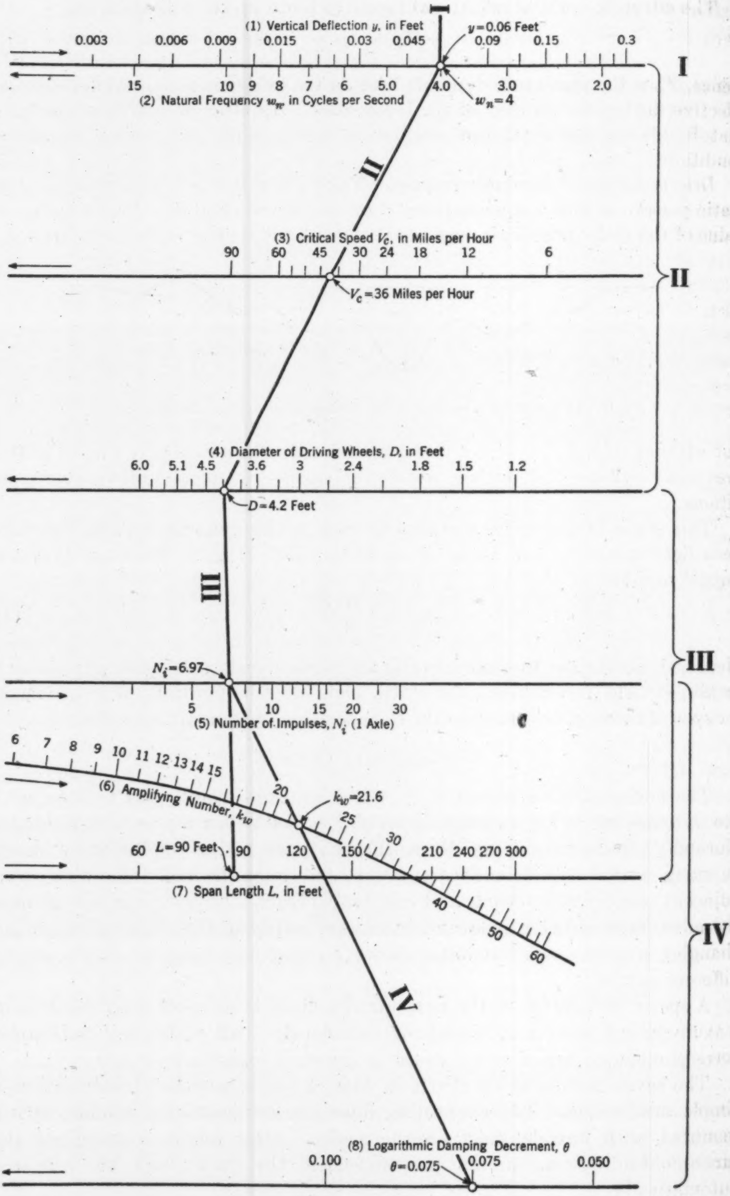


FIG. 4.—NOMOGRAPHS I TO IV

Certain cases may yield a dynamic impact number as great (for example) as $\phi = 3$, which is higher than that required by standard specifications. It must be borne in mind, however, that such a high dynamic impact number is not necessarily dangerous. Assume an allowable stress of 16,000 lb per sq in., an impact value, corresponding to the standard specification, of 1.4, and the same train loads as in the static design. The live load, including $I = 140\%$, may cause five eighths of the allowable stress ($\frac{5}{8} \times 16{,}000 = 10{,}000$ lb per sq in.). Hence, the combined stress of dead and live loads will be increased to $6{,}000 + 10{,}000 \times \frac{3}{1.4} = 27{,}400$ lb per sq in. This value is lower than the yield strength, but might be higher than the endurance limit of the steel (punched or

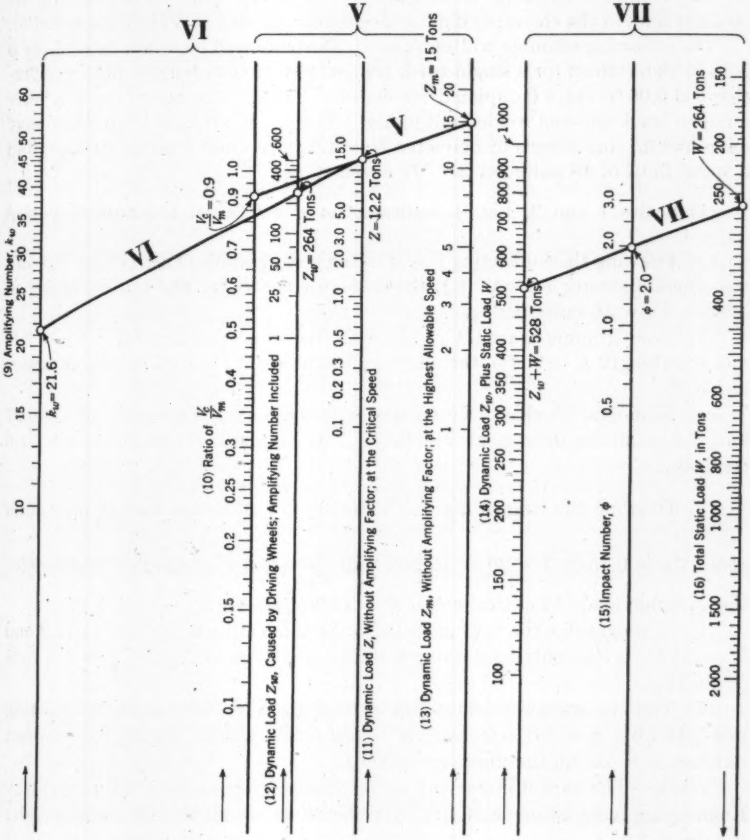


FIG. 5.—NOMOGRAPHS V TO VII

drilled structural steel). However, the bridge will stand the traffic for many years without failure, especially if the traffic is light (effect of repose).

NUMERICAL EXAMPLE

The complete dynamic investigation of a bridge consists in drawing six straight lines in the corresponding nomographic charts I to VII (Figs. 4 and 5).

The following example will serve as an illustration: The dynamic number ϕ is to be determined for a single-track bridge of 90-ft span length, with a deflection y of 0.06 ft, and a damping decrement θ of 0.075. The structure may have an open trackway and welded rail joints. The passing freight locomotive may have five driving wheels of diameter $D = 4.2$ ft, a wheel load of 20 tons, and a speed limit of 40 miles per hr. In seven steps:

(1) Scales 1 and 2, Fig. 4, indicate, for $y = 0.06$ ft, the natural period $\omega_n = 4$.

(2) Drawing the connecting line II in the nomographic chart II ($\omega_n = 4$ and the wheel diameter $D = 4.2$ ft), the indicator shows for the critical speed in scale 3: $V_c = 36$ miles per hr.

(3) Nomographic chart III yields, with the connecting line III ($D = 4.2$ ft and span length $L = 90$ ft), the number of impulses $N_i = 6.97$ on the indicator (scale 5).

(4) Nomographic chart IV indicates, with the connecting line IV ($N_i = 6.97$ and the damping decrement $\theta = 0.075$), an amplifying number: $k_w = 21.6$ (scale 6).

(5) Drawing the connecting line V in Fig. 5 $\left(\text{dynamic load at maximum speed } Z_m = 0.15 \times 5 \times 20 = 15 \text{ tons and } \frac{V_c}{V_m} = 0.9\right)$, the indicator shows for the dynamic load: At critical speed $Z = 12.2$ (scale 11).

(6) Nomographic chart VI gives, with the connecting line VI ($Z = 12.2$ and $k_w = 21.6$), the amplifying number included, the dynamic load: $Z_w = 264$ tons (scale 12).

(7) The final answer in chart VII is determined by drawing the connecting line VII ($W_{90\text{ ft}} = 264$ tons and $W + Z_w = 528$ tons), yielding the impact number: $\phi = 2.0$ on the indicator (scale 15).

Result.—The impact increment, corresponding to the old American Railway Engineering Association (A.R.E.A.) specifications, would be 1.79 and is smaller than $\phi = 2.0$.

The maximum stress caused by the live load, including $I = 179\%$, may be five eighths of the total stress $= \frac{5}{8} \times 16{,}000 = 10{,}000$ lb per sq in., or, taking ϕ into consideration, the stress is $10{,}000 \times \frac{2.0}{1.79} = 11{,}200$ lb per sq in. The stress caused by the dead load is three eighths of the total load; hence, $\frac{3}{8} \times 16{,}000 = 6{,}000$ lb per sq in., and the total stress, therefore, becomes, 17,200 lb per sq in.

The dynamic investigation proves in this case that the endurance limit for punched or drilled structural steel, which may be assumed as one fourth of the ultimate tensile strength ($0.25 \times 65{,}000 = 16{,}250$ lb per sq in.), is about 1,000 lb per sq in. smaller than the maximum dynamic stress.

CONCLUSIONS

The example presented herein can be supplemented to include span lengths from 60 ft to 300 ft and three different and comparatively light two-cylinder steam locomotives. The three locomotives are: One high-speed locomotive with three pairs of driving wheels of 6.9-ft diameter, 20-ton wheel load, and an allowable speed of 75 miles per hr; one medium-speed locomotive with three pairs of driving wheels of 5-ft diameter, 15-ton wheel load, and an allowable maximum speed of 55 miles per hr; and the aforementioned freight locomotive. The freight locomotive, with its five pairs of small drivers and relatively slow allowable speed, but comparatively large wheel loads, causes the highest dynamic impact.

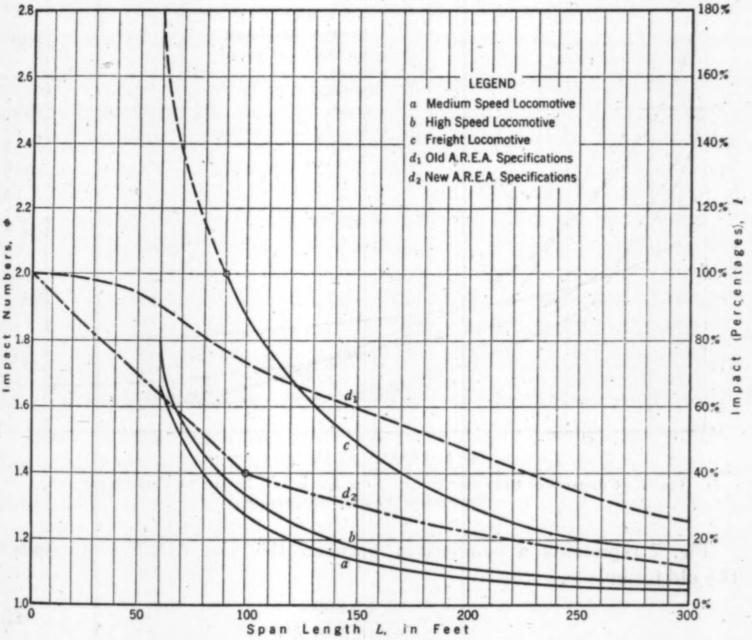


FIG. 6.—COMPARISON BETWEEN IMPACT NUMBERS ϕ OF THREE DIFFERENT TYPES OF LOCOMOTIVES AND A.R.E.A. SPECIFICATIONS

This result has often been verified in field tests.[5,7,9,13] It is not the two-cylinder high-speed and medium-speed steam locomotives, but nearly always two-cylinder freight engines with small driving wheels and relatively slow allowable speeds, which cause the greatest dynamic stresses. The numerous impulses on the bridge, and the centrifugal forces growing rapidly to their maximum value, are predominant.

For high-speed passenger service, steam locomotives with three or more cylinders, electric engines, or light-weight motor-driven engines may come more and more into favor. For freight traffic, the heavy two-cylinder locomotive with its high impact influence cannot be replaced so easily.

Fig. 6 shows clearly the increasing value of ϕ for bridges with shorter span lengths (60 ft to 120 ft). Similar results are reported by other investigators;[4,7] for relatively long bridges (more than 300 ft), as well as for short bridges (less than 90 ft), the dynamic influence caused by resonance between the unbalanced masses of steam locomotive parts and the natural frequency of the bridge decreases considerably. The tendency of the ϕ-curves, which decrease gradually with the span length (over 90 ft), is the same as in the impact curves of standard specification.

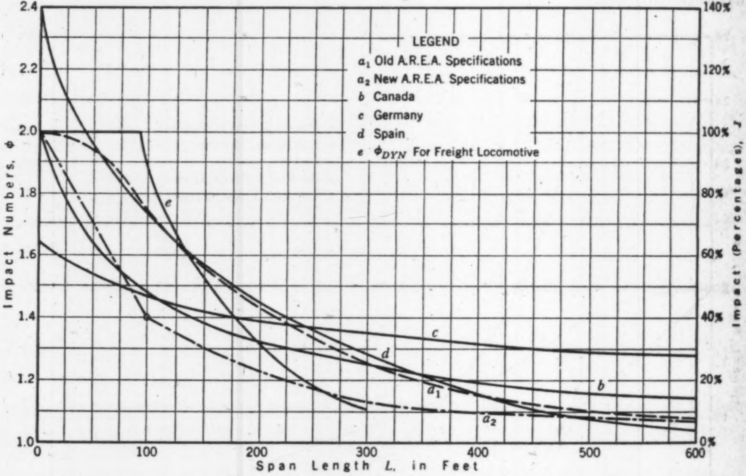


FIG. 7.—COMPARISON BETWEEN IMPACT NUMBERS ϕ AND IMPACT IN PERCENTAGE FOR BRIDGES IN VARIOUS COUNTRIES

Fig. 7 represents a comparison between the two A.R.E.A. formulas:[5] The old formula—

$$I = \frac{30{,}000}{30{,}000 + L^2} \qquad (12)$$

and the new formulas—

$$I = 100 - 0.60\,L\ (L \leq 100 \text{ ft}) \qquad (13a)$$

and

$$I = \frac{1{,}800}{L - 40} + 10\ (L \geq 100 \text{ ft}) \qquad (13b)$$

as well as the impact specifications of other countries.

It must be borne in mind, however, that, for span lengths less than approximately 90 ft, the dynamic influence will increase considerably for other reasons. The wheel effect is no longer important, because the critical speed becomes higher than the speed limit. On the other hand, the direct shocks are more effective since, for example, the distance from the exciting (nonperiodic) forces on the rail to the shorter girders, especially track girders, becomes smaller.

For span lengths between 200 ft and 300 ft, the impact number ϕ, as computed dynamically, follows the new A.R.E.A. specification, even under the most unfavorable dynamic conditions. In case of span lengths less than 200 ft, the values of ϕ may become higher than those of the new A.R.E.A. formula, although following rather closely the old A.R.E.A. specifications.

SUMMARY

(1) A dynamic stress analysis of railroad bridges becomes advisable in certain cases, especially if the impact value required by the standard specifications is smaller than the actual value.

(2) A simple and approximate dynamic analysis is possible for a considerable number of medium-size bridges by the use of nomographic charts which may avoid laborious computations.

(3) A dynamic analysis is possible only if certain dynamic qualities of the bridge are known. Furthermore, the dynamic properties of the different moving loads must be taken into consideration separately for each type of passing vehicle. The largest dynamic stresses are caused in many cases by relatively slow freight locomotives and not by high-speed or medium-speed engines.

(4) The method outlined in this paper should be verified further by tests on old and new bridges; that is, it should be clearly understood that the assumptions made in simplifying the dynamic computation require experimental checking. The dynamic properties of bridges, as well as of vehicles, both while standing and while moving, should be investigated—that is, their natural frequency, damping capacity, vibrating masses, deflection and stress diagrams, dynamic wheel-load diagrams, and the influence of the motion of the loads.

APPENDIX

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials compiled by a Committee of the American Standards Association[14] with Society representation, and approved by the Association in 1932:

A = centrifugal force vector in Fig. 2;
a = major axis of ellipse;

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[14] ASA–Z10a–1932.

B = centrifugal force vector in Fig. 2;
b = minor axis of ellipse;
D = diameter of driving wheels of locomotive;
F = amplifying factor for resonance case;
g = acceleration due to gravity;
I = impact, in percentage;
k = substitution constant (see Eq. 6):
    k_w = amplifying number (see Eq. 8);
L = length of span, in feet;
N = number:
    N_i = number of impulses;
    N_t = number of periods necessary to build up oscillation;
    N_w = number of driving wheels of one locomotive;
P = concentrated load:
    P_A = axle load = load imposed by locomotive drivers;
    P_a = wheel load;
V = velocity; speed in miles per hour:
    V_c = critical speed;
    V_m = highest allowable speed of locomotive;
W = total static load; weight;
x = abscissas;
y = ordinate = vertical deflection of bridge;
Z = dynamic load at critical speed:
    Z_m = maximum dynamic load without amplification factor;
    Z_w = maximum dynamic load including amplification factor;
θ = logarithmic damping decrement;
ϕ = impact number;
ω = frequency:
    ω_n = natural frequency.

DISCUSSION

R. S. Chew,[15] M. Am. Soc. C. E. (by letter).—The profession is keenly interested, at present, in the forced vibratory effect on structures due to wind and earthquake, so that Professor Bernhard's paper treating of the effect on bridges from moving train loads is timely. In his treatment, the locomotive unbalanced wheel load is presumed to build up a vibrational resonance that is more destructive than the dynamic impact that has heretofore been assumed.

In discussing the subject it is necessary to define the terms "dynamic impact" and "vibrational effect," as used by the writer:

(*a*) "Dynamic impact" indicates a blow that, if repeated, recurs at an interval that is widely removed from the natural period of the structure struck; and

(*b*) "Vibrational effect" is the result of a blow that recurs in an interval closely related to the natural period so that the initial effect is accumulative.

In the case of vibration, then, the effect is intimately connected with the period of the structure; but in any problem, including forced vibration, it will be found that the calculation of this period (except for the simplest type, such as a rod) is ambiguous, so that it becomes necessary to test the finished structure in order to obtain a period of any value.

To illustrate, the writer will set up the theoretical equation for the period of a bridge which is excited by the application of recurring movement at regular intervals of the load system shown.

Referring to Fig. 8, let: p_1 = train wheel load; p_2 = locomotive wheel load; w = uniform dead load per linear foot; y = deflection at the point of load from live load, in feet; and T_1 = the period of the bridge. Then,

$$T_1 = \frac{2\,\pi}{\sqrt{g}} \sqrt{\frac{\Sigma\,(w\,p\,y^2)}{\Sigma\,(w\,p\,y)} + \frac{\Sigma\,(p_1\,y^2)}{\Sigma\,(p_1\,y)} + \frac{\Sigma\,(p_2\,y^2)}{\Sigma\,(p_2\,y)}} \qquad (14)$$

Eq. 14 indicates:

(1) Certain values of y which include shear and flexure (if y_f = flexure deflection due to live load, and y_s = shear deflection due to live load,

$$y = y_f + y_s \qquad (15a)$$

In order to find deflection values from the elastic curve y_s must be in terms of y_f. If it is assumed that $y_s = R\,y_f$

$$y = y_f\,(1 + R) \qquad (15b)$$

in which values of y_f are determined from statics, with live loads in the position given);

(2) That this value of T_1 is true only for the loads acting in position on bridge as shown and any shifting of loads changes values of y and T_1;

(3) For forced vibration the same loading must recur in the same position during regular intervals; and

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[15] Cons. Engr., San Francisco, Calif.

(4) The value of T_1 is the period imposed on an elastic structure of known value of $E\,I$ and dimensions, so that the calculated value must assume these characteristics together with R and is thus of little value.

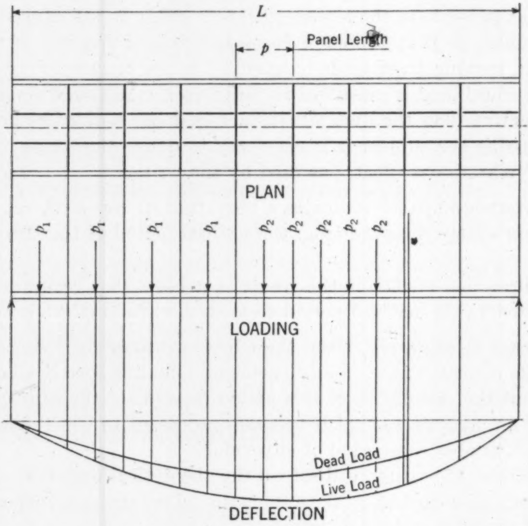

Fig. 8

Items 1 to 4 appear to indicate that a moving locomotive and train load will tend to produce a chaotic vibration, which will have an exciting but no accumulative effect.

When the locomotive has passed from the bridge, the structure becomes uniformly loaded and the action of this excited load will tend to produce forced vibration.

For this condition of uniform loading, a simple formula can be written, as follows:

$$\Delta = \frac{5}{384}\,\frac{w\,L^4}{E\,I} = \frac{5}{384}\,\frac{W\,L^3}{E\,I} = \frac{5}{8}\,W\left(\frac{L^3}{48\,E\,I}\right) \qquad (16)$$

in which W is acting through an average deflection of $\frac{5}{8}\,\Delta$. Therefore,

$$\frac{5}{8}\,W\,\Delta = \left(\frac{5}{8}\right)^2 \frac{4\,\pi^2\,\Delta^2}{g\,T^2_1}\,(1 + R)\,W \qquad (17)$$

from which the natural period of the bridge is found to be

$$T_1 = 2\,\pi\sqrt{\frac{5}{8}\,\frac{\Delta}{g}\,(1 + R)} \qquad (18)$$

in which Δ = maximum deflection due to live load. The natural frequency is:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{8}{5}\frac{g}{\Delta\,(1+R)}} \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

If the uniform live load is moving with a speed such that the loads recur at panel points in regular intervals, forced vibration will occur due to an impulse created by the deflection of stringers.

For example, if: V_s = speed of the train, in miles per hour; p = panel length in feet; and T = time of recurrence = period of excitement,

$$T = \frac{0.68\,p}{V_s} \ldots\ldots (20)$$

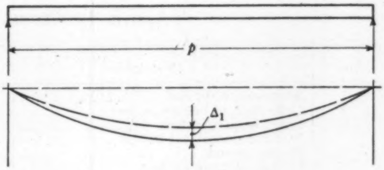

FIG. 9

which gives the period of excitement; and when $T = T_i$ resonance occurs.

In order to find the effect at this impulse (F), let Fig. 9 indicate a stringer carrying a uniform load of w pounds per linear foot moving with velocity of $v = \frac{p}{T}$. Then

$$F = \frac{5}{8}\,w\,p\,\frac{4\,\pi^2\,\Delta_1\,(1+R)}{g\,T^2} = \frac{0.8\,\Delta_1\,(1+R)}{T^2}\,w\,p \ldots\ldots\ldots (21)$$

in which Δ_1 = deflection due to live load. If N = number of impulses, U = length of train, in feet, p = panel length in feet, and L = length of bridge—

$$N = \frac{U - L}{p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Let V_c = the speed of a train in miles per hour such that $T_i = \frac{0.68}{V_c}$; then

$$T^2 = T^2{}_i\left(\frac{V_c}{V_s}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

which substituted in Eq. 21 gives the forced vibration at the panel in terms of the panel load:

$$F = \frac{0.8\,N\,\Delta_1\,(1+R)\,w\,p}{T^2{}_i}\left(\frac{V_s}{V_c}\right)^2 \ldots\ldots\ldots\ldots\ldots (24)$$

This value of F will be limited by frictional damping.

From the foregoing it can be seen that the writer is not in accord with the author in regard to the locomotive unbalanced wheel loads producing accumulative vibrational effect on the bridge as a whole. In reference to the value of z given in Eq. 9 and assuming that it holds, still its application for each impulse is πD removed, which means that it does not accumulate but rather acts as a dynamic force.

The writer approves of Fig. 3, if z is substituted for z_w.

It appears to the writer that the value z, plus the effects of the sudden application of load, is a measure of the dynamic impact to be used for girders and shear members. If Eq. 24 is adjusted to include the damping decrement K_w, as given by the author:

$$F_d = K_w \left[\frac{0.8\,\Delta_1\,(1 + R)\,(w\,p)}{T^2_i} \right] \frac{V^2_s}{V^2_c} \ldots\ldots\ldots\ldots\ldots \quad (25)$$

Two examples illustrating the application of Eq. 25 are given in Table 1, which is based on the premise that all values of $w\,p$ are identical, a condition that is very improbable. However, the formula does emphasize the fact that forced vibration is a factor of the ten items given in Table 1, so that, as in buildings subjected to earthquake, it is impracticable to establish a constant vibrational factor. After completion the bridge should be investigated for the items in Table 1 and from this the factor for the structure should be established.

TABLE 1.—APPLICATION OF EQ. 25

No.	Description	Symbols	Example 1	Example 2
1	Span length, in feet	L	90	200
2	Deflection of bridge, in feet	$\Delta\,(1 + R)$	0.06	0.20
3	Panel length, in feet	p	20	20
4	Stringer deflection, in feet	$\Delta_1\,(1 + R)$	0.01	0.01
5	Bridge period	T_i	0.214	0.395
6	Length of train, in feet	U	4,000	4,000
7	Critical speed, in miles per hour	V_c	63	35
8	F at a speed of V_c	F_d	6.5	1.65
9	Limiting speed, in miles per hour	V_s	31	27
10	F at a speed of V_s	F	1.0	1.0

It may be noted further that Eq. 25, with adjustments, will indicate the vibrational condition on the suspension bridge under wind. If, from any cause, a differential vertical deflection between cables recurs in time T_i, the formula will indicate the forced vibration, in which: $w\,p$ is the load on the suspender; and T_i is the natural period of the bridge.

Then F indicates approximately the condition that existed on the Tacoma-Narrows bridge in Seattle, Wash., when V_s has equaled V_c.

In conclusion the writer believes the author's paper to be of value and trusts that it will receive the attention it deserves.

R. K. BERNHARD,[16] M. AM. SOC. C. E. (by letter).—Mr. Chew's discussion certainly represents a welcome addition to the paper.

In order to simplify the setup of the nomographic charts, the writer tried deliberately to avoid more complicated formulas. The accumulative effect of unbalanced wheel loads of steam locomotives has been proved experimentally. Numerous stress-time diagrams as given, for example, by J. B. Hunley in 1935,[5] or as early as 1928 in the report of the Bridge Stress Committee,[7] give sufficient evidence.

It would be rather simple to change the nomographic charts slightly in order to adapt them for transverse torsional vibrations. This case corresponds to self-sustained vibration due to aerodynamic instability of suspension bridges excited by horizontal wind loads.[17]

It must be emphasized, however, that investigations on actual bridges to determine the damping values more accurately are essential.

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[16] Prof. and Head, Dept. of Eng. Mechanics, The Pennsylvania State College, State College, Pa.

[17] "The Failure of the Tacoma Narrows Bridge," report of board of engineers, O. A. Ammann, T. von Karman, and G. B. Woodruff, March 28, 1941.

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Paper No. 2135

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

By Homer M. Hadley,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Thomas E. Stanton, W. F. Way, Glenn S. Paxson, Lester C. Hammond, Ladis H. Csanyi, G. M. Williams, Harry E. Squire, J. W. B. Blackman, E. C. Jack, Alfred M. Freudenthal, T. B. Rights, and Homer M. Hadley.

Synopsis

Results of extended observations of concrete marine structures, along the Pacific Coast of the United States and Canada, are presented in this paper. No evidence is found of sea-water (sulfate of magnesium) attack on the concrete; where deterioration has occurred, it has been due to other causes. It is concluded from the uniform absence of sulfate attack in the wide range of concretes and cements involved in these structures that special precautions against sulfate attack are needless. It is concluded, however, that in sea-water concrete every care should be exercised to use sound materials, to obtain uniform density and impermeability of concrete, and to protect reinforcement from corrosion.

Introduction

The final criterion by which the behavior of anything in nature is to be judged is not the behavior of some similar thing; it is not the advance opinion of authority; and it is not the preliminary findings of laboratory investigation and research. The final criterion of behavior is no more or less than the actual behavior itself, and it is by their conformity and agreement with this actual behavior that the preliminary bases of judgment are to be appraised and assessed. If there is agreement, well and good. If there is disagreement, then it is the preliminary findings that must be adjusted and revised; it is not the actual behavior that must be denied.

This paper attempts to summarize the actual behavior of concrete marine structures along the Pacific Coast. How can that behavior be truly presented? When the mountain did not come to Mohammed, Mohammed went to the

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Note.—Published in January, 1941, *Proceedings*.

[1] Regional Structural Engr., Portland Cement Assn., Seattle, Wash.

mountain. Nothing similar is possible here in the case of the average reader. The best that can be done is to submit representative views of structures taken in the very act of "behaving," in place, and to invite consideration of these views in combination with other stated facts, all of which evidence, of course, is subject to quite independent check and examination by those to whom access to the structure in question may be available.

The writer is quite aware that there is a large body of opinion at variance with the views set forth in this paper. He regards it desirable, therefore, to focus attention on the question at issue—the behavior of concrete marine structures in service and the existence, in any form, of deterioration in such structures, apart from the enumerated kinds, which is definitely and peculiarly attributable to the sulfate action of sea water upon the hydrated cement of the concrete. Otherwise stated, the question is: Does any form of deterioration occur in the concrete in sea water that would not develop if the exposure were changed to similarly agitated, similarly fluctuating, fresh water? In the considerable body of evidence that exists along the Pacific Coast, the writer is aware, with noted exceptions, of none whatever. It is also to be observed that it is with sea water in its normal range of concentrations—not with other sulfate waters—that the paper is concerned.

THE PREVAILING VIEWPOINT

> "Many engineers who are most experienced in marine construction are of the opinion that all concrete structures in salt water, and a considerable proportion of those near salt water in warm climates, deteriorate steadily and surely."[2]

These words express a widely accepted opinion that deterioration is inevitable. It is supposed to result from the combination of sulfates of magnesium in the sea water with free lime in the hydrated cement and from the consequent formation of calcium sulfate which either by dissolution or expansion disrupts the concrete and effects its destruction. Periodically, in the technical press, reference is made to the sulfates of magnesium in sea-water attack on concrete in the most casual, matter-of-fact manner, as if it were definitely proved—an established reality. Remedies are proposed for it.

Under these conditions it is, to say the least, extraordinary that nowhere along the Pacific Coast, from Canada to Mexico, has the writer been able to find any evidence in concrete marine structures that an attack of such a nature occurs. Deterioration of various kinds assuredly is to be found. There is much of it in certain localities. Sometimes it is very serious; sometimes it is minor and trivial. None of it, however, is of a character to satisfy the sulfate of magnesium-free lime explanation which Messrs. Atwood and Johnson state[3] was advanced by L. J. Vicat. They quote E. Candlot as follows:

> "According to Vicat, it is the action of this salt [sulfate of magnesia] to which must be attributed the rapid decomposition of lime and of certain cements by sea water; the sulphate of magnesia combines with the free lime of the mortar and is transformed into sulphate of lime; the magnesium is precipitated. If the current of the water which traverses the mortar be

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[2] "The Disintegration of Cement in Sea Water," by William G. Atwood, M. Am. Soc. C. E., and Arthur A. Johnson, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 205.

[3] *Loc. cit.*, p. 207.

> somewhat rapid, the sulphate of lime is carried out, a new quantity of lime enters into solution, is in turn transformed into sulphate, and the *gangue* thus decomposes continuously and finishes by forming only a sandy mass. When the current is weaker or is produced only at rather long intervals, the lime sulphate may crystallize and thus cause the expansion of the mortar. It is to this cause that we may attribute the phenomenon of expansion of mortars decomposed by sea water."

From the foregoing it appears that there should be either a progressive leaching out of the calcium sulfate with a correspondingly developed porosity and weakening of the concrete, or the concrete should become swollen, disrupted, and disintegrated by crystallization of the calcium sulfate. Furthermore, although there is no definite statement of the time rate at which these changes occur, it is to be assumed that they take place at a rate sufficiently rapid to show some indication of this deterioration after a reasonable period of time—say twelve or fifteen years. Certainly one should not be compelled to wait longer than that to find the first traces of it if it is a matter of any practical consequence.

With the understanding, then, that the characteristic form and manifestation of sulfate of magnesium attack on concrete is either a developed porosity or a swollen disruption of the mass and, after extensive inspections and examinations of many structures which uniformly fail to reveal deterioration of the requisite type, it is repeated that there is no evidence in concrete exposed to sea water along the Pacific Coast to substantiate the sulfate of magnesium-free lime theory. The confirmation is not to be found, however diligent the search. On the other hand, when a large amount of concrete (made with many different brands of portland cement) is seen that has been subjected to full sea-water action for fifteen, twenty, or more years and still has the original wood grain and form marks showing in its surfaces; still has clean, sharp corners and offers flinty, hard resistance to hammer blows; and is utterly lacking in deterioration, there is no avoiding the conclusion that the sulfate of magnesium-free lime deterioration theory is deficient in that indispensable necessity of all valid theories—proof. If, as is said, sea water attacks concrete, why does not all concrete deteriorate? Why, even in deteriorated structures, is most of the concrete free from deterioration in spite of having the same identical exposures as the deteriorated parts?

CONCRETE A GENERIC TERM

It would be well at the outset to consider what concrete is and to recognize that the term is a broad and generic one. Concrete is the name given to any hardened mixture of portland cement, water, and fine and coarse aggregate. If the coarse aggregate is lacking, the mixture is called mortar, but otherwise the name concrete is given, regardless of wide variations in the proportions of the several components of the mixture and regardless of the manner and method by which it is mixed and placed in its forms. Whatever it is, it is called concrete as long as its material components are as stated. The consequence of this convention is that the behavior of concrete under any particular set of conditions may vary widely, depending upon the particular concrete that is subjected to the exposure. Quite similar would be the variable behavior of rock which, if it were hard granite, would behave in a markedly different manner than rock which was a soft sandstone.

It is to be expected that a wide range of mixes has been used in concrete marine work, and such indeed is the case. Likewise variable have been the methods of placement. It is apparent in many structures that care has been exercised to place the concrete uniformly and without segregation. It is apparent in other structures that little or no concern was felt for the attainment of these objectives: The concrete was dumped into the forms in a carefree, happy-go-lucky fashion, contentment abiding in the knowledge that "sufficient unto the day is the evil thereof." In reinforced structures the thickness of the protective covering over the bars varies in different jobs as does the quality of concrete in the covering. Out of all these variables, manifestly, there are opportunities for diverse behavior of "concrete in sea water," and it is not surprising that deterioration of various kinds has developed.

The Sea-Water Exposure

The sea-water exposure itself varies widely. In the most sheltered and protected locations there is the rise and fall of tides with alternate wetting and drying of the concrete. Waves, swells, and the wash of ships frequently amplify the tidal wetting. Flotsam and drift are tossed about by the water and deliver mechanical blows. On the open ocean coast, storm waves themselves deliver tremendous thrusts and blows against the structure and may roll and grind loose boulders against it; moreover, along the Pacific Coast, from San Francisco (Calif.) north, the forest mantle of the country yields great quantities of drift which sometimes may be whole logs and trees that form heavy battering material. Additional to this along some coasts are the disruptive effects of ice and freezing. All in all it must be recognized that, quite as a marine exposure is a poor place for rock of a soft shale, indurated mud type, so, likewise, is it unsuited for lean, porous concrete of segregated, random structure.

Deterioration by Mechanical Attack

Indeed the most striking and characteristic forms of deterioration to be found in concrete exposed to sea water are those due to deficiencies in the quality and structure of the concrete itself: Weak concrete is abraded, honeycombed areas become full cavities, laitance seams become extended voids, construction joints and pour planes tend to chip and widen at their edges, and sharp corners and projections become broken and knocked off. The processes of attrition and erosion by which, in the long reaches of time, all things are reduced to fragments and to dust are in aggressive action on the ocean's shores, and that which is vulnerable to attack is speedily revealed. On the other hand, strong, sound concrete that is uniformly dense and impermeable shows but slight change in a quarter century or more and quite merits the term "permanent" in the time scale of human affairs.

Such contrasting aspects of concrete in sea water obviously result from variations in the concrete itself. They have nothing to do with sulfates of magnesium in the sea water, nor with the sulfate-resistant qualities of cement. Deterioration such as that described is due to physical and mechanical causes and could, and would, occur as well if the ocean were of fresh water instead of salt water. Unfortunately, a great deal of writing has indiscriminately lumped all forms of deterioration together as "sea-water attack," whence the conclusion is readily drawn that it is the sea water itself that exercises a peculiarly malign effect upon the concrete. For resistance to this, the commonest form of deterioration in concrete exposed to sea water, it is safe to say that making concrete which is fresh-water resistant or frost resistant elsewhere will furnish full and adequate protection.

Rusting of Reinforcement

In reinforced structures, the second most common form of deterioration is that due to rusting of reinforcement, disruption of the covering concrete, further corrosion of steel with loss of section, loss of bond, etc. It develops most frequently in members of small section—piles and the stems of beams and girders. Unprotected steel rusts. It rusts very slowly in a dry climate and more rapidly in a moist climate. It rusts most rapidly and heavily of all along the sea coasts where the air may carry minute drops of salt water in suspension, caught up from waves and surf. In such an atmosphere, rust will form quickly on reinforcing bars wherever the covering concrete is porous or absent. If possible it will develop progressively along the bar and with its inevitable expansion will split and disrupt concrete. The primary remedy is to provide dense, impermeable concrete and adequate cover. Around salt water is a poor place for scant cover over reinforcement, for misplaced reinforcement, or for lean concrete. However, it is to be noted that, although the rusting of reinforcement is indeed aggravated and accelerated by the salt-water exposure, it is wholly independent of the sulfate-resisting properties of the cement used in the concrete. Consequently, here again, primary importance attaches to density and impermeability and to the thickness of the protective concrete rather than to other things. Additional protective measures that may be accepted as thoroughly effective are the asphaltic impregnation of concrete piles, extensively used by the Los Angeles (Calif.) Harbor Board,[4] and the ingenious system of applying an asphaltic coating to the underside of a concrete deck, developed and used by the engineers of the San Francisco Harbor Board.[5] To the best of the writer's knowledge, these methods have been 100% effective in accomplishing their respective objectives.

Solvent Action

Occasionally there is evidence of a slight solvent action of sea water on concrete. The original surface film of cement has disappeared, or a honeycombed spot weakens and loosens some pieces of aggregate. In time, porous concrete may appear to become increasingly porous. Such changes are not peculiar to sea-water exposure, however. They can occur equally in fresh water. Long-time tests made in the testing laboratory of the City of Seattle, Wash., about 1925, by T. H. Carver, M. Am. Soc. C. E., clearly showed a minute but definite dissolution of concrete in flowing fresh water, which dissolution did not occur in still water. In Fig. 1(*a*), the cylinders on the left were stored in a tank through which a slow steady flow of Cedar River water was maintained. Those on the right (Fig. 1(*b*)) were stored in still water in sealed jars. The mixes were, left to right, 1–1–2, 1–2–4, and 1–3–6, respectively, in both Fig. 1(*a*) and Fig. 1(*b*). After five years the standard-cured 2-in. by 4-in. cylinders, in Fig. 1(*a*), all showed evidence of attack. The specimens of 1–3–6 mix had entirely lost their surface films of cement paste; the 1–1–2 specimens could have theirs removed easily by wire brushing. The companion specimens, in Fig. 1(*b*), stored in still water in sealed jars, were unaffected. The loss of strength, running-water storage compared with still-water storage, was markedly greater with the lean mixes than was the case with the richer mixes. The desirability of an impermeable concrete was clearly shown. In the case of sea water, there is a continual change of water with tides, currents, surf, etc. There is ample evidence from both laboratory and field investigations of the need for density and impermeability in concrete exposed to sea water, and also in concrete exposed to fresh water.

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[4] *Civil Engineering*, November, 1931, p. 1245.

[5] *Engineering News-Record*, June 8, 1939, p. 771.

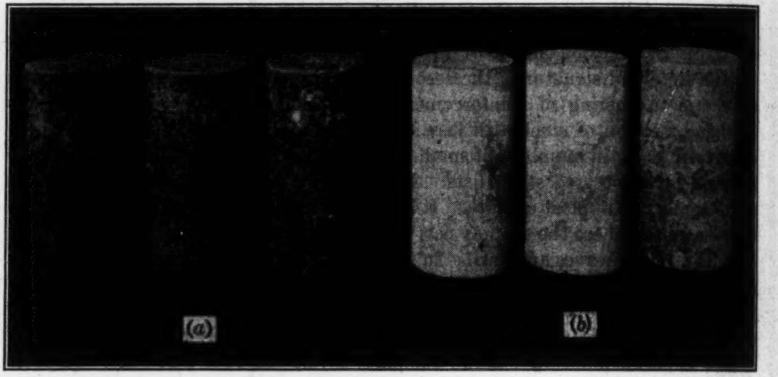

FIG. 1.—WIRE-BRUSHED, 2-IN. BY 4-IN. CYLINDERS AFTER FIVE YEARS IMMERSION IN FRESH WATER

FREEZING

Disruption of porous concrete by the freezing of absorbed water does not occur along the Pacific Coast of the United States, not because there is no porous concrete to be found, but because freezing seldom, if ever, occurs. Such disruption of porous concrete by freezing is quite independent of sea water, however. It occurs with both fresh and salt water.

EVAPORATION

Disruption may likewise develop with porous concrete in the tidal zone or above if evaporation produces seriously harmful concentrations of sea water in the pores. The late C. S. Pope,[6] M. Am. Soc. C. E., has presented a view of the top of a sea wall that shows scaling and deterioration which were undoubtedly due to concentrated saline solutions and crystallization produced by evaporation. This is a form of deterioration which is indeed peculiar to sea water and not to fresh water. In general, however, it is of limited occurrence, since the usual freedom of drainage appears to prevent it. The flat top of the sea wall cited by Mr. Pope acted to trap and hold a certain amount of spray thrown upon it, and with rather porous concrete and aggregate the consequences were as shown in Fig. 1.

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[6] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 511.

Unsound Materials

Still another type of deterioration—that due to the use of unsound materials in the manufacture of concrete—is to be found. This is characterized by a pronounced general expansion of the entire body of the concrete and much cracking. There are two distinct varieties of this classification. The common one is that in which the concrete remains hard and is entirely unaffected by the sea water. In the other variety the concrete softens into a mushy mass practically devoid of cohesion.

Concrete is composed of cement, water, fine aggregate, and coarse aggregate. The coarse aggregate is a highly unlikely cause of a general and all-pervasive expansion in a concrete structure free from local poppings and pittings and may be dismissed from consideration in all but exceptional cases. So, likewise, are differences in mixing waters, although, as the agent initiating the hydration of cement, mixing water may be regarded as a cause. Perhaps strongly alkaline waters under some conditions may be unfit for use, although sea water itself, notably in the case of the Florida East Coast Railway Extension to Key West, has been used as mixing water in a number of instances with no appreciable ill effects. There remain the cement and fine aggregate. The latter is almost always a natural sand produced by weathering and erosion of a great variety of rocks, and is transported, intermingled, and deposited by streams. Depending upon their origin, sands from the same pit may be of definitely uniform or of utterly diverse and random mineralogical characteristics. It is a common and usually valid assumption that natural sands are stable, sound, and chemically inert, but this assumption, easy though it is to make, is not necessarily true. It is cement and sand, individually or in combination, that must be considered as possible causes of the expansion referred to.

Improperly manufactured cement may indeed be the cause of the trouble. Nevertheless, the adoption of modern improvements in manufacturing and testing procedure should give the engineer increased confidence in the capability of portland cement to meet fully all reasonable construction requirements. More careful grinding and particularly blending of the raw materials insure more uniform and correct burning and therefore a more dependable product. New and more severe tests for soundness detect any tendency toward delayed unsoundness. However, the cases of deterioration in existing marine structures in which serious expansion has occurred are not numerous, and those in which this expansion is properly to be charged to the cement alone—that is, wherein the aggregate is not partly or completely responsible—have yet to come to the writer's attention.

Expansion of the type in which the structure cracks but the concrete remains hard and not observably affected by sea water is discussed and illustrated in a series of valuable papers by Thomas E. Stanton,[7] M. Am. Soc. C. E. The original paper, presented at the annual meeting of the Society in January, 1940, describes a typical case of such expansion in a sea wall in Ventura County,

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[7] "Influence of Cement and Aggregate on Concrete Expansion," by Thomas E. Stanton, *Engineering News-Record*, February 1, 1940, Fig. 2, pp. 59–61; see also *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781

California. In March, 1940, the writer discussed[8] the condition of this sea wall and others nearby. There appears to be no reasonable doubt that the nature and cause of deterioration of this kind have been definitely established by the notable work of Mr. Stanton and his associates. This form of expansion has developed in bridge piers and abutments in fresh-water streams—for example, the highway bridge across the Salinas River at King City, Calif.

A. A. M. Russell, Assoc. M. Soc. C. E., has made some tests which brought into definite and almost dramatic focus hitherto confused and obscure causes of trouble of the expansion-softening type. From a pit from which had come the sand used in a waterfront structure that developed very serious deterioration of this type in a few years' time, he obtained sand samples, and for comparison he used sand from another pit, several hundred miles distant from the first pit, and of known soundness. Petrographic examination of the questioned sand revealed a heavy percentage (more than 40%) of feldspar grains which proved to be of a highly unstable nature.

As an accelerated test, equal samples of both sands were put in beakers of sea water, which were then placed on an electric plate and held continuously at a temperature slightly below boiling. In four days about half of the questioned sand had broken down and changed completely to a mud, whereas the other sand sample showed only a thin film of mud across its top surface. Then, as a direct sea-water test, briquet specimens were made up of the questioned sand and of standard Ottawa sand. The latter, consisting entirely of particles passing No. 20 and retained on No. 30 sieves, suggested a similar grading for the other sand. Consequently, it was sieved into three gradings and three different mixes were prepared. In one mixture only particles passing the No. 10 and retained on No. 20 sieves were used; the second had only particles passing No. 20 and retained on No. 30 sieves—that is, the Ottawa sand grading; the third mixture had a grading with particles ranging uniformly from fine to coarse. After tension tests were made, the briquet fragments were placed in jars of sea water and were kept at room temperature, the sea water being renewed occasionally as evaporation took place. In less than a year the Ottawa sand specimens were wholly unaffected, having hard sharp edges and no trace of deterioration. The same was true of the specimens of the questioned sand that had the fine-to-coarse grading and were therefore of reasonably dense structure. The specimens made with the No. 10 to No. 20 mesh particles were beginning to soften slightly on their edges but were in fair shape. The No. 20 to No. 30 mesh samples, on the other hand, were in an advanced state of disintegration. They had swelled, cracked, were slimy on their surfaces, and their edges were soft and crumbly. These specimens, being directly comparable in every respect to the specimens made with Ottawa sand and with the same cement from the same sack (which Ottawa sand specimens were wholly unaffected), showed conclusively, in conjunction with the preceding accelerated test on the sand, that the unsoundness in this case was in the sand and not in the cement.

Inasmuch as the deterioration produced closely resembled that in the structure built with this sand, it appears to be established, at least in this case, that the sea-water attack was on the sand and not on the cement. It is scarcely conceivable that this trouble could have been avoided by special sulfate resistance or other special qualities in the cement. It is a fact that serious deterioration of this type appears to have developed in a section of the coast of rather marked geographical limits. It should be added that it is only in sea water that the writer knows of the occurrence of this form of deterioration, and indeed it may be that this feldspathic sand is particularly unstable in sea water. Its use for any concrete anywhere, however, appears to be questionable.

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[8] *Engineering-News Record,* March 14, 1940, p. 45.

There are a few cases known to the writer where concrete in exposed locations appears to have developed expansion troubles somewhat intermediate between the two types—that is, there has been cracking of the type described in Mr. Stanton's papers with apparently a certain softening of the concrete. This has resulted in the complete destruction of parts of the structures. Extensive cracking of exterior surfaces due to expansion of the interior of the mass, of course, greatly weakens resistance to flexural stresses; similarly, such complete prevention of longitudinal expansion in certain sections as to lead to crushing at the ends would practically destroy their resistance to the pounding of waves at such points. Whatever the exact cause of the trouble in these cases, there is no uncertainty about abnormal expansion; it is to be avoided and eliminated. It is believed that the preceding covers all the main types of deterioration that are to be found in Pacific Coast concrete marine structures.

SUBJECTIVE DETERIORATION

Another trouble of concrete in sea water has nothing to do with the sea water. This trouble arises from hasty observation and incomplete reporting of conditions. In glowing zeal to establish either broad generalizations or the urgent need for the adoption of certain safeguards, only as much of the evidence is adduced as supports the contention. This frequently creates in readers' minds an entirely erroneous idea of conditions.

Fig. 2 shows a view of an old quay wall at the Puget Sound Navy Yard, Bremerton, Wash., built in 1901 and known as quay wall D. It extends from the stepped-off joint at the reentrant angle to the corner at the extreme left. At the right of this section and in the right foreground of the view is a portion of quay wall E, constructed in 1896. The deep cavities and extremely bad conditions at the base of wall D are clearly apparent. Likewise clearly apparent is the condition of the wall above the base portion which is extremely hard and today (1940) shows no trace of deterioration, the wood grain and form marks being discernible through the coating of fuel oil which now covers it. An official statement regarding this condition is the following:

> "The bottom part of the west wing wall from the stepped off joint to the south corner was poured in 1901 with a tremie up to a point where the water could be expelled from the open cofferdam, probably three feet higher than the water level in the photograph."

Regarding this same wall, the following is the complete report,[9] published in 1924 by the National Research Council:

> "Quay Wall D—This is a gravity type wall built in 1901–02. Local aggregates and condon [Condor] cement were used in the proportions of

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[9] "Marine Piling Investigations," National Research Council, 1924, p. 403.

1 : 2½ : 5. This concrete was hand mixed and deposited below water level by tremie and above with wheel barrows.

"The inspection of Mar. 8, 1923, showed this wall to be badly deteriorated."

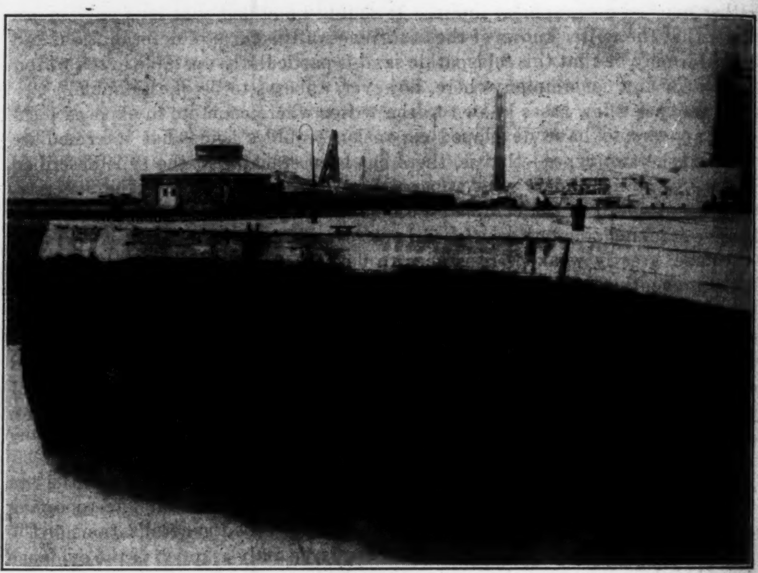

FIG. 2.—QUAY WALL D, PUGET SOUND NAVY YARD, BREMERTON, WASH.

Although this report is literally true, it nevertheless creates a wholly erroneous impression since no reader would know that the upper part of the wall, also in sea water, made with the same cement as the base, was practically free from deterioration on March 8, 1923. With the same construction methods, any and every cement regardless of type or composition that might have been used in this wall would have been similarly attacked or not attacked by the sulfates of magnesium in the sea water, depending on whether it was dumped through a tremie tube or deposited by wheel barrows. Many reports of sulfate-of-magnesium attack on concrete in sea water are of the foregoing character.

SUMMARY

The harm resulting from misunderstanding a natural phenomenon arises from the likelihood that measures adopted to effect changes and alterations in it will be misdirected. There is no particular reason for research and for grave alarm over the strong links of a chain so long as there are weak links that need strengthening. Not until the real hazards of sea-water use are definitely foreseen and are guarded against is there need for such extreme refinements as protecting against sulfates of magnesium—and no particular harm will result if this safeguarding is completely neglected.

FIG. 3.—TYPICAL VIEW OF SUBSTRUCTURE OF FERRY PIER, NORTH VANCOUVER, B. C., BUILT IN 1909

FIG. 4.—DETAIL OF OLD WATERFRONT WALL, SEATTLE, WASH., BUILT IN 1913

This paper is based upon visual observations of concrete structures in sea water along the Pacific Coast and is concerned with sea water only. It is intended that its statements and conclusions apply to sea water only—not to those sulfate waters and exposures elsewhere which assuredly require special measures and treatments. The dearth of supporting laboratory data may be regarded as somewhat unusual, but it is hoped that the references to the work of Messrs. Carver, Stanton, and Russell will supply this deficiency in some measure. After all, it really is not necessary to have laboratory confirmation of that which is spread obviously before one's eyes.

FIG. 5.—BROADWAY PIER, SAN DIEGO, CALIF., BUILT IN 1912

Probably no better ending could be made to this paper than to append a short list of some representative older concrete-in-sea-water structures, variously located, which exemplify the behavior of sound, impermeable concrete and which may be examined by any one. Several views are shown in Figs. 3 to 7. These structures are not without flaw or blemish. Some of them have bad spots such as patches of honeycomb dating from original construction, or have been worn by abrasion. However, they have had twenty or more years of sea-water exposure, and their concrete is substantially free from trace of sea-water attack. They were built at different times, with different brands of

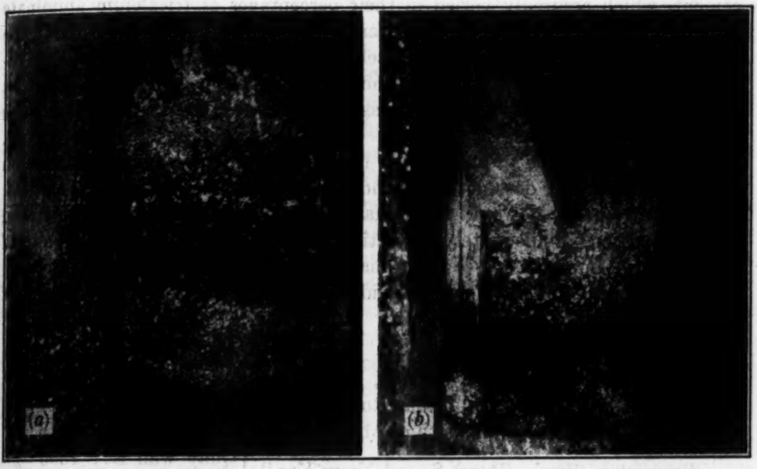

FIG. 6.—PIER ON SAN FRANCISCO WATERFRONT, BUILT IN 1909

FIG. 7.—DETAIL OF ABUTMENT OF EAST NEAPOLITAN BRIDGE IN NAPLES DISTRICT, LONG BEACH, CALIF. OFF ALAMITOS BAY, ERECTED IN 1913

cement, which assuredly varied in their percentages of tricalcium aluminate and other compounds, and with different construction methods. After the examination let the observer ask himself the question: Where is the sulfate-of-magnesium attack? Assuming that there is only one conclusion—namely, that he has not seen any—it is then fair to ask why it is not to be found. It is to be hoped that he will look for it elsewhere—not in books, but along the waterfront. This paper is intended to present the idea, definitely, that such an attack on concrete in sea water does not occur and therefore need not be guarded against. On the other hand, it would re-emphasize what many observers have stated before—the need of adequately protecting reinforcement from corrosion and of having dense, impermeable concrete made of sound and enduring materials.

Representative older structures evidencing no sulfate-of-magnesium attack upon their concrete are the following:

Location:		Built:
North Vancouver, B. C., Canada, Ferry Pier		1909
Victoria, B. C., Canada, Dallas Road sea wall		1911
Seattle Wash., sea wall south of Madison Street		1916
Bremerton, Wash. (Puget Sound Navy Yard)	Quay wall E.	1896
	Quay wall D.	1902
San Francisco, Calif.	Cylinder substructure, pier 38	1909
	Pre-cast pile substructure, pier 17	1912
Los Angeles, Calif.	Lighthouse base, San Pedro breakwater	1910
	Piers of Southern Pacific Railway bascule bridge at West Basin	1912
San Diego, Calif., Broadway Pier (see Fig. 5)		1912

Reports of 27-yr tests of concrete specimens, made with five different brands of cement immersed in sea water on the San Pedro breakwater at Los Angeles,[10] were published in 1932. These specimens were made in 1905 by the U. S. Engineer Department and were exposed in the tidal zone. No evidence of sulfate-of-magnesium attack was found in them nor any deterioration other than abrasion due to mechanical causes.

Fig. 4 is a view of an old sea wall in Seattle, Wash., built in 1913. Part of the wall shows what is commonly called "sea-water attack," whereas most of the wall shows no attack. The cause of the trouble is self-evident.

In Fig. 6(*a*), honeycombed concrete and sound concrete are shown side by side in the same cylinder, and both conditions occur at the water level. Fig. 6(*b*) shows another cylinder that is honeycombed at the high-tide line and also above that line.

CONCLUSIONS

Because of complete lack of evidence in concrete marine structures of sulfate-of-magnesium attack upon them, there appears to be no valid basis for believing that such attack occurs.

The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement.

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[10] *Western Construction News*, June 25, 1932, p. 367.

DISCUSSION

Thomas E. Stanton,[11] M. Am. Soc. C. E. (by letter).—Mr. Hadley is to be complimented for his presentation of the results of his extended observations of concrete marine structures along the Pacific Coast of the United States and Canada and for his emphasis on density and uniformity. The writer is in accord with Mr. Hadley's conclusions that if, in concrete exposed to sea water, every care is exercised to use sound materials, to obtain uniform high density and impermeability of concrete, and to protect reinforcement from corrosion, there need be little fear of subsequent disintegration due to sea-water attack, at least in so far as cements similar to those under observation are concerned, which probably include all of the standard cements manufactured on the Pacific Coast. That this is true everywhere, however, can be ascertained only through similar observations conducted throughout the world and with all brands of cement. Certainly the literature on cement and concrete is replete with "alleged" instances of salt-water disintegration.

The writer checks Mr. Hadley's observation of numerous concrete structures, subject to sea-water attack along the Pacific Coast, which are at least 25 years old and apparently as sound today as when constructed. This observation, combined with confirmatory laboratory tests, was the basis for the writer's conclusions, published[12] in 1938, that density and uniformity are the first considerations for a durable concrete, assuming that sound ingredients are used.

It was definitely observed, however, based on laboratory tests, that porous mortars of unquestionably sound fine aggregates, but with a high C_3A portland cement, are non-durable[13] when subjected to attack by sea water of normal concentration, but that mortars using a low C_3A cement are highly resistant even in 1 : 3 ungraded Ottawa sand mortars.

The conclusion was reached, therefore, that, for insurance against disintegration due to sea-water attack in the case of porous concrete, at least a moderate sulfate-resistant (8% or less of C_3A) cement might well be specified, particularly if such a cement can be secured without paying a premium. It is freely conceded, however, that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement" and that, if these conditions are met, little concern need be felt relative to the cement component, provided the cement is sound in every respect and of the general composition of those manufactured by Pacific Coast mills. The exception to the foregoing conclusions, of course, is the condition described by the writer in the paper to which Mr. Hadley refers.[7]

W. F. Way,[14] M. Am. Soc. C. E. (by letter).—It is the writer's impression that Mr. Hadley's paper is timely. Too long it has been thought that all concrete, good or bad, deteriorates in sea-water structures, the deterioration being due to chemical reaction between the sea water and the various compounds in the hydrated cement of the concrete. The writer agrees quite heartily with the conclusions in the paper. In fact, one cannot come to any other conclusion when observing in the same structure both sound and unsound concrete in close proximity. Concrete that was placed, probably, from the same bucket or buggy—that is, concrete that has been under the same exposure—is found partly sound and partly unsound. Surely the difference is not due to chemical action. True it is that concrete structures in sea water do not enjoy a flawless record; but it is also true that the real causes of unsound concrete have not been explained by the allegations of sea-water attack.

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[11] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

[12] "Resistance of Cements to Attack by Sea Water and by Alkali Soils," by Thomas E. Stanton and Lester C. Meder, *Journal*, A. C. I., Vol. 9, March–April, 1938, pp. 433 *et seq.*

[13] *Loc. cit.*, supplement, September, 1938, p. 464–7.

[14] Project Mgr., Constr., Basic Magnesium, Inc., Las Vegas, Nev.

The writer once had occasion to repair the substructure of a reinforced concrete pier whose supports were cylinders 4 ft in diameter, belling out to bases of 13-ft diameters. In the original cylinder construction, a timber shell form was made that was placed over a pile cluster previously driven, and when in place the marine ooze was pumped out and the base was sealed with a tremie plug. The next step was to dewater the form and to place the reinforcing steel. The base and shaft were completed with a concrete supposedly placed "in the dry." For various reasons a good job was not obtained, and years later after the teredos had eaten away the wooden forms (which had all been left in place) the concrete began to ravel away. The repair contract required all work to be done in the dry, which necessitated the use of a pneumatic caisson. This caisson enclosed the cylinders successively, one at a time, from high water to the mud line—about 45 ft. Thereby an excellent opportunity was afforded to study this concrete after its fifteen years of service. The typical case showed lenses of mud through the base of tremie concrete, above which was a very lean concrete in the shaft capped off with a plug of solid laitance varying from 2 to 4 ft thick, or sections of lean concrete separated by bands of laitance 6 to 12 in. thick. Many local areas of good solid concrete were encountered. It can be appreciated that this "failure of concrete" was due solely to poor workmanship, although the impression gained by many was that an inherent weakness of concrete when exposed to sea water had something to do with this failure.

The design of marine structures is a most important factor in determining the amount of maintenance that later will be required. Generally, designers have the option of using concrete piles or concrete cylinders in supporting pier decks. With concrete piles it is almost certain to be specified that all cracked piles shall be pulled and replaced with sound piles. Considerable controversy may arise over this point and therefore a brief description of a project in which 3,500 piles were driven may be apropos.

The piles on this job were 16 in. by 16 in. for lengths as great as 60 ft, and 18 in. by 18 in. for 60-ft to 80-ft lengths. All piles were driven with a double acting hammer on a floating rig. These piles were carefully designed; the materials used were certainly of the highest quality; and the workmanship throughout, including curing, was in accordance with the very best practice (1933). When the piles were placed in the leads of the driver, no signs of cracks were visible. After they were driven each of the 3,500 piles was inspected by inspectors in diving suits, and it was found that practically every one had developed cracks during driving. Only at the inshore and in shallow water where the piles were shorter (about 40 ft), and the unsupported length during driving was 20 ft or less, were no cracks observed. Since it would have been beyond reason to expect the contractor to pull these piles and drive others, a distinction was made on this project between a "cracked" pile and a "broken pile," under which it was considered that only such piles as were incapable of acting as elastic members would be rated "broken." Of the total number driven, about 60 piles were definitely "broken" and had to be replaced.

In reaching this decision about "cracked" piles an investigation was made of the piles in an adjacent existing structure, built fifteen years previously. The marine growth was removed and the faces of the piles were inspected carefully in an effort to locate cracks similar to those developing on the present job. As expected they were found; the concrete was then chipped back along the cracks, and in all cases the cracks were found to be opposite the reinforcing ties binding the vertical bars. They were all tight, and no signs of corrosion of either the bands or the main steel were observed.

Since it was definite that these cracks, which occurred opposite the bands, had developed during the driving it was decided that the new piles should be watched during the actual driving process. Because the water was warm, a shallow-water helmet was used, being safer and permitting freer movements around the pile. Furthermore, the large face plate affords excellent vision. After a pile was in the leads, the inspector, in a shallow-water suit, climbed on the pile, keeping under the barge as much as possible to protect himself from falling spalls. After a few blows the cracks would be observed forming, and as driving proceeded the cracks would start to "spit" out the crushed material ground up by each blow. In many of the piles there would be a crack opposite each reinforcing band. The conclusion was that as long as the bond on the main vertical steel was practically unimpaired there would be sufficient ground-up material remaining in each crack to re-cement itself and to protect the steel, and that the pile was as good as could be obtained. It is the writer's opinion that it is impossible to drive a pre-cast reinforced concrete pile, as now being made, without developing these small fine cracks, especially when the length is around 35 to 40 ft from the mud line to the water surface. It is his further opinion that such fine cracks are not harmful if the concrete of the pile is of good quality, but that they are dangerous if the concrete is of poor quality. It all depends on "what kind" of concrete cracks.

Many instances of poor concrete have come to the writer's attention, occurring both in fresh and salt water substructures, and also many examples of good concrete have likewise been observed. One that might be mentioned is a small sea wall built along the beach of Elliott Bay at Seattle, Wash. This wall is 5 in. thick at the top and 7 in. at the base, and 5 ft high. The elevation of the top is at extreme high water. The wall is reinforced at the center with ⅜-in. round bars 15 in. on centers both ways. The concrete was made with the sand and gravel shoveled up from the beach, which at times contained a considerable quantity of sea water. The cement content was 6 sacks per cubic yard. The exposure is very severe and the wall has had to stand much pounding from both waves and drift. The wall is now (1941) thirteen years old and the only signs of distress are those due to erosion; no chemical deterioration can be observed.

GLENN S. PAXSON,[15] ASSOC. M. AM. SOC. C. E. (by letter).—Mr. Hadley's excellent paper on the resistance of concrete to disintegration in sea water should find an appreciative audience among engineers who are working along the seacoasts. He should, indeed, be commended on this summary of his observations and its clear presentation.

It is noted that the list of representative structures given in the paper includes some in British Columbia, Washington, and California. The writer would add to this list the structures along the entire 400-mile length of the Oregon Coast Highway. Frequent inspections of all highway structures are made for maintenance purposes, and no example of disintegration due to chemical action of sea water has been found. These structures are not as old as those cited by Mr. Hadley—the oldest dating from 1920—but, if chemical action is taking place, some evidence should be apparent after 20 years of exposure.

Every indication points to abrasion rather than chemical action as the cause of any disintegration. It is noted that concrete members in deep water, where little or no sand is carried by wave action, retain the surface film of rich mortar and show the form marks. Piers and walls in shallow water or in the tidal range have this film worn away and show a granular texture. The concrete is hard and firm, and it can be assumed that this slight wear is due to the sand carried in the water rather than to any solvent or disruptive action of the sea water itself.

The enlargement of gravel pockets, laitance seams, and construction joints is the result of this same abrasive action. Examples of such structural defects that have not changed after 20 years of exposure to quiet sea water are common.

The excessive rusting of inadequately protected reinforcing steel in sea water with the concomitant spalling of the concrete incasement is often apparent. Adjoining bars with the same exposure, but with adequate protection, show no rusting. This disintegration is certainly not chargeable to any solvent or disruptive action in the concrete itself.

The writer is in complete accord with Mr. Hadley's conclusions that there is no evidence of any solvent or disruptive attack by sea water on properly made and placed concrete.

LESTER C. HAMMOND,[16] M. AM. SOC. C. E. (by letter).—In his "Conclusions" the author states that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement."

Although he agrees heartily with this statement, the writer cannot agree that the lack of evidence Mr. Hadley has observed, of sulfate attack in concrete marine structures, warrants him in assuming that sulfates in sea water do not reduce the life of the concrete. The writer observed concrete in sea water that was constructed in 1911, which in 1924, after 13 years of service, showed no signs of deterioration; yet the same concrete, when inspected in 1934, showed signs of surface disintegration.

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[15] Bridge Engr., State Highway Comm., Salem, Ore.

[16] Chf. Engr., Borough of Manhattan, New York, N. Y.

The concrete viaducts of the East River Drive in New York City were built in sea water. The East River is a tidal estuary connecting the Long Island Sound and New York Bay. During construction, Ladis H. Csanyi, M. Am. Soc. C. E., made an extensive series of tests on twenty-nine concrete cylinders, of which all but nine were samples of the actual concrete used in the structures. These tests consisted of the complete submersion of the samples in a 43% solution of Epsom salts, which is equivalent to a 20% solution of anhydrous magnesium sulfate. This solution is estimated to be about fifty times as strong as the sulfate concentration in the waters of the East River. The test was limited to purely chemical action. The result of these tests proved that, within 92 days, imperfect concrete would show surface attack, and that even the best concrete would not withstand this chemical action after 189 days. Many of the tests extended over a period of 250 days. This proves to the writer that the sulfates in sea water have some chemical action on any concrete. The visible effects on good concrete are considerably less than on poor concrete. Since the tricalcium aluminate (C_3A) content of the cement has a bearing on the resistance of the concrete to sulfate action, it is advisable to use sulfate-resistant cement. The tests confirmed this view and it was noted that concrete having a computed C_3A percentage between 6% and 7% showed best results.

It is not proper to assume that because the deterioration of the concrete observed in Pacific Coast waters can be largely explained by the fact that the concrete was improperly constructed, that the sulfates in the sea water did not contribute to this deterioration. In 1917, Rudolph J. Wig and Lewis R. Ferguson,[17] Members, Am. Soc. C. E., demonstrated that chemical action was noted all along the Pacific Coast. Furthermore, the report of Thomas E. Stanton,[18] M. Am. Soc. C. E., in 1938, indicates that chemical action has been cause for concern along the California Coast.

The reasons for concrete failure in sea water, stated in their relative importance, are: Improper placing, and forms; improper grading of aggregates; insufficient cement; excess of water; improper character of aggregates; and too much C_3A in the cement.

The causes are: Scouring action, frost action, and the chemical action of the sulfates in the water.

With the foregoing possibilities of failure, it is reasonable to claim that the chemical action of the sea water is not the most important cause of its failure; but it seems unreasonable to believe that it is not a contributing factor.

LADIS H. CSANYI,[19] M. AM. SOC. C. E. (by letter).—During the spring of 1939, nineteen concrete structures situated in the harbor of New York City were examined, and all pertinent information concerning the design, mixing, and placing of the concrete was compiled and analyzed. The results of this study confirm the findings of Mr. Hadley, in that a dense, uniform concrete properly placed is essential wherever sea-water exposure is encountered. Only three of the structures inspected showed a resistance to disintegration for periods of twelve years. Of these three, two utilized cements, approaching a moderate sulfate-resistant cement, by containing a computed C_3A-value of 9%. The other, in service for thirty years, yielded no information concerning the source or composition of the cement.

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[17] *Engineering News-Record*, Vol. 79, Nos. 12, 14, 15, 16, and 17.

[18] "Testing Cement Mortars in Sea Water," by Thomas E. Stanton, *Engineering News-Record*, March 17, 1938, p. 400; also "Expansion of Concrete Through Reaction Between Cement and Aggregate," by Thomas E. Stanton, *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

[19] Engr. in Charge, Asphalt Plant, Dept. of Borough Works of Manhattan, New York, N. Y.

In an inspection such as this and the one conducted by Mr. Hadley, the area open to visual examination is wholly above dead low water, wherein the differentiation of physical and chemical reactions is extremely difficult, due to the violent effect of mechanical action. The part below this area, which is totally and continuously submerged, where mechanical and physical action is greatly reduced, and where chemical action is increased, cannot be examined without diving gear. Thus, no conclusive evidence could be obtained by this examination to support, satisfactorily, the contention that cement is not a factor in the resistance of concrete in sea water.

In order to obtain further information concerning the behavior of concrete in that part totally and continually submerged, a series of tests was undertaken by the writer.[20]

The test consisted of subjecting twenty-nine standard concrete cylinders to total continuous submersion in a solution of Epsom salts. During the test the concentration of the solution averaged 27.2° (Baumé), or a 43% solution, which is slightly below saturation. Magnesium sulfate was selected as the test medium because: (1) Its concentration and composition can be controlled easily; (2) it is the main corrosive agent found in sea water; and (3) it is used in checking the soundness of aggregates used in concrete. The concentration was set high primarily to accelerate the test, and secondarily because the soundness test for aggregates used a saturated solution. "What is sauce for the goose is sauce for the gander." The concrete used for the cylinders was obtained in all except eight instances directly from the chutes leading into the forms of the bulkhead walls being constructed for the East River Drive (New York City) in order to provide concrete actually used in the structure for the test. The cements used were all of the sulfate-resistant type, varying in computed C_3A-content from 4.1% to 6.6%. The mixes tested were 1 : 1.8 : 3.2 and 1 : 2.9 : 5.1. In order to check cements of higher C_3A-content, the remaining eight specimens were prepared in the laboratory, having a mix of 1 : 2.2 : 3.9 and using cements having computed C_3A-contents of 9% and 16%.

The results of the test are briefly as follows:

(*1*) *Effect of Sulfate Action on Compressive Strength.*—The compressive strength of the concrete under the test increased generally to 30% above the standard 28-day check test after extended immersion in spite of surface disintegration. The increase stopped and decrease started upon approximately 50% of surface disintegration when further increase in strength was offset by disintegration.

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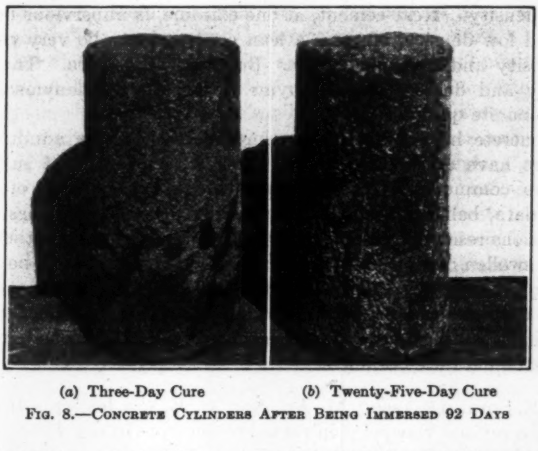
[20] "Resistance of Concrete to Magnesium Sulphate Solution," by Ladis H. Csanyi, with the cooperation of the Dept. of Borough Works of Manhattan, New York, N. Y., submitted to the Polytechnic Institute of Brooklyn, N. Y., in 1940, in partial fulfilment of the requirements for the degree of Master of Civil Engineering.

(*2*) *Effect of Curing on Rate of Disintegration.*—Samples cured for 3 days (1 day water, 2 days wet sand) showed a marked difference in the form of surface disintegration from those cured for 25 days (1 day water, 24 days wet sand). In the 3-day cure (Fig. 8(*a*)), the attack penetrated the grout or carbonate skin and progressed beneath it, flaking off large sections at a time, and thus rapidly exposing the aggregates of the concrete. In the 25-day cure (Fig. 8(*b*)), disintegration was isolated to the points of attack, giving the concrete a blotchy, pimply appearance. In this case exposure of the aggregates was limited and progress of disintegration materially curtailed.

(*3*) *Relation of Design of Concrete Mixture to Rate of Disintegration.*—In every case the richer mixes possessing the greater density showed better resistance for the same type of cement. Mixes of similar density with cement of 16% computed C_3A-content showed less resistance than one having a 9% computed C_3A-content.

(*4*) *The Influence of Cements Upon the Rate of Disintegration.*—The comparison based on the computed C_3A-content of the cement clearly showed that reduction of C_3A-content from 16% to 9% to 6% materially improved the resistance of the concrete. Reduction of C_3A-content below 6% to 4.5% and 4.1% showed material loss of resistance. Therefore, a range between 6% and 7% computed C_3A-content of the cement is indicated as yielding the maximum sulfate resistance to the concrete as far as the effect of the cement is concerned.



(*a*) Three-Day Cure (*b*) Twenty-Five-Day Cure

FIG. 8.—CONCRETE CYLINDERS AFTER BEING IMMERSED 92 DAYS

These studies, therefore, further confirm the essential need of good, dense, uniform concrete, carefully and properly placed and adequately cured, if good sea-water resistance is to be attained. They also refute Mr. Hadley's contention, and confirm the work and belief of others,[21,22,23] that cement is important in imparting some resistance to the concrete when subjected to sea water.

The writer's studies revealed two questions that concern engineers: "What is sea water?" and "What does it contain?" In harbors it may vary from brackish water to high concentrations of sewage frequently containing corrosive industrial wastes. Therefore, let the engineer study the sea water in which he must place his structure, and provide adequate protection and properly designed mixes placed with especial care when the sea water requires it.

G. M. Williams,[24] M. Am. Soc. C. E. (by letter).—The conclusion in Mr. Hadley's paper that there is no evidence of attack or disintegration due to sulfate action could be accepted more readily if chemical analyses of concretes were offered in support. Although developed porosity or disruption may not be apparent, this is not necessarily good evidence that deterioration due to sulfate action is not in progress. Rich concrete mixtures may fail by cracking without prior indication of surface swelling, spalling, or disruptions which are characteristic of failures of medium or lean concretes.

Chemical analyses of small samples taken at several points in a structure such as between low and high tide, parts exposed only to spray, and parts never in contact with sea water should indicate the extent of sulfate action, if any, as well as measure its rate of progress.

Concrete of low permeability is desirable for permanence and resistance to weathering or possible sulfate action, but such concrete is generally not one of greatest density. Neat cement, at one extreme, is impervious but has high porosity and low density, whereas a lean concrete may be very pervious but low in porosity and high in density. Both are absorptive. The terms impermeability and density are antonyms rather than synonyms as used to designate concrete qualities.

That concrete in sea water sometimes deteriorates is admitted. Other investigators have attributed this at times to the action of sulfates. The author, who comments upon the somewhat unusual dearth of supporting laboratory data, believes that no sulfate action has occurred, basing his conclusion upon the results of visual inspection and the absence of either developed porosity or swollen disruption of the mass which he assumes to be the characteristic form and manifestation of sulfate of magnesium attack. Since sulfate content of concrete can be determined so readily by chemical analyses of easily obtained samples, it would seem that valid conclusions can be drawn only after such data are obtained.

Harry E. Squire,[25] Assoc. M. Am. Soc. C. E. (by letter).—In purporting to advocate a revised viewpoint in regard to concrete in sea water, Mr. Hadley

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[21] "Testing Cement Mortars in Sea Water," by Thomas E. Stanton, M. Am. Soc. C. E., *Engineering News-Record*, March 17, 1938, p. 400; also "Expansion of Concrete Through Reaction Between Cement and Aggregate," by the same author, *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

[22] "Influence of Composition on Volume Constancy and Salt Resistance of Portland Cement Pastes," by R. H. Bogue, William Lerch, and W. C. Taylor, 1934.

[23] "The Chemistry of Cement and Concrete," by F. M. Lea and C. H. Desch, *24 Concrete Manual*, U. S. Dept. of the Interior, 1938.

[24] Prof., Civ. Engr., Univ. of Saskatchewan, Saskatoon, Saskatchewan, Canada.

[25] Asst. Chf. Engr., Board of State Harbor Commrs., San Francisco, Calif.

has actually presented a carefully prepared and plausible argument in favor of one of the oldest traditional viewpoints of this controversial subject—namely, that sea water does not attack "sound" concrete, or embedded steel that is adequately covered with "sound" concrete. The contrasting viewpoint is that sea water does attack portland cement concrete and embedded steel whenever it comes in intimate contact with the cement paste or embedded steel, and that the only difference in durability between "sound" and "unsound" concrete is the time factor and intensity of exposure.

Before little if any concrete had been subjected to the sea water of the Pacific Coast, European engineers with a background of more than fifty years experience in marine concrete construction had likewise divided along almost identical lines of opinion.[26] On the Pacific Coast, the forms had barely been stripped from the early concrete before the same cleavage developed among those charged with responsibility for waterfront structures. The extravagant claims of advocates of the newly introduced "permanent" construction faded under the searching forces of sea-water erosion which laid bare all manner of defects ignored under ordinary atmospheric exposure. Nevertheless, enough resistant concrete remained to encourage the proponents of "sound" concrete in their claim that all deterioration could be explained by the term "defective concrete." Improved understanding of concrete technology combined with improved methods of manipulation and manufacture have eliminated almost entirely the defects of the earlier structures; and these methods insure a service life of present-day concrete equivalent to the commercial life of most marine structures built. The controversy becomes more and more a question of academic rather than practical interest and, in the writer's opinion, might be automatically ended by setting up a definition of "sound" concrete in terms of sea-water exposure—material which would withstand for a stated time (say forty years) a certain type of exposure (say tidal or complete immersion) with a stated loss of calcium (say 5%) or, if preferred, an equivalent gain in magnesium. Every one acknowledges that such concrete has been, and is being, manufactured, and most authorities will agree that such concrete is satisfactory for 90% of the structures built. The energy and enthusiasm of those proponents who refuse to acknowledge the mortality of concrete in sea water could then be concentrated on extending the frontiers of durability to really debatable periods beyond fifty years.

In studying the behavior of "sound" concrete in sea water, the writer agrees that there is no better place to go than to the structures which have stood successfully for periods of from thirty to fifty years. However, the writer suggests that the next time the author emulates the prophet and "goes to the mountain," he take a core boring machine and a laboratory assistant with him. There have already been too many surface inspections of both sound and disintegrated structures and too much speculation as to what is going on under the outer skin of the concrete. This is well illustrated by one of the instances of "sound" concrete cited in the paper—namely, the breakwater test specimens exposed at San Pedro.[10] These experimental blocks are almost an exact replica of similar test specimens exposed by Russian engineers in the breakwater at Libau a decade earlier.[27] The Libau blocks were found to be sound under surface inspection but they were, nevertheless, blasted open and samples of concrete from the outside and interior were analyzed and tabulated to show variation from the original cement. Quoting the concluding sentence of the report:

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[26] "Treatise on Dock Engineering," by Brysson Cunningham, 1904, Chapter 4, pp. 123–130.

> "One fact, nevertheless, was certain—that the process of deterioration had already commenced and that it progressed, slowly perhaps, but none the less surely in the case of the mortars considered."

The writer had the good fortune to make a visual inspection of the San Pedro specimens on the contractor's barge during coring operations. The concrete was sound but saturated even in the interior of the blocks. The cores were to be forwarded to the National Bureau of Standards for testing and analysis, but as yet (1941) no record of these tests comparable to the Russian tabulation has come to the writer's attention. The article referred to by the author[10] is merely a report of the surface inspection together with some photographs of the core specimens nicely crated for shipment—hardly sufficient information to warrant the author's rather positive statement that there was "no evidence of sulfate of magnesium attack." Physical and chemical tests of the core samples would give what the lawyers call "the best evidence" as to what attack, if any, has taken place and, if repeated on future cores, will give valuable data on the rate of attack.

The writer hopes that one of the organizations having funds available for research will find it advisable to make a systematic investigation of structures, successfully exposed to sea water, by means of core borings and laboratory analysis. The list of structures selected by Mr. Hadley would be satisfactory to begin with. No doubt, most waterfront engineers would like to add some pet structure to the list, in which case the writer would select the Ferry Building Foundation Wharf at San Francisco, Calif. This well-preserved structure, approaching its 50th anniversary and outliving its utility as a passenger terminal, is remarkable in that it violates some of the choicest taboos of sea-water concrete practice. For one thing, the immersed concrete was mixed approximately one to eight and can hardly be considered impermeable; for another, the Dykerhoff cement used was of a reasonably high alumina content (approximately 11%) in contrast to the special cements of low alumina content advocated for sulfate resistance.

Most of the analyses available have been made of concrete that was badly disintegrated. Investigators seem to take a ghoulish interest in these misfortunes but have little time for successful structures. Invariably, these analyses show a partial replacement of calcium by magnesium. This has lead to the hypothesis, questioned by the author, that the disintegration is the result of sulfate of magnesia reaction with the calcium hydrate of the set cement.

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[27] *Proceedings*, International Assoc. for Testing Materials, 6th Cong., New York, N. Y., 1912, Second Section, XVII.

Analyses of disintegrated concrete taken from a recent and somewhat disconcerting failure of good concrete in sea water (that of the concrete piles of the Ford Motor Company Plant at Long Beach, Calif.) indicated not only the usual increase in magnesia content at the expense of the calcium, but in addition a marked increase in the alumina content. Search for the source of the alumina led to the discovery of variable percentages of soluble alumina and active silica in samples of fine aggregate taken directly from the gravel pit. Submitted to the state mineralogist, the samples were identified as mixtures of quartz particles with varying percentages of feldspar particles, the feldspar being partly kaolinized.

The disintegration was confined to that part of the pile below the line of immersion, and was spotted in a most irregular manner over the entire wharf structure. The conclusion was inescapable that it was due to chemical reactions caused by the sea water; the variation in the behavior of the piles could readily be attributed to variations in the percentage of deleterious feldspar in the aggregate. A marked characteristic of this disintegration was swelling in volume of the concrete affected. The formation of sulfo-aluminates (alums) with their great capacity for increase in volume by taking up water of crystallization has been suggested as the primary cause of this type of breakdown. No matter what the theory, whether due to replacement of calcium by magnesia, to the formation of sulfo-aluminates, or of complex silicates from the active silica of the sand, the fact remains that exactly the same phenomenon of disintegration was simulated in the laboratory by A. A. M. Russell, Assoc. M. Am. Soc. C. E., using the suspected sand mixed with local cement and immersion in San Francisco Bay water. The breakdown of the mortar was similar to what occurred in the Long Beach piles exposed to sea water.

The writer questions the author's opinion that disintegration is primarily the result of physical and mechanical forces rather than chemical, but he agrees that most breakdowns occurring in less than 25-yr exposure may be attributed to two kinds of defective concrete:

(*a*) The use of mixes having a deficiency of cement; and
(*b*) The placing of mixes in excessive water (not merely wet mixes but concrete drowned by leaky forms or by an accumulation of water in tight forms).

Both are probably phases of the same fundamental defect—concrete in which the water-cement ratio is so high that water voids or loose crystallization admits the sea water into ready contact with the cement paste.

After studying the breakdown occurring in the piles at Long Beach and also experiments made by Mr. Russell on a large number of puzzolan admixtures which showed that many substances containing active silica or alumina produce unsound briquets, the writer is convinced that a number of failures previously attributed to improperly proportioned or placed concrete have been accelerated, if not primarily caused, by aggregates which yield alumina and silica capable of reacting with the substances in the set cement paste and with the sea water; and that the superior behavior of some successful structures is due in large measure to the inert nature of the aggregate. In this type of failure, the expansion resulting from internal chemical activity seems, progressively, to open up the structure of even superior concrete to the sea-water salts.

In regard to the other ailment which afflicts concrete in sea water—the corrosion of embedded steel—experience has revealed one optimistic fact. Below the level of immersion, corrosion appears to be completely inhibited and the embedded steel appears to be quite as permanent as wood below the groundwater level. In tidal waters this immunity extends to about the mean high-tide line.

Above the high-tide line a very different condition prevails, and when the surface is subjected to intermittent splashing and drying out, superior concrete in thicknesses ordinarily specified in reinforced concrete practice will not protect against cracking and spalling from expansion of the corroded steel. Greater depths of cover may prevent this type of damage because there is a depth at which the shearing resistance of the concrete cover exceeds the pressure of the corroding materials; but the absence of cracking or spalling is no assurance that corrosion is not in process. Where small bars are used, they will frequently rust away entirely without showing surface indication. Furthermore, corrosion is sometimes localized, the same rod rusting entirely away at one point and having full section not more than a few feet away. The writer never has been able to satisfy himself that if the corrosion of a 1-in. rod cracks 1 in. of cover in from ten to fifteen years, it may not crack 2 in. to 3 in. in thirty to forty years or it may even rust off entirely. Here again is a place for field investigation to determine the relation between depth of cover, impermeability, and rate of corrosion. Perhaps the investigators will conclude, as the writer has, that it is easier to seal the concrete off from sea-water moisture than to obtain a convincing answer.

In the "Summary" Mr. Hadley makes a significant statement which rings responsively in the writer's mind:

> "The harm resulting from misunderstanding a natural phenomenon arises from the likelihood that measures adopted to effect changes and alterations in it will be misdirected."

There are few branches of engineering construction in which methods are more divergent and in which those engaged in the work more individually opinionated than in marine concrete work. Under the guise of producing resistant concrete, engineers resort frequently to the most extreme measures in proportioning, in the use of water, in surface treatments, in manipulation, and in the use of admixtures—measures which are frequently so onerous that they defeat their purpose by diverting attention from simple but necessary requirements. Much of this effort must be misdirected because the writer has rarely seen more than one or two of these extreme requirements applied simultaneously by any one engineering organization.

In addition, many practices are contradictory. Thus, in one case there is the exhibition of two groups of eminent engineers charged with the same problem of constructing bridge piers in harbor waters—the one group, adopting a special cement with a specially low alumina content, and the other prescribing a special puzzolan cement in which an activated alumina silicate is added to the normal alumina content; in another instance, the paradoxical practice of two large harbor organizations in which one protects the concrete under water to the extent of impregnating it under pressure with asphalt but ignores the surfaces above water, whereas the other carefully seals the above-water surfaces but exposes the under-water surfaces without protection to the ocean sea waters.

The best proof of the ability of ordinary, good concrete and standard portland cement to withstand the usual sea-water exposures is the fact that so many structures have given a good account of themselves in spite of extraordinary procedures resorted to in the name of producing resistant concrete. The writer agrees that it is highly desirable to clarify the concrete technologist's viewpoint and attitude when he comes down to the sea; but he believes that the way to do it is to probe into existing structures and discover what changes in structure and chemical content have occurred. Only by knowing what has happened in the first fifty years can one judge what may be impending in the next fifty or hundred.

J. W. B. BLACKMAN,[28] M. AM. SOC. C. E. (by letter).—Referring to Mr. Hadley's paper, it is noted that observations on marine structures are confined to the Pacific Coast. It is further noted that the paper deals only with disintegration of portland cement structures from the action of sulfate of magnesium. This very much limits the discussion.

It must be stated that there are very few portland cement concrete structures north of San Francisco. Since the seaports of the Pacific Northwest—in Washington, Oregon, British Columbia, etc.—are in a timber country, creosoted piling is used extensively for construction of their marine works. The San Francisco Bay area and the Los Angeles and San Diego harbors are where the greatest number of concrete structures exist.

Portland cement concrete, when exposed to sea water and sea air, is subjected to many causes of deterioration and disintegration other than the action of sulfate of magnesium (which is generally considered as secondary by engineers engaged on the construction of marine work). Mr. Hadley has mentioned the many other attacks of various kinds.

It may be well to summarize briefly some of the various causes of action that the writer has encountered. In driving pre-cast concrete piles, many hazards are met before the pile reaches its final position. Assume that the concrete mixture has been carefully designed, the mineral aggregates tested, the concrete compacted by careful vibration, a minimum quantity of water used, constant inspection by experienced men, and not less than 3 in. of cover for the steel. Then, with a perfectly impervious and well-protected pile cast ready for transportation, there are yet many hazards to guard against. It must be lifted and carefully handled at the point of driving. It must be placed in the pile-driver leads and driven with a hammer whose weight is proportioned to the size and weight of the pile. The amount of driving that the pile is to be subjected to is to be governed generally by the experience of the engineer and the type of soil it penetrates. Where piles are held entirely by friction, some engineers prefer to jet most of the way down, only driving the last few feet, whereas others prefer to drive the entire distance, if possible.

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[28] Cons. Engr., Long Beach, Calif.

There is a difference of opinion between some engineers as to the amount of driving that a pile should receive. It is not necessary to elaborate in detail on Mr. Hadley's paper with regard to the many types of failure to which concrete is subjected, but briefly they appear to be as follows: Poor workmanship, faulty mineral aggregates, poorly designed mixtures, too much water, insufficient cover of reinforcement, cracks due to shrinkage, handling, and driving. Porosity is one of the greatest causes of failure. It allows the moisture to gain access to the steel reinforcement, causing ready oxidation and steel expansion which, in turn, spalls the concrete. Many writers have demonstrated that there are two distinct causes of failure—one due to mechanical action and the other due to chemical decomposition.

Porous concrete can be disintegrated by forces exerted by the crystallization of salts in its pores. This condition is accelerated by the constant wetting and drying of concrete exposed to the wave action of sea water. The continuation of this process creates surface cracks, which are caused by this building up of crystals and concentration of the salts, and will in time cause deterioration of the concrete both by mechanical and chemical action. It is apparent, therefore, that all concrete exposed to sea water and sea air must be as impermeable to water as it is possible to make it.

The writer was asked to inspect a wharf built some few years ago in Long Beach (Calif.) Harbor. A large number of the concrete piles were in a serious stage of disintegration. The concrete in which disintegration showed was swollen and disrupted, and this condition appeared to be more acute on the upper side of the batter piles. Repairs to the structure cost about $250,000. In the opinion of some engineers the disintegration was due to feldspar in the small aggregates. In the opinion of others the damage was due to other causes. It is interesting to note that a large number of the piles adjacent to those disintegrated showed no apparent decomposition. Is it to be assumed that the cause of the disintegration was due to faulty workmanship? For instance, some of the piles may have been compacted carelessly, there may have been too much water, and the porosity may have been much greater than that of the adjacent piles that were not injured. This raises a question that is difficult to answer.

The Portland Cement Association has, with wisdom, recommended[29] as follows: "For unusually severe exposures, as in sea water, etc., a clear cover of 3″ is recommended at least for the portion of the pile where such exposure is encountered." Some cement companies have placed on the market brands of cement such as "sulfate resisting cement," "high-silicate cement," etc. The chemists of cement companies have been endeavoring to find a remedy for attacks on cement concrete exposed to sea water and sea air.

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[29] "Concrete Piles," Portland Cement Assn., November, 1939, p. 28.

Concrete in the soffits of slabs and beams is just as vulnerable as other parts of concrete structures to disintegration or deterioration from the various causes mentioned. Hence, various methods of protection have been tried, some of which have proved successful.

Reference is made by Mr. Hadley to the small ferry wharf at North Vancouver, B. C., Canada. The writer knows this structure well and has inspected it. He feels that where such examples are cited, for comparison, the following additional information should be supplied:

(1) The structure is built on columns (not piles). Had piles been used instead of columns, they would have been subjected to the stresses of handling and driving, which in many cases creates cracks. This in time readily admits sea air and salt water.

(2) The salinity of the Burrard Inlet (North Vancouver) is much less than that of the open sea because large bodies of fresh water are discharging all the year round from the creeks upstream and downstream from the ferry wharf.

Mr. Hadley's statement that the protective measures of asphalt impregnation of concrete piles, as used in Los Angeles Harbor, are thoroughly effective as to impermeability and density is interesting. A recent inspection of the same type of piles in Richmond Harbor, Calif., and San Diego Bay, shows that these piles are in excellent condition. Some of the piles in Los Angeles Harbor have been in use for twenty years.

Mr. Hadley should be thanked for bringing this interesting matter to the attention of engineers.

E. C. JACK,[30] ESQ. (by letter).—Any viewpoint of the subject of concrete in sea water should be somewhat philosophical, in that disintegration or deterioration may or may not be serious, depending upon the rate, and keeping in mind the anticipated life of the structure. So many of the construction people with whom the writer does business consider 10 or 20 years a long time, and anything that might happen after that they believe is immaterial. However, one needs only to look back over the centuries to realize that this matter of ships is a long-range, continuing one extending far into the future. The U. S. Navy is primarily interested in ships, and waterfront improvements to serve the ships should be provided with an eye to a life of at least 100 years. Thus, what to one person might be a negligible breaking up of the structure, to another would be a serious indication when a long life had been planned.

It has been noted that, in almost all cases where disintegration of one kind or another has taken place, there are contributing factors upon which the onus of the situation can be placed, thus clearing sulfate attack of the charge. Nevertheless, it is agreed with Mr. Hadley that the weakness of concrete toward sulfate attack has perhaps been over-stressed. Chemists have their catalytic agents and metallurgists their eutectics; and, in a similar manner, is it not probable that the weakness of concrete toward sulfate and acid attacks makes possible the disintegration by other means, such as wave and frost action, abrasion, and chemical action? In several places in Alaska, notably at

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[30] Senior Civil Engr., Puget Sound Navy Yard, Bremerton, Wash.

Kodiak and at Dutch Harbor, concrete has met with progressive disintegration between tide lines, whereas the remainder of the structure has withstood weathering very well for 25 years. Surely these cases must be a combination of sea-water attack plus frost and wave action.

Cement is a wonderful material that has been improved remarkably through the years, and its comparatively low cost, together with the rather universal distribution of the mills throughout the United States, makes possible the building of many structures that would have been impracticable years ago. It is far from perfect, however, and it may be well to review the most well-known weaknesses of cement and concrete.

Aggregate.—The quality and usefulness of concrete, with present-day cements, demand that aggregate be sound, chemically neutral or basic, and free of oil and other deleterious substances, in addition to being well graded. Thus, in some localities, concrete is very costly because of the distance from which aggregate must be brought. On St. Paul Island, in Bering Sea, it was necessary, one time, to carry gravel by ship from San Francisco, Calif. If cement acted slightly different chemically, then a wider range of aggregate might be used.

Proportioning.—The mixture must be a proper one with sufficient cement, and this item of cement, too, is relative: Witness the amount of cement used per cubic yard today compared with the amount used 50 years ago.

Compounding.—The mixture must be put together in the right order, well worked, with one's weather eye open for numerous technical pitfalls.

Chemically Basic.—Cement and concrete in the green stage are strongly basic, and even when well hardened they are slightly basic. They are thus particularly vulnerable to all substances of an acid nature, such as humous, coffee, industrial wastes, sea water, etc. Just what goes on within the mass during hardening is still very much a mystery. A more thorough and complete chemical combination very probably would improve not only the chemical weakness but also reduce the troubles from erratic hardening, and actually reduce the shrinkage.

Porosity.—Concrete made in the best manner known is quite porous under some conditions, and is thus vulnerable to wave and frost action and penetration of salt water to embedded steel, causing spalling and other such deterioration.

Time of Set.—The time of set is not subject to very good control, which very much limits the use of concrete under some circumstances. In fact, this lack of control can be very embarrassing at times, being either too slow or too fast, and this also nearly prohibits its use for repair work between tides.

Rate of Hardening.—This property naturally varies greatly with different kinds of cement, but in all cases it is absolutely dependent upon the temperature—too much so for comfort in cold climates.

Heat of Crystallization.—This factor has been partly taken care of with the modified cements, but it is still difficult to disperse in spite of concrete being a relatively good conductor.

Workability.—In spite of great care taken to proportion cement and aggregate, the resulting mixtures are almost universally harsh; that is, the cement paste is not as smooth as one would like to have it.

Shrinkage With Hardening.—In practically all cases, concrete shrinks with hardening, thus causing troublesome construction joints.

Volume Change.—This phenomenon occurs, with addition or removal of moisture.

The foregoing résumé includes only the more obvious weaknesses. The items are interrelated in many cases. The first three will probably be present always as they are inherent in the universal use of any material of so cosmopolitan a character as cement. Modified cements and the use of admixtures are the result of trying to obviate some of these deficiencies. If a cement could be manufactured that would combine with all of the water ingredient, it would solve many of these problems.

In conclusion, the analysis of the many combinations of circumstances surrounding any one incident of deterioration is a complicated problem, and it has not been found possible in many cases to put one's finger definitely on any single underlying cause. Generally, it is a combination of poor circumstances, or a poor combination of circumstances, which made the deterioration possible. Therefore, sea-water attack by itself, it is believed, is seldom the cause of progressive disintegration, but in combination with other weaknesses it probably is a contributing factor.

Alfred M. Freudenthal,[31] Assoc. M. Am. Soc. C. E. (by letter).—In attempting to present the problem of resistance of concrete in marine work "in a nutshell," and in stressing the overwhelming importance of resistance against mechanical attack, Mr. Hadley is to be highly commended.

However, there are few engineering problems less suited to the dogmatic approach than that dealt with by the author, and if the question—"Does any form of deterioration occur in the concrete in sea water that would not develop if the exposure were changed to similarly agitated, similarly fluctuating, fresh water?"—is answered definitely in the negative, this appears to be overstating the point.

The deteriorating action of the sea is of two kinds: Mechanical and chemical. The lack of distinct evidence of chemical corrosion in concrete of adequate design and composition (on which evidence the author rests his conclusions) indicates that two "lines of resistance" may be assumed to exist, the "advance line" of the two being the resistance of the structure to mechanical attack. However, there is definite evidence of chemical attack of sea water on the cement and the conclusion that "* * * there appears to be no valid basis for believing that such attack occurs" (see heading "Conclusions") is in contradiction to the results of very extensive and very careful research in America as well as in different countries of Europe.[32]

It is not only the sulfate-of-lime attack that leads to disintegration of concrete; the tricalcium aluminate-hydrate is by far more injurious, although less frequent. Evidence of other corrosive effects of sea water, as well as of the interrelation between the specific cement qualities and the intensity and speed of deterioration, has remained inconclusive, and additional research is needed urgently.

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[31] Res. Engr., Marine Trust, Tel. Aviv Port, Tel. Aviv, Palestine.

[32] "Resistance of Cements to Attack by Sea Water and by Alkali Soils," by Thomas E. Stanton and Lester C. Meder, *Journal*, A. C. I., Vol. 9, 1938, pp. 433–464; "Report of International Congress for Testing Materials," London, 1937 (see especially reports by H. Kuebl, F. Ferrai, and A. Steopoe; and "Deterioration of Structures of Timber, Metal and Concrete Exposed to the Action of Sea Water," Dept. of Scientific and Industrial Research, H. M. Stationery Office, London, 1935 (report by R. E. Stradling).

Questions of serious import are: "Is the effect of puzzolanic admixtures physical (that is, do they increase the density), or is it chemical?" The use of quick-setting cement in marine work creates an early mechanical resistance to external forces. "Do the mechanical advantages outweigh the chemical disadvantages?" The writer has observed conclusive evidence that aluminous cements are very resistant to the action of sulfur compounds in fresh ground water but has further observed that they are definitely weak in sea water. "Why then do they prove definitely weak in marine structure?" "Is this weakness due to physical or chemical action?" "Can pre-cast members be protected against corrosion by applying a bituminous coating to the surface?" Such investigations were begun at Tel-Aviv Port in 1939 in connection with the driving of concrete piles.

The foregoing questions are of considerable importance, and the opinion expressed by the author (see heading "Summary")—that "There is no particular reason for research * * *" is scarcely justified. If the metaphor used by Mr. Hadley were correct, the resistance against chemical corrosion might be considered a comparatively "strong link" in the "chain" of total resistance. This point is supported only by the author's testimony, since the structures referred to do not demonstrate more than the fact that, in all cases where sufficient resistance of the concrete against mechanical attack has been insured, the amount of chemical corrosion has appeared insignificant. The metaphor of the "chain" and its "links" is not a happy one, since the wearing down of both kinds of resistance—the mechanical and the chemical—is not simultaneous but rather consecutive. Considerable deterioration is always preceded and initiated by deficiencies in the mechanical resistance, and there is nothing to support the opinion that after the breakdown of this "first line" of mechanical resistance the subsequent deterioration is not considerably intensified and accelerated by the chemical attack.

Observation of marine structures shows that the main damage to the concrete occurs within the tidal range as well as on horizontal surfaces above sea level (such as flat tops of breakwaters and sea walls) subject to constant spray. It is due to the evaporation and consequent concentration of the attacking saline solution. The fact that the strongest mechanical action of the waves usually occurs near the mean sea level, and thus within the tidal range (wearing down the resistance of the concrete against mechanical attack more rapidly than elsewhere), makes this range particularly susceptible to the concentrated chemical attack beginning in the eroded cavities and voids. Since the corrosion of the permanently submerged parts of marine structures is almost nil, due to the formation of a siliceous gel film that protects the surface, it is not the actual saline concentration of the sea water that is relevant, but the high concentration of the saline solution caused by evaporation. The circumstance that the eroding effect of the initial concentration of the sea water might be regarded theoretically as rather weak thus has no bearing upon the actual intensity of the attack upon concrete marine structures.

It is evident that the density and the impermeability of the concrete, particularly near the surface, are the principal qualities constituting the mechanical resistance of the concrete against attack by the sea. Theoretically, these qualities may be obtained by adequate grading of the aggregates, rich mixes, and (if this is not thought sufficient) puzzolanic admixtures or the admixture of other dispersing agents. Practically, however, the placing of concrete and its early resistance to mechanical attack is of overwhelming importance in attaining the required density and impermeability, and the best mix will be entirely deprived of its merits if there is even slight neglect in the procedure of placing. The considerable hazards encountered in the construction of marine works are sufficiently known and may not always be forestalled. In spite of all precautions it often may not be feasible to secure practically perfect conditions of placing and to prevent local damage of the concrete surface. Thus its capacity to resist mechanical attack is affected. If no resistance against chemical attack is provided for, relatively rapid deterioration of the structure will start locally, and it will extend gradually over parts initially unaffected. Adequate capacity to resist chemical attack, however, will counteract the injurious effects of local erosion to a considerable degree.

Although the author's statement that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement" may be fully indorsed, his opinion that "* * * there appears to be no valid basis for believing that such attack occurs" is definitely contradicted by actual evidence (see heading "Conclusions"). The conclusion (see heading "Synopsis") "* * * special precautions against sulfate attack are needless" is not justified, and its application in practice would deprive concrete marine structures of a highly valuable "second line of resistance."

T. B. RIGHTS,[33] ASSOC. M. AM. SOC. C. E. (by letter).—Experience has led the writer to believe that concrete should never be used in sea water between high-tide and low-tide elevations. If there is disintegration in concrete in sea water, above the high-tide level, the factor of the brand of cement used in the construction should be investigated carefully.

In 1925, the engineers of an eastern railroad built a bridge across an estuary near New York, N. Y. At the eastern end of this bridge, two test piers were constructed. Pedestal piers that supported the steel work of the bridge protected these test piers, so that mechanical abrasion due to floating ice or logs was reduced to a minimum.

The test piers were identical except for the cement. The bottoms were about 2 ft below mean low water and the tops 2 ft above mean high water. No reinforcement was placed in either pier. Fresh water was used for mixing water. Both test piers were constructed "in the dry" with accurately measured water, and proper batching of aggregates. One test pier was built with the same brand of cement as the remainder of the bridge; the other was constructed with quick-setting high-strength cement.

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[33] Roselle, N. J.

Within two years after construction, the test pier made of the quick-setting cement began to disintegrate and within five years had disintegrated so much that it could only be seen at low water.

Twelve years after construction, the test pier of regular cement was sound, and apparently unaffected by years of exposure to the tidal range of sea water. At that time, one would draw the conclusion that the failure of the one test pier was due only to the cement. At present (only sixteen years after construction), the regular cement test pier is beginning to disintegrate.

Engineers of this railroad company find that, as a general rule, concrete of the better grade will not show disintegration for at least twelve years. Consequently, on important bridge piers built in salt water, it is their general practice to face the piers between a plane about 2 ft below low water to a height of 2 ft above high water, with either granite or hard paving blocks. The railroad bridge cited herein, where the test piers are situated, has its many piers constructed in this manner, with a belt of granite blocks between extreme high and low water. There is no apparent disintegration in any piers thus protected. The concrete above the granite blocks seems (1941) to be in excellent condition. However, the mortar between the granite blocks, on the horizontal layers, is missing in a number of these piers. In fact, at some piers it seems as if only the bond at the backs of the granite blocks holds them in place. This railroad company has not experienced trouble with any of its concrete piers poured in fresh water. The only zone of attack and disintegration appears to be in sea water between the tidal range.

Figs. 2 and 4 apparently show disintegration at the base of the wall. If a belt of granite had been used, the structure might be in good condition today, as the remainder of both walls above high tide appears sound.

The writer does not know what caused this disintegration—whether it is sulfate-of-magnesium attack, freezing, or crystallization of the salt in the water—but he does know that if he were to construct piers in salt water, the concrete would be protected by a belt of granite at the tidal zone.

Homer M. Hadley,[34] Assoc. M. Am. Soc. C. E. (by letter).—It is the privilege of the writer to answer, or to attempt to answer, the objections that have been raised to his paper. On the moot subject of concrete in sea water there would probably have been a wider discussion and more objections raised if times were less troubled and more leisurely than now. However, a sufficient amount of skepticism has been presented, directly, indirectly, and by reference, to stimulate the writer to further and closing endeavors, as follows:

First be it said that the paper attempted to give a summarized but complete statement of facts and conditions as they exist; then an interpretation of those conditions. As to the statement that there are a large number of concrete structures of considerable age in sea water along the Pacific Coast, which are free from deterioration, no denials were entered. On the contrary Messrs. Stanton, Way, and Paxson confirm and indorse these statements, and Mr. Squire, although favoring the chemical attack theory, specifically declares that the substructure of the Ferry Building at San Francisco, "approaching its 50th anniversary," is free from deterioration. Therefore, the existence of the numerous old and unaffected structures, not having been challenged or denied, may be accepted as reality by every one. This may be done the more readily because beyond any one's assertion, pro or con, are the structures themselves, standing in place and giving their own testimony to whoever examines them. Mr. Squire appears to feel that conclusions based on a mere quarter century's performance are too hasty and precipitate and that good behavior for the first 25 years is no assurance as to what may happen in an ominous future; but apart from his warning no dissent on the widespread immunity from sea-water attack has been expressed. It is largely in the interpretation of the deterioration which has occurred that variance of opinion is most pronounced.

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[34] Regional Structural Engr., Portland Cement Assn., Seattle, Wash.

Before coming to the other matters, however, the question of the significance of these old and unaffected structures may well be asked. If sea water attacks them why are they immune? If they are immune—and they are—what is the matter with the sea-water attack? Where is it? Why doesn't it attack? In short, where is the reality—in the structures or in the theory that they contradict and refute? Under these conditions the theory manifestly becomes untenable. There is no satisfaction in retaining and cherishing a theory that does not work, however plausible it may be. A prediction of future occurrences must be duly fulfilled if faith and credence are to be maintained in the prediction. Therefore, it appears advisable to the writer that the inevitable sea-water attack theory be abandoned because of altogether too much conflicting evidence. However, should any one be minded like Mr. Squire to retain it for another 10 or 20 years, let him feel wholly free to do so. Time is not an end. Nevertheless, he should bear in mind what was said of old: "Hope deferred maketh the heart sick."

Also deserving of fullest consideration are the partly deteriorated structures—those which combine disintegrated parts with perfectly sound and unaffected parts. Why do they do this? How can such things be? If the sulfates of magnesium, or what-have-you, attack the concrete in some places, why not in others? Attention is directed to Figs. 2, 4, and 6 as food for thought for every one, but particularly for those who yearn after the chemical and the obscure.

It is gratifying to find that there is general agreement on the need for proper mixtures, minimum water content, careful placement, etc. Since the importance of these matters is now widely recognized, it follows that concrete in sea water will have a far better record in the future than it has had in the past.

The dearth of supporting laboratory data in the paper receives adverse comment in two discussions. That the submittal of such data is most conventional is admitted. It is orthodox, regular, "almost always done." Yet is there need for it as an aid in permitting one to believe the existence of that with which he is confronted? Assuming that the observer stands before concrete that has been exposed to sea water for many years and that he not only sees that it is sound and unaffected but on striking it with a rock or a hammer he likewise feels that it is hard and unyielding and hears it ring to the blow as only hard, sound concrete does, then on this evidence of three of his five senses—all that are involved—it is submitted that without more ado and without waiting for core borings he may well believe that it is indeed unaffected by the sea water. Such may seem a daring, risky, hazardous thing to do on one's own initiative and "without benefit of clergy," so to speak, but inasmuch as there is a wealth of corroborative experience to be had, the act is recommended. If there were not the additional and supporting evidence, physical and chemical analyses would assuredly be most desirable. An extraordinary anomaly deserves the fullest investigation; but this is no rare thing, this unaffected concrete. It is everywhere, even constituting the major part of badly deteriorated structures. Relevant, therefore, appear the words of Lewis Mumford, who can see no need for research to prove that water is wet or that pepper tastes hot on the tongue. There is nothing particularly wrong with such research, of course, but why, laboriously and scientifically, demonstrate the obvious? What is needed far more than any laboratory research or cutting of cores is a comprehensive and detailed survey and report on conditions as they exist, supported with ample photographic evidence. Every one could then form his own conclusions from the photographs—and there would be little to argue about. In general, the cause of sea-water attack is no more than this: Too much water in too lean mixtures frequently assisted by careless placement. Change these conditions and the attack disappears. Retain these conditions and the attack follows. "Pay your money and take your choice."

Several advance the suggestion that where deterioration does occur the sea-water sulfates hasten and accelerate the destruction. Possibly this is true—possibly it isn't. What quantitative data exist to support the hypothesis? For many years the writer has looked at concrete—good, bad, and indifferent—not only in sea water but in fresh water as well, and he can only say that his impression is that the deterioration is substantially alike, allowance being made for the greater water movement and greater mechanical destruction along the seacoasts. Following Mr. Hammond's reference to them, the writer reread the articles[17] by Messrs. Wig and Ferguson published in 1917, and he looked at the illustrations of these articles. Then he looked through a dozen or so volumes of the *Proceedings* of the American Concrete Institute (A. C. I.), which have carried many a paper and report about durability of concrete, deterioration of concrete, frost-resistant concrete, etc., principally illustrated with examples in the Great Lakes region. For every Wig and Ferguson illustration of sea-water deterioration—reinforcement troubles excepted—he thought he found its counterpart illustration of structures around the Great Lakes, far from the sulfates of magnesium. To any one interested in seeing for himself, these several writings are commended.[35] Also, let every one who would make sea-water resistant concrete read a paper by R. B. Young,[36] published in 1940. It presents the recipe although not written with that thought in mind.

Captain Blackman and Mr. Freudenthal advance the view that the making of proper concrete for sea-water use is a matter of delicacy and extreme refinement. "A hair, perchance, divides the false from true." From which view the writer must dissent. Given reasonable proportions and control there is no cause for believing that a slight departure from the established quantities is

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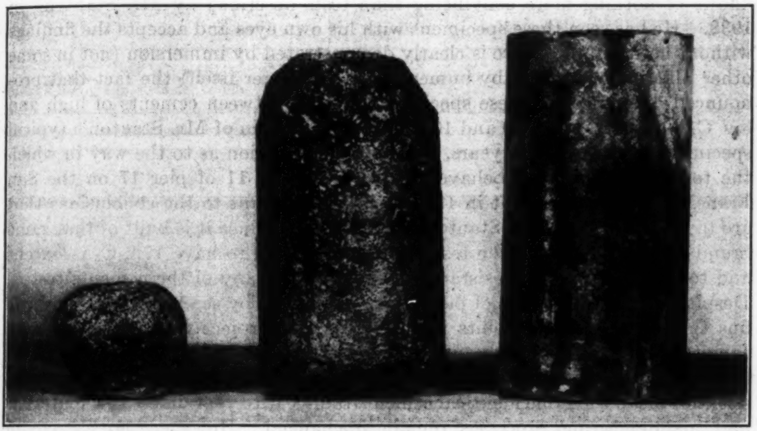
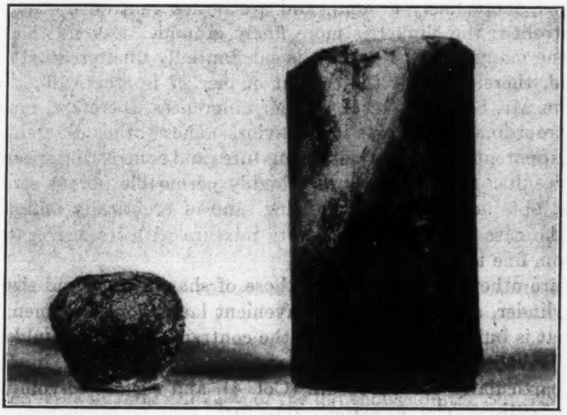
[35] See particularly illustrations in *Proceedings*, A. C. I., 1925, pp. 271–279; 1929, pp. 65 and 66; 1937, pp. 1055 and 1068; and 1940, p. 479.

[36] "Frost Resistant Concrete," by R. B. Young, *loc. cit.*, 1940, p. 477



(a) Standard Ottawa Sand; 1 : 3 Mix (b) Standard Ottawa Sand; 1 : 2 Mix (c) Graded Russian River Sand; 1 : 3 Mix

FIG. 9.—VARIATIONS OF MIX; CEMENT SPECIMENS AFTER FOUR YEARS IN SEA WATER (17.2% C_3A



(a) 17.2% C_3A (b) 7.2% C_3A

FIG. 10.—VARIATIONS OF C_3A-CONTENT; CEMENT SPECIMENS AFTER FOUR YEARS IN SEA WATER (STANDARD OTTAWA SAND; 1 : 3 MIX)

fraught with dire consequences. If water is kept to the minimum which placement will permit, and if the cement content is not less than 1.50 bbl per cu yd, considerable fluctuations can occur without ill effects. They are not recommended, of course, but neither should they be appalling to contemplate if they do occur.

Of all the evidence in support of the sea-water attack theory, the writer knows of nothing more convincing than tests described by Mr. Stanton[21] in 1938. He has seen these specimens with his own eyes and accepts the findings without hesitation. There is clearly demonstrated by immersion (not in some other alkaline water but by immersion in sea water itself) the fact that pronounced differences in these specimens develop between cements of high and low C_3A-content. Figs. 9 and 10 are views of certain of Mr. Stanton's typical specimens at the age of 4 years. There is no question as to the way in which the test specimens have behaved. However, Fig. 11 of pier 17 on the San Francisco waterfront, built in 1912, is highly relevant to the conclusions that are to be drawn from Mr. Stanton's investigations since it is built of that same brand of cement used in the tests which was found to have 17.2 C_3A-content and to have the poorest resistance to sea water of any of the cements tested. Despite this fact, the piles of pier 17 are unaffected by sea water and the little fins that formed at the joints in the forms still project from their surfaces. Inevitably the question arises: "So what?"

It is to be recognized by every one that the same name on a sack of cement does not mean the same identical cement from year to year unless raw materials and manufacturing processes remain unchanged. However, no change of major consequence has occurred with this particular brand of cement. It uses the same limestone and same clay and essentially the same manufacturing process today as it did in 1911 and 1912. Its raw materials are more carefully proportioned, more finely ground, and are better controlled; the burning is better controlled; the clinker is more finely ground. Beyond these changes, however, the manufacturing process is substantially unaltered. It can safely be assumed, therefore, that the cement of pier 17 is practically the same as that used in Mr. Stanton's tests. Other differences, therefore, must account for the extraordinary contrasts in behavior. These other differences are immediately apparent in the contrasting mixtures and contrasting specimen structure that results. In the one case a highly permeable porous structure was sought and obtained with mortars using sand of practically uniform particle size. In the case of pier 17 a concrete mixture with its aggregate particles grading from fine to coarse was used.

There are other differences also—those of shape, form, and size. A 2-in. by 4-in. cylinder, although a very convenient laboratory specimen, resembles nothing that is built in structures; on the contrary, it is quite unlike anything so built. How great these differences are is shown in Table 1, in which are to be noted particularly the curvature (Col. 4), and surface-to-volume (Col. 5) and edge-to-volume (Col. 6) ratios. The little cylinder is much more unfavorably shaped than is the corresponding round pile or square pile section. The square pile has more edge length—usually chamfered—but has flat plane surfaces.

There is not only pier 17 at San Francisco in conflict with Mr. Stanton's and others' findings regarding C_3A-percentages. Mr. Squire records the 11%

TABLE 1.—RELATIVE SHAPE AND SIZE OF CONCRETE SPECIMENS

Description	Volume (cu in.) (1)	Surface (sq in.) (2)	Edge length (in.) (3)	Curvature[a] (4)	$\frac{\text{Col. 2}}{\text{Col. 1}}$ (5)	$\frac{\text{Col. 3}}{\text{Col. 1}}$ (6)
2-in. by 4-in. cylinder	12.57	31.416	12.57	57.30	2.50	1.00
4-in. section of 16-in. round pile	804.04	201.06	0	7.16	0.25	0
Ratio, 2-in. by 4-in. cylinder to pile	.....	......	....	8	10	∞

[a] Curvature of edge and side, in degrees per inch.

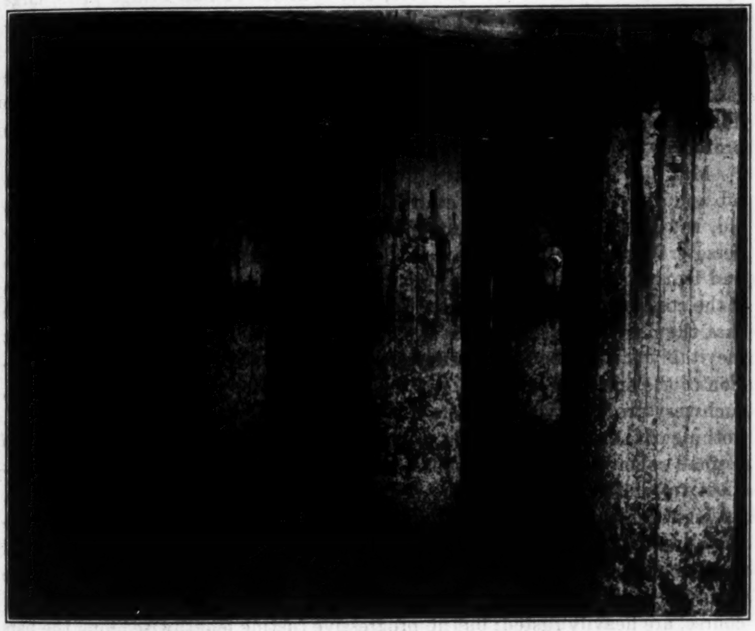


FIG. 11.—PRECAST CONCRETE PILE JACKETS; SAN FRANCISCO PIER BUILT IN 1912

C_3A of the unaffected San Francisco Ferry Building substructure, which has been nearly 50 years in sea water. The cylinder substructure of pier 38 at San Francisco built in 1909, the base of the lighthouse on the San Pedro breakwater at Los Angeles built in 1910, the cylinder shells of pier 8, Puget Sound Navy Yard at Bremerton, built in 1911 are all 30 or more years old, are all perfectly sound and unaffected by sea water, although the cylinders of pier 38 contain numerous honeycombed areas dating from their original construction. They were built of that brand of cement which in Mr. Stanton's tests was found to have 13.2% C_3A-content. Average analyses of bin samples of this cement for the years 1909 to 1913 ranged from a low of 14.5% to a high of 15.9% C_3A-content. Moreover Mr. Stanton advises that his specimens made with high C_3A cement but with the graded Russian River sand still showed no signs of attack after seven years of the same complete immersion in sea water that proved so disastrous to the specimens made with Ottawa sand. It is for these reasons that the writer is unable to attach more than minor significance to the C_3A-content of cement for sea-water use. It certainly is important in 2-in. by 4-in. cylinders made of Ottawa sand; but what further claims are (by evidence) warranted concerning it? The disparity between 2-in. by 4-in. cylinders and ordinary piles is great, but how much greater is the disparity between 2-in. by 4-in. cylinders and 4-ft cylinders, 8-ft cylinders, small bridge piers, large bridge piers! The differences may become of wholly different orders of magnitude. The matter of concern is the behavior of the structure and the prototype. Therefore distinctions which, by a process of magnification, are made manifest in a laboratory appear to the writer to have little significance unless they are also found in the prototype; and if they reveal no effect in the prototype, what practical importance attaches to them?

Mr. Way's detailed account of experiences with a large new concrete pile job and of what was found on examination of a near-by pile structure, 15 years old, is a valuable one, particularly in its bearing on the importance of transverse cracks in concrete piles. Frequently a great deal of alarm, agitation, and trouble to all concerned result from fine shrinkage cracks in piles which, if the concrete is of proper quality, cause no harm whatsoever. It is not true that the mere admission of sea water through fine cracks to the reinforcement inevitably brings about progressive rusting of that reinforcement and disruption of the enclosing concrete. If the concrete is (or was) of a lean, wet mix, such very probably will be the result; if the concrete was not lean and not wet, nothing of the sort will happen. A few miles west of Olympia, Wash., in 1920 a small concrete bridge was built across the head of Mud Bay, the southernmost tip of Puget Sound. The columns of the bents were poured in place. At high tide they are almost completely immersed in sea water. At low tide they are completely bare. At a number of places on the square, chamfered corners of these columns, drift logs have broken off the concrete, exposing the main vertical reinforcing bars for several inches. These exposed lengths, of course, are heavily rusted; but no progressive rusting lengthwise along the bars or splitting of the concrete has ever resulted. The rust formed to the edges of the concrete and stopped there. The writer makes this statement with the greater assurance because he broke off some of the abutting concrete, further exposing the reinforcement, and saw that the rust stopped at the edge of the original fracture. Why should this be, if the admission of sea water through the least fine shrinkage crack means doom and perdition? Well, it means no such thing.

An interesting case showing the effect of too much water is in the San Francisco district where considerable corner cracking over the main longitudinal bars occurred in a large pile job, necessitating rather difficult and extensive repairs. The piles, as is almost always the case, were cast in a horizontal position. Three of their sides were thus poured against forms; their top sides were troweled. When the cracks developed, in practically every instance they occurred on the troweled side of the pile—that is, on what had been the top side. "Water gain"—a fine film of water on the under side of the top longitudinal bars, which water film later left an equally fine void space wherein rust could form and progress—appears to be the explanation of this case. Bond tests by H. J. Gilkey, M. Am. Soc. C. E., S. J. Chamberlin, Assoc. M. Am. Soc. C. E., and R. W. Beal[37] and by Carl A. Menzel[38] show strikingly the effect of casting position—"orientation." It is also worth recording here that the very unfortunate and widely celebrated early experiences with concrete piles in the work of the Los Angeles Harbor Commission were had with piles of 1 : 2 : 4 mix, some of which at least (as photographs attest) were cured by a man sprinkling them with a hose; how continuously or how intermittently is not revealed by the photographs.

Mr. Paxson's comments on sea-water concrete along the Oregon coast are appreciated. His observation that concrete in deep water shows less deterioration than that on the shore where abrasive and scouring action occurs is highly significant and amusing as well: The more completely the sulfates of magnesium surround the concrete the freer it is from deterioration.

To Mr. Hammond the writer would refer his preceding discussion of Mr. Stanton's tests. He would question Mr. Hammond's statement that Messrs. Wig and Ferguson "demonstrated" that chemical action was noted all along the Pacific Coast. Their papers published in 1917 reveal that they found a considerable amount of deteriorated concrete. The interpretation they placed upon it was that the deterioration resulted from sulfate attack. Presumably they examined all of the old concrete structures shown or referred to by the writer since they state[39] that their examination had been made in the preceding 2 years and that they had made "personal examination of nearly every concrete [marine] structure * * * in the United States." Nevertheless, having reread their papers, the writer cannot see that they "demonstrated" sulfate attack. They attributed the deterioration to sulfate attack. It would be very interesting, however, to see how well their theory of immunity to sea-water attack achieved by the formation of lime carbonate near the outer surfaces of the concrete would be sustained by little 2-in. by 4-in. Ottawa sand specimens that were given proper initial curing followed by generous air curing, and what, under these circumstances, the effect of varying C_3A would be.

Mr. Csanyi's discussion, reporting the condition survey of nineteen structures in New York Harbor, is very interesting. Regarding the continuously submerged parts of concrete structures that cannot readily be examined, it is to be said that little or no question has ever been raised about them. It appears to be the general experience that this concrete does not deteriorate. When the piers of the Seaside Bridge of the Union Pacific Railroad were removed from Cerritos Channel, Los Angeles Harbor, in 1934, after 26 years of immersion, they were found to be wholly unaffected. They were built of a 1½ : 3 : 5 mix, not 1 : 3 : 5 as so frequently and so unfortunately has been done in the past. "Not a bad spot in them," declared the superintendent of Merritt-Chapman-Scott Corporation who took them out. The concrete fragments from the piers certainly confirmed this statement. When the old bridge at the site of the new Purdy Bridge, Pierce County, Washington,[40] was taken out, its foundations had been in sea water for 15 years. One of the piers had to be removed completely. Its concrete throughout was found to be in perfect condition and its reinforcement clean and without rust.

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[37] "Bond Between Concrete and Steel," by H. J. Gilkey, S. J. Chamberlin, and R. W. Beal, *Bulletin No. 147*, Eng. Experiment Station, Iowa State College, Ames, Iowa, 1940.

[38] *Proceedings*, A. C. I., 1939, p. 517.

[39] *Engineering News-Record*, September 20, 1917, p. 532.

The rigor of Mr. Csanyi's refutation of the writer's contention is lessened and abated by the fact that his studies, experiments, and conclusions are based not on behavior of test specimens in sea water but on their behavior in a 43% solution of Epsom salts. It would appear to the writer that they are highly relevant to concrete that is to be given such exposure, but that the relationship of the 43% Epsom salts solution to sea water has not been established. Pending that determination the writer will continue "in the error of his ways," if such it be.

Professor Williams' brief discussion is charged with numerous doubts. Regarding chemical analyses, the subsequent discussion of two subjects referred to by Mr. Squire will be pertinent. As to Professor Williams' statement that "Rich concrete mixtures may fail by cracking without prior indication of surface swelling, spalling, or disruptions," the writer would inquire if he (Professor Williams) has ever seen this happen with rich concrete mixtures, not in other alkaline waters, but in sea water. If he has, it is to be hoped that full particulars may be reported, for assuredly it is a highly unusual, not to say unique, occurrence, the like of which has not come under the writer's observation. The terms dense and impermeable in the paper were not used academically but as they are commonly employed. The writer disclaims any responsibility for establishing the criteria by which the sulfate attack of sea water upon concrete is supposed to evidence itself. Messrs. Atwood and Johnson[2] quoted E. Candlot who attributes the sea-water sulfate attack theory and its supposed manifestations to L. J. Vicat. The writer merely assumed (see heading "The Prevailing Viewpoint") that if the theory were valid the evidences of the attack should be discoverable. The paper purports to be no more than an unequivocal statement that such evidence is not to be found along the Pacific Coast and a report of what deterioration is to be found. Also, it was stated at the outset: "* * * it is with sea water in its normal range of concentrations—not with other sulfate waters—that the paper is concerned" (see "Introduction"). It was not without knowledge of much to the contrary that has been published earlier that this paper was written. Despite that knowledge, it was written, quite deliberately.

Mr. Squire raises a number of points. One of these concerns the San Pedro breakwater specimens referred to in the paper, the cutting of cores from which, and the saturated condition of the interior concrete of which, on the basis of his "visual" and therefore, by his stern standards, very questionable inspection only, are attested to by Mr. Squire. What the result of the Bureau of Standards' tests and analyses might be was naturally an interesting question. The blocks gave every outward evidence of being all right but were they so in full reality? "Gilded tombs do worms enfold." One of Mr. Squire's fellow citizens in the Bay region who had read the magazine article cited[10] wrote to the U. S. Engineer office at Los Angeles and duly received a reply concerning the findings. When the writer made personal inquiry at that office, he was shown a copy of this letter, which was quite in accord with other information he had received. There was nothing about the findings to make them of a confidential nature, and there appears to be no good reason why they should not be summarized herein from notes made at the time.

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[40] *Engineering News-Record*, March 3, 1938.

The Bureau of Standards felt that the tests were inconclusive, principally because the cores were of small diameter (3 in.). Tests were made of eleven cores—five of 1 : 2 : 4 mix and 6 of 1 : 3 : 6 mix. Six brands of cement had been used—three European and three American. The maximum size of aggregate was 2 in. The cores were sawed into cylinders, 3 in. high, which were first tested for strength and then analyzed chemically. Strengths varied widely. Values with the 1 : 2 : 4 mix ranged from 6,650 to 3,110 lb per sq in. Values with the 1 : 3 : 6 mix ranged from 7,560 to 1,630 lb per sq in. No signs of disintegration were noted in the broken cylinders. The report pointed out how the low strengths and erratic values were readily accounted for by the varying position and size of the coarse aggregate particles in the cores. It did not attempt to explain how, after 27 years in the tidal zone surrounded by seawater sulfates, the high values found could occur. Chemical tests were made with specimens of five of the six cements, two of which were in 1 : 2 : 4 mix cores, the other three being in 1 : 3 : 6 mix cores. There were six samples of the former and nine of the latter. The tests were to determine percentages of magnesia, sulfuric anhydride, and chlorine. Chemical analyses were stated to be inconclusive because of inability to secure representative samples from the small quantity of material in the cores. The samples of 1 : 2 : 4 concrete showed a 74% increase in magnesia content accompanied by a 19% decrease in SO_3-content, which decrease is not in good form chemically when concrete is disintegrating. The samples of 1 : 3 : 6 concrete showed a 358% increase in MgO accompanied by a 51% increase in SO_3. In considering the numerical magnitude of these percentages it must not be overlooked that they are percentages of original very small quantities. Regarding the samples of two of the three cements used in the 1 : 3 : 6 mix specimens the report stated: "While B-1 and B-4 would seem to indicate that some magnesia from sea water has replaced part of the original lime of the cement, this fact may be solely one due to the question of sampling. There apparently has been no increase in the SO_3 content, which is what should accompany this disintegration." The view was expressed (let borers of cores and takers of small samples give heed) that 2 to 3 cu ft of material were needed to make a satisfactory sample.

Inasmuch as there was no definite evidence of physical or chemical deterioration—which is entirely in accord with the many structures referred to—the writer feels quite warranted in stating that there was "no evidence of sulfate of magnesium attack." He feels further confidence in "what is going on under the outer skin of concrete" if this "outer skin" itself withstands scrutiny.

Mr. Squire introduces evidence of concrete in sea water in Europe by reference to Russian tests in the waters of the Baltic Sea at Libau, together with a choice quotation from the report[27] concerning them: "* * * slowly perhaps, but none the less surely" the processes of deterioration proceed, etc., presumably demonstrating that the San Pedro blocks were "an almost exact replica" of the Russian blocks. Thanks to the reference, the report on the tests was obtained and read. The highlights of, and additional quotations from, it are as follows:

Tests were made in June and July, 1905. Eight test blocks—four masonry and four concrete—were sunk in 1891 and 1892. The concrete blocks measured 6 ft by 6 ft by 10 ft. The concrete mix, by volume, measured separately, was: 1.0 Portland cement, 2.5 sand, 1.5 gravel, and 4.6 "rubble stone"—that is, 1 part of cement to 8.6 parts aggregate, measured separately. "Rubble stone" was stated to be granite "in pieces 10 to 14 cubic inches"—in other words 2⅛-in. to 2½-in. cubes. Quoting:

> "The greater part of the Portland cement came from Russian Works, foreign cement was also used, but no chemical analysis was made. * * * The sand was taken from the sea on the spot."

Concrete blocks Nos. 8, 9, and 10 were immersed with about 4½ ft of water above their tops. The fourth, No. 15, was set on the breakwater with its bottom at water surface. Incidentally, the Baltic is tideless, and the inflow of fresh water from numerous rivers exceeds evaporation:

> "* * * hence an enormous surplus must be got rid of by an outflowing current which is named the 'Baltic stream' * * *. The fresher Baltic water can be traced far into the North sea especially along the Norway coasts even as far as North Cape."[41]

On removal from the water the blocks were found to be "in good state of preservation" but with various defects: "White exudations," "liquefied mortar," "white liquid derived from deteriorated mortar" in seams and cavities, etc.

> "As a whole the three cements used, slag, Portland, and quartzose cements, were found practically equal in quality."
>
> "The physical tests having given but indifferent results in regard to the determination of the change which the mortar undergoes in the blocks, a chemical analysis of the various kinds of mortar was made.
>
> "The analysis made afforded bases for the following conclusions: If after the determination of the principal elements * * * of the mortar tested and after elimination of the elements proceeding from the Libau sand, the results be compared with the elements forming the cements used (Portland, slag, and quartzose cements) it is easy to ascertain the extent to which the elements forming the cements have undergone variation by action of the seawater.
>
> "* * * The transformation was characterized principally by the freeing of the lime * * * and by the absorption of the oxide of magnesia and of the sulphuric acid contained in the seawater" (refer to Le Chatelier, Michaelis, Feret *et al*).
>
> "The same tables show that the calculated proportion in weight of alumina and iron oxide exceeds in some instances the proportion of the same elements which were in the cements used in making these mortars. This is probably the case because the average composition of the Libau sand reckoned upon does not always compare completely with the composition of the sand actually used. The Libau sand has in fact but little

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[41] *The Encyclopædia Britannica*, 14th Ed., 1929, p. 1016.

> homogeneity. The chemical analyses of five samples showed the amount in weight of Al_2O_3 and Fe_2O_3 to vary within very wide limits viz. from 0.10 to 0.70 percent. This is due to the presence of particles of earth and clay; these are found especially when the sand is taken in quarries on the downs and not at the sea shore.
>
> "It should be remarked also that although in the mortar made with quartzose cement the quantity of Portland cement was half that in the mortar made with Portland cement, the chemical analyses of the mortar made with quartzose cement was not inferior to that of the Portland cement one.
>
> "One fact, nevertheless, was certain—that the process of deterioration had already commenced and that it progressed, slowly perhaps, but none the less surely in the case of the mortars considered."

There appears to be little need for comment on the preceding testimony. The specimens were made 50 years ago, and the control conditions were not those regarded as necessary today; but as an instance of the highly extolled European practise and experience the reference has its interest. One further matter has its interest also. The English magazine "Port and Harbour Authority," for June, 1941, reprinted the present paper in its entirety with editorial comment. Some of this editorial comment was to the effect that here was surprising news: Concrete in sea water in America that had not deteriorated! True, it did not generally deteriorate in sea water in Britain, but practically all they had ever heard of sea-water concrete in America was that it deteriorated and "went to pot." Apparently while they were receiving such information from this side of the Atlantic, Americans in turn were hearing of the superior and long-established European practise that would not think of using Portland cement in sea water without treatment or process of some kind or other. What in current slang is called "the run-around" seems to have prevailed over a long term of years.

After so much disagreement with Mr. Squire, it is a pleasure to concur with him about the case of the Ford Motor Company Plant at Long Beach (except in minor detail) and to again express warm and sincere admiration for Mr. Russell's work in testing in connection with it. This is the case the writer referred to under the heading "Unsound Materials." It is undoubtedly the one Captain Blackman refers to in his discussion. It appears to be shown conclusively that the felspathic aggregate was the cause of the trouble. The concrete used in the piles deserves Mr. Squire's characterization as "good concrete" in every respect, except that all unwittingly the unsound aggregate was incorporated in it. Thereupon it ceased to be "good." As for Mr. Squire's "chemical reactions caused by the sea water," the writer would gladly subscribe to them while directing attention to Mr. Russell's demonstration, previously cited, of the "chemical reactions caused by the sea water" upon this aggregate all alone and without any cement present while the reactions were in progress. Since the aggregate alone is most unstable in the presence of sea water, it seems scarcely necessary to speculate upon the effects of the addition of cement. It is a not uninteresting fact that while this trouble was occurring at Long Beach the Ford plants at Richmond, Calif. (on San Francisco Bay), and at Seattle, Wash. (in the strongly saline waters at the mouth of the Duwamish

River), built at the same time as the Long Beach plant, were—and still are—free from any trouble in their concrete piles.

So far as the writer knows concrete piles are impregnated with asphalt primarily to protect the pile from low tide to its top, rather than to "protect the concrete under water." This latter protection is incidental to the manufacturing process. If this belief is correct, it is probably more fitting to so state.

With his friends, Captain Blackman and Mr. Squire, the writer has had some stirring arguments in the past. On this occasion may he differ with Captain Blackman regarding the number of Portland cement structures north of San Francisco by saying that, although most large structures are in the south, there is a wealth of lesser structures in the north—bridge piers, pile trestles, sea walls, culverts, outfall sewers, etc., together with a considerable number of larger works. It is not necessary to build a cathedral to have fine architecture reveal itself. Neither is it necessary to have a 1,000-ft long pier to determine the behavior of concrete in sea water.

The writer is made happy by Captain Blackman's statement that deterioration of concrete in sea water by the action of sulfate of magnesium is generally considered as secondary by engineers engaged in harbor work. It is hoped that Captain Blackman reads Mr. Way's discussion, particularly those parts dealing with the concrete pile jobs, cracks, etc. It is believed that the little bridge near Olympia, with certain parts of its reinforcing steel exposed, may also yield him cheer should the hazards to which unimpregnated concrete piles are subject weigh him down unduly.

He refers to the 3-in. cover over reinforcement recommended by the Portland Cement Association[29] and more recently by the 1940 Joint Committee Report.[42] Although this is needed cover with some kinds of concrete, it is not an unmixed blessing, as Captain Blackman knows full well, particularly as it affects the weight of a slender reinforced member subject to bending both in installation and in subsequent service. The writer is of the opinion that the suggestion of J. H. Wasson[43] advanced in 1929 that the tops of piles coming above low tide be made of a particularly good quality of concrete would warrant the omission of extra cover. As a matter of fact, it is doubtful in view of the performance of the 1918 ships and barges, with their limited cover over reinforcement, whether with richer mixes 3 in. of cover is not a wholly unnecessary precaution. Furthermore it would appear that the newly developed vacuum process might have very valuable application in the field of concrete piles in improving the density of the surface concrete and in permitting the reduction of cover over reinforcing bars to 1 or 1½ in.

Captain Blackman delicately refers to special sulfate resisting cements. He states that "The chemists of cement companies have been endeavoring to find a remedy for attacks on cement concrete exposed to sea water and sea air." Undoubtedly this is, or has been, true. Without knowing the exact basis of their action, the writer would assume it to result from the plethora of past writings about what is the best European practise, about sulfates of magnesium in sea water, and about all calamitous, inevitable things that befall concrete in sea water. Probably these special cements are highly efficacious in genuinely severe sulfate exposures.

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[42] *Proceedings*, Am. Soc. C. E., June, 1940, Pt. 2.

[43] *Proceedings*, A. C. I., 1929, p. 760.

The writer gladly declares that the North Vancouver Ferry Pier is built upon columns, not piles. Fig. 3 is a view of part of its substructure. Considering the diagonal bracing shown in Fig. 3, it is very unlikely that any would think it was built on piles, but if they do, may this statement set them right. The beach beneath the pier is cemented hardpan, and piles could not have been driven.

Considering Captain Blackman's familiarity with the North Vancouver Ferry Pier, it was amazing to the writer to read his statements regarding Burrard Inlet, Vancouver's magnificent landlocked harbor which, with its north arm, covers an area of 31.6 sq miles. True, a number of small streams and creeks or little rivers flow into it from the mountains on the north side. The largest of these, the Capilano River, has its discharge greatly reduced by the use of its water in the water system of Vancouver. Sometimes these streams flood, sometimes they grow small; but at all times, compared with the mighty volume of sea water passing in and out of the harbor with every changing tide (the tides average 12 ft), the inflow of fresh water is insignificant indeed. The harbor authorities report a salinity of harbor water of 1,023 compared with 1,026 for ocean water and 1,000 for fresh water. Consequently "upstream and downstream from the ferry wharf," although a fanciful whimsy, seems more reminiscent of Captain Blackman's old home at New Westminster, British Columbia, on the Fraser River than applicable to Vancouver Harbor. The fact is that, unimpregnated though it be, the North Vancouver Ferry Pier is a very fine, very successful little sea-water structure now aged 32 years. Across the harbor on the city side, where no streams discharge, the Great Northern Pier built in 1913, the Ballantyne Pier[44] built in 1923, the Canadian Pacific Pier B–C,[45] and various other structures are free from deterioration and sulfate attack.

Regarding Mr. Jack's suggestion that sea water may "speed the parting guest," the earlier part of this closure presents the writer's opinion. Mr. Jack's résumé of the various deficiencies of cement is acknowledged as essentially correct, without the writer's having knowledge of how the deficiencies may be overcome. For many uses it would also be desirable to have cement and concrete of half their present weight. To have definite control over the time of set appears to the writer to be particularly desirable for sea-water concrete.

Certain of Mr. Freudenthal's points and objections have been answered previously. His stated experience with aluminous cements in sea water is not confirmed locally. The pier at the Puget Sound Navy Yard, in Bremerton, Wash.,[46] is now (1941) 15 years old, and its cylinders built with high alumina cement are, after 15 years in sea water, in absolutely perfect condition. On the other hand the writer heard of some bridge in Chesapeake Bay built about the same time with this same type of cement whose piers disintegrated very quickly. Probably this was not in any way due to the sea water but rather to inadequate provision for the dissipation of the large quantities of heat generated in this cement's setting process. Seattle has quite a number of street pavement patches made with high alumina cement. Some of the early ones crumbled and broke up. After it was learned that such patches had to be continuously sprinkled or ponded all trouble ended. It is obvious that sea water had nothing to do with these street pavement failures.

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[44] *Engineering News-Record*, May 24, 1923, p. 908.

[45] *Loc. cit.*, December 16, 1926, p. 984.

[46] *Proceedings*, A. C. I., 1927, p. 79.

The effect of puzzolanics is speculated upon by Mr. Freudenthal. The south main pier of the Golden Gate Bridge at San Francisco was built with a high-silica puzzolanic cement, the north main pier with ordinary Portland cement. Perhaps some time in the future distinct differences will be observed between them, although, as Mr. Paxson notes, in deep water changes are much slower than at the shore. Bonneville Dam on the Columbia River was built with a puzzolanic cement. It was completed in 1937. The resistance of its concrete to abrasion and cavitation has not been wholly satisfactory, but it is believed that the number of shrinkage cracks in the dam are fewer than if it had been built with Portland cement, a very comforting thought.

Mr. Rights' discussion lacks sufficient detail to permit much comment on the failure of the railroad test piers he describes. The size and shape of these "piers," the cement content of their concrete, and the exact character of the change that is beginning to manifest itself in the remaining pier (described by the words "beginning to disintegrate") are either not stated or are indefinite. The year the test was initiated (1925) suggests the possibility that the "quick-setting high-strength cement" referred to by Mr. Rights may have been the same as that used in the Seattle street pavement patches previously mentioned. This is only conjectural on the writer's part, however. Under these conditions there does not appear to be much basis for comment on the failure referred to by Mr. Rights.

There is no question, however, that frost adds greatly to the severity of the sea-water exposure, and it may be that concrete of a quality that would perform satisfactorily in sea water where freezing conditions are absent would not be satisfactory where freezing conditions prevail. On the other hand, the writer sees no reason for believing that unfaced concrete of a quality adequate to resist the sea-water exposure under freezing conditions cannot be readily obtained.

In 1923 Meyer Hirschthal,[47] M. Am. Soc. C. E., reported sound 16-yr old concrete in a wall at Hoboken, N. J. The North River is not heavily saline, however. Nevertheless it is probably comparable to the estuary cited by Mr. Rights. It would be interesting to know the present condition of this unfaced wall. In his private files the writer has correspondence regarding two test prisms, 12 in. square and about 8 ft long, set in the tidal zone at Saint John, New Brunswick, Canada, in 1913. Freezing conditions prevail at Saint John and the specimens had four square corners, which do not help any toward resisting either abrasion or weathering. In 1933 one of these specimens (made of 1 : 3 : 5 mix, gravel aggregate, and "fairly wet mix") was reported to be entirely disintegrated. The mortar surface had disappeared and the specimen was in broken pieces. The other specimen, of 1 : 2 : 4 mix, crushed rock coarse aggregate, and of "about 2–3 slump," was reported to show "only slight wear just about high water."

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[47] *Engineering News-Record*, January 17, 1924, p. 127.

It is to be hoped that a further report may sometime be made regarding the Aberthaw tests of unfaced concrete specimens placed in sea water in 1909 at the U. S. Navy Yard at Charlestown, Mass. The last published report was made in 1920. In 1940 the writer sought additional information regarding them, learned that twelve specimens of the original twenty-four still remained, and was unable to learn anything more. Since a very good record of the manufacture of these pile specimens exists, it is to be hoped that their condition can be reported. They are now (1941) over 30 years old and have a background of experience that will not be available to test specimens made today until thirty future years have passed.

Granite facings are both admirable and costly. So far as the walls shown in Figs. 2 and 4 are concerned, a solution to their troubles, simpler and less expensive than granite, would have been to have changed the careless construction practises plainly evident in them. There is plenty of unaffected concrete in the tidal zone in both of these views to demonstrate the nature and cause of the deterioration. The wall on the right side of Fig. 2, starting at the stepped-off joint at the reentrant angle is Quay Wall E, built in 1896, and still showing no need of granite facing. The only recent use of granite facing on the Pacific Coast that the writer knows of is the protective course in the tidal zone in the piers of the new Lions Gate Bridge at the entrance to Vancouver Harbor. There is no freezing or ice or unusual condition of abrasion to be concerned about at present, but if the glacial age ever returns these piers are foresightedly prepared to withstand it.

The Dallas Road sea wall at Victoria, listed in the paper as built in 1911, has a belt course of granite 2 or 3 ft high at the beach line, which has undoubtedly been helpful in protecting the concrete from abrasion. These are the only Pacific Coast uses of granite facing on concrete that the writer can think of, although there are quite numerous cases where concrete structures have had granite or other stone blocks dumped loosely around and against them. Granite of good quality, of course, is one of the finest and most resistant materials of construction that there is, and the writer would say nothing to impugn it. As a necessity for the preservation of concrete in the tidal zone, however, he entertains what he at least would call reasonable doubts concerning it. According to reports reaching him, a certain resin either interground with the cement, or introduced into the concrete as an admixture, gives most interesting promise of being a substitute for granite facings when the urge for special treatment in the tidal zone is strong upon one.

To sum up, the situation appears to be this: The sea water exposure is a very severe one. It has been sufficiently demonstrated that concrete can be made which will give a highly unsatisfactory performance in sea water. Undoubtedly bad concrete can be made with any brand of cement and with a wide variety of admixtures and treatments. The disintegrated concrete is, in general, characterized by a lean, porous, and frequently segregated structure. Too little cement, too much water, too little care in placement are the chief causes of trouble. Poor aggregate is sometimes a further contributing cause. On the other hand there are a large number of concrete structures of considerable age in sea water which are substantially free from deterioration or any evidence of sea-water attack. These have been made with many different cements, differing widely in composition and in percentage of tricalcium aluminate. This immunity occurs with, and is characterized by, concrete of dense, impermeable structure. There is no sea-water deterioration to be found, other than that due to rusting of reinforcement, that is distinctly different from fresh-water deterioration. There is no peculiar sulfate attack. For these reasons it appears to be highly important to use fairly rich mixtures, minimum mixing water consistent with good placement, care in placement, and all reasonable measures which result in density and impermeability. Special safeguards against sulfate attack can be reserved for use where they are needed—that is, elsewhere than in sea water.

To all who contributed discussions the writer would express his thanks and his hope that all differences of view are dispassionate and impersonal. Out of the reporting of various experiences comes increased knowledge and better understanding. It is to be hoped that such may be the outcome here with respect to concrete in sea water.

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Paper No. 2136

ANALYSIS OF STATICALLY INDETERMINATE TRUSSED STRUCTURES BY SUCCESSIVE APPROXIMATIONS

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WITH DISCUSSION BY MESSRS. FRANCIS L. CASTLEMAN, JR., CHARLES A. ELLIS, DAVID J. PEERY, L. E. GRINTER, AND O. T. VOODHIGULA.

SYNOPSIS

In this paper the writer has attempted to simplify the classical method of analyzing statically indeterminate trussed structures, and to develop a physical concept whereby an analysis may be made rapidly by successive approximations.

INTRODUCTION

So far as the writer knows, there are two direct methods of successive approximation, applied to indeterminate trussed structures. Both methods were developed in England; the one by R. V. Southwell and the other by J. F. Baker, Assoc. M. Am. Soc. C. E., and A. J. Ockleston.[2] Due to the complicated nature of statically indeterminate trussed structures, the direct method of distributing stress becomes tedious. Professor Pippard states[3] that "It is clear that less labor was involved in determining the forces in the members of the framework * * * by strain energy methods than by either of the successive approximation methods * * *." The method of work still seems to be the best. The only objection lies in the solution of the simultaneous equations since the labor involved increases rapidly with the number of equations.

METHOD

The structure is made statically determinate by cutting the redundant bars and replacing each of them with a pair of equal and opposite forces, as shown in Fig. 1. When the loads are applied, there will be relative movements at different cuts due to the distortions of the bars; the ends of the cut bars will move away from or toward each other, depending on the nature of the distor-

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NOTE.—Published in January, 1941, *Proceedings*.

[1] Municipal and Public Works, Bangkok, Thailand.

[2] "The Analysis of Engineering Structures," by A. J. S. Pippard, M. Am. Soc. C. E., and J. F. Baker, New York, 1936.

[3] *Loc. cit.*, p. 108.

tions. The relative movement of the two ends can be found by the method of work. Let the relative movements due to the distortion of the statically determinate structure, of the two ends at a, b, and c, be δ_{a0}, δ_{b0}, δ_{c0}, respectively; thus

$$\delta_{a0} = \sum \frac{s\,u_a\,L}{A\,E} \tag{1a}$$

$$\delta_{b0} = \sum \frac{s\,u_b\,L}{A\,E} \tag{1b}$$

and

$$\delta_{c0} = \sum \frac{s\,u_c\,L}{A\,E} \tag{1c}$$

in which: s is the stress in each bar due to the applied load; u_a, u_b, and u_c are the stresses in each bar due to a unit resistance to movement at points a, b, and c, respectively; L = the length of the bar; A = the cross-sectional area of the bar; and E = the modulus of elasticity.

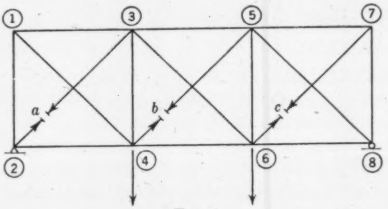

FIG. 1

The gaps must be closed by applying the forces X_a, X_b, X_c simultaneously. The relative movements at a, b, and c due to these three forces must be equal to the relative movements δ_{a0}, δ_{b0}, and δ_{c0} caused by the loads on the base structure; that is:

$$- \sum \frac{s\,u_a\,L}{A\,E} = X_a \sum \frac{u_a\,u_a\,L}{A\,E} + X_b \sum \frac{u_a\,u_b\,L}{A\,E} + X_c \sum \frac{u_a\,u_c\,L}{A\,E} + \cdots + X_n \sum \frac{u_a\,u_n\,L}{A\,E} \tag{2a}$$

$$- \sum \frac{s\,u_b\,L}{A\,E} = X_a \sum \frac{u_b\,u_a\,L}{A\,E} + X_b \sum \frac{u_b\,u_b\,L}{A\,E} + X_c \sum \frac{u_b\,u_c\,L}{A\,E} + \cdots + X_n \sum \frac{u_b\,u_n\,L}{A\,E} \tag{2b}$$

and

$$- \sum \frac{s\,u_c\,L}{A\,E} = X_a \sum \frac{u_c\,u_a\,L}{A\,E} + X_b \sum \frac{u_c\,u_b\,L}{A\,E} + X_c \sum \frac{u_c\,u_c\,L}{A\,E} + \cdots + X_n \sum \frac{u_c\,u_n\,L}{A\,E} \tag{2c}$$

or they may be written in the following forms:

$$- \delta_{a0} = X_a\,\delta_{aa} + X_b\,\delta_{ab} + X_c\,\delta_{ac} + \cdots + X_n\,\delta_{an} \tag{3a}$$

$$- \delta_{b0} = X_a\,\delta_{ba} + X_b\,\delta_{bb} + X_c\,\delta_{bc} + \cdots + X_n\,\delta_{bn} \tag{3b}$$

and

$$- \delta_{c0} = X_a\,\delta_{ca} + X_b\,\delta_{cb} + X_c\,\delta_{cc} + \cdots + X_n\,\delta_{cn} \tag{3c}$$

In Eqs. 3 all the terms except X_a, X_b, and X_c are known or can be computed; δ_{a0}, δ_{b0}, and δ_{c0} are dependent on the loads, their positions, and the given structure; and δ_{aa}, δ_{ab}, δ_{cc}, δ_{bb}, etc., are dependent on the given structure only and can be considered as the constants. This knowledge at once suggests that the equations can be solved by means of successive approximations. In Eq. 3*a* assume X_b and X_c equal to zero to obtain the approximate value of X_a. With X_a known and assuming $X_c = 0$, X_b can be computed from Eq. 3*b*. The approximate value of X_c can be solved from Eq. 3*c*. By repeating the foregoing process, successive values of X_a, X_b, and X_c will be obtained and these will approach the solution of Eqs. 3, if there is convergence. This is the solution of simultaneous equations by iteration.

In this problem a technique is needed in order to simplify the process of approximation to reveal the physical significance. Rearrange Eqs. 3 in the following forms:

$$X_a = -\frac{\delta_{a0}}{\delta_{aa}} - X_b\frac{\delta_{ab}}{\delta_{aa}} - X_c\frac{\delta_{ac}}{\delta_{aa}} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (4a)$$

$$X_b = -\frac{\delta_{b0}}{\delta_{bb}} - X_a\frac{\delta_{ba}}{\delta_{bb}} - X_c\frac{\delta_{bc}}{\delta_{bb}} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (4b)$$

and

$$X_c = -\frac{\delta_{c0}}{\delta_{cc}} - X_a\frac{\delta_{ca}}{\delta_{cc}} - X_b\frac{\delta_{cb}}{\delta_{cc}} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (4c)$$

They can be still further simplified as follows:

$$X_a = -s_a' - X_b K_{ab} - X_c K_{ac} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (5a)$$

$$X_b = -s_b' - X_a K_{ba} - X_c K_{bc} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (5b)$$

and

$$X_c = -s_c' - X_a K_{ca} - X_c K_{cb} \cdots \text{(etc.)} \ldots\ldots\ldots\ldots (5c)$$

The new notations may be defined as follows:

$s_a' = \frac{\delta_{a0}}{\delta_{aa}}$ = single redundant stress; that is, the stress in the redundant *a* when all the other redundants are not acting; and

$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}}$ = stress in bar *a* due to a unit axial load in bar *b*; it may be called a carry-over factor of stress of *a* due to a unit axial load acting in the redundant *b*.

Eqs. 5 are easier to follow than the previous equations, and the physical significance of every term is clear and enables one to visualize the stress distribution in the structure. It may be stated that the stress in any redundant is equal to the single redundant stress of the bar plus the carried over stresses from the other redundants. One may begin with the single redundant stresses, like fixed-end moments in the moment-distribution method,[4] and then carry over the stresses from the other single redundants according to their carry-over factors. After the first approximation is complete, use the new values of the redundants and make the second approximation and so on. The convergence is very rapid as will be seen in the examples. This process of approximation is continued until the successive values of X become constant. In practical problems the approximations do not have to be extended so far; it is up to the designers to decide when to stop, depending on what degree of precision is required. This method is general and can also be extended to the external redundants.

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The single redundant stresses s' can be computed readily by the method of work and so can the carry-over factors K. The computation of K is not tedious. Maxwell's law of reciprocal deflection is helpful for this purpose, and by inspection one can see what bars contribute to the deflection; and thus much unnecessary work can be avoided. After the constants K and s' are determined and tabulated, the computation is simple. The method can be best illustrated by examples.

A single redundant stress in the redundant a is expressed by

$$s_a' = \frac{\delta_{a0}}{\delta_{aa}} = \frac{\sum \frac{s\, u_a\, L}{A\, E}}{\sum \frac{u_a\, u_a\, L}{A\, E}} \quad \text{......................} (6)$$

in which: δ_{a0} and δ_{aa} are the relative movements of the cut a due, respectively, to the distortion of the statically determinate structure and an axial load of unity applied at the cut. The first subscript defines the position of movement or stress, and the second defines the position of the applied load. Note that Eq. 6 is dependent on the loads and the given structure.

The carry-over factor of the redundant a from redundant b is expressed as

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = \frac{\sum \frac{u_a\, u_b\, L}{A\, E}}{\sum \frac{u_a\, u_a\, L}{A\, E}} \quad \text{......................} (7)$$

Unlike Eq. 6, this formula is independent of the loads.

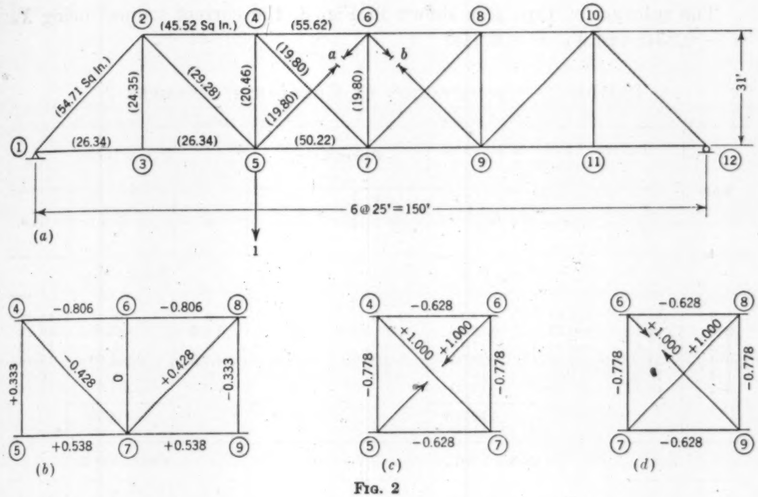
TABLE 1.—COMPUTATION OF CONSTANTS, EXAMPLE 1

(a) PANEL 3						(b) PANEL 4[a]				
Bar (Fig. 2)	$\frac{L}{A}$	s	u_a	$\frac{s\, u_a\, L}{A}$	$\frac{u_a^2\, L}{A}$	Bar (Fig. 2)	s	u_b	$\frac{s\, u_b\, L}{A}$	$\frac{u_b^2\, L}{A}$
4–6	5.39	−0.806	−0.628	+ 2.74	+ 2.13	6–8	−0.807	−0.628	+ 2.74	+ 2.13
5–7	5.97	+1.074	−0.628	− 4.02	+ 2.36	7–9	+0.538	−0.628	− 2.02	+ 2.36
4–5	18.15	+0.333	−0.778	− 4.70	+11.00	8–9	−0.333	−0.778	+ 4.70	+11.00
6–7	18.76	0	−0.778	0	+11.35	6–7	0	−0.778	0	+11.35
4–7	24.10	−0.428	+1.000	−10.30	+24.10	7–8	+0.428	+1.000	+10.30	+24.10
5–6	24.10	0	+1.000	0	+24.10	6–9	0	+1.000	0	+24.10
				−16.28[b]	+75.04[c]				+15.72[d]	+75.04[c]

[a] L/A is the same for the bars in Panel 4 as in the bars of Panel 3. [b] $\delta_{a0}\, E$. [c] $\delta_{aa}\, E$ in Panel 3 and $\delta_{bb}\, E$ in Panel 4. [d] $\delta_{b0}\, E$.

EXAMPLE 1.—TWO REDUNDANTS

The relative movement of the cut a (Fig. 2(a)) depends on the distortions of the bars in panel 3 only, and similarly the relative movement of cut b is due to the distortions of the bars in panel 4.



(45.52 Sq In.)
(55.62)
(54.71 Sq In.)
(24.35)
(29.28)
(20.46)
(19.80)
(19.80)
(19.80)
a
b
31'
(26.34)
(26.34)
(50.22)
6 @ 25' = 150'
1
(a)
−0.806
−0.806
+0.333
−0.428
0
+0.428
−0.333
+0.538
+0.538
(b)
−0.628
−0.778
+1.000
+1.000
−0.778
−0.628
(c)
−0.628
−0.778
+1.000
+1.000
−0.778
−0.628
(d)

FIG. 2

Constants for substitution in Eqs. 5 are computed as in Table 1, with the result that:

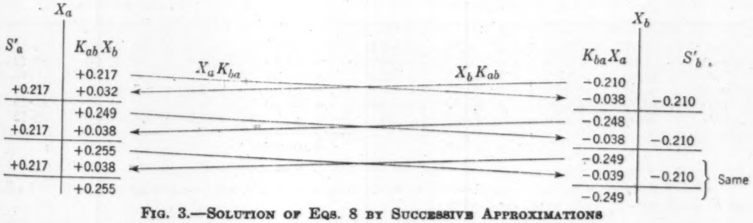
$$s_a' = \frac{\delta_{a0}}{\delta_{aa}} = (-)\frac{-\ 16.28}{+\ 75.04} = +\ 0.217;$$

$$s_b' = \frac{\delta_{b0}}{\delta_{bb}} = (-)\frac{+\ 15.72}{+\ 75.04} = -\ 0.210;$$

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = (-)\frac{(-\ 0.778)\ (-\ 0.778)\ 18.76}{+\ 75.04} = -\ 0.152;$$

and

$$K_{ba} = \frac{\delta_{ba}}{\delta_{bb}} = K_{ab} = -\ 0.152.$$



X_a
S'_a
$K_{ab}X_b$
+0.217
+0.217
+0.032
+0.249
+0.217
+0.038
+0.255
+0.217
+0.038
+0.255
X_aK_{ba}
X_bK_{ab}
X_b
$K_{ba}X_a$
S'_b
−0.210
−0.038
−0.210
−0.248
−0.038
−0.210
−0.249
−0.039
−0.210
−0.249
Same

FIG. 3.—SOLUTION OF EQS. 8 BY SUCCESSIVE APPROXIMATIONS

Note that, in the determination of K_{ab} and K_{ba}, only bar 6–7 enters. The sign of all the single redundant stresses and of the carry-over factors must be changed, because every term on the right-hand side of Eqs. 5 has a negative sign; thus

$$X_a = -\ s_a' - X_b\ K_{ab} \qquad (8a)$$

and

$$X_b = -\ s_b' - X_a\ K_{ba} \qquad (8b)$$

The solution of Eqs. 8 is shown in Fig. 3, the correct values[5] being $X_a = +0.254$; and $X_b = -0.248$.

TABLE 2.—COMPUTATION OF CONSTANTS, EXAMPLE 2

Bar[a]	PANELS[a] LEFT AND RIGHT SYMMETRICAL			PANEL LEFT		PANEL RIGHT		
	$\frac{L}{A}$	u_x	$\frac{u_x^2 L}{A}$	s	$\frac{s\,u_x L}{A}$	s	$\frac{s\,u_a L}{A}$	Bar[a]
PANEL ONE						PANEL EIGHT		
1–3	14.40	−0.60	+ 5.18	−1.5	+12.98	0	0	15–17
2–4	14.40	−0.60	+ 5.18	0	−0	+4.5	−38.90	16–18
1–2	17.45	−0.80	+11.15	−2.0	+27.90	0	0	17–18
3–4	32.00	−0.80	+20.05	−2.0	+51.20	+6.0	−153.60	15–16
1–4	19.20	+1.00	+19.20	+2.5	+48.00	0	0	17–16
3–2	19.20	+1.00	+19.20	0	+ 0	−7.5	−144.0	15–18
			+79.96[b]		140.0[c]		−336.50[c]	
PANEL TWO						PANEL SEVEN		
3–5	8	−0.60	+ 2.88	−3.0	+14.40	−4.5	+ 21.60	13–15
4–6	8	−0.60	+ 2.88	+1.5	− 7.20	+9.0	− 43.20	14–16
3–4	32	−0.80	+20.05	−2.0	+51.20	+6.0	−153.60	15–16
5–6	32	−0.80	+20.05	−2.0	+51.20	+6.0	−153.60	13–14
3–6	24	+1.00	+24.00	+2.5	+60.00	0	0	15–14
5–4	24	+1.00	+24.00	0	0	−7.5	−180.00	13–16
			+93.86[b]		169.60[c]		−508.80[c]	
PANEL THREE						PANEL SIX		
5–7	5.76	−0.60	+ 2.08	−4.5	+ 15.57	−9.0	+ 31.10	11–13
6–8	5.76	−0.60	+ 2.08	+3.0	− 10.38	+7.5	− 25.90	12–14
5–6	32.00	−0.80	+ 20.05	−2.0	+ 51.20	+6.0	−153.50	13–14
7–8	32.00	−0.80	+ 20.05	−2.0	+ 51.20	−2.0	+ 51.20	11–12
5–6	34.30	+1.00	+ 34.30	+2.5	+ 85.80	0	0	13–12
7–6	34.30	+1.00	+ 34.30	0	0	+2.5	+ 85.80	11–14
			+112.86[b]		+193.39[c]		− 11.30[c]	
PANEL FOUR						PANEL FIVE		
7–9	4.97	−0.60	+ 1.79	−6.0	+ 17.90	−7.5	+ 22.40	9–11
8–10	4.97	−0.60	+ 1.79	+4.5	− 13.40	+6.0	− 17.90	10–12
7–8	32.00	−0.80	+ 20.05	−2.0	+ 51.20	−2.0	+ 51.20	11–12
9–10	32.00	−0.80	+ 20.05	−2.0	+ 51.20	−2.0	+ 51.20	9–10
7–10	40.00	+1.00	+ 40.00	+2.5	+100.00	0	0	11–10
9–8	40.00	+1.00	+ 40.00	0	0	+2.5	+100.00	9–12
			+123.68[b]		+206.90[c]		+206.90[c]	

[a] See Fig. 4, $u_a = u_h$, $u_b = u_g$, $u_c = u_f$, $u_d = u_e$; also, $x = a, b, c$, etc., according to panel. [b] $\delta_{aa} E$, $\delta_{bb} E$, $\delta_{cc} E$, etc. [c] $\delta_{a0} E$, $\delta_{b0} E$, $\delta_{c0} E$, etc.

EXAMPLE 2.—EIGHT REDUNDANTS

Similarly to Example 1, constants for substitution in Eqs. 5 are computed as in Table 2, for use in determining the single redundant stresses

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[5] "Theory of Statically Indeterminate Structures," by W. M. Fife and J. B. Wilbur, Assoc. Members, Am. Soc. C. E., McGraw-Hill Book Co., Inc., 1937, p. 166.

(see Fig. 4):

$$s_a' = \frac{\delta_{a0}}{\delta_{aa}} = -\frac{+\,140.08}{79.96} = -\,1.750; \quad s_b' = \frac{\delta_{b0}}{\delta_{bb}} = -\frac{+\,169.60}{93.86} = -\,1.810;$$

$$s_c' = \frac{\delta_{c0}}{\delta_{bb}} = -\frac{+\,193.39}{112.86} = -\,1.715; \quad s_d' = \frac{\delta_{d0}}{\delta_{dd}} = -\frac{+\,206.90}{123.68} = -\,1.675;$$

$$s_e' = \frac{\delta_{e0}}{\delta_{dd}} = -\frac{+\,206.90}{123.68} = -\,1.675; \quad s_f' = \frac{\delta_{f0}}{\delta_{ff}} = -\frac{-\,11.30}{112.86} = +\,0.100;$$

$$s_g' = \frac{\delta_{g0}}{\delta_{gg}} = -\frac{-\,508.60}{93.86} = +\,5.425;$$

and

$$s_h' = \frac{\delta_{h0}}{\delta_{hh}} = -\frac{-\,336.40}{79.96} = +\,4.210.$$

As all the verticals are of the same dimensions $\delta_{ab} = \delta_{ba} = \delta_{bc} = \delta_{cb} = \delta_{cd} = \delta_{dc} = \delta_{de} = \delta_{ed} = \delta_{ef} = \delta_{fe} = \delta_{fg} = \delta_{gf} = \delta_{gh} = \delta_{hg} = (-)\,(-\,0.80)\,(-\,0.80) \times 32 = -\,20.45$.

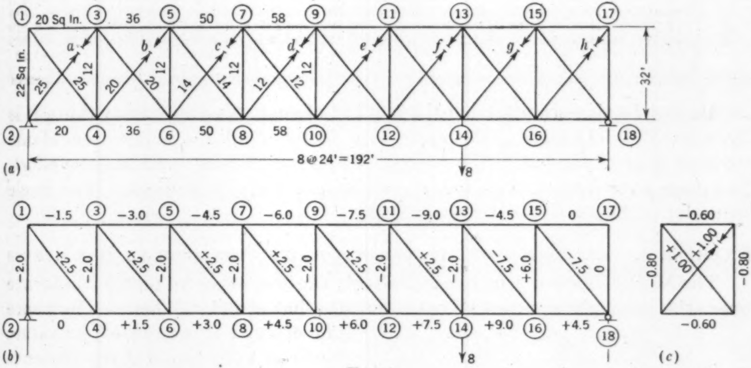
① 20 Sq In. ③ 36 ⑤ 50 ⑦ 58 ⑨ ⑪ ⑬ ⑮ ⑰
a
b
c
d
e
f
g
h
22 Sq In.
25
25
12
20
20
12
14
14
12
12
12
12
32'
② 20 ④ 36 ⑥ 50 ⑧ 58 ⑩ ⑫ ⑭ ⑯ ⑱
8 @ 24' = 192'
8
(a)
① −1.5 ③ −3.0 ⑤ −4.5 ⑦ −6.0 ⑨ −7.5 ⑪ −9.0 ⑬ −4.5 ⑮ 0 ⑰
−2.0
+2.5
−2.0
+2.5
−2.0
+2.5
−2.0
+2.5
−2.0
+2.5
−2.0
+2.5
−2.0
−7.5
+6.0
−7.5
0
② 0 ④ +1.5 ⑥ +3.0 ⑧ +4.5 ⑩ +6.0 ⑫ +7.5 ⑭ +9.0 ⑯ +4.5 ⑱
8
(b)
−0.60
−0.80
+1.00
+1.00
−0.80
−0.60
(c)

FIG. 4

Therefore, the carry-over factors are:

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = \frac{(-\,20.45)}{79.96} = -\,0.256;$$

$$K_{ba} = K_{bc} = \frac{\delta_{ba}}{\delta_{bb}} = \frac{(-\,20.45)}{93.86} = -\,0.218;$$

$$K_{cb} = K_{cd} = \frac{\delta_{cb}}{\delta_{cc}} = \frac{(-\,20.45)}{112.86} = -\,0.181;$$

$$K_{dc} = K_{de} = \frac{\delta_{dc}}{\delta_{dd}} = \frac{(-\,20.45)}{123.68} = -\,0.165;$$

$$K_{ed} = K_{ef} = \frac{\delta_{ed}}{\delta_{dd}} = \frac{(-\,20.45)}{123.68} = -\,0.165;$$

$$K_{fe} = K_{fg} = \frac{\delta_{fe}}{\delta_{ff}} = \frac{(-\,20.45)}{112.86} = -\,0.181;$$

$$K_{gf} = K_{gh} = \frac{\delta_{gf}}{\delta_{gg}} = \frac{(-\,20.45)}{93.86} = -\,0.218;$$

and

$$K_{hg} = \frac{\delta_{hg}}{\delta_{hh}} = \frac{(-20.45)}{79.96} = -0.256.$$

The solution for the redundant stresses by Eqs. 8 is shown in Table 3. This is a classic problem solved by Messrs. Johnson, Bryan, and Turneaure.[8]

TABLE 3.—COMPUTATIONS OF

(In s_x', the Subscript $x = a$,

Cycle No.	PANEL ONE		PANEL TWO			PANEL THREE			PANEL FOUR		
	$K_{ab}X_b$	X_a	$K_{ba}X_a$	$K_{bc}X_c$	X_b	$K_{cb}X_b$	$K_{cd}X_d$	X_c	$K_{dc}X_c$	$K_{de}X_e$	X_d
s_x'	−1.750			−1.810			−1.715			−1.675	
K	−0.256			−0.218			−0.181			−0.165	
		−1.750			−1.810			−1.715			−1.675
1	+0.463	−1.347	+0.293	+0.374	−1.143	+0.207	+0.303	−1.205	+0.199	+0.278	−1.198
2	+0.292	−1.458	+0.318	+0.263	−1.229	+0.222	+0.217	−1.276	+0.210	+0.246	−1.219
3	+0.314	−1.436	+0.313	+0.278	−1.219	+0.221	+0.221	−1.273	+0.210	+0.226	−1.239
		−1.44			−1.21			−1.25			−1.23

Moving Loads.—In applying the method of successive approximations, it is obvious that determining the carry-over factors and the single redundants requires most of the labor; the process of approximations requires little effort. The carry-over factors, once found, can be used for any loadings. The single redundants change with changes in loads.

EXAMPLE 3.—EXTENSION OF MÜLLER-BRESLAU'S PRINCIPLE TO BAR STRESS

The Müller-Breslau principle is probably the best means of plotting influence lines; the principle can also be extended to a bar stress. When the influence line of any bar is required, introduce a pair of axial forces at the two ends of the bar so that a unit distortion is produced. The deflected load line of the structure is the influence line for the stress of that bar.

Suppose the influence line of the stress in the bar 2–4, Fig. 5, is required. Introduce a unit negative distortion in bar 2–4; in this case the distortion of the bar does not affect the stress in the other bars. The angle change $\Delta\theta = \frac{1}{1} = 1$; the moment curve of the imaginary beam is the deflected load line and it is the influence line for the stress in the

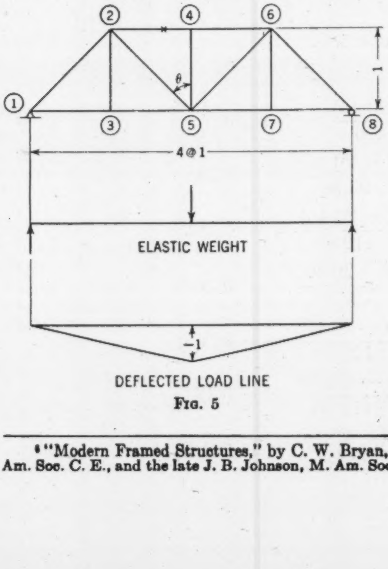

FIG. 5

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[8] "Modern Framed Structures," by C. W. Bryan, Jr., M. Am. Soc. C. E., F. E. Turneaure, Hon. M. Am. Soc. C. E., and the late J. B. Johnson, M. Am. Soc. C. E., Pt. II, 10th Ed. (1929), p. 310.

bar 2–4. This can be verified readily by statics.

The following conventions must be observed: A negative angle change is an elastic load acting downward. The positive moment curve must be drawn below the reference line, because positive moment causes the beam to deflect downward. It does not matter whether negative distortion or positive distortion is applied to the bar, the sign of stress caused by a load can be determined easily by inspection. Suppose the load is applied at point 7 of the equivalent beam; it is obvious that the beam will deflect down farther and it means that the assumed compression stress is correct. If positive distortion is applied in the bar 2–4, the deflected load line is above the reference line. In this case, when the load is applied at point 7, the load tends to decrease the deflection which means that the sign is opposite to what it was assumed to be.

REDUNDANT STRESSES, EXAMPLE 2
b, *c*, Etc., According to the Panel)

PANEL FIVE			PANEL SIX			PANEL SEVEN			PANEL EIGHT		Cycle No.
$K_{ed} X_d$	$K_{ef} X_f$	X_e	$K_{fe} X_e$	$K_{fg} X_g$	X_f	$K_{gf} X_f$	$K_{gh} X_h$	X_g	$K_{hg} X_g$	X_h	
	−1.675			+0.100			+5.425			+4.210	s_x'
	−0.165			−0.181			−0.218			−0.256	K
		−1.675			+0.100			+5.425		+4.210	
+0.198	−0.017	−1.494	+0.271	−0.982	−0.611	+0.133	−0.916	+4.642	−1.189	+3.021	1
+0.201	+0.101	−1.373	+0.245	−0.840	−0.495	+0.108	−0.658	+4.875	−1.250	+2.960	2
+0.204	+0.082	−1.388	+0.251	−0.883	−0.532	+0.116	−0.645	+4.896	−1.270	+2.940	3
		−1.37			−0.53			+4.86		+2.93	

EXAMPLE 4.—DETERMINATION OF THE INFLUENCE LINE OF THE SINGLE REDUNDANT STRESS s_d' OF THE TRUSS IN EXAMPLE 2

When a distortion is applied to a single redundant, the problem is not as simple as in the previous case, because stresses will be set up in some other bars. However, it can be solved conveniently by the method of elastic weight. The constants for substitution in Eq. 5 are determined by setting up a computation form similar to Table 2, Panel 1, from which $s_d' = \dfrac{s\,u_d\,L}{A} = \dfrac{-83.68}{123.68} = 0.677$.

Stresses.—The force required to shorten bar 8–9, Fig. 6(*a*), 1 in. is $X = \dfrac{1.00}{0.677} \times \dfrac{12 \times 30{,}000{,}000}{40 \times 12} = 1{,}107{,}000$ lb.

Angle Changes.—Formulas for the angle changes (see Fig. 6(*d*)) are:

$$\Delta\gamma = \frac{1}{E}\left[(f_c - f_b)\cot\alpha + (f_c - f_a)\cot\beta\right]\ldots\ldots\ldots\ldots(9a)$$

$$\Delta\beta = \frac{1}{E}\left[(f_b - f_c)\cot\alpha + (f_b - f_a)\cot\gamma\right]\ldots\ldots\ldots\ldots(9b)$$

and

$$\Delta\alpha = \frac{1}{E}\left[(f_a - f_c)\cot\beta + (f_a - f_b)\cot\gamma\right]\ldots\ldots\ldots\ldots(9c)$$

The solution of Eqs. 9, applied to Fig. 6(*e*), is as follows:

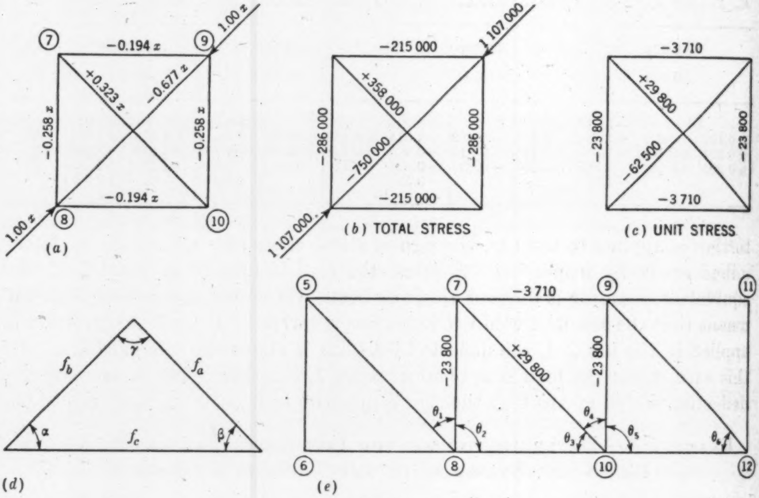
Joint 8—

$$\Delta\theta_1 = \qquad\qquad 0$$

$$\Delta\theta_2 = \frac{1}{E}\left[(+29{,}800 + 23{,}800)\frac{32}{24} + (+\,29{,}800 + 3{,}710)\frac{24}{32}\right]$$

$$= \frac{1}{E}[+\,71{,}500 + 25{,}100] \qquad = \frac{+\,96{,}600}{E}$$

Total angle change at joint 8, Fig. 6(*e*) $= \dfrac{+\,96{,}600}{E}$



7
9
−0.194 *x*
1.00 *x*
+0.323 *x*
−0.677 *x*
−0.258 *x*
−0.258 *x*
−0.194 *x*
1.00 *x*
8
10
(*a*)
−215 000
1 107 000
+358 000
−286 000
−286 000
−750 000
−215 000
1 107 000
(*b*) TOTAL STRESS
−3 710
+29 800
−23 800
−23 800
−62 500
−3 710
(*c*) UNIT STRESS
γ
f_b
f_a
α
f_c
β
(*d*)
5
7
9
11
−3 710
−23 800
+29 800
−23 800
θ_1
θ_2
θ_4
θ_3
θ_5
θ_6
6
8
10
12
(*e*)

FIG. 6

Joint 10—

$$\Delta\theta_3 = \frac{1}{E}\left[(-\,23{,}800 - 29{,}800)\frac{32}{24} + (-\,23{,}800 + 3{,}710)\,0\right] \qquad = \frac{-\,71{,}500}{E}$$

$$\Delta\theta_4 = \frac{1}{E}\left[(-\,3{,}710 + 23{,}800)\,0 + (-\,3{,}710 + 29{,}800)\frac{24}{32}\right] \qquad = \frac{-\,25{,}100}{E}$$

$$\Delta\theta_5 = \frac{1}{E}(0 + 23{,}800)\frac{32}{24} \qquad = \frac{+\,31{,}700}{E}$$

Total angle change at joint 10 $= \dfrac{-\,64{,}900}{E}$

Joint 12—

$$\Delta\theta_6 = \frac{1}{E}(-\,23{,}800 - 0)\frac{32}{24} \qquad = \frac{-\,31{,}700}{E}$$

Total angle change at joint 12 $= \dfrac{-\,31{,}700}{E}$

Influence ordinates are then computed, thus (see Fig. 7): For a unit load at joint 8, $s_d' = \dfrac{+16{,}100 \times 3 \times 24 \times 12}{30{,}000{,}000} \times 0.677 = +0.314$; for a unit load at joint 10, $s_d' = \dfrac{(+16{,}100 \times 4 - 96{,}600 \times 1)\,24 \times 12 \times 0.677}{30{,}000{,}000} = -0.210$; and, for a unit load at joint 12, $s_d' = \dfrac{-16{,}100 \times 3 \times 24 \times 12}{30{,}000{,}000} \times 0.677 = -0.314$. To check these results assume a load of 8 units at joint 14, Fig. 7: By computation, $s_d' = -1.675$; and from the influence line, $s_d' = -0.314 \times \frac{2}{3} \times 8 = -1.675$, which checks. Note that the deflection of the imaginary beam, which is equal to the moment of the beam at that point, is equal to the relative movement of the joints 8 and 9. The actual stress set up in the bar 8–9 is only 0.677 of the force applied.

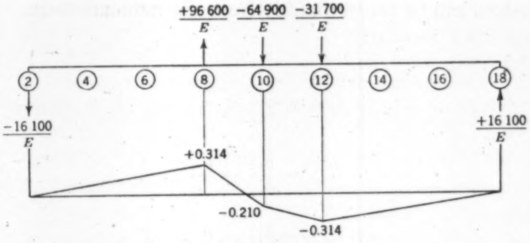

+96 600 / E
−64 900 / E
−31 700 / E
2
4
6
8
10
12
14
16
18
−16 100 / E
+16 100 / E
+0.314
−0.210
−0.314

FIG. 7.—INFLUENCE LINE FOR s_d'

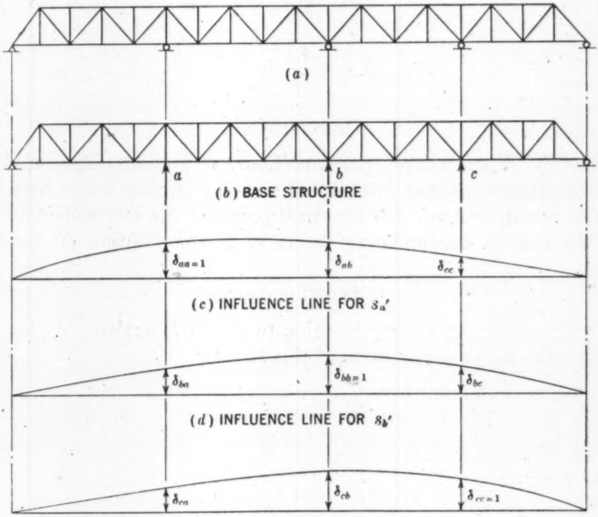

(a)
a
b
c
(b) BASE STRUCTURE
δaa = 1
δab
δac
(c) INFLUENCE LINE FOR sa'
δba
δbb = 1
δbc
(d) INFLUENCE LINE FOR sb'
δca
δcb
δcc = 1
(e) INFLUENCE LINE FOR sc'

FIG. 8

EXTERNAL REDUNDANTS

The same principle can be applied to external redundants. Single redundants and carry-over factors must be found for the process of approximations. In the case of external redundant structure shown in Fig. 8, it is advantageous to draw the influence lines for the single redundants, because the carry-over factors can be taken directly from the influence lines.

For example (see Eqs. 5):

From Fig. 8(*c*)—

$$K_{ab} = \frac{\delta_{ab}}{\delta_{aa}} = \frac{\delta_{ab}}{1} \qquad \text{(10a)}$$

and

$$K_{ac} = \frac{\delta_{ac}}{\delta_{aa}} = \frac{\delta_{ac}}{1} \qquad \text{(10b)}$$

from Fig. 8(*d*)—

$$K_{ba} = \frac{\delta_{ba}}{\delta_{bb}} = \frac{\delta_{ba}}{1} \qquad \text{(10c)}$$

and

$$K_{bc} = \frac{\delta_{bc}}{\delta_{bb}} = \frac{\delta_{bc}}{1} \qquad \text{(10d)}$$

and from Fig. 8(*e*)—

$$K_{ca} = \frac{\delta_{ca}}{\delta_{cc}} = \frac{\delta_{ca}}{1} \qquad \text{(10e)}$$

and

$$K_{cb} = \frac{\delta_{cb}}{\delta_{cc}} = \frac{\delta_{cb}}{1} \qquad \text{(10f)}$$

CONCLUSION

The method of successive approximations, as presented herein, should be useful to structural analysts in effecting a saving of time and a better understanding of the structure. For practical purposes the slide-rule may be used without the fear of accumulating errors as in the solution of simultaneous equations.

ACKNOWLEDGMENT

The writer expresses his appreciation to Albert Haertlein, M. Am. Soc. C E., for guidance in the preparation of this paper.

DISCUSSION

FRANCIS L. CASTLEMAN, JR.,[7] ASSOC. M. AM. SOC. C. E. (by letter).—The term "classical method" is a misnomer when used to describe the general method to effect a solution for the internal redundants of frames, such as are shown in Figs. 1, 2, and 4. Classical methods, in such cases, are modern methods, since basically for problems of this type, a solution must always be effected by the method of deflections. Whether recourse is had in the detail of solution to least work, dummy unit loading, virtual work, etc., is a matter of individual choice; all lead to the same end. All analytical methods are based on the elastic energy of the system and can be derived from the general principles laid down for such methods.

Eqs. 3 express the gist of the matter. Once these equations are properly set up and their constants correctly determined the engineering is done. They have been known for a long time; they are still mandatory for the solution of many problems, and hence are modern in the sense that they are indispensable. Thus the writer questions if Mr. Voodhigula has simplified, in any way, the so-called classical method of analyzing the statically indeterminate structures in question. What he has really done, in the writer's opinion, and very ably, is to devise an extremely ingenious method for the detail of the solution of Eqs. 3.

After trying this method on several "pet problems" it was found to be quite helpful in some respects, but no faster than the ordinary method of iteration without the graphical interpretation such as given in Fig. 3. It is more compact and offers a visual interpretation of what is believed to happen. However, in all fairness, it must be said that perhaps complete facility was not acquired in its use. The implied visual interpretation of stress distribution is really a "second sight" as the true physical and visual interpretation is implied in Eqs. 3 and their proper development.

Errors of analogy are often introduced in an analysis when operations performed are remindful of those executed in a like manner in a previous but non-similar situation. In moment distribution there is an entirely different situation and one operation in the solution of this situation involves an action the end result of which is termed "carry-over." In the situation under consideration, which by no stretch of the imagination is similar to moment distribution, a name for an operation is literally "carried over" from a non-similar situation. On reflection the writer believes the term "stress factor" would be more appropriate. Certainly, if such were the case, excuse could never be offered (by analogy) for dividing the carry-over factor by 2 (or any other value).

Mr. Voodhigula dismisses the problem of external redundants with far from the proportionate space devoted to internal redundants; yet the former is generally the problem confronted, as structures of the latter type are few (at least in American practice) as compared with the former. This would seem the most fruitful field for the method under discussion. However, a more than cursory examination of such structures leads to some very interesting observations as regards the accuracy of solution by any method. Sometimes it is quite advantageous to choose a member or members of such a structure as the redundants rather than the reactions. For example, in Fig. 8 the reactions "*a*," "*b*," and "*c*" are treated as redundants. This presupposes for the statically determinate base system the full span acting as a simple truss under the appropriate loadings. Equations can be set up similar to Eqs. 3 and reduced with proper manipulation and interpretation such that, by their use, influence lines can be constructed for each of the external redundants, and subsequently those for the individual members in the truss. The latter step can be achieved by cutting the lower chord at "*a*" and the upper chord over "*b*" and "*c*" and treating the cut members as redundants. The statically determinate base system then becomes three simple-span trusses. Comparable errors will be greatly reduced, therefore, with consequent reduction in the errors of the required redundant values. American textbooks have paid too little attention to this fact. By way of illustration, consider:

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[7] Head, Dept. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

$$X = \frac{\begin{vmatrix} \delta_m' & \delta_{mn} \\ \delta_n' & \delta_{nn} \end{vmatrix}}{\begin{vmatrix} \delta_{mm} & \delta_{mn} \\ \delta_{nm} & \delta_{nn} \end{vmatrix}} = \frac{\delta_n' \, \delta_{nn} - \delta_n' \, \delta_{mn}}{\delta_{mm} \, \delta_{nn} - \delta_{mn}^2} \qquad \text{(11)}$$

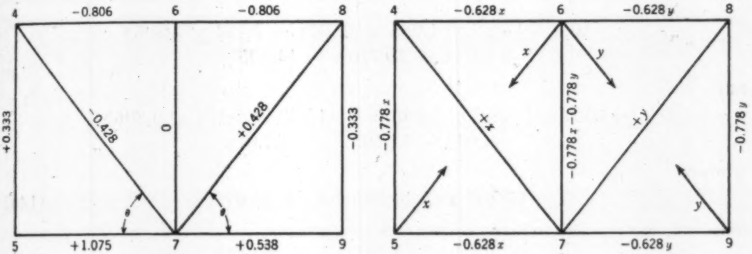
and with the following numerical values inserted, $X = \dfrac{50 \times 100 - 90 \times 60}{40 \times 100 - (60)^2}$ $= -1$. If the errors of determining the δ-values vary by $\pm 2\%$ then, in the extreme, $X = \dfrac{51 \times 102 - 88.2 \times 58.8}{40.8 \times 102 - (58.8)^2} = 0.225$. Thus a tremendous error has been introduced from small, possibly unavoidable errors in the basic quantities. Through use of a properly chosen statically determinate base system such errors can generally be held to a minimum. Exact solutions, computing machines, methods of iteration, and even Divine Providence cannot eliminate such errors once they are present in the fundamentally basic quantities.

In the interest of rigor it would seem logical to emphasize that the theoretically correct values indicated by Eqs. 3 have some properties that can be profitably studied before Eqs. 3 are thrown into the form of Eqs. 5 for use in the method of iteration. For a solution, Eqs. 3 must conform to the following: There must be as many equations as unknowns; the equations must be independent of each other; and possible conditions must be represented. Solving Eqs. 3 in determinant form and representing expressions for numerator and denominator by N and D, respectively, $x_a = \dfrac{N_a}{D}$, etc. If $D = 0$, the left-hand side of the equations are functionally related. If $N = 0$ also, the right-hand sides are related functionally in the same manner. Under the aforementioned conditions the equations are dependent on each other and by themselves will not lead to a solution. If $D = 0$, $N \neq 0$ the equations do not represent possible conditions.

In using the method of iteration once a definite trend has been established, by plotting known values against cycles the range can be extended graphically. If this is carefully done and good judgment is exercised the method of iteration itself will be considerably shortened.

Charles A. Ellis,[8] M. Am. Soc. C. E. (by letter).—There are three principles by which a statically indeterminate structure may be analyzed: (1) Principle of work; (2) principle of least work; and (3) principle of deflections.

The first two are essentially mathematical abstractions of horse-and-buggy days. They do not permit physical conception or space perception, so helpful in the comprehension of stress analysis. The principle of deflections is far superior, and analysis may be made by at least three methods: (*a*) Area-moments; (*b*) expression of moments in terms of angular rotations of joints and members (generally known as the "slope-deflection method"); and (*c*) moment distribution. The two latter methods are most easily derived by the area-moment method which the writer considers the closest to fundamentals.



(*a*) (*b*)

Fig. 9

The principles of work and least work were the only ones available for an algebraic solution of statically indeterminate trusses until the "Williot Strain Equations"[9] were discovered by the writer, when computing the tower stresses in the Golden Gate Suspension Bridge. These equations will now be used in solving the author's Example 1.

The stresses in the necessary members (redundant members removed) are shown in Fig. 9(*a*); and the stresses due to stresses x and y, respectively, in the redundant members are shown in Fig. 9(*b*). The combined stresses for each member are given in Table 4, in which s represents the stress and Δ the strain. Hence,

$$E\,\Delta = \frac{s\,l}{A} \quad \ldots\ldots (12)$$

TABLE 4.—Combined Stresses for Members in Fig. 9

Member	s	$\frac{l}{A}$	$E\,\Delta = \frac{s\,l}{A}$
4–6	$-0.806 \quad -0.628\,x$	5.39	$-\ 4.344 \quad -\ 3.385\,x$
5–7	$+1.075 \quad -0.628\,x$	5.97	$+\ 6.418 \quad -\ 3.749\,x$
4–5	$+0.333 \quad -0.778\,x$	18.15	$+\ 6.044 \quad -14.121\,x$
6–7	$-0.778\,x -0.778\,y$	18.76	$-14.595\,x -14.595\,y$
4–7	$-0.428 \quad +\ x$	24.10	$-10.315\,x +24.100\,x$
5–6	$+\ x$	24.10	$+24.100\,x$
6–8	$-0.806 \quad -0.628\,y$	5.39	$-\ 4.344 \quad -\ 3.385\,y$
7–9	$+0.538 \quad -0.628\,y$	5.97	$+\ 3.212 \quad -\ 3.749\,y$
8–9	$-0.333 \quad -0.778\,y$	18.15	$-\ 6.044 \quad -14.121\,y$
7–8	$+0.428 \quad +\ y$	24.10	$+10.315 \quad +24.100\,y$
6–9	$+\ y$	24.10	$+24.100\,y$

In any rectangular panel,[9] the sum of the strains in the diagonals multiplied by cosec θ equals the sum of the strains in the horizontals multiplied by cot θ

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[8] Prof., Structural Eng., Purdue Univ., West Lafayette, Ind.

[9] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *Engineering News-Record*, April 26, 1934, p. 534; also "Williot Equations for Statically Indeterminate Structures," by Charles A. Ellis, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580.

plus the sum of the strains in the verticals, or

$$(\Delta_{47} + \Delta_{56}) \operatorname{cosec} \theta = (\Delta_{57} + \Delta_{46}) \cot \theta + \Delta_{45} + \Delta_{67} \ldots\ldots (13a)$$

and

$$(\Delta_{69} + \Delta_{78}) \operatorname{cosec} \theta = (\Delta_{79} + \Delta_{68}) \cot \theta + \Delta_{67} + \Delta_{89} \ldots\ldots (13b)$$

Substituting from Table 4,

$$(-10.315+48.2\,x)\,1.285 = (2.074 - 7.134\,x)\,0.8065 + 6.044 - 28.716\,x - 14{,}595\,y$$

and

$$(+10.315 + 48.2\,y)\,1.285 = (-1.132 - 7.134\,y)\,0.8065 - 6.044 - 14.595\,x - 28.716\,y$$

whence

$$96.407\,x + 14.595\,y = +\,20.972 \ldots\ldots (14a)$$

and

$$96.407\,y + 14.595\,x = -\,20.212 \ldots\ldots (14b)$$

or $x = +\,0.2551$; and $y = -\,0.2483$. Eqs. 13 may be written in the form

$$x = +\,0.2175 - 0.1514\,y \ldots\ldots (15a)$$

and

$$y = -\,0.2097 - 0.1514\,x \ldots\ldots (15b)$$

Any one with leisure time may solve for x and y by successive approximations, as follows: Assume $y = 0$ in Eq. 15a and $x = 0$ in Eq. 15b; then: $x = +\,0.2175$; and $y = -\,0.2097$. Substitute $y = -\,0.2097$ in Eq. 15a and $x = +\,0.2175$ in Eq. 15b; whence: $x = +\,0.2492$; and $y = -\,0.2426$. Repeat this process three times and the following successive values appear:

x	y
+0.2175	−0.2097
+0.2492	−0.2426
+0.2542	−0.2474
+0.2550	−0.2482
+0.2551	−0.2483

These same numerical values appear in Fig. 3.

Comment on solutions by successive approximations seems relevant. This process has received much attention since Professor Cross published his moment-distribution method[4] and used successive approximations as a tool; and it is an exceedingly efficient tool, especially when the unknowns are numerous. Moment distribution has no monopoly, however. Slope-deflection equations invariably have the same peculiarity that renders them susceptible to the same rapidity of convergence when solved by successive approximations, as when moment distribution is used.

Consider any problem in which rotation of joints, but no rotation of members, is assumed and solve by moment distribution. Then solve by slope deflection using successive approximations in the solution of the equations and note that identical numerical values appear in both methods.

When the unknowns are numerous, the method of successive approximations is an exceedingly efficient tool; but for many writers it has become a toy to be used whenever and wherever possible. For example: In a box culvert having large fillets with increasing moments of inertia at the ends of the members, wherein the carry-over factor is as large as three fourths, resulting in relatively slow convergence, the simultaneous solution of four slope-deflection equations is the method to be preferred.

Perhaps the most risible example is the so-called rigid frame—the two-hinged rectangular reinforced concrete arch. One and only one elastic equation[10] is required for finding the corner moment for each condition of loading, whether dead or live. The solution of such a problem, containing but one unknown quantity by any method where successive approximations may or can be used, is something fancy, and, if one reveres mathematical accuracy, the elastic equations can take into account the curvature of the deck axis, whereas the slope deflection and moment distribution are impotent to do so.

Finally the writer expresses doubt in the author's conclusion that the principle of work combined with successive approximations is useful in a better understanding of the structure.

David J. Peery,[11] **Jun. Am. Soc. C. E.** (by letter).—A method of analyzing statically indeterminate trussed structures is presented in this paper that is similar to modern methods of analyzing continuous beams and rigid-frame structures. The author points out that for trussed structures it seems better to solve simultaneous equations set up by classical strain-energy methods than to use direct methods of successive approximation in distributing stresses. The simultaneous equations are set up by customary methods, therefore, and a method of successive approximations is applied to their solution.

The method of solving the simultaneous equations is essentially the method of iteration, or successive substitutions. It has been in use for some time by structural engineers for the solution of similar types of equations obtained by the use of three-moment equations, slope-deflection equations, or moment distribution.[12,13] For equations in which the convergence of the proposed method is rather slow, the method proposed by George H. Dell,[14] Assoc. M. Am. Soc. C. E., in which a series summation is used after the first few cycles of substitution, will materially reduce the labor involved.

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[10] "Two Hinged Rectangular Arches Having Variable Moments of Inertia," *Bulletin No. 79*, Eng. Experiment Station, Purdue Univ., Lafayette, Ind.

[11] Engr., Curtiss-Wright Corp., Robertson, Mo.; in Chg. Eng. Defense Training Courses in Airplane Stress Analysis, Washington Univ., St. Louis, Mo.

[12] "On a Simple Method for Solving Simultaneous Linear Equations by a Successive Approximation Process," by J. Morris, *Journal*, Royal Aeronautical Soc., April, 1935, p. 349.

[13] "Analysis by Moment Distribution Aided Through Use of Iteration," by A. Floris, *Engineering News-Record*, June 25, 1936, p. 922.

[14] "Solution of Equations in Structural Analysis by Converging Increments," by George H. Dell, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1543.

The examples chosen by the author yield systems of simultaneous linear equations in which the coefficients belonging to the principal diagonal of the matrix are much greater than the other coefficients. The solution of such equations by successive approximation converges rapidly. Most problems in indeterminate analysis are of this type, or can be made so by a proper selection of the redundants. In Example 2, for instance, the redundants are such that the equations are much simpler than for the general structure with eight redundants. If the expressions of Eqs. 3 are expanded, they yield, for the general case of a structure with eight redundants:

$$\left.\begin{aligned}
-\delta_{a0}&=X_a\delta_{aa}+X_b\delta_{ab}+X_c\delta_{ac}+X_d\delta_{ad}+X_e\delta_{ae}+X_f\delta_{af}+X_g\delta_{ag}+X_h\delta_{ah}\\
-\delta_{b0}&=X_a\delta_{ba}+X_b\delta_{bb}+X_c\delta_{bc}+X_d\delta_{bd}+X_e\delta_{be}+X_f\delta_{bf}+X_g\delta_{bg}+X_h\delta_{bh}\\
-\delta_{c0}&=X_a\delta_{ca}+X_b\delta_{cb}+X_c\delta_{cc}+X_d\delta_{cd}+X_e\delta_{ce}+X_f\delta_{cf}+X_g\delta_{cg}+X_h\delta_{ch}\\
-\delta_{d0}&=X_a\delta_{da}+X_b\delta_{db}+X_c\delta_{dc}+X_d\delta_{dd}+X_e\delta_{de}+X_f\delta_{df}+X_g\delta_{dg}+X_h\delta_{dh}\\
-\delta_{e0}&=X_a\delta_{ea}+X_b\delta_{eb}+X_c\delta_{ec}+X_d\delta_{ed}+X_e\delta_{ee}+X_f\delta_{ef}+X_g\delta_{eg}+X_h\delta_{eh}\\
-\delta_{f0}&=X_a\delta_{fa}+X_b\delta_{fb}+X_c\delta_{fc}+X_d\delta_{fd}+X_e\delta_{fe}+X_f\delta_{ff}+X_g\delta_{fg}+X_h\delta_{fh}\\
-\delta_{g0}&=X_a\delta_{ga}+X_b\delta_{gb}+X_c\delta_{gc}+X_d\delta_{gd}+X_e\delta_{ge}+X_f\delta_{gf}+X_g\delta_{gg}+X_h\delta_{gh}\\
-\delta_{h0}&=X_a\delta_{ha}+X_b\delta_{hb}+X_c\delta_{hc}+X_d\delta_{hd}+X_e\delta_{he}+X_f\delta_{hf}+X_g\delta_{hg}+X_h\delta_{hh}
\end{aligned}\right\}\ldots(16)$$

Substituting the numerical values from Example 2:

$$\left.\begin{array}{lllllllll}
-140.08= & 79.96\,X_a & +20.45\,X_b & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0\\
-169.60= & 20.45\,X_a & +93.86\,X_b & +\ 20.45\,X_c & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0\\
-193.39= & 0 & +20.45\,X_b & +112.86\,X_c & +\ 20.45\,X_d & +\ 0 & +\ 0 & +\ 0 & +\ 0\\
-206.90= & 0 & +\ 0 & +\ 20.45\,X_c & +123.68\,X_d & +\ 20.45\,X_e & +\ 0 & +\ 0 & +\ 0\\
-206.90= & 0 & +\ 0 & +\ 0 & +\ 20.45\,X_d & +123.68\,X_e & +\ 20.45\,X_f & +\ 0 & +\ 0\\
+\ 11.30= & 0 & +\ 0 & +\ 0 & +\ 0 & +\ 20.45\,X_e & +112.86\,X_f & +20.45\,X_g & +\ 0\\
+508.60= & 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +\ 20.45\,X_f & +93.86\,X_g & +20.45\,X_h\\
+336.40= & 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +\ 0 & +20.45\,X_g & +79.96\,X_h
\end{array}\right\}(17)$$

It should be noticed that 42 of the 64 coefficients of the unknowns are zero, resulting in an enormous simplification of the solution.

The solution of Eqs. 17 is quite easy by the common methods of elimination or determinants, whereas, if none of the coefficients of Eqs. 16 were zero, the solution would be quite laborious by successive approximations, and practically impossible by elimination or determinants. It seems obvious, therefore, that it is more important to select the redundants in such a manner as to simplify Eqs. 3 than it is to simplify the actual solution of the equations.

If the redundants are selected so that all the coefficients δ_{mn} having unlike subscripts are zero, Eqs. 3 will have the following solutions:

$$X_a = -\frac{\delta_{a0}}{\delta_{aa}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18a)$$

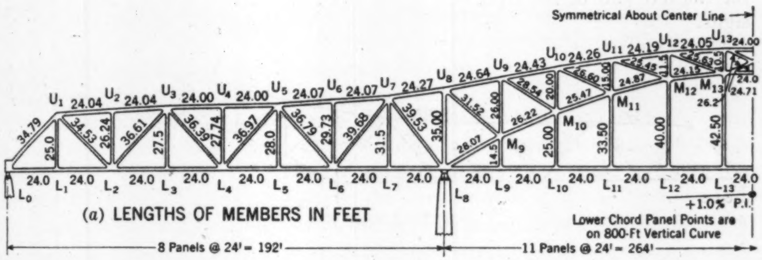
$$X_b = -\frac{\delta_{b0}}{\delta_{bb}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18b)$$

and

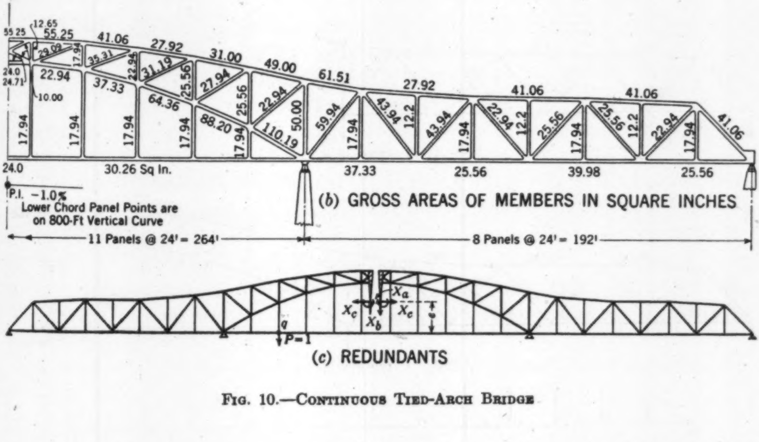
$$X_c = -\frac{\delta_{c0}}{\delta_{cc}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18c)$$

This simplification eliminates the necessity for solving simultaneous equations, and makes it possible to obtain the influence lines for the redundants directly as simple deflection curves. The influence lines shown in Figs. 7 and 8 are not for the redundants X_a, X_b, and X_c, but give only the terms S_d', δ_{aa}, and δ_{ab} to substitute in the simultaneous equations, which must then be solved for the redundants.

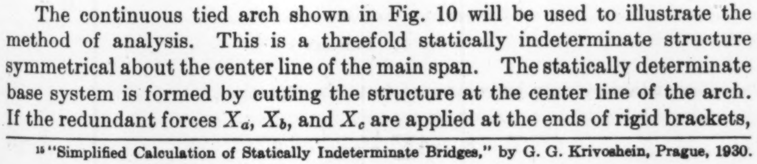
It is possible to select the redundants for any statically indeterminate structure in such a manner that Eqs. 3 are satisfied.[15] Although this procedure is often used for the analysis of fixed arches, many engineers do not realize that it may be used for the analysis of statically indeterminate trussed structures. The influence lines for the redundant forces may be found directly by the method of elastic weights, without solving simultaneous equations.


Symmetrical About Center Line
U1 24.04 U2 24.04 U3 24.00 U4 24.00 U5 24.07 U6 24.07 U7 24.27 U8 24.64 U9 24.43 U10 24.26 U11 24.19 U12 24.05 U13 24.00
34.79
25.0
34.53
26.24
36.61
27.5
36.39
27.74
36.97
28.0
36.79
29.73
39.68
31.5
39.53
35.00
31.52
26.00
28.07
14.5
26.22
M9
28.54
20.00
25.00
M10
25.47
26.60
15.00
33.50
M11
24.87
40.00
M12
M13
24.15
42.50
26.2
24.0
24.71
24.0 L0 L1 24.0 L2 24.0 L3 24.0 L4 24.0 L5 24.0 L6 24.0 L7 24.0 L8 24.0 L9 24.0 L10 24.0 L11 24.0 L12 24.0 L13 24.0
+1.0% P.I.
Lower Chord Panel Points are on 800-Ft Vertical Curve
8 Panels @ 24' = 192'
11 Panels @ 24' = 264'

(a) LENGTHS OF MEMBERS IN FEET


55.25
12.65
55.25
41.06
27.92
31.00
49.00
61.51
27.92
41.06
41.06
29.09
17.94
35.31
22.94
31.19
25.56
27.94
25.56
22.94
50.00
59.94
43.94
12.2
43.94
22.94
12.2
25.56
25.56
12.2
22.94
41.06
24.0
24.71
22.94
10.00
37.33
64.36
88.20
110.19
17.94
17.94
17.94
17.94
17.94
17.94
17.94
17.94
17.94
17.94
24.0
30.26 Sq In.
37.33
25.56
39.98
25.56
P.I. -1.0%
Lower Chord Panel Points are on 800-Ft Vertical Curve
11 Panels @ 24' = 264'
8 Panels @ 24' = 192'

(b) GROSS AREAS OF MEMBERS IN SQUARE INCHES


X_a
X_c
X_c
X_b
e
q
P=1

(c) REDUNDANTS

FIG. 10.—CONTINUOUS TIED-ARCH BRIDGE

The continuous tied arch shown in Fig. 10 will be used to illustrate the method of analysis. This is a threefold statically indeterminate structure symmetrical about the center line of the main span. The statically determinate base system is formed by cutting the structure at the center line of the arch. If the redundant forces X_a, X_b, and X_c are applied at the ends of rigid brackets, as shown in Fig. 10(*c*), the conditions,

---

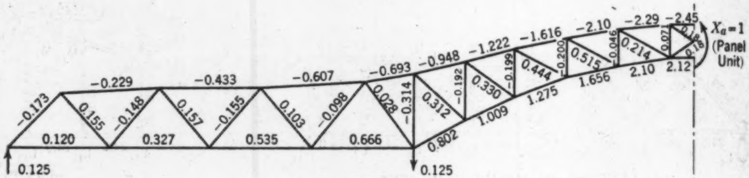
[15] "Simplified Calculation of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, 1930.

$$\delta_{ab} = \sum \frac{u_a\, u_b\, L}{A\, E} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

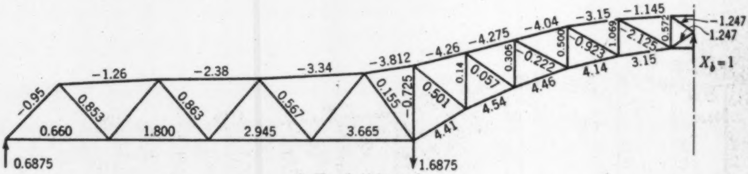
and

$$\delta_{bc} = \sum \frac{u_b\, u_c\, L}{A\, E} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

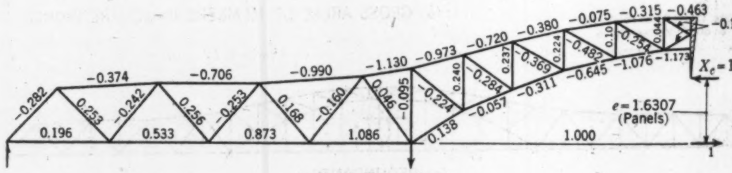
are evidently satisfied. The stresses u_a and u_c, resulting from $X_a = 1$ and $X_c = 1$, respectively, are symmetrical with respect to the center of the structure. The stresses u_b, resulting from $X_b = 1$, are antisymmetrical. The summations of Eqs. 19*a* and 19*b*, therefore, will have the same absolute values for the left half of the structure as for the right half, but will have different signs.


-0.173
-0.229
0.155
-0.148
-0.433
0.157
-0.155
-0.607
0.103
-0.098
-0.693
0.028
-0.314
-0.948
0.312
-0.192
-1.222
0.330
-0.199
-1.616
0.444
-0.200
-2.10
0.515
-0.046
-2.29
0.214
0.071
-2.45
0.18
0.18
$X_a = 1$
(Panel Unit)
0.120
0.327
0.535
0.666
0.802
1.009
1.275
1.656
2.10
2.12
0.125
0.125

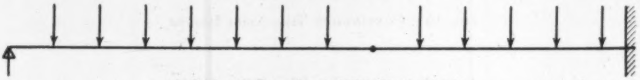
(*a*) $X_a = 1$ (Values of u_a)


-0.95
-1.26
0.853
-2.38
0.863
-3.34
0.567
-3.812
0.155
-0.725
-4.26
0.501
0.14
-4.275
0.057
0.305
-4.04
0.222
0.500
-3.15
0.923
1.069
-1.145
2.125
0.572
-1.247
1.247
$X_b = 1$
0.660
1.800
2.945
3.665
4.41
4.54
4.46
4.14
3.15
0.6875
1.6875

(*b*) $X_b = 1$ (Values of u_b)


-0.282
-0.374
0.253
-0.242
-0.706
0.256
-0.253
-0.990
0.168
-0.160
-1.130
0.046
-0.095
-0.973
-0.224
0.240
-0.720
-0.284
0.237
-0.380
-0.369
0.224
-0.075
-0.482
0.10
-0.315
-0.254
0.044
-0.463
-0.158
$X_c = 1$
0.196
0.533
0.873
1.086
0.138
-0.057
-0.311
-0.645
-1.076
-1.173
$e = 1.6307$
(Panels)
1.000
1

(*c*) $X_c = 1$ (Values of u_c)



(*d*) Conjugate Beam Loaded With Elastic Weights

FIG. 11.—STRESSES

The distance *e* may be computed from the equation,

$$\delta_{ac} = \sum \frac{u_a\, u_c\, L}{A\, E} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19c)$$

Substituting the numerical values of the stresses u_a (Fig. 11*a*) and u_c (Fig. 11*c*), in terms of *e*, into the summation of Eq. 19*c*, the following values are obtained: $\sum \frac{u_a\, u_c\, L}{A} = 46.570\, e - 75.942 = 0$; or, $e = 1.6307$ panels.

Since Eqs. 19 are now satisfied, the value of the couple X_a for a load $P = 1$ at any point *q* on the structure may be found as follows:

$$X_a = -\frac{\delta_{aq}}{\delta_{aa}} = -\frac{\sum \frac{u_a\, u_a\, L}{A\, E}}{\sum \frac{u_a{}^2\, L}{A\, E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

The numerator of Eq. 20*a* represents the deflection of any point *q*, resulting from a couple $X_a = 1$. This numerator is evaluated for all positions of the load *P* by the method of elastic weights. The elastic weights, or angle changes, for the load $X_a = 1$ panel unit are computed as shown in Table 5(*a*), Col. 1. The shears and bending moments in the conjugate beam of Table 5(*a*), Cols. 2 and 3, represent the slopes and deflections of the actual structure when it is loaded with the couple X_a. The terms used in Eq. 20*a* may be in any units as long as the same units are used in the numerator and in the denominator. In this discussion the moment arms are used in panel units, in order that the moments in the beam of elastic weights (Table 5(*a*), Col. 1) may be obtained by a summation of the shears. The numerator and denominator of Eq. 20*a* both may be multiplied by *E*.

The denominator of Eq. 20*a* represents the rotation of the cut ends of the structure relative to each other, resulting from the couple $X_a = 1$. This term may be evaluated from the shear in the conjugate beam loaded with the elastic weights for $X_a = 1$. Thus, from Table 5(*a*), Col. 2: $\delta_{aa} = 2 \times 97.054 = 194.108$.

Therefore, the influence line for X_a is found by dividing the bending moments in the beam of elastic weights (Table 5(*a*), Col. 1) by 194.108, as shown in Table 5(*a*), Col. 4. This influence line is symmetrical with respect to the center line of the structure. The influence-line ordinates are shown in panel units, or they represent the value of X_a in panel-kips for a load of one kip. The moments may be converted to foot-kips by multiplying by 24, since the panel length is 24 ft.

The value of X_b for a unit load at any point *q* is found from the equation:

$$X_b = -\frac{\delta_{bq}}{\delta_{bb}} = -\frac{\sum \frac{u_b\, u_q\, L}{A\, E}}{\sum \frac{u_b{}^2\, L}{A\, E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

The numerator of this equation represents the deflection of point *q* resulting from a unit value of X_b, and is evaluated for all positions of *q* by the method of elastic weights. These computations are shown in Table 5(*b*), Col. 1. The denominator of Eq. 20*b* is found as the deflection in the direction of X_b, for a load of $X_b = 1$; or as twice the bending moment in the conjugate beam at the center line of the structure. The influence-line ordinates for X_b are given in

TABLE 5.—COMPUTATION OF INFLUENCE ORDINATES (IN PANEL UNITS)

Panel point	Elastic weights[a] (1)	CONJUGATE BEAM Shears (2)	CONJUGATE BEAM Moments[b] (3)	Influence ordinate[c] (4)	Panel point	Elastic weights[a] (1)	CONJUGATE BEAM Shears (2)	CONJUGATE BEAM Moments[b] (3)	Influence ordinate[c] (4)
		(*a*) ORDINATES FOR X_a					(*b*) ORDINATES FOR X_b		
0	−5.776	+5.776	0	0	0	−31.918	+31.918	0	0
1	0.399	+5.377	+5.776	−0.0297	1	2.180	+29.738	+31.918	−0.0221
2	1.068	+4.309	+11.153	−0.0575	2	5.892	+23.846	+61.656	−0.0428
3	1.372	+2.937	+15.462	−0.0797	3	7.550	+16.296	+85.502	−0.0593
4	1.658	+1.279	+18.399	−0.0948	4	9.380	+6.916	+101.798	−0.0706
5	4.487	−3.208	+19.678	−0.1014	5	24.707	−17.791	+108.714	−0.0754
6	3.484	−6.692	+16.470	−0.0849	6	19.187	−36.978	+90.923	−0.0631
7	3.084	−9.776	+9.778	−0.0504	7	16.965	−53.943	+53.945	−0.0374
8	−0.676[d]	−9.100	0[d]	0[d]	8	4.553[d]	−58.496	0[d]	0[d]
9	4.028	−13.128	−9.100	0.0469	9	19.948	−78.444	−58.496	0.0406
10	7.780	−20.908	−22.228	0.1145	10	28.107	−106.551	−136.940	0.0950
11	17.474	−38.382	−43.136	0.2222	11	46.088	−152.639	−243.491	0.1688
12	32.522	−70.904	−81.518	0.420	12	58.192	−210.831	−396.130	0.2750
13	26.15	−97.054	−152.422	0.785	13	17.064	−227.895	−606.961	0.4210
C[e]	....		−200.949	1.035[f]	C[e]	....		−720.908	0.5000
				(*c*) ORDINATES FOR X_c					
0	−9.422	9.422	0	0	7	5.030	−15.942	15.963	−0.1263
1	0.650	8.772	9.422	−0.0746	8	2.893[d]	−18.835	0[d]	0[d]
2	1.743	7.029	18.194	−0.1440	9	2.184	−21.019	−18.835	0.1491
3	2.236	4.793	25.223	−0.1996	10	1.852	−22.871	−39.854	0.3155
4	2.705	2.088	30.016	−0.2376	11	−0.896	−21.975	−62.725	0.4965
5	7.317	−5.229	32.104	−0.2543	12	−11.766	−10.209	−84.700	0.6705
6	5.683	−10.912	26.875	−0.2126	13	−10.210	0	−94.909	0.7510
7	5.030		15.963	−0.1263	C[e]	....		....	....

[a] Elastic weights for X_a, X_b, or $X_c = 1$. [b] Values of δ_{aq}, δ_{bq}, or δ_{cq}. [c] Values of $-\frac{\delta_{aq}}{\delta_{aa}}$, $-\frac{\delta_{bq}}{\delta_{bb}}$, or $-\frac{\delta_{cq}}{\delta_{cc}}$. [d] Hinged. [e] Center cross. [f] Slope of the influence line at this point $= \theta = \tan\theta = 0.5$.

Table 5(*b*), Col. 4, for half the structure. They are antisymmetrical with respect to the center line.

The value of X_c for a unit load at any point q of the structure is found from the equation:

$$X_c = -\frac{\delta_{cq}}{\delta_{cc}} = -\frac{\sum \frac{u_c\, u_q\, L}{A\, E}}{\sum \frac{u_c^2\, L}{A\, E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

The numerator of Eq. 20*c* may be evaluated by the method of elastic weights, as in the computation of the influence lines for X_a and X_b. The denominator, however, represents the horizontal deflection of the point of application of the redundants, and must be evaluated by summation. Substituting the values of u_c from Fig. 11(*c*), and the lengths and areas from Fig. 10, in this summation:

$$\sum \frac{u_c^2\, L}{A\, E} = \frac{126.34}{E}\left(\frac{\text{panels}}{\text{kip}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

If the values shown in Table 5(*c*), Col. 3, are divided by 126.34, the ordinates of the influence line for X_c are obtained. This influence line is symmetrical with respect to the center line of the structure, and is shown for half the structure in Table 5(*c*), Col. 4.

The stresses in all the members of the structure may be found from the influence lines of Cols. 4, Table 5, by simple statics. Obviously, it is not necessary to solve simultaneous equations at any point in the analysis of this structure. G. G. Krivoshein[15] shows that all statically indeterminate structures may be analyzed without the solution of simultaneous equations for the redundants. Although the symmetrical continuous tied arch is readily analyzed in this manner, there are some types of structures in which it may be as laborious to select the redundants so that they satisfy Eqs. 18 as it is to solve the simultaneous equations. The redundants for the structure in Example 2, for instance, may be chosen by inspection so that 42 of the coefficients vanish, and it probably would not be desirable to compute the redundants so that 56 of the coefficients vanish in order to satisfy Eqs. 18.

Acknowledgment.—The continuous tied-arch bridge analyzed in this discussion is probably the first structure of its type in America. This bridge was constructed in 1940 over the Meramec River near St. Louis, Mo., by the Missouri State Highway Department. The data on this structure were furnished through the courtesy of Howard Mullins, designer of the bridge, and N. R. Sack, bridge engineer of the Missouri State Highway Department.

L. E. Grinter,[16] M. Am. Soc. C. E. (by letter).—This is the first paper on the analysis of trussed structures by successive approximations that has appealed to the writer as both interesting and practical. As mentioned by the author, several methods of successive approximations have been presented that complicated analysis instead of simplifying it. Actually, of course, the possibility of an important simplification does not exist for trussed structures because the usual procedure involving Maxwell's equations is not very complicated itself. There is a great difference between the difficulty involved in applying methods of deflection or work to a truss and to a rigid frame. In the first place, the truss is composed of members that are straight and are assumed to resist only direct stress. The application of virtual work for the computation of a deflection involves merely a simple summation of terms, and the calculations are readily tabulated. For a frame, integrations are always necessary and these may be complicated by variation of moment of inertia within each span, by distributed loadings, and even by a curved axis for one or more members. The other simplification that applies to trussed structures is that there are seldom more than three redundants to be considered, and, therefore, no more than three simultaneous equations need to be solved. The case of six or eight redundants in an indeterminate truss is almost limited to the case analyzed by the author as Example 2.

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[16] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

The derivation of basic relationships in the paper is made dependent upon Maxwell's Eqs. 3. Actually, one might write out Eqs. 5 directly as a statement of a self-evident fact. That is, the final stress in any redundant member a must be equal to the stress existing in that member when the other redundants are eliminated, plus the changes in stress that occur when these other redundants come into play in sequence.

Hence, one may write at once without reference to sign, $X_a = s_a' + X_b K_{ab} + X_c K_{ac} \cdots$ (etc.); but it must not be overlooked that the terms s_a', K_{ab}, and K_{ac} demand the same deflection computations by virtual work required for the writing of Maxwell's equations.

The paper has been shortened to the point where certain fundamental considerations are not completely clarified. The computation of the deflection δ_{aa} is obtained by virtual work $\left(\sum \frac{S\,u\,L}{A\,E} \right)$ for six members only—those encompassed by a single panel. This is true because the virtual forces are unit

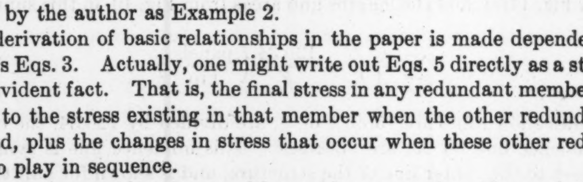

FIG. 12.—ACTION OF VIRTUAL FORCES

stresses in the cut diagonal member, which are represented by a pair of equal and opposite forces having the same line of action as shown in Fig. 12(a). Since two such forces are in equilibrium, the end reactions of the truss are zero, and there are no stresses in other panels.

The deflection δ_{ab} is computed even more simply than δ_{aa}. In Fig. 12(b), there are shown two adjacent panels of a truss with virtual forces (1 lb) acting in the redundant diagonals. The only member that receives stress from each of these virtual pairs of forces is the central vertical post *C D*. Since its stresses s and u are each -0.8 and its length is 32.0, the deflection δ_{ab} is computed as $0.8 \times 0.8 \times 32 \div A\,E$.

Many readers undoubtedly will question how the author simplified Eqs. 5 when they are applied to Example 2 in which there are eight redundants. One would assume that each of the eight equations should contain all such factors as K_{ab}, K_{ac}, K_{ad}, K_{ae}, K_{af}, K_{ag}, and K_{ah}, representing the interaction of the panel a with all other panels of the truss. Actually, all of these factors except K_{ab} are zero. The reason again may be found by reference to Fig. 12. For example, $K_{ac} = \delta_{ac} \div \delta_{aa}$. The deflection δ_{ac} would have to be computed by virtual work with virtual forces applied in the redundant diagonals of the panels a and c. In neither case would these redundant forces stress the same members and, therefore, the term $\frac{S\,u\,L}{A\,E}$ would be zero for each member. It is unfortunate when such matters as these are not explained in a technical paper since they save the reader much wasted time.

The writer has been greatly interested in the influence of errors in carry-over factors upon final results in any process of successive corrections. In Example 2, the carry-over factors are about 0.25 toward the end panels and roughly 0.20 in all other cases. In the latter instance, the actual variation of carry-over factors is from 0.165 to 0.218. In Table 6 the successive corrections of Table 3 are solved by use of the approximate carry-over factors 0.25 and 0.20. It is interesting that the maximum error for a redundant stress is 9% although the maximum error in the approximate carry-over factors was 18%. It is evident, therefore, that, whereas a small error in the solution of simultaneous equations is often very serious, small errors in the process of successive corrections tend to reduce themselves. One is often able to estimate carry-over factors with sufficient accuracy for preliminary studies. It must not be forgotten that the design of an indeterminate truss requires at least a preliminary analysis followed by a final analysis after the corrected areas have been chosen.

TABLE 6.—USE OF APPROXIMATE CARRY-OVER FACTORS FOR EXAMPLE 2

Description	Panel 1	Panel 2	Panel 3	Panel 4	Panel 5	Panel 6	Panel 7	Panel 8
1st $X = s'$	−1.75	−1.81	−1.71	−1.67	−1.67	+0.10	+5.42	+4.21
$K_{ab}\,X_b$, etc.	+0.45	+0.34	+0.33	+0.33	−0.02	−1.09	−0.84	
$K_{ba}\,X_a$, etc.		+0.26	+0.24	+0.23	+0.22	+0.30	+0.14	−1.18
2d X	−1.30	−1.21	−1.14	−1.11	−1.47	−0.69	+4.72	+3.03
$K_{ab}\,X_b$, etc.	+0.30	+0.23	+0.22	+0.29	+0.14	−0.94	−0.61	
$K_{ba}\,X_a$, etc.		+0.29	+0.26	+0.25	+0.23	+0.26	+0.12	−1.23
3d X	−1.45	−1.29	−1.23	−1.13	−1.30	−0.58	+4.93	+2.98
Correct X	−1.44	−1.21	−1.25	−1.23	−1.37	−0.53	+4.86	+2.93

The method presented by the author is interesting and also useful if there are many redundants. The usual procedure by the direct solution of Maxwell's equations is so simple in itself that the use of successive corrections could offer little advantage for two or three redundants. Nevertheless, the author is to be complimented for presenting the most usable device yet described for analyzing indeterminate trusses by successive corrections.

O. T. VOODHIGULA,[17] JUN. AM. SOC. C. E. (by letter).—Professor Castleman points out the advantage of choosing members as redundants in the case of a continuous truss. It is obvious that much work can be saved in computing the carry-over factors and the single redundants, whereas in the case of the reaction redundants every member must be included in every computation.

The writer has no objection to Professor Ellis' preference for "Williot Strain Equations." All the methods used in analyzing statically indeterminate structures are fundamentally the method of distortions or deflections.

Mr. Peery's discussion is very interesting. The writer fully agrees with his statement that "It * * * is more important to select the redundants in such a manner as to simplify Eqs. 3 than it is to simplify the actual solution of the equations." In the example chosen by him, it is possible to obtain a direct solution, because the redundants act at the elastic center, where a redundant has no effect on the others; in other words, the carry-over factors are zero.

Dean Grinter's comments are very pleasing.

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[17] Municipal and Public Works, Bangkok, Thailand.

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CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

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With Discussion by Messrs. Jerome Fee, V. E. Leman, P. S. O'Shaughnessy and E. S. Randolph, Carroll F. Merriam, Hunter Rouse, G. H. Hickox, J. M. Mousson, J. H. Douma, and Harold A. Thomas and Emil P. Schuleen.

Synopsis

Occurrence of severe cavitation damage to the concrete surfaces of the outlet conduits of the Madden Dam, in the Panama Canal Zone, supplied the incentive to carry on extensive studies by means of laboratory models to investigate the cavitation potentialities, if any, in the conduits of the Tygart River Dam, near Grafton, W. Va., and to develop methods of eliminating or minimizing future damage in the conduits of the Madden Dam. Similar studies, on a less elaborate scale, were conducted in connection with the design of the Bluestone Dam in West Virginia, Hiwassee Dam in North Carolina, and Redbank Creek Dam in Pennsylvania.

The development of cavitation-testing facilities of two types, known respectively as the "enclosed-tank apparatus" and the "diverging-tube apparatus," is described, and the hydraulic theory pertinent to the making of cavitation tests in these facilities is presented. A description is given of cavitation studies conducted on models of the conduit entrances of the Madden Dam.

Introduction

In the late summer of 1935 the pouring of concrete in the lower portions of Tygart River Dam was in progress under the direction of the district engineer, U. S. Engineer Office, Pittsburgh, Pa., when word was received from the Panama Canal authorities that the concrete in the entrance portions of the conduits in Madden Dam had been seriously damaged by cavitation. Since the conduits to be placed in Tygart River Dam were identical in cross section to those in Madden Dam, and differed only slightly in entrance curvature,

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Note.—Published in November, 1940, *Proceedings*.

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concern was naturally felt regarding the possibility of cavitation also occurring in the Tygart River Dam conduits. To test the design for such a possibility, and to provide means for analyzing possible alterations in design to eliminate cavitation, should such be found to exist, arrangements were made with the Hydraulic Laboratory of the Carnegie Institute of Technology, in Pittsburgh, to construct and test a model of the Tygart conduit entrance.

The tests to be undertaken under this arrangement were of an unusual nature for which there was little precedent, since a search of records of cavitation tests disclosed none of models of existing or proposed conduits designed for high velocities. It was necessary, therefore, to design a complete testing apparatus in order that correct scale reproductions of the absolute pressures operating in the prototype could be developed in the model. The pioneering nature of these tests led to some uncertainty regarding the ability to secure an accurate reproduction in the model of cavitation effects which might appear in the prototype if the entrance were improperly designed. As a means of verifying the accuracy with which prototype phenomena were reproduced, a model of the conduit entrance in Madden Dam was constructed and tested, and the results compared with data on cavitation and damage in the prototype as reported by the Panama Canal authorities.

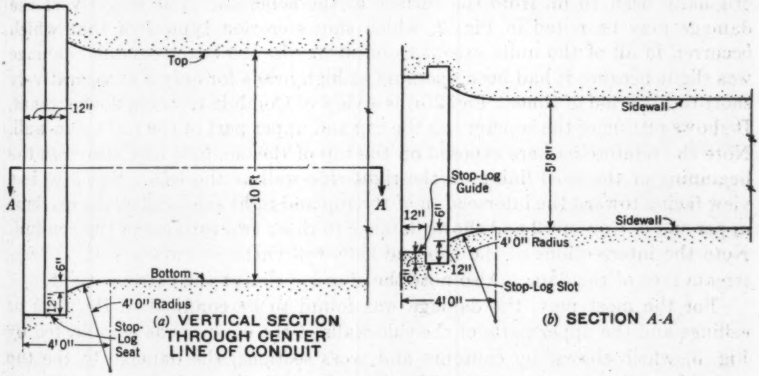
As the tests progressed, the Governor of the Panama Canal Zone took an increasing interest in the work and in the early spring of 1936 officially requested that tests be conducted to develop a feasible means of eliminating or alleviating the cavitation action and resulting damage in the conduits of Madden Dam. By this time, tests of the conduits designed for Tygart River Dam had been substantially completed, so that the testing apparatus could then be made available almost exclusively for tests of models of the conduits in Madden Dam. Various designs of entrance curves for conduits were tested in developing a design that might be readily installed in the prototype at a minimum cost. A suitable design was evolved and a report was in the final stages of preparation when the Governor of the Panama Canal Zone requested a series of additional tests. These additional tests involved: (*a*) A check of the previously developed design to determine its adequacy at somewhat higher heads as well as its adequacy when the two conduits of a pair were operated together (previous tests had disregarded the effect of an adjacent conduit in operation); (*b*) further study of a bellmouth entrance designed by the engineers at the Panama Canal; (*c*) studies of the degree of gate opening that could be permitted without destructive cavitation occurring in the conduit entrance; (*d*) various schemes for increasing the pressure in the critical area by constrictions within the conduit or at its discharge end; and (*e*) various studies involving fillers for stop-log slots, venting of the conduits, etc. The second series of tests was completed during the winter of 1937–1938 and a report was made the following summer.

During and subsequent to the tests at Madden Dam, the availability of the cavitation apparatus at the Hydraulic Laboratory of the Carnegie Institute of Technology permitted the testing of conduit entrances for several other dams, among which may be mentioned the Bluestone, Hiwassee, and Redbank Creek dams.

A description of the details of the numerous tests on the Madden models, and of cavitation studies on models of the outlet conduits of the Tygart, Bluestone, Redbank Creek, and Hiwassee dams, is included in the original manuscript from which this paper is condensed, the original being on file in Engineering Societies Library.[3]

CAVITATION EFFECT IN MADDEN DAM CONDUITS AS ORIGINALLY DESIGNED AND CONSTRUCTED

The Madden Dam,[4] a concrete gravity structure comprising part of the Panama Canal Project, is pierced by six rectangular conduits, located as shown subsequently in Fig. 12. These conduits are grouped in pairs and are controlled by service and emergency slide gates. Each is 5 ft 8 in. wide and operates under a maximum head of 168.3 ft. The upstream or intake ends of the conduits, which are the points of particular interest in this paper, were flared in the original construction on a radius of 4 ft as indicated by Fig. 1 (which also shows the design and location of the stop-log slots). The conduits are steel-lined for a reach of 48 ft in the vicinity of the slide gates but are unlined above and below this reach. The intake ends of the conduits, therefore, are unprotected by metal lining over the initial 12.89 ft, this measurement including the stop-log slot and guide.



(a) VERTICAL SECTION THROUGH CENTER LINE OF CONDUIT

(b) SECTION A-A

FIG. 1.—ORIGINAL CONDUIT ENTRANCE, MADDEN DAM

The reservoir was placed in operation in 1934, and the conduits were in use at various times during the process of filling the reservoir. By March 12, 1935, the water level had been lowered to an elevation one foot above the spillway crest by discharging over the crest while making spillway tests, and the conduits were then placed in operation to continue the release of impounded water. It was observed at once that the operation of the conduits was accompanied by a loud crackling and popping noise which was particularly noticeable in the slide-gate galleries. However, operation was continued at the full capacity of the conduits until March 15, 1935, when a porous tile drain in the partition wall between Conduits 1 and 2 began to leak water into the gallery. It was then concluded that the crackling noise and the leak which had developed were the result, respectively, of cavitation and its damaging effect in the entrance portion of the conduit. It was decided to reduce the cavitation action by throttling the discharge by partial closure of the slide gates. The crackling noise or crepitation ceased when the gates were closed to the 90% open position, and the gates were left in that position during the remaining process of draining the reservoir.

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[3] 29 West 39th Street, New York, N. Y.

[4] "Hydraulic Tests on the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1080.

After the reservoir had been completely drained, the conduits were inspected and the damage that had resulted from cavitation was recorded. The inspection revealed the fact that the cavitation action had seriously eroded the walls and tops, or ceilings, of the conduits by pitting the concrete to depths of as much as almost 2 ft and in many places by exposing reinforcing bars that had originally been 10 in. from the surface of the concrete. The severity of the damage may be noted in Fig. 2, which shows erosion typical of that which occurred in all of the units except Conduit 3. In the latter conduit, damage was slight because it had been operated at high heads for only a comparatively short total period of time. Fig. 2(*a*) is a view of Conduit 1, facing downstream. It shows pitting of the concrete on the top and upper part of the right side-wall. Note the reinforcing bars exposed on the top of the conduit, and also note the beginning of the steel lining on the right side-wall at the left. Fig. 2(*b*) is a view facing toward the intersection of the top and right side-wall at the extreme upper end of Conduit 2 and shows damage to those two surfaces of the conduit. Note the intersections of the top and side-wall entrance curves with the upstream face of the dam. Also note the stop-log slot at the extreme right.

For the most part, the damage was found to be confined to the tops or ceilings and the upper parts of the side-walls of the conduits as is indicated by Fig. 3, which shows, by contours and cross sections, the damage to the top and side-walls of Conduit 5. This also is typical of the damage to all units except Conduit 3.

A résumé of the number of hours and minutes that each conduit was operated at various heads during the first season of operation, and the resulting damage as indicated by the maximum depth of pitting and the volume of concrete disintegrated and removed, is given in Table 1.

Disregarding Conduit 3, where damage was comparatively slight, analysis of the data in Table 1 indicates that the concrete on the tops of the conduits was pitted to maximum depths of more than 1.0 ft to 1.5 ft, whereas the side-walls were pitted to maximum depths of more than 0.5 ft to 1.75 ft. Neglecting the time in which the conduits were operated at 90% gate opening (when, as previously explained, cavitation was reduced to a minimum), the data for Conduits 1, 2, 5, and 6, as given in the table, indicate a progressive increase in the volume of concrete disintegrated and removed with increased hours of operation at heads greater than 45 ft. However, comparison of the data for

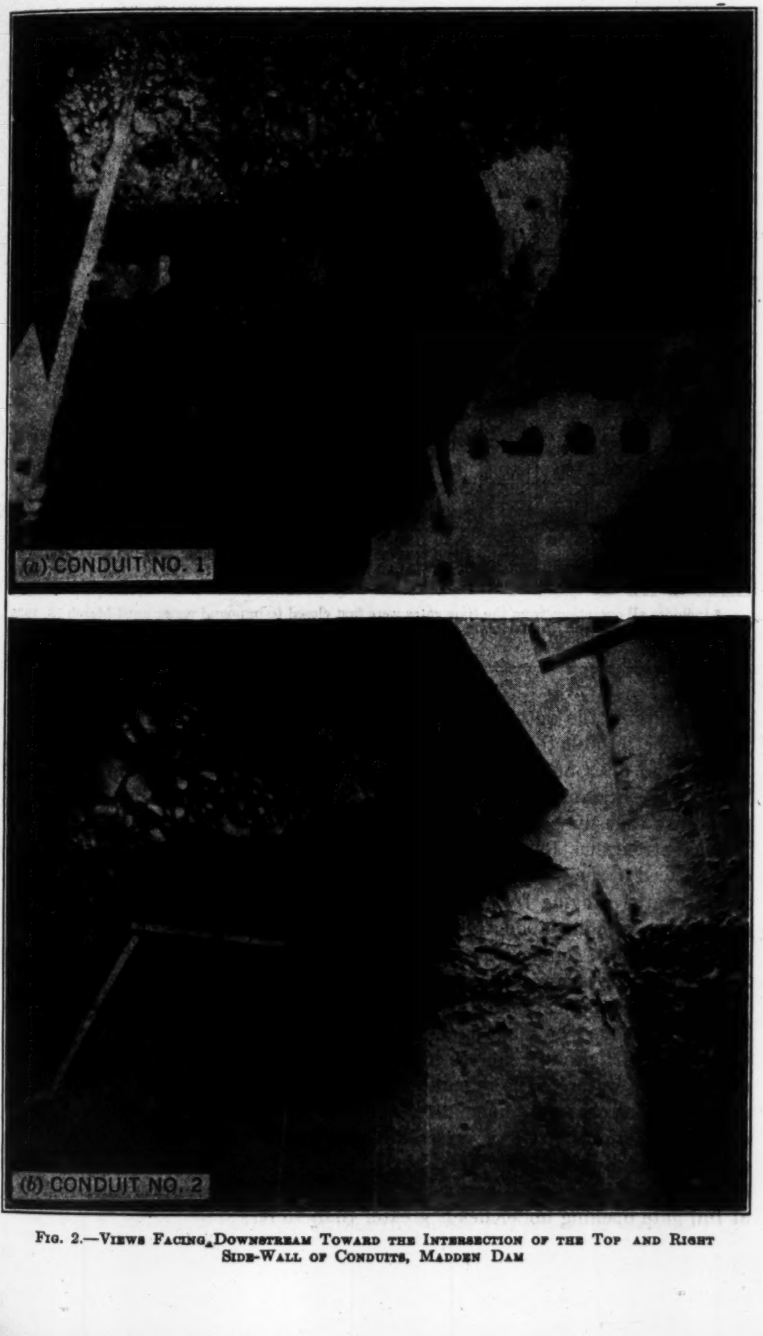

(a) CONDUIT NO. 1
(b) CONDUIT NO. 2

FIG. 2.—VIEWS FACING DOWNSTREAM TOWARD THE INTERSECTION OF THE TOP AND RIGHT SIDE-WALL OF CONDUITS, MADDEN DAM

Conduits 1 and 2 with those for Conduits 5 and 6 shows a tendency for the rate of disintegration and removal of concrete to become less as the conduits are subjected to greatly increased hours of operation at high heads. This may be explained by the tendency for the erosion on the top of the conduit to become stabilized but to continue to progress down the side-walls, as will be discussed more fully in comparing the cavitation region or pocket with the eroded area in the section, "Behavior of Original Madden Entrances in Models."

TABLE 1.—Operation Data at the End of the First Season

Conduit	Hours and Minutes of Operation[a]							Approximate Maximum Depth of Pitting[b] (Ft)			Volume of Concrete Removed (Cu Ft)			
	At Full Gate Opening				At 90% Gate Opening									
	Head Range (Ft)			Total 45–145	Head Range (Ft)		Total 45–145							
	0–45	45–95	95–145		45–95	95–145		Top	Right wall	Left wall	Top	Right wall	Left wall	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	26:03	1:53	136:08	138:01	....	29:08	29:08	1.50	1.25	1.75	27.3	14.2	11.5	53.0
2	84:58	3:41	126:42	130:23	80:01	131:05	211:06	1.25	0.75	1.50	26.4	5.5	9.6	41.5
3	60:06	43:22	18:57	62:19	79:39	119:52	199:31	0.25	0	0	0.4	0.0	0.0	0.4
4	150:58	67:02	165:08	232:10	79:19	131:06	210:25	1.00	1.00	0.50	23.9	4.0	2.3	30.2
5	62:56	231:58	152:59	384:57	78:25	131:02	209:27	1.50	1.50	1.00	33.2	19.8	7.8	60.8
6	196:43	32:35	203:21	235:56	77:08	131:00	208:08	1.25	1.50	1.50	29.3	9.8	16.0	55.1

[a] Includes all operations from the time gates were first closed to impound water until March 28, 1935.

[b] Values given correspond to maximum depth contours shown on drawings for each conduit similar to those of Fig. 3.

In studying the data of Table 1, it may be noted that Conduit 4 was apparently better able to withstand the ravages of cavitation than were Conduits 1, 2, 5, and 6. Conduit 3 was in operation under high heads for only a few hours. No explanation can be given for this inconsistency other than the possibility that the concrete in the region of the entrance to Conduit 4 was of unusual strength. Records of compression tests made on samples of the concrete placed in the regions of the various conduit entrances gave 28-day breaking strengths ranging from about 3,230 lb per sq in. to about 4,680 lb per sq in., and averaging about 3,800 lb per sq in. The results of tests of individual cylinders tend to confirm the suggestion that concrete in the walls of Conduits 3 and 4 may be of unusual strength.

As a further indication of the quality of the concrete in the conduit entrances in general, it may be mentioned that the inspection revealed all loose or disintegrated particles of concrete to have been removed by the terrific scouring action of the water. The remaining concrete in all damaged areas was ragged and sharp but otherwise in sound condition. It is evident, therefore, that the cavitation action must have exerted terrific forces on the particles of concrete making up the conduit surfaces, particularly if consideration is given to the comparatively short total periods at which the various conduits were operated at full gate opening under heads greater than 45 ft.

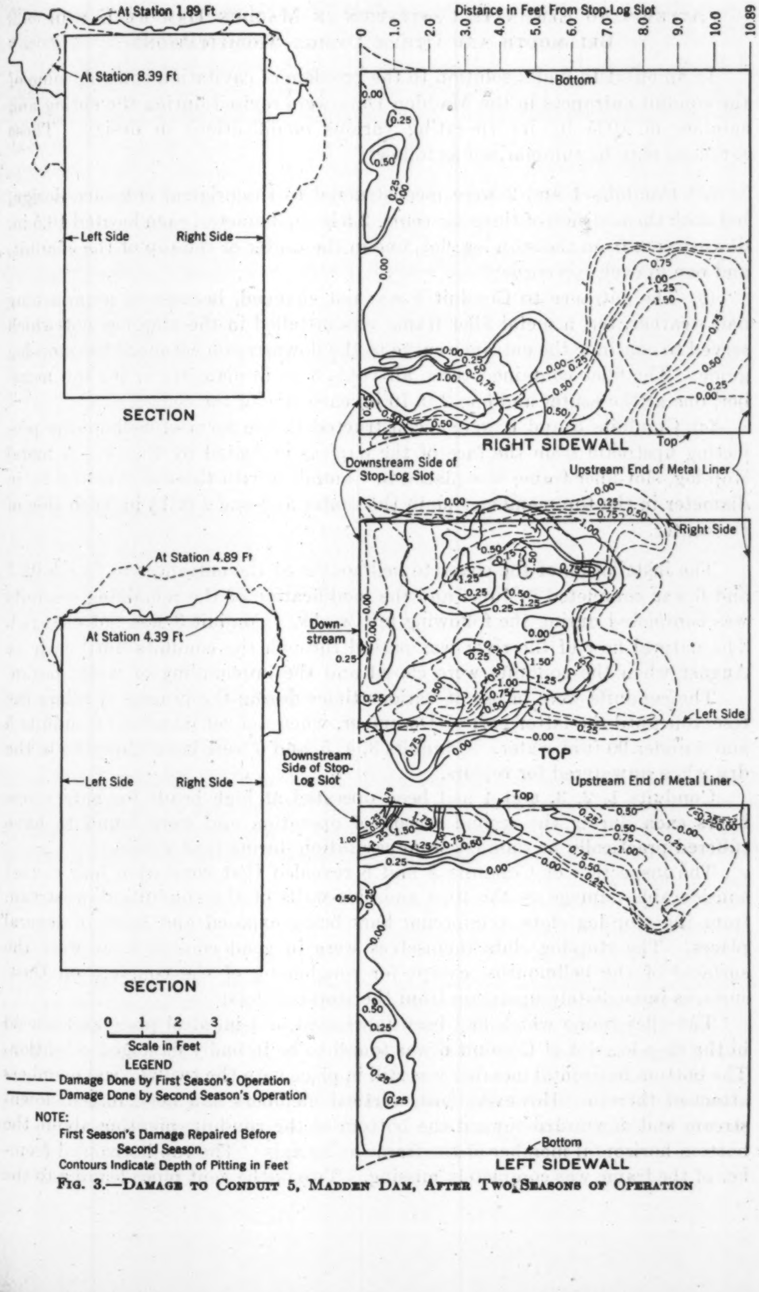

At Station 1.89 Ft
At Station 8.39 Ft
Left Side
Right Side
SECTION
At Station 4.89 Ft
At Station 4.39 Ft
Left Side
Right Side
SECTION
0 1 2 3
Scale in Feet
LEGEND
Damage Done by First Season's Operation
Damage Done by Second Season's Operation
NOTE:
First Season's Damage Repaired Before Second Season.
Contours Indicate Depth of Pitting in Feet
Distance in Feet From Stop-Log Slot
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
10.89
Bottom
RIGHT SIDEWALL
Top
Downstream Side of Stop-Log Slot
Upstream End of Metal Liner
Right Side
Down-stream
Left Side
TOP
Downstream Side of Stop-Log Slot
Upstream End of Metal Liner
Top
Bottom
LEFT SIDEWALL
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75

FIG. 3.—DAMAGE TO CONDUIT 5, MADDEN DAM, AFTER TWO SEASONS OF OPERATION

ATTEMPT TO ELIMINATE CAVITATION IN MADDEN DAM BY USE OF BELLMOUTH AND OTHER DESIGN MODIFICATIONS

In an effort to find a solution to the problem of cavitation control, some of the conduit entrances in the Madden Dam were revised during the spring and summer of 1935 by incorporating various modifications in design. These revisions may be summarized as follows:

(*a*) Conduits 1 and 2 were reconstructed to the original entrance design, but with the addition of three air vents, 2.5 in. in diameter, each located 10.5 in. downstream from the stop-log slot, one in the center of the top of the conduit, and one in each top corner.

(*b*) The entrance to Conduit 4 was not changed, because of approaching bad weather, but a metal filler frame was installed in the stop-log slot which served to continue the entrance curve to the downstream corner of the stop-log guide. The filler contained three air vents 3 in. in diameter in the top member, one at the center and one 2 ft 10 in. each side of the center.

(*c*) Conduits 5 and 6 were reconstructed in the form of bellmouths projecting upstream from the face of the dam as indicated by Fig. 4. A metal stop-log, slot-filler frame was placed in Conduit 6 with three air vents 3 in. in diameter in the top member, one in the center and one 2 ft 1½ in. each side of the center.

The installation of the concrete bellmouths at the entrances to Conduits 5 and 6 was completed in June, and the modification of the remaining conduits was completed during the following few weeks. Conduit 3 was not changed. The natural flow of the river was passed through the conduits until early in August when the conduits were closed and the impounding of water began.

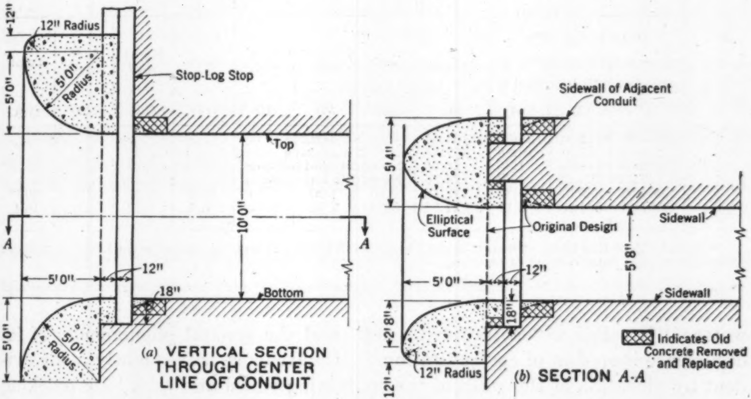
The conduits were in use at various times during the process of filling the reservoir until the latter part of December, when a diver inspected Conduits 5 and 6 under 90 ft of water. Conduits 3, 4, 5, and 6 were later inspected in the dry when unwatered for repairs.

Conduits 1, 2, 3, and 4 had been operated at high heads for only a few hours each during the second season of operation and were found to have suffered practically no damage from cavitation during that season.

The inspection of Conduits 5 and 6 revealed that cavitation had caused considerable damage to the tops and side-walls of the conduits downstream from the stop-log slots, reinforcing bars being exposed and loose in several places. The stop-log slots themselves were in good condition, as were the surfaces of the bellmouths, except for roughening of the concrete on these surfaces immediately upstream from the stop-log slots.

The filler frame which had been fabricated of ½-in. steel plate and placed in the stop-log slot of Conduit 6 was found to be in badly damaged condition. The bottom horizontal member was still in place with the two vertical members attached thereto. However, both vertical members had been forced downstream and downward toward the bottom of the conduit, pivoting about the bottom horizontal member of the frame as an axis. The top horizontal member of the frame was completely missing. Two of the vent pipes leading to the filler frame were snapped off about 7 ft above the top of the frame, and the third was snapped off at the frame.

The location and extent of pitting as a result of cavitation in Conduit 5 during the second season of operation are indicated in Fig. 3. The pitting in Conduit 6 was quite similar. Superimposed on the contours showing this damage are contours indicating the damage to this conduit as a result of operation during the first season—that is, before the bellmouth entrances had been installed. It may be noted that the general proportions and locations of the pitted areas on the top of the conduit were approximately the same in the case of both seasons of operation, although the damage resulting from operation during the second season extended a slightly lesser distance into the conduit than did that resulting from operation during the first season. The deepest pitting of the side-walls occurred 1 to 4 ft downstream from the stop-log slots in the case of the second season of operation, whereas during the first season of operation the deepest pitting of the side-walls occurred 8 to 10 ft downstream from the stop-log slots. During the second season of operation pitting occurred to a minor degree over the full height of the side-walls immediately downstream from the stop-log slots. In these areas pitting was entirely absent, except near the tops of the conduits, at the close of the first season of operation, as indicated by broken lines in Fig. 3.



(*a*) VERTICAL SECTION THROUGH CENTER LINE OF CONDUIT

(*b*) SECTION *A-A*

FIG. 4.—BELLMOUTH ENTRANCE AT CONDUITS 5 AND 6, MADDEN DAM

For comparison with Table 1 and the degree and extent of pitting resulting from operation during the respective seasons, as indicated by Fig. 3, Table 2 gives a résumé of the number of hours and minutes each conduit was operated at various heads during the second season of operation, together with the resulting damage as indicated by the maximum depth of pitting and the volume of concrete disintegrated and removed. The values given for depths of pitting correspond to the maximum-depth contours shown for the respective conduits in Fig. 3.

NATURE AND CAUSES OF CAVITATION

It is well known that under certain conditions of hydraulic flow a persistent void space or cavity will occur at some given location in the stream of moving water. If this cavity is not connected with the atmosphere, and therefore contains practically nothing but water vapor and flying particles or slugs of water, it is called a "cavitation pocket" and the general phenomenon of its occurrence is spoken of as "cavitation." Its existence is frequently made evident by vibration of the walls of the containing vessel and by a loud cracking sound known as "crepitation." Where a cavitation pocket occurs in contact with a solid object, such as the concrete surface of a conduit or the metal blade of a turbine runner, the solid material eventually becomes damaged by destructive disintegration commonly known as "pitting."

TABLE 2.—MADDEN DAM CONDUITS, RESULT OF SECOND SEASON OF OPERATION

Conduit	Hours and Minutes of Operation at Full Gate Opening				Approximate Maximum Depth of Pitting (Ft)			Volume of Concrete Removed (Cu Ft)			
	Head Range (Ft)			Total 45–145	Top	Right wall	Left wall	Top	Right wall	Left wall	Total
	0–45[a] (1)	45–95 (2)	95–145 (3)	(4)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	2,174:28	113:31	20:30	134:01	No damage			....	....	....	....
2	2,178:44	87:57	19:58	107:55	No damage			....	....	....	....
3	3,224:52	119:00	1:35	120:35	No additional damage			....	....	....	....
4	3,189:32	122:48	1:38	124:26	No additional damage			....	....	....	....
5	1,061:07	389:08	859:35	1,248:43	0.75	1.00	1.75	12.6	9.5	7.4	30.5[b]
6	1,607:11	551:44	870:11	1,421:55	0.50	1.00	0.75	6.1	6.7	3.6	16.4

[a] 85 to 97% of this time, operation was at heads from 0 to 25 ft to pass natural river flow. [b] Includes 1.0 cu ft removed from bottom of conduit.

Cavitation pockets of the type occurring in diverging tubes are characteristically traversed from end to end by a succession of swiftly moving slugs of water separated by vapor cavities. Prof. H. E. Edgerton has taken remarkable moving pictures,[5] exposed at 3,000 per sec, showing the water slugs and vapor cavities traversing pockets of this type. One group of twenty five of these exposures[6] shows, with marked clearness, the formation, growth, and collapse of a moving vapor cavity during the one hundred and seventy fifth of a second required for it to traverse the length of the region designated as the "cavitation pocket."

Some years ago it was thought that the pitting of turbine runners, ships' propellers, and pump impellers was caused by the chemical action of nascent oxygen released in the cavitation pockets; but this theory has been discarded. It is now generally recognized that damage to materials in contact with a cavitation pocket is caused by the hammering or penetrating effect of extraordinarily high water pressures generated in local regions in connection with the collapse of vacuum pockets. Although to the uninitiated it may seem surprising that a moving mass of water can strike a blow sufficiently severe and concentrated, not only to disintegrate concrete but to indent the hardest metals, abundant experimental evidence proves that such action is possible and of common occurrence.

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[5] "Cavitation Research," by J. C. Hunsaker, *Mechanical Engineering*, Vol. 57, April, 1935, pp. 211–216.

[6] "Applied Fluid Mechanics," by M. P. O'Brien and G. H. Hickox, Members, Am. Soc. C. E., p. 34.

An analytical demonstration of the hammering power of water under cavitation conditions may be made in two steps: (1) Proof that the collapse of vapor cavities can produce extremely high local velocities; and (2) proof that the sudden checking of such velocities can produce extremely high local pressures. The first step may be explained most conveniently in connection with an arbitrary system in which the dimensions of the vapor cavities can be expressed by a simple formula. Consider for this purpose that Fig. 5 represents a prismatic space of square cross section with diameter d, terminated at one end by a pyramid of altitude a. First, consider this space to be bounded by solid walls, and later as representing a cavitation pocket for which three of the four boundary walls are of water. Suppose that two slugs of water of equal volume and equal original kinetic energy are traveling down this space, the one in the prismatic portion having a volume $b\,d^2$ and a velocity v_0, and the one in the pyramidal portion having the same volume but velocities of v_1 and v_2 at its forward and rear ends, respectively. The three vapor cavities adjacent to these two slugs of water are indicated in Fig. 5. If w represents the weight of a unit volume of water, the kinetic energy of a lamina of volume $y^2\,dx$ having a velocity v is

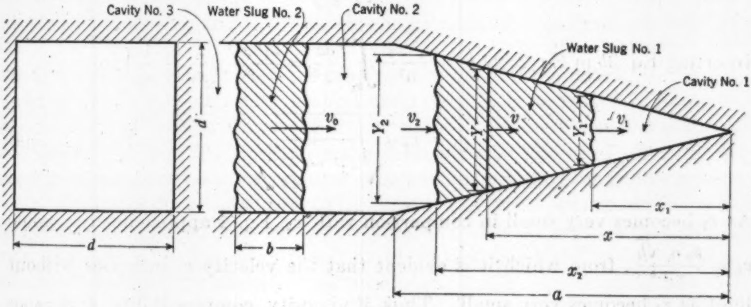

Fig. 5.—Movement of Water Slug into the Confined Region of Diminishing Cross Section

$$E_k = w\,(y^2\,dx)\,\frac{v^2}{2\,g} = w\left(\frac{x\,d}{a}\right)^2\frac{v^2}{2\,g}\,dx \ldots\ldots\ldots\ldots\ldots (1)$$

Therefore, the kinetic energies of the two water slugs in Fig. 5 are given respectively by the first and second members of the following equation:

$$w\,b\,d^2\,\frac{v_0^2}{2\,g} = \int_{x_1}^{x_2} w\left(\frac{x\,d}{a}\right)^2\frac{v^2}{2\,g}\,dx \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, in addition to symbols shown in Fig. 5: w = unit weight of water and g = acceleration of gravity.

In order to justify equating these two expressions for kinetic energy, it is necessary to neglect the dissipation of energy by hydraulic friction or viscous resistance to deformation, the storage of energy by elastic compression of the water or the conduit walls, and the release of energy by condensation of the water vapor in the cavities as the volumes of the latter decrease. This analysis thus assumes frictionless non-viscous flow, incompressible materials, and negligible vapor pressure. The effect of these assumptions will be discussed subsequently.

In Fig. 5, imagine the cross-sectional plane of abscissa x and area y^2 to consist of a weightless elastic membrane which separates the slug of water into two parts and moves forward with the velocity v. Since the volume of water must thus remain constant between the sections whose respective abscissas are x and x_1:

$$v\,y^2 = v_1\,y_1^2 \qquad \text{(3a)}$$

Since $y = \frac{x\,d}{a}$ and $y_1 = \frac{x_1\,d}{a}$, Eq. 3a becomes $v\left(\frac{x\,d}{a}\right)^2 = v_1\left(\frac{x_1\,d}{a}\right)^2$, or

$$v^2 = v_1^2\left(\frac{x_1}{x}\right)^4 \qquad \text{(3b)}$$

Inserting Eq. 3b in Eq. 2: $v_0^2\,b = \frac{v_1^2\,x_1^4}{a^2}\int_{x_1}^{x_2}\frac{dx}{x^2} = \frac{v_1^2\,x_1^4}{a^2}\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$; or,

$$v_1 = v_0\,a\sqrt{\frac{b}{x_1^3 - \frac{x_1^4}{x_2}}} \qquad \text{(4)}$$

As x_1 becomes very small in comparison with x_2, Eq. 4 approaches the value: $v_1 = \frac{v_0\,a\,\sqrt{b}}{x_1^{1.5}}$, from which it is evident that the velocity v_1 increases without limit as x_1 becomes very small. Thus, if viscosity, compressibility, and vapor condensation did not influence the rate of movement of the water, the velocity at the tip of the slug would become infinite at the final instant of the collapse of the vapor cavity at the apex of the pyramid. It is possible also to prove by similar methods that at this same instant, and under the same assumptions, the velocity v_2 becomes zero. In other words, all of the original kinetic energy of the slug would still exist in the kinetic form; but it would be practically all concentrated into the fluid particles that make the final contact with the solid surface at the point where the last vestige of the collapsing cavity disappears, with the result that these particles would acquire enormous velocity.

It is evident that the latter statement could be proved for spaces of other shapes than the square pyramid in Fig. 5. In fact, if incompressible materials, frictionless flow, and negligible vapor pressure are assumed, the statement is perfectly general and applies to spaces of all shapes, whether bounded by solids or fluids. Under these assumptions, if a slug of moving liquid is brought to rest by traveling into a confined vacuum space of any shape, the velocity of the last particles making the contact must become infinite.

Although the factors specifically neglected in the foregoing demonstration undoubtedly have important effects, the analysis reveals the fundamental reason why intense local concentrations of energy are associated with the collapse of vacuum cavities. From the foregoing it is apparent that if cavitation occurs in large outlet conduits of high dams there is abundant opportunity for tremendous concentrations of energy to occur at local spots along the conduit surfaces.

Under the foregoing assumptions, the pressure developed at the point where the two incompressible materials finally meet with infinite relative velocity would evidently be "super-infinite." The problem of determining the pressure developed by the collision of compressible materials moving at finite relative velocity remains to be discussed. This problem comprises the second part of the analytical demonstration of the hammering power of water, and requires a development of the proof that the sudden checking of high-water velocities can produce extremely high pressures. This proof is analogous to that which shows that high pressures in a pipe can be produced by the sudden closure of a valve. When a rapidly moving mass or slug of water strikes a rigid plane surface normal to the direction of motion, the water particles adjacent to the contact surface and near to the center of the contact area have their velocity suddenly reduced to zero and their kinetic energy suddenly changed to energy of elastic compression. By equating the values of these two kinds of energy, per pound of water, the amount of the pressure rise may be computed.

In order to make this computation accurately, it is necessary to have information regarding the compressibility of water under the unusual conditions considered. At very high pressures the volumetric coefficient of elasticity of water increases to several times the ordinary value published in textbooks.

In discussing the effects of rapid compression of water, the results of ordinary slow-compression tests cannot be used directly. In slow or isothermal tests, the heat of compression has time to be dissipated to the surrounding objects or mediums, practically uniform temperature being maintained in the specimen at the times when readings of deformation are taken. Under such conditions water at 50° F solidifies into ice at a pressure of about 110,000 lb per sq in. In rapid or adiabatic compression, the heat generated has no opportunity to escape to the surrounding objects or mediums, and therefore the temperature of the water rises as the pressure is applied, thus preventing solidification into ice.

The results of an elaborate investigation on the compressibility of water were published in 1912 by P. W. Bridgeman,[7] who presents a diagram which, when translated from metric to English units, gives the volume of one pound of water at all pressures from 0 to 180,000 lb per sq in. in combination with all temperatures from 4° to 176° F, except where the water is in the solid state. He also presents a diagram giving, for the same range of pressures and temperatures, the adiabatic rise in temperature per unit rise in pressure, as determined, not from actual quick-compression tests, but from a thermodynamic analysis of thermal measurements during slow-compression tests. By the use of these two diagrams it is possible to start with one pound of water at atmospheric pressure and any given temperature, and compute its volume and temperature after any number of successive increments of adiabatic compression. It is also possible to compute the total external work or energy contributed to the pound of water during the adiabatic compression, since the work done during any one increment of pressure is equal to the product of the mean pressure and the decrement in volume. The results of such a computation by the writers, starting with one pound of water at 50° F and extending to the limits of Bridgeman's diagrams, are given in the first eight columns of Table 3: Col. 1 gives the pressure in increments of 14,223 lb per sq in. (1,000 kg per sq cm); Cols. 2 and 4 are obtained from Bridgeman's diagrams, with the values converted to English units; Col. 3 is an accumulative summation of the increments in Col. 2; Col. 5 consists of the differences between successive items in Col. 4; Col. 6 contains the averages of successive items in Col. 1; Col. 7 gives the products of the items in Cols. 5 and 6; and Col. 8 is obtained by summing the items in Col. 7.

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[7] "The Thermodynamic Properties of Water," by P. W. Bridgeman, *Proceedings*, Am. Academy of Arts and Sciences, Vol. 48, 1912, pp. 307–362.

TABLE 3.—COMPUTATION OF PRESSURE RISE IN WATER DUE TO DESTRUCTION OF VELOCITY BY SUDDEN IMPACT (BASED ON INITIAL WATER TEMPERATURE OF 50° F)

Pressure (lb per sq in.)	Adiabatic temperature rise (degrees F)	Temperature (degrees F)	Volume of 1 lb of water (cu in.)	Decrement in volume of 1 lb of water (cu in.)	Average pressure (lb per sq in.)	Increment of work (in-lb)	Total energy of compression per lb of water (in-lb)	VELOCITY (FT PER SEC) REQUIRED TO PRODUCE A GIVEN PRESSURE AGAINST:	
								Rigid surface	Water surface
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0		50.0	27.68				0	0	0
	2.3			1.10	7,112	7,820			
14,223		52.3	26.58				7,820	205	237
	4.1			0.83	21,335	17,710			
28,446		56.4	25.75				25,530	370	427
	4.7			0.64	35,558	22,750			
42,669		61.1	25.11				48,280	510	589
	4.9			0.54	49,781	26,880			
56,892		66.0	24.57				75,160	636	735
	4.9			0.47	64,004	30,080			
71,115		70.9	24.10				105,240	752	869
	5.2			0.40	78,227	31,300			
85,338		76.1	23.70				136,540	856	989
	5.8			0.36	92,450	33,270			
99,561		81.9	23.34				169,810	955	1,103
	6.1			0.31	106,673	33,080			
113,784		88.0	23.03				202,890	1,043	1,205
	6.0			0.27	120,896	32,640			
128,007		94.0	22.76				235,530	1,124	1,298
	6.0			0.24	135,119	32,420			
142,230		100.0	22.52				267,950	1,199	1,385
	6.0			0.22	149,342	32,850			
156,453		106.0	22.30				300,800	1,270	1,467
	6.0			0.20	163,565	32,700			
170,676		112.0	22.10				333,500	1,338	1,545

The kinetic energy of one pound of water moving at velocity v feet per second is $12\,\frac{v^2}{2\,g} = \left(\frac{v}{2.318}\right)^2$ in-lb. Each item in Col. 9, Table 3, gives the initial velocity required to develop kinetic energy equal to the energy of elastic compression at the pressure indicated in Col. 1. This is found by multiplying the constant 2.318 by the square root of the corresponding item in Col. 8. In other words, any pressure value in Col. 1 is that generated when water moving at the velocity given by the corresponding figure in Col. 9 strikes a rigid plane surface normal to its path. For example, if water is moving at 1,338 ft per sec, its impact on such a surface will develop a pressure of 170,676 lb per sq in. Under ordinary conditions, a velocity as high as 1,338 ft per sec could be acquired only in the case of flow into a converging space, according to principles previously explained. Thus, if the aforementioned rigid plane surface were in a position to block the end of such a converging space, the pressure of 170,676 lb per sq in. would be developed against its face. However, if the surface were not a true plane, but contained crevices, depressions, or intracrystalline cracks such as to cause small portions of the water to acquire velocities much higher than the general velocity of 1,338 ft per sec before being finally brought to rest, local pressures much higher than the general pressure of 170,676 lb per sq in. would be developed.

The law governing the foregoing process of pressure multiplication undoubtedly changes when the crevices become so small as to approach molecular dimensions. However, what has been given contains a suggestion as to the source of pressures known to be sufficient to penetrate or deform the hardest metals.

In the foregoing analysis, when the slug of water hits the rigid surface, a pressure wave of intensity p is generated at the contact surface and travels out into the liquid at the acoustic velocity

$$u = \sqrt{\frac{g\,e}{w}} \qquad \text{(5)}$$

the phenomenon being similar to the generation of a water-hammer wave in a pipe by the sudden closure of a valve. However, when the slug of water strikes a water surface normal to the direction of motion, two pressure waves travel out from the contact surface in opposite directions, each moving at the acoustic velocity. This action may be described in more detail by reference to Fig. 6, which shows a slug of water moving at velocity v and striking a water surface. Consider within the slug a rectangular prism of water of cross-sectional area $(dx)^2$, with its sides normal to the contact surface. The enlarged section of a portion of this prism in Fig. 6 indicates that the contact surface is being forced down with a velocity v_c. At an instant dt seconds after contact, the row of particles which were originally in surface $C'C'$ have moved down to CC, and the particles which were in $A'A'$ at the instant of contact have moved down to AA. A pressure wave front having the acoustic velocity u relative to the surrounding fluid has traveled upward from $C'C'$ to AA, and another has moved downward from $C'C'$ to BB. The shaded volume is under a pressure p and has a unit weight w and a coefficient of elasticity E. The volume corresponding to the unshaded area is still under its original pressure. Since the force required to retard or accelerate a mass of water is equal to the mass of water multiplied by the rate of change in velocity: Force on CC = (Mass of $A'A'C'C'$) the rate of reduction in its velocity = (Mass of $C'C'BB$) the rate of increase in its velocity; or

$$p\,(dx)^2 = \left(\frac{w}{g}\,u\,dt\right)(dx)^2\left(\frac{v - v_c}{dt}\right) = \left(\frac{w}{g}\,u\,dt\right)(dx)^2\left(\frac{v_c}{dt}\right) \quad \ldots\ldots(6)$$

Therefore, $v_c = 0.5\,v$.

In other words, the velocity of particles at the contact surface is one half the initial velocity of the slug of water; therefore, one fourth of the original kinetic energy of these moving water particles is still in the kinetic form, and three fourths has been converted into energy of elastic compression. Thus, if v_r and v_w are the initial velocities required to develop a given pressure p in the respective cases of impact against a rigid surface and against a water surface:

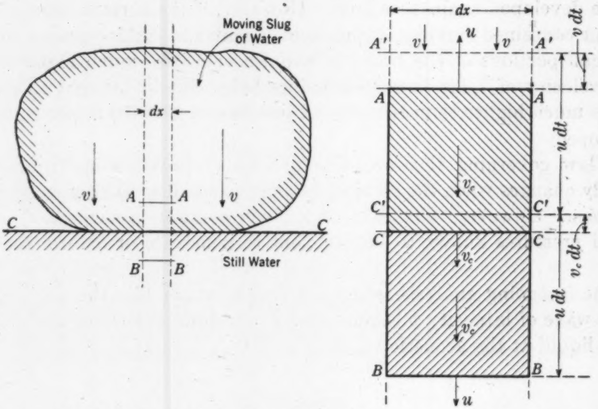

FIG. 6.—MOVING SLUG OF WATER STRIKING STILL WATER SURFACE NORMAL TO THE DIRECTION OF FLOW

$$\frac{3}{4}\,\frac{(v_w)^2}{2\,g} = \frac{v_r^2}{2\,g} \quad \ldots\ldots\ldots\ldots(7)$$

and from this it is seen that $v_w = v_r\sqrt{\frac{4}{3}} = 1.155\,v_r$. In other words, a velocity 15.5% higher is required to produce a given pressure if the impact is on a water surface than if it is on a rigid surface. Col. 10 of Table 3 is computed on the basis of this principle.

From the foregoing it is seen that when a cavity in water collapses, the pressure developed at the final point of contact may be very high if this point is in contact with a solid surface and partly surrounded by water, although not quite as high as if it were fully surrounded by solid materials.

The familiar effects of water hammering sufficiently intense to indent and disintegrate the surfaces of hard metals furnish abundant proof that pressures approaching or exceeding the higher values of Table 3 actually occur in engineering machines and structures.

In the case of metal surfaces the development of deep pitting is a slow process, usually requiring months of continuous operation. The long-continued intense hammering on tiny local spots overstresses the surface of the metal until fatigue cracks appear. The enormous pressure developed in such cracks, in accordance with the foregoing theory, then quickly enlarges and spreads them. In the case of concrete surfaces slightly permeable to water, the penetration of enormous pressures into the pore spaces easily breaks the bond between the aggregate particles, so that extremely destructive effects can occur within comparatively few hours.

Description of Apparatus for Cavitation Tests

Cavitation may occur at any point in a hydraulic system when the pressure at that point is reduced to the vapor pressure of the liquid flowing in the system. This requirement for the cavitation phenomenon to occur, therefore, precludes the study of its possible occurrence in hydraulic structures by means of small undistorted models of the usual laboratory type in which air at ordinary barometric pressure and water at room temperature are used and relative headwater and tailwater depths are represented to scale. In such a model, the whole atmospheric pressure is effective on the free surfaces of the liquid used in the model and is transmitted throughout that liquid so that the absolute pressure in the regions of the model corresponding to the cavitation regions of the prototype is far greater than the vapor pressure of the liquid in the model. In order that the basic conditions which produce cavitation in certain regions of the prototype may be correctly represented in the model, it is necessary that the pressures in these regions of the model be reduced to the vapor pressure of the liquid used in the model.

In planning the cavitation tests of the conduits at Tygart River and Madden dams, modification of the vapor pressure of the liquid by thermal or chemical means was eliminated from consideration because of its inconvenience from the standpoint of laboratory practice. Therefore, two possible means of simulating the basic conditions which produce cavitation in the critical regions of the prototype presented themselves—namely: (1) The design of a testing apparatus in which absolute pressure could be reduced in such a way as to simulate reduction of the atmospheric pressure on the headwater and tailwater surfaces in the model; and (2) the design of a testing apparatus such that, by distorting the headwater pressure and the discharge scale, the correct cavitation potentiality would be produced in the critical area of the conduit. The first of these testing facilities was used exclusively in the earlier tests, but for the later tests the second was also used, particularly to facilitate tests involving the admission of air into the conduits to alleviate or cushion the shock of cavitation. For the purpose of this paper these two testing devices are designated "enclosed-tank cavitation apparatus" and "diverging-tube cavitation apparatus," respectively.

Enclosed-Tank Cavitation Apparatus.—A general understanding of the design of the enclosed-tank cavitation apparatus may be had from Fig. 7.

In testing models in this apparatus, the absolute pressures on the surfaces corresponding to the headwater and tailwater surfaces in the prototype had to be reduced in almost direct proportion to the model scale. Consequently, the entire fluid medium had to be subjected to subatmospheric pressure, thus necessitating that the apparatus constitute an enclosed system in which the water supply was continuously circulated. As indicated by Fig. 7, the apparatus consisted essentially of a head tank (*A*), 32 in. in diameter and flattened on the top at the outlet end (*B*), where it was welded to a rectangular section (*C*) containing the model, the outlet end of the rectangular section in turn being connected by an 8-in. pipe line through a centrifugal pump (*D*) and control valve (*E*) to the inlet end of the head tank. An air trap (*F*) was placed in the top of the head tank and connected to a vacuum pump (*G*) which supplied the necessary reduction in pressure to the system of the testing apparatus. An opening was placed in the top of the rectangular section of the apparatus to facilitate the insertion and removal of models. A one-inch plate of annealed glass (*H*) served as a cover for the opening and also as an observation window to facilitate study of flow conditions and the action of cavitation in the model. The glass plate was protected against direct contact with the concrete of the model by a rubber gasket and was sealed against leakage by an application of sculptor's plastic waterproof clay between the edge of the glass plate and the edge of the opening in the tank. For tests involving measure-

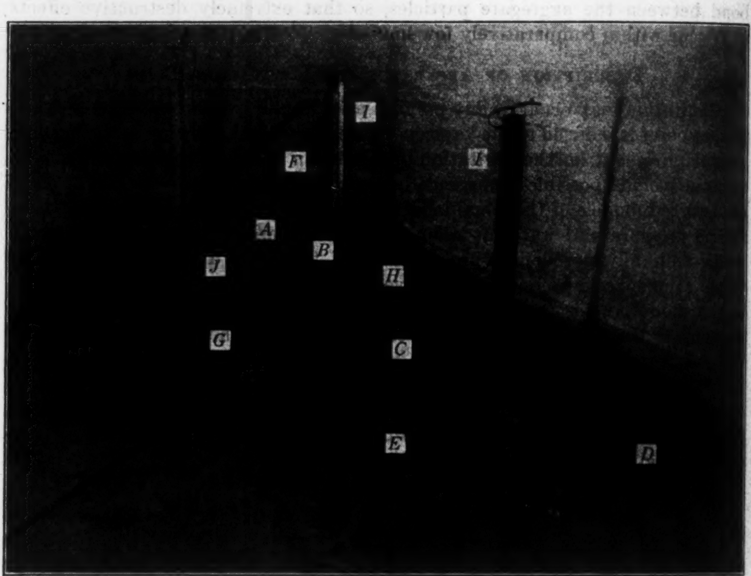

FIG. 7.—ENCLOSED-TANK CAVITATION APPARATUS

ments of the pressure distribution over a longitudinal section of the conduit, the annealed plate glass cover was replaced by a brass plate containing a network of piezometer openings. Mercury gages (*I*) of the cistern manometer type were provided at each end of the model to determine pressures at those points (corresponding to points *H′* and *R′* of Fig. 8). Baffles were provided

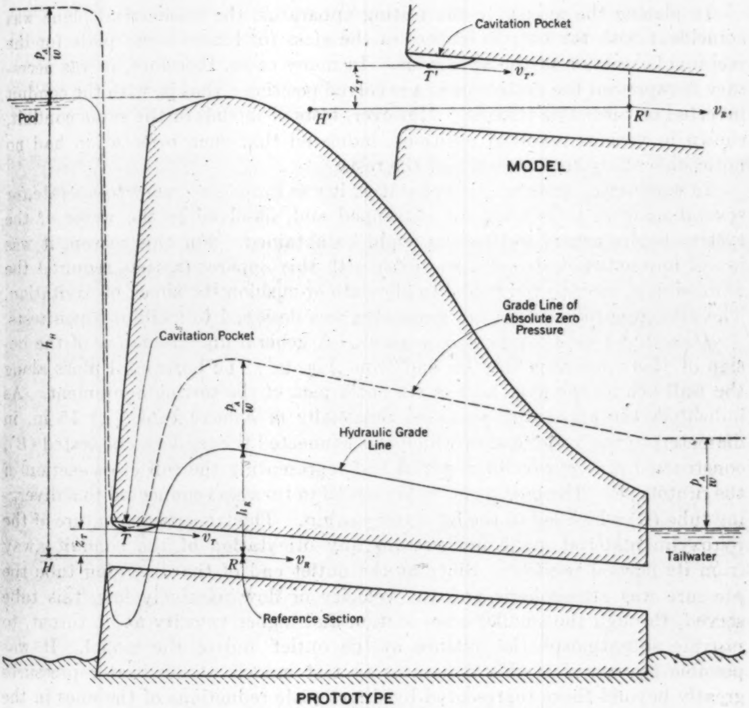
Cavitation Pocket
T'
$v_{T'}$
$z_{T'}$
H'
R'
$v_{R'}$
MODEL
Pool
$\frac{p_a}{w}$
h_H
Cavitation Pocket
Grade Line of Absolute Zero Pressure
$\frac{p_a}{w}$
Hydraulic Grade Line
h_R
$\frac{p_a}{w}$
z_T
T
v_T
H
R
v_R
Reference Section
Tailwater
PROTOTYPE

FIG. 8.—TYPICAL DAM SECTION SHOWING PRINCIPLES OF MODEL OPERATION TO SIMULATE CAVITATION

at the inlet end of the head tank to eliminate excessive turbulence in the water and to provide good approach conditions to the model. Provision for measuring the discharge through the model was made by a pitot tube (*J*) in the 8-in. pipe line, the pitot-tube installation having been calibrated by traversing the cross section of the pipe.

The models tested in this apparatus were generally constructed of concrete, with such appurtenances as stop-log slots and guides constructed of wood or sheet metal. In tests to study pitting, inserts of a specially designed concrete of low strength (see section "Experiments to Find a Material to Simulate Pitting") were cast in the critical regions of the model.

To facilitate observation and study, it was necessary with this testing apparatus to construct and test models representing only one half of the prototype cross section; that is, the prototype was represented in the model as halved by a diametrical plane, the inclination of the plane being chosen to represent the prototype to the best advantage for the purpose of the particular test.

In placing the models in the testing apparatus, the diametrical plane was coincident with the bottom surface of the glass (or brass) cover plate for the rectangular section of the apparatus. In many cases, therefore, it was necessary to represent the prototype in a revolved position—that is, with the conduit inverted or turned on its side. However, tests of models of the same conduit, shown in various revolved positions, indicated that such orientation had no noticeable effect on the results of the tests.

In conducting tests in this apparatus, it was found necessary to operate for several minutes to exhaust air entrapped and dissolved in the water of the system before actual test results could be obtained. For this reason, it was found impracticable to conduct tests, with this apparatus, that required the admission of air into the model to alleviate or cushion the shock of cavitation. The diverging-tube cavitation apparatus was designed to facilitate such tests.

Diverging-Tube Cavitation Apparatus.—A general understanding of the design of this apparatus may be had from Fig. 9. The horizontal pipes along the wall behind the gage boards are not a part of the testing equipment. As indicated, the apparatus consisted essentially of a head tank (*A*) 15 in. in diameter, to the outlet end of which was connected the model to be tested (*B*), constructed of a pyroxylin material and representing the full cross section of the prototype. The outlet end of the model in turn was connected to a diverging tube (*C*) which led to the laboratory sump. The transparent nature of the pyroxylin material made unnecessary any orientation of the conduit away from its normal position. Since at the outlet end of the diverging tube the pressure was atmospheric and the velocity of flow relatively low, this tube served, through the smaller cross section and higher velocity at its throat, to provide sub-atmospheric pressure at the outlet end of the model. It was possible by means of this apparatus to test models at headwater pressures greatly beyond those represented by direct scale reductions of the ones in the prototype, with the result that velocities were correspondingly exaggerated. Such exaggeration of the head and velocity scales was necessary in order that absolute pressures in the critical regions of the conduit be in correct relation to the absolute pressures which would exist in the prototype. Provision for measuring the pressures at each end of the model (corresponding to points *H'* and *R'* of Fig. 8) was made by mercury gages (*D*) of the U-tube type.

Connection of the gage to the lower end of the model was by means of a piezometer opening on each of the four sides of the conduit, the individual openings being connected through a common header to a single gage. An air cushion consisting of a glass jar (*E*) was provided in the line connecting the piezometer opening in the head tank with the mercury gage for determining the pressure at that point. Suitable baffles were provided in the head tank to eliminate excessive turbulence and to provide smooth approach conditions to the model. The rate of discharge of water through the model was measured by a venturi meter in the pipe line leading from the pump.

In operating this testing apparatus, water was drawn from the laboratory sump and returned to it, and therefore the elimination of entrapped and dissolved air in the system was greatly simplified. It was possible, thus, to study the effect on cavitation of venting the conduits without encountering the difficulty of having the air that was admitted to the model reappear in the head tank. To facilitate the tests of venting the conduits, provision was made for introducing air into the model at the inlet end of the conduit (*F*) and also immediately below both the emergency (*G*) and service (*H*) slide-gate slots However, since the emergency slide gates would be operated only on rare occasions, only the service slide gate was simulated in the model; but slots for both were included. The rate of flow of air into the model during venting tests was measured by means of circular, round-edged, brass orifices (*I*) provided with U-tube water manometers (*J*) to measure the head loss. The valve (*K*) in the upper end of the diverging tube served as a bleeder to adjust the pressure at the lower end of the model.

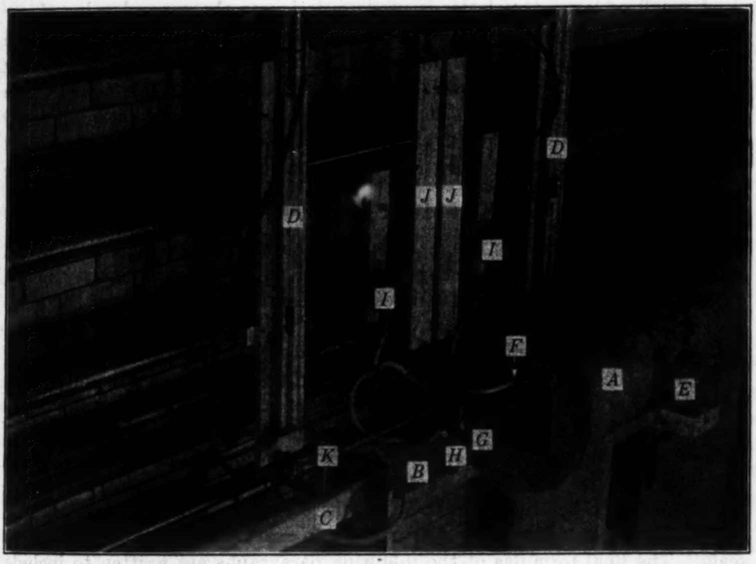

FIG. 9.—DIVERGING-TUBE CAVITATION APPARATUS

NOTATION

The notation adopted in this paper conforms essentially with American Standards Symbols for Hydraulics[8] compiled by the American Standards Association with Society representation, and approved by the Association in 1929.

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[8] ASA—Z10b—1929.

Frequent mention will be made of three points whose locations in the prototype are indicated in Fig. 8. The point R is at the center of a reference cross section of the conduit, H is in the quiet water of the headwater reservoir at the same elevation as R, and T is on the streamline portion of the boundary of any given cavitation pocket. The points in the model corresponding geometrically to R and T are designated as R' and T', whereas a point in the quiet water of the headwater tank at the same elevation as R' is designated as H'. The elevation of T above R is designated as z_T and the elevation of T' above R' is designated as z_T'. It is to be noted that if the orientation of the model with respect to the direction of gravity does not duplicate that of the prototype, the ratio of z_T' to z_T will not necessarily be equal to the linear scale L used in constructing the model.

A small or lower-case letter is used to designate the general value of any given quantity pertaining to the prototype structure or water, and the same letter primed is used to denote the general value of the corresponding quantity pertaining to a model of this structure or the liquid used in this model. The same capital letter is used to denote the corresponding scale ratio. For example: If l designates the length of any line in the prototype, then l' designates the length of the corresponding line in the model, and L denotes the linear scale ratio $\frac{l'}{l}$. The value of a quantity at any particular point in the prototype or model is designated by using the letter denoting that point as a subscript on the lower-case letter denoting the general value of that quantity in the prototype. For example: p_R' denotes the pressure at the center of the reference section in the model conduit.

All models described herein are geometrically similar to the prototype, in so far as their solid parts are concerned, and therefore have a definite linear scale L. However, "gravitational distortion of scale" occurs when the orientation of the model with respect to the direction of gravity does not duplicate that of the prototype, and "operational distortion of scale" occurs when the absolute pressures due to headwater and tailwater are not reduced in exact accordance with the linear scale L, or where the liquid in the model is not chilled to reduce its absolute vapor pressure in exact accordance with the same scale. The final formulas of the following paragraphs are written to include cases where these distortions are used.

In accordance with Torricelli's theorem, the velocity of flow in the prototype at point T is given by

$$v_T = c_T \sqrt{2\,g\left(\frac{p_H}{w} - z_T - \frac{p_T}{w}\right)} \ldots\ldots\ldots\ldots (8a)$$

and the velocity at point T' in the model is given by

$$v_T' = c_T' \sqrt{2\,g\left(\frac{p_H'}{w} - z_T' - \frac{p_T'}{w}\right)} \ldots\ldots\ldots\ldots (8b)$$

in which: v_T = the velocity of a streamline filament at point T; c_T = coefficient of velocity for flow from point H to point T written to give the filament velocity

v_T; and z_T = the elevation of point T above point R. At first it will be assumed: (*a*) That the model is built to a definite linear scale L; (*b*) that it is set in the apparatus without gravitational distortion; (*c*) that the headwater and tailwater depths are adjusted to the linear scale; (*d*) that the water in the model is chilled until its vapor pressure is reduced in the same scale; and (*e*) that the atmospheric pressure on the headwater and tailwater surfaces is reduced in the same scale. If these conditions are satisfied, Eq. 8*b* can be written in the form,

$$V\,v_T = c_T'\sqrt{2\,g\left(\frac{L\,p_H}{w} - L\,z_T - \frac{L\,p_T}{w}\right)} \qquad \text{(8c)}$$

in which: V = scale ratio, for velocity $= \frac{v'}{v}$; L = scale ratio for length $= \frac{l'}{l}$; p_T = absolute pressure at point T, or vapor pressure of water at the given temperature; and p_H = absolute pressure at point H, given by

$$p_H = w\,h_H + p_A$$

in which: h_H = head of water above point H, p_A = absolute pressure of air on the free surfaces of the liquid, and w = unit weight of the liquid. If it is assumed that the roughness of the model surface has been adjusted until $c_T' = c_T$ and that the values of g and w are alike in the model and prototype, the expression, $V = \sqrt{L}$ (known as Froude's law) may be obtained by dividing the two members of Eq. 8*c* by the corresponding members of Eq. 8*a*. This result means that if the model is operated according to Froude's law (that is, with all the aforementioned adjustments) the velocity at any point in the model will become equal to that of the corresponding point in the prototype multiplied by the square root of the linear scale, and the absolute pressure at the point T' will become equal to the vapor pressure of the water used in the model. If this is true for all points in the model corresponding to points on the boundary of the cavitation pocket in the prototype, a cavitation pocket must open up in the model, and will correspond exactly to the prototype pocket in shape, size, and location.

In the foregoing it was assumed that the coefficient of velocity c_T' for flow from H' to T' could be considered equal to c_T. For pockets near the conduit entrance where the friction losses are small in comparison with the total head, this assumption is not likely to be appreciably in error. For pockets farther from the inlet to the conduit, where the friction losses are larger, the problem of adjusting the roughness of the model surfaces to make c_T' equal to c_T presents difficulties if a high-precision solution is desired. Similar difficulties are encountered in all hydraulic model systems where the effects of gravity and viscosity are both important, and therefore require the satisfaction of the two mutually conflicting laws of Froude and Reynolds. Fortunately, a highly precise duplication of this coefficient is not required in typical cavitation studies, since a design which approaches even near the cavitation limit may be classed as unsatisfactory.

In the aforementioned method of model operation in accordance with Froude's law, the absolute pressure in the model not only at T' but at every other point in the liquid is equal to the absolute pressure at the corresponding point in the prototype multiplied by the linear scale ratio L. In spite of the simplicity of this method of operation, numerous occasions arise for deviating from it in conducting miscellaneous cavitation studies by means of models. These occasions are usually created by the limitations of the available apparatus. Because of the location of an observation window, a conduit of rectangular cross section in the prototype may have to be inverted or laid on its side in the model; or because of the inconvenience of chilling the water in the model to reduce its vapor pressure, a slight variation from the linear-scale representation of the atmospheric pressure may be made; or the suction apparatus may not develop sufficiently low pressure to permit undistorted scale testing. When such departures are made from operation in accordance with a single linear scale for all dimensions and pressure heads, the absolute pressures are no longer to scale over the entire region occupied by the liquid in the model. This means that a group of several cavitation pockets in the model cannot be studied simultaneously. Attention must be focused on a limited region in which the occurrence of a cavitation pocket is known or suspected; and the general pressure and flow conditions of the model must be adjusted to produce the correct cavitation potentiality in that limited region. A readjustment of the general conditions must be made when a pocket in another location is studied. The equations for making these adjustments are given subsequently.

Referring to Fig. 8, it is assumed that the "reference section" R in the prototype corresponds to the location of an accurate mercury piezometer in the model. In most of the cavitation tests at Pittsburgh, the models did not represent the entire conduit, but only the upstream portion, and the piezometer was placed near the downstream end of the model. In the application of these formulas it is assumed that, for a given water elevation in the reservoir, either the mean velocity V_R or the pressure head h_R at the prototype reference section is known by computation or by actual experiments on the prototype. For reasons already mentioned, a direct determination of both of these quantities by model studies is not feasible, because the effects of gravity and viscosity are both involved to an important degree.

Referring again to Fig. 8, the mean velocity at the prototype reference section may be expressed by:

$$v_R = c_R \sqrt{2\,g\left(\frac{p_H}{w} - \frac{p_R}{w}\right)} \qquad (9a)$$

and the corresponding mean velocity in the model by:

$$v_R' = c_R' \sqrt{2\,g\left(\frac{p_H'}{w} - \frac{p_R'}{w}\right)} \qquad (9b)$$

in which: v_R = mean velocity in a conduit cross section at point R; c_R = coefficient of velocity for flow from point H to point R, written to give the mean velocity v_R; and p_R = absolute pressure at point R, equals

$$p_R = w\,h_R + p_A$$

in which: h_R = pressure head at point R. By assuming that the model roughness is adjusted so that c_R' and c_R are equal and eliminating them between these equations, considering again that g and w are alike in model and prototype, there is obtained:

$$\frac{v_R'}{v_R} = \sqrt{\frac{p_H' - p_R'}{p_H - p_R}} \qquad \text{(9c)}$$

If m represents the ratio of v_T to v_R, Eqs. 8a and 8b may be expressed in the form:

$$m\,v_R = c_T\sqrt{2\,g\left(\frac{p_H}{w} - z_T - \frac{p_T}{w}\right)} \qquad \text{(10a)}$$

and

$$m'\,v_R' = c_T'\sqrt{2\,g\left(\frac{p_H'}{w} - z_T' - \frac{p_T'}{w}\right)} \qquad \text{(10b)}$$

Considering that m and m' must be identical if the flow patterns for prototype and model are to be geometrically similar, and assuming as before that c_T' and c_T are made identical by proper adjustment of the model roughness, and eliminating these quantities from Eqs. 9 and 10, there results:

$$\frac{v_R'}{v_R} = \sqrt{\frac{p_H' - w\,z_T' - p_T'}{p_H - w\,z_T - p_T}} \qquad \text{(11)}$$

Eqs. 9c and 11 may be combined in various ways to determine the proper methods of model operation to develop a given pressure p_T' at any given location in a model, such as point T' in Fig. 8. If this value of p_T' is the vapor pressure corresponding to the temperature of the water in the model, and if point T is on the boundary of a cavitation pocket in the prototype, then point T' must also be on the boundary of a corresponding cavitation pocket in the model.

Suppose that the values of p_H, p_R, z_T, and p_T are known for the prototype, that cavitation in the model is to be investigated in a limited region for which z_T' is known, that the vapor pressure p_T' for the liquid of the model is known, and that by means of the vacuum pump or other suction device a known low absolute pressure p_R' can be maintained at the reference section in the model. To operate the model properly, it would be required to determine the absolute pressure p_H' which must be carried in the model headwater tank at the point H'. The desired value may be obtained by eliminating $\frac{v_R'}{v_R}$ in Eqs. 9c and 11, resulting in the following equation:

$$p_H' = \frac{p_R'\,(p_H - w\,z_T - p_T) - (w\,z_T' + p_T')\,(p_H - p_R)}{p_R - w\,z_T - p_T} \qquad \text{(12)}$$

If the model is correctly built and its surface roughness correctly adjusted, simultaneous setting of the two control piezometers at the values of p_H' and p_R' prescribed by Eq. 12 will automatically cause the velocity in the conduits to assume the value v_R', and the pressure at point T' to drop to the vapor pressure p_T', thereby causing the model cavitation pocket to appear in its proper form and location.

If it is desired to regulate the model by controlling the discharge with the aid of a valve and a venturi meter or other meter in the supply pipe, instead of regulating the pressure at point H' in the model head tank (the known quantities being the same as in the preceding paragraph except that v_R is given instead of p_H), the equation for determining the necessary model conduit velocity v_R' may be found by eliminating p_H' between Eqs. 9c and 11, resulting in the following:

$$v_R' = v_R \sqrt{\frac{p_R' - w\,z_T' - p_T'}{p_R - w\,z_T - p_T}} \qquad (13)$$

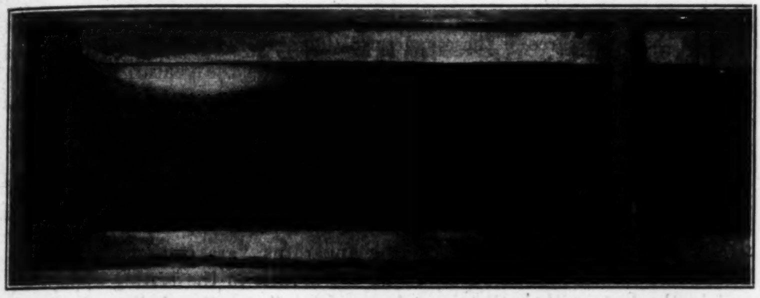
If the model is correctly built and adjusted for roughness, the setting of v_R' and p_R' at the values prescribed by Eq. 13 should give exactly the same flow conditions as those described in the preceding paragraph. Actually, however, due to natural and unavoidable errors in the meters and gages and to small imperfections in the model and its adjustment, minor discrepancies are likely to occur in the results of the two methods of operation. A perfect roughness adjustment is not usually feasible in actual laboratory practice, as the highest polish which can be produced on the available materials is insufficiently smooth.

In connection with the discussion of principles of model operation, mention may be made of the comparative advantages of the diverging-tube and enclosed-tank types of apparatus. The relative merits of the former are as follows: (1) It is much cheaper, provided that a good-sized pump or other source of fairly liberal flow under moderate head is already available; (2) its operation is much simpler, quicker, and more convenient, since it dispenses with the procedure for freeing the tank from entrained air; (3) it lends itself to studies of problems of cavitation mitigation by air admission; and (4) its higher throat velocity is advantageous in conducting actual pitting tests on models. The principal disadvantage of this apparatus is that feature which limits the conduit velocity to values greater than the spouting velocity of water at a head equivalent to atmospheric pressure. Unless a liberal flow of water is available, this requires that the models be quite small. Furthermore, unless an enclosed water storage tank is provided, this apparatus cannot be used advantageously in studies in which close control of the air content of the water is required. The principal feature of the enclosed-tank apparatus which justifies its use, in spite of its greater cost and lesser convenience, is that it permits the use of much lower conduit velocities. For a pump of given discharge capacity, a lowering of conduit velocity results in an increase in the model conduit cross section, thus allowing the use of larger models.

A third type of apparatus could also be built, in which the necessary degree of vacuum would be obtained by a vertical non-flaring draft tube about 34 ft high, but this was not feasible in the laboratory.

Computation of Gage Settings for Cavitation Tests on a Model Conduit

Models of various conduit entrances on several different scales have been built and tested at the laboratory in Pittsburgh in either the enclosed-tank apparatus or the diverging-tube apparatus. The first step in making such a



(*a*) Model Scale, 1 : 15, Enclosed-Tank Apparatus

(*b*) Model Scale, 1 : 20, Enclosed-Tank Apparatus

(*c*) Model Scale, 1 : 40, Diverging Tube Apparatus

FIG. 10.—ORIGINAL CONDUIT ENTRANCE, MADDEN DAM, OPERATING AT A 168-FT HEAD

test is to compute the necessary settings of the control gages. In each apparatus there are three of these gages: (1) The headwater pool gage, (2) the conduit reference-section gage, and (3) the gage on the venturi meter, pitot tube, or other flow meter in the supply pipe. If the prototype quantities p_H, p_R, and v_R are known, the setting for any one of these gages may be assumed, and the settings for the other two computed from them by Eqs. 11 and 12. However, the assumption must be such as to make all three settings fall within the capacity of the apparatus. The various models tested were geometrically similar to the prototype in their solid portions. However, in practically all the tests in the enclosed-tank apparatus, "gravitational distortion" was introduced by tipping or inverting the models in the testing apparatus; and in the tests with both types of apparatus "operational distortion" was introduced by omitting to chill the liquid until its vapor tension was reduced in proportion to the linear scale, or by the arbitrary choice of one of the three aforementioned gage settings. Under such conditions, as previously explained, the pressure scale would not be constant at all points in the model liquid, and the same is true of the velocity scale. The adjustment of gage settings was made to give the correct pressure condition only in a small local region containing the upstream boundary of a single known or suspected cavitation pocket.

The following example shows a typical computation for testing a 1 : 20 scale model of a pair of conduits of the Madden Dam in the enclosed-tank apparatus. Distortion of both of these types is used. A photograph of this model is reproduced in Fig. 10(*b*). The model conduits are split along the horizontal plane passing through their axes, the upper halves of the conduits being reproduced in the model and tested in the inverted position, and the top of the prototype conduit being represented in the bottom of the model. The purpose of the inversion is to make visible the large cavitation pocket known to form against the top of the conduit just inside the entrance. The computation establishes the gage settings necessary to develop the pressure corresponding to the vapor pressure of the water in this critical region of the model. The dimensions of the model were chosen to utilize most of the capacity of the pump when operating under the condition corresponding to full head on the conduits. In this example a pump discharge of 2.343 cu ft per sec is assumed. This fixes the necessary setting of the flow-meter gage, leaving the readings of the other two gages to be computed. The prototype dimensions required in the computation are shown in Fig. 11; the headwater is assumed at El. 204.7. This value of headwater elevation is used in the example since it represents the greatest head under which the conduit discharge has been verified by actual discharge measurements on the prototype. While conducting the tests on the Madden models, the feasibility of including certain refinements in the settings of the apparatus did not become apparent until near the close of the work. For this reason the computation procedure in the example is not exactly the same as that used in the actual tests.

The assumptions used in this example may be listed as follows:

Applying in General.—The headwater elevation is 204.7 ft and the tailwater elevation is at a level below the bottom of the conduit; water temperatures, in degrees Fahrenheit, are, for the prototype, 77, and for the model, 68; and the barometric pressures, in inches of mercury, are, for the prototype, 29.500, and in the laboratory at the time of the model tests, 29.655. During the test the discharge setting of the pump is 2.343 cu ft per sec; and the rating equation for the prototype conduit (based on pitot-tube tests) is

$$v_R = 7.06 \sqrt{h_H} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The model setting was such as to produce the correct cavitation potentiality at the top of the conduit, just inside the entrance.

Applying to the Prototype.—The total head on the conduit, h_H, is 110.0 ft (see Fig. 11); and the corresponding velocity v_R at the conduit reference section is $7.06 \sqrt{110.0} = 74.0$ ft per sec. Since the water temperature is 77° F, the vapor pressure p_T is 65 lb per sq ft; and, for a barometric pressure of 29.500 in. Hg, the atmospheric pressure p_A equals $\frac{29.500}{12}(13.59 \times 62.4) = 2{,}085$ lb per sq ft. Since the elevation of point T, Fig. 11, is 100.00, the value of $z_T = 100 - 94.7 = 5.3$ ft. The absolute pressure at point H is $p_H = w\,h_H + p_A = 62.4 \times 110.0 + 2{,}085 = 8{,}949$ lb per sq ft.

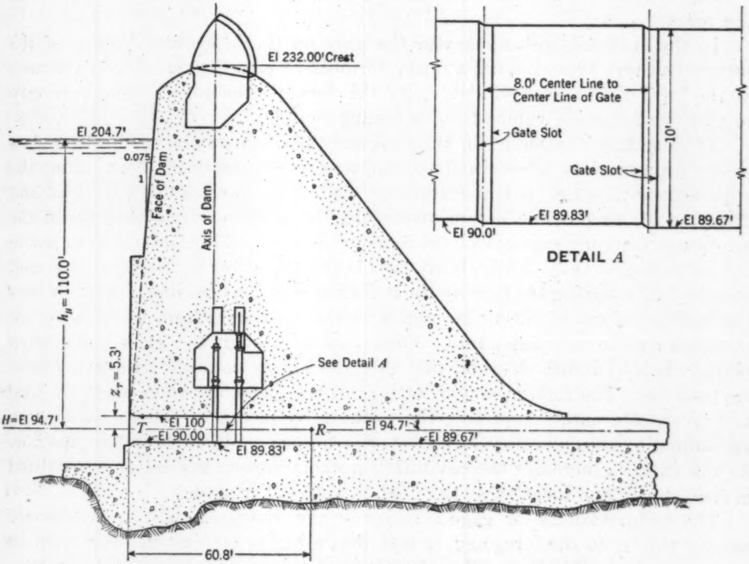

El 232.00' Crest
El 204.7'
0.075
1
Face of Dam
Axis of Dam
$h_H = 110.0'$
$z_T = 5.3'$
H=El 94.7'
T
El 100
El 90.00
El 89.83'
R
El 94.7'
El 89.67'
See Detail A
60.8'
8.0' Center Line to Center Line of Gate
Gate Slot
Gate Slot
10'
El 89.83'
El 90.0'
El 89.67'
DETAIL A

FIG. 11.—SECTION OF MADDEN DAM SPILLWAY AND CONDUIT

Applying to the Model.—The linear scale ratio $L = \frac{l'}{l} = \frac{1}{20}$. With the axis of the conduit horizontal, the model is inverted in the apparatus; therefore, $z_T' = \frac{-5.3}{20} = -0.26$ ft. For a water temperature of 68° F, $p_T' = 48.8$ lb per sq ft; and corresponding to a barometric reading of 29.655 in. Hg, p_A' = 2,096 lb per sq ft. With the cross-sectional area of the two half conduits equal to 0.1417 sq ft, the velocity $v_R' = \frac{2.343}{0.1417} = 16.53$ ft per sec. Finally, the absolute pressure required in the headwater tank at point H' (at the level of the glass window) may be obtained from Eq. 11, thus:

$$\left(\frac{16.53}{74.0}\right)^2 = \frac{p_H' - [62.4 \times (-0.26)] - 48.8}{8{,}949 - (62.4 \times 5.3) - 65}$$

from which $p_H' = 459$ lb per sq ft. This corresponds to a vacuum of 2,096 − 459 = 1,639 lb per sq ft, or 11.35 lb per sq in. less than the atmosphere of the room.

In the enclosed-tank apparatus the gage on the headwater tank is of the mercury-cistern type, having a rating formula to give the pressure in the tank at the level of the glass window. By this formula the gage setting necessary to give the foregoing value of p_H' is readily found.

After setting the model in the enclosed-tank apparatus, sealing the glass observation plate (window) in place with waterproof plastic clay, and filling the apparatus with water, it is necessary to remove as much of the entrained and dissolved air as possible before attempting to make accurate settings of the headwater mercury gage and of the flow-meter gage. This is done by applying the vacuum gradually, while slowly circulating the water in the apparatus, and occasionally stopping the flow by the main valve in the pipe line. This permits the bubbles of entrained air to collect in the air dome, from which they are removed by the vacuum pump. Observation through the glass observation plate (window) indicates when the water no longer contains visible traces of air bubbles. The final accurate settings of the gages are then made by trial and error, alternately adjusting the vacuum by means of the vacuum pump and adjusting the conduit velocity by means of the main valve in the pipe line. As the vacuum increases the cavitation pockets appear and become stabilized in size, shape, and location as the final adjustment is made.

The computations for gage settings in the diverging-tube apparatus are exactly similar to the foregoing, except that a higher conduit velocity must be assumed. In a diverging tube of infinite divergence and negligible friction, the ideal throat velocity required to raise the pressure from a vapor pressure of one foot of water at the throat to an atmospheric pressure of 14.7 lb per sq in., or 33.9 ft of water, at the exit would be $\sqrt{2\,(32.2)\,(33.9-1.0)}$ or 46.1 ft per sec. Making some allowance for friction and exit losses requires a velocity somewhat higher than this—say about 50 ft per sec—in the model conduit, since the latter is used as the venturi throat.

In many cavitation tests it is not necessary to compute the gage setting with as much refinement as that used in the foregoing illustration. For example, in the enclosed-tank apparatus it is possible to produce so low a vacuum at the conduit reference section as to cause long cavitation streamers to open up in the body of the water, usually along the axis of the conduit and in contact with the glass window. Under such conditions, the cavitation potentialities simulated at all places along the model conduit surfaces are far more severe than those of the prototype. If a design shows no cavitation pockets in contact with the conduit surfaces under such operation, it can be classed as excellent from the cavitation standpoint. In the diverging-tube apparatus, a similar exaggeration of the severity of the prototype cavitation conditions can be secured by increasing the velocity in the model conduit.

BEHAVIOR OF ORIGINAL MADDEN DAM CONDUIT ENTRANCES IN MODELS

Those dimensions of the original entrances of the Madden conduits which are significant from a hydraulic standpoint are shown in Figs. 1 and 11. Fig. 11 also indicates at point *T* the approximate location of the pitting in the prototype. As noted in Fig. 1, the conduit entrances are not symmetrical about a horizontal plane through the conduit axes, but the bottoms of the stop-log recesses terminate in a horizontal step or shelf 18 in. below the level of the conduit bottom. This shelf suppresses the contraction of the jet to such an extent that no pitting on the bottom occurs in the prototype or models, and no visible cavitation pockets appear in contact with the bottom in the models.

Fig. 10(*a*) shows a 1 : 15 scale model of one of the original Madden conduits, split along a vertical diametrical plane. Fig. 10(*b*) shows a 1 : 20 model of a pair of the same conduits split along a horizontal diametrical plane, the model being inverted so that the portion reproducing the top of the conduit is seen through the glass window. Fig. 10(*c*) shows a 1 : 40 scale transparent model of one of the original Madden conduits. Figs. 10(*a*) and 10(*b*) show the first two models being tested in the enclosed-tank apparatus and the third (Fig. 10(*c*)) in the diverging tank apparatus under pressures equivalent to a 168-ft head in the prototype. A photograph showing pitting in one conduit of the twin model of Fig. 10(*b*) is reproduced in Fig. 12. In all these views the large cavitation pocket against the top of the conduit just inside the entrance is very conspicuous. No other cavitation pockets are present anywhere in the entrance region, except the faint streamers just visible near the conduit bottom in Fig. 10(*a*). These streamers do not come in contact with any solid material, and therefore do not cause pitting. In a test of a 1 : 15 scale model of one of the Madden conduits after installation of the bellmouth, the size of the cavitation pocket was seen to be nearly as large as in the original conduits.

One of the most puzzling problems encountered during the conduct of the tests described herein was to find a reason for the fact that in the original Madden conduits, and in those models of these conduits which were actually subjected to pitting tests, deep erosion of the side-walls of the conduits occurred at places where absolutely no cavitation could be observed visually in the uneroded models. Contours of the side-wall erosion in Madden Conduit 5 after the first season's operation are shown in Fig. 3, whereas the photographs reproduced in Fig. 10 indicate the area covered by the aforementioned large cavitation pocket. The edges of this cavitation pocket are in contact with the side-walls of the conduit near the top, but, as noted in Fig. 3, the side-wall erosion not only occurs in the area touched by this pocket, but also extends a considerable distance downstream and downward toward the bottom of the conduit. The point of deepest side-wall erosion is entirely outside the area touched by the cavitation pocket in Fig. 10(*a*).

The explanation of this phenomenon is as follows: There is an outstanding difference between the process that controls the growth of the pit eroded in the top of the conduit and that which controls the growth of the pits eroded in the two side-walls. However, the excavation of all three of these pits is started by the action in the same large cavitation pocket. In the case of the pit in the top of the conduit, the enlargement and deepening of the eroded region provides a stilling pool which helps to absorb the energy of the rapidly moving water filaments and to cause the vapor cavities to collapse in water rather than in contact with solid material. Eventually, this pit tends to attain a certain stability in dimensions. The action is closely analogous to the erosion of a pit in the bed of a river below a dam whose apron is not specifically designed to prevent such erosion. The scouring out of such a pit is very rapid until it becomes large enough to stabilize the hydraulic jump and form a natural stilling pool where the energy of the spillway jet is dissipated. As this condition is attained, the growth of the excavation becomes slower and finally ceases.

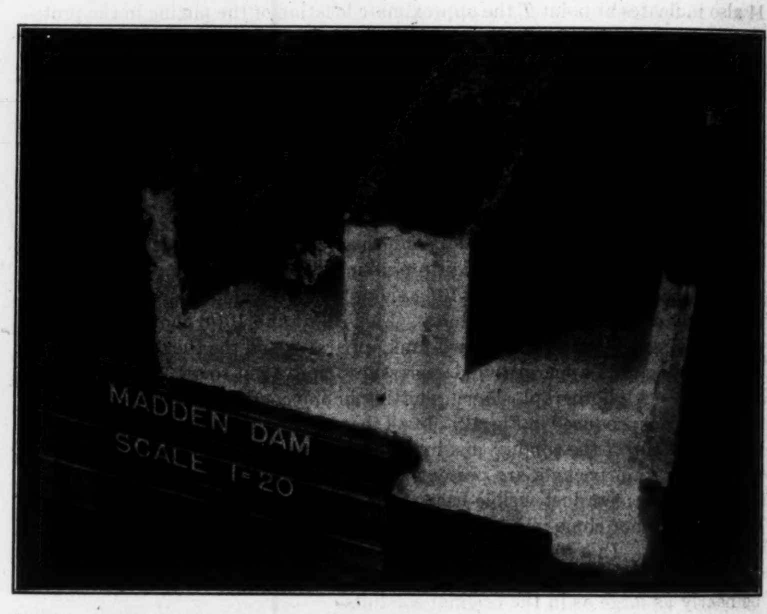


Fig. 12.—Original Conduit Entrance, Madden Dam, Showing the Pitting in a 1 : 20 Scale Model After One Hour of Operation (Prototype Velocity, One Conduit, 128 Ft per Sec)

In the case of the pits in the side-walls of the Madden conduits, the process of growth is radically different from the foregoing. As shown in Fig. 12, the pit first develops along the edge of the large cavitation pocket, where especially intense impacts are caused by collisions between the swiftly moving water of the streamline filaments and the broken masses of water and vapor hurled against these filaments from the interior of the pocket. As the pit develops, it becomes in itself a source of cavitation. As the swift water of the streamline filaments passes over the edge of the pit, vapor cavities form beneath or pass in from the main cavitation pocket, and the violent collapse of these vapor cavities enlarges the excavation. This growth of the pit in the downward direction, or toward the bottom of the conduit, causes it to eat in under the swift water of the streamline filaments and thereby tap new sources of destructive energy. Along this edge of the pocket there is no opportunity for the impact to become cushioned by enlargement of the excavation, but swift water is constantly fed in over the edge of the pit, inducing violent collapse of vapor cavities at a place in immediate contact with solid material. This action causes the excavation to grow rapidly in a direction toward the bottom of the conduit and downstream, and thus the pit eventually extends itself into a region far beyond the boundary of the original cavitation pocket.

The value of model studies of cavitation by visual observation of the occurrence of cavitation pockets is not destroyed by the fact that such observation does not furnish full information regarding the final location of the pitting. If any cavitation pockets are observed in contact with the surfaces of a model conduit, it is safe to assume that destructive effects will be induced in the prototype, although the exact nature of such effects may be uncertain.

Experiments to Find a Material to Simulate Pitting

In spite of theoretical difficulties, it was thought that there might be a possibility of developing, by empirical methods, some material that would pit at a rate determinable experimentally in terms of the intensity of cavitation. It would have been very convenient to make a model conduit of such a material, run it in the cavitation apparatus for a definite number of hours, remove it and measure the depth of pitting, if any, and then state that the same degree of pitting must occur at the same location in the prototype after a definite period of operation. A considerable amount of thought and work was expended in efforts to develop such a material, but these efforts were not rewarded with complete success.

The accurate measurements available of the depth and location of destructive pitting in the original conduit entrances of the Madden Dam furnished an excellent opportunity to check the pitting in a model against that of a prototype. Materials were found which (after a reasonable number of hours of operation in models of the original Madden conduits) would undergo pitting simulating that of the prototype quite accurately in depth and location. However, full success was not obtained in controlling the uniformity of manufacture of these materials to such a degree that they would always pit to the same depth in the same number of hours.

Portland cement concrete or mortar, of approximately the same strength as that of the prototype, is altogether too hard for use in pitting tests in the enclosed-tank apparatus. In a 1 : 15 scale model of a Madden conduit constructed of this material, the depth of pitting after more than a hundred hours of operation amounted to less than an eighth of an inch, whereas the desired depth of cutting to simulate the prototype pitting was more than an inch.

In order to obtain more rapid rates of pitting, various materials were tried in the model conduits, the best results being obtained with mixtures of plaster of Paris, portland cement, and sand. When a mixture of these materials was made sufficiently weak to pit to the required depth in two or three hours of operation, there was a general tendency for the surface to soften or dissolve and wash away. This action was more rapid in places where the velocity was high, and cutting from this cause could not be distinguished easily from that due to cavitation. This difficulty was overcome by painting the surface of the material with a grout of portland cement and water. After setting twenty-four hours or more this gave a brittle skin surface which afforded perfect protection against erosion by smoothly flowing water, but was easily disintegrated by the hammering effect of cavitation.

The following material when used in a 1 : 15 scale model of the original Madden conduits was found to reproduce the pitting approximately to scale during a test period of about two to four hours: 1 part portland cement, 12 parts plaster of Paris, and 16 parts sand, mixed with 7.23 cc of water per ounce of dry material. The wash used on the surface consisted of a portland cement grout containing 17 cc of water per ounce of cement. The pitting obtained by operating a model made of this material for one hour is shown in Fig. 12.

Although skin formed by the portland cement wash was quite successful in eliminating general erosion and localizing the pitting to the cavitation areas, much difficulty was found in duplicating the thickness and total strength of this skin in models built at different times. In fact, this was one of the principal difficulties that prevented the attainment of full success in securing sufficient uniformity in the manufacture of model conduits to justify the placing of complete reliance on the results of pitting tests on such models.

MODEL STUDIES OF VARIOUS METHODS PROPOSED TO ELIMINATE OR MINIMIZE CAVITATION DAMAGE IN THE CONDUIT ENTRANCES OF MADDEN DAM

In the course of the model studies leading toward a solution of the cavitation problem at Madden Dam, tests were made of various methods proposed to eliminate or minimize damage in the conduit entrances. These various methods or measures may logically be arranged in two general groups—namely: (1) Those which have as their purpose the provision of conduit boundaries conforming more nearly to the paths naturally sought by the boundary filaments of the flowing water, and (2) those which have as their purpose a general adjustment of pressure within the entire conduit or upstream part of it. Among the methods tested in the laboratory may be mentioned the following:

Group 1.—

(*a*) Reconstruction of the conduit entrance to provide more liberal curvature;

(*b*) Extension of the conduit entrance upstream from the face of the dam to provide a bellmouth entrance (with and without stop-log slot filler or cover); and

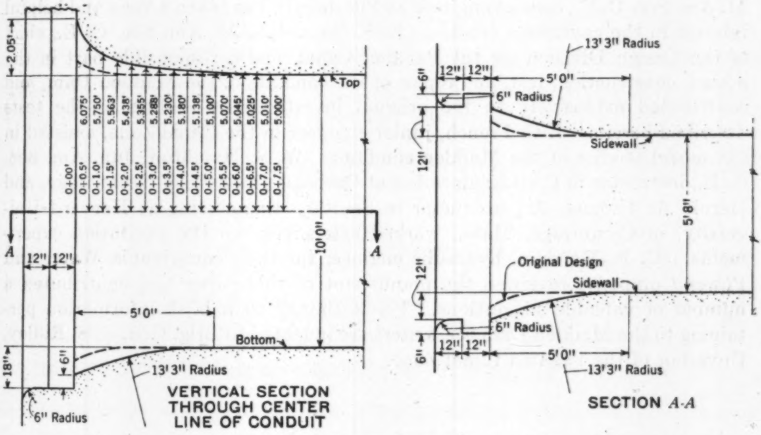
(*c*) Construction of a venturi throat in the critical region of the conduit.

Group 2.—

(*a*) Operation of the conduits at partial gate closure;
(*b*) Provision of a constriction or nozzle at the outlet end of the conduit;
(*c*) Venting of the conduit by admission of air at the stop-log slot; and
(*d*) Venting of the conduit by admission of air at the slide gates.

In addition to studies of these various methods proposed for solving the problem, tests also were made to indicate what effect debris accumulations on the trash racks would have in altering flow conditions at the entrances to the conduits, and what effect such altered flow conditions would have on cavitation within the entrances.

In the case of the Madden Dam, all of the measures of Group 2 were found to have the disadvantage of reducing the discharge capacity of the conduits, and were therefore considered applicable only as temporary expedients. In Group 1, under item (*a*), extensive tests on streamlining the conduit entrances resulted in the development of "Design No. 2," shown in Fig. 13. This was recommended for actual incorporation in all the units except Conduits 5 and 6. The model studies showed this design to be free from cavitation even under a head much greater than the maximum possible in the prototype. Under item (*b*) it was found that in Conduits 5 and 6, which were already provided with bellmouths extending upstream from the entrance, cavitation could be eliminated or minimized by providing the stop-log slot with a strong cover or filler. Under item (*c*), the venturi throat was found to have no advantage over "Design No. 2" to compensate for its greater cost. In the case of the Tygart River Dam, the models showed only faint or incipient flashes of cavitation at the conduit entrances. However, since the conduits were to be steel-lined and the entrance castings already had been fabricated and delivered, it was decided


VERTICAL SECTION THROUGH CENTER LINE OF CONDUIT

SECTION A-A

2.05'
Top
6.075'
5.750'
5.563'
5.438'
5.355'
5.288'
5.230'
5.180'
5.138'
5.100'
5.070'
5.045'
5.025'
5.010'
5.000'
0+00'
0+0.5'
0+1.0'
0+1.5'
0+2.0'
0+2.5'
0+3.0'
0+3.5'
0+4.0'
0+4.5'
0+5.0'
0+5.5'
0+6.0'
0+6.5'
0+7.0'
0+7.5'
A
12''
12''
10'0''
A
5'0''
Bottom
18''
6''
13'3'' Radius
6'' Radius
13'3'' Radius
6''
12''
12''
5'0''
6'' Radius
Sidewall
12''
5'8''
Original Design
12''
Sidewall
6'' Radius
6''
12''
12''
5'0''
13'3'' Radius

FIG. 13.—DESIGN NO. 2, MADDEN DAM CONDUIT ENTRANCE, ADOPTED FOR CONDUITS 1 TO 4

to complete the installation in accordance with the original design, but to observe carefully the effect of operation by regular inspection of the conduits, and to make such repairs as necessary if pitting of the entrance casting and lining is found to occur. The entrance bells of the Hiwassee, Bluestone, and Redbank conduits, designed with exceptionally liberal curvature in the light of information regarding the Madden difficulties and the model studies, proved to be entirely free from cavitation, but in the cases of the latter two of these three dams the models detected quite serious cavitation at gate slots in the preliminary designs, which was later eliminated through revision of these designs.

ACKNOWLEDGMENTS

Various engineers from the Panama Canal Office, the Pittsburgh and Huntington (W. Va.) District Offices of the United States Engineer Department, and the Hydraulic Research Laboratory of Carnegie Institute of Technology have made contributions toward solving the problems of controlling or eliminating cavitation in the structures which have been mentioned. W. E. R. Covell, M. Am. Soc. C. E., district engineer at Pittsburgh, has taken a keen and helpful interest in the cavitation studies. E. S. Randolph, M. Am. Soc. C. E., chief of the Design Division for the Panama Canal, had a responsible part in the design, construction, test, and repair of the conduits of the Madden Dam, and contributed extensively to the original investigations leading to the tests described herein. J. C. French, junior engineer in the Canal Zone, assisted in the model studies of the Madden conduits. W. S. Hamilton, Jun. Am. Soc. C. E., instructor in Civil Engineering at Carnegie Institute of Technology, and Harold A. Thomas, Jr., instructor in Sanitary Engineering at Harvard University, in Cambridge, Mass., worked extensively on the cavitation experiments. C. F. Merriam, hydraulic engineer for the Pennsylvania Water and Power Company, reviewed the manuscript of this paper and contributed a number of valuable suggestions. For authority to publish information pertaining to the Madden Dam the writers are indebted to Brig. Gen. C. S. Ridley, Governor of the Panama Canal Zone.

DISCUSSION

JEROME FEE,[9] ASSOC. M. AM. SOC. C. E. (by letter).—A subject of great practical importance is presented in this paper. It deals with a difficult problem in physics, challenging modern devices of experimental technique. The authors have made available a valuable record of experience with cavitation phenomena, both at Madden Dam and from their extensive model tests.

The writer wishes to discuss the analytical methods whereby the authors demonstrate: (1) The occurrence of high local velocities; (2) the production of high intensity pressures; and (3) the percentage relation between impact forces on water and rigid surfaces, respectively.

Eq. 1 is based on the assumption that the "slug" of water travels with constant kinetic energy. In the following development, the writer assumes, instead, that the momentum remains constant. Using the same notation, the momentum of a lamina of volume $y\,dx$, having a velocity v, is (compare Eq. 1):

$$M = w\,(y^2\,dx)\,\frac{v}{g} = w\left(\frac{x\,d}{a}\right)^2 \frac{v}{g}\,dx$$

Therefore, the total momentum of each of the slugs (a regrettable term applied to anything with the properties of a fluid) is (compare Eq. 2):

$$w\,b\,d^2\,\frac{v_0}{g} = \int_{x_1}^{x_2} w\left(\frac{x\,d}{a}\right)^2 \frac{v}{g}\,dx \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Also, from Eq. 3b, $v = v_1\left(\frac{x_1}{x}\right)^2$. Therefore, $v_0\,b = \frac{v_1\,x_1^2}{a^2}\int_{x_1}^{x_2} dx = \frac{v_1\,x_1^2}{a^2} \times (x_2 - x_1)$; or:

$$v_1 = \frac{v_0\,b\,a^2}{x_1^2\,(x_2 - x_1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Eq. 16 should now be compared with Eq. 4, which may be written

$$v_1 = v_0\sqrt{\frac{b\,a^2\,x_2}{x_1^3\,(x_2 - x_1)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

It is clear that Eqs. 16 and 4 are incompatible. Hence, if the kinetic energy is constant, as the authors assume, the momentum must change; but this means that some external force is acting of which the authors have taken no account. Their entire argument must therefore be regarded as unsound. Of course, it has no bearing on the question to say that the authors' conclusion is correct. The occurrence of high velocities in connection with cavitation is known from other evidence; but it might well be doubted if it rested on the analysis presented in this paper.

What would be the fate of this slug of water, for example, if the pyramidal section of the narrowing passage, shown in Fig. 5, had suddenly ceased to taper at a certain point, and had continued, instead, as a constant rectangular section of indefinite length? The writer can only imagine that the slug of water would tear itself to pieces in an effort to conform to the assumption that the kinetic energy is constant.

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[9] San Francisco, Calif.

The neglect of viscosity forces is a serious defect of another character. With diminishing volumetric dimensions, approaching the apex of the pyramid in Fig. 5, the viscosity forces would become an important factor.

Referring next to Table 3, there are two very different phenomena that should be distinguished. Case (1) is illustrated by a steady stream of water, issuing from a nozzle and striking a plate normal to the direction of flow. The distinguishing feature is that the flow, although non-uniform, is steady and continuous. Case (2) is represented by the impact of a column of water flowing in a confined space, such as the inside of a pipe, and being suddenly stopped, as by the closure of a gate. In this case, the flow is neither steady nor continuous. From the instant that impact begins, ordinary flow ceases and the water particles merely oscillate about an equilibrium position, while a wave of pressure travels through the liquid.

The authors apparently have no difficulty in transferring quantitative results from one case to the other. In fact, they seem to make little distinction between them. Presumably, they have the first case in mind when they restrict the water particles to those "adjacent to the contact surface and near to the center of the contact area" (see heading "Nature and Causes of Cavitation"). Their conclusions, however, are expressed in general terms: Water moving with the velocities shown in Col. 9 of Table 3 will produce the pressure shown in Col. 1, on striking a rigid plane surface normal to its path.

The writer believes that the maximum pressure usually will be different in the two cases, for the same velocity; and that the pressure in Case (2) is the upper limit in Case (1), which may be approached but not exceeded, even in the central region of the contact area.

Case (2) is of particular interest to hydraulic engineers from its connection with water hammer. The problem is readily transformed into that of a still body of water in a pipe, which is suddenly struck by a plane surface, such as the end of a piston. An account of tests made under such conditions was given in 1929 by H. Diederichs and W. D. Pomeroy.[10]

Although the results have long been known, for moderate velocities, from the theory of sound, it is believed that the following analysis, suggested by the method of Messrs. Diederichs and Pomeroy, is somewhat simpler than other methods of development that have appeared. Let: A = area of piston, in square feet; v = velocity of piston, in feet per second; p = pressure against piston, in pounds per square foot; m = density of water, at 62.4 lb per cu ft; g = acceleration of gravity (32.2 ft per sec per sec); C = velocity of sound, in feet per second; W = work, in foot-pounds; and K = modulus of elasticity for water (43,200,000 lb per sq ft). For moderate velocities, this modulus will serve for both adiabatic and isothermal compression.

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[10] "The Occurrence and Elimination of Surge or Oscillating Pressures in Discharge Lines from Reciprocating Pumps," by H. Diederichs and W. D. Pomeroy, *Transactions*, A. S. M. E., Vol. 51, 1929, p. 9.

The piston is originally at rest at the end of a pipe full of water; the static pressure is negligible. Suddenly, the piston starts to move with constant velocity, v. (Actually, some finite time, however short, is required for acceleration. This has only a transient effect.) A pressure equal to p is produced in the layer of water adjacent to the piston, and a pressure increase travels down the pipe with the speed of sound. After t seconds, the wave front has covered a distance, $C\,t$ feet, and a column of water of length $C\,t$ is now under pressure, p, and is moving at a velocity v. The work done by the piston during time, t, is: $W = A\,p\,v\,t$. This work is expended in two ways: (a) In increasing the kinetic energy of a water column of length $C\,t$ by changing its velocity from 0 to v; and (b) by increasing the potential energy of the same column, owing to the increase of pressure from 0 to p. This means that

$$W = A\,p\,v\,t = \frac{1}{2} A\,C\,t \frac{m}{g} v^2 + \frac{1}{2} A\,C\,t \frac{p^2}{K} \quad \text{(18)}$$

or

$$\frac{C}{2\,K} p^2 - v\,p + \frac{C\,m}{2\,g} v^2 = 0 \quad \text{(19)}$$

From the theory of sound

$$C = \sqrt{\frac{g\,K}{m}} \quad \text{(20)}$$

which, substituted in Eq. 19, gives

$$\frac{1}{2}\sqrt{\frac{g}{m\,K}}\,p^2 - v\,p + \frac{1}{2}\sqrt{\frac{m\,K}{g}}\,v^2 = 0 \quad \text{(21)}$$

Multiplying by $2\sqrt{\frac{m\,K}{g}}$ gives

$$p^2 - 2\,v\sqrt{\frac{m\,K}{g}}\,p + v^2\frac{m\,K}{g} = 0 \quad \text{(22)}$$

or $\left(p - v\sqrt{\frac{m\,K}{g}}\right)^2 = 0$; whence

$$p = \sqrt{\frac{m\,K}{g}}\,v \quad \text{(23)}$$

The foregoing shows that the pressure p is proportional to the velocity v. Substituting numerical values, $p = \frac{1}{144}\sqrt{\frac{62.4 \times 43{,}200{,}000}{32.2}}\,v = 63.5\,v$.

The values of p for various values of v are shown in Fig. 14 which shows also the authors' values. The two curves agree very well to about 205 ft per sec, but for higher velocities there is a marked difference.

Although it is interesting to have the data of Table 3 for excessively high velocities, it should be recognized that the authors present no quantitative evidence to demonstrate that velocities of this order do occur, in connection with cavitation. The maximum pressure of Table 3 is about ten times the maximum internal pressure in a 12-in. gun. Under such extreme conditions, the usual theory of sound no longer applies. Indeed, tests have shown that the sound from such a gun starts with about twice its ordinary velocity, but drops to normal in the short space of 300 ft or 400 ft. Returning to Table 3, it is not surprizing to find that the pressures determined by the theory of sound are too low.

Following Table 3, the authors develop a numerical ratio between the pressure of impact on water, and that on a "rigid" surface, illustrating their remarks by Fig. 6. The writer feels certain that the analysis is unsound.

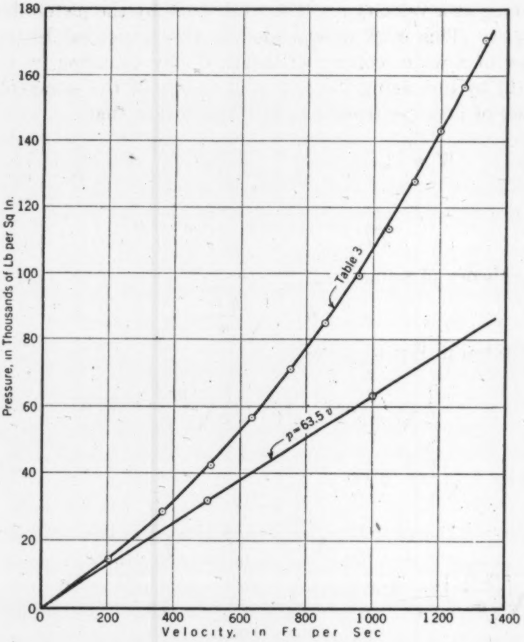

FIG. 14

The conclusion (see paragraph following Eq. 6) that "the velocity of particles at the contact surface is one half the initial velocity of the slug of water" is not merely incorrect, as to the magnitude of this velocity. It is based on an idea that is wholly unreal. There are no "particles at the contact surface" that can be distinguished in this manner from other particles, and whose velocity is susceptible of measurement or analysis.

Eq. 6, which the authors intend as a proof of their proposition, is nothing more than an arbitrary statement that assumes, to begin with, that the impinging particles, and those impinged upon, move during a finite interval of time with a common velocity v_c, which is constant. In other words, the impact between these particles is assumed to be totally inelastic. It might be well to mention that when a velocity is constant during a differential time interval it must necessarily be constant during a finite time.

In discussing these three topics, the writer is conscious that they are not indispensable to the main topic. In so far, however, as they may be subject to correction, it is important that they should be clarified.

V. E. **Leman**,[11] Esq. (by letter).—Efforts, made in recent years, to arrive at a clear conception of the processes relating to the behavior of water in the cavitation zone, have not met with much success. J. C. Hunsaker[5] has referred to this condition, as follows: "We cannot describe the exact mechanism by which wall damage results, but there must be some connection with the fluctuation of pressure resulting from the collapse of water vapor which periodically becomes unstable in such vapor phase."

In the light of such developments, Messrs. Thomas and Schuleen have rendered a most valuable service in giving an analytical demonstration of the hammering power of a slug of water traveling into a confined space.

The writer feels, however, that the full scope of the analysis has not been exploited by the authors, who limited themselves to a single aspect of the process—namely, to the action of a slug of water sandwiched in between two vapor cavities, without taking into consideration other factors instrumental in producing pitting in the cavitation zone.

Under the heading, "Behavior of Original Madden Dam Conduit Entrances in Models," the authors describe the character of flow at the collapse zone as one "where especially intense impacts are caused by collisions between the swiftly moving water of the streamline filaments and the broken masses of water and vapor hurled against these filaments from the interior of the pocket."

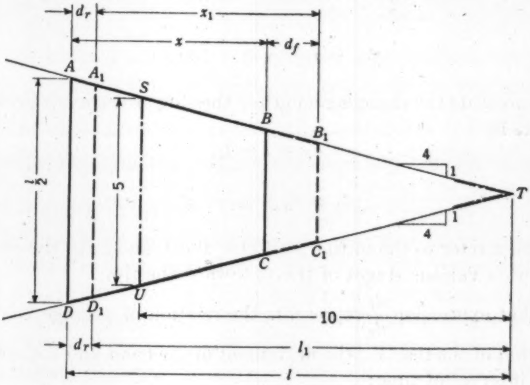

Fig. 15

It seems to be an established fact that, when a vapor cavity collapses, water rushes into it, eventually reaching the wall, not in the shape of broken masses but usually retaining, in substance, its pattern of continuity. It is the writer's contention that both the broken masses and the streamline filaments contribute to the destruction of the wall.

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[11] Civ. Engr., New York, N. Y.

To substantiate his point of view, the writer presents a "physical picture" of what, he considers, happens when water enters a crevice, either in the form of an isolated slug or of a solid column. The picture may be helpful also in visualizing the processes inherent in the travel of water into converging spaces under ideal conditions (as assumed by the authors), these processes being, ordinarily, somewhat puzzling. The case of a slug, in accordance with the authors' analysis, will be first presented.

For the sake of simplicity the crevice is assumed to be not a pyramid, as used by the authors, but wedge shaped, as shown in Fig. 15. To facilitate the calculations the sides of the wedge are assumed to slope at a rate of 1 on 4, which makes the spread of the wedge equal to one half the distance to the tip T.

A slug of water (of a width 1 normal to the plane of the drawing) moves from its position $A\ B\ C\ D$ to a new position $A_1\ B_1\ C_1\ D_1$, with its rear traveling a distance d_r, and its front a distance d_f. Due to convergence of space, d_f is larger than d_r, as volume $A\ A_1\ D_1\ D$ must be equal to volume $B\ B_1\ C_1\ C$. This faster movement of the front in relation to that of the rear becomes more and more pronounced as the slug travels toward the tip of the wedge. The following analysis will express this algebraically:

Suppose the slug $A\ B\ C\ D$ moves from a position, where its rear is l units away from the tip T, to its final position when its front hits the tip—this final position being represented by a triangle $S\ T\ U$, with a base 5 and a height 10. As the volume $A\ B\ C\ D$ is equal to the volume of the triangular prism $S\ T\ U$:

$$\tfrac{1}{2}\left(\frac{l}{2} + \frac{l - x}{2}\right) x = \frac{5 \times 10}{2} = 25;$$

or

$$x = l - \sqrt{l^2 - 100} \quad \text{.....................} \quad (24)$$

in which x represents the changing length of the slug as it travels toward the tip.

From Fig. 15:

$$d_r = l - l_1 \quad \text{.........................} \quad (25a)$$

and

$$d_f = d_r + x_1 - x \quad \text{....................} \quad (25b)$$

in which l and x refer to the initial positions, and l_1 and x_1 to the new positions of the slug in its various stages of travel toward the tip.

Noting that expression $\frac{d_f}{d_r}$ represents the relation of average velocity of the front V_f to that of the rear V_r, the movement of the front and the rear occurring in the same interval of time:

$$\frac{V_f}{V_r} = \frac{d_f}{d_r} = \frac{l - l_1 + x_1 - x}{l - l_1} \quad \text{...................} \quad (26)$$

and, as the energy changes with the square of velocity:

$$\frac{E_f}{E_r} = \left(\frac{V_f}{V_r}\right)^2 \quad \text{.......................} \quad (27)$$

in which E_f and E_r represent the energies of particles at the front and the rear of the slug, respectively.

Eqs. 24, 26, and 27 have been used to plot the curves in Fig. 16, the values of l being selected in such a way as to give the more characteristic points shown. One can readily see that, for an ideal case of motion (no viscosity, no elastic deformations and absolute vacuum), the relations of velocities and energies of front and rear may become infinite, with practically all energy concentrated at the tip of the crevice. This is in complete agreement with the authors' statement to that effect in presenting Eqs. 3 and 4.

The case of streamline flow involves conditions differing in many ways from those of a slug; whereas the latter is comparable to a flying projectile, the former is characterized by a water column reaching back to the free water surface of the reservoir.

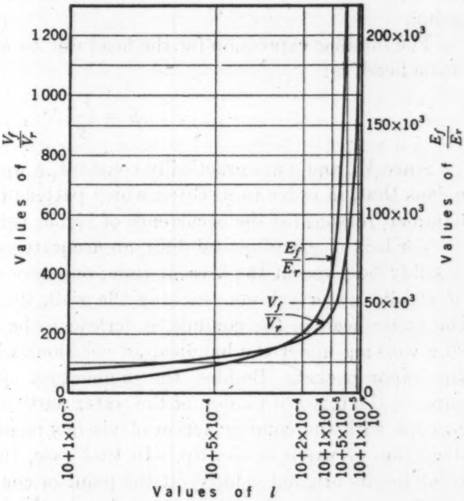

FIG. 16

The purely kinetic energy formulas (Eqs. 1 and 2) must now yield to the total energy equations that would include the pressure energy as well, no matter how small this factor appears to be in the low-pressure zone.

Fortunately, however, these considerations need not be taken in account as far as the "physical picture" of the process within the crevice is concerned. Referring again to Fig. 15, imagine that the space to the left of line $A\,D$ is not a vacuum but is filled with water extending to the free surface of the reservoir, the line $A\,D$ being merely a dividing line, separating a volume $A\,B\,C\,D$ from the column of water to the left. The reasoning used in establishing the curves in Fig. 16 can now be applied, without reservations, to the case of a solid water column entering a crevice; in spite of a difference in absolute values of velocities and energies in the two cases, the process of their respective increase remains identical. The final result (accumulation of energy at the tip) is also identical.

A comparison of the actions of the two flow patterns from the point of view of destruction may be proper.

The writer is unable either to agree with the authors' theory of water-hammer action in respect to the slug, or to deny it. He would not be willing to go far beyond the assumption that, a pressure wave within the slug itself being a certainty, its propagation farther out, over the vapor cavities, remains an open question.

This uncertainty, coupled with insufficient information as to the actual velocity of the slug, precludes any accurate estimation of the relative destruction capacities of the two flow patterns. The following, therefore, is merely an attempt to evaluate such capacities in a rather general manner, under the assumption that some kind of water-hammer action does exist in the case of a slug.

The familiar expression for the head due to water hammer (in excess of the static head) is:

$$h = \frac{V_w}{g} V \qquad (28)$$

Since V_w and g are practically constant, h would depend on V alone. This means that, in order to discover which pattern of flow creates a greater water hammer, reasons for the occurrence of higher velocities should be investigated.

For lack of experimental data, an accurate solution of this problem cannot possibly be given at the present time; one may feel, however, that, in the case of a solid water column reaching the wall, its velocity, being dependent on the static head in the conduit, is certain to be higher than that of a minute slug working under the handicap of collisions with broken masses of water in the vapor pocket. Besides, the momentum of such a column is certain to support the forward motion of the water particles in the crevice, helping them to cope with the counter action of viscous resistance, elastic stresses, or condensation of vapor in the tip. In that case, the flow has a better chance of retaining its original velocity at the point of contact with the wall.

These considerations lead the writer to the belief that, provided the water column has a chance to reach the wall as a solid mass, its eventual effect is certain to be more violent than that of a slug. This does not necessarily mean that all destruction is the result of such action. The fact that the collapse of cavities occurs periodically, whereas the slugs are continuously at work, makes the problem rather indeterminate. Experiments by P. De Haller,[12] in Switzerland, are worth being mentioned in this connection: Metal specimens have been subjected to heavy hammering by water slugs. The results are indicative of the fact that, to a high degree, such slugs are instrumental in producing erosion which, in its sponge-like appearance, is strikingly similar to the pitted surfaces resulting from collapse of cavities.

The writer has one more remark to make: The authors state (under the heading "Model Studies of Various Methods Proposed to * * *: Group 2") that the reason for not using vent pipes in their final design was their unwillingness to reduce the discharge capacity of the conduits. No matter how important this consideration, the writer is under the impression that vent pipes have not been given a chance to prove their worth. The three 3-in. pipes, installed in connection with the modified design of 1935, could not help being more of a nuisance than a remedy, owing to their quite insufficient size. Judging by the standards of the U. S. Bureau of Reclamation,[13] the cross-sectional area of the pipes should have been about 8 times larger than the one used.

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[12] *Schweizerische Bauzeitung*, May, 1933, p. 243.

[13] "Dams and Control Works," U. S. Dept. of the Interior, 1938, p. 180.

P. S. O'Shaughnessy,[14] Esq., and E. S. Randolph,[15] M. Am. Soc. C. E. (by letter).—In 1928, when plans were first made for the Madden Dam, sluiceways near the level of the Chagrés River bed were specified. These outlets were to permit lowering of the reservoir for purposes of inspection, cleaning out debris, and routine maintenance and repairs. The sluiceways became a part of the flood-control plan, to be used when and if floods occurred in excess of the spillway capacity. During construction of the dam, the six 5-ft 8-in. by 10-ft sluiceways proved to be a valuable aid in diverting the flow of the river.

The flare provided at the sluiceway entrances in the original design was limited to permit the construction of slots to accommodate stop logs of reasonable size. When the laboratory tests were made, one of the objects was to preserve the stop-log slots and thereby confine any required modifications to the part of the conduit downstream from the slots. This was accomplished in the case of sluiceways 1 to 4, where the model studies showed that the entrance could be reconstructed in such a manner that cavitation was eliminated with very slight changes in the stop-log slots. However, for sluiceways 5 and 6, the most satisfactory means developed to reduce cavitation consisted of providing covers at the stop-log slots.

Between September and December, 1934, as part of a comprehensive program of hydraulic tests on the Madden Dam, the sluiceways were operated at various heads from 71.5 to 138.5 ft during the rise of the reservoir. After December, 1934, the sluiceways (except sluiceway 3 in which the testing equipment had been installed) were used at times to control the reservoir level.

From the beginning of the sluiceway operation, it was noticed that the flow was accompanied by popping and crackling noises that were very loud within the sluice-gate galleries and were audible even on the parapet of the dam. Apprehension was felt at the time that damage was occurring, but definite evidence was lacking until the cavitation had progressed to such a depth that a porous tile drain began to discharge into the dam on March 15, 1935 (see heading "Introduction," in the paper). The knowledge that cavitation had occurred coincided closely with the appearance of the paper by Mr. Hunsaker,[5] which was of assistance in understanding the matter. Remedial measures were undertaken immediately. It was hoped that small air vents would prevent or diminish the cavitation by serving as a cushion, but they did not produce the desired result of eliminating cavitation completely.

After the collapse of the air-vented filler in sluiceway 6, which had been fitted with a bellmouth inlet, attention was directed to the great intensity of the force acting downward through the slot. Consideration was given to a stronger filler for closing the four sides of the slot, but sufficient confidence existed in the efficacy of a tight-fitting cover, alone, to result in its selection as a means of closure. This was done prior to making the model tests, which eventually justified the selection of this method. The covers were placed on the two sluiceways provided with bellmouths—5 and 6. These covers were designed to withstand full hydrostatic head in addition to atmospheric pressure.

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[15] Cons. Engr., The Panama Canal, Balboa Heights, Canal Zone.

Steel cables were attached to the covers to permit their removal without the services of a diver.

During the second season of operation of sluiceways 5 and 6, after installation of the bellmouths, there was a noticeable difference in the volume of concrete removed. As shown in Table 2, the operating periods at high heads were, respectively, 1,248 and 1,421 hours. The volumes of concrete removed were, respectively, 30.5 and 16.4 cu ft. It appears that the vented stop-log slot filler in sluiceway 6 remained in place for some time after its installation but collapsed at some unknown date. Vacuum measurements were made at the entrance to sluiceway 6 on December 11 and 16, 1935. On the former date, with the sluiceway operating under a head of 134.3 ft, a vacuum of 24 in. of mercury was recorded. On December 16, under a head of 124.3 ft, no vacuum was developed.

It seems reasonable to assume that the collapse of the frame occurred between these two dates. Since most of the sluiceway operations, both in point of time and of high heads, had occurred by December 16, it would appear that the lesser damage to sluiceway 6 may have been due to the beneficial action of the vented filler frame before its collapse. That the frame did not collapse under the higher heads prevailing at the beginning of operation can be explained by assuming that the force acting upon it was not constant but of a pulsating character. It was noticed throughout the sluiceway tests that the stream issuing from the conduits was of this type.

During the first season of operation, sluiceways 1 and 2 were badly damaged by 138 and 130 hours, respectively, of operation under high heads. After these sluiceways were repaired, they were operated 134 and 107 hours, respectively, at high heads without suffering damage. It is probable that the concrete used in the repairs was denser and, since it was mixed and placed in smaller batches, its placing was done more carefully than that of the original concrete. It is possible, also, that the air vents installed after repairs were made were of some use in decreasing the effect of cavitation collapse.

It is not made clear in the paper how the air measured in the model tests on the diverging tube apparatus can be related quantitatively to the air that would be required in venting the prototype. Even if the quantity of air can be expressed in terms of the prototype, it appears doubtful that venting is the best approach to most problems of eliminating cavitation in outlet conduits.

Mr. Hunsaker[6] showed that for a given hydraulic system the product of the frequency and the length of the cavitation pockets is equal to the product of the throat velocity and a substantially constant non-dimensional number. If the velocity remains constant, venting will lengthen the cavitation pocket and decrease the frequency in direct proportion. In this case, the pockets will form farther downstream and the immediate effect of cavitation will be decreased. However, this is not a satisfactory solution to the problem since cavitation is not eliminated unless a large volume of air is introduced; and, in this case (as stated by Messrs. Thomas and Schuleen), a reduction in discharge will result.

Difficulties similar to those at the Madden Dam may have been encountered in other structures. These, if reported and discussed, may serve to focus attention on the importance of considering cavitation in designing.

Since the completion of the model tests, it has been determined to limit sluice-gate openings on sluiceways 1 to 4 to 70% until they have been repaired. Sluiceways 5 and 6, provided with bellmouths and cover plates over the stop-log slots, may be operated at full capacity for the passage of extreme floods.

Acknowledgments.—The designs for the Madden Dam were prepared by the U. S. Bureau of Reclamation, in collaboration with engineers of the Panama Canal. The chief engineer and the chief designing engineer of the Bureau of Reclamation were engaged as consulting engineers by the Governor of the Panama Canal Zone. The bellmouth entrances that were attached later to sluiceways 5 and 6 were designed also by engineers of the Bureau of Reclamation.

CARROLL F. MERRIAM,[16] ESQ. (by letter).—There is much food for thought in the similarities and the differences to be noted in the work of experimenters in the field of cavitation. The striking point is that interest in cavitation is not confined to the builders and users of hydraulic machinery. It proves that the research conducted primarily through interest in the mechanical aspects finds application in a problem that is essentially structural. At the present stage of development at least, there is one outstanding difference in approach. In the case of discharge conduits, which may be operated at full capacity but a relatively short period of time, the idea of the designer is to avoid all danger of cavitation, whereas the builder of high-speed hydraulic machinery is seeking the economic balance between operating revenue and cost of repairs. Consequently, he must strive for quantitative measurement, always seeking to evaluate risk of damage and resistance of materials, together with the value of increased capacity. The former is more interested in qualitative tests which give reasonable assurance that the design proving best in the model is the most suitable for the prototype. Under these conditions it would appear that the authors' experimental results have greater immediate practical application than their mathematical analysis.

The mathematical analysis affords an interesting approach that is not to be discredited by any means, for, even though exact agreement with practice can never be expected, it is true that only through mathematics does the investigator gain insight into the complicated play of factors which influence the results.

Obviously, the action that takes place is far too involved to permit prediction of the exact pressure, at any given region, in model or prototype, under any given conditions. However, the authors' development of the conception of a slug of water striking a converging passage explains, excellently, why it is that, when pitting has once started and has gained a little foothold, further damage is likely to progress at an accelerated rate. The test pieces removed from the Holtwood stand[17] showed deep pitting along the edge where the sides of the flume make a right angle.

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[16] With Pennsylvania Water & Power Company, Baltimore, Md.

[17] "Pitting Resistance of Metals Under Cavitation Conditions," by J. M. Mousson, *Transactions*, A. S. M. E., July (1939), Vol. 59, No. 5, pp. 399–408.

It should be emphasized in addition that this multiplication of impact leads to tremendously high concentration of stress over an exceedingly limited area. This is well illustrated by the experiments cited in the paper,[5] in which ordinary methods of measuring high pressures such as are used in internal combustion engines, gun barrels, etc., failed completely as criteria of cavitation severity. Finally, the use of thin diaphragms, backed up by perforated plates, proved in vain because they were punctured merely by accidental hits like bullets rather than by a widespread blow.

The principal point of disagreement with the authors lies in their not entertaining the theory advanced by W. Watters Pagon,[18] M. Am. Soc. C. E., in 1935, at least as an alternative explanation. His conception of eddies as the fundamental form of the cavities demonstrates not only the possibility of extremely high local pressure concentrations, resulting from bombardment of the walls by particles propelled at terrific velocity generated by progressive but sudden collapse, but also explains the location of the more seriously pitted areas with relation to the form of the cavitation pocket.

The authors recognize that any mathematical treatment is beset with factors that are extremely difficult to evaluate since they enumerate the many things that must be neglected in order to justify equating kinetic energies. To this number should be added the rotational energy in the eddies.

Although the action within the cavitation pocket, particularly at the downstream fringe, is extremely complicated, as shown by the high-speed moving pictures to which the authors refer,[5] the reader should not gain the impression of a confusion of slugs of water being hurled about promiscuously, these slugs striking with destructive violence upon the walls of the conduit. Careful study of these moving pictures, together with stop-motion stills and examination of the models in operation illuminated by ordinary light as well as by electric spark, reveals that at the upstream edge there seems to be an orderly leaping of the water away from the walls. Behind this region, when cavitation is well developed, there is undoubtedly a part of the channel boundary that is never wetted. The most important part of the pocket, from the point of view of damage, is where the stream apparently returns to the walls. If the phenomenon were perfectly continuous and could take place smoothly and uniformly, the stream would merely deliver a glancing blow with little probability of doing any harm.

Instead, the action is far from stable and is characterized by an alternate lengthening and shortening of the pocket so that the downstream boundary is a fringe of extreme turbulence. In this region slugs of water can be imagined as being hurled through an environment of water vapor; or the opposite, which would mean bubble-like cavities filled only with condensable water vapor tossed about in a seething mass of turbulent currents of water. The chances are that neither of these conceptions is entirely correct.

The point is that eddies are set up by viscous sheer and are shed by passing around the lip at the entrance where the water finally leaves the surface of the conduit. It has been shown mathematically, by Mr. Pagon and others, that at the heart of such eddies the pressure is much lower than in the surrounding environment. Consequently, when the absolute pressure drops, these eddies are open and can remain as open cavities even after the pressure is partly restored; but they are extremely unstable. It seems more likely that the final collapse of these cores is the direct cause of the destruction rather than any pelting by slugs of water as assumed by the authors. In support of this idea there is the observation that in two-dimensional, venturi-shaped passages the damage comes where the axes of the eddies would terminate against the side walls rather than on the profiles, which would be more likely to receive the onslaught of slugs of water.

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[18] "Cavitation and Erosion Investigated as a Problem in Fluid Mechanics," by W. Watters Pagon (unpublished papers, A. S. M. E., 1935). On file for reference in Engineering Societies Library.

Regardless of the theory of the collapse, there is an excellent field for further research. In the first place the authors' pressure gradient curve in Fig. 8 is well confirmed by other experiments and shows a rising pressure in a channel of uniform cross section. This cannot be explained by the principle of Bernoulli. What is the origin of this pressure increase?

Furthermore, a strong similarity has been noted between this pressure rise and the surface of the hydraulic jump. The action appears to be similar, at least from visual observation. In addition, it should be noted that in expanding passages both phenomena are stable. In passages having parallel sides both are particularly unstable. In one model which consisted of a venturi-shaped passage, terminating in a parallel section, there was a sharp increase in noise as well as damage just as soon as the cavitation pocket was made to extend into the parallel section. This phenomenon was reported by W. Spannhake.[19] The disruptive effort seems to be definitely associated with the instability of flow.

A most important precaution taken by the authors was to free the water from dissolved air as much as possible. It is doubtful whether they were completely successful although the small volume of air remaining probably had no appreciable effect upon their results. It should be emphasized, however, that even a small amount of dissolved air is sufficient when very low pressures are encountered to upset the laws of hydraulic similitude. Under these conditions the water expands as an emulsion with the air coming out of solution. The result is that Q is no longer equal to $A \times V$. In hydraulic model testing, in which at any point there are pressures less than a half an atmosphere, there is danger of trouble from this source.

With further reference to the aforementioned hydraulic grade line, detailed exploration will show that the valley has a flat bottom rather than rounded as reported.

Although the authors are correct in stating that in practice it usually requires months to develop deep pitting in metal surfaces, it should not be overlooked that in certain laboratory tests it is possible to see the first signs of pitting after several seconds and to produce satisfactory results in the course of an hour.[20] Nevertheless, attempts to force metal to pit in models as it would be expected to do in service have not been particularly successful. Perhaps one of the most fruitful fields for further investigation is to establish relations between actual service and the results of various accelerated tests.

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[19] "Cavitation Research," Progress Report, by Carroll F. Merriam based on work of Wilhelm Spannhake, New England Hydraulic Power Committee of the New England Electrical Utilities Engrs., 1933. Filed for reference in Engineering Societies Library.

Many investigators have been forced to rely upon indications of surface damage, such as the removal of paint, and consequently the authors are to be congratulated for attaining any success at all in finding a material to simulate pitting. The principal difficulty would seem to lie in there being a threshold value of severity below which no damage is done but above which failure takes place with surprising rapidity. Quantitative measures of the extent of pitting either by loss of weight or volume are also very difficult.

In indicating an interest in cavitation outside of the field of hydraulic machinery, this paper suggests the possibility of even more extended application. So far, cavitation has always been regarded as the "villain in the plot"—the bane of the designer. The word cavitation itself carries a bad connotation. It would be interesting if some day a useful purpose could be found for it. Experimenters have noticed a similarity between cavitation and the hydraulic jump; furthermore, the hydraulic jump has been used to produce an intimate mixture of liquids.[21] Do not these two statements suggest that cavitation might be used to secure an intimate mixture of two chemicals when, for some reason or other, it was desired to have the time of mixing reduced to an extremely small fraction of a second? Another possible application may be found in the fact that cavitation is capable of extracting dissolved gases from liquids. Air content in water is reduced in the cavitation zone faster than it can be dissolved in the following turbulent zone. When such gases have harmful effects, such as the oxidation of fats in milk, there is at least a possibility of the content being quickly and materially reduced by cavitation.

HUNTER ROUSE,[22] ASSOC. M. AM. SOC. C. E. (by letter).—Cavitation research has progressed to a comparatively refined stage in several of the engineering professions, but Messrs. Thomas and Schuleen appear to be the first to have investigated, by laboratory experiments, the occurrence of cavitation in masonry structures. Although it might be argued that the phenomenon is the same regardless of the surroundings in which it occurs, the writer feels that much credit is due the authors, both for the development of the experimental technique which they describe and for calling the attention of the civil engineer so forcibly to the dangers of poor hydraulic design.

While the writer therefore voices only commendation for the major part of the paper under discussion, he believes that the authors' comments on the "Nature and Causes of Cavitation" leave much to be desired. The attempted proof of the existence of enormous local intensities of pressure seems particularly subject to criticism, owing to the artificial (if not physically impossible) nature of the assumption of constant kinetic energy in the specimen slug of water. Perhaps quite as artificial, although considerably more elegant, would have been the assumption of a momentary spherical bubble of water vapor within an initially stagnant, incompressible liquid of arbitrary volume; the work done by external pressure in accelerating the liquid toward this center would then yield the ultimate kinetic energy as the bubble collapsed, leading in a more reasonable manner to the same conclusion of an infinite local velocity and an infinite local pressure intensity at the point of collapse. Indeed, one need only hear the violent shocks which are produced by discharging steam under water to concede the issue that the collapse of such vapor pockets might well lead to exceedingly high local stresses. Of particular interest, in the adaptation of this approach to the case of a compressible liquid, is the fact that a two-dimensional or three-dimensional positive pressure wave must invariably be followed by a negative wave of similar order of magnitude.[23] Such conditions would seem to point even more strongly to the stress-reversal theory of boundary failure.

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[20] "Progress Report on Cavitation Research at Massachusetts Institute of Technology," by J. C. Hunsaker, *Transactions*, A. S. M. E., Vol. 57, 1935, pp. 423–424; and "Determination of the Relative Resistance to Cavitation Erosion by the Vibratory Method," by S. Logan Kerr, M. Am. Soc. C. E., *loc. cit.*, Vol. 59, 1937, No. 5, pp. 373–397.

[21] "Computation of the Tailwater Depth of the Hydraulic Jump in Sloping Flumes," by Robert W. Ellms, *loc. cit.*, HYD–50–5.

[22] Prof., Fluid Mechanics, State Univ. of Iowa; Cons. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

On the other hand, high-speed motion pictures by H. Mueller[24] of the collapse of vapor bubbles produced in cavitation would indicate that the bubbles tend to flatten on the downstream side rather than to dwindle concentrically. In other words, the impact area is then of finite magnitude rather than infinitesimal, and the computed terminal velocities are therefore finite even if the fluid is assumed incompressible. Conclusions of this nature would tend to invalidate much of the authors' basic hypothesis, as do piezoelectric measurements by J. Ackeret[25] of only moderately high instantaneous stresses in zones of cavitation. Moreover, the effect of pitting has been noted in Pelton-wheel buckets, where there is no reason to expect the formation of vapor pockets, and the method of passing test specimens repeatedly through free jets of water has been used by Professor Ackeret in comparing the resistance to pitting of various materials.

Although the authors' assumption of unbelievably high intensities of pressure has long been a favorite explanation of the pitting effect, it is possible that the theory of fatigue through repeated impact of not-too-great magnitude will eventually be more generally accepted. In other words, whereas vaporization of the liquid is the essential feature of cavitation itself, the existence of the vapor state apparently is not essential, but merely contributory, to the phenomenon of pitting through liquid impact. The term "impact," however, is so often used erroneously in connection with liquid motion that its true import has become obscured. For instance, a continuous jet of liquid deflected by a vane is frequently cited in hydraulics to illustrate the "impact" of a liquid upon a solid boundary, and the stagnation pressure at the point of zero velocity is even more frequently called the "impact pressure." This is in no way related to the case of impact dealt with in theoretical or applied mechanics, for no discontinuity or unsteadiness of the liquid is involved. It is not surprising, therefore, that such a fictitious case of impact—the steady deflection of a continuous liquid jet—produces no sign of pitting in the solid boundary. On the other hand, if the free jet is not continuous, but composed of drops or even slugs of liquid, pitting of the solid by true liquid impact is readily obtained.[26]

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[23] "Hydrodynamics," by H. Lamb, Cambridge University Press, 6th Ed., 1932, pp. 491, 492, and 525–527.

[24] "Hydromechanische Probleme des Schiffsantriebs," edited by G. Kempf and E. Foerster, Hamburg, 1932, pp. 311–314.

[25] *Loc. cit.*, p. 239.

Nevertheless, it is difficult for the hydraulic engineer to conceive of liquid impact as having essentially the same capacity for destruction as the impact of one solid upon another. Yet this was simply demonstrated by H. Föttinger[27] through the use of a glass tube partly filled with liquid, evacuated, and sealed at both ends. By inverting the tube, the slug of liquid within was made to drop to the bottom, producing a shock akin to that of a metal rod falling the same distance; indeed, the shock was once so great as to break the lower end of the tube, atmospheric pressure then driving the liquid upward with such force as to break the upper end as well. Needless to say, air left within such a tube would tend to cushion the blow, just as the presence of air in any zone of liquid or solid impact tends to reduce the destructive effect of repeated momentary stresses.

G. H. Hickox,[28] M. Am. Soc. C. E. (by letter).—The authors of this paper have done the engineering profession a service in setting before it an analysis of some of the problems connected with cavitation in sluices of high dams. The writer wishes to add a few comments relative to the cause of cavitation in such sluices and some criticism and questions regarding the authors' methods of testing and calculation.

Cavitation will occur whenever and wherever the pressure at a point in a liquid is reduced to the vapor pressure of the liquid. At the vapor pressure of the liquid, evaporation occurs and cavities form which are filled with vapor and dissolved gases. These cavities pass downstream with the liquid, eventually entering a region of higher pressure where they collapse, generating extremely high pressures. If the cavities collapse in the vicinity of a solid wall, there is a considerable destructive effect known as pitting. It should be noted that cavitation and pitting are not synonymous terms. It should also be noted that, whereas the phenomenon of cavitation is always due to a lowering of liquid pressure to the vapor pressure, the corresponding pitting always occurs somewhere downstream in a region of considerably higher pressure.

The reduction of pressure may be due to two causes: (1) A general lowering of the pressure caused by an increase in the average velocity of flow or a rise in the conduit; or (2) a "local" pressure reduction caused by curvature of flow. A siphon spillway offers examples of all of these phenomena. The pressure at the throat is low because the throat is the highest part of the siphon. Frequently its area is less than that of either the intake or discharge legs, which causes an increase in velocity and a corresponding reduction in pressure. There is also curvature of flow that results in a further reduction of pressure on the inside of the curve. The cumulative result of these effects may be to lower the pressure to such a point that cavitation may exist.

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[26] "Hydromechanische Probleme des Schiffsantriebs," edited by G. Kempf and E. Foerster, Hamburg, 1932, p. 238.

[27] *Loc. cit.*, pp. 251–254.

[28] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

A number of sluice conduits through high dams have been built in such a manner that cavitation occurred at their inlets. In a large conduit, the friction loss is not excessive and the hydraulic grade line may not be far above the roof, particularly if the conduit is inclined downward. The shape of the entrance is very important. If the entrance is gradually curved, or bell-mouthed, the local pressure reductions are low, and there may be no cavitation. If the curvature of the entrance is sharp, however, the local pressure reduction is much greater and cavitation may be suspected. In the case of the original Madden entrance shown in Fig. 1, there was an angle at the intersection of the conduit with the upstream face of the dam. Water flowing along the surface was required to make a sudden change of direction, necessitating an infinite acceleration at that point. The probable actual occurrence was that water flowing around the corner described as sharp a curve as could be effected by the maximum force available—that is, the difference between the pressure in the sluice and the vapor pressure of the water on the inside of the curve. In other words, the angular corner caused a region of extremely low pressure immediately below the entrance. Vapor cavities formed in this region and traveled downstream to a region of higher pressure where they collapsed, causing the pitting shown in Fig. 2.

A sharp-cornered entrance is not the only cause of local pressure reduction that may cause cavitation. Fig. 17 shows pitting that occurred below weep holes in the liner of the Norris sluice conduit during the first year of operation. The probable flow condition existing at these holes during operation is shown diagrammatically in Fig. 18. The downstream edge of the hole deflected the high velocity current away from the wall toward the center of the conduit. The flow returned to the wall almost immediately but the velocity and radius of curvature of flow were such that the pressure next to the wall was reduced to the cavitation point. Cavities forming in this region collapsed in the higher pressure area immediately downstream and attacked the liner with the result shown. Repairs were made by welding up the pitted areas, filling the holes with steel rods, welding them in place, and grinding the joints smooth. No further trouble is anticipated.

The writer is not quite in agreement with the authors' definition of a "cavitation pocket" as a void space filled with flying slugs of water. The photographs referred to[6] show clearly that there is not a cavity traversed by random slugs of water but rather a region in which cavities form, travel with the stream, and finally collapse. The rapid succession of these cavities has the appearance shown by the light areas in Fig. 10; but it is erroneous to refer to these regions as "pockets." More correctly, they should be called "cavitation regions" or "cavitation areas." The term "pocket" should be reserved for the individual cavities that traverse this region.

The writer is not convinced that it is necessary to produce cavitation in models in order to design structures that will be free from cavitation. If the method proposed by the authors is to be followed, however, its limitations should be clearly stated. In general, the treatment is fairly clear, but there are a few places where the meaning is ambiguous and seems to require further explanation. At the end of the first paragraph in the section "Description of Apparatus for Cavitation Tests" it is stated:

FIG. 17.—CAVITATION PITTING BELOW WEEP HOLES IN NORRIS SLUICE CONDUIT LINER

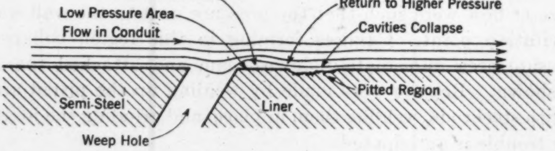
Cavities Form
Low Pressure Area
Flow in Conduit
Return to Higher Pressure
Cavities Collapse
Pitted Region
Semi-Steel
Liner
Weep Hole

FIG. 18.—REGIONS OF LOW PRESSURE AND PITTING IN NORRIS SLUICE CONDUIT

> "In order that the basic conditions which produce cavitation in certain regions of the prototype may be correctly represented in the model, it is necessary that the pressures in these regions of the model be reduced to the vapor pressure of the liquid used in the model."

This statement is correct but the reader who attempts to follow the authors' methods of representing prototype cavitation conditions in the model is likely to become confused. The cases in which such a model may be operated in accordance with Froude's law while at the same time reducing the absolute pressures to the linear scale as indicated by Eq. 8*c* are the rare exceptions rather than the rule. Such operation can be accomplished only if the ratio of the vapor pressure of the model liquid to the vapor pressure of the prototype liquid is equal to the model scale. The authors have pointed this out in a discussion of "operational distortion of scale." It is also included in the assumptions immediately preceding Eq. 8*c*.

It is very seldom possible to adjust the model vapor pressure to a value that will be in the correct ratio to the prototype vapor pressure. The limitations of model testing make the largest practical model scale about 1 : 15. The water temperatures in the lower parts of deep reservoirs are relatively low. At Norris Dam, the average temperature of the water discharged through the sluices is approximately 50° F. The vapor pressure of water at this temperature is 0.36 in. of mercury. The vapor pressure at 32° F is 0.18 in. of mercury. It is evidently impossible to reproduce the vapor pressure to scale in a model of a Norris sluice using water as the model liquid. With water at 32° F for the model liquid at a scale of 1 : 15, the corresponding prototype vapor pressure is 2.70 in. of mercury, which occurs at 112° F. This temperature seldom occurs in prototype structures. The difficulty is further aggravated by the enclosed-tank apparatus. It is extremely difficult to work with low temperatures using this apparatus, as energy at the rate of several kilowatts is constantly being added to the system by the pump.

It might be assumed from the authors' remarks following Eq. 8*c* that operation of a model in accordance with Froude's law requires that the ratio of absolute pressures in model and prototype be equal to the model scale. This is not correct. It is sufficient that the ratio of pressure differences between corresponding points be equal to the model scale. This can be illustrated by writing Eq. 8*c* in the form

$$V\,v_T = c_T'\sqrt{2\,g\left[L\left(\frac{p_H - p_T}{w}\right) - L\,z_T\right]} \ldots\ldots\ldots\ldots (29)$$

In Eq. 29 p_H and p_T are not necessarily equal to p_H'/L and p_T'/L but may have any convenient values as long as $\frac{p_H - p_T}{w} = \frac{p_H' - p_T'}{w\,L}$. In fact this is the manner in which hydraulic models are usually operated. The results are completely satisfactory except in those cases where the model indicates pressures which would be below the vapor pressure in the prototype. Even in these cases, similarity can be obtained if the model pressure is adjusted so that the boundary of the vapor pressure region in the prototype corresponds with the boundary of the vapor pressure region in the model. It should be noted that point *T* is on the boundary dividing the vapor pressure region from the region of normal flow. This is necessary since the pressure gradient is discontinuous at this boundary and Eqs. 8 are not applicable across it. Hence, in order to produce similar performance in model and prototype, it is necessary to know the boundary of the vapor pressure region. Unfortunately the need for model tests of any particular structure means that the vapor pressure region in the prototype is not known. Therefore, such adjustment, while theoretically possible, cannot be made by the experimenter.

The foregoing discussion has been confined to models geometrically similar to their prototypes and properly oriented. Rotating a vertical section into a horizontal plane would seem to introduce still further difficulty. The exact location of the cavitation region under the roof of a conduit must be influenced by the increasing pressures at greater depths. In a section rotated to a horizontal position, the pressures in the section cannot increase with depth, and consequently correct reproduction of the cavitation region cannot be expected. The relationships expressed in Eqs. 9 to 12 represent an attempt to overcome this difficulty. As pointed out by the authors, however, the best result that can be obtained is correspondence at only one point. For this reason, tests made with a rotated section of model are open to question, particularly with respect to the location of the pitted area.

It is noted that the use of Eq. 13 usually fails to result in model operation similar to that obtained by the use of Eqs. 9 to 12. This may be explained by the fact that in Eqs. 9 to 12 the velocity ratio is fixed in accordance with Froude's law, as may be seen by inspection of Eqs. 9. Eq. 12 expresses the adjustment of p_H' necessary to maintain this velocity ratio. Eq. 13, on the other hand, expresses the ratio of velocities that will exist for a given set of pressures and elevations in model and prototype. It would be a remarkable coincidence if this ratio did satisfy Froude's law.

The application of the equations to an actual model test raises further questions. In the example given, it is stated that a discharge of 2.343 cu ft per sec was assumed. It is doubtful if the discharge can be assumed arbitrarily. In order that the model flow pattern shall approximate that of the prototype, there should be at least a semblance of agreement with Froude's law. The discharge used gives a model velocity of 16.53 ft per sec which, compared with the prototype velocity of 74.0 ft per sec, gives a velocity scale of 1 : 4.47. This is exactly the square root of the length scale, 1 : 20, and indicates perfect agreement with Froude's law. One is led to believe that the authors meant to include this explanation since such good agreement could hardly be fortuitous.

The analysis following Eq. 14 is based on the assumption that the pressure at point T in the prototype for the reservoir elevation shown is the vapor pressure of the water. This was probably true in the case of the original Madden Dam sluices where there was an angle at the entrance. The application cannot be general, however, for the object of successful design is to build a conduit in which cavitation will not occur. The entire discussion is based on the assumption that the pressure at T is known, either in model or prototype, or both. When cavitation occurs, the pressure at this point is obviously the vapor pressure of the liquid and is thus fixed. When cavitation does not occur, as in a properly designed entrance, the pressure is much higher, and is not determinable by any of the equations of this paper. If the model is adjusted to operate in conformity with Froude's law, and the absolute pressures in reservoir and conduit barrel are adjusted to conform to the linear scale ratio, the model may indicate cavitation that would not occur in the prototype because of the lack of conformity of the vapor pressure ratio. Assuming a vapor pressure of 1 ft of water in both model and prototype, cavitation would occur in the model at an absolute pressure corresponding to 15 ft of water in the prototype, a condition that would be quite safe. It is needless to state that the cavitation region would not be correctly reproduced. The authors seem to present no method of adjusting the model to insure correspondence if the pressures in the region where cavitation is suspected are not known in advance. Of course, if the pressures are known beforehand, the reason for experimental determination becomes rather obscure. The writer would like to know how the methods of the paper were applied to a rotated vertical diametrical section of the entrance of the Hiwassee sluice, for example, which was found to be entirely free from cavitation.

It is believed that Eqs. 8*a* and 8*b* are not in strict accordance with Torricelli's theorem. This theorem merely states that the velocity of efflux from an orifice under a given head is equal to the velocity that would be attained by a body falling freely through a distance equal to the head, and may be written as $V = \sqrt{2\,g\,h}$. Eqs. 8*a* and 8*b* are derived from Bernoulli's equation, of which Torricelli's theorem is a special case. The additional terms representing pressure were introduced by Bernoulli much later.

It is not intended that these criticisms should detract from the value of the paper as a whole. The authors have done the profession an invaluable service by pointing the way toward a solution of a very troublesome problem in the design of high dams. It is probable, however, that the experience of other designers and other hydraulic laboratories with similar problems may eventually lead to simpler techniques that will accomplish the same purpose.

J. M. MOUSSON,[29] M. AM. SOC. C. E. (by letter).—This paper is very timely in many respects. It is of particular interest to hydraulic engineers because the form-giving criteria for water passages are often overlooked and, as in other fields of endeavor, it needs an extreme case of unfavorable conditions entailing expensive repairs to bring the matter to the renewed attention of the profession as a whole. With no physical damage to structures, and with inadequacies limited to disappointing coefficients (that is, low efficiency), usually no one ever hears anything of it.

It is surprising how little thought is often given to entrance conditions on conduits or intakes for turbine installations. Although a negligible sum would provide ideal conditions free from vortices, the crudest forms of transitions persist at many new power plants; and they affect the efficiencies of the wheels adversely, due to trains of vortices or uneven flow distribution. The accrued deficiencies in power output, even if only 1% or 2% are involved, entail large monetary losses unnecessarily wasted to the detriment of the national economy.

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[29] Project Engr., Rustless Iron & Steel Corp., Baltimore, Md.

As to the specific theories on cavitation proposed by the authors, some caution would seem advisable, particularly in the light of other researches. For instance, it is stated that (see heading, "Nature and Causes of Cavitation"):

> "The familiar effects of water hammering sufficiently intense to indent and disintegrate the surfaces of hard metals furnish abundant proof that pressures approaching or exceeding the higher values of Table 3 actually occur in engineering machines and structures."

(Table 3 gives values to 170,676 lb per sq in.)

It is well to remember that cavitation is a high-frequency phenomenon to the effect that any breakdown of material exposed to this punishment is primarily due to fatigue. The numbers of stress applications even in most severe cases may run into the tens of thousands,[30] and under these conditions it is reasonable to assume that the lower values in Table 3 (say 14, 223 ± lb per sq in.) may be entirely sufficient to cause severe damage rather than those near the upper limit. Furthermore, values in the upper part of the range are not likely to occur since damage would be so severe in a matter of minutes as to endanger any structure within the same time interval.

Additional items of evidence against the high pressures thought likely by the authors are the results obtained by means of a cavitation test stand at Holtwood, Pa., where steels and alloys of a large variety and in various conditions were exposed to cavitation.[31] The most resistant materials could be attacked within 16 hr with the adopted standard throat velocity of 265 ft per sec corresponding to the lower limit of the range in Table 3.

It could be argued, of course, that the throat velocity is not the primary criterion but that the velocity of cavity collapse and subsequent impact of water on water, or water on solid matter, is only to be considered. If this were true, then the degree of pitting found in any test stand would be little influenced by the throat velocity so long as cavitation exists. This, however, has not been indicated by any of the researches in the United States and abroad. The throat velocity was found to be one of the most important variables with respect to the severity of pitting.

Regarding the process of destruction by cavitation, it may be stated that the detailed microphotographic analyses of a large number of metallurgical specimens revealed that surface deformation with subsequent fatigue failures is but one form of disintegration. In many cases the breaking up or disintegration starts from within rather than from without, which again suggests caution with respect to the theory proposed by the authors as to the importance of cracks. Some of the reasons conducive to breakdown from within are the following: Uneven shape of crystals or grains, flake formations of metallic or nonmetallic substances in base matrix, metallic or nonmetallic inclusions, impurities, unfavorable stratifications, etc. All of these factors lead to possible failure due either to stress concentrations, or to surfaces providing little bond or poor interlocking. These same factors are certainly coming into play with concrete where fatigue cracks may develop due to unfavorable stress concentrations at sharp and pointed surfaces of the coarse or fine aggregate, or where failure may occur along the flat surfaces of sand particles or stones, providing little bond, or "break-throughs" of cracks may develop between inclusions. Although possible surface deformations on concrete should not be ignored, nevertheless a microphotographic analysis would seem advisable to prove this point, particularly because the plasticity of concrete is low when compared with certain alloys.

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[30] "Ueber die Zerstoerung von Werkstoffen durch Tropfenschlag und Kavitation," by J. Ackeret and P. De Haller, *Schweizerische Bauzeitung*, September 5, 1936.

[31] "Practical Aspects of Cavitation and Pitting," by J. M. Mousson, Edison Electric Inst. *Bulletin*, September and October, 1937.

Another factor that would appear to discredit the importance of surface cracks is the experience gained on turbine installations. For the repair of a badly pitted cast-iron turbine runner a straight stainless chrome steel of low ductility was used some years ago. In the course of welding, hair cracks developed, checking the entire welded areas. These cracks were so deep that they could not be smoothed off by grinding; yet despite these cracks no pitting whatever has developed, although stainless chrome nickel steel at the same location showed surface deformation. Similarly, on a turbine throat ring, with appreciably wider surface cracks due to contraction on an area welded with the same straight stainless chrome steel referred to, no cracks were found after a number of years.

In view of the foregoing, great caution seems advisable with respect to certain theories advocated by the authors regarding the necessity of stresses of extreme magnitude and the importance attributed to surface cracks and crevices.

J. H. DOUMA,[32] JUN. AM. SOC. C. E. (by letter).—Undoubtedly, this paper has been read with much interest by hydraulic engineers who have been concerned with the cavitation problem. It is especially noteworthy for being one of the first in this relatively new field of model investigation. In addition to a detailed outline of the procedure in conducting a model study for an investigation of cavitation, the authors have presented data that are useful in some degree to designing engineers. No attempt, however, has been made to establish a design criterion for outlet conduit entrance curves. Such a criterion would eliminate the seeming necessity for a model study of every proposed design.

For any given high velocity of flow, the boundary conditions of a closed system may be so varied that a pressure intensity equal to the vapor pressure will be approached at some point on the boundary. When the pressure intensity is reduced to the vapor pressure at any point, the liquid will just begin to vaporize. The region of vaporization will grow in size as the pressure intensity is further reduced, and cavities composed of water vapor and flying particles of water will form. The abrupt collapse of these cavities as they are carried into regions of higher pressure intensities causes the destructive disintegration of boundary surfaces known as pitting.

To prevent the formation of cavities the boundary conditions must be such that the outermost streamlines of the resulting flow conform approximately with the boundary profile over their entire length. This requirement can be fulfilled by proportioning any conduit inlet passage to fit the shape of a jet issuing from a sharp-edged orifice of the same cross section as the maximum inlet section. The effect, on the flow, of entrance surfaces so designed is to modify the outermost streamlines so that positive pressure intensities exist at all points on the boundary surfaces.

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In Fig. 19, the upper surfaces of free jets issuing from a sharp-edged circular orifice and from a long narrow slit are shown plotted for a unit *vena contracta* thickness. With the origin of coordinates at the orifice edge, x and y are the horizontal and vertical distances from the origin, and D is the jet thickness at the *vena contracta*.

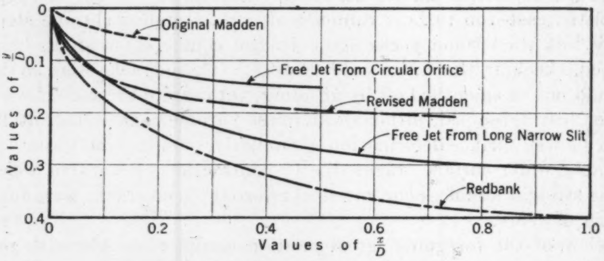

FIG. 19.—COMPARISON OF UNIT ENTRANCE CURVES

For comparative purposes, similar unit top curves of the original and revised entrance sections of the Madden Dam conduit, and of the final design of the entrance section of the Redbank Dam conduit, are also plotted in Fig. 19. The original Madden Dam curve falls far short of the free-jet curves, a condition conducive to the development of cavitation, which is borne out by the severe pitting of the prototype conduits. The revised entrance curve for Madden Dam, shown to have no cavitation potentialities by the model study, approximates the free-jet curve for a slit at its upstream end and lies between the free-jet curves for the circular orifice and the slit at its downstream end. The Redbank curve lies well beyond the free-jet curves, and a model study indicated satisfactory performance with regard to cavitation.

As noted in Fig. 1, the entrances of the Madden Dam conduit are not exactly symmetrical about a horizontal plane through the conduit axis, but the bottom of the stop-log recess terminates in a horizontal step some distance below the beginning of the entrance curve. Lowermost streamlines obviously begin to curve at the upstream end of this shelf. By plotting the circular orifice and slit free-jet curves with their upstream ends at this point, the original Madden Dam bottom entrance curve was found to lie between the free jet curves. No visible cavitation pockets appeared in the stop-log recess during model tests, and, as shown in Fig. 3, no cavitation damage occurred on the bottom of the prototype conduit. Apparently, the recess was filled with liquid, which was effective in easing the streamlines through this region to the entrance curve.

A similar comparison of the side-wall entrance curves with the free-jet curves indicated that they were inadequate to preclude cavitation potentialities, which is substantiated by the cavitation damage occurring at the prototype side-walls. In plotting unit entrance curves for the rectangular conduit, the conduit height was taken as *d* for the top and bottom curves, and the width as *d* for the side curves.

Inasmuch as the jet issuing from a square orifice approaches the shape of the circular orifice jet, and as the opening changes to a narrow rectangular cross section the jet approaches the shape of the slit jet, unit curves for a properly designed entrance to any given rectangular conduit would lie between the circular orifice and slit unit curves. A model investigation of free jets issuing from rectangular sharp-edged orifices, covering a useful range of height to width-of-opening ratio, would provide a basis for determining the exact shape of entrance curves to any rectangular conduit.

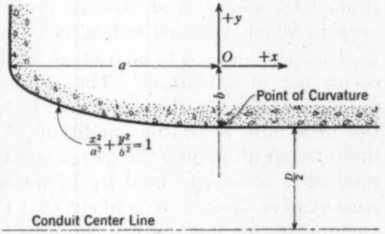

FIG. 20.—ENTRANCE CURVE SHAPE

To evolve a conservative basis of design until definite data relative to rectangular orifice jets are available, the entrance passages to circular conduits might be proportioned to approximate the shape of the free jet issuing from a sharp-edged circular orifice and for rectangular conduits the shape of the jet from a slit. Referring to Fig. 20, the free-jet surface curves approximate an ellipse in shape, expressed in the general form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The *vena contracta* of the circular orifice jet is located 0.5 D from the aperture, and the jet diameter at this point is 0.77 D_o, in which D is the jet diameter at the *vena contracta* and D_o the orifice diameter. This gives a semi-major axis equal to 0.5 D and a semi-minor axis equal to 0.15 D for the surface curve. The design equation of the entrance curve for a circular conduit becomes

$$4\,x^2 + 44.4\,y^2 = D^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The location of the *vena contracta* for the slit jet is approximately equal to the *vena contracta* jet thickness from the opening, and the jet thickness at the *vena contracta* is equal to 0.62 times the slit opening. These dimensions give a semi-major axis equal to D, and a semi-minor axis equal to 0.31 D for the surface curve. The design equation for the entrance curves of rectangular conduits becomes

$$x^2 + 10.4\,y^2 = D^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which D is the conduit height for the top and bottom curves and the width for side curves.

The writer contends that Eqs. 31 and 32 can be used reliably, in designing entrances to conduits, with reasonable assurance that cavitation would not occur in the prototype, except in the vicinity of gate slots, if they are not properly designed. As a matter of fact, the adequacy of Eq. 31 already has been verified by model studies of the entrances to the circular outlets of Grand Coulee Dam. These entrance curves fit closely that given by Eq. 31, and the model study indicated satisfactory performance relative to cavitation by showing positive pressures at all points.

Although, as the authors state, the cavitation phenomenon cannot be studied by means of small-scale undistorted models of the usual laboratory type in which no scale reduction is made in the atmospheric pressure, such models may be used to advantage in checking any conduit design for possible occurrence of cavitation. The method is to locate piezometer openings at numerous points in the region in which cavitation might occur. Then, under the maximum operating conditions, if at least a slight positive pressure is indicated at all points, the design is satisfactory for averting cavitation. This method is ordinarily used by hydraulic laboratories, and, since it results in conservative designs, it is likely that this procedure rather than the so-called vacuum testing method will continue to be favored. A comparison of the required entrance for any given conduit as determined by each of the two methods would be of interest.

Harold A. Thomas,[33] **M. Am. Soc. C. E., and Emil P. Schuleen,**[34] **Assoc. M. Am. Soc. C. E.** (by letter).—That so many distinguished engineers have seen fit to contribute to the discussion of this paper is a source of gratification to the writers, and they are glad to acknowledge the constructive nature of the comments and suggestions received in this way. Their appreciation should not be obscured by the argumentative tone of this final closure, which, of necessity, must be devoted largely to answering specific questions or criticisms.

Mr. Fee makes a number of criticisms of the analytical methods of the paper, but the writers cannot concede the validity of any of his arguments. In his discussion of Fig. 5 and Eqs. 1 to 4, Mr. Fee discards the writers' assumption that the slug of water travels with constant kinetic energy and assumes instead that its momentum remains constant. He develops Eqs. 15 to 17 on the latter basis, and, because Eq. 17 does not agree with Eq. 4, he states that the writers' "entire argument must therefore be regarded as unsound." The writers would like to point out that in this problem the assumption of constant momentum is quite inapplicable. In accordance with Newton's laws of motion, the momentum of a mass of material remains constant only if no unbalanced external forces act on the material. In this problem a large unbalanced force acts on the slug of moving water—namely, the upstream component of the pressure exerted on the slug by the walls of the tapering channel. Under the action of this force, the momentum of the slug is quickly reduced to zero. Therefore, to assume that the momentum is constant in this problem is entirely incorrect.

To confirm his statement that the writers' assumption of constant kinetic energy is unsound, Mr. Fee includes the following sentences in his discussion:

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[34] Senior Engr., U. S. Engr. Office, Pittsburgh, Pa.

> "What would be the fate of this slug of water, for example, if the pyramidal section of the narrowing passage, shown in Fig. 5, had suddenly ceased to taper at a certain point, and had continued, instead, as a constant rectangular section of indefinite length? The writer can only imagine that the slug of water would tear itself to pieces in an effort to conform to the assumption that the kinetic energy is constant."

It is not necessary to use imagination to determine that the slug will "tear itself to pieces" under these conditions, as this fact can be readily verified by computation or experiment. An application of the method used in deriving Eqs. 1 to 4 will demonstrate that the pressure in the water of the slug becomes negative just after it enters the smaller rectangular section, thus causing the slug to break into fragments as it enters the smaller section. A simple experimental demonstration of this action may be made with a quart bottle whose form approximates two cylindrical tubes connected by a tapering section. If the bottle is filled about a quarter full of water and is then held mouth upward in the hands and is jerked suddenly downward, the jet issuing from the mouth of the bottle will not be solid but will be broken into spray. The action can be shown to even better advantage in a piece of apparatus consisting of a 1-in. glass tube about 1 ft long, drawn down to a diameter of about $\frac{3}{8}$ in. in its middle third. About an ounce of water may be placed in the tube, and the ends sealed after exhausting the air with a vacuum pump. When this apparatus is jerked back and forth, the disintegration of the slugs of water as they enter the constricted section is clearly apparent. The existence of this phenomenon is a confirmation rather than a contradiction of the correctness of the writers' assumption of constant kinetic energy.

Mr. Fee's discussion contains the following sentences: "Referring next to Table 3, there are two very different phenomena that should be distinguished" —steady continuous flow and impact. "The authors apparently have no difficulty in transferring quantitative results from one case to the other. In fact, they seem to make little distinction between them." The writers feel that the comment contained in these sentences is quite unjustified. The entire phenomenon under discussion is one of impact caused by a detached mass of water striking a stationary solid or liquid. In preparing the condensed column headings for Table 3 and the accompanying explanation, the writers felt that this point would be so thoroughly appreciated by the reader that it would be unnecessary to include specific statements that the data were inapplicable to cases of steady continuous flow.

Mr. Fee's discussion devotes more than a page to a derivation (suggested by the method of Messrs. Diederichs and Pomeroy[10]) of the ordinary formulas for water-hammer pressure in a pipe. The writers do not consider that this material is pertinent to the discussion of the paper or that any point is gained by plotting the results of an ordinary water-hammer computation in Fig. 14. However, they will refer to this derivation again in discussing a criticism of Eq. 6.

The following sentences are contained in Mr. Fee's discussion:

> "The writer feels certain that the analysis [leading to Eq. 6] is unsound. The conclusion (see paragraph following Eq. 6) that 'the velocity

> of particles at the contact surface is one half the initial velocity of the slug of water' is * * * based on an idea that is wholly unreal.* * *
>
> "Eq. 6, which the authors intend as a proof of their proposition, is nothing more than an arbitrary statement that assumes, to begin with, that the impinging particles, and those impinged upon, move during a finite interval of time with a common velocity v_c, which is constant."

Perhaps the significance of the analysis leading to Eq. 6 could be made clearer by considering conditions within a closed pipe, rather than those at the stagnation point on the contact surface where an irregular slug of moving water meets a horizontal water surface as in the diagram at the left of Fig. 6. Mr. Fee's derivation of the water-hammer formulas, previously referred to, assumes a still body of water (in a cylinder or pipe) that is suddenly struck by a (rigid) plane surface such as a piston, and leads to the maximum pressure rise given by Eq. 23. Now, instead of this, assume a still body of water in a cylinder or pipe to be suddenly struck by an elastic piston consisting of water. Imagine the contact surface between the water which was originally still and that which was moving to be defined by a thin film or web, say tissue paper. If one applies himself to the solution of this new problem, he will recognize the fundamental nature of the requirement that in each of the two elastic bodies the front of a pressure wave must travel away from the tissue-paper contact surface at the acoustic velocity. Starting from this fact and proceeding by irresistibly logical steps, using Mr. Fee's method or any other of the ordinary derivations of the water-hammer formulas, one is led to the necessary conclusion that (before reflected waves return) the tissue-paper contact surface must move with a velocity equal to half the original velocity of the water piston, exactly as expressed in Eq. 6, and that the pressure rise must be exactly as expressed by Eq. 7. It is unnecessary to repeat the algebraic details of this proof, since the case is precisely similar to that shown in Fig. 6 and explained in connection with it, the vertical lines *AB* being interpreted as representing the walls of the pipe.

Mr. Leman reveals the fact that high pressures can be produced by the sudden checking of the velocity not only of a detached slug of water, but also of a larger mass of water extending continuously to the source, as in the more familiar case of water hammer in an ordinary pipe. Although the writers agree with him fully on this point, they chose a detached slug for demonstration of the fundamental principle of velocity concentration because the mathematical analysis seemed simpler and because close observation of cavitation phenomena under many different experimental conditions seemed to emphasize the importance of the part played by flying fragments of detached or semi-detached water.

Messrs. O'Shaughnessy and Randolph contribute additional data of much interest in connection with the use of fillers for the stop-log slots in the Madden Dam, and the admission of air through vents in the stop-log slot fillers. They offer the comment that "It is not made clear in the paper how the air measured in the model tests * * * can be related quantitatively to the air that would be required in venting the prototype." In reply to this comment, it may be stated that the analysis of the experiments did not attempt to establish such a quantitative relation. Since air entrainment is a function of bubble size, which in turn is primarily dependent on surface tension, it follows that Froude's law is entirely inadequate to relate the percentages of air entrained in the respective waters of a model and its prototype. During the model tests on the Madden conduits, the quantity of air drawn into the vents was measured as a control for repeating experiments, but it was not considered feasible to relate this air flow to that of the prototype.

Mr. Merriam's discussion contributes interesting information on the objectives and methods for cavitation tests on metal in hydraulic machinery as contrasted with the writers' tests on concrete surfaces in the outlet conduits of dams. The writers appreciate Mr. Merriam's clear statement of the fact that

> "In the case of discharge conduits, which may be operated at full capacity but a relatively short period of time, the idea of the designer is to avoid all danger of cavitation, whereas the builder of high-speed hydraulic machinery is seeking the economic balance between operating revenue and cost of repairs. Consequently, he must strive for quantitative measurement * * *. The former is more interested in qualitative tests which give reasonable assurance that the design proving best in the model is the most suitable for the prototype."

Considerable difficulty has been experienced in making this point clear to various engineers visiting the laboratory during the tests described in the paper, or in convincing them that, in the case of cavitation tests on models of fixed concrete structures, it is a waste of time and money to duplicate the elaborate technique used by hydraulic machinery cavitation laboratories for controlling the air and gas content of the water.

Mr. Merriam states that his principal point of disagreement with the writers lies in their not entertaining the theory advanced by Mr. Pagon[18] in 1935. This theory is discussed by the writers with some detail in their original full-length manuscript, which is on file in the Engineering Societies Library.[3] In essence, Mr. Pagon emphasizes the importance of the low-pressure region at the center of an eddy as a primary source of a "vapor void," and attributes a sudden collapse of the tubular vapor void along the eddy axis to dynamic instability of the eddying motion. In condensing the paper for publication, the writers decided to omit the discussion of this theory because, in their opinion, its tube-collapse feature did not seem to offer an adequate or complete explanation of the principal experimental facts in the type of cavitation under consideration. Granting the importance of curvilinear or eddying flow as a source of low pressure for opening up vapor voids, the writers nevertheless believe that in the types of cavitation under discussion the most important cause of the sudden collapse of vapor voids is simply the impact of swiftly moving water containing such voids against less swiftly moving water or solid objects, and that such collapse will occur whether or not the water is eddying. An important light is thrown on this topic by the analogy (mentioned by Mr. Merriam) between the downstream region of a cavitation pocket where such impacts are most active and the turbulent region in an ordinary hydraulic jump. Moreover, Professor Edgerton's motion pictures[6] of cavitation convey no suggestion of the existence of tubular eddy-voids.

In support of the eddy-tube-collapse theory, Mr. Merriam mentions that

> "* * * there is the observation that in two-dimensional, venturi-shaped passages the damage comes where the axes of the eddies would terminate against the side walls rather than on the profiles, which would be more likely to receive the onslaught of slugs of water."

As mentioned in the paper, in introducing Fig. 12, a similar phenomenon was observed in the tests on models of the Madden conduits, the side-wall pitting being shown, and its cause explained by the words

> "* * * the pit first develops along the edge of the large cavitation pocket, where especially intense impacts are caused by collisions between the swiftly moving water of the streamline filaments and the broken masses of water and vapor hurled against these filaments from the interior of the pocket."

The writers believe that this explanation of side-wall pitting conforms with the physical facts more accurately than that based on the eddy-tube-collapse theory.

The writers appreciate Professor Rouse's commendation of the paper in general. However, they disagree with the reasoning by which he attempts to disprove the analytical and experimental evidence that the essential cause of cavitation damage is the production of very high pressures by the collapse of vapor voids. It does not seem probable that Professor Rouse means to deny the possibility that high local pressures occur, since his discussion contains the sentence: "Indeed, one need only hear the violent shocks which are produced by discharging steam under water to concede the issue that the collapse of such vapor pockets might well lead to exceedingly high local stresses." It seems more likely that he merely intends to take issue with the writers regarding the actual numerical values of the maximum pressures. It is to be noted that the writers have not established any such numerical values, but have simply carried the computation of Table 3 to the maximum value permitted by the best available compression tests, those of Bridgeman.[7]

Professor Rouse bases his criticism of the writers' analytical work on the "artificial * * * nature of the assumption of constant kinetic energy in the specimen slug of water," presumably referring to the neglect of the items of viscosity, vapor pressure, and rotational energy. However, the writers believe that in the types of cavitation encountered in the large stationary structures under consideration, the preponderance of evidence indicates strongly that these neglected items are of secondary importance in comparison with the main item of kinetic energy. The writers' analysis is not intended to establish precise values of maximum pressure, but merely to furnish a qualitative conception of the fundamental cause of pressure concentration. They feel that the assumption of constant kinetic energy is satisfactory for this purpose, and that the "artificial nature" of this assumption is not of sufficient consequence to otherthrow this position.

Professor Rouse mentions that the assumption of a moving slug of water in a tapering channel is less elegant than that of a momentary spherical bubble of water vapor within an initially stagnant, incompressible liquid of arbitrary volume. The writers feel that, although the latter would be a reasonable assumption to illustrate the shock caused by discharging a steam bubble under water, it has little pertinence in connection with cavitation of the type under discussion, where the kinetic energy of the flying masses of water is very large in comparison with their potential energy of position with respect to the centers of the adjacent voids.

This point may be explained in more detail as follows: The kinetic energy developed by the collapse of a vacuum bubble in initially stationary water under atmospheric pressure is equal to the volume of the bubble times the atmospheric pressure, the latter corresponding to a water column about 34 ft high. The kinetic energy of a slug of water moving at a given velocity is equal to the volume of the slug times the pressure developed by a column of water of height equal to the velocity head. In the case of flow in an outlet conduit of a dam 350 ft high, the latter column might be about 340 ft high. Professor Edgerton's moving pictures[6] of cavitation in a diverging tube show a succession of vapor bubbles separated by moving slugs of water having approximately the same size as the bubbles. Thus, if cavitation of this kind occurred in the conduits of a dam 350 ft high, the initial velocity of the moving slugs would provide about ten times as much kinetic energy for destructive cavitation effects as would be provided by the release of potential energy during the collapse of the voids from initially static conditions.

Professor Rouse mentions that "high-speed motion pictures by H. Mueller[24] of the collapse of vapor bubbles produced in cavitation would indicate that the bubbles tend to flatten on the downstream side rather than to dwindle concentrically," and that "Conclusions of this nature would tend to invalidate much of the authors' basic hypothesis, as do piezoelectric measurements by J. Ackeret[25] of only moderately high instantaneous stresses in zones of cavitation." The writers have already pointed out that Professor Rouse's conception of cavitation shock as arising from the collapse of spherical vapor bubbles in a static liquid is not applicable to the types of cavitation under consideration. The flattening of the vapor cavities is also observed in Professor Edgerton's photographs.[6] This flattening strongly disproves the spherical-bubble-collapse theory and confirms the writers' thesis that the collapse of a void under typical cavitation conditions occurs when the void is caught and compressed between swiftly moving water on its upstream side and stationary or less swiftly moving water or solid material on its downstream side.

With regard to J. Ackeret's piezometric measurements of pressures in cavitation zones, attention may be called to the utter lack of success of elaborate attempts by other experimenters to attain direct measurements of cavitation pressures. This point is mentioned by Mr. Merriam, who states that it should be emphasized that the multiplication of impact leads to tremendously high concentrations of stress over an exceedingly limited area, and that this is well illustrated by the experiments cited in the paper by J. C. Hunsaker,[5] in which ordinary methods of measuring high pressures such as are used in internal combustion engines, gun barrels, etc., failed completely as criteria of cavitation severity. Mr. Merriam also states that the use of thin diaphragms, backed up by perforated plates, proved in vain because they were punctured merely by accidental hits rather than by a widespread blow. The impossibility of obtaining consistent piezometric measurements in, or closely adjacent to, cavitation pockets was amply verified during the writers' experiments on models of the outlet conduits of the Madden Dam and other dams.

The writers do not disagree with Professor Rouse's point that a positive wave must invariably be followed by a negative wave of equal magnitude, and that fatigue due to the resulting repeated stress reversal is of importance in causing the ultimate failure of the material. This fact, together with the fact that the negative pressure is especially important in destroying concrete because the latter is much weaker in tension than in compression, is implied in a paragraph of the paper preceding "Description of Apparatus for Cavitation Tests." Moreover, the writers feel that their main thesis is not jeopardized by Professor Rouse's prediction that the "theory of fatigue through repeated impact of not-too-great magnitude will eventually be more generally accepted" as a cause of cavitation. Experiments to determine the fatigue of metals in indentation tests are apparently not available, but from a knowledge of the great pressure intensity required to produce a perceptible permanent indentation under the Brinnell ball or Rockwell diamond in hardness tests, and from a knowledge of the relationship of the stress-reversal fatigue limit to the ordinary elastic limit in tensile or cross-bending tests, it does not seem at all unlikely that the indentation fatigue limit in the harder metals known to be attacked by cavitation may be at least of the order of the highest values of Table 3. If Professor Rouse's words "not-too-great magnitude" refer to stresses of an order considerably lower than this, the writers would take issue with him. In this connection, the writers are not referring to soft or mild metals of the ordinary structural grade, but to harder materials having an elastic limit of the order of several times that of ordinary structural steel. Since no upper limit has been found to the hardness of metals pitted in accelerated cavitation tests, and in view of the intense forces required to produce any type of permanent displacement among the molecules of the hard metals by either single loading, repeated loading, or successively reversed loading, it seems impossible to escape the conclusion that extremely high pressures on at least pin-point areas can be produced by the action of water under cavitation conditions. Further considerations bearing on this topic will be mentioned in the writers' reply to Mr. Mousson's discussion.

The writers are pleased that Mr. Hickox agrees with their findings and conclusions in so many respects, a considerable number of these conclusions being restated in his discussion. What seems to be an essential disagreement between Mr. Hickox and the writers is stated as follows in his derivation of Eq. 29:

> "It might be assumed from the authors' remarks following Eq. 8c that operation of a model in accordance with Froude's law requires that

> the ratio of absolute pressures in model and prototype be equal to the model scale. This is not correct."

The source of this disagreement apparently results from a difference in the definition used to extend Froude's law to cover phenomena affected by pressures due to other sources than the action of gravity. In the sentences preceding and following Eq. 8c the writers have defined Froude's law as stating essentially that the ratios of all heads including vapor pressure heads in the model and prototype must vary as the square of the ratio of the velocities in model and prototype, whereas the definition used by Mr. Hickox omits the words "including vapor pressure heads." The actual physical facts, independently of this definition, are expressed correctly by the writers' equations. The same remarks apply to Mr. Hickox's comments on Eq. 13, the apparent difference of opinion lying merely in the definition used for Froude's law. The discrepancy that Mr. Hickox thinks would occur between the physical results of tests based on Eqs. 12 and 13 does not actually exist, both of these equations being derived from Eqs. 9c and 11 by simple algebraic transposition.

It might be argued that the best method of extending the definition of Froude's law to cover phenomena not investigated by Froude himself is a matter of personal opinion. However, the writers feel that their extension of this definition has important advantages from the standpoints of logic and convenience.

Mr. Hickox is not quite in agreement with the writers' description of a cavitation pocket "as a void space filled with flying slugs of water," but regards it rather as "a region in which cavities form, travel with the stream, and finally collapse." The latter description is especially applicable to the cavitation phenomenon that occurs in a gradually diverging tube, as illustrated by Professor Edgerton's photographs.[6] However, the former description is better fitted to the cavitation occurring behind sharp corners or projections. Although the writers feel some sympathy toward Mr. Hickox's statement of the reasons why he dislikes the use of the term "cavitation pocket," they feel constrained to use this term until a better one is suggested. "Cavitation area" is objectionable for the designation of a volumetric space, and "cavitation region" is more properly applied to the entire region in which the phenomenon is occurring, including not only the space actually traversed or occupied by vapor voids, but also that of the surrounding flow pattern.

In the fourth paragraph following Eq. 29 Mr. Hickox states that "It is doubtful if the discharge can be assumed arbitrarily. In order that the model flow pattern shall approximate that of the prototype, there should be at least a semblance of agreement with Froude's law." The writers disagree completely with this statement. Works on hydrodynamics contain numerous illustrations of streamline patterns that are fixed by the physical boundary conditions and are independent of the velocity, that of the ordinary jet from a sharp-edged orifice being a familiar example. In the same paragraph, Mr. Hickox calls attention to the fact that the writers have used a numerical example in which the velocity scale is exactly equal to the square root of the head scale, and states that "One is led to believe that * * * such good agreement could hardly be fortuitous." Since the enclosed tank apparatus was originally designed for the accommodation of models of scale conforming to the foregoing relationship, it was often found convenient to assume a pump discharge that would fit this relation exactly. This had the advantage of making the velocity scale come out in even figures. The discharge used in this particular example was assumed in this way. However, other arbitrary pump settings could have been used as well (and were actually used in many tests) without distorting the flow pattern. In operating the diverging-tube apparatus, great deviations from the Froude's law setting were used normally, since a velocity scale far in excess of the square root of the linear model scale was required to produce the necessary pressure reduction in the critical cavitation regions.

In the third from the last paragraph of his discussion, Mr. Hickox states or implies that, if a model were operated according to the equations given by the writers, it might be possible for the model to indicate cavitation that would not occur in the prototype. The writers would like to emphasize that Mr. Hickox is mistaken on this point. His misunderstanding of their equations in this connection probably arises again from the lack of an accepted generalization of Froude's law, applicable in cases where pressure heads other than those due to gravity are involved. Consider a model of the conduit entrance of a dam on which the head is variable, and suppose this to be operated without gravitational or operational distortion and in accordance with the writers' equations. At low head, there will be no cavitation in the model or prototype. At a certain head, incipience of cavitation will appear at some point in the prototype, and the writers' equations provide that the adjustment of pressures and velocity in the model will cause incipient cavitation to appear in the model at exactly the head corresponding to this prototype head and at exactly the corresponding point. At higher heads, stronger cavitation will appear in both model and prototype at this point and other pockets may possibly open up. There is no possibility for cavitation to occur in the model where it does not appear in the prototype, or vice versa. The analytical reasoning establishing the correctness of the foregoing statement is given in the paper in such detail that it seems unnecessary to repeat it. The sentence on which Mr. Hickox's argument is based is stated in the following words by him:

> "If the model is adjusted to operate in conformity with Froude's law [as defined by Mr. Hickox this does not require vapor pressure head scale to be proportional to the square root of the velocity scale], and the absolute pressures in reservoir and conduit barrel are adjusted to conform to the linear scale ratio, the model may indicate cavitation that would not occur in the prototype * * *."

This statement is perfectly ture, but the fallacy of the argument lies in the fact that the words quoted do not state the same thing as the writers' equations.

As stated in the paper, if the model is operated with gravitational or operational distortion, the gage settings to produce incipience of cavitation in the model at exactly the same comparative head and at the same point at which it occurs in the prototype cannot be made to apply to the entire fluid system of the model, but a separate setting must be used to test each point at which it is desired to know whether cavitation would occur in the prototype. For example, Mr. Hickox states that he "would like to know how the methods of the paper were applied to a rotated vertical diametrical section of the entrance of the Hiwassee sluice, for example, which was found to be entirely free from cavitation." The Hiwassee cavitation tests are described in the original manuscript on file in the Engineering Societies Library.[3] Since the entrance bell of this circular conduit is nearly a surface of revolution, the first and most likely point to be tested for possible cavitation was at the conduit roof just inside the entrance, where the general pressure was lower than at the floor, because of shallower submergence, and where the velocity was as high as anywhere else. The gage settings were made according to the equations of the paper and the model showed no cavitation at this point, thus proving that there would be no cavitation at the corresponding point in the prototype. Other suspected points, near the rails carrying the closure bulkhead, etc., were investigated in a similar manner but no cavitation was found. By exaggerating the general cavitation-severity condition in the model, it was easily proved that no doubtful points had been overlooked.

The writers appreciate the constructive criticisms by Mr. Mousson, and in particular they are glad to accept his word of caution regarding the importance of the opening up of fatigue cracks as a feature in the mechanism of cavitation damage to metals. As they have stated in the paragraph preceding Eq. 5, the writers realize that "The law governing the * * * process of pressure multiplication undoubtedly changes when the crevices become so small as to approach molecular dimensions." Since the theoretical investigation of the action taking place in very small crevices is definitely a problem for the molecular physicist rather than for the engineer; since direct experimental measurement of pressures in such crevices appears to be totally lacking; and since Mr. Mousson is able to cite practical cases, where cavitation damage to turbine runners was not increased by the presence of hair cracks, and laboratory tests, where subsurface disintegration occurred without surface cracking, the writers are quite willing to hold in abeyance further speculations regarding cavitation action in fine cracks.

Mr. Mousson joins with Professor Rouse in questioning the probable occurrence of pressures approaching or exceeding the higher values of Table 3, and in believing that cavitation damage to metals can be explained satisfactorily as a fatigue phenomenon due to stresses comparable to the lower values of the table. Because of the failure of the most painstaking experiments to give any inkling of true cavitation pressures, this entire topic lies in the realm of speculation. However, the writers believe that the preponderance of evidence points to the conclusion that the harder metals cannot be destroyed in the absence of local pressures of at least the order of magnitude of the higher values in Table 3. A striking illustration of the high intensity of pressure that can be sustained on a local area of a plane surface of a hard metal without producing a permanent deformation is found in the classic series of tests of the bearing of locomotive wheels on steel rails, reported by the late C. L. Crandall,[35] M. Am. Soc. C. E., and in the discussion by the late J. B. Johnson,[36] M. Am. Soc. C. E. In these tests pressures of 164,000 lb per sq in. produced no permanent set.

After reading Mr. Mousson's discussion, one of the writers made some experiments with the Brinnell-ball apparatus to check the pressure intensity required to produce a perceptible permanent indentation in the surface of a hard metal. The first steel specimen selected at random for this test had a Brinnell hardness number of 524, which is about "file hard." Under a load of 500 kg on the Brinnell ball, a slight but definitely perceptible indentation 1.1 mm in diameter was produced on the surface of this specimen. A little arithmetic shows the average pressure on the contact area to have been just less than 750,000 lb per sq in. It is probable that the maximum pressure intensity at the center of the indentation was materially greater. A similar test was run on a piece of metal from an ordinary structural steel angle, the Brinnell hardness number of this steel being 90.4, and a perceptible indentation of 1.40 mm in diameter being produced by a load of 150 kg on the Brinnell ball. The mean pressure under the latter loading thus was about 139,000 lb per sq in., whereas the maximum at the center of the indentation was probably higher. These values carry an implication that, in view of the enormous local pressures required to produce slight permanent indentations (or permanent displacements of the molecules) on the surfaces of hard metals, it might reasonably be expected that pitting would not take place on such metals unless the pressures developed within the cavitation pocket were at least as high as the larger values of Table 3. Although no cavitation tests have been made on the particular pieces of steel mentioned, experience with such tests indicates definitely that extreme hardness or toughness of the metal is no barrier to ultimate pitting and disintegration under severe cavitation conditions.

Analogy with various types of fatigue tests makes it highly probable that the fatigue limit or repeated pressure intensity required to produce a permanent disintegration of the metal would be at least a large fraction of the pressure producing the first permanent set. Moreover, numerous short-time cavitation tests of metals are on record[18] where fatigue has not had time to become effective and where the first evidences of damage are a few scattered tiny pits or prick-punch indentations due to individual blows. Because of the minuteness and the scattered nature of the individual pits or areas in which the extreme pressures act, the writers do not concur with Mr. Mousson's argument that pressure "values in the upper part of the range [of Table 3] are not likely to occur since damage would be so severe in a matter of minutes as to endanger any structure within the same time interval."

Mr. Mousson states that another item of evidence against the theory of local high-pressure points in cavitation regions is the fact that, in tests on metals in cavitation stands, throat velocity has been found to be one of the most important variables with respect to pitting. The writers do not recognize the soundness of this argument, since the rate of pitting would obviously increase with the energy content of the individual blows and their frequency, even if the maximum pressure at the center of each affected spot did not increase. However, in this connection it seems desirable to repeat that the writers' analytical work in Eqs. 1 to 7 and in Table 3 was not intended as a complete analysis of the mechanism by which high-pressure points are developed in cavitation regions, but was intended merely to explain or illustrate one of the most salient features of this mechanism—namely, the tendency toward energy concentration at the final point of a void collapse. It should by no means be thought that the writers regard the neglected items of viscosity, vapor pressure, and rotational energy as so unimportant that the maximum intensity of pressure developed by a vapor pocket collapse is entirely independent of the total quantity of energy in the moving water that is brought to rest.

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[35] "Friction Rollers," by the late C. L. Crandall and A. Marston, *Transactions*, Am. Soc. C. E., Vol. XXXII, July-December, 1894, p. 99.

[36] *Loc. cit.*, p. 270.

The discussion by Mr. Douma calls attention to the desirability of establishing methods or criteria, based on free-jet profiles, for designing conduit entrances that will be free from cavitation. The writers are much interested in Mr. Douma's ideas on this topic, since they have done considerable analytical and experimental work along somewhat similar lines (described in some detail in the original manuscript on file in the Engineering Societies Library[3]). The rigid space requirements for publication made it necessary to eliminate somewhat more than three fifths of the original subject matter, the section on the design of entrance bells being one of those deleted. The writers are in full agreement with Mr. Douma on the principle that the nappe of a free jet issuing from a sharp-edged orifice has the ideal shape for a cavitation-free entrance bell, and that entrance bells having solid parts projecting inside the location of this nappe surface are especially liable to cavitation. However, it may be remarked that it is not feasible to use this principle in designing an entrance bell unless the upstream face of the dam in the vicinity of the entrance is a plane normal to the conduit axis. Moreover, entrance bells designed with the "ideal" profile will usually have a considerable factor of safety, since the pressure on all surfaces of the bell is equal to that in the upstream end of the conduit proper, and therefore considerably above atmospheric pressure (if the conduit is level or nearly so), whereas the pressure head required to induce cavitation is about 33 ft below atmospheric pressure. This fact may be utilized to achieve economy in the design of entrance bells for conduits under comparatively low head. In the original manuscript, a table of coordinates for the profile of an ideal entrance bell is given, based on G. Kirchoff's exact equations for the free-jet profile[37] rather than on the ellipse used as an approximation by Mr. Douma. Also, a table is included, based on original experiments, giving the end and side contractions of the jet for conduits of rectangular cross sections having various slenderness ratios.

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[37] "Hydrodynamics," by H. Lamb, Cambridge University Press, 6th Ed., 1932.

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DESIGN OF ACCELERATION AND DECELERATION LANES

By ADOLPHUS MITCHELL,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. H. F. Holley, D. W. Loutzenheiser, Hawley S. Simpson, Milton Harris, T. F. Hickerson, W. L. Waters, R. A. Moyer and Donald T. Davidson, Stephen E. Butterfield, and Adolphus Mitchell.

Synopsis

Mark Twain once said: "Everybody talks about the weather, but nobody does anything about it." The design of acceleration and deceleration lanes is in a position similar to that of the weather. This paper is an attempt to do something about it. The writer hopes discussers will present data sufficient to preclude argument on some of the controversial factors involved.

Introduction

The purpose of a deceleration lane is to provide a means for vehicles to leave the highway at cruising speed and come to a stop or a lower speed before entering a street, filling station, or other destination. The purpose of the acceleration lane is to provide a means by which slow-moving traffic can reach the cruising speed of a highway before entering its lanes. The deceleration lane should be long enough to provide for a "weaving distance," motor deceleration to a speed at which braking can be done safely, and a comfortable braking distance from that point on. ("Weaving distance" is used herein to describe the transverse movement from one lane to another whether converging to, or diverging from, the traffic stream.) The acceleration lane should provide for full acceleration to cruising speed, and a weaving distance sufficient for the driver to enter the highway, provided there is a space between cars into which he can turn. In the event no such space is available, or made available by cars on the outside lane moving over, sufficient perception and brake reaction distance should be allowed in addition to a braking distance which would not necessitate pitching the passengers to the floor of the car. A sketch of the elements of the deceleration and acceleration lanes, as proposed, is shown in Fig. 1. The function of the insulation strip is for channelization, to encourage

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Note.—Published in March, 1941, *Proceedings*.

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proper use of the speed-change lanes, and to eliminate points of conflict. As will be seen in the illustration, the side entrance to the lanes is tapered so as to permit gradual merging with, or diverging from, the traffic stream on the highway.

Deceleration Lanes

The first element of the deceleration lane as detailed in Fig. 1 is the weaving distance. This distance should be sufficient to allow safe turning at cruising speed from the highway into the deceleration lane. The curves in Fig. 2 give the weaving distance for any cruising speed with or without an insulation strip. (An insulation strip—see Fig. 1—is any physical barrier separating the deceleration lane from the highway proper. This barrier guarantees correct use of the deceleration lane.) The curves are based upon the formula derived in the Appendix. The width w (see Fig. 1) varies with the speed of the vehicle as follows:

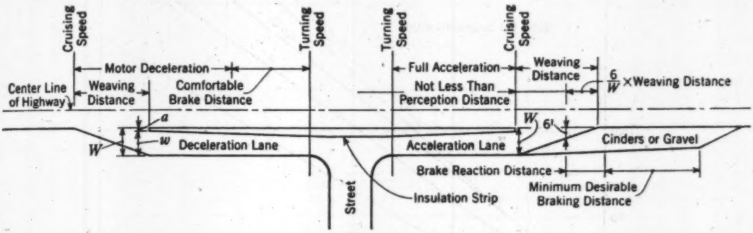
Cruising Speed
Turning Speed
Turning Speed
Cruising Speed
Motor Deceleration
Full Acceleration
Weaving Distance
Center Line of Highway
Weaving Distance
Comfortable Brake Distance
Not Less Than Perception Distance
$\frac{6}{W}$ × Weaving Distance
a
W
w
Deceleration Lane
Acceleration Lane
W
6'
Cinders or Gravel
Brake Reaction Distance
Minimum Desirable Braking Distance
Street
Insulation Strip

Fig. 1

Speed, in miles per hr	Width, w, in ft
60	14.6
50	13.8
40	13.0
30	12.2
20	11.4
Minimum	11.0

The assumption of an S-path of travel is a theoretical expedient entirely, and there is no doubt that drivers would find a spiral path more natural than a reverse circular path. The formula derived on the basis of the S-path contains all the variables involved in the natural path. For this reason it is necessary only to determine suitable constants for this formula. Grade is a negligible factor as shown in theoretical studies. Field studies of the weaving distance as measured in terms of time and speed lead to an average coefficient of friction of 0.16. This value is identical with that used everywhere as a comfortable factor for determining the degree of curvature for highways and should not be confused with the maximum obtainable coefficient of friction between tire and pavement. A comfortable coefficient of friction is one that does not make it difficult to keep one's seat while traversing a curve. A comparison of results shown by the curves in Fig. 2 with accepted values of weaving distances is very favorable. Most studies of weaving distances have been made in perfecting the operation of traffic circles where sharp curvatures make the maneuver difficult.

Motorists traveling at high speeds do not like to apply their brakes. This reluctance on the part of the driver must be considered in the determination of the distance required for deceleration, or "slowing" on the highway will result. Values of motor deceleration have been determined by Donald W. Loutzenheiser,[2] Jun. Am. Soc. C. E., for passenger cars on a level grade.

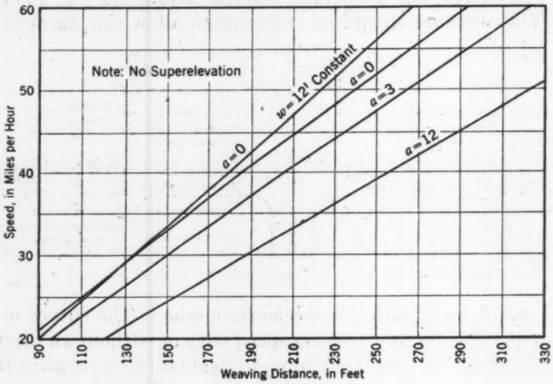

FIG. 2

"Motor deceleration" is the process of slowing the car in gear, with the foot off the accelerator. Fig. 3 gives motor deceleration distances for passenger cars on grades from 3% upgrade to 3% downgrade. These curves are based upon an extension of the results found by Mr. Loutzenheiser. The derivation of the mathematical formulas to include the effect of grade is presented in the Appendix. It was found that motor deceleration from a speed of 40 miles per hr or faster to a speed of 25 miles per hr or less, without resorting to braking, severely taxed the driver's patience. Drivers interviewed by the writer indicated that a preponderance of them think they do not apply brakes until they reach 35 miles per hr or 30 miles per hr, unless it is necessary to do so. Studies, by John Beakey,[3] of normal deceleration from speeds of 50 to 60 miles per hr by the public on approaching a "stop" sign at a highway intersection indicate that there is very little braking at 40 miles per hr, and to all practical purposes substantiate the belief of the persons interviewed. Best results should be obtained by varying the speed to which motor deceleration is assumed with the design speed of the highway (that is, assume 40 miles per hr for a 70-mile-per-hr highway and 30 miles per hr for a 50-mile-per-hr highway). This would be in keeping with the sensation of speed as felt by the driver as well as the time element involved.

The last part of the deceleration lane is that required for a comfortable braking distance from the end of the motor deceleration period to the speed at which it is desired to bring the motorist at the end of the lane. Fig. 4 shows curves for the comfortable braking distance from a given speed to a full stop

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[2] "Speed-Change Rates of Passenger Vehicles," by Donald W. Loutzenheiser, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1938, p. 90.

[3] "Acceleration and Deceleration Characteristics of Private Passenger Vehicles," by John Beakey, *loc. cit.*, p. 81.

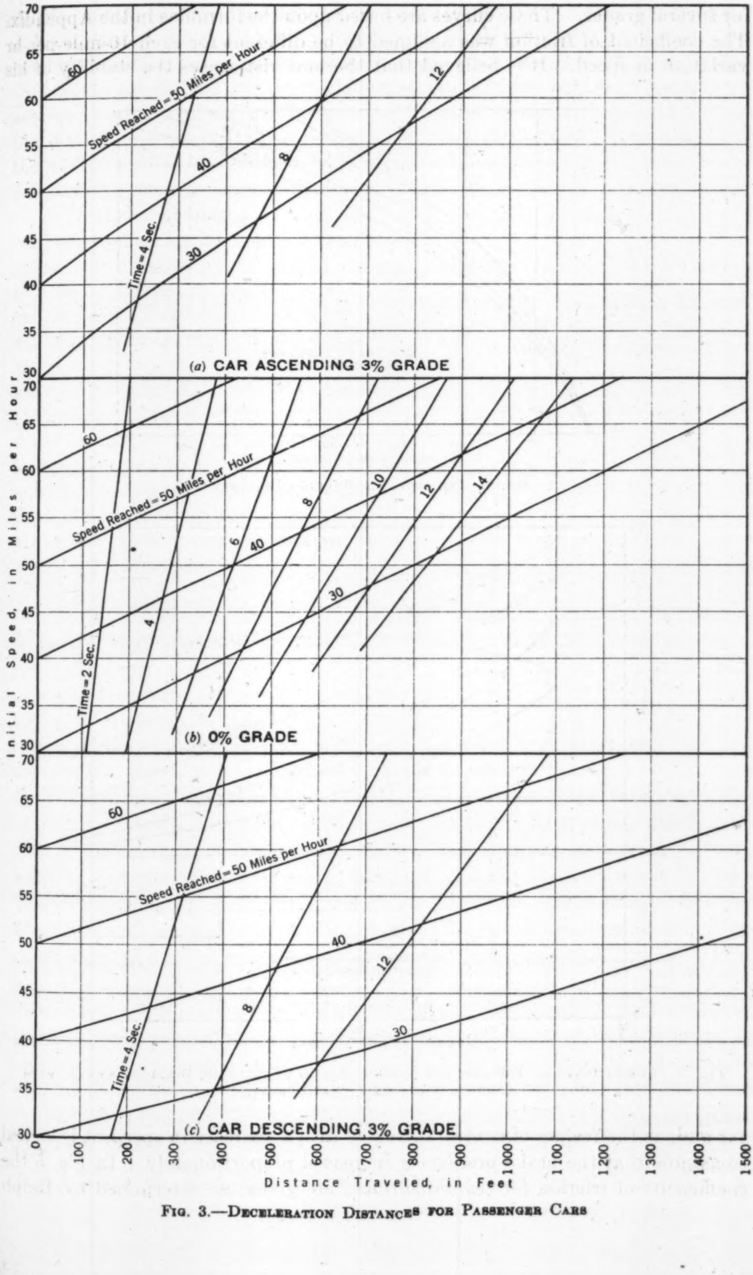

FIG. 3.—DECELERATION DISTANCES FOR PASSENGER CARS

for several grades. These curves are based upon the formulas in the Appendix. The coefficient of friction was assumed to be different for each 10-mile-per-hr variation in speed. It is believed that the motorist senses the stability of his car and, as the feeling of control increases with a decrease in speed, it is logical to assume that the brake pressure is increased proportionately. In Fig. 5 the coefficients of friction for several surfaces are given, as determined by Ralph

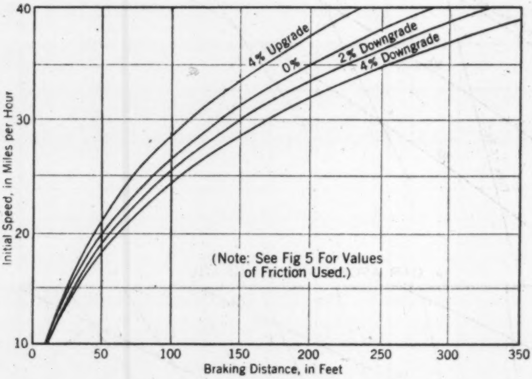

FIG. 4.—COMFORTABLE BRAKING DISTANCE

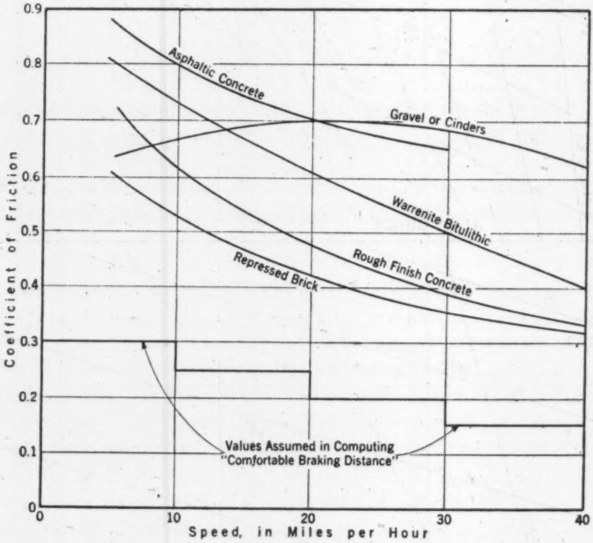

FIG. 5.—COEFFICIENTS OF FRICTION FOR VARIOUS SPEEDS (FULL SKID, STRAIGHT AHEAD, WITH SMOOTH TIRES ON A WET SURFACE)

A. Moyer,[4] Assoc. M. Am. Soc. C. E. Mr. Beakey concluded that a rate of deceleration corresponding to a coefficient of friction of about 0.4 is the maximum without discomfort to passengers. As it was desired to use values that would correspond closely to the usual habits of the average driver, it was assumed that a coefficient of friction of 0.15 at 40 miles per hr would increase to 0.3 at 10 miles per hr. The distances indicated in Fig. 4 checked closely with those observed by Mr. Beakey, which substantiate the assumptions made.

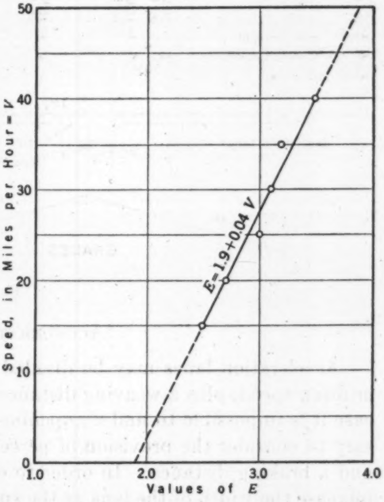

FIG. 6.—AVERAGE DISTANCE, *E*, OF RIGHT FRONT WHEEL FROM EDGE OF 22-FT CONCRETE PAVEMENT, IN FEET

A further refinement of the deceleration lane can be made by funneling. By funneling is meant to provide a decreasing width of lane as the speed decreases. Operators will not drive as fast on narrow as on broad pavement. J. T. Thompson and Norman Hebden[5] determined the average distance of the right front wheel from the edge of the pavement for speeds from 40 miles per hr to 15 miles per hr. Fig. 6 shows results of their observations with an extrapolation sufficient to cover most requirements. In this curve,

$$E = 1.9 + 0.04\,V \ldots\ldots (1)$$

is the average distance, in feet, of the right front wheel from the edge of a 22-ft concrete pavement. Until better information is available, it is suggested that these results be used as a basis for the variation in width of deceleration lanes, provided the minimum is kept at 11 ft.

To illustrate the application of the foregoing fundamentals to the approach of a filling station, attention is directed to Fig. 7. The cruising speed of the highway is assumed to be 50 miles per hr. Entering Fig. 2, it is found that a weaving distance of 265 ft will be required if a 3-ft insulation strip is used. The car is going up a 3% grade as it approaches the filling station, and (see Fig. 3(*a*)) a distance of 550 ft will be required for motor deceleration to a speed of 30 miles per hr, at which speed brakes are assumed to be applied. From Fig. 4 it is found that 120 ft would be required for a comfortable braking distance to a full stop. The 50-ft radius of the turn-in will permit a 15-mile-per-hr speed very comfortably. Therefore, it is not necessary to bring the motorist to a complete stop. From Fig. 4 it is found that the comfortable braking distance from a speed of 15 miles per hr to a complete stop is approximately 20 ft. The distance required for comfortable braking from a speed of 30 miles per hr to one of 15 miles per hr, therefore, is 120 − 20 = 100 ft. In this manner, and with these assumptions, it is found that a total distance of 650 ft is required to bring a car from a cruising speed of 50 miles per hr to a turning speed of 15 miles per hr.

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[4] "Skidding Characteristics of Automobile Tires on Roadway Surfaces, and Their Relation to Highway Safety," by Ralph A. Moyer, *Bulletin No. 120*, Iowa State College Eng. Experiment Station, Ames, Iowa, 1934.

[5] "A Study of the Passing of Vehicles on Highways," by J. T. Thompson and Norman Hebden, *Public Roads*, U. S. Dept. of Agriculture, Bureau of Public Roads, 1937, pp. 121–137.

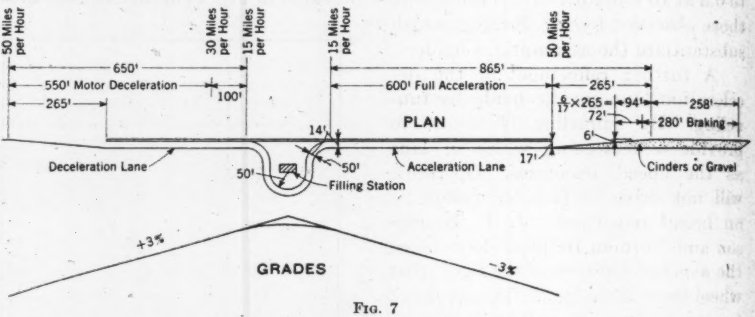

Fig. 7

Acceleration Lanes

Acceleration lanes may be divided into a distance for full acceleration to cruising speed, plus a weaving distance to permit entering the traffic lane. In case it is impossible to find an opening in the highway traffic, it will be necessary to consider the provision of perception distance, brake reaction distance, and a braking distance. In order to encourage acceleration, it is desirable to increase the width of the lane as the speed increases. The use of an insulation strip is advisable to make certain that the drivers cannot enter traffic in less than a distance sufficient to eliminate dangerous speed differentials.

Values of full acceleration distance were determined by Mr. Loutzenheiser.[8] He defined full acceleration as a maximum pickup possible under the control of typical drivers. The model cars tested varied from 1935 to 1937, and tests were made on flat grades. An extension of these results to include grades from 3% upgrade to 3% downgrade is given in Fig. 8. These curves are based upon the mathematical formula derived in the Appendix.

The weaving distance is the same as that for the deceleration lane. (In the treatment of acceleration and deceleration lanes, the weaving distance has been assumed the same whether for cars diverging from, or converging with, the traffic stream. Mathematically, the diverging weaving distance could be slightly smaller, provided that the accelerator was released on leaving the traffic lane, but, since the decrease in speed would be so small, no difference of practical importance would result.) It is further stipulated, however, that the weaving distance should not be less than perception distance plus a distance depending upon the ratio of the width of a car to that of the acceleration lane at the beginning of the weaving distance, as shown in Fig. 1.

Perception distances can be read directly for any speed from Fig. 9(*a*).

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[8] "Speed-Change Rates of Passenger Vehicles," by Donald W. Loutzenheiser, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1938, p. 92.

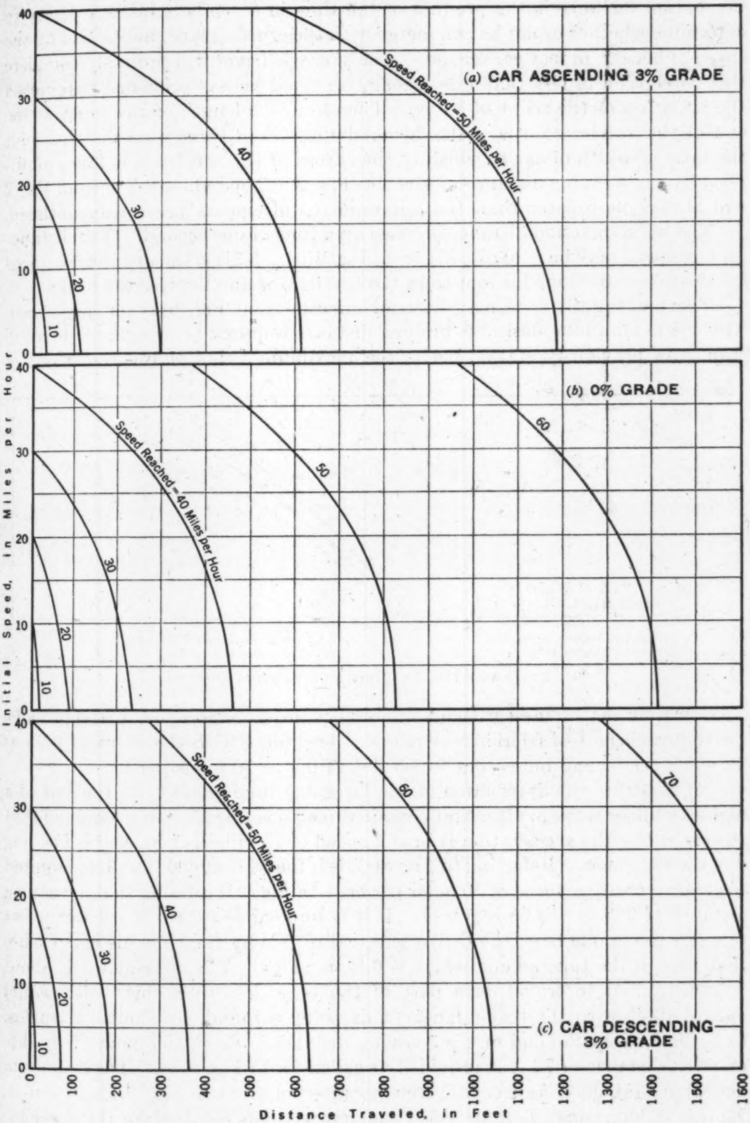

FIG. 8.—ACCELERATION DISTANCES FOR PASSENGER CARS

Perception distance is the product of the time in seconds it takes a driver to determine whether or not he can merge with the cruising lane, multiplied by the speed of his car, in feet per second. The average driver will probably complete this perception in less than one second, provided he has previously appraised the situation at the point of entry. Therefore, the length of the taper at the end of the acceleration lane should not be less than perception distance plus the ratio of width of car to width at the throat of the acceleration lane, multiplied by the weaving distance as given in Fig. 2. The values taken from Fig. 2 will always be greater than the minimum requirements previously outlined.

The brake-reaction distance is based on a time of one second. The distance for any speed may be read directly from Fig. 9(*a*). This is the distance required for the driver to move his foot from the accelerator and depress the brake.

The braking distance may be read directly from Fig. 9(*b*) for any speed. This is the minimum desirable braking distance required to bring a car to a full stop. As previously stated, 0.4 is the maximum value of the coefficient of friction that can be used without discomfort to passengers. A variable value for the coefficient of friction was adopted beginning with the value of 0.25 at 60 miles per hr and increasing to a value of 0.4 at 30 miles per hr.

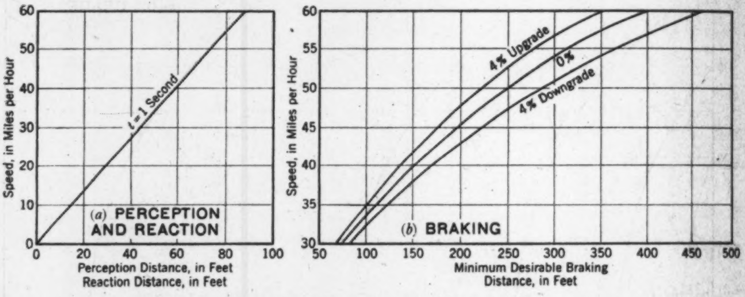

FIG. 9.—PERCEPTION, REACTION, AND BRAKING DISTANCES

To illustrate the application of the foregoing fundamentals to the exit of a highway filling station, attention is again directed to Fig. 7. It is assumed that the car enters the acceleration lane at a speed of 15 miles per hr, and is heading down a 3% grade. Referring to Fig. 8(*c*), it is found that 600 ft will be required to attain a cruising speed of 50 miles per hr. In Fig. 2 it is found that a weaving distance of 265 ft will be required. If it is impossible for the accelerating car to enter the traffic lane, the driver will realize this by the time his car reaches that part of the tapered end which is 6 ft in width. The average car is about 6 ft wide, and to travel on a part of the taper less than this width would mean entry on to the traffic lane. This point is found, by similar triangles, to be 94 ft from the end of the weaving distance. From this point, a brake-reaction distance of 72 ft is provided as called for in Fig. 9(*a*). The minimum desirable braking distance, as given by interpolation in Fig. 9(*b*), is 280 ft. Thus it is determined that an 865-ft concrete apron is required for the acceleration lane. It is found further that an additional 258 ft of the improved shoulder should be provided for emergency stops.

JUSTIFICATION

The provision of adequate acceleration and deceleration facilities at the intersection of a high-speed with a low-speed roadway or turn may be justified by accident experience. Of the accidents on the Schuyler Merritt Parkway, in Connecticut, 71% have been due to internal stream friction. Most of these were further classified as rear-end collisions. Since actual trials indicate that many of the exits and entrances to this highway permit speeds no higher than 20 miles per hr, there remains little doubt as to the location of a sizable percentage of the rear-end collisions, although locations given on accident reports have been too indefinite to be positive in this matter. In cases of this kind, which involve state highway intersections and state-owned roadside parks, there is manifestly a responsibility falling on the state to provide adequate facilities.

The entrances and exits of private property and small business establishments present a problem. It has been estimated that there is a filling station for each 1½ miles of highway. The cost of providing adequate acceleration and deceleration lanes would be several times the value of most of these stations. The problem is even greater when all the eating houses, open-air theaters, etc., are added to the filling stations. Regulation by the state department of traffic control can do much to improve existing conditions on the basis of cooperation between the state and the individual, but the complete solution advocated in this paper appears to be economically justifiable only where the right of access of the abutters is purchased along with the right of way, as has been done in the building of parkways and freeways. It has been predicted that the next decade will bring 25,000 miles of new parkways and freeways, which in itself promises a sizable interest in the problem of speed-change lanes. Once the abutter has no right of access, it is possible to space filling stations so as to leave no doubt of the warrants for the provision of correctly designed acceleration and deceleration lanes.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation to all members of the staff of the Yale Bureau for Street Traffic Research for their encouragement and sound suggestions. Special credit is due Leslie Williams for his painstaking assistance in improving the manuscript.

CONCLUSION

It is concluded that acceleration and deceleration lanes can be reduced to component parts that can be individually analyzed and determined with sufficient accuracy for design purposes. Existing highways have not been provided with adequate acceleration and deceleration lanes. Failure to make such provision has necessitated deceleration to a low speed in the main traffic lanes. As speeds have increased, accident records have disclosed the danger in creating a large differential in speed between traffic on the highway and traffic in the process of leaving or entering it. In designing entrances and exits for an express highway, the principal pitfall to be avoided is insufficient lengths. The motorist will not leave the highway at cruising speed if he knows

it will be necessary to bring himself to a violent stop on a deceleration lane that is much too short. He cannot avoid dangerous differentials in speed if the acceleration lane is too short.

APPENDIX

DERIVATION OF FORMULAS

***Weaving Distance*.—From geometry, "the sum of the squares of the legs is equal to the square of the hypotenuse"; hence (see Fig. 10),**

$$\left(\frac{L}{2}\right)^2 + \left(r - \frac{W}{2}\right)^2 = r^2 \text{..(2a)}$$

or

$$L = \sqrt{W\,(4\,r - W)} \text{..(2b)}$$

Standard texts develop the formula for superelevation:

$$r = \frac{V^2}{15\,(f + e)} \text{....(3)}$$

in which: V = speed in miles per hour; f = coefficient of friction; e = superelevation in feet per foot; and r = radius in feet.

The edge clearance,[5] in feet, should be equal to $1.9 + 0.04\,V$ (see Fig. 6); hence

$$W = a + b + 3.8 + 0.08\,V \text{.(4)}$$

in which: a = width of insulation strip in feet; and b = width of car in feet.

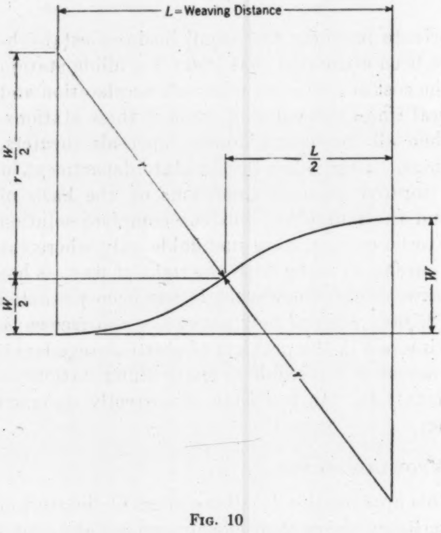

FIG. 10

***Motor Deceleration Formulas*.—From Newton's Second Law of Motion and Fig. 11, the effect of grade on motor deceleration is:**

$$\text{Deceleration} = -\,g\,\frac{G}{100} \text{..................(5)}$$

in which g = acceleration of gravity in feet per second[2] and G = per cent grade. The motor deceleration[2] is as shown in Fig. 12 for flat grades; hence

$$\text{Motor deceleration} = -\,a = \frac{3}{100}\,(v + 15) - g\,\frac{G}{100} \text{.......(6)}$$

Integrating and converting v to miles per hour:

$$\left(V + 10 - g\,\frac{G}{4.5}\right) e^{(3/100)t} = \bar{V} + 10 - g\,\frac{G}{4.5} \text{..........(7)}$$

in which t = time in seconds; $\bar{V}$ = initial speed; and V = final speed. Integrating again, the deceleration distance is

$$s = 50\left[-\left(10 - g\frac{G}{4.5}\right)\log_e\left(\frac{\bar{V} + 10 - g\dfrac{G}{4.5}}{V + 10 - g\dfrac{G}{4.5}}\right) + (\bar{V} - V)\right] \quad (8)$$

in which G changes sign to minus for uphill travel.

Braking Distance.—Let G = per cent grade; F = force in pounds; $\bar{V}$ = initial speed in miles per hour; V = final speed in miles per hour; s = braking distance in feet; W = weight of vehicle in pounds; g = 32.2 ft per sec²; and f = coefficient of friction. From energy conservation:

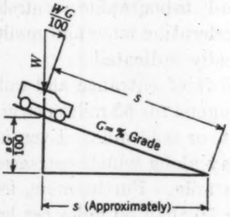

FIG. 11.—EFFECT OF GRADE

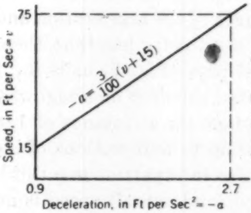

FIG. 12

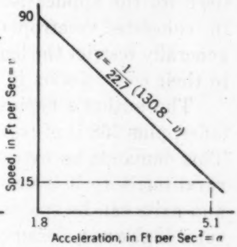

FIG. 13

$$F \times s = \frac{1}{2}\frac{W}{g}(1.47)^2(\bar{V}^2 - V^2) + \frac{WG}{100}s = \frac{W}{g}(fg)s \quad (9)$$

For downhill travel (solving Eq. 9 for s):

$$s = \frac{\bar{V}^2 - V^2}{30f - 0.3G} \quad (10a)$$

and, for uphill travel:

$$s = \frac{\bar{V}^2 - V^2}{30f + 0.3G} \quad (10b)$$

Full Acceleration.—From Newton's Second Law of Motion and Fig. 11, the effect of grade on full acceleration is:

$$\text{Acceleration} = g\frac{G}{100} \quad (11)$$

Full acceleration² is as shown in Fig. 13 for flat grades; hence,

$$\text{Full acceleration} = a = \frac{1}{22.7}(130.8 - v) + \frac{gG}{100} \quad (12)$$

Integrating twice, the acceleration distance is:

$$s = 34.05\left[(87.2 + 0.1513\,gG)\log_e\frac{87.2 - \bar{V} + 0.1513\,gG}{87.2 - V + 0.1513\,gG} - (V - \bar{V})\right] \quad (13)$$

In Eq. 13, G is positive for downhill travel and negative for uphill travel.

DISCUSSION

H. F. Holley,[7] **M. Am. Soc. C. E.** (by letter).—The design of acceleration and deceleration lanes as presented in this paper clarifies a theory that engineers engaged in highway design have been applying in a more or less rule-of-thumb manner for many years.

Acceleration and deceleration lanes are fast becoming the rule rather than the exception in modern highway construction, especially in the design of motorways in metropolitan areas. However, the opportunity rarely presents itself for the application of theory in laying out motorway entrances and exits in congested metropolitan areas. Land values and topographic controls generally restrict the length of the acceleration and deceleration lanes and result in their construction in lengths far less than theoretically indicated.

The author's typical plan (Fig. 7) calls for 1,665 ft of entrance and exit lanes, plus 258 ft of treated shoulder on a highway designed for 50 miles per hr. This demands an extra lane for a distance of 1,923 ft, or 0.36 mile. Even in rural territory it is unusual to find sections of highway along which entrances and exits can be spaced as far apart as one third of a mile. Furthermore, for rural highways, designed (and actual) speed is nearer 70 than 50 miles per hr. Entrance-exit treatment as proposed by Mr. Mitchell on most 60-mile rural highways would call for a continuous extra lane. It was this condition that led to the practice of establishing "service roads" on either side of high-speed rural highways in territory subject to development—first into residential, then small business, and finally into "shoestring" business use.

In the writer's judgment, Mr. Mitchell has been ultraconservative in his requirements for motor deceleration and braking distances. Extensive observations on driver habits in the Los Angeles (Calif.) metropolitan area indicate that deceleration by engine compression is seldom practiced. California drivers tend to approach even a boulevard "stop" intersection under power, and apply their brakes immediately after releasing the throttle. Upon approaching a signalized intersection, the tendency is to speed up in the hope of finding the signal on the "go" upon arrival at the intersection.

TABLE 1.—Braking Distances, in Feet

Agency	Year	Speed in Miles per hr		
		20	30	40
National Safety Council	1935	21	47	84
Aetna Casualty and Surety Company	1935	22	49.5	88
"Traffic Engineer and the Police"; consulting bureau	1938	26	57	101
National Bureau of Standards	1935	28	62	109
Automobile Club of Southern California	1930	22.2	50	89

The "comfortable braking distance," shown graphically in Fig. 4, indicates about 50 ft to stop from a speed of 20 miles per hr, 135 ft from a speed of 30 miles per hr, and 280 ft from a speed of 40 miles per hr. Table 1 shows the braking distances determined from experiments by the agencies listed. It is admitted that these braking distances are more in the nature of emergency braking, rather than "comfortable" braking distances. Also, they do not include "reaction-time distance" which, using the accepted reaction time of 0.75 sec, would add 22, 33, and 44 ft, respectively, to the braking distances shown at 20, 30, and 40 miles per hr.

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[7] Asst. Chf. Engr., Automobile Club of Southern California, Los Angeles, Calif.

Mr. Mitchell's paper provides an objective toward which designing engineers should strive. However, in metropolitan motorway design the ideal entrance and exit ramp rarely can be achieved.

The Arroyo Seco Parkway, a motorway extending from the heart of metropolitan Los Angeles to the neighboring City of Pasadena, completed in 1940, involved twenty-two vehicular crossings in a distance of 6 miles. It was impossible, due to constriction caused by the Arroyo Seco Channel and other topographic features, to build conventional "clover leafs" at points of vehicular interchange. It was also impossible, due to the large number of intersecting streets, to design entrance and exit lanes of desired length. However, nonstop motorway features were adhered to, the design calling for two 35-ft roadways separated by a planting strip, no intersections at grade, no left turns, and the prohibition of ingress and egress from abutting property. To compensate for the lack of the desired length of the acceleration and deceleration lanes, a "compressed clover leaf" was designed at most interchange crossings. Generally (see Fig. 14), the exit and entrance ramps at intersecting streets are approximately parallel to the motorway; and, due to the fact that the motorway roadway is depressed, the exit ramps are on ascending grades, whereas the

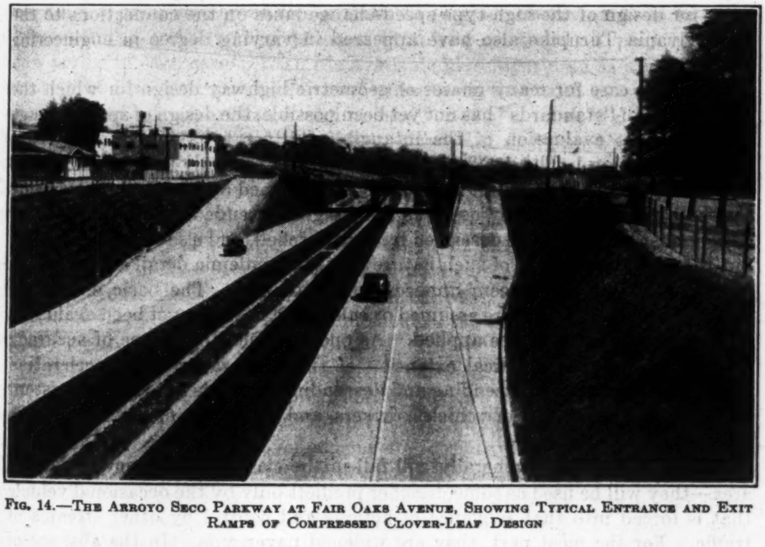

FIG. 14.—THE ARROYO SECO PARKWAY AT FAIR OAKS AVENUE, SHOWING TYPICAL ENTRANCE AND EXIT RAMPS OF COMPRESSED CLOVER-LEAF DESIGN

entrance ramps are on descending grades, permitting relatively fast deceleration and acceleration, with the path of the entering or leaving car roughly parallel to the main roadway. In spite of the fact that entrance and exit lanes are much shorter than indicated as desirable under Mr. Mitchell's theory, traffic using the Arroyo Seco Parkway (which averages about 40,000 vehicles per day) experiences little difficulty in entering or leaving the motorway, and there has been a very favorable accident experience at interchange points.

In the case of an elevated motorway, with intersecting streets crossing under the structure, an unfavorable deceleration and acceleration situation would be encountered at interchange points, and longer exit and entrance lanes should be provided.

Mr. Mitchell is to be commended for reducing to scientific form a matter that highway engineers have been inclined to treat in a haphazard manner.

D. W. Loutzenheiser,[8] Jun. Am. Soc. C. E. (by letter).—The subject of speed-change lanes is an important phase of highway design, to which considerable research and study should be applied. Although the author infers that his paper is the initial attempt to rationalize design of speed-change lanes, it appears to be an academic discussion, somewhat extended in detail, of a design approach made in a paper[9] published elsewhere, in 1940. The details of the bases for design of the high-type speed-change lanes on the connections to the Pennsylvania Turnpike also have appeared in varying degree in engineering publications.

As is the case for many phases of geometric highway design for which the preparation of "standards" has not yet been possible, the design of speed-change lanes involves evaluation of the intangible "driver behaviors" to evolve a practical solution. The result must be simple and perfectly understandable to those using it, but above all it must be so dimensioned and located that drivers can, and will, automatically use it as the designer intended. The approach for design must be of the type described by Mr. Mitchell, and elsewhere previously, but the entire problem is of such nature that the academic detail and precision of values in this paper seem unnecessarily extensive. The basic data upon which the various factors are assumed or calculated have not yet been evaluated to the degree that they are applied. As one example, the degree of accuracy inferred in the mathematical extension of the acceleration and deceleration rates (Figs. 3 and 8) for ascending and descending grades is not at all consistent with the limited number of vehicles, drivers, and conditions of test from which the rates were determined.

Speed-change lanes generally will fall in the class of emergency pavement area—they will be used as some designer predicts only by the occasional vehicle that is forced into that sequence of actions and control by other streams of traffic. For the most part, they are widened pavements. In the absence of other traffic, drivers will enter or leave the outer lane with considerable variance in position and speed, as found necessary at the particular instant. One large field of application of speed-change lanes is at intersections on two-lane and three-lane roads. As shown in Figs. 1 and 7, Mr. Mitchell obviously excludes these cases in order to introduce the insulation strip. The writer feels that in most instances, even along express highways, the flow of traffic will not be facilitated to the extent possible by considering long insulation strips as an essential part of the design. Their chief function is to prevent improper left turns, but as far as use of the speed-change areas is concerned they are better omitted or limited to short lengths. It is stated that the function of the insulation strip is to eliminate points of conflict. The introduction of such strips adds an island point that may be more hazardous than beneficial.

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[8] Associate Highway Engr., Div. of Design, Public Roads Administration, Washington, D. C.

[9] "A Policy on Intersections at Grade," Am. Assoc. of State Highway Officials, 1940.

In reference to Fig. 6, Mr. Mitchell states that operators will not drive as fast on narrow pavement as on broad pavement, citing data reported by Messrs. Thompson and Hebden as proof.[5] This is questionable usage—conditions other than pavement width usually determine speed of travel on any highway. The important point is that faster traffic generally will drive a greater distance from the edge of pavement than will slower traffic on the same pavement. The placement data quoted[5] were observations on the average position of a "critical vehicle"—that is, placement on a two-lane road as the vehicle was passed by another vehicle. This may or may not be representative of placement on a one-way road, or on roads of other widths; but it does not prove that narrower pavements result in lower speed operations. A study in Missouri[10] reported in 1940 showed that high-speed traffic traveled just as fast on an 18-ft as on a 20-ft pavement, with average speeds of 53 miles per hr, and a top speed of 88 miles per hr in each case. With the available placement data as a basis there seems to be little justification for any variation in the width of speed-change lane, provided it is wider than a minimum of 11 or 12 ft. A desirable width for a one-way lane is 14 ft, barely permitting passage at slow speed past a vehicle stalled on the lane. In the case of the deceleration lane it is fundamentally incorrect to provide narrower widths as the sharp radius curve (50 ft) is approached. The lane width around this curve should be 14 to 17 ft as a minimum.

The evaluation of weaving distance might be clearer if also expressed as a time interval. The speed-distance values calculated from a friction factor of 0.16 approximate that for about a 3-sec interval, which method of presentation is not as suggestive of great refinement in the accuracy of the calculation bases.

A comprehensive study of the speed, placement, driver behavior, and all other pertinent factors in the operation of vehicles on existing speed-change lanes is sorely needed as a basis for proper design of such lanes. Until such data are secured any great refinement for design details based largely on assumptions for the various factors should be considered only as of academic value. Even a short length of added lane will facilitate the flow of vehicles turning sharp curves. Although the benefits of longer lengths of added lane may be anticipated (and doubtless correctly so), it cannot yet be implied or stated that they are proven. It is to be hoped that this paper will focus attention on the need for field data to be used as a basis for proper design of speed-change lanes.

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[10] "Speed of Motor Vehicles on Missouri Highways," Missouri State Highway Dept., 1940.

HAWLEY S. SIMPSON,[11] M. AM. SOC. C. E. (by letter).—Mr. Mitchell has opened a very interesting subject, and one that has been largely ignored in the past. Entrances and exits to and from high-speed highways have been mainly designed by the "catch-as-catch-can" method. As more express highways in rural districts are built, greater attention must be given to reducing the effect of conflicts caused by vehicles entering and leaving the highway.

With the general principle of the deceleration lane the writer is thoroughly in accord. Regardless of whether traffic is light, or whether it is heavy, vehicles about to leave an express highway may be followed closely by others moving at the same speed. To move the turning vehicle out of high-speed traffic without slowing following vehicles and to reduce the probability of rear-end collisions are the purposes of the deceleration lane. The design proposed by Mr. Mitchell fulfils these requirements quite adequately. The only valid criticism is that it may be too adequate. The author has treated the motorist with greater consideration than the economics of the situation warrants. With the little that is known about habits of drivers it seems quite possible that they would react as favorably to a design that does not involve the rather long distances proposed by Mr. Mitchell. From his own driving experience, the writer sees no reason why such a lane should not be designed to permit deceleration, without braking application, to 40 miles per hr instead of 30 miles per hr and then require a much harder brake application than that proposed by Mr. Mitchell to reduce the speed of the vehicle to a safe turning speed.

The deceleration lane designed by Mr. Mitchell will give the impression of extreme length, and many, if not most, drivers will continue at "cruising" speed for some distance down the deceleration lane. Then they will coast to about 40 miles per hr, and make a much harder brake application than that required by the use of a coefficient of friction of 0.16.

The design of an acceleration lane, however, is in a decidedly different category. Nevertheless, Mr. Mitchell has applied practically the same principles in reverse order. Here the vehicle is assumed to enter the acceleration lane at a relatively low speed and accelerate exactly to cruising speed. The driver reaches this speed at precisely the point where he is expected to begin "weaving" into the cruising lane. At this exact time he must find a gap in traffic sufficient to accommodate his vehicle, and one that will provide clearance space between preceding and following cars, or he must abandon the maneuver and make a hard brake application in the emergency stopping space. There he will await an opening in traffic and enter just as if no acceleration lane had been provided.

Whether such a lane is proper is a moot question. Consider the two extremes of traffic—(1) extremely light, and (2) extremely heavy. In the first situation, there is a high probability that the required opening will be available upon arrival at the weaving section. If this is so, why is an acceleration lane necessary? Would it not be just as feasible and much less expensive to require the vehicle to come to a full stop at a normal right-angle intersection and await an opportunity to enter which, theoretically, would present itself almost immediately?

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[11] Research Engr., Am. Transit Assn., New York, N. Y.

At the other extreme, when traffic on the main highway is very dense, the probability that the driver, having reached cruising speed, will be able to enter the traffic stream at the exact moment he arrives will be so remote that a very large percentage of the entering vehicles will be forced to use the emergency stopping area. Thus, they will be required to enter the traffic stream exactly as if there had been no acceleration lane; in other words, as if the intersection had been designed as a normal right-angle entrance.

For traffic volumes between these two extremes the condition will partake of the nature of either one or the other of the two cases just described.

Thus, it seems more logical to forego the construction of an acceleration lane, and to require traffic to stop at the entrance to a high-speed highway and await an opportunity to enter without interfering with other vehicles.

From the standpoint of accidents it seems likely that this solution is preferable to the construction of a deceleration lane. A driver in a car moving on to a main highway, and at the same speed as the traffic on it, is in a most difficult position to see and appraise the speed and position of vehicles at his left rear. His chances of being able to enter that traffic at the precise point where there happens to be an opening is so slight as to insure a marked increase in traffic accidents. Construction of such lanes should proceed cautiously until an opportunity has been afforded to learn the outcome more accurately.

Commenting upon the accident experience on the Merritt Parkway in Connecticut under the heading "Justification," Mr. Mitchell states, from rather inconclusive evidence, that: "there remains little doubt as to the location of a sizable percentage of the rear-end collisions * * *." The writer doubts if this conclusion is warranted by the facts and strongly urges a more intensive accident study of highways of this kind before exact conclusions are attempted. Many hours of driving on the Merritt Parkway have led the writer to conclude that there is a substantial potential hazard in the inadequacy of the deceleration lanes, and relatively little hazard at entrance points which, though providing a short length of additional roadway width, partake more nearly of conventional design.

MILTON HARRIS,[12] ASSOC. M. AM. SOC. C. E. (by letter).—The "Design of Acceleration and Deceleration Lanes" has been treated admirably by Mr. Mitchell in segregating the design requirements into their component parts. This approach allows one to weigh the relative importance of each element and to give consideration to the data upon which the design is founded.

Following this line of thought, there seems to be little to criticize in application of the tables and curves if they are assumed to be inherently correct and applicable to this problem. The only exception in application seems to lie in advocating an improved shoulder for braking distance beyond the exit from the acceleration lane. It would seem that a car using the acceleration lane could see parallel moving vehicles and so adjust its speed, while still within the lane, to permit it to mix without stoppage when the exit was reached. This presumes that the insulating strip is a low curb or fin across which a motorist can see. An insulating strip is an excellent device to channelize traffic.

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[12] Associate Highway Engr., State Div. of Highways, Sacramento, Calif.

One element of design that does not seem to be included in the paper is the relation of volume to lane length and width. This is of particular importance when designing acceleration and deceleration lanes for large turning movements and storage where at least two lanes are required to handle the volume efficiently. The problem grows still more complicated when consideration is given to the necessity of crossing deceleration volumes with the acceleration stream at an intersection and signalization is introduced.

One is led to believe that traffic engineers are prone to accept formulas and experimental data without due regard to their practical application, both from an economic as well as a traffic standpoint. The data presented in this paper in the form of tables and curves have been collected over a period of time under conditions that were hardly similar to those presented. Although it cannot be said that such data are not applicable to this design problem, the fact still remains that much research must be conducted to ascertain if fundamental data are applicable to a variety of conditions.

With an unlimited laboratory at the "front door," it is suggested that new designs for traffic use be subjected to actual test under a variety of conditions before adoption. In the case of acceleration and deceleration lanes, an improved shoulder adjacent to an intersecting road could be used and sandbags placed to simulate the insulating island. The speed, volume, and wheel placement patterns could be collected and analyzed for various lengths of lane under similar conditions, thus proving or disproving the theoretical deductions in so far as that one locality was concerned.

T. F. HICKERSON,[13] M. AM. SOC. C. E. (by letter).—One of the hazardous and annoying features of highway transportation to which even the most careful drivers are subjected is that incident to traffic unexpectedly entering or leaving the highway at speeds different from that of the traffic stream. For the freeway, express highway, parkway, or super-highway of tomorrow, where abutting traffic has no right of access except at widely separated intervals, the foregoing hazards may be greatly minimized if entrance and exit lanes are built in accordance with the general considerations brought out so thoroughly by Mr. Mitchell.

The deceleration equation, Fig. 12, may be simplified perhaps if written as follows:

$$a \text{ (or } -a) = 1.35 + 0.03\,v \quad \ldots\ldots (14)$$

Likewise, the acceleration equation, Fig. 13, might be written

$$a = 5.76 - 0.044\,v \quad \ldots\ldots (15)$$

In changing from v (feet per second) to V (miles per hour) it is a little confusing to the reader to note the ratio 1.47 (the strictly correct value) in Eq. 9, whereas the approximate value of 1.50 is used in Eqs. 7, 8, 12, and 13. Obviously the difference is of no practical significance.

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[13] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

Since $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$; by division, $\frac{v}{a} = \frac{ds}{dv}$; or, $ds = v\,\frac{dv}{a}$. Hence, Eq. 8 may be obtained directly from Eq. 6, by only one integration, instead of from Eq. 7. In a similar manner, Eq. 13 may be derived from Eq. 12 by one integration instead of two, as suggested by the author. These latter comments are trivial and are merely offered for what they may be worth as constructive criticism of a timely and valuable paper.

W. L. WATERS,[14] M. AM. SOC. C. E. (by letter).—Although this paper is very interesting, it is somewhat academic and it is based on assumptions, so that the tables and formulas given may create an unwarranted feeling of confidence in a designer who uses them.

The paper may be criticized for neglecting the practical features of the problem as distinct from the academic ones. In any highway where high speed (say, 40 miles per hr) is permitted on the outside lane, and where the traffic is fairly dense (say, 1,000 cars per hour in that lane), the only safe procedure is for the incoming motorist to come to a full stop and await an opening in the traffic. From the converging acceleration lane good vision of the oncoming traffic is impossible, and any attempt to break into a line of oncoming high-speed traffic at this point is dangerous.

The author's method of designing the deceleration lane is very safe; but the entry of cars into high-speed traffic is always a danger point. For the latter, the most practical and safe plan seems to be to limit the speed in the outside lane, place conspicuous warning signs one-quarter mile ahead of any entrance, and make all incoming traffic come to a full stop.

R. A. MOYER,[15] ASSOC. M. AM. SOC. C. E., AND DONALD T. DAVIDSON,[16] JUN. AM. SOC. C. E. (by letter).—In the paper on the design of acceleration and deceleration lanes, Mr. Mitchell has focused attention on certain relatively new design details which highway engineers are slowly recognizing as worthy of consideration in the design of the modern highway. The planning of details in the method of handling traffic at intersections and entrance and exit lanes will always be a challenge to the highway designer's ingenuity. There are also certain fundamental principles in design involving vehicle behavior with its limitations, and driver behavior with the driver's limitations, which are gradually being recognized by engineers and for which standards of performance are being established. As the results of field tests and related research yield new and more exact information on these subjects, it is to be expected that closer agreement in these standards will result.

In general, the writers are in agreement with the standards that Mr. Mitchell has recommended, and consider them quite reasonable on the basis of the results of tests conducted by the writers in 1941. In this discussion certain refinements in the design of the curves are offered, with comments in regard to certain other elements in the design proposed by the author which appear to be worthy of consideration.

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[14] Cons. Engr. (Bury & Waters), New York, N. Y.

[15] Research Associate Prof., Highway Eng., Iowa State College, Ames, Iowa.

[16] Research Graduate Asst.-Highway Engr., Eng. Experiment Station, Iowa State College, Ames, Iowa.

The author computed the values for weaving distance in both the deceleration and acceleration lanes on the basis of a reverse curve composed of two symmetrical circular curves. He states (see heading "Deceleration Lanes"): "The assumption of an **S**-path of travel is a theoretical expedient entirely, and there is no doubt that drivers would find a spiral path more natural than a reverse circular path." He further states: "The formula derived on the basis of the **S**-path contains all the variables involved in the natural path." The writers have taken exception to the latter statement because an examination of the author's formulas, and the very fact that no provision is made for the natural spiral path in the formulas, indicates that the important variable C, the rate of change of acceleration, is not considered. The value of C reflects the driver's ability or skill in steering a car along a definite path from a tangent to a curve or from one curve to any other curve with a reasonable degree of accuracy in a given distance or length of time. The use of the reverse circular curve or of the straight-line transition from one lane to the other, as shown in Fig. 1, adds little to the economy of construction or to the appearance of the design. This type of construction is almost certain to provide too abrupt a change in direction and too short a length in the weaving distance because it does not provide a natural path that the driver can follow easily.

To clarify this important phase of design of exit and entrance lanes, values of weaving distance were computed on the basis of a natural path composed of four equal spiral curves transitional throughout using the same speeds and values of W as in Fig. 2. In addition to the mathematical analysis, a series of carefully controlled road tests were run to determine minimum lengths of weaving distances using a test car whose characteristics were well known from previous tests on highway curves.

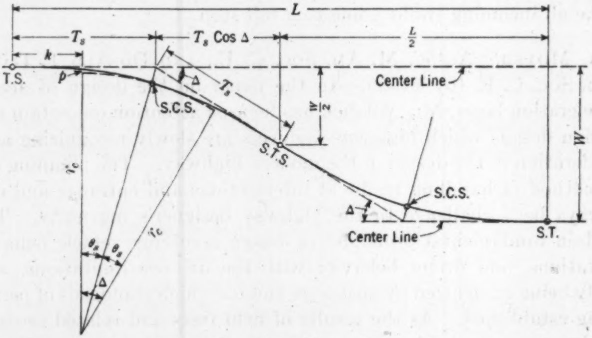

FIG. 15.—REVERSE CURVE COMPOSED OF FOUR EQUAL SPIRALS TRANSITIONAL THROUGHOUT

In the mathematical analysis for the determination of the weaving distance, the standard spiral formulas, as recommended by the Iowa Engineering Experiment Station [4] and by the U. S. Public Roads Administration,[17] were used.

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[17] "Transition Curves for Highways," by Joseph Barnett, Bureau of Public Roads, U. S. Dept. of Agriculture, U. S. Government Printing Office, Washington, C. D., 1938, pp. 7-26.

The proposed curve layout is shown in Fig. 15, and from this it is evident that the weaving distance L may be computed as follows:

$$L = 2\,[T_s + T_s \cos \Delta] = 2\,[T_s\,(1 + \cos \Delta)] \quad \text{.......... (16)}$$

Since the tangent distance T_s and the length of each spiral L_s are approximately equal,

$$L = 2\,[L_s\,(1 + \cos \Delta)] \quad \text{.......... (17)}$$

The standard formula for length of spiral L_s is

$$L_s = \frac{3.16\,V^3}{C\,r_c} \quad \text{.......... (18)}$$

in which V is the speed in miles per hour, C is the rate of acceleration in feet per second^3, and r_c is the radius of the circular curve in feet. Substituting this value for L_s in Eq. 17,

$$L = 2\left[\frac{3.16\,V^3}{C\,r_c}\,(1 + \cos \Delta)\right] \quad \text{.......... (19)}$$

The standard curve formula for the determination of the radius r_c is

$$r_c = \frac{0.067\,V^2}{f + e} \quad \text{.......... (20)}$$

in which f is the coefficient of friction and e is the superelevation in feet per foot.

Substituting the value for r_c in Eq. 19, and using a value of e equal to zero, will reduce the value of L to

$$L = 2\left[\frac{47.1\,f\,V}{C}\,(1 + \cos \Delta)\right] \quad \text{.......... (21)}$$

For a spiral curve that is transitional throughout, the intersection angle Δ is equal to the sum of the two spiral angles θ_s. Since the standard formula for the spiral angle θ_s is

$$\theta_s = \frac{L_s\,D_c}{200} \quad \text{.......... (22)}$$

in which D_c is the degree of the circular curve, then

$$\Delta = \frac{L_s\,D_c}{100} \quad \text{.......... (23)}$$

The spacing W between the center line of the highway and of the exit or entrance lane can be computed for any value of L by using the formula

$$W = 2\,L_s \sin \Delta \quad \text{.......... (24)}$$

A summary of the computed values for weaving distances L and lane spacings W for speeds from 20 to 60 miles per hr for values of friction f and rates of change of acceleration C in the usual operating range is given in Table 2. The values of coefficient of friction used in these computations ranged from 0.10 to 0.20. A value of $f = 0.20$ is generally accepted as a reasonable design value for speeds of 20 miles per hr or less, and a value of 0.16 is the standard value for speeds from 30 to 60 miles per hr. For speeds greater than 60 miles per hr a value of $f = 0.10$ is preferred. The values of C can be treated in like manner—that is, use a value of $C = 4$ ft per sec^3 for speeds of 20 miles per hr or less, $C = 3$ ft per sec^3 from 30 to 60 miles per hr, and a value of $C = 2$ ft per sec^3 for speeds greater than 60 miles per hr.

TABLE 2.—"WEAVING DISTANCE" VALUES FOR REVERSE CURVES COMPOSED OF FOUR EQUAL SPIRALS, TRANSITIONAL THROUGHOUT

Speed, in miles per hr	f	R_c, in ft	D_c, in degrees	$C = 2$			$C = 3$			$C = 4$		
				L_s, in ft	W, in ft	L, in ft	L_s, in ft	W, in ft	L, in ft	L_s, in ft	W, in ft	L, in ft
20	0.20	134.0	42.8	94.5	122.6	333.0	63.0	57.2	238.0	47.0	32.4	182.0
20	0.16	167.5	34.2	75.5	65.6	287.0	50.3	29.8	197.0	37.7	16.8	149.0
20	0.12	223.5	25.6	56.5	28.2	222.0	37.8	12.8	150.0	28.0	7.1	113.0
20	0.10	268.0	21.4	47.2	16.6	187.5	31.4	7.4	125.0	23.6	4.2	94.0
30	0.20	301.5	19.0	141.5	128.0	535.0	94.5	58.4	369.0	70.8	33.0	280.0
30	0.16	376.0	15.2	113.5	67.4	444.0	75.6	30.2	300.0	56.8	17.0	226.0
30	0.12	503.0	11.4	85.0	28.6	338.0	56.5	12.6	226.0	42.4	7.1	169.0
30	0.10	604.0	9.5	70.6	16.6	282.0	47.2	7.4	188.5	35.3	4.2	141.0
40	0.20	536.0	10.7	188.5	130.0	730.0	126.0	58.7	496.0	94.2	33.0	374.0
40	0.16	670.0	8.6	151.0	68.0	596.0	100.7	30.4	400.0	75.5	17.1	302.0
40	0.12	895.0	6.4	113.0	28.2	450.0	75.4	12.6	301.0	56.5	7.1	226.0
40	0.10	1,070.0	5.4	94.5	16.8	377.0	63.0	7.4	252.0	47.3	4.3	189.0
50	0.20	837.5	6.8	236.0	131.0	926.0	157.0	58.4	624.0	118.0	32.8	470.0
50	0,16	1,047.0	5.5	189.0	68.4	750.0	126.0	30.2	503.0	94.3	17.2	376.0
50	0.12	1,395.0	4.1	141.5	28.6	565.0	94.4	12.8	377.0	70.8	7.2	283.0
50	0.10	1,675.0	3.4	118.0	16.4	471.0	78.5	7.4	314.0	59.0	4.4	236.0
60	0.20	1,210.0	4.7	282.0	129.0	1,110.0	188.0	58.0	748.0	141.0	32.4	562.0
60	0.16	1,510.0	3.8	226.0	67.0	899.0	151.0	30.0	603.0	113.0	17.0	451.0
60	0.12	2,010.0	2.9	170.0	29.0	679.0	113.0	13.2	453.0	84.8	7.3	338.0
60	0.10	2,415.0	2.4	141.2	16.6	565.0	94.4	7.6	377.0	70.6	4.2	282.0

As a result of the mathematical analysis, it was apparent that for any given values of f and C only one value for length of weaving distance L and one value for spacing W was possible if four equal spiral curves, transitional throughout, were used. To determine the length L for any given value of W, changes in the design values of f or C are required. To obtain smaller values of W than those determined using the recommended design values for f and C, a reduction in the value of f is the safest procedure. To obtain larger values of W than those determined using the recommended design values for f and C, a reduction in C is the safest procedure. It should be noted in Table 2 that, with $W = 16.8$ ft for a speed of 20 miles per hr, $f = 0.16$, and $C = 4$, the length L is 149.0 ft; whereas with very nearly the same distance $W = 16.6$ ft at the same speed, $f = 0.10$, and $C = 2$, the length L is 187.5 ft. This increase of 38.5 ft in the length of L is an increase of only 25% and indicates that the changes in weaving distance within the ordinary range in design values of f and C will not vary by more than from 25% to 30%.

On the basis of the foregoing analysis Fig. 16 for weaving distance was prepared using a constant value of $W = 12$ ft and values of a equal to 0, 3, 6, and 9 ft for speeds ranging from 20 to 60 miles per hr. When comparing the values of L in Fig. 16 with similar values in Fig. 2, it will be noted that the latter are approximately 40% less than the former. This difference in the lengths of L is the logical difference between circular reverse curves of the same radius and spiral curves with radii varying from an infinite radius to that used by the author for the same conditions of speed, lane spacing, and values of f.

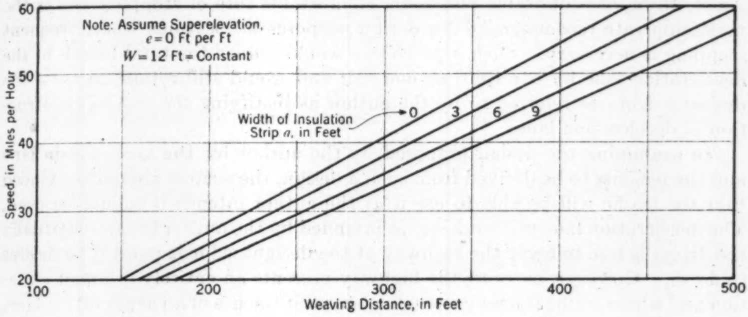

FIG. 16

As a check on the mathematical analysis, road tests were run with a test car of known steering characteristics on curves. This car has been used on many tests on curves and, on the basis of the results of tests by the General Motors Corporation on many different makes and year models of cars,[18] may be considered as an average type of car in respect to steering and handling on curves. The car was equipped with a ball bank indicator that was used to aid the driver in holding to the maximum specified side friction and the maximum curvature at each speed. By holding the side friction to the maximum value of $f = 0.16$ recommended by the author, the minimum lengths in the weaving distance obtained in these tests checked rather closely with the values given in Fig. 2. Since the test values were the minimum lengths that could be obtained without exceeding the value of $f = 0.16$, it follows according to the mathematical analysis for the determination of L using spiral curves that large values of C were developed in these tests. With relatively small intersection angles Δ, accuracy in the steering of the car was not a critical item, which indicates why values of C equal to 6 and 8 ft per sec^3 were possible. For design purposes values of C equal to 3 or not greater than 4 ft per sec^3 are to be preferred and this greatly simplifies the steering of the car from one lane to the other.

The design proposed by the author may be criticized on the basis that the lengths of the deceleration and acceleration lanes are unnecessarily long. The introduction of spirals as the natural path that the cars will follow will further increase these lengths and, where local conditions require it, some modification in the design standards would have to be made. Under these conditions a reduction in the cruising speed or the design speed at the entrance to the exit lanes would be an important consideration. Also, motor deceleration might be used only for speeds greater than 40 or 50 miles per hr for a fixed distance such as one half the weaving distance to be followed by higher rates of stopping by the use of brakes than are recommended by the author. A stopping rate of 8 ft per sec^2, which corresponds to a value of f equal to 0.25, has been established on the basis of extensive tests at Iowa State College, Ames, Iowa, as a desirable maximum comfortable rate of stopping and is the maximum rate recommended for design purposes in locations where frequent stopping is necessary. Such a procedure would reduce the total length of the deceleration lane by one third or one half and would still retain many of the desirable features referred to by the author as justifying the cost of construction of deceleration lanes.

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[18] "Marking Highway Curves with Safe Speed Indications," by R. A. Moyer and D. S. Berry, *Proceedings*, Highway Research Board, National Research Council Vol. 20, 1940, pp. 399–428.

In examining the design proposed by the author for the acceleration lane and the benefits to be derived from such a design, the writers are not convinced that the traffic will be able to use it as the author intends it should be used. The deceleration lane will work out as intended by the author because normally the driver is free to leave the highway at the designated exit point if he desires to do so. However, entering the highway presents an entirely different situation and where traffic is heavy enough to warrant the use of an acceleration lane, the opportunities to enter the highway freely without interference are likely to be available only for a limited percentage of the traffic entering the highway. This is a phase of traffic behavior that should be investigated to determine the extent to which traffic entering from an acceleration lane can do so freely with varying volumes of traffic on the highway.

Although an acceleration lane designed solely as such has the advantage in reducing delay both for the turning traffic and the through traffic on the highway, the writers contend that an important function of the acceleration lane should be to provide ample space in which the turning traffic can merge safely into the through traffic lane without requiring the turning traffic to stop or the through traffic to reduce speed seriously. To satisfy this requirement a greater length should be provided in which the traffic can merge by reducing the length of the insulation strip and by replacing the cinders or gravel braking section (which traffic would not use) by a paved section of the same length which the traffic would use. Full acceleration to the cruising speed is not necessary when entering the highway and a reduction in the length of the insulation strip to one half the length shown in this section of the acceleration lane should be quite satisfactory, especially under conditions where a reduction in the total length of the acceleration lane recommended by the author is necessary. Observations of traffic for many years have revealed to the writers that traffic which is not legally required to stop by signs or similar devices will stop only as a last resort and for this reason it is questionable if there is any justification for a cinder or gravel braking section. An improvement here would be to cover this section with a distinctive type of low-cost paved surface that could be used either for braking or for merging the turning traffic with through traffic. Such a treatment would provide greater flexibility in the use of the acceleration lane and would reduce delays and traffic interference by an appreciable amount as compared with that possible if the design proposed by the author is used.

There is little doubt but that the proposals made by Mr. Mitchell are in the right direction. The trend in the control of traffic today is to provide a smooth continuous flow of traffic along highways and through intersections in such a way that delays, large speed differentials, and unnecessary stops are eliminated, and so that traffic can leave and enter through highways freely. The savings in the cost of vehicle operation and in time occasioned by the elimination of delays and traffic stops mount rapidly as the volume of traffic increases. Road tests conducted by the writers have revealed that when cars are stopped every other block the gasoline mileage is cut in half, the average speed is reduced by one third, and the tire wear is more than six times as great as when no stops are required. Furthermore, the design of highways to provide a smooth continuous flow of traffic will not only reduce operating costs and save time but, as the author has clearly stated, it should reduce highway accidents.

The author is to be congratulated for having made a careful analysis of the many details in vehicle and driver behavior which are certain to play a more prominent part in the design of highways in the future. It is hoped that his paper will stimulate research in field observations of traffic behavior and in the development of standards of design that will be adopted by highway departments.

STEPHEN E. BUTTERFIELD,[19] ASSOC. M. AM. SOC. C. E. (by letter).—State traffic engineers, engineers dealing with the control of rural and high-speed traffic, and students of rural accident statistics are well aware of the large number of accidents that result directly from the conflict arising between slow-moving, decelerating, or accelerating vehicles and through-moving vehicles, traveling at a constant, high rate of speed. Of equal economic significance is the accompanying congestion, resulting in time losses and lower over-all speeds, which occurs at points of egress from or access to high-speed highways carrying substantial volumes of traffic. It is apparent that many highway designers have not been equally cognizant of the importance of providing a facility that permits a vehicle to diverge safely from, or merge with, the fast through-moving stream of traffic. In many of these cases where highway designers have attempted to provide accelerating and decelerating facilities, it is apparent that insufficient thought has been given to their functional design or to the study of their component parts. The exit and entry ramps to the Merritt Parkway, in Connecticut, and to the Hutchinson River Parkway, in Westchester County, New York, are examples of these earlier and inadequate facilities.

In view of these facts and in view of the vast program of parkway, freeway, and dual highway construction upon which the United States is engaged, Mr. Mitchell's treatise is extremely timely. The author has clearly outlined the essential elements that should be considered in the design of accelerating and decelerating lanes. Although this subject has been studied previously in some detail,[20] Mr. Mitchell has now successfully presented a concise treatment of the component elements in the design of accelerating and decelerating lanes. These elements are treated in a logical sequence and in a strictly analytical manner. Also, the author has carefully assembled and interpreted, in the light of their value to the design of accelerating and decelerating lanes, the available research material applicable to this subject.

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[19] Traffic Engr., State Highway Dept., Hartford, Conn.

[20] "Accelerating and Decelerating Lanes," by Wortham W. Dibble, Harvard Bureau for Street Traffic Research, June, 1938; also "Accelerating and Decelerating Roadway Areas," by Stephen E. Butterfield, Yale Bureau for Street Traffic Research, June, 1939.

From a review of this paper, it should now be clear to most highway engineers that there is no longer need for making major assumptions or hypotheses in the designing of accelerating and decelerating lanes. Further research on some of the minor controversial matters relating to this subject doubtless will throw much-needed light on certain phases of the problem, such as to what extent motor deceleration should be considered and at what point motor deceleration gives way to brake deceleration.

Mr. Mitchell has presented a sound mathematical approach to the matter of weaving distance, a problem which formerly has been subject to the greatest diversity of opinion. An insulation strip, in the form of a true physical barrier, is of the greatest importance to insure the correct use of the decelerating lane and to insure that the decelerating process occurs entirely off from the through-moving lanes. This insulation strip should form a sufficient barrier so that no motorist will be tempted to ride over it in order to enter the exit ramp. Except for a short distance at its approach end, this insulation strip should not be a low mountable curb. The color scheme of this barrier area, and of the decelerating lane itself, should offer such contrast to the highway proper that the point of divergence will be clear to the approaching motorist.

In his presentation, the author has allowed the motor deceleration distance to overlap the weaving distance, thus permitting a part of the through-pavement area to be used for the act of deceleration. This means, in the example presented, that with a cruising speed of 50 miles per hr, a vehicle under motor deceleration will be traveling at approximately 41 miles per hr at the end of the weaving distance. This is the point at which a vehicle is finally entirely off from the through-moving lanes. Should not the deceleration distance be treated separately from, and introduced at the end of, the weaving distance? This would insure freedom from any possible conflict between the decelerating vehicle and a following through-moving vehicle. This would be consistent with the design of the accelerating lane in which no attempt at merging with the through stream of traffic is made until the full cruising speed has been attained.

In the use of certain values as the cruising speed for the through-moving traffic, one should not lose sight of the fact that the existing cruising speed or the design cruising speed may not be the one that will be adhered to in the not too distant future. Lest the highway under construction become antiquated soon after its completion, it is wise to consider the cruising speed as that average speed to be expected on the projected highway at a time ten or fifteen years in the future.

Some highway designers are still reluctant to admit the necessity of providing accelerating lanes, even though they do admit the very real need of decelerating lanes. It is generally conceded that if either the decelerating lane or the accelerating lane must be sacrificed, it should be the accelerating lane in favor of the decelerating lane. The crux of the argument revolves around whether full security is to be provided to the following through-moving motorist at the point of exit, and yet no security provided to him against a suddenly entering vehicle at a point of entry. Since vehicles are likely to be grouped, even with a light volume of traffic on the through lanes, an entering vehicle may encounter merging conditions not drastically different from those encountered under high volume conditions.

The inability of the motorist of average intelligence and long driving experience to judge accurately his distance from, and the speed of, an approaching vehicle is well known. Because of his inability to judge the speed of an approaching vehicle and because of his normal impatience to delay, is it not far better to remove the possibility of a human error by providing a facility which largely removes the human element? With the provision of grade separations, freedom from access to abutting properties, decelerating lanes, and the other natural attributes of a freeway, a sense of security is afforded the motorist that does not actually exist unless merging lanes are provided which permit vehicles to enter the highway at the cruising speed. It follows naturally that the higher the speed of the through route the greater is the need of accelerating lanes and the greater is the inability of the motorist who is stopped at the entry to the expressway to judge the speed of the approaching vehicles.

So-called accelerating lanes in the form of extra pavement widths have been provided at the entry point to certain parkways located in Eastern United States. The failure of vehicles to use these extra pavement widths as accelerating lanes should not serve as a measure of their value; rather it should serve as an indictment of their inadequate design. When carefully designed, and in accordance with the general principles outlined by Mr. Mitchell, accelerating lanes will be found to be a highly functional and an urgently needed parkway facility.

This paper should be welcomed by all engineers and highway designers confronted with the problem of parkway and expressway design. Mr. Mitchell has so segregated the design elements that it should be a relatively simple matter to adapt the general method to the treatment of any individual problem or to substitute for the submitted data later data derived from further research on the individual elements.

ADOLPHUS MITCHELL,[21] ASSOC. M. AM. SOC. C. E. (by letter).—In preparing the paper on acceleration and deceleration lanes it was recognized that the available data on which the limit of motor deceleration and other constants were based was not sufficient to allow the selection of final values. For this reason, it was hoped that those discussing the paper would do so in a constructive manner and present more conclusive evidence for the determination of these factors. It is gratifying that practically all discussions were kept on this high plane.

In summarizing the discussions submitted, an effort will be made to isolate the controversial elements and to modify the recommendations of the paper where it appears to be justified.

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[21] Senior Traffic Engr., State Highway and Public Works Comm., Raleigh, N. C.

Deceleration Lanes.—The weaving distance was given particular attention by Messrs. Loutzenheiser and Moyer and Davidson. It is gratifying to note that they all agreed with the writer that his values were acceptable as minimums. Messrs. Moyer and Davidson appear to have gone further into the problem than any one else and in a very scientific manner. They state that "By holding the side friction to a maximum value of $f = 0.16$ recommended by the author, the minimum lengths in the weaving distance obtained in these tests checked rather closely with the values given in Fig. 2." However, on the basis of a spiral shown in Fig. 15, they have arrived at a length somewhat in excess of those given in Fig. 2, and this bears some comment.

In stating that "The assumption of an S-path of travel is a theoretical expedient entirely," and that "there is no doubt that drivers would find a spiral path more natural than a reverse circular path," the writer did not have reference to the spiral employed by Messrs. Moyer and Davidson. This spiral was developed by railroad engineers on the basis of providing for a transition from a tangent to a circular curve in a manner that would increase the centrifugal force acting on the vehicle uniformly from zero at the end of the tangent to that determined by the circular curve at the end of the transition. This would permit the elimination of all "pants friction" provided that the superelevation was increased uniformly from zero at the end of the tangent to that of the circular curve at the end of the transition, on the condition that a constant speed was maintained. In the present problem it will be impracticable to superelevate the pavement in this manner and to assume that the speed will not vary throughout the weaving distance. For these reasons, Messrs. Moyer and Davidson not only failed to present a formula for the "natural path" but also failed to present one resting on a sound theoretical basis. Although the discussion of the weaving distance gives the impression of providing a basis for longer minimum lengths, examination of the throw (p) of the spiral shows this to be purely theoretical (amounts to $p = 0.35$ ft in the problem of Fig. 7, although L is equal to 505 ft, using $f = 0.16$, $W = 17$, $V = 50$, and $C = 3$) and not at all commensurate with the accuracy with which automobiles are steered. Their comparison of Fig. 16 with Fig. 2 is misleading. All curves in Fig. 2, except one, are based on a width of lane varying with the speed as listed under the heading "Deceleration Lanes." The values of Fig. 2 are approximately 70% less than those of Fig. 16 by Messrs. Moyer and Davidson.

Their remarks regarding C in Eq. 18 were not clear since C is a constant, the evaluation of which is not final. They themselves recommend using C as a constant for any given design speed.

Mr. Simpson suggests that motor deceleration be assumed equal to 40 miles per hr, and Messrs. Moyer and Davidson suggest 40 or 50 miles per hr. The writer suggested that "40 miles per hr for a 70-mile-per-hr highway and 30 miles per hr for a 50-mile-per-hr highway" be assumed. Since nearly all of those taking part in the discussion agreed that deceleration lanes determined on this basis are too long, it is suggested that the assumption of motor deceleration to 50 miles per hr be used for a 70-mile-per-hr highway and 40 miles per hr for a 50-mile-per-hr highway.

Most of those discussing the braking distance agreed that a somewhat higher coefficient of friction should be adopted than those shown in Fig. 5. Messrs. Moyer and Davidson suggested a value of $f = 0.25$, on the basis of actual tests. The writer is inclined to agree with them and suggests that this value be used. This would result in the following equation:

$$s = \frac{\bar{V}^2 - V^2}{7.5 \pm 0.3\,G} \quad \text{.......................}(25)$$

There was very little comment on funneling or providing a decreasing width of lane as speed decreases. Apparently no one knows anything about this subject at the present time.

Very little was written about the insulation strip for the deceleration lane. The writer is inclined to shorten this strip to a length determined by the emergency stopping distance so as to provide a more open throat than that theoretically required. The insulation strip is necessary to insure correct use of the deceleration lane, as has been proved by experience. Proper design will eliminate the hazard of introducing an island point.

The changes agreed upon would result in a length of deceleration lane of 465 ft instead of 650 ft for the problem of Fig. 7.

Acceleration Lanes.—It appears that there is no definite agreement of the distance required for full acceleration. Mr. Loutzenheiser does not think the writer was justified in extending the data for level ground to include the effect of gradients. It has been the experience of Ernest E. Wilson, director of the General Motors Proving Ground, that "The hill climbing abilities of a passenger car can be very closely determined from theoretical equations if acceleration data taken on a level road are used in the calculations."

The insulation strip along the acceleration lane was an item of considerable interest. Mr. Loutzenheiser would either shorten it or omit it altogether. Mr. Butterfield thinks it is all right as presented. Messrs. Moyer and Davidson suggest that it be shortened to one half of the length of the acceleration lane. The writer suggests that it be shortened to the point where it is possible for entering traffic to attain the average speed of the adjacent highway.

There was considerable diversity of opinion as to the braking section provided at the end of the lane. Mr. Harris does not think it is necessary, and Messrs. Moyer and Davidson would use a low-cost paved surface. Since this section amounts to no more than improved shoulder, the writer is inclined to agree with Messrs. Moyer and Davidson.

Messrs. Simpson and Moyer and Davidson would construct deceleration lanes only, whereas the others appeared to think that the acceleration lane had some merit, particularly Messrs. Harris and Butterfield. After studying the various comments on the subject, the writer still fails to understand why acceleration lanes are not important for dual-lane highways. Where traffic is heavy, it is even more important than where it is light, because it would be necessary to use signals to enable traffic to enter. The only problem seems to be that of persuading traffic to move from the outside to the inside lane in order to allow traffic to enter from the acceleration lane (signs, signals, markings, and devices are available for this purpose). Since it is relatively easy to enter a traffic stream when approximating the speed of the automobiles thereon, it would require a smaller distance between vehicles on the highway in order to enter it than would be the case when an automobile had to enter at a very low speed as would be the case at a T-intersection. The operation of "stop" lights would introduce delays on the highway that engineers have come to recognize as equivalent to a monetary loss. The cost of such delays should be estimated to determine if they would justify the construction of the acceleration lane.

Messrs. Loutzenheiser, Harris, and Waters mentioned the need for additional data on acceleration and deceleration lanes, some of which were furnished by Messrs. Moyer and Davidson. Data are needed on the form of the "natural path," the maximum permissible differential in the speed of cars on the highway with that of merging traffic, and other factors involved in the design of acceleration and deceleration lanes.

Conclusion.—Mr. Holley's example of the Fair Oaks Avenue crossing of Arroyo Seco Parkway was most interesting; however, its value was greatly reduced by his failure to give speeds, grades, and dimensions. It is significant that he states, "Mr. Mitchell's paper provides an objective toward which designing engineers should strive."

Mr. Loutzenheiser appears to believe that the writer's paper is "an academic discussion, somewhat extended in detail, of a design approach made in a paper[9] published elsewhere, in 1940." The writer's paper was written in 1939 and submitted to Yale University, at New Haven, Conn., in 1940, through the Yale Bureau for Street Traffic Research, in partial fulfilment of the requirements for a certificate in street traffic control. There is a marked lack of similarity between the two treatments.

The writer is indebted to Professor Hickerson for checking the development of the equations used.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2139

CRACK PREVENTION PROGRAM, HIWASSEE DAM

BY O. LAURGAARD,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. DOUGLAS MCHENRY, REGINALD H. THOMSON, W. R. WAUGH, AND O. LAURGAARD.

SYNOPSIS

As completed, Hiwassee Dam, a Tennessee Valley Authority (TVA) structure in western North Carolina, with a height of 322 ft from the lowest rock foundation to the roadway, was the highest overflow gravity dam. Its concrete, manufactured from the local graywacke rock, had to be placed largely during the warm summer months. To insure that the dam would be impervious to the passage of water and its surface would be weather-resistant, it was important that the mass should be free from major cracks. This required a concrete with a gradual temperature rise, one that would harden slowly, and one that would permit considerable expansion before its ultimate strength was reached.

To achieve these results the program included: (1) Use of low-heat cement; (2) a low cement content; (3) thin casting lifts; (4) long exposure periods; (5) artificial cooling for mixing water; (6) washing, rinsing, and cooling the aggregate; (7) artificial cooling of the concrete in place; (8) cleanup of horizontal joints between lifts; (9) the use of steel reinforcement; (10) diagonal keyways on bulkhead joints; and (11) curing and winter protection.

This paper shows how the program progressed and how cracking has been practically eliminated by rigid control and inspection. Actually, a saving in cement resulted, which more than offset the additional cost of the crack prevention program.

INTRODUCTION

Hiwassee Dam (Fig. 1) is one of the important units in the comprehensive development of the Tennessee Valley. It is situated in the very westerly corner of North Carolina, adjacent to Georgia and Tennessee, on the Hiwassee River, which flows about 50 miles westerly and slightly north into the Ten-

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NOTE.—Published in March, 1941, *Proceedings*.

[1] Care, Bethlehem Alameda Shipyard, Alameda, Calif. (formerly Construction Engr., Hiwassee Dam).

nessee River. This overflow structure has its spillway at El. 1,503.5 above mean sea level.[2] Average monthly temperatures ranged from a summer high of about 85° F during the working shift from 6:00 a.m. to 3:00 p.m., to a winter low of about 26° F for the shift from 3:00 p.m. to 12:00 midnight.

From the beginning of operations it was an important objective to produce a monolithic structure—one that would conform closely to the design requirements, would be impervious to the passage of water under high pressure, and would resist weathering. A strict program of concrete control therefore was necessary, on account of (1) the material of which the aggregate (including the sand) was composed; (2) the difficulty of controlling the river during construction without using separate by-pass channels or tunnels; and (3) the rather warm climate, especially in the summer time.

FIG. 1.—GENERAL VIEW, HIWASSEE DAM, LOOKING UPSTREAM FROM THE SOUTHWEST

Special consultants were appointed by the TVA to review conditions and submit a report. Two of these consultants, R. W. Carlson, Assoc. M. Am. Soc. C. E., and R. E. Davis, M. Am. Soc. C. E., held several conferences with engineers of the Authority and submitted a program. Quoting from their report: "To obtain water tightness the structure should not only be free from major cracks, but the concrete mix should be plastic, workable and free from water gain, and the workmanship should be such as to insure the bonding of each lift to the lift below"; and again "To insure against devastating action of weathering and erosion, the exposed faces should be of high strength concrete at later ages." The consultants summarized their conclusions and recommendations to achieve the required results in a report of April, 1938.

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[2] For a general description of the project, see papers by Cecil E. Pearce, Assoc. M. Am. Soc. C. E., on "Design of Hiwassee Dam," in *Civil Engineering*, covering "Basic Considerations," June, 1940, p. 340, and "Engineering Details," July, 1940, p. 433.

By following the suggested program, Hiwassee Dam has been constructed relatively free from cracks, and capable of withstanding weathering for a long time. These results have been accomplished by careful inspection and testing of the cement, by rigid control of the concrete, by selection of proper equipment for manufacturing aggregate, mixing, placing and curing of concrete, and by cooperation between engineering and construction forces. Practically all of the work has been done by the Authority's forces.

Until recently, cracking of mass concrete was regarded as inevitable. As concrete control has developed, certain contributing factors for cracking have been eliminated and thus the mass has been greatly improved. For the recommended program submitted by the consultants, supplemented by such additional features as were believed desirable and worthy by the engineers of the Authority, a number of provisions to reduce or eliminate cracks in Hiwassee Dam were adopted: (1) The use of low-heat cement; (2) a low cement content; (3) thin casting lifts; (4) long exposure periods; (5) artificial cooling of the mixing water; (6) washing, rinsing, and cooling of the aggregate; (7) artificial cooling of the concrete in place; (8) cleanup of horizontal joints between lifts; (9) the use of steel reinforcement; (10) diagonal keyways on bulkhead joints; and (11) curing and winter protection.

None of these methods is novel or original, but the combination of all of them on one job is believed to be unique. After two years of concrete placement, there appear to be no cracks on the upstream face, none of any consequence on the downstream face, and only one small crack that can be dignified by that name in the galleries. Although a few small cracks did appear on the contraction joint faces between blocks during construction, these were unavoidable because of large differences in elevation between the tops of adjacent blocks (caused by closure procedure) at a time of great variations in temperature.

ROUTINE CONCRETE CONTROL AND PROCEDURE

Low-heat cement was furnished by five manufacturers from six different mills, with acceptance tests by the Authority. Rigid control was exercised in the manufacture of aggregate including sand, and in the mixing and placing of concrete. Frequent screen analyses served as a basis for controlling the aggregate grading. Tests were made during the first few months of concreting to determine the effect of grinding in the mixers, which were used to correct the combined aggregate grading of the material going into the mixers and to obtain a desirable grading in the fresh-mixed concrete. All concrete aggregates except sand were thoroughly washed to remove extraneous material including fines and thus aid to maintaining a reasonably constant combined aggregate grading. Rapid moisture tests on all aggregates at the mixing plant served to control the water-cement ratio. Similarly, vigilant inspection of concrete placing in the forms assured thorough compaction and proper placement.

Other measures to insure uniformly high quality included particular attention to the cleanup of old concrete surfaces and adequate curing of all concrete. Routine tests of specimens made from daily samples served as a check on the compressive strength, and other desirable properties, of the concrete. Special tests on mixer grinding, bleeding, workability, and strength relationship for variable water-cement ratios produced additional valuable information. Pertinent data for all the concrete mixes and test specimens are given in Table 1.

TABLE 1.—TYPICAL CONCRETE MIXES—HIWASSEE DAM

Item	6-in.[a] mass	6-in.[a] face	3-IN. CONCRETE WITH LIGHT REINFORCING				1½-in.[a] concrete with heavy reinforcing	¾-in.[a] concrete, extra heavy reinforcing
			3-in.[a] face	3-in.[a] face	3-in.[a] mass	3-in.[a] mass		
(a) POUNDS OF MATERIAL IN A CUBIC-YARD BATCH								
Aggregate:								
Sand	838	771	887	867	986	996	1,017	1,474
Rock (No. 4 to ¾ in.)	511	525	682	667	705	639	954	1,475
Rock (¾ in. to 1½ in.)	620	631	853	833	845	924	1,209	....
Rock (1½ in. to 3 in.)	693	700	989	967	986	996	....	....
Rock (3 in. to 6 in.)	985	876	....	....	....	....	....	....
Total aggregate	3,647	3,503	3,411	3,334	3,522	3,555	3,180	2,949
Water	241	251	273	291	273	265	327	381
Cement	301	433	470	501	342	331	564	658
Total batch weight	4,189	4,187	4,154	4,126	4,137	4,151	4,071	3,988
(b) COMPRESSIVE STRENGTHS								
Field Mix:								
7 days	900	1,713	1,526	1,530	700	700	1,371	1,480
28 days	2,509	4,254	3,916	3,873	2,177	1,947	3,567	4,013
90 days	4,078	6,168	6,101	6,010	3,435	4,015	5,493	6,060
6 months	4,569	6,560	6,360	7,025	4,266	4,255	....[c]	....[c]
1 year	4,718	7,218	6,708	....[c]	....[c]	4,405	....[c]	....[c]
Concrete Cores:								
313 days	3,620	....	....	....	....	....	....	....
321 days	3,470[b]	....	....	....	....	....	....	....
217 days	4,170	....	....	....	....	....	....	....
225 days	3,560[b]	....	....	....	....	....	....	....
203 days	4,450	....	....	....	....	....	....	....
211 days	3,520[b]	....	....	....	....	....	....	....
(c) CEMENT PER CUBIC YARD								
Bbl	0.80	1.15	1.25	1.33	0.91	0.88	1.50	1.75
(d) WATER-CEMENT RATIO *W/C*								
By weight	0.80	0.58	0.58	0.58	0.80	0.80	0.58	0.58

[a] Maximum size of aggregate. [b] Wet. [c] Tests not completed at these ages.

USE OF LOW-HEAT CEMENT

Contracts with several cement companies had been awarded for modified cement when it was concluded to use low-heat cement. Therefore, it was necessary to pay a premium of from 5 to 10 cents per bbl for the new material. To comply with the specifications this material was called "modified cement for summer use at Hiwassee Dam," although in reality similar specifications governed the low-heat cement used at Marshall Ford Dam and elsewhere under the United States Bureau of Reclamation. Specifications for "summer cement" were similar to the modified or "Type B" cement, except that the tricalcium aluminate (C_3A) was limited to a maximum of 7%, tricalcium silicate (C_3S) to a maximum of 35%, dicalcium silicate (C_2S) to 40% minimum and 65% maximum, and tetracalcium alumino ferrite (C_4AF) to a maximum of 20%. The "summer cement" specifications also required the fineness to be between 1,700 and 2,200 sq cm per g as against 1,600 to 2,200 for "Type B" cement.

Strength requirements are also somewhat lower for the "summer" than for the "Type B" cement. While exact data are not available to compare concrete strengths for the two types of cement, relative results for Norris and Hiwassee concretes are shown in Table 2. Although the greater strength shown on the Norris Dam concrete is due to some extent to the lower water-cement ratio (W/C), it is also known that the type of aggregate was responsible for some additional strength.

TABLE 2.—COMPARISON OF CONCRETE STRENGTHS, IN LB PER SQ IN.

Age at test	Face concrete, Norris[a]	Face concrete, Hiwassee[b]	Mass concrete, Hiwassee[c]
7 days	3,617	1,723	900
14 days	4,646	2,750	1,438
28 days	5,500	4,254	2,509
90 days	7,332	6,168	4,078
6 months	7,768	6,560	4,569
1 year	8,609	7,218	4,718

[a] 1.2 bbl "Type B" (modified) cement per cu yd at *W/C* ratio of 0.57 by weight.

[b] 1.15 bbl "summer" cement, at *W/C* ratio of 0.58 by weight. [c] 0.8 bbl "summer" cement, at *W/C* ratio of 0.8 by weight.

For concrete using low-heat cement it was expected that the stripping of forms in the winter time would have a deterimental effect on slowing up the work, but as a matter of fact no serious difficulty arose. The principal consideration favoring low-heat cement was the reduction in temperature rise during the heat of hydration and, consequently, all other things being equal, the corresponding reduction in cracking due to temperature changes. The advantages of insuring the continuity of the structure outweighed the obvious objections of the slight additional cost of cement, the requirement of longer curing, the reduction in early strength, and possibly the necessity for more protection against damage in winter.

LOW CEMENT CONTENT OF CONCRETE

The concrete for Hiwassee Dam and spillway was composed of the two types listed in Table 2. The consultants believed that concrete must contain sufficient cement to provide strength and to insure impermeability, but at the same time that the lower its cement content, the less the tendency would be to crack. In general, dams with a larger percentage of cement have exhibited a greater than average degree of cracking and, in many cases, excessive leakage. On the other hand, some older dams, for which the cement content was less than 0.75 bbl per cu yd, were comparatively free from major cracks and yet were exceedingly watertight. With the recent tendency toward better cement, better gradations of aggregate, the use of drier mixes placed by vibration, and better control in all respects, it seemed reasonable to expect that a low cement constant would minimize cracking.

At Hiwassee Dam the aggregate was considered suitable for the use with lean mixes. Although more water was required for a satisfactory consistency than with other aggregates, the experiments before concreting showed that a mix containing 0.75 bbl per cu yd was plastic and workable and showed almost no water gain. The consultants recommended that a W/C ratio of not more than 0.85 should be used for mass concrete and that the cement content be not more than 0.8 nor less than 0.75 bbl per cu yd. They also recommended that within a distance of, say, 8 ft from exposed faces, the W/C ratio should be approximately 0.6 and the cement content about 1.10 bbl per cu yd. With the W/C ratio of 0.6, using low-heat cement, a strength of 2,500 lb per sq in. was expected at the end of 28 days and a strength of 5,000 lb per sq in. at the end of a year.

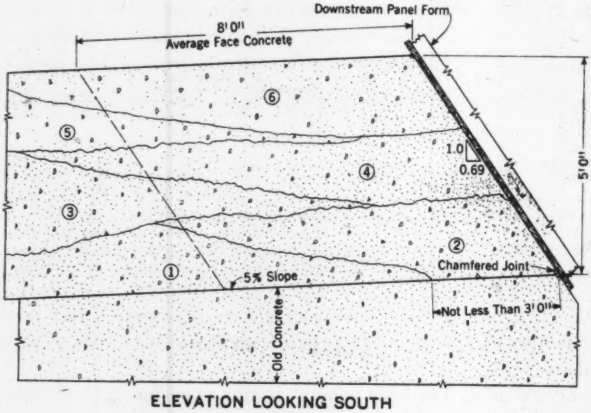


Fig. 2.—Sequence of Pours (Circled Numerals) for Concrete on Downstream Face of Dam

After due consideration, the cement content for the mass concrete was set at 0.80 bbl per cu yd, and for the face concrete at 1.15 bbl with W/C ratios of 0.80 and 0.58, respectively. Near both faces of the dam the concrete was placed in sequence between the mass concrete and the face concrete (Fig. 2) so that no well-defined joint existed between the two types. For the reinforced sections, concretes with maximum aggregates of 3 in. and of $1\frac{1}{2}$ in. were used with a cement content of 1.33 and 1.50 bbl per cu yd, respectively, and a W/C ratio of 0.58. For mass concrete a slump of one inch or less was maintained; for face concrete, $1\frac{1}{2}$ to 2 in.; and for both types of reinforced concrete, 2 to 3 in. Typical data for one class of concrete obtained from 17-in. drilled cores are given in Table 3. The dam was constructed using blocks between vertical expansion joints which were, in general, 50 ft apart (less in the spillway section). Numbering started with block 0 at the north (right) abutment, extending to block 8 next to the spillway; blocks 9 to 15 in the spillway section; block 16 near the power house and so on to the extreme south bank at block 27.

TABLE 3.—Test Results from Concrete Drill Cores, by U. S. Bureau of Reclamation

(Mass Concrete with 6-In. Maximum Aggregate; 0.8 Bbl Cement per Cu Yd; W/C, 0.8)

Block No.	Hole No.	Age (days)	Strength, in Lb per Sq In.[a]		Elastic modulus, in 10^6	Poisson's ratio	Unit weight,[b] in lb per cu ft
			Dry	Wet			
6	A	313	3,620		2.7	0.13	155.74
6	B	321		3,470	2.6	0.20	154.94
16	A	217	4,170		2.9	0.13	154.85
16	B	225		3,560	2.6	0.18	156.75
23	A	203	4,450		2.7	0.13	155.93
23	B	211		3,520	2.6	0.13	155.68

[a] Strength corrected for H/D (ratio of height to diameter) by American Society for Testing Materials (A.S.T.M.)—Designation C 42–31. [b] Unit weight by nominal dimensions.

LOW-CASTING LIFTS

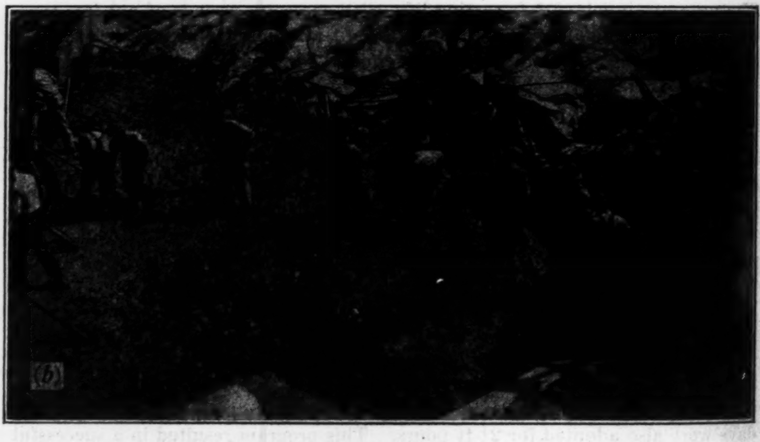
By careful control the dam was built up slowly over its full length by reasonably thin lifts and with long exposure periods to enable the "heat of hydration" to be dissipated before the lift was covered with another pour. The program involved 5-ft casting lifts with exposure periods of not less than five days, or equivalent to a one-foot height of concrete placed per day. It was believed that the additional time required for pouring thin lifts would not materially increase the cost.

Exceptions, however, were made to the general rule, particularly during the beginning of concrete placement on foundation rock or the resuming of concrete operations on old concrete; in such cases 2½-ft pours were standard practice. In general, 2½-ft pours were placed until approximately 60% of the foundation rock of each block was covered, except that on the steep foundation rock on the abutments practically the entire foundation was covered by using 2½-ft pours (Fig. 3). Further exceptions were made in some locations; for instance, in blocks 4 and 5 on the north abutment and blocks 23, 24, and 25 on the south abutment, marked changes in the regularity of the foundations formed abrupt slopes or humps. Where humps were found it was necessary to pour a rather large number of 2½-ft lifts before the top of the hump was covered. In order to permit the concrete to harden as long a period as possible and thus cool to about the temperature of the adjoining rock, exposure periods of four and five days were also adopted for 2½-ft pours. This program resulted in a successful treatment of these irregularities. Only one crack was observed as a result of the humps. This occurred in block 4 at a very sharp break in the foundation rock and where the concrete had to be relatively thin over the rock and exposed for almost a year.

From Table 4 it may be noted that, from the lowest area in the foundation in block 11 (El. 1,215.5) to the roadway elevation at the top of the dam (El.

1,237.6), a maximum difference in elevation of 322 ft is recorded and a total of sixty-nine pours was required giving an average of 4.66 ft. Sixty three of these were 5 ft high; two were 2½-ft pours on the rock foundation; and four 2½-ft pours were made on intermediate old concrete surfaces. In block 8 with an approximate height of 270 ft, a total of twenty six 2½-ft pours was made, giving an average of 3.91 ft per pour. Similar data are given for all the blocks with totals for the whole dam.

(*a*) Beginning to Vibrate the Pile



(*b*) Vibrating Completed

FIG. 3.—SHALLOW CONCRETE LIFTS (2½ FT) ON FOUNDATION ROCK

The small number of cracks is due in part to the care used in casting the concrete on the foundation in 2½-ft lifts. The considerable expense involved on account of practically doubling the cleanup cost seemed to be justifiable. Although the exposure time was permitted at two and a half days for the 2½-ft lifts, usually two 2½-ft pours were cast per week. Studies cited subsequently show the advantages of casting thin lifts with long exposure on foundation rock, thus reducing temperature rise.

TABLE 4.—SUMMARY REPORT—CONCRETE PLACEMENT, HIWASSEE DAM
(Number of Pours by 2½-ft and 5-ft Lifts)

Block No.	Approximate height of block	No. of 5-ft pours	No. of 2½-ft pours	No. of intermediate 2½-ft pours	Total No. of pours	Total No. of 5-ft lifts on 5% slope upstream
0	25	4	2	....	6	5
1	35	4	6	....	10	7
2	60	6	12	....	18	12
3	82.5	16	1	....	17	16.5
4	115	17	12	....	29	23
5	155	21	20	....	41	31
6	190	31	10	4	45	38
7	225	37	10	8	55	46
8	270	43	26	....	69	56
9	295	51	16	4	71	61
10	295	55	16	....	71	63
11	322	63	2	4	69	66
12	308	66	4	....	70	68
13	310	60	6	2	68	64
14	305	60	8	....	68	64
15	305	56	6	6	68	63
16	300	53	4	12	69	62
17	290	52	6	12.5	70.5	61
18	280	52	2	7	61	58
19	275	50	6	8	64	57
20	245	43	6	8	57	50
21	215	35	4	14	53	44
22	195	31	10	8	49	40
23	170	24	12	10	46	35
24	125	13	22	4	39	26
25	90	10	8	8	26	18
26	70	7	8	6	21	14
27	35	4	6	....	10	7
Totals		964	251	126	1,341	....

LOW CASTING TEMPERATURES

It was known that the casting temperature of the concrete would be rather high, especially during the summer, unless something was done to reduce it artificially. Two expedients were adopted, (1) using refrigeration of the mixing water and (2) rinsing and cooling of the aggregate. The first was the more important. Water batches were taken from the insulated mixing plant surge tank that was supplied by an insulated pipe line from the refrigeration plant. An insulated overflow line returned the excess water to a tank at the refrigeration plant from which it was fed by gravity to sprinkler pipes discharging over the outside of the water cooler coils, thus securing second cooling contact. The water dripped off the cooler coils and ran into the pump sump ready for circulation. Approximately 50 gal per min of river water was cooled for concrete mixing from a usual temperature of about 80° F to as low as 35° F.

The construction of the refrigeration plant was begun in August, 1938, and was completed and put into service on September 2, but was shut down on

TABLE 5.—AVERAGE MONTHLY TEMPERATURES IN DEGREES FAHRENHEIT, CONCRETE AND CONCRETE MATERIALS

Month	NUMBER OF DAYS MIXING WATER WAS:			TEMPERATURES[a]																
				Air		Cement		River water	MIXING WATER						CONCRETE PLACING TEMPERATURE WITH:					
	Not treated	Heated	Cooled						Not treated		Heated		Cooled		Mixing water not treated		Mixing water heated		Mixing water cooled	
				1	2	1	2		1	2	1	2	1	2	1	2	1	2	1	2
1938:																				
April	7	....	....	70	....	74	....	60	63	....	....	....	....	....	69	....	....	....	....	....
May	21	....	....	76	65	80	85	65	66	66	....	....	....	....	71	69	....	....	....	....
June	22	....	....	80	77	98	95	72	72	72	....	....	....	....	76	74	....	....	....	....
July	20	....	....	79	72	102	106	69	72	73	....	....	....	....	79	80	....	....	....	....
Aug.	23	....	....	83	76	104	105	75	75	76	....	....	....	....	81	81	....	....	....	....
Sept.	10	....	11	76	66	90	92	68	68	68	....	....	53	53	73	73	....	....	73	75
Oct.	....	....	21	67	58	92	93	59	....	....	....	....	42	43	....	....	....	....	63	61
Nov.	18	3	....	54	48	76	78	52	50	53	112	106	....	....	55	57	46	49	....	....
Dec.	6	10	....	44	35	68	71	43	46	44	85	89	....	....	49	50	50	51	....	....
1939:																				
Jan.	13	8	....	46	40	72	75	42	45	45	80	87	....	....	51	47	50	51	....	....
Feb.	16	4	....	53	46	75	80	43	47	49	115	107	....	....	52	53	53	53	....	....
Mar.	22	....	....	60	50	80	82	52	52	54	....	....	....	....	56	57	....	....	....	....
Apr.	20	....	....	64	54	74	75	56	58	58	....	....	....	....	59	60	....	....	....	....
May	10	....	11	73	65	101	101	65	62	60	....	....	42	42	65	67	....	....	67	68
June	5	....	17	83	74	107	107	75	76	76	....	....	44	45	81	81	....	....	74	74
July	....	....	19	85	75	104	105	77	....	....	....	....	43	46	....	....	....	....	75	75
Aug.	....	....	23	81	73	92	95	78	....	....	....	....	42	46	....	....	....	....	73	74
Sept.	....	....	20	79	69	96	105	77	....	....	....	....	41	43	....	....	....	....	72	70
Oct.	....	....	22	70	58	89	92	62	....	....	....	....	39	43	....	....	....	....	63	61
Nov.	13	....	8	52	45	82	81	55	49	50	....	....	39	38	51	48	....	....	51	50
Dec.	8	10	....	49	39	79	80	43	44	45	84	89	....	....	50	50	53	53	....	....
1940:																				
Jan.	....	22	....	....	26	....	46	36	....	....	....	134	....	....	....	....	....	54	....	....
Total	234	57	152	[a] Divided into columns marked 1 and 2 and referring to: Shift 1, 6:00 a.m.–3:00 p.m.; shift 2, 3:00 p.m.–12 midnight																

October 23, 1938, due to cold weather. In Table 5 are given various temperature data. During winter operations, the mixing water was heated by steam, its temperature being raised from approximately 40° F to 130° F. During a cold spell, the temperatures of the water ran as high as 180° F.

Washing, Rinsing, and Cooling the Aggregate

To control the grading, rinsing screens were installed to wash all aggregate except sand. In this process the aggregate was thoroughly wetted and subsequently cooled by a blast of cold air while it was dropping from the rinsing screens to the conveyer belt. This had the effect of cooling it approximately 3°. The effective reduction in concrete placing temperature by refrigerating the mixing water was about 6°. For a 5-ft lift during an exposure of five days the effective reduction in maximum temperatures due to refrigeration of the mixing water was calculated at about 3°, and due to the washing and cooling of the aggregate about 1°. The placing temperature of the concrete in the summer months averaged about 70° F, and for the winter months every effort was made to secure a temperature between 45° F and 50° F. In view of the small reductions in maximum temperature due to refrigerating the mixing water, it seems that better results might be obtained by installing an effective system of cooling for the aggregate and other ingredients.

Artificially Cooling the Concrete in Place

In general, resort was made to artificial cooling of the concrete in place (1) to reduce temperature cracking that might occur on high exposed contraction joint faces; (2) where the exposure period on the usual 5-ft pours was reduced on account of the exigency of the schedule; (3) to equalize the temperatures between the concrete on the foundation and the adjoining rock as quickly as possible; and (4) to reduce the temperature of concrete in closed recesses to bring it as quickly as possible to that of the adjoining concrete. In block 19 (Fig. 4) about 15 gal per min of river water flowing through each of three loops composed of one-inch iron pipes were used to artificially cool a 5-ft lift of concrete in place. Better results were obtained when the water was of low temperature. The difference in temperature between inflowing and outflowing water ranged from 4° to as high as 10° F. The flow as a rule was regulated daily and the cooling at each level was continued for fifteen days or more.

In Table 6 are given applicable data on artificial cooling. The cofferdam bulkheads in blocks 17 and 18 were poured with slightly thicker lifts than usual so cooling pipes were laid and water circulated in three lifts. This served to (1) expedite the pouring of concrete in those two temporary spillway blocks, (2) eliminate cracking because these bulkheads were designed as integral parts of the dam structure, and (3) offset the constant danger from destructive floods. The concrete in the assembly recesses around the ring seal gates was heavily reinforced and artificially cooled. Thus the maximum time was saved in the pouring schedule by equalizing the temperatures of the green with the older concrete; and high temperatures were prevented in parts of the gate castings when other parts were relatively cool, thus preventing warping in the castings and the gate. The first two 5-ft lifts of closure blocks 10–12–14 (spillway section) were artificially cooled in order to eliminate the necessity of making these pours in 2½-ft lifts. As a large structural steel gate was utilized in the closure, a program of pouring in 2½-ft lifts would have been awkward. The concrete in block 11 was cooled to prevent cracks due to irregularities in the foundation and in the shape of the block itself. Nearly all of the concrete in block 3 was cooled so that a more rapid pouring schedule might be adopted; this block contained the transfer track and, therefore, was not poured until many of the blocks in the dam and the adjoining blocks had almost reached the

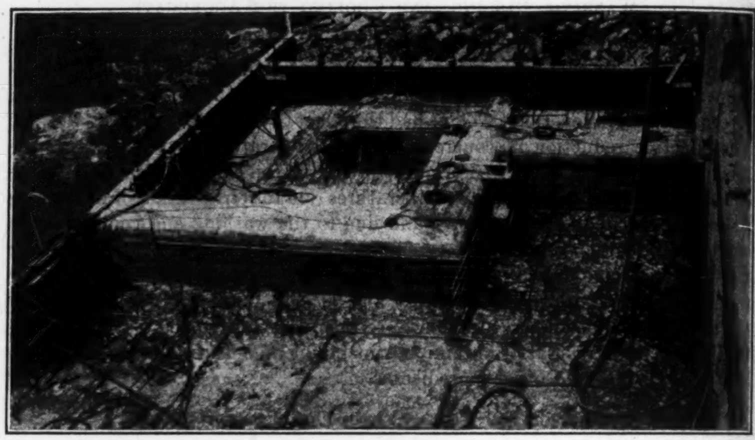


FIG. 4.—GALLERY FORMS AND COOLING PIPES IN BLOCK 19

TABLE 6.—SUMMARY OF CONCRETE LIFTS ARTIFICIALLY COOLED

Block No.	Lifts Cooled		Vertical Location, by Numbered Lifts[a]		Remarks
	No.	Thickness, in ft	From	To	
3	12	5	13	2	Pouring 2 lifts per week
9	5	2½[b]	51	49	Ring seal gate recess only
10	2	5	54	53	In lieu of four 2½-ft lifts
11	5	5	64	60	Due to foundation conditions
11	6	2½[b]	51	49	Ring seal gate recess only
12	2	5	54	53	In lieu of four 2½-ft lifts
13	6	2½[b]	51	49	Ring seal gate recess only
14	2	5	54	53	In lieu of four 2½-ft lifts
15	6	2½[b]	51	49	Ring seal gate recess only
17	3	7	54	50	Bulkhead only
17	3	5	53	51	Adjacent to bulkhead
18	3	7	54	50	Bulkhead only
18	3	5	53	51	Adjacent to bulkhead
19	17	5	46	30	High bulkhead wall

[a] Cooling pipes were laid on top of one lift and covered by the next. Lift numbers refer to covering lifts. Numbering of 5-ft lifts started with No. 1 just below roadway level and extended downward to No. 68 at maximum depth. [b] Approximately.

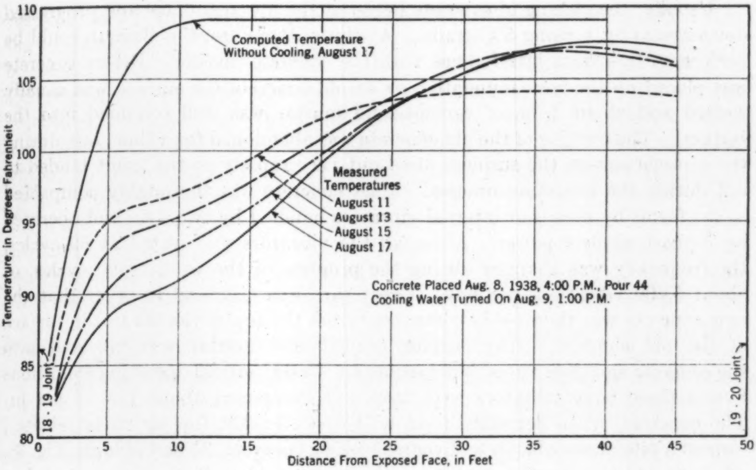


Fig. 5.—Temperature Distribution Parallel to Axis of Dam; Block 19, Bottom of Pour 44, El. 1,320.7

ultimate top elevation of the dam. Therefore, it was poured, utilizing 5-ft lifts with two pours weekly and artificial cooling.

The curves in Fig. 5 show the temperature distribution parallel to the axis of the dam, at the bottom of pour 44 in block 19, both with and without artificial cooling by the circulation of water. It is apparent that the artificial cooling of the concrete after placing was justified.

Cleaning Horizontal Construction Joints Between Lifts

Great care was exercised in preparing old concrete surfaces and foundation rock before pouring in order to secure a good bond, and thus prevent cracking and percolation along the joint with the resulting high uplift pressure. For the first three months a wet sandblast only was used, but on account of the high cost, due partly to the large number of 2½-ft lifts, other methods were attempted.

The method which was finally adopted consisted of an initial "green cut," with a jet of air and water, of the entire surface from 6 to 8 hr after the concrete had been poured in the summer, and from 8 to 20 hr in the winter; and then a final cleanup over the entire area with air and water, which also included a wet sandblast cut of that area from the upstream face of the dam to a line approximately 5 ft below the 8-in. formed drains, with special sandblasting treatments of large areas of laitance. The cutting with a high velocity jet of air and water was performed with an air pressure of about 50 lb and a water pressure of about 100 lb. The final cleanup usually came just prior to resuming concrete placement in the block. It was essential that the initial and final cleanup be followed by a thorough washing and a clean-off by air to remove loose material and water. The cleanup on the job as a whole was excellent.

Usually, the placing of concrete began at the upstream face and progressed downstream on a rising 5% grade. At times, therefore, the cleanup could be performed free from interference with the concrete placing. Before concrete was placed on the rock foundation or on old concrete the surface was usually wetted and about ¾ in. of cement-sand mortar was well broomed into the surface. The wetting of the old concrete was abandoned for a time, but during the summer season the surfaces dried out very rapidly so the grout tended to roll during the brooming process. Mass concrete was thoroughly compacted in the forms by means of internal vibrators handled by two men and operated by 3-phase electric power. Although the vibrators were rated at 60 cycles, the frequency was changed during the progress of the work to 90 cycles, or about 5,400 vibrations per min. Great care was exercised in order that the new concrete was thoroughly compacted from the top of the lift to the surface of the old concrete. Any surplus cement-sand mortar was vibrated into the concrete and thus a good bond secured. Under normal operating conditions five of these large vibrators were used and compacted about 125 yd per hr. The concrete, when deposited from a 7½-cu-yd bucket (by air valve release), formed a pile of very harsh and coarse looking concrete, 3½ or 4 ft high (Fig. 3), which would be vibrated down to a thickness of 1½ or 2 ft. Care was taken that the concrete be not dumped from an elevation higher than 15 ft above the casting grade.

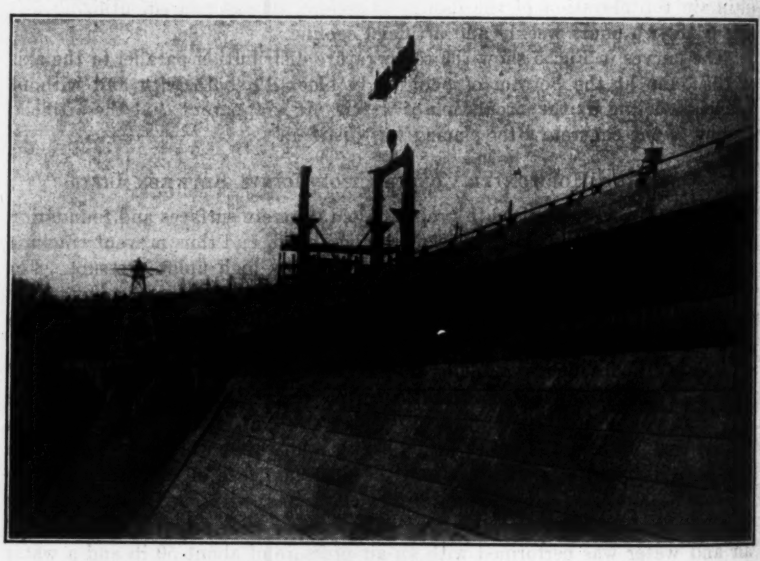


FIG. 6.—DOWNSTREAM FACE, SOUTH ABUTMENT, SHOWING HORIZONTAL CHAMFERED JOINTS, AND CRANE ASSEMBLY ON TOP

To improve the appearance of the dam, to break the usual monotony of a bare concrete face to such great height, and to eliminate any conspicuous appearance of cracks at the joints between pours, a system of horizontal and vertical chamfered joints at lift and block lines was inaugurated. This gives the appearance of a dam with blocks 50 ft wide by 5 ft high (see Fig. 6). The horizontal chamfered joints were not placed on the downstream surface of the spillway section, but they were placed on the upstream face, as well as on the adjacent abutments, above the drawdown level (El. 1,415). The vertical chamfered joints on the abutments were 3¾ in. wide and 1¼ in. deep as compared with 2 in. by 0.75 in. for the spillway. The horizontal chamfered fillets were 3¾ in. by 1¼ in. On blocks where a large number of 2½-ft pours were scheduled, 5-ft pours were cast for a distance of approximately 20 or 30 ft from the downstream face and then the surface was sloped down to make a 2½-ft pour through the balance of the block. The distances of 20 and 30 ft from the downstream faces were staggered so as to eliminate the possibility of a vertical crack forming parallel to the axis of the dam. As far as is known cracking was obviated by this procedure.

To prevent the percolation of water along the contraction joints from the upstream face, vertical 6-in. drains were constructed, with water stops of stainless steel sheets extending through the drains and 12 in. on both sides into the concrete.[2] These extended from the rock foundation to the top of the dam. Additional strips of stainless steel sheets 8 in. wide and 3 ft long were welded to the stainless steel water stops on each side of the contraction joint, and extended 4 in. into the top of each lift, thus bonding the two pours and preventing the seepage of water along the horizontal pour joints with resultant cracking.

Except for the closure blocks 10, 12, and 14 and for some slight adjustments in blocks 17 and 18 due to their use as a temporary spillway and to complications during the construction of the penstock, concrete was poured on a 5% grade (Fig. 2) sloping upward from the upstream face toward the downstream face. No joints or breaks in these lifts were permitted except when they were unavoidable on account of breakdowns in the plant or on account of a flood, as in the case of a closure block. Only six of these inside joints were made in the total number of 1,341 pours in the dam. Under no consideration were these

TABLE 7.—INTERRUPTIONS IN CONCRETE POURING

Date	Shift	Duration	Block No.	Cause
8-4-38	Night	5 hr	23	Excessive rain flooded out freshly poured concrete causing abandonment of form
12-20-38	Night	34½ hr	7	Conveyer pulley shaft broke at head house
7-12-39	Night	11½ hr	18	Changing endless cable on cableway
3-23-39	Night	6½ hr	18	Block too large to complete in regular shift
3-28-39	Night	8 days[a]	8	A 2½-ft pour left incomplete, cableway trouble
12-14-38	Day	20½ hr	14	Head pulley and shaft broken on conveyer belt to mixing plant

[a] Did not complete half lift.

joints permitted closer than 20 ft from either face of the dam; and where the exposure period was in excess of approximately 6 hr, the surfaces were cleaned up and covered with grout the same as a new pour. It is certain that no longitudinal crack appeared in the dam as a result of these joints. Irregularities in pouring are listed in Table 7.

STEEL REINFORCEMENT TO PREVENT CRACKING

Careful inspection of the 1,827.6 lin ft of galleries in the dam has disclosed very few cracks. This is due to several contributing circumstances, one of which is the placing of heavy steel reinforcement, from 1-in. to $1\frac{1}{4}$-in. square bars, in a horizontal position 5 in. above the roof of the gallery (Fig. 7). No doubt other contributing conditions included the use of low-heat cement; the low cement content of mass concrete that was placed all around the inspection galleries regardless of its close proximity to the upstream face of the dam; the long periods of exposure between pours; the pre-cooling of the mixing water in warm weather; and the use of flat roof with corner fillets on a 12-in. radius rather than a full arch.

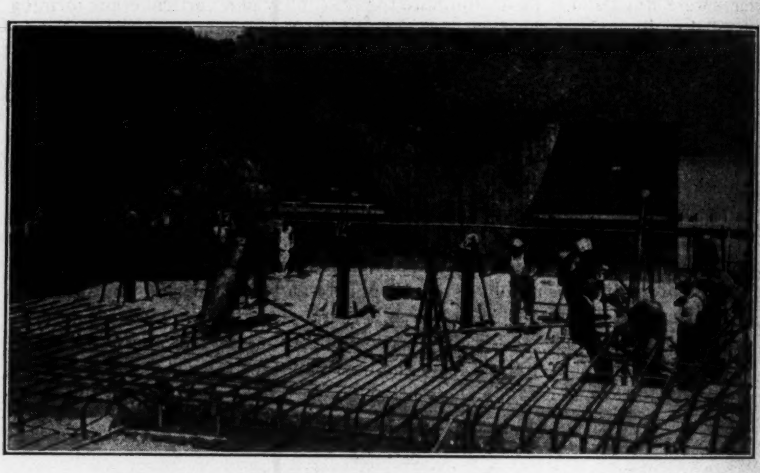


FIG. 7.—TYPICAL REINFORCEMENT OVER ROOF OF INSPECTION GALLERY

Steel reinforcement was also used in the recesses provided for the placement of the ring seal gates in order to prevent cracking in the comparatively thin walls of concrete between the gates and the contraction joints and to bind the concrete firmly to the regulating conduits and the ring seal gate castings. A few cracks were found in the bulkhead faces of blocks 4, 6, 7, 15, 19, and 20, and several of these cracks extended through more than one lift. The first crack was discovered (September 23, 1938) on the 19–20 contraction joint face of block 20. This was caused, no doubt, by a rather large height differential between the two blocks (about 30 ft), and by a rather sudden drop in temperature from a low of 60° at night to a low of approximately 40°. Two cracks were found on the bulkhead face of block 19, which also were caused by the excessive difference (maximum 130 ft) between the top of blocks 18 and 19.

It was anticipated that the high face in block 19 would be subject to some cracking. Therefore, sixteen 5-ft lifts were cooled by pipes for a total height of 80 ft. Two cracks occurred in this bulkhead face, the largest being 0.1 in. It was interesting to note that this crack terminated after it penetrated the cooled portion of the block.

The largest crack occurred in block 15 on the 14–15 contraction joint about November 28, 1938. This crack was influenced, no doubt, by the large ring seal gate recess in the center of block 15, but the 60-ft difference in height between the tops of blocks 14 and 15, coupled with a sudden drop in the minimum air temperatures, probably caused it. This crack reached a maximum opening of ⅛ in. and extended to El. 1,298 and also across the top of the lift parallel to the axis and into the ring seal gate recess. As it was more nearly of structural significance than any other crack, it was carefully explored and full data obtained as to the location, depth, and width inside the concrete. An attempt was made to stop this crack from projecting into the pours that would be made above El. 1,298. Another crack was discovered in block 4, which was caused by the existence of a hump in the rock under the concrete, a considerable height differential, and a long exposure of the block.

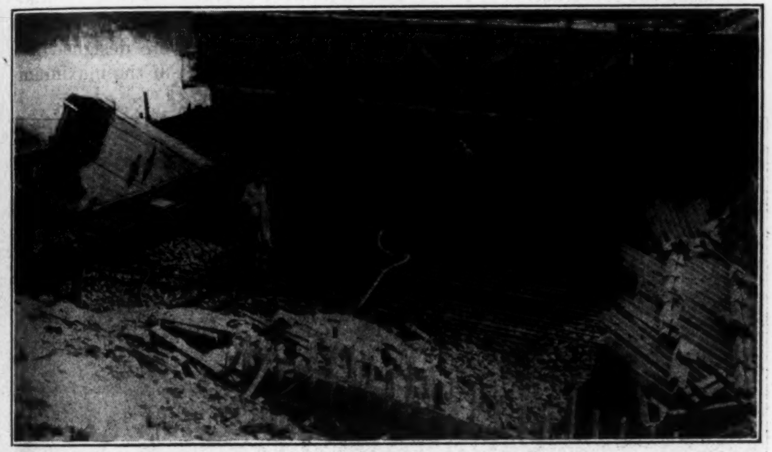


Fig. 8.—Mat of Steel Reinforcement Placed in Block 4 to Prevent Upward Extension of Crack

The crack in block 15 occurred on the side of the bulkhead wall, where it was possible to observe it from time to time. It was decided to place heavy mats of reinforcing steel on the top of the lift and across the block in an attempt to keep these cracks from extending upward. In general, it may be stated that the mat in block 15 was composed of 1¼-in. square bars placed 9 in. center to center, both vertically and horizontally, in cross sectional areas parallel to the axis, and $\frac{3}{4}$-in. spacer bars 9 in. vertically and 2 ft 6 in. horizontally. A mat for block 4 was similar (Fig. 8). These mats proved very effective, as it was possible to observe that the crack in block 15 did not extend beyond the top of the block at El. 1,298 where the steel was placed.

In addition, a decision was made to treat cracks wider than 0.03 in. by covering them with tar impregnated canvas to prevent the entrance of mortar and foreign matter. Horizontal holes were then drilled into the concrete of the bulkhead wall to intersect the crack about a foot from the surface, and pipes were laid for grouting these cracks at a later date.

DIAGONAL KEYWAYS ON BULKHEAD JOINTS

During the construction of Norris Dam a number of cracks on the bulkhead walls extended to the downstream face of the dam following the vertical keyways. To overcome this tendency, diagonal keyways were designed for Hiwassee Dam on the bulkhead walls following the direction of the maximum stresses (practically parallel to the downstream face, see Fig. 9). As a rule, the metal forms used for these keyways were tacked securely to the face of the bulkhead panels to keep them attached during the stripping operations.

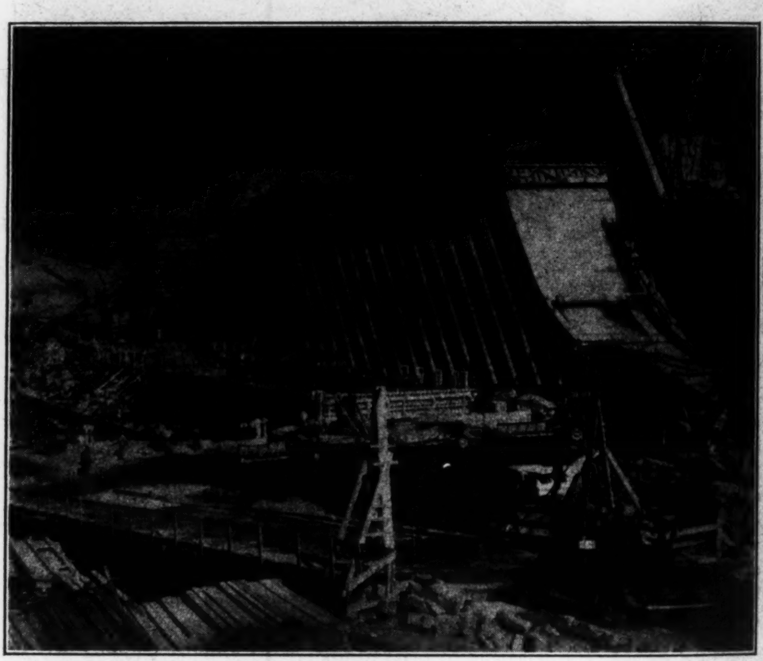


FIG. 9.—DIAGONAL KEYS ON BULKHEAD WALL OF BLOCK 15 (SPILLWAY SECTION)

The keys have a trapezoidal cross section, 6 in. deep and 4 ft wide at the bottom with 12 in. for each chamfered slope.[3] They were constructed of 14-gage and 16-gage steel plate, and on an inclination of $7\frac{5}{32}$ in. horizontally to 12 in. vertically. It is difficult to form any positive conclusion as to the effectiveness of these keyways to prevent cracking on the downsteam face because there were so few cracks at Hiwassee Dam and none of the cracks on the bulkhead faces extended to the downstream face. On the contraction joints, however, it was noticed that fine cracks would follow the keyways, but in almost all instances cracks that extended through several lifts crossed the keyways.

CURING AND WINTER PROTECTION

A program of curing was adopted for Hiwassee Dam at the outset. It consisted of wetting the top of the blocks and the sides for a period of twenty-one days. This did not include the bulkhead walls nor was it done as a rule when the air temperature was 40° F or lower. For a time, sprinklers were used on top of the blocks, and then hand sprinkling. Iron pipes with small holes about 6 in. center to center were fastened to the panel forms to spray both upstream and downstream faces of the dam.

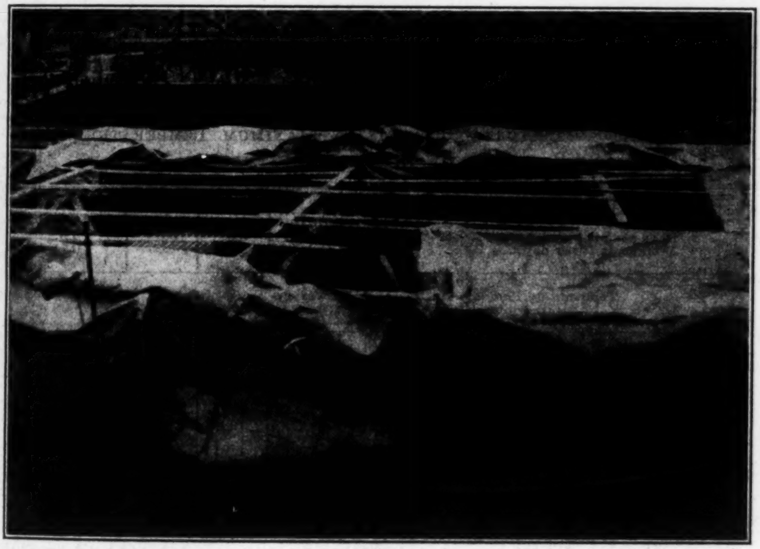


FIG. 10.—TO PROTECT CONCRETE IN FREEZING WEATHER A TARPAULIN COVER WAS SUPPORTED ON HORSES, WITH STEAM COILS UNDERNEATH

Efficient curing by sprinkling with water for long periods undoubtedly produced a continuous gain in strength; and this prevented major cracking as well as fine checks or hair cracks. These hair cracks, when subjected to sudden and wide variations in temperature, might develop into real cracks. It is believed, therefore, that an adequate program of curing did materially assist in the crack prevention program.

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[3] "Design of Hiwassee Dam—Engineering Details," by Cecil E. Pearce, *Civil Engineering*, July, 1940, p. 435.

Due to the slower hardening process of low-heat cement, it was necessary to install rather elaborate equipment for winter protection not only for the aggregate but also for the mixing and placing of concrete. The gates in the aggregate reclaiming tunnel were surrounded with steam pipes and the tunnel itself was heated by steam. The rinsing screens were protected by wooden housing built around the entire equipment and heated by electricity. The Mixer Building was heated, also the aggregate in the bins, by means of steam coils. The mixing water tank was provided with steam coils. After the concrete had been mixed and delivered to the forms, it was protected from freezing by coverings of heavy tarpaulins or canvas (Fig. 10) and by the use of large steam coils placed sufficiently above the concrete to prevent its damage due to drying out. Much cold weather was experienced, but no serious damage was done to the dam or spillway structure.

TEMPERATURE STUDIES

An attempt has been made to determine by what amount the maximum temperature in the dam has been reduced by the various expedients adopted. From temperature observations it was ascertained that the maximum temperature rise in the dam was 40°. Calculations based on observations at Hiwassee Dam, Norris Dam, and elsewhere, seem to indicate that the use of low-heat cement, low cement content (0.8 bbl per cu yd), low casting lifts (2½ ft) with three and a half days exposure, artificial cooling of mixing water, aggregate,

TABLE 8.—ESTIMATED EFFECTS ON MAXIMUM TEMPERATURE RISE DUE TO VARIOUS ADJUSTMENTS

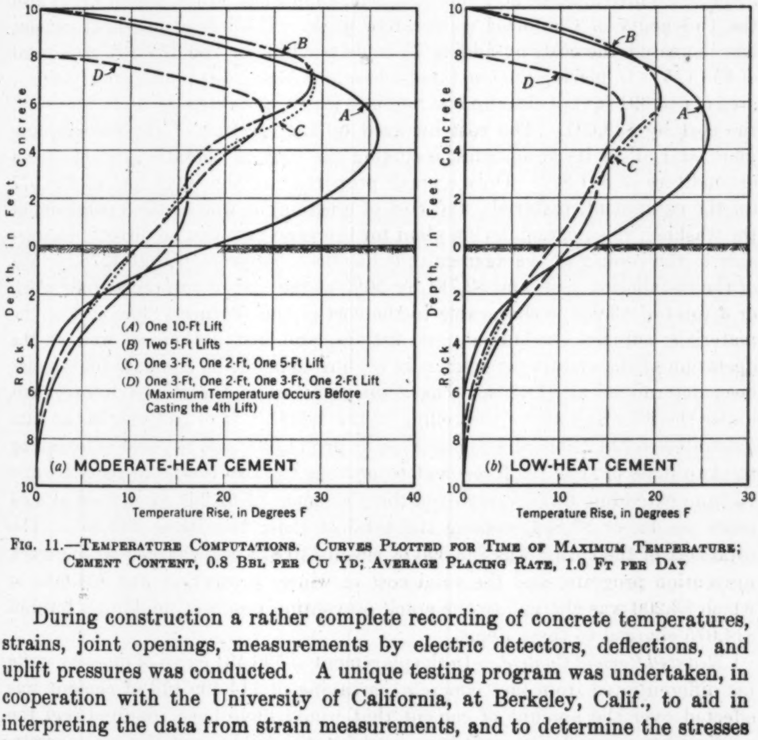
No.:	Normal Schedule:					
1	Lifts, in ft	10.0	10.0	5.0	5.0	2.5
2	Days per lift	3.0	5.0	3.0	5.0	3.5
3	Average, in ft per day	3.33	2.00	1.67	1.00	0.71

Temperature Reduction in Degrees F for Various Types of Cement
(S = Standard; M = Modified; L = Low Heat)

		S	M	L	S	M	L	S	M	L	S	M	L	S	M	L
	Reduction:															
4	Normal schedule	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	Normal schedule, but L cement	15	7	..	13	6	..	12	5	..	9	4	..	7	3	..
6	Normal schedule, but 5-ft lifts every five days	13	11	8	9	7	5	7	5	4	..	..	..	−5[a]	−4[a]	−3[a]
7	Normal schedule, but 2.5-ft lifts every 3.5 days	18	15	11	14	11	8	12	10	8	5	4	3	..	..	..
	Combinations (with L Cement):															
8	5-ft lifts every five days	22	15	8	18	11	5	16	9	4	9	4	..	4	0	−3[a]
9	2.5-ft lifts every 3.5 days	26	18	11	21	14	8	19	12	8	12	7	3	7	3	..

[a] Minus sign indicates an increase in temperature.

and concrete in place have reduced the maximum temperature approximately 31° below that which could have been expected had "Type B" (modified cement) been used, with a cement content of one barrel per cubic yard, and 5-ft lifts and five-day exposure periods. Of course, this would apply only to those parts of the dam constructed in the 2½-ft lifts and using artificial cooling of the mass. Table 8 shows estimated changes in the maximum temperature rise due to the placing rate and type of cement used; it gives temperatures for concrete, poured at Hiwassee Dam, containing 0.8 bbl of cement per cubic yard, based on preliminary estimates of heat generation for low-heat cement. The cement actually used generated more heat than was estimated. In addition, Figs. 5 and 11 show temperature variations, both recorded and computed.



(a) MODERATE-HEAT CEMENT (b) LOW-HEAT CEMENT

FIG. 11.—TEMPERATURE COMPUTATIONS. CURVES PLOTTED FOR TIME OF MAXIMUM TEMPERATURE; CEMENT CONTENT, 0.8 BBL PER CU YD; AVERAGE PLACING RATE, 1.0 FT PER DAY

During construction a rather complete recording of concrete temperatures, strains, joint openings, measurements by electric detectors, deflections, and uplift pressures was conducted. A unique testing program was undertaken, in cooperation with the University of California, at Berkeley, Calif., to aid in interpreting the data from strain measurements, and to determine the stresses by laboratory methods rather than by mathematical analyses. The entire scientific program was directed by the TVA materials engineering organization, except the experiments at the university. This scientific program was not necessary at Hiwassee Dam in order to prosecute the crack prevention program. That is, when the results were finally interpreted by the experiments and calculations, it was too late to apply them; but they may be of service to other engineers and on other large projects.

Cost of Program

A comparison of the cost of the program of crack prevention at Hiwassee Dam, with a program that would have used a predicted cement content of one barrel per cubic yard and without all of the crack prevention features, has been made. In reviewing the cost, the additional life of the structure, which has no doubt been increased because of the relative absence of cracks, should also be considered and credited, especially when the crack prevention program has actually resulted in a saving.

Additional Costs.—To conduct the scientific program cost $29,000, which includes all instruments, equipment, special shop work, labor, salaries, and also the University of California cooperative work. The use of low-heat cement has increased the cost, by adding 7½ cents per bbl for 766,375 bbl, or a total of $58,115. In placing the two hundred and fifty-one 2½-ft pours, an additional area of 643,691 sq ft of cleanup was required which, at 6 cents per sq ft, increased the cost by $38,621. The cost incurred by the erection of the refrigerating plant and all of its accessories, including the cost of operation, involved an expenditure of $41,820. Only a small proportion of the total cost of $48,677 for the equipment, materials, and cost of erection, as well as the operation, of the washing, rinsing, and drying plant for the aggregate can be directly chargeable to the cooling of the aggregate; it has been determined that $278, or 3% of the installation cost, and $2,781, or 50% of the cost of compressed air used, or a total of $3,059 is chargeable to the cost of this feature. The cost of the materials, supplies, including pipe, fittings, and installation, as well as the operation of the system for artificially cooling some of the concrete blocks, has been determined as $11,864. The additional cost of the steel reinforcement under the floor and above the ceiling of the 1,828 lin ft of galleries in the dam amounted to 238,210 lb at 4.64 cents per lb, or $11,033; and the cost of installing the two mats of steel reinforcement to prevent the two cracks in blocks 4 and 15 from extending upward involved the placement of 52,201 lb of steel at 4.64 cents per lb, or $3,814, making the total of these two items $14,857. The total cost of the curing was $43,500 of which $10,875 was charged to the crack prevention program, and the total cost of winter protection was $31,600, of which $3,200 was charged to the crack prevention program, making a total of $14,075 charged to these items.

Reduced Cement Content.—In the placement of 596,431 cu yd of mass concrete (as differentiated from face concrete), a saving of 114,499 bbl of cement was effected over the amount of cement that would have been required had the predicted one barrel of cement been used for each cubic yard of mass concrete. When a check was made by deducting 0.20 bbl of cement per cubic yard for the 596,451 yd of mass concrete, a saving of 119,290 bbl was obtained. In calculating the saving in cement, however, the lower number (114,499 bbl) was used; at $2.14 per bbl, which is the cost at the mixing plant, a total saving of $245,028 is indicated.

The cost of the crack prevention program may be summarized as follows: Additional cost of—

Scientific program	$ 29,000
Use of low-heat cement	58,115
Thin casting lifts	38,621
Low casting temperatures	41,820
Cooling the aggregate	3,059
Artificially cooling the concrete in place	11,064
Use of steel reinforcement	14,867
Curing and winter protection	14,075
Total additional cost of crack prevention (794,439 cu yd at 26½ cents)	$210,621
Reduction in cost for saving in cement (114,499 bbl at $2.14)	245,028
Surplus or saving	$ 34,407

ACKNOWLEDGMENTS

During construction I. L. Tyler, M. Am. Soc. C. E., Douglas H. McHenry, and W. R. Waugh, Assoc. M. Am. Soc. C. E., have had charge of the Materials Engineering Section in the order named under the immediate direction of the writer. These men supervised the work in the concrete laboratory, the technical control of the aggregate manufacture, and the concrete mixing and placing; conducted the scientific program; and directed much of the work on the crack prevention program. The results achieved are largely due to their efforts.

Hiwassee Dam was designed and built by the TVA under the general direction of: A. E. Morgan as chief engineer and Carl A. Bock as assistant chief engineer until May 1, 1938, and since that date, Theodore B. Parker, chief engineer; B. W. Steele until November 20, 1936, as chief design engineer; Barton M. Jones as acting chief design engineer from November 20, 1936, to January 16, 1939; H. A. Hageman since that date as chief design engineer; Theodore B. Parker to July 1, 1938, and A. L. Pauls, since that date, as chief construction engineer; C. E. Blee, project engineer at the dam; and the writer as construction engineer. All the foregoing are members of the Society. Lex Phifer was acting construction superintendent to January 1, 1938; from that date to March 8, 1940, J. E. Walters was construction superintendent, and since the last date E. C. McClenagan was acting construction superintendent.

DISCUSSION

Douglas McHenry,[4] **Esq.** (by letter).—The author has presented a straightforward objective account of the procedures which were successfully used to control cracking during the construction of a large concrete dam. It might be well to emphasize the thought that Hiwassee, like every other major dam, represents a particular case in respect to both design and construction, and that the control methods which were satisfactory in this case may be insufficient or uneconomical elsewhere. On this account the paper would not have lost in value or interest had it included more discussion of the reasoning which led to the adoption of certain procedures and the rejection of others, together with a final evaluation of the effectiveness of the various items listed in the cost schedule.

On future projects where cracking control is required, the Hiwassee experience may well serve as a guide, but scarcely as a model to be followed exactly. The list of factors to be considered in conducting a control program may reach rather formidable dimensions. It will include such items as properties, topography, and structure of the foundation, properties of the materials of construction, details of design, river diversion scheme, climate, and (not the least important) the personnel of the design and construction groups. All of these items can, and should, be taken into account. Many of the details can be accorded only qualitative consideration, and must be handled in accordance with common sense and experience. Many cannot be covered by advance specifications because the conditions requiring treatment are often unpredictable.

A successful crack control program must combine (very much after the fashion of a successful program of foundation treatment) a rigid general specification, based upon preliminary study, with flexible detail specifications which may be adapted to various unpredictable conditions.

The writer was in close contact with the Hiwassee program from the time of its inception until the completion of the dam, and gained a definite impression that the relative values of the different phases of the program were about proportional to their relative costs, with the exception of the item "low casting temperatures." It appears, now that the job is completed, that the expenditure for cooling the mixing water would have been better justified had it been increased sufficiently to permit the use of slush ice in the mixers (as at Friant Dam, in California); or, if it had been distributed among the other phases of the program. The advantage gained by pre-cooling is dissipated to some extent during the period of exposure between lifts; so pre-cooling will be most effective with a rapid placing schedule.

The evidence favoring some type of low-heat cement for massive concrete construction is now established rather definitely. Former fears regarding construction difficulties due to its slower rate of hardening have been largely dispelled, and experience has shown that it need cost little or no more than modified cement. Superficial temperature studies may be deceptive in regard to the effect of type of cement. Under certain construction conditions it is possible that low-heat cement will produce a greater temperature rise than modified cement. The advantage lies chiefly in the slower early rate of heat generation (especially advantageous with artificial cooling), the more favorable shape of the temperature gradients, and the greater extensibility of the concrete.

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[4] Engr., U. S. Bureau of Reclamation, Denver, Colo. (formerly Engr., TVA).

The decision to use thin lifts on the foundation and on old concrete surfaces was based upon theoretical studies of temperature distribution, and the resulting stresses. Such studies were necessarily incomplete because of the present lack of a satisfactory means of relating strains and stresses during the early life of concrete. It is gratifying that field performance indicated that this phase of the control program was fully justified.

The cost item of $11,064 for cooling the concrete in place artificially represents, in the writer's opinion, one of the most worth-while items of expenditure in the program. Among other things, it has indicated the complete feasibility of applying embedded-pipe cooling to local regions where a degree of volume constancy is required, either to prevent cracking or for other reasons. It is of interest to compare the cost of the adopted control methods with the cost of an alternative scheme, probably equally effective, involving artificial cooling of the entire dam. With complete cooling the items of thin casting lifts, low casting temperatures, cooling the aggregate, and local artificial cooling might have been eliminated, but all others would have been largely, if not wholly, retained. The minimum cost of a complete embedded-pipe cooling system may be taken as $0.144 per cu yd of concrete (the cost for Grand Coulee Dam), or $115,000 for the entire structure. The total cost under this scheme would have been $231,000, compared with the actual cost of $210,621. The difference is scarcely great enough to be significant as an indication of the relative economies of the two schemes because of insufficient data on their relative merits. In this particular case, grouting the contraction joints before putting the structure into service was not considered necessary. In cases where early grouting is essential, complete artificial cooling is definitely required.

The author's balancing of saving in cement against the cost of the control program to indicate a net saving is most interesting. This balance apparently carries an inference that the dam as actually constructed, with a low interior cement content and relatively free from cracks, is equal in quality to the same structure with higher strength interior concrete and a normal amount of cracking. The writer is inclined to concur in this view, but recognizes it as an excellent subject for speculation.

The inclusion of the $29,000 item for the scientific program is likewise of interest; this inclusion acknowledges a debt to the similar research investigations of the past that are to a large extent responsible for the success and economy of the Hiwassee control program.

REGINALD H. THOMSON,[5] HON. M. AM. SOC. C. E. (by letter).—This is a paper of unusual excellence because it gives a clear and concise report on the numerous different operations that had as their common objective the prevention of cracks and the construction of the most durable wearing surfaces in the Hiwassee Dam. Furthermore, a financial statement on the program is submitted, and the reader is afforded an opportunity of appraising this phase of concrete dam construction on a dollars-and-cents basis. Although he differs with the author on some matters, the writer would nonetheless express his appreciation of this distinctly superior paper. The fact that the total cost of the dam is nowhere given renders percentage studies based upon total cost impossible.

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[5] Cons. Engr., Seattle, Wash.

During recent years the building of concrete dams has become a matter of increasing care and concern to those charged with the responsibility of their construction and maintenance. For this there is good reason, since some of the older dams have failed to possess either the watertightness or "permanence" so confidently and enthusiastically attributed to them when they were first built. Dams are expensive structures at best, and heavy maintenance and repair bills following heavy initial investments are always unsatisfactory. Particularly are they so when unexpected.

The engineers furnishing the ten or eleven formulas, under which this construction was to be completed, showed by their provisions that they fully realized that water was one of the greatest of solvents, that in passing through cracks either in concrete or granite it dissolved and carried away some of the substance, and in so doing not only enlarged the crack but weakened the structure. The rules for the distribution of the cement throughout the structure showed an intelligent comprehension of the relation of heat in the newly made blocks of concrete to their expansion and contraction in the process of curing.

There was also a definite comprehension of the fact that either rock or masonry exposed to weather is subject to elements provided by nature for the destruction of masonry by the aid of water in connection with heat and cold. It was also finally detected and falteringly admitted that water is a destructive agent in the concrete mix itself, and that its proportionate quantity must be regulated carefully.

Engineers finally came to a realization that excessive mixing water was the cause of many troubles. Dams and other structures built with sloppy mixes were leaking, weathering, and eroding, whereas older dams of the "dry-tamped" period were not. With flawless logic it was reasoned that a marked return toward former practice would largely restore the former qualities. It also was observed that concrete mixtures with more cement in them resisted weathering and the penetration of water better than the leaner mixes. Methods of mixing so as to combine the aggregate into a somewhat unified body were studied also.

Out of the accumulated experiences has come a very marked improvement in modern dam construction that permits a central mass of concrete of reasonably low cement content, a water content as low as possible, and an aggregate gradation of maximum density. Having established these values for the particular project, thenceforth it is demanded to maintain them and secure uniformity in batching, mixing, placement, and curing. The final result comes from the minute attention given to the adopted formula—or to its misuse.

The tables in this paper, beginning with Table 1, constitute valuable data for the student of concrete behavior. Here are given the weights of seven different batch mixes, and the weights of cement, sand, and large aggregates of four different sizes, together with the pounds of water used. This is followed by records of compressive strength achieved by the various field mixes running up to one year. This provides a field for specific investigation for inquisitors through many years to come.

Table 2, comparing concrete to certain composition in the Norris Dam with concrete of different composition in the Hiwassee Dam, shows the persistence of study and forward look on the part of the Hiwassee constructer. Table 5 gives the temperatures prevailing when the various masses were deposited. Certainly, the care and watchfulness taken in securing and recording the facts given in these tables mark a class of construction personnel far ahead of the ordinary, and, through such careful work and intelligent study of results, concrete so made should enjoy the word "permanent." How much of the special work done under this $210,621 contract is at this date specifically unusual it is difficult to affirm definitely.

The use of low lifts (approximately 5 ft) with long hours given for each lift to enable the concrete to adjust its chemical combination and return to quite nearly normal temperature, using a minimum of cooling pipes throughout, as against the common practice of 10-ft lifts with an abundance of cooling pipes, is a process that can be commented upon only with data furnished from structures of similar size, and this comparison the writer is not able to make.

It is true that the three casting lifts are charged with only $38,621 of the total increased cost, and that at 6¢ per sq ft for the cost of cleaning up, and with no knowledge as to the price paid per hour, the 6¢ cost carries little information for a separate job. On such works as the writer has been engaged upon, time of completion of the structure has been of great value. Here, with 5-ft casting lifts, five days (or one day per ft) is given for curing of concrete. This was done to avoid cumulative heat and resultant expansion. Since all this work was done on a cost-plus arrangement, and since the writer is ignorant as to the terms of the agreement, he cannot give any opinion as to the probable final cost compared with the cost that would have followed with 10-ft lifts and standard cooling pipes.

The report on diagonal keyways is very interesting and indicates that the usual belief that stresses occur normally at right angles to the face of a structure is reasonably correct.

The paper refers to tests made during the first few months of concreting to determine the effect of grinding in the mixers, which tests were used to correct the combined aggregate grading of the material going into the mixers, and to obtain a desirable grading in the fresh mixed concrete. This process is one which the writer does not quite comprehend. Was it to determine the number of revolutions to be made in the mixing drum, or was the rock aggregate so soft that the reduction made in the size in the mixers had to be accounted for? This is a refinement to which the writer is not accustomed.

Engineers who follow contractors' methods of "getting things done," rather than laboratory methods, will no doubt question the value or the worth of the $210,621 cost added to the possible lesser cost that might have been incurred with less attention to details.

One can only "guess at" or roughly estimate the probable cost of the dam, except by listing the quantities of materials used in its construction, but cannot place the probable cost at less than, say, $12,000,000. Now, an insurance cost of $210,621 placed upon a $12,000,000 structure, to insure longer life, is a very small sum. Personally, the writer feels that the need of refacing the face and spillway of this dam has been postponed by this care for such a number of years that it will make this sum a ridiculously small one to have been incurred in order to produce the added life.

It is understood that there is a committee of engineers in the employ of the federal government which is now examining every known dam in the United States, and preparing a report in which all salient points relating to construction will be set out. When that has been published, many theories will no doubt be dissolved, and a better light given by which to design; but the tables given in this paper will continue to be of value.

W. R. WAUGH,[6] ASSOC. M. AM. SOC. C. E. (by letter).—In recent years more and more study has been given to the problem of temperature cracking in mass concrete. An analysis of the causes surrounding the occurrence of a crack usually has suggested a possibility of eliminating the recurrence of a crack under similar conditions by controlling the circumstances that were known to have caused the original crack. A diligent record, made during construction, of the location and circumstances of occurrence of cracks, is invaluable as an aid in studying the causes, and suggesting means, of controlling cracking. At Norris Dam such a record was made as a part of a program of special-instruments investigation which included the measurement of concrete temperatures at selected points in the dam. Much of the experience gained from a study of these data, along with knowledge gained from the experience of other agencies, was translated into the special program adopted for Hiwassee Dam. The success of the program is attested by the small number of cracks that developed.

As indicated by the author, some cracks did occur and careful records were made of locations and circumstances of occurrence. These indicate the need for continued study of causes and improvements in the technique of prevention. Refinements in the construction program offer a means of eliminating circumstances under which cracking is likely to occur. The elimination of large differentials in height between adjacent blocks is one means of avoiding circumstances that are conducive to cracking. High contraction-joint faces exposed for long periods nearly always result in cracking; however, the construction program can usually be arranged economically to eliminate most of these high exposed faces. In some instances it is necessary to leave certain blocks low for stream diversion during construction. When such a situation arises, special measures can be taken to control the conditions of temperature in the adjacent blocks with the high exposed faces so that cracking will not occur. The mass was cooled artificially in a circumstance of this nature at Hiwassee Dam, with success.

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[6] Materials Engr., TVA, Hiwassee Dam, N. C.

In the closing paragraphs of the paper under the heading "Cost of Program: Reduced Cement Content"), the author has presented a summary in which he purports to show that an actual saving in total cost of the structure was achieved as a result of the program. In several respects the writer cannot agree with the author's method of computing what he calls "Additional Cost," and there also arises a question as to the validity of the credit claimed for a saving in cement.

Considering first the summary of additional cost and using the author's nomenclature and explanation under the heading "Additional Cost," the estimate in Table 9, Col. 3, is submitted as the writer's estimate of additional cost. Comparing these values with the author's summary (Col. 2, Table 9), it will be noted that three items (Nos. 1, 5, and 8, Table 9) have been omitted and one item (No. 7) changed. First, the cost of the scientific program (No. 1) has been omitted because this program was conducted to gain additional knowledge concerning stress, strain, contraction joint openings, foundation uplift pressures, and interior temperatures during construction and with the structure in service. It is true that some of the thermometer installations aided in the control of artificial cooling in one block, and this was a benefit to the program; but largely the scientific program did not benefit the crack prevention program directly and, therefore, should not be charged against it. Second, some small benefit, no doubt, was gained from the cooling of the aggregate (No. 5, Table 9) after the rinsing process was completed due to evaporation of free water. The rinsing process, however, had an entirely different function, and the benefit which did accrue was small and purely incidental; and, therefore, this charge should not be made. Third, the charge for "curing and winter protection" (No. 8, Table 9) should be omitted because these are normal functions of the concreting program that would have been

TABLE 9.—Comparative Cost Estimates, Crack Prevention Program, in Dollars

No.	Description (1)	Author (2)	Writer (3)
1	Scientific program	29,000	....
2	Use of low-heat cement	58,115	58,115
3	Thin casting lifts	38,621	38,621
4	Low casting temperatures	41,820	41,820
5	Cooling the aggregate	3,059	....
6	Artifically cooling the concrete in place	11,064	11,064
7	Use of steel reinforcement	14,867	3,814
8	Curing and winter protection	14,075	....
9	Total additional cost of crack prevention	210,621	153,434
10	Reduction in cost for saving in cement	245,028	123,000
11	Surplus or saving	34,407	....
12	Increase in cost of project	....	30,434

conducted regardless of whether or not any special measures for controlling temperature cracking had been adopted. Fourth, as to the "use of steel reinforcement" (No. 7), only that which was used in the special mats intended for the purpose of preventing the extension of two cracks which had already occurred should be charged to the program, because the steel in the roof of the galleries would normally have been used in any event.

In proof that an actual reduction in cost to the structure resulted from the program, the author has taken a credit for cement saved. The cement content of 0.80 bbl per cu yd for interior (mass) concrete was a natural outcome of having selected a maximum allowable water-cement ratio which resulted in this cement requirement since for the desired consistency the water requirement was a rather definite quantity. In assuming what he calls "the predicted one barrel of cement" per cubic yard as a basis for the saving computed, the author has used a value that is subject to question since this prediction was made during the very early investigations and before a maximum allowable water-cement ratio was selected. The saving he has computed, therefore, is subject to question. The value of using the lowest possible cement content commensurate with desired properties of the hardened concrete both from the standpoint of pure economy and the reduction in tendency to crack is beyond question, but in this case the manner of computing the saving is questioned. At Norris Dam a cement content of 0.90 bbl per cu yd for the interior concrete was adopted. Using this value in computing the credit for cement saved would seem more logical. Thus the credit would be about half of that used by the author, or roughly $123,000 (No. 10, Table 9). Applying this credit against the suggested "Additional Cost" (No. 9), an estimated increase in cost to the structure of $30,434 would result.

It does not seem logical that a program so valuable to the finished structure could be conducted without cost. This would seem contrary to the old adage that, "You can't eat your cake and have it too." Even if the program had cost the project all of the $153,434 proposed as the "Additional Cost" (which is probably not true) and if no credit was taken for cement saved (which would indeed be an extreme viewpoint), the value to the structure in reduced maintenance and increased life should be much more than the additional amount expended on the program. In the preliminary planning the improved quality of the structure was the consideration that prompted the adoption of the program. It was anticipated that the program would cost something, but it was fully appreciated that the improvement in quality of the structure would be worth far more than the additional cost incurred.

O. LAURGAARD,[7] M. AM. SOC. C. E. (by letter).—In addition to the formal written discussions of this paper, certain phases have been discussed in conversation and by correspondence with other engineers. The writer was pleased with the favorable comments by all the discussers, because they are familiar with the subject and qualified to express their opinions. In general, he agrees with most of them, but in some instances questions will be answered, the text clarified, or explanations made.

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[7] Care, Bethlehem Alameda Shipyard, Alameda, Calif.

The contribution by Mr. McHenry is interesting and valuable to the general discussion regarding flexible detail specifications for a successful crack prevention program; the use of low-heat cement and temperature studies; the use of thin concrete lifts during construction, based upon theoretical studies of temperature distribution and resulting stresses; the comparative value of the item of low casting temperatures; the relative value of cooling the concrete, in place, artificially by embedded water pipes; and the balancing of the saving in cement against the cost of the crack prevention program.

The writer agrees with Mr. McHenry that the paper would have lost none of its value or interest had it included more of the discussion or reasoning that led to the adoption of certain procedures and the rejection of others, together with a final valuation of the effectiveness of the various items listed in the cost schedule; but such data are so numerous and formidable (as expressed subsequently by Mr. McHenry) that time and space would not permit the inclusion of all of them. The items enumerated in the second paragraph of Mr. McHenry's discussion were considered by the engineers of the TVA in connection with the crack prevention program as adopted.

Although it may be admitted that the pre-cooling of the mixing water was not as effective as had been hoped, the writer cannot agree with Mr. McHenry that this item was not as justifiable in proportion to cost as the other items. From Table 5 it may be seen that for the five months from May to September, inclusive, in 1939, for shift 1, the air had an average temperature of 80.2°, the cement 100°, the river water 74.4°, the cooled mixing water 42.4°, and the concrete placing temperature 72.2°. It will be noted that for June, 1939, for a five-day period when the water was not artificially cooled for the same shift, the average temperatures were: Air 83°, cement 107°, river water 75°, and the concrete casting temperature 81°. Thus it will be noted that the casting temperature of the concrete for June, 1939, was reduced about 9°. In the paper it was stated (see heading "Washing, Rinsing, and Cooling the Aggregate") that "The effective reduction in concrete placing temperature by refrigerating the mixing water was about 6°." Of course, it must be admitted that, if a temperature of 32° had been maintained for the pre-cooled water, the casting temperature would have been reduced another 2°, which would have been justifiable. The pre-cooling of the mixing water at Hiwassee Dam was an afterthought and was not effective until September, 1938. An old refrigerating plant that was available was installed so the minimum temperature for the mixing water was set at 35° or 36° to prevent the freezing of the pipes. In the case of a slow placing schedule, something less than 1 ft per day as was adopted for Hiwassee Dam, the advantage gained by pre-cooling may be lost to a limited extent.

The writer agrees with Mr. McHenry in regard to cooling the concrete in place artificially. He advocated the placement of cooling pipes and the circulation of cool water through the concrete of the dam before construction was begun, but this plan was not adopted by those in higher authority. The embedment of cooling pipes in local regions to reduce cracking, maintain uniform volume, and for other purposes resulted, and was fully justified by the results obtained.

The comparison of costs, as made by Mr. McHenry, between complete artificial cooling of the concrete in the dam and the combined cost of thin casting lifts, low casting temperatures, cooling the aggregate, and local cooling, is interesting. The estimates made by the writer for plant and operation and for embedding cooling pipes were about 17¢ to 18¢ per cu yd, which is higher than the cost of 14.4¢ at Grand Coulee Dam, primarily on account of the smaller yardage and the remoteness of the work. Pre-cooling the mixing water in conjunction with artificial cooling of the concrete in place would be much more effective because the low casting temperature would reduce the temperature during the early setting stage, and the circulation of cool water would prevent the dissipation of this pre-cooling during the long exposure period.

The discussion by Mr. Thomson is of great interest and of especial value concerning: The increasing care and concern to those in charge of concrete dams; the heavy maintenance and repairs of poorly constructed dams; the distribution of cement in the dam and its relation to heat, contraction, and expansion; the destructive effect of water and exposure; the danger of excessive use of water; the use of thin casting lifts; the use of diagonal keyways; and the writer's interpretation of the tables submitted with the paper.

To answer the inquiry of Mr. Thomson concerning the use of 10-ft lifts with complete artificial cooling, several assumptions must be made. If the costs of thin casting lifts, low casting temperatures, cooling the aggregate, and artificially cooling the concrete in place are eliminated in the tabulation of the cost of the crack prevention program, and the other four items retained, then savings could be made in addition to the cleanup of the pours. If the cleanup should be made at the same cost as at Hiwassee Dam (6¢ per sq ft) and the cost of circulating cooling water is placed at 17¢ per cu yd (as estimated in this discussion), then the cost of the crack prevention program using 10-ft lifts would be about 21¢ per cu yd compared with 26.5¢ at Hiwassee Dam. In the judgment of the writer the use of 10-ft lifts with the foregoing program would not be as effective for crack prevention as that used at Hiwassee Dam; and in addition other factors should be considered also.

Mr. Thomson refers to tests made to determine the effect of grinding in the mixers, and desires to know the purpose of the tests. They were made to determine the grading of the aggregate in the concrete after the mixing, in order that the engineers could make the necessary changes and adjustments to the grading of the aggregate as it was fed into the mixers. It was discovered early that the graywacke rock was inclined to break down in the mixers and produce an excess of fine material. The grinding of the aggregate in the mixers, including the tests made and conclusions drawn, is important and could easily be made the subject of a paper, so time and space will not permit complete discussion in great detail. The writer feels, however, that a brief résumé may be of interest. The rock and sand seemed to break down to a minimum size, between 200-mesh and 325-mesh, but there was practically no aggregate of a smaller size. During the summer of 1938 a large number of small "grab" samples (from 100 to 125 lb) were taken from the concrete at the mixer discharge, which were wet-screened to give an average grading from the mixer, and were compared with the average theoretical grading going into the mixer. The tests indicated a large increase in the – 100-mesh material, so this size was reduced from 16% to 12% in the finished sand by wet classifying. These small tests showed increases in the – 100 fines from 100% to 200%. The tests also showed an increase of the material between 28-mesh and 100-mesh during the mixing, which suggested a reduction of the quantity of sand. It may be noted in passing that no precedent was available as a guide to determine the most desirable grading or to make adjustments to compensate for the grinding. It was found that other aggregate material, which was believed to be stable, showed considerable change due to grinding, but not perhaps to the same extent as the graywacke.

In order to obtain more reliable data than those from the small "grab" samples, it was decided to analyze some full 2½-cu-yd batches. In the test batches it was necessary to omit the cement because the time for wet-screening a batch of that size was about a week for several men. In place of the cement, – 100-mesh classifier fines were substituted by absolute volume in order to simulate as nearly as possible the mixing action when cement was used. Six 2½-cu-yd batches were analyzed, three of which were charged and mixed for 2½ min in the regular manner and three of which were charged on what was designated a "split batch" charging, and mixed for a total of 2½ min. The "split batch" cycle was charged in two parts: First, the water, cement (or – 100-mesh fine substitution), sand, and two smaller sizes of rock were charged into the mixer and mixed for 1¼ min; and second, the coarse rock and cobbles were charged, and mixing continued for another 1¼ min. The objective, of course, was to reduce the amount of grinding, since the grinding, it was assumed, was largely a result of "ball mill" action with the coarse aggregate grinding the sand. This method of charging and mixing was adopted for "face" concrete after July 17, 1939. Changes in grading of the coarse aggregate—particularly cobbles, coarse rock, and medium rock—were small. The amount of material larger than 4-mesh changed only slightly after mixing, and a comparison of these amounts (see Table 10) indicates that very little material is lost from the coarse aggregate into the sand classification. The results for the full batch tests altered the conclusions reached from the small "grab" samples. The large tests showed an average increase of about 50% for the – 100 classification.

Mr. Thomson raises a question as to the cost of the dam. The approximate costs kept at the dam are $13,285,000, to which should be added $2,379,000, representing other general accounts, and $1,150,000 for land acquisition, making a total of $16,814,000. If it were possible to separate, physically, the power house, penstocks, and power equipment from the dam, the $13,285,000 could be broken down further into these items: Dam, $10,226,000; power house, substation, turbine, generator, and miscellaneous power-house equipment, $2,001,000; reservoir relocations, $765,000; and reservoir clearing, $295,000. Mr. Thomson fixed the cost of the dam at $12,000,000, whereas the dam and power house cost $12,226,000; therefore the expenditure of $210,621, or 1.72%, as a factor for crack prevention and longer life seems reasonable.

Mr. Waugh takes exception to the writer's claim for reduction in the cost of the crack prevention program by the saving in cement. He also deducts the cost of the scientific program, $29,000; the cost of cooling the aggregate, $3,059; the cost of curing and winter protection, $14,075; and a part of the cost of steel reinforcement, $11,053, from the estimated cost of $210,621 for the entire program. The net result of the deductions by Mr. Waugh is to show an increased cost of $30,434 for the program, whereas the writer showed a saving of $34,407. These items will be taken up briefly in the foregoing order, which is the same as that used by Mr. Waugh.

TABLE 10.—COMPARISON OF AGGREGATE GRADING FOR MASS CONCRETE BEFORE AND AFTER MIXING[a]

(Expressed as a Percentage Change Based on Volume of Material Before Mixing)

Test No.	Standard Screen Size						
% passing: % retained:	.... 4-mesh	4 8	8 14	14 28	28 48	48 100	100 Pan
			(*a*) REGULAR CHARGING				
6	−1.0	−26.8	−11.5	−12.2	0	+41.9	+78.1
2	−3.0	− 6.8	−12.2	− 7.9	+41.2	+13.9	+47.2
1	−5.0	−20.0	−12.5	− 7.7	+27.6	+61.9	+58.2
			(*b*) SPLIT CHARGING				
3	+2.0	−31.0	−11.5	− 7.3	+11.1	+23.7	+22.6
4	0	−29.7	−10.4	−10.5	+19.5	+25.0	+25.0
5	0	−27.4	−12.2	−12.8	+10.8	+ 5.7	+67.7

[a] Minus (−) indicates that the quantity after mixing is less than it was before mixing; plus (+) indicates that the quantity after mixing is greater than it was before mixing.

In the paper, the writer used the phrase "predicted cement content of one barrel per cubic yard" advisedly and after careful consideration. In other words, it seemed that there was no hope of using less than 1.00 bbl of cement for the interior mass concrete and 1.25 bbl for the face, using modified cement (Type B) and graywacke rock and sand. During the early period of the Hiwassee project, before the type of cement or cement content had been decided upon, 1.05 bbl of cement per cubic yard for the mass concrete in the interior of the dam was generally considered, especially using sand manufactured from graywacke rock. Consideration was even given to the importation of quartz sand from Georgia because of the breaking down of the graywacke sand. Exhaustive tests were made in the laboratory to determine the suitability of the various types of available aggregate, with the result that it was determined to use both graywacke rock and sand. Engineers of the design department and others in authority advocated 1.05 bbl of cement for the interior mass concrete of the dam. At several conferences the writer advocated 1.0 bbl per cu yd of modified cement (same as Norris Dam) for the mass concrete, but it was not adopted.

It was only after the consultants (Messrs. Carlson and Davis) had recommended[8] low-heat cement and low cement content that any cement content less than 1.0 bbl per cu yd was considered (to the knowledge of the writer). In their report they suggested that the desired results would be most nearly obtained as follows:

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[8] "Problems Pertaining to the Concrete for Hiwassee Dam," Report by Roy W. Carlson and Raymond E. Davis, Cons. Engrs., to Carl A. Bock, Asst. Chf. Engr., April 11, 1938, pp. 2 and 7.

(1) By using a low-heat cement;

(2) By using for the interior of the mass a concrete of low cement content, and for the exposed faces a concrete of normal cement content;

(3) By a controlled placing schedule, using thin lifts immediately above foundations and immediately above surfaces of concrete that had been placed for a considerable period; and

(4) By maintaining low casting temperatures.

In their report they further stated:

> "A trial mix containing 0.75 bbl. per cu. yd. has been shown to be very plastic and workable and to exhibit almost no water gain. * * * preliminary investigations would seem to indicate that a cement content of 0.75 bbl. per cu. yd. might be employed with a water-cement ratio of not greater than 0.85, * * *. It is recommended that for the mass of the dam there be employed a concrete for which the water-cement ratio is about, but not more than, 0.85 and for which the cement content is not more than 0.8 nor less than 0.75 bbl. per cu. yd."

Table 11, from the official records of TVA, is submitted to show the cement content of the dams designed and constructed previous to the Hiwassee Dam.

TABLE 11.—CEMENT CONTENT OF TVA DAMS

Description	Pickwick	Wheeler	Guntersville	Norris	Chickamauga	Hiwassee
Volume (cubic yards)	626,919	612,857	286,312	968,119	489,169	783,154
Cement Content:						
Barrels	693,956	774,054	369,442	1,061,190	642,783	741,127
Barrels per cubic yard	1.105	1.260	1.290	1.096	1.310	0.946

The amount of saving by the reduction in cement used can be obtained by three methods: First, by the actual number of barrels of cement used for the 596,431 cu yd of interior mass concrete, compared with the predicted one barrel per cubic yard, which gives 114,499 bbl at $2.14, or $245,028; second, by applying the saving of 0.20 bbl per cu yd for the 596,431 cu yd, which gives 119,290 bbl of cement at $2.14, or $255,280; and third, by taking the total yardage of concrete at Hiwassee Dam of 794,439 cu yd and applying the difference between the average cement content at Hiwassee Dam and the average at Norris Dam (as shown in Table 11) at 0.15 bbl per cu yd, which gives 119,166 bbl at $2.14, or $255,015. After all, the amount of saving in cement is a matter of judgment. The writer believes the lowest sum ($245,028), which was used in the paper, to be fair. Mr. Waugh states that "At Norris Dam a cement content of 0.90 bbl per cu yd for the interior concrete was adopted." According to the record, that is correct, but the adopted content was not realized in practice. It is found from the record that, during the first two months of concreting at Norris Dam, 1.10 bbl was used; then the content was changed to 0.95 bbl and later adopted to 0.90 bbl. Evidently the actual average of placement was in excess of 1.00 bbl because the average of all mass concrete interior and face (for 871,763 cu yd) was about 1.061 bbl per cu yd and the average for 968,119 cu yd was 1.096 bbl.

The discussion by Mr. Waugh, of studies of temperature cracking in mass concrete, is valuable.

Mr. Waugh admits that some benefit was received during construction from the scientific program, and the writer admits that the benefit did not equal the total cost of $29,000, but the clearest explanation is made by Mr. McHenry when he states

> "The inclusion of the $29,000 item for the scientific program is likewise of interest; this inclusion acknowledges a debt to the similar research investigations of the past that are to a large extent responsible for the success and economy of the Hiwassee control program."

Mr. Waugh admits a small benefit from the cooling of the aggregate, but the charge (explained in detail in the paper) also is small. On the other hand, the reduction in casting temperature of the concrete amounted to about one degree The design of the inspection tunnels (being flat on the top and bottom with chamfered corners and heavy horizontal steel reinforcement above the roof and below the invert), and the placement of interior mass concrete around the exterior of the tunnels, produced a job practically free from cracks; so the inclusion of the cost of the steel reinforcement in the crack prevention program seems reasonable. Only that part of $74,600 (the cost of the curing and winter protection, which could be attributed to the slow setting tendency of low-heat cement), or $14,075 as explained in the paper, was charged to the crack prevention program. All of the items of cost were obtained from the records of the cost engineer, and the final figures reflect his judgment, and that of several others familiar with the work.

The writer desires to express his appreciation to those participating in the discussion of this paper.

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TRANSACTIONS

Paper No. 2140

FORMULAS FOR THE TRANSPORTATION OF BED LOAD

BY H. A. EINSTEIN,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOE W. JOHNSON, A. A. KALINSKE, O. G. HAYWOOD, JR., SAMUEL SHULITS, JOHN S. MCNOWN, AND H. A. EINSTEIN.

SYNOPSIS

A method for the representation of bed-load data is given in this paper. The method is based on the conception that bed-load transportation is the movement of bed particles, as governed by the laws of probability. By means of this method an equation is obtained, which describes a great number of experiments in channels with uniform beds. A group of experiments conducted on sand mixtures provides material for describing another application of the method.

INTRODUCTION

In the past the problem of bed-load transportation has been studied mostly by empirical methods. More recently, there has been a tendency to base transportation formulas on the new theories of turbulence.[2, 3] It is the writer's belief that an approach to the problem of transportation can be made by a combination of the empirical and rational methods and that the results can be expressed by dimensionless plots.

Before proceeding with the development of these formulas, it is necessary to discuss briefly two important considerations: (1) The difficulty or impossibility of defining, accurately, the so-called "critical" values; and (2) the possibility of correlating bed-load movement with local fluctuations in water velocity along the bed.

(1) Attempts have been made in the past to derive an expression for the "initial movement" that is governed by certain definable "critical" conditions to be used as the first step toward the solution of the transportation problem.

NOTE.—Published in March, 1941, *Proceedings*.

[1] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

[2] "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Heft 26, Berlin, 1936.

[3] "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," by Hunter Rouse, Assoc. M. Am. Soc. C. E., SCS, Sedimentation Div., SCS-TP-25, Washington, D. C., July, 1939 (mimeographed).

In interpreting the results of many experiments on bed-load movement, and in comparing them with those obtained by other experimenters, the writer has concluded that a distinct condition for the beginning of transportation does not seem to exist. It is just as impossible to determine the limit of initial movement as to determine the maximum possible flood of a river. Just as the engineer is able to predict the probable maximum flood of a river to be expected within a given range of years, however, so is he able to define the hydraulic conditions in a stream that will produce any given small rate of movement, which might be called the limit. This value can be chosen without any restriction. It is difficult to believe that the hydraulic conditions that will produce such movement could have any special meaning in the problem of transportation as a whole. Therefore, the writer will not use the conception of critical tractive force, or any other critical value, when the term "critical" pertains to the flow where transportation begins.

(2) In general, transportation of bed load has been described as follows:[4] A particle of the bed moves when the pushing force or lifting force of the water overcomes the weight of the particle. This push or "lift" is expressed in terms of the average flow. The usual conception is that transportation begins when the velocity increases enough to overcome the weight, and that, with further increasing velocity of the water, the rate of transportation will also increase, following a certain law that is found empirically. To prove that this conception is misleading, assume that the force acting on a particle could be described by means of the average flow alone. If the velocity of the water is increased gradually to a point at which the first particle would just be moving, the force acting on all the other particles of the same kind and size would move those too. Therefore, in a uniform bed where all particles have the same size and shape, all would start moving together; they would be unable to settle again because at all points the water velocity is just sufficient to start movement. If this were true, there could be no law governing the rate of transportation, but only a critical velocity. At all undercritical velocities, there would be no transportation, whereas, at all supercritical velocities, the rate of transportation would be limited only by the number of particles available. Therefore, it cannot be presumed that the rate of bed-load transportation is a function of the average flow. Instead it is proposed to express it in terms of the fluctuations of the water velocity near the bed.

Results of previous studies[5] describing the movement of a bed-load particle by means of statistical methods are to be used in an attempt to coordinate the rate of transportation with the fluctuations of the water velocity near the bed. The results of these studies can be summarized as follows:

(*a*) These flume studies dealt with the movement of rather coarse particles along a bed consisting of the same kind of grains. Being coarser than 1 in. these particles always remained near the bed, rolling, sliding and, sometimes, in saltation according to the normal description of bed-load movement. It was found that the moving bed load and the bed on which it was moving formed a unit, inasmuch as there was a steady and intensive exchange of particles between the two. Thus it is concluded that all the particles of the bed, down to a certain depth, take equal part in the movement, alternatively moving and returning into the bed.

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[4] "The Force Required to Move Particles on a Stream Bed," by William W. Rubey, *Professional Paper No. 189-E,* U. S. Geological Survey, Washington, D. C., 1938.

[5] "Der Geschiebetrieb als Wahrscheinlichkeitsproblem," von H. A. Einstein, *Mitteilung der Versuchsanstalt für Wasserbau an der Eidgenössische technische Hochschule in Zürich*, Verlag Rascher & Co., Zürich 1937.

(*b*) Bed-load movement is to be considered as the motion of bed particles in quick steps with comparatively long intermediate periods of rest. Thus bed-load movement is a slow downstream motion of a certain top-layer of the bed.

(*c*) The average step of a certain particle seems always to be the same even if the hydraulic conditions or the composition of the bed changes; and

(*d*) Different rates of transportation are produced by a change in average time between two steps and by a change in the thickness of the moving layer.

These concepts permit the development of a formula in general terms. The rate of transportation will be described by means of this average "step."

NOTATION

The letter symbols in this paper are defined where they first appear and are assembled for convenience of reference in Appendix I.

DERIVATIONS

This paper will treat only the bed-load movement of uniform sediment and mixtures acting like uniform sediment. In both cases it is possible to describe the sediment by a representative diameter D and its density ρ_s. The expression "acting like uniform material" means that both bed material and moving material have the same composition and, therefore, the same representative diameters. It is possible in this case to describe transportation at a certain point of the bed by one symbol; namely, the rate of transportation q_s. Two special cases of transportation, both of which have been observed and described in engineering literature, may be excluded from treatment in this paper: (1) Bed material moving in suspension; and (2) bed-load movements, in which the composition of the bed is essentially different from the composition of the transported material.

A bed-load formula is an equation linking the rate of bed-load transportation with properties of the grain and of the flow causing the movement. A formula of this kind can be derived by expressing in an equation the fact that all particles passing the unit width of a section as bed load are just on the way to perform one of these steps of the constant length $L = \lambda_0 D$. Fig. 1 shows the cross section and the rectangular area with the length L and with unit width, where all particles start the steps that together form the rate of transportation q_s. Eq. 1 expresses the condition that the total volume passing the unit width of the section per second (q_s divided by the specific gravity of the particles, both under water)

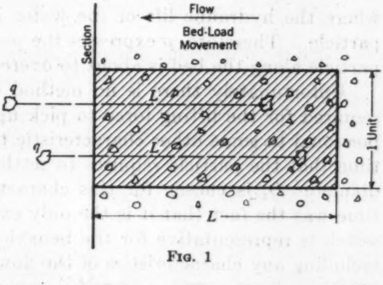

FIG. 1

equals the total volume of all the particles starting a step during a second in the aforementioned rectangular area. This volume is obtained by multiplying the number of particles in the surface of the area by the probability that a particle in the bed surface will start moving during a given instant and with the volume of a given particle. Eq. 1 follows:

$$\frac{q_s}{(\rho_s - \rho_f)\, g} = \frac{L}{A_1 D^2}\, p_s\, A_2\, D^3 = \frac{A_2}{A_1}\, \lambda_0\, p_s\, D^2 \qquad (1)$$

in which: q_s = the rate of transportation, in weight (under water), per unit of width, per second; ρ_s and ρ_f = density of particle and fluid, respectively; g = acceleration due to gravity; D = representative diameter of the particles; A_1 and A_2 = dimensionless ratios such that $A_1 D^2$ = the area that the grain covers in the bed and $A_2 D^3$ = the volume of the particle; p_s = the probability that a particle will start moving in any given second; and λ_0 = the dimensionless measure for the length of the single step.[5] It must be kept in mind, however, that λ_0 may or may not be a constant; that is, it has not been proved to be constant.

If A and D are transposed to the left side of Eq. 1, that side will include all terms pertaining to the grain, whereas the right side $\lambda_0\, p_s$ is still an unknown function of the flow—that is:

$$\frac{q_s}{(\rho_s - \rho_f)\, g\, D^2}\, \frac{A_1}{A_2} = \lambda_0\, p_s \qquad (2)$$

In this equation A_1, A_2, and λ_0 are dimensionless, but p_s has the dimension sec^{-1}. In order to make the right side of Eq. 2 dimensionless, p_s must be multiplied by a given time. The most reasonable time to use is the average time, t, required for the water to remove one particle from the bed. If $p = t\, p_s$, then p is dimensionless and gives the number of steps that start from any given place during the time it takes to remove one particle. The maximum value of p is 1, and indicates that at all times and at all points particle by particle starts to move. The minimum value of p is zero; therefore, p expresses the probability that a step is about to begin at a given place. These steps start everywhere on the bed. Therefore, p can be interpreted as the probable part of the bed area in which steps are starting. A step is started only at a point where the hydraulic lift of the water is able to overcome the weight of the particle. Therefore, p expresses the probability that the hydraulic lift on any particle along the bed is about to overcome the weight of the particle.

Unfortunately there is no method of expressing or measuring the time t required for the lifting force to pick up a particle. It is assumed to be proportional to some other characteristic time of the particle in the water. The time that the particle requires to settle in water, a distance equal to its own diameter D, is chosen for this characteristic. The reason for choosing this time was the fact that it is the only expression with the dimension of a time, which is representative for the behavior of the particle in the liquid without including any characteristics of the flow. This time can be expressed as

$$\frac{D}{v_f} = \frac{1}{F} \sqrt{\frac{D\, \rho_f}{g\, (\rho_s - \rho_f)}} \qquad (3)$$

in which: v_f = the velocity of a particle settling in water; and F = a parameter for settling velocity. In Eq. 3, $F = 0.816$ for particles greater than 1 mm, settling in water of normal temperature. Fig. 2 shows the values of F for smaller grain sizes. Characteristics of the materials in Fig. 2 are: Kinematic viscosity $\nu = \frac{\mu}{\rho_f}$ equals, for water, 0.012, and, for air, 0.16, cm² per sec; and specific densities, $\frac{\rho_s - \rho_f}{\rho_f}$, are as follows—

Material	Specific density
Barite in water	3.22
Gravel in water	1.65
Coal in water	0.25
Gravel in air	2,210.0

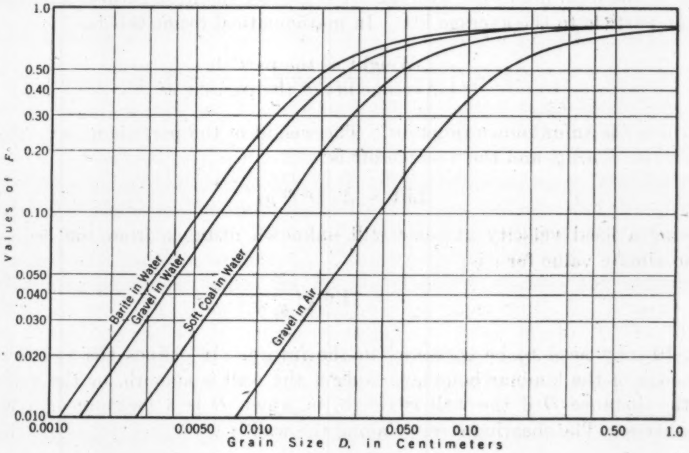

FIG. 2.—PARAMETER F FOR DETERMINING THE SETTLING VELOCITY OF VARIOUS MATERIALS

In Fig. 2 the following equation for the settling velocity derived by William W. Rubey[6] has been used for the determination of F:

$$v_f = \sqrt{\frac{2}{3} g \frac{\rho_s - \rho_f}{\rho_f} D + \frac{36\,\mu^2}{\rho_f^2 D^2}} - \frac{6\,\mu}{\rho_f D} = F \sqrt{D\, g \frac{\rho_s - \rho_f}{\rho_f}} \ldots\ldots (4)$$

In this formula all terms are measured in centimeter-gram-second units. Hence, F will be

$$F = \sqrt{\frac{2}{3} + \frac{36\,\mu^2}{g\, D^3 \rho_f (\rho_s - \rho_f)}} - \sqrt{\frac{36\,\mu^2}{g\, D^3 \rho_f (\rho_s - \rho_f)}} \ldots\ldots\ldots (5)$$

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[6] "Settling Velocities of Gravel, Sand, and Silt," by William W. Rubey, *American Journal of Science*, Vol. 25, No. 148, April, 1933.

The time t required to remove a particle from its place in the bed then will be

$$t = \frac{A_3}{F}\sqrt{\frac{D\,\rho_f}{g\,(\rho_s - \rho_f)}} = \frac{p}{p_s} \qquad (6)$$

in which A_3 is still an unknown constant. Eq. 1 can now be changed to the form

$$p = \frac{A_1\,A_3}{\lambda_0\,A_2}\left[\frac{1}{F}\,\frac{q_s}{(\rho_s - \rho_f)\,g}\sqrt{\frac{\rho_f}{\rho_s - \rho_f}}\,\frac{1}{g^{0.5}\,D^{1.5}}\right] \qquad (7)$$

In an attempt to express p as the probability of the local hydraulic lift[4] to overcome the weight of the particle, p refers to the part of the bed in which locally (at a certain moment) the lifting force is greater than the weight of the particle. It can be stated that p refers to the part of the bed in which the ratio of the local lift to the average lift is greater than the ratio of the weight of the particle to the average lift. In mathematical terms this is

$$p = f\left(\frac{\text{Weight of the particle}}{\text{Average lift of the particle}}\right) \qquad (8)$$

in which f is an unknown function. The weight of the particle under water is $A_2\,D^3\,(\rho_s - \rho_f)\,g$, and the average lift is

$$\text{Lift} = A_4\,D^2\,v^2\,\rho_f \qquad (9)$$

v being a local velocity at some still unknown distance from the bed. An approximate value for v is:

$$v = 11.6\sqrt{\frac{\tau}{\rho_f}} \qquad (10)$$

Eq. 10 may need to be corrected in the future. It defines the velocity[3] at the edge of the laminar boundary layer if the wall is smooth, or the velocity at the distance D if the wall is rough, in which D is a measure of average roughness. The shearing stress τ along the wall is

$$\tau = S\,R\,\rho_f\,g \qquad (11)$$

in which S is the slope and R is the hydraulic radius; therefore

$$v = 11.6\,\sqrt{S\,R\,g} \qquad (12)$$

Eq. 8 can now be written

$$p = f\left[\frac{A_2\,D^3\,(\rho_s - \rho_f)\,g}{(A_4\,D^2\,\rho_f)\,(135\,S\,R\,g)}\right] = f\left[\frac{A_2}{135\,A_4} \times \frac{(\rho_s - \rho_f)\,D}{\rho_f\,S\,R}\right] \qquad (13)$$

By assuming that Eq. 10 gives the correct value for the velocity, Eqs. 7 and 13 can be combined and a new transportation formula formed:

$$A\left\{\frac{1}{F}\left[\frac{q_s}{(\rho_s - \rho_f)\,g}\right]\sqrt{\frac{\rho_f}{\rho_s - \rho_f}}\,\frac{1}{g^{0.5}\,D^{1.5}}\right\} = f\left[B\left(\frac{\rho_s - \rho_f}{\rho_f}\,\frac{D}{S\,R}\right)\right] = p. \qquad (14)$$

in which

$$A = \frac{A_1 A_3}{\lambda_0 A_2} \qquad (15a)$$

and

$$B = \frac{A_2}{135\, A_4} \qquad (15b)$$

are constants which, however, may vary with different shapes of the particles. Whether λ_0 and A_4 really are constant under all conditions must be determined later. Introducing

$$\phi = \frac{1}{F} \frac{q_s}{(\rho_s - \rho_f)\, g} \sqrt{\frac{\rho_f}{\rho_s - \rho_f}} \frac{1}{g^{0.5}\, D^{1.5}} \qquad (16a)$$

and

$$\psi = \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{S\, R} \qquad (16b)$$

Eq. 14 can be written in the short form:

$$A\, \phi = f(B\, \psi) = p \qquad (17)$$

The function f as well as the two constants A and B must be determined empirically. Data from a great number of measurements using various materials have been analyzed, and values of ψ and ϕ computed. A semilogarithmic plot of these values is shown in Fig. 3(*a*). The grain sizes range from 0.315 to 28.65 mm in diameter, the water depth from 18 to 1,100 mm, and the specific gravity of the particles from 1.25 to 4.22. All these experiments are performed in flumes with uniform sediment. The experiments marked "Zürich" are described briefly in a paper by E. Meyer-Peter[7] whereas the remaining experiments are taken from the generally known paper by G. K. Gilbert.[8] It seems that all these points follow, reasonably, a single curve. It should be noted that from these data not a single experiment has been omitted even when the measurement appeared questionable. It might be mentioned also that the hydraulic radius R is computed by a method[9] that eliminates the effect of side-wall friction and gives results comparable to a channel of infinite width (see Appendix II).

In Fig. 3(*a*) all the points with values of ϕ less than 0.4 seem to follow the straight line, curve (1), the equation of which is

$$0.465\, \phi = e^{-0.391\, \psi} \qquad (18)$$

If Eq. 18 is assumed to represent the law of transportation:

$$\left.\begin{aligned} A &= 0.465 \\ B &= 0.391 \\ f(x) &= e^{-x} \end{aligned}\right\} \qquad (19)$$

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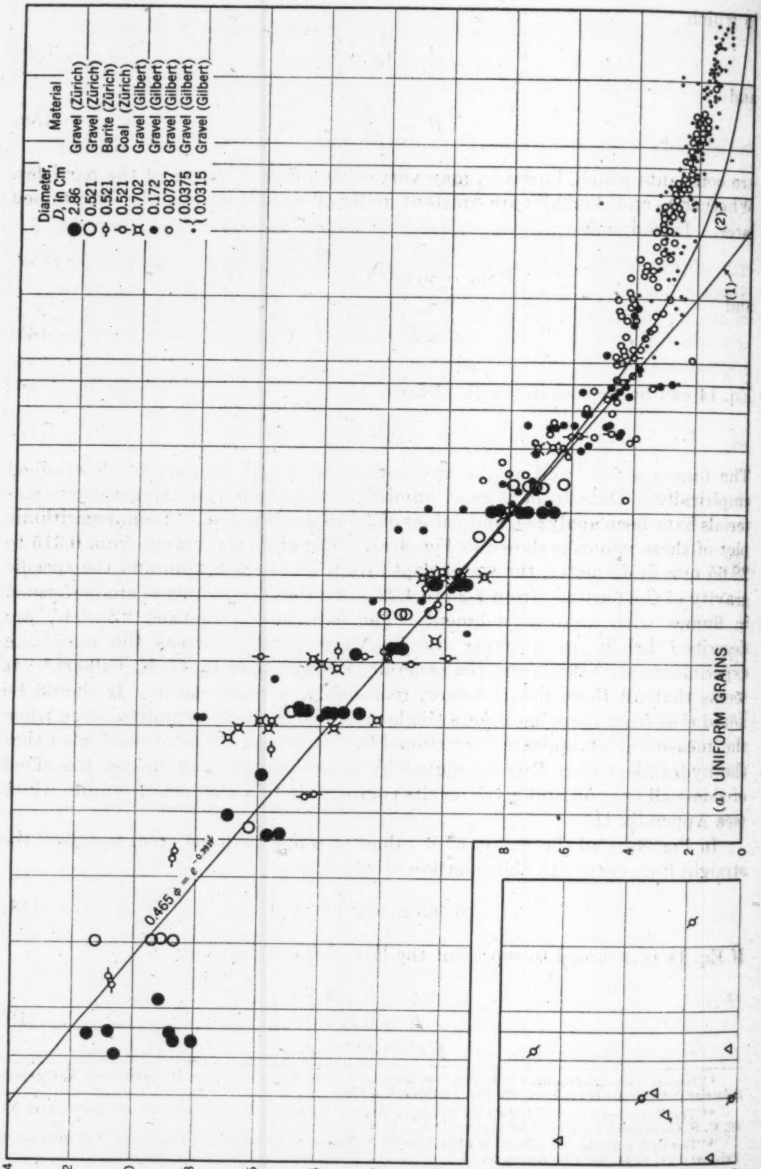
[7] "Neuere Versuchsresultate über den Geschiebetrieb," by E. Meyer-Peter, H. Favre, and A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 13, March, 1934.

[8] "The Transportation of Debris by Running Water," by Grove Karl Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, D. C., 1914.

[9] "Der hydraulische oder Profil-Radius," by H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 8, February 24, 1934; see also Appendix II.


Diameter, D, in Cm
Material
● 2.86 Gravel (Zürich)
○ 0.521 Gravel (Zürich)
0.521 Barite (Zürich)
0.521 Coal (Zürich)
¤ 0.702 Gravel (Gilbert)
• 0.172 Gravel (Gilbert)
∘ 0.0787 Gravel (Gilbert)
· { 0.0375 / 0.0315 Gravel (Gilbert)
0.465 φ = e^{-0.391ψ}
(1)
(2)
(a) UNIFORM GRAINS
Values of ψ
24
22
20
18
16
14
12
10
30
28
26
24
22
0
2
4
6
8

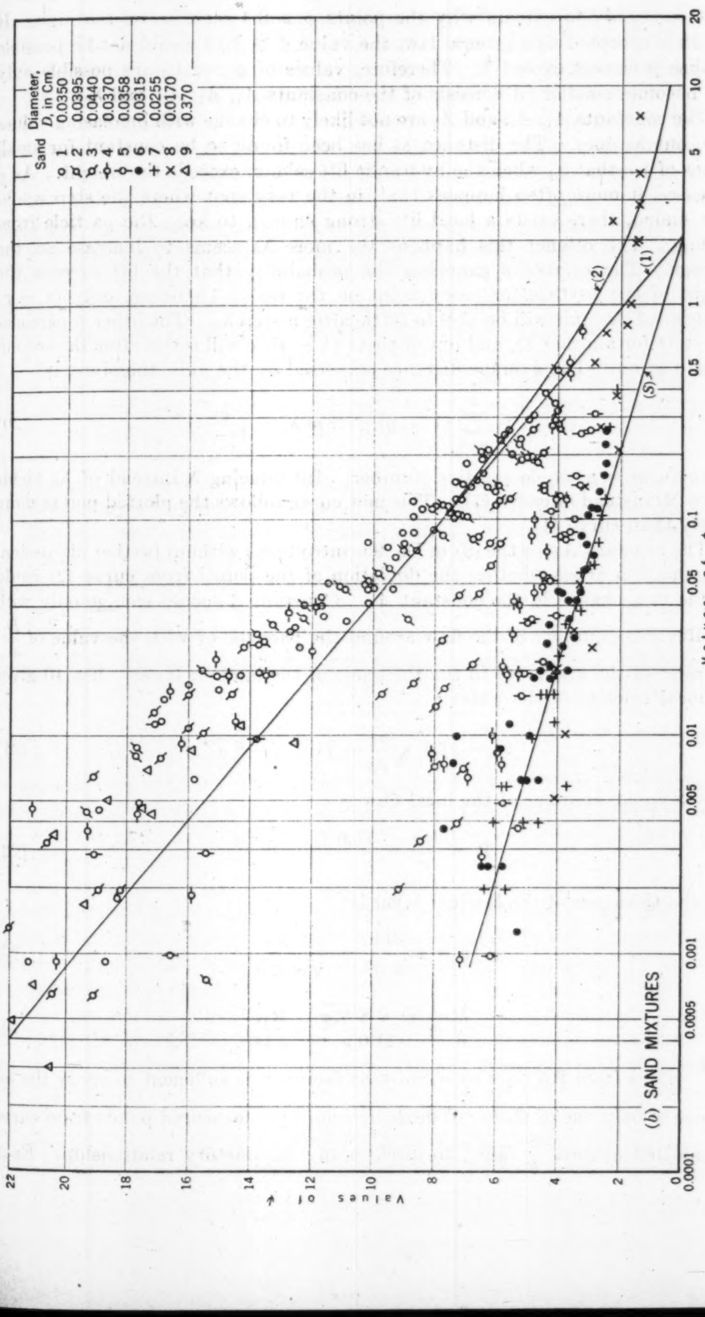


FIG. 3.—BED-LOAD EXPERIMENTS SHOWING THE RELATION BETWEEN φ AND ψ

it remains only to explain why the points $\phi > 0.4$ seem to be too high. If Eq. 18 is accepted as a general law, the value $\phi > 2.15$ would not be possible because p cannot exceed 1. Therefore, values of $\phi > 2.15$ are possible only if A becomes smaller (A consists of the constants A_1, A_2, A_3, and λ_0).

The constants A_1, A_2, and A_3 are not likely to change with increasing values of p, but λ_0 does. The distance λ_0 has been found to be constant for small values of p—that is, when the hydraulic lift seldom exceeds the weight. As p increases, it more often happens that, in the very spot where the step would have ended, there exists a local lift strong enough to keep the particle from settling. The oftener this happens the more λ_0 seems to increase on the average. The symbol p expresses the probability that the lift exceeds the weight of the particle for every point on the bed. Therefore, only $(1 - p)$ particles of the unit will be able to settle after a step λ_0. The other p particles will start for another λ_0, and out of these $(1 - p)\ p$ will settle after the second λ_0, and so on. The average distance traversed by the unit, therefore, is:

$$\lambda = \sum_{m=0}^{\infty} (1 - p)\, p^{m-1}\, m\, \lambda_0 = \frac{\lambda_0}{1 - p} \quad \text{..............} \quad (20)$$

in which m = a whole positive number. Introducing λ instead of λ_0 yields curve (2) instead of curve (1). This new curve follows the plotted points more closely than curve (1).

The constant A_4 for the lift in Eq. 9 is introduced without further discussion. The question arises whether the deviation of the points from curve (2) could be due to a change in the constant A_4. Constant A_4 could change only with the Reynolds number of the flow around the particle, or with the value of $\frac{D}{\delta}$, the ratio of the grain size to the thickness of the laminar layer. Eq. 10 gives the local velocity of the water:

$$v = 11.6 \sqrt{\frac{\tau_0}{\rho_f}} = 11.6 \sqrt{S R g} \quad \text{...............} \quad (21)$$

The Reynolds number of the local flow is

$$\mathbf{R} = \frac{D v}{\nu} = \frac{11.6\, D \sqrt{S R g}}{\nu} \quad \text{..................} \quad (22)$$

and the thickness of the laminar layer is

$$\delta = \nu \frac{D v}{\tau_0/\rho_f} = \frac{11.6\, \nu}{\sqrt{S R g}} \quad \text{..................} \quad (23)$$

or

$$\frac{D}{\delta} = \frac{D \sqrt{S R g}}{11.6\, \nu} = \frac{\mathbf{R_D}}{134} \quad \text{...............} \quad (24)$$

As $\frac{D}{\delta}$ differs from $\mathbf{R_D}$ only by a constant factor, it is sufficient to study the influence of only one of them. The deviation of the measured points from curve (2) plotted against $\frac{D}{\delta}$ failed to disclose any satisfactory relationship. Each grain size appears to follow a separate curve; therefore, it appears much more probable that A_4 is a constant, but that the exponential law for p does not extend down to $\psi = 0$. Another explanation for the deviation of the points may be that part of the grains have been transported in suspension. In this case those experiments would be outside the field of application of Eq. 17. More experiments, however, will be required in this range to determine the exact shape of the curve. It may be emphasized that in most American rivers the bed load is largely transported under conditions pertaining to this part of the curve, and, therefore, further study in this direction is most urgent.

The title of this paper was chosen specifically to avoid the impression that any attempt was being made to discover "the law of bed-load transportation," because it is the writer's belief that such universal law does not exist in a simple mathematical form. Just as it is necessary to distinguish between friction in smooth and rough pipes or channels, so it is necessary to distinguish between different kinds of transportation. Nevertheless, the distinction between friction along rough, wavy, and smooth walls was only possible on the discovery of the general method of plotting the friction factor against Reynolds' number. An attempt is made in this paper to determine a corresponding method of presenting transportation data by introducing the quantities ψ and ϕ, both of which are derived by pure speculation, using only generally known facts.

As an example, the method is used to discuss the results of experiments with sand mixtures, conducted at the U. S. Waterways Experiment Station, at Vicksburg, Miss.[10] Fig. 3(*b*) gives the results of these experiments as a ϕ-ψ graph. The method described in Appendix II has been used to determine the effective hydraulic radius. Curves (1) and (2) are transferred from Fig. 3(*a*), and all the various sand mixtures have been assigned different symbols.

The first problem was to determine the effective diameter of the mixtures—that is, the value of D that would represent the mixture in the formulas. Experience gained in previous studies has convinced the writer that the most usable value for this effective diameter is the grain size of which 35% to 45% of the material is finer. This value is readily obtained from the cumulative size-frequency curve of the mixture. The 40% value was used for Fig. 3(*b*), although it is realized that the use of a 35% value would tend to bring the high points closer to curves (1) and (2).

The distribution of the points in Fig. 3(*b*) is very interesting to note. At first glance one observes that the points for sands 1, 2, and 9 distinctly follow curves (1) and (2). Sands 3, 4, and 5 follow the curves in the upper part only. Sands 6, 7, and 8 fall below the curves at all points, but a distinct grouping of points along a line curve (*S*) is noticed.

The two curves, (2) and (*S*), seem to represent limits of maximum and minimum transportation for a given value of ψ. In searching for an explanation of this the three following questions naturally arise: (1) Is there any relationship between the position of the points and friction loss? (2) Is there any relationship between the position of the points and the condition of the bed?

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[10] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935.

(3) Would a similar distribution be possible also in experiments with uniform material, or is it characteristic only of mixtures?

With regard to question (1), it was found that Manning's n, without exception, increased suddenly when the points leave curves (1) and (2). This means that the bed becomes rougher than the original material as soon as the rate of transportation decreases below that shown by curves (1) and (2). The reverse is also true—that is, the rate of transportation will decrease as soon as the roughness of the bed increases.

The reason for this increased roughness is of interest. As a rule, riffles begin to form precisely at the place where the points depart from curve (1). If riffles are the reason for the increase in Manning's n, this value must always increase when riffles are formed. Sand 1 does not show this increase and sand 2 only very slightly—although these sands develop general riffles like all the other mixtures. Therefore, the riffles are not the reason for the increased roughness, but merely happen to develop simultaneously. This answers question (2) in the negative.

Question (3) suggests that perhaps some kind of sorting of the grains is the reason for the deviation from curve (1) and for the increase of roughness at the same time. It is the writer's belief that this is true, but unfortunately it is not subject to direct proof. This sorting would be caused by the lack of material in the upper end of the flume. If the sand is fed in at a smaller rate than the stream is able to transport it, the bed starts to build some kind of a protective layer of coarse grains on the surface and buries all the finer grains beneath. The average grain size in this coarse layer is much greater than the average grain size in the bed, and scour is either reduced or completely prevented. For this reason, it is impossible, during an experiment, to detect a lack of feeding merely by watching the position of the bed. Curve (1) gives the results obtained when the highest quantity of sand is fed in that can be transported without deposition, and curve (S) gives the smallest amount that will be transported without scour on the protecting layer. If D is the effective diameter of the original bed material, it is reasonable to believe that these two limits coincide for uniform material and that curve (S) falls more and more below curve (1) as the material decreases in uniformity.

These are merely some suggested methods of studying bed-load transportation. It would be very easy to determine, by experiment, the validity of such reasoning. If the interpretations are correct, it should be possible to determine all points between the two limiting curves by merely changing the rate of sand feed. It would also be very instructive to conduct a similar group of experiments in the opposite sequence—that is, by beginning with high discharges and progressively decreasing the discharge and load. If the explanation is correct, one will probably not return to the same curve obtained with increasing flow. This would also answer question (3). This ϕ-ψ method is offered as a new procedure for studying bed-load problems. It may be possible to refine the method by introducing corrections for the velocity v, and various constants; but as a whole it seems to be satisfactory in its present form.

SUMMARY

In concluding it may be stated that the treatment of transportation problems by means of statistical methods, made possible by the use of large-scale experiments, led to the proposed method of representation:

Two dimensionless functions ψ and ϕ have been developed theoretically, ψ as the ratio of the forces acting on the particle, and ϕ including the rate of transportation and the size and settling velocity of the particle. The interrelation between these two functions expresses the law of transportation, and at the same time expresses the statistical distribution p of the velocity of the liquid close to the laminar boundary layer. The transportation law is derived by use of a great many experiments with uniform sediment, and is then used in discussing published results of experiments with sand mixtures.

ACKNOWLEDGMENTS

The writer is greatly indebted to Prof. E. Meyer-Peter of Zürich, Switzerland, for having permitted the use of valuable data, even though the results of the original work have not as yet been published in full.

APPENDIX I

NOTATION

The following notation conforms essentially with American Standard Symbols for Hydraulics compiled by a Committee of the American Standards Association[11] with Society representation, and approved by the Association in 1929:

A = a dimensionless constant, defined in Eq. 15*a*:
    $A_1 D^2$ = the area that the grain covers in the bed; and
    $A_2 D^3$ = the volume of the grain;
    A_3 = a dimensionless unknown constant in Eq. 6;
    A_4 = a dimensionless unknown constant in Eq. 13, the expression for the hydraulic lift of a particle;
    A_b = the area with reference to the bed;
    A_m = the area with reference to part m of a cross section;
    A_w = the area with reference to the wall;
B = a dimensionless constant, defined in Eq. 15*b*;
b = surface width of a stream;
D = diameter; the representative diameter of a particle;
d = depth; depth from the water surface to a particle;
e = base of Napierian logarithms;
F = a factor defined by Eq. 5;
f = function of; see also f_m in Eq. 25:
    f_b = function for friction along the bed;
    f_w = function for friction along the wall;

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[11] ASA-Z10b-1929.

g = acceleration due to gravity;

L = length of the steps taken successively by particles in a river bed = $\lambda_0 D$, the average of all the steps taken by a single particle;

m = a whole number;

N = a number of units;

n = Manning's roughness coefficient:

- n_b = roughness of the river bed;
- n_m = roughness of part m of the perimeter;
- n_w = roughness of the side-wall;

P = wetted perimeter:

- P_b = wetted perimeter along the bed;
- P_m = wetted perimeter of part m of the circumference;
- P_w = part of the wetted perimeter along the wall;

p = probability that, at a given point in the bed, the lifting force required to overcome the weight of the particle has been generated:

- p_s = the probability of a particle starting movement at a given second;

q = rate of flow or discharge per unit width:

- q_b = discharge pertaining to the unit width of bed;
- q_s = the rate of transportation, in weight (under water), per unit of width, per second;

R = hydraulic radius:

- R_b = hydraulic radius pertaining to the bed;
- R_m = hydraulic radius pertaining to the part m of the cross section;
- R_w = hydraulic radius pertaining to the wall;

$\mathbf{R_D}$ = Reynolds' number of the grain D;

S = hydraulic slope;

t = time required for the water to build up sufficient force to pick up a particle;

V = average velocity in the cross section of a stream;

v = filament velocity at a given point in the cross section and at an unknown distance from the bed:

- v_f = settling velocity;

δ = thickness of the laminar layer (see Eq. 23);

λ = average distance traveled by a unit of bed load:

- λ_0 = the dimensionless measure of a single step taken by a particle;

μ = absolute viscosity of a fluid;

ν = kinematic viscosity = $\frac{\mu}{\rho_f}$;

ρ = density:

- ρ_f = fluid density;
- ρ_s = density of a particle;

τ = shearing stress;

ϕ = a function defined by Eq. 16*a*;

ψ = a function defined by Eq. 16*b*.

APPENDIX II

METHOD OF CALCULATING THE HYDRAULIC RADIUS IN A CROSS SECTION WITH DIFFERENT ROUGHNESSES

Water flowing through a certain profile continuously transforms its potential energy into heat by means of friction.[9] This friction, along smooth or rough walls, follows certain laws. Formulas expressing these laws contain the velocity of the water V, slope of the energy grade line S, acceleration of gravity g, roughness of the wall, length of wetted perimeter P, and the cross-section area A. The latter two are commonly combined into the hydraulic radius $R = \frac{A}{P}$. Some of these variables enter the equation explicitly, and others, like g and the roughness, are sometimes hidden in other constants.

In this connection only turbulent flow is of interest. A thin layer of laminar flow exists along the wall, which may or may not be able to hide the irregularities of the wall. In both cases most of the energy of the flow is destroyed by transformation into turbulence—that is, by an irregular deflection of the different flow filaments, caused by instabilities of the laminar layer (smooth wall) or by roughnesses extending out of the layer (rough wall). The filaments (which have been flowing only in the direction of the general flow) are deflected without gaining energy. Therefore, the average velocity in the direction of the flow of the newly established eddy is smaller than the average of the velocities of all the particles composing the eddy prior to its formation. The slow-moving eddy is pulled into the fast flow and accelerated again by other particles by use of part of their flow energy. At the same time, eddy energy is destroyed by laminar friction and the entire process begins over again.

In order to discuss the distribution of friction over a cross section in mathematical terms, two assumptions are necessary. The first assumption is that the entire cross section can be divided into units that will correspond to similar units of the wetted perimeter so that the potential energy of the area unit is transferred into eddy energy along the corresponding surface unit, and these eddies are accelerated and destroyed again in the former area unit. It is evident that the ratio of all area units to the corresponding surface unit will be the same in a cross section with uniform roughness. If the average velocity is assumed to be the same for all units of a simple cross section, every friction formula applicable to the entire section will also be applicable to each unit.

The second assumption is that the same friction formula will be applicable to a certain unit, even though the roughness of other parts of the cross section is changed. This second assumption enables one to make computations on any cross section with composite roughness, as, for instance, a flume with smooth side-walls and granular bed, a river with rough banks or partly sandy and partly rocky bed, etc. The procedure is as follows: For each part (P_m) of the wetted perimeter having a certain roughness, one can find the corresponding area, A_m, to be combined into a variable hydraulic radius $R_m = \frac{A_m}{P_m}$. The roughness in this unit may be described by the factor n_m. The slope S and the average velocity V are constant for all units. If there are N units, it is possible to establish $N + 2$ equations; that is, N equations of the form—

$$V = f_m(S, g, n_m, \text{ and } R_m) \ldots\ldots\ldots\ldots\ldots (25)$$

one equation of the form—

$$A = \sum_{m=1}^{N} A_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

which shows that all the area units add up to the total area, and one equation of the form—

$$P = \sum_{m=1}^{N} P_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

which shows that all the wetted-perimeter units add up to the total wetted perimeter.

Consider, as an example, a flume experiment in which the section is divided into two units—bed and side-walls. In this case four equations may be stated. The roughness of the side-walls will be obtained from preliminary experiments. The function f_w and the constant n_w are known. The values of P_w, P_b, V, and S will be measured in each individual experiment. Eq. 27 is meaningless, in this case, because all values of P_m are measured. Eqs. 25 and 26 enable the investigator to compute A_w, A_b, and n_b, if he assumes the function f_b suitable for the character of the bed.

This method can be checked very easily by applying it to a flume with different roughness in the side-wall and a stable bottom. If it is possible to find two roughness factors n that will satisfy measurements having different water depths, the method is satisfactory. The writer has checked the method several times and always has found it to be correct within the accuracy of the measurement. As an example, this procedure is well illustrated when both bed and side-wall friction can be expressed by means of Manning's formula:

$$V = \frac{1.486}{n} S^{0.5} R^{0.67} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

As previously stated, V, S, width b, and depth d are measured, and n_w is known; therefore, it is possible to determine R_w directly by using Manning's formula for the wall:

$$R_w = \left(\frac{n_w}{1.486} \frac{V}{S^{0.5}} \right)^{1.5} \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

The area with reference to the wall is

$$A_w = 2\, d\, R_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30a)$$

the area with reference to the bed is

$$A_b = d\,(b - 2\,R_w) \qquad \text{(30}b\text{)}$$

and the corresponding hydraulic radius is

$$R_b = \frac{A_b}{b} = d\left(1 - 2\,\frac{R_w}{b}\right) \qquad (31)$$

By using the radius R_b, the roughness factor n_b is then obtained, thus,

$$n_b = \frac{1.486}{V}\,S^{0.5}\,R_b{}^{0.67} \qquad (32)$$

and the discharge corresponding to the unit width of the bed is

$$q_b = V\,R_b \qquad (33)$$

The values R_b, q_b, and n_b have been used in the paper for the evaluation of ψ for all experiments.

To consider a numerical example,[12] let $b = 2.416$ ft; $d = 0.276$ ft; $q = 1.000$ cu ft per sec; and $S = 0.0010$. For turbulent flow[13] the average value of Manning's n, applied to the side-walls, is 0.0097. From this information the following computations can be made: $A = b\,d = 0.667$ sq ft; $V = \frac{q}{A} = 1.50$ ft per sec; $R_w = \left(\frac{n_w}{1.486}\,\frac{V}{S^{0.5}}\right)^{1.5} = \left(\frac{0.0097}{1.486} \times \frac{1.50}{0.0316}\right)^{1.5} = 0.172$ ft; $R_b = d\left(1 - 2\,\frac{R_w}{b}\right) = 0.276\left(1 - 2\,\frac{0.172}{2.416}\right) = 0.237$ ft; $n_b = \frac{1.486}{V}\,S^{0.5}\,R_b{}^{0.67} = \frac{1.486}{1.50}\,0.0316 \times 0.384 = 0.012$; and $q_b = V\,R_b = 1.50 \times 0.237 = 0.356$ cu ft per sec per ft of width.

Values R_b, n_b, and q_b do not contain the influence of the side-wall and, therefore, can be used in any formula for bed-load transportation, the results being expressed for a channel of infinite width.

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[12] Data from "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935, Table 11.

[13] *Loc. cit.*, Tables 8, 9, and 10.

DISCUSSION

Joe W. Johnson,[14] Assoc. Am. Soc. C. E. (by letter).—Compliments are due the author on his approach to the problem of bed-load transportation, especially in his dimensionless expressions and the absence of any "critical" term whose value, in general, is difficult to determine accurately.

In connection with investigations[15,16] by the Soil Conservation Service on the transportation of sediment in natural streams, most of the published and unpublished data from the United States and Europe on the transportation of bed load in laboratory flumes have been assembled. For these various data, computations have been made and the results tabulated in a consistent system of units as follows:

(1) Discharge (cu cm per sec)
(2) Flume width (cm)
(3) Mean water depth (cm)
(4) Water-surface slope
(5) Water temperature (°C)
(6) Mean velocity (cm per sec)
(7) Bottom velocity (cm per sec)
(8) Hydraulic radius of bed (cm)
(9) Friction velocity (cm per sec)
(10) Reynolds' number
(11) Manning's n for the bed ($ft^{1/6}$)
(12) Unit discharge (cu cm per sec per cm)
(13) Rate of bed movement (g per sec per cm)
(14) Duration of run (minutes), and
(15) Bed condition.

Items 1 to 7, inclusive, and 13 to 15, inclusive, are observed factors and have merely been converted from the various units of observation to the foregoing units. The hydraulic radius of the bed (item 8) has been calculated by the author's method of eliminating the effects of the flume side-walls. The other derived terms—that is, friction velocity, Reynolds' number, Manning's n, and unit discharge—are calculated by using the hydraulic radius of the bed.

An off-hand assumption might be that there is available a set of data reduced to a consistent system of units and applying to channels of infinite width. With the effect of flume width eliminated it would appear possible to obtain a relation between the rate of transportation or bed roughness and certain physical and hydraulic factors. However, it is scarcely possible to obtain such a relationship because of certain important factors inherent in the apparatus used in each separate set of experiments. These factors vary from experiment to experiment, and unfortunately cannot be reduced to a definite value as in the case of side-wall effect.

In studying the summary of the various bed-load data, several objections to the experimental apparatus and procedure are apparent and appear to account for the limiting value of the data. The most important objections are: (*a*) Variations in the type and manner of sand feed; (*b*) short flumes; and (*c*) short periods of sand collection. The author has already referred briefly to the problem of sand feed and has stressed its importance. The use of short flumes has the twofold disadvantage that: (1) It is extremely difficult to obtain an accurate measurement of the slope; and (2) the distribution of velocity and flow conditions do not become stabilized in such short reaches. Of interest in respect to this latter factor are certain experiments conducted in the United States, as well as in Europe, with a bed of movable material placed in a recess only a few feet in length located near the center of a flume in which the remainder of the bed is fixed at a different roughness than the sand bed. In such cases the vertical velocity distribution measured above the movable bed is not the distribution resulting from flow over the movable bed but more closely approximates the distribution for the relatively smooth bed upstream. No relation between velocity distribution and bed movement or roughness could be expected in such cases.

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[14] Hydr. Engr., Sedimentation Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[15] "Studying Sediment Loads in Natural Streams," by Gilbert C. Dobson and Joe W. Johnson, *Civil Engineering*, February, 1940, pp. 93–96.

[16] "A Distinction Between Bed Load and Suspended Load in Natural Streams," by H. A. Einstein A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, 1940, Pt. 2, pp. 628–633.

The duration of run is of importance in giving information on the relative value of various experiments. In most of the past experiments where short collection periods were used, a considerable scattering of data is noticeable. The author previously[17] has stressed the importance of using relatively long collection periods in studying the rates of bed-load transportation in order to eliminate the short-period fluctuations that are characteristic of most bed-load movement. Also, with short runs the bed condition may not have time to become so adjusted that a calculated roughness factor is indicative of conditions that would prevail after a longer period of operation.

Data from laboratories in the United States as well as in Europe all appear to be limited in value by one or more of the aforementioned factors. The distinct limitations of the various data became evident to the writer when an attempt was made to use certain data to obtain information on the correct value of the grain diameter to use in the author's ψ-functions and ϕ-functions when mixtures are considered. (The author has used the grain diameter at the 40% value in discussing the movement of United States Waterways Experiment Station mixtures, but admits that the 35% value would have tended to cause better agreement.) Mr. Gilbert's data[8] on the transportation of synthetic mixtures, for instance, should be of value in providing such information, but unfortunately he neglected to observe the depth of flow in these particular experiments. In this case, as well as in many others, extremely important observations have not been recorded because the experimenter apparently failed to realize their importance, or was confining his attention to some other phase of the problem. All pertinent data should be recorded whether or not the observer can see any immediate use of certain observations.

It is strongly urged, therefore, that, before future extensive flume experiments on bed-load transportation are undertaken (with the possible exception of experiments intended for demonstration or qualitative information), the experimenter "take stock" of the limitations of past studies and design his experiments to eliminate most of these objections. Little progress in the technique and range of conditions of laboratory bed-load investigations appears, in general, to have been made since the classic experiments of Mr.

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[17] "Calibrating the Bed-Load Trap as Used in the Rhine," by H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 110, No. 14; also, see *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1293.

Gilbert.[8] Bed-load experiments are both expensive and time consuming and future experiments to improve on those made in the past should be directed to give more accurate data throughout a wider range of conditions, especially in the range $\phi > 10$ where transportation in large streams is perhaps most likely to occur.

A. A. Kalinske,[18] Assoc. M. Am. Soc. C. E. (by letter).—Any attempt to rationalize the problem of bed-load transportation by using known basic physical phenomena is most welcome, especially if the expressions developed are correlated with experimental data. Although the author's work in this direction is most enlightening, there are present certain ambiguities and some indefinite statements and assumptions that make the writer question the physical picture of bed-load transportation as presented, and the generality of the final expressions. If these questionings are due to misinterpretations on the part of the writer, it is hoped that the author will attempt to clear up these points.

One confusing item to the writer is in regard to exactly what the author considers as being bed load. In the "Introduction" he states that bed-load movement is confined to those bed particles moving in quick steps and with long rest periods. Does this only include particles that slide and roll or does it also include those that jump up? If a particle rolls along fairly continuously, or with only short rest periods, is it excluded from the author's definition? Under "Derivations" the author states definitely that his treatment excludes "Bed material moving in suspension." However, in his derivations he considers the lifting of particles off the bed and the effect of the turbulence on the particles. What the line of demarcation is between bed load and suspended load is nowhere definitely stated. In a previous paper[19] the author with A. G. Anderson, Jun. Am. Soc. C. E., and J. W. Johnson, Assoc. M. Am. Soc. C. E., gives a definition for bed load as follows: "Bed-load is that part of the total sediment-load composed of all particles greater than a limiting size whether moving on the bed or in suspension, and includes all bed-material in movement." Does this definition apply to the analyses in the present paper? Would it not be better to define or differentiate between various types of sediment transportation by consideration of the different forces causing the movement instead of considering its visual appearance? It seems that this is what must be done if a physically correct analysis is to be made of any type of sediment movement.

The writer cannot agree that the concept of critical tractive force has no meaning in bed-load transportation. Naturally, for any given size of material, there is no single value of the tractive force below which not a single particle moves, and above which all particles of that size do move. First of all, the tractive force is not a constant value but tends to fluctuate about some average value. Second, the dragging and pushing forces tending to move the bed particles depend on the position a particle happens to assume and how much influence its neighbors exert. Nevertheless, statistically a value of the tractive force can be defined such that below it any movement will be practically negligible. To use the author's terminology it is possible to define a tractive force such that the probability of movement of any particular particle is very remote indeed for any practical considerations. Certainly the fine work of A. Shields[2] in regard to the critical tractive force for different sizes and types of particles indicates that this concept has physical meaning and practical usefulness.

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[18] Asst. Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[19] "A Distinction Between Bed-Load and Suspended Load in Natural Streams," by H. A. Einstein, A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, 1940, Pt. II, p. 628.

The transportation of sediment by the process of saltation, so clearly described and analyzed by R. A. Bagnold[20] for the movement of desert sand by wind, also occurs to a certain extent in water. The physical processes causing such movement are closely related and even similar to those for bed load. In fact, experimentally, separate determination of bed load and saltation load is almost impossible. The author's description and analysis of the movement of bed load seem very much similar to movement by saltation, particularly his statements regarding step movement and the lifting process. Mr. Bagnold differentiates between bed load and saltation load by calling the former those particles which creep, slide, and roll along the bed without any appreciable rise from the bed. For desert sand this differentiation is fairly definite, since the particles moving by saltation rise an appreciable distance above the sand surface, many times as high as 4 ft to 6 ft. In ordinary rivers it is doubtful if the rise of particles off the bed in the process of movement by saltation is more than perhaps a fraction of an inch or a few inches at the most. Therefore, any distinction between bed load and saltation load in rivers and canals becomes difficult.

Physically, the bed load might be considered as those particles which do not have sufficient lift exerted on them to raise them off the bed; instead they can only roll or slide. It appears that saltation occurs when the forces causing the rolling and sliding become large enough to exert a "lift" on the particles and thus raise a particle up into the flowing fluid. The raised particles immediately drop down since there is no force to support them in the fluid. Actually, of course, turbulence is present and upward velocity components tend to keep the raised particles up longer than they otherwise would stay. However, it is not considered that transportation by suspension comes into the picture unless vertical velocity components are present that are larger than the fall velocity of the particles a sufficiently large proportion of the time. Mr. Bagnold definitely concluded that the turbulence in the air had a negligible effect in transporting the sand of the size he was considering. In fact, from his description of the process of saltation, this type of movement can occur without the presence of turbulence if a sufficiently high drag and forward velocity could be developed without inducing turbulence. Naturally, this is not possible with air or ordinary liquids. In regard to this particular point, Mr. Shields[2] showed experimentally that bed-load movement takes place at a Reynolds number when the viscous boundary layer still covers the particles. All this leads to the conclusion that bed-load movement, in the sense considered by Mr. Shields, and movement by saltation can be brought about by forces that do not necessarily arise due to the presence of turbulence. Of course, if turbulence is present, particle movement by these methods may be profoundly affected.

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[20] "Transport of Sand by Wind," by R. A. Bagnold, *Geographical Journal*, Vol. 89, 1937, p. 409.

In the writer's opinion, this is the intrinsic difference between bed-load and saltation-load movement and transportation by suspension; the latter type of movement is due entirely to turbulence.

In his classic experiments and observations, Mr. Bagnold defines a so-called "threshold" wind velocity and also relates the total sand movement to the mean flow conditions. This definitely contradicts the author's statements relating to critical conditions and the possibility of defining bed-load transportation as a function of mean flow.

In the "Introduction" the author lists four axiomatic statements on which he bases all of his analyses. According to him these statements are the results of previous studies; however, since the results referred to are published in a paper not readily available to American engineers in general, it would have been better to give a little more background material and present the basis for these fundamental statements. For instance, the concept of an "average step" certainly needs further elaboration.

In the "Derivations" the factor p is defined as giving "* * * the number of steps that start from any given place during the time it takes to remove one particle." Exactly what is meant by "place" and the "time to remove one particle"? Is "place" the area covered by one particle? In order to make p, in Eq. 2 dimensionless the author introduces, very arbitrarily, a time t, which he chooses to call the time required for a particle to settle a distance equal to its own diameter. This particular t certainly could have been defined in many other ways. Therefore, it appears that at this point in particular the author departed from a theoretical analysis of the physical processes and stepped over into the realm of abstract dimensional analysis.

In the "Derivations" the author refers to bed-load movement as being due solely to the "lift" on a particle. The lifting forces that act on a particle resting on the bed are certain definite hydrodynamic forces caused by the flow over the particle. However, bed-load movement is not due entirely to such lifting forces; impact and pushing forces are certainly also present. Of course, all such other forces are also probably proportional to $D^2 v^2 \rho_f$ as given by the author for the lift forces, although at low velocities the liquid viscosity may be of importance. Eq. 10 is not correct for the velocity at the distance from the wall of the particle diameter. J. Nikuradse's velocity equation for sand-roughened pipes gives for the value of v at the edge of the protuberances the value $8.5 \sqrt{\frac{\tau}{\rho_f}}$.

The data presented by the author seem to be functionally related in the manner indicated, particularly for the smaller sediment loads. However, for the larger sediment loads it is extremely doubtful if the measurements included solely bed load in the sense that the writer interprets bed load. In regard to the sand mixtures, such mixtures cannot be described completely by any mean effective diameter unless the size-frequency curve is of the same type and shape for all such mixtures.

The writer agrees with the author that there is no single, universal law which will describe bed-load transportation. The reason for this is that different sets of forces come into play for different stages of movement. For small ratios of sediment size to laminar boundary layer thickness, the forces causing movement are certainly different from conditions when the laminar layer is no longer present. As shown by Paul Nemenyi,[21] if any analysis or correlation of experimental data is to be really useful for extrapolation, the entire "picture" must be kept in mind. The complete picture of bed-load transportation is probably divided into a number of phases, each having a slightly different set of predominating forces. The various phases are probably separated by critical regions. The transition from pure rolling and sliding to combined rolling, sliding, and "jumping" or saltation is such a critical region. The coming into play of the vertical velocity component of the turbulence is another such critical region. All these various phases of sediment transportation will become immensely clarified if some parameter can be found against which data can be plotted for all the phases of sediment transportation, from the simple rolling and sliding of the particles, when the grains are still covered by a laminar boundary layer, to when the major part of the material is being moved in true suspension. Mr. Shields made a start in this direction by use of the parameter $v_f\, D/\nu$, in which v_f is the friction velocity, $\sqrt{\frac{\tau}{\rho_f}}$; D is the particle diameter; and ν is the kinematic viscosity. Perhaps the parameter $\frac{v_s}{v_f}$ (in which v_s is the settling velocity) may serve this all-important function. This particular parameter is a function of the Reynolds and Froude numbers of an individual particle.

This paper should certainly serve the very important function of stimulating thought in regard to the problems of sediment transportation, and particularly so-called bed-load movement, along directions different from those followed by engineers in the past years. The paper should also show definitely that the innumerable, purely empirical bed-load equations now scattered throughout the literature are probably of limited application and certainly should not be used blindly for conditions widely different from those for which the data used in developing the equation were obtained. Eq. 18 certainly seems quite generalized; nevertheless, the limitations of its applicability should be carefully noted. The important question about this equation is in regard to what exactly was taken or considered as "bed load" in the experiments on which it is based. This point should be more definitely clarified both for Mr. Gilbert's data[8] and that obtained by the author in Zürich.

O. G. HAYWOOD, JR.,[22] JUN. AM. SOC. C. E. (by letter).—An interesting analytical derivation of a bed-load formula is presented in this paper, which has the decided advantage that all of the arbitrary constants introduced have a definite physical meaning. However, proceeding backward from his final empirical expression to the evaluation of some of these constants leads to results sufficiently doubtful to reflect upon its validity.

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[21] "The Different Approaches to the Study of Propulsion of Granular Materials and the Value of Their Coordination," by Paul Nemenyi, *Transactions*, Am. Geophysical Union, 1940, Pt. II, p. 633.

[22] Capt., Corps of Engrs., U. S. Army, 27th Engr. Battalion, Fort Buchanan, Puerto Rico.

From Eqs. 15*a* and 19, $A = 0.465$. The order of magnitude of constants A_1 and A_2 may be estimated by assuming spherical grains, thus giving $A_1 = \frac{\pi}{4}$ and $A_2 = \frac{\pi}{6}$. The general magnitude of λ_0, if it is a constant as assumed, may be estimated from observations of bed-load movement. For a grain of 1 mm diameter, a travel distance in a single step of 10 cm is not an unreasonable estimate, or $\lambda_0 = 100$. Then $A_3 = \frac{0.465 \times 100 \times \frac{\pi}{6}}{\frac{\pi}{4}} = 31$; and, from Eq. 6, $t = \frac{31}{0.75}\sqrt{\frac{0.1 \times 1}{980 \times 1.65}} = 0.32$ sec, which appears to be an absurdly high value for the time to remove one particle of 1 mm diameter from the bed. From the definitions of t, p, and p_s, t is evidently that almost instantaneous transition period when a particle passes from a state of rest to a state of motion, and which the writer would consider more properly of the order of a few thousandths of a second.

A similar analysis of the other equation constant, $B = \frac{A_2}{135\, A_4} = 0.391$, leads to equally disturbing results. Again assuming spherical grains in order to approximate the values of the constants, $A_2 = \frac{\pi}{6}$ and A_4 then computes to be 0.01. Thus, the coefficient of lift is 0.01 and constant for all conditions satisfying the author's basic equations. It must be noted, however, that Eq. 9 is identical to the generally accepted equation for the drag on a particle. Moreover, numerous experiments have shown that the coefficient for drag varies decidedly with the turbulence around the particle (Reynolds' number), and that the coefficient has a minimum value of the order of 0.15. It is evident that the lift must exceed the weight of the particle before it will be lifted from the bed. It appears decidedly unlikely that particles could withstand a dragging force of at least fifteen times their weight before moving. In other words, if the coefficient of lift is that indicated by the author's results, the particles composing the bed would all be shoved along the bed long before the lift forces could become of sufficient magnitude to be effective, and the derivation based on this lift equation fails.

Returning to a more general view of the paper, the writer does not concur in the statement in item 2(*c*) of the "Introduction" that "The average step of a certain particle seems always to be the same even if the hydraulic conditions or the composition of the bed changes." This statement is important, as it forms the "backbone" of the analytical procedure, and its validity appears to be confirmed, as the value of λ_0 is found empirically to be constant for most of the data. From the writer's observations of bed-load movement, it appears that the steps increase in length as the velocity of flow increases. Although most of these observations were on light-weight materials, the phenomenon of equal steps was never noted by the writer and was never mentioned by any of his co-workers at the U. S. Waterways Experiment Station, Vicksburg, Miss., who had done extensive experimentation with sand movement. The fact that λ_0 actually does appear to be constant indicates to the writer not that the assumption of equal steps is correct but instead that λ_0 is an arbitrary constant that does not have the physical meaning attributed to it.

Finally, the evident discrepancy of certain of the data presented in Fig. 3 is explained as a result of differential sorting of the grain mixture forming the bed. The author states that this hypothesis is not subject to direct proof. The U. S. Waterways Experiment Station, during the conduct of the investigations analyzed by Mr. Einstein, made periodic analyses of the material moving into the sand trap. It would appear that these data should furnish sufficient direct proof as to whether or not appreciable sorting is occurring. Data for one sand are reproduced in Fig. 4. The curve is typical, showing that the larger grains move first, but that, as movement increases in intensity, the material in motion becomes representative of the material in the bed.

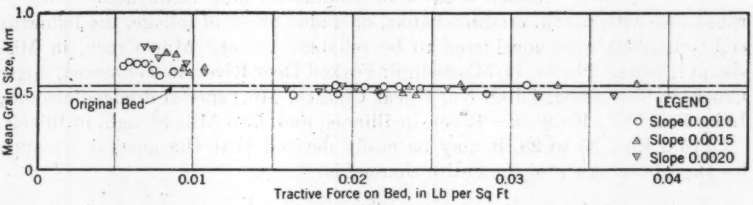

FIG. 4.—MEAN GRAIN SIZE OF MATERIAL IN MOTION; EXPERIMENTATION STATION, SAND 3

However, even without these data, the explanation appears faulty from the plotted data presented in the paper. It appears that, if the hypothesis is correct, the grouping of points along curve (*S*) in Fig. 3 is caused by the formation of a coarse protective layer of grains, and the plotted points would fall along curves (1) and (2) if the grain size of this protective layer were used in the formulas instead of the mean grain size of the original bed material. The same analysis may be applied in the reverse direction. Knowing the location of the plotted data and the bed-load equation, the grain size of the protective layer may be determined.

From Eqs. 16*a* and 16*b*, $\phi \sim \frac{1}{F} \times \frac{1}{D^{1.5}}$; and $\psi \sim D$. Taking sand 6, for example, there is a grouping of points at about $\phi = 0.0025$ and $\psi = 6.0$. By cut and try, it may be found that the points fall on curve (1) if $D = 0.125$ cm, giving $\phi = 0.00025$ and $\psi = 24$. Thus, if the author's explanation is correct the protective layer is composed of grains of an average diameter of 0.125 cm.

However, the reference from which Mr. Einstein obtained these data gives a grain-size distribution curve of this sand. There are no particles as large as 0.125 cm in the sand mixture, and less than 2% by weight of the particles composing this sand mixture are even half as large. Obviously, sorting alone is not the answer.

The writer has also noted, with interest, the contents of Appendix II regarding elimination of side-wall influence, as he has made a somewhat extensive investigation of this subject.[23] Although the author's method has been used by many investigators, published data actually verifying the method are rather rare. Most data on open channel flow in laboratory flumes have an experimental error of such magnitude as to obscure small variations in roughness. Natural channels, on the other hand, do not tend to have a marked change of roughness at some particular depth. The ideal channel to investigate would be one of sufficient size so that errors in measurement have negligible weight, and which has channel boundaries that have a definite change of roughness at some stage, with uniform roughness below this stage, and uniform but different roughness above. Some drainage ditches approximate this condition, and an analysis has been made of data published in the Department of Agriculture's bulletin, "Flow of Water in Drainage Channels."[24]

After rejecting channels because of insufficient data, insufficient change of roughness with depth, erodible banks, or undue effect of foliage, the following eight channels were considered to be reliable: Twenty Mile Creek, in Mississippi; Bogue Phalia, in Mississippi; Forked Deer River, in Tennessee; Sugar Creek, in Tennessee; Ditch No. 1 near Chaffee, Mo.; special dredged channel, Lake Fork, Ill.; Kaskaskia River, in Illinois; and Two Mile Slough, in Illinois.

From Eqs. 25 to 28, it may be easily derived that the general equation for the Manning's n of the entire channel is:

$$n^{1.5} = \frac{\sum_{m=1}^{N} n^{1.5}{}_m P_m}{P} \qquad (34)$$

Most of the dredged drainage channels have slight flow for the major part of the year, and a greatly increased flow for short periods of time. Thus, the lower part of the channels tends to be free of vegetation, whereas the banks are more or less thickly vegetated. If it is assumed that the perimeter is composed of sections of two uniform roughnesses (that is, the bottom and the banks) the general equation may be written

$$n^{1.5} P = n^{1.5}{}_1 P_1 + n^{1.5}{}_2 P_2 \qquad (35)$$

in which P_1 is the wetted perimeter of the lower section of the channel, P_2 the remainder of the wetted perimeter, and n_1 and n_2 their respective roughness coefficients.

The value of P, of course, is dependent on the depth of flow. For flow in the lower part of the channels, the value of P_1 varies with the depth of flow, and P_2 is zero. For greater depths of flow P_1 is constant and P_2 varies with stage. Thus, Eq. 35 may be written in two parts:

$$n^{1.5} P = n^{1.5}{}_1 P_1 \qquad (36a)$$

if the flow is in the lower part of the channel and

$$n^{1.5} P = \text{constant} + n^{1.5}{}_2 P_2 \qquad (36b)$$

if the vegetated banks form part of the wetted perimeter.

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[23] "Flume Experiments on the Transportation by Water of Sands and Light-Weight Materials," by O. G. Haywood; submitted in 1940 to the Massachusetts Institute of Technology, at Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science.

[24] "Flow of Water in Drainage Channels," by C. E. Ramser, *Technical Bulletin No. 129*, U. S. Dept. of Agriculture, November, 1929.

When the value of $n^{1.5} P$ is plotted against wetted perimeter, the points therefore will lie on two straight lines. One line passes through the origin, with its slope determined by the value of "n" for the lower part of the channel. The slope of the other line is determined by the roughness coefficient of the banks, and intersects the lower line at a value of perimeter corresponding to that stage at which flow against the vegetated banks first occurs.

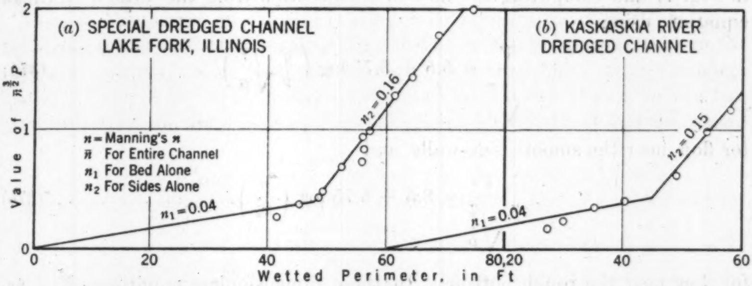

FIG. 5

Two of the graphs made by the writer are reproduced in Fig. 5. It will be noted that the data check the predicted straight lines very closely. All of the aforementioned drainage ditches were plotted in similar graphs, and none were

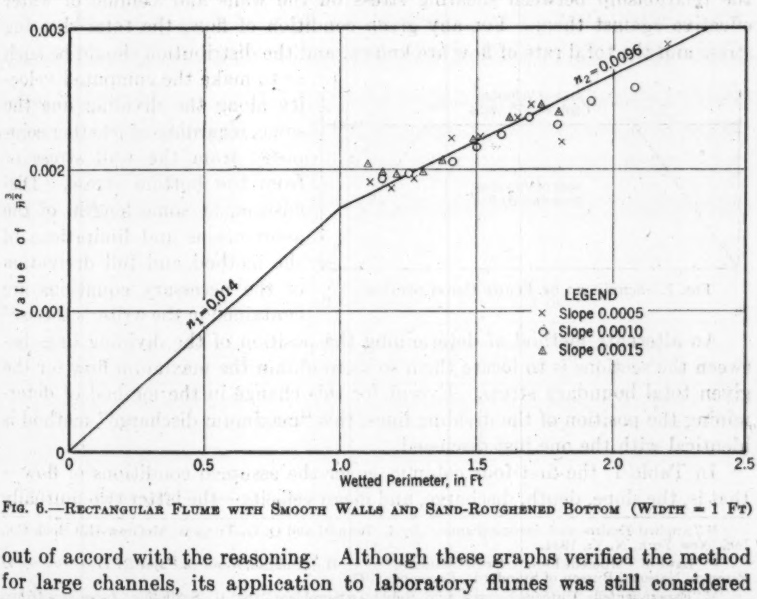

FIG. 6.—RECTANGULAR FLUME WITH SMOOTH WALLS AND SAND-ROUGHENED BOTTOM (WIDTH = 1 FT)

out of accord with the reasoning. Although these graphs verified the method for large channels, its application to laboratory flumes was still considered questionable. Certain of the author's assumptions appeared valid for large values of the Reynolds number, but more questionable for lower values such as those obtained in flumes. Some experimental data taken by the author in a flume with smooth walls and sand-roughened bottom are shown in Fig. 6.

As a further check on the method, the writer attempted an entirely different approach based on the Prandtl-Kármán theory of flow,[25] following to some extent the reasoning of G. H. Keulegan[26] and H. Schlicting.[27] The basis of this method is the division of the flume channel into three sections as shown in Fig. 7, and computing the flow in each section from the Prandtl-Kármán equation, using

$$\frac{V}{\sqrt{\frac{\tau}{\rho}}} = 5.5 + 5.75 \log \left(\frac{y}{\nu} \sqrt{\frac{\tau}{\rho}} \right) \ldots\ldots\ldots\ldots\ldots\ldots (37a)$$

for flow near the smooth side-walls, and

$$\frac{V}{\sqrt{\frac{\tau}{\rho}}} = 8.5 + 5.75 \log \left(\frac{y}{k_s} \right) \ldots\ldots\ldots\ldots\ldots\ldots (37b)$$

for flow near the rough bottom. In these dimensionless equations, V = velocity at any point at distance y from the boundary, τ = shearing stress on boundary, ρ = fluid density, ν = fluid kinematic viscosity, and k_s = "equivalent sand roughness" of boundary.

The position of the dividing lines between the sections was determined by the relationship between shearing stress on the walls and volume of water effective against them. For any given condition of flow, the total shearing stress and the total rate of flow are known, and the distribution should be such as to make the computed velocity along the dividing line the same, regardless of whether computed from the wall stress or from the bottom stress. Discussion, at some length, of the assumptions and limitations of the method and full derivation of the necessary equations are contained in the writer's thesis.[23]

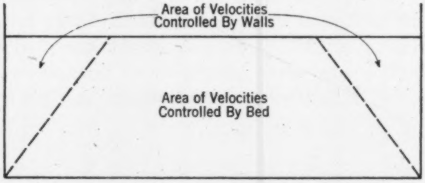
Area of Velocities Controlled By Walls
Area of Velocities Controlled By Bed

FIG. 7.—SCHEMATIC OF FLUME CROSS SECTION

An alternate method of determining the position of the dividing lines between the sections is to locate them so as to obtain the maximum flow for the given total boundary stress. Except for this change in the method of determining the position of the dividing lines, this "maximum discharge" method is identical with the one just discussed.

In Table 1, the first four columns cover the assumed conditions of flow—that is, the slope, depth, discharge, and mean velocity—the latter two mutually dependent quantities being dependent on the roughness of the bed.[23] The effective discharge is that part of the discharge that is effective against the bed; that is, the total discharge minus that part dissipating its energy against the walls. Computations for the effective discharge under the various conditions of flow were made by two methods, as shown in the table, the "writer's method" being that based on the Prandtl-Kármán method, and the "author's method" being that described in the paper.

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[25] "Applied Hydro- and Aeromechanics," by L. Prandtl and O. G. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

[26] "Laws of Turbulent Flow in Open Channels," by G. H. Keulegan, *Research Paper RP1151*, *Journal of Research*, National Bureau of Standards, December, 1938.

[27] "Experimentelle Untersuchungen zum Rauhigkeitsproblem," by H. Schlicting, *Ingenieur-Archiv*, Vol. VII, No. 1, February, 1936 (abridged translation in *Proceedings*, Am. Soc. C. E., November, 1937, Translations, p. 16.)

The tractive force on the bed is the total boundary shearing stress minus the stress acting against the walls. In addition to computations based on the "writer's method," the "author's method," and the "maximum discharge method" (described briefly herein), computations are also shown for the two limiting values, the first based on the walls offering no resistance and the second on the walls offering resistance equal to that of the bed. The first is the familiar du Boys tractive-force equation,

TABLE 1.—Comparison of Side-Wall Correction Methods

Slope	Depth, in ft	Discharge, in ft per sec	Mean velocity, in ft per sec	Effective Discharge, in Cu Ft per Sec		Tractive Force on Bed, in Lb per Sq Ft				
				Writer's method	Author's method	du Boys $\gamma D S$	$\gamma R S$	Writer's method	Author's method	Maximum discharge method
(a) Glass-Walled Flume, Width 1.0 Ft, Walls Smooth										
0.0001	0.4	0.167	0.42	0.125	0.119	0.0025	0.0014	0.0017	0.0018	0.0017
0.0005	0.1	0.060	0.60	0.054	0.051	0.0031	0.0026	0.0026	0.0027	0.0027
0.0005	0.1	0.031	0.31	0.030	0.029	0.0031	0.0026	0.0029	0.0029	0.0029
0.0005	0.4	0.491	1.23	0.316	0.275	0.0125	0.0069	0.0073	0.0070	0.0078
0.0005	0.4	0.416	1.04	0.311	0.275	0.0125	0.0069	0.0083	0.0083	0.0084
0.0005	0.4	0.416[a]	1.04[a]	0.310[a]	0.275[a]	0.0125[a]	0.0069[a]	0.0081[a]	0.0083[a]	....
0.0005	0.4	0.296	0.74	0.253	0.236	0.0125	0.0069	0.0097	0.0100	0.0098
0.0005	0.8	1.246	1.55	0.422	0.478	0.0250	0.0096	0.0102	0.0096	0.0114
0.0005	0.8	0.849	1.06	0.597	0.553	0.0250	0.0096	0.0159	0.0163	0.0155
0.0015	0.4	0.772	1.93	0.577	0.494	0.0375	0.0208	0.0249	0.0240	0.0243
0.0015	0.4	0.556	1.39	0.475	0.434	0.0375	0.0208	0.0290	0.0292	0.0280
0.0015	0.8	2.024	2.53	0.969	0.935	0.0750	0.0288	0.0370	0.0346	0.0355
0.0015	0.8	1.576	1.97	1.110	0.996	0.0750	0.0288	0.0476	0.0474	0.0421
(b) Concrete Flume, Width About 2.3 Ft, Walls Smooth										
0.0020	0.4	2.413	2.62	2.13	1.96	0.0500	0.0371	0.0407	0.0404	....
0.0020	0.4	1.51	1.64	1.34	1.36	0.0500	0.0371	0.0425	0.0434	....

[a] Water temperature 16° C (60° F); all other computations are for a water temperature of 24° C (75° F).

$$\tau = \gamma D S \qquad (38a)$$

in which τ = drag, or boundary shearing stress, γ = unit weight of fluid, D = depth of water, and S = slope of energy gradient or, for uniform flow, slope of water surface. The second,

$$\tau = \gamma R S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

is the same equation with the hydraulic radius substituted for the depth.

The substantial agreement of the writer's and the author's methods, especially for the values of bed tractive force, will be noted. Although it is fully realized that this agreement does not definitely prove that either method gives correct results, the two methods are based on quite different assumptions and analytical procedures, and their substantial agreement is encouraging. As extreme accuracy in analyzing bed-load data is not necessary because of the large experimental error always present, it is concluded that either of the two methods, and probably any other method based on rational consideration of velocity and shear distribution throughout the cross section, is sufficiently accurate. However, it should be noted that, if the average tractive force on the bed is computed by Eq. 38(*a*), serious error is introduced, particularly in deep, narrow channels. It is not intended herein to question the validity of the du Boys expression for large, natural channels where the depth is small in comparison with the width of the channel.

Samuel Shulits,[28] Assoc. M. Am. Soc. C. E. (by letter).—This scholarly probe into the realm of a universal law for the transportation of bed load is inspiring and merits the careful consideration of researchers in solids transportation. Since the ultimate end of all such analytical inquiries is a rational and practical implement for engineers, it may be interesting to conjecture what the reaction of a river engineer might be to the proposed method.

The largest part of available experimental data on bed-load movement is that measured by G. K. Gilbert, A. Schoklitsch, E. Meyer-Peter, and the U. S. Waterways Experiment Station[4]—and perhaps F. Schaffernak. Others have made a few scattered flume tests and then produced a bed-load formula, based on their own tests and those of the aforementioned authorities. All the formulas are different, with varying claims for their applicability, although some can be shown to resemble each other very closely. The author presents a law of transportation, the empirical coefficients for which are determined from the Zürich[7] (Meyer-Peter) and Gilbert[8] data and then finds that the U. S. Waterways Experiment Station measurements do not fit very well. Professor Meyer-Peter and the U. S. Waterways Experiment Station have offered their own formulas for their measurements, whereas Mr. Gilbert arrived at no generally usable equation. Professor Schoklitsch calculated another formula with his own and the Gilbert data. Of what avail is all this research if different investigators can create different results from the same basic data? Can the river engineer take one formula and use it with assurance or with some indication of its inherent error? The maze becomes worse when one realizes that not all the proposed formulas have been mentioned in this paragraph.

The author does not favor the concept of a critical value pertaining to the flow at which transportation begins. This factor is of physical significance, since there are flow conditions at which no bed-load movement occurs—a demonstrable fact with which river engineers like to deal. Little is gained by discarding the idea of a critical value, as the complicated formulas of this paper reveal. Why discard it when it is usually of little influence in practice? To illustrate this view, consider merely as an example the Schoklitsch bed-load formula,[29] derived from simple rational premises:

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[28] Mississippi River Commission, Vicksburg, Miss.

$$G = C\,(Q - Q_0) \quad \text{(39a)}$$

In Eq. 39a G is the bed load, Q the instantaneous discharge, and Q_0 the critical discharge at which movement begins—all in cubic feet per second; whereas C is the dimensionless bed-load coefficient. Eq. 39a is also dimensionally pure. In actual calculations, Q_0 is negligible in comparison with Q, and its exclusion or inclusion is simply a matter of judgment, still necessary regardless of the bed-load formula preferred. The formula then becomes

$$G = C\,Q \quad \text{(39b)}$$

There is no valid logical objection to the concept of such a critical parameter. Its magnitude is not yet definitive, to be sure, but that is the problem of research and does not vitiate the significance of this physical concept. Similarly, the author refers to grain diameter and settling velocity, which are two parameters still far from being "nailed down" in a utilitarian sense.

The difference between the Schoklitsch formula, for example, and the Einstein law is, briefly: The former predicts the mathematical relationship between certain parameters, and then strives by means of experimental data to prove the basic reasoning and to determine certain coefficients; whereas the author devises two dimensionless functions and the determines empirically (from experimental data) their mathematical relationship (f) and certain coefficients (A and B). The first method is usually accepted as the best research procedure. As far as usefulness is concerned, it cannot be claimed that the second method is superior, although it may not be inferior. The river engineer may like to use Eq. 18, which states that ϕ is a constant times ψ; but he will then have to use constants and a function (f) which, strictly, fit only the Zürich and Gilbert data. Thus, there is no assurance of the universality of the constants or function, even if dimensionless characteristics are desirable.

Since Eq. 18 can be written:

$$\phi = \frac{\psi}{0.465\, e^{0.391}} \quad \text{(40)}$$

the possibility of a straight-line plot is indicated, rather than the semilogarithmic presentation in Fig. 3. Has the author investigated this possibility?

John S. McNown,[30] **Jun. Am. Soc. C. E.** (by letter).—The equation presented by the author, for bed-load movement, marks a forward step in the solution of this complex problem. A phenomenon that is subject to so many variables does not lend itself readily to a general analysis. For this reason the tendency of the experimental points to approach a single curve for movement of a uniform bed load, as shown in Fig. 3(*a*), is remarkable. The principal variables included in the experimental data used in this paper are amount of bed load, depth (or hydraulic radius), and particle density. Of necessity other variables such as water discharge, water velocity, and channel slope are included because of their dependence on the principal variables.

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[29] "The Schoklitsch Bed-Load Formula," by S. Shulits, *Engineering*, June 21 and 28, 1935, p. 644.

[30] Instr. of Math., Mechanics and Hydraulics, Univ. of Minnesota, Minneapolis, Minn.

Some factors of probable importance are apparently omitted in the analysis. That the effect of a nonuniform bed is not included is obvious from a comparison of Fig. 3(*a*) and Fig. 3(*b*). Whether or not the shape of particles used in the two sets of experiments has an effect is not clear as no mention is made of this factor by the author. The plotted data show no trend that could be accounted for in this way. The application of this equation to the data obtained by Ho Pang-Yung[31] using irregularly shaped particles would be of assistance in settling this point. Also, the writer is of the opinion that the solution of the bed-load movement will be incomplete until the degree of turbulence is included as a variable. This cannot be done, of course, until an instrument for measuring turbulence conveniently is devised.

The agreement obtained is all the more remarkable as some of the underlying assumptions are admittedly uncertain. The existence of a constant ratio between the length of step and the size of particle seems improbable to the writer. Under this condition all particles in a uniform bed would have to take steps of the same size. Also a particle of a certain size would have to make the same size of steps under all conditions of movement. This implies a regularity that the writer has never observed in bed-load movement. If the same result were to be obtained by the assumption of a variable step length the plausibility of the derivation might be increased. This can be done by a slight change in the method of obtaining Eq. 1.

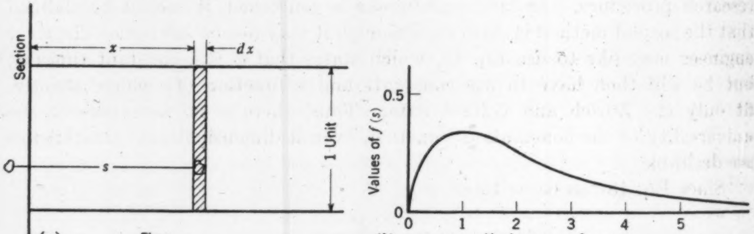

FIG. 8

The number of particles per second that cross the line labeled "Section" in Figs. 1 and 8(*a*) must be equal to the number of particles starting a step of variable length in one second that is long enough to cross the line. The two sets of particles are not identical but the equality must exist for continuous flow. The number of particles per unit width, beginning a step in one second from the differential area, which is a distance x from the line, is given by

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[31] "Abhängigkeit der Geschiebebewegung von der Kornform und der Temperatur," by Ho Pang-Yung, *Mitteilungen der Preussischen Versuchsanstalt für Wasser-, Erd- und Schiffbau*, Heft 37, Berlin, 1939.

$$N = \frac{dx}{A_1\, D^2}\, p_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

as shown in Fig. 8(a). The probability that a step will cross the line is designated as p_x. The functional relationship defining the frequency distribution of the variable-step length s should satisfy the following conditions: (1) The probability of making a step of zero distance is zero, (2) the probability of making a step of infinite distance is also zero, (3) the function must exist and be positive at all points between zero and infinity, and (4) the total probability or area between the distribution curve and the s-axis must be equal to one. One function (see Fig. 8(b)) that satisfies these conditions is

$$f(s) = s\, e^{-s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

This curve may be thought of as a special case of the family of skewed frequency curves generally designated as Pearson's Type III. The probability that $s > x$ is given by

$$p_x = \int_x^\infty s\, e^{-s}\, ds = (x + 1)\, e^{-x} \ldots\ldots\ldots\ldots\ldots (43)$$

The total number of particles beginning steps that are long enough to reach or cross the line per unit width is then

$$\int \frac{dx}{A_1\, D^2}\, p_s\, p_x = \frac{p_s}{A_1\, D^2} \int_0^\infty (x + 1)\, e^{-x}\, dx = \frac{p_s}{A_1\, D^2}\, 2 \ldots\ldots (44)$$

From the integral the term 2 is seen to have the dimension of a length and is approximately three fourths of the mean step length for the distribution that was assumed. By considering this value as simply a characteristic length, the length, L, may be substituted for it by a change of units and the author's expression, $\frac{p_s\, L}{A_1\, D^2}$, is obtained for the number of particles crossing the line, per second per unit width. The introduction of a constant would have no effect on the final result so that L could be taken as the average length of step. The author's assumptions on step length would then be applied to the mean value rather than to a single step.

In Eqs. 9 and 10 the effective velocity in determining the lift is assumed to be the wall velocity. The criticism that the author applies to the concept of critical movement would apply here also. Were the lifting force dependent on the wall velocity alone a critical velocity would necessarily exist. The wall velocity is probably not as important in the movement of particles as is the vertical component of the instantaneous velocity. An increase in the velocity criterion used should indicate an increase in the probability that the lift would exceed the weight of the particle rather than an increase in the lift itself. Using a turbulence criterion such as the root mean square of the vertical component of the instantaneous velocity, a gradual transition would result because of the nature of turbulence. Under comparable experimental procedures the wall velocity and the turbulence velocities may have a nearly constant ratio. Should this be the case no serious error would be introduced by the use of the wall velocity. This question, like so many others, must await a more complete experimental study of turbulence.

The critical values that have been used particularly in formulas for bed-load movement based on the du Boys concept are more likely misnamed than incorrectly conceived as the author maintains. Their use can be interpreted as simply the introduction of a constant to fit the general equation to the experimental data. The measurements corresponding to this particular value mark the end of the transition between no bed load and conditions that fit the particular formula. Some such value is necessary just as the quantity A in Eq. 17 is necessary in the author's derivation.

The value τ in Eqs. 10 and 11 designates the boundary shear, not the variable internal shear. If τ were replaced by τ_0 this fact would be shown more clearly.

This paper should certainly serve as a valuable analysis on which further developments in this intricate problem can be based. The methods and equations used by the author are somewhat complex but, in general, basically sound. The author is to be complimented for his excellent paper.

H. A. EINSTEIN,[32] ASSOC. M. AM. SOC. C. E. (by letter).—All discussions of this paper definitely show how little is known about bed-load movement. Mr. Shulits, for instance, asks of what avail is all this research if different investigators can derive different results (formulas) from the same basic data. This is the fate of all empirical research, because, when one tries to fit a mathematical curve through a group of scattering points, it is always possible to use fundamentally different functions that will all fit the points equally well within a certain range. Only by enlarging the range of measurements into a region where the different curves diverge can one choose among them. In this respect the writer was extremely fortunate in having the Swiss experiments[5] available, since they extend the range of experimental data considerably toward the greater depths and coarser grains.

In 1934 the writer developed[7] what is today generally known as the Meyer-Peter formula. This formula was based on the same experiments as the ϕ-ψ formula and satisfied a great number of them, especially those with coarse sediment. Accordingly, it proved very useful and dependable in its application to mountain streams with steep slopes and coarse sediment. It is well known that this formula will not describe the transportation of fine sediment, such as Gilbert's sands, smaller than 1 mm in diameter.

The ϕ-ψ method of describing bed-load transportation, and the Meyer-Peter formula, are both based on similar principles, but the ϕ-ψ method is more flexible. For this reason it can be correlated to all known experiments. The parameter ϕ is a measure of the intensity of transportation and seems to be of special importance, as most known bed-load formulas are valid only in certain ranges of ϕ. However, even if this method permits all measurements to be represented, it is still doubtful if any one mathematical curve will describe the entire range of known experiments. Conditions in large rivers with fine sediment are characterized by still higher values of ϕ, for which no flume experiments have ever been made. Any final curve should certainly embrace this range of higher values.

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[32] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

The general pattern of research in the bed-load problem includes, since the first work of Gilbert[8] and Schoklitsch,[33] the following three steps: (1) Flume studies on beds of uniform grain; (2) extension of the results to mixtures; and (3) application to rivers under natural conditions. The ϕ-ψ method of representation is developed from the first stage alone. Its application to problems of the second step has been tried by the writer, using the experiments[10] made at Vicksburg; but he was forced to the conclusion that some kind of sorting must have occurred, contrary to Captain Haywood and the testimony of Fig. 4. Bed-load investigations conducted by the U. S. Soil Conservation Service in a small creek seem to give an explanation for this discrepancy. In these experiments the ϕ-ψ graph also shows points between curves (2) and (*S*) (Fig. 3(*b*)), and the composition of the bed as a whole is definitely the same as that of the bed load moving on it. In the top layers of the bed, however, a definite sorting took place, accumulating the coarse grains in a certain layer and the fine grains above and below. Depending on the extent to which this coarse layer is exposed, the rate of transportation may change for any one flow. Very likely the same thing has happened in the Vicksburg flume, and therefore could not be detected by samples as represented in Fig. 4. Investigations by the Soil Conservation Service on this problem are still in progress (November, 1941) and may soon yield definite results. If the writer's explanation is correct, the entire space between curves (2) and (*S*), Fig. 3(*b*), would constitute a possible range for non-silting and non-scouring channels. Elaboration on the extreme importance of this question is probably not necessary.

The average single step as introduced in statement (*c*) of the "Introduction" has been determined in a great number of measured distributions similar to the one presented by Mr. McNown in Fig. 8(*b*). Their averages, not the steps themselves, are independent of the flow, but seem to vary considerably with form and roundness of the particles. Further discussion of this question does not seem desirable at this time, as the complicated formulas were presented in 1937,[5] and relatively few readers would be interested in their derivation.

Professor Kalinske asks what particles are included in the description of bed load given in the "Introduction." This is very easy to answer: All particles in any flume experiment with uniform material, rolling or in saltation. It just happened that under the conditions of all these experiments particles did not seem to go into suspension. There is no single flume study with uniform grain available in which bed load and suspended load have been measured simultaneously.

The term "critical tractive force" is so familiar to every one that it seems almost impossible to do without it. Unfortunately, it is a condition that does not exist in nature. If it did, the points somewhere near the upper end of curve (2) would approach a horizontal line. The top points, representing a rate of transportation of some few pounds per hour in a 6-ft wide flume, would definitely be called zero transportation by all observers. If this small rate is measured, however, the data are found to fit curve (2). The writer would readily change his opinion about this point if any one could cite some experiments that show this break in curve (2); but apparently the break does not seem to exist.

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[33] "Ueber Schleppkraft und Geschiebebewegung," by A. Schoklitsch, Verlag Wilhelm, Engelmann, Leipzig & Berlin, 1914.

It is clear that for the practical engineer the concept of a critical tractive force or critical value of ψ is very useful, since it indicates the flow under which a natural or artificial bed will be stable. For this purpose one must remember that an exact limit of transportation does not exist, but that the erosion can be reduced to any safe amount by choosing the proper ψ-value according to the expected duration of the critical flow.

The final important point under discussion is the influence of turbulence. R. A. Bagnold[34] shows indirectly that turbulence is greatly responsible for the transportation of bed load. Describing his transportation experiments conducted in a special wind tunnel, he states:

> "Sand placed at the mouth itself was never disturbed even at the highest speeds used, despite the fact that the drag and the normal velocity gradient must be a maximum there."

He does not give any explanation for this fact, but it is obvious that at the mouth of the wind tunnel turbulence has not developed yet and that this lack of turbulence can be the only reason for the failure of the sand to move.

The term turbulence includes all pulsatory velocities in the flow. The influence of vertical turbulence velocities on a moving grain is the action typical for suspension and has been excluded from the problem as such. Such vertical velocities might to some extent be responsible for the deviation of the finest grain sizes in Fig. 3(*a*) by increasing λ. Any influence of vertical velocities on grains in the bed, however, is impossible, as those vertical turbulence velocities do not exist in the immediate proximity of the bed (the water would have to stream out of the bed). Therefore, only horizontal velocities are available to move particles out of the bed. Since Mr. Bagnold proved that the average velocity without turbulence is definitely not able to cause the movement, it must be concluded that the horizontal turbulence or pulsations are responsible for the movement. (Mr. Kalinske may remember that in Mr. Shields' experiments the flow outside the laminar sub-layer was definitely turbulent and that the disturbances due to the eddies reached well into this sub-layer.)

Mr. Rubey[4] chose as the title of his paper: "The Force Required to Move Particles on a Stream Bed," thus indicating that his investigations deal with the forces acting on particles, like particle *a* in Fig. 9, that are on the bed rather than on particles like *b* which are in the bed. Close observation of bed-load movement shows that most moving particles are dislodged from positions in the bed. Thus, it is easily understood why Captain Haywood found a constant $A_4 = 0.01$ for the lift on the particles, and this is also the reason why the writer always refers to a "lifting" force. The low value of A_4 in itself excludes the possibility of any impact or push, such as would take place in the case of particles in position *a*, Fig. 9. Captain Haywood's rather high value of $t = 0.32$ sec may be correct. An explanation could be that before removal the particles first must be freed from the surrounding particles by small vibrating movements. In motion pictures of bed-load movements these vibratory movements can be seen readily.

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[34] "The Movement of Desert Sand," by R. A. Bagnold, *Proceedings*, Royal Soc. of London, Series A, Vol. 157, 1936, p. 600.

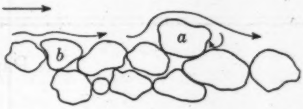

FIG. 9

The writer believes that he has answered the most important questions raised in discussion. Much could be added. In conclusion he would like to emphasize the fact that the problem is far from being solved and that a great number of the most important questions pertaining to sediment transportation cannot be answered because the necessary measurements do not exist or because they are incomplete. Every one investigating problems of sediment transportation would do well to keep Mr. Johnson's remarks in mind.

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VALUE OF PUBLIC WORKS

By J. P. Hallihan,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Clarence W. Post, Uel Stephens, William J. Wilgus, Bernard L. Weiner, Albert Ed. Scheible, H. B. Cooley, Philip W. Henry, Baxter L. Brown, Evan S. Martin, Isador W. Mendelsohn, W. W. Crosby, and J. P. Hallihan.

Synopsis

It has long been the theory in the United States that cyclic depressions in industry, due to lack of a market for the goods produced, resulting in large-scale unemployment, could be alleviated materially by increased activity in the field of public works—in the construction of facilities requiring no market except the approval of the people. The opinion was also widely held that the transfer of the man power released by industry into the operations of public works would be a simple process performed with no great loss of time, and that the equivalent employment would sustain the purchasing power of the nation until industry returned to a normal basis.

The nine years 1931 to 1939 have furnished an opportunity to test these theories, and it may be useful to review the record to determine to what extent they have been supported in practice.

The Volume of Public Works

There are only two sources of public works—those incident to the development or protection of the national resources or the requirements of national defense, defined as federal works, and those required by states and lesser political subdivisions to meet the needs of their respective communities, defined as non-federal works. In the larger sense, the public interest is dominant in all these works, since any construction or employment adds to the national income, but the federal works can be initiated and financed only by federal action, whereas the non-federal works are wholly dependent on the willingness of the people directly benefited to assume the necessary financial obligations.

Financing of Public Works

When it became apparent, late in 1930, that extraordinary efforts must be undertaken to find employment for the millions of workers dropped from the

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Note.—Published in February, 1941, *Proceedings*.

[1] Director, Municipal Eng. Section, WPA, Washington, D. C.

industrial rolls, the potentialities of state and municipal public works were explored. A survey conducted at the request of the Hoover Committee on Employment, by the American Engineering Council, through Engineering Committees voluntarily serving in each state,[2] reported in April, 1932, that a great reservoir of state and municipal projects for basic improvements existed in each state, typified by an estimate of $490,000,000 in the state of Michigan on a 5-yr program, and that this general proportion of needs to population was characteristic of the entire forty-eight states. The Committee further reported that, although a considerable proportion of the estimated volume had been in contemplation for some time and was in line as to general plans for early construction, it could not be financed through community sources because of dwindling tax revenues and a reluctant bond market. A considerable stimulus in the form of federal aid would be required to bring it within the range of availability.

FEDERAL AID

Federal aid was authorized by the Emergency Relief and Construction Act of 1932, through the Reconstruction Finance Corporation (RFC), in the form of loans on self-liquidating projects. As that definition admitted only projects for water supply and sanitation covered by rates to users, vehicular bridges and tunnels covered by tolls, and public transportation facilities covered by fares, it was not utilized in sufficient measure to assist the situation materially and was terminated on June 26, 1933. The National Industrial Recovery Act (NIRA) of June 16, 1933, creating the Federal Emergency Administration of Public Works (PWA) and authorizing loans on useful projects at 5% interest to the extent of 70% of the cost and a federal grant for 30%, brought more of the basic projects of municipalities into the construction stage.

The continuing increase in unemployment and in the number of persons coming on the relief rolls made it apparent that a more liberal stimulus must be provided to overcome the inertia of municipalities in the planning and presentation of public works. In consequence, the grant was raised to 45%, and the interest rate on loans was reduced to 4%.

In the meantime, as an emergency measure, the operations of the Federal Emergency Relief Administration (FERA), which made grants to states for direct relief and cultivated relief work projects, were supplemented by the Civil Works Administration (CWA), which put men at work on any kind of project that would provide work and succeeded in employing more than 4,000,000 workers in the winter of 1933–1934.

As a measure to furnish more immediate employment for unskilled labor, the Emergency Relief Appropriation Act (ERA) of 1935 authorized the creation of the Works Progress Administration (WPA) to take over the work programs of CWA and FERA and to operate as a general contractor in furnishing relief labor and a part of the material on projects designed by state, county, and municipal sponsors, suitable for force-account administration.

Thus, although the necessity of immediate expansion of public works was voiced in 1930, and became imperative in 1933, it was not until 1935 that it began to operate effectively in the works program. In the meantime, heavy expenditures for direct relief and work expedients started the national income up from a low of 40 billion dollars in 1932 to 42.3 billions in 1933, and 50.1 billions in 1934. Reflecting the influence of advance in public works, it rose to 55.2 billions in 1935, 63.5 billions in 1936, and 71.9 billions in 1937. A reduction in the national income to 64.0 billions in 1938 is generally attributed to a reduction of federal aid in the latter half of 1937.

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[2] President's Emergency Committee for Employment, Government Printing Office, Washington, D. C., 1931.

The federal aid extended to construction in the years from 1933 to 1939 is shown in Table 1. This table includes sponsor's contributions, which were an integral part of PWA and WPA projects, as compiled by the Construction Statistics Section, WPA. The PWA operated four months in 1933, CWA operated two months in 1933, and WPA operated five months in 1935 (see Table 1).

TABLE 1.—EXPENDITURE OF EMERGENCY FUNDS IN CONSTRUCTION, AND EMPLOYMENT CREATED

No.	Description	1933	1934	1935	1936	1937	1938	1939
	(a) EXPENDITURES, IN MILLIONS OF DOLLARS							
1	Total, from emergency funds	400	1,696	1,547	3,203	2,434	2,284	2,905
2	Non-federal projects	301	1,187	925	1,870	1,455	1,559	1,803
3	Federal projects	40	363	304	450	243	184	208
	Sponsor's Funds:							
4	Works Progress Administration[a]	....	....	17	227	334	343	336
	Public Works Administration—							
5	National Industrial Recovery Act[b]	9	36	126	347	104	30	494
6	Emergency Relief Administration Act	....	....	20	213	184	166	
7	Loans[b]	50	110	155	96	114	69	198
	(b) EMPLOYMENT, IN THOUSANDS OF PERSONS							
8	Non-federal employment	285	1,484	1,241	1,853	1,278	2,209	2,100
9	Federal employment	21	123	110	244	178	149	150
10	Total employment created	306	1,607	1,351	2,097	1,456	2,358	2,250
	(c) COST, IN DOLLARS PER PERSON EMPLOYED							
11	All agencies	1,307	1,055	1,145	1,527	1,672	969	1,291

[a] From "Summary Charts and Statistics," WPA. [b] Does not include railroad nor limited-dividend housing loans.

FINANCIAL LIMITATIONS OF MUNICIPALITIES

The delay in organizing a public works program was not due to any lack of useful projects. Many other considerations intervened between promise and performance, such as (1) the rapidly increasing poverty of municipalities, (2) statutory legislation, and (3) established procedure.

(*1*) *Rapidly Increasing Poverty of Municipalities.*—Cities derive their principal revenue from property taxes. In 1932 the assessed valuation of all real property in the United States was estimated at 163 billion dollars. In 1922, the corresponding value was 125 billion dollars and in 1912, 70 billion dollars (Bureau of the Census, 1932). In 1937 it was estimated by experienced analysts to have dropped to 90 billion dollars. In the business district of a typical large city the drop was 40%. This condition made it impossible for many cities to furnish the share of the cost required by the federal agencies, either from tax resources or by borrowing.

(2) *Statutory Legislation.*—In times of prosperity there is a tendency for communities to over-extend themselves in construction and to incur financial obligations beyond their ability to carry when tax revenues fall off. This tendency is controlled by statutory limitation of use of community credit and tax resources to a definite percentage of the current assessed valuation, generally about 10%. Exemptions from the bond limitations are made in the case of municipally owned public utilities that are made self-supporting by service charges to the users—as water, light, power, and public transportation facilities. Further restrictions on municipal borrowing, quite as effective as direct statutory and charter limitations, are found in state laws limiting investment of funds of savings banks, life insurance companies, trust companies, and sinking funds to bonds meeting the test of rigid financial standards. In many states municipal bond issues are automatically prohibited when tax delinquencies exceed a definite ratio of the current tax levy.

(3) *Established Procedure.*—Municipal improvement bonds must be submitted to the people at a general election and must be approved by at least a majority, and usually 60% of the total vote on the question. Even in times of prosperity, projects of recognized necessity may have to be submitted several times before approval is secured. In recent elections, in cities having more than 50,000 population, statistics secured by the Department of Commerce show that 65% of the bond issues were disapproved.

From the experience gained in this period of experimentation, it may be fairly stated that the theory that public works may be depended upon in times of depression to take up the slack in industrial employment is tenable only in part.

Effect on Construction Industry

The quick action necessary to make the theory effective practically is not possible of attainment under the foregoing conditions. Comparatively few of the projects presented to PWA in 1933 could be brought into line with respect to legal and financial requirements in time for approval under the authorization of funds for that program. Defects in bond issues required new legislative action in many states to make the bonds acceptable in the bond market. In this connection, the powers, limitations, and scope of action of the federal agencies conducting works programs of non-federal sponsorship should be understood.

Differences in Authority of Relief Agencies

Although both PWA and WPA are designed to stimulate the production of jobs for the unemployed, there is a considerable difference in their powers and limitations and, in consequence, in their scope of action.

The PWA may grant 45% of the cost of a public works project only when the sponsor has first furnished the remainder of the cost in the form of sequestered funds, bank credits, or definite commitment to purchase the securities to be issued; but it may further assist the sponsor by direct purchase of the securities issued for his share which are stipulated to bear 4% interest with provision for amortization in thirty years or less. The work must be performed under contracts acceptable to PWA and under its engineering supervision, and must employ not less than 25% of relief labor. The direct authority of PWA thus extends down to the job.

The operations of WPA are on a different basis. It has no authority to make grants of money. It may meet the entire payroll on public works projects designed by the sponsor on which at least 95% of the federal funds expended for labor are applied to payment of labor certified from the relief rolls, and may also contribute toward the rental of specified equipment and purchase of materials in definite proportion to the volume of employment created. The sponsor designs and supervises the work and accepts full responsibility for completion in the event that the relief labor finds other work or WPA comes to the end of its appropriation. The WPA executes the work through its state administrators who have sole authority to select for operation, from a general list of projects approved by the President of the United States, those projects that are most suited to the current needs of the local relief program.

TABLE 2.—CONSTRUCTION

No.	Description	PRE-DEPRESSION				
		1925	1926	1927	1928	1929
	(a) CONSTRUCTION EXPENDITURES,					
1	Total private construction	7,978	8,260	8,523	8,576	7,751
2	Total public construction	2,181	2,137	2,373	2,484	2,415
3	Grand total construction	10,159	10,397	10,896	11,060	10,166
	Comparison Between Normal and Depression Periods:					
4	All construction	Average, 10,536				
5	Private construction	Average, 8,218				
6	Public construction	Average, 2,318				
	(b) AVERAGE EMPLOYMENT, IN					
7	In construction Annual	2,125	2,197	2,352	2,396	2,265
8	For the period	Average, 2,267				
9	In private construction Annual	1,689	1,762	1,859	1,862	1,736
10	For the period	Average, 1,782				
11	In public construction Annual	436	435	493	534	529
12	For the period	Average, 485				
	(c) NATIONAL INCOME,					
13	Totals[a]	72.7	74.9	73.7	77.6	81.1

[a] From "Construction Expenditures and Employment 1925–1936 and 1937 Compared with 1936," by 1939, in "National Income Produced"; U. S. Department of Commerce. National income for six years prior to for 1924, 67.9.

These differences in limitations and restrictions have resulted in a division of the public works assisted by these agencies along lines more or less determined by the quantity of material required by the project. Public buildings, schools, hospitals, water purification plants, sewage treatment plants, power plants, bridges, grade separations, and subways, which require a large proportion of manufactured material and skilled labor, averaging approximately 40% labor and 60% material, constitute the majority of PWA projects.

Because one of the determining rules of eligibility for WPA expenditures is that federal funds should be conserved for wages in the greatest degree consistent with proper execution of the job, the work undertaken runs largely to construction of roads, streets, dams, water mains, storm and sanitary sewers, airports, armories, landscaping of school grounds and parks, recreational facilities, drainage, and mosquito elimination, in which the labor-material proportion averages approximately 70% and 30%, respectively.

The works developed, however, were highly effective in retarding the disintegration of the construction industry, which was very seriously affected by the great drop in private construction. The highlights of the construction situation from 1925 to 1939 and the effect of construction employment on the national income are shown in Table 2.[3]

EXPENDITURES AND EMPLOYMENT

Depression					Post-Depression				
1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
in Millions of Dollars									
5,379	3,422	1,411	1,175	1,363	1,982	3,005	3,589	3,268	3,687
2,726	2,512	1,878	1,827	2,619	2,579	3,261	3,034	3,373	4,091
8,105	5,934	3,289	3,002	3,982	4,561	6,266	6,623	6,641	7,778
Average, 4,862; average deficiency, 5,674; and total deficiency, 28,370.					Average, 6,374; average deficiency, 4,162; and total deficiency, 20,810.				
Average, 2,550; average deficiency, 5,668; and total deficiency, 28,340.					Average, 3,106; average deficiency, 5,112; and total deficiency, 25,560.				
Average, 2,312; average deficiency, 6; and total deficiency, 30.					Average, 3,268; average excess, 950; and total excess, 4,750.				
Thousands of Persons									
1,968	1,593	1,001	1,107	2,185	2,149	3,174	2,719	3,493	3,266
Average, 2,265; average deficiency, 2; and total deficiency, 20									
1,334	938	420	348	366	533	814	876	802	905
Average, 734									
634	655	581	759	1,819	1,616	2,360	1,843	2,691	2,361
Average, 1,531									
in Billions of Dollars									
68.3	53.8	40.0	42.3	50.1	55.2	63.5	71.9	64.0	68.5

Peter A. Stone, Construction Statistics Section, Works Progress Administration. Corrected to December 31, 1925 was reported as follows: For 1919, 67.3; for 1920, 68.1; for 1921, 50.7; for 1922, 58.6; for 1923, 68.0; and

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[3] "Construction Expenditures and Employment 1925–1936 and 1937 Compared with 1936," by Peter A. Stone, Construction Statistics Section, WPA. Corrected to December 31, 1939, in "National Income Produced," U. S. Dept. of Commerce.

It may be observed from Table 2 that public works construction constitutes only about 20% of the total volume of construction in normal times (1925–1929), the remaining 80% being made up by private construction. In the depression period from 1930 to 1939, the total volume of construction fell to 53% of normal, but of this reduced volume 50% (2.3 times its normal proportion) was furnished by public construction. Taking the average expenditure in the 5-yr period from 1925–1929 as a base, the cumulative deficiency in private construction since 1929 is more than 53 billion dollars, whereas the cumulative excess in public construction is more than 4 billion dollars. Employment from private construction averaged only 41% of its former value, whereas public construction employment averaged 316% of normal.

It will be noted in Table 2 that in the 5-yr period, 1930 to 1934, inclusive, there was a continuous drop in both public and private construction to a low in 1933; but at the end of 1933, public construction was still 80% of its normal volume, whereas private construction was less than 15%. The rise in volume thereafter was principally due to public construction which arrived, in 1939, to 176% of normal, whereas private construction is still only 45% of normal.

Possibilities of Public Works

It appears, therefore, that, although the expansion in public works fell considerably short of expectations, in point of immediate availability, the results obtained in the maintenance of the construction industry fully demonstrate the high value of public works in the national economy.

It appears quite evident, also, that, in similar circumstances in the future, the immediate effect sought cannot be achieved unless there is present an ample reservoir of projects that have gone through the time-consuming preliminaries of engineering examination and legal and financial authorization, and are ready to go into operation as soon as financial arrangements can be perfected. It is not easy to develop such a program in advance of need. The restrictions attending advance planning in municipalities and the development of construction programs are not likely to be materially reduced in the future. Accordingly, dependence must be placed on projects of national scope or local projects in which the national interest warrants federal participation in planning and financing.

There is no paucity of such projects. The nation has a definite interest in the elimination of 232,000 railroad grade crossings, most of which are in cities and are daily adding to the sum of fatalities and injuries from motor accidents. First-stage improvement on 2,000,000 miles of rural roads to make them passable in all weathers would affect, beneficially, the fortunes of 20,000,000 people and would be a sound investment of federal funds. It might be found justifiable, in view of their importance in national defense, to assist the railroads in catching up with their deferred maintenance of roadway and track, estimated by railroad authorities to require an investment of $400,000,000 annually on a 5-yr program.

Construction of wide thoroughfares through slum districts of large cities and the creation or betterment of public transportation facilities would merit federal assistance. The engineering surveys precedent to developing a program for such projects should properly be under federal control and direction through the appropriate works agency. The consolidation, in 1939, of government construction agencies under a single control in the Federal Works Agency, an administrative betterment long urged by engineers, and a similar consolidation of government loaning agencies under the Federal Loan Agency, have made possible a continuous coordination of effort with the National Resources Planning Board in the preparation and financing of a program of federal public works designed to permit the orderly development of the nation's resources.

The machinery is available, therefore, to inventory and classify the public works of the nation and to determine in what degree the national benefit derived from basic non-federal projects justifies their inclusion in a federally controlled and federally aided program of such volume, variety, and territorial coverage as to be really serviceable in taking up the load of industrial unemployment.

Many of the most important civic projects are unable to proceed because funds are not available for the purchase or condemnation of the lands or right of way required, as with rehabilitation of blighted or slum areas, elimination of grade crossings, and widening of important traffic arteries. The powers of the loaning agencies might reasonably be adjusted to meet such emergencies by long-term, low-interest loans, or the federal government might acquire the lands directly and lease them to the communities on a long-term recapture basis.

Another handicap to development of a program of first-grade projects is the limitation of federal aid to an annual basis, leaving a community with no assurance that a project may be prosecuted continuously to completion as a usable asset. Such handicaps can readily be removed by suitable legislation, but while they exist they operate to prevent the development of a public works program of the first order of usefulness.

CONCLUSION

On a foundation of preparedness and with administrative machinery geared for prompt action and control, there is every reason to believe that public works can be brought much closer to expectation as a compensative employment element. To make a program of public works fully effective in assisting to take up the slack in industry, its full power must be thrown in promptly upon the inception of the depression. For that reason, federal works, which are immediately controllable upon congressional authorization, must be prepared to furnish the greater volume of projects.

Under the most favorable conditions, a certain time is required to bring non-federal projects into operation. In times of industrial depression, cities are always in a straitened financial position from loss of revenues and impaired banking credit. To cut down this time, arrangements for federal loans on a long-term, low-interest basis for programmed projects should be perfected in advance of need.

ACKNOWLEDGMENT

Acknowledgment is made to Peter A. Stone, chief of Construction Analysis Unit, WPA, for valued assistance in compilation of the construction and employment statistics used in this analysis.

DISCUSSION

Clarence W. Post,[4] M. Am. Soc. C. E. (by letter).—In this paper, Major Hallihan places sewage treatment plants and water purification plants in the category of projects that naturally fall into a class handled by the PWA. In New York State this has not been true; some of the largest and best projects that the WPA has constructed in the state have been for water treatment, sewer systems, and treatment plants. These range from a small complete sewer system and treatment plant at the Village of Ft. Ann (population in 1940, 437) to the interceptors and treatment plant for the Ley Creek Sanitary District, Syracuse, which cost more than $3,000,000, and which was designed for a population of 30,000 and a daily average flow of 4½ million gal.

Under the WPA program, thirty-four treatment plants have been constructed and twenty-seven plants enlarged and improved. Projects for interception and treatment plants have also been started in many more communities. In addition to these plants, 1,900 miles of sanitary and storm sewers have been installed. Two storm relief tunnels driven through rock have been built in the City of Rochester, and a third submitted for approval.

The plans and specifications for this type of work must be prepared by either regular civil service employees of the engineering bureaus of the various cities, or by consulting engineers. All plans and specifications must be approved by the Division of Sanitation, New York State Health Department, before any work may be performed.

When a project is placed in operation the selection of a superintendent to have charge of the operation is of utmost importance. Applicants' personnel records and construction background are examined and the final determination and approval for employment comes from the director of operations or the chief engineer of the New York State Work Projects Administration.

One other factor helps in this type of work in New York State: Each community signs an agreement to furnish the necessary skilled labor if such skills are not available from the relief rolls. Small buildings have been, and are being, built successfully by the cooperation received from the sponsors who, in many instances, have furnished the bricklayers and any other skilled artisans who are lacking from the relief rolls. In every instance, however, the federal expenditures on these projects are only about 30% to 40% of the total cost.

The plants can be built with relief labor when care is exercised over the selection of key personnel to supervise the work.

Uel Stephens,[5] M. Am. Soc. C. E. (by letter).—The subject which Major Hallihan has so ably presented is most appropriate at this time when the entire attention of the United States is being centered on its defense requirements. Now is the proper time for the engineering profession to formulate plans for "carrying on" after the flush of defense construction subsides, and buying begins to decline, and finally reduces to a minimum.

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[4] Deputy State Administrator & Chf. Engr., New York State WPA, Albany, N. Y.

[5] Asst. Regional Director, Dept. of Public Works, Defense Public Works Div., Federal Works Agency. Fort Worth, Tex.

It is a well-established fact that every period of unprecedented construction and spending activity must be followed by a period of below-normal activity that usually will exceed both in duration and intensity the preceding period of above-normal activity. It appears that the amplitude or distance between the crest of the wave of prosperity and the bottom of the valley of the depression increases in direct proportion to the degree of industrialization. The capability of industry and small political subdivisions to combat the periods of depression, and to keep within reasonable bounds the extent of expansions during periods of abnormal development, appears to become less and less, and the tendency to look to the federal government for the solution increases proportionately. These facts being accepted, it then behooves the federal government through its proper agencies to begin at once, and rapidly develop, a vast plan that will contain a large reservoir of worth-while public projects that can be placed under construction rapidly and economically in accordance with adequate engineering designs.

The engineers who experienced the many phases of public building from the Emergency Relief and Construction Act of 1932 to the defense construction programs of 1940 and 1941 should realize, better than members of any other profession, the dire needs for adequate and proper planning before the depression comes. Their experience in rushing into great construction programs in 1933 and 1934 clearly demonstrates the waste and inefficiency that cannot be avoided unless some one central agency has planned and can be given the necessary authority to execute some well-developed and properly coordinated plan for construction projects of all types. Probably one of the most important duties of such a federal planning agency would be the coordination of the activities of various agencies. Competition between governmental agencies, such as existed between the PWA and the WPA and now (1941) is occurring between the WPA and the PRA (Public Roads Administration), should be made impossible.

If the federal government, through its National Resources Planning Board, could provide funds in the form of grants to municipalities to pay the engineering cost, in part or in whole, for the preparation of preliminary plans and specifications covering such projects as could be undertaken for construction, there would be an incentive for private engineers to plan a program that could be constructed with little delay when the necessity arose. Such an organization must be local to the extent that the study of prevailing local conditions can be made. Such work could not be done successfully by a central organization in Washington, D. C., for the reason that local conditions could not be comprehended readily. By decentralizing and placing complete responsibility in the hands of state organizations, the local engineers would be more prone to plan and study projects if they thought that final consideration could be given locally rather than being reserved by a central office—some located great distances away.

In conclusion, Major Hallihan is to be complimented for the very able paper he has presented. Based on experience and on the records that have been kept, it is evident that the present is the proper time to begin planning for the next depression, which is sure to follow in the wake of the present unprecedented construction program.

WILLIAM J. WILGUS,[6] HON. M. AM. SOC. C. E. (by letter).—In these days of peril it is noteworthy that Major Hallihan's thoughtful paper should have appeared simultaneously with the report[7] of the National Resources Planning Board in favor of the "storing up [of] a reservoir of non-defense public works which can be loosed when the pace of rearmament slackens." There would seem to be no room for argument that the United States now should prepare for the defense of its institutions from attacks from within as well as from without. It is not difficult to visualize what will occur if countless of its citizens are to be permitted again to suffer from unemployment when the war boom ends. In a word, the two kinds of preparedness must go hand in hand if wisdom is to prevail, even though many may feel that preparedness for internal peace should "take a back seat" while the nation is engaged in preparedness for external war. To win in one and not in the other will spell ruin.

It is widely urged that the furtherance of projects "which are either economically non-productive or of a sort which displaces private enterprise in its own legitimate field" will mark "a plunge headlong into state socialism and totalitarian government."[8] There is indeed risk of this in a world afire; but if large-scale employment ceases, and with it the buying power of multitudes thrown out of work, what will enable free enterprise to take up the slack any more than in 1933? Idle money means idle men—the death not only of free enterprise but of all else that Americans hold dear.

No one knows what new order of society will come forth from the existing world chaos. Whatever it may be, men and women must be given opportunities to earn their daily bread in the production of things worth while, whether economically productive or desirable in a social sense. Now is the time, as proposed by Major Hallihan, to prepare plans in advance, rather than, by neglect in this respect, to invite a return of the soul-crushing, leaf-raking days of 1933.

The field for advance planning is well-nigh limitless. Many projects at once come to mind, such as water supply and sanitation; the control, purification and shore protection of streams; conservation of natural resources; light, heat, and power; housing and hospitals; parks and parkways and the beautification of the roadside; and transportation by rail, highway (including back roads), water, air, and pipe line. What a land this would be could a fraction of the money expended for destruction be devoted to construction of such a high order. Idealistic? Yes; but were people ever more in need of ideals to buoy their hopes for a better day? With it, however, must go the practical to save civilization when the present false prosperity ends.

It is not alone for the future postwar era that the planning of public works is needed. Right now much of it is required for wartime defense, as stated by

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[6] Weathersfield, Ascutney P. O., Vt.

[7] "Development of Resources and Stabilization of Employment in the United States," National Resources Planning Board, January, 1941.

[8] *The Wall Street Journal*, March 20, 1941.

Major Hallihan. Perhaps this is most true with the railroads. Deferred maintenance in that field gives ground for alarm; and, too, the lowest inventory ratio to operating revenue in thirty years, barring three, reported in the *Railway Age* of March 29, 1941, which in case of invasion or a similar emergency might spell disaster to the Nation.

Obsolescence of railroad equipment is marked, the Federal Reserve System's bulletin of October, 1940, reporting that in the period between 1930 and 1940 the percentage of locomotives less than ten years of age had dropped from 20 to 6, and of freight cars from 34 to 14. This condition is emphasized in the *Railway Age* of January 4, 1941, which, in prophesying car shortages, calls attention to the locomotive situation in which 68.5% in service in 1940 were twenty-one or more years old against 57% five years ago. Planning coupled with action in this field is needed immediately, not only for normal peacetime traffic, plus an evenly distributed comparatively small increase for defense traffic, but also for the many exigencies of war in case of home invasion or military operations beyond the borders in Canada or Mexico, or in more distant lands.

In their fixed properties the railroads also call for immediate planning and corresponding action in order that they may not obstruct the progress to be expected in equipment, service, and operating efficiency. The strengthening and refinement of many thousands of miles of track, including bridges, the repair and modernization of yards, locomotive terminals, shops and shop tools, fuel and water facilities, work equipment, signals and interlocking, and freight and passenger stations; numerous other improvements such as by-passes around centers of congestion and the separation of grades; and the making good of deferred maintenance—all these call for several years of expenditures amounting to something like a billion dollars annually, in which is included provision for equipment. The aforementioned issue of the *Railway Age* goes into this question in great detail, as does its issue of January 7, 1939.[9]

The urgency of the need for doing something about the railroad situation is made startlingly apparent in the *Railway Age*, which states in its issue of January 11, 1941, that "to be frank, the financial results of the operation of the railways lend some color to the the charge of 'decadence,'" and that "the Nation cannot be restored to economic health and military invincibility if it continues to nurse along a transportation cancer within its economic body." That "military invincibility" in this respect is not to be expected from the railroads, unaided by federal intervention, is indicated by these quotations from an authoritative railroad source:

> "War participation * * * is outside of a consideration of the railroads' ability to carry on under conditions properly incident to that of National defense."
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> "It would be unreasonable to anticipate the financing of equipment for such purposes from funds collected for the transportation of passengers and goods in a competitive transportation endeavor."

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[9] See also "Financial Requirements of Railways," Interstate Commerce Commission *Statement No. 3911*, March, 1939; and "Report of Committee Appointed September 20, 1938, by the President of the United States to Submit Recommendations Upon the General Transportation Situation," December 23, 1938.

> "Enemy invasion of the United States * * * does not appear likely in the near future."
>
> "Plans on the part of the Government for activities under National Defense * * * have not heretofore been revealed to railroad managers."

In the light of these circumstances the value of public works on which Major Hallihan dwells would appear to be clearly evident; likewise, the wisdom (as he states under the heading "Possibilities of Public Works") of having "present an ample reservoir of projects that have gone through the time-consuming preliminaries of engineering examination and legal and financial authorization," coupled with prompt action in such instances as that of the railroads.

Bernard L. Weiner,[10] **M. Am. Soc. C. E.** (by letter).—Considered merely as a tabulation of statistics, the author's data may probably be accepted as substantially correct—even though definitions of such terms as "national income" are not given. It is obvious that, unless care is taken, duplication may easily occur. For the purpose of this discussion, however, it is not necessary to go into refinements. It will be sufficient for the purpose even if the data are accepted as being merely indexes of relative values. In any case, it seems to the writer that the conclusions drawn from them are not justified. It is only necessary to plot the statistics to see that the conclusions as to cause and effect are unwarranted. Fig. 1 shows that, in relation to national income, the variation in public construction is so slight that it is unreasonable to suppose that the latter is the cause of the variation in the former. The author states that the 1938 drop in income is generally attributed to the reduction of federal aid in the latter half of the preceding year. The chart shows that, while there is a slight wiggle in the public construction curve, the total construction curve remains practically level. The ratio curves show further that, just prior to 1932, construction was decreasing faster than income and then increased slower than income. One is led to believe that construction and national income are both affected by factors other than those treated in this paper—factors that are of a magnitude comparable to the results they are supposed to control.

The author shows that in the ten-year period, 1929 to 1939, there was a loss of 53 billion dollars in private construction, whereas public works showed a gain of a mere four billions. During the same period, the loss in national income based on the 1929 income was about 250 billion. The four-billion dollar increase in public construction can hardly be credited with having more than a very minor effect on these large sums; the increase shrinks into insignificance by comparison. It might be argued that the 1929 income is not a just base for comparison since it is a maximum; but the fact remains, nevertheless, that the same industrial and agricultural capacity, as well as the manpower that was available and capable of producing the maximum amount in one year, had the same potential physical ability to produce a like amount in any other year. Whatever it was that prevented the national economy from functioning at the same level—or at even a higher level, as might reasonably be expected in a dynamic economy—was also responsible for the lag in construction. The "drop in the bucket" that constitutes public construction cannot, it seems to the writer, be the cause of the rise in the national income after 1933. The true controlling causes or factors, however, must be found; and it is perhaps no exaggeration to state that it constitutes the most important problem confronting the United States and the world today.

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[10] Designer, Air Depot Archt. Engrs., New York, N. Y.

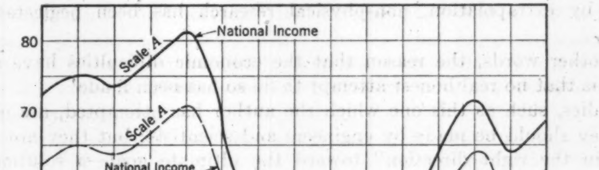

FIG. 1

To declare that the most important problems that confront the nation are immediate recovery and long-range planning for the stabilization of industry is to state the obvious. The fact that the nation is now engaged in an undeclared war—or is in state of non-belligerent intervention, if this characterization is preferred—does not change matters. It is the opinion of the writer that the so-called domestic problem cannot be solved without taking the international situation into account and the two problems are so inextricably intertwined that they are really one and the same problem. Many panaceas have been offered for the solution of both problems treated separately, no two being alike. The conclusions drawn by the author are merely the echos of a political party pointing with pride to its supposed achievements—and at best, the paper is merely another panacea. The fact remains that the world is confronted with a failure of more than major importance—civilization is threatened. All methods from the appeal to the brotherhood of man to self-interest have been tried—and all have failed. In the domestic field, industry has been spending vast sums of money on physical research with the result that, as the products of industry approach perfection, the economic crisis becomes more and more acute. At the same time, wars are becoming ever more frequent and more destructive. On the other hand, except for such superficial studies as the charting of market trends in an effort to forecast the future by extrapolation, non-physical research has been neglected almost entirely.

In other words, the reason that the economic difficulties have not been solved is that no real honest attempt to do so has been made!

Studies, such as this one which the author has attempted, are necessary, and they should be made by engineers and scientists; but they are merely a "step in the right direction" toward the ultimate goal—a solution to the problem of war and poverty. That a solution is possible should be obvious. There is no reason to suppose that this problem with which the world is faced is somehow so peculiar that it will not yield to the same scientific treatment that has solved so many other problems. The same mind of man that is capable of producing the truly great marvels of modern civilization should also be capable of producing the solution for this non-physical problem. Engineers and scientists have been sadly remiss in their duties as citizens in this respect. They are both peculiarly fitted by the very nature of their training to tackle the economic problem.

A little consideration should show that the engineer in the rôle of economist is not at all a strange or far-fetched idea; in fact, it is very logical that he should be peculiarly fitted for it. What else is the work of the engineer—in the broad sense—but bringing order out of chaos? How else but by research to determine the facts, and by analysis to determine the solution from the facts, does the engineer work? Most important of all, the engineer has another qualification. It has been remarked often that the engineer acts for both his client, the owner, and for the contractor, and the disinterestedness of his judgment is never questioned. In other words, the engineer has the recognized distinction of being free from bias in forming his judgments. The very fact that those who have attempted to solve the economic problem have been led astray by their biased, preconceived ideas is one of the principal reasons why failure has always resulted. Those who succeed finally in finding a solution must be able to follow their studies through with scientific rigor, no matter where they lead—a specification that fits the engineer more than it does any other group. Finally, the same methods that made possible the physical basis of modern civilization should and could, logically, be applied to achieve the primary purpose of all human endeavor: To make the necessities and even the luxuries of life available to all mankind.

It might really be said that it is not so much a question of the engineer's becoming an economist, as it is one of applying engineering methods and procedure to economics.

Albert Ed. Scheible,[11] Assoc. M. Am. Soc. C. E. (by letter).—The subject is both interesting and timely. It is interesting because there has not been much published on the subject in the form in which it is presented in this paper; and it is timely because many persons are already beginning to think about the future—what will happen after the present conflagration has burned itself out. One evening the writer listened to an address by a noted sociologist on the subject, "After the War—What Then?" Engineers of every branch would do well to study this question. If the engineer does not do this, members of some other profession probably will. That is what happened during the past depression when many avoidable mistakes were made. Those who had instigated the program failed to realize that such an undertaking could be handled best by engineers.

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[11] Prin. Asst. Engr., R. F. MacDowell, Cleveland, Ohio.

It is the writer's opinion that here is a chance for professional and technical societies to take an active part in that "human engineering" which one hears so much about. Technical societies, working with the local professional groups, could do much to promote thought, and in some cases, could initiate a program of planning for the future.

As the author states, public works is not the entire answer to unemployment, but it can, and should, take the strain from other methods that will have to be adopted when a similar situation has to be met in the future. Based, then, on the experience gained in the recent past, it should be possible to make many improvements. To that end the organized engineering profession should bend every effort.

Having been directly connected with non-federal projects, both as a government employee and later as an employee of a consulting engineer, the writer would like to present some views regarding the subject.

The hectic days at the beginning of the program in 1933 may be discounted. The present WPA program seems to have reached a point of more or less stability and, of course, PWA has ceased to function, at least in the matter of accepting new projects.

Those persons entirely unfit for manual labor, including the aged, should not be employed on construction projects. Some attempt has been made to employ only men who are physically fit. For some reason, the government rules applying to physical fitness have not been enforced as stringently as have some of the other rules, many of which are not nearly so important. More recently, in an apparent attempt to keep the WPA alive, this regulation seems to have been "winked at."

A planning program is of the utmost importance. Although the federal government "foots the bill," or most of it for labor at least, it should not expect financially harried municipal officials to "jump through the hoop" every time the unemployment curve takes a sharp rise. Most municipalities are always understaffed, particularly with respect to engineering personnel. Indeed, many small municipalities have no engineering departments, which may be one of the reasons for such poorly planned projects. The time to plan projects is now. As emphasized by the author, municipalities are becoming poorer for obvious reasons. They have no money with which to plan projects that may not be executed in the near future. The federal government should make funds available now, by grants to municipalities, for this work.

Under the present program (referring particularly to the WPA), it seems that all kinds of construction work, with the possible exception of paving, must be done in the winter time. An effort to find more protected work in the winter time would be very helpful. A program planned far enough in advance might aid in this respect, and perhaps a frank admission that most construction work, now being done or attempted, is out of place in the winter time. However, this reverts to the need for some other type of program during the winter months when the unemployment curve seems to rise. The writer believes that this frank admission should be made and that a new approach to the problem should be sought. The present program seems to have been started and then left hanging.

The contractor, it appears, is the forgotten man in the entire program. He did have some work under PWA but the program was so short that the average small contractor was lucky to secure one or two jobs, and at a ridiculously low sum. Every WPA form requires that the sponsor make a statement to the effect that the work proposed is not a normal function of the sponsoring body, and yet the writer has before him the report of a department of a certain state giving the amount of sewer and water work done with WPA aid during 1940, work that heretofore was done mostly by contract. If this condition continues, the contractor will soon take the place of the American Indian as the Vanishing American.

Due to rules and regulations, local offices must take a very arbitrary stand on all matters of policy. In the business world there is a saying, "the customer is always right." In the case of the WPA it is very often difficult to tell which is the customer. The local offices should have a certain degree of authority and there should be a certain degree of elasticity in the actual operation of a project. The writer hazards a guess that never has a construction project been executed in which everything was completed to the smallest detail as planned; and yet this is what is expected under the present system.

The need for the program is obvious. The administration is to be commended for its courage in initiating it. There is still much room for improvement. To that end every effort should be made by those on whom so much of the future will depend—the engineers.

H. B. COOLEY,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a]—Many of the difficulties that faced the execution of the public works program in the depression period are stated clearly by Major Hallihan. The writer feels that it is not too early to begin to lay the ground work and prepare for the inevitable adjustment period that is bound to occur with the cessation of hostilities and the armament program—whenever that may occur. Major Hallihan mentions some of the many difficulties that should be overcome and outlines remedial steps; but will the next occasion find the United States as unprepared as did the last period?

It should be remembered that the enlargement of public works as a method of alleviation of cyclical depression is merely one theory for the accomplishment of this purpose and is not "The" theory, as was stated in the "Synopsis" of the paper. Consequently, the advantages and disadvantages of the public works theory must be weighed against the advantages and disadvantages of various other theories. That there is any single remedy for our economic ills is doubtful and the increased expenditure for public works in times of decreased spending by private enterprise is undoubtedly an aid, but this increased spending should be accompanied by other measures that will supplement the program rather than work against it.

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[12] Asst. Prof. of Economics and Business Administration, West Virginia Univ., Morgantown, W. Va.

It seems to the writer that there are a number of reasons which may be given in justification of an expanded public works program in times of depression. By such a program is meant one that will take up the slack in normal construction—which norm would in itself be a varying term and would probably refer to the average volume of construction existing in a period of years immediately before the particular depression. Thus a program to relieve conditions in the depression of the 1930's would refer to the period of the late 1920's as normal, whereas a program in the 1940's would refer to the level established just before that period.

One justification for such a program might be the maintenance of the existing construction industry—the equipment, financial resources, workers of all types, the suppliers of materials and equipment—in a condition as similar as possible to that existing before the depression. Such an objective would presuppose the use of existing methods of letting contracts and the use of existing organizations, contractors, equipment manufacturers and distributors, and material producers and dealers, wherever possible. This would assume costs on a comparable basis with the pre-depression costs, with, roughly, the same volume of business, the only differences being a shift from private construction to public construction and a change in emphasis from types of construction generally undertaken by private industry to types which could be undertaken by a public agency. In such a program, the flow of money through the various channels would be largely the same as the flow in the so-called "normal" period before the depression.

Another justification may be found by considering the increased public works as an offset to a decrease in private industrial activity. This would include not only normal construction activity but also the decline in manufacturing and distributing functions. If such a purpose is to be achieved the construction industry not only must be kept at its pre-depression level but it must be expanded to provide employment for those released in activities outside the construction industry and its related fields. This purpose may be accomplished by increasing the dollar volume of construction at the same unit costs, by maintaining the same dollar volume at decreased unit costs, or by altering the methods of construction so that a larger number of men may be employed on the same physical volume of work.

A third justification may be to find employment in the construction industry for all unemployed laborers who are capable of working and hence are employable. It seems to the writer that such a proposal is not feasible as many of the unemployed are not suited to construction activities. This applies to women, to the elderly worker, and to the majority of the so-called "white collar" workers, among others, although places can be found for some of these on construction projects. To attempt to place labor, which is not suited to such work, at work in a construction program must result in the creation of made work and "boon-doggling" projects, the value of which may well be questioned.

The values given in Table 2 indicate that the enlarged public works program failed to take up all the slack in the previous program when only the dollar expenditures are considered. Although public construction increased from an average expenditure in the pre-depression period of $2,318,000,000 to an average amount in what Major Hallihan terms the "post-depression" period of $3,268,000,000, there was actually a decrease in total expenditures from an average of $10,536,000,000 in the pre-depression period to $6,374,000,000 in the post-depression period, and from a high annual amount before the depression of $11,060,000,000 to a high annual amount after 1934 of $7,778,000,000. If the values are at all comparable it is apparent that the enlarged program of public works failed to meet the needs of the construction industry as it existed in pre-depression years and certainly could not have contributed toward the elimination of the slack in other industries not related to the construction industry. If costs were materially reduced per unit of physical work produced, due to a general decline in labor and material prices, then it is possible that the same physical volume was maintained even though the dollar expenditure was less. Such a decrease in costs, however, would have to be about 40% less than the pre-depression costs, based upon the total values.

Table 2 does show, however, that the average employment increased, as a result of the program, from 2,396,000 persons in 1928 to 3,493,000 persons in 1938, with a low of 1,001,000 in 1932. Was this increase of approximately 1,000,000 persons between 1928 and 1938 due to a change in methods of executing construction work or are the numbers comparable in any way? Due to the smaller number of hours worked per man each month, it is natural to suppose that the increase in the average number of men employed is due to this factor as much as to the increase in public expenditures from $2,484,000,000 in 1928 to $3,373,000,000 in 1938, especially when the decline in private construction from $8,576,000,000 in 1928 to $3,268,000,000 in 1938 is considered.

To judge the works program only on the basis of dollars expended or the number of men employed would seem to be fallacious. To what extent did it replace the existing industry of the 1920's; or did the shift from projects with a higher percentage of payment for material to the later projects with a higher percentage for labor tend to create other conditions which decreased the benefits flowing from the expenditure of the individual dollar in the first instance? In the former case the money was more widely circulated, and hence benefited more industries and more workmen, than was the result when a large part of the construction dollar was spent for construction labor and was immediately expended for consumption goods.

What was the ultimate value to society of the money spent on construction projects? Did the results obtained compare favorably or unfavorably with the results achieved in the preceding period in the 1920's? It seems to the writer that the cost of the works program should be evaluated from two viewpoints. One is the social view; and consideration must be given to the fact that large numbers of people were given employment who would otherwise have been idle.

The other consideration is the cost of the construction under such a program when compared to the cost of the same improvements by using, what might be termed for want of a better name, more orthodox methods. Men may be kept employed by having them spend their time on useless projects, "boondoggling," or they may be selected and employed so as to achieve the maximum efficiency for their labor without regard to the number employed. Also, an intermediate course may be pursued that will afford employment to larger numbers than in the second case but at the sacrifice of efficiency. The latter course was the one pursued and may have been the best course but it would seem desirable to know just what was achieved by such a program, both from the human viewpoint and also from a construction efficiency viewpoint, and just how much was sacrificed on the one hand in order to achieve the benefits on the other. Until this information is obtained, publicized, and discussed, it would seem that any plans based upon past experience rest upon a weak foundation. The two objectives are confused and merged into one another so that it is difficult to determine just what benefits have been derived from the program in the past.

To illustrate this confusion reference may be made again to the expenditure and employment values of Table 2. It was noted that the total employment increased by a yearly average of about 1,000,000 persons between 1928 and 1938, whereas the total expenditures for construction decreased $4,419,000,000 per year in the same period. Assuming that construction costs decreased during this period, that living costs also decreased and that, due to a change in types of projects and methods of executing the work, there was a larger percentage of the construction dollar paid to labor, what does this indicate? Was the individual workman benefited in proportion to the amount expended or were his annual earnings reduced? It may be that his standard of living was materially lowered because of a reduction in his actual annual wage. On the other hand, the steadiness of his employment throughout the year, even though for fewer days in the month, may have increased the average annual wages per worker. Which is correct and how does this benefit to the individual (after all there was a benefit to some when total unemployment was the alternative) compare with the cost to society for this social benefit to the human being and the physical benefit as measured by the actual value (not the cost) of the physical structures?

The situation may well be likened to that of a manufacturer producing a number of different products. His over-all operations result in a profit and for this reason he may be lulled into a feeling that all is well with the business. More detailed information, however, might reveal the fact that certain operations are unprofitable and that the entire enterprise would be benefited by discontinuing certain activities. Such an analysis might well be undertaken with relation to the public works program.

PHILIP W. HENRY,[13] M. AM. SOC. C. E. (by letter).—Congratulations are due the author for presenting so clearly and concisely the expenditures made by the federal government during the recent depression in order to aid state, county, and municipal construction projects and to afford employment for those who were out of work. The details given in Tables 1 and 2 present a clear picture of the situation; and the inclusion in Table 2 of the national income for the years 1925 to 1939, inclusive, is pertinent. A statement of the yearly direct federal debt for the same period would also have been appropriate. Therefore, the writer has taken the liberty of including it in Table 3 for the purpose of comparison with the cost of public construction and national income, which are taken from Table 2.

From this statement it appears that in the depression period, when presumably governmental help was most needed, there was actually a slight decrease in the yearly average of public construction, notwithstanding an increase of 1.772 billion dollars (9.5%) in the federal debt, and a decrease of 25.1 billions (33.0%) in the national income.

In the post-depression period there was an increase of 0.950 billion dollars (40.9%) in the yearly average of public construction compared with the pre-depression period, an increase of 16.661 billions (89.4%) in the federal debt, and a decrease of 11.4 billions (15.0%) in the national income. Evidently the federal government was slow in coming to the rescue, and its greatest expenditures, shown in public construction and increased debt, were in the post-depression period instead of during the depression itself. The author, noting this, states (see heading "Conclusion"): "To make a program of public works fully effective in assisting to take up the slack in industry, its full power must be thrown in promptly upon the inception of the depression."

TABLE 3.—COMPARISON OF NATIONAL FINANCES, 1925 TO 1939

Year	YEARLY AVERAGE, IN BILLIONS OF DOLLARS					
	Public construction	Increase %	Federal debt [a]	Increase %	National income	Decrease %
Pre-depression, 1925–1929	2.318	0.0	18.641	0.0	76.0	0.0
Depression, 1930–1934	2.312	0.0	20.413	9.5	50.9	33.0
Post-depression, 1935–1939	3.268	40.9	35.302	89.4	64.6	15.0

[a] For fiscal years ending June 30; values are from the *Monthly Bulletin* of the National City Bank of New York for February, 1941.

Although no doubt most, if not all, of this public construction was well planned, well executed, of permanent value, and desired by the community, was it advisable for the federal government to assist state, county, or municipal projects by donating part of their cost? The local people know whether such projects—no matter how desirable—are really needed. If so, they should pay for them in full. The federal government could properly purchase local securities issued for construction (if sound) during the period of a reluctant bond market. Later on there would be no difficulty in selling them to the public, as has been demonstrated by the sales made in recent years by the Reconstruction Finance Corporation of such securities.

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[13] Cons. Engr., New York, N. Y.

As far as public construction is concerned, the writer is not in accord with the author when he says that the "* * * heavy expenditures for direct relief and work expedients started the national income up from a low of 40 billion dollars in 1932 to 42.3 billions in 1933, and 50.1 billions in 1934." Table 2 shows a decrease in public construction from 1.878 billions in 1932 to 1.827 billions in 1933 and an increase to only 2.619 billions in 1934. More likely the increase in the national income was due to the recovery of the people from their state of panic that started in 1929. In the opinion of the writer the recent depression was similar to the major depressions of 1837, 1857, 1873, and 1893, all of which showed distinct recovery in six years or less with little help from the government.

Just what the federal government should do during major depressions is a moot question. In the depression of 1893 which, in the memory of the writer was just as serious as that of 1930–1934, the President of the United States took the position that it was not the function of the government to support the people, but that it was the function of the people to support the government. At any rate, the depression of 1893 was over in four years with an increase of only $262,336,000 in the federal debt (for the purchase of gold), whereas, in the depression of 1930–1934, it was increased from 16.185 billion dollars in the fiscal year, June 30, 1930, to 40.440 billions in 1939, a net increase of 24.255 billions. Therefore, this new method of dealing with depressions cost the people of the United States as much as did World War I, when the debt increased from 1.225 billions in 1916 to 25.482 billions in 1919, a net increase of 24.257 billions.

Not all, by any means, of this increased debt between 1930 and 1939, was due to public construction, for, although there was an increased debt from 1930 to 1939 of 24.255 billions, the author shows in Table 2 that the total excess of public construction during that period amounted to only 4.720 billions, or about 20%. However, the total excess of public construction for the post-depression years (1935–1939), which amounted to 4.750 billion dollars, was 35% of the increase in the federal debt of 13.387 billions, during that period, when the debt ascended from 27.053 billions in 1934 to 40.440 billions in 1939.

The writer has no complaint to make of the good intentions back of the public construction policy, and indeed back of all the other expenditures undertaken to counteract the effects of the depression; but he points out that the effect of any legislation is the same, regardless of the motives back of it. No individual can pursue a policy of borrowing continuously for ten years without experiencing the consequences, but his policy will affect only a few people; but when a government pursues such a policy, however, all the people are affected. Of course, in times of war greatly increased governmental expenditures are necessary, and have been incurred in all our wars, regardless of consequences; but, inasmuch as in previous depressions the country has recovered with little or no help from the government, the writer is not at all convinced that the increased expenditures for public construction, noted by the author, and other expenditures, represented by the increase of 24.255 billion dollars in the federal debt between 1930 and 1939, were justified. This seems particularly true when the similar depression of 1893 caused a rise in the public debt of only 1% of that amount.

BAXTER L. BROWN,[14] M. AM. SOC. C. E. (by letter).—Major Hallihan's paper may be termed prophetic for the reason that at the time he prepared it (early in 1940) there did not exist the movement for the tremendous expenditures for national defense that now (June, 1941) is in progress.

The demand for public work will be enormous on account of the large number of men who will be out of employment after the defense program is concluded. Therefore, it is incumbent not only for the government to prepare for federal work, but for the states, counties, and municipalities to do likewise in preparing for non-federal work. There will be a still greater demand for public work when the army is demobilized. Looking to the future, under these conditions, the writer's judgment is that plans should be prepared for work of a non-defense character, which can be postponed until the necessity arises. Naturally, there will be some work that will have to be undertaken as emergency work.

An interesting item that can be used to illustrate this proposition is the project to beautify Memorial Plaza in St. Louis, Mo. The money for this work is available, but it would be of no value in the defense program and, therefore, should be deferred until a later date. The City of St. Louis also has some betterments and enlargements of city-owned hospitals and institutions that should be undertaken with a view to the possible necessity of taking care of sick or injured should the United States enter the war.

On account of prosperity brought about by expenditures for national defense, it probably will be much easier to vote bonds for public works than it has been in the past few years. For instance, in St. Louis, bonds were voted in 1920 and 1923 by much more than the required two-thirds vote.

EVAN S. MARTIN,[15] ASSOC. M. AM. SOC. C. E. (by letter).—Planning public works to be undertaken primarily to furnish employment during disturbed and disordered economical periods, it is to be hoped, is no longer imperative. There is only one sound and sufficient reason for public works—and private enterprise as well—namely, to serve the public in the way it wishes to be served. Although private enterprise must be attuned to voluntary public demand for its market and success, there is the danger that some public promotion may be forced which does not serve the public to a degree comparable to its cost. Private misadventures are discerned by their financial failure, but not so with public developments. The writer has classed the New York Barge Canal as a public benefit not equal to its cost. Then, too, there are examples of developments resulting in partial losses through arbitrary change of public policy. Such are the Grand Trunk Pacific Railway in Canada, which was to facilitate the development of a large new area but suffered because such development was arrested by the Great War and the subsequent drastic restriction of immigration; and the Soo Line Railway, planned to carry international trade, which was estopped by a protective tariff policy.

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[14] Cons. Civ. Engr., St. Louis, Mo.

[15] Toronto, Ont., Canada.

Major Hallihan sets forth the narrow domain of public works. It is rather the narrow-minded notions held in the past regarding public service. Rivers, harbors, and perhaps mineral deposits, it is generally agreed, are natural resources. Well, what are the forests, soil, and crops of the fields and the inhabitants? Anything that serves the public is certainly public service regardless of its ownership and management. If the use of one's mind is to be hamstrung by precedent, he will continue to be inconveniently handicapped in thinking.

One of the restraints mentioned in this paper is the requirement of self-liquidation of an investment. During the past two centuries of almost continuous and intensive development, man has accustomed himself to "getting his money out of an investment." This is a phrase totally unrelated to the merits of an investment. One liquidates an investment by means of outright sale or by accumulating a surplus of receipts over expenditures during a term of years. No one can collect more money than he pays out except as others pay out more than they get. This means that investments are capable of being liquidated only when, and to the extent that, others make investments—that is, as development continues.

When the investment dollar stopped circulating in 1931–1932, it did so because further investment seemed unsound. Investment was unsound because a long, active period of development had increased the capacity of national economy much above present standards of living, which provide the market. The proper remedy was, and continues to be, a much higher wage level or lower interest rate—the latter having the same effect. No artificial public works program can give relief for long from making the proper adjustment.

Isador W. Mendelsohn,[16] **M. Am. Soc. C. E.** (by letter).—Major Hallihan presents succinctly a timely subject that affects the foundations of the future national existence of this country and that merits, of the engineering profession most of all, sober judgment divorced from partisan politics and personal jaundice. Public works as related to the entire construction field and unemployment and its far-flung effect upon the economic and social structure of the nation has received surprisingly little, and generally superficial, attention. So delineated, the value of public works still remains to be determined.

It is the purpose of this discussion to assist with a number of brief pertinent observations in the equable comprehension of the significance and the rôle of public works in national economy and welfare.

1. Statistical data presented in the paper's two interest-arresting tables concern such terms of widely varying meaning (as "construction," "employment," "national economy," "post-depression," etc.) that it would have aided in their understanding had appropriate definitions been included. Government and private economists and agencies differ in their use of these terms. Statisticians annually revise their construction statistics in the light of more complete and accurate data.

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[16] Capt., Corps of Engrs., U. S. Army, Schenectady, N. Y.

2. There is reason to question the implication of Table 2 that the depression is ended. Unemployment is still prevalent on an extensive scale. Piecemeal construction programs provide piecemeal employment, whether in public works or national defense. Upon completion of a few months' construction project, the same workers are looking for other employment. Entrance into the present war will only postpone the inevitable struggle with this problem. Thorough understanding of the fundamental causes of unemployment and the depression has not as yet been attained because the necessary analysis has not been carried through to completion. When such analysis is made, far-reaching changes in mode of life may be necessary to curb heedless "rugged individualism," whether of man or corporation, in behalf of the better public welfare necessary for enduring democratic national existence.

3. As presented in Table 2, the data do not warrant the presumption of marked correlation between public works construction and employment and national income. Although construction is a significant factor of national economy, there are many others, some of which are of greater importance. A comparison[17] of several major branches of economic activity in the United States during the period 1920–1930 shows the following estimated average annual production—

Estimated average annual production for all:	Billions of dollars
Manufactured products	59.7
Agricultural products	11.2
Retail sales	34.9
Goods marketed at wholesale	74.9
Foreign trade (imports plus exports)	8.8
Construction, of all types, including maintenance and repairs	11.6

The same authority gives the number of workers directly dependent upon the respective industries mentioned for their livelihood (annual averages, in millions of workers) as 11.2, 10.6, and 6.5 for retail and wholesale trade combined; and 3.1 for construction. Fluctuations in national income as presented in Table 2 may very well be due largely to these other factors instead of to construction. In a report[18] of the National Resources Planning Board, the definite statement is made that "present levels of public construction expenditure cannot be considered the major determinant of the level of business activity and national income * * * the determination of the national income is the product of many variables."

4. The term "construction" as used in the paper refers to new construction only. This is verified by comparison of the data with the table on "Estimated On-Site Employment on New Construction, by Sources of Funds, 1925–38."[19] As a matter of fact, there is considerable annual construction for maintenance of fixed works and structures. For the period 1926–1939 inclusive, the estimated value of maintenance construction[20] varied from 1,543 to 3,112 millions

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[17] "Construction Activity in the United States, 1915–37," U. S. Dept. of Commerce, 1938, pp. 27–28.

[18] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 36.

[19] *Loc. cit.*, p. 40.

[20] "Survey of Current Business," U. S. Dept. of Commerce, September, 1940, Reprint No. 17685.

of dollars annually, representing 20% to 38% of total annual new, maintenance and work-relief construction, private and public.

5. The paper refers only to on-site employment created by new construction, by its comparison with the table of the reference[21] mentioned in the preceding paragraph. Construction of any kind also provides for employment which is directly attributable to the construction project but which is provided away from the site, primarily in the manufacture and transportation of the materials.

> "On-site and off-site employment together constitute the identifiable effect of public construction on the volume of employment. They do not constitute the total effect of such construction, since additional employment is generated by the expenditure of the wages and incomes received by those engaged in construction or the supplying of materials. These further effects are not statistically identifiable." [22]

6. Public works place less emphasis on on-site employment; much of the expenditure is for materials that provide a considerable volume of off-site employment; a comprehensive volume of heavy construction is undertaken, and, typically, the work is performed under contract. This has been the procedure with PWA projects. The main purpose of work relief is to provide the maximum of direct, or on-site, employment to needy unemployed. Usually this work is done by force account and not by contract. These differences are not too sharply defined, but the broad purposes of public works and work relief make such a distinction necessary.

TABLE 4.—ON-SITE AND OFF-SITE MAN-HOURS PER MILLION DOLLARS OF PWA CONSTRUCTION CONTRACTS AWARDED[a]

(A = On-Site; B = Off-Site)

Description	BUILDINGS				WATER SUPPLY		SEWAGE DISPOSAL		STREETS AND ROADS		RECLAMATION		POWER AND LIGHT				NAVAL VESSELS	
	Non-Residential		Residential										Steam		Diesel			
	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Man-hours[b]	344	657	410	615	262	876	358	667	475	669	402	645	161	620	161	641	491	589
Total,[b] A+B	1,001		1,025		1,138		1,025		1,144		1,047		781		802		1,080	
Ratio, B : A	1.91		1.50		3.34		1.86		1.41		1.60		3.85		3.98		1.20	
Cost per man-hour	$1.00		$0.98		$0.88		$0.98		$0.87		$0.95		$1.28		$1.25		$0.93	

[a] Source: Bureau of Labor Statistics, U. S. Department of Commerce. [b] Thousands of man-hours.

7. Estimates of the volume of off-site employment created by the various federal construction programs in relation to the volume of on-site employment indicate[23] that the ratio of off-site man-hours to on-site man-hours was 1.0 for

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[21] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940.

[22] *Loc. cit.*, p. 38.

[23] *Loc. cit.*, p. 109.

PWA Federal projects, 2.0 for PWA non-federal projects, 1.0 for regular federal projects, 0.6 for projects conducted by federal departments under the works program, and 0.16 for WPA projects. These estimates may represent more nearly the maximum than average ratios. The variations in the over-all ratios of the several programs, excepting work relief, result primarily from differences of project composition. For similar types of projects, the off-site ratios are practically identical. In this connection, it is of interest to note in Table 4 the list[24] of on-site and off-site man-hours per million dollars of PWA construction contracts awarded, by types of projects.

8. Public works construction has an important effect upon the materials output of certain industries (see Table 5).[25]

9. In various periods different types of new construction projects are favored. In the period 1934–1940,[20] with public works construction aided by federal funds, highways, sewage disposal, water supply, public buildings, educational buildings, hospitals and institutions, and social, recreational, conservation, and development projects received more impetus than in the preceding years. In 1917–1918, as at present, military and defense construction projects received major consideration. During 1926–1929, commercial, factory, private residential, and public utility construction were preeminent. According to the National Resources Planning Board report[26]

TABLE 5.—MATERIALS PURCHASED FOR PWA CONSTRUCTION PROJECTS (RATIO OF ORDERS TO TOTAL PRODUCTION, 1934–1937[a])

Type of material	1934	1935	1936	1937[b]
Brick and hollow tile	23.5	26.9	42.7	27.7
Cement	73.6	36.8	16.8	13.2
Structural and reinforcing steel	39.8	35.9	23.8	12.3
Cast-iron pipe and fittings[c]	35.0	30.0	31.8	17.7
Sand and gravel	37.7	26.6	16.6	9.9

[a] PWA orders include certain distribution costs. [b] PWA expenditures curtailed in 1937. [c] Estimates for cast-iron pipe and for structural and reinforcing steel from data supplied by National Resources Committee; all others from Bureau of Labor Statistics.

> "* * * as the proportion of public works increases relative to that of private construction, we tend to get, at any level of total construction, relatively fewer factories, office buildings, public utilities, and housing, and relatively more highways, schools, recreational facilities, and conservation work. Furthermore, the trend of recent years seems to indicate that, within the field of public works, expenditures for conservation, the development of natural resources, and, as yet in only embryonic form, low-cost housing, are increasing in significance."

It is worth considering whether such planless construction as the foregoing, an outgrowth of heedless individualism, works to the best interests of the nation as a whole.

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[24] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 54.

[25] "America Builds—The Record of PWA," Public Works Administration, 1939, p. 27.

[26] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board, November, 1940, p. 23.

10. Public works in the depression years have been of inestimable value[27] in numerous other ways. For many years practices obstructing best construction have been multiplying, such as the wage "kickback," misclassification of laborers, lax inspection, political or personal favoritism in award of contracts, collusive bidding, unbalanced bidding, restrictive specifications, and inadequate performance bonds by certain contractors. All these have been guarded against and practically eliminated in the strictly public works programs. Public works have raised the standards of communities in public construction by: Insistence upon employment of competent engineers and architects, and prompt payment to them of reasonable fees; the institution of acceptable accounting practices in small communities; aiding in revamping the financial structure of small communities; improving municipal credit and reduced bond interest; furthering the incorporation of local districts and the creation of public authorities, thus enabling the construction of numerous public works; developing revenue-bond financing, which made possible hundreds of public works projects; and furthering the preparation of thorough project plans and specifications.

11. The letter of transmittal accompanying the report[28] of the National Resources Planning Board to the President of the United States contains pungent paragraphs, on the rôle of public works in construction, which merit the attention of the engineering profession.

12. As mentioned so aptly in the paper, public works have sustained the construction industry during the depression years, preserved contracting organizations, maintained consulting engineer staffs, and revitalized construction material industries. Now in time of increasing emergency, the nation is reaping the benefit of this indirect aid in increasing measure. Many of these consulting engineer firms and contracting organizations have designed and built new Army and Navy camps, airfields, sea bases, and munitions plants, and the supply industries have furnished the vast quantities of defense construction materials and equipment.

13. Engineers, lawyers, statisticians, accountants, clerks, and typists who were employed and trained by federal agencies on public works during the period 1933–1940 are now being employed in the War and Navy departments and other federal defense organizations, applying the skill, technical information, and training acquired in public works construction to serve the nation in defense construction.

14. On June 5, 1941, the President stated:

> "Our defense production is a gigantic assembly line. Transportation is its conveyor belt. If raw materials cannot flow freely to our great industrial plants and the products cannot move continuously to the front, defense breaks down. Bottlenecks in transportation are as serious as shortage of power."

The tremendous federal highway construction programs during depression years will prove timely in moving troops, materials, and supplies in these emergency days. Educational buildings constructed in recent federal programs now (1941) are useful for training technicians and skilled-trades operators, and educating hundreds of thousands of others in defense tasks that are in the making. Hospitals and institutions so sorely needed heretofore to serve civilian populations will provide improved medical facilities for the military wants of the emergency. Waterworks and sewerage systems sponsored by federal aid during the period 1933–1940 have increased public health safeguards as no similar construction work in any 25-yr period in the nation's past, and thereby will indirectly further national defense. Conservation and power projects such as Hoover Dam, Bonneville, Grand Coulee, Santee-Cooper, the All-American Canal, Tennessee Valley Authority projects, the Nebraska projects, and a host of others are developing hitherto forgotten sections and resources of the United States and are furnishing power to industries serving national defense.

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[27] "America Builds—The Record of PWA," Public Works Administration, 1939.

[28] "The Economic Effects of the Federal Public Works Expenditures, 1933–1938," National Resources Planning Board," November, 1940, p. iii.

15. Table 1 shows that federal funds expended for emergency construction exclusive of loans for the period 1933–1939 totaled $10,892,000,000, of which PWA[27] funds were $3,143,000,000. This federal aid has been criticized profusely, at times somewhat vituperatively, as to its advisability, the efficiency of the expenditures, the permanent value resulting therefrom, and whether greater benefits could not have been obtained through other and less costly measures. The Great War, to April 30, 1919, cost the United States $21,850,000,000 of federal funds alone, exclusive of loans to Allies. How many Americans have wondered as to the value of participation in that war, its advisability, the efficiency of expenditures, the permanent value resulting therefrom, and whether greater benefits could not have been obtained through other and less costly measures? Already federal appropriations for the present military emergency amount to $42,000,000,000. How much more will be required is unknown, but the total sum will be stupendous. How many Americans are considering the value of participation in the present war, its advisability, the efficiency of expenditures, the permanent value resulting therefrom, and whether greater benefits cannot be obtained through other and less costly measures?

Must the path to national democratic existence lead through vales of wars and depressions in rapid succession? In view of national experiences since 1917, do Americans feel greater gratification with expenditures for war than for public construction?

16. The United States is facing a war that may test the nation's very foundations. During World War I, for the period April 6, 1917, to July 1, 1919, with combined armed forces of 4,800,000, the total deaths of United States citizens numbered 125,500, of which approximately 43% were due to battle in a period of less than nineteen months.[29] It is within the realm of possibility that the coming war may last five years, may require the combined armed forces of more than 10,000,000, and may entail deaths of more than 1,000,000, both military and civilian. Great as was the dislocation of national life resulting from World War I, it is likely that the aftermath of the coming war will present a far greater cataclysm. It is none too soon to plan and prepare for the ex-

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[29] "The War with Germany," by Col. Leonard P. Ayres, War Dept., August 1, 1919.

pected rehabilitation work. World wars may be necessary in this century to maintain free nations, but of equal if not greater importance for enduring national existence is proper after-war recovery. Planning for utilization of national resources in men, materials, money, and morale in behalf of national interests is a vital necessity. If Germany at war can find time for planning the subjugation of conquered nations, it behooves the United States to plan for maintaining its democratic ideals of national existence after war as well as during war. Public works that have sown the seeds of planning in thousands of communities in depression years should play an important rôle in planning for national recovery and the better American way of life after war.

W. W. Crosby,[30] M. Am. Soc. C. E. (by letter).—Utterly allergic, from his breeding, education, and experience, as the writer admits himself to be, to all such hypotheses as "spending for prosperity," "the end justifies the means," and the "survival of the unfit," he nevertheless wishes to avoid in his remarks political argument or even discussion of the political economy involved and to offer only such comment as might be helpful to those of another persuasion who may really and sincerely believe that such postulates can be established as principles of the "American way of life."

Therefore, he now passes without further comment the wastes of "leaf-raking" and "boon-doggling," claimed to have been justified by some of the early "emergencies," to the practical defects as observed by him in the substantial work later attempted to be marketed for the "approval of the people."

The writer enjoys a ringside seat at a relatively small (in area) but active arena where a dozen public, and several different corporate, authorities contend for results from large funds apparently regarded as limitless. The wastes, the lack of cooperation, and the want of coördination evident are reminiscent of a "battle royal." They do not make for "public approval" when the early glamor of the frenzied activities has worn off.

Public discrimination is increasingly keen. Engineers have encouraged its development. They must keep ahead of it. This is not done merely by "making one dollar do the work of two"; nor by "Heath Robinson" or "Rube Goldberg" schemes to effect a temporary result.

Investigation here reveals an apparent cause for the conflicts of efforts and results. It lies in the "tunnel vision" (hemianopia) of the separate authorities. The planners have failed, or been unable, to regard considerations outside of each one's immediate, individual goal. In some cases each one seems to have disregarded consideration of contiguous interests.

The more obvious cases have given the public the idea that the projects were badly planned and wasteful; and the writer believes that the American people are not yet fundamentally wasteful and extravagant, although they may be induced to be at times.

The local public of any locality is inclined to credit the higher authority with greater abilities and perspective. The local planners and executives are too familiarly known and often do have a lack of outside knowledge and experience to offset their familiarity with local conditions. However, it does not follow that the county or state planners prove more competent. Often they show a neglect of points known to be of local importance.

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[30] Cons. Engr., Coronado, Calif.

It is when the planning gets into bureaucratic control that the value of the product in the market is often most questionable—if only from the aspects of narrowness of design and uncoordinated response to the needs in the wider field visible to the public. The public has learned that "made in Washington" does not guarantee satisfaction.

If deficiencies in locally made plans could be removed as they went up, and their final approval should signify that the project did actually fit in the entire picture, the market values would be greater and the demand then might stimulate local support.

Bureaucracy seems too often incapable of cooperation and coordination with others. Whether or not this defect in the setup can be overcome remains to be seen. If all such work is to be done under "emergency," "unlimited emergency," or "super de luxe emergency" conditions, and without a competent coordinating head, the prospect does not seem inviting. It may yet be that an earlier statement, reputed to have been made once by a most eminent authority, to the effect that "this country is too big and too diversified to be run [and, inferentially, to be planned] by one man or one group of men" will be proved and remembered.

The writer means no criticism of the motives of those in authority. Their limitations are generally those of law and environment. However, there is a need for a "catalyst" or some kind of an efficient agent to bring order out of the confusion if the general approval of the public is to endure.

The writer thinks that Major Hallihan has most conscientiously and illuminatingly set forth the case of the proponents of his theory. His recognition of the difficulties of localities in meeting the conditions imposed by higher authority, of the local limitations to action, of the objections found to bond issues by disappointed electorates, and of the belated (if not lacking) establishment of a coordinating or reviewing authority seems to indicate some agreement with the writer's comments.

J. P. HALLIHAN,[31] M. AM. SOC. C. E. (by letter).—The writer is gratified that his attempt to make an analysis of the value of public works has met with such generous discussion. That was his hope, as it is evident that, when an emergency problem of this character is presented, an analysis of the actual results obtained under experimental processes and the troubles encountered in the pursuit of a relief works program will be of the greatest possible aid in determining the possibility of future betterment in methods. For all practical purposes, the paper might just as well have been entitled "The Value of Time," since the results indicate that lack of preparedness was a very important factor in delaying the process of getting the program of public works under way.

The analysis of the situation in the United States as a whole, under the conditions of stress and disturbance of the national economy that were made evident in 1930, required much the same collection of facts as would be presented to a receiver of any large private property that suddenly found itself unable to meet its current necessities. For example, one becoming unexpectedly heir to a great ranch property in a run-down condition would find his primary consideration directed to an inventory of his assets—the condition of the ranch in regard to summer and winter feed, the water supply and its possible distribution to bring it within easy reach of the grazing herds, the creation of adequate reserves against long droughts, and the question of whether the cattle stocked were of breeds best suited to the ranch conditions. All would be basic situations to be considered before entering upon a program of rehabilitation of the property.

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[31] Director, Municipal Eng. Section, WPA, Washington, D. C.

Similarly, there existed in 1930 only very sketchy information on the condition of the natural resources of the United States, its forests, lands, water resources, and in particular its mineral resources in the light of the possible demands of national defense. Progress has since been made toward catching up on this deficiency. In many cases it has been found that great losses have been sustained by failure to have reasonably complete information available and to determine whether properties previously regarded as assets were still in that category or were in an advanced stage of depletion requiring long periods of conservation and building up processes before they could again be considered as assets.

The writer did not intend to convey the impression that the development of a public works program was the only thing necessary to do in combating an industrial depression. The record shows that it proved to be the most readily available measure to provide the necessary employment quickly and to secure the greatest salvage in the creation of needed facilities against practically the same expenditure on direct relief pending what was thought by many to be the certain revival of industry if let alone.

In the period beginning in 1930, no effort of consequence was made by the vast industrial machine to set up a program that would take the place of a federal-aid public works program. The privately owned industries are not to be blamed for reluctance to gamble their reserves, which would certainly be needed to rebuild obsolete plants on the resumption of industrial activity, in the expenditure of such sums before the trend of recovery became clearly defined. The same hesitancy and doubt caused the hoarding of private capital rather than expenditure in private construction, such as housing, not wholly dependent upon industrial recovery but facing the imperative necessity of taking up obsolescence. In this field the deficiency was so great that it would have required something like three times the volume of public construction to approximate the same relative volume of total construction. In the face of this condition, the writer is of the opinion that the program of public works so painfully developed was the only measure that had any effect in stopping the descending spiral caused by the combined effects of acceleration of unemployment and diminishing purchasing power. In considering its beneficial effects, the collateral employment created in the industries supplying materials and in the service industries should not be overlooked. The United States as a whole perhaps may be grateful at some future date that attention was thus brusquely called to its own deficiencies and that through the experience of corrective efforts and mistakes a plan may be devised that will find the Nation, in the event of recurrence of similar conditions, with a comprehensive plan of operations that will make it able to meet the same kind of emergency with measures adequately prepared in advance of need.

The writer is impressed with the comments of Mr. Henry in comparing the depression of 1893 with the depression of 1930 to 1939, and the thought that the quick recovery from 1893 to 1895 (which was followed by the silver campaign depression, equally quick in recovery, and a continued long period of prosperity to 1907, a temporary money panic) could have been duplicated in 1930. In 1893, the United States was still only partly developed, and the opportunity to initiate individual enterprises and to develop lands, mining, and transportation offered quite a different set of conditions to those confronting the country beginning in 1930, when the depression was affected not only by the conditions in the United States but by world-wide conditions.

In a chart[32] prepared by Brig.-Gen. Leonard P. Ayres the line of normal conditions shows depressions at various intervals, dropping in 1893 to a maximum of 20 points below the economic line (which was the greatest drop since 1808) and to 26 points below normal in the primary postwar depression of 1921. What is termed by General Ayres as a secondary postwar depression, commencing in 1930, fell in 1932 to a low of 48 points below normal and, after a brief temporary rise above normal in 1937, dropped again in 1938 to 36 points below the normal line, from which it ascended to 4 points above normal at the end of 1939, dropping again in 1940 to 14 points below, and is now (1941) again on the road up due to the defense boom.

General conditions in the United States were also different in 1893. It was not considered unusual for men to move from a place of unemployment to a place of employment where they worked in camps, and to follow the harvest. The country was in the process of terminating the work on its great trunk line railroads and feeders, where the method of operation was to draw raw labor from the congested centers, transport it to the job, and train it on the job to do the work required. That was the same kind of operation that has been required in the six years 1935 to 1941 under the relief works program, except that the relief program was obliged to find work at the point where the excessive unemployment existed. The writer believes, therefore, that the possibilities of early recovery that were present in 1893 did not exist in 1930. Conditions had become too crystallized. The temper of the people was less resilient. Time was lost from 1930 to 1933 by waiting for some evidence of the spontaneous recovery indicated by Mr. Henry, and it appears doubtful that in any future similar situation it would be logical to expect any such recovery by private initiative under the conditions now prevailing.

In Captain Mendelsohn's discussion the point is made in paragraph 7 that the practice of the PWA in contracting the work on public works financed by loan and grant of federal funds produced a greater proportion of off-site employment than the WPA methods, leaving the inference that the indirect payroll through high mechanization was of greater importance than the direct payroll in relieving the unemployment and in increasing purchasing power. The writer is unable to agree with this view as the direct payroll produces an almost immediate effect in circulation through the channels of trade, whereas there is a considerable lag in the final distribution of the indirect payroll. The quantity of materials purchased for the same type of structure, of course, would be the same, regardless of the method of construction.

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[32] "Annual Business Activity Since 1790," by Leonard P. Ayres, 14th Ed., October, 1940.

In Table 5, Mr. Mendelsohn presents the ratio of PWA orders for structural materials to the total production of such materials, indicating the beneficial effect on the suppliers by reason of such orders. The effect would be more clearly shown had he included the substantial volume of orders for materials placed by the WPA in the same period. Purchases of principal construction materials by the WPA and sponsors (not including purchases by other federal agencies financed by allocation of WPA funds) for the period from July 1, 1935, to December 31, 1940, aggregated $987,563,052.

The writer does not consider it within the scope of the paper to discuss questions that have been posed concerning the comparative cost of work done under contract methods and under the limitations of the laws governing the operation of relief agencies. One statement may be made, however, toward clarifying the thought on this subject. The cost of a public works job performed by contract methods, where the contractor may mechanize the operation to its fullest extent and exercise unrestricted freedom of selection of the cream of the labor from the general labor pool, does not include one important element of cost, which is the cost of maintenance, in condition ready to serve, of the general labor pool. The objective of the laws governing the operation of relief agencies in aiding public works projects with federal funds is that primary consideration should be given to the greatest possible employment of labor. In this operation, the proportion of cost attaching to the project for the maintenance of the labor pool is measurably included. It has been made clear in this emergency that this cost is a fundamental element that has heretofore been sustained by the entire Nation. It may be best that it should be so sustained because of the obvious difficulty in designing a method to apportion the collateral costs equitably to the particular work involved. It has been made equally clear that no equitable comparison between the cost of contract work and the cost of work performed under the relief program can be made without consideration of this factor.

The writer expresses his appreciation of the high quality of the comments and papers presented in the discussion and invites attention to the creation on June 24, 1941, by the Federal Works Agency, in cooperation with the National Resources Planning Board, of a division of planning entitled the Public Works Reserve, which presents great potentialities as a measure of preparedness.

The primary purpose of this division is stated to be the promotion of comprehensive studies of the situation in municipalities preliminary to the planning of improvements covering at least a 6-yr period with general plans and preliminary estimates of over-all cost, and the selection from such plans of projects deemed most necessary by the communities concerned for inclusion in a tentative construction program. Thus such projects may be ready for immediate consideration with respect to the financing and development of working plans for contract bidding or assignment of labor forces promptly upon the coming of emergency conditions.

It is apparent that in such studies any deficiencies in statutory or charter legislation will be brought to light and measures for correction prepared for action by the appropriate legislative bodies. These studies and plans involve no financial commitments but are designed to present an adequate picture of possibilities and to clear the field for action by reducing to the lowest point all legal and procedural retarding elements.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2142

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

By Katharine Clarke-Hafstad[1]

With Discussion by Messrs. Paul V. Hodges, C. S. Jarvis, Howard W. Brod, Merrill Bernard, Charles F. Ruff, Eugene L. Grant, Waldo E. Smith, Robert L. Lowry, Jr., Herbert C. S. Thom, and Katharine Clarke-Hafstad.

Synopsis

The accuracy of rainfall frequency values should be considered carefully in the design of flood and erosion control structures. This paper is concerned with the factors affecting the accuracy of rainfall frequency determinations. A method involving a statistical test for persistence is suggested for estimating the reliability of frequencies calculated by the station-year method.

Introduction

Determinations of the frequencies with which rainstorms of certain extent and intensity occur, together with measures of the reliability of these determinations, are essential to the solution of problems of flood control and of soil and water conservation. In the construction of soil-conserving devices such as terraces, check dams, and contour furrows, designed to decrease runoff and soil loss, it is necessary to know the range of stresses to which rainfall will subject these various devices, and the frequency with which these stresses will occur.[2] The size of dam or other control structure which it is economical to build on a given watershed depends chiefly upon the size and frequency of floods in that watershed. In order to predict the frequencies of various amounts of runoff, a knowledge of the frequencies of rainstorms of various sizes is required.

In the present state of the knowledge of the physical controls of weather and climate, the only feasible method for predicting the probable frequencies of certain amounts of rainfall is one based on a statistical study of precipitation history. The record of past precipitation, available in the form of measure-

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Note.—Published in November, 1940, *Proceedings*.

[1] Asst. Climatologist, Section of Climatic and Physiographic Research ,Div. of Research, Soil Conservation Service, Washington, D. C.

[2] "Climatic Research in the Soil Conservation Service," by C. Warren Thornthwaite, Benjamin Holsman, and David I. Blumenstock, *Monthly Weather Review*, U. S. Weather Bureau, Vol. 66, 1938, pp. 351–368.

ments by rain gages, however, provides an extremely inadequate statistical basis for making predictions of future precipitation. Rain gages set up to sample rainstorms have been very widely spaced, an insufficient number have been of the continuously recording type, and relatively few years of record have been obtained. As a result of the inadequate sampling, very little quantitative and reliable information is available concerning the areal extent of rainstorms, their duration, intensities, and frequencies.

Primarily to mitigate the limitation of the paucity of years of record to be used in calculating probable rainfall intensities and frequencies, the station-year method was devised. This method makes possible the determination of the frequency of those very intense rainfalls which occur so infrequently that only a few observations of their occurrence will be found among the records of a group of stations. It is particularly useful for determining the average number of occurrences of a given amount in a certain length of time when this period is longer than the actual number of years of rainfall record.

Although first applied to records of stream gages,[3] the station-year method was used as early as 1917 by the engineers of the Miami Conservancy District in their elaborate study of storm rainfall of Eastern United States.[4] Their technique has been used in its general outline by all subsequent investigators who have desired to determine for any locality the maximum rainfall to be expected once on the average in any given length of time.

The method consists essentially of collecting for an area all the rainfall records of whatever length, and treating the sum of all the records for all stations (called the "station-year record") as if it were a single record for the midpoint of the area under consideration. A certain time interval is then selected, which may be anything from minutes to months, depending upon the information desired and the data available. All rainfall quantities for this time interval appearing in the aggregate record are then arranged in order of magnitude. The highest amount is considered to be the maximum to be expected in the area, on the average, once in the period of years represented by the station-year record; the second highest amount may be expected once in one half the aggregate record, and so on for smaller quantities. The combined experiences of all stations in the area are assumed "* * * to give a weighted average which may be regarded as the probable average experience for any one point within that area."[5]

The maximum rainfall to be expected with a certain frequency is termed the "pluvial index" for an area. These pluvial indexes are plotted on charts and iso-pluvial lines drawn by interpolating between the indexes. In drawing these lines the assumption is made that the pluvial index refers to the midpoint of the area for which it was determined rather than to any point. This may or may not be a correct assumption, depending (as will be shown) upon the distribution of stations and the homogeneity of rainfall characteristics of the area.

Analysis of the station-year method shows that by this technique one is determining, in effect, the reciprocal of the average number of occurrences of a given amount of rainfall, per station per year. If 500 station years of record yield 5 occurrences in excess of a certain amount, the average number of occurrences per station per year will be $\frac{1}{100}$. This is equivalent to an average frequency of 100 years per occurrence, which is the same result that would be obtained by taking the fifth highest amount in a station-year record of 500 years.

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[3] "Flood Flows," by W. E. Fuller, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII, December, 1914, pp. 564–617.

[4] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed., 1936, Dayton, Ohio.

[5] *Loc. cit.*, p. 63.

It should be noted that the station-year method is a simple statistical procedure for arriving at an approximate answer to a problem which it is hoped may some day be solved more satisfactorily either from a consideration of the meteorological processes producing rainstorms, or through the accumulation of more detailed data and a study of rainstorm morphology. Ideally, one would like to know the frequency of rainstorms of certain area, intensity, and duration—not simply the frequency of a certain amount of rainfall for a point in an area. Aside from the restriction due to length of record, the chief limitation in the usefulness of the station-year method is its failure to indicate the size of area which may be expected to receive the given amount of precipitation. At present very little information is available on the intensity-area relations of storms, and this is due to the fact that, until recently, recording rain gages have been so widely spaced that storm areas could not be determined with any degree of accuracy. This deficiency of gages is gradually being remedied.

In certain areas of Eastern United States many more rain gages have been installed within recent years. In the watersheds of the Susquehanna River and the Ohio River above Pittsburgh, Pa., in September, 1940, there were about 100 automatic recording gages in operation, approximately 1 for every 400 sq miles. No single agency assumes responsibility for maintaining all these gages. The Pennsylvania Water and Power Resources Board; the United States Geological Survey, Weather Bureau, Forest Service, and Soil Conservation Service; local power companies; and other agencies have each placed their gages in locations most advantageous to themselves. The collection and compilation of the data from all these gages, however, will contribute fundamental information on rainstorm frequency and intensity that will be extremely valuable to all agencies concerned with the precipitation on this watershed. Similarly, in the Tennessee Valley,[6] a region of approximately 40,000 sq miles, there were, in July, 1940, about 460 rain gages maintained by various agencies, of which approximately 70 were of the recording type. Most of these gages have been in operation 5 years or more. The maintenance of such networks as these, together with the very dense net of automatic gages, 500 in 8,000 sq miles, established by the Soil Conservation Service on the Muskingum Watershed in Ohio, will make possible the determination of the frequencies of rainstorms of given area and intensity.[7] The reliability of the frequency data from these networks of stations will increase rapidly as the years of record increase.

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[6] "Precipitation in Tennessee River Basin," Tennessee Valley Authority, Hydraulic Data Div., *Annual*, 1938.

[7] "Microclimatic Studies in Oklahoma and Ohio," by C. Warren Thornthwaite, *Science*, Vol. 86, 1937, pp. 100–101.

FACTORS AFFECTING THE ACCURACY OF FREQUENCY DETERMINATIONS

The simple computation of average frequency by the station-year method requires no information regarding the relation of rainfall amount to the associated storm, but a knowledge of the characteristics of rainstorms is necessary for the correct interpretation of the results.

If there is a closely spaced, well-distributed net of recording stations in an area, and a large number of years of record—not station years, but actual years of time—the average frequency of a specified amount of rainfall computed by the station-year method is a reliably representative one for the midpoint of that area for which it was determined. The relationship between the length of record and the accuracy of the average frequency is the chief concern of this paper. The arguments will be based on the fact that the statistical theory of probability makes it possible to indicate, by a percentage of error in the average, how the reliability of a frequency value is related to the length of independent record.[8]

Number and Distribution of Stations.—Some of the ways in which the accuracy of the average frequency will be affected by the number and distribution of recording stations will be briefly mentioned before treating the effect of the length of record, although it should be emphasized that much more data and study on intensity-area relationships are required before any improvement can be made in the accuracy of frequency determinations.

C. Warren Thornthwaite[9] has shown that with a widely spaced net of stations it is highly improbable that the maximum rainfall occurring in a storm will be recorded at any station. This is illustrated by the fact that no gage recorded the rainstorm which produced a flood on the Republican River in May, 1935. If maximum quantities are not recorded for all rainstorms, the computed maxima to be expected with a certain frequency certainly will be too low.

An average frequency value for an area calculated from the data for several stations will be representative for any point in the area only if the rainfall characteristics are fairly uniform throughout the area. Although no area in the United States as large as a quadrangle of 2° of latitude and longitude is homogeneous with respect to its rainfall characteristics, the engineers of the Miami Conservancy District stated that the station-year method, as used in their study, presupposes

> "* * * that the rainfall characteristics, especially with regard to high rates of precipitation, are essentially uniform at all points within the area of a 2-degree quadrangle of the earth's surface."[10]

They recognize, of course, that this supposition is not strictly true because on the charts of pluvial indexes[11] they have drawn iso-pluvial lines which indicate transitions from one pluvial index to another across a 2° quadrangle.

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[8] "A Statistical Method for Estimating the Reliability of a Station-Year Rainfall-Record," by Katharine Clarke-Hafstad, *Transactions*, National Research Council, Am. Geophysical Union, Vol. L, 1938, pp. 526–529.

[9] "The Reliability of Rainfall Intensity-Frequency Determinations," by C. Warren Thornthwaite, *loc. cit.*, Vol. 2, 1937, pp. 476–484.

[10] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed. ,1936, Dayton, Ohio, p. 67.

[11] *Loc. cit.*, Figs. 13–36, pp. 69–80.

Since it is obvious that no 2° quadrangle of the earth's surface will have uniform characteristics of rainfall, other methods of selecting areas should be considered. More representative frequency-intensity values could probably be obtained by calculating the averages for overlapping circular areas whose radii would be equivalent in length to one side of a 1° quadrangle and whose centers would be corners of 1° quadrangles.

More accurate values might be obtained, also, by selecting the homogeneous areas on the basis of regions having the same pluvial indexes as shown by the iso-pluvial lines on the Miami charts. Eugene L. Grant,[12] M. Am. Soc. C. E., has proposed the use of the χ^2-test to delimit a homogeneous area whose pluvial index would be representative of any point in the area.

In regions with mountains or other features that result in great differences in rainfall characteristics in short distances, it is practically impossible to obtain average frequencies that have any real significance for large areas. A very dense and uniformly spaced net of rain gages would be necessary to record all the areal differences in order that the average value would represent the area as a whole, and even this value would depart greatly from the true value for certain parts of the area. Average frequencies for mountainous parts of the United States are especially unreliable because in these regions the rain gages are insufficient in number and are located chiefly in the valleys.

An irregularity of station spacing will seriously affect the values of average frequency in areas with non-uniform rainfall characteristics. If there are 2 gages, in any area, several miles apart, one may record 3 occurrences of a certain amount per year, and the other gage 6 such occurrences; but if there had been a third gage so close to the gage recording 6 occurrences that it gave an identical record, the average for the area would be distorted in favor of the higher number of occurrences. Since there is probably no area with absolutely uniform rainfall, an irregularity of station spacing will affect all frequency determinations to some extent.

The accuracy of frequency values depends to a very great degree upon the adequacy of the sampling of rainstorms. It is logical to question whether, from a given net of recording stations, fair and adequate samples of the various rainfall quantities are being obtained. This depends not only upon the number and distribution of the stations, but upon the relation of this distribution to the distribution of precipitation.

In general, rainstorms are of two types: (*a*) Those of wide extent (sometimes hundreds of thousands of square miles, and characterized by a small range of intensities, which results in a rather uniform rainfall pattern); and (*b*) those limited in area to a few hundred or thousand square miles (characterized by a wide range of intensities resulting in a very spotty distribution pattern). Gradations from one type to the other occur, and combinations of the two types result in very complex patterns.

The fundamental distinction between these two patterns is based on the type of physical process that causes the precipitation. The spotty rainfall pattern results primarily from rain originating through a convective process; the widespread homogeneous pattern almost always results from frontal

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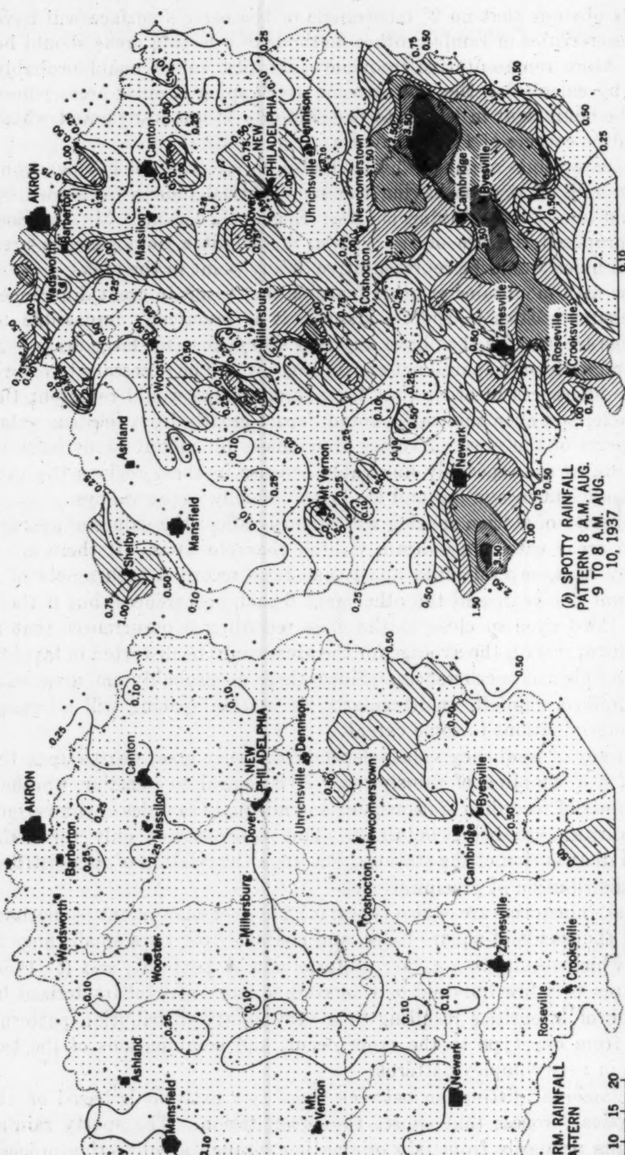
[12] Discussion by Eugene L. Grant of "Rainfall Intensities and Frequencies," by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 384.


(a) UNIFORM RAINFALL PATTERN
(b) SPOTTY RAINFALL PATTERN 8 A.M. AUG. 9 TO 8 A.M. AUG. 10, 1937
0 5 10 15 20
Scale in Miles
LEGEND (Inches)
0 0.10 0.50 1.00
0 0.10 0.50 1.00 1.50 2.50 3.50 UP
AKRON
Barberton
Wadsworth
Canton
Massillon
Wooster
Ashland
Mansfield
Shelby
Millersburg
Dover
NEW PHILADELPHIA
Uhrichsville
Dennison
Newcomerstown
Coshocton
Mt. Vernon
Newark
Zanesville
Cambridge
Byesville
Roseville
Crooksville

(Time, 8 a.m., November 12, 1937, to 8 a.m., November 13, 1937)

FIG. 1.—TOTAL PRECIPITATION ON THE MUSKINGUM WATERSHED, OHIO

activity whereby moist, less dense air is cooled adiabatically by ascent over colder, more dense air to the point where the moisture condenses.

The two general types of precipitation patterns are illustrated by the storms of November 12–13 and August 9–10, 1937, which occurred in the Muskingum Watershed in Ohio (Fig. 1). Rainfall measurements from the 500 automatic recording gages installed by the Soil Conservation Service on the 8,000 sq miles of this watershed provide the data for the maps of these storms. The November storm illustrates the uniform pattern in which a large area received very nearly the same rainfall in a 24-hr period; and the August storm illustrates a spotty pattern with large ranges in amount in small distances.

A wide spacing of stations would suffice to give a fairly accurate value of the average frequency of rainfall quantities if all storms were of the large-area, homogeneous type shown in Fig. 1(*a*); but a more dense net would be necessary to obtain the same degree of accuracy for the quantities associated with a spotted pattern like Fig. 1(*b*). Since the storms of more uniform intensities, in general, are those of low intensities, the frequencies will be more accurately given for the smaller amounts. Very high rainfall may occur over large areas and give a fairly uniform pattern. Such storms are rare, however, and produce the widespread major floods such as that experienced in the Ohio Valley in January, 1937. A few stations would suffice to give a good sample of the intensities in such storms, but very many years of record are required to give reliable values of their frequency. On the other hand, there are many occurrences of high intensities equivalent to those of the widespread storms, but associated with the spotty type of rainfall, in which there may be several centers of small area receiving very heavy rainfall. The many flash floods that occur throughout the United States every year, washing out soil conserving terraces and small dams, result from this spotty type of rainfall. Although more frequent than the storms producing the major floods, their area is so small relative to the average spacing of stations that many occurrences are not observed and consequently their frequencies cannot be determined accurately.

When several years of record are available from a battery of closely spaced stations, such as that operated in Ohio by the Soil Conservation Service, it will be possible to make comparisons of the average frequencies derived from various spacings and locations of stations, and consequently to provide corrections to be applied to the frequencies obtained from widely spaced, poorly distributed sets of recording stations, in regions with rainfall characteristics similar to the Ohio area.

The Length of Independent Record.—The second factor to be considered in estimating the reliability of an average frequency of specified rainfall is the length of record from which the average was determined. It is obvious that the reliability of a mean frequency based on data from a single station is a function of the length of record for that station, because the longer the record the more occurrences there will be upon which to base a frequency calculation. This must be true, also, for a mean frequency based on the data from several stations in an area. The station-year method simulates a long record for a single station by placing contempory records, from several stations, end to end.

Although by this method more storm experiences may be obtained than in a record for a single station, it obviously cannot be assumed that more actual years of time are being sampled by this expedient. A record from 10,000 rain gages operating on an area for one year cannot replace 10,000 actual years of record for one station although both records could conceivably include the same number of storm experiences. The reliability of an average frequency computed from these two 10,000-station-year records might be the same, but only if each rainstorm of given intensity affected only one station.

The use of additional station records does not increase the reliability of the frequency determinations in direct proportion to the number of station records that are used. An average frequency for an area calculated from a station-year record for several stations, however, is more reliable than one calculated from the record of one of them. Because of the spottiness of rainfall, certain precipitation amounts will be recorded at some stations and not at others during a period of years, and the inclusion of records from many stations will increase the likelihood of including in the calculations of the mean frequency these more rare events.

Regardless of how many station years of record are used, there will always be a certain degree of unreliability in the frequency determinations simply because the actual years of record are not sufficient to sample all the possible annual variations in the rainfall characteristics. If every year had the same characteristics and if there were no abnormally wet or abnormally dry years, then, of course, any one year of record would give as reliable values as any other year. Furthermore, if there are cyclic or secular changes in rainfall, the frequencies from data obtained during a part of a cycle or period may not be applicable to other parts of the cycle. The Miami engineers recognized that the station-year method was valid only if "there are no permanent or cyclic climatic changes affecting the occurrence of high rates of rainfall."[10] This might have been expressed in another way by saying that there will be a high degree of unreliability in the frequency values if there are cyclic or permanent changes in the occurrences of high rates of rainfall.

A quantitative measure of the reliability of average frequencies is needed in order to compare the reliability of the frequency value obtained at a single station with that derived from the composite record of several stations.

In any sampling process the reliability of an average is proportional to the square root of the number of observations upon which the average was determined. The conventional method of expressing the reliability of a mean is in terms of its expected fluctuations, these fluctuations being measured by the standard deviation. The standard deviation of a distribution of means is called the standard error of the mean, and for a normal distribution it is equal to the standard deviation of the sample divided by the square root of the number of values entering the mean. Certain definite relations exist between the size of the standard error and the number of values in the distribution that differ from the mean for the entire population by certain specified amounts. From these established ratios it is possible to indicate the extent to which additional samples may be expected to vary from the average obtained from a single sample. Suppose, for example, that in a 500-yr rainfall record for a single station, 100 occurrences of a certain amount are observed, giving an average of one occurrence in 5 years, and that the standard error of this mean is 10% of 5 years. Then, knowing the relationships between the size of the standard error of the mean and the distribution of values in the normal curve, one could say that there are 99.7 chances out of 100 that the mean computed for these 500 years (if they are random) is not more than three times the standard error or 30% away from the true average for many thousand years; and there are 68.3 chances out of 100 that this mean will not exceed or fall short of the true mean by more than the standard error, 10%.

The usual formula for the standard error of the mean cannot be applied to frequencies of rainfall because they do not form a normal distribution but rather a discontinuous one of the Poisson type. For this distribution type the standard error expressed as a percentage of the mean is equal to $\frac{1}{\sqrt{n}}$, in which n is the number of independent occurrences.[13]

For the foregoing example, the calculation of the average frequency and its reliability would be as follows: $\frac{500}{100} \pm \frac{1}{\sqrt{100}}$; or 5 years ± (10% of 5); or 5 years ± 0.5 year.

The maximum rainfall that has occurred in any record will have an average frequency (if an average can be said to be determined from a single observation) of once in the total years of record. When the number of observations, N_o, upon which the mean is based is small, generally less than thirty, the usual $\frac{1}{\sqrt{N_o}}$-law fails to give a reasonable estimate of the error, and $\frac{1}{\sqrt{N_o - 1}}$ provides a better though not rigorous approximation (it is inappropriate to discuss herein the difficulties involved in determining the errors in means derived from a few observations. Detailed consideration of the problem has been given by W. N. Bond[14] in 1935 and by G. Udny Yule[15] in 1936). The error of the frequency of the maximum rainfall amount, therefore, would be given by $\pm \frac{1}{\sqrt{1 - 1}}$ or ∞. This can be interpreted only as meaning that a single observation gives no clue to the size of its error; in other words, nothing actually is known concerning the frequency of a rainfall quantity from a single observation of its occurrence. Very high degrees of unreliability are encountered in the expectancies of all high rainfall quantities, inasmuch as the greater the depth the smaller will be the number of occurrences.

Determining the reliability of an average frequency for a single station record is a relatively simple statistical procedure. The next problem concerns its determination for a composite station-year record.

Referring again to Fig. 1(*b*), one sees that during this storm period 5 stations received 3.5 in. or more of rain. If each of the 500 rain gages in this area continued operation for one year, there would be 500 station years of record. Now, assume that 100 storms like that of August 9–10 occurred in this watershed during one year. There would be 500 occurrences (100 storms each affecting 5 stations) in 500 station years, giving an average frequency of one occurrence of 3.5 in. per year. If in this one year of record, instead of 100 storms, there were 500 separate storms each affecting only one station, or if there were one large storm covering the entire area and giving 3.5 in. at all 500 stations, the total number of occurrences of 3.5 in. would be the same, and likewise the average frequency of such an amount would be the same. The station-year method will give the same average frequency for a given number of station years and total occurrences regardless of the combination of numbers of storms and numbers of stations receiving the given amount which makes up the total occurrences.

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[13] "Statistical Methods for Research Workers," by R. A. Fisher, London, 6th Ed., 1936, pp. 56–59.

[14] "Probability and Random Errors," by W. N. Bond, London, 1935, pp. 45–65.

[15] "An Introduction to the Theory of Statistics," by G. Udny Yule, London, 1936, pp. 277–278, and 353.

In so far as the average frequency will indicate how often, on the average, one may expect storms of 3.5 in. in a 24-hr period, however, it would probably be agreed that more reliance could be placed in an average frequency determined from 500 storms than in one based on only one storm. A single observation of a storm in a period of record, regardless of how many stations recorded this storm, tells nothing about its frequency. The storm that resulted in the great Ohio flood of March, 1913, provided 11 of the 16 six-day amounts of 8.7 in. or more of rain in a 1,650 station-year record.[16] The average frequency of this 8.7-in. rain is given as once every 100 years; but the fact that this storm occurred once in 1,650 station years (or even 16,500 station years, if there had been 10 times as many gages in the area) does not indicate whether it can be expected once every 100 years, or once every 1,000 years. In other words, the reliability of an average frequency, expressed as a standard error, should not be calculated from the total occurrences of a given amount in a station-year record, but rather from the number of storms.

In studies involving the station-year method, no reference has been found concerning the reliability of the pluvial indexes; in fact, the number of storms of given intensity is not recorded, and hence standard errors for the average frequencies, based on the number of storms, cannot be determined.

The rainstorm itself is really the independent event whose frequency one would like to know, but since the available data are not in such a form that storm frequencies can be calculated, it is assumed in this paper that the 24-hr rainfall period constitutes the storm, and is the independent event.

If it were true that all storms that result in a given rainfall at any point in an area were of the same areal extent, and if there were equal and fixed distances between recording stations so that the same number of stations recorded the specified amount in every such storm that affected the area, then one could simply divide the total occurrences of that amount in the station-year record by that number of stations and find the number of storms—that is, the number of independent events upon which the reliability of the average frequency should be based. Since storms of a given intensity do not always affect the same number of stations, the average number of stations affected by storms of this intensity may be used instead to determine the number of storms. This average number of stations will be designated simply as N_a herein.

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[16] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed., 1936, Dayton, Ohio, p. 66.

Consider again the case of 100 occurrences of rainfall of certain depth, this time occurring in a station-year record of 500 years derived from 20 years of record for 25 stations. Suppose, in this case, that the average number of stations recording the given amount in each rain period of 24 hours were five. There were, then, $\frac{100}{5}$ or 20 independent rain periods. The average frequency and its standard error would be given by $\dfrac{25 \times 20}{100 \pm \frac{1}{\sqrt{20}}}$; or 5 years ± 22%; or 5 years ± 1.1 years.

Compared with the average frequency for the 500-yr record for a single station, which had a standard error of ± 0.5 year, one notes that, although the frequency remains the same, the reliability is less when determined from the number of storms in a station-year record than when determined from the single station record of equivalent length.

It has been shown that the reliability of the average frequency should be determined from the number of independent events (that is, from the number of storms), and that this number may be obtained by dividing the total number of occurrences in the station-year record by N_a. The fact that frequently more than one station records the same storm may be described as dependence between stations. One phase of this problem of determining the reliability of the average frequency of rainfall quantities becomes, then, one of calculating the dependence between recording stations.

DETERMINATION OF DEPENDENCE BETWEEN STATIONS

Although it is possible to use the average number of stations per storm receiving the given rain quantity as the index of dependence between stations, another measure of dependence will be described herein that is an even better index upon which to base the reliability determinations. This index is calculated by a statistical technique, devised by J. Bartels,[17,18] and proposed originally for testing persistence in observations taken at equal time intervals. (A series of numbers having "persistence" is called a correlative series. There is a tendency in such a series for high values to be followed by high values, and low values to be followed by low values.) In the case of a time sequence successive observations are affected by a single event because of its extension in time; in the station-year record of rainfall amounts, adjacent stations may be affected by a single event because of the areal extent of the storm.

The technique for testing persistence is based on the fundamental theorem of probability that in a random series of numbers the ratio of the standard deviation of the means of equal-sized groups of these numbers to the standard deviation of the individual numbers of the series is equal to unity divided by the square root of the number of values in the groups ("randomness," as used herein, means that, on the basis of the numbers up to a certain point, the chance for the next number to be higher or lower than the mean for a long series is equal or unpredictable. A series of random numbers can be most

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[17] "Zur Morphologie Geophysikalischer Zeitfunktionen," by J. Bartels, Sonderausg. aus den *Sitzber*, der Preussischen Akad. der Wiss. Phys.-Math., Klasse 30, 1935.

[18] "Geomagnetism," by S. Chapman and J. Bartels, Oxford, Vol. II, 1940, pp. 582–585.

easily obtained from a table of "Random Sampling Numbers," by L. H. C. Tippett[19]).

This may be explained more clearly by carrying through the calculations on a series of random numbers. One first marks off groups of a certain number of ordinates and forms the average for each of these groups. These group averages will approximate the normal more closely than the individual values. If, for example, groups of 25 ordinates were used, the scattering of the group averages measured by their standard deviation, σ_{25}, will be less than that indicated by the standard deviation, σ_1, for the individual numbers; but $\sigma_{25} : \sigma_1 = 1 : \sqrt{25}$; or $\sigma_{25} = \frac{\sigma_1}{\sqrt{25}}$. In general, if groups of s numbers, instead of 25, are formed, $\sigma_s = \frac{\sigma_1}{\sqrt{s}}$.

Returning to the theoretical case of 25 stations, each having 20 years of record, and storms of given depth always affecting 5 stations in each storm: From the 20 years of record for these 25 stations one may express the relationship between the standard deviations of the total occurrences for each year in the groups of 25 stations, and the standard deviations of the number of occurrences at the individual stations each year as

$$\sigma_{25} = \frac{\sigma_1}{\sqrt{\frac{25}{5}}} \qquad (1)$$

Substituting s for 25 and N_d for 5 in Eq. 1:

$$N_d = \left[\frac{\sigma_s}{\frac{\sigma_1}{\sqrt{s}}}\right]^2 \qquad (2)$$

in which N_d equals the number of stations on the average receiving the given amount in each storm day.

To show how this technique may be applied to a set of rainfall data, assume that one had 10 years of concurrent record for 10 stations. The total occurrences of a certain rainfall amount at each station for each year could be set up in a table of 100 values, with the 10 columns representing stations and the 10 rows representing years. If there is no dependence between stations or years, these 100 values will constitute a random series. Any departure from randomness will be a measure of the amount of dependence between the stations, or between years.

Artificial tables of rainfall data have been set up by using random numbers and introducing a known degree of dependence between stations.[20] Eq. 2 was tested by calculating N_d from such tables. In every case the correct value of the dependence within the standard error of N_d was obtained (the standard error of N_d was calculated on the assumption of a normal distribution since any error due to this assumption will be of second order. A more detailed consideration of the calculation of the standard error of a non-normal distribution is out of place in a non-mathematical paper such as this).

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[19] "Random Sampling Numbers," by L. H. C. Tippett (Tracts for Computers 15, London Univ., Dept. of Applied Statistics, 1927).

[20] "On the Bartels' Technique for Time-Series Analysis, and Its Relation to the Analysis of Variance," by L. R. Hafstad, *Journal*, Am. Statistical Assoc., Vol. 35, 1940, pp. 347–361.

The χ^2-test and analysis of variance were tried on the table of random numbers, but since they gave only the information that the numbers in the rows were more alike than one would expect in a random sample, and that the numbers in the columns were about as in a random sample, these tests were discarded as inadequate for the determination of N_d, the number of dependent ordinates, or stations.[20]

If the average number of stations recording the given depth in a storm day were the same for every year, and if the numbers of storm days per year constituted a random series, then the value of N_d would be equal to N_a for the total years of data.

In actual rainfall data, a variability exists from year to year in the average number of stations affected per storm, and a greater fluctuation exists in the number of storm days than might be expected in a random series. The value of N_d is determined from the sums of the occurrences for each year, and there are wide fluctuations in these annual sums because they combine the fluctuations in the average for each year of the number of stations affected per storm, and the annual fluctuations in the number of storms. If there are wide fluctuations from year to year in the number of storms, then it is necessary to have more years of data to give an average frequency that is as reliable as would be obtained from a few years which had practically the same number of storms each year.

In addition to measuring the persistence between stations (average number of stations affected per storm), N_d measures also these fluctuations from year to year, and it is therefore a more conservative, but also a more accurate, index to use in calculating the reliability than simply the average number of stations affected per storm. The standard errors will be higher if determined from N_d than if determined from either the total occurrences or the number of storms.

USE OF N_d FOR ACTUAL RAINFALL DATA IN DETERMINING STANDARD ERRORS OF FREQUENCIES OF RAINFALL AMOUNTS

The calculation of N_d has been made from rainfall data for two areas in the United States in different climatic regions. The method of tabulation and calculation will be described in detail for one of these areas.

Twenty-one stations in the quadrangle between meridians 93° and 95° west longitude and parallels 41° and 43° north latitude in central Iowa, corresponding to Quadrangle 14-D of the Miami study,[4] were selected to represent that region. Tabulations were made of all daily rainfall depths of one inch or more at every station for 32 years, 1905 through 1936. Only stations at which observations were made in the evening, and only those stations having continuous records for the entire period, were used.

Tables were made of the total number of occurrences at each station during each year of daily rains of one inch or more, 2 in. or more, 3 in. or more, and 4 in. or more. Table 1 is a sample tabulation and compilation

TABLE 1.—THE NUMBERS OF OCCURRENCES OF DAILY RAINS OF ONE INCH OR MORE AT 21 STATIONS IN CENTRAL IOWA FOR THE YEARS 1905–1936, AND DATA FOR CALCULATING N_d

Year	Ames	Afton	Baxter	Belmond and Britt	Chariton	Cumberland	Monroe and Des Moines	Newton and Grinnell	Guthrie Center	Hampton and Allison	Humboldt	Jefferson	Knoxville	Lacona	Perry	Pocahontas	Rockwell and Sac City	Waukee	Webster	West Bend	Winterset	Σ	$\frac{\Sigma}{N}$	$\left(\frac{\Sigma}{N}\right)^2$
1905	9	11	9	5	15	15	10	10	7	10	9	5	12	8	9	16	8	15	9	8	10	210	10.000	100.000
1906	9	4	3	8	5	7	2	5	5	7	11	2	8	3	7	8	12	5	8	11	8	138	6.571	43.178
1907	4	7	10	6	9	10	4	10	4	9	7	7	6	9	8	7	5	13	6	5	6	152	7.238	52.389
1908	12	14	6	11	6	17	9	6	7	8	12	5	8	10	10	10	20	13	7	10	11	212	10.095	101.909
1909	14	18	8	10	10	9	7	10	9	12	12	7	10	6	19	10	16	10	11	11	5	224	10.667	113.785
1910	5	7	5	3	1	9	5	2	7	3	4	1	5	2	3	1	8	2	4	2	4	83	3.952	15.618
1911	9	3	5	5	9	4	6	11	4	10	5	2	6	5	4	6	6	5	9	5	8	127	6.048	36.578
1912	8	10	8	6	6	6	6	9	13	9	1	6	6	9	6	9	11	6	3	7	7	152	7.238	52.389
1913	5	10	11	5	6	10	1	11	3	11	6	6	5	7	4	5	14	6	6	10	8	150	7.143	51.022
1914	9	5	11	12	5	5	10	10	9	9	5	11	7	6	9	9	7	8	6	8	9	170	8.095	65.529
1915	11	11	8	8	14	11	12	13	9	8	10	5	13	12	10	8	8	10	9	9	8	207	9.857	97.160
1916	4	4	7	10	7	2	6	6	8	7	3	8	5	6	5	7	4	1	5	6	5	116	5.524	30.515
1917	12	6	9	6	11	6	10	10	9	9	6	9	9	6	9	5	5	6	8	9	8	168	8.000	64.000
1918	6	6	10	11	5	5	11	11	5	8	3	7	3	5	4	6	6	6	9	8	6	141	6.714	45.078
1919	9	13	8	9	12	10	13	12	12	9	5	9	12	12	13	3	8	14	10	4	12	209	9.952	99.042
1920	6	10	5	9	9	7	15	4	8	9	11	13	9	9	7	5	7	9	3	9	5	169	8.048	64.770
1921	7	6	4	4	12	5	12	13	7	12	6	10	8	10	7	7	5	6	2	7	8	158	7.524	56.610
1922	6	9	11	4	9	8	10	5	5	8	8	8	12	12	4	5	8	8	8	5	13	166	7.905	62.489
1923	5	3	9	4	3	6	9	4	9	6	5	8	8	2	3	5	5	4	3	6	3	110	5.238	27.437
1924	6	10	8	8	4	11	6	8	9	6	7	5	6	9	2	6	5	9	8	5	9	147	7.000	49.000
1925	8	8	6	7	9	3	6	7	5	8	3	3	8	9	7	2	4	8	6	2	3	122	5.810	33.756
1926	7	9	6	8	10	5	10	6	8	3	6	8	10	10	8	4	6	6	4	7	5	146	6.952	48.330
1927	3	4	5	4	6	3	4	4	3	4	5	4	4	9	4	1	2	5	3	3	2	82	3.905	15.249
1928	10	11	7	11	8	9	13	10	10	11	9	9	8	14	12	7	6	8	5	5	8	191	9.095	82.719
1929	7	6	5	4	4	7	6	2	3	8	6	2	1	5	5	4	4	8	8	5	3	103	4.905	24.059
1930	3	8	4	4	5	2	6	5	2	7	4	3	6	5	2	4	8	1	6	4	8	97	4.619	21.335
1931	9	11	10	7	16	9	10	8	9	8	4	10	10	15	7	7	13	16	5	7	12	203	9.667	93.451
1932	8	9	9	8	5	13	9	10	8	6	9	6	6	14	6	10	8	6	8	8	5	171	8.143	66.308
1933	3	7	6	7	6	3	4	3	4	4	8	6	4	7	5	8	7	3	7	8	3	113	5.381	28.955
1934	7	5	6	6	4	3	5	8	4	8	5	6	7	4	4	7	6	6	7	4	3	115	5.476	29.987
1935	10	11	8	6	14	7	14	12	6	7	8	7	16	15	7	4	4	9	7	8	14	194	9.238	85.341
1936	6	5	8	9	7	8	9	7	7	6	6	5	5	8	7	3	2	8	4	5	8	133	6.333	40.107
Totals (Sum of the squares of the individual values = 42,041)																						4,879	232.333	1,798.095

sheet, this one being for rains of one inch or more in 24 hours. By substituting the values from Table 1 in the formulas, the values of N_d were computed as follows:

$$\sigma_{21} = \sqrt{\frac{1{,}798.095}{32} - \left(\frac{232.333}{32}\right)^2} = \sqrt{3.4771}$$

$$\sigma_1 = \sqrt{\frac{42{,}041}{672} - \left(\frac{4{,}879}{672}\right)^2} = \sqrt{9.8476}$$

$$N_d = \left(\frac{\sqrt{3.4771} \times \sqrt{21}}{\sqrt{9.8476}}\right)^2 = 7.415.$$

The corresponding standard error equals 1.854. Calculations of N_d were made for rains of various intensities only up to 4 in. or more per day. Heavier rains were so few that N_d could not be determined with any degree of reliability.

Similar tabulations and calculations were made from 20 years (1917 through 1936) of rainfall data for 15 stations in the quadrangle between parallels 33° and 35° north, and meridians 79° and 81° west, comprising parts of North and South Carolina. This area is Quadrangle 7-H of the Miami study.

Table 2 gives the values of N_d and of N_a derived from the data for the areas in Iowa and in South Carolina. The values in the table show that, in general, the more intense storms cover a smaller area than those yielding lower quantities. When N_d or N_a approximate unity, the occurrences may be considered to be random events. On the average an occurrence of 3 in. of rainfall will affect only one station per storm day in Iowa, but in South Carolina about four stations would be affected, and this in spite of the fact that the stations are more widely spaced in South Carolina.

TABLE 2.—VALUES[a] OF N_d AND N_a, WITH THEIR STANDARD ERRORS, FOR VARIOUS RAINFALL DEPTHS, IN A 2° QUADRANGLE IN IOWA AND IN NORTH AND SOUTH CAROLINA

Rainfall, in inches per day	IOWA (21 STATIONS, 32 YEARS)		NORTH AND SOUTH CAROLINA (15 STATIONS, 20 YEARS)	
	N_d	N_a	N_d	N_a
One or more	7.42±1.85	3.52±0.58	8.49±2.68	3.10±0.57
Two or more	4.20±1.49	2.05±0.47	6.04±1.90	1.95±0.47
Three or more	1.46±0.73	1.72±0.40	3.90±2.08	1.72±0.41
Four or more	1.00 (approximately)	1.37±0.31	1.70±1.07	1.68±0.78

[a] N_d equals number of stations to be used in calculating the reliability of rainfall frequencies, and N_a equals average number of stations affected by storms of a given intensity.

There are three bases upon which one might compute a standard error of the average frequency of specified amounts of rainfall. One of these is the sum of the occurrences, which is the one assumed by the station-year method. Another is the number of storms, which is equivalent to the total occurrences divided by the average number of stations affected. The third uses N_d in place of average number of stations. The differences in standard error obtained by the first and the second of these methods have been shown earlier in the paper for a theoretical case. In Table 3 these comparisons are made for the actual rainfall data for the areas in Iowa and the Carolinas, the computations being:

TABLE 3.—COMPARISON OF THE STANDARD ERRORS OF RAINFALL FREQUENCIES BASED ON TOTAL OCCURRENCES, ON NUMBER OF STORM DAYS, AND ON N_d

Rainfall, in inches per day	(a) IOWA				(b) SOUTH CAROLINA			
	Frequency (years)	Standard Error (%) by:			Frequency (years)	Standard Error (%) by:		
		Station-year method	Storm days	N_d		Station-year method	Storm days	N_d
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
One or more...	0.14	1.4	2.7	3.9	0.08	1.6	2.8	4.7
Two or more...	0.80	3.4	4.9	7.1	0.38	3.6	5.1	8.8
Three or more..	3.10	6.8	8.8	8.2	1.47	7.0	9.2	13.9
Four or more..	10.67	12.6	14.7	14.0 (approximately)	3.80	11.2	14.1	14.7

Column Nos.	Computation
1	$\dfrac{\text{Stations} \times \text{years}}{\text{Occurrences}}$
2	$\pm \dfrac{1}{\sqrt{\text{Occurrences}}}$
3	$\pm \dfrac{1}{\sqrt{\left(\dfrac{\text{Occurrences}}{\text{Average number of stations}}\right)}}$
4	$\pm \dfrac{1}{\sqrt{\left(\dfrac{\text{Occurrences}}{N_d}\right)}}$

The availability of either the value of N_a or N_d makes possible an approximate answer to the question proposed by C. S. Jarvis,[21] M. Am. Soc. C. E., and others regarding the equivalence of a station-year record to a record of the same length for a single station. In the Carolina quadrangle, for rainfalls of one inch or more per day, on the average 3 stations record that amount in a rain day. As a result there are only $\frac{15}{3}$ or 5 independent stations. The station-year record for this quadrangle of 300 station years (15 stations for 20 years each) could not be considered equivalent to 300 years for a single station, but would be more nearly equivalent to 5 × 20 or 100 years; one might call this the valid station years of record based on the average number of stations affected per storm. For the foregoing reasons it would be still better to use the value N_d rather than N_a. This would give 36 valid station years for rainfalls of one inch for the Carolina quadrangle.

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[21] Discussion by C. S. Jarvis of "Rainfall Intensities and Frequencies," by A. J. Schafmayer and the late B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 394.

STANDARD ERRORS FOR FREQUENCIES OF PLUVIAL INDEXES OF THE MIAMI STUDY

Although the true reliability of frequency values should be determined on the basis of the degree of persistence in the data, it is possible to give some indication of the reliability from the total number of occurrences of the given rainfall depth appearing in the station-year record.

A practical application of the standard error as a measure of the reliability of rainfall frequencies may be made in connection with the charts of pluvial indexes for quadrangles in Eastern United States, published by the Miami Conservancy District.[11] For each 2° quadrangle these charts give the maximum rainfall (one day to six days) to be expected once in 15 years, 25 years, 50 years, and 100 years. There is given, also, for each quadrangle the total number of station years used in the computations.[22] The number of station years divided by 100 gives the rank of the 100-yr pluvial index in the series of rains arranged in order of depth. For example, for Quadrangle 8-E (see Fig. 2(*d*)) in western Ohio, the pluvial-index of 5.2 in. for the 100-yr frequency has a rank of 15, obtained by dividing 1,524 (station years) by 100. In other words, this pluvial index with an expectancy of once in 100 years was based on 15 occurrences. A percentage standard error in the frequency may then be given as $\frac{1}{\sqrt{15}}$, or 26%, and the average frequency should be written as 100 years ± 26 years. A percentage standard error on the basis of all occurrences has been calculated for all quadrangles of the Miami study for the 100-yr, 50-yr, 25-yr, and 15-yr frequencies, and these percentages are shown in Fig. 2. Errors were calculated from the total occurrences with no correction for dependence in the data.

Since the number of occurrences will depend upon the intensity, the number of years of record, and the number of stations, the reliability of the frequency will be least for high rainfall amounts in quadrangles with the fewest station years of record. It will be seen that about one fifth of the quadrangles have standard errors of 50% for the 100-yr pluvial index. For these quadrangles the expectancy should be given as once every 100 years ± 50 years.

SUMMARY

It has been shown that two factors affect the accuracy of average frequency values of precipitation determined by the station-year method—the number and distribution of stations and the length of independent record. The second of these factors has been discussed in some detail. It has been shown

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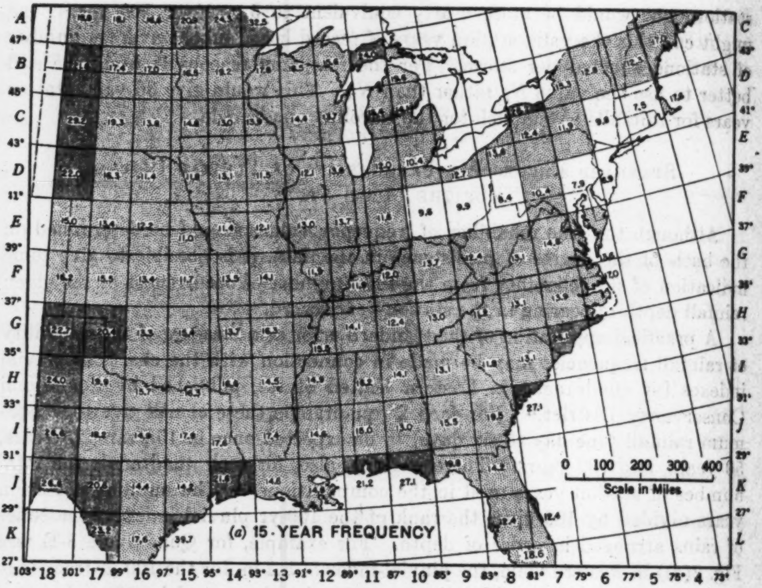
[22] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed., 1936, Dayton, Ohio, Fig. 37, p. 81.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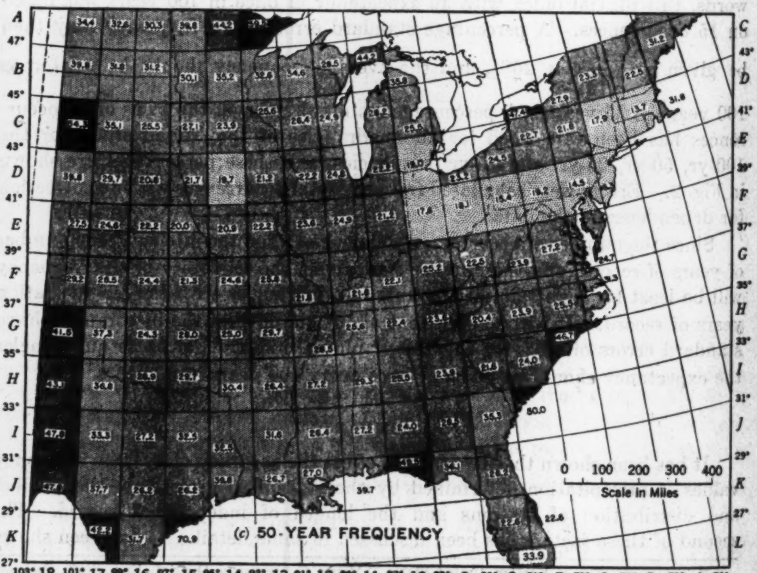
0 100 200 300 400
Scale in Miles

(a) 15-YEAR FREQUENCY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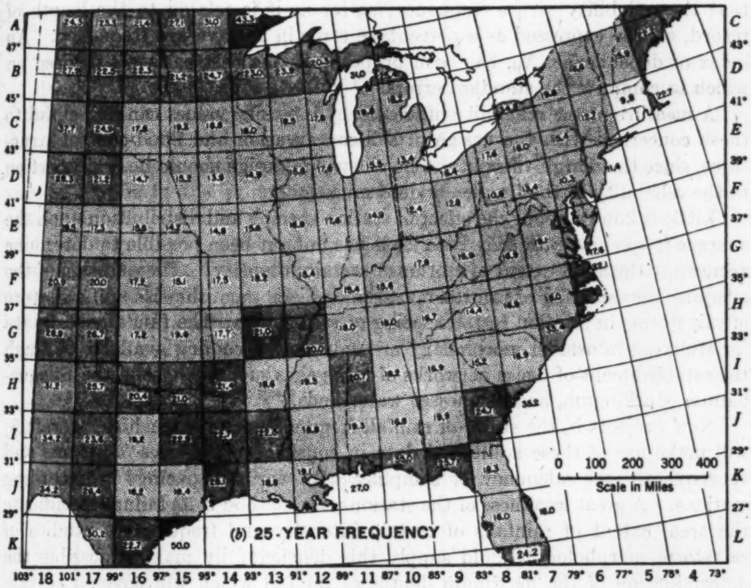
Scale in Miles

(c) 50-YEAR FREQUENCY

FIG. 2.—PERCENTAGE STANDARD ERRORS IN STORM-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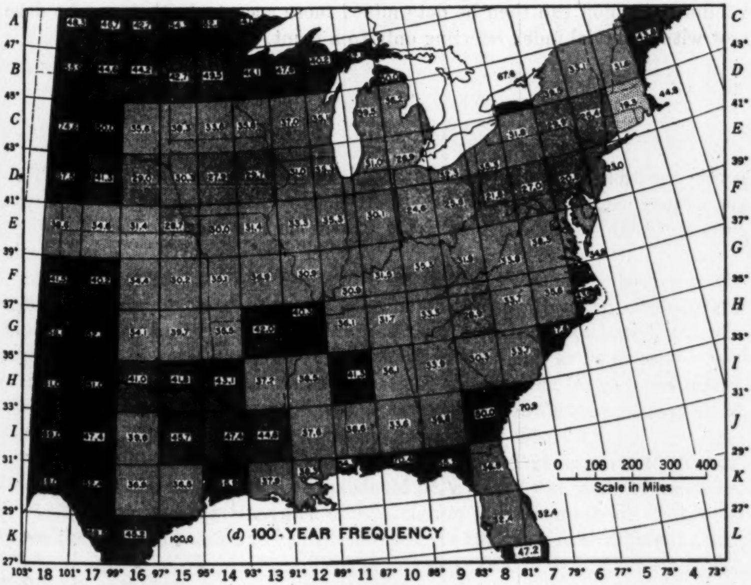
Scale in Miles

(*b*) 25-YEAR FREQUENCY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0 100 200 300 400
Scale in Miles

(*d*) 100-YEAR FREQUENCY

FREQUENCY DETERMINATIONS OF MIAMI STATION-YEAR STUDY that the reliability of the frequency, as far as it is related to the length of record, can be expressed as a percentage error in the average frequency. An index of dependence, N_d, has been proposed as the best basis yet devised on which to compute the standard errors.

A measure of the reliability of frequency-intensity values can be helpful to those concerned with the design of soil-conservation and flood-control structures, since the margin of safety to be allowed in design should be in proportion to the reliability of the frequency determinations.

Little is known about the effect of station spacing and distribution upon the average frequency, primarily because it has not yet been possible to determine accurately the areal extent of storms of certain intensity. The question of the adequacy of sampling of rainstorms, also, depends upon the size and structure of rainstorms in relation to the spacing of rain gages. The fundamental data for studying rainstorm morphology are only now becoming available through the establishment of dense networks of rain gages such as those in the Susquehanna, Muskingum, and Tennessee watersheds.

New projects in the study of rainfall frequencies and intensities, therefore, will make use of these additional data in considering the sizes of storms and consequently the adequacy of sampling with various spacings of recording stations. A great weakness of the station-year method is its failure to indicate the areal extent of rainfalls of certain intensity and frequency. Studies of rainstorm morphology should supply this deficiency by making possible the determination of the maximum depths of rainfall which will cover a certain area and occur with a given frequency. The probable frequency of specified volumes of runoff can then be determined more accurately than can be done now with a pluvial index referring only to a point.

DISCUSSION

Paul V. Hodges,[23] **M. Am. Soc. C. E. (by letter).**—In the determination of the probable maximum floods to be expected in different parts of the country, the pluvial indexes and their standard errors give a basis from which comparisons and computations can be made. This paper is of considerable assistance and value in the proper appraisal of records of storm rainfall over the eastern half of the United States.

In an unpublished study of the Grand River, in South Dakota, by the writer, to determine the probable maximum flood, for a spillway design, the pluvial index of Quadrangle 18–B was used (see Fig. 2). The pluvial indexes, in inches, for this quadrangle are given in Table 4*A*. The indication of the standard error as it affects the frequency in years is as follows:

Frequency, in years	Pluvial index (in inches)
15± 3.3	2.7
25± 7.0	4.2
50±19.9	4.4
100±55.9	4.8

The extremes of these periods were plotted against the pluvial index. By connecting the plotted points the range in the pluvial index for any frequency is obtained as follows:

Frequency, in years	Range in the pluvial index (in inches)
15	2.3 to 3.5
25	3.5 to 4.31
50	4.3 to 4.85
100	4.54 to 5.27

The errors of these various quantities are reflected in their plotting. With more Weather Bureau stations, well distributed, and with longer records, the points, no doubt, would plot on smooth curves and the results would be more consistent.

Maximum Rainfall to be Expected Once in 100 Years.—To determine the probable maximum flood to be expected on the Grand River in northwestern South Dakota the "Hydrograph Method" was used and compared with the largest floods of nearby streams. The maximum rainfall to be expected once in 100 years for storms of different durations was derived for different areas. The pluvial index for quadrangles of the Grand River drainage area is given in *Technical Reports*, Part 5, of the Miami Conservancy District.[4]

The pluvial index gives the rainfall to be expected at any point. To determine what the depth over an area would be, a study was made of a number of storm areas and the time-area-depth relation was determined. Table 4*B* shows the depth of rainfall to be expected once in 100 years over different areas.

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[23] Hydr. Engr., U. S. Indian Irrig. Service, Los Angeles, Calif.

The quantities given in this table were derived from the pluvial indexes of two quadrangles, 18–B and 18–C, in the northwestern part of South Dakota.

TABLE 4*A*.—PLUVIAL INDEXES, IN INCHES

TABLE 4*B*.—INDEXES OF RAINFALL TO BE EXPECTED ONCE IN 100 YEARS

Duration of storm, in days	FREQUENCY, IN YEARS				TIME-AREA-DEPTH RELATION			
	15	25	50	100	At the point	4,085 sq miles	5,664 sq miles	6,000 sq miles
1	2.0	2.5	3.4	4.0	4.50	3.20	3.03	3.00
2	2.7	3.3	4.1	4.2	5.10	3.60	3.43	3.40
3	2.7	4.0	4.3	4.5	5.50	4.10	3.93	3.90
4	2.7	4.2	4.4	4.8	5.70	4.30	4.05	4.00
5	2.7	4.2	4.4	4.8	5.80	4.40	4.15	4.10
6	2.7	4.2	4.4	4.8	5.80	4.40	4.15	4.10
%*	21.8	27.9	39.8	55.9	* Standard error (percentages)			

Flood Runoff, Grand River Near Wakpala, S. Dak.—The runoff of the Grand River near Wakpala to be expected once every 100 years is computed by using a depth of rainfall of 4.15 in. and estimating that 30% will appear as runoff:

Rainfall of 4.15 in. = 0.346 ft;
0.346 × 5,664 × 640 = 1,255,000 acre-ft of precipitation; and
30% runoff = 377,000 acre-ft of runoff = 190,000 sec-ft-days of runoff.

Flood Runoff, Grand River at Blue Horse Dam Site.—The runoff of the Grand River at the Blue Horse Dam site to be expected once every 100 years is computed by using a rainfall depth of 4.40 in. and estimating that 30% will appear as runoff:

Rainfall of 4.40 in. = 0.367 ft;
0.367 × 4,085 × 640 = 960,000 acre-ft of precipitation; and
30% runoff = 288,000 acre-ft of runoff = 145,000 sec-ft-days of runoff.

Unit Hydrographs.—From runoff records of the Grand River near Wakpala, hydrographs for a number of high-water periods were plotted, and the total discharge for the periods was computed and compared with the precipitation. A unit hydrograph was determined by computing the average daily percentages of the total flow during the periods. In order to determine a unit hydrograph for the Grand River at the Blue Horse Dam site, the peak flow was based on a comparison with the probable maximum peak of Grand River near Wakpala, and the base of the hydrograph was shortened to agree with the hydrographs of nearby streams having drainage areas of nearly the same size.

The discharge during a maximum flood period was computed to be as shown in Table 5. The plotting of these daily discharges shows that the peak discharge or flood peak would reach about 60,000 cu ft per sec for each of the locations on the river.

Comparative Method.—A study of recent floods, showing maximum peaks of record, occurring in South Dakota, Montana, and Wyoming, together with the expected Big Horn River flood from a hypothetical transposition of the 1923 Powder River storm to the Big Horn Basin, has been made. The aforementioned floods were exceptionally high and probably represent about the maximum for those streams.

TABLE 5.—MAXIMUM GRAND RIVER FLOOD

Day	NEAR WAKPALA		AT BLUE HORSE DAM SITE	
	Percentages	Average discharge (cu ft per sec)	Percentages	Average discharge (cu ft per sec)
1	1.2	2,280	3.0	4,350
2	6.3	12,000	16.1	23,300
3	18.8	35,700	35.8	51,900
4	27.8	52,800	24.1	35,000
5	20.1	38,200	11.9	17,260
6	12.7	24,100	5.4	7,830
7	6.3	12,000	2.4	3,480
8	3.7	7,030	1.3	1,880
9	1.9	3,610	....	....
10	1.2	2,280	....	....
Total	100.0	190,000	100.0	145,000

The formula

$$Q = 933\,A^{0.491} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

shows the maximum flood to be expected, if such flood is of the same magnitude as shown by maximum recorded or computed hypothetical floods. Using Eq. 3, and computing for the maximum flood to be expected on the Grand River, the following quantities are obtained:

Location	Drainage area, A, in sq miles	Maximum discharge, Q, in cu ft per sec
Grand River near Wakpala	5,664	64,800
Grand River at Blue Horse Dam site	4,085	55,300

The station-year method of determining the accuracy of rainfall frequency gives the solution of the standard error of the frequency of rainfall as it occurs at particular stations. The "dependence between stations" is an attempt to show the areal effect, or to consider contemporary rainfall at nearby stations. It does not, of course, give the error of frequency of areal storm depths.

The results of this paper can be used and are adaptable for the areal determination of the limits of storm rainfall to be expected, as it appears certain that the standard error of storm frequency as computed by the station-year method is considerably larger than the standard error of frequency of areal storm depths.

The comparison between the standard error of frequency of rainfall and the approximate error in depth of rainfall is given for Quadrangle 18–B as follows:

Frequency		Depth of Rainfall	
Years	Standard error (%)	Inches	Error (%)
15	21.8	2.7	22.0
25	27.9	4.2	9.6
50	39.8	4.4	6.4
100	55.9	4.8	7.6

In the study of flood runoff the areal extent of storm rainfall is desired.

The flood study, from the pluvial index and the indicated standard error, was made to show its application to relatively large areas, and an allowance for the indicated error was considered, or in other words, a larger pluvial index was used to cover the possibility of the occurrence of a more intense storm.

The sizes of the probable maximum flood to be expected, as determined by using the pluvial index, adjusted for the probable error, and compared with the maximum floods of nearby streams indicate a fairly close agreement and show that the available data give a fair criterion for similar studies.

C. S. JARVIS,[24] M. AM. SOC. C. E. (by letter).—The station-year approach to rainfall frequencies, magnitudes, and normal patterns of distribution, intensity, and recurrence has long been accepted with some reservations and misgivings. Its use persisted because of the obvious advantages which it offered in cementing short segments and forming therefrom a synthetic record of impressive length and plausibility, with evident claims to reliability that could not be readily disproved.

The service performed by the author in making the exhaustive analysis, rigorous tests, and logical findings cannot be appraised adequately until one explores the labyrinths of disjointed, intermittent, and fragmentary records that have heretofore been set aside, much as is the custom for segregating refractory ores, in the hope that processes may yet be devised for their reduction and the sure recovery of their intrinsic values. Such a process is now available, largely as a result of this paper.

One must recognize, however, that more than one method is used for deriving frequency of rainfall. This subject is discussed under the heading "Introduction," where five occurrences in excess of a given amount within a record comprising 500 station years are regarded as "equivalent to an average frequency of 100 years per occurrence, which is the same result that would be obtained by taking the fifth highest amount in a station-year record of 500 years." Here is where another method has been used, along the following lines of reasoning: The maximum value occurred once in 500 years; the second from the top value, once in 250 years; the third is likely to occur once in 125 years; the fourth, once in 62.5 years; and the fifth, once in 31.25 years. The question is, which of the two methods should prevail, or is there a third approach? Should one recognize the zone between the values of 100 and 31.25 as being the range of uncertainty regarding probable frequency?

As it happens, nearly a comparable variation in results occurs under the heading, "Use of N_d for Actual Rainfall Data in Determining Standard Errors of Frequencies of Rainfall Amounts," in answer to the writer's question of some years ago. There, the

> "* * * 300 station years (15 stations for 20 years each) * * * would be more nearly equivalent to 5 × 20 or 100 years; one might call this the valid station years of record based on the average number of stations affected per storm. For the foregoing reasons it would be still better to

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[24] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

> use the value N_d rather than N_a. This would give 36 valid station years for rainfalls of one inch for the Carolina quadrangle."

Does this mean that the best one can do is to bracket the most probable values between those limits?

It was the writer's good fortune to view the early results of the author's current research, some years ago, and to utilize some of the findings to good advantage. Thereafter, the writer urged the need not only for further exploration and tests, but also for their prompt release to the profession through technical publications, such as have been chosen. The paper seems to present much data, many tests, and a certain maturity of concept beyond the tentative and informal accounts of research to which the writer had access some years ago. Now, having observed the modest beginnings, the step-by-step development, and the seemingly authoritative analyses and solutions of successive problems, and having tested them by the practical standards that have been acquired throughout a varied experience in hydrologic fields, the writer indorses the paper as a distinct scientific achievement.

Perhaps a lack of perception will account for the writer's inability to reconcile some minor details thus far, but he anticipates that these matters will be cleared up in the author's closing comments.

Howard W. Brod,[25] **Jun. Am. Soc. C. E.,** (by letter).—It is often more important to understand the limitations of a law than to know the law itself. The author unmistakably implies that the station-year method is applicable to large drainage areas. It is significant to point out that the Miami Conservancy District confined the use of the station-year method to the "* * * design of sewer systems, bridge and culvert openings for small drainage basins, and in the design of dams, levees, and channel improvements where the watersheds involved are not more than a few square miles in area."[26] Furthermore, the practice of grouping stations with markedly different rainfall characteristics is questionable.[27]

There are two fundamental weaknesses in the station-year method which restrict its applicability to a very limited class of data: (*a*) The variation of seasonal rainfall; and (*b*) the dissimilarity of rainfall stations.

(*a*) *Variation of Seasonal Rainfall.*—First, consideration will be given to the variations in seasonal rainfall quantities. From the interpretations that have been made of the longest available records of floods (Nile River, Egypt), it is evident that no secular or long-time changes in climatic conditions are taking place. Cyclic variations, however, are an integral characteristic of seasonal rainfall. There seems to be an inertia in the unbalance of the atmospheric forces which causes prolonged series of wet seasons and dry seasons to take place. The results that would be obtained if the station-year method were applied to a set of data which lies wholly within a wet phase or wholly within a dry phase might lead to tragic consequences, such as a dam failure or a shortage in water supply. Reference is made to the case cited in a National Resources Committee report[28] of a federal project which was based on a 20-yr hydrologic record. "Unhappily, the years of record seemed to have coincided with a wet period in the precipitation cycle. When the dry period came the project could be developed to only one-third its intended size."

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[25] Engr., The Panama Canal, Diablo Heights, Canal Zone.

[26] *Technical Buletin No. 5*, Miami Conservancy District.

[27] "Rainfall Intensities and Frequencies," by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 344.

The relevancy of the cyclic variations of seasonal rainfall to the occurrences of high rates of rainfall is readily apparent to those who have tried to correlate major floods with annual rainfall. Although it is possible to have a major flood during a comparatively dry year, it would be the exception rather than the rule; by far the greater number of major floods occur in comparatively wet years. Similarly, a greater number of high rates of rainfall will take place during wet years than during dry years and, therefore, such occurrences will parallel the cyclic variations of seasonal rainfall.

(*b*) *Dissimilarity of Rainfall Stations.*—On a purely theoretical basis, the station-year method is invalid because it assumes that there is equal opportunity for every station to receive an equal quantity of rain. Of course, this is not true since the characteristics of rainfall stations are similar to those of fingerprints in that no two of them are alike. No two rainfall stations can have the same conditions of geographic location, elevation, slope, exposure, distance from the ocean or other large body of water, etc., and, therefore, the rain-catching potentialities of any two rain gages cannot be exactly alike.

The station-year method is valid only when the data do not conflict with the limitations made in items (*a*) and (*b*). Condition (*a*) may be satisfied by developing an "index" of mean seasonal rainfall for the region for a sufficiently long period of time to assure a reliable approximation of the mean seasonal rainfall. This may be accomplished satisfactorily on the basis of a single long-term record, but preferably more. If no long-term records are available in the region, a meteorologic study of the storm paths that are characteristic of the region should be made, and the characteristics of the "index" of mean seasonal rainfall of the adjacent regions which influence the rainfall of the region under consideration should be transferred to that region. Condition (*b*) may be considered negligible in small areas of comparatively uniform meteorologic characteristics and in certain regions where the differences in the areal distribution of rainfall quantities are equal to or less than the error with which a rain gage represents the average rainfall over the area in the vicinity of the rain gage.

There are several other phases of the subject of rainfall probabilities upon which the author touches which provoke discussion. Among these the determination of dependence or independence of the events or observations is of prime importance. Eugene L. Grant, M. Am. Soc. C. E., has pointed out that one of the weaknesses of the flood-probability method lies in the fact that "the successive floods on a given stream in a given year are certainly not independent events." The same is true regarding successive rainfall events of the frontal type. When a series of fronts (for example, cold fronts) invade a region in which warm air has been stagnating, the first cold front meets a great opposition in meteorologic characteristics. With the passage of subsequent fronts, the meteorologic characteristics of the region become modified and the differences between the invading air and the residual air become less. The phenomena are similar to the change in saturation conditions for successive floods. The meteorologic conditions often associated with major floods consist of a series of fronts passing over a region. The successive storms associated with the passage of these fronts are certainly not independent events.

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[28] "Deficiencies in Basic Hydrologic Data," National Resources Committee, September, 1936.

The principal reasons for the superiority of rainfall probabilities over flood probabilities are that the aforementioned dependency is true for only a small part of a rainfall record and that rainfall is a function of much fewer variables and therefore requires less data to establish the position of the frequency curve.

The inadvisability of using the nominal, or published, 24-hr rainfalls as the basis of a frequency study is indisputable. This practice violates the requirement of independency of events and also brings up the question of "How great, in the long run, is the difference between nominal and actual maximum 24-hr rainfall?" The independence requirement of the rainfall events is violated when the end of the nominal 24-hr rainfall comes within the storm period and a single event is indicated as two occurrences. The differences in the results that may be obtained using nominal or published 24-hr rains are shown in Fig. 3. The example has been selected at random, and therefore it is probable that even more serious differences exist.

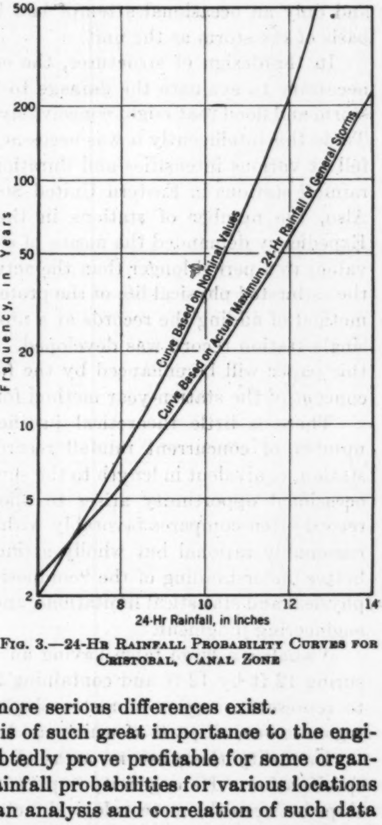

FIG. 3.—24-HR RAINFALL PROBABILITY CURVES FOR CRISTOBAL, CANAL ZONE

The subject of rainfall probabilities is of such great importance to the engineering profession that it would undoubtedly prove profitable for some organization to conduct an investigation of rainfall probabilities for various locations throughout the world. The results of an analysis and correlation of such data should prove of unlimited value. As the author points out, the introduction of the area factor into frequency studies of rainfall has not yet been made and is an outstanding deficiency in hydrologic research. The work of the hydrology organizations of the U. S. Soil Conservation Service and Tennessee Valley Authority is directed toward overcoming this deficiency and should represent a great advance in the application of probability technique to rainfall data.

MERRILL BERNARD,[29] M. AM. SOC. C. E. (by letter).—Efforts to inject mathematical precision into the treatment of hydraulic problems that rely upon precipitation for solution are timely. Too often progress in fields of applied science depends upon the ability to bolster an inadequate sampling medium with assumptions that are never wholly satisfying. Such was the necessity faced by the Miami Conservancy District in 1916. Until that time rainfall records were considered to be so many independent point experiences, and only an occasional attempt had been made to organize the data on the basis of the storm as the unit.

In the design of structures, the engineers of the Miami project found it necessary to evaluate the damage to be expected from the highly infrequent storm and flood that might conceivably damage or destroy the protection works. To do this intelligently it was necessary to know the frequency with which rain fell at various intensities and durations. At the time there were only a few rainfall stations in Eastern United States with records greater than 30 years. Also, the number of stations in the network was admittedly inadequate. Expediency demanded the means of extrapolating the frequency of the greater values to a period longer than the actual record and in reasonable accord with the estimated physical life of the protection system. Thus, the "station-year" method of adding the records at a number of stations to obtain an equivalent single station record was developed. It is believed that an understanding of this paper will be enhanced by the following attempt to present a simplified concept of the station-year method for determining rainfall frequency.

There is little theoretical justification for combining an indiscriminate number of concurrent rainfall records to arrive at the record for a single station, equivalent in length to the sum of the station years; and yet, where the occasional opportunity arises to allow such a comparison, the synthesized record often compares favorably with the actual. The writer will resort to a reasonably rational but wholly artificial means of attempting to arrive at a better understanding of the "composite record" or "station-year" method, its physical and statistical limitations, and any success it may have had in guiding engineering judgment.

Visualize a large table having an area confined within a raised edge measuring 12 ft by 12 ft and containing 20,736 sq in. The table top is intended to represent a region of meteorological homogeneity. By definition, such a region is one, all parts of which can be said to have the same rainfall regimen.

In reality there are no areas of appreciable extent that conform to this specification. Homogeneity implies, first, a complete stability in the relationships between the meteorological factors involved in the mechanism of a storm and, second, a consistent action from storm to storm of surface influences, such as mountain barriers, abrupt changes in elevation, and the proximity of appreciable water surfaces. It is understood, then, that the suggested demonstration "region" is homogeneous only within the tolerances imposed by the foregoing considerations.

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[29] Hydrologic Director, U. S. Weather Bureau, Washington, D. C.

Next, prepare three sets of concentric rings of light metal, symmetrically spaced and held rigidly in place according to the specification in Col. 1, Table 6. For the moment, ignore the artificiality of the medium, and visualize, in the sets of concentric rings, the isohyetal pattern of rainfall idealized as to shape.

TABLE 6.—ANALYSIS OF STORM VISITATION

Ring diameter, in in. (1)	Area, in sq in. (2)	Incremental area, in sq in. (3)	Average depth, in in. (4)	No. of times contained in total area (5)	Average No. of throws to make one hit (6)	Minutes at end of which one hit is made (7)	Average No. of hits at the time first hit in smallest circle is made (8)	No. of hits at end of 17,800 min (9)	ΔA (10)	ΔB (11)	ΔC (12)	Summation of Cols. 10, 11, and 12 (13)
(*a*) STORM *A* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 12-MIN PERIOD)												
3	7	7	13.0	2,960	1,480	17,800	1	1	1	...	...	1
5	20	13	9.3	1,600	800	9,600	2	2	2	...	...	2
8	50	30	7.5	690	345	4,150	4	4	4	6	...	10
12	113	63	4.0	329	164	1,970	9	9	9	29	41	79
16	201	88	1.0	236	118	1,420	13	13	13	70	341	424
(*b*) STORM *B* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 4-MIN PERIOD)												
3	7	7	8.3	2,960	1,480	5,920	1	3	3	3	...	6
5	20	13	7.3	1,600	800	3,200	2	6	4	6	...	10
8	50	30	6.3	690	345	1,380	4	12	5	12	...	17
12	113	63	4.3	329	164	656	9	27	8	27	35	70
18	254	141	1.7	147	74	296	20	60	12	60	215	287
24	452	198	0.5	105	52	208	28	84	14	84	539	637
(*c*) STORM *C* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 1½-MIN PERIOD)												
3	7	7	5.7	2,960	1,480	2,220	1	8	6	15	8	29
5	20	13	4.9	1,600	800	1,200	2	15	8	21	15	44
8	50	30	4.3	690	345	513	4	35	...	...	...	...
12	113	63	3.3	329	164	246	9	72	10	37	72	119
18	254	141	2.2	147	74	111	20	160	12	52	160	224
26	530	276	1.1	75	38	57	39	312	13	68	312	393
36	1,017	477	0.5	43	22	33	67	539	...	...	...	...

Usually such a pattern is elongated and irregularly elliptical. It is believed little would be added to the experiment if differences in pattern shape were included as a variable.

There are innumerable shades of difference in the characteristics of storms, and only a generalized classification is possible. A principal distinction lies in the inverse relation between intensity and the duration and areal extent of the storm. Three generalized storm types are pictured in the depth-area curves shown in Fig. 4. Storm *A* is the highly intense downpour of limited extent that produces the phenomenal depths of rainfall recorded in certain parts of the United States. Storm *C* is the typical "warm front" storm producing moderate intensities over considerable area; and storm *B* is of intermediate type, often combining the characteristics of types *A* and *B*.

In Table 6, consider the differences in the diameters to fix the widths of the concentric bands between the rings (isohyets) of each "quoit" (storm). It is realized that the duration of storms *A*, *B*, and *C* will vary considerably. It will not detract too greatly from the reality of the experiment if an average duration is adopted for all.

Although no attempt is made to simulate quantitative relationships found in actual storms, it will be reasonable to consider square inches in the model as equal to square miles of the prototype. Depths have been computed from Fig. 4 that are to be taken as averages for the concentric bands between the isohyets.

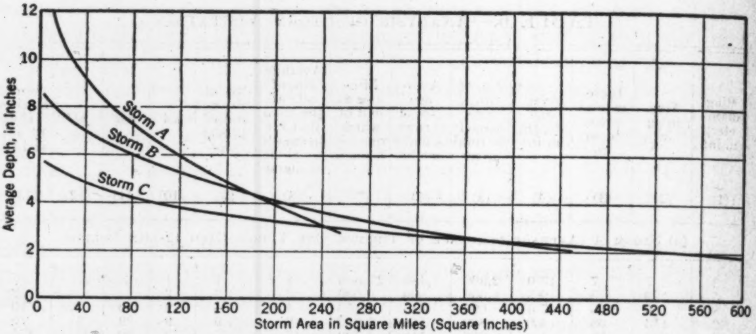

FIG. 4.—AREA-DEPTH CURVES, HYPOTHETICAL STORMS

A region of meteorological homogeneity also can be defined as one in which a storm, having entered it, can center over any point within its limits with equal probability. Without going into mechanical detail, assume the means of injecting the storm quoits to, and on, the table with the impersonal result of mechanically thrown dice, or under the assumption that the quoit, having had energy applied to it, is under perfectly balanced compensative influences so that it can come to rest in any position on the table with complete fortuity. Attention is now directed to Table 6, in which Cols. 1 and 2 are isohyetal diameter and enclosed area, respectively. Col. 3 is the area of the concentric band between a given ring and the next smaller one. Col. 4 is the assigned average depth of the concentric bands between isohyets. Col. 5 is the number of times the respective incremental areas are contained in the entire table area. (The circles could be adjusted actually to space by changing the shape or overlap, but this is not considered necessary to the purpose of the discussion.)

The following sequence of average storm visitation to the table (region) is adopted:

Minutes	Storm	Minutes	Storm
1	*C*	9	*B*
2	*C*	10	*C*
3	*B*	11	*C*
4	*C*	12	*A*
5	*C*	1	*C*
6	*B*	2	*C*
7	*C*	3	*B*
8	*C*	etc.	etc.

At the beginning of the initial minute (year) of the record, a small upright peg (station) is placed in an arbitrary position on the table (region). The experiment now begins under the foregoing schedule by putting the mechanical quoit thrower into action. The peg can be taken to mark the center of one of the 2,960 areas measuring 3 in. in diameter contained in the entire table top (Col. 4). It is possible that the first quoit thrown could center over it, or the entire 2,960 opportunities could be exhausted before the peg (station) was encircled by the 3-in. ring. If an indeterminate number of runs are made, it will be found that the peg will be ringed as often in the first half as in the last half of the series so that the most probable number of throws necessary to ring the peg would be the average, or 1,480.

With only one peg on the table, by far the greater number of throws will miss the peg entirely (no rain recorded at the station). The number of times the peg falls between the isohyets of greater diameter will be in proportion to the opportunity afforded by the greater area of the concentric bands between the rings (isohyets).

Col. 6, Table 6, gives the average number of throws under the schedule to bring the peg within the areas fixed by the various isohyetal diameters. Col. 7 gives, for each storm type, the average number of minutes at the end of which one hit was made in each of the concentric bands. Col. 8 gives the number of hits for each isohyet diameter within the average number of minutes taken to ring the peg in the smallest isohyet of each storm type. Col. 9 gives the number of hits within the average number of minutes taken to ring the peg with the smallest isohyet of the least frequent storm *A* (17,800 min).

It is now seen that the peg (station) has experienced through the record period of 17,800 min (months, years) the visitation of three types of storms in varying sequence, producing varying depths of rainfall. Although storm *A* contributed the two greatest depths, the remainder of the depth-range was represented generally by all three storm types. In order, then, to organize the record at the peg (station), it is necessary to assemble the number of occurrences for each average depth from each type of storm, as shown in Cols. 10, 11, 12, and 13. These values are arranged in order of depth-magnitude

TABLE 7.—ANALYSIS OF STORM FREQUENCY

Average depths, in in.	No. of hits	FREQUENCIES, *F*				Average depths, in in.	No. of hits	FREQUENCIES, *F*			
		One station	Two stations	Twenty stations	Thirty-six stations			One station	Two stations	Twenty stations	Thirty-six stations
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
13.0	1	17,800	8,900	890	494	4.3	70	254	127	13	7
9.3	2	8,900	4,450	445	247	4.0	79	225	112	11	6
8.3	6	2,960	1,480	148	82	3.3	119	150	75	8	4
7.5	10	1,780	890	89	49	2.2	224	79	40	4	2
7.3	10	1,780	890	89	49	1.7	287	62	31	3	2
6.3	17	1,050	525	52	29	1.1	393	45	22	2	1
5.7	29	614	307	31	17	1.0	424	42	21	2	1
4.9	44	404	202	20	11	0.5	637	28	14	1	1

in Col. 1, Table 7. The frequency with which one peg (station) will experience various depths of rainfall is given in Col. 3, Table 7.

The demonstration is thus complete for a record cycle of 17,800 min at the peg (station). If the cycle is representative, and if 13 in. is considered to be the maximum depth for the region, even if it were experienced in the first minute (month, year) of record, the peg (station) experience must continue throughout the course of the cycle if it is to be confirmed as a maximum value.

The analogy between the table top and the region is strengthened somewhat if the former is considered covered with green baize and the outline of each quoit (storm pattern) is made with chalk as each throw comes to rest. The result at the end of a complete series of throws will be a completely covered table top (regional map) with superimposed storm patterns giving the same impression as that gained by superimposing maps prepared by the Miami Conservancy District.[30]

If station 1 is 100 ft from station 2, it will be impossible for station 1 to have, say, 3 in. of rainfall without station 2 experiencing a fall somewhat near this quantity. In other words, the record accumulating at station 2 is not independent of that at station 1. As station 2 is moved away from station 1, its dependency upon the latter will become less and less until the two records no longer represent that of concurrent rainfall, although statistically both may have the same frequency curves.

Dependency may be illustrated on the table top and with the hypothetical storms *A*, *B*, and *C*. With station 1 at the center of storm *A*, the dependency of the record at station 2 for a storm of this type will reduce in proportion to the lengthening radius of storm *A* to its outer limit of 8 in. (miles). With the peg (station) at the outer limit of the storm, a distance between stations 1 and 2 must be equal to at least the diameter (16 in.) to insure complete independence of record. Under these requirements the minimum distance for storm *B* would be 24 in., and for storm *C* 36 in.

With this fact in mind locate peg (station) 2 at some arbitrary point on the table and at a greater distance than 36 in. from peg (station) 1. Now subject the table (region) to a series of storm visitations according to the established schedule. Because of the wholly fortuitous delivery of the quoits (storms), one station or the other will ring (record) the 3-in. isohyet of storm *A* for an average series of throws in a time period, equal, on the average, to one half of that taken to make the same hit with only peg (station) 1 on the table. If other pegs (stations) are added, the average time taken to ring the 3-in. isohyet of storm *A* in any series of throws will be equal to the result of dividing the period for one peg (station) by the number of pegs (stations).

The dependency relationship between stations is affected also by the relative importance of depth. Considering the diameter of the isohyets assigned to the various depths to be the minimum distance between stations, it is seen from Fig. 5 that, if complete independency is to be assured, no more than 20 pegs (stations) can be utilized in record combination if 0.5 in. is considered;

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[30] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Ed., 1936, Dayton, Ohio, Figs. 54 through 65.

no more than 36 if 1.0 in. or more is taken into account; and no more than 300 if a depth of 6 in. or more is to make up the significant record.

Again, it is necessary to call attention to the artificiality of the synthesized rainfall region and the limitations of the assumptions under which the demonstration has been made. One can draw from it, however, certain conclusions

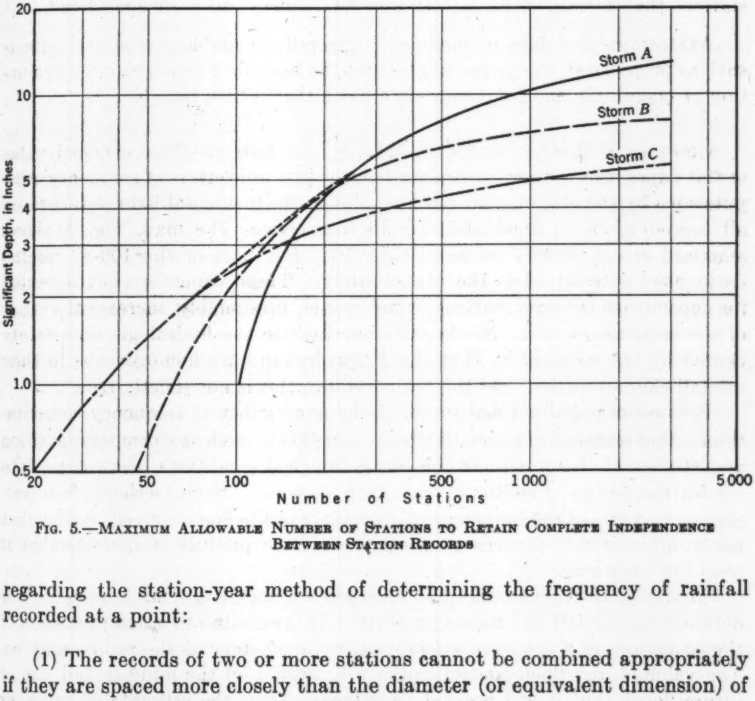

FIG. 5.—MAXIMUM ALLOWABLE NUMBER OF STATIONS TO RETAIN COMPLETE INDEPENDENCE BETWEEN STATION RECORDS

regarding the station-year method of determining the frequency of rainfall recorded at a point:

(1) The records of two or more stations cannot be combined appropriately if they are spaced more closely than the diameter (or equivalent dimension) of the storm area of significant rainfall depth.

(2) If a number of stations are so spaced, an approach to the ultimate record for one station, representative of the area, can be accelerated by combining their records, if they are of reasonable length—say greater than 5 or 10 years. From the synthetic example presented as Table 6, it is seen that 13 in. are reached at one of 36 stations in an average time of 494 min, whereas, at the end of this time, one of 20 stations would have registered about 10.0 in., and a single station only about 5 in. under the regimen of storm visitation adopted as typical.

(3) There is no basis in the combined records of a number of stations for the determination of the areal extent of storm rainfall. Records cannot be combined unless they are so spaced that only one station falls within the extent of the individual storm. As the station can occupy an infinite number of positions

within the storm pattern, its rainfall depth cannot be considered an index to depths at other points within the storm area.

(4) The apparent practical success of the station-year method lies in the fact that rainfall studies, such as that of the Miami Conservancy District, have been concerned primarily with the greater depths of rainfall; and therefore (as illustrated in Fig. 5) independency can prevail among a greater number of stations than if total storm rainfall and total storm area were considered.

As storm rainfall data are made more generally available, statistical methods such as proposed in this paper will become the basis for a much sounder evaluation of magnitude and frequency than has hitherto been possible.

Charles F. Ruff,[31] M. Am. Soc. C. E. (by letter).—The principal value of this paper is in the emphasis placed on the probable errors of frequency computations by the station-year method, and in the incidental light it throws on all computations of flood and rainfall frequency. The map, Fig. 2, shows standard errors of 30% to more than 50% for most of the 100-yr rainfall frequencies determined in the Miami study. These values are not corrected for dependence between stations, which would, presumably, increase the range of the statistical error. Neither do they include the hydrologic uncertainty caused by the assumption that the 2° quadrangles are homogeneous in their rainfall characteristics. As the author states, this is not strictly true.

Because of a general realization of the uncertainty of frequency computations, other methods of estimating maximum floods, such as storm transposition and studies of the worst possible meteorological conditions, have come into use for the design of spillways and other purposes. Such methods, however, give no measure of the frequency of occurrence of the flood derived, except that the number of coincidences assumed to occur to produce it indicate that it would be very rare.

Mrs. Clarke-Hafstad has not enlarged on the reason and necessity for determining rainfall and flood frequency. It is essential to a determination of the economics of a flood-control project, unsatisfactory as the results may be. The value of any flood-control works will depend on the number and size of future floods that it will prevent or reduce. Thus, the estimate of value, or "benefits" as it is termed, involves errors of the same order as occur in the frequency determinations. When the benefits far exceed the cost of the control works, such errors are not significant; but the writer has seen projects recommended in which the benefits were said to exceed the cost by as little as 10%. If a probable error of 30% in the frequency on which these benefits were computed exists, such a statement becomes misleading. The probable benefits would be roughly from 80% to 140% of the cost. It follows logically that in such a case the probable error of the frequency becomes critical in deciding whether the project is economically sound and should be built or not. The expenditure for such a project is not like the purchase of an automobile, but resembles more the purchase of a majority of the chances on a car which is to be raffled. The extent of the "gamble" involved is indicated by the standard error of the frequency determination.

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[31] San. Engr., Caribbean Arch't-Engr., New York, N. Y.

Mrs. Clarke-Hafstad's paper deals only with rainfall frequency. For small works such as contour plowing and terracing, the rainfall can probably be used directly; but for larger areas, the methods used to transform the rainfall into flood runoff involve the assumption of values for percentage of runoff or infiltration rates. These values vary from time to time and constitute a series of independent events that combine with the rainfall to produce the flood. Thus, the flood from a 50-yr rainfall is not necessarily a 50-yr flood. The writer feels that comment on this phase of the problem would be helpful.

One of the most serious defects of the station-year method is the lack of areal significance, as mentioned by the author. For a watershed containing several stations the simultaneous rainfall at all these stations is needed to estimate the flood. If there were absolute dependence between stations, the 50-yr rain would occur at all of them at the same time. If they were entirely independent, however, such an occurrence would have a very long frequency, of the order of 50^n years, in which n is the number of stations. The writer wonders if Mrs. Clarke-Hafstad's measure of dependence between stations would permit estimating this frequency, or preferably the frequency and amount of rain at surrounding stations when a given station had its 50-yr rainfall.

The writer is familiar with a project in which the 50-yr rainfall was estimated for each of a group of stations in a watershed, of which nearly all had less than 20 years of record. This 50-yr rainfall was then assumed to occur at all the stations simultaneously and translated into a flood. The enormous damage done by this flood was then divided by the original 50 years of the rainfall frequency at one station as a part of the calculation of annual benefits. The writer cannot but feel that such a computation contains a very large probable error. This example illustrates the practical nature of the problems with which Mrs. Clarke-Hafstad's paper deals, and the importance, not only of reliable frequency determinations, but of a measure of their reliability.

The writer is in agreement with the author's statement that new projects of rainfall study should deal with storms rather than station rainfall readings. The latter depend on both the size of the storm as a whole and the location of the gage with respect to the storm, and are thus more complex quantities than the storm itself. Once the frequency of the storms of a region is determined, the frequency of rainfall at either a single station or the average over a given area can be computed.

EUGENE L. GRANT,[32] M. AM. SOC. C. E. (by letter).—The value of this excellent paper lies not only in its specific contributions but also in its implied suggestion that engineers dealing with problems of hydrology should be interested in statistical measures of the reliability of the conclusions of their studies. The station-year method presents two questions which constitute a challenge to the student of statistical technique. One question is whether or not the stations used are too far apart to be subject to the same cause system with regard to storms of the duration and intensity under consideration. The other question is whether the stations used are so close together that the same storm has been counted more than once. The Bartels' technique, explained in this paper, provides a satisfactory method of answering this second question.

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[32] Prof. of Economics of Eng., Dept. of Civ. Eng., Stanford Univ., Stanford University, Calif.

One generalization which seems justified on the basis of the evidence now available is that the station-year method is particularly well adapted to dealing with high-intensity storms of short duration. The greater the intensity and the shorter the duration, the larger will be the area that seems to be statistically homogeneous and the smaller will be the area covered by an individual storm. It has already been shown[27] that a large area of Mid-Western United States is statistically homogeneous with regard to the frequency of certain extreme storms of 5-min to 120-min duration, although not so with regard to the frequency of less intense storms of the same duration. Table 2 shows how, in the Iowa and Carolina quadrangles studied by Mrs. Clarke-Hafstad, the value of N_d decreases rapidly with an increase in intensity; although a 1-in., 24-hr storm in Iowa affected more than 7 stations, a 4-in., 24-hr storm affected only one station. An examination of storm maps such as Fig. 1 serves to confirm the view that, the greater the intensity, the smaller will be the area that a storm is likely to cover.

As the author states, the dense networks of rain gages now established in several areas may be expected to provide better estimates of the areas associated with storms of various degrees of intensity. With regard to the very intense storms that cover only a few square miles, these networks will provide many independent station years of record in a very short time.

WALDO E. SMITH,[33] M. AM. SOC. C. E. (by letter).—A welcome addition to the literature relating to the frequency of rainfall is found in this paper, which provides a statistical analysis of the frequently accepted and widely used station-year approach. It gives an index to the probable reliability of rainfall frequencies determined by that method and provides further an outline of a new method utilizing the basic statistical record. Mrs. Clarke-Hafstad is to be commended for her splendid presentation.

As the paper has its background in the mathematics of statistics, the major part of which the author has intentionally omitted, there are questions which arise that the reader is unable to answer. One of these is in connection with the statement under "Factors Affecting the Accuracy of Frequency Determinations: The Length of Independent Record" that "* * * there are 99.7 chances out of 100 that the mean computed for these 500 years * * * is not more than * * * 30% away from the true average * * *; and there are 68.3 chances out of 100 that this mean will not exceed or fall short of the true mean by more than * * * 10%." The writer believes that a little more explanation on the normal curve leading to the 99.7 and the 68.3 would not be out of place.

It appears that this method should have application to flood frequencies based on the historical record at a given gaging station. Inasmuch as this is a matter of treatment, station by station, to a large extent, data are even more meager, in a way, than for the determination of rainfall frequencies. With 100 years of historical record, it has been customary to call the largest flood within this period the 100-year flood. It is indeed quite disconcerting to find that this is correct to within the limits of $\pm \infty$, or (see heading mentioned previously) that "nothing actually is known concerning the frequency * * * from a single observation * * *." This approach does not take cognizance of the fact that this is not truly a single observation, but bears a certain relationship to others of lesser magnitude. Similarly, the second highest flood in such a record, usually given with some confidence as the 50-year flood (although doubt is acknowledged on the frequency of the greatest flood), has a standard error of frequency of ± 100% (this cannot be − 100%). Inasmuch as the hydraulic engineer must, perforce, deal with the historic record and must base his designs on these elusive frequencies of great floods, this conclusion is likewise disturbing. It would seem more reasonable for this purpose to use the equation first given—namely, the standard error equals $\frac{1}{\sqrt{n}}$. This is probably nearer the truth. It acknowledges the possibility of a 100% error in the first value and about 70% in the second.

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[33] Head, Hydrologic Land Use Section, Hydrologic Div., SCS, Washington, D. C.

The symbol N_a is defined as the average number of stations affected by a storm of given intensity, whereas N_d equals the number of stations on the average receiving a given amount in each storm day. Assuming that a day is the period for the N_a-values, the definitions would appear to the writer to be identical. The author indicates that there are very limited conditions only for which this would be true, which the writer has been unable to follow. The wide disparity between the findings of the two values indicates a wide difference in concept that should be more fully explained. The fact that either N_a or N_d is used in evaluating the equivalent of a station-year record to a record of the same length, for a single station, would imply that the values should not be greatly divergent.

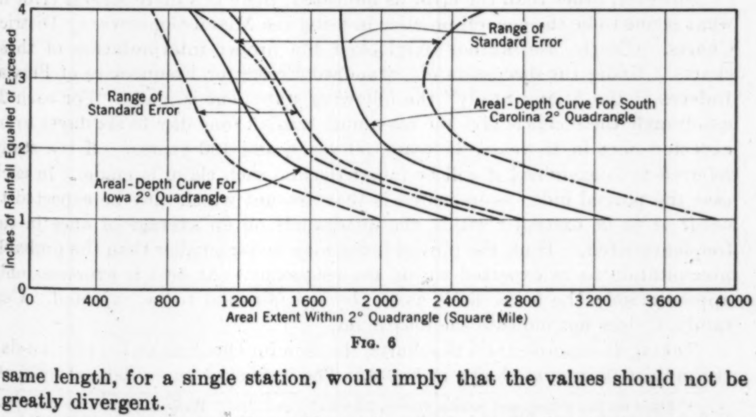

FIG. 6

An interpretation of N_a or N_d, with regard to regions and seasons for a given region, is that they may be, to a limited extent, indexes of the rainfall characteristics. On the South Carolina area, on which there is one station for each 1,055 sq miles, the average extent of 1-in. storms within the 2° quadrangle is 3.10 ± 0.57 stations or 3,270 ± 600 sq miles. In Iowa there is one station for each 677 sq miles and the average 1-in. storm within the quadrangle covers 2,200 sq miles ± 390 sq miles. Determining these areas for the other daily rainfall rates given, and plotting them, define the curves shown in Fig. 6. Inasmuch as heavy rains are of more general extent in South Carolina than in Iowa, one may conclude, at least tentatively, that they are affected less often by the purely convective influence. The plotting of the spread of the standard error gives an idea as to reasonableness of this value, determined on a statistical basis. It would appear in Fig. 6 that the standard error as found for 4 in. for South Carolina is definitely out of line with the others found, and one may feel some justification in applying an adjustment to the standard errors determined by the statistical method.

The author makes reference to the Republican River flood of May, 1935 (see heading "Factors Affecting the Accuracy of Frequency Determinations"), and states "that no gage recorded the rain storm which produced" the flood. An inspection of the published data[34] for this flood shows that several rain gages measured the rainfall, although it is true that no one of them secured the maximum catch indicated by certain unofficial measurements.

Robert L. Lowry, Jr.,[35] M. Am. Soc. C. E. (by letter).—It is fairly easy to prepare a diagram showing the "standard error" in relation to the number of observations for various frequencies. This has been done in Fig. 7, in which the equations for the standard error plat as straight lines on double logarithmic paper. From such a chart the probable errors in frequency for any given conditions can be determined without computation. The chart shows at a glance the magnitude of the errors when only short records are considered.

However, other than the error as indicated, there is a more serious error in what seems to be the general practice in using the Miami Conservancy District Charts. Clearly, the author overlooked the proper interpretation of these charts. Under the discussion of "Standard Errors for Frequencies of Pluvial Indexes of the Miami Study" the following statement is made: "For each 2° quadrangle these charts give the maximum rainfall (one day to six days) to be expected once in 15 years, 25 years, 50 years, and 100 years." If the study referred to is examined it will be found that no such claim is made. In each case the pluvial index as indicated is that amount which may be expected to occur or to be exceeded within the quadrangle on an average of once in the frequency cited. Thus, the pluvial index may be far smaller than the probable precipitation to be expected within the frequency. At best it expresses only approximately the lower limit as to the precipitation to be expected. Certainly, it does not indicate the maximum.

Taking an example from the charts, the data for Quadrangle I–14 for a 3-day storm were plotted as shown in Fig. 8. The frequency curve defined by these data has been extended in both directions purely for illustration. The interpretation of this curve is that the precipitation indicated for any frequency is that which may be expected to occur or be exceeded. By how much it may be exceeded is not known. That is where the trouble lies. Presumably, it could be anything up to the probable maximum that may be expected; but this probable maximum is also unknown. It is known, however, that for a 3-day storm in that quadrangle as much as 19.7 in. of precipitation has been recorded. Obviously, the maximum is some greater value.

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[34] "Flood on Republican and Kansas Rivers, May and June, 1935," *Water Supply Paper No. 796-B*, U. S. Geological Survey, pp. 26–27.

[35] Care National Resources Planning Board, Denver, Colo.

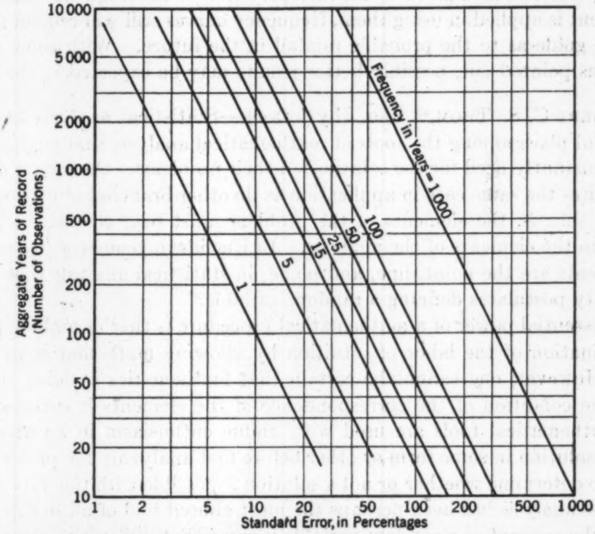

FIG. 7.—RELATION BETWEEN NUMBER OF OBSERVATIONS (IN THIS CASE, YEARS OF RECORD) AND THE STANDARD ERROR FOR GIVEN FREQUENCIES

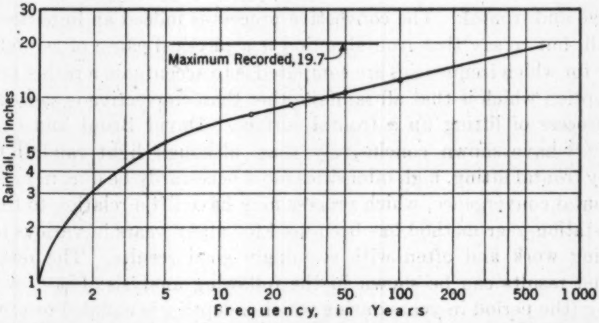

FIG 8.—DATA FROM MIAMI CHARTS FOR QUADRANGLE I-14 FOR A 3-DAY STORM

From the charts it is not possible to determine what the maximum rainfall for any frequency may be. Yet it is the maximum that is required in hydraulic studies rather than the value indicated by the charts. Therefore, when the maximum is unknown, it seems that the probable error in the lower limit is of rather small consequence.

If more attention is given to the interpretation of the frequency curves and if judgment is applied in using them, frequency curves still will remain the best available guide as to the probable rainfall in the future. With some of their limitations pointed out, possibly better results may be expected in the future.

HERBERT C. S. THOM,[36] ESQ. (by letter).—Statistical analysis has taken its rightful place among the tools of mathematical analysis that engineers and others constantly need for the solutions of their problems. As such it deserves and requires the same care in application as do other branches of mathematical analysis; that is, the elements of the problem must bear the closest possible relation to the elements of the analysis. In Euclidean geometry, for example, the elements are the point, line, and plane; in statistical analysis they are the probability postulates defining a random variable.[37]

The essential power of a mathematical procedure is that of making possible the elimination of the labor of intuition by allowing mathematics to do this work. However, one cannot be certain that mathematics is doing the work unless the condition of the correspondence of the elements is satisfied. Too often mathematical tools are used with undue enthusiasm in an attempt to obtain a solution in some form or other before first analyzing the problem sufficiently to determine whether or not a solution is possible with the data at hand. Statistical analysis has been perhaps the most abused tool of all in this respect because the concept of a random variable is somewhat difficult to comprehend. Nevertheless, it is a definite branch of mathematics and must be applied only to such problems for which it can give solutions—problems involving random variables.

The author classifies rainstorms into two types and then states that their fundamental distinction is based on the types of physical processes—namely, convective and frontal. The convective process is indeed an important cause of rainfall, but to say that frontal action is a physical cause of rainfall of the intensity for which frequencies are computed is to accentuate a rather prevalent misconception which is that all rainfall other than convective is caused by the simple process of lifting up a frontal surface. David Brunt and C. K. M. Douglas[38,39] have shown conclusively that, although light rainfall may be caused by frontal lifting, high intensities must necessarily be due, in most cases, to horizontal convergence, which process may have little relation to fronts.

The station-year method has been used for many years in various forms of engineering work and often with seemingly good results. The reasons for these good results can be shown in the following analysis: Let T = rainfall frequency (the period in years during which a depth y is equaled or exceeded); R = the number of years of record; $\bar{N}$ = average number of storms per year;

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[37] "On the Statistical Analysis of Rainfall Data," *Transactions*, Am. Geophysical Union, 1940, p. 490.

[38] "Physical and Dynamical Meteorology," by David Brunt, Cambridge University Press, 1939, p. 354 *et seq.*

[39] *Memoirs*, Royal Meteorological Soc., Vol. III, No. 22, p. 34 *et seq.*

p = the probability of an event equaling or exceeding y; q = the probability of an event being less than y; and N_p = the number of occurrences equaling or exceeding y. Now

$$T = \frac{R}{N_p} \qquad (4a)$$

and

$$N_p = \frac{R}{T} \qquad (4b)$$

so that

$$p = \frac{N_p}{R\bar{N}} \qquad (5)$$

It can be readily seen that if station records are to be combined by means of the station-year method, the frequency curves must be the same at every station. Under this condition $\bar{N}$ will be the same at all stations. When more stations are added, N_p and R will vary directly; hence p will remain practically constant regardless of how many stations, independent or not, are added. This does not mean, however, that new information proportional in amount to the increased number of stations has been added.

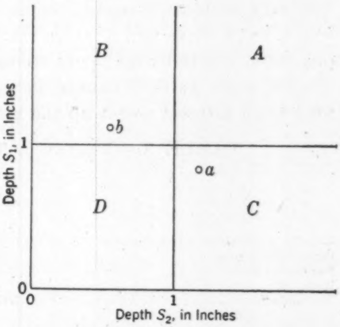

FIG. 9.—CORRELATION TABLE

The author purports to determine the dependence or correlation between stations by Bartels' technique[17] for obtaining the effective number of random components. Bartels claims his technique to be useful only in problems where the analogy with the casting of dice or drawing of random numbers is good. However, in a correlated multi-variate population such as is produced by a group of stations in a 2° quadrangle, the analogy is very weak and consequently Bartels' statistic is rather inefficient. The author has ignored much of the dependence by considering only storm events at the stations that exceeded 1 in. or some other base value. This can be seen clearly in Fig. 9, which shows a correlation table for depth at station S_1 against depth at station S_2. Thus, an event producing 0.8 in. at S_1 and 1.2 in. at S_2 would plot as point a in Fig. 9, whereas another event producing 1.2 in. at S_1 and 0.6 in. at S_2 would plot as point b. Since there are twenty-one stations in Table 1, there will be ${}_{21}C_2 = 210$ correlation tables. If the author's data were plotted in Fig. 9, only square A would contain observations. This procedure ignores any dependence introduced by the simultaneous occurrence of events in squares B and C as well as in D. The computation of the author's value of N_d for various other frequencies does not overcome this difficulty. A further and more serious criticism arises from the fact that the observations in square A do not constitute a random sample and, therefore, should not be operated on with statistical methods. The Bartels' statistic applied to completed tables of the form shown in Fig. 9 would be identical with the square correlation ratio. It is well known that the correlation ratio is an especially poor statistic when computed from tables with only two class intervals—for example, storms producing rains less than 1 in. and storms producing rains of more than 1 in.

R. A. Fisher[40] has decried the computation of a standard error for the correlation ratio because "even with indefinitely large samples the distribution of η for undifferentiated arrays does not tend to normality unless the number of arrays is also increased without limit." The standard error has little significance if the distribution is not normal.

Emphasis should be placed on the fact that only a single rainfall value at a particular station may be used from each storm; otherwise the successive events would not be mutually exclusive and no form of statistical analysis would then apply. If the author did not choose storm events so that they were mutually exclusive, then the results of her analysis would be very uncertain. The lack of mutual exclusiveness may be illustrated by the casting of a die which has a three and a six on the same face so that when that face turned up one could not tell whether to consider the event as a three or a six.

Although Table 1 cannot be analyzed by the analysis of the variance, it may be of interest to set up the variance table because the values can be used in other computations. The variance of a set of values is the expected value of the squared deviations from their mean. In the case of samples of greater than 30, it is equivalent to the squared standard deviation.

TABLE 8.—VARIANCE TABLE (MEAN FREQUENCY, 7.27)

Variation (1)	Sum of squares (2)	Degrees of freedom (3)	Variance (4)
Between stations	254	20	12.7
Between years	2,334	31	75.2
Residual	4,029	620	6.50
Total	6,617	671	9.85

Table 8, Col. 2, gives the sum of squares of the differences of the frequencies from the appropriate mean frequency, stations, or years. Col. 3 is simply a number by which the "sum of squares" is divided in order to obtain the expected value of the variance. In this table "degrees of freedom" (Col. 3) is one less than the number of observations in the columns or rows for stations and years, one less than the total number of blocks for the total, and the remainder for the residual. Since the residual variance 6.50 represents the random variation in the table, it would be expected that, if samples were independent, the residual would be equal to, or greater than, the Poisson sampling variance, which is equal to the mean and less than the Bernoullian variance. The latter represents independent sampling conditions. Inasmuch as the residual is 10% less than the mean, it may be concluded that there is dependence within the table. It can be readily seen that this is true in Table 1.

This dependence upsets the application of the analysis of variance because the essence of this analysis is to compare the residual, to the explained or year and station, variances; but as the residual variance is subnormal and the between-year variance is therefore supernormal, no comparison can be made. A further criticism of the application of the analysis of variance is that the distribution within the table is not normal and a transformation would have to be made to make it normal. However, by comparison with an analysis of a similar table,[41] it may be concluded, from comparing the station variance to the residual by means of the z-distribution, that there is a significant variation from station to station and that, therefore, the stations with greater frequency have been similar in the different years, at least indicating that the station classification is important. As to the highly supernormal variation between years, nothing can be said. It will be noted that the author's σ_1^2 is the total variance of Table 8, whereas σ_{21}^2 is approximately the variance between years divided by 21. If the "sum of squares" between years had been divided by 32 instead of by 31 it would have been exactly equal to it. A better estimate of the variance is obtained, however, by dividing by the "degrees of freedom" rather than by the number of observations.

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[40] "Statistical Methods for Research Workers," by R. A. Fisher, 4th Ed., 1932, p. 235 *et seq.*

It is seen further from Table 8 that the author's value of N_d is simply the between-years "sum of squares" divided by the total and multiplied by 21, or 0.353 times the number of stations. In fact, the author could have obtained N_d more easily in this way. However, the interesting fact is that 0.353, the explained "sum of squares" divided by the total, is like a squared correlation ratio of years on stations. The importance of this fact is that N_d is as inefficient as η^2 and further, having been estimated from a supernormal between-years "sum of squares" and a subnormal total "sum of squares," it is considerably too small. The squared correlation ratio can be interpreted only as the percentage of the total variation introduced by the year classification. It cannot even be interpreted as the percentage of common causes unless the causes are linearly related and *a fortiori*, not as the number of independent events. Since the distribution of η would be still skewed for thirty-two classifications,[40] its standard error has little meaning.

A word of caution might be inserted here about the standard error. The statistic "standard error" has sometimes been interpreted to be some standard or even mystical measure of error like the old probable error that has practically gone out of use. Neither the standard error nor the probable error, unfortunately, has any such connotations. The standard error is simply the standard deviation of a statistic, and the probable error is 0.6745 times the standard deviation. The standard deviation is a measure of the dispersion of the measures about their mean and only in the case of a normal distribution does it have any significance as a measure of error. Thus $a \pm \sigma$ means that the chances are about two to one that a measure or statistic will lie between $a - \sigma$ and $a + \sigma$, and $a \pm 2\,\sigma$ means that the chances are nineteen to one that it will lie between $a - 2\,\sigma$ and $a + 2\,\sigma$. Likewise, the probable error r is interpreted as even chances that a measure of the statistic will lie between $a + r$ and $a - r$ and nine to two that the measure lies in $a \pm 2\,r$. In modern statistical analysis the odds are usually taken as nineteen to one (that is, $a \pm 2\,\sigma$); thus only one observation in twenty will fall outside of $a \pm 2\,\sigma$ due to random causes. Even if the various standard errors that the author has computed had significance, they would still only give odds of two to one against observations exceeding them, or one out of three observations would lie outside the limits set by the author.

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[41] "Statistical Methods for Research Workers," by R. A. Fisher, 4th Ed., 1932, p. 214 *et seq.*

In computing the standard deviation of the frequencies, the author has followed the questionable reasoning of Professor Grant[12] in assuming without *a priori* justification that the station frequencies follow a Poisson (exponential) distribution. The fact that large rainfalls are rare is not *a priori* justification for assuming that these events are distributed in a Poisson (exponential) distribution or the so-called "Law of Small Numbers." It can be shown[42] readily that if $p\ (= 1 - q)$ is the probability of a storm equaling or exceeding y then the frequency N_p will be distributed in a binomial distribution $(p + q)^n$, in which n is the total number of occurrences.

By Eq. 5, $p = \frac{N_p}{R\,\bar{N}}$; and

$$q = 1 - p = 1 - \frac{N_p}{R\,\bar{N}} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Since it is well known that for the binomial distribution $\sigma^2 = n\,p\,q$, and by definition $n = R\,\bar{N}$, substitution gives:

$$\sigma^2 = R\,\bar{N}\,\frac{N_p}{R\,\bar{N}}\left(1 - \frac{N_p}{R\,\bar{N}}\right) = N_p\left(1 - \frac{N_p}{R\,\bar{N}}\right) \ldots\ldots\ldots (7)$$

Substituting:

$$\frac{1}{T} = \frac{N_p}{R} \quad \text{and} \quad N_p = \frac{R}{T}$$

$$\sigma^2 = \frac{R}{T}\left(1 - \frac{1}{T\,\bar{N}}\right) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

If $\bar{N} > 50$ and $T > 1$, then $\sigma^2 \cong \frac{R}{T}$. Thus, in Table 3, $T < 1.00$, and $\bar{N}$, the (average annual) number of mutually exclusive and independent events, is not much more than 50, so that the standard errors for the two upper rows are not correct. The standard errors given in Fig. 2, however, are correct.

It is well known that the skewness of the binomial distribution[43] does not tend to zero as n increases if $p = 0\left(\frac{1}{n}\right)$, or if p is the order of magnitude of $\frac{1}{R\,N}$ or less. Under these conditions standard deviations cannot be interpreted as errors, as they can be with the normal distribution. Since $p = \frac{N_p}{R\,\bar{N}}$ (Eq. 5), this happens only when $N_p \leqq 1$, or when the frequency is defined by one storm.

Had the author computed the standard errors by the foregoing procedure there would have been less difficulty in interpreting the standard error for

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[42] "Frequency Curves and Correlation," by W. P. Elderton, 3d Ed., 1938, p. 185.

[43] "Statistical Mathematics," by A. C. Aitken, Edinburgh, Oliver and Boyd, 1939, p. 58.

$N_p = 1$. Since this involves a binomial distribution with $p = \frac{1}{R\bar{N}}$, the standard error simply is the dispersion about the expected value of N_p, or $n\,p$.

KATHARINE CLARKE-HAFSTAD[44] (by letter).—The discussions have demonstrated a recognition on the part of engineers and hydrologists of a need for more reliable estimates of the intensities and frequencies of rainstorms through adequate analyses of existing rainfall data. Although this closing discussion must deal chiefly with points in the paper on which the writer and the discussers disagree, the writer wishes to express here her appreciation of the interest in the paper and of the constructive comments that will lead to a better understanding of the station-year method.

Because of the nature of the publication and the interests of the readers, it was felt that the mathematics and statistics should be simplified and confined to a minimum. As Mr. Smith states, "* * * the paper has its background in the mathematics of statistics, the major part of which the author has intentionally omitted * * *." Therefore, the statistical part of the paper is unsatisfactory to the statisticians.

At the present stage of development of statistics there is no method for the rigorous determination of the amount of dependence in data that are non-random in both time and space, such as station-year data. Nevertheless, the widespread use of the station-year method by hydrologists necessitated some means of obtaining a measure of persistence in such data and consequently of the reliability of the frequencies determined by the method, and led to the adaptation of the technique described in this paper.

Some of the questions raised in the discussions, such as the one pertaining to the relation between "analysis of variance," discussed by Mr. Thom, and Bartels' method of measuring dependence, had been carefully studied by the writer but were considered inappropriate for discussion in such a paper. They are covered more specifically in a paper[20] by L. R. Hafstad.

The writer protests the statement by Mr. Thom that "Statistical analysis * * * must be applied only to such problems for which it can give solutions—problems involving random variables." Although the philosophies and methods of modern statistics were developed on the premise that only random variables are involved, completely random or independent data are seldom found in natural phenomena. The outlook for progress in climatic and hydrologic research by the use of statistical analysis would be dark indeed if this tool could not be used on observations possessing persistence. It seems to the writer that it is imperative, first, to determine by statistical methods whether or not the data are non-random in character, and then either to eliminate any persistence before using the statistics applicable only to random data, or to devise new techniques for dealing with non-random observations.

Evidently Mr. Thom misunderstood the manner in which the data in Table 1 were derived. Every occurrence of 1 in. or more of rainfall in a day at every station, regardless of the amounts the other stations recorded, was tabulated. If these data were plotted on a correlation table such as Fig. 9, not all the points would fall in square *A*. Thus the data which the writer used are not comparable to those of Mr. Thom and the comparison he made of the square correlation ratio and Bartels' statistic is irrelevant.

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Mr. Thom's criticism, that much of the dependence is ignored in taking only daily amounts of 1 in. or more, is not clear, since the purpose was to determine the amount of persistence only for those storm days on which 1 in. or more fell at one or more stations. If a lower limit, say 0.80 in., had been selected, one would, of course, expect to find more persistence in the data than in the case of the 1-in. amounts.

Throughout the paper the writer was careful not to use the word "storm" with reference to the daily occurrences of rain, because storm data are not available; and if they were, the storms would be of various lengths and statistical analysis very much complicated by this fact. Mr. Thom states "that only a single rainfall value at a particular station should be taken from each storm." Probably Mr. Thom means that only one daily amount should be used from each storm. The assumption has been made in the paper that the daily interval constitutes the single event, and it is known that these events are not mutually exclusive, or independent. The result of this temporal dependence is to make the data, such as those in Table 1, less random than if there were areal persistence only. Although the effects of these two types of persistence cannot be separated by the method proposed, at least the final measure of dependence, N_d, upon which the reliability of the frequency depends, does include all the persistence in both time and space which tends to decrease the reliability of the frequency.

The writer stated (see heading: "Factors Affecting the Accuracy of Frequency Determinations: Number and Distribution of Stations") that two general patterns of rainfall distribution result from two general types of physical processes taking place in the atmosphere—convection and frontal activity. Mr. Thom assumes that frontal activity consists only in the "simple process of lifting up a frontal surface," and on the basis of this assumption criticizes the writer for stating that frontal action is one of the chief causes of intense precipitation. Mr. Thom would attribute most high intensity rainfall to horizontal convergence.

The writer disagrees with Mr. Thom's concept of "frontal activity." Frontal action should include all processes that may take place in the atmosphere in and near frontal surfaces—vertical motion, convergence, and convection. This concept is supported by Sverre Petterssen, who states that:[48]

> "This kind of precipitation [intermittent or continuous from a continuous cloud cover of the alto-stratus or nimbo-stratus type] is caused by the slow upward movement of a large mass of air, due to convergence in the horizontal motion of the air. * * * this type of precipitation is called frontal precipitation."

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[48] "Introduction to Meteorology," by Sverre Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1941, p. 38.

Messrs. Brunt and Douglas state,[39] "It is now widely recognized that continuous rain is usually associated with 'fronts,' or lines of separation of air masses of different temperature."

Convergence is a part of frontal activity. It is, also, an important factor in releasing and maintaining convection. The intensity of rain at frontal surfaces does not depend entirely on the amount of convergence and the upslope motion. "It depends also on the slope of the frontal surface, the moisture content, and the stability of the air."[46] Although high intensity rains have occasionally been observed to result from horizontal convergence alone with no clearly defined fronts in the vicinity, by far the greater number of occurrences of high intensity rains are associated with the passages of fronts or with convective activity.

In his discussion, Mr. Hodges has attempted to make a practical application of errors in pluvial indexes to the problem of flood forecasting. Unfortunately, the writer cannot agree to the transformation from error in frequency to error in pluvial index suggested by Mr. Hodges. The standard errors given in Fig. 2 are simply errors in the spacing in time of any given amount of rain, based on the assumption that the number of occurrences of that amount in the station-year record is known, and that the frequency distribution of these amounts follows the Poisson law. Since the analyst does not know the error in the number of occurrences of the given amount from which number the pluvial index is derived, nor the form of the distribution of rainfall amounts against number of occurrences, he is not justified in assuming that he knows anything about the error in the pluvial index.

The writer had never heard of the method for deriving rainfall frequencies from a station-year record mentioned by Mr. Jarvis—that of assuming that the second highest value in a station-year record has a frequency of once in one half the total years of record, the third highest a frequency of once in one fourth the total years, etc. Indeed, there is no logical reason for determining an average by such a method. If the procedure suggested by Mr. Jarvis were extended, the fifteenth highest value in a 500-station-year record would have an average frequency of once in 11 days. The usual method would give an average frequency of 33.3 years. The difference in these two values could scarcely be considered as the "range in uncertainty regarding the probable error."

Mr. Brod questions the application of station-year pluvial indexes to large drainage basins. As was stated in the paper (see heading "Factors Affecting the Accuracy of Frequency Determinations: Number and Distribution of Stations"): "no 2° quadrangle of the earth's surface will have uniform characteristics of rainfall"; and, as Mr. Bernard emphasizes, the station-year method should be applied only to a region having in all its parts the same rainfall regime. Although no detailed studies have been made to determine the size of such areas, it is certain from meteorological considerations that they are of extremely limited extent, certainly not as large as the area included within a 2° quadrangle of latitude and longitude. Professor Grant's study[12] concerned only the determination of areas homogeneous with respect to the frequency of very high rates of rainfall. The writer may perhaps be pardoned for emphasizing once more that areal dependence in the station-year data is just as important a factor detracting from the value of the method as is the dependence between successive years and successive storms and the dissimilarity of rainfall stations mentioned by Mr. Brod.

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[46] "Weather Analysis and Forecasting," by Sverre Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 425.

It may be of interest to engineers to know that the Climatic and Physiographic Division of the Soil Conservation Service is conducting an investigation of rainfall probabilities. At present (1942) the project is confined to the development of an improved statistical technique for determining the probabilities of various rainfall intensities for selected stations in the United States. The data being used are the actual maximum amounts for selected time intervals, read from the triple-register sheets of the recording rain gages.

Mr. Bernard has contributed a very clear explanation of the manner in which a station-year record is synthesized. One reading it cannot fail to understand why and how there is introduced into the composite record a certain amount of "dependence" between the station records. From Mr. Bernard's theoretical considerations of the morphology of rainstorms it is evident that the hydrologist needs to know much more about the size of areas receiving various depths of precipitation in rainstorms, and about the relations between the intensities and total amounts that actually occur in the storms and those that are recorded by various spacings of rain gages. Until some studies of these features are made one cannot hope to obtain reliable values of average frequencies of various amounts of rain, or of total actual precipitation falling on drainage basins.

Mr. Ruff's question as to whether, from a measure of the dependence between stations, one could estimate the frequency and amount of rain at surrounding stations when a given station has its 50-yr rainfall, must be answered in the negative. If the form of the mean depth-area distribution curve for rainstorms were known, one could calculate the probability that an area of a certain size received the 50-yr amount when it was observed at a single station; but as Mr. Bernard points out in his conclusion, the position of the station receiving the 50-yr amount with respect to the pattern of the rainfall for this single storm will not be known from the observations at this one station, and therefore its observed depth will give no clue to the depth or frequency of the rain occurring at surrounding stations.

Mr. Smith has compared the standard errors for the mean areal extent of rainstorms in South Carolina and Iowa (see Fig. 6 and supporting text) and concludes "that the standard error as found for 4 in. for South Carolina is definitely out of line with the others * * *." One is not justified, however, in comparing in this manner the standard errors in average extent of rainstorms for the two areas, since the length of record and the spacing of stations is not the same for the Iowa quadrangle and for the Carolina quadrangle. The Iowa rainfall data are for twenty-one stations for 32 years, or 672 station years, whereas the Carolina data are for fifteen stations for 20 years, or 300 station years. Since the size of the standard error of the mean number of stations receiving a given amount of rain in a day is a function solely of the number of observations of that amount in the total record, it is not unreasonable to find that the error for the 4-in. amounts in the Carolina quadrangle is larger than for the Iowa quadrangle which has the longer record.

Mr. Smith has indicated, also, that there is some difficulty in distinguishing between the definitions for N_a and N_d used in the paper. Symbol N_a is the notation for the average number of stations receiving a given amount of precipitation in a calendar day, and N_d is an index of the number of statistically dependent ordinates. In the theoretical case described under the heading "Determination of Dependence Between Stations," the writer set up an artificial situation with a known amount of dependence between stations by assuming that a certain amount of rain always fell at exactly the same number of stations (5) on each day of rain, and that the days on which this amount occurred were distributed at random among the years of record. Some misunderstanding of the definitions may have resulted because in this theoretical case $N_a = N_d$. In the application of the test for dependence to actual rainfall data, such as Table 1, there will be dependence both between years and between stations and $N_a \neq N_d$, as was stated under the heading "Determination of Dependence Between Stations."

Mr. Smith is correct in stating that several rain gages measured the rainfall associated with the flood on the Republican River in May, 1935. However, it is doubtful whether this flood would have been expected from the measurements of these gages alone. Although unofficial measurements in stock tanks, cans, etc., in the area of highest intensity reached 13 in. (one even 24 in.), the highest amount recorded by a Weather Bureau gage was 5.05 in.

It is unfortunate that the writer defined the pluvial index (see "Introduction") as "the maximum rainfall to be expected with a certain frequency." As Mr. Lowry states, the correct definition as used in the Miami study is "that amount which may be expected to occur or to be exceeded * * * on an average of once in the frequency cited." In the case cited by Mr. Lowry for Quadrangle I-14 of the Miami Charts, 12.6 in. is the pluvial index corresponding to an average frequency of 100 years. However, the "most probable value" of a distribution is usually defined as the mode, and if this definition is used it is not certain that the most probable maximum value to be expected in a time interval of 100 years will be higher than the pluvial index of 12.6 in. Mr. Lowry probably referred to the mean maximum value, which will be somewhat larger.

Although very little is known concerning the distribution of rainfall amounts, there is evidence to support the assumption that they form some type of exponential distribution. In the following example (contributed by George Blumenstock, Jr., of the Climatic and Physiographic Division of the Soil Conservation Service), a simple exponential distribution

$$f(x) = e^{-x} \qquad (9)$$

is used to clarify the distinction between the value of the variable x (the pluvial index) corresponding to a mean recurrence interval, T, and the most probable maximum and mean maximum values of x in an equivalent interval.

First, determine the value, K, of the variable, x, for which the mean recurrence interval is T (that is, the value of x which may be expected to be equaled or exceeded on the average once in every T trials). This value, K, is defined by the relationship:

$$\int_0^\infty e^{-x}\,dx = T\int_K^\infty e^{-x}\,dx \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

from which:

$$K = \log_e T \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

In a random sample of T events drawn from the population $f(x)$, the probability distribution of the largest value will be given by the relationship:

$$\phi(x) = T\,F^{T-1}(x)\,f(x) \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

The mode, u, of this distribution is the most probable maximum value, and will be given by the solution of the equation:

$$\frac{T-1}{F(x)}f(x) + \frac{f'(x)}{f(x)} = 0 \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

This leads to the result:

$$u = \log_e T \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

It is to be seen, then, that at least for the type distribution $f(x) = e^{-x}$ for any given recurrence interval, T, the corresponding value, K, is exactly equal to the mode, or most probable value, of the probability distribution of the largest value of x for an equivalent time interval.

In the case of rainfall data, however, the mean of distribution (Eq. 12) will often be a more significant measure of the maximum value of x to be expected in T trials. R. A. Fisher and L. H. C. Tippett[47] have shown that for an exponential distribution similar to Eq. 9, the mean, $\bar{u}$, of Eq. 12 for large values of T is given by:

$$\bar{u} = u + \frac{c}{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

in which u is the mode; c is Euler's constant, 0.57721+; and α is defined by:

$$\alpha = \frac{f(u)}{1 - F(u)} \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

For the distribution defined by Eq. 9, it follows from Eq. 16 that α is equal to 1, and, since $K = u$:

$$\bar{u} = K + c \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

The distinction between the value K corresponding to a mean recurrence interval, the most probable maximum u, and the mean maximum $\bar{u}$ is now clear. For the original distribution $f(x) = e^{-x}$ one may now say:

(1) The value K corresponding to a mean recurrence interval, T, is (Eq. 11): $K = \log_e T$. This is also the most probable maximum value, u.

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[47] "Limiting Forms of the Frequency Distribution of the Smallest and the Largest Member of a Sample," by R. A. Fisher and L. H. C. Tippett, *Proceedings*, Cambridge Philosophical Soc., Vol. 24 (1928), pp. 180–190.

(2) The mean maximum value $\bar{u}$ in T trials is

$$\bar{u} = \log_e T + c \qquad (18)$$

For example, where T is assumed equal to 100 (a mean frequency of 1 in 100) the value of K and u is $\log_e 100$, or 4.605, but the mean maximum, $\bar{u}$, in 100 trials is 5.182.

It should be emphasized that these relationships have been selected only to show that a distinction between the three values does exist. In the case of rainfall data the exact character and extent of the distinction depend upon the distribution $f(x)$ which may be selected as representative of the total population of rainfall amounts from which the sample is drawn.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2143

TUNNEL CONSTRUCTION, SIXTH AVENUE SUBWAY, NEW YORK, N. Y.

By Jacob Feld,[1] M. Am. Soc. C. E.

With Discussion by Messrs. John H. Myers, and Jacob Feld.

Synopsis

As part of the contract to construct Section 10 of the Sixth Avenue Subway, between 40th and 47th streets, in New York City, one double-track, and two single-track, tunnels were driven for three blocks under a busy city street, under the foundations of an operating elevated railroad, very close to buildings varying from two to sixteen stories in height, through a variety of rock formations, and with scant cover. This paper describes how the work was planned and organized to provide efficient and continuous progress, how it was executed, and at what cost; and it describes the type and adaptability of the equipment used. The major factor in the execution of this work was the necessity for safety.

General Description of the Contract

The contract included the construction of a four-track subway for a distance of 1,800 ft in Sixth Avenue, which is 100 ft wide, and crossed seven intersecting streets. The southerly part of the work, including a station, was built in open cut. The northerly end of the contract, about 260 ft long, was also built in open cut. Between these two areas the four tracks were built in tunnel without disturbing the street surface. The work was done under a unit price contract, the tunnel work being included in the total contract items. As far as plant and equipment were concerned, the construction of the tunnels was merely part of the total contract, except as certain special equipment was found necessary for the tunnel operation (see Fig. 1). The contract plans and specifications were prepared by the Board of Transportation of the City of New York. The contract was awarded on February 6, 1936, and the tunnel work described herein was executed from June, 1937, to April, 1938.

General Plan and Specifications

The plan of the east half consisted of a double-track tunnel running in a straight line under the easterly half of Sixth Avenue, from a point 50 ft south

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Note.—Published in April, 1941, *Proceedings*.

[1] Cons. Engr., New York, N. Y.

of 46th Street for a distance of 720 ft. At approximately the middle of this length there is a ventilating and emergency exit shaft serving both of these tracks and running to the street surface. At two other points, in 44th Street and in 45th Street, the easterly tunnel is widened out to provide space for a cable-duct manhole connected to the street surface with a cable-pull pipe 10 in. in diameter. Starting at the 46th Street end, the third track from the east is in a single-track tunnel that dips sharply downward to the west emergency exit shaft (see Fig. 1) and a sump to which the drainage of all the tunnels is connected, and then is carried upward at the same grade, making a total distance of 814 ft, emerging into an open-cut area south of 43d Street at the west side of the street. This tunnel is built on a curve to change the track position from the third to the fourth lane. South of 45th Street, at about its lowest point, this tunnel connects to a ventilation and emergency exit shaft carried to the street surface. From the southerly face of this shaft the fourth tunnel starts near the west building line of Sixth Avenue directly over the third tunnel and crosses from the fourth to the third lane, entering the open-cut area at 43d Street. The double-track tunnel required an excavation approximately 18 ft high and 32 ft wide. Each of the single-track tunnels required an excavation approximately 18 ft high and 18 ft wide.

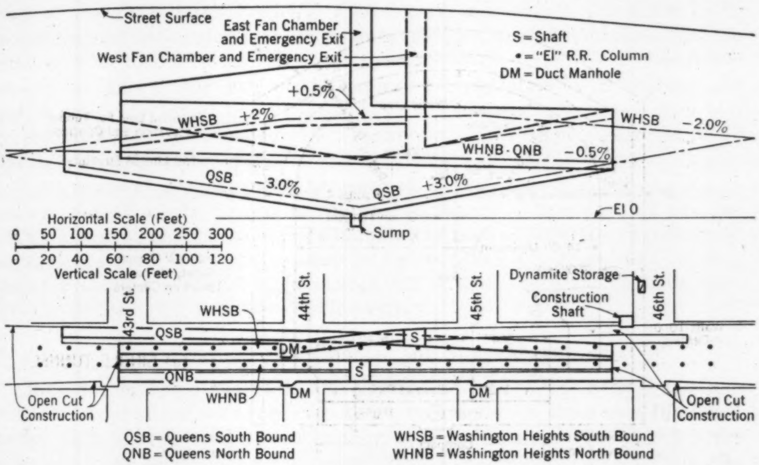

Street Surface
East Fan Chamber and Emergency Exit
West Fan Chamber and Emergency Exit
S = Shaft
• = "El" R.R. Column
DM = Duct Manhole
+0.5%
WHSB
+2%
WHSB
−2.0%
WHNB · QNB
−0.5%
QSB
−3.0%
QSB
+3.0%
El 0
Sump
Horizontal Scale (Feet)
0 50 100 150 200 250 300
0 20 40 60 80 100 120
Vertical Scale (Feet)
43rd St.
44th St.
45th St.
46th St.
Dynamite Storage
Construction Shaft
WHSB
QSB
DM
S
S
QNB
WHNB
DM
DM
Open Cut Construction
Open Cut Construction
QSB = Queens South Bound
QNB = Queens North Bound
WHSB = Washington Heights South Bound
WHNB = Washington Heights North Bound

FIG. 1.—PLAN AND PROFILE OF TUNNELS

Specifications provided that the methods used for tunneling should be suitable to the local conditions, and must not cause any injury to the foundations, walls, or other parts of adjacent buildings, structures, or surfaces, such methods to be subject to the approval of the engineers before work was begun. Such methods were to be changed from time to time as local conditions required, but at no time was the approval of the engineers to relieve the contractor of his responsibility for the proper execution of the work. The contractor had the option to design and submit for approval any method of temporary timbering for the support of the rock before the concrete lining was placed. As local rock conditions made it advisable, the engineer had the right to order the tunnel lining placed before the excavation was completed.

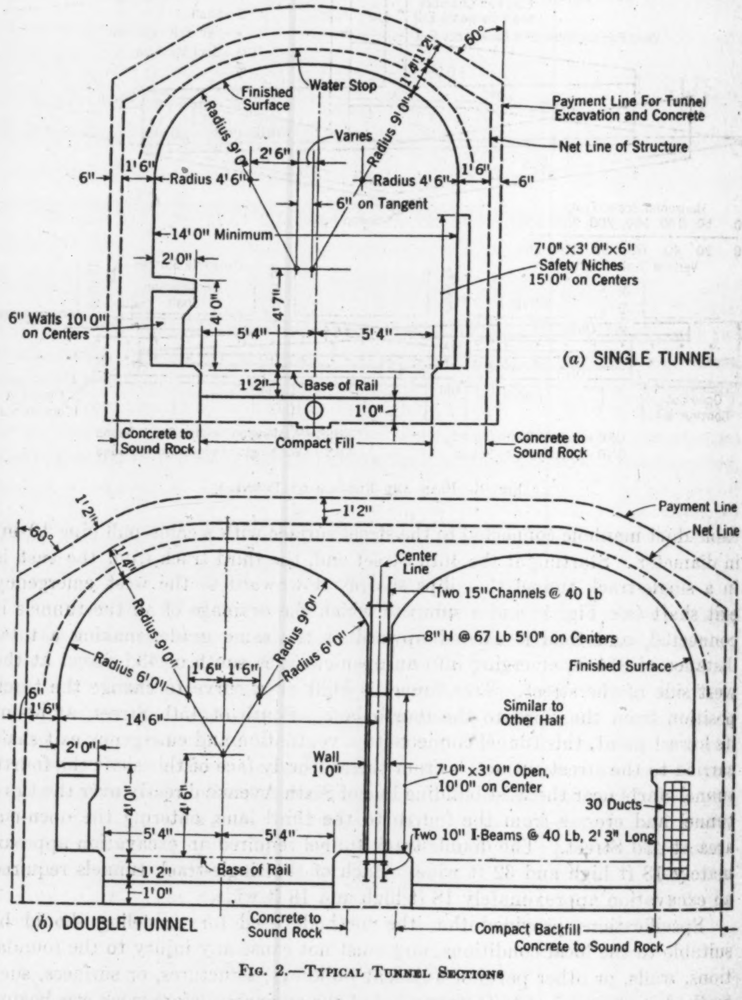

1'4"
1'2"
60°
Water Stop
Finished Surface
Radius 9'0"
Radius 9'0"
Payment Line For Tunnel Excavation and Concrete
Vanes
Net Line of Structure
2'6"
6"
1'6"
Radius 4'6"
Radius 4'6"
1'6"
6"
6" on Tangent
14'0" Minimum
7'0" ×3'0"×6" Safety Niches 15'0" on Centers
2'0"
6" Walls 10'0" on Centers
4'0"
4'7"
5'4"
5'4"
(a) SINGLE TUNNEL
1'2"
Base of Rail
1'0"
Concrete to Sound Rock
Compact Fill
Concrete to Sound Rock
1'2"
1'2"
60°
1'4"
Payment Line
Net Line
Center Line
Radius 9'0"
Radius 9'0"
Two 15" Channels @ 40 Lb
Radius 6'0"
Radius 6'0"
8" H @ 67 Lb 5'0" on Centers
1'6"
1'6"
Finished Surface
6"
1'6"
14'6"
Similar to Other Half
2'0"
Wall 1'0"
7'0" ×3'0" Open, 10'0" on Center
4'0"
4'6"
30 Ducts
5'4"
5'4"
Two 10" I-Beams @ 40 Lb, 2'3" Long
1'2"
Base of Rail
1'0"
(b) DOUBLE TUNNEL
Concrete to Sound Rock
Compact Backfill
Concrete to Sound Rock

FIG. 2.—TYPICAL TUNNEL SECTIONS

The single-track tunnels were to be lined with concrete as shown in Fig. 2, such lining to include no reinforcing or structural steel. The double-track tunnel was also lined. The two arches met in an umbrella section, in which a series of bents was concreted, consisting of double channels supported on two columns. The center wall between the two tunnels in which these bents are concreted has a series of cross-connecting openings spaced 15 ft apart. In each of the single-track tunnels there was a series of man safety niches in one side, spaced 15 ft apart. On one wall of the double-track tunnel, and on one wall of one single-track tunnel, a line of clay cable ducts was to be built, consisting of thirty individual 3.5-in. ducts connecting the manholes. On the other wall of the double tunnel, and in the other single tunnel, a raised walkway was to be built at the level of the subway car platforms for emergency and patrol use. The invert of all tunnels is 12 in., with cast-iron drain pipes laid in certain locations, cross-connected between the two tunnels near the north end, all draining to the sump.

The contract provided that the surface of the concrete lining was to be smooth and of a type known as "finished surface." The contractor had the option to use steel forms so as to eliminate the need for rubbing the concrete. Grout holes were left in the concrete lining for filling any spaces left between the rock and the concrete. The installation of track, cables, and other operating equipment was not part of this contract.

BORINGS AND GEOLOGY OF SUBSURFACE

Prior to beginning the work, the city had several sets of borings made in the area under which the tunnel was to go, and reports of these were furnished to the contractors. Four of the borings were driven almost 200 ft below the street surface, chiefly to check what leakage could be expected from the Catskill Aqueduct approximately that distance below Sixth Avenue. These borings were one in each block (260 ft apart) and showed a great variety of rock recoveries, the samples removed ranging from 1% to a maximum of 93% of the boring depth. In addition, there were seven other borings taken to rock surface with a 10-ft core into the rock, and the recoveries varied from 1.4 to 7.7 ft of that distance.

In addition to the borings several maps were also available showing the accumulation of previous information, none of which, however, indicated the complexity of rock formations actually encountered. For instance, a field report of June, 1937, described

> "* * * rock in both tunnels of great variety, from soft, disintegrated schist to an extremely hard igneous syenite with seams of feldspar, which in some cases is so disintegrated that it can be taken out by hand. Seams are generally almost vertical, slight tip to the West and running almost to South slightly to East."

In general, the basic rock was schist, but in the volume occupied by the tunnels there were a great number of intrusions in the form of bands running southward about 20° to the east and almost vertical, sometimes tipped 5° to 20° to the west. At the north end of the tunnels a band of very hard syenite with quartzite intrusions was found crossing the double tunnel. In the vicinity of 45th Street a very hard and fine-grained syenite crossed all four tunnels. To the south of this band was a layer of disintegrated syenite which was so soft that it could be broken easily by hand, and, in the terms of the blasters, was "dead." South of this point there was more mica schist, which again changed to disintegrated syenite a few feet in thickness, in contact with several layers of chlorite and serpentine, interspaced with disintegrated syenite. These latter layers were encountered at the extreme south end of the double tunnel, affecting the westerly part only, and crossed the south end of the Washington Heights southbound single tunnel near its southerly end, and also crossed the Queens southbound tunnel about 300 ft from its southerly end. Keeping in mind that these layers were encountered practically on edge and running diagonally across the face of the headings, the complexity of the drilling and blasting operations can be pictured easily. Practically no water was encountered at any point.

Surface and Subsurface Conditions

The street width is 100 ft, and buildings occupy every available piece of ground the full length of the tunnels. These buildings vary from altered four-storied brownstone residences with stores on the ground floor, built about 1880, to modern, fireproof, seventeen-story office buildings. In the middle of the street there was a two-track elevated railway, supported on wrought-iron columns spaced on 43-ft centers longitudinally, with two columns 23 ft apart in each bent. Train operation of this railway was continuous, with the average headway about 5 min each way. The columns rested on brick piers, some of which were carried to rock, and others were in the nature of spread footings several feet above rock elevation.

Prior to the beginning of any construction, a careful examination was made of each building on Sixth Avenue, and also the buildings within a distance of 100 ft east and west of Sixth Avenue. These examinations were made by representatives of the Board of Transportation and the contractor, and usually the owner of each building. Stenographic notes were taken during the examination, and the report was signed by all those present. Where unusual conditions were found, the contractor also took photographs for his records. At the completion of the work these buildings were again examined to determine whether any changes had occurred.

To avert breakage of windows as far as possible, a wire cable and timber trussing were placed over each window with either dimension greater than 6 ft. This included all of the show windows. These "spiders" were kept in repair during the entire operation, and not one of the show windows along the line of the tunnel was broken.

In the avenue itself, below the surface, there was an old brick sewer in the center about 14 ft down, built in 1890, or before, in a rock trench. At higher levels there were two cast-iron water mains, one 20 in. and one 12 in., with 12-in. lines crossing the avenue at each street. Two steel gas mains were constructed in the avenue under the subway contract, prior to any tunnel work, to replace a great variety of old cast-iron gas mains that were found. In some points as many as twelve old gas mains were thus replaced. In the easterly part of the roadway there was a large bank of telephone cables laid in clay ducts, containing approximately 25,000 telephone circuits. There were also two banks of clay ducts containing low-tension and high-tension power cables, one at each curb, with sub-branches and service lines. Similar subsurface equipment also existed in each of the cross streets.

Approximately 200 ft below the street surface there is a concrete-lined aqueduct tunnel of 15 ft inside diameter, bringing water under pressure into the city. The fear that subway construction would affect this tunnel delayed the construction of the Sixth Avenue Subway link until such time as the second aqueduct tunnel was completed in 1934. Such fear was unfounded, as proved by the record that no serious leakage was encountered at any point along the Sixth Avenue Subway construction.

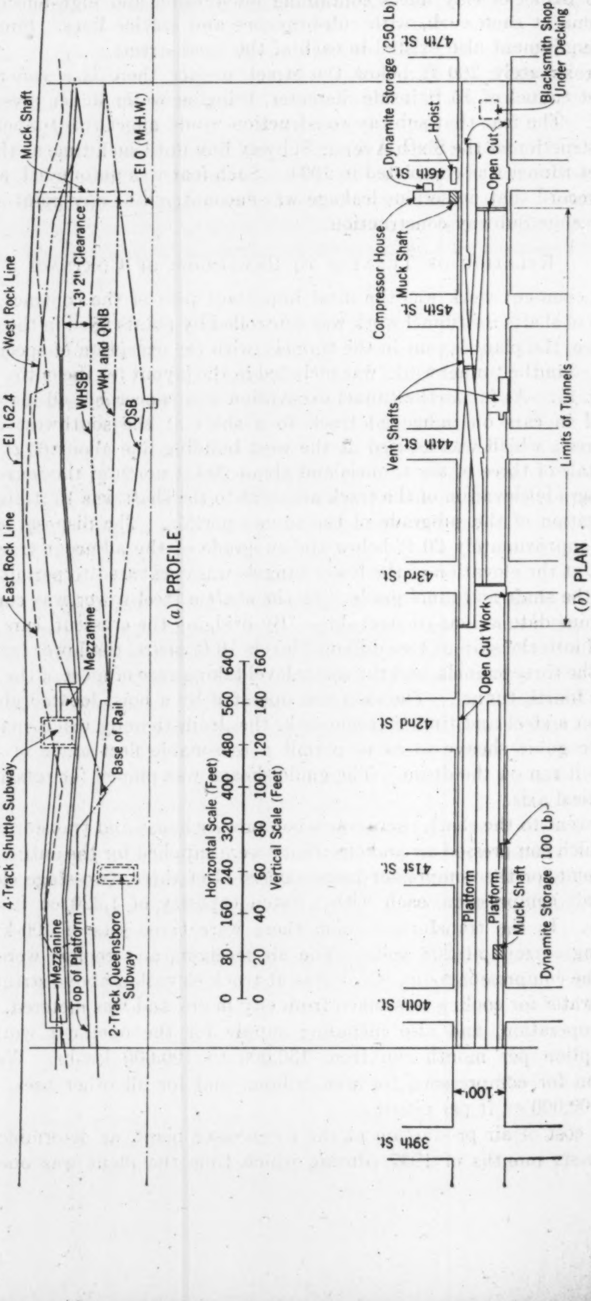
Relation of Tunnels to Remainder of Contract

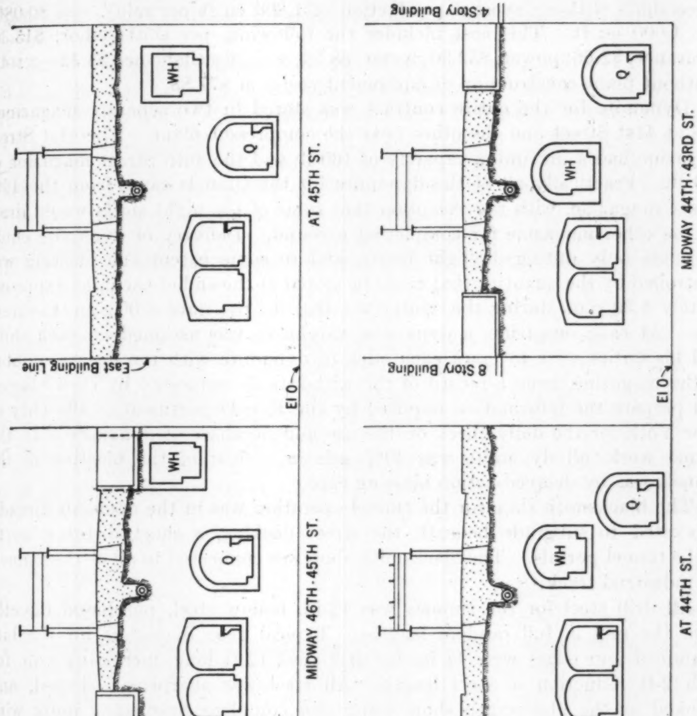
The open-cut work was the most important part of the contract, and the location of shafts for tunnel work was controlled by points fixed in the contract. Therefore, the plant layout in the tunnels, with the exception of special equipment used in the tunnel work, was included in the layout for the entire contract (see Fig. 3). As far as the tunnel excavation was concerned, all material was removed in cars on industrial track to a shaft at the southwest corner of 46th Street, which was located at the west building line about 60 ft north of the portals of three of the tunnels and about 200 ft north of the fourth portal. The subgrade elevation of the track adjacent to the shaft was 10 ft higher than the elevation of the subgrade of the tunnel portals. The disposal shaft was carried approximately 20 ft below the subgrade of the adjacent track, and a crosscut at the elevation of the lower tunnels was excavated to permit bringing cars to the shaft at tunnel grade. In the shaft a steel tower was constructed to accommodate a 6-cu-yd steel skip. By bridging the crosscut, cars could be dumped into the skip at two different levels 10 ft apart, the lower level taking care of the three tunnels, and the upper level taking care of most of the open-cut and the fourth tunnel. The skip was operated by a hoist located above 46th Street on a steel and timber framework, the drum thereby being spaced 50 ft from the guide sheave so as to permit a reasonable fleet angle of the hoist cable as it ran on the drum. The guide sheave was pinned for rotation about the vertical axis.

Adjacent to the shaft, there was a compressor house and transformer vault, from which compressed air and electricity were supplied for the entire contract. Equipment for the compressor house consisted of three, two-stage electrically driven air compressors, each with a rated capacity of 1,320 cu ft at 120-lb pressure. In the transformer room there were three sets of 100-kva units, providing current at 208 volts. The air receivers and coolers were directly below the compressor room, which was at truck elevation above street surface.

All water for cooling was taken from city mains and was metered. During tunnel operation, and also including supply for the open-cut work, power consumption per month ran from 130,000 to 190,000 kw-hr. Water consumption for compressors, for wet drilling, and for all other uses, averaged about 400,000 cu ft per month.

The cost of air production at the compressor plant, as determined during the last six months of 1937 (during which time the plant was operated on


4-Track Shuttle Subway
East Rock Line
El 162.4
West Rock Line
Muck Shaft
Mezzanine
Top of Platform
2-Track Queensboro Subway
Base of Rail
Mezzanine
WHSB
WH and QNB
13' 2" Clearance
OSB
El 0 (M.H.W.)
0 80 160 240 320 400 480 560 640
Horizontal Scale (Feet)
0 20 40 60 80 100 120 140 160
Vertical Scale (Feet)
(a) PROFILE
39th St.
40th St.
41st St.
42nd St.
43rd St.
44th St.
45th St.
46th St.
47th St.
100'
Platform
Platform
Muck Shaft
Dynamite Storage (100 Lb)
Open Cut Work
Vent Shafts
Compressor House
Muck Shaft
Dynamite Storage (250 Lb)
Hoist
Open Cut
Blacksmith Shop Under Decking
Limits of Tunnels
(b) PLAN


17-Story Building
Q = Queens
WH = Washington Heights
Shaft
Q
WH
Muck Shaft
4-Story Building
AT 46TH ST.
MIDWAY 46TH-45TH ST.
East Building Line
AT 45TH ST.
4-Story Building
Shaft
MIDWAY 45TH-44TH ST.
El 0
AT 44TH ST.
8 Story Building
El 0
MIDWAY 44TH-43RD ST.
4-Story Building

(c) SECTIONS FACING SOUTH

FIG. 3. CONTRACT PLAN AND STATIONS

three shifts with an average production of 1,800 cu ft per min), was $0.0865 per 1,000 cu ft. This cost includes the following, per shift: labor, $13.30; insurance, $2.66; power, $52.00; water, $8.50; and miscellaneous, $1.42—a total (without plant construction or equipment costs) of $77.88.

Dynamite for the entire contract was stored in two separate magazines, one at 41st Street and the other near the compressor plant. The 41st Street magazine had a maximum capacity of 100 lb and the 46th Street magazine of 250 lb. Practically all of the dynamite for the tunnels came from the 46th Street magazine, with the exception that some of the night shifts would draw on the other magazine for completing a round. Delivery of dynamite could be made only during daylight hours, and to some extent the blasting was controlled by the quantity that could be stored at the end of each day (approximately 4:30 p.m. during the winter months) for use until 8:00 a.m. the next day. At each magazine, a dynamite watchman was assigned on each shift, and his duties were to mark each stick of dynamite with the license number of the magazine, keep a record of the withdrawals requested by each blaster, and prepare the information required by the Fire Department of the City of New York for the daily check on the use and purchase of dynamite. In the tunnel work, all dynamite was 40% gelatin. Most of the blasting in the tunnels was by delayed-action blasting caps.

The blacksmith shop for the tunnel operation was in the open-cut already excavated to subgrade beneath the street decking, a short distance north of the tunnel portals. The blacksmith shop was connected to all of the tunnels by industrial track.

All drill steel for the tunnels was 1¼-in. hollow steel, purchased directly from the mill in full random lengths. Typical gage diameters for the bits (standard four edge) were 1⅞ in. for drill steel 12 ft long, increasing ⅛ in. for each 2-ft reduction in steel length. All steel was sharpened, shaped, and shanked at the blacksmith shop, using two complete sharpening units with oil-fired furnaces. Two of these units were operated on one shift, with only one unit on each of the other two shifts. This continuous use required relining of the furnaces twice during the entire contract. This excellent service was due mostly to the use of a special firebrick and latite mortar. During the last six months of 1937 the drill-sharpening cost per cubic yard of rock was as follows: labor, $0.313; insurance, $0.04; fuel, etc., $0.038; and water, $0.006—a total cost per cubic yard of $0.40 (plus equipment costs). The average steel required from the blacksmith was 3.64 pieces per cubic yard of rock, making an average total cost of 11 cents per drill sharpened. A blacksmith unit, consisting of three men, produced 350 pieces of drill steel, sharpened, per shift. The extra gang of men on one shift took care of preparing new steel and reshanking old steel as required.

LABOR REQUIREMENTS, RATES, AND ORGANIZATION IN THE FIELD

Under the terms of the contract, labor requirements, work assignment, and the minimum rates were fixed by the State Labor Law. Under such rules, with the exception of the foremen, no man could work more than five days in one week nor more than eight hours in one day, except in cases of emergency.

In addition, except in cases of emergency, no Sunday work was permitted by city ordinances.

The job was organized on a three-shift basis, with eight hours in each shift, including one-half hour for lunch, payment being made on a straight basis for eight hours each shift. The foremen, including the blasters, worked six shifts a week, and the other men were rotated so that no man worked more than five days in one week. The exception to this rule was the "graveyard" shift, which omitted the shift starting midnight Sunday and only worked five days a week. Tunneling operation was continuous, therefore, from 8:00 a.m. Monday morning until midnight Saturday. However, concrete was poured only during the day shift, except that a part of the day shift was called at 6:00 a.m. to prepare the forms, and in cases of necessity the 4:00-o'clock shift would complete the concreting at the expense of their regular work.

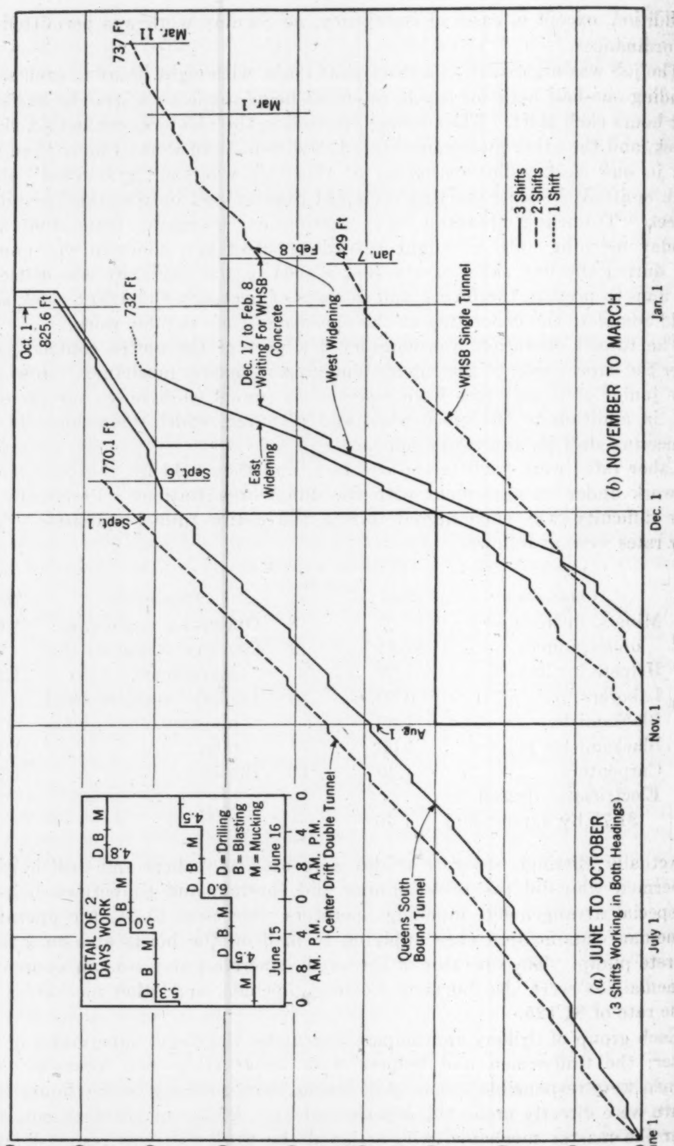
The tunnel operation was considered a part of the entire contract, and under the supervision of the superintendent and master mechanic. However, three junior civil engineers were assigned to tunnel work only, one for each shift, in addition to the office work and planning, which were done by the engineering staff for the entire contract.

Labor rates were fixed in the contract, and they held for the duration of the work under an agreement with the union organizations. Practically no labor difficulty was encountered during the entire tunnel operation. The labor rates were as follows:

Item No.	Classification	Rate
1	Miners, drillers, and underpinners	$1.25
2	Helpers for item 1	1.00
3	Laborers	0.80
4	Blacksmiths	1.40
5	Blacksmith's helpers	1.00
6	Carpenters	1.40
7	Electricians (raised to $1.70 by agreement)	1.40
8	Operating enginemen	$1.65
9	Conway operators (by agreement)	1.25
10	Dinkey runners and maintenance enginemen	1.125
11	Blasters	1.65

Actually, distinct, separate trades were used as drillers and drill helpers, timbermen who did the underpinning and shoring, and timbermen helpers. By special arrangement, mucking operators were paid $1.25, the operating engineman classification only applying to men on the hoists and on a large concrete pump. The operator of the small concrete pump was an apprentice engineman, as were also burners, welders, pipemen, and other mechanics, all at the rate of $1.125.

Each group of drillers and helpers was under the direct supervision of the blaster; the timbermen and helpers were under their own foremen. The foremen were responsible to two shift bosses, each covering twelve hours, who in turn were directly under the superintendent. All the mechanical men were under the master mechanic, who assigned the work and was responsible for the performance of the equipment as well as the operators.



(a) JUNE TO OCTOBER
(3 Shifts Working in Both Headings)

(b) NOVEMBER TO MARCH

Fig. 4.—Excavation Progress

SEQUENCE OF OPERATIONS

The general sequence of operations, as planned and as actually performed, was to start excavation of the Queens southbound single tunnel and the center cut of the double tunnel at the same time at the north end, and proceed south. The rock was shored simultaneously with the excavation during the drilling periods, except that, where bad rock was encountered, drilling was postponed until the rock was shored to the face. For the single tunnel, the concrete operation was first to pour the two benches and follow with the complete arch, using the benches to support the form. Concreting the invert was postponed until the remainder of the tunnel was completed.

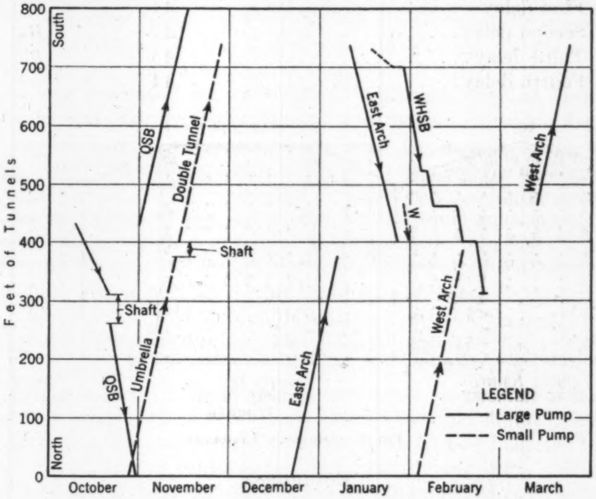
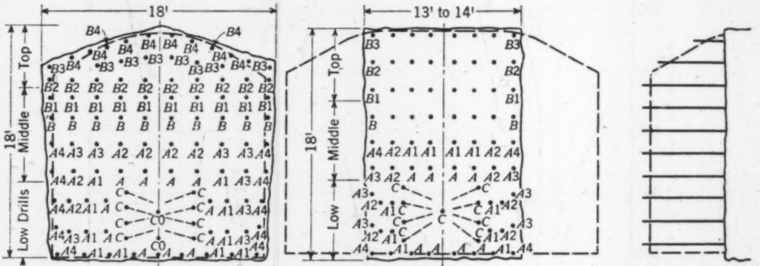


FIG. 5.—PROGRESS, POURING OF ARCH CONCRETE

In the center cut of the double tunnel after completion of excavation, the next step was ring drilling for the widening on each side of the center cut; then followed the concreting of the center bench of the "umbrella," erecting the structural steel columns, and concreting the umbrella section to the roof so that the side supports of the roof bracing could be eliminated. The posts of the roof bracing were then used to line the two faces of the umbrella concrete while the ring-drilled rock on each side was shot out and removed. This operation, in turn, was followed by concreting each of the benches, completing the arch, grouting, and concreting the invert. The excavation progress and timing are illustrated in Fig. 4. The corresponding concrete progress is shown in Fig. 5.

Although the original plan was to place the cut shot at the bottom of each heading, it was found too difficult in the field to drill diagonal holes nearer subgrade than about 15 in. The cut holes were raised about 18 in. above subgrade (see rounds C, Fig. 6). The typical hole locations were varied daily to suit rock and seam conditions in the heading, the normal loading schedule (Fig. 6) being as follows:

Descriptive	Round *C*	Round *A*	Round *B*
Cut shot	*C*	....	....
Blank	....	*A*	*B*
First delay	....	*A*1	*B*1
Second delay	....	*A*2	*B*2
Third delay	....	*A*3	*B*3
Fourth delay	....	*A*4	*B*4



SINGLE TUNNEL
90 TO 103 HOLES

CENTER DRIFT OF
DOUBLE TUNNEL
77 HOLES

SIDE ENLARGEMENT
10 HOLES, 2 FT APART

Fig. 6.—Drilling Diagrams

Equipment

To a large extent equipment purchased for the entire contract was used in the tunnels, except that the muckers were purchased chiefly for tunnel operation but were used partly in open-cut work. The drill jumbo and steel-erection jumbo were made especially for the tunnels; the concrete pumps were used chiefly in the tunnels; and the smaller unit was used in some open-cut work. The cars, track, dinkeys, pumps, and other small equipment were adapted for use both in the tunnels and in the open cut.

The drilling jumbo was fabricated at the site from drawings made to fit conditions of the job. The first design called for mounting six drills, and this was later changed for eight drills, seven of which were used at all times and one for a standby. In general, the idea was to have drills at three levels, dividing the 18 ft of total height into approximately three equal sections. Hinged side platforms at the intermediate and top levels permitted the use of the jumbo in the 18-ft single-track tunnel as well as in the 14-ft center cut of the double tunnels. To compensate for the lack of dead weight, the structure being made as light as possible for easy shifting, screw jacks were placed at the top and bottom of the vertical columns for wedging against the roof and the floor of the excavated tunnel. The mounting was on trucks from small muck cars used on a previous contract.

Tunnel drilling operation was started with drills with 3-in. pistons. As soon as it was found that drilling time was slowing up the expected schedule of a round per shift, these drills (which had been used on a previous contract) were replaced by drills with 3½-in. pistons. With these larger drills a round was drilled out in three hours. When the drill jumbo was changed from six to eight drills, the two additional drills installed were equipped with 4-in. pistons.

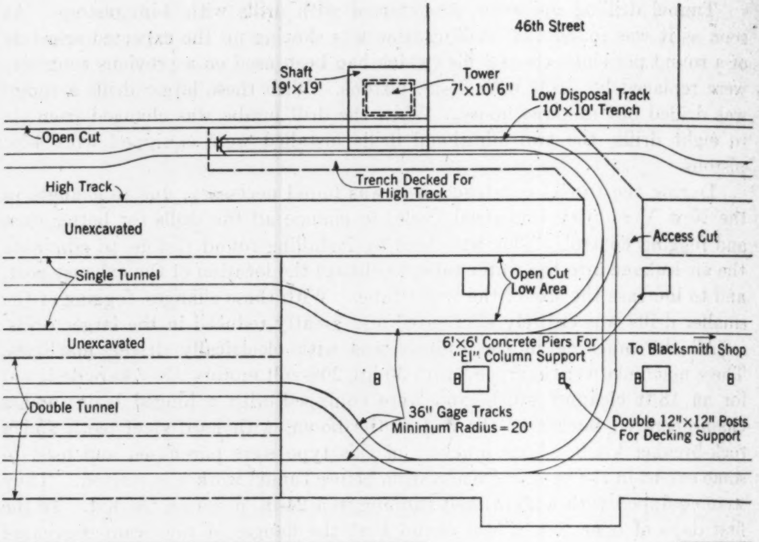
During the tunnel construction it was found necessary, due to changes in the New York City Industrial Code, to change all the drills for better dust and fogging control. This was done by installing round pistons to eliminate the air leakage into the water tube, to change the location of the exhaust port, and to increase the size of the water tube. With these changes, fogging of the smaller drills was entirely eliminated and greatly reduced in the larger drills.

All the mucking in the tunnels was with electrically driven machines. These machines were equipped with 50-hp, 208-volt motors; they were designed for an 18-ft cleanup width, and were equipped with a hinged boom and a chain-operated bucket at the end of the boom, with hard steel teeth and a rock-breaker knob. Four muckers of this type were purchased and used to some extent in the open-cut excavation before tunnel work was started. They were equipped with a 20-in. belt running in a 24-in. beam on the flat. In the first days of operation it was found that the flanges of the beam prevented proper contact between the mucked material and the belts. The upper flanges of the beam were cut away and rewelded to the web of the beam to form a sloping trough. This correction was made on the first two machines in the field and on the other two, which had not yet been shipped, in the shop. It was also noticed that considerable of the slip could be eliminated by a small stream of water running continuously down the sloping belt. Such slip was caused chiefly by the large quantity of mica in the mucked rock.

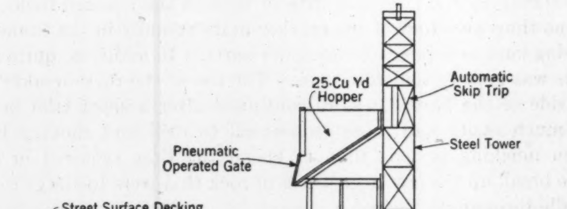
At no time were four of the mucker units actually in the tunnel, the fourth unit being kept as a reservoir for spare parts. In addition, quite a large stock of parts was kept in the storeroom. The use of the rock-breaker knob on the under side of the bucket was discontinued after a short trial because it was found much easier and more economical to drill and shoot a large boulder between mucking periods than to spend the time required in the mucking cycle to break up the few large pieces of rock that were too large for the mucker to handle through its throat.

Considerable study was given to the belts used in the mucking machines, not only because of the high cost of such belts, but also because of the time wasted when a belt broke and required replacement. The belts originally supplied by the manufacturer had been developed as the result of considerable research in the California tunnels. However, the rock conditions encountered on this contract were much more severe, especially because of the sharp edges and large quantity of mica. The average life of a belt was about 2,500 cu yd of rock removed.

All track was of standard 36-in. gage, composed of 40-lb industrial rail, with standard switch points and frogs. Dump cars were 5-cu-yd, water-level, steel body, side dump, with side top hinged. They were dumped into the skip


46th Street
Shaft
19'×19'
Tower
7'×10' 6"
Low Disposal Track
10'×10' Trench
Open Cut
Trench Decked For
High Track
High Track
Unexcavated
Access Cut
Single Tunnel
Open Cut
Low Area
Unexcavated
6'×6' Concrete Piers For
"El" Column Support
To Blacksmith Shop
B
B
B
B
B
Double Tunnel
36" Gage Tracks
Minimum Radius = 20'
Double 12"×12" Posts
For Decking Support

(*a*) PLAN OF DISPOSAL TRACKAGE


25-Cu Yd
Hopper
Automatic
Skip Trip
Steel Tower
Pneumatic
Operated Gate
Street Surface Decking
Air Cylinder
For Car Dump
54' 6"
Double Tunnel
Single
Tunnel
11' 6"
10'
Clean-Out
Sump

(*b*) SECTION AT PORTALS

Fig. 7.—Disposal Trackage

at the 46th Street shaft by the use of a pneumatic cylinder 12 in. in diameter with sufficient stroke to unload the car surely by gravity. Two dump levels were provided directly over each other, and by operating the dumping with only one cylinder, to which the car at either level could be attached, the danger of some one attempting to dump two cars at one time was eliminated (see Fig. 7).

The dinkeys for pulling the cars, the muckers being self-propelling, were electric-battery locomotives of 6-ton rated capacity, equipped with 42-plate batteries of the high cap type. Each dinkey had two sets of batteries, and four dinkeys in all were available for the tunnel work. The batteries were charged with a 30-kw, 108-volt, 1,750-rpm, 278-ampere generator direct-connected to a 50-hp, three-phase, 60-cycle, 220-volt motor. The batteries were charged at the upper level beneath the decking near the 46th Street shaft. By means of an overhead trolley hoist and beam track, batteries were raised from the dinkeys and placed into charging position quite efficiently.

Due especially to the long wheelbase of the muckers, it was found necessary to use a minimum radius of 40 ft on the track and to maintain the gage of the track with standard railroad track-gage rods made over to the 36-in. gage. Wood ties were used, spaced about 30 in. on centers; usually 6-in. by 12-in. timbers were available from decking lumber, or 6-in. by 8-in. timbers were purchased specially.

Practically all the concrete was placed by two concrete pumps. A small, gasoline-driven unit, with a 6-in. pipe, and with a conical hopper and integral agitator, was used in pouring benches. This smaller pump was also used on the umbrella and a part of the arch of the double tunnel, when the larger pump was being used elsewhere. The larger pump was the same make as the smaller, with a 7-in. pipe. It was propelled by a gasoline motor but of a model antedating the time when integral agitators were installed. The smaller pump gave an actual production of from 20 to 25 cu yd per hr, compared with 30 to 35 cu yd per hr for the larger pump.

Some of the smaller items in the tunnel, such as catwalk and duct-protection incasement, were poured manually from a special car with a high box body and side chute. However, about 98% of the concrete volumes, including the inverts, were placed with the pumps.

Miscellaneous equipment also used included two blowers, each of 2,000-cu-ft-per-min capacity, interconnected with each other and with the ventilator pipe in each tunnel, so that a maximum capacity of 4,000 cu ft for either blowing or sucking could be provided in either tunnel. The vent pipe was composed of 8-in. thin-shell steel pipe with dresser couplings. Two short lengths of flexible tubing were provided, but after a trial it was found just as easy to remove one or two sections of the steel pipe from the heading as to use the flexible pipe.

For trimming the roof, a stoper drill, with automatic feed, for 1-in. hexagonal shankless steel, was used. All other drill steel was 1¼-in. round.

The drill-sharpening plant (which was situated just north of the tunnels beneath the decking and connected by rail with all the tracks in the tunnels) consisted of two complete units for making up and sharpening drilling steel, as well as a small blacksmith shop. Each drill-sharpening unit consisted of a drill sharpener, with clamping dies for both 1¼-in. round steel and 1-in. hexagonal steel. The equipment for each set comprised: A collaring device for the hexagonal steel and lug shanking device for the round steel, gaging dies, fuller dies, dollies for sizes from 1⅝ in. to 3 in. inclusive, a shank and bit punch, an oil forge, and fuel-oil tank. No trouble was encountered with the operation of the blacksmith and drill-sharpening plants beneath the decking; the natural draft as a result of the difference in temperature between the street and the confined volume beneath the decking were found sufficient to take care of ventilation.

EXCAVATION OF SINGLE-TRACK TUNNELS

The nature of the rock exposed in the open-cut work made it necessary to begin tunnel excavation 10 ft north of the theoretical tunnel portal and only after a concrete beam protection had been placed above the tunnel portal (see Fig. 8). The purpose of the beam was to retain any loosened rock and thereby prevent damage not only to the street surface but to the subsurface structures, as well as the foundations beneath the elevated columns a short distance to the south of the tunnel portal. In the preliminary excavation, it was found that, because of the seamy nature of the rock, carrying the drilling holes 6 in. outside of the net lines on all sides, including the bottom, was necessary in

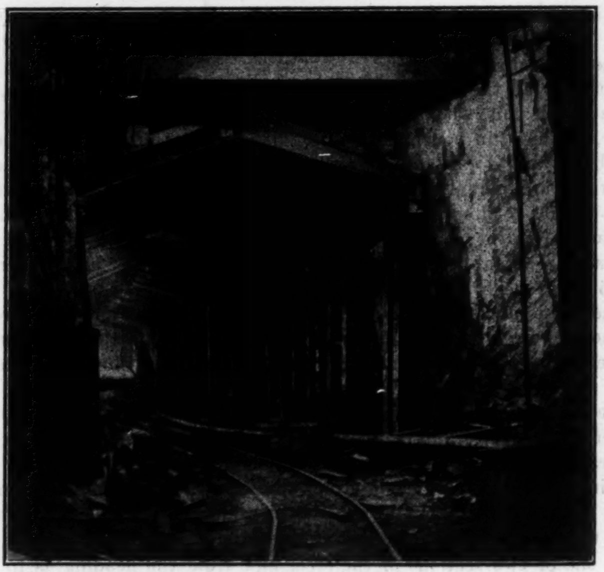


FIG. 8.—ENTRANCE AT NORTH PORTAL OF QUEENS SOUTHBOUND TUNNEL

FIG. 9.—EFFECT OF WILD CUT SHOT ON BRACING

order to reduce the amount of trimming. The cut shot normally did not follow the cut holes but broke away along the nearest seam (see Fig. 9).

The general plan of operation was to drill one round sufficiently to give an advance of 6 ft and then blast and muck all in one shift. The typical cycle of operations in both the single and double tunnels is shown in Fig. 4. It must be kept in mind that the blasting operation was governed by contract requirements that not more than 100 lb of dynamite could be used in any one round fired. As a result, it was necessary to load and shoot three times in each cycle of operations. The normal procedure was to shoot the cut by itself, using one delay in some instances; the second firing took care of the relievers and some of the trimmed holes with five delays; and the final round cleaned the entire section, using from three to five delays (see Fig. 6).

The Queens southbound single tunnel was the first of the single tunnels excavated. The record for August, 1937, the first full month of tunnel operation, shows 34 rounds averaging 7.62 ft. It was found that, with seven drills operating and with a cross section giving 12 cu yd per ft of advance, an 8-ft advance completely filled the entire shift of seven and one-half working hours. If a greater advance was drilled and shot, the entire cycle of operation was interfered with. On August 4 and August 26 seams of rock were encountered in this tunnel that were so badly shattered as to require steel bracing directly to the face before drilling could continue. This was necessary as a safety precaution not only for the men in the tunnel but to avoid failure of street surface and elevated columns above the tunnel.

During the entire month of September, 1937, this tunnel was in soft and seamy rock, the seams containing a slippery chlorite. Even with steel frames placed 3 ft on center, the pressure of the rock deformed the steel bracing.

In a number of areas the heading was progressed only after steel outriggers had been extended to the face at the roof and full timber lining was placed on both the roof and the sides. During the month the total progress was 170 ft, with the rounds averaging from 4 to 8 ft, some of them being greater than the actual depth of holes drilled. This tunnel was holed through on October 1, 5 ft south of the theoretical portal and in rock so soft that the normal quantity of dynamite was cut to less than one half because a large area outside of the cut would move and fall when the cut was shot. When shot, this material fell vertically, with practically no displacement beyond the limits of the drill-hole length.

The Queens southbound tunnel ran parallel to a part of the west track previously completed in the open cut in seamy rock. During the open-cut work the seamy rock was doweled by drilling across the seams and by grouting in 1-in. square rods approximately 4 ft long. Some of these rods were encountered in the tunnel excavation where breakage exceeded the drilled area by more than 1 ft. However, no damage was noticed at any time in the completed open-cut structure.

At the south end, the seams in the rock were so soft, and in a number of instances actually open voids, that shooting was reduced to a few holes at a time to reduce the wild breakage. In this area six 6-in. H-beams were carried as cantilever lagging ("crown bars") above the steel ribs up to the face, and after the top holes were shot the lagging was pulled ahead and blocked before the excavation was continued. During this work the advance in the single-track tunnel was given precedence over advance in the center cut of the double tunnel, and no attempt at any definite cycle was considered.

In the other single tunnel (Washington Heights southbound track), although much higher with less cover than the Queens southbound tunnel, rock conditions at the beginning were much better and holes were collared 3 in. outside of the net lines. However, after advancing about 100 ft, the rock turned to a fine-grained syenite with soft mud seams and chlorite jointing. Large quantities of this material would fall from the roof during blasting, and the bracing steel showed definite signs of rock pressure (see Fig. 10). Where this tunnel came beneath an "el" column footing, steel bracing bents were placed on 5-ft centers for a distance of 25 ft in the schist rock. In the seamy rock, steel bents were placed 5 ft on centers continuously. Where the rock breakage was irregular, steel framing was altered to fit the actual breakage, especially on the side adjacent to the double-track tunnel which was already completed and which was only 2 ft or 3 ft away from the actual breakage. Near the end of this tunnel the rock was so poor that no attempt was made to make more than one round in twenty-four hours, especially since additional steel framing was found necessary as intermediate bents when the original framing started to show a sag. Soft ground methods could not be used for tunneling because the individual syenite rocks between the joints were hard and irregular. In January, 1938, the last 50 ft of progress was made in one week, and the remainder of the month was used in rebracing to clear the concrete lines and to place concrete in back of the side-wall sheeting to prevent movement of the rock.

The proximity of adjacent tunnels was shown clearly when a duct manhole was excavated in the side of the Washington Heights southbound tunnel where a theoretical clear distance between the tunnels was 5 ft. Because of the over-breakage of the two adjacent tunnels, and the fact that concrete very often ran through the seams, the concrete of the Washington Heights southbound tunnel manhole was encountered in three places in the double tunnel excavation.

FIG. 10.—ROCK BREAKAGE AT SOUTH END OF TUNNELS AT CHLORITE SEAMS

The Washington Heights southbound single tunnel was started on November 8, 1937, with special shift periods. It was the only tunnel being excavated and was planned for 2 rounds in twenty-four hours, the drilling and blasting crews coming to the job at 8:00 a.m. and 8:00 p.m. and the mucking crews at 2:00 p.m. and 2:00 a.m. At first it was found difficult to push the drillers so that blasting was completed before the mucking crew came to the job. However, when the drilling crew was given an assignment to drill for a 6-ft round and the entire crew with the exception of one scaler was permitted to go home after the entire round had been shot out, the required progress was obtained easily. As a matter of fact, it was determined by the blaster that the one scaler who remained for the full shift was the driller who finished last. When this rule was instituted, the remainder of the crew usually went home in six to six and a half hours. Generally, between 100 and 110 holes were drilled, each 6.5 ft long, except the ten cut holes which were between 7 and 7½ ft long. Seven drills were operated on the jumbo, using the 4-in. drills for the cut holes and the 3.5-in. drills for the remainder. The average production per twelve hours was 5.42 ft up to December 20 when the seamy and blocky rock was encountered.

EXCAVATION OF THE DOUBLE TUNNEL

The general procedure in the double tunnels was to excavate the center drift for the full height, and about 14 ft wide, as a separate tunnel operation. Bracing was designed for future extension to incorporate the bracing of the enlargements on each side. After the center tunnel excavation had been completed, the two sides were ring drilled, and this was followed with concreting of the center wall footing. The structural steel of the center wall was then placed, and the umbrella, consisting of the center wall and parts of the two roof arches, was poured as a unit, incasing the steel beams of the temporary roof support. The side enlargements were then made by shooting the ring holes previously drilled, taking each side as a separate operation.

The center cuts of the double tunnel and the Queens southbound tunnel were started simultaneously from the same face in the open-cut excavation. During August, 1937, 36 rounds were taken with an average progress of 7.34 ft in fairly hard mica schist with some quartz intrusions. Near the south end of the tunnel the rock became quite seamy, and tunneling was stopped 40 ft south of the theoretical tunnel portal, although it had been planned to carry this tunnel 80 ft into the open-cut area not yet started south of 43d Street. Ring drilling was begun immediately thereafter, using the drillers who were working in the Queens southbound tunnel at such times as the latter operation permitted. Ring drilling was completed on October 16 and was followed by steel erection and concrete center bench construction, working from 46th Street southward.

FIG. 11.—EXCAVATION OF THE WIDENING IN THE DOUBLE TUNNELS

FIG. 12.—WEST WIDENING IN DOUBLE TUNNEL

Blasting for the east enlargement for the center tunnel was started at the north end on November 1, with a small crew, including two drillers for trimming in addition to the normal mucking crew (see Fig. 11). This work was done on a two-shift basis until the progress reached a point at which it was too close to the already excavated Washington Heights southbound tunnel, at which time the east widening was continued on a one-shift basis and the west widening was stopped until the adjacent tunnel was concreted and grouted.

Progress in the east widening during the first month was 288 ft on a two-shift basis. During the same month, beginning November 15, the west widening was extended 210 ft on a two-shift basis with separate and distinct crews and equipment. The intent was to keep the two widenings moving simultaneously but at least 75 ft apart north and south so that the blasting of one would not interfere with work on the other. Because of the dip of the rock seams, the east widening required much more additional longitudinal drilling than the west widening.

As each widening progressed, the timber supports placed in the center cut were removed and used as a protection for the concrete umbrella (see Fig. 12). It also was found that an excellent protection for the upper edge of the concrete umbrella from flying rock was the cushioning effect of discarded rubber belts hung from the temporary roof steel and almost in contact with the concrete.

The east widening of the double tunnel was completed on December 28, progress in December being 316 ft. Until December 22 work was done on three shifts, and after that one shift, per day, making an average progress per eight hours of 4.71 ft. The west widening of the double tunnel was closed

FIG. 13.—INCIDENTAL SEAM BRACING IN THE DOUBLE TUNNEL; WALL BENCH CONCRETED

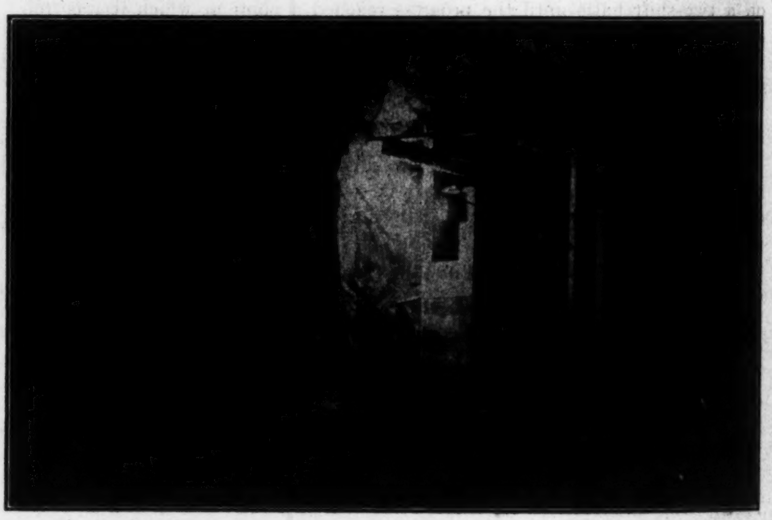


FIG. 14.—STEEL AND WALL FORMS AT DUCT MANHOLE IN TUNNEL

on December 17 with 193 ft still to go because of the proximity of the adjacent tunnel. In this tunnel progress of 334 ft in December showed an average of 7.79 ft per shift. The much better progress on the east side shows the favorable direction of the rock seams.

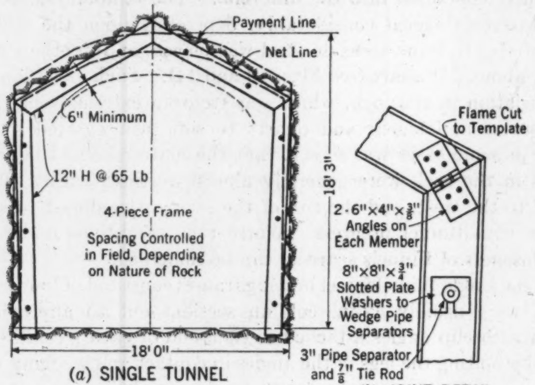
In both of the widening operations some steel bracing was found necessary because of the open seams, and such bracing was tied in to the roof steel previously concreted into the umbrella. The temporary steel bracing placed in the tunnels varied considerably, depending upon the rock conditions encountered. In some areas no steel was placed, and in others steel frames were placed about 2 ft apart (see Figs. 13 and 14). Such variation corresponded to the variation in the rock, which was from an extremely hard igneous syenite with seams of feldspar and quartz to soft disintegrated schist and chlorite which in some cases was so soft that the material could be dug out by hand. Seams in the rock were generally almost vertical, with a slight dip from 5° to 15° to the west, and the run of the seams was almost to south, or directly in line with tunnel progress. Worse rock conditions under the small cover and closeness of tunnels scarcely can be imagined.

In the single tunnels steel bracing frames consisted of four pieces of H-beams, which were usually 12-in. column sections cut to approximate shape and length with clip angles attached to each end of each piece. Connections were made by bolting the legs of the angles in contact and wedging with steel wedges as found necessary to take up the shape of the excavated tunnel. Typical sections of the type of bracing used, and details, are shown in Fig. 15. The steel bracing as well as temporary posts and lagging was placed by timbermen crews brought into the tunnel as found necessary (see Fig. 15). During July, 1937, working seventy-four shifts, 70 bents were placed in the single tunnel and 71 bents in the double tunnel. These bents were placed in the total tunnel length of 479 ft.

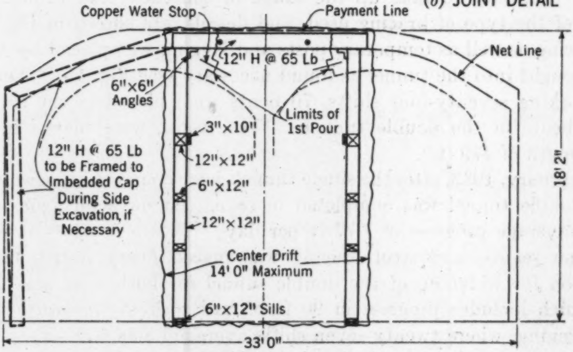
In February, 1938, after the single tunnels were concreted, the west widening of the double tunnel was completed in seventeen days on a one-shift basis with an average progress of 7.77 ft per day. The rock was soft mica schist with open seams, and steel bracing was placed every 5 ft. The average production for widening of the double tunnel for both sides was 6.50 ft per shift, which includes progress in 90 ft of bad rock at the south end of the west widening, where twenty-seven shifts averaged 3.36 ft.

At approximately the middle of the double tunnel where the vent shaft and emergency exit were to be built, 30 ft of the center umbrella arch was omitted and a crossover track was installed to connect the operations in the east and west widenings. This crossover expedited the mucking problem when both of the widening operations were south of this future shaft. To eliminate any possible damage to buildings in the vicinity of the shaft, the plan was not to break through to the street surface until all tunnel operations were completed.

In the three manholes excavated by widening the tunnels, 10-in. steel pipe connections were provided with the street surface to act as cable feed pipes for signal and other cables to be pulled in the ducts placed along the east sides of the Washington Heights southbound tunnel and of the double-track tunnel (see Fig. 14). These pipes were inserted in 12-in. cored borings taken from the street surface into the excavated tunnels. Two of these holes measured from rock elevation about 6 ft below street surface, and were each 22½ ft long; the third hole was 13½ ft long. A shot drill operated by a gasoline engine was started on December 14 at the first hole and all work was completed by January 10 for all three holes. Considerable difficulty was encountered because of the large amount of the shot that was lost in the seams of the rock.


Payment Line
Net Line
6" Minimum
12" H @ 65 Lb
4-Piece Frame
Spacing Controlled in Field, Depending on Nature of Rock
18' 3"
18' 0"
Flame Cut to Template
2-6"×4"×⅜" Angles on Each Member
8"×8"×¾" Slotted Plate Washers to Wedge Pipe Separators
3" Pipe Separator and ⅞" Tie Rod

(*a*) SINGLE TUNNEL (*b*) JOINT DETAIL


Copper Water Stop
Payment Line
12" H @ 65 Lb
Net Line
6"×6" Angles
Limits of 1st Pour
3"×10"
12"×12"
12" H @ 65 Lb to be Framed to Imbedded Cap During Side Excavation, if Necessary
6"×12"
12"×12"
Center Drift 14' 0" Maximum
6"×12" Sills
18' 2"
33' 0"

(*c*) DOUBLE TUNNEL

FIG. 15.—DETAILS OF TIMBER BRACING

However, the operation was quite successful, and the total cost of all labor, materials, and equipment was $11 per foot of core removed. There was no difficulty in placing a 10-in. (inside diameter) galvanized steel pipe in the cored holes. These pipes were grouted in and capped at the street surface.

LABOR SCHEDULES, PROGRESS, AND COSTS

The typical labor setup for tunnel excavation is shown in Table 1. The weekly summary and averages of tunnel progress, consisting of the completion of the Queens southbound and the completed center cut for the double-track tunnel, are shown in Table 2 (see Fig. 4(*a*)).

TABLE 1.—LABOR SCHEDULE ON TUNNEL EXCAVATION

Description (1)	Single tunnel[a] (2)	Queens southbound[b] (3)	Washington Heights southbound[c] (4)	Double tunnel widening[d] (5)	Remarks[e] (6)
Blasters	1	1	1	1	....
Drill runners	9	9	2 at 7 1 at 3	2	Col. 4: 7 runners each for 2 shifts 3 runners for 1 shift
Drill runners' helpers	8	9	2 at 7	....	Col. 4: 7 runners each for 2 shifts
Apprentice engineman	1	1	1	1 at 1	Col. 5: One shift only
Powder monkey	1	1	1	1	....
Muck foreman	1	1	2 at 1	1	Col. 4: Two shifts, 1 foreman each
Tunnel labor	7	7	2 at 9 1 at 5	6	Col. 4: 9 laborers, 2 shifts 5 laborers, 1 shift
Laborers	3	3	3	2	....
Hoist engineman	1	1	1	1 at 1	Col. 5: One shift only
Apprentice engineman	1	2	1	1	Col. 1: 1 extra on day shift only
Dinkey operator	2	2	2	1 at 2	Col. 1: Only one during first month
Electrician	1	1	1	1 at 1	All columns: On day shift only; part time on others
Tunnel boss	2	2	2	0	Cols. 2, 3, 4: Two bosses covered all 3 shifts
Powder watchman	1	1	1	1	All Columns: One per shift and additional men for Sundays
Totals for three shifts	113	117	92	35	

[a] Also center drift of double tunnel; three shifts at one round per shift. [b] Three shifts. [c] Also drilling two widening; three shifts. [d] Two shifts, 8:00 a.m. to 4:00 p.m. and 2:30 p.m. to 11:00 p.m. [e] Does not include timber crew, which usually comprised a foreman, 3 timbermen, and 3 helpers, for one or two shifts, as the tunnels required it.

TABLE 2.—SUMMARY OF TUNNEL EXCAVATION PROGRESS

Week ending (1937):	QUEENS SOUTHBOUND TUNNEL		WASHINGTON HEIGHTS AND QUEENS, NORTHBOUND		Totals, in ft	Average progress, in ft per shot
	No. of shots	Progress, in ft	No. of shots	Progress, in ft		
June 5	3	16.6	4	22.0	38.6	5.51
June 12	7	38.5	8	41.1	79.6	5.38
June 19	8	41.5	9	48.9	90.4	5.32
June 26	6	35.0	8	49.0	84.0	6.00
July 3	9	47.5	8	42.0	89.5	5.27
July 10[a]	8	44.0	8	49.0	93.0	5.81
July 17	7	45.0	8	56.0	101.0	6.73
July 24	9	59.0	9	61.0	120.0	6.66
July 31	8	56.0	9	61.0	117.0	6.88
August 7	8	60.0	8	57.0	117.0	7.31
August 14	8	57.0	9	65.0	122.0	7.18
August 21	9	69.0	8	60.5	120.5	7.62
August 28	8	66.0	8	63.0	129.0	8.06
September 4	7	45.0	9	64.5	109.5	6.84
September 11	4	22.0	1[b]	7.0	29.0	4.15
September 18	7	36.0	....	....	36.0	5.14
September 25	6	40.0	....	....	40.0	6.67
October 2	7[b]	41.0	....	....	41.0	5.85
Total	129	819	114	747	1,566	....
Average progress, in ft per round		6.35	....	6.55	....	6.45
No. of calendar shifts		294	....	237	....	294[c]
Progress, in ft per shift:						
To September 7, 1937		2.90	....	3.15	....	6.05
After September 7, 1937		2.34	....	....	....	....
Total[d]		2.76	....	....	....	....

[a] Five days. [b] Completed. [c] From June 1. [d] Beginning September 7, 1937, all three shifts, ring drilling in two tunnels during spare time averaged 1.08 shifts per round.

Costs.—The average labor cost of all the tunnel rock removed in 1937 was $5.75 per cu yd. During August, with two tunnels being excavated, the total labor cost was $5.11, distributed as follows:

Operation	Dollars per cu yd
Blasting and mucking	4.23
Compressor plant operation	0.12
Blacksmith shop operation	0.35
Equipment repair	0.25
Cleaning up	0.16

The labor cost of ring drilling the double tunnel widenings was $26.20 per lin ft of tunnel, ring drilled on two sides.

A study was made of the comparative costs of tunnel excavation for the full face running two tunnels at one time, as occurred when the Queens southbound and the center cut of the double tunnel were worked, and the cost of excavating the widening tunnels of the double tunnel in the manner in which it was done with ring drilling, or as separate tunnels with longitudinal drilling. In general, the labor cost of driving two tunnels together is practically the same as the labor cost of the drilling and mucking in widening the double tunnel (see Table 3). However, considerable time was saved by the ring-drilling process. There is an indirect saving (which is difficult to estimate) in the continuous use of drillers while the Queens southbound tunnel was being finished, and there is a saving of 50% in the dynamite cost in the excavation of the widening cuts over that used in the other tunnels.

TABLE 3.—LABOR COSTS OF RING DRILLING VERSUS DRIFTING, IN DOLLARS PER CU YD

Operation	ACTUAL[a]		ALTERNATE[d]		
	Rock excavation[b]	Ring drilling[c]	1.5 shots per shift	1 shot per shift	Two, 6-ft shots per shift
Drillers required for 4.5 shots at 6 ft, in three shifts	....	....	6	4	....[e]
Drilling	4.23	1.82	5.08	5.86	3.82
Blasting, mucking, and trimming		2.46			
Compressor plant operation	0.12	0.12	0.12	0.12	0.12
Sharpening steel	0.35	0.35	0.35	0.35	0.35
Repairs to equipment	0.25	0.25	0.25	0.25	0.25
Total	4.95	5.00	5.80	6.58	4.54

[a] Cost of two tunnels as driven. [b] Comparative labor cost of rock excavation. [c] Ring drilling and widening as of December 13, 1937. [d] Probable costs of driving two drifts instead of using the ring-drilling method. [e] Very unlikely condition.

For the entire tunneling operation, the total labor costs were as follows: Charged to excavation of 30,000 cu yd, $190,000; charged to timber, placing 155 million fbm, $16,224; charged to steel, placing 578 tons, $23,306; and charged to grouting 2,300 bbl of cement, $3,235. To the foregoing should be added labor costs in the blacksmith shop; equipment repair outside of drills;

compressor-plant operation; the proportion of the cost for setting up the compressor plant and the mucking shaft; and the pay roll of the master mechanic, superintendent, timekeepers, storeroom keeper, and engineering overhead.

DUST CONTROL

After excavation was begun, instructions were received to comply with the then latest regulation of the New York State Industrial Code concerning dust control in rock excavation work. Such a code did not exist at the time the contract was awarded, and, with the exception of normal ventilation and wet drills, no special provisions had been made for such compliance. On June 16, 1937, the New York State Industrial Commission sent representatives, together with observers from the New York Labor Department and the U. S. Bureau of Mines, to take dust count samples during the drilling operation. Although no definite results were reported to the contractor, it was made known indirectly that compliance with the Code would be required and that some method must be submitted for approval unless the strict requirement of the Code (which at that time permitted only dust collectors) was provided.

Immediately thereafter, tests were run in the excavation with wet drills of various types (ventilation being both by blowing and by sucking within the maximum capacity of 4,000 cu ft per min available). Tests were also made to determine the effect of several types of wetting agents mixed in the drill water. Private dust counts were made under each of several combinations of the foregoing possible remedies.

As a result of all of this research, a method or formula for dust control was submitted to the New York State Industrial Commission and after approval was incorporated in the Code. The rules set up for compliance with the amended regulation included the following seven items:

(1) Wet drifters shall be fitted for 1½ to 2 gal per min of water applied to the drill steel;
(2) The exhaust port of the drifter shall be directed away from the face;
(3) A minimum amount of air shall pass through the hole in the drilling steel;
(4) No holes shall be directed upward except a minimum at the top of the tunnel;
(5) Air shall be exhausted at the rate of 1,200 cu ft per min per each drifter —maximum size, 4 in.;
(6) The half-throttle, air-bleeder port shall be plugged;
(7) All holes must be collared wet.

The quantity of water that flowed through the drill needles was tested under various pressures, both for the $\frac{1}{16}$-in. standard needle and for the ⅛-in. special needle. It was found that the rate of flow varied between 1 and 2 gal per min between the pressures of 20 and 70 lb per sq in. for the ⅛-in. needle, which was adopted for all drills.

Before any special control methods were used, dust counts were made of the air in the tunnel without drilling and also on the street surface. It was not unusual to find a dust count more than twice as high in the air at the street surface as in the tunnel without drilling.

A complete report of all these tests has been published by F. B. Flinn and P. S. Miller,[2] Assoc. M. Am. Soc. C. E.

In connection with compliance with the Code (which provides one permissible dust concentration for rock containing less than 10% free silica and a different permissible concentration for rock containing more than 10% free silica), a study was made to determine whether the average rock encountered came in the classification of more than, or less than, 10% free silica. Except for the approximate petrographic methods, the only reliable chemical method for this determination, found after a thorough detailed study by the experimental laboratories of the DuPont Company, was that described by A. Shaw[3] in 1934. This method is so detailed that only the best equipped laboratories could perform the test and then only after about a week of work. Another method reported by Sartorius and Jotten[4] is a combination of chemical separation and centrifuging the free silica from the solution. Neither of these methods was considered practicable enough for attempting a determination of the true amount of free silica in the rocks encountered. When the question was presented to the New York State Industrial Commission, and samples of rock were sent for inspection, it was reported that the rocks contained more than 10% free silica without giving the exact percentage or advising what method had been used.

THE EFFECT ON THE "EL" STRUCTURE

During tunnel operations, daily readings were taken on level points fixed on all elevated columns above the excavated parts of the tunnels and for some distance beyond the headings. Practically no deviation in elevations was found in any of the columns except the one column directly above the Washington Heights southbound single-track tunnel, near the south end. During excavation a slight settlement was noticed in this column. Since the excavation was through soft material that broke very poorly, steel bents were placed to support the roof. After the tunnel was completed, some of this steel bracing required readjustment to clear the concrete lines, and during this operation a considerable mass of loose material that had been resting on the roof steel fell down. This happened directly below the elevated column footing, which, however, was not a dangerous condition because the elevated column load had been jacked up on two temporary steel beams above the street surface as soon as the settlement in the column had reached 1 in. After all the loose material had been allowed to fall, exploratory drill holes were started from the street surface, and it was found that this footing was resting on a layer of rock about 5 ft in thickness with soft material below it. This rock was consolidated by grouting from the street surface, as well as through the concrete after it was poured. The column itself was loosened from its footing and jacked up to proper elevation, and the intervening gap was filled with a dry-packed grout.

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[2] *Engineering and Mining Journal*, Vol. 139, July, 1938, pp. 38–43.

[3] *The Analyst*, Vol. 39, 1934, p. 449.

[4] *Zentralblatt für Gewerbehygiene*, Vol. 21, p. 65.

DYNAMITE

Contract specifications limited the use of dynamite to not more than 100 lb of 60% gelatin strength in any one blast, including delayed explosions. As stated previously, the rock was of such nature that 40% gelatin dynamite was found more suitable.

The total quantity of dynamite used for excavating these tunnels was 2.98 lb per cu yd, net measurement, of 40% gelatin strength. Based on actual excavation, due to over-breakage, the total was actually about 2¾ lb per cu yd. On the average one cap was used for each six sticks of dynamite, or 3 lb.

The weight of dynamite per cubic yard moved varied from month to month, depending upon the type of rock encountered. The maximum was in June, 1937 (3.12 lb); the minimum was in January, 1938 (1.04 lb). The average dynamite consumption for straight tunnel excavation was approximately 3.10 lb per cu yd, and the average for shooting ring holes was 1.65 lb per cu yd.

All detonation was electrical, a separate blasting line being carried into each heading from a switch located at the portal.

CONCRETING

Concrete in all arches, including the umbrella portion in the double tunnel, was placed by concrete pumps, using steel forms. Total forms purchased for the job included a 30-ft section of umbrella form, a 30-ft section of side arch that was usable in either of the enlargements for the double tunnel, and a 30-ft section of single complete arch tunnel. The umbrella form consisted of two side panels and part of the arch with a trussed framework that supported the wheels. The form collapsed by sliding it on the wheel shaft. This was done by turning the shaft with a ratchet handle, thereby pulling the form away from the concrete. By means of screw jacks built into the wales, the form was collapsed vertically. In this manner the form tended to lean toward the concrete, and rollers were provided to run on the concrete while the form ahead was moved. Bulkheads at each stage were built up with wood. For the remainder of the double tunnel a usual type of traveling form was used, with jacks at the bottom, and it covered approximately three quarters of the arch. In detail it was very similar to the complete tunnel form.

The complete form for the single-track tunnel was 30 ft long built up in multiples of 6 ft. Two openings in the side-wall were spaced 15 ft apart for niches. The trussing and framework for the form, including the traveler, were so arranged that a muck car could be run through the form, and in a number of locations such a procedure was necessary. Usually, timber braces were placed across the legs of the traveler to increase the stiffness of the form. Because of the fact that the tunnel is wider on curves, a slip lap joint was provided for the top plate which, in the wider part, resulted in a flat spot at the top of the arch. This panel was really the key plate, placed last as the concrete came up to each section. The form was held in place by means of screw anchors embedded at accurate spots in a short height of the wall built with the benches. The same rail was used as for mucking, additional rail lengths being moved ahead to accommodate the wide gage of the traveler. Collapsing of the form was by lowering the screw jacks on to the axles. The concrete record and progress are shown in Fig. 5 and the location of equipment in Fig. 16.

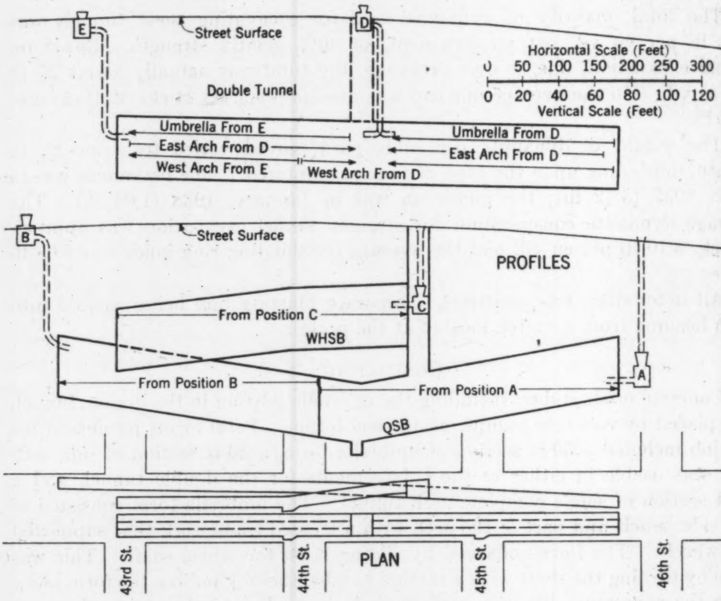
E
Street Surface
D
Horizontal Scale (Feet)
0 50 100 150 200 250 300
0 20 40 60 80 100 120
Vertical Scale (Feet)
Double Tunnel
Umbrella From E
East Arch From D
West Arch From E
West Arch From D
Umbrella From D
East Arch From D
B
Street Surface
PROFILES
From Position C
C
WHSB
From Position B
From Position A
A
QSB
PLAN
43rd St.
44th St.
45th St.
46th St.

Fig. 16.—Concreting Setups

In the single-track tunnel, concrete arches were poured at the rate of 30 ft each day, using the larger pump. In the double tunnel, the umbrella section was poured at the rate of 30 ft a day with the smaller pump, such a pour being between 40 and 45 cu yd. The complete arch pour was about 135 cu yd.

No difficulty was experienced in pouring once every day. The early morning shift collapsed the form and moved it ahead, and carpenters starting about 6:00 a.m. would build the bulkhead and prepare for the next pour to start at 10:00 a.m. After the first few days of operation, and after all the idiosyncrasies of the concrete pump apparatus were fairly well understood, the 135 cu yd of pour would be finished by 4:30 p.m. The concrete pumps were placed at approximately subgrade level, and the concrete was chuted down from mixing trucks at the street surface through circular 8-in. elephant trunk chutes. The rate of delivery was controlled by bell signals between the pump operator and the street surface. Pump operation was controlled by the engineers stationed at the pump in accordance with electric bell signals from the foreman at the form. Very little difficulty was encountered because of the rule that all signals, when received, must be repeated by the receiver, and the little additional expense of providing return signals from each line was found much worth while.

In some instances in the double tunnel it was found possible to place the pump in the shaft area at approximately the middle of the length of the tunnel and at the street surface, using a down-feed pipe to the tunnel level. With fairly uniform delivery of concrete, and starting each day's pour with approximately 3 cu yd of cement grout to lubricate the pipe, no difficulty was found in pumping concrete approximately 500 ft horizontally and 20 ft vertically.

At a point about 30 ft from the form an air-connection tee was added to the concrete pipe to be used as a booster for filling up the form. When the concrete had filled up the entire form and the end bulkhead had been completed, the short pipe sections inside the form were removed as the concrete filled up the volume. Air at 100-lb pressure was admitted into the concrete pipe, and the fluid concrete ahead of the air inlet, about 30 ft in length, was shot into the concrete mass inside the form. This operation, it was found, could not be repeated safely until the normal pumping had again filled up the entire pipe length; otherwise the back pressure clogged the pipe between air connection and pump.

With this procedure the concrete could be pushed up 6 ft above the outlet pipe inside the form where rock crevices were that high; and, as far as visual inspection was concerned, the concrete seemed to be tight against the rock. During pumping operations, it was found practically safe (although not unduly comfortable because of the high humidity and temperatures) for men to work inside the form when placing and tying grout pipes, spading concrete, and operating vibrators.

The typical labor setup for concreting, including stripping and shifting steel form, building bulkheads, setting and brazing copper water stops, setting up concrete pipe, setting and making up grout pipes, revising steel bracing, concreting, and cleaning up pipe and concrete pump, was as given in Table 4.

TABLE 4.—LABOR SCHEDULE FOR CONCRETE WORK

Description	Timbermen	Timbermen helpers	Laborers	Carpenters	Concrete foremen	Concrete laborers	Maintenance enginemen
Single Tunnel Form:							
Queens southbound tunnel	4	4	5	10	1	14	3
Washington Heights southbound tunnel	5	6	....	14	1	11	3
Double tunnel, west arch	6	7	2	16	1	11	5

The labor cost for stripping, moving, and resetting forms and building bulkheads was $0.23 per sq ft contact area of tunnel surface (not including areas of bulkheads).

The total labor cost of tunnel concrete in 1937 was $3.80 per cu yd, to which must be added the cost of the forms, insurances, overhead, and concrete, including the value of the excess concrete outside of pay lines. This labor cost includes all of the men involved in preparing, setting, and removing the forms, and the placing of the concrete inside the form, placing the grout pipes

FIG. 17.—SOUTH END OF DOUBLE TUNNEL

FIG. 18.—DOUBLE TUNNEL COMPLETED EXCEPT FOR CATWALK

and water stops, and any incidental work in connection with the concrete both for the arches and the benches.

STEEL ERECTION

In the double tunnel, structural steel columns and base and top grillages were erected in the center wall. Top grillages were continuous over two columns in the form of bents independent of each other. The structural steel was brought into the tunnel on the mucking tracks and set at the side of the tunnel until all the steel had been delivered. Erection was by hand, using the roof bracing for attaching block and falls. As the structural steel was set in place, the track had to be removed. A total of 102 tons of steel was brought in with a total labor cost of $22.37 per ton, including supervision, erection, labor, riveting, and trucking from lighters about 2 miles away.

COMPLETION OF TUNNELS

After the main arches were poured, in which grout pipes had been installed, the track remaining in place, a grouting gang was sent through to grout all the holes that would take it, at 50-lb to 60-lb pressures. This was followed by a second run at 60-lb pressure (and in some cases more), after which the grout holes were filled. At the same time, any finishing that needed to be done on the concrete, such as removal of fins, was completed (see Figs. 17 and 18). Grout pipes were 2 in., with a coupling set flush on the form. Grout cocks were connected by nipples, which were removed after the completion of grouting.

The grout used was approximately of 1 : 1 mix. The grouting machine ran on the track, and the sand and cement were brought up in bags on flat cars. The Queens southbound single tunnel, 700 ft long, required 1,210 bbl of cement, or approximately 1¾ bbl of cement per linear foot of tunnel. The total cost of this grouting was as follows: Labor, $1.24; insurance, $0.24; cement, $2.10; ⅛ cu yd of sand, $0.25; sales tax, $0.05; and air and equipment, $0.12—a total of $4 per bbl of cement. In the other tunnels the labor cost ran quite uniformly the same, at $1.25 per bbl of cement. The average day's production was 250 bags of cement, and the maximum day's production was 356 bags.

At convenient times the inverts were graded by hand, removing the track after grading was completed. The invert concrete slab was 12 in. thick, and in some locations covered cast-iron drain pipe was set slightly below the invert. This concrete was placed by pump, keeping the pipe on the side bench. One shift could easily do 250 ft of invert, or 125 cu yd, using the small pump.

In two of the tunnels a catwalk, or emergency walkway, was built in sections of about 30 ft, re-using a set of forms for the entire length of two tunnels. In the other two tunnels a bank of 30 clay ducts was laid against the wall and concreted in, the top becoming the catwalk.

SHAFTS

After the tunnels were concreted completely, two shafts were excavated down from the top. The west shaft required removal of only the thickness of rock between the two tunnels which crossed each other at that point—approximately 5 ft. Mucking was by crane and buckets from the street surface.

In the east shaft, where the cover was about 20 ft above the tunnels, earth and some of the rock were removed from the street surface to expose and protect the water main and utility ducts, and also to deck the opening in the street surface; then by means of long drilling, holes were driven through to the tunnel. The rock was then blasted into the tunnel by loading the bottom of the holes. Mucking, again, was by crane and buckets from the street surface.

In the east shaft niches were cut to provide for the exit stairways connecting the catwalk with the street surface, in this way avoiding open excavation immediately adjacent to the building. In spite of the seamy rock in the east shaft, steel bracing in the tunnel and in the shaft prevented any movement of the rock, and the operation was completed without hazard to the pedestrian traffic as well as the buildings immediately adjacent to the shaft.

ACCIDENTS

In general, the tunnel operation was remarkably free of accidents, although there was one fatal case. As far as equipment was concerned, at one time a mucking machine tipped over in the tunnel causing a loss of three hours in schedule but no injuries. At another time a piece of roof fell out on the boom of the mucking machine and broke the beam connection, without any personal injuries. The fatal accident occurred on the last night of tunneling when a small piece of rock fell from the roof and injured the mucking machine operator, who at the time was watching the track gang install the last piece of track for the final cleanup; the man died some days later.

To reduce accidents, every man in the tunnels was required to wear a safety hat and safety boots that were provided with nonskid bottoms and aluminum tips. The foremen were held personally responsible for accidents to the men in their crews. No separate record was kept of accidents in the tunnels as against those on the remainder of the contract. The total record of lost-time accidents from July, 1937, to April 1, 1938 (during which period most of the tunnel work was done), shows a total of thirty-seven accidents for the nine months, of which thirty-two were lost-time accidents. The number of men engaged in that period averaged between 400 and 500 per day. In this period of nine months the maximum number of lost-time accidents per month was eight in the month of July and the minimum was one in the month of February. There was no accident involving any one outside of regular employees. Alleged damage to neighboring property along the line of the tunnels was practically nil.

PERSONNEL

Contract for the construction of Section 10 of the Sixth Avenue Subway, which included these tunnels, was awarded to the Rosoff-Brader Construction Corporation, which, in August of 1938, became the Brader Construction Corporation. The chief engineer of the Rosoff-Brader Construction Corporation was Fred W. Stiefel, Assoc. M. Am. Soc. C. E.

The work was done under the supervision, and in accordance with the design, of the Board of Transportation of the City of New York, of which Jesse B. Snow, M. Am. Soc. C. E., is chief engineer. During the tunnel-construction period, the late Byron Houghtaling, M. Am. Soc. C. E., was first division engineer, with Charles M. Madden, M. Am. Soc. C. E., as section engineer at first. Later, Mr. Madden replaced the late A. M. Mayell as subdivision engineer. This work was completed under Francis V. Hayes, Assoc. M. Am. Soc. C. E., as section engineer. The design was prepared by Albert Goertz as design engineer for this section.

In the contractor's organization, all of the construction procedure and operations were under the control of the writer, who was in charge of the construction. The engineering staff not only designed the methods of procedure but ordered the materials, laid out the work for the various shifts, and in general scheduled the operation of construction. In the field one engineer was assigned to each shift in the tunnels, the three shifts being covered by S. M. Marks and Sidney Philip, Juniors, Am. Soc. C. E., and James Lyttle. The dust-control work, as well as some of the office work in connection with the planning of the drilling, equipment, etc., was done by Philip S. Miller, Assoc. M. Am. Soc. C. E., aided by Sol Rusitzky and W. Mortensen in the office work.

The superintendent was Jacob Siebert, to whom too much credit cannot be given for the success of the operation. Joseph Tinley was master mechanic and Walter Reed was master electrician. Shift bosses were Harvey Urey and Due Carroll. Credit must be given not only to these men but to all the loyal mechanics and laborers in their respective gangs.

DISCUSSION

JOHN H. MYERS,[5] M. AM. SOC. C. E.—There was a time when "rock men" had in mind only progress. Inasmuch as their work was done largely in the wilderness, it was not necessary to give much attention to safety. The rock men of that day were adepts in blowing down mountains and incidentally scattering spalls all over the landscape, or in making, with the machinery then available, what was considered remarkable progress in tunnel excavation. However, when engineers began to build large structures within existing cities, progress, tempered with the necessary precautions for the safety of life and of existing structures, was inevitable and as a result a new breed of rock men has been developed. Mr. Feld gives much evidence of this fact and he has done something very worth while in recording the methods used in the construction of the Sixth Avenue Subway.

The so-called "umbrella section" of double-track tunnel, described by Mr. Feld, has been used for many years in the tunnel work of the New York Rapid Transit Railroads. To the writer's knowledge, it was used in many places on the New York City subway work, among them being the connection between the Lexington Avenue Subway and the existing tunnels under Park Avenue; and also at many locations along Lexington Avenue north of the point where the subway emerges from under the Commodore Hotel. In fact, some of the tunnels under Lexington Avenue are two tracks high. The upper half of the tunnel is of the "umbrella section" and rests on a steel floor which is at the same time the roof of the lower half.[6] The fact that the "umbrella section" is still being used gives proof that although, when completed, it furnishes an adequate encasing structure for the Rapid Transit Railroad tracks, it is, at the same time, a practical structure to build under many difficult conditions. In the writer's experience with it, the difficult conditions were brought about by the rock encountered, and he has known the "umbrella section" to be constructed by many different methods. At times it was built by excavating a central drift, placing the concrete center bench on the floor of this drift, erecting the permanent steel work, bracing the rock roof from this steel work and then concreting the "umbrella." Following this step, the tunnel for one track was excavated without further timbering, the arch over the track was concreted, and temporary bracing was placed against the unexcavated rock on the other side of the center line to take up the unbalanced thrust of the completed half tunnel. The remaining half was then excavated and the concrete poured. In Mr. Feld's experience, however, he had to cope not only with bad rock conditions but, in addition, with buildings alongside the tunnel work and subsurface structures, and an elevated railroad above it. All in all, it serves to demonstrate that the "umbrella" form of tunnel is a flexible design and that it is possible to erect it under many and various difficult conditions. The designers of the subway construction have no reason to be ashamed of it.

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[5] Div. Engr., Board of Transportation (*Retired*), White Plains, N. Y.

[6] "Construction Problems of the Manhattan-Bronx, and Lexington Avenue Subway Junction and Queensborough Tunnel Connections," by George Perrine, *Transactions*, Am. Soc. C. E., Vol. LXXXII, December, 1918, p. 278; see also "Tunnel Work on Sections 8, 9, 10, and 11, Broadway-Lexington Avenue Subway, New York City," by Israel V. Werbin, *ibid.*, Vol. LXXXI, December, 1917, p. 341.

Jacob Feld,[7] **M. Am. Soc. C. E.**—At the time of organizing the construction procedure and methods for the tunnel sections of the contract for the construction of Section 10 of the Sixth Avenue Subway, a number of questions arose as to the most advisable procedure and methods, expected progress, as well as expected costs of each proposed method. Because of insufficient literature on the subject in attempting to determine what had been done under similar circumstances and what had resulted from the methods used, the writer decided that after this construction job was finished he would publish a paper, and list his own experience and the answers to the questions that arose. This paper is the result of that decision.

It might be worth while to enumerate some of the problems that arose, and their solutions:

1. Proper protection of soft and seamy rock at the tunnel portals, keeping in mind the close proximity of elevated column footings, street sewers, and utility lines, was provided by the construction of a reinforced concrete combination retaining wall and girder spanning across the tunnels. This method was very satisfactory and eliminated the expected trouble from the raveling of the rock at the portals.

2. The great variety of rocks encountered required a flexible drilling diagram, as well as flexible bracing methods. Especially helpful was the detail decided upon for connecting the individual units of the bracing frames.

3. The use of a full face drilling and shooting procedure—an unusual method in constructing tunnels within city limits—was found to be advisable and economical, and in no way increased the hazards of the work.

4. A flexible shooting cycle was necessary to coordinate the amounts of dynamite necessary at each face with the restrictions placed upon blasting within the city limits.

5. The method of mucking required a study of different strengths of dynamite for the type of rock encountered, to avoid the necessity for breaking up rock too large for the mucking machine.

6. The lack of large areas, or easy access to the tunnel work, required an unusual disposal method; and the double-deck dumping board was found satisfactory and safe if only one dumping mechanism was provided.

7. The elimination of hauling the drill steel from the shaft for sharpening purposes was definite when traffic conditions on the street were considered and the decision to place the drill-sharpening plant beneath the decking at subgrade level proved not only economical but also safe. The predicted complaints from neighboring properties because of this operation did not materialize.

8. The special control of air and dust, resulting from the drilling operation, required considerable emergency research to avoid delay in the construction schedule. The method used has become standard and was accepted by the New York State Industrial Commissioner as an alternate to the only approved method at the time the rules were issued. Incidentally, the rules were issued after this contract was started.

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[7] Cons. Engr., New York, N. Y.

9. The provision of small diameter shafts for inserting cable feed pipes to connect the street surface with the manholes adjacent to the tunnels by core drilling from the street surface was a substantial economy over the normal shaft-sinking method previously used.

10. The use of a concrete pump for placing all of the concrete within the tunnels, with the addition of an air booster to fill the irregular rock roof completely, was considered a novelty and was instituted in spite of the objections of the field organization. However, this method proved satisfactory and resulted in the use of the same equipment in many later jobs.

Many of these methods are new if compared with the reports of the rock-tunnel operations in the subway sections referred to by Mr. Myers in his discussion, even though the design of the subway tunnels has not changed.

This report proves that, contrary to usual feeling in the construction industry, satisfactory and uniform progress can be obtained in a short job by proper planning and, at the same time, can result in a low accident record. The accident frequency of one lost time accident per 25,000 man-hr is far below any record on similar work in previous years.

It is hoped that the data in this paper will be useful to engineers finding themselves in a similar position of planning and coordinating tunnel work within congested areas.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2144

FORT PECK SLIDE

By T. A. Middlebrooks,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Jacob Feld, Joel D. Justin, William Gerig, Alfred J. Ryan, Glennon Gilboy, Frank E. Fahlquist, Irving B. Crosby, and T. A. Middlebrooks.

Synopsis

Tests and analyses conducted on the soils and rocks encountered at the Fort Peck Dam (in northeastern Montana, on the Missouri River) which have a bearing on the slide that occurred on September 22, 1938, are the basis of this paper. Since very little information has been published on the investigations conducted prior to the slide, the first part of the paper has been devoted to that subject. The major part is devoted specifically to the studies conducted after the slide, to determine its cause and to outline a method of repair.

Introduction

After preliminary investigations were completed, it was realized that, in addition to the difficulties normally expected from the construction of a full hydraulic fill dam of this magnitude, the foundation conditions were complicated (see Fig. 1). A thick plastic clay stratum approximately 60 ft below the natural ground surface extended over the center two thirds of the valley and well outside the upstream and downstream toes of the dam. Overlying, underneath, and outside the limits of the clay stratum, there were stratified pervious materials varying from fine sand to gravel. At each abutment the shale was either exposed or lay under a relatively shallow overburden. An extensive program of foundation exploration and laboratory investigations was planned to obtain the necessary information for design purposes. A soil testing laboratory was established and equipped to perform all types of soil tests, including the determination of photoelastic stress distribution. In addition to the regular shear, consolidation, and permeability tests, numerous special tests and experiments were performed, such as: (1) Expansion tests on shales and tills; (2) large-scale shear and consolidation tests; (3) clay model tests to observe action during plastic deformation; (4) photoelastic tests on gelatin models to develop a rational means of computing the stress distribution in the embankment and

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Note.—Published in December, 1940, *Proceedings*.

[1] Prin. Engr., Corps of Engrs., War Dept., Soil Mechanics Section, Washington, D. C.

foundation; (5) earth-dam models of the main dam on approximate scales of 1 to 50 and 1 to 200 and earth models of dike sections on a scale of 1 to 2; and (6) sand models to obtain information on the effect of consolidation of the core of the shells.

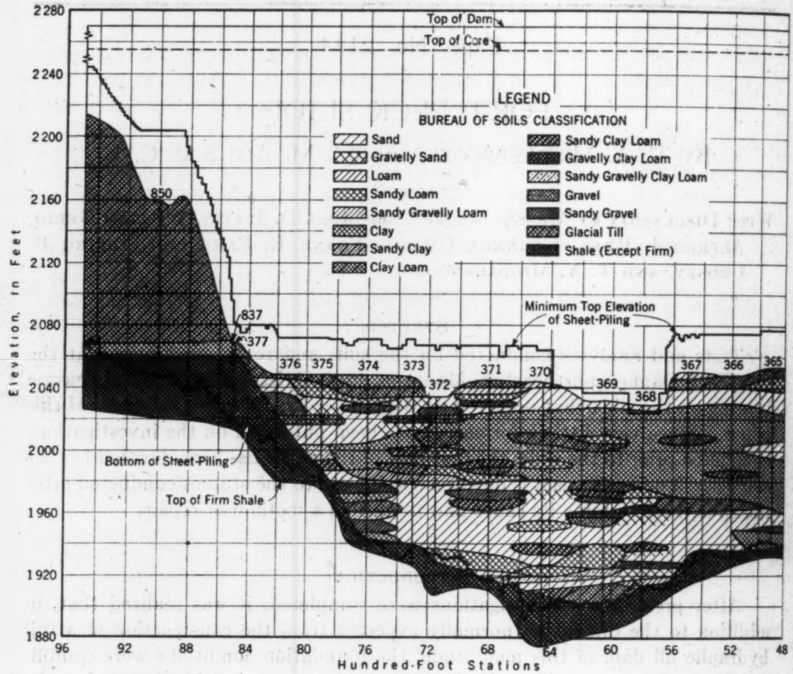


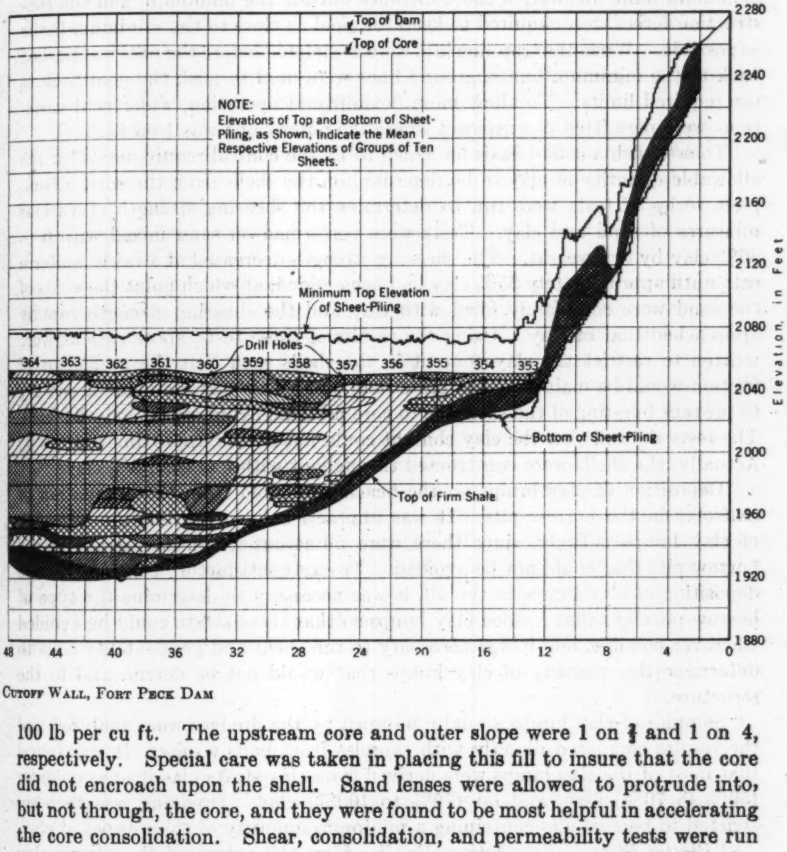
FIG. 1.—SOIL PROFILE ALONG

In order to keep a close check on the settlement and seepage during and after construction, settlement plates and perforated pipes were installed in the fill on several cross sections of both the dam and dike sections. In addition, hub stakes were set outside both toes of the dam and on the surface of the completed part of the slopes. The elevation and location of these hub stakes and plates were determined periodically. They were usually read monthly, but in some cases critical locations were observed daily.

TEST AND ANALYSIS PRIOR TO SLIDE

Investigations were conducted prior to and during construction to insure a safe structure with reference to the following possible sources of trouble: (*a*) Bursting of the shell due to excessive core pressure; (*b*) slides in the shells due to excessive deposition of core material and clay balls; (*c*) plastic flow of the foundation clay; (*d*) slides in the foundation sands or silts; and (*e*) slides on the shale surface.

"Bursting of Shell."—The possibility of the core bursting the upstream shell was investigated prior to and checked during construction, using the method proposed by Glennon Gilboy,[3] Assoc. M. Am. Soc. C. E., in 1934. It was found that the dam was safe even when the core was assumed to be a liquid, weighing 100 lb per cu ft. The upstream core and outer slope were 1 on $\frac{2}{3}$ and 1 on 4, respectively. Special care was taken in placing this fill to insure that the core did not encroach upon the shell. Sand lenses were allowed to protrude into, but not through, the core, and they were found to be most helpful in accelerating the core consolidation. Shear, consolidation, and permeability tests were run on undisturbed samples taken from the core during construction. Density samples were taken every 5 ft in elevation through the core on each 500-ft station at the center line every three months in orden to check closely the amount and rate of consolidation. These tests indicated that the core material consolidated at an exceedingly rapid rate.



Cutoff Wall, Fort Peck Dam

"Slides in Shell."—Consideration was given to the fact that slides in the shells might occur from two conditions: (1) Deposition of core material on the beaches due to wide pools; and (2) deposition of large pockets of clay lumps from the pocket discharge lines. Daily control samples, as well as permanent

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[3] "Mechanics of Hydraulic-Fill Dams," by Glennon Gilboy, *Journal*, Boston Soc. of Civ. Engrs.. July, 1934, p. 185.

record samples, at fixed intervals were taken to check on the core limits and the type of material being placed in both the core and the shells. It was found necessary to establish maximum and minimum core limits, since it was obvious that a fixed limit was not practical in construction. Upstream, the allowable maximum limit was only a short distance outside the minimum, and the construction forces were required to keep the pool as close to the minimum limits as possible. When the trap-line lifts were completed before the pool was pushed back to the minimum limits, ground lines were used to push the pool back to the required limits. To allow more flexibility of operation, wider pool variations were permitted downstream where the outer slope was 1 on 8½.

To establish a sound basis on which to fix the control requirements for the allowable quantity of clay to be deposited on the shells from the core, a complete series of tests were run to determine the shearing strength of various mixtures of sand and clay. Tests were performed on sand mixed with 0 to 30% clay by dry weight. The shearing strength decreased at a fairly uniform rate until approximately 25% clay had been added, at which point the voids of the sand were completely filled with clay and the shearing strength rapidly approached that of clay. Using the results of these tests, specifications were written to restrict the clay content in the shells such that the coefficient of friction would be maintained at more than 0.6. This was the value necessary to prevent bursting of the shell due to core pressure assuming no consolidation. The tests showed that the clay content could not exceed approximately 8.0%. Actually, the shells were constructed throughout with less than 3% clay.

Deposition of clay lumps on the beaches resulted from the excavation of stiff clay in the borrow pits. It was impracticable to prevent the pumping of clay lumps entirely, since there were numerous small clay strata in the borrow pits that could not be avoided. To exert satisfactory control over the deposition of clay lumps in the fill, it was necessary to determine the type of borrow-pit clays that formed clay lumps so that these strata could be avoided wherever possible, and it was necessary to run shear and permeability tests to determine the quantity of clay lumps that would not be detrimental to the structure.

Samples of clay lumps actually pumped by the dredges were analyzed and the results compared directly with samples from borrow pits. It was found that most of the clay lumps were derived from clay strata classified as medium (50% to 70%) clay, and fat (70% to 100%) clay. Dredging was then restricted to borrow pits containing a minimum quantity of these types of clay.

"*Plastic Flow of Foundation Clay.*"—After the extent of the plastic clay in the foundation had been outlined by the preliminary explorations, an extensive investigation was made to check the stability of the proposed sections both for the completed dam and for the 1-on-3 construction slopes in the closure section.[3] The special foundation exploration consisted of ten 6-in. borings through the clay strata. Continuous undisturbed samples were taken for the full thickness of the clay strata, and shear and consolidation tests were run on representative samples from each hole. Since considerable deformation occurs in a clay prior to ultimate failure in shear, it was feared that sufficient deformation might occur in the thick plastic clay stratum (60 ft maximum thickness) to cause failure in the dam before the full strength of the clay could be mobilized. After a careful study of this condition in clay models constructed in the laboratory, it was decided the shearing strength should be taken at the "yield point" instead of the point of "ultimate failure." The result was a decrease in the deformation of approximately 75% from the ultimate without decreasing the strength more than approximately 25%. Strength of the clay was determined from consolidated and unconsolidated shear tests by taking into consideration the percentage of consolidation during construction.

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[3] "Foundation Investigation of Fort Peck Dam Closure Section," by T. A. Middlebrooks, *Paper G-3*, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Cambridge, Mass., 1936.

Since failure of clay would occur due to plastic flow, the Swedish slide method[4] for determining the stability was not considered applicable. The elastic-theory method was used for determining the stress distribution in the clay strata. The works of J. Boussinesq,[5] S. D. Carothers,[6] Leo Jürgenson,[7] and others were utilized to develop a rational method for determining these stresses. Photoelastic experiments on a gelatin model were performed to check the mathematical application of the elastic theory to this type of structure. As a result of this investigation,[8] the construction slopes in the closure section were flattened from 1 on 3 to 1 on 5, and a berm was added upstream for the full length of the clay stratum, Station 30+00 to Station 70+00, Fig. 1.

"Slides in Foundation Sands and Silts."—During the investigation of the clay stratum sufficient undisturbed samples of the foundation sands and silts were obtained to determine their strength. It was found that there was ample factor of safety against failure of these materials.

"Slides on Shale."—Explorations in the right abutment disclosed that beneath several feet of exposed disintegrated shale, there was a zone of blocky weathered shale approximately 30 ft thick. There was a gradual transition from the blocky weathered shale into subfirm and firm shale. In the valley adjacent to the abutment a pervious overburden averaging 30 ft in thickness overlays a thin stratum of clay 1.5 ft thick. This clay rested directly on subfirm shale.

The possibility of failure in the shale in this vicinity was considered as highly improbable, since subfirm shale of similar classification excavated in other locations on the project had shown that the subfirm shale and the bentonite seams contained therein possessed ample strength to prevent failure. Detail investigation of this zone after the slide showed that the upper part of stratum classified as subfirm shale was actually weathered shale. It was also found that the clay stratum adjacent to the abutment would consolidate 100% during construction. For this reason, all shear tests on the clay were performed on consolidated samples. Stability analyses made on the basis of the results obtained from the tests indicated that the dam was safe against failure from this source.

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[4] "Erdstatische berechnungen mit reibung und kohäsion (adhäsion) und unter annahme kreiszlindrischer gleitflächen," by W. Fellenius, W. Ernst (publisher), Berlin, 1927.

[5] "Application des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques, principalement au calcul des déformations et des pressions que produisent, dans ces solides, des efforts quelconques exercés sur une petite partie de leur surface ou de leur intérieur; mémoire suivi de notes étendues sur divers points de physique mathématique et d'analyse," by J. Boussinesq, *Mémoires de la Société des Sciences, de l'Agriculture et des Arts de Lille*, Vol. 13, 1885, p. 1.

[6] "The Elastic Equivalence of Statically Equipollent Loads," by S. D. Carothers, *Proceedings of the International Mathematical Congress*, Vol. II, Toronto, Canada, 1924, p. 519.

[7] "The Application of Theories of Elasticity and Plasticity to Foundation Problems," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 206.

[8] "Foundation Investigation of Fort Peck Dam Closure Section," by T. A. Middlebrooks, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., June, 1936.

TEST AND ANALYSIS AFTER SLIDE

Directly after the slide occurred near the right abutment on September 22, 1938 (see Fig. 2), all previous test data and analyses were reviewed to assist in arriving at the most logical probable causes of the slide. Preliminary investigations indicated that the slide may have been caused by one or a combination of the following conditions: (1) Movement along some weak zone in the shale; (2) movement along the shale surface; (3) bursting of the shell due to excessive core pressure; or (4) temporary liquefaction of the shell or foundation sand, or both.

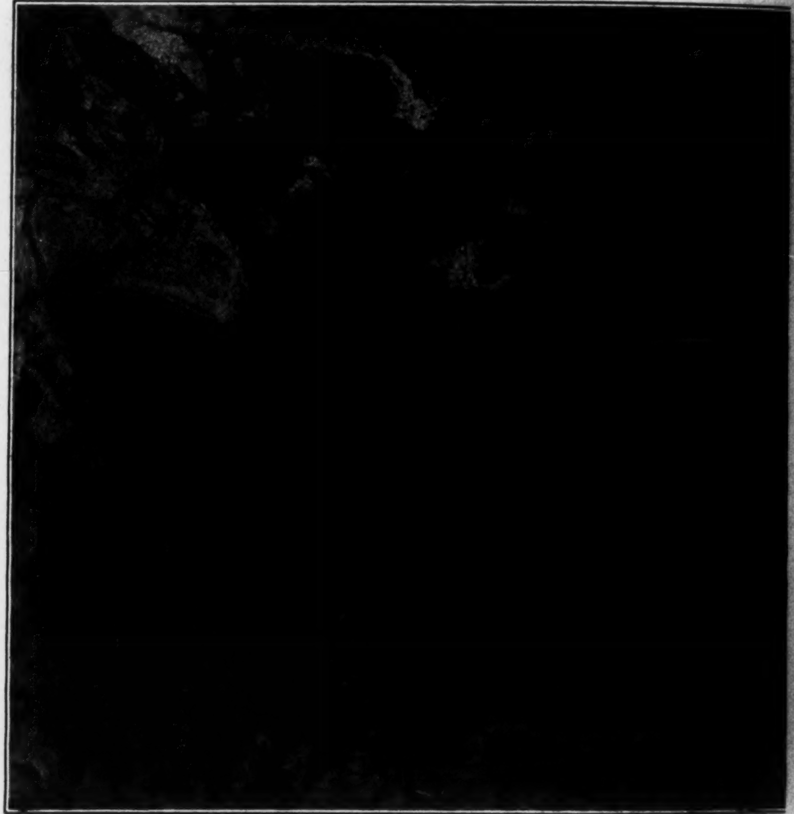
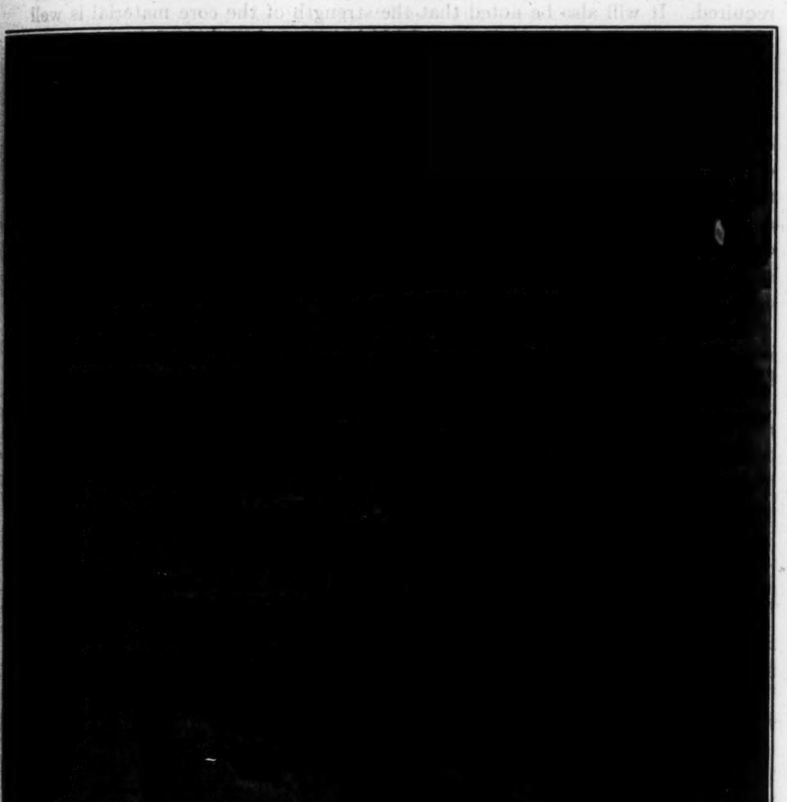


FIG. 2.—AERIAL PHOTOGRAPH OF

To determine the extent of additional laboratory work necessary for a complete investigation, tests were run by the latest approved methods to check previous testing, so that there would be no unnecessary duplication. The standard direct shear apparatus was used for most of the testing, but the shearing strength of the core and the critical density of the shell material were determined by the latest developed triaxial shear machine.

Movement in Shale.—In order to investigate possible movement along some weak zone in the shale, undisturbed samples were taken from test pits, drift, and large-diameter (12-in. to 36-in.) core borings. A considerable number of small (3-in. to 6-in.) borings were made through the slide material and overburden and 40 ft into the shale to outline the distribution of the various materials. Continuous drive samples were taken of the slide material and overburden and cored samples from the shale. Undisturbed samples of clay, weathered shale, and bentonite from the test pits, drifts, and large-diameter borings were tested for shear and consolidation characteristics in the laboratory.



DAM IMMEDIATELY AFTER SLIDE

In addition, large-scale field tests were run on undisturbed weathered bentonite in the drifts driven into the weathered shale. Representative results of these tests are shown in Fig. 3 which also shows the required strength to give a safety factor of 1.0 for the section prior to failure. It will be noted that most of the tests on bentonite show a strength below the required strength line, whereas most of the tests on clays and weathered shale show a strength above that required. It will also be noted that the strength of the core material is well above that required, the coefficient of friction averaging about 0.6.

Results of these tests indicate clearly that the bentonite and weathered shale were the weakest materials in the foundation, and that undoubtedly the weathered bentonite was by far the weakest material encountered. This conclusion is substantiated by all the explorations. Numerous movement zones or slide planes were found in the weathered shale by the drive samples taken from the small borings. In practically all cases bentonite was mixed with gouge material in these zones of movement. In some cases, the large-diameter cores showed that bentonite seams had been completely removed by the slide; some were thoroughly mixed with shale along a gouge zone, others remained as definite seams but were drawn out showing considerable movement.

Observations made on holes drilled into the shale revealed another very important factor that was difficult to take into consideration in the analyses but which, it is reasonable to assume, had a major effect on stability. Holes drilled into the shale downstream of the slide and on the west abutment showed that there was a high hydrostatic pressure in the shale, which was considerably in excess of the maximum lake level or the saturation line in the dam. In Table 1, which shows the water levels obtained in the various drill holes, it will be seen that one hole developed a pressure head equal to that of a water column extending to El. 2,310, approximately 60 ft above the maximum height of the fill and 90 ft above the saturation line. Other holes developed pressures of more than 100 ft above the lake level or the saturation line in the dam. Borings prior to the construction of the dam showed no artesian water in the

TABLE 1.—HYDROSTATIC PRESSURE IN FOUNDATION SHALE
(Elevations, in Feet Above Sea Level)

LOCATION OF HOLE		Natural ground	Embankment	Saturation line	Equivalent column[b]	LOCATION OF HOLE		Natural ground	Embankment	Saturation line	Equivalent column[b]
Station	Range[a]					Station	Range[a]				
4.92	5.16U	2,175	2,259	....	2,195	76.02	2.81U	2,046	2,211	2,105	2,183
11.25	3.750U	2,100	2,189	2,155	2,175	78.94	0.50U	2,048	2,252	2,220	2,310
11.30	6.67U	2,078	2,175	2,155	2,145	79.01	2.80U	2,042	2,216	2,115	2,256
13.50	8.92U	2,046	2,156	2,115	2,115	80.00	0.37D	2,048	2,247	2,220	2,310
13.75	3.75U	2,046	2,183	2,162	2,118	81.77	2.78U	2,041	2,219	2,113	2,213
16.25	3.75U	2,044	2,182	2,174	2,074	81.94	0.50U	2,048	2,251	2,220	2,232
18.75	8.50U	2,048	2,158	2,122	2,170	84.94	0.50U	2,073	2,250	2,220	2,260
13.50	1.10D	2,060	2,185	2,154	2,128	84.98	2.72U	2,057	2,217	2,115	2,190
15.00	4.00D	2,049	2,232	2,125	2,210	87.94	0.50U	2,156	2,250	....	2,100
20.00	4.00D	2,046	2,229	2,127	2,226	90.93	0.50U	2,176	2,250	2,219	2,104
75.94	0.50U	2,048	2,250	2,220	2,242	93.94	0.50U	2,214	2,250	2,220	2,181

[a] U = upstream; D = downstream.

[b] Elevation of a column equal to hydrostatic pressure.

shale. All artesian wells in this vicinity are drilled through the Bearpaw shale to a depth of from 600 to 1,000 ft, and none of these wells developed a pressure even closely approximating those obtained from these later holes drilled through the dam.

The most logical reason for the existence of such a high hydrostatic pressure in the shale is that trapped water was squeezed out of the shale, due to the superimposed load of the dam, most likely along weathered bentonite seams.

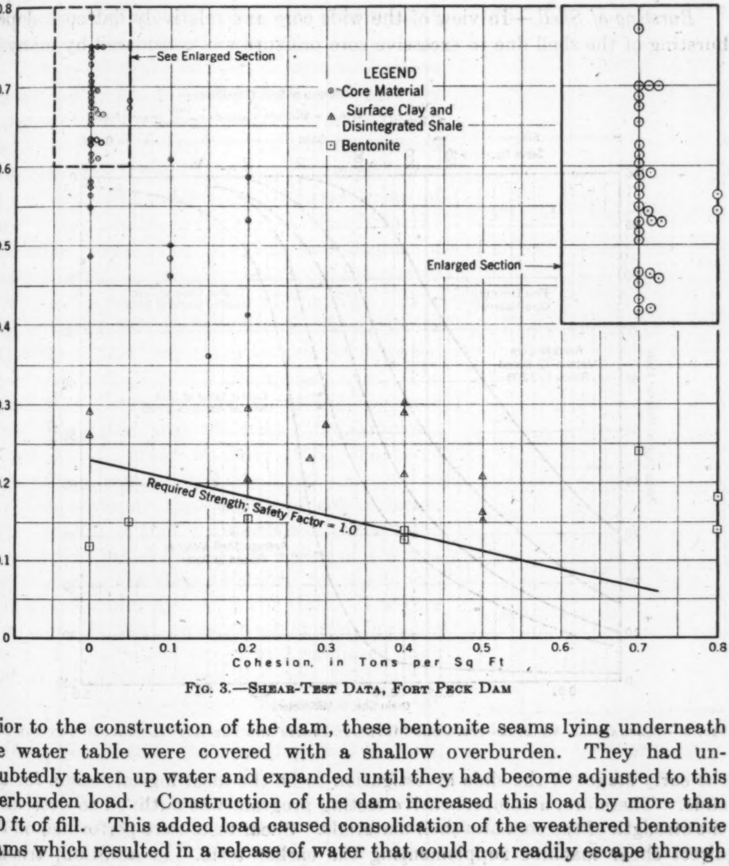


FIG. 3.—SHEAR-TEST DATA, FORT PECK DAM

Prior to the construction of the dam, these bentonite seams lying underneath the water table were covered with a shallow overburden. They had undoubtedly taken up water and expanded until they had become adjusted to this overburden load. Construction of the dam increased this load by more than 200 ft of fill. This added load caused consolidation of the weathered bentonite seams which resulted in a release of water that could not readily escape through the overlying impervious shale. It was trapped along the seams, developing the high hydrostatic pressures which were later observed. It is the firm conviction of the writer that this excess hydrostatic pressure which was found to exist in the shale was one of the major causes of the slide and accounts in large part for the speed at which, and the distance to which, it moved.

Movement on Shale.—In the slide area there was approximately 1.5 ft of clay and disintegrated shale overlaying the subfirm shale. This clay was in direct contact with a free draining foundation sand. Additional tests confirmed earlier investigations and showed that the thin clay strata would consolidate rapidly with the application of load. Consolidated shear tests were used for determining the shearing strength of this material. All of the strengths so determined were well above the required strength to give a factor of safety of 1.0.

Bursting of Shell.—In view of the wide core and relatively flat core slopes, bursting of the shell due to excessive core pressure was considered by many in the early stages of the slide investigation to be the most logical reason for the slide. Therefore, a comprehensive testing program was outlined to determine the strength of the core and shell materials. Shear tests were performed on the direct shear machine supplementing the earlier tests. In addition, triaxial tests were run at Harvard University, Cambridge, Mass., at the U. S. Waterways Experiment Station, Vicksburg, Miss., and in the laboratory of the Fort Peck District. Results of some of these tests on the core material are shown in Fig. 3. The coefficient of friction of the core material averaged approximately 0.6, with only a small degree of cohesion. The triaxial tests showed the

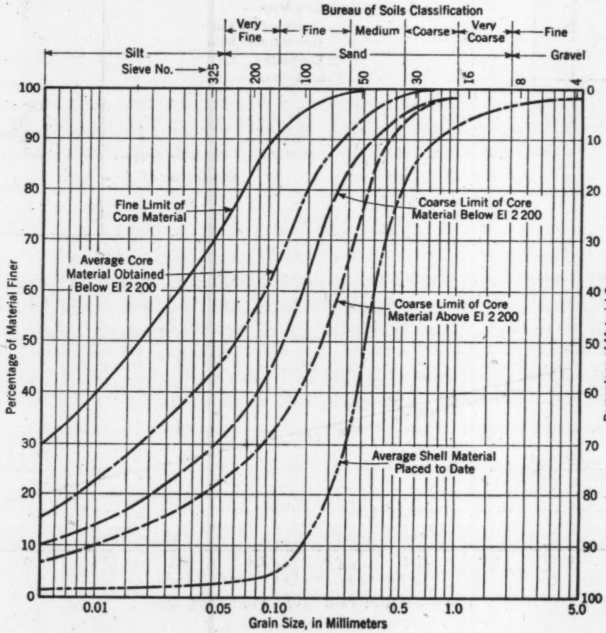


FIG. 4.—MECHANICAL ANALYSES FOR CORE-MATERIAL LIMITS AND THE ACTUAL MATERIAL IN THE FILL

core material to be even stronger than the direct shear tests indicated. After watching some triaxial tests run and studying the results, one engineer remarked that "the core material appeared to brace itself against failure." It was concluded from all the tests that the core material was practically as strong as the shell material.

An attempt was made to measure the horizontal pressure exerted by the core by the use of Goldbeck earth pressure cells attached to a long pile. The high degree of consolidation of the core material made it impossible to set the pile in the core without impairing the accuracy of the cells.

Composition of the average core material is shown in Fig. 4. Limits of the core as actually constructed did not vary greatly from the design limits, except for sand lenses which extended into, but not through, the core. Consolidation of the core was accelerated by these lenses of sand which protruded into the core from the transition zone. Fig. 5 shows the degree of consolidation for core material on the center line for different elevations of the fill. These data were obtained about two months prior to the slide. The tests comprise samples between Station 15+00 to 85+00, along the axis. The void ratios (averages plotted in Fig. 5) were determined volumetrically from drive samples 2 in. by 18 in., taken in July, 1938. The core pool varied from El. 2,213 to 2,200.

The relatively high degree of consolidation and strength of the core, as shown by the tests, was further borne out by the action of the material during the slide as shown by the topography and exploration after the slide. The core remaining in place on the west end of the slide stood on very steep slopes and exhibited high resistance to scour from the core pool water. Borings showed that the downstream part of the core did not move out with the slide, but remained intact while the overlying shell material moved upstream over it. Boring No. 4 at Station 14+75, approximately on the axis, showed that a large hole had developed during the slide down to natural surface. Core material did not squeeze into it as might have been expected, but it was filled with sand.

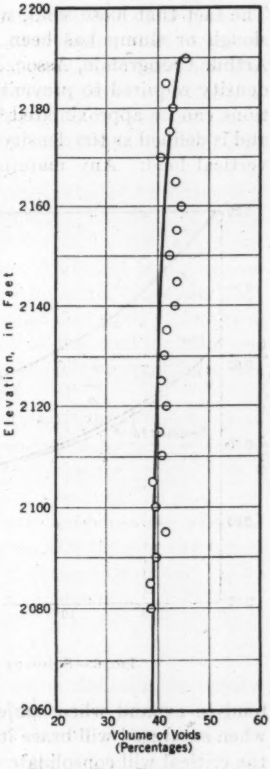


Fig. 5.—Core Consolidation

The upstream toe trestle was pushed out ahead of the slide. If the shells had burst due to core pressure, the most likely path of failure would have been over the trestle. Failure did not occur where the core was the least consolidated, which was the closure section. This part of the dam was raised approximately 150 ft in four months, whereas the dam at the east abutment was raised to this height over a period of twenty-eight months.

Therefore, it was concluded from the test data, explorations, and surface observations made directly after the slide that the core was decidedly stronger than anticipated in the original design and consequently possessed a good safety factor against bursting the shells.

Temporary Liquefaction of Sand.—"Liquefaction" of the sand shell or foundation sand (or both) was advanced as another possible cause of the slide. The fact that loose sand, when subjected to a shock or change in stress, will slough or slump has been recognized by engineers for ages past. In 1936 Arthur Casagrande, Assoc. M. Am. Soc. C. E., showed a method whereby the density required to prevent sloughing for different sands and different conditions can be approximated.[9] It has been designated as the "critical density" and is defined as the density at which no change in volume will occur at a given vertical load. Any material having a density greater than the critical will tend to expand when subjected to shearing stresses; therefore, this material when saturated will brace itself against failure; whereas, a material looser than the critical will consolidate when subjected to shearing stresses, thereby allowing the load of a saturated material to be partly carried by the water which reduces its shearing strength.

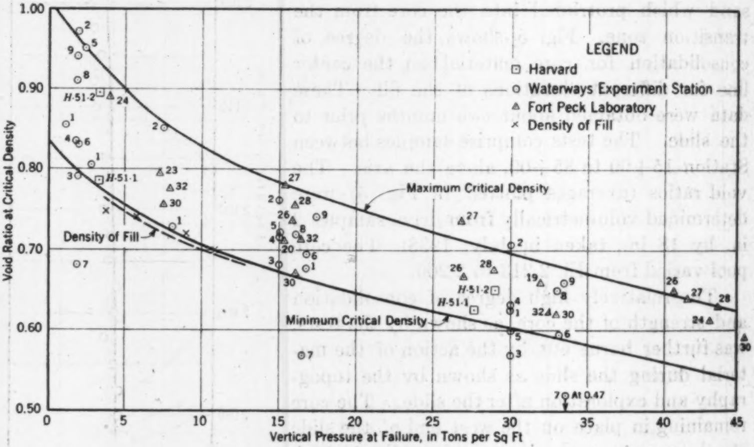


Fig. 6.—Summary of Critical Density Tests on Shell Material

It has been concluded that a material denser than the critical is definitely safe against flow. On the other hand, it does not follow that all materials looser than the critical density are unsafe. This looser material will definitely have some reserve strength, depending on its density. This problem did not enter into the slide investigation since it was found that density of the shell material was equal to, or greater than, the critical. It is a problem, however, that deserves careful consideration by investigators in the future.

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[9] "Characteristics of Cohesionless Soils Affecting the Stability of Slopes and Earth Fills," by Arthur Casagrande, *Journal*, Boston Soc. of Civ. Engrs., January, 1936, p. 13.

In order to determine the actual density of the shell material and to obtain samples for tests, several test pits were excavated in the undisturbed upstream shell. Continuous 6-in. undisturbed samples were taken, and the density and mechanical composition determined. Critical density tests using the triaxial machine were run on representative samples at Harvard University, U. S. Waterways Experiment Station, and Fort Peck. Results of these tests are shown in Fig. 6. Density of the Fort Peck shell material is shown as a heavy broken line. These tests show that the hydraulic fill is denser than required. In addition to the critical density tests on disturbed shell materials shown in Fig. 5, tests on undisturbed samples to determine the effect of stratification were run in the Fort Peck Laboratory. These test results showed that there was an even greater margin of safety against flow of the shell material than was indicated by the other tests.

The conclusions drawn from the test data were substantiated by inspection of the undisturbed 36-in. cores. The inspection disclosed that definite stratification, as it existed in the hydraulic fill, was undisturbed by the movement except for numerous shear planes. This stratification might not be destroyed by streamlined flow of the shell material, but it can be stated definitely that there would be no shear planes in the sand if flow had occurred. Therefore, it can be concluded from the tests and explorations that the shell material was of sufficient density to prevent it from becoming liquefied even under the severe stress conditions that occurred during the slide.

The distance that the slide moved upstream may have been due to some liquefaction of the slide materials, as has been concluded by the Board of Consultants. It is the opinion of the writer that the evidence proves such a liquefaction must have occurred along the bentonite seams in the shale, especially after sufficient movement had occurred to mix the bentonite with the trapped water. The inlet channel, cut approximately 800 ft from the toe of the dam, also had a major influence on the distance that the slide moved. The cut is approximately 500 ft wide and 30 ft deep (bottom elevation, 2,030.0).

General Questions

Before proceeding to the stability analysis, it seems desirable to discuss briefly a number of questions raised during the investigation which, directly or indirectly, affect any stability analyses that might be made. The questions are listed below and discussed in subsequent paragraphs:

(1) What effect did filling and lowering of the reservoir have on the slide?

(2) Was the construction rate too rapid?

(3) Did excessive settlement occur in the slide area prior to the slide?

(4) Did seepage away from the abutment through the fill have any appreciable effect on the slide?

(5) Was the core at the east abutment finer or coarser than the remaining portion of the dam?

(6) What effect did the slide have on the adjacent abutment and dam?

Effect of Reservoir Water.—At the time of the slide the reservoir was at El. 2,117.5, having been drawn down from El. 2,137.0 in approximately 60 days. The shale and bentonite in which the slide occurred were below the natural water table and were therefore saturated before the dam was constructed. Since the load of the dam was much greater than the reservoir pressure, the water was actually being squeezed out of the foundation. The water in the reservoir, therefore, could not enter the shale and bentonite to weaken it. The reservoir water at El. 2,117.5 decreased the stresses in the foundation from what they would have been if it had been empty. This decrease in stress, however, was in part counterbalanced by the increase in stress due to the drawdown; but, on the whole, the water in the reservoir had very little effect on the stability of the structure. Although the effect was small, these conditions were given full consideration in the stability analyses.

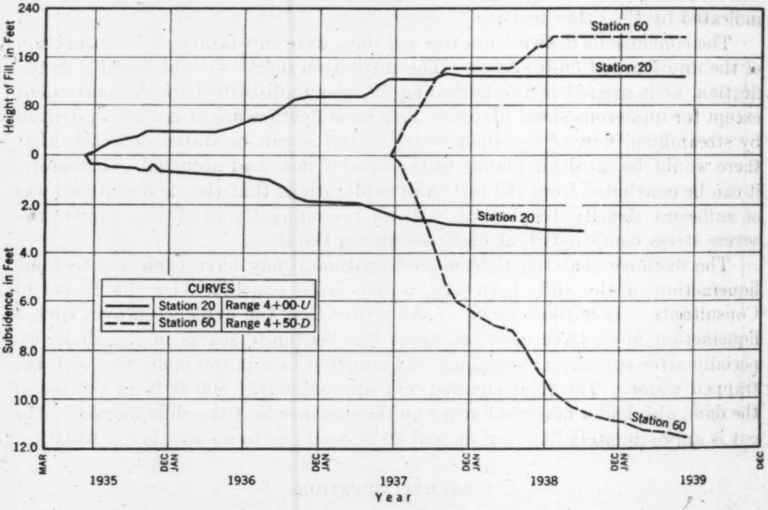


FIG. 7.—RECORD OF FOUNDATION SUBSIDENCE AND FILL HEIGHT, STATIONS 20+00 AND 60+00

Rate of Construction.—The rate of construction for the part of the dam in the slide was comparatively slow. The rate of construction is shown as a curve in Fig. 7 in connection with the subsidence. It will be noted that the fill was brought up in stages. The first five months of construction brought the fill up approximately 40 ft, followed by a winter rest period of five and one half months. Then in seven months the fill was raised approximately 55 ft, and another rest period of five months occurred. This was followed by about two months of operation, prior to closure, in which the fill was raised approximately 30 ft. Closure operations allowed this fill to rest another four months, after which approximately 15 ft of fill was placed in one month. Following a rest period of five months, the fill was then raised approximately

60 ft in the five months prior to the slide. Increase in the fill height averaged approximately 5 ft per month including the shutdown periods. The maximum rise per month in any one period was approximately 15 ft.

In the slide area the construction rate was extremely slow compared with the closure section. The fill in the closure section rose 160 ft in four months, rested five months, and then rose 60 ft more in five months. The average rise per month was nearly 15 ft and the maximum was 40 ft. Therefore, if trouble was to be experienced in the embankment due to rapid construction, surely it would have occurred in the closure section.

Any reasonable extension of the construction period could not have been expected to increase materially the strength of the weathered shale or bentonite or to release the pressure in the relatively impervious shale where the source of the trouble existed.

Settlement.—The question concerning excessive settlement can be easily answered by reference to Fig. 7. It will be noted that the settlement at a point about the middle of the upstream slope had practically stopped. The settlement in this area amounted to only about 2% of the fill height, whereas in the closure section where the plastic clay is approximately 60 ft thick, the settlement amounted to more than 10 ft, or more than 5% of the fill height. Settlement in the slide area occurred at a rate of less than 0.08 ft per month; but in the closure section this rate was approximately 0.6 ft per month. Obviously then, it is most unlikely that excessive settlement contributed to the failure.

Seepage.—Seepage moving outward from the abutment undoubtedly raised the saturation line in the dam slightly, but the additional stress added to the foundation was negligible when compared with the total stress set up by the embankment.

Texture of Core Material.—During the preliminary stages of the investigation some engineers thought that the core was composed of finer material at the abutment and that greater core pressure was exerted there. Actually the major portion of the core was coarser at the abutments, a condition which it was impractical to prevent.

Effect of Slide on Adjacent Abutment and Dam.—The slide had no material effect on either the abutment or the dam outside the slide area. A small surface slough occurred in the abutment due to the toe support being removed quickly. However, it was of no importance.

STABILITY ANALYSES

Stability analyses were made, using the Fort Peck elastic theory method[3] and the static slide method.[4]

Section Prior to Slide.—Several cross sections were analyzed in the slide area. Station 15+00 has been selected because it is a representative cross section and because the slide started at approximately this location. Figs. 8 to 10 show the stability analyses of the section by the elastic theory method, prior to failure. Distribution of shearing stresses along the rigid boundary was computed and plotted in Fig. 8. The effect of seepage forces, the difference in weight of shell and core material, and the elevation of the lake were taken into consideration in determining the stress distribution. All fill materials were assumed to be homogeneous and isotropic, and had the following weights:

Material	Weight (lb per cu ft)
Sand material above the saturation line	105
Sand material below the saturation line	60
Core material (buoyed)	65
Disturbed shale (buoyed)	78

Using values of tan $\phi = 0.185$ (ϕ = angle of internal friction) and cohesion $C = 0.2$ tons per sq ft, the shearing strength of the shale (broken line in Fig. 8) was plotted so that a direct comparison could be made with the shearing stress. It will be noted that there was an overstressed zone from range 4+00 to 6+25. Failure of the weathered shale and bentonite undoubtedly started in this overstressed zone and gradually increased until a sufficiently large area had been overstressed to produce failure of the entire structure. This is borne out by statements of the eye witnesses concerning the fact that the shell moved nearly straight out and the core pool and adjacent shells moved down as if dropping into a hole.

Attention is directed to the fact that in this analysis no consideration is given to the strength of the fill material. Where the weak material is in the foundation, failure starts in this material and progresses back into the embankment. It follows, therefore, that failure of the fill material in this case is incidental to the prior failure of the foundation material. This is one of the fundamental weaknesses of any static slide method, since progressive failure is not taken into consideration.

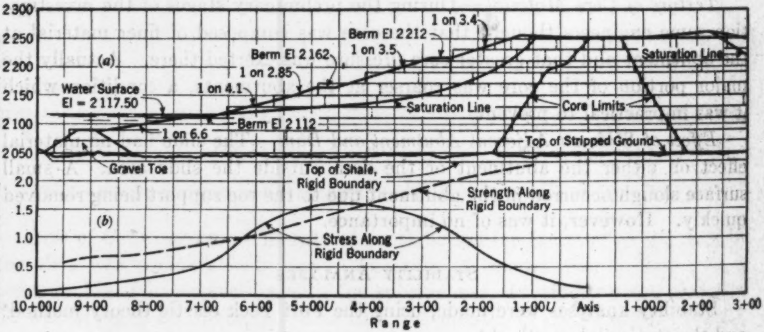


FIG. 8.—STRESS AND STRENGTH (TAN $\phi = 0.185$, $C = 0.2$ TON PER SQ FT) ALONG RIGID BOUNDARY

In this method of analysis the over-all safety factor (Fig. 9) is computed by taking the ratio of the accumulated shearing strength and stress by starting at the toe (Fig. 8) and progressing back to a point where it becomes a minimum. It will be noted in Fig. 10 that the safety factor drops below 1.0 at approximately range 4+00 and remains below to approximately range 2+00.

Attention is also directed to the fact that the test (Fig. 10) data, except for the weathered shale and bentonite, lie above the required strength line for a safety factor of 1.0.

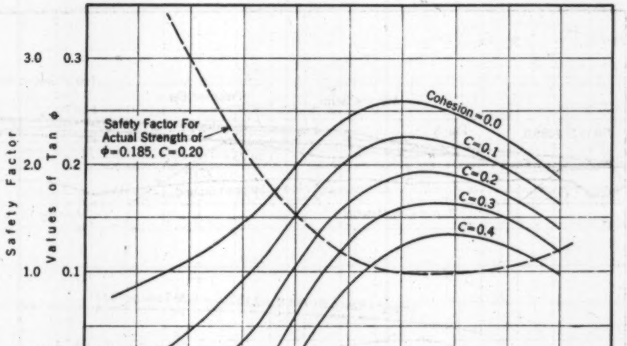


FIG. 9.—RELATION OF COHESION AND TAN φ ALONG THE RIGID BOUNDARY FOR A SAFETY FACTOR OF 1.0

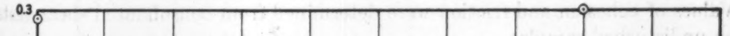


FIG. 10.—VALUES OF COHESION AND FRICTION FOR SOIL ALONG RIGID BOUNDARY FOR A SAFETY FACTOR OF 1.0

A stability analysis was made on the same cross section using the static slide method. Since the weakest plane across the dam passes through very little shell material, the strength of the core is the governing factor. If a value of tan $\phi = 0.3$ is used for the core material, this method checks the elastic theory method. On the other hand, if a value of tan $\phi = 0.6$, which is more in line with the test data, is used, the section is shown to be stable by this method. The failure plane as shown in the shale for all stability analyses was determined from the foundation explorations after the slide.

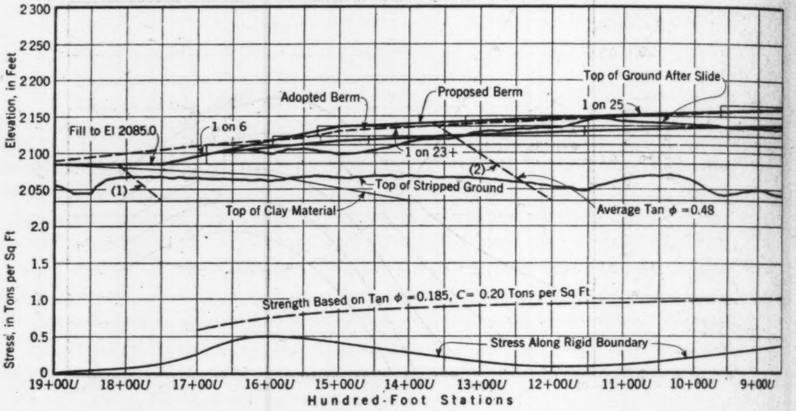


FIG. 11.—STRESS AND STRENGTH ALONG A

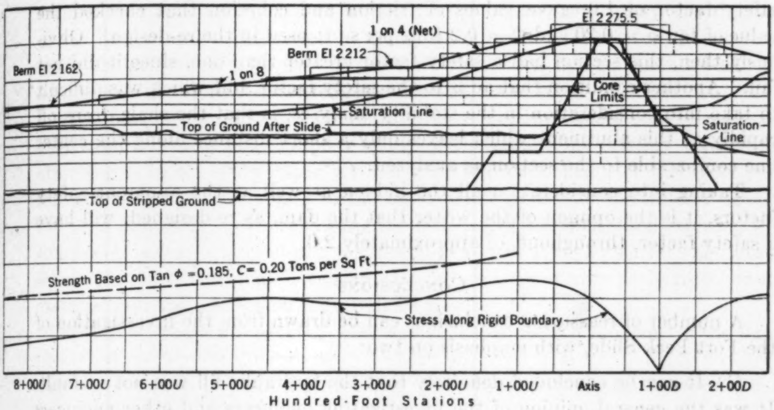
Values of cohesion and friction were determined from consolidated shear tests on undisturbed samples.

The results of these analyses on the sections prior to the slide checked against the test data further show that the failure did occur in the weathered shale and bentonite, and the results also furnish values for the coefficient of friction and cohesion which take into consideration all the variables including the excess hydrostatic pressure. In this way, a reasonable over-all value of the shearing strength of the foundation was determined for use in the re-design.

Reconstruction of the Slide Area.—In all stability analyses for the re-design of the dam in the slide area, values of $\tan \phi = 0.185$ and $C = 0.2$ tons per sq ft were used as determined from the analyses of the dam section prior to the slide. The re-design was based on two criteria established by the Board of Consultants: (1) That the factor of safety of the re-designed section should be greater than 1.5, using values of $\tan \phi = 0.185$ and $C = 0.2$ tons per sq ft; and (2) that no point in the foundation should be overstressed as determined by the elastic theory method.

Figs. 11 and 12 show the stability analysis by the Fort Peck elastic theory method for the re-designed section at Station 15+00. It will be noted in Fig. 11 that in all cases the shearing stress in the shale is well below the shearing strength at any point, and the safety factor against failure of the main dam is 1.95 (Fig. 12) and against failure of the outer slope of the berm is 1.85. The saturation line shown in Fig. 11 represents the worst condition in the upstream shell during dredging operations. As before, all the fill materials were assumed to be homogeneous and isotropic and the effect of the seepage forces was included in the analysis.

Part of Dam Not Affected by Slide.—In checking the design of the dam that was unaffected by the slide, the following criteria were established: (*a*) The factor of safety should be greater than 1.5, using the minimum shearing strength as determined from previous investigation; and (*b*) no point in the foundation should be overstressed.



Rigid Boundary, Reconstruction Plan

In analyzing the center section of the dam over the plastic clay, the results of "quick" or unconsolidated shear tests were used since they gave minimum values. No credit was taken for the increase in strength due to the consolidation in the clay strata which had occurred during construction. A maximum settlement of the embankment had already exceeded 12 ft. Most of this settlement is due to consolidation of the plastic clay. The minimum safety factor as determined for this section of the dam, therefore, is less than those that will actually exist in the completed structure. Using the minimum shear values, the minimum safety factor was found to be 1.5.

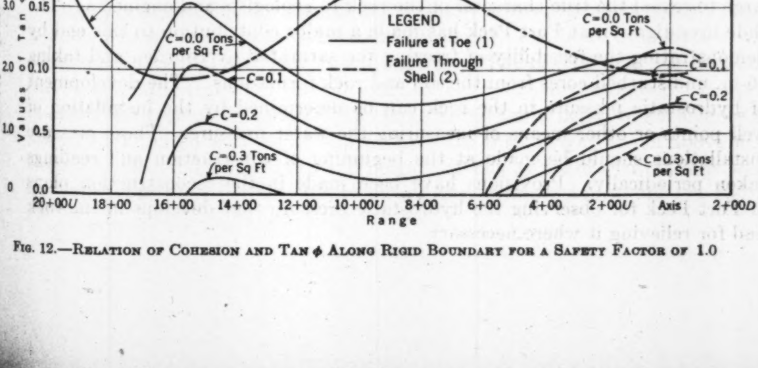


Fig. 12.—Relation of Cohesion and Tan ϕ Along Rigid Boundary for a Safety Factor of 1.0

A stability analysis was made for possible failure occurring in the shale on the west abutment at Station 85+00. The minimum safety factor was found to be 1.5. This analysis also has a hidden safety factor. An analysis of this section prior to construction of the berm, assuming that it only had a safety factor of 1.0, gave values of friction and cohesion that checked the value of $\tan \phi = 0.20$ and $C = 0.2$ tons per sq ft used in the re-design. Obviously then, this section had a safety factor greater than one, since it did not fail. Another condition that adds to the safety factor, but which was difficult to take into consideration in the stability analyses, is that the shale drops off rapidly at this abutment which leaves only a short distance along the center line comparable to the section as analyzed.

Taking into consideration all the hidden as well as the apparent safety factors, it is the opinion of the writer that the dam, as re-designed, will have a safety factor, throughout, of approximately 2.0.

CONCLUSIONS

A number of reassuring conclusions can be drawn from the investigation of the Fort Peck Slide, with emphasis on two:

(1) It can be concluded definitely that the hydraulic fill was not at fault. It was the general opinion of the investigating engineers and other engineers having an intimate knowledge of the project that the fill material performed excellently even under the most difficult circumstances existing during the slide.

(2) Furthermore, it is encouraging to note that no major changes in soil testing or methods of stability analysis were considered necessary as a result of the slide investigation, although comparison of the "static slide method" with the "elastic theory method" of stability analysis did indicate that the latter was to be preferred for the conditions encountered at Fort Peck.

The paramount question in every one's mind may be stated as follows: How can similar conditions be avoided or anticipated in the future? It is apparent, of course, since the slide did occur, that the foundation explorations and investigations, extensive as they were, did not indicate to the geologist and engineer the extent of the weathering in the shale and bentonite seams underlying this part of the dam; nor did they indicate that high hydrostatic pressures would develop in the bentonite seam due to the superimposed load of the dam. Therefore, the major problem in this connection is to always obtain representative undisturbed samples of this type of rock, sufficiently large to reveal the true character of the rock to geologists and engineers. The slide investigation at Fort Peck has made a major contribution to this end by demonstrating the feasibility of freezing the saturated overburden and taking 36-in. undisturbed cores from the soil and rock formations. The development of hydrostatic pressure in the rock can be determined by the installation of well points or other means of measuring the water pressure. These pressure installations should be made at the beginning of construction and readings taken periodically. Provisions have been made in the reconstruction plans of Fort Peck for observing the hydrostatic pressure that develops in the rock and for relieving it where necessary.

DISCUSSION

Jacob Feld,[10] **M. Am. Soc. C. E. (by letter).**—Much more can be learned from a complete description of a single failure than from many descriptions of the design and construction of stable structures. Yet, seldom is a failure made the subject of a paper. From that viewpoint, this paper is a welcome addition to the store of knowledge on the subject of soils.

The fact of the failure is common knowledge; the causes can be deduced only from a careful and complete study of the conditions existing before and after the failure. This paper gives a complete description of the tests performed on the materials used in the dam, before and after the failure. Since the tests definitely indicated strengths in excess of the computed stresses, the conclusion is drawn that sliding (or shearing failure) along subsurface bentonite seams in the "firm shale" caused the failure. Bentonite is a treacherous material and the conclusion may well be true.

However, there are several other suspicious conditions shown by the data presented by Mr. Middlebrooks. Failure occurred near Station 15, nowhere near the location of maximum loading, or maximum computed stress on the firm shale layer. Yet, there are two phenomena that occur only near Station 15, which might be a clue to the cause of the failure: (1) The bottom of the sheet piling between stations 13 and 16 is below the "top of firm shale"; and (2) the full height of fill from the original grade of 2,050 to top of dam 2,250 occurs from Station 12 to Station 16, with soil cover over the shale only 20 ft thick. These values refer to the profile along the cutoff wall, as shown in Fig. 1. Both of these facts are definite indications of possible trouble when compared to the conditions along the remainder of the profile, where the sheet piling stopped in the region designated "except firm shale" and the cushion of natural soil between the fill and the firm shale varied from 20 ft at Station 16 to 170 ft maximum at Station 64 and to a second minimum of 70 ft at Station 84.

The penetration of the sheet piling into firm shale rock makes one question whether the boring indication of firm rock is correct. If the rock could be cut by sheet piling driven through 10 ft of sandy loam and gravelly sand, and through more than 10 ft of rotten shale, then the bentonite seams that were discovered later must have been expected. The seams were probably under pressure and the firm shale was in thin layers.

The large variation in natural soil cushion always causes trouble. That is known in all foundation design and frequently has been explained as the cause for differential settlements in buildings. The natural soil was certainly less consolidated (further from the critical density) than the prepared mixture in either the core or the shells. The large variation in depth of subsidence at stations 20 and 60 is much less apparent if the total height of soil above firm shale is considered. Although the heights of fills at these two stations were the same, the total depths of rock below the ultimate top of fill are 250 ft and 370 ft, of which 230 ft is the additional fill. At the fill height of 150 ft, the total depths to rock were 170 ft and 290 ft; the corresponding subsidences (as given in Fig. 7) are 3 ft and 6 ft.

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[10] Cons. Engr., New York, N. Y.

The tests of the shell materials show a high shear strength; this corresponds to a small value of shear strain at failure. The writer believes that both the core and the shells were too rigid to take up the differential subsidence or settlement without failure. Once the shear cracks had developed in the body of the fill, lateral pressure against the upstream shell, founded as it was on a water-bearing seamy rock, caused outward movement.

The effect of the surcharge load on the liquid bentonite in the seams was to increase the uplift pressure on the base of the fill. The tendency for the pressure to be relieved by escape of the liquid was nullified by the fact that bentonite does not give up its water easily and by the compression of the seams under the added load. The seamy shale might have been strong enough when not compressed and also when compressed with a sufficiently large layer of porous cover. The high hydrostatic pressure in the shale was noted in the core holes (see Table 1 and supporting text). The actual loading on the shale near Station 15 was much more than the weight of the material directly above it because of the lower subsidence at this station as compared to the remainder of the fill. Such a condition explains the findings for a hole in the fill near Station 14 + 75, down to natural surface; it is similar to the open crack which develops in a wall founded on varying depths of unconsolidated soil or in a wall founded partly on rock and partly on soil.

In the opinion of the writer, this failure is another example to prove the correctness of the rule that unequal settlement is more to be guarded against than overload. Excessive settlement did not contribute to the failure—quite the contrary. Lack of settlement due to the thin cushion, and probably also the high resistance of the relatively sudden slope in the right-hand bank, caused internal failure, and the earth pressure on the shell did the rest. To complete the paper, it is hoped that Mr. Middlebrooks will define "factor of safety" in the closing discussion.

To follow customary practice in structures, the dam should have been built in one of two ways: (1) Vertical cleavage joints, with proper shear dowels, at about stations 20 and 85; and (2) construction of the dam between stations 20 and 85 first, closing the ends after the consolidation and subsidence were substantially complete. A combination of the two methods can be developed.

The reconstructed area that failed will not be exposed to the same transfer of load, nor the same shearing failure, since the main part has substantially reached its final position. In the writer's opinion, the failure was caused by the "erection" stresses due to differential vertical shrinkage, and, once that state has been overcome, the structure as designed by either the elastic theory method or the static slide method is stable. The paper is additionally valuable in outlining in so clear a fashion all the factors requiring investigation in the design of large earth dams.

Joel D. Justin,[11] M. Am. Soc. C. E. (by letter).—There is only one good thing about a slide such as occurred at Fort Peck. If it is thoroughly investigated and studied, the engineering profession is thereby better equipped to prevent similar slides in the future. Therefore, Mr. Middlebrooks' paper is worthy of careful study by all engineers interested in the design and construction of earth dams. It should help to clear away the misconceptions of many engineers who have not had the opportunity to study the investigation that followed. Thus, one engineer recently told the writer that he was opposed to using a hydraulic-fill dam at a certain location because hydraulic-fill dams had a habit of sliding upstream!

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[11] Cons. Engr., Philadelphia, Pa.

The investigation of the Fort Peck Slide was undoubtedly the most thorough and extensive that has been made in connection with any earth-dam slide during construction. The importance and size of the structure and the investment involved justified the extent to which the investigation was carried. Any one who takes the trouble to study the complete record of the investigations[12] thoroughly will reach the conclusion that the cause of the slide was insufficient shear strength in the foundation to resist the stresses imposed by the dam as originally designed at the section where failure occurred.

The main dam is approximately 10,000 ft long, but the slide was limited to the upstream part of the dam, approximately between stations 4 and 30. The investigation gave no indication that the foundation under the downstream part of the dam between stations 4 and 30 was any better than the foundation under the upstream part of the dam in this section, but there was no movement in the downstream part of the dam at any point. In this connection, it is significant to note that, whereas the average upstream slope in the section where the slide took place averaged 1 on 4, the slope of the downstream face in the same section averaged 1 on 8.5. It is also significant that immediately north of the slide area there was an extensive upstream berm at El. 2,112 extending from Station 30 to Station 75 (see Fig. 13).

The flatter downstream slope resulted in lower intensity of shear stress both in the foundation and at the base of the dam. Similarly on the upstream side, the berm north of Station 30 resulted in lower intensity of shear stress in the foundation just north of Station 30, where there was no movement, than in the foundation just to the south of Station 30 where the slide took place.

Thus from the foregoing it is evident that at the section where the slide took place, not only was the foundation weaker, as shown by Mr. Middlebrooks, but also the unit shearing stresses in the foundation were higher than in the adjoining section.

As a result of the investigations and studies conducted after the slide, and in accordance with the decisions of the Board of Consultants appointed for the purpose, the design was modified and the sections shown in Fig. 14 were adopted and executed for the reconstruction.[12]

Hydraulic-fill methods were used in placing the larger part of the additional shell material within the slide area and also for the extensive additional upstream berm throughout the remainder of the dam. The new core (which was a highly impervious glacial till from the north abutment) and the topping off of the dam, were constructed by rolled-fill methods. The upper part of the

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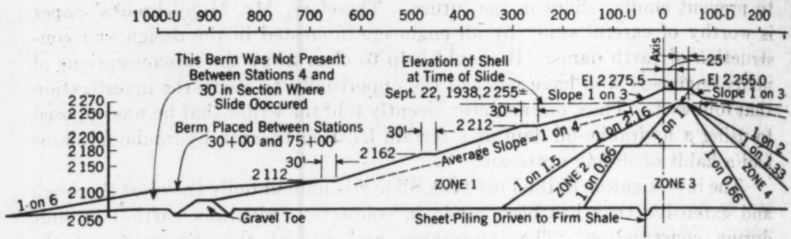
[12] See "Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam," Corps of Engrs., U. S. Army, July, 1939.


1 000-U
900
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Axis
This Berm Was Not Present Between Stations 4 and 30 in Section Where Slide Occured
Elevation of Shell at Time of Slide Sept. 22, 1938, 2 255±
25'
El 2 275.5
El 2 255.0
Slope 1 on 3
Slope 1 on 3
2 270
2 250
2 200
2 180
2 150
2 100
2 050
30'
2 212
30'
Berm Placed Between Stations 30+00 and 75+00
Average Slope = 1 on 4
2 162
30'
2 112
1 on 6
1 on 1.5
ZONE 1
ZONE 2
1 on 0.66
1 on 2.16
1 on 1
ZONE 3
1 on 2
1 on 1.33
ZONE 4
1 on 0.66
Gravel Toe
Sheet-Piling Driven to Firm Shale

FIG. 13.—TYPICAL CROSS SECTION OF FORT PECK DAM AS

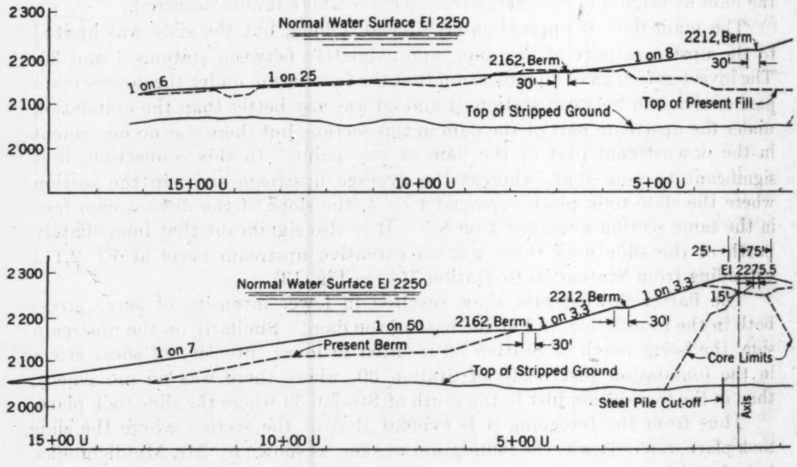
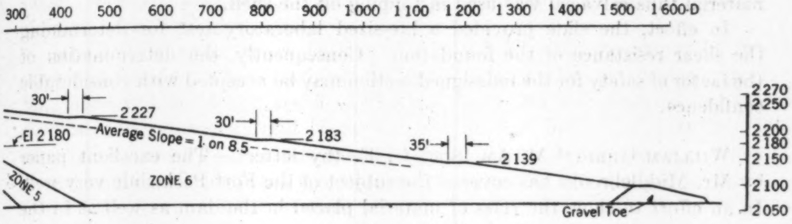
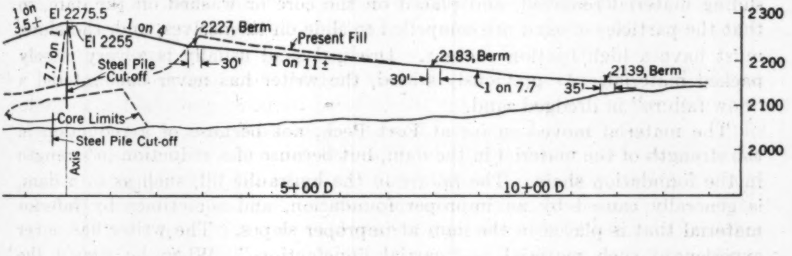

2 300
2 200
2 100
2 000
Normal Water Surface El 2250
2212, Berm
1 on 8
30'
2162, Berm
1 on 6
1 on 25
30'
Top of Present Fill
Top of Stripped Ground
15+00 U
10+00 U
5+00 U
25'
75'
El 2275.5
2 300
2 200
2 100
2 000
Normal Water Surface El 2250
2212, Berm
1 on 3.3
15'
2162, Berm
1 on 3.3
30'
1 on 50
Present Berm
1 on 7
Core Limits
Top of Stripped Ground
Steel Pile Cut-off
Axis
15+00 U
10+00 U
5+00 U
El 2275.5
2 300
2 200
2 100
2 000
Normal Water Surface El 2250
1 on 3.3
El 2255
2212, Berm
2162, Berm
1 on 3.3
30'
1 on 50
30'
Present Surface
1 on 7
Core Limits
Top of Original Ground
Steel Pile Cut-off
Axis
15+00 U
10+00 U
5+00 U

FIG. 14.—TYPICAL CROSS SECTIONS OF FORT PECK DAM AS


300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
1600-D
30'
2 227
30'
2 183
35'
2 139
El 2 180
Average Slope = 1 on 8.5
ZONE 5
ZONE 6
Gravel Toe
2 270
2 250
2 200
2 180
2 150
2 100
2 050

ORIGINALLY DESIGNED AND CONSTRUCTED PRIOR TO THE SLIDE


1 on 3.5+
El 2275.5
7.7 on 1
1 on 4
2227, Berm
Present Fill
El 2255
Steel Pile Cut-off
30'
1 on 11±
2183, Berm
30'
1 on 7.7
2139, Berm
35'
Core Limits
Steel Pile Cut-off
Axis
5+00 D
10+00 D
2 300
2 200
2 100
2 000
2227, Berm
1 on 4
Present Fill
30'
1 on 11±
2183, Berm
30'
1 on 7.7
2139, Berm
35'
1 on 7.7
2095, Berm
25'
1 on 5
Gravel Toe
5+00 D
10+00 D
15+00 D
2 300
2 200
2 100
2 000
2227, Berm
1 on 4
Present Fill
30'
1 on 11±
2183, Berm
30'
1 on 8+
2139, Berm
35'
5+00 D
10+00 D
15+00 D
2 300
2 200
2 100
2 000

REDESIGNED AND RECONSTRUCTED AFTER THE SLIDE downstream face was steepened to 1 on 4, as indicated in Fig. 14, and the material thus salvaged was used in topping off the shell.

In effect, the slide provided a life-sized laboratory test for determining the shear resistance of the foundation. Consequently, the determination of the factor of safety for the redesigned section may be accepted with considerable confidence.

WILLIAM GERIG,[13] M. AM. SOC. C. E. (by letter).—The excellent paper by Mr. Middlebrooks has covered the subject of the Fort Peck slide very well. In an effort to show the class of material placed in the dam as well as in the foundation, the author shows that much work in securing borings and other data was done in a thorough manner.

The writer has never experienced a case in which a "flow failure" has occurred in the dredge material. One who is familiar with dredges should know that sand handled by a dredge has practically all the clays and other sliding material removed, and placed on the core or washed off the dam, so that the particles of sand are compelled to slide on themselves, and, therefore, must have a high friction content. Dredged sand usually is a very closely packed material. As previously stated, the writer has never encountered a "flow failure" in dredged sand.

The material moved so far at Fort Peck, not because of a reduction in the strength of the material in the dam, but because of a reduction in strength in the foundation shale. The failure in the hydraulic fill, such as in a dam, is generally caused by an improper foundation, and sometimes by inferior material that is placed in the dam at improper slopes. The writer has never experienced such material as "partial liquefaction." When he signed the Report of the Board of Consultants,[14] he had in mind that the foundation material contained some clay, which might have been in a "state of liquefaction." In fact, when the word was first used in the Report, he did not know what it meant; but after the word was defined as being similar to what is usually called "quicksand," he thought the clays might flow.

In the Zanesville (Ohio) District (U. S. Engineer Office), for example, a part of a river was unwatered, and it was found that the material in the bed consisted of pebbles, small boulders, and sand, and that this was in a state similar to "quicksand." One cannot imagine that these boulders, in this condition, would liquefy in the sense that Webster defines it. The writer, therefore, fully indorses Mr. Middlebrooks' conception that the material in any part of the dam was not liquefied, and that the slide at Fort Peck Dam occurred in the foundation.

The conditions were quite similar (as far as could be seen) on both abutments, except that the downstream slope was 1 on 10, whereas that of the upstream was designed for 1 on 4.5 at the east abutment. The design was similar at the west abutment. There was little evidence of disintegrated shale at this abutment, however, because evidently there was a stronger current during high water, and this current had carried off the disintegrated shale. The west abutment, furthermore, was composed of a larger percentage of more or less firm glacial till.

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[13] Cons. Engr., Arkadelphia, Ark.

[14] Report on the Slide on a Part of the Upstream Face of the Fort Peck Dam, War Dept., Corps of Engrs., U. S. Army, 1939.

Mr. Middlebrooks also mentions the 1-on-3 construction slopes in the closure of the dam where the river was used as an outlet for surplus water. This slope was not found stable in the test dams and was used only in the river section proper, so that when the closure was made the quantity of material required for the closure would be less than on a flatter section. When it was definitely decided that this slope was too steep, it was changed to about 1 on 4.5 and gave no more trouble.

If the failure on the east end of the dam was due to anything except a poor foundation (and probably the increased slope on the upstream side when the location of the dam was changed), it would have been evident on the downstream side. The bentonite seams were found in the tunnels, and they were not discharging a large quantity of water; in fact, there was no trouble with water in these tunnels. If the water in the bentonite seams were a contributing cause of the slide, it would have been shown in these seams, where they extended farther than the downstream side of the dam.

The author submits a statement showing the large number of borings and other data that were obtained prior to the construction of the dam and also the large total number that were obtained. Many of these data were duplicated after the slide occurred and were obtained chiefly as a check. The greatest possible care was taken to obtain the data correctly, and tests were carefully made and studied for this purpose. Often the data obtained by means of tests in a laboratory are not satisfactory. To illustrate: Obtaining the friction through various sizes of smooth pipe (approximately 8 in., 12 in., 20 in., and 30 in. in diameter), the frictional resistance per hundred feet was found to be different for the different sizes of pipe, although the velocities were kept the same in all sizes. This statement is inserted to show that entire dependence cannot always be given to the results of laboratory tests. This is true in many of the tests made in soil mechanics, and for that reason and others the writer views with doubt some of the results in soil mechanics obtained in this manner. Earth dams were built many years ago and some are still standing, although the theory of soil mechanics was not known to those builders.

ALFRED J. RYAN,[15] ASSOC. M. AM. SOC. C. E. (by letter).—The only true test of any method of analysis of earth embankments is to apply that method to the failure of an actual structure. Such an application is dependent upon knowledge of all of the facts pertaining to the failure. Mr. Middlebrooks' frank and complete description of the failure, and of the apparent reasons for the failure, at Fort Peck Dam is a valuable contribution to this phase of engineering.

The conclusion that the failure was the result of a weakened foundation seems sound beyond any reasonable doubt. If it is true that the static slide method of analysis, properly applied, would not indicate this failure, it is a matter of grave concern to those who have analyzed existing structures by that method. Any method of analysis, when applied to an embankment composed of more than one material, has some doubtful features. The static slide method is no exception, but it is no less applicable to the plastic flow condition than to any other condition.

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[15] (Crocker & Ryan), Denver, Colo.

The essential difference between the "elastic state" and the "plastic state" of soil is the existence of excessive hydrostatic pressure of the fluid contained in the voids of the latter. The fluid consists of water or air, or a mixture of the two. When a load is applied to a soil that has not been sufficiently consolidated, the soil particles attempt to adjust themselves to this load. This requires a reduction in the volume of voids. In a soil that resists drainage, the fluid in the voids cannot escape quickly enough to allow this reduction and part of the load must be carried by hydrostatic pressure of the fluid. It is commonly called "pore pressure" and cannot produce friction. As the fluid escapes, the volume of voids and the intensity of the pore pressure are reduced. The rate of reduction is a function of the permeability of the soil and of the distance to a drainage face. That part of the load not carried by the fluid (that is, the total load minus the pore pressure) is carried by the grain-to-grain contact of the soil particles and does produce friction. Therefore, the static slide method is adapted to a plastic condition by making the proper corrections for pore pressure. Methods of evaluating this pressure have been developed by Karl Terzaghi,[16] M. Am. Soc. C. E., and by J. H. A. Brahtz.[17]

The basic concept of the static slide method is that of an instantaneous failure along the entire sliding surface. Observed failures of both models and actual embankments have proved that this is not the case. Instead, the failure is progressive and the failure surface indicated by the analysis represents the approximate limit of this progression. A careful check as to the effect of this inconsistency reveals that the summation of the sliding forces and of the resistance can be represented by the basic concept as long as stability is maintained. Consequently, the fact that the conditions of an earth embankment are such that any failure will obviously be progressive does not exclude this method of analysis.

The writer has made a static slide analysis of the section with pore pressures computed by the method developed by Professor Terzaghi. The analysis was made at Station 15+00, as shown in Fig. 8, and was based upon information given by the author and in the report of the slide.[18] The consolidation characteristics of the soils were taken from the results of tests of representative samples of the various materials as shown in the report. The sample of foundation material was from the Merriman Drift and was classified as undisturbed weathered shale. The core sample was from Hole C–7. The shell material was assumed to be consolidated to the imposed load and to have no pore pressure. Due to lack of sufficient information, a check of the computed pore pressures by Mr. Brahtz' method was impossible. However, approximate agreement of these pressures with those shown in Fig. 1 was considered an adequate check. The computations indicated pore pressures in the weathered shale of sufficient intensity to make the shale, as well as the bentonite, susceptible to failure.

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[16] "Theorie der Setzung von Tonschichten eine Einführung in die Analytische Tonmechanik," von Karl Terzaghi und O. K. Fröhlich, Leipzig, Deuticke, 1936.

[17] "Notes on Soil Mechanics," *Technical Memorandum No. 592*, Bureau of Reclamation, U. S. Dept. of the Interior.

[18] Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam, Corps of Engrs., U. S. Army.

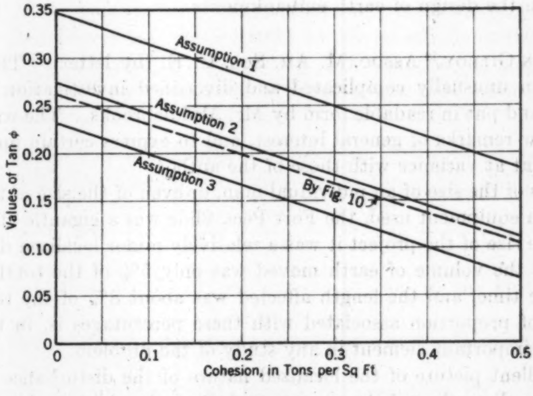

FIG. 15.—STRENGTH REQUIRED FOR STABILITY

Computations of the pore pressures in the core were complicated by the fact that the location and extent of the sand lenses were not known. Analyses were made for three assumptions regarding the effect of these lenses: (1) Neglecting the effect of the lenses; (2) assuming lenses such as to provide a 50% decrease in the distance between drainage faces; and (3) assuming lenses such that the pore pressures were completely dissipated at the time of the failure. The first and third assumptions were obviously used to establish limits and the second to indicate the intermediate trend. Comparison of computed consolidations along the center line of the core with those given in Fig. 5 indicate that the true condition lies somewhere between the first and second assumptions. Results of an analysis based upon the second assumption would be liberal instead of conservative.

Results of the analyses are shown in Fig. 15. The requirements for stability are slightly higher for the second assumption than those found by the author by the elastic theory method. In the writer's experience, this has usually been the case when comparing the results of the two methods. The author states that a static slide analysis will not indicate failure if a value of tan $\phi = 0.6$ is used for the core material. But this analysis does indicate failure. With the exception of the pore pressures, the analysis was based entirely upon information given by the author or in the report, including the value of tan $\phi = 0.6$ for the core material. Only two differences can exist: (1) The generalization of using the values of one representative sample for each material; and (2) the pore pressures. All embankment analyses are based upon a degree of generalization, so any difference from that source should be small enough to be negligible. Apparently then, the adaptation of the static slide method of analysis to this type of dam is dependent upon the correct application of the pore pressures, and these pressures were the only conditions not fully covered by the author.

Therefore, it is felt that if more complete information along this line were included in the concluding discussion, it would be of definite value to all of those interested in the design of earth embankments.

GLENNON GILBOY,[19] ASSOC. M. AM. SOC. C. E. (by letter).—The principal results of an unusually complicated and diversified investigation have been condensed and put in readable form by Mr. Middlebrooks. The writer wishes to add a few remarks of general interest, and to express certain opinions that are somewhat at variance with those of the author.

In terms of the size of an individual man, or even of the size of the massive construction equipment used, the Fort Peck Slide was a gigantic affair; but in terms of the size of the project it was a relatively minor localized disturbance. Specifically, the volume of earth moved was only 5% of the total volume in place at the time, and the length affected was about 8% of the total length. The sense of proportion associated with these percentages is, in the writer's opinion, an important element in any study of the problem.

An excellent picture of the localized nature of the disturbance is given in Fig. 2. The distortion of the upstream shell at the right (westerly) edge of the break shows clearly that the dam at this point had sufficient reserve strength to offer effective resistance against further propagation of the movement. The cross section here was no stronger than that in the slide area. Hence arises the logical inference that the dam itself was stable under the existing conditions, and that a weakness of the foundation, strictly localized in the vicinity of the right abutment, must have been responsible for the movement.

Fig. 1 illustrates the manner in which the foundation near the right abutment differed materially from that in the major part of the valley. The shale surface rises to a flat bench at a relatively shallow elevation, extending approximately from Station 13 to Station 19. A considerable depth of weathering is shown and the depth of penetration of the sheet piling indicates low resistance in the surface shale. Furthermore, the shale is interspersed with seams of bentonite, which tend to decrease its shearing resistance, especially in a horizontal direction.

Mr. Middlebrooks also mentions the intake channel, 800 ft upstream from the toe, and cut into the shale to El. 2,030. A significant detail in this connection is that the channel was cut approximately parallel to the dam from the intake structure, Station 11, out to Station 19, from which it veered sharply upstream.

Thus it is clear that a substantial length of the dam near the right abutment rested on a shelf of weathered shale and bentonite which, on account of its shallow depth, was subjected to the full intensity of horizontal shear induced by the embankment, and which was horizontally unsupported as a result of the channel excavation.

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[19] Cons. Engr., Falls Church, Va.

It is the writer's belief that the shearing strength of the material composing this shelf, though undoubtedly increasing somewhat under the weight of the dam, did not increase fast enough to keep pace with the increasing horizontal shear; that the margin of safety was thus decreased as the height of the dam increased; that the balance point was reached when there was still about 25 ft to go; and that movement was initiated by an essentially horizontal shear failure of the shelf material in the direction of its unsupported face.

The observed facts were consistent with this hypothesis. Among the more significant are:

(1) The failure was not sudden, but gradual. Distortions visible to the unaided eye were noted more than three hours before the slide took place. How much strain had developed previously it is impossible to say; but a considerable amount could have gone undetected due to the very nature of the operation.

(2) The distortions were maximal between Station 15 and Station 17, the precise center of the critical zone previously described.

(3) The first indication of the rapid final phase of the slide was a settling of the core pool in this zone.

(4) Initially, the upstream shell moved out practically all at once, like a huge gate hinged at the abutment.

(5) Subsequent investigations showed masses of weathered shale to be partially or wholly absent from their positions as determined by earlier borings.

So much for the initial phase. It should be noted here that the foregoing concept is the writer's personal explanation, and that, although the Board of Consultants agreed unanimously on the weathered shale zone as the source of the trouble, the detailed concepts of the individual members may differ materially from that presented here.

As to the progress of the slide after the initial failure, no unanimity of opinion was obtainable, and the Board was obliged to leave the question undecided in its final report. Since the writer's opinion on this point does not coincide with that expressed by the author, an explanation of the reasoning behind it seems indicated.

The upstream movement of material was truly remarkable. The intake channel, 800 ft upstream from the toe and 500 ft wide, was completely filled, to depths as great as 40 ft. Fanning out of material toward the center of the valley was on a comparable scale. Intact masses of the upstream shell moved more than a thousand feet from their original positions. The author's explanation of this extensive movement is that "liquefaction" took place in the weathered shale and bentonite zone, due to trapped water. This is equivalent to stating that the shearing strength of this zone reached a maximum at the instant of initial failure, and decreased to a considerably lower value after the peak point was passed. The writer does not dispute the probability of such reduction in shearing strength, but submits that by itself it is not sufficient to account for the observed behavior of the slide.

Suppose the dam had been constructed by rolled-fill methods. Horizontal base shears would have existed, of course. Suppose further that the dimensions of the rolled-fill structure and its rate of construction had been such as to bring about a balance of shearing stress and shearing strength in the weathered foundation zone when the dam was within 25 ft of the top. Shear failure in the foundation would have taken place. Reduction of shearing strength beyond the peak point would have exaggerated the movement. Considerable damage would have resulted, and expensive repairs would have been required. However it is inconceivable, to the writer's mind at least, that the rolled-fill material would have moved to anything like the extent observed in the actual slide. Outward movement of the toe, severe slumping in the main body of the dam, cracking, fissuring, faulting, all these things the writer can visualize; but nothing like an upstream movement of more than a thousand feet.

If this visualization is sound, it follows that the progress of the actual slide was intimately associated with the peculiarities of hydraulic-fill construction. Among these are: Complete saturation of the central part of the dam, with an excess of water on top; segregation of progressively finer materials from the outer slopes toward the core; and no artificial compaction. When construction is progressing normally, these items are merely part of the program; but if a disturbance occurs, they can unleash latent forces within the body of the dam which will tend to exaggerate the results of the disturbance.

Consider the effect of the core pool alone. If a shear failure of the foundation occurs, removal of support from the core will cause it to slump. A rolled-fill core would slump under similar circumstances; but when a hydraulic core slumps, the water of the core pool piles up at the low spot and produces an additional push on top of the core, thereby exaggerating the movement of the slide. This action will continue until the shell is fissured, whereupon the accumulated pool water will rush out through the fissures, carrying masses o core and shell material with it.

Segregation of fine, saturated, sandy material in the vicinity of the core—in the section of the dam often termed the "transition zone"—is another potential source of hazard. The coarser parts of the shell are inherently stable, or can be made so by artificial compaction; but the transition zone, half in and half out of water, is difficult to treat. A violent disturbance will tend to produce a temporary "quick" or "liquefied" condition in this zone, with a consequent increase of internal pressure and a further tendency to exaggerate the effect of the disturbance.

The writer submits that both the foregoing influences are present in any hydraulic-fill dam; that they were present in the Fort Peck Dam; that the initial disturbance due to foundation failure was violent enough to bring them fully into play; and that they were primarily responsible for the great distance covered by the slide after the initial failure.

The author dismisses the possibility of liquefaction of the shell on the ground that subsequent tests (Fig. 6) showed that the shell was denser than the critical value obtained for most of the samples tested. The writer agrees that most of the shell, certainly the outer parts, behaved very well; but this is no proof that liquefaction of the transition zone did not take place. On the contrary, at the time of the writer's first inspection, ten days after the slide, much of the surface material in the slide zone, especially the fine sand, was still in a "quick" condition, and it is difficult to imagine that it was not so during the occurrence of the slide. The amount of material which liquefied might have been small in comparison with the volume of the slide, yet might have played a very important rôle in keeping the more stable masses on the move.

As far as the critical density tests are concerned, it will be noted that the density of the fill coincides closely with the "minimum critical density" curve of Fig. 6. This means that the margin of safety was not very great. Although it is true, as the author states, that sands looser than critical may exhibit some residual shearing strength when disturbed, the writer believes that the nature and violence of the disturbance are of great importance, and that saturated sands somewhat denser than critical may, if sufficiently disturbed, pass through a temporary condition of liquefaction before attaining a new condition of equilibrium. Therefore, the writer contends that the critical density tests do not prove the absence of liquefaction; and that the balance of other evidence is strongly in favor of the view that some liquefaction took place, especially in the transition zone, and assisted in the movement of the slide.

In view of these considerations, the writer is now of the opinion that his method for computing shell stability does not contain as great a hidden factor of safety as might at first glance be expected. To provide absolute safety under all conditions, the parts of the shells which are inherently stable when deposited, or can be made so by artificial compaction, should be strong enough to withstand possible liquid pressure from the transition zone which might be produced by subsequent disturbances. While this may seem an unnecessarily severe criterion, it must be remembered that as improvements in design methods permit increasing economies in cross section, the influences of previously negligible factors become increasingly important.

In conclusion, the writer wishes to endorse the views of the author concerning the importance of thorough quantitative investigations of the properties of weak rocks such as those encountered at Fort Peck. These materials fall into a sort of twilight zone, where soil mechanics ends and geology begins, and the combined efforts of workers in both subjects should go far toward reducing the uncertainties now existing in this field.

FRANK E. FAHLQUIST,[20] ASSOC. M. AM. SOC. C. E. (by letter).—The engineering profession is indebted to Mr. Middlebrooks for his thorough analysis of the causes and behavior of the Fort Peck Dam slide. To the writer, at least, the crux of the entire problem is indicated in the author's discussion of the rock foundations, as stated under the heading "Test and Analysis Prior to Slide: Slides on Shale" (first thirteen lines). Also of particular interest are the first ten lines of the author's concluding paragraph.

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[20] Senior Geologist, U. S. Engr. Office, Providence, R. I.

During the summer of 1931, the writer had an opportunity to study the geology of several potential dam sites in the Missouri River Valley of Montana. Although the time available for this work was limited, and the area covered was great, sufficient information was obtained to indicate that, before any final selection of sites, designs, or constructions could be made, consideration should be given to several important problems bearing on the geology of the Missouri River Valley and the design and construction of dams. These geologic problems have not been adequately described in the paper nor in any other accounts of the Fort Peck Dam slide that have come to the writer's attention.

The Missouri River Valley, from the North Dakota state boundary line to Great Falls, Mont., is formed in an alternating series of inclined shale and sandstone formations. These rocks are of Cretaceous age and, therefore, in a geologic sense, are comparatively young. As a result, but chiefly because there has been only a slight burden of overlying sediments, subsequently removed by erosion, these rocks have behavior characteristics that are not ordinarily a part of rock formations found in other parts of the United States. The several sandstone members of this sedimentary series, throughout the eastern half of the valley, in general, are poorly cemented and friable. The shale members (that at the Fort Peck Dam being known as "Bearpaw" shale) are only partly consolidated or indurated. However, where this formation occurs buried beneath an overburden of soil or water, and, further, where it has not been affected by any geologic disturbances, the rock is firm and stable. These shales, however, have two undesirable characteristics or properties: (1) They contain stratifications of bentonite ranging from a fraction of an inch to several inches in thickness, and (2) when uncovered and exposed to the air, they readily disintegrate or slack into innumerable small pieces.

In his reconnaissance of the valley, the writer was impressed at the evidences of landslides of both recent and old age. Several areas were noted where large masses of shale and sandstone had broken off of the inner valley cliffs and slid on to the valley bottom. Some of these landslides were pronounced and easily identified, whereas others, being eroded and probably of greater age, were difficult to recognize. At one dam site, located at a considerable distance above the Fort Peck Dam, but within the reservoir, landslide activity had extended over a large area, causing the writer to recommend elimination of the site from further consideration.

With these extraordinary geologic conditions in mind, the writer, in submitting his report,[21] outlined several broad concepts that should be considered in the investigation, design, and construction of any dams located in the Missouri River Valley:

(1) That the air-slacking shale presented a special construction problem in rock excavation work. It was not anticipated, however, that this property would create any serious difficulties, as the condition could be easily remedied by forced applications of portland cement grout on freshly excavated rock surfaces. Actually, in the construction of the Fort Peck Dam, asphalt was used, which is equally suitable. In addition, the ease with which the shale

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[21] Report to the U. S. Engineers, Kansas City District, Kansas City, Mo., November, 1931.

disintegrated precluded its use as riprap or in rock fills. Obtaining more suitable and durable rock from distant igneous rock areas was recommended. Actually, hard, durable boulders were shipped in to the site from a remote area.

(2) That the occurrence of bentonite in the shale formation and evidence of landslide activity presented the most serious problem. The writer judged that the bentonite stratifications could be, and probably were, the cause of many of the landslide effects. Because the Bearpaw formation extends over such a large area, the writer was led to report that:

> (*a*) Landslide activity should be investigated at any sites finally selected;
>
> (*b*) If any landslide blocks of large proportions were found to occur at any of the selected sites, the question of further study and construction or abandonment was a matter of cost versus benefits, compared with other sites; and
>
> (*c*) In the case of all dams in the valley, particular care should be exercised in foundation and abutment constructions. If surface and subsurface investigations revealed occurrence of landslides or other disturbance in the foundation rock, this condition, together with occurrence of bentonite stratifications, would be serious, and probably would warrant excavation and removal of all rock affected by these deformations. This precautionary measure would be necessary to assure founding the dam in the abutment areas on undisturbed bedrock.

The author's discussion of the rock foundations, under the heading "Tests and Analysis Prior to Slide: Slides on Shale" (particularly his description of conditions disclosed by later investigation), leads the writer to believe that the 30-ft thick zone of blocky weathered shale that gradually passes into a subfirm and firm shale is quite likely an eroded remnant of an old landslide block. The observations as disclosed by detailed investigations tally exactly with conditions observed by the writer in the field. The author's conclusion that the solution to similar problems may be found by obtaining representative undisturbed samples does not go far enough. The very first essential in a problem of this kind, or in any foundation or abutment problem, is an understanding of the geologic history of the area or region. Such an understanding, no matter how meager the evidence is at first, will help one to formulate a method of attack for accumulating more pertinent information and to separate the more serious and important problems from those of less importance. Undisturbed samples are fine and necessary, as is also the analysis of stress and strain, but the writer can easily imagine cases in which any amount of undisturbed sampling and mathematical analysis could not fully develop a complete geologic picture. For example, consider the blocky weathered condition described by the author. In this case there is a disrupted mass of rock, a shale which disintegrates and slacks readily, which contains stratifications of bentonite, and which, when weathered and wet at the surface, forms a sticky and slippery mass of soil. When such conditions are recognized, the only adequate exploration to confirm or disprove preliminary geologic conceptions is twofold: (1) A few well-selected and carefully executed borings to outline the extent of such geologic conditions, and (2) real life-size excavations to explore the condition fully, in area and in depth. When this exploratory work is completed, all testing done, and all factors of safety computed, one should bear in mind that there may still be some worse condition not yet disclosed, and then one should consider carefully whether it would not be best to remove the entire questionable mass and build on undisturbed material.

Irving B. Crosby,[22] **Affiliate Am. Soc. C. E.** (by letter).—The causes of the Fort Peck slide have been discussed by Mr. Middlebrooks in his very useful paper, and he has shown that the slide was due to weakness in the foundation shale and not to any weakness in the dam itself. The writer believes, however, that certain geologic factors have not been brought out by the author and that an erroneous impression has been given by some of the statements.

The Board of Consultants unanimously agreed that the slide "was due to the fact that the shearing resistance of the weathered shale and bentonite seams in the foundation was insufficient to withstand the shearing force to which the foundation was subjected,"[23] but as to the relative importance of the various factors the Board could not reach agreement. The writer is solely responsible for the following ideas, some of which may be at variance with those of other members of the Board:

The Bearpaw shale in the vicinity of Fort Peck Dam has been intersected by numerous faults or fractures on which movement has taken place. Many of these faults were due to deep-seated causes, but some of them were caused by landslides in the bluffs of the Missouri Valley at a time when the river was flowing at a lower level. The principal results of the faults were the production of numerous planes of weakness and of channels for ground-water movement. Study of the numerous boring cores and records made after the slide indicated that there was horizontal movement on a bed of bentonite and that this plane of movement reached the surface largely through weathered shale. The writer believes that the bentonite beds and faults played a very important part in causing this and previous slides.

Mr. Middlebrooks has described the hydrostatic pressure found in many borings in the shale and has ascribed to this pressure an important part in causing the slide. He states that prior to the construction of the dam the bentonite seams were covered by only a shallow overburden, that they had expanded and taken up water, and that then the superimposed load of the dam caused high hydrostatic pressures in the bentonite, which facilitated the slide. No such condition (water taken up by the bentonite) existed, however, in the case of the older slides in the bluffs. In those slides the bentonite was still under a thick load, and it did not have opportunity to take up water and expand. These slides were due to undercutting of the bluff by the river with consequent indirect and very gradual overloading. Similar slides on a small scale have occurred in the banks of the reservoir since the water was raised and lowered, and in every case observed by the writer there was a bed of bentonite. Similar slides occurred at Snake Butte, Mont., associated with beds of bentonite in the shale, but in this case the peculiar situation of thin overburden and sudden overload assumed by the author did not exist and yet slides occurred.

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[22] Cons. Eng. Geologist, Boston, Mass.

[23] "Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam," Corps of Engrs., U. S. Army, July, 1939, p. 8.

The writer believes that the water squeezed out of the shale in the various borings came largely from the intricate network of fractures in the shale and to only a small extent from the pores of the shale and bentonite, and that "liquefaction" of the bentonite previous to the slide did not exist. Once movement occurred, however, water from the fractures may have been mixed with the bentonite and facilitated the extent of movement. The presence of flat bentonite beds and of fault planes inclined toward the valley provided planes of low resistance on which movement occurred when the superimposed load became sufficiently great.

Mr. Middlebrooks concludes that the best means of avoiding similar slides in the future "is to always obtain representative undisturbed samples of this type of rock, sufficiently large to reveal the true character of the rock tc geologists and engineers." Numerous undisturbed samples of this shale and bentonite were taken, however, and numerous tests were made on them prior to the slide; but the true condition of the shale was not understood and the slide was not foreseen.

This writer doubts that more and larger samples would have disclosed the dangerous foundation condition unless the geologic history of the region had been correctly interpreted first. The history of past slides is written physiographically on the face of the bluffs. Nature has been conducting large-scale tests on the shale, over a long period of time, in a manner that cannot be duplicated in the laboratory. Correct understanding of these natural tests will often reveal what laboratory tests fail to show. The tests, by nature, in the bluffs have shown that the bentonite is the weakest natural material in this region and that slides frequently occur on it. Correct interpretation of the physiography and stratigraphy shows that great masses of shale have slid from their original positions and that the shale in the bluffs may be intersected by innumerable zones of weakness.

With the geologic and physiographic history of the region correctly interpreted, the possibility that a dangerous situation may exist becomes apparent. Then clues that may have been overlooked would be recognized and warning given, and the actual conditions could be disclosed by special investigations. Had the true conditions been understood, the dam could have been designed to meet those conditions safely.

T. A. Middlebrooks,[24] Assoc. M. Am. Soc. C. E. (by letter).—Mr. Feld declares that "large variation in natural soil cushion always causes trouble." The writer cannot agree with this statement. If trouble was to be experienced from a wide variation in the thickness of the natural overburden, the slide should have occurred on the left abutment instead of the right abutment. Between stations 12+00 and 19+00, the thickness varied from zero to about 25 ft, a change of approximately 3.5 ft in 100 ft; whereas, between stations 77 and 84, the thickness varied from 100 ft to 10 ft, a change of approximately 13.0 ft in 100 ft. It is evident, therefore, that this change had nothing to do with the failure. The writer has investigated numerous earth slides and has found none that could be remotely attributed to a change in thickness of the overburden in the manner described by Mr. Feld.

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[24] Prin. Engr., Corps of Engrs., War Dept., Soil Mechanics Section, Washington, D. C.

Mr. Feld's conclusion that the fill was too rigid to take up the differential settlement can scarcely be substantiated. Actually, differential settlement between stations 10+00 and 20+00 was quite small—approximately 0.25 ft (3 in.) in 100 ft. Such a small differential movement could not cause shear failure in either the sand shells or the core.

He concludes correctly that the surcharge load increased the hydrostatic pressure in the shale and caused uplift (excess hydrostatic) pressures in the base. The fact that the surcharge load, as pointed out by the writer, was the sole cause of the excess hydrostatic pressure in the shale has been definitely proved by further investigation since the slide.

The writer is at a loss to understand how Mr. Feld would increase the stability of the structure by the use of "Vertical cleavage joints, with proper shear dowels," and the "construction of the dam between stations 20 and 85 first." It is customary practice to place cleavage joints and shear dowels in concrete structures. However, their use in the Fort Peck Dam would have served no useful purpose and would have been entirely impracticable to construct. Observations on the excess hydrostatic pressure in the shale, since the slide, have shown conclusively that any reasonable extension of the construction period would not have materially increased the strength of the shale and bentonite, or reduced the excess hydrostatic pressure.

In Mr. Feld's closing paragraph he has evidently confused stress with strength in his reference to "'erection' stresses." The stress in a given earth structure will be practically the same directly upon completion as it will after the shrinkage in the dam and settlement in the foundation have been taken up. The greatest variable is the strength of materials involved, which reaches a maximum only when the materials are 100% consolidated.

The writer was pleased to have Mr. Justin's discussion and his agreement as to the primary cause of the slide. As Mr. Justin states, considerable confidence can be placed in the redesigned section since the slide was used as a full-scale test for the determination of the over-all strength of the materials involved.

Mr. Gerig's statements concerning the stability of dredged sand, and his conclusion that the sand shell did not flow or liquefy, should be noted carefully. Since Mr. Gerig's experience checks with the conclusions drawn from tests conducted on the shell material and observations of the undisturbed cores, it can be concluded, without question, that the sand shell did not flow or liquefy.

The writer agrees fully with Mr. Gerig's conclusion that "entire dependence cannot always be given to the results of laboratory tests." However, laboratory tests are essential where there is no past experience upon which to rely, and they are a necessity to the practical engineer in all other cases to supplement his judgment and experience.

Mr. Ryan's clear description of the action of excess hydrostatic pressure in the voids of a soil is most interesting. The writer agrees that for most cases the static slide method will give satisfactory results provided the distribution and amount of excess hydrostatic pressure and the reduction in strength due to progressive movement are known. An investigation reported in 1941 by the U. S. Waterways Experiment Station (Engineer Department)[25] shows that, with the proper determination of the excess hydrostatic pressure and remolding effect due to progressive movement during construction, the circular slide method will give reasonable results.

Evidently Mr. Ryan found in his stability analysis (Fig. 15) the same thing as the author—namely, that by proper manipulation of the shearing strength of the core he could get the two methods to check. It is well to emphasize that for a correct analysis by any method the distribution and amount of excess hydrostatic pressure and the decrease in strength due to progressive movement must be known. In the redesign of the section these factors were taken into consideration by using over-all strength values determined from the actual slide. The writer regrets that there are no additional data on the excess hydrostatic pressure that would be helpful in a stability analysis over that given in the paper.

Mr. Gilboy has given a clear résumé of the action of the slide, and it is noted that he is in general agreement with the writer in all main features. He disagreed, however, on the action of the shell and transition after the initial failure had occurred. The action of this material after the initial failure is of minor importance in analyzing the cause of the Fort Peck slide; however, it is of major importance in the design of future hydraulic-fill dams in earthquake regions. Mr. Gilboy concludes definitely that there was liquefaction of the transition zone material and possibly the shell material during the slide.

The density of fill does coincide closely with the "minimum critical density" as pointed out by Mr. Gilboy. The margin of safety, however, is much greater than this comparison would indicate, since the critical density varied with every change in the material in the hydraulic fill. It is not an exaggeration to state that the critical density actually varied from the minimum to the maximum within a few inches of fill. Liquefaction over a sufficiently large area to cause trouble would have to include the strong with the weak. It is obvious, therefore, that the average critical density should be compared with the average fill density.

There are several other pertinent factors to this question, which were not mentioned by Mr. Gilboy, that should be emphasized. Undisturbed (frozen) 36-in. cores taken from the shell and from transition material after the slide showed no evidence of liquefaction, which is the major one of these factors. The fact that the "quick" condition on the surface, noted by Mr. Gilboy, was in the area where the transition zone and shell material sloughed into the settling core pool, as the shell moved out in a body, is another important factor. The core pool water trapped in the fissures and crack of the material was forced up through the mass, and a "quick" condition resulted at the surface. The remaining shell material, even that resting on the transition zone, moved out as a solid mass with very little disturbance on the surface except for tension cracks. In view of the overwhelming evidence that shows that the dredged sand did not liquefy and the obvious lack of any positive evidence that it did, it must be concluded that there was no liquefaction of the dredged sand.

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[25] "Investigation of the Pendleton Levee Failure," report by the U. S. Waterways Experiment Station, Vicksburg, Miss., 1941.

The writer agrees fully with Mr. Gilboy that the water in the core pool settling into the crack left by the outward movement of the shell probably gave a little extra push that would not have been present in the rolled fill. However, the writer is confident that there would have been no basic difference in the nature of the slide except that the maximum distance that it moved might have been somewhat less (probably 900 ft instead of 1,000 ft).

Landslide activity stressed so strongly by Mr. Fahlquist had no bearing on the true cause of the slide. However, since this condition has been brought into the discussion, with the inference that it was not investigated, some comments are essential. Old landslides and faulting were quite extensive in the Bearpaw shale, and it was evident that a thorough investigation was absolutely necessary. Therefore, one of the first steps in the investigation of the foundation condition was to outline in detail all the faults and landslides. These conditions in the slide area were mapped very much in detail prior to the slide by explorations, including a tunnel into the abutment, shafts, and trenches, as well as core boring, and by actual excavation of the shafts, tunnels, and portal. It was concluded from the voluminous data available prior to the slide that the faulting and old landslide activity in this area would not cause any trouble, and the detail investigation after the slide substantiated this earlier conclusion. Even though there was considerable faulting in this area, as shown in the full report[12] on the slide, the Report of the Board of Consulting Engineers did not mention it as a possible cause of the slide. They did consider the possibility of grouting the so-called "A" fault on the left abutment, but after a thorough study had been made no grouting was considered necessary.

The 30-ft thick zone of blocky weathered shale found on the abutment, and referred to by Mr. Fahlquist, played only a secondary part in the slide. Movement definitely started around Station 15+00, due to the weak zone in the foundation, and as the mass moved out it carried a large part of this weathered shale with it. The writer is confident that the weathered shale in the abutment had no influence on the initial movement of the slide.

Mr. Fahlquist leaves the impression that the slide was due to some mystic geological condition involving old landslide activity that cores and undisturbed samples of the shale would not disclose, but that geological study of the region would. If there is any one point that stands out clearly from the writer's experience at Fort Peck, it is that a general geological study of the region is worthless for the investigation of any definite area for probable slides, unless it is accompanied by undisturbed samples of the rock in questionable areas. In any such investigation, the strength of the rock is essential, and it cannot be determined by a geological study of the region or even the immediate vicinity.

If, in all cases, the entire questionable mass was removed, as recommended by Mr. Fahlquist, there would, of course, be no need for undisturbed samples of the rock. However, at Fort Peck this procedure definitely was not economically feasible, and it bordered on the impossible.

In his closing statement, Mr. Fahlquist makes the following recommendations:

> "When such conditions are recognized, the only adequate exploration to confirm or disprove preliminary geologic conceptions is twofold: (1) A few well-selected and carefully executed borings to outline the extent of such geologic conditions, and (2) real life-size excavations to explore the condition fully, in area and in depth."

These recommendations are inadequate since they do not provide a means of determining whether the rock is strong enough to be stable. The Fort Peck slide is ample testimony to this fact, since the only thing missing from the original geological investigation was sufficiently large, undisturbed samples to determine the true character and strength of the upper 10 ft of the rock. This is the outstanding lesson to be learned by geologists and engineers from the Fort Peck slide.

Mr. Crosby minimizes the importance of the excess hydrostatic pressure in causing the slide, because other natural slides in this area occurred without this condition existing. The writer had no intention of leaving the impression that all slides were caused by the presence of excess hydrostatic pressure. Old slides along the river were definitely caused by undercutting which is an entirely different situation from that which existed in the Fort Peck slide. The slide at Snake Butte, Mont., was an old one that started moving again when the upper part was heavily loaded with quarry waste. These slides present no evidence from which one could logically conclude that excess hydrostatic pressure in the Fort Peck slide was not important.

It is agreed that the water that was present to develop the excess hydrostatic pressure did not come from the pores of the firm or sub-firm shale and bentonite seams. It came from the fractured and weathered zones in the shale and bentonite.

The writer agrees with Mr. Crosby in his conclusion "that 'liquefaction' of the bentonite previous to the slide did not exist." However, after a slight movement had occurred, the bentonite and shale were undoubtedly mixed with water and their strength considerably reduced. This, the writer believes, was the prime reason for the distance that the slide moved.

Mr. Crosby erroneously states that:

> "Numerous undisturbed samples of this shale and bentonite were taken, however, and numerous tests were made on them prior to the slide; but the true condition of the shale was not understood and the slide was not foreseen."

There were no undisturbed samples taken of the rock in the slide area, since the small core boring showed only sub-firm shale in this area. Sub-firm shale was found in numerous excavations to be a structurally strong rock. The geological explanation for the lack of weathered shale over the sub-firm shale in this area was that the river had scoured it off leaving only sub-firm shale. This conclusion at the time (without the benefit of hindsight) appeared entirely reasonable. Having been directly connected with the project for four years prior to the slide, the writer is firmly convinced that the actual strength and condition of the rock in the slide area could have been determined only by large undisturbed samples. It is rather far fetched to assume, regardless of how carefully the geology of the region was studied, that the true character and strength of the rock at any given location could be determined by this procedure.

The writer fully agrees that "Had the true conditions been understood, the dam could have been designed to meet those conditions safely." This is merely another way, however, of saying that hindsight is better than foresight

It has not been the intent of the writer in this discussion to minimize the importance of thorough geological investigations. However, it is certainly false security to rely too much on a geological study of the region as proposed by Mr. Fahlquist and Mr. Crosby. In the future, under such circumstances, the writer will take large undisturbed samples of all weak rocks regardless of what the geologist might find in a study of the region.

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ON THE METHOD OF COMPLEMENTARY ENERGY

AND ITS APPLICATION TO STRUCTURES STRESSED BEYOND THE PROPORTIONAL LIMIT, TO BUCKLING AND VIBRATIONS, AND TO SUSPENSION BRIDGES

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WITH DISCUSSION BY MESSRS. I. K. SILVERMAN, GEORGE R. RICH, R. V. SOUTHWELL, AND H. M. WESTERGAARD.

SYNOPSIS

The method of complementary energy is a general method of structural mechanics. The basic law was stated by F. Engesser in a paper in 1889. He extended Castigliano's law of least work to apply beyond the range of Hooke's law by replacing work by complementary work, which is an integral of distance times increment of force. Engesser's paper is little known.

The purpose of the present paper is to give proof and demonstration of the method. The proof goes back to fundamentals and includes a re-examination of the fundamentals; this is needed to remove doubts about the ranges of applicability. The demonstration consists of representative applications and may be interpreted as an exploration of the field.

HISTORICAL NOTES

The method of complementary energy is an extension of Castigliano's method of least work. Alberto Castigliano[2] published his method during the Seventies in papers and a treatise. His principle of least work applies to statically indeterminate structures stressed within the range of Hooke's law and subject to the restriction that all significant deformations must be linear homogeneous functions of the loads. Castigliano showed that among all the

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NOTE.—Published in February, 1941, *Proceedings*.

[1] Dean, Graduate School of Eng., and Gordon McKay Prof. of Civ. Eng., Harvard Univ., Cambridge, Mass.

[2] Thesis to obtain diploma as engineer, Torino, 1873; two papers in *Atti della Reale Accademia delle Scienze di Torino*, Vol. 10, 1875, p. 380, and Vol. 11, 1876, p. 127; and "Théorie de l'équilibre des systèmes élastiques," Torino, 1879, 480 pp., translated into English by E. S. Andrews under the title "Elastic Stresses in Structures," Scott, Greenwood & Son, London, 1919.

statically possible states of stress in such a structure the correct one is that which makes the energy of the internal stresses a minimum. This state of stress satisfies automatically not only the requirement of equilibrium but also the requirement of geometrical continuity. L. F. Ménabréa[3] had stated this principle clearly for trusses in 1858, but his proof contained misunderstandings, and the method is credited justly to Castigliano.

Castigliano[4] himself gave the method its first extension; he stated a revised expression that must be made a minimum if imperfect fits of redundant members create initial stresses; and he applied this procedure to temperature stresses in a general discussion and in six examples.[5] H. Müller-Breslau[6] improved the procedure for temperature stresses and contributed much toward making Castigliano's method known. A useful and dependable critical account of the original works in the field was given by M. Grüning[7] in 1912. In the twentieth century Castigliano's method has become stock in trade; it holds a position today as one of several useful general procedures of structural mechanics. It is worthy of note that in a book published in 1936 R. V. Southwell[8] of Oxford University, Oxford, England, gave an attractive original derivation of Castigliano's principle, based on a discussion of self-strains.

The contribution that has the greatest interest for the present study was published by Fr. Engesser[9] in a paper in 1889. He derived the basic law of the method of complementary energy. It is a modification of Castigliano's law of least work in which work is replaced by complementary work or complementary energy. As work is an integral of force times increment of distance or of stress times increment of deformation, so is complementary work an integral of distance times increment of force or of deformation times increment of stress. Engesser's theory applies beyond the range of Hooke's law; it includes not only Castigliano's method but also Müller-Breslau's procedure for temperature stresses as special applications. In his review of the field in 1912 Grüning[10] quoted and discussed Engesser's contribution, but otherwise it has received little attention. A plausible explanation is that structural analysis has been concerned mainly with stresses below the proportional limit, and the applicability to buckling and vibrations had not been realized.

The method of complementary work or complementary energy is analogous to another method which has become important in structural statics; namely, the method based on the "principle of minimum of the potential energy by variation of the shape." It is advantageous to consider the two methods in conjunction. The basic principle of the second method is sometimes stated as a direct consequence of a general law of dynamics, but it can be derived from the simplest laws of statics. Daniel Bernouilli and Leonhard Euler[11] stated and used this principle in a special form in the first half of the eighteenth century. The method played a part in the nineteenth century. A mathematical paper published by W. Ritz[12] in 1908 gave new impetus to its use, which has been widespread since then. A number of applications are found in the writings of S. Timoshenko.[13]

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[3] "Nouveau principe sur la distribution des tensions dans les systèmes élastiques," by L. F. Ménabréa, *Comptes Rendus*, Paris, Vol. 46, 1858, pp. 1056–1060.

[4] "Théorie de l'équilibre des systèmes élastiques et ses applications," Torino, 1879, p. 39.

[5] *Loc. cit.*, pp. 39, 317, 324, 332, 347, 428, and 442.

[6] "Der Satz von der Abgeleiteten der ideellen Formänderungs-Arbeit," by H. Müller-Breslau, *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 30, 1884, columns 211–214; "Die neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen," Leipzig, 1886, 5th Ed., 1924, pp. 74–79; "Graphische Statik der Baukonstruktionen," Vol. 2, Subvolume 1, Leipzig, 1892, p. 49, 5th Ed., 1922, p. 47.

[7] "Theorie der Baukonstruktionen I: Allgemeine Theorie des Fachwerks und der vollwandigen Systeme," by M. Grüning, *Encyklopädie der mathematischen Wissenschaften*, Vol. 4, Subvolume 4, Leipzig, 1907–1914, pp. 419–534, especially pp. 437–454.

[8] "An Introduction to the Theory of Elasticity," by R. V. Southwell, Oxford Univ. Press, 1936, p. 91.

[9] "Ueber statisch unbestimmte Träger bei beliebigem Formänderungs-Gesetze und über den Satz von der kleinsten Ergänzungsarbeit," by Fr. Engesser, *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, Vol. 35, 1889, columns 733–744, especially 738–744.

[10] *Loc. cit.*, p. 454.

THE LAWS OF LEAST ENERGY AND OF LEAST COMPLEMENTARY ENERGY

Since it is desirable that no doubts shall remain about the ranges of applicability, all the steps of the derivations of the basic theorems will be shown.

The Structure.—Fig. 1 represents a structure of general type. The black parts are joints, which, by a definition adopted here, are rigid bodies. The shaded parts are deformable members, which are attached to the joints. External forces and reactions are assumed to act on joints only. Since the joints may be three-, two-, or one-dimensional or may be without extension, and any number of joints may be assumed to exist, it is difficult to conceive of a structure to which this picture could not be adapted.

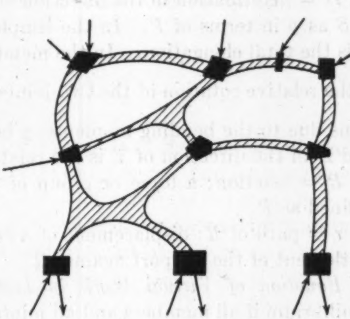

FIG. 1.—STRUCTURE OF GENERAL TYPE

Notation.—The notation applied to the structure in Fig. 1 requires explanations, which follow:

P = external load. The load P may be a single force P, or it may consist of a group of forces $c_1 P$, $c_2 P$, $c_3 P$, $\cdots$, all proportional to P and varying together when P varies. This conception makes it possible to consider a whole dead load or a whole live load as "a load"; which is in accordance with common usage of the word "load."

p = path of the load P during the deformation from a starting shape of the structure to an assumed shape. If P is a single constant force, p is the displacement of the point of application in the direction of P, and Pp is the work of P. If the load P is a group of forces, then p is defined by the statement that with the value of P remaining constant during the assumed deformation, the work of P is Pp. For example, if P is a total load uniformly distributed over the length of a beam, p is the increase of the average deflection.

S = stress in a member. The stress in the sense adopted here is interpreted as a load exerted by joints on a member, and is defined in the same way as P except for the restriction that all the forces constituting a particular stress S must be in equilibrium. If the member is a simple tension member in a truss, S may be taken as the total tension, and the load on the member is the two equal and opposite pulls, each equal to S, at the ends. There can be more than one stress in a member. For example, in a beam flat joints may be assumed at two adjacent cross sections a distance dx apart. The member between these two joints has two stresses: One is the bending moment M, which consists of two equal and opposite couples exerted by the joints; the other is the transverse shear V, which consists of two equal and opposite shearing forces and a supplementary balancing couple Vdx at one of the joints. Instead one may in this case assume two members occupying the same space between the cross sections, one resisting M and the other V.

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[11] "De curvis elasticis," by Leonhard Euler, 1744; annotated translation into English by W. A. Oldfather, C. A. Ellis, M. Am. Soc. C. E., and Donald M. Brown, in *Isis*, Vol. 20, 1, November, 1933, pp. 72–160, especially p. 78.

[12] *Crelles Journal*, Vol. 135, 1908.

[13] See especially "Theory of Elasticity," 1934, "Theory of Elastic Stability," 1936, and "Theory of Plates and Shells," 1940, by S. Timoshenko, McGraw-Hill Book Co., Inc.

D = deformation in the direction of S; D is the path of S, defined in terms of S as p in terms of P. In the simple tension member with total tension S, D is the total elongation. In the member in the beam D in the direction of M is the relative rotation of the two joints, that is, $-\frac{d^2y}{dx^2}\,dx$ if y denotes the deflections due to the bending moments, y being measured from the starting shape; and D in the direction of V is the relative sliding of the joints.

R = reaction; a force or group of forces exerted by a support, otherwise defined as P.

r = path of R; displacement of a support in the direction or R; $-r$ is the settlement of the support against R.

Equation of Virtual Work in Infinitesimal Form.—The structure is in equilibrium if all members and all joints are in equilibrium. The members are loaded only by the stresses S, and are automatically in equilibrium by the definition of the stresses. A joint, by definition, is a rigid body. The equilibrium of a rigid body can be investigated by assuming an arbitrary infinitesimal movement of it while the forces remain constant. If during any such movement the sum of work of all the forces is zero except for an infinitesimal quantity of second or higher order, the rigid body is in equilibrium. Assume an arbitrary infinitesimal movement of each joint, whereby the deformations p, D, and r increase by the amounts δp, δD, and δr. The forces acting on all the joints are the forces constituting the loads P, the stresses reversed or $-S$, and the reactions R. The condition that the sum of work of all the forces on all the joints must be zero is expressed by the equation

$$\sum P\,\delta p - \sum S\,\delta D + \sum R\,\delta r = 0 \quad \text{.................} (1)$$

This equation will be recognized as the equation of virtual work in infinitesimal form; it states a principle that has played a great part in mechanics. Eq. 1 is a general condition of equilibrium, unrestricted by any requirement of linearity of the relations between the deformations, stresses, and loads; that is, unrestricted by any law of superposition.

Initial Assumption.—Changes of temperature can be considered to have taken place in advance, before the forming of the starting state. Then it becomes feasible to assume, and it will be assumed, that in all operations that need be considered in the analysis each stress S is a definite continuous function of the deformations D of the member in which it belongs, and conversely each deformation is a definite continuous function of the stresses in the member.

Energy.—Now consider a series of changes of shape, a variation of the shape, during which the loads and the positions of the supports remain constant; that is, the values of P and r remain constant. Each joint is moved from the starting position into an arbitrarily assumed position; thereby the structure is changed from the starting shape to an assumed shape. The members remain attached to the joints, so that continuity is maintained, but the requirements of equilibrium are ignored in this operation; that is, the geometrical requirements are respected, while the statical requirements are abandoned temporarily. The stresses S will be the proper definite functions of the deformations D. Then, for each assumed shape the structure and the loads can be said to have a potential energy equal to

$$T = \sum \int_{D_0}^{D} S\,dD - \sum P\,p \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the lower limits D_0 are chosen arbitrarily and the summations include all stresses and loads. The phrase "potential energy" can be used here because the stresses are definite functions of the deformations in the operations of the analysis. An infinitesimal variation of the assumed shape causes an increment, a first variation, of the potential energy equal to

$$\delta T = \sum S\,\delta D - \sum P\,\delta p \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Let the requirements of equilibrium be imposed again. Then Eq. 1 must be satisfied. Since $\delta r = 0$ in the variation considered, it follows that

$$\delta T = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

That is, if not only all the geometrical requirements but also all the statical requirements are to be satisfied, the first variation of the potential energy must be zero; that is, the potential energy must be a minimum or a maximum. Further arguments, omitted here, show that a stable equilibrium requires that T be a minimum. The statement

$$T = \min \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

is interpreted then with the reservation that T is normally a minimum, but under some circumstances the minimum may be replaced by a maximum. Eqs. 5 and 2 together state the "law of minimum of the potential energy by variation of the shape." Eq. 5 like Eq. 1 is unrestricted by any law of superposition.

Assumption of Superposition of Deformations, and the Equation of Virtual Work in Finite Form.—It will now be assumed that the law of superposition (or linearity) applies in a limited form; namely to deformations only, without involving loads, stresses, and reactions: it is assumed that if p', D', r' and p'', D'', r'' are two geometrically possible sets of deformations p, D, r, then $p'+p''$, $D'+D''$, $r'+r''$ are a possible set, provided that all the deformations involved are within some range. If this law is assumed, δp, δD, and δr in Eq. 1 may be replaced by p', D', and r'; so that

$$\sum P\,p' - \sum S\,D' + \sum R\,r' = 0 \qquad (6)$$

This is the equation of virtual work in finite form, which has been important in structural mechanics since the Seventies, when Otto Mohr applied it to trusses. It is noted that p', D', r' are not the deformations produced by the loads P.

Eq. 6 may be restated in the form

$$\sum P'\,p - \sum S'\,D + \sum R'\,r = 0 \qquad (7)$$

in which P', S', R' are a statically possible set of values of the loads, stresses, and reactions. A possible form of Eq. 7 is

$$\sum p\,\delta P - \sum D\,\delta S + \sum r\,\delta R = 0 \qquad (8)$$

in which δP, δS, δR are a statically possible set of increments of P, S, R.

Complementary Energy.—The preparations have now been made for the study of the second fundamental type of variation; namely, a variation of the state of stress, during which the statical requirements of equilibrium are the ones that remain satisfied, while the geometrical requirements of continuity are abandoned temporarily. As before, P and r are considered to remain constant, but now the deformations D are interpreted as functions of the stresses S, and values S_0 are arbitrarily chosen lower limits of the stresses. Then the complementary energy will be defined by the expression

$$U = \sum \int_{S_0}^{S} D\,dS - \sum r\,R \qquad (9)$$

An infinitesimal variation from any one of the statically possible states of stress to an adjacent statically possible state gives U an increment, a first variation, equal to

$$\delta U = \sum D\,\delta S - \sum r\,\delta R \qquad (10)$$

If the set of values of D and r happen to satisfy the geometrical requirements of continuity, then these values must satisfy Eq. 8 with $\delta P = 0$. It follows that

$$\delta U = 0 \qquad (11)$$

That is, the first variation of U vanishes; the complementary energy, U, is a minimum or a maximum, like the energy T ordinarily a minimum. The statement

$$U = \min \qquad (12)$$

is interpreted with the same reservation that was applied to Eq. 5: under abnormal circumstances the minimum may be changed into a maximum. Eq. 12, with U defined by Eq. 9, is the law of least complementary energy. This minimum is produced by a variation of the state of stress.

Analogy.—A comparison of Eqs. 1 to 5 with Eqs. 8 to 12, in order, shows that a complete analogy exists between the two laws of least energy and least complementary energy, with a one to one correspondence of the quantities, as follows: in the shifting from one principle to the other the quantities P, S, R, T, U, p, D, r are replaced by the same quantities in the reversed order.

Castigliano's Principle as Special Case.—When Hooke's law applies and a stress-less state can be and is chosen as starting state, and the lower limits S_0 are chosen as zero, the part of the complementary energy contributed by the stresses, the first sum in Eq. 9, becomes

$$U_i = \tfrac{1}{2} \sum D\,S \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

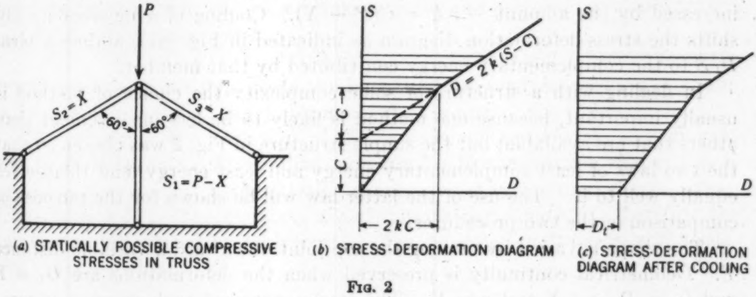
which is the same as the internal energy of stresses. If the supports have not moved, then $r = 0$. In this special yet general case the law of least complementary energy becomes

$$U_i = \min \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

which is Castigliano's principle of least work, or, the principle of minimum of the internal energy by variation of the state of stress.

APPLICATION TO A SIMPLE STATICALLY INDETERMINATE STRUCTURE STRESSED BEYOND THE PROPORTIONAL LIMIT

The simple truss in Fig. 2(*a*) will serve as an example. The stresses are total stresses, positive in compression. The state of stress is varied by varying X. The three members are assumed to be alike, with the stress-deformation diagram shown in Fig. 2(*b*). The supports are assumed not to have moved.



(*a*) STATICALLY POSSIBLE COMPRESSIVE STRESSES IN TRUSS (*b*) STRESS-DEFORMATION DIAGRAM (*c*) STRESS-DEFORMATION DIAGRAM AFTER COOLING

FIG. 2

When none of the stresses exceeds the proportional limit $2\,C$, the complementary energy is the same as the stress energy, which is

$$U = \tfrac{1}{2}\,k\,(P - X)^2 + k\,X^2 \ldots\ldots\ldots\ldots\ldots (15)$$

In Eq. 15, U must be a minimum by Castigliano's principle, which gives $X = \dfrac{P}{3}$.

When $P > 3\,C$, the stress in the vertical member will exceed the proportional limit $2\,C$. The complementary energy is determined by adding areas such as the shaded area in Fig. 2(*b*). If the proportional limit is not exceeded in the inclined members, the total complementary energy is

$$U = k\,C^2 + k\,(P - X - C)^2 + k\,X^2 \quad\ldots\ldots\ldots\ldots\ldots(16)$$

In Eq. 16, U becomes a minimum when

$$X = \frac{P - C}{2} = S_2 = S_3 \quad\ldots\ldots\ldots\ldots\ldots(17a)$$

and

$$S_1 = \frac{P + C}{2} \quad\ldots\ldots\ldots\ldots\ldots(17b)$$

Eqs. 17 apply when $3\,C < P < 5\,C$. If $P > 5\,C$, the complementary energy is

$$U = 3\,k\,C^2 + k\,(P - X - C)^2 + 2\,k\,(X - C)^2 \quad\ldots\ldots\ldots(18)$$

which becomes a minimum when

$$X = \frac{P + C}{3} = S_2 = S_3 \quad\ldots\ldots\ldots\ldots\ldots(19a)$$

and

$$S_1 = \frac{2\,P - C}{3} \quad\ldots\ldots\ldots\ldots\ldots(19b)$$

Computations such as these are simplified in less simple applications by expressing the derivatives that determine the minimum, without computing the value of the complementary energy itself.

If the support of the vertical member in Fig. 2(a) settles a distance c downward, the path of the corresponding upward reaction R is $r = -\,c$, and U is increased by the amount $-\,r\,R = c\,(P - X)$. Cooling of a member merely shifts the stress-deformation diagram as indicated in Fig. 2(c), adding a term $D_t\,S$ to the complementary energy contributed by that member.

In dealing with a structure of some complexity the choice of method is usually important, because one method is likely to be less inconvenient than others that are available; but the simple structure in Fig. 2 was chosen so that the two laws of least complementary energy and least energy lend themselves equally well to it. The use of the latter law will be shown for the purpose of comparison of the two procedures.

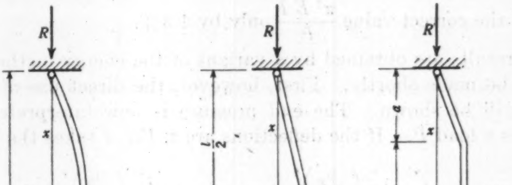
The shape is varied by moving the top joint downward a variable distance Y. Geometrical continuity is preserved when the deformations are $D_1 = Y$ and $D_2 = D_3 = \frac{1}{2}\,Y$ and $p = Y$. The strain energy of each member is measured by an area under the lines in Fig. 2(b). It is justifiable to call this quantity energy because the stress is a definite function of the deformation in the operations of the analysis; the quantity need not be energy in every physical sense; it is energy in the analysis. If the proportional limit has been exceeded in the vertical member only, the energy in Eq. 2 becomes

$$T = \left(\frac{Y^2}{4\,k} + C\,Y - k\,C^2\right) + \frac{1}{k}\left(\frac{Y}{2}\right)^2 - P\,Y \quad\ldots\ldots\ldots(20)$$

In Eq. 20, T is a minimum when $Y = k\,(P - C)$, which reproduces the stresses in Eqs. 17.

BUCKLING OF COLUMN WITH HINGED ENDS

When a column buckles under a critical load, the movement of the loaded end, the shortening of the chord, is proportional to the square of the lateral deflections; this is a departure from linearity and from the law of superposition; but the law of superposition of deformations will hold with good approximation in the significant applications if the starting shape is chosen close to the final shape.



(a) (b) (c)

FIG. 3.—STARTING SHAPES OF COLUMN

Fig. 3(*a*) shows a plausible starting shape for a hinged-ended column, held between two fixed blocks. The blocks give the end pressure the character not of a load but of a reaction R.

Let: z = deflection in the starting shape; y = additional deflection from the starting shape to the final shape, making the total final deflection $y + z$; and EI = modulus of elasticity times moment of inertia of the cross section.

In computing the variation of the complementary energy according to Eq. 10, S is taken as the bending moment Rz in the starting shape; this is approximately correct if the starting shape is close to the final shape. Then δS becomes $z\,\delta R$. The members are of infinitesimal length dx; therefore the summation is replaced by an integral. The deformation D becomes the relative change of slope

$$-\frac{d^2y}{dx^2}\,dx = \left(\frac{R\,z}{E\,I} + \frac{d^2z}{dx^2}\right)dx \qquad (21)$$

The displacement r is zero since the end blocks do not move. Thus one finds

$$\delta U = \int_0^l \left(\frac{R\,z}{E\,I} + \frac{d^2z}{dx^2}\right)dx\;(z\,\delta R) = 0 \qquad (22)$$

which gives the approximate formula

$$R = -\frac{\displaystyle\int_0^l z\,\frac{d^2z}{dx^2}\,dx}{\displaystyle\int_0^l \frac{z^2\,dx}{E\,I}} \qquad (23)$$

By choosing as a starting shape the parabola

$$z = 4\,c\,l^{-2}\,(l\,x - x^2) \qquad (24)$$

and assuming $E\,I$ constant, one finds by easy integrations

$$R = \frac{10\,E\,I}{l^2} \qquad (25)$$

which exceeds the correct value $\frac{\pi^2\,E\,I}{l^2}$ only by 1.3%.

The same results are obtained by a variant of the energy method to which reference will be made shortly. First, however, the direct use of the law of least energy will be shown. The end pressure is now interpreted not as a reaction but as a load P. If the deflections are z, Eq. 2 takes the form

$$T = \tfrac{1}{2}\int_0^l E\,I\left(\frac{d^2z}{dx^2}\right)^2 dx - P\cdot\tfrac{1}{2}\int_0^l\left(\frac{dz}{dx}\right)^2 dx \qquad (26)$$

The variation δT vanishes only when P has the critical value that makes $T = 0$. This gives

$$P = \frac{\displaystyle\int_0^l E\,I\left(\frac{d^2z}{dx^2}\right)^2 dx}{\displaystyle\int_0^l\left(\frac{dz}{dx}\right)^2 dx} \qquad (27)$$

With z as in Eq. 24 and $E\,I$ constant, Eq. 27 gives $P = 12\,E\,I\,l^{-2}$ which is 22% too great. The method of complementary energy gives the same result when z is taken as in Fig. 3(b); the numerator in Eq. 23 then being computed as c times the change of slope at the middle of the column. With the starting shape in Fig. 3(c) the method of complementary energy gives $R = \frac{6\,E\,I}{2\,a^2 + 3\,a\,b}$, which has its smallest and therefore most plausible value, $10.67\,E\,I\,l^{-2}$, when $a = \frac{3\,l}{8}$.

The comparison just made indicates superiority of the method of complementary energy over the direct application of the method of the potential energy to columns; but, as has been indicated, a variant of the energy method, applicable to columns, is available. S. Timoshenko[14] has described and used it. This improved variant can be explained by noting that the numerator in Eq. 27 is improved by replacing $\frac{d^2z}{dx^2}$ by $-\frac{P\,z}{E\,I}$. Then the formula becomes

$$P = \frac{\displaystyle\int_0^l\left(\frac{dz}{dx}\right)^2 dx}{\displaystyle\int_0^l\frac{z^2\,dx}{E\,I}} \qquad (28)$$

Eq. 28 can be rewritten in the form of Eq. 23. That is, Timoshenko's variant of the energy method for columns gives the same results as the method of complementary energy.

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[14] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, p. 81.

COLUMN WITH INITIAL CURVATURE

If the column that has been under discussion is slightly curved before loading, with the initial deflections z_0, and if z still denotes the total deflections in the starting shape, Eq. 22 will be changed to

$$\delta U = \int_0^l \left(\frac{R\,z}{E\,I} + \frac{d^2z}{dx^2} - \frac{d^2z_0}{dx^2} \right) dx\,(z\,\delta R) = 0 \dots\dots\dots\dots (29)$$

Assume that the shape before loading makes it plausible to choose z proportional to z_0. Denote by Q the critical value of R defined by Eq. 23 for $z_0 = 0$. Then Eq. 29 gives

$$R - \left(1 - \frac{z_0}{z}\right) Q = 0 \dots\dots\dots\dots (30)$$

or,

$$\frac{z}{z_0} = \frac{Q}{Q - R} \dots\dots\dots\dots (31)$$

Eq. 31 defines a magnification factor for the deflections. Such factors will be discussed again later.

GREATEST LOAD ON AN INITIALLY CURVED COLUMN STRESSED BEYOND THE PROPORTIONAL LIMIT

If the column in Fig. 3(*a*) is stressed beyond the proportional limit, it may be plausible to assume a law of deformations of the type,

$$\text{Change of curvature} = \frac{1}{E_r\,I}\,(M + F^{-2}\,M^3) \dots\dots\dots\dots (32)$$

in which E_r and F are functions of the end pressure R and dependent on the material and the shape of the cross section; E_r being a reduced modulus of elasticity, and F a coefficient measurable in inch-pounds. The theory developed by Engesser (from 1889), A. Considère (1891), F. S. Yasinsky (1895), and Theodor von Kármán, M. Am. Soc. C. E., (1910),[15] furnishes a method of computing E_r; for example, this method gives for a rectangular cross section

$$E_r = \frac{4\,E\,E_t}{(\sqrt{E} + \sqrt{E_t})^2} \dots\dots\dots\dots (33)$$

in which E_t is the tangent modulus of elasticity. The theory also will make it possible[16] to establish reasonable values of F. Moreover, it should be possible to determine both E_r and F by tests of short specimens under eccentric pressure.

Let z_0 denote, as before, the initial deflections before loading, and z the total starting deflections under the pressure R. With Eq. 32 accepted, Eq. 29

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[15] For explanation of the basis of the theory and for references, see "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, pp. 157–159.

[16] See, for example, the paper "Strength of Steel Columns," by H. M. Westergaard and William R. Osgood, Members, Am. Soc. C. E., *Transactions*, A. S. M. E., Vol. 50, 1928, No. 17, p. 65.

changes into the following expression of the law of least complementary energy:

$$\frac{dU}{dR} = \int_0^l \left[\frac{1}{E_r\, I} (R\, z + F^{-2}\, R^3\, z^3) + \frac{d^2 z}{dx^2} - \frac{d^2 z_0}{dx^2} \right] z\, dx = 0 \quad \text{.....(34)}$$

Let z_0 be assumed and z chosen as follows:

$$z_0 = c_0 \sin \frac{\pi\, x}{l} \quad \text{.....(35a)}$$

and

$$z = c \sin \frac{\pi\, x}{l} \quad \text{.....(35b)}$$

and denote,

$$Q = \frac{\pi^2\, E_r\, I}{l^2} \quad \text{.....(36)}$$

Then Eq. 34 gives

$$R\, c + \frac{3}{4} \frac{(R\, c)^3}{F^2} = Q\, (c - c_0) \quad \text{.....(37)}$$

In the course of a test in a laboratory, as the distance between the end blocks is reduced, the maximum deflection c will increase gradually and may serve as a measure of the progress of the test. The end pressure R will be a function of c, dependent on the initial deflection c_0. It is desired to determine the maximum value of R as a function of c, for different definite values of c_0 and l.

It is noted that E_r in Eq. 36 is a function of R; therefore Q in Eqs. 36 and 37 is a function of R. With this in view Eq. 37 shows that in the special case of $c_0 = 0$, the maximum value of R occurs when $c = 0$, and is equal to Q, as it should be in accordance with the theory initiated by Engesser. With $Q = R$ and R chosen, Eq. 36 can be solved for l, thus serving as a formula for the idealized case of an initially straight column.

When the column is curved before loading (that is, when $c_0 > 0$), Q loses its significance as a maximum load and becomes merely a quantity introduced for convenience and defined by an equation. To make R a maximum the condition $\frac{dR}{dc} = 0$ is imposed. Since Q is a function of R, it follows that $\frac{dQ}{dc} = 0$. Under these conditions, by applying the operators $3 - c \frac{d}{dc}$ and $c \frac{d}{dc} - 1$ to Eq. 37, one finds

$$2\, R\, c = Q\, (2\, c - 3\, c_0) \quad \text{.....(38a)}$$

and

$$\frac{3\, (R\, c)^3}{2\, F^2} = Q\, c_0 \quad \text{.....(38b)}$$

respectively. Elimination of c from Eqs. 38 gives

$$c_0 = \frac{4\, F}{9\, Q} \left(\frac{Q}{R} - 1 \right)^{3/2} \quad \text{.....(39)}$$

or, with Q substituted from Eq. 36,

$$c_0 = \frac{4\,F\,l^2}{9\,\pi^2\,E_r\,I}\left(\frac{\pi^2\,E_r\,I}{R\,l^2} - 1\right)^{3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Eq. 40 is not a design formula, but may lend itself to interpretations of tests. If E_r and F are numerically known functions of R, the values of R and l may be chosen, and the greatest initial deflection c_0 permitting these values may be computed by Eq. 40.

If Eq. 32 is replaced by the simpler but probably less plausible formula,

$$\text{Change of curvature} = \frac{1}{E_r\,I}\,(M + F^{-1}\,M^2) \ldots\ldots\ldots\ldots (41)$$

the same procedure of analysis gives, instead of Eq. 40,

$$c_0 = \frac{3\,F\,l^2}{32\,\pi\,E_r\,I}\left(\frac{\pi^2\,E_r\,I}{R\,l^2} - 1\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Buckling of a Thin Circular Elastic Plate with Simply Supported Edge Under a Uniform Pressure at the Edge

A solution of this problem was indicated by G. H. Bryan[17] in 1891. The differential equation for the slope in the radial direction is solved by a Bessel function of order one. When Poisson's ratio is 0.3, this exact solution gives the following value of the critical pressure per unit of length:

$$R = \frac{4.62\,E\,I}{a^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

in which: I = moment of inertia of the cross section per unit of width; and a = radius of the plate.

The application of the method of complementary energy to this problem is not more convenient than the use of Bessel functions, but it will serve the purpose of an illustration. The procedure can be used in more complicated related problems, involving, for example, a varying thickness.

The following additional notation is used:

r = radial distance from the center of the plate.

m_r = bending moment in the radial direction per unit of width of the circumferential section on which it acts.

m_θ = bending moment in the circumferential direction per unit of width of the radial section on which it acts; both m_r and m_θ are measurable in inch-pounds per inch, or, in pounds.

z = starting deflection.

ζ = supplementary deflection, making the final deflection $z + \zeta$.

c = z at center.

$f = f(r)$ = stress function, measurable in inch-pounds.

X = constant to be determined.

μ = Poisson's ratio.

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[17] *Proceedings*, London Mathematical Soc., Vol. 22, 1891, p. 54; also, "Theory of Elastic Stability," by S. Timoshenko, 1936, pp. 367–369.

It is permissible to assume that z, ζ, and f are functions of r only. When z is chosen so that the values of ζ will be insignificant relative to the values of z, the bending moments can be expressed by the formulas,

$$m_r = R\,z + \frac{f}{r} \quad \text{(44a)}$$

and

$$m_\theta = R\,z + \frac{df}{dr} \quad \text{(44b)}$$

A simple examination shows that no matter what differentiable function f is chosen, the moments in Eqs. 44 maintain equilibrium. The variation of the state of stress therefore is reduced to the variation of R and f. When the edge is simply supported, $z = 0$ and $f = 0$ at the edge.

As in the study of columns, R is interpreted as a reaction exerted by a strong support in a fixed position. The changes of curvature in the directions of m_r and m_θ, measured from the starting shape to the final shape, are $-\dfrac{d^2\zeta}{dr^2}$ and $-\dfrac{1}{r}\dfrac{d\zeta}{dr}$, respectively. Accordingly the law of least complementary energy takes the form

$$\delta U = \int_0^a 2\,\pi\,r\,dr \left[-\frac{d^2\zeta}{dr^2}\,\delta m_r - \frac{1}{r}\frac{d\zeta}{dr}\,\delta m_\theta \right] = 0 \quad \text{(45)}$$

To obtain an approximate solution the following expressions are chosen:

$$z = c\left(1 - \frac{r^2}{a^2}\right) \quad \text{(46a)}$$

and

$$f = c\,(X - R)\left(r - \frac{r^3}{a^2}\right) \quad \text{(46b)}$$

By Eqs. 44 the corresponding moments are

$$m_r = c\,X\left(1 - \frac{r^2}{a^2}\right) \quad \text{(47a)}$$

and

$$m_\theta = c\,X\left(1 - \frac{3\,r^2}{a^2}\right) + \frac{2\,c\,R\,r^2}{a^2} \quad \text{(47b)}$$

The bending moments define the corresponding changes of curvature:

$$-\frac{d^2\zeta}{dr^2} = \frac{1}{E\,I}\,(m_r - \mu\,m_\theta) + \frac{d^2z}{dr^2}$$

$$= \frac{c}{E\,I}\left\{ X\left[1 - \mu - (1 - 3\,\mu)\,\frac{r^2}{a^2}\right] - \frac{2\,\mu\,R\,r^2}{a^2} \right\} - \frac{2\,c}{a^2} \quad \text{(48)}$$

and

$$-\frac{1}{r}\frac{d\zeta}{dr} = \frac{1}{E\,I}\,(m_\theta - \mu\,m_r) + \frac{1}{r}\frac{dz}{dr}$$

$$= \frac{c}{E\,I}\left\{ X\left[1 - \mu - (3 - \mu)\,\frac{r^2}{a^2}\right] + \frac{2\,R\,r^2}{a^2} \right\} - \frac{2\,c}{a^2} \quad \text{(49)}$$

By substituting from Eqs. 47, 48, and 49 in Eq. 45, and carrying out the integrations one finds

$$\frac{3\,E\,I}{\pi\,a^2\,c^2}\,\delta U = \left[\,4\,X - (3+\mu)\,R\,\right]\delta X$$

$$+\left[\,-\,(3+\mu)\,X + 4\,R - \frac{6\,E\,I}{a^2}\,\right]\delta R = 0 \quad \text{(50)}$$

Eq. 50 gives

$$X = \frac{3+\mu}{4}\,R \quad \text{(51}a\text{)}$$

and

$$R = \frac{24\,E\,I}{(7+\mu)\,(1-\mu)\,a^2} \quad \text{(51}b\text{)}$$

With $\mu = 0.3$, Eqs. 51 give $R = 4.70\,\frac{E\,I}{a^2}$ which is only 1.7% greater than the correct value in Eq. 43.

By the choice,

$$z = c\cos\frac{\pi\,r}{2\,a} \quad \text{(52}a\text{)}$$

and

$$f = c\,(X - R)\,r\cos\frac{\pi\,r}{2\,a} \quad \text{(52}b\text{)}$$

instead of Eqs. 46, the slightly better value $R = 4.66\,\frac{E\,I}{a^2}$ was obtained with $\mu = 0.3$.

APPLICATION TO VIBRATION OF BEAMS

If an elastic beam vibrates freely in one of its modes, the deflections at the time t may be stated as

$$\eta = (z + y)\cos\omega\,t \quad \text{(53)}$$

in which ω is a constant defining the period as $\frac{2\,\pi}{\omega}$, and z and y are functions of the distance x measured along the beam. The function z will be chosen so that the values of y are relatively small. Then z may be interpreted as a starting deflection and y as a supplementary deflection, both referring to the times when $\cos\omega\,t = 1$.

Let X denote the bending moment at any point due to an imagined static load defined as the product of z times the weight w per unit of length; w may be constant or a function of the distance x; X will be a function of x and will be measurable in lb-in.^{2} When $\cos\omega\,t = 1$, the inertia force per unit of length is $\frac{\omega^2}{g}\,w\,(z + y)$, in which g is the acceleration due to gravity; $\frac{w}{g}$ is the mass per unit of length. When the contributions from y can be ignored, the bending moment due to the inertia forces becomes

$$M = \frac{\omega^2}{g}\,X \quad \text{(54)}$$

The use of the inertia forces converts the problem into one of statics. The quantity $\frac{\omega^2}{g}$ now may be interpreted as an adjustable reaction, playing the same rôle as R in the application to columns. Then

$$\delta M = X\,\delta\left(\frac{\omega^2}{g}\right) \qquad (55)$$

and the variation of the complementary energy becomes

$$\delta U = \int\left(\frac{M}{E\,I} + \frac{d^2z}{dx^2}\right) dx\,\delta M = \delta\left(\frac{\omega^2}{g}\right)\int\left(\frac{\omega^2 X}{E\,I\,g} + \frac{d^2z}{dx^2}\right) X\,dx = 0 \qquad (56)$$

Eq. 56 gives

$$\omega^2 = -\frac{\int X\,\frac{d^2z}{dx^2}\,dx}{\int \frac{X^2\,dx}{E\,I\,g}} \qquad (57)$$

from which the period may be computed as $\frac{2\,\pi}{\omega}$.

It is of interest to compare Eq. 57 with the next equation, which represents Rayleigh's[18] method. The left side of the equation is twice the kinetic energy when $\cos\omega\,t = 0$; the right side is twice the internal potential energy when $\cos\omega\,t = 1$; and either side is twice the total mechanical energy:

$$\omega^2\int\frac{w}{g}\,z^2\,dx = \int E\,I\left(\frac{d^2z}{dx^2}\right)^2 dx \qquad (58)$$

When the deflection z is stated approximately, Eq. 58 gives an approximate value of ω^2. The example that follows will show that Eq. 58 requires a more accurate statement of z than does Eq. 57 for the same degree of approximation in the value of ω.

A cantilever of length l, with a fixed support at $x = 0$, and with w and $E\,I$ constant, is chosen as example. An exact solution[19] of the proper differential equation gives an expression for the extreme deflections $z + y$ in terms of two hyperbolic and two trigonometric functions. Thereby the value of ω in the first mode of vibration is found to be

$$\omega = 3.516\,\omega_0 \qquad (59)$$

in which ω_0 denotes the quantity

$$\omega_0 = \sqrt{\frac{E\,I\,g}{w\,l^4}} \qquad (60)$$

In an approximate solution by the method of complementary energy the starting curve may be taken as a parabola,

$$z = \frac{c\,x^2}{l^2} \qquad (61)$$

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[18] "Theory of Sound," by Lord Rayleigh, 1877, 2d Ed., MacMillan & Co., Vol. 1, 1894, pp. 109 and 257; see also "Vibration Problems in Engineering," by S. Timoshenko, Van Nostrand Co., 1928, pp. 55–60.

[19] See "Vibration Problems in Engineering," by S. Timoshenko, Van Nostrand Co., 1928, p. 234.

Then

$$X = -\frac{w\,l^2\,c}{12}\left[\left(\frac{x}{l}\right)^4 - \frac{4\,x}{l} + 3\right] \dots\dots\dots\dots (62)$$

When these functions are substituted in Eq. 57, one finds

$$\omega = \omega_0\sqrt{\frac{162}{13}} = 3.530\,\omega_0 \dots\dots\dots\dots (63)$$

which differs only by 0.4% from the correct value in Eq. 59.

Exactly the same result can be obtained by Rayleigh's method, with Eq. 58, but the function z must be chosen much closer to the true deflections; the function z equal to the deflection under a uniform load will serve the purpose.[20] If z were taken from Eq. 61, Eq. 58 would give $\omega = \omega_0\,\sqrt{20} = 4.47\,\omega_0$, which is 27% too great.

If a straight line, $z = \frac{c\,x}{l}$, is chosen as starting shape instead of the parabola in Eq. 61, the method of complementary energy gives $\omega = \omega_0\sqrt{\frac{140}{11}} = 3.57\,\omega_0$. Rayleigh's method gives the same result when z is chosen as the deflection due to a concentrated load at the free end.[21]

Stabilizing Loads

When a load producing buckling is reversed, it becomes a stabilizing load; but it can still be studied by the theory of buckling. The general theorems which are needed in subsequent applications of the method of complementary energy are derived most conveniently by considering the energy as a function of the shape. The derivations will be shown in brief form.[22]

It is assumed that any shape of the structure that is geometrically possible, when continuity is maintained, can be defined by assigning a definite set of values to a set of parameters $u_0, u_1, u_2, \cdots u_n, \cdots$. It is assumed that these parameters define any internal deformation by a linear equation of the form

$$D = D_0\,u_0 + D_1\,u_1 + D_2\,u_2 + \cdots + D_n\,U_n + \cdots = \sum_{0,1,2,\dots}^{n} D_n\,u_n \dots (64)$$

Besides, it is assumed that each member obeys Hooke's law. Then each stress will be a linear function of the parameters, and the strain energy will be a quadratic function of the parameters.

Two loads, P and Q, are considered. It is assumed that the path p of P can be stated adequately as a linear function of the parameters,

$$p = \sum_{1,2,\dots}^{n} p_n\,u_n \dots\dots\dots\dots (65)$$

but that a quadratic function of the parameters is required for an adequate statement of the path q of Q. It will be assumed that a special choice of parameters has made it possible to state the path q as

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[20] "Vibration Problems in Engineering," by S. Timoshenko, Van Nostrand Co., 1928, p. 59.

[21] *Loc. cit.*, p. 58.

[22] A fuller account of the theory represented by Eqs. 64 to 73 and 78 to 80 and explanations of the special terminology are given in the paper, "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 576, especially pp. 604–637.

$$q = -q_0 u_0 - \tfrac{1}{2} \sum_{1,2,\ldots}^{n} q_n u_n^2 \ldots\ldots\ldots\ldots\ldots\ldots (66)$$

and the strain energy as

$$V = \tfrac{1}{2} V_0 u_0^2 + \tfrac{1}{2} \sum_{1,2,\ldots}^{n} Q_n q_n u_n^2 \ldots\ldots\ldots\ldots\ldots (67)$$

with all the coefficients q_0, q_n, V_0, and Q_n positive and all of the values Q_n different from one another. That this combination of relations is not only possible, but typical of buckling and stabilizing loads, will be brought out by the discussion that follows.

With Eqs. 65 to 67 accepted, the principle of minimum of the potential energy by variation of the shape takes the form,

$$T = V - Q\,q - P\,p = \tfrac{1}{2} V_0 u_0^2 + \tfrac{1}{2} \sum_{1,2,\ldots}^{n} Q_n q_n u_n^2$$

$$+ Q \left(q_0 u_0 + \tfrac{1}{2} \sum_{1,2,\ldots}^{n} q_n u_n^2 \right) - P \sum_{1,2,\ldots}^{n} p_n u_n \ldots\ldots\ldots (68)$$

$$\delta T = 0, \quad \text{or,} \quad \frac{\partial T}{\partial u_n} = 0 \qquad \text{for } n = 0, 1, 2, \cdots \ldots\ldots\ldots (69)$$

Eqs. 68 and 69 will be applied to investigate a series of four actions, in which u_n will be denoted successively u_n, $\bar{u}_n$, $\bar{\bar{u}}_n$, and u_n. Table 1 will serve as a summary of the results.

TABLE 1.—SUMMARY OF THEORY OF BUCKLING AND STABILIZING LOADS

Action	Definition	Value of parameter u_n for $n = 1, 2, 3, \cdots$	Equation No.
Astatic	$P = 0, Q = -Q_n$	u_n = any value, others $= 0$	71
Orthostatic	$P \neq 0, Q = 0$	$\bar{u}_n = \dfrac{P\,p_n}{Q_n\,q_n} = \dfrac{Q}{Q_n}\,\bar{\bar{u}}_n$	73, 77
Relaxed	$P \neq 0, Q > 0$, V reduced	$\bar{\bar{u}}_n = \dfrac{P\,p_n}{Q\,q_n} = \dfrac{Q_n}{Q}\,\bar{u}_n$	77, 73
Heterostatic	$P \neq 0, Q \neq 0$, V restored	$u_n = \dfrac{Q_n}{Q_n + Q}\,\bar{u}_n = \dfrac{Q}{Q_n + Q}\,\bar{\bar{u}}_n$	80, 81

Astatic Action.—The first action is defined by

$$P = 0, \qquad Q = -Q_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

Inspection of Eq. 68 shows that $\delta T = 0$ when

$$u_0 = \frac{Q_n q_0}{V_0}, \quad u_n = \text{any value, all other parameters } u_m = 0 \ldots\ldots (71)$$

Since the parameter u_n may pass through a continuous range of values, defining a continuous range of shapes of the structure, the equilibrium is called neutral, and the action is described as buckling under the critical load $-Q = Q_n$. Since all other parameters remain zero except u_0, this buckling is called pure buckling or astatic action. The load $-Q$ is called an astatic load; the parameters $u_1, u_2, \cdots u_n, \cdots$ are called astatic parameters; and u_0 is called the orthostatic parameter of the astatic action. If T is a maximum by variation of some of the parameters, the equilibrium is unstable, though neutral by variation of u_n. Further examination[23] of Eq. 68 shows that if the quadratic expressions for V and q are written with mixed terms included, such as $V_{mn} u_m u_n$ and $q_{mn} u_m u_n$, knowledge of the existence of two astatic actions, one with $u_m =$ any value, $u_n = 0$ and the other with $u_m = 0$, $u_n =$ any value, at two different values of the astatic load, leads to the conclusion that $V_{mn} = q_{mn} = 0$. It follows that if the states of pure buckling can be ascertained, q and V can be written in the relatively simple forms of Eqs. 66 and 67; and thereby the nature of the astatic parameters is also ascertained.

Orthostatic Action.—The second action to be examined is defined by

$$P \neq 0, \qquad Q = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

Eqs. 68 and 69 give $u_0 = 0$ and the following value of u_n for $n = 1, 2, \cdots$:

$$\bar{u}_n = \frac{P\, p_n}{Q_n\, q_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

The parameters are proportional to the load P; the same applies to the deformations and stresses, which are linear functions of the parameters. No buckling is involved; therefore this is the usual action dealt with in structural mechanics. In the terminology of the theory of buckling, this action, in which the astatic load is absent, is called orthostatic action. The load P is called the orthostatic load. Eq. 73 defines the values of the astatic parameters in the orthostatic action.

Relaxed Action.—In the third action to be examined the structure is assumed to have been modified by a relaxation of stiffness. The strain energy V in Eq. 67 is reduced in the relaxed structure by removing all terms except the first, so that now

$$V = \tfrac{1}{2} V_0\, u_0^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

It will be assumed that

$$P \neq 0, \qquad Q > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

When the proper terms are omitted in Eq. 68 one finds

$$u_0 = -\frac{Q\, q_0}{V_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

and the following value of u_n for $n = 1, 2, \cdots$:

$$\bar{\bar{u}}_n = \frac{P\, p_n}{Q\, q_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

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[23] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 576, especially pp. 611–612.

Eq. 77 defines the values of the astatic parameters in the relaxed action. The equilibrium is stable, and Q is a stabilizing load.

Heterostatic Action.—In the fourth and last action to be considered the structure is assumed to have recovered its stiffness. Both the orthostatic and the astatic loads are present, that is,

$$P \neq 0, \qquad Q \neq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

This general type of action is called heterostatic action; the combined load P, Q is a heterostatic load, with the orthostatic component P and the astatic component Q. By Eqs. 68 and 69 one finds u_0 as in Eq. 76, and for $n = 1, 2, 3, \cdots$:

$$u_n = \frac{P\,p_n}{(Q_n + Q)\,q_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

A comparison of Eq. 79 with Eqs. 73 and 77 shows that

$$\frac{u_n}{\bar{u}_n} = \frac{Q_n}{Q_n + Q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

and

$$\frac{u_n}{\bar{\bar{u}}_n} = \frac{Q}{Q_n + Q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

When Q is positive, Eq. 80 defines the reduction factor of the astatic parameter u_n by transition from the orthostatic action to the heterostatic action under the influence of the stabilizing load Q; and Eq. 81 defines the reduction factor of the same parameter by transition from the relaxed action to the heterostatic action by restoration of the stiffness. When Q is negative, the factors in Eqs. 80 and 81 become magnification factors. It may be noted in passing that $\frac{Q}{Q_n}$ is the transition factor $\frac{\bar{u}_n}{\bar{\bar{u}}_n}$ by change from the relaxed to the orthostatic action.

Example.—A slender horizontal tension member bending under its own weight will serve as illustration. The ends are assumed to be hinged, and the end load central. The total horizontal tension is the stabilizing load Q. The weight, w per unit of length, is the orthostatic load. Assume that the cross section is constant, with area A and moment of inertia I. Let E = modulus of elasticity, l = length, x = distance measured from one end. Then the nth critical value of the astatic load $-Q$ is defined by Euler's formula

$$Q_n = \frac{n^2\,\pi^2\,E\,I}{l^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

and the deflections maintained by it are

$$y = u_n \sin \frac{n\,\pi\,x}{l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

in which u_n is a proper astatic parameter. As orthostatic parameter of the orthostatic action one may introduce the total shortening of the center line,

$u_0 = -\dfrac{Q\,l}{E\,A}$, which makes $q_0 = 1$. The parameter u_0 presents no difficulties in this problem.

In the relaxed action the member is changed into a cable, which deflects according to the law

$$-\frac{d^2y}{dx^2} = \frac{w}{Q} \qquad \text{(84)}$$

Eq. 84 may be rewritten so that the right side expresses a sum of corresponding effects in astatic actions, as follows:[24]

$$-\frac{d^2y}{dx^2} = \frac{4\,w}{\pi\,Q} \sum_{1,3,\ldots}^{n} \frac{1}{n} \sin \frac{n\,\pi\,x}{l} \qquad \text{(85)}$$

When the stiffness is restored, each term in the summation in Eq. 85 is multiplied by a reduction factor defined by Eqs. 81 and 82. After multiplying by $-E\,I$, one obtains in this way the bending moments in the heterostatic action,

$$M = \frac{4\,w\,l^2}{\pi^3} \sum_{1,3,\ldots}^{n} \frac{Q_1}{n\,(Q_n + Q)} \sin \frac{n\,\pi\,x}{l} \qquad \text{(86)}$$

The series in Eq. 86 converges and defines the stresses.

Eq. 86 may also be derived by way of the orthostatic action. When $Q = 0$,

$$\frac{d^4y}{dx^4} = \frac{w}{E\,I} = \frac{4\,w}{\pi\,E\,I} \sum_{1,3,\ldots}^{n} \frac{1}{n} \sin \frac{n\,\pi\,x}{l} \qquad \text{(87)}$$

After two integrations one finds the bending moment in the orthostatic action

$$\bar{M} = \frac{4\,w\,l^2}{\pi^3} \sum_{1,3,\ldots}^{n} \frac{1}{n^3} \sin \frac{n\,\pi\,x}{l} \qquad \text{(88)}$$

When Q is introduced, and the reduction factors are applied according to Eq. 80, Eq. 86 is reproduced.

The general theory may be applied to the problem in other ways. For example, the theory will furnish the series needed first, such as those in Eqs. 85 and 87; this is important for the more complicated problems in which the corresponding series are not simple trigonometric series. The computations may be made as follows: The contribution of the deflections $y = u_n \sin \dfrac{n\,\pi\,x}{l}$ to the path of the distributed load w (which takes the place of P in Eq. 68) is

$$p = p_n\,u_n = \int_0^l y\,dx = \frac{2\,l\,u_n}{n\,\pi} \qquad \text{(89)}$$

if n is uneven, and zero if n is even; and the corresponding contribution to the path of Q is

$$q = -\tfrac{1}{2}\,q_n\,u_n^2 = -\tfrac{1}{2} \int_0^l \left(\frac{dy}{dx}\right)^2 dx = -\frac{n^2\,\pi^2\,u_n^2}{4\,l} \qquad \text{(90)}$$

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[24] See "A Short Table of Integrals," by B. O. Peirce, 2d Ed., Ginn & Co., 1910, formula 808.

The last two equations define p_n and q_n directly, and thereafter $\bar{u}_n$ through Eqs. 73 and 82. Thus one finds the deflections in the orthostatic action

$$y = \sum_{1,3,\ldots}^{n} \bar{u}_n \sin \frac{n \pi x}{l} = \frac{4 w l^4}{\pi^5 E I} \sum_{1,3,\ldots}^{n} n^{-5} \sin \frac{n \pi x}{l} \ldots\ldots\ldots (91)$$

The deflections in the heterostatic action are obtained by applying the reduction factor in Eq. 80 to each term in Eq. 91. Eq. 86 is reproduced easily by this method. Another route to the same result is to determine $\bar{\bar{u}}_n$ by Eq. 77, and thereby the deflections in the relaxed action; from these one finds the deflections in the heterostatic action by applying the reduction factor in Eq. 81 to each term in the series.

The method of complementary energy is applicable to problems of stabilizing loads through its applicability to astatic actions.

STIFFENING OF SUSPENSION BRIDGES

That a suspension bridge is stiffened by its own weight was brought out in an analysis published by Joseph Melan[25] in 1888. Leon S. Moisseiff, M. Am. Soc. C. E., developed the theory further for the design of the Manhattan Bridge (1909); and F. E. Turneaure,[26] Hon. M. Am. Soc. C. E., investigated this application and contributed to, and presented, the theory. Since then this principle of stiffening has become well appreciated in America; it has been utilized in the design of the great bridges, and has been discussed in a notable American technical literature on the subject.[25, 26, 27] The dead load has the character of a stabilizing load.

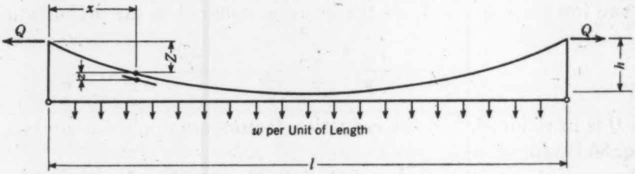

FIG. 4.—MAIN SPAN OF SUSPENSION BRIDGE

It is not intended here to present a full analysis of the stiffening of suspension bridges. It is desired only to show the applicability of the method of complementary energy to this problem through the theory of stabilizing loads and buckling.[28] The analysis therefore is limited to a simplified case, which is suggested in Fig. 4. The following actions are left out of account, as matters that can either be incorporated in a more extensive analysis by the same principles or be referred to independent supplementary analyses: elongations of the cable beyond the initial stretching; movements of the points of the cable at the tops of the towers; and elongations of the suspenders beyond the initial stretching. The suspenders are assumed to be spaced closely. The initial curve of the cable is assumed to be a parabola with a sag that is fairly small compared with the span. The following notation is used:

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[25] "Theorie der eisernen Bogenbrücken und Hängebrücken," by Joseph Melan, *Handbuch der Ingenieurwissenschaften*, Vol. 2, Subvolume 4, Leipzig, 1888, pp. 1–144, especially p. 38 (English translation of 3d Ed. (1906) entitled "Theory of Arches and Suspension Bridges," by D. B. Steinman, M. Am. Soc. C. E., The M. C. Clark Publishing Co., 1913).

[26] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., Vol. 2, 9th Ed., 1911, pp. 276–318.

[27] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, John Wiley & Sons Co., Inc., 1929, pp. 246–282; Leon S. Moisseiff, *Journal of the Franklin Institute*, October, 1925. *Transactions*, Am. Soc. C. E., Vol. 91 (1927), J. A. L. Waddell, pp. 884–910, especially pp. 893–895; Vol. 94 (1930), S. Timoshenko, pp. 377–391; Vol. 97 (1933), O. H. Ammann, pp. 1–65, especially pp. 39–44; Vol. 100 (1935), D. B. Steinman, pp. 1133–1170; and Vol. 104 (1939), Shortridge Hardesty and Harold E. Wessman, pp. 579–608.

[28] The application of the theory of buckling of "the suspension bridge upside-down" to suspension bridges was suggested by H. M. Westergaard in a discussion, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1021.

x = horizontal coordinate.

Z = initial vertical coordinate, positive downward, of any point of the cable.

z = addition to Z by the change from the initial shape to the starting shape; with the restrictions imposed, z is also the starting deflection of the stiffening truss.

y = addition to z by the change from the starting shape to the final shape.

Y = total deflection, addition to Z, in a relaxed action in which neither the cable nor the truss contributes stiffness against bending.

η = total deflection, addition to Z, in the heterostatic action.

l = span.

$h = Z_{\max}$.

Q = horizontal component of the total tension in the cable; Q is interpreted as a measure of the stabilizing load.

Q_n = critical value of $-Q$ in an unstable astatic action.

w = uniformly distributed vertical load per unit of length; interpreted in this analysis as a reaction adjusting itself to Q as if it were a hydrostatic pressure.

P = orthostatic load.

K = force defined by Eq. 95.

M = sum of the bending moments in the stiffening truss and the cable at any value of x.

EI = measure of the combined stiffness of the stiffening truss and the cable against bending.

k = ratio defined by Eq. 117.

The initial parabolic curve of the cable has the equation

$$Z = 4\,h\,l^{-2}\,(l\,x - x^2) \quad \text{.....................(92)}$$

If the shape is changed by infinitesimal increments δZ, the potential energy of the uniformly distributed load w will be changed by the amount $-\,w \int_0^l \delta Z\,dx$. This amount is infinitesimal of second order because it is the total change of energy if the cable is unstiffened, and the parabola is the curve of equilibrium of the unstiffened cable. Therefore, when small quantities of second order, which will be without importance in the subsequent computations, are ignored, and under the simplifying restrictions that were imposed, and as long as the deflections z, y, Y, and η remain small, it can be asserted that

$$\int^l z\,dx = \int^l y\,dx = \int_0^l Y\,dx = \int_0^l \eta\,dx = 0 \quad \text{.........(93)}$$

The first task is to investigate the astatic actions at critical negative values of the loads in Fig. 4. Under the loads in Fig. 4, when the deflections are z, the combined bending moment in the cable and the truss is

$$M = \tfrac{1}{2} w (l x - x^2) - 4 Q h l^{-2} (l x - x^2) - Q z \quad \text{(94)}$$

It is expedient to introduce a force K defined by the equation

$$w = 8 (Q + K) h l^{-2} \quad \text{(95)}$$

Then

$$M = - Q z + 4 K h l^{-2} (l x - x^2) \quad \text{(96)}$$

and the increment of M by variation of the state of stress is

$$\delta M = - z \, \delta Q + 4 h l^{-2} (l x - x^2) \, \delta K \quad \text{(97)}$$

When z is chosen so that the supplementary deflections y are small, the relation of deformations to stresses

$$- \frac{d^2 y}{dx^2} = \frac{M}{E I} + \frac{d^2 z}{dx^2} \quad \text{(98)}$$

may be used with M as in Eq. 96.

The variation of the complementary energy may now be written:

$$\delta U = \int_0^l \left(\frac{M}{E I} + \frac{d^2 z}{dx^2} \right) dx \, \delta M$$

$$= \frac{1}{E I} \int_0^l \Big[- Q z + 4 K h l^{-2} (l x - x^2)$$

$$+ E I \frac{d^2 z}{dx^2} \Big] \Big[- z \, \delta Q + 4 h l^{-2} (l x - x^2) \, \delta K \Big] dx = 0 \quad \text{(99)}$$

This equation is converted into the following two, in which all integrations are from 0 to l:

$$E I \frac{\partial U}{\partial Q} = Q \int z^2 \, dx - 4 K h l^{-2} \int (l x - x^2) z \, dx$$

$$- E I \int z \frac{d^2 z}{dx^2} dx = 0 \quad \text{(100)}$$

$$E I \frac{\partial U}{\partial K} = - 4 Q h l^{-2} \int (l x - x^2) z \, dx + 16 K h^2 l^{-4} \int (l x - x^2)^2 \, dx$$

$$+ 4 E I h l^{-2} \int (l x - x^2) \frac{d^2 z}{dx^2} dx = 0 \quad \text{(101)}$$

Eqs. 100 and 101 are satisfied by the trivial solution $z = 0$, $K = 0$, $Q =$ any value. Non-trivial solutions exist, however. To find them, the following forms of z are suitable:

$$z = z_n \sin \frac{n \pi x}{l} \qquad \text{for } n = 2, 4, 6, \cdots \quad \text{(102)}$$

and

$$z = z_n \left(\sin \frac{n \pi x}{l} - \frac{1}{n} \sin \frac{\pi x}{l} \right) \qquad \text{for } n = 3, 5, 7, \cdots \ldots \ldots (103)$$

Both satisfy the requirement in Eqs. 93.

With z as in Eq. 102 the solution is

$$K = 0, \qquad - Q = Q_n = n^2 \pi^2 \frac{E I}{l^2}, \qquad \text{for } n = 2, 4, \cdots \ldots \ldots (104)$$

This result could have been found more easily, without invoking the method of complementary energy, by noting that Eq. 102 is the solution of Eq. 98 with $y = 0$, and with K and Q as in Eq. 104.

It is when the diagram of deflections is symmetrical, as by Eq. 103, that the method of complementary energy shows merit; because Eq. 103 will be close, although not identical, to an exact solution of Eq. 98 with $y = 0$. With z as in Eq. 103, Eqs. 100 and 101 become, after multiplication by $\frac{2}{l}$:

$$(1 + n^{-2})\, z_n^2\, Q + 32\, (n^{-1} - n^{-3})\, \pi^{-3}\, h\, z_n\, K$$
$$+ (n^2 + n^{-2})\, \pi^2\, E\, I\, z_n^2\, l^{-2} = 0 \ldots \ldots \ldots \ldots \ldots (105)$$

and

$$32\, (n^{-1} - n^{-3})\, \pi^{-3}\, h\, z_n\, Q + \frac{16}{15} h^2\, K = 0 \ldots \ldots \ldots \ldots (106)$$

Eq. 106 shows that K is proportional to Q but also proportional to the small ratio $\frac{z_n}{h}$; therefore K is a small supplementary force, although its influence may not be small. Elimination of K gives

$$[1 + n^{-2} - 960\, (n^{-1} - n^{-3})^2\, \pi^{-6}]\, Q = -\, (n^2 + n^{-2})\, \pi^2\, E\, I\, l^{-2} \ldots (107)$$

Since π^6 is very close to 960, and n^{-6} will be insignificant, the left side of Eq. 107 can be restated as $(1 + 2\, n^{-4})\, Q$. A further simplification, representing no significant loss of accuracy, reduces the solution to the form

$$- Q = Q_n = (n^2 - n^{-2})\, \pi^2 \frac{E I}{l^2}, \qquad \text{for } n = 3, 5, 7, \cdots \ldots \ldots (108)$$

Exact values of Q_n for $n = 3, 5, 7, \cdots$ can be found by solving Eq. 98 with $y = 0$ and M as in Eq. 94. It is found that these values are defined by the equations

$$\tan \alpha = \alpha + \frac{\alpha^3}{3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (109a)$$

and

$$Q_n = 4\, \alpha^2 \frac{E I}{l^2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots (109b)$$

Furthermore, it is found that Eq. 108 gives a very close approximation to these exact values.

This completes the study of the astatic actions, but the goal is the heterostatic action. This goal may be reached either by the route of the orthostatic action or by the route of the relaxed action, in which all stiffness identified with elasticity is removed. The latter route is chosen.

The stabilizing load Q consists of the two horizontal forces Q in Fig. 4 and the distributed load w which is proportional to Q. An added vertical load is an orthostatic load if it requires no change of Q in the relaxed action. Let M_0 denote the bending moments that such a load would produce in a simple beam replacing the bridge. Simple considerations of statics show that in the relaxed action this load produces the deflections

$$Y = \frac{M_0}{Q} \dots\dots\dots\dots\dots\dots\dots\dots (110)$$

By referring to Eqs. 93 one obtains the condition

$$\int_0^l M_0 \, dx = 0 \dots\dots\dots\dots\dots\dots\dots\dots (111)$$

The loads in Fig. 5 are ascertained as two examples of orthostatic loads by noting that the moments which they would produce in a simple beam satisfy Eq. 111. It is observed further that any vertical load can be resolved into two component parts: one, an orthostatic load with moments satisfying Eq. 111; and the other a uniformly distributed load contributing to the astatic load Q. Therefore the function Y can be determined readily for any vertical load.

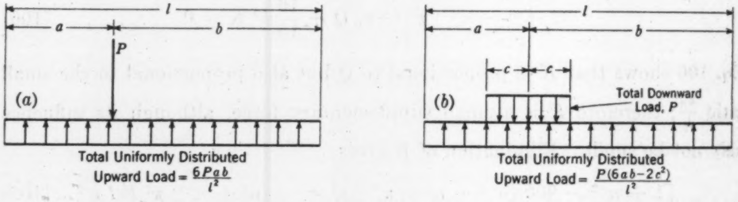
l
a
b
P
(a)
Total Uniformly Distributed
Upward Load = 6Pab/l²
l
a
b
c
c
(b)
Total Downward
Load, P
Total Uniformly Distributed
Upward Load = P(6ab−2c²)/l²

FIG. 5.—TWO EXAMPLES OF ORTHOSTATIC LOADS ON SUSPENSION BRIDGE

The next step is to express Y as a sum of functions representing deflections in astatic actions. The functions in Eqs. 102 and 103 are sufficiently close to the exact functions to serve this purpose. A suitable procedure is to expand the function Y into a Fourier series of the form

$$Y = \sum_{1,2,3,\dots}^{n} Y_n \sin \frac{n \pi x}{l} \dots\dots\dots\dots\dots\dots (112)$$

with the coefficients

$$Y_n = \frac{2}{l} \int_0^l Y \sin \frac{n \pi x}{l} \, dx \dots\dots\dots\dots\dots (113)$$

It may be convenient instead to begin by expressing the orthostatic load by a Fourier series, and then integrating twice to obtain M_0. In either case the series in Eq. 112 is obtained by a straight-forward and well-established process.

It is assured in advance that $\int_0^l Y\,dx = 0$. It follows that

$$Y_1 = -\sum_{3,\,5,\,\ldots}^{n} n^{-1}\,Y_n \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(114)$$

Then Eq. 112 may be rewritten in the desired form

$$Y = \sum_{2,\,4,\,\ldots}^{n} Y_n \sin\frac{n\,\pi\,x}{l} + \sum_{3,\,5,\,\ldots}^{n} Y_n\left(\sin\frac{n\,\pi\,x}{l} - n^{-1}\sin\frac{\pi\,x}{l}\right)\ldots(115)$$

The transition from the relaxed to the heterostatic action is made by applying the reduction factor in Eq. 81 to each term in Eq. 115, with Q_n defined by Eqs. 104 and 108. This reduction produces the deflections in the heterostatic action,

$$\eta = \sum_{2,\,4,\,\ldots}^{n} \frac{Q}{Q_n + Q}\,Y_n \sin\frac{n\,\pi\,x}{l}$$

$$+ \sum_{3,\,5,\,\ldots}^{n} \frac{Q}{Q_n + Q}\,Y_n\left(\sin\frac{n\,\pi\,x}{l} - \frac{1}{n}\sin\frac{\pi\,x}{l}\right)\ \ldots\ldots\ldots(116)$$

By introducing the notation

$$k = \frac{Q\,l^2}{\pi^2\,E\,I}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(117)$$

and referring to Eqs. 104 and 108, which define the critical values Q_n of $-Q$, the final deflection, η in Eq. 116, can be restated in either of the following two forms:

$$\eta = -\sin\frac{\pi\,x}{l}\sum_{3,\,5,\,\ldots}^{n} \frac{k}{n(n^2 - n^{-2} + k)}\,Y_n$$

$$+ \sum_{2,\,4,\,\ldots}^{n} \frac{k}{n^2 + k}\,Y_n \sin\frac{n\,\pi\,x}{l} + \sum_{3,\,5,\,\ldots}^{n} \frac{k}{n^2 - n^{-2} + k}\,Y_n \sin\frac{n\,\pi\,x}{l}\ldots(118)$$

or,

$$\eta = Y + \sin\frac{\pi\,x}{l}\sum_{3,\,5,\,\ldots}^{n} \frac{n^2 - n^{-2}}{n(n^2 - n^{-2} + k)}\,Y_n$$

$$- \sum_{2,\,4,\,\ldots}^{n} \frac{n^2}{n^2 + k}\,Y_n \sin\frac{n\,\pi\,x}{l} - \sum_{3,\,5,\,\ldots}^{n} \frac{n^2 - n^{-2}}{n^2 - n^{-2} + k}\,Y_n \sin\frac{n\,\pi\,x}{l}\ldots(119)$$

After a certain value of n the series in Eq. 118 converge more rapidly than those in Eq. 119. For numerical computation the choice between the two formulas depends on the values of k and Y_n and on the accuracy that is desired.

INFLUENCE OF STABILIZING LOADS ON VIBRATIONS

A stabilizing load stiffens a structure. Therefore it will tend to reduce the periods of free vibration.

A mode of free vibration is characterized by the condition that all the significant deformations are proportional at each instant to a single parameter v which varies with the time t according to the law $v = u \cos \omega_n t$; u may have any value, is independent of t, and is a parameter corresponding to v but defining the extreme deformations.

It happens in some significant cases that the parameter u can be taken as any one of the astatic parameters u_n in Eq. 68. Then, by referring to Eq. 68, it will be seen that the part of the potential energy that varies during the vibration is $\frac{1}{2}(Q_n + Q)\, u_n^2 \cos^2 \omega_n t$. Since the velocities are proportional to $\omega_n u_n \sin \omega_n t$, the kinetic energy can be written as $\frac{1}{2} K_n \omega_n^2 u_n^2 \sin^2 \omega_n t$. Constancy of the combined potential and kinetic energy requires that

$$K_n \omega_n^2 = Q_n + Q \qquad (120)$$

If the stabilizing load Q is removed without changing the masses, ω_n and the corresponding period $t_n = \dfrac{2\pi}{\omega_n}$ will assume different values ω_{0n} and t_{0n}. Eq. 120 shows that the relations of the values are

$$\frac{t_n^2}{t_{0n}^2} = \frac{\omega_{0n}^2}{\omega_n^2} = \frac{Q_n}{Q_n + Q} \qquad (121)$$

That is, in the transition from a free vibration without the stabilizing load to one with the stabilizing load a reduction factor is applied to the square of the period equal to the reduction factor in Eq. 80, representing the transition from the orthostatic action to the heterostatic action under the stabilizing load Q.

FREE VIBRATIONS OF A SUSPENSION BRIDGE

While the astatic actions of a suspension bridge may be visualized by imagining gravity reversed or the bridge turned upside down, correspondingly, the free vibrations without stabilizing loads may be visualized by imagining gravity removed temporarily or the bridge merely keeled over so that cable and stiffening truss are placed in a horizontal plane.

The discussion will be limited again to the simplified case that has been under consideration. Furthermore, only vibrations in the plane of the cable and stiffening truss will be considered; and energies due to components of velocity in the direction of the span will be assumed to be relatively so insignificant that they can be ignored. Then the modes of vibration will be represented adequately by Eqs. 102 and 103. When the procedure leading to Eq. 57 is applied to Eq. 103, one finds that in the modes of $n = 3, 5, 7, \cdots$ the bending moments due to the load $w\,z$ are represented not exactly but with good approximation by the formula

$$X = \frac{w\, u_n\, l^2}{n^2 \pi^2} \left(\sin \frac{n \pi x}{l} - \frac{1}{n} \sin \frac{\pi x}{l} \right) \qquad (122)$$

By referring to Eq. 57, one finds, for $n = 3, 5, 7, \cdots$,

$$\omega_{0n}^2 = -\frac{\displaystyle\int X \frac{d^2 z}{dx^2}\, dx}{\displaystyle\int \frac{X^2\, dx}{E\, I\, g}}$$

$$= \frac{n^2\, \pi^4\, E\, I\, g \displaystyle\int_0^l \left(\sin \frac{n\, \pi\, x}{l} - \frac{1}{n} \sin \frac{\pi\, x}{l} \right) \left(n^2 \sin \frac{n\, \pi\, x}{l} - \frac{1}{n} \sin \frac{\pi\, x}{l} \right) dx}{w\, l^4 \displaystyle\int_0^l \left(\sin \frac{n\, \pi\, x}{l} - \frac{1}{n} \sin \frac{\pi\, x}{l} \right)^2 dx}$$

$$= \frac{(n^6 + n^2)\, \pi^4\, E\, I\, g}{(n^2 + 1)\, w\, l^4} \ldots\ldots\ldots\ldots\ldots\ldots (123)$$

or, by substitution from Eq. 108 and with a further permissible approximation,

$$\omega_{0n}^2 = \frac{n^2\, \pi^2\, g}{(1 + n^{-2} - 2\, n^{-4})\, w\, l^2}\, Q_n \qquad \text{for } n = 3, 5, 7, \cdots \; \ldots . (124)$$

A similar but simpler computation based on Eq. 102 gives

$$\omega_{0n}^2 = \frac{n^2\, \pi^2\, g}{w\, l^2}\, Q_n \qquad \text{for } n = 2, 4, 6, \cdots \; \ldots\ldots\ldots (125)$$

When the suspension bridge is again placed in its natural position and gravity contributes stiffness, the values of ω_{0n}^2 in Eqs. 124 and 125 will be increased to ω_n^2 in accordance with Eq. 121 by replacing the factors Q_n by $Q_n + Q$. Thereafter the periods of free vibration are obtained as $t_n = \dfrac{2\,\pi}{\omega_n}$. With the critical loads Q_n substituted from Eqs. 104 and 108, and the ratio k introduced from Eq. 117, the periods of free vibration become

$$t_n = \frac{2\, l^2}{n\, \pi} \sqrt{\frac{w}{(n^2 + k)\, E\, I\, g}} \qquad \text{for } n = 2, 4, 6, \cdots \; \ldots\ldots (126)$$

and

$$t_n = \frac{2\, l^2}{n\, \pi} \sqrt{\frac{(1 + n^{-2} - 2\, n^{-4})\, w}{(n^2 + n^{-2} + k)\, E\, I\, g}} \qquad \text{for } n = 3, 5, 7, \cdots \; \ldots . (127)$$

CONCLUSION

It has been shown that the method of complementary energy can be applied with advantage to a variety of problems in structural statics and dynamics.

DISCUSSION

I. K. SILVERMAN,[29] ASSOC. M. AM. SOC. C. E. (by letter).—The ideas presented in this paper throw a great deal of light on a concept that seems to have become axiomatic in many minds. This concept is one that deals with the economy of Nature, and it is expressed in textbooks dealing with the theory of structures by means of the "Principle of Least Work." Perhaps it is because the engineer, in dealing with forces, has come to have a respect for the way Nature behaves. The behavior of liquids, the action of falling bodies, the structure of organic materials such as bones and feathers,[30] the apparent minimal surfaces that may be obtained with thin films, cells of honeycombs, etc., all seem to be conclusive evidence of the economy of Nature.

It is a far cry from the Ptolemaic concept of the universe to the Principle of Least Work of modern structural analysis, but the conflict between the theological and mechanical concepts of the universe, which raged around the Ptolemaic theory, imparted a metaphysical twinge to the ideas of the philosophers who followed Galileo.

In their studies of the phenomena of Nature the investigators of the seventeenth and eighteenth centuries attempted to explain them from the inherent perfection of the Creator. It is not strange, therefore, to find the concepts of minimum arising from the efforts of the philosophers of this age to explain the universe. So great has been the influence of these precursors of modern science that their "minimum principles" have come down through the centuries to be included in modern textbooks with even the argument of the wisdom of the Creator in calling for economy in Nature. Such is the background of the principle of the minimum in the theory of structures.

Jules Henri Poincaré declared that any generalization which is based on experimental results has no right to be called a principle, and an examination of the assumptions which led to Castigliano's second theorem and thence to what is generally, but incorrectly, called the Principle of Least Work will bear this out. The basic assumptions are as follows, as applied to engineering structures:

1. The deformations produced are infinitesimal and thus it follows that the law of superposition holds; and
2. A linear relation exists between the forces and displacements.

From these two assumptions the energy stored in a body can be expressed as a quadratic in the forces or the displacements. Castigliano's first theorem follows: The partial derivative of the expression for energy containing the forces with respect to any force gives the displacement of the force in the direction it acts; and, for those forces which do not move because of their geometrical or kinematical restraints, the partial derivatives are zero. This latter result can be expressed as a theorem interpreting the vanishing of the derivative as a condition for a minimum. This theorem of "minimum" will not hold for the cases in which (*a*) the law of superposition is no longer true, and (*b*) the material does not follow Hooke's law.

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[29] Care, Panama Canal, Canal Zone.

[30] "The Science of Mechanics," by E. Mach, The Open Court Publishing Co., Chicago, Ill., 1893, p. 452.

These conditions are purely experimental, and in the light of Poincaré's specifications the term "principle" is inadmissible. Most of the materials used in structural engineering may be considered to satisfy the conditions under which the theorem holds, and the results obtained in practise agree essentially with those predicted by the application of the theorem.

The author has introduced a new term, complementary energy, which is certainly not of anthropomorphic origin. It is nothing more than a mathematical expression involving stresses or forces which, upon differentiation with respect to the stress or force, gives the corresponding strain or displacement. When the forces involved are constrained kinematically or geometrically the ensuing derivative is set equal to zero, which is interpreted as a minimum. The author has shown that in the case of Hooke's law "complementary energy" coincides with "elastic energy."

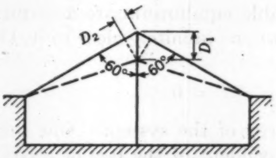

FIG. 6

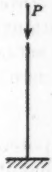

FIG. 7

The only check on the results obtained by using either the "law of least complementary energy" or the "law of least elastic energy" is a geometrical one. In both cases the equations of equilibrium are first applied and when dealing with a statically indeterminate structure a "minimum principle" is used to determine the statically indeterminate quantities. In the last analysis the results obtained can be checked only by noting that the conditions of continuity are preserved. Thus the three solutions for S_1 of Fig. 2(*a*) can be shown to be correct as follows:

From Fig. 6 the deformation D_1 is connected with the deformations D_2 and D_3 by the following equation:

$$D_1 = \frac{D_2}{\cos 60^\circ} = 2\,D_2 \ldots\ldots\ldots\ldots\ldots\ldots (128a)$$

For $P < 3\,C$,

$$D_1 = (P - X)\,k = 2\,k\,X = 2\,D_2; \quad \text{and} \quad X = \frac{P}{3} \ldots\ldots (128b)$$

For $3\,C < P < 5\,C$,

$$D_1 = 2\,k\,(P - X - C) = 2\,k\,X = 2\,D_2; \quad \text{and} \quad X = \frac{P - C}{2} \ldots (128c)$$

and, for $P > 5\,C$,

$$D_1 = 2\,k\,(P - X - C) = 4\,k\,(X - C) = 2\,D_2; \quad \text{and} \quad X = \frac{P + C}{3} \ldots (128d)$$

The fundamental character of a solution based on consistent deflections is apparent; but it must be admitted that a solution based on a "minimum" principle as presented by the author will prove more attractive to analysts because of its economy of thought.

Stability Problems.—The essential feature in the study of stability problems is that the structure in question is assumed to have a geometrical configuration entirely different from its original state. The equations of equilibrium are set up from this strained state rather than from the unstrained state as is done in all other problems of equilibrium. The question then becomes: What must be the value of the forces so that the strained state may be maintained? Two general methods are available that can be shown to be identical.

The first method involves the problem of the elastica—that is, the solution of the differential equation that describes the possible states of equilibrium. The second method, the energy method, is based on the fact that, for unstable equilibrium, the potential energy of a system is a maximum and any virtual variation from that state involves a decrease in potential energy. The criterion by which the conditions of unstable equilibrium are determined is that, in the transition from the actual state to one infinitely close to it, the variation of the potential energy is zero; that is,

$$\delta T = 0 \dots\dots\dots\dots\dots\dots\dots\dots (129)$$

in which T is the potential energy of the system. One important feature of this statement is that it is independent of the law of elasticity and holds for large displacements.

Instead of using the potential energy of the structure, the author has substituted for it a "complementary energy" and claims superiority of the method of "complementary energy" over the potential energy by showing that, by the use of Eq. 23, based on "complementary energy," a closer value to the buckling load can be obtained than by using Eq. 27, which is based on potential energy.

The reason for this apparent superiority lies in the fact that the author has satisfied all boundary conditions by using the expression $M = P z$ for the moment at any point whereas Eq. 27 utilizes the more general term $M = E I \frac{d^2 z}{dx^2}$. When Eq. 27 is to be used the approximating function must satisfy all the boundary conditions and using Eq. 24 would naturally give a poor result. The Timoshenko variant takes this fact into account by writing

$$P z = M = E I \frac{d^2 z}{dx^2} \dots\dots\dots\dots\dots\dots (130)$$

If an expression that satisfies all boundary conditions is used, Eq. 27 furnishes a more accurate result than does Eq. 23 when Eq. 24 is used. For example, assume

$$z = c \,(l^3 x - 2\, l\, x^3 + x^4) \dots\dots\dots\dots\dots\dots (131)$$

which satisfies all boundary conditions. From Eq. 27

$$P = \frac{168}{17} \frac{E I}{l^2} = 9.8824 \frac{E I}{l^2}.$$

Substituting this expression in Eq. 23 or Eq. 28 will yield as a result

$$P = \frac{306}{31}\frac{E\,I}{l^2} = 9.8797\,\frac{E\,I}{l^2}\ldots\ldots\ldots\ldots\ldots\ldots(132)$$

As a matter of fact Eq. 27 has a marked superiority over Eq. 23 in that it is perfectly general and will hold under all boundary conditions, whereas Eq. 23 will have to be modified according to the boundary conditions. Consider the case shown in Fig. 7.

Assume a starting shape

$$z = c\,(3\,l\,x^2 - x^3)\ldots\ldots\ldots\ldots\ldots\ldots(133)$$

which satisfies all boundary conditions. The exact solution for this case is $P = \frac{\pi^2}{4}\frac{E\,I}{l^2}$. From Eq. 27, $P = 2.5\,\frac{E\,I}{l^2}$, whereas Eq. 23 gives the impossible value of $P = -\frac{14}{11}\frac{E\,I}{l^2}$. Evidently Eq. 23 must be modified to take the free end into account.

Eq. 27 was derived by equating Eq. 26 to zero, but an equivalent and (in the writer's opinion) more fundamental expression may be derived by the application of Eq. 129.

From Eq. 26

$$T = \frac{1}{2}\int_0^l \left[E\,I\left(\frac{d^2z}{dx^2}\right)^2 - P\left(\frac{dz}{dx}\right)^2\right] dx\ldots\ldots\ldots\ldots(134)$$

The variation of this integral is a problem in the Calculus of Variations, and on performing the variation the following result is obtained:

$$\delta T = \int_0^l \left[E\,I\,\frac{d^4z}{dx^4} + P\,\frac{d^2z}{dx^2}\right]\delta z\,dx - \left[\left(P\,\frac{dz}{dx} + E\,I\,\frac{d^3z}{dx^3}\right)\delta z\right]_0^l$$

$$+ E\,I\left[\frac{d^2z}{dx^2}\,\delta\left(\frac{dz}{dx}\right)\right]_0^l = 0\ldots\ldots\ldots\ldots\ldots\ldots(135)$$

The terms in the brackets are called "the terms at the limits," and when they are zero Eq. 135 becomes

$$\int_0^l \left[E\,I\,\frac{d^4z}{dx^4} + P\,\frac{d^2z}{dx^2}\right]\delta z\,dx = 0\ldots\ldots\ldots\ldots\ldots(136)$$

Since the variation δz is arbitrary, Eq. 136 is fulfilled when

$$E\,I\,\frac{d^4z}{dx^4} + P\,\frac{d^2z}{dx^2} = 0\ldots\ldots\ldots\ldots\ldots\ldots(137a)$$

Eq. 137*a* may be integrated to

$$E\,I\,\frac{d^2z}{dx^2} + P\,z = 0\ldots\ldots\ldots\ldots\ldots\ldots(137b)$$

which is nothing more than the differential equation given by Leonhard Euler. From this demonstration may be seen the connection between the problem of the elastica and the energy method.

The approximate solution of all problems in mechanics based on the variation of an integral can be simplified greatly, as follows: When the boundary conditions are such that the "terms at the limits" are zero, Eq. 135 becomes Eq. 136. In general, Eq. 136 may be written[31] as

$$\int_0^l [\text{Differential equation of equilibrium}] \times [\text{Virtual change of the function describing the state of equilibrium}]\, dx = 0 \quad \text{(138)}$$

When the "terms at the limits" are not zero an equation of the form of Eq. 135 must be dealt with. It must be remembered that Eq. 135 is the result of the variation of the integral of potential energy of a straight column subjected to end loads. Any other condition of loading will require a slightly different expression.

Consider the case shown by Fig. 3(*a*). The application of Eq. 135 to this case is as follows:

Since the ends are held, the terms at the limits vanish leaving Eq. 136 or 138. Assuming the value of z in Eq. 131, which satisfies all boundary conditions:

$$\frac{dz}{dx} = c\,(l^3 - 6\,l\,x^2 + 4\,x^3);\quad \frac{d^2z}{dx^2} = 12\,c\,(x^2 - l\,x) \quad \text{(139a)}$$

$$\frac{d^3z}{dx^3} = 12\,c\,(2\,x - l);\quad \frac{d^4z}{dx^4} = 24\,c \quad \text{(139b)}$$

and,

$$\delta z = \delta c\,(l^3\,x - 2\,l\,x^3 + x^4) \quad \text{(139c)}$$

Substituting in Eq. 136

$$\int_0^l [E\,I\,24\,c + 12\,P\,c\,(x^2 - l\,x)][l^3\,x - 2\,l\,x^3 + x^4]\,\delta c\,dx = 0 \quad \text{(140)}$$

Performing the integrations it will be found that

$$P = \frac{168}{17}\,\frac{E\,I}{l^2} \quad \text{(141)}$$

The differential equation of the strained column is given by Eq. 137*b*. From Eq. 138

$$\int_0^l \left[E\,I\,\frac{d^2z}{dx^2} + P\,z \right] \delta z\,dx = 0 \quad \text{(142)}$$

Substituting Eqs. 139 and performing the integrations will lead to

$$P = \frac{306}{31}\,\frac{E\,I}{l^2} \quad \text{(143)}$$

For the case shown by Fig. 7, Eq. 138 cannot be used and Eq. 135 including the "terms at the limits" must be used.

The same general method can be applied to vibrating systems.[32]

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[31] "Eine Wichtige Vereinfachung der Methode von Ritz zur angenaherten Behandlung von Variationsproblemen," by H. Hencky, *Zeitschrift für angew. Mathematik und Mechanik*, Vol. 7, 1927, p. 80.

[32] "On Forced Pseudo-Harmonic Vibrations," by I. K. Silverman, *Journal of the Franklin Institute*, June, 1934, p. 743.

One point which should be emphasized in the foregoing approximations is that the critical loads and their analogous quantities in vibrating systems are always larger than the actual values and, therefore, are necessary but not sufficient magnitudes. Therefore, these approximate values are on the unsafe side and other methods[33] should be used to determine the lower limit.

Fundamentally there is no difference between the method presented by the author as expressed by Eq. 11 and that as expressed by Eq. 138. The writer believes that the differential equation or the energy expression of a given problem can be "set up" with greater ease than the deflections, displacements, and changes of curvature required by Eq. 10. Furthermore, all reference volumes on the theory of structures present the general theory of equilibrium via the differential equation and the investigator will find Eq. 138 quite easy to handle.

The method based on Eq. 138 also may be used in the approximate solution of problems of stable equilibrium (for example, in the determination of the deflections of beams, etc.). One important problem to which it has been applied is that dealing with the action of a suspension bridge subjected to lateral forces and to the determination of the natural frequencies of this type of structure when vibrating in a horizontal plane.

Under the action of a uniform wind load the deflection of a suspension bridge can be expressed as

$$\delta = \sum_{n=1,3,5\cdots} \frac{a_n\, l^4}{E\, I} \sin \frac{n\, \pi\, x}{l} \qquad \text{.................. (144)}$$

The moment and shear in the horizontal system may be determined from $M = E\, I \frac{d^2\delta}{dx^2}$; and shear $= E\, I \frac{d^3\delta}{dx^3}$. Eq. 138 may be used to determine the values of a_n. A set of simultaneous equations is obtained given by the general formula

$$a_n\, [(\pi\, n)^4 A_1 + (\pi\, n)^2 A_2 - A_3] - \sum_i a_i \frac{A_4\, (n\, i)^5}{(n - i)^2\, (n + i)^2}$$
$$= \frac{A_5}{(n\, \pi)^3} - \frac{A_6}{n\, \pi}, \qquad \text{for } n = 1, 3, 5, \cdots \text{ .... (145)}$$

in which i may have any odd integral value except n, depending upon the number of terms assumed in the series, Eq. 144; and A_1 to A_6 are constants of the structure and loading. Thus a direct solution is furnished by use of Eq. 138 to replace the tedious trial and error solution now available.[34]

George R. Rich,[35] M. Am. Soc. C. E. (by letter).—Dean Westergaard has prepared a valuable and interesting paper which, for maximum benefit, should be read as a sequel to his earlier paper on the "Buckling of Elastic Structures,"[36] from which the somewhat formidable terms "orthostatic action,"

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[33] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, p. 84; also "Rayleigh's Principle," by G. F. J. Temple and W. G. Bickley, Oxford Press, 1933, p. 29 and p. 72.

[34] "Suspension Bridges Under the Action of Lateral Forces," by Leon S. Moisseiff and Frederick Lienhard, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1080.

[35] Chf. Design Engr., TVA, Knoxville, Tenn.

[36] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

"astatic action," and "heterostatic action" emerge as the more familiar expressions "simple stress or bending," "pure buckling," and "mixed buckling."

In determining critical buckling loads or natural periods of vibration, it appears from the examples adduced by the author that the method of complementary energy affords much greater latitude than the Rayleigh method in the selection of functions to represent the deflected elastic line. This is particularly striking in the author's use of parabolic loci in the cases of the buckling of hinged-end columns and the lateral vibration of prismatic beams. In the Rayleigh type of investigation, parabolic curves do not ordinarily yield dependable results for the reason that the resultant expression for curvature of the member, $\frac{d^2z}{dx^2}$, is a constant, whereas the curvature should vary with the bending moment. In problems similar to the analysis of vibrations in suspension bridges, in which the choice of functions naturally gravitates to certain standard trigonometric forms or Fourier expressions, the Rayleigh method appears to have the advantages of greater simplicity and directness without any considerable sacrifice of accuracy.

For example, Eqs. 125 and 126 may be verified exactly, Eq. 124 checked very closely, and Eq. 127 checked within 10% by direct substitution, in a single operation, of the functions

$$z = z_n \sin \frac{n \pi x}{l} \sin \omega t, \qquad \text{for } n = 2, 4, 6, \cdots \ldots\ldots\ldots (146a)$$

and

$$z = z_n \left(\sin \frac{n \pi x}{l} - \frac{1}{n} \sin \frac{\pi x}{l} \right) \sin \omega t, \qquad \text{for } n = 3, 5, 7, \cdots \ldots\ldots (146b)$$

in the standard Rayleigh form of the energy equation:

$$\int_0^l \frac{w}{2\,g} \left(\frac{dz}{dt} \right)^2 dx = \int_0^l \frac{E\,I}{2} \left(\frac{d^2z}{dx^2} \right)^2 dx + \int_0^l \frac{Q}{2} \left(\frac{d^2z}{dx^2} \right) z\, dx \ldots (147)$$

In fact, Eqs. 125 and 126 may be verified exactly by using the somewhat rougher form of Rayleigh expression:

$$\int_0^l \frac{w}{2\,g} \left(\frac{dz}{dt} \right)^2 dx = \int_0^l \frac{E\,I}{2} \left(\frac{d^2z}{dx^2} \right)^2 dx + \int_0^l \frac{Q}{2} \left(\frac{dz}{dx} \right)^2 dx \ldots (148)$$

in which the term $\int_0^l \frac{Q}{2} \left(\frac{dz}{dx} \right)^2 dx$ reflects the assumption that the increased length of cable due to the vibratory motion is equal to the increased length of elastic line of the deflecting stiffening truss. In using the Rayleigh method for this problem, the x-axis is taken as the undeflected axis of the stiffening truss, and all ordinates z are measured to the deflected elastic line of the truss. No ordinates are measured to the cable. The functions adopted for z simply represent the idea that the stiffening truss deflects during vibration into either an even number of alternating equal bay waves or an odd number of alternating unequal bay waves, the configurations in either case being such that the net departure of the cable during vibratory motion from the parabolic form in equilibrium under the dead load is a minimum. This action is implied by

Eqs. 93. In addition, these functions represent the familiar normal modes in which the oscillations of all elements of the elastic system are either in phase or in opposition, and the functions chosen have the convenient conjugate orthogonal property

$$\int \sin m\,x \sin n\,x\,dx = 0, \qquad \text{for } m \neq n \text{............} \quad (149)$$

In this connection, it is recognized that the author's primary purpose is to give examples of sufficient variety to demonstrate the power and range of the method of complementary energy. He is well aware, of course, that in certain instances alternative methods may prove equally convenient, as is shown by his statement that the problem of buckling of a thin circular disk supported at the edges and loaded by a normal edge load may be solved with equal ease by the standard method of attack using Bessel functions.

Dean Westergaard has rendered continued service to practicing engineers by demonstrating the great economy of time and labor, and the penetrating insight into structural action, that are afforded by the use of the more incisive methods of advanced analysis. Without attempting the mastery of existence theorems, reasonable facility in the use of the variational principles may be acquired for a surprisingly small expenditure of time; and, as remarked by Lord Rayleigh sixty years ago,[18] the difficulty in connection with Fourier series lies not in its practical physical applications, but only in a rigorous demonstration of what the professional mathematician means by the statement that the expansion represents the function "almost everywhere."

R. V. **SOUTHWELL**,[37] **ESQ.** (by letter).—By this communication the writer wishes to record his appreciation of Dean Westergaard's very clear presentation of the "Method of Complementary Energy." The use of a quasi-energy function, defined as in Eq. 9, was propounded in one of the contributions to a symposium of engineering mechanics held at Ann Arbor, Mich., in the summer of 1935. The writer does not remember, however, that the notion was then attributed by any speaker to Engesser, and he was not aware that it had been propounded already. It must be of interest to all teachers who are concerned to understand the fundamentals of stress-strain theory.

Whether it will have equal appeal to the engineer whose first concern is to calculate correctly, only time can show. Use has so familiarized the concepts of kinetic and potential energy that as an investigator one is kept straight, so to speak, in one's working by a kind of physical intuition, and as a teacher one is likely to regard those concepts as natural and easy. It is salutary, perhaps, to find how groping are one's first attempts to use the complementary notion: One may thereby be made more sympathetic to the difficulties of students! Clearly, time must be found for a serious attempt to master the new technique, and to assess its sphere of usefulness.

Meanwhile, the writer can offer only one comment in regard to the comparison of methods given under the headings "Buckling of Column with Hinged Ends" and "Application to Vibration of Beams." Dean Westergaard has given two examples in which the error resulting from a use of Rayleigh's principle exceeds 20%. The writer suggests that these examples are not really fair to the principle, for the reason that modes such as Eqs. 24 and 61 are excluded (tacitly) in the argument by which it is derived. That argument is based on the concept of conservation of energy, and a formula such as Eq. 27 presumes that no energy can enter or leave the system. A mode in which both slope and curvature are non-zero at the ends of a strut, however, must entail there either input or output of energy. Neglecting this "leakage," it is not surprising that one should obtain seriously inaccurate results. Similar remarks apply to the second example.

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[37] Prof. of Eng. Science, Oxford Univ., Oxford, England.

The writer's reason for emphasizing this point is that an analogous condition for close accuracy should presumably be recognized when the complementary method is used. It would be worth while to examine whether this is so, and whether (for example) it excludes the use of a mode as shown in Fig. 3(*b*). In fact, when two methods have such close similarity, it is to be expected that every precaution that must be taken in using the first has its analogue in a precaution that must be taken in using the second. Provided that such precautions have been stated and are generally recognized, the method of complementary energy seems likely to play an important part in the study of plastic distortions, where the notion of elastic strain-energy has such limited utility.

H. M. WESTERGAARD,[38] M. AM. SOC. C. E.—The writer is indebted to Mr. Rich and Professor Southwell for their comments, but he cannot accept the arguments presented by Mr. Silverman.

For example, Mr. Silverman notes that Eq. 23 does not apply well to a cantilever column with the axis of x along the original center line; but one would not write Eq. 23 for that case. The bending moment for the starting shape in this case is not $P\,z$ but $P(z - z_{\max})$, which, with z as in Eq. 133, gives

$$P = \frac{42}{17}\frac{E\,I}{l^2} \qquad (150)$$

The value in Eq. 150 differs only by 0.13% from the true value,

$$P = \frac{\pi^2}{4}\frac{E\,I}{l^2} \qquad (151)$$

and is, in fact, a satisfactory result. Mr. Silverman uses the calculus of variations to derive Eq. 137*b*, but it should be observed that this equation does not apply to the cantilever column with the axis of x along the original center line. In Mr. Silverman's Eq. 138, "Differential equation of equilibrium" is a meaningless factor. If one guesses that "one side of a differential equation" is intended instead of "differential equation," the statement in Eq. 138 still would require a clear explanation of the relation of the mathematical forms to the actualities of structural mechanics. Mr. Silverman's Eq. 145 for vibrations of a suspension bridge contains undefined constants and is only a vague suggestion of a solution. His comment that Castigliano's laws are not principles can be dismissed as untenable. The entire discussion by Mr. Silverman can be dismissed; it throws no light on the method of complementary energy.

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[38] Dean, Graduate School of Eng., and Gordon McKay Prof. of Civ. Eng., Harvard Univ., Cambridge, Mass.

Referring to the understanding comments by Mr. Rich, the writer only wishes to state that he does not advocate the supplanting of other methods by the method of complementary energy; the writer does suggest the inclusion of the method of complementary energy among the tools of structural analysis.

The remarks by Professor Southwell on the importance of avoiding leakage in the account of energy and on the nature of analogies are much to the point. Illustrations surely can be found in which there is greater danger of leakage in the account of the complementary energy than in the corresponding account of the energy. As to analogies, it should be mentioned that they exist in different degrees. In the basic equations involving the energy T and the complementary energy U, in changing from the use of T to the use of U, the statical terms, load P, stress S, and reaction R, are replaced in order by the geometrical terms, path r of a reaction, deformation D, and path p of a load, and vice versa. This statement applies not only to the equations expressing the fact that T and U are minima when the quantities P and r are constant, but applies also to two further equations which can be derived from the principles of minimum and represent the extension of Castigliano's first and second laws. These two further equations apply when the quantities r are admitted as variables defining T, or the quantities P are admitted as variables defining U; and they are:

$$R = \frac{\partial T}{\partial r} \quad \text{.........................} (152)$$

and

$$p = \frac{\partial U}{\partial P} \quad \text{.........................} (153)$$

Thus an analogy is established in which there is a one-to-one correspondence of terms in two pairs of basic equations, but this analogy has not been extended to such a degree of completeness that, given a structure, one could always visualize an analogous structure in which the statical quantities of the original structure appear as geometrical quantities, and vice versa. Some of the well-known specific analogies possess this degree of completeness; for example, the soap-film analogy by which torsional stresses are represented by slopes of a film (which is a structure of a kind); or the analogy by which curvatures of a plate represent a two-dimensional state of stress in the plane of a slice. The comments made by Professor Southwell suggest the observation that the old field of structural mechanics is still open for profitable new exploration.

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SOME ECONOMICS OF AIRPORTS

By W. WATTERS PAGON,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. WILLIAM E. RUDOLPH, DONALD M. BAKER, AND W. WATTERS PAGON.

SYNOPSIS

So much has been written regarding the location and the engineering design of airports that the writer has chosen rather to discuss some of the economic factors relating to the growth in size and in density of use of existing ports. This paper also deals largely with ports intended for transport planes. Until recently, the number of airports had reached and declined from a saturation point. Under the stimulus of war there is now (1941) a renewed growth, and a higher saturation value, although at present there is no reliable indicator to serve as a guide. For example, many state planning bodies are compiling lists of airports to be built progressively as the demand requires, and these will be correlated into a nation-wide plan; but the problem facing the present-day designer is primarily one of improvement and enlargement of present facilities.

GROWTH OF AIRPLANE USES

***The Entire Country.*—Naturally the first statistical indicator is the growth in use of planes, and for determining this growth the designer must rely on the rates of growth of other means of transportation. Such means fall roughly into two classes: (*a*) The common carrier, and (*b*) the private conveyance. Indicators for these classes cannot be determined upon comparable bases, for lack of detailed information, but they will be set up from available information and utilized to the extent of their evident limitations.**

In order to correlate such widely different sources of data one may adopt a Pearl biometric type of curve as indicative of growth of all means of transportation, as well as of biometric growth. Let:

f = numerical factor used to multiply u (having the dimension 1/time);
u = age in any units of time;
k = ratio of number of growth units to the (assumed) saturation number, with values between 0 and 1;

NOTE.—Published in February, 1941, *Proceedings*.

[1] Cons. Engr., Baltimore, Md.

v, y, y' = number of growth units, measured to different base lines, as described below;

b = number of growth units at saturation; that is, at the time of maximum growth, which is (mathematically speaking) at infinite time;

$\log_e c$ = time distance between two origins, as described subsequently;

a = number of growth units (from a previous life) at inception of the growth function that is now operating.

If the origin is chosen at that year in which the number of growth units equals one half of the number at saturation, the result is a symmetrical S-shaped curve

$$f\,u = \log_e \left(\frac{k}{1-k}\right) \qquad (1)$$

The abscissa, u, is measured plus from this origin with increasing years; ordinates, v, measured from the origin, are plus when u is plus, and ordinates, y', measured from a base line, are always plus; the factor, k, is the ratio of y' to the total number, b, at saturation. To another origin, at distance $\log_e c$ to the left of this midpoint origin the equation is

$$f\,x = \log_e c \left(\frac{k}{1-k}\right) = \log_e c + \log_e \left(\frac{k}{1-k}\right) \qquad (2)$$

The ordinate $y' = v + \frac{b}{2}$. If the curve begins tangent to a base line that is above the zero line by the distance, a, such that $y' = y - a$, Eq. 2 may be written

$$y = a + \frac{b}{1 + c\,e^{-fx}} \qquad (3a)$$

but in all that follows a equals zero, and $c = 1$; that is,

$$\frac{y'}{b} = k = \frac{1}{1 + e^{-fu}} \qquad (3b)$$

To use such a formula it is necesssry to know the saturation number, b, which of course it is impossible to know until all growth has ceased. However, if a smoothed curve through plotted data indicates a reversal of curvature, it may tentatively be assumed that the point of contraflexure is the aforementioned first origin; that is, $\frac{b}{2}$ above the base line. At the midheight point the slope of the tangent $\left(\text{that is, } \frac{dk}{dx}\right)$ is maximum, and is $\frac{1}{4}f$. In general, the slope is equal to $f\,k\,(1 - k)$, and therefore varies as a parabola referred to k. The annual increment, when $f = 0.3$, is $\frac{k_n\,(1 - k_n)}{k_n + 2.86}$, and the annual growth ratio is $\frac{k_{n+1}}{k_n} = \frac{1}{0.26\,k_n + 0.74}$, and therefore declines every year.

In Fig. 1 such a curve is shown, marked a, as well as others to be described. The vertical scale represents k; the horizontal scale represents $f\,u$ to the arbi-

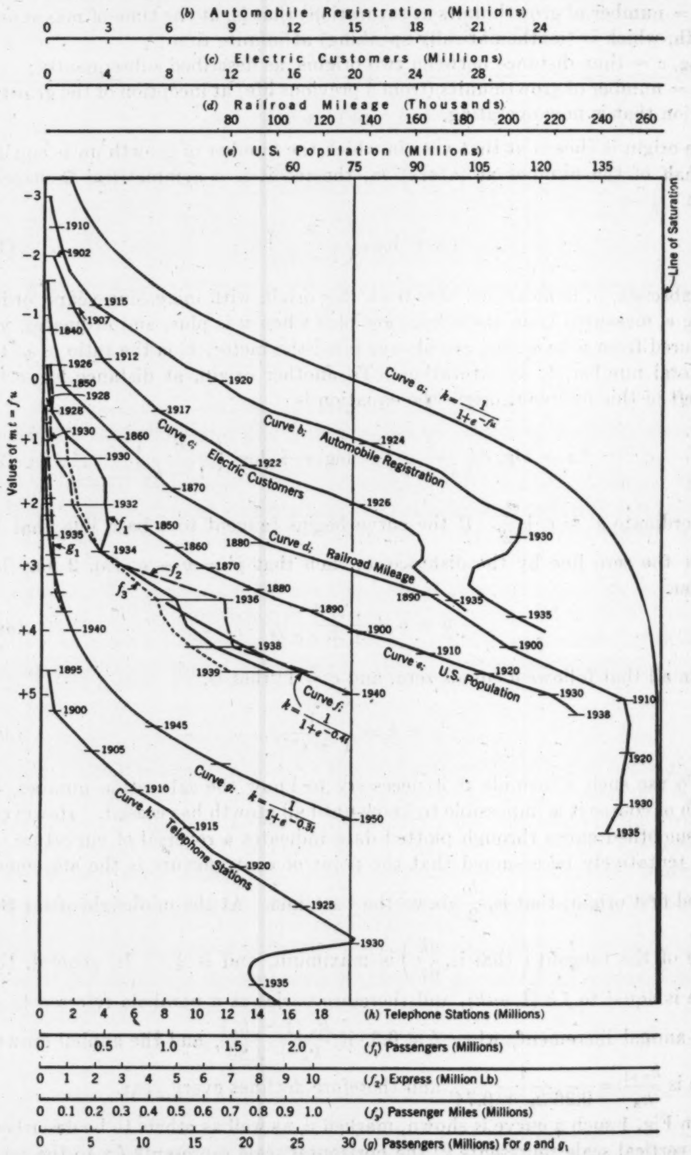

FIG. 1.—GROWTH INDICATORS (t EQUALS TIME IN YEARS, AND m IS A NUMERICAL FACTOR OF THE SAME TYPE AS f)

trary values noted at the bottom. The other curves represent the growth of various indicators, with abscissas as marked on each curve. The first of these (curve *b*) shows automobile registration in the United States, based on the year 1924 as the midheight point. Curve *c* (which is foreign to a discussion of transportation except in so far as it deals with distribution) represents the number of electrical customers, based upon the year 1926. Curve *d*, which probably is a complete one, represents the railroad mileage of the United States, based upon the year 1885. Already this curve has declined from its saturation point. The next curve, *e*, is the population of the United States.

To the right of these curves are three broken lines that represent, respectively, the annual number of airplane passengers, the number of passenger miles flown, and the pounds of air express, to include 1940. Through these curves is drawn an S-curve as a frame upon which to extend the data, the saturation number being assumed at about three times the present count. Beside it is a similar S-curve (curve *g*) upon which is redrawn, to much smaller scale, the number-of-passengers curve, g_1.

The horizontal scale for each of these curves is indicated by the years marked on them. The first curve (curve *f*) is merely indicative of the slope, because the data are so irregular as to permit almost any curve to be drawn through them, the irregular effects of the business depression being clear. The second curve (curve *g*) is based upon a saturation number of passengers equal to 12% of the total number of passengers (through and commutation) on the steam railroads of the United States in 1939.

The total future growth of air travel and air use is any one's guess. The data accumulated are clearly a poor indicator. If comparison is made first with the rate of growth of railroad mileage, then the engineer may count upon a continually increasing rate of growth for about fifty years, or say thirty-five years longer, before the midheight point is reached; but this comparison seems unfair, because in a little more than ten years the airline network has grown to a mileage that is comparable with one hundred years of growth of railroad mileage.

Seemingly, on the other hand, automobile registration is a good indicator, although it must be recognized that here the student is dealing with the private conveyance largely, whereas the commercial use of airplanes relates to common carriers. However, the automobile developed mostly during the short space of about fifteen years, and after the major expansion of the country had taken place (therefore at a time of slower growth in population, although, of course, some growth in registration has come from increased population). If the automobile indicator is used, one must assume, tentatively, that the period of half-growth will be approximately fifteen years, instead of fifty years.

The first airplane curve (curve *f*) is based upon a half-growth period of about fifteen years, and would yield as saturation values: 4,800,000 passengers, 24,000,000 lb of express, and 1,600,000 passenger miles. The equation (for the number of passengers) is

$$y' = \frac{4,800,000}{1 + e^{-0.4t}} \qquad (4)$$

in which *t* is the time in years from 1940.

The second curve (curve g) is based upon a half-growth of about twenty-five years, and a saturation number of 12% of the railroad passengers—that is, 60,000,000 passengers. The equations for express and passenger miles would be related. For this curve the abscissas and ordinates have been altered, as shown, the ordinates being one twelfth, and the abscissas four fifths.

These curves (curves f and g, Fig. 1) cover a wide range. The one reflects individual desire for transportation to a great extent; and the other is based upon common carrier transportation, which is influenced by a long period of growth in population, and a paucity of population in the early years. Because of the irregularity of the data it is impossible to fit any curve that is not open to serious objection, but since some crude basis of prediction is needed, the writer offers curve g for discussion, as being more likely of attainment. If it be correct there will be a half-growth period of about twenty-five years. In further discussion, therefore, this latter curve is used. Its equation is

$$k = \frac{1}{1 + e^{-0.3t}} \qquad (5)$$

in which t is the time in years, measured from 1950. The assumed saturation value for passengers is 60,000,000 per yr; and, for express, 180,000,000 lb per yr. From this curve, the growths in Table 1(b) are derived.

TABLE 1.—Growth Studies Applying to Air Transportation

(a) Annual Traffic at Newark, N. J.[a]				(b) Future Growth in the United States (Eq. 5)				
Year	Passenger fares	Freight, in Lb		Year	Value of k	Ratio[b] $\frac{k_{n+1}}{k_n}$	No. of passengers	Express, in lb
		Mail	Express					
1929	4,000	322,000	0	1939	0.0355	1.34	2,130,000	6,390,000
1930	30,000	905,000	47,000	1940	0.0475	1.33	2,860,000	8,580,000
1931	90,000	2,061,000	64,000	1941	0.0630	1.32	3,780,000	11,340,000
1932	91,000	1,730,000	213,000	1942	0.0840	1.31	5,040,000	15,120,000
1933	117,000	1,603,000	457,000	1943	0.1090	1.30	6,540,000	19,620,000
1934	123,000	1,511,000	758,000	1944	0.1420	1.29	8,520,000	25,560,000
1935	205,000	2,744,000	1,151,000	1945	0.1825	1.27	10,950,000	32,850,000
1936	265,000	3,800,000	2,175,000	1946	0.2320	1.25	13,920,000	41,760,000
1937	299,350	4,766,719	2,488,580	1947	0.2890	1.23	17,340,000	52,020,000
1938	355,123	5,259,013	2,783,667	1948	0.3550	1.20	21,300,000	63,900,000
....	......	....	....	1949	0.4255	1.18	25,530,000	76,590,000
....	......	....	....	1950	0.5000	1.15	30,000,000	90,000,000

[a] Prior to opening La Guardia Field. [b] Where $\frac{k_{n+1}}{k_n} = \frac{1}{k_n(1 - e^{-f}) + e^{-f}} = \frac{1}{0.26\,k_n + 0.74}$, for $f = 0.3$. k_{n+1} = value of k for the $(n + 1)$th year, etc. For $f = 0.3$ this ratio gives 35% annual growth at the start, and 0% at the end of the life, and for $f = 0.4$, 50% and 0% respectively.

The Individual Airport.—The foregoing values apply to the United States as a whole. There are no means for predicting the volume of traffic for each individual airport, because competition between nearby airports, local business conditions, etc., make a prediction far more hazardous. For the sake of discussion, however, the writer will apply the 1950 ratio—14 : 1—to individual airports, meaning that by the end of a period of eleven years the volume of traffic will be fourteen times as great as in 1939. After five years Baltimore (Md.) exceeds such a curve by about 15%.

As indicative of such growth the traffic per annum at Newark, N. J., is given in Table 1(*a*). Transfer of the airlines to La Guardia Field caused an abrupt drop after 1938.

Density of Traffic

There is little experience to guide a designer in determining how many planes can be landed and discharged from an airport.[2] Such experience as there is points to a minimum time between take-offs of about 2½ min. This would indicate a rate of 24 planes per hr, if systematically scheduled. Probably if double-width runways are provided this time can be reduced materially —perhaps by one half; but this is questionable.

If transport planes in the next ten years average 20 passengers each, landing or taking off from terminal airports, one may consider the probable passenger load to be roughly 500 per rush hr. For intermediate airports one can assume a capacity of fourteen times the values for 1939 as being the growth up to about the year 1950.

Size of Airplanes

There is no basis for predicting the growth in the size of aircraft that is not subject to much criticism. The growth curve may quite likely be an **S**-curve such as has been analyzed already; but the constants which determine the curve are indeterminate for want of basic data. Perhaps one may draw some conclusions from the rate of growth in speed, as indicated by the Schneider Cup Races, but it should be noted that in many cases the winner was a seaplane. For all winning planes, whether land or seaplanes, the horsepower increased roughly along a biometric curve. Similarly the top speed, when plotted, approximates such a curve and shows clearly the flattening off of the curve as it approaches what (in the light of present knowledge of the compressibility of air) is probably the maximum speed attainable in flight.

Airplane dimensions are more difficult to plot because they depend so largely upon the individual ideas of the designers of the differing types manufactured. It is probable that the rate of growth in size of land planes is approaching a maximum, but it is also probable that seaplanes will continue to grow beyond the limits of the land planes. The restriction imposed by the size of available landing areas is already affecting the first class, whereas there are areas along the ocean coasts and the Great Lakes in which it is possible to take off and land a large seaplane within reasonable access to important business centers.

In 1936 the writer made the prediction for seaplanes given in Table 2. The Pan American Clipper ships operating across the North Atlantic, for instance, have the actual dimensions listed under 1939. However, airline operators consider these estimates to be too conservative.

One American seaplane manufacturer has constructed his factory with a 300-ft clear span, and 40-ft clear height, anticipating the need for this added space in the not distant future. A proposed German design has a wing span of

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[2] Some pertinent information will be found in "Forschungsvergebnisse des verkerswissenschaftlichen Instituts für Luftfahrt an der Technischen Hochschule Stuttgart," by Carl Pirath, Verlag von R. Oldenbourg, Berlin, 1930.

289 ft, a carrying capacity of 170 passengers and crew, length 158 ft, gross weight 250,000 lb. Mr. I. Sikorsky[3] has stated his belief in a ship of 1,000,000-lb gross weight as an engineering possibility. Such a plane might have a wing span of 450 ft. An English writer envisages a 500-passenger seaplane of 400-ft wing span, which will cross the North Atlantic in 12 hr.

TABLE 2.—PREDICTED GROWTH IN SEAPLANES

Year	Span of wings, in ft	Over-all length, in ft	Height, in ft	Gross weight, in lb	Weight per sq ft, in lb	Tail span, in ft	Passengers (sleeping)
1935	130	90	25	52,000	23.5	40	....
1939[a]	152	106	27.57	82,500	28.7	45.77	40
1945	175	110	32	100,000	31	42	50
1950	200	130	34	140,000	35	43	75
1955	250	160	37	250,000	40	45	100

[a] Actual values for Pan American Clipper ships.

SIZE OF AIRPORTS

Mathematical Analysis.—In this paper attention is centered on landing fields for landplanes. The size of an airport depends, first, upon the obstructions in the vicinity. Until such obstructions are removed the landing area is reduced by some multiple of the height of the obstruction. In 1925 it was officially ruled that no area could be used for take-off or landing purposes whose distance from an obstruction was less than seven times the height of the obstruction. This rule was supplanted by a ratio of 10 : 1 very shortly; then by 15 : 1; and at present by 20 : 1; but the new (tentative) obstruction ratios are 30 : 1 on runways not to be used for instrument landings,[4] and 40 : 1 for those on which instrument landings are proposed. In this paper, therefore, it will be assumed that all such obstructions have been removed, and only the net dimensions will be considered available.

The length of a runway is determined primarily by two factors: (*a*) The take-off (or landing) run, while the plane is still on the ground or is just hovering; and (*b*) the length required for a path that will just clear an officially defined obstacle. In general, with brakes and wing flaps, the take-off run exceeds the landing run, and the latter need not be considered. The actual ground run is very much less than the total length of runway.

Although all pilots are required to warm up their engines and test them before taking off, there have been numerous cases in which the engines failed before any considerable height was attained, and cases in which the plane was so heavily loaded that it was unable to take off at all. In the single-engine plane the pilot must do the best that he can if the engine fails, and select a spot on which to land; in the multi-engine plane some power is still available to lengthen the landing glide, and therefore to multiply the chances for a safe re-landing. In any event, the pilot must choose between a straight glide, in the same direction as the take-off, or an attempt to turn in the hope of again regaining the airport. In the turn he may also suffer, or gain, from side slip, which in effect increases the gliding angle.

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[3] *Journal of The Aeronautical Sciences*, December, 1936, Vol. 4, No. 2, p. 75.

[4] Airport Approach Standards (Tentative), *Civil Aeronautics Administration*, August 26, 1940

No one can predict at what time the engine, or engines, will fail; hence the following analysis is predicated upon determining the size of field required such that a pilot may re-land by effecting a turn (of 180° or less), regardless of the point at which failure occurs.[5] Side slip is assumed to be zero. The take-off angle, the glide angle, the speeds, and other factors in relation to size and power, are related to certain fundamental characteristics of airplanes. The relations are not identical for all makes of plane; and hence the resulting answer is more a generalized than a specific one.

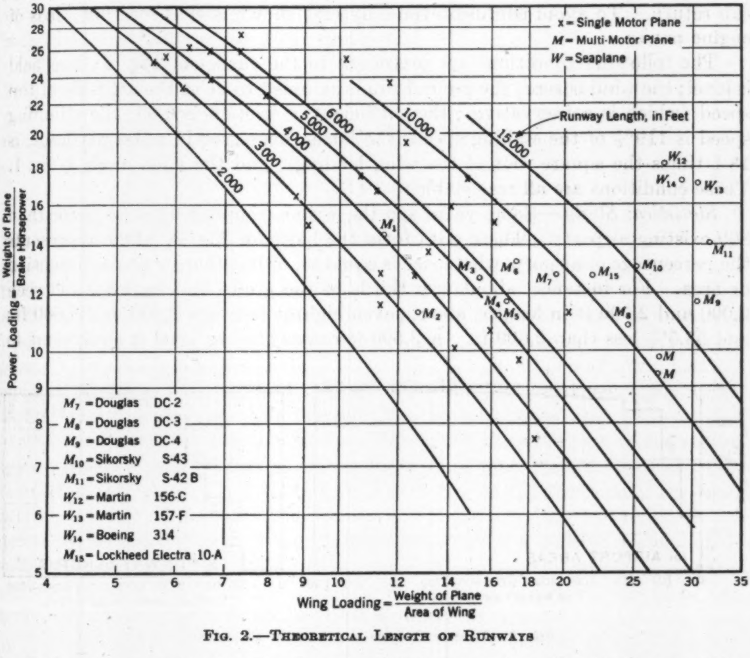

FIG. 2.—THEORETICAL LENGTH OF RUNWAYS

This answer appears in Fig. 2. The abscissas are the wing loading, and the ordinates the power loading. Wing loading is merely the weight of the plane per square foot of wing area; the power loading is the weight of the plane divided by the brake horsepower; both are factors that are known for various models, and are annually published in aviation magazines. In Fig. 2 the curves represent the length of runway on a square landing area within which a plane may re-land, with curving glide, based upon the two factors described. On this figure are also shown a number of present models, plotted according to their wing and power loadings. The points are classified as: Single engine planes, denoted by the letter, x; multi-engine planes, denoted by the letter, M; and seaplanes, denoted by the letter, W. Certain of these planes are identified by manufacturer's name and the model number; others can be identified by reference to published lists in aviation magazines.

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[5] "On the Necessary Size of Aerodromes in Order that a Landing May Be Made if the Engine Fails When Getting Off," by H. Glavert, *British Reports & Memoranda No. 996*, January, 1926.

It will be noted, first, that for fields larger than 5,000 ft the spacing of the curves becomes very small, so that there is little gain by enlarging the field beyond 5,000 ft. A second point to be noted is that there is relatively little gain from a 4,000-ft field to a 5,000-ft one. It is also evident, third, that none of the multi-engined planes can return safely if all engines fail, but that with one half, two thirds, three quarters, or more of the engine power available, they can return. To avoid confusion the only case shown is that for 100% loss of engine power.

The following conditions are precedent to the curves of Fig. 2: The field is level; the wind is zero; the ground run is assumed to be at the attitude of top speed (which is conservative); the coefficient of friction is 0.05; the climbing speed is 119% of the stalling speed (the latter, expressed in miles per hour, is 18.1 times the square root of the wing loading); and the glide angle is 5 : 1. These conditions are all reasonable.

Statistical Study.—Some years ago the writer compiled data on more than 800 existing airports. These data form the basis for Fig. 3, which represents the percentage of airports whose size is equal to, or less than, a given dimension or area. For instance, about two thirds of the group had runways between 2,000 and 2,500 ft in length; about seven eighths between 2,500 and 3,000 ft; and 98.5% less than 5,000 ft. A 2,500-ft runway at sea level is equivalent to about 3,000 ft at El. 3,000. In Fig. 3, fields are included regardless of their elevation above sea level.

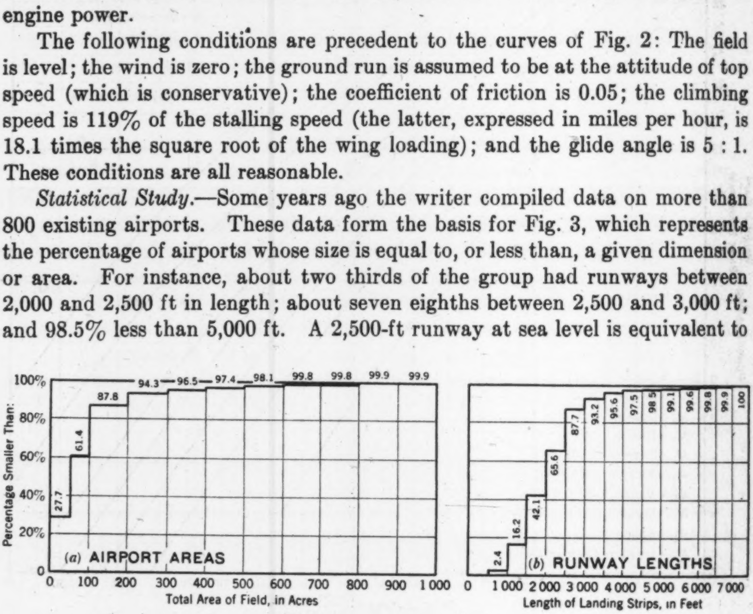

FIG. 3.—DESIGN CURVES FOR AIRPORT PLANNING

Probable Maximum Size.—In view of the large investment represented in airports it is scarcely likely that the size can be increased beyond 5,000 ft for even important ports, and it is certainly true that for many the size will not be enlarged beyond present dimensions because they cannot be enlarged.

In view of the utilization of multi-engined planes, the prevalence of wing flaps, and the possibility of other improvements, it seems that a fair conclusion to be drawn is that 5,000 ft probably will be the maximum size required in the future for airports intended for airplanes. Plane designers will be forced to design planes that can use fields of such size safely. For seaplanes, however, two to four miles is the size under consideration.

For lighter-than-air ships, fields on which a hangar is to be provided will also require approximately 5,000-ft dimensions. For a hangar 1,000 ft long, with a 2,000-ft approach at either end, 5,000 ft will be required on the axis of the hangar. If, now, the hangar is rotatable, a 5,000-ft diameter will suffice. If means are developed for handling a ship into, and out of, a fixed hangar regardless of the velocity of cross winds, then the field may be restricted to 5,000 ft on the hangar axis, and perhaps 3,000 ft at 90° to the axis. Of course on such a field other space is needed for auxiliary buildings and storage.

The combined conclusion, then, is that probably a 5,000-ft square field, free of obstructions, at sea level (or of larger size for high altitudes) will be the maximum size required for major airports.

Density of Traffic.—Such a field will handle traffic up to the limit of perhaps 24 planes per hr on a single-width runway. Should double-width runways prove to be successful in operation the capacity will be increased. When congestion becomes too great a new field must be developed.

FIELD ARRANGEMENT

The question of field arrangement largely falls outside of the purview of this paper. The size of the airport, however, is determined in part by the presence of buildings upon it. The buildings should be located with reference to the runways so as to allow not less than 1,400 ft of clear width, centered on each runway, and if space permits, this width should be increased to avoid the presence of disturbing eddies whose intermittent character can create a severe hazard to planes landing or taking off. Such eddies may persist for a distance equal to 10, 20, or more times the height of the building, and they slowly widen as they move away from their source. Nearby hills, trees, and other outside obstructions are also a source of such eddies, and if the eddies are of the rolling type, with their axis horizontal, as results, for instance, from a grove of trees, their effect is to cause a sudden and rapid deviation in the direction of the wind near the ground surface which may cause the plane to appear as though forced down, or suddenly lifted. A field located adjacent to a body of water is also subject to variations in the rate of ascent of the rising, heated air, giving the same effect as a rolling eddy. The size of the field, therefore, should be determined with such eddies, and vertical currents, in mind.[6]

AIRPORT FACILITIES

Administration.—The building that houses all means for serving the public will be called, for convenience, the "air station." Such a building will contain the airport administration, the airline ticket and accounting offices, the personnel for flight control, the personnel for meteorological service, the postal, express and such services, and finally (where required), customs, public health, and immigration services. Each of these will be considered in the light of present knowledge.

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[6] "Wind Velocity in Relation to Height Above Ground," by W. Watters Pagon, *Engineering News-Record*, May 23, 1935.

The airport administration offices will probably be restricted to a space not greatly exceeding present requirements. Each airline will require not less than 12 ft of counter length and 200 sq ft of space behind it for the sale of tickets. If the airport is an intermediate one, an additional 600 sq ft of space will be required for accounting work, for teletypewriters, etc. If it is a terminal port, and, of course, if the main offices of the airline are at such a terminal airport, a large area may be required, or even an entirely separate building.

The size of the airport control room is a debatable matter. Opinions range from 144 sq ft to 256 sq ft, or more. An additional space, about equal in size, is needed for housing the equipment and accessories that need not be within the control room itself.

The space required for airway flight control and for meteorology increases from year to year. Only in recent years has there been need for control of traffic on the airways, but each year there will be increasing density of traffic, and increasing space in the station will be required for control. At present there is evidence that about 5,000 sq ft should be available, on one floor, close below the control room, for these facilities. In addition, "meteo" will require outside space, on the ground or on the roof, for instrument shelters. In Europe air police must also be accommodated.

The space required for postal and express services is dependent largely upon the type of operation as well as upon the volume of traffic. Where truck deliveries can be made, profitably, directly to and from a plane, little or no space is required in the station. With increasing volume the necessity will arise for inter-transit storage of express (between planes, or between planes and trucks); and in course of time the postal authorities will probably begin to demand space. These areas are completely indeterminate. The writer has attempted to obtain some clue from the express companies as to their requirements, but without success. Hence, the designer of an air station must rely entirely upon his imagination, and—having done so—should locate postal and express space so as to permit of a great expansion.

The space required for customs, immigration, and public health services will be considered only where there is likelihood of international airplane traffic. The writer has provided an area of 1,350 sq ft for these combined services at the Transatlantic Seaplane Base of the Baltimore (Md.) Municipal Airport[7] which, for the present, at least, has been satisfactory. The counter for customs inspection occupies 60 ft. When seaplanes that will carry more than 100 passengers are introduced, no doubt there will be need for additional space.

The spaces that occupy the greater part of the air station are those that serve the public generally, rather than the air traveler. Just as in the early days of the railroad, the general public has an intense interest in watching the operations of an airport. On Sundays and week ends, on holidays, on race days or days of exhibition flying or when some distinguished visitor may be seen, extraordinary crowds mill about the airport and into the station. Space for shelter, sanitary facilities, restaurants, etc., must be provided within reasonable limits.

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[7] *Proceedings*, Am. Soc. C. E., October, 1940, p. 1488.

At present no one knows what are reasonable limits. During the week there are almost no visitors—only the traveling public; on week ends come the crowds. Table 3 gives some estimates of the number of persons who visit airports in different cities of the United States. In Continental Europe and in England the number is comparable and even larger. Many of the estimates are pure guesses, but they are still indicative.

TABLE 3.—THE NUMBER OF PERSONS VISITING AIRPORTS[a]

City[b]	An ordinary summer day	A summer week end	Largest recorded in one day
Akron, Ohio	1,500[c]	....	45,000
Washington, D. C. (Hoover)	500 to 600	1,000 to 1,200	1,500 to 2,000
Atlanta, Ga.	500	6,000[c]	50,000
Boston, Mass.	500 to 1,000	....	10,000
Buffalo, N. Y.	400 to 500	15,000[c]	75,000
Camden, N. J.	500	2,000 to 5,000	25,000
Chicago, Ill.	100	1,000[c]	200,000
Cincinnati, Ohio	200	5,000[c]	35,000
Cleveland, Ohio	3,000 to 4,000	28,000[c]	140,000
Columbus, Ohio	2,000 to 5,000	10,000 to 15,000	50,000
Dallas, Tex. (Love Field)	1,500 to 2,500	15,000 to 25,000	80,000
Dayton, Ohio	300	5,000	15,000
Des Moines, Iowa	100	2,000	30,000
Detroit, Mich.	....	10,000	75,000
Garden City, L. I., N. Y.	1,000	10,000	150,000
Grosse Ile, Mich.	....	5,000[c]	12,000
Hartford, Conn.	....	5,000[c]	75,000
Indianapolis, Ind. (Hoosier)	100	10,000 to 15,000[c]	75,000
Jacksonville, Fla.	100 to 500	10,000[c]	30,000 to 40,000
Kansas City, Kans.	5,000	10,000 to 30,000	259,221[d]
Kansas City, Mo.	....	5,000	20,000
LeRoy, N. Y.	100	20,000	86,000
Memphis, Tenn.	500	30,000	80,000
Miami, Fla. (Pan-American)	....	....	30,000
Minneapolis, Minn.	400 to 500	10,000	75,000
Newark, N. J.	2,500 to 3,500	5,000 to 10,000	60,000
Philadelphia, Pa.	200	1,000 to 10,000	10,000
Pontiac, Mich.	100	500 to 1,000[c]	50,000
St. Paul, Minn.	200	3,000	75,000
Syracuse, N. Y.	200 to 500	5,000	30,000
Terre Haute, Ind.	500	5,000	25,000
Toledo. Ohio	....	20,000	40,000
Utica, N. Y.	500	3,000 to 5,000	27,000
Wichita, Kans.	500	10,000	20,000

[a] "Airports," by Henry V. Hubbard, Miller McClintock, and Frank B. Williams, Harvard Univ. Press, 1930. [b] Special name of airport in parentheses. [c] Sunday only. [d] Estimate made on basis of 3 persons per automobile.

In his search for an indicator of the required extent of these public facilities the writer turned to railroad experience. Some years ago the American Railway Engineering Association collated information as to railroad stations throughout the United States, and derived diagrams[8] based upon the number of "rush-hour passengers." Of course, the rush hour is congested largely by commuters, for whom the station serves mostly as a gateway from train to street, and for whom little waiting space and few sanitary or restaurant facilities are required. Of course, also, the railroad station must be of a size to accommodate crowds assembled to greet friends or notable persons, without undue congestion.

It is probable that for ten years to come the crowds at an airport will far exceed those at a railroad station, in relation to the number of passengers; it is certain that for ten years to come there will be little business of commuting by airplane. As a result, the indicators described herein are all dependent upon a crude estimate, or even guess, as to the basic factor underlying them all;

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[8] *Bulletin*, A.R.E.A., Vols. 24, 25, 26, 1923–1925.

but they possess this important characteristic: They are all consistent among themselves. If, then, the assumed basic factor proves to be correct, the station will be consistent in design; and in order to allow for error, the basic assumption should be underestimated. Then the design can be fortified by such planning that the facilities may be enlarged pro rata as the need arises.

From the railroad data the writer derived formulas for each of the public services. All depend upon the estimated number of rush-hour passengers (designated as p); some being in linear relation, others as the square root. Each curve may have a fixed, or initial, value as follows:

Concourse, in square feet	4,000 +	$4.4\,p$
Terrace, or outside area, in square feet		$300\sqrt{p}$
Men's smoking room, in square feet	500 +	$0.3\,p$
Women's rest room, in square feet	500 +	$0.6\,p$
Total inside seats, number		$17\sqrt{p}$
Men's toilets, in square feet	500 +	$0.5\,p$
Water closets, number		$0.55\sqrt{p}$
Urinals, number		$0.35\sqrt{p}$
Lavatories, number	4 +	$0.0044\,p$
Women's toilets, in square feet	400 +	$0.33\,p$
Water closets, number		$0.40\sqrt{p}$
Lavatories, number	6 +	$0.004\,p$
Telephone booths, number	1 +	$0.006\,p$
Telegraph office, in square feet	60 +	$3.5\sqrt{p}$
Dining room and restaurant, in square feet	500 +	$2\,p$
Seats, number	15 +	$0.08\,p$
Kitchen, in square feet	200 +	$1.2\,p$
News stands, in square feet		$11.5\sqrt{p}$

On the Continent it is customary to provide hotel facilities for the traveler, but this is not usual in the United States. However, some space should be provided, with appropriate toilet facilities, for visiting pilots, male and female, and there should be sleeping quarters for airport operating and for "meteo" personnel, these being dependent entirely upon the local conditions at the airport. If there are already such accommodations of suitable quality in the near vicinity they may be omitted, of course, except perhaps for operating and "meteo" personnel.

As time goes by an urgent need will be found for garaging automobiles of the traveling public, and such a garage should provide moderate accommodations for the airport personnel. In addition, large open areas should be set aside for parking the cars of the visiting public, with adequate roadways designed to avoid the congestion on visiting days.

HANGARS

The area and size of hangars are problems that are peculiar to each airport. For intermediate airports only one such area is needed, with hangar storage space as required for the local planes that base there. For terminal airports there must be hangars of number and dimensions suited to the requirements of the airlines basing there, in addition to the small local planes.

For important intermediate and terminal fields there is probably need for hangar area of 50,000 to 75,000 sq ft; for smaller airports, proportionately less. In one hangar space should be provided for repair shops and storage areas, and the writer suggests areas (in square feet) approximately as follows:

Engine overhaul shop	3,600
Engine storage	1,200
Wood shop	1,500
Paint shop	1,200
Metal shop	2,000
Propeller shop	800
Fabric shop	900
Machine shop	700
Forge shop and heating plant	1,000
Radio shop	500
Instrument shop	500
Parts storage	4,700
Accessory shop	300
Shop offices	1,000
Toilets, locker room, etc	2,000
Field equipment storage	3,000
	24,900

The ceiling height of the hangar should be from 30 to 35 ft, and of the shops preferably 18 to 20 ft. If the shops are situated within the hangar the offices, locker and toilet rooms, etc., may be placed in a second story. Some of the first-story shops may require additional height, and should be placed so as to utilize a part of the second story area.

FUELING

It is unnecessary to attempt a formula for the capacity of the fueling system. If servicing can be secured readily from nearby sources, it may be entirely proper to fuel directly from tank trucks; otherwise there should be underground storage of sufficient volume, and pumps of sufficient capacity, to minimize or to avoid delays in fueling. The problem is local to each airport.

SUMMARY

In setting down the statistical and approximately mathematical data of this paper the writer (to use a slang expression) is "putting his neck out." Some of the predictions are based upon reasonable factual assumptions, some upon comparison with railroad and other experience, and some upon airport experience and the judgment that derives from such experience. Predictions of growth are naturally very hazardous, because the growth of the air transportation industry has not yet progressed far enough for the determination of the trends; but, of even more importance, predictions are difficult because the irregularity of the data (resulting from the effect of the business depression) is such as to prevent the fitting of any accurate growth curve. Such curves have much value, however, in that they have been found to express growth relationships for other variables with considerable success; but they always must be projected with due caution, because it is noteworthy that sudden changes in conditions postulate the ending of the previous growth curve, and the inception of a new one. This is illustrated well in the curve, *h*, of Fig. 1, showing the number of telephone stations in the United States, in which there first appears a change of the curve during the World War, and then the slump due to the depression. It is highly unsafe, therefore, to project any such curves more than ten years in advance, and the result even at the end of ten years may be seriously in error.

The writer offers the data, and the deductions therefrom, for the sole purpose of inviting constructive criticism in order that, by concerted action, the industry may derive some idea, even though approximate, of the rate of growth, and the resulting requirements that airports must meet in the near future. He expressly omits a discussion of the engineering features of design because so much has already been written on the subject, much of it now obsolete, to be sure.

It is hoped, therefore, that the profession will criticize, freely, all the statements made, and in so doing will supplement the data and the deductions to the same end that the writer has in mind.

DISCUSSION

William E. Rudolph,[9] **M. Am. Soc. C. E.** (by letter).—Mr. Pagon has presented most interesting data for determining the rate of growth of air travel and resulting requirements that airports must meet in the future. However, the writer would invite attention to another important factor that is certain to influence the rate of increase of air travel, although not inherently part of airport design—that of speed and comfort of travel between airport and business center.

At present the traveler between Chicago, Ill., and St. Louis, Mo., or between New York, N. Y., and Philadelphia, Pa., actually spends as much time in motor buses to and from airports as he does aboard the plane. Necessarily, these airports are at considerable distances from business centers, with long and not too comfortable trips through crowded city traffic lanes the unavoidable consequence. Whereas air traffic conditions entering and leaving airports may be improved by building longer and wider runways as discussed in Mr. Pagon's paper, surface traffic conditions in crowded city streets will require special and far more expensive remedies. The business center of the future cannot move to the airport as it once encircled the railroad stations of the past—because business requires high buildings, which are not desirable near airports. Hence, there is the obstacle of distance to be surmounted at airports, and the best solution of this problem may be in trunk line motor travel arteries to adjacent business centers, a major construction feat.

The rapid growth of air travel would appear to be predicated upon two major advantages over other methods of transportation—time saving and greater comfort.

From the standpoint of time saving, the future will determine where the line should be drawn between those trips that may better be made by airplane, and those falling within the zone of the railroad and motor vehicle. As the distance that may be advantageously traveled by air becomes shorter and shorter, so will the volume of air travel increase by leaps and bounds, because there are far more passenger trips and passenger miles in short journeys than in long journeys. Here the factor of lost time at air terminals looms up as important, inasmuch as its proportion of the total travel time is considerably larger for short trips than for long trips.

From the standpoint of greater comfort, air travelers whom the writer has met agree almost without exception that the airplane is far more comfortable than the railroad or motor car. The one refreshes, the other fatigues. The writer encourages his staff men to use the airways because he considers this to make for higher efficiency; if a man's work is of sufficient value to his employer to justify his time and expenses in travel, it is equally important that this employee be at his best for performing such work upon arrival at his destination. Again, people of the present day are not content with the travel conditions to which their parents were accustomed, and the next generation will look askance at that which is considered comfortable today—another reason why air traffic should grow.

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[9] With Anaconda Copper Mining Co., New York, N. Y.

Particularly in mountainous regions does the differential on time saving, and decreased "wear and tear" upon the individual, become apparent. Here the railroad must adopt a circuitous route, whereas the plane flies directly over Nature's obstacles. The railroad pays endless toll in fuel and in carrying charges upon large capital investment in such sections, which places the airplane at a decided advantage. Thus a greater rate of increase might be expected in airplane passenger travel in mountainous regions of dense population than in flat regions where railroad locations and motor roads follow direct courses. In the development of South America, which would seem to be the principal task of the future, air travel should play an important part, in view of that continent's rugged terrain.

Donald M. Baker,[10] M. Am. Soc. C. E. (by letter).—The use of the type of curve suggested by the author in forecasting future air traffic is based upon a sound theoretical premise, since such traffic will increase in magnitude with population. This soundness is further borne out by examples of the growth of other activities and their similarity in growth pattern to the theoretical curve given in Fig. 1.

Uncertainties as to the probable location of the point of inflection on the air-traffic curve, and consequently of the probable saturation point, can be expected with the relatively small amount of basic data yet available, but the reasoning followed, coupled with the fact that the author calls attention to the danger of attempting to forecast for a period longer than ten years, is sound, and allows a proper safety factor to be used when results of the forecast are used. The formulas presented for computing areas necessary for various facilities are of much value.

The reasoning and supporting data from which the conclusion is reached that a runway 5,000 ft long represents the probable maximum that will ever be required carry conviction, and should be welcomed by those responsible for acquiring and planning airports. Heretofore lengths of runways have shown a constant increase, and no sooner had a port been constructed according to the latest standards than such standards were raised, and inadequacy resulted. From the present time forward the length of runway will control the plane designers, instead of the latter controlling the length of runway. The extremely large proportion of airports with runways much shorter than 5,000 ft is undoubtedly due to the fact that most of the land for airports now in existence was acquired in the late 1920's or early 1930's, when U. S. Department of Commerce ratings for a Class 1 airport stipulated two runways at least 2,500 ft long, at an angle greater than 60°; and for a Class 2 airport one runway 2,500 ft long, in the direction of the prevailing wind, or two runways 2,000 ft long, at an angle greater than 60°. Instrument flying or landing was then practically unknown, and freedom from fog was a very important factor. Planes flew at elevations of from 5,000 to 6,000 ft, landed or took off at an angle of 1 in 7. Two or three transport landings a day were big business.

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[10] Cons. Engr., Los Angeles, Calif.

Any flat open area of about 160 acres, even if surrounded by residential development, was suitable for an airport (the term "transport terminal" was unknown), and the owner of such a property, provided he had suitable political connections and did not ask an exorbitant price for his land, was usually able to dispose of it for use as the local airport. Possibilities of future expansion were given little thought.

With present stratosphere flying, instrument landings, greater speed, weight, and size of ships, obstruction ratios increased to 1 in 40, and with reasonably definite information as to ultimate maximum size of fields required, it is possible to consider intelligently the question of the adequacy of existing fields, and to plan for ultimate extensions in the length of runways and the size of fields.

The data given by the author indicate that, in the near future, many fields will have to be increased in size. Additional adjacent land will have to be acquired, or the existing sites abandoned and new and larger sites purchased. When faced by this situation, the agency owning and operating the site must consider the economics of the situation.

Only large metropolitan centers require that the site to be used as a transport terminal have 5,000-ft runways; but, although open unimproved areas of 160 acres (which will accommodate 2,500-ft runways) occur with reasonable frequency at fairly close-in locations in such areas, 640-acre tracts of this character are scarce.

The major factor in encouraging air travel is the time saved in comparison to other methods of travel, usually rail travel. Already fares for air travel have been reduced to a point that, between termini farther apart than overnight trips by rail, a saving can be shown over rail travel if but a nominal value is placed upon the traveler's time.

In rail travel the time spent between the points of origin and destination and the railroad depot is a negligible factor, both because such depots are usually located close to the business district, and also because such travel time is an extremely small proportion of the total time spent on a rail trip. Time spent in ground travel between points of origin and destination and the airport is a much more important factor, particularly when the trip is relatively short. In 1928 the writer presented a paper before the City Planning Division of the Society,[11] the conclusion of which was that there was little time saving in air over rail travel in distances of much less than 500 miles, when considerable time was required for ground travel at the beginning and end of the trip. This was based upon scheduled air and rail speeds of 100 and 33.3 miles per hr, respectively. It was borne out by the fact that there was then very little "short-haul" passenger traffic.

At present these speeds have increased to approximately 250 and 55 miles per hour, respectively, and "short-haul" passenger traffic has increased materially. When a one-way air trip of 500 miles involved a total time between points of origin and destination of from 6 to 6½ hr, usually with daytime flying, at least two (and sometimes three) working days were spent away from the place of business, whereas overnight rail trips allowed a complete working day at the destination with but one day away from the traveler's place of business. Present-day speeds of 250 miles per hr by air allow a person nearly as much time at his destination, and he can sleep at home. This has greatly increased the "short-haul" traffic.

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[11] "Factors Governing the Location of Airports," by Donald M. Baker, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 269.

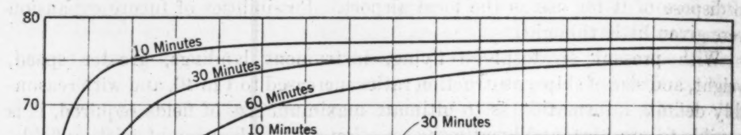

FIG. 4.—TIME SAVED BY AIR OVER RAIL TRAVEL

Fig. 4 shows the saving in time of air travel over rail travel, based upon air and rail speeds of 100 and 250 miles per hr, and 33.3 and 55 miles per hr, respectively, with varying times of ground travel to and from the airport. Present higher speeds in air travel increase the percentage of time saved from 10% to 12% over former speeds, but likewise show a greater effect in time of ground travel.

With air speeds of 100 miles per hr, percentage of time saved in air over rail travel increases 9% with a decrease in time distance of from 60 min to 10 min in one-way ground travel for 500-mile trips, and 17% when air speeds are increased to 250 miles per hr. For 1,000-mile trips, the time saved is 5% at 100 miles per hr, and 9% at 250 miles per hr as compared with comparable time periods of ground travel.

Cost of land in metropolitan centers varies inversely with the distance from the business center. Improved land will cost from 4 to 10 times more than unimproved acreage will cost in the same location. If the size of an existing airport must be increased, and such increase is prohibitive in cost because of the price of additional land, the question of acquiring a cheaper outlying site will depend upon the additional cost of maintaining reasonably similar time of ground travel; and this in turn involves a problem of determining the future magnitude of traffic to and from the airport and the business district or higher class residential district, unless the new site can be located near an existing high-speed rail line or highway.

Data given by the author indicate a volume of air traffic in 1940 amounting to 2.2% of the national population, and an estimated volume in 1950 amounting to about 22% of the then estimated population, or a per capita increase of 10 times. Based upon population of metropolitan centers, these percentages will be much greater. The air traffic at the principal Los Angeles air terminal for 1940 was 210,000 passengers, an increase of 80% over that for 1939, and represented between 7% and 8% of the metropolitan population. If such traffic were to increase in accordance with the rate estimated by the author for the nation, air traffic in Los Angeles would be of the order of 2½ millions in 1950. Such traffic would, without question, justify considerable expenditures for improving the accessibility of any major air terminal, these expenditures being in excess of savings made through acquisition of outlying land at a price lower than that for land closer in.

The writer believes that, in the past, inadequate consideration has been given to the economics of the location of airports, but feels that with increasing information and basic data becoming available, such as that contained in the paper, there should be no excuse for such lack of consideration in the future.

W. Watters Pagon,[13] M. Am. Soc. C. E. (by letter).—Mr. Rudolph stresses the relation of ground time to air time for short hops between near-by cities. Mr. Baker deals mathematically with the same subject, by the reconstruction of his former figure to bring it into harmony with modern air speeds. The writer suggests that such discussions should emphasize that it is the difference in ground time to the air terminal as against the rail terminal, rather than the actual time. For example, the writer can reach the Baltimore (Md.) Municipal Airport in 15 to 18 min from his office, as opposed to 10 min or more to either of the rail terminals, and this ratio applies to almost all of the Baltimore business district. For the short hop from Baltimore to Philadelphia, Pa. (about 96 rail miles), the ground time in the two cities is 50 to 55 min to the air terminals as opposed to 25 to 30 min to the rail terminals; and the traveling times are slightly more than 0.5 hr and 1.5 hr, respectively, with an approximate saving of half an hour.

The City of Los Angeles, Calif., has studied the problem of providing air taxi service from one or more central airports to scattered "satellite" airports of small size. Small airplanes or autogyros will undoubtedly be used in the future to minimize the ground time to remote airports.

Mr. Rudolph's comments on the relative comfort of the airplane in mountainous regions are worthy of inclusion in the prediction of the growth of air travel. The writer is indebted to both discussers for their comments.

When a growth prediction has been made, from time to time it is desirable to check the actual progress against the prediction, to detect any changes of trend. This is especially true in the present matter because of the erratic growth curve from which the writer's curve has been projected. Table 4 brings the trends up to July, 1941, and for convenience the results for seven months in 1941 have been amplified in the same ratio that prevailed in 1940 to obtain the figures for 1941. The number of passengers is slightly in excess of the prediction; the pounds-of-express curve has been much accelerated for eighteen months by war conditions. The data are taken from the "Survey of Current Business."[13]

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[13] Cons. Engr., Baltimore, Md.

TABLE 4.—COMPARISON OF ACTUAL WITH PREDICTED GROWTHS

Year	PASSENGERS CARRIED		PASSENGER MILES	EXPRESS (POUNDS)	
	Actual	Predicted	Actual	Actual	Predicted
1938	1,343,457	....	557,721	7,335,967	....
1939	1,876,051	2,130,000	749,785	9,514,300	6,390,000
1940	2,959,480	2,860,000	1,117,447	12,506,576	8,580,000
1941[a]	4,020,000	3,960,000	1,450,000	19,200,000	11,880,000

[a] Data complete to include July, 1941; the total for 1941 is estimated on that basis.

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[13] "Survey of Current Business," Bureau of Foreign and Domestic Commerce, U. S. Dept. of Commerce, the following Annual Review Nos.: February, 1939, Vol. 19, No. 2, p. 78; February, 1940, Vol. 20, No. 2, p. 77; February, 1941, Vol. 21, No. 2, p. 84; and October, 1941, Vol. 21, No. 10, p. S-21.

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TRANSACTIONS

Paper No. 2147

METHOD OF PREDICTING THE RUNOFF FROM RAINFALL

BY RAY K. LINSLEY, JR.,[1] AND WILLIAM C. ACKERMANN,[2]
JUNIORS, AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. BERTRAM S. BARNES, RICHARD VAN VLIET, LEROY K. SHERMAN, FRANKLIN F. SNYDER, AND RAY K. LINSLEY, JR., AND WILLIAM C. ACKERMANN.

SYNOPSIS

The problem of estimating runoff accurately and quickly from reported rainfall is the most difficult phase of river forecasting. The writers have analyzed hydrological and meteorological records of the Valley River Basin in North Carolina to develop a rational method of predicting runoff based on average rainfall and evaporation from a standard land pan.

DESCRIPTION OF THE BASIN

The Valley River above the U. S. Geological Survey gaging station at Tomotla, N. C., drains an elliptical basin of 104 sq miles. Tomotla is near the lower end of the river, which follows roughly the long axis of the ellipse. Topographically the basin consists of a gently rolling valley floor bounded on the north by the Snowbird Mountains and on the south by the Valley River Mountains. Elevations within the basin range from 1,600 to 1,900 ft above sea level on the valley floor and from 3,000 to 4,500 ft above sea level along the surrounding rim. The channel slopes of the river average about 30 ft per mile in the upper half of its course and about 15 ft per mile in the lower reaches.

The rock underlying the Valley River Basin is metamorphic, consisting mainly of gneisses and mica schists. The structure is rather complicated as the rocks are extensively faulted. A thin belt of marble underlies almost the entire basin. There is an extensive local alluvial fill from 20 to 30 ft deep over part of the basin that results in sustained ground-water flows. There is no evidence of loss of water from the Valley Basin by deep seepage.

NOTE.—Published in June, 1941, *Proceedings*.

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The City of Andrews with a population of about 2,000 is located near the center of the basin. The remainder of the area is rather sparsely settled. The valley floor, comprising about 60% of the total area, is dominantly cleared and about one half of this is in cultivation. The remaining area is forested, with mixed hardwoods and long-leaf pine at intermediate levels and hardwoods at the higher elevations.

DATA USED

Since appreciably different runoff characteristics exist in the various seasons of the year, it is essential to segregate data by seasons before attempting an analysis. In this study only the flood season, which begins about the middle of December and extends to the middle of April, is considered. This period falls within the dormant season and coincides with the period of general cyclonic rains. The limits of this season for a particular year can be determined by inspection of the temperature and stream-flow records. The study is based on the three flood seasons of 1934–1935 to 1936–1937.

Average rainfall was computed by the weighted-area method, using the records at Andrews, Hyatt Creek, Tatham Gap, and Old Road Gap in North Carolina. Time of rainfall was determined by inspection from the various recording gages (Fig. 1) and from the reports of the observers. Records of evaporation and temperature were obtained from the Class *A* evaporation station maintained by the Tennessee Valley Authority (TVA) at Murphy, N. C. A continuous stream-flow hydrograph was developed from the recorder charts of the rated U. S. Geological Survey recording stream gage at Tomotla.

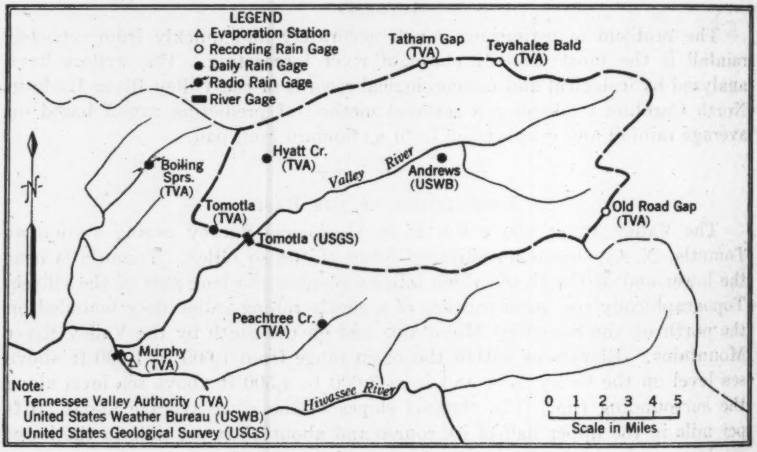
LEGEND
△ Evaporation Station
○ Recording Rain Gage
● Daily Rain Gage
● Radio Rain Gage
▬ River Gage
N
Tatham Gap (TVA)
Teyahalee Bald (TVA)
Boiling Sprs. (TVA)
Hyatt Cr. (TVA)
Valley River
Andrews (USWB)
Old Road Gap (TVA)
Tomotla (TVA)
Tomotla (USGS)
Peachtree Cr. (TVA)
Murphy (TVA)
Hiwassee River
Note:
Tennessee Valley Authority (TVA)
United States Weather Bureau (USWB)
United States Geological Survey (USGS)
0 1 2 3 4 5
Scale in Miles

FIG. 1.—MAP OF VALLEY RIVER IN NORTH CAROLINA

THEORY

Rainfall occurring over the natural river basin is assumed to be disposed of as follows:

(*1*) *Surface Loss.*—Surface loss is that part of the rain which is intercepted by the vegetal cover and natural or artificial retention basins from which it eventually evaporates and is thus prevented from entering the stream channels. This loss has been found to vary with the rainfall, being approximately constant for rains of the same amount. In other words, the greater the depth of rain the greater will be the quantity lost at the surface until a practical maximum is reached at some extremely high rainfall. Observation indicates that the full value of the potential surface loss is again available within a few hours after the end of a storm; therefore, whenever rains are separated widely enough to be analyzed independently, the full loss may be assumed for each rain. There is some indication that the amount of the surface loss is affected by the intensity of rainfall; but the complexity of the problem prevents a solution for the laws governing the variation. Storms occurring during flood season, which are considered herein, rarely reach extreme rates of rainfall and little error is introduced by the use of average values.

(*2*) *Field-Moisture Loss.*—Field-moisture loss is that part of the rain which is absorbed in the upper layers of the soil where it remains until removed by evaporation or transpiration. The potential field-moisture loss at any time is equal to the net amount of moisture taken from the soil by evaporation and transpiration since the last time the soil was saturated. The writers have used the accumulated evaporation from a standard evaporation pan as an index of field-moisture deficiency. It has been demonstrated that the moisture loss from the soil is greater during the first few days immediately after a rain than during the succeeding days, but the complexity of the problem precludes the determination of a variable coefficient for converting the pan evaporation to evaporation from the soil. There is a practical maximum that the field-moisture deficiency cannot exceed, and this value can be determined. Inasmuch as the storms of the flood season come at relatively short intervals, the use of an average coefficient introduces little error and reduces, considerably, the work involved in the use of the method.

(*3*) *Surface Runoff.*—Surface runoff is that part of the rain which travels across the ground surface to the nearest stream channel and thence to the main river system. The subsurface flow that travels through the upper layers of the soil to the stream channel is included with the surface runoff.

(*4*) *Ground-Water Accretion.*—The ground-water accretion is that part of the rainfall which percolates through the soil to the ground-water level. The resultant rise in the ground-water level causes an increased flow from the ground-water storage to the stream channel, until an amount equal to the accretion has drained into the stream channel.

ANALYSIS OF THE HYDROGRAPH

Stream-Flow Separation Charts.—The total loss for each storm studied was computed by subtracting measured runoff from observed average rainfall. Continuous discharge hydrographs were plotted to a scale 2,000 cu ft per sec to the inch vertically and two days to the inch horizontally (Fig. 2, at reduced scale). The average rainfall was plotted in block form on the same sheet, together with notes regarding snow on the ground or sub-freezing temperatures. These hydrographs were analyzed and the runoff measured by storms.

To aid in the complete separation of the hydrographs by storms, the following curves and relations were developed:

Ground-Water Recession Curve.—Fig. 3(*a*) is a ground-water recession curve plotted as the relation between ground-water flow at a given instant and the flow twenty-four hours later.[3] The curve is a composite of a large number of

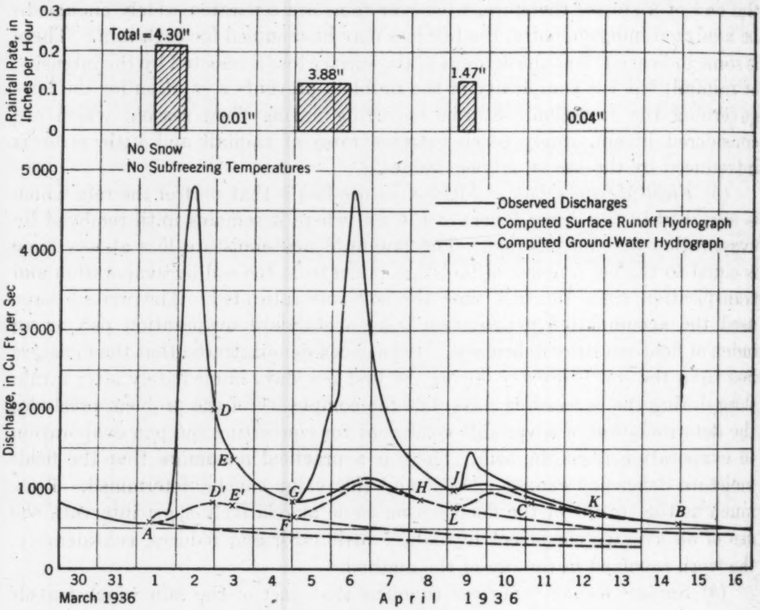

FIG. 2.—TYPICAL HYDROGRAPH

points from selected data for five years of record. The relation plots as a straight line on logarithmic paper and, for the range shown as a dashed line, represents an extension beyond the limit of these data. Care must be taken that the data selected for this curve are free of surface runoff. The duration of surface runoff may be taken as the length of the unit graph base or may be computed.[4]

Surface-Runoff Recession Curve.—Fig. 3(*b*) is a surface-runoff recession curve in a form similar to the ground-water recession curve. In this case the time increment was taken as twelve hours because of the higher rates of change involved. Data for this curve were obtained by drawing the ground-water hydrograph under the receding limbs of selected storm hydrographs by using

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[3] "Some Channel-Storage Studies and Their Application to the Determination of Infiltration," by W. B. Langbein, *Transactions*, Am. Geophysical Union, 1938, Pt. 1, p. 435.

[4] "Synthetic Unit-Graphs," by F. F. Snyder, *loc. cit.*, Pt. 1, p. 447.

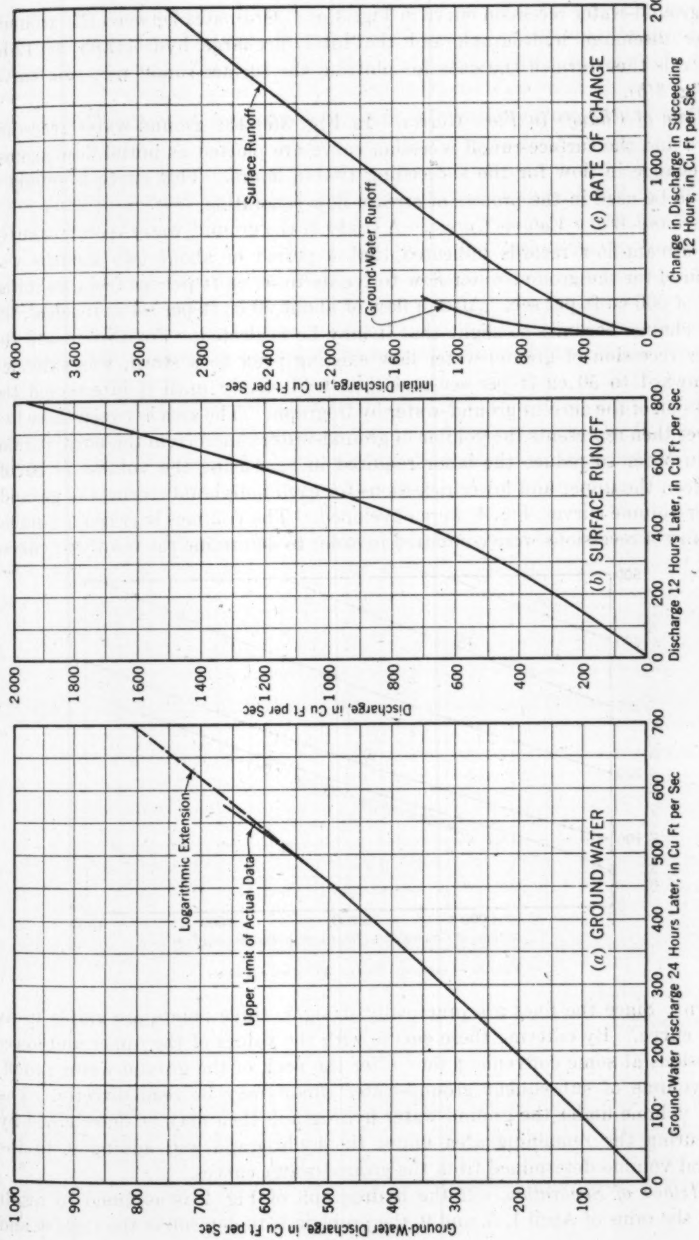

FIG. 3.—RECESSION CURVES

the ground-water recession curve in Fig. 3(*a*). Ordinates between the ground-water discharge hydrograph and the total discharge hydrograph at 12-hr intervals then furnish the data for plotting the surface-runoff recession curve in Fig. 3(*b*).

Rate of Change in Flow Curves.—In Fig. 3(*c*) the ground-water recession curve and the surface-runoff recession curve are plotted as initial flow against the change in flow for the succeeding twelve hours. This curve is purely a tool to be used in the process of stream-flow separation.

Ground-Water Volume Curves.—A study of the ground-water recession curve and stream-flow records indicated that a period of about two months was required for the ground-water flow to recede to 50 cu ft per sec from an initial flow of 500 cu ft per sec. After a flow of about 50 cu ft per sec is reached, the flow change becomes so slight that it may be neglected. For this reason the lower recession of ground-water flow existing prior to a storm was extended downward to 50 cu ft per sec and then held steady until it intersected the recession of the current ground-water hydrograph. The area between these two curves then represents the volume of ground-water runoff from the current rain.

In order to reduce the labor required in measuring the volume of runoff between the upper and lower recessions for each individual storm, the ground-water volume curves, Fig. 4, were developed. The volumes between a number of pairs of recessions were computed in order to determine the family of curves

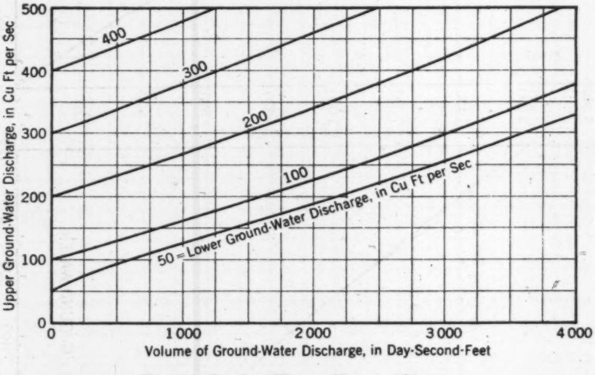

FIG. 4.—GROUND-WATER VOLUME CURVES

shown. Since the lines are practically straight, three points are ample to fix each curve. By entering these curves with the values of the upper and lower recessions at some convenient time after the peak of the ground-water runoff, the volume of subsequent ground-water runoff may be read directly. The total volume under the ground-water hydrograph then may be determined by measuring the remaining area under the hydrograph and adding it to the partial volume determined from the ground-water curves.

Method of Separation.—If the hydrograph of Fig. 2 is assumed to result from the rains of April 1, 5, and 9, the problem is to determine the surface and ground-water runoff for each rain. The ground-water recession that would have occurred if there had been no rain may be drawn by applying the ground-water recession curve (Fig. 3(*a*)), starting at point *A*. In a like manner the ground-water recession curve may be applied starting at point *B*, where the shape of the hydrograph indicates that the flow is entirely from ground water, and carried backward until it departs from the stream-flow hydrograph and reaches a point *C* somewhere near the estimated point of inflection of the falling limb of the total ground-water hydrograph. The remaining part of the ground-water hydrograph between *A* and *C* must be computed by another method.

The change in flow for the 12-hr period *D E* on the stream-flow hydrograph is the sum of the change in flow *D′ E′* of the ground-water recession and the concurrent change in the surface-runoff recession. If the ground-water flow at point *D′* is assumed, an approximate value for the change in ground-water discharge in the succeeding twelve hours may be obtained from Fig. 3(*c*). If this value is subtracted from the observed change in discharge, *D E*, the result is the change in surface runoff in twelve hours. By re-entering Fig. 3(*c*) with this rate of change of the surface runoff, the initial surface discharge, *D D′* may be determined. The value of ground-water flow at point *D′*, computed by subtraction, should agree with the assumed value. If this value is far different from the preliminary estimate, a recomputation is indicated. In this manner that part of the total ground-water hydrograph under the observed stream-flow recessions may be computed. Since the ground-water discharge comes to a peak considerably after the peak surface runoff, this method permits the computation of the peak ground-water flow. With this flow known, the rising limb of the ground-water hydrograph may be drawn in arbitrarily without introducing any appreciable error. The receding limbs of the ground-water hydrographs for the storms of April 1 and 5 may be drawn in with the ground-water recession curve starting at points *F* and *L* respectively. The computed ground-water flows for the storm of April 9 will join the recession near point *C* and the ground-water separation is complete.

Starting with the surface-runoff ordinate *G F*, the ordinates of the surface-runoff recession for the storm of April 1 may be computed from the surface-runoff recession curve (Fig. 3(*b*)) and added to the total ground-water discharge to give the recession *G H*. In a similar manner the recession *J K* for the storm of April 5 can be computed. The volume of ground water may be computed as outlined under the discussion of the ground-water volume curves. The volume of surface runoff may be measured on the hydrograph.

This method of stream-flow separation is based on the time characteristics of the stream flow. This is the classification of stream flow in common use today.

Supplemental Relations.—If selected isolated storms are attacked first with the preceding method of separation, certain data may be developed that will be helpful in the solution of the more complicated cases. The writers have developed several relations in the course of this study that can be used for further separation work or to predict the ground-water hydrograph in forecasting work. These relations are:

Total Runoff Versus Ground-Water Runoff.—Fig. 5 shows the relation between total runoff and ground-water runoff for the storms included in this study. A curve similar to this has been presented by other writers, but as yet no explanation or logical basis for the relation has come to the writers' attention. An attempt was made to determine an infiltration rate or rate of ground-water accretion in inches per hour, but the results varied so widely as to lead to the belief that no consistent rate existed. This belief was further strengthened when the relation shown in Fig. 5 was developed.

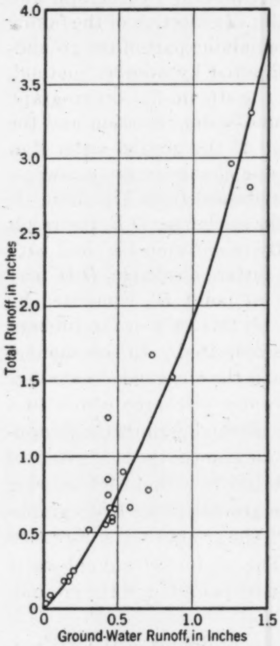

Fig. 5.—Total Runoff Versus Ground-Water Flow

Ground-Water Runoff Versus Net Peak Ground-Water Flow.—Fig. 6 presents the relation between the volume of ground-water runoff and the net peak height of the ground-water hydrograph.

Duration of Rainfall Versus "Time to Ground-Water Peak."—The relation between the duration of rainfall and time-to-peak ground-water discharge is shown in Fig. 7, as indicated by the data for the Valley River. "Time to peak" was computed from the beginning of rainfall.

Analysis of Rainfall Losses

Method.—The analysis of stream flow having been completed, it was possible to compute the total runoff from each individual storm and to determine the total rainfall lost. As outlined under the heading "Theory," this total loss is to be separated into surface loss and field-moisture loss. The problem of dividing this total loss into two parts, each governed by different factors and with all the governing relations unknown, does not lend itself to a direct mathematical solution. Therefore, a more philosophical method was used.

Based on the writers' observations in forecasting work and their knowledge of the area under consideration, preliminary assumptions were made for the variation between rainfall and surface loss. First assumptions for the relation between pan evaporation and soil evaporation were taken from standard texts on hydrology. Using these assumptions, the total losses for several entire flood periods were studied and the indicated adjustments were made to the basic assumptions so that the computed losses approximated the measured total losses for the periods.

With these revised assumptions the study of individual storms was begun. Immediately after a rain of several inches, or a series of lesser rains totaling several inches in a few days, the accumulated pan evaporation between successive rains was computed and multiplied by the assumed ratio between pan and soil evaporation. Adding to this value an estimated surface loss based on the amount of rainfall gave a computed value of the theoretical total loss for the storm. By comparing this computed value with the observed loss, in the light of all the available data, further readjustments of the assumed relations were made and certain other governing rules were established until, by successive approximations, relations that seemed to give the most consistent results were developed. The final relations developed were as follows:

Surface Loss.—As outlined herein under "Theory" the surface loss was found to be proportional to the amount of rainfall. Fig. 6 shows the relation developed for the Valley River Basin. This curve represents the final result of successive approximations which in combination with the final relations for field-moisture loss gives the lowest standard error of estimate for the data studied. The points plotted on the curve represent, for each storm studied, the difference between observed total loss and estimated field moisture loss as computed from the relations outlined under "Analysis of Rainfall Losses: Field Moisture Loss."

Field-Moisture Loss.—As stated under the heading "Theory" the writers have used the pan evaporation as an index of the field-moisture deficiency.

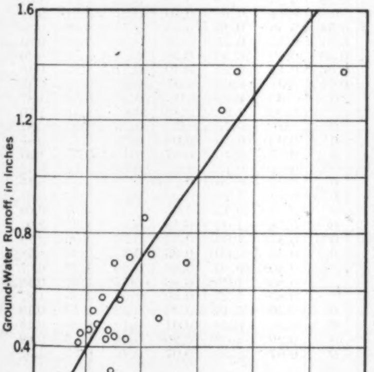

FIG. 6.—GROUND-WATER RUNOFF VERSUS NET PEAK GROUND-WATER FLOW

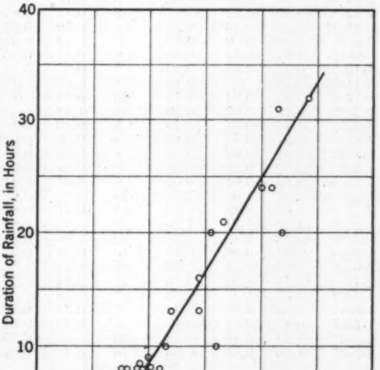

FIG. 7.—DURATION OF RAINFALL VERSUS TIME TO GROUND-WATER PEAK

It was found that the field-moisture deficiency at any time was equal to 0.9 times the total pan evaporation since the ground was last saturated, less any additions made to the field moisture by intervening rains. Studies of storms occurring after protracted dry spells showed that the total observed loss less surface loss averaged very close to 0.8 in. This value was taken as the maximum possible value of field moisture deficiency.

In the course of the study it was observed that for storms of short duration the observed loss less the estimated surface loss was often less than the available field-moisture deficiency despite the fact that there was an adequate volume of rainfall to satisfy the available deficiency. A further study of these cases showed that the difference between observed loss and estimated surface loss could be expressed very closely by 0.05 times the duration of the rain in hours.

TABLE 1.—TABULATION OF DATA AND COMPUTATION OF LOSS AND RUNOFF
(Quantities Expressed in Inches Except Duration, Which Is in Hours)

Date	DATA					FIELD-MOISTURE LOSS							
	Rainfall (1)	Surface runoff (2)	Ground-water runoff (3)	Duration of rainfall (4)	Measured total loss (5)	0.9 × accumulated evaporation (6)	Residual deficiency (7)	Total deficiency (8)	Maximum possible absorption (9)	Estimated absorption (10)	Surface loss (11)	Computed total loss (12)	Computed total runoff (13)
2-9-35	0.96	0.05	0.20	8	0.71	0.68	0	0.68	0.40	0.40	0.30	0.70	0.26
2-13-35	1.19	0.14	0.42	4	0.63	0.14	0.28	0.42	0.20	0.20	0.34	0.54	0.65
2-14-35	1.66	0.37	0.53	8	0.76	0.01	0.22	0.23	0.40	0.23	0.42	0.65	1.01
2-22-35	0.42	0.01	0	2	0.41	0.44	0	0.44	0.10	0.10	0.21	0.31	0.11
2-25-35	1.19	0.14	0.46	7	0.59	0.22	0.34	0.56	0.35	0.35	0.34	0.69	0.50
3-5-35	1.19	0.09	0.58	5	0.52	0.34	0.21	0.55	0.25	0.25	0.34	0.59	0.60
3-6-35	1.01	0.21	0.45	6	0.35	0.06	0.30	0.36	0.30	0.30	0.31	0.61	0.40
3-10-35	0.77	0.03	0.19	8	0.55	0.35	0.06	0.41	0.40	0.40	0.27	0.67	0.10
3-11-35	0.36	0	0.18	13	0.18	0	0.01	0.01	0.65	0.01	0.19	0.20	0.16
3-12-35	2.41	0.94	0.73	13	0.74	0	0	0	0.65	0	0.54	0.54	1.87
1-2-36	3.15	1.02	0.70	24	1.43	....	....	0.80[a]	1.20	0.80	0.65	1.45	1.70
1-4-36	1.15	0.32	0.43	9	0.40	0	0	0	0.45	0	0.33	0.33	0.82
1-6-36	1.22	0.43	0.43	8	0.36	0.03	0	0.03	0.40	0.03	0.34	0.37	0.85
1-8-36	1.82	0.82	0.44	8	0.56	0.07	0	0.07	0.40	0.07	0.45	0.52	1.30
1-9-36	0.70	0.20	0.32	2	0.18	0.02	0	0.02	0.10	0.02	0.26	0.28	0.42
1-15-36	0.68	0.01	0.02	8	0.65	0.24	0	0.24	0.40	0.24	0.25	0.49	0.19
1-18-36	4.41	1.93	1.38	24	1.10	0.22	0	0.22	1.20	0.22	0.82	1.04	3.37
2-17-36	1.06	0.13	0.46	8	0.47	0.05	0	0.05	0.40	0.05	0.32	0.37	0.69
4-1-36	4.30	1.71	1.24	20	1.35	0.58	0	0.58	1.00	0.58	0.81	1.39	2.91
4-5-36	3.88	1.42	1.38	32	1.08	0.34	0	0.34	1.60	0.34	0.75	1.09	2.79
4-9-36	1.47	0.09	0.70	10	0.68	0.33	0	0.33	0.50	0.33	0.39	0.69	0.78
12-6-36	1.93	0.14	0.57	16	1.22	....	....	0.80[a]	0.80	0.80	0.46	1.26	0.67
12-18-36	2.19	0.22	0.73	21	1.24	....	....	0.80[a]	1.05	0.80	0.51	1.31	0.88
1-1-37	3.35	1.30	0.50	27	1.55	....	....	0.80[a]	1.35	0.80	0.68	1.48	1.87
1-5-37	0.53	0.03	0.16	8	0.34	0.20	0	0.20	0.40	0.20	0.23	0.43	0.10
1-6-37	0.33	0.01	0.17	5	0.15	0.01	0	0.01	0.25	0.01	0.19	0.20	0.13
2-6-37	1.34	0.19	0.48	20	0.67	0.29	0	0.29	1.00	0.29	0.36	0.65	0.69
2-9-37	1.90	0.66	0.86	10	0.38	0.07	0	0.07	0.50	0.07	0.46	0.53	1.37

[a] Maximum possible deficiency.

Application of Method.—Table 1 presents the data used in the study and a computation of the losses and runoff for these storms. Cols. 1 to 5 contain observed data obtained from rainfall records and the analysis of the hydrograph. The accumulated evaporation between storms multiplied by 0.9 is entered in Col. 6. The residual field-moisture deficiency existing at the end of the previous rain is given in Col. 7. The sum of these two values gives the total field-moisture deficiency at the beginning of the current storm in Col. 8. The maximum possible absorption by the field moisture in Col. 9 is equal to 0.05 in. multiplied by the duration in Col. 4. The estimated absorption in Col. 10 is equal to the total deficiency except where this is greater than the maximum possible absorption, in which case the latter value is used. The surface loss in Col. 11 is determined from the relation in Fig. 8 by entering with the rainfall for the storm shown in Col. 1. The sum of Cols. 10 and 11 gives the computed total loss in Col. 12 that may be compared with the actual loss in Col. 5. Total runoff in Col. 13 is obtained by subtracting the computed total loss from the rainfall in Col. 1.

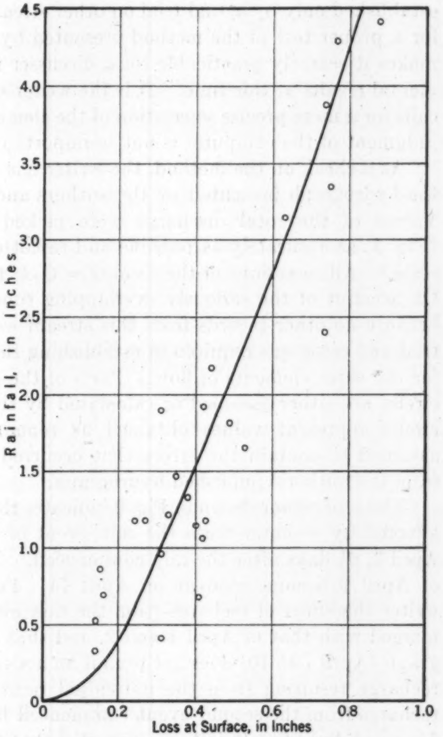

FIG. 8.—SURFACE LOSS CURVE

CONCLUSIONS

(1) The methods of stream-flow separation presented herein are satisfactory for the Valley River. By reducing the degree of personal judgment required in stream-flow analysis to a minimum, consistent results were obtained from all data studied.

(2) Accumulated evaporation from a standard evaporation pan is an index of field-moisture deficiency.

(3) Surface loss varies approximately with the amount of rainfall and appears to approach a practical maximum for high rainfalls.

(4) The methods of estimating runoff presented in this paper represent a distinct refinement over rainfall-runoff relations in which such third variables as initial flow, ground-water flow, or days to the last rain are used as criteria of runoff.

(5) Further study of a parallel nature is indicated in order to determine how these relations vary for other types of basins and for other seasons of the year. Perhaps further study on other basins will furnish data that will permit a greater refinement of the method. However, the average difference between measured and computed losses for the storms studied herein is only + 0.01 in., whereas the standard error of estimate is 0.10 in., which represents rather satisfactory accuracy for this problem.

DISCUSSION

Bertram S. Barnes,[5] Assoc. M. Am. Soc. C. E.—The general applicability of a forecasting procedure developed on one type of stream can be established only by actual trial on other streams. The length of time required for a proper test of the method presented by Messrs. Linsley and Ackermann makes it scarcely practicable for a discusser to present any conclusive experimental results at this time. It is the writer's belief, however, that the scheme calls for a more precise separation of the elements of flow, in which the personal judgment of the computer is not so important a factor.

As a check on the method, the writer has separated the elements of flow in the hydrograph presented by the authors and the results are shown in Fig. 9. Values of the total discharge were picked from the authors' hydrograph (Fig. 2) as accurately as possible and replotted, using a logarithmic discharge scale.[6] All recessions of the form $Q = Q_0 k^t$ therefore appear as straight lines. On account of the seriously overlapping runoff during this short period, and because no other records from this stream were at hand, a certain amount of trial and error was required in establishing the values of the depletion factor k for the three elements of flow. Parts of the graphs that are not marked with circles are either assumed or calculated by the use of k. Points indicated by circles represent values obtained as remainders, by subtraction, and are assumed to contain the errors that occurred in picking the discharge values from the authors' published hydrograph.

The separation shown in Fig. 9 indicates that the ground-water flow was not affected by recharge from the rain event of April 1 and 2 until midnight of April 7, 6½ days after the rain commenced. The recharge from the rain event of April 9 become evident on April 14. For some reason unknown to the writer the effect of recharge from the rain event of April 5 and 6 has become merged with that of April 1 and 2, and that part of the ground-water hydrograph (April 7 to 10) does not permit an accurate separation of the amounts of recharge resulting from the individual rains. It appears that the effect of recharge from the second event commenced between midnight of April 8 and 6 a.m. of April 9. It is to be expected that the higher ground-water level resulting from the first recharge would shorten the time lag of runoff from the second. However, the time lag for the beginning of runoff from the rain of April 9 was about 5½ days, with a higher initial ground-water level. As will be shown, the apparent total recharge was greater than could have been produced by either one of the two rain events alone unless the actual rainfall was materially greater than the value that the authors have obtained by Thiessen's method. The graph of net values of the ground-water event of April 7 to 10 is shown in Fig. 9 as a broken line. The curve of the first part of this graph can be extended to separate the two runoff events. If drawn with the maximum plausible curvature, it would give to the first event about one fourth of the total ground-water runoff for both events, and with a minimum curvature, the first event would receive about half the total, or 1.2 in. When comparing rainfall, runoff, and losses, it was found that, if the first event amounted to less than 0.92 in. or 38% of the total ground-water runoff for the two events, the second rain event would show no loss at all. With the assumed maximum value of 1.2 in. for the first event, the loss from the second rain would be about ½ in. or one third of the total for the two events. It is evident, therefore, that the greater loss occurred from the first rain.

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[5] Regional Hydrologic Engr., Region 3, U. S. Weather Bureau Office, Iowa City, Iowa.

[6] "The Structure of Discharge Recession Curves," by B. S. Barnes, *Transactions*, Am. Geophysical Union, 1939, Pt. IV.

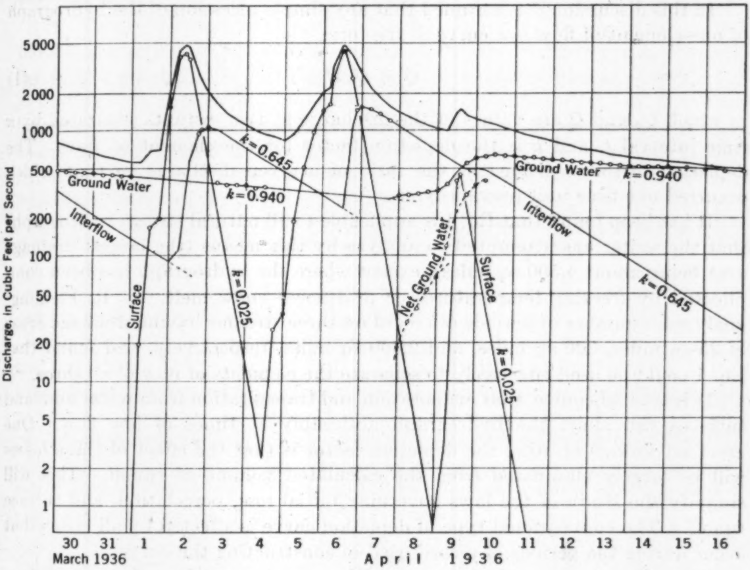

FIG. 9.—SEPARATION OF ELEMENTS OF FLOW, VALLEY RIVER AT TOMOTLA, N. C.

The names used by various writers to designate flows that are neither of ground water nor of surface origin have not been received with general approval. There is also some difference of opinion regarding the cause and nature of such flows. The writer now proposes the name "intermediate flow" or "interflow" and will use the latter term in the present discussion. It was the expressed intention of the authors to combine the elements of interflow and surface flow in the separation that they presented. A comparison of Fig. 2 with Fig. 9 indicates that they have actually included a large part of the interflow in their ground-water hydrograph. It also appears from Fig. 9 and the subsequent tabulation of data that the runoff from interflow is greater than that from surface flow.

It is the writer's opinion that interflows of appreciable amount should be given separate treatment in the forecasting of runoff, for two reasons. In the first place, the volume of interflow appears to be more closely related to the duration of rainfall than is the volume of surface flow or that of ground-water flow. Secondly, the inclusion of variable amounts of interflow with the other elements of flow interferes with the precise analysis of the hydrograph by the unit graph method, by the use of standard depletion curves, or by using the depletion factor.

In this discussion it is assumed that any simple recession of the hydrograph of one element of flow is a curve of the form

$$Q = Q_0 k^t \qquad (1)$$

in which Q_0 and Q are values of the discharge at two instants separated by a time interval t, and k is the depletion factor for the element of flow. The depletion factor is defined as the ratio of a given discharge to that which occurred one time unit previously.

It has been found that Eq. 1 is applicable to all natural stream hydrographs that the writer has attempted to analyze by this means (the largest drainage area being about 4,500 sq miles), except where the hydrograph has been complicated by freezing temperatures or protracted snow melt. J. R. Fleming[7] analyzed a number of periods of record on three streams having drainage areas of 25 sq miles, 320 sq miles, and 2,300 sq miles, respectively, and found that Eq. 1 could be used effectively to separate the elements of flow of all three.

It is true, of course, that evaporation and transpiration from water and land surfaces will affect the hydrograph noticeably at times of low flow. One great advantage of using the depletion factor is that the effect of these losses will be largely eliminated from the calculated volume of runoff. This will simplify the study of the laws governing initial loss, percolation, and surface runoff. The conventional type of depletion curve is affected by all losses that occur during the periods of record used in constructing the curve.

The volume of runoff, R, measured on a recession of an element of flow, is expressed as

$$R = \int Q \, dt \qquad (2)$$

Substituting the value of Q from Eq. 1, integrating, and applying the limits zero and T,

$$R = \int_0^T Q_0 k^t \, dt = \frac{Q_0 (k^T - 1)}{\log_e k} \qquad (3)$$

When Q_0 is the initial discharge and T is the duration of the period, Eq. 3 is the general expression of the volume of runoff during any period of the recession. Let an infinite time be substituted for T, and Q for Q_0, in Eq. 3. Since k is always smaller than unity, the equation becomes

$$R = \frac{-Q}{\log_e k} \qquad (4)$$

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[7] "Analysis of Discharge-Recession Curves for Three Iowa Streams," by J. R. Fleming, unpublished thesis, State Univ. of Iowa, 1941.

Eq. 4 is the expression of the quantity of water (of one element—surface, interflow, or ground water) remaining in storage at any instant during the recession. It is solved by assigning to k its proper value for the element of flow that is being considered, and evaluating Q.

The total volume of water that a given rain event contributes to one element of flow may be calculated from the hydrograph as follows:

(1) Determine the hydrograph of the element of flow by graphical analysis, tabular computation, or a combination of the two. This graph should be drawn to a point that is past the peak and far enough down the recession limb so that Eq. 1 is applicable. Subtract the amounts of flow that are the result of previous rain events. These may be calculated by the use of logarithms, or the previous recession may be prolonged as a straight line on semilogarithmic paper and the values picked off directly. The depletion factor k is evaluated by taking values of Q at any two points on the recession.

$$\text{Log}\, k = \frac{(\log Q_1 - \log Q_2)}{T} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which T is the elapsed time between the two points selected. In this discussion, one day will be used as the unit of time.

(2) Select a point on the hydrograph that is well past the peak of the event and definitely on the recession limb. Calculate the net runoff up to that point in the usual manner by the summation of values of $Q\,t$ for periods of one hour, one day, or any other convenient basis. In Eq. 4, substitute for Q the ordinate of the discharge hydrograph at the point selected and determine the value of R. Add the result to the volume of runoff previously computed for the initial part of the event. The sum will be the total volume of water that the storm has contributed to that element of flow.

The results of the foregoing analysis, compared with that of the paper, are presented in Table 2 with those obtained by the authors. The values of the depletion factor k were found to be as follows: Surface flow, $k = 0.025$; interflow, $k = 0.645$; and ground-water flow, $k = 0.940$. It is probable that a more accurate determination of these values could have been made if a longer period of record than 18 days had been used. The consistent plotting of the remainders in Fig. 9 indicates that the adopted values are sufficiently accurate for the present purpose.

TABLE 2.—TABULATION OF DATA AND COMPUTATION OF LOSS

(Quantities of Water are Expressed in Inches)

Date (April, 1936)	Rainfall	Duration (hours)	Runoff, surface (writer)	Runoff, interflow (writer)	RUNOFF, GROUND WATER		RUNOFF, SURFACE PLUS INTERFLOW		RUNOFF, TOTAL		LOSS	
					Author	Writer	Author	Writer	Author	Writer	Author	Writer
1...........	4.30	20	0.98	0.96	1.24	(1.1)	1.71	1.94	2.95	(3.04)	1.35	(1.26)
5...........	3.88	32	1.17	1.23	1.38	(1.3)	1.42	2.40	2.80	(3.70)	1.08	(0.18)
Total, 1 and 5....	8.18	52	2.15	2.19	2.62	2.40	3.13	4.34	5.75	6.74	2.43	1.44
9...........	1.47	10	0.07	0.08	0.70	0.19	0.09	0.15	0.79	0.34	0.68	1.13
Total, 1, 5, and 9.	9.65	62	2.22	2.27	3.32	2.59	3.22	4.49	6.54	7.08	3.11	2.57

Quantities enclosed in parentheses depend upon an assumed amount of 1.1 in. for the ground-water runoff resulting from the rain of April 8 and 9. This value has previously been shown to lie between 0.92 in. and 1.2 in. The total amounts of runoff and loss shown in Table 2 are those resulting only from the three rain events listed. Two other small rains occurred during the period but are assumed to have produced no runoff or recharge to ground water. There was also runoff resulting from precipitation that occurred previous to March 30. At midnight of March 29 there was an interflow of 330 cu ft per sec, corresponding to an amount of 0.25 in. in storage. It seems likely that this flow resulted from the thawing of frost in the soil or of snow in gullies and depressions. There was a ground-water flow of 480 cu ft per sec, corresponding to an amount of 2.61 in. in storage.

The storage equation may be applied to these data, using the form:

$$\text{Rainfall} - \text{runoff} - \text{loss} = \text{change in contents} \qquad (6)$$

The total loss shown in Table 2, therefore, may be checked with reasonable accuracy by the following summary, using storage values determined by Eq. 4:

Description	Loss (inches)
Total rainfall, March 30 to April 16, inclusive	9.70
Total runoff, March 30 to April 16, inclusive	7.42
Amount in storage at midnight, March 29	2.86
Amount in storage at midnight, April 16	2.54
Decrease in storage, March 30 to April 16, inclusive	0.32
Total loss, March 30 to April 16, inclusive	2.60

The writer believes that the suggestions contained in this discussion will not only simplify the work required in forecasting by methods similar to that presented by the authors, but will also eliminate, to a large extent, the errors arising from personal judgment in separating the elements of flow. The precise separation of the three elements should make each of them more readily predictable in amount. Whenever two elements of flow are added together in unknown proportions, the forecasting of their total amount and rate of depletion is necessarily complicated and uncertain, especially since a set of natural conditions favorable to a large flow of one element will not necessarily favor a large flow of either of the others.

Specifically, the application of the principle of the depletion factor would make straight lines of the curves given in Fig. 3. The slope of each line would be equal to the reciprocal of the appropriate depletion factor. It has been the writer's experience that ground-water curves of the type shown in Fig. 3(*a*), when extended downward to zero discharge, do not ordinarily pass through the origin. This fact suggests that if today's ground-water flow were zero, tomorrow's would be negative; that is, if any water were to be introduced into the stream channel tomorrow, it would be subject to a loss into the stream bed.

Likewise, Fig. 4 would become a single straight line with values of the net increment of ground-water discharge for its ordinates and recharge to ground-water storage for abscissas. Similar diagrams could be drawn for surface flow and interflow, but such diagrams would have to be used with care and skill in order that the proper allowance might be made for runoff occurring previous to the peak.

The authors' procedure of separating initial loss into two parts—surface loss and field-moisture deficiency—and comparing the latter with losses from an evaporation pan, is a distinct departure from any previous practice known to the writer. It is believed that the plan merits trial under various conditions of climate, soil, agricultural development, and topography. The degree of development of growing plants, especially annual crops, may have to be considered when comparing evaporation and transpiration from a natural watershed with the measured evaporation from a standard pan.

Richard Van Vliet,[8] **Esq.**—Some of the factors that enter into the so-called water-loss curve have been evaluated in this paper. Water-loss curves based on monthly or seasonal rainfall have been used for a number of years by hydrologists in studying the yield of drainage basins. The authors have developed such a curve for the Valley River, based on individual stream rises. The writer wishes to comment on a few of their conclusions.

The authors refer to one of the ways by which rainfall is disposed of as "surface loss" which, as defined by the authors, is in reality the sum of two quantities—interception and depression storage.[9] In order to determine these quantities separately, a study of the storm rainfall pattern is required. It cannot be done with any degree of success by considering only the total amount of storm rainfall and duration. The volume of interception and depression storage depends on rain intensity. Interception occurs at all rainfall intensities, whereas depression storage cannot occur unless the rate of rainfall is greater than the infiltration capacity. The authors state that "surface loss" becomes constant at some extremely high rainfall, but this is true only if the particular "high rainfall" is equal to the infiltration capacity. Excluding infiltration, interception becomes constant at very moderate rain intensities after interception storage is filled. This paper is a study of surface runoff phenomena, and in such a study the rainfall must be considered in detail, in regard to both its intensity and duration.

The authors state that the duration of surface runoff may be taken as the length of the unit graph base. From analyses of various stream rises, together with the corresponding channel inflow graphs, it has been found that direct surface runoff ends at about the point of inflection on the recession side of the hydrograph of a stream rise.[10] Beyond the point of inflection the recession side of the hydrograph is composed of two items—channel storage and ground water. The area under the recession side of the hydrograph beyond the point of inflection and above the ground-water flow curve is usually referred to as "channel storage." The use of the term "surface-runoff recession curve" by the authors to express this quantity is deemed unfortunate, as the term "channel storage recession curve" is generally used to express this quantity, since surface runoff has ended. Attention is called to the fact that the method of developing channel storage recession curves was first published by Robert E. Horton,[10] M. Am. Soc. C. E.

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[8] Prin. Asst. Engr. to Robert E. Horton, Cons. Hydr. Engr., Voorheesville, N. Y.

[9] "Hydrologic Interrelations of Water and Soils," by Robert E. Horton, *Proceedings*, Soil Science Soc. of America, Vol. 1, 1937, pp. 401–429.

[10] "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935.

In Fig. 7 the authors correlate "duration of rainfall, in hours," with "time to ground-water peak, in hours." A study of the paper leads to an understanding that duration of rainfall is the total duration of storm rainfall, including any residual rainfall (rainfall in which the intensity is less than the infiltration capacity). The writer cannot agree with the authors that this relation holds true for all cases if total rainfall is accepted under their definition. For example, consider two storms, each having identical periods of rainfall excess of 1-hr duration; that is, with rainfall intensity greater than infiltration capacity. In one storm, however, assume also a period of residual rainfall in which the intensity is, say, 0.05 in. per hr, or well below the infiltration-capacity rate, and lasting for 4 hr. In the first storm the duration of rainfall is 1 hr and in the latter storm 5 hr. Other factors being equal, the time at which the ground-water flow will reach its peak will be practically the same for both storms. It seems to the writer that any attempt to correlate total duration of rainfall with time for ground water to reach its peak will give results of little value if the total duration of rainfall includes the period of residual rainfall.

LeRoy K. Sherman,[11] **M. Am. Soc. C. E.**—The quantity of moisture in the soil materially governs the intake of rainfall and consequently affects the quantity of surface runoff. Several empirical procedures have been used to approximate this important antecedent soil-moisture index. The writer[12] and others have based their approximations on percentage of antecedent rainfalls together with the intervening amounts of "drying out" time. The authors have estimated the amount of "drying out" with the logical and more specific use of an evaporation factor.

TABLE 3.—CHECK COMPUTATIONS OF LOSS AND RUNOFF[a]

Date	Rainfall (1)	− Surface runoff (2)	= Infiltration and initial loss (A)	Ground-water runoff (3)	+ Absorption (10)	+ Surface loss (11)	= Infiltration and initial loss (B)	Difference (Col. A minus Col. B)
2-13-35	1.19	0.14	1.05	0.42	0.20	0.34	0.96	−0.09
2-14-35	1.66	0.37	1.29	0.53	0.23	0.42	1.18	−0.11
3- 5-35	1.19	0.09	1.10	0.58	0.25	0.34	1.17	+0.07
3- 6-35	1.01	0.21	0.81	0.45	0.30	0.31	1.06	+0.25
3-10-35	0.77	0.03	0.74	0.19	0.40	0.27	0.86	+0.12
3-12-35	2.41	0.94	1.47	0	0.73	0.54	1.29	−0.20
4- 1-36	4.30	1.71	2.59	1.24	0.58	0.81	2.63	+0.04
4- 5-36	3.88	1.42	2.46	1.38	0.34	0.75	2.47	+0.01

[a] The column numbers correspond with those in Table 1.

In Table 1 the usual practice of segregating base flow, or ground-water flow, and surface runoff has not been followed. Apparently the authors assume that ground-water runoff (Col. 3) is due to infiltration, not absorbed

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[11] Cons. Engr., Chicago, Ill.

[12] "The Unit Hydrograph and Its Application," by L. K. Sherman, *Journal*, Associated State Eng. Societies, April, 1941, p. 4.

(Col. 10), and that this ground-water runoff all comes from the given storm on the basin. This may or may not be true. Ground-water runoff may originate, in part, from territory outside of the basin. A part of the ground-water flow may be measured by the stream gage and a part of this flow may by-pass the gage in transit through the soil. It is possible to check the assumption made in this case.

In Table 3 the writer has selected, at random, eight storms from Table 1. If the ground-water runoff is all accounted for in the manner assumed by the authors, then the figures in Col. B will equal the corresponding figures in Col. A. The column of differences A − B is the error in the authors' assumption, predicated upon correctness of computed absorption and surface loss. For a pioneering job this showing is not bad. It indicates that in this particular basin the authors' assumption as to ground-water flow and origin is correct. The authors have not made use of the more recent concepts of the infiltration theory, such as rainfall excess, the recession curve, and the mass curve of infiltration capacity. The writer used a few of the authors' antecedent soil moisture figures in such tests, but the number so used was not sufficient to draw any conclusions.

Further research, on the derivation of initial soil moisture along the lines suggested by the authors, is much needed in applied hydrology.

Franklin F. Snyder,[13] **Assoc. M. Am. Soc. C. E.**—A concise description of detailed studies of rainfall and runoff relations is presented in this paper. The methods followed give computed values of runoff that agree quite favorably with the observed values for the storms studied.

The methods of analysis followed and relationships developed are quite similar to those used in the Pennsylvania hydrologic investigations[14] for development of flood forecasting procedures and used by the writer[15] to compute a year's record of runoff from rainfall for a 1,147-sq-mile drainage area with no reference to observed stream flow. The authors have used pan evaporation as an index of soil moisture variation, whereas time and temperature were used for that purpose in the studies just mentioned.

There is one striking difference either in conception or in nomenclature: The writer believes that the author's "surface loss," which consists of by far the greater part of the difference between rainfall and runoff for the storms listed in Table 1, is really accretion to soil moisture and that the loss classified as addition to field moisture is probably an initial or surface loss. The most convincing indication of this is a comparison of pan evaporation and surface loss for periods of time for which complete data are given in Table 1.

For all except four of the storms listed the surface loss (Col. 11) is greater than the values in Col. 6 (0.9 × accumulated evaporation). For the period including the storms of January 4, 1936, to February 17, 1936, the total surface loss is 2.77 in. and the total evaporation is 0.70 in. $\left(\frac{0.63}{0.9}\right)$. Surface loss is defined in the paper (see heading "Theory: (1) Surface Loss") as "that part of the rain which is intercepted by the vegetal cover and natural or artificial retention basins from which it eventually evaporates and is thus prevented from entering the stream channels." It is difficult to understand how this surface loss by evaporation can be four times the observed evaporation from a pan or how there can be enough natural or artificial surface retention basins to store 2.07 in. (2.77 − 0.70) for later evaporation. For a longer or a more rainy period, the procedure followed might call for a still larger amount of surface storage. The writer believes that a considerable part of this rainfall became soil moisture.

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[13] Associate Hydrologic Engr., U. S. Weather Bureau, Washington, D. C.

[14] "Report of Cooperative Hydrologic Investigations," by Pennsylvania Dept. of Forests and Waters, U. S. Geological Survey, and U. S. Weather Bureau, Commonwealth of Pennsylvania, August, 1939.

[15] "A Conception of Runoff Phenomena," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, 1939, Pt. 4, pp. 725–738.

In 1937 the writer made extensive studies of rainfall and runoff relations for storms occurring in the period October, 1934, to September, 1936, on the Hiwassee River Basin above Reliance, Tenn., which includes the Valley River studied by the authors. Using what seemed to be rational values of evaporation and transpiration, a maximum value of 4.0 in. of accumulated soil moisture above a base of 0.00 was obtained in March for the winter of 1934–1935 and 7.5 in. in February for the winter of 1935–1936. At the time it was concluded that the water went to soil moisture rather than deep seepage since the method of computing evaporation and transpiration reduced the accumulated values of soil moisture to the assumed base of 0.00 in. in what appeared to be a reasonable manner during the succeeding summers. These values of variation in soil moisture are much greater than the maximum possible value of 0.80 in. field-moisture deficiency assumed by the authors. This value of 0.80 in. would appear to be a more reasonable maximum value of surface loss.

There is also some question as to the general applicability of the empirical value of 0.05 in. per hr maximum possible absorption. This criterion controls the values of estimated absorption for seven of the first eight storms listed in Table 1, all in February or March of 1935, and does not control in any of the other storms. Better results would have been obtained if the assumed maximum rate of absorption had been larger for the first three storms concerned (the storm of February 14, 1935, is not involved) and smaller for the last four storms. This indicates that the rate decreased with a continuation of frequent rains.

The question raised as to nomenclature and definitions does not invalidate the very good results obtained by the authors, but it would be of considerable importance to someone trying to apply the procedures to other areas.

Ray K. Linsley, Jr.,[16] **and William C. Ackermann,**[17] **Juniors, Am. Soc. C. E.**—The response to this paper presenting a new method of runoff prediction is gratifying. Several helpful suggestions have been made.

The writers are quite in agreement with Mr. Barnes' statement that a more precise determination of the runoff associated with a given storm event would aid considerably in the solution of the entire problem. It is unfortunate that no better data were available for Mr. Barnes to check the writers' figures than the small-scale hydrograph published with the paper. The differences shown in Table 2 between his values of runoff and those computed by the writers may result from slight errors in the published hydrograph as a result of several drafting operations incidental to publication.

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[16] Asst. Hydrologic Engr., U. S. Weather Bureau, Sacramento, Calif.

[17] Asst. Hydr. Engr., TVA, Knoxville, Tenn.

The writers have attempted to analyze hydrographs of several streams in the Tennessee Valley by the procedure suggested by Mr. Barnes. The results were not sufficiently consistent to justify the inclusion of a third runoff element.

Mr. Barnes suggests that the depletion curve of Fig. 3(*a*) should not pass through zero. The curve actually becomes tangent to a 45° line at 50 cu ft per sec, indicating that for ground-water flows of 50 cu ft per sec or less there is no change in flow from day to day.

Mr. Van Vliet questions the writers' failure to include rainfall intensity as a factor in their analysis. This is due primarily to the requirement that river forecasting procedure be simplified to the point where the maximum accuracy is achieved in the time available to the user. The lengthy job of working up hourly rainfall amounts is beyond the scope of such work. Moreover the determination of hourly average rainfall amounts for basins of a hundred square miles or larger is difficult and possible of only approximate solution at best. The use of a limiting rate of absorption in inches per hour, however, does recognize the importance of considering duration or average rate of rainfall for each storm event.

Mr. Van Vliet is correct in stating that depression storage cannot occur unless the rate of rainfall exceeds the infiltration capacity. In a natural stream basin the infiltration capacity will vary from point to point with values of zero at some points. Depression storage therefore can begin to accumulate almost with the first drops of rain. As the rain continues the depressions will gradually fill, and if a sufficient volume of rain falls in a single storm they will fill to a point where no additional water can be accommodated. It was on this basis that the writers state that surface loss—that is, depression storage and interception loss—increases with increasing volume of rainfall until, for high amounts of total rainfall, a practical maximum possible detention is reached.

The writers' statement (see "Analysis of the Hydrograph: Ground-Water Recession Curve") that "The duration of surface runoff may be taken as the length of the unit graph base * * *" is not correctly worded. It should have read "The duration of outflow of surface runoff at the gaging station may be taken as the length of the unit graph base." The term "surface runoff recession" was chosen because the recession curve so designated includes only that portion of the channel storage which, in the writers' opinion, was derived from surface runoff. Undoubtedly some improvement in standard terminology would be of great value to the entire problem of rainfall and runoff study.

Throughout the paper the duration of runoff used was an estimated effective value derived by averaging the approximate durations of the mass of the rainfall at the various stations. Insignificant amounts before or after the main storm were not included in the determination.

Mr. Snyder's comments on the definition of surface loss given by the writers are partly correct. Evaporation from depression storage and interception could only account for a portion of the computed surface loss and the balance must go into soil moisture. The value of 0.8 in. for the maximum possible soil moisture deficiency was based on data from relatively few storms. However, additional studies of numerous summer storms indicate a value of 1.0 in. which is a rather close check on the original figures. This should be considered as a maximum soil moisture deficiency possible through evaporation rather than as a total possible volume of soil moisture that the basin is capable of retaining.

The method of determining the surface and field moisture loss was by trial and error to find the values which best fit the theory. The exact distribution of the losses from the storms studied will probably never be known. That portion of the loss which appears to vary with volume of rainfall does not seem to be attributable logically to field moisture absorption. Conversely it would not appear logical that evaporation is an index of interception capacity of depression storage.

Subsequent to the publication of the paper, the study was duplicated independently using data from a large number of summer storms on the Valley River Basin. The results obtained agreed quite closely with those of the original study. The results for the four important factors used in the proposed method are as follows:

Evaporation Factor.—The factor of 0.9 was not changed.

Surface Loss.—The surface loss curve derived was shifted laterally about 0.05 in. to the right. This change may indicate more interception during the summer months although the change is too small to be conclusive.

Maximum Possible Field Moisture Deficiency.—As stated elsewhere in this discussion, a value of 1.0 in. was determined for this factor. This value is probably more reliable than the earlier value of 0.8 in. since more data were available for study.

Maximum Possible Rate of Absorption.—The rate of absorption seems to be about 0.10 in. during the first hour of a storm, decreasing until the average rate of 0.05 in. per hr applies for storms of 12 hr or longer. Improvement in the definition of this rate results from the large number of short-duration storms included in the summer data. There is some likelihood that these rates change from storm to storm as Mr. Snyder suggests. The writers made no attempt to determine this rate quantitatively since the change seems to be rather small in any case.

In closing, the writers wish to thank those who contributed to the discussion of this paper. It seems likely that a real test would come only from carefully conducted experiments on additional watersheds with adequate rainfall and runoff records, a central evaporation station, and sufficient field surveys to observe the soil conditions thoroughly.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2148

THE SUSPENSION BRIDGE TOWER CANTILEVER PROBLEM

BY BLAIR BIRDSALL,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. FRANCIS P. WITMER, JOHN I. COPP, AND BLAIR BIRDSALL.

SYNOPSIS

A generalized theory is offered in this paper for the study of the behavior of suspension bridge towers under the action of forces in a plane parallel to the axis of the bridge.[2] Reduced to its essentials, the structure under consideration is a vertical cantilever, of variable cross section, with a fixed base. The top of the cantilever is free to rotate, and is deflected by means of a horizontal and a vertical load. The usual problem is to find the horizontal tower-top force required to produce a certain deflection in conjunction with a given vertical tower-top load. As soon as the horizontal force has been found, it is a simple matter to find any other properties of the deflected tower, such as the elastic curve or the stresses.

The theory for the so-called "General Case" includes the effect of the weight of the tower as a producer of bending moment in the deflected tower, as well as the effect of an eccentrically placed tower-top load and eccentric loads at the roadway level. (In a mathematical sense, the true general case has not been used in the body of this paper, as the emphasis is on practical application. Thus, the so-called "General Case" presents the tools by means of which the true general case may be approached as nearly as necessary to produce what the writer considers to be a sufficient degree of accuracy for the most complicated case ordinarily encountered in practice.) Simplifications of the "General Case" for many special cases are either given in detail or outlined.

The results of applying the theory to three tower shafts are used as a means of indicating the accuracy of several approximate methods which may be used as practical tools for solving this problem if the precision of the mathematical treatment is not required.

NOTE.—Published in April, 1941, *Proceedings*.

[1] Asst. Chf. Engr., Bridge Dept., John A. Roebling's Sons Co., Trenton, N. J.

[2] The more detailed study from which this paper was prepared has been placed on file for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

INTRODUCTION

The mathematical principles underlying the development of this theory are not new. In 1882 the late S. W. Robinson,[3] M. Am. Soc. C. E., published the general solution of the differential equation for bending moment in a beam or column caused by combined transverse and axial loads. The method of solution for this type of equation (linear differential equation with constant coefficients) may be found in any book on differential equations. The salient features of the application of the general solution to the problem of flexure and direct stress have been summarized by Francis P. Witmer,[4] M. Am. Soc. C. E. The applications of the general solution to some of the cases included in this paper have been published[5] in a standard textbook.

In this paper, the writer has attempted to make a comprehensive presentation of the subject, combining the fundamental theory for handling the problem under any combination of loads with a description of some practical short-cut methods, for the use of both the student and the practicing engineer.

OUTLINE OF THE PROBLEM AND DEFINITION OF SYMBOLS

The problem consists of finding the horizontal load required to produce a desired deflection at the top of a vertical, fixed-base cantilever which is acted upon by certain known vertical loads. The conditions of the problem are illustrated by Fig. 1. All loads are expressed in pounds, all dimensions in feet, and all moments in pound-feet. Other definitions are:

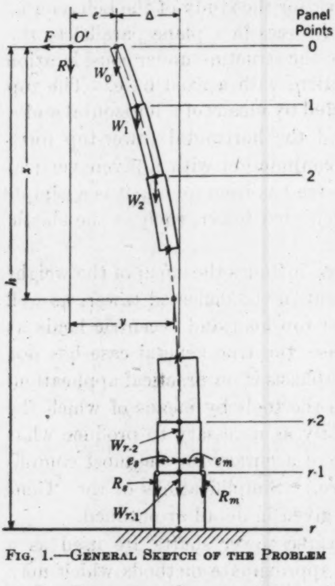

FIG. 1.—GENERAL SKETCH OF THE PROBLEM

F = desired horizontal tower-top load;

R = known vertical external load (cable reaction) on tower top;

e = eccentricity of R with respect to center line of top of tower;

Δ = known required deflection of tower top;

$0, 1, 2 \cdots (r-1), r$ = panel points along the center line of tower at which there are changes in moment of inertia, or known concentrated loads, or both;

$W_0, W_1, \cdots W_{r-1}$ = parts of tower weight assumed to be concentrated at the panel points indicated by the subscripts;

R_s, R_m = reactions on tower at roadway level;

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[3] "Strength of Wrought Iron Bridge Members," by S. W. Robinson, Van Nostrand Science Series, No. 60, D. Van Nostrand, Publisher.

[4] "Basic Formulas for Combined Flexure and Direct Stress," by Francis P. Witmer, *Civil Engineering*, December, 1937, p. 855.

[5] "Modern Framed Structures," by the late J. B. Johnson, C. W. Bryan, Members, Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Pt. II, 10th Ed., John Wiley & Sons, Inc., p. 278.

e_s, e_m = eccentricity of roadway reactions with respect to center line of tower at that point;

x_1, x_2, $\cdots$ x_{r-1}, x_r = known vertical distances from tower top to panel points;

y_1, y_2 $\cdots$ y_{r-1}, y_r = ordinates to panel points from line of action of R;

S_a = length of panel between panel points $a - 1$ and a;

h = total height of tower;

I_a = moment of inertia (in²-ft²) of the tower shaft between panel points $a - 1$ and a;

E = modulus of elasticity of tower material (pounds per square inch);

M = bending moment;

A, B, D, and C = constants determined from the loading conditions; and

G_a, K_a = integration constants for the panel between points $a - 1$ and a.

Basic Assumptions.—Six basic assumptions should be kept in mind:

1. All vertical distances remain constant;
2. Shear deformation is zero;
3. The tower base is rigidly fixed;
4. The tower weight is applied as a series of concentrated vertical loads;
5. The tower shaft is made up of a series of panels in each of which the moment of inertia is constant; and
6. The moment arm of R does not change due to rotation of the tower top.

The deflections and deformations are so small in comparison with the vertical dimensions that the error introduced by Assumption 1 is negligible. The heaviest concrete pier will yield to a certain extent, but the deflecting load and tower moments based on Assumption 3 will be on the side of safety. In Assumption 4 the concentrations are placed at the points of change in moment of inertia. However, the weights can be applied at any points without affecting the method of solution. The labor of computation would be increased, however, as each concentration as well as each change in moment of inertia requires an additional pair of integration constants unless they occur at the same point.

The Problem.—

Given: All external loads except deflecting load F; all physical characteristics of the tower shaft; and all dimensions, including the deflection Δ, except the ordinates to the elastic curve.

To Find: The deflecting load F, the ordinates to the elastic curve, and any other desired information, such as the bending moments, the stresses, or the tangents to the elastic curve.

THE DIFFERENTIAL EQUATION

From elementary mechanics, the general differential equation for bending moment is

$$\frac{d^2y}{dx^2} = \frac{M}{E\,I} \qquad (1)$$

The moment due to the type of loading contemplated in this paper may be expressed as follows:

$$\frac{M}{E\,I} = \pm A\,y \pm B\,x \pm D\,x^2 \pm C \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the sign depends on the direction of rotation. The function of x represents the moment due to transverse loads or any other moment which is independent of the ordinate y, and no combination of transverse loads can lead to a higher power of x than the second. The product $A\,y$ represents the moment due to axial loads. There will be no higher power of y than the first as long as concentrated loads only, and not distributed loads, have moment arms that depend on the ordinate y.

In order to determine the proper signs, assume that the cantilever of Fig. 1 is rotated 90° in a counterclockwise direction, and place the origin of coordinates at O'. By the usual convention, it is apparent that the column carries negative moment. Any load to the left of a section, which has a counterclockwise direction of rotation about the neutral axis of the section, helps to produce the negative moment.

Thus, since the predominating axial loads have counterclockwise moments about sections of the tower to the right of the points of application of these loads, the resulting sign of $A\,y$ must be negative. Since y is positive, however, A must be negative, and the term, as it appears in Eq. 2, is $-A\,y$. Since x is always positive, the same reasoning applies to all other terms, and Eq. 2 becomes:

$$\frac{d^2y}{dx^2} = \frac{M}{E\,I} = -A\,y - B\,x - D\,x^2 - C \ldots (3)$$

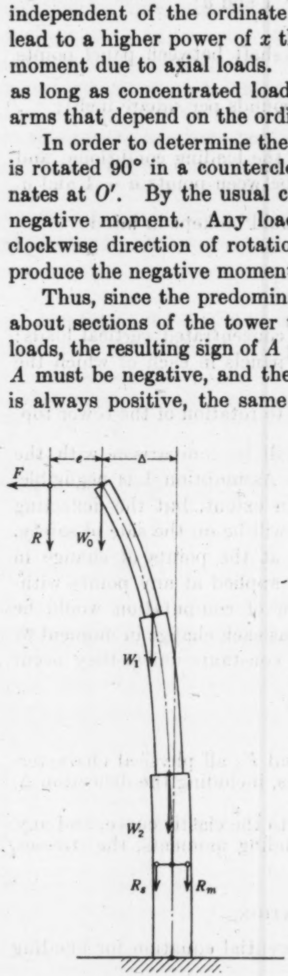

FIG. 2.—THE "GENERAL" CASE

It is shown elsewhere[4] that, in general, the term for the axial loads is negative for loads producing compression, and positive for loads producing tension, and that the signs of the terms for the transverse loads depend on the direction of rotation. The general solution of Eq. 3 is:

$$y = -\frac{K}{A}\sin x\sqrt{A} - \frac{G}{A}\cos x\sqrt{A} - \frac{D}{A}x^2 - \frac{B}{A}x + \frac{2\,D}{A^2} - \frac{C}{A} \ldots\ldots (4)$$

The formal derivation of this solution is given in Appendix I of the complete paper.[2] Expressions for slope, bending moment, and shear may be found in terms of the independent variable x by taking successive derivatives of Eq. 4; thus—

Slope of the elastic curve:

$$\frac{dy}{dx} = -\frac{K}{\sqrt{A}}\cos x\sqrt{A} + \frac{G}{\sqrt{A}}\sin x\sqrt{A} - \frac{2\,D}{A}x - \frac{B}{A} \ldots\ldots\ldots (5)$$

bending moment:

$$E\,I\,\frac{d^2y}{dx^2} = E\,I\left(K\sin x\sqrt{A} + G\cos x\sqrt{A} - \frac{2\,D}{A}\right)\ldots\ldots\ldots(6)$$

shear:

$$E\,I\,\frac{d^3y}{dx^3} = E\,I\,(K\sqrt{A}\cos x\sqrt{A} - G\sqrt{A}\sin x\sqrt{A})\ldots\ldots\ldots(7)$$

THEORY

The "General" Case; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight and Eccentric Roadway Loads Considered.—To simplify the presentation, a tower shaft having three different values of I is used. The roadway loads are placed at a section that lies within the lowest of the three tower divisions. The problem is illustrated by Fig. 2.

For this case, four equations of the following form are obtained from Eqs. 3 and 6:

$$\frac{d^2y}{dx^2} = -\,n_a^2\,y - B_a\,x - C_a = K_a\sin n_a\,x + G_a\cos n_a\,x\ldots\ldots(8)$$

The conditions of the problem yield the following values for the loading constants of Eq. 3: With $D = 0$ in all panels, let $A = n^2$; then,

$$\left.\begin{aligned}
A_1 &= \frac{R + W_0}{E\,I_1} = n_1^2, \quad B_1 = \frac{F}{E\,I_1}\\
A_2 &= \frac{R + W_0 + W_1}{E\,I_2} = n_2^2, \quad B_2 = \frac{F}{E\,I_2}\\
A_3 &= \frac{R + W_0 + W_1 + W_2}{E\,I_3} = n_3^2, \quad B_3 = \frac{F}{E\,I_3}\\
A_4 &= \frac{R + W_0 + W_1 + W_2 + R_s + R_m}{E\,I_3} = n_4^2, \quad B_4 = B_3\\
C_1 &= -\,\frac{W_0\,e}{E\,I_1}\\
C_2 &= -\,\frac{W_0\,e + W_1\,y_1}{E\,I_2}\\
C_3 &= -\,\frac{W_0\,e + W_1\,y_1 + W_2\,y_2}{E\,I_3}\\
&\text{and}\\
C_4 &= -\,\frac{W_0\,e + W_1\,y_1 + W_2\,y_2 + R_s\,y_3 - R_s\,e_s + R_m\,y_3 + R_m\,e_m}{E\,I_3}
\end{aligned}\right\}\ldots(9)$$

By Eq. 5, for this case, there are four equations of the following form for the slope of the elastic curve:

$$\frac{dy}{dx} = -\,\frac{B_a}{n_a^2} - \frac{K_a}{n_a}\cos n_a\,x + \frac{G_a}{n_a}\sin n_a\,x\ldots\ldots\ldots\ldots(10)$$

The eight equations of the form of Eqs. 8 and 10 (one formula for moment and one for slope in each panel) contain twelve unknowns, including the eight constants of integration K_1, K_2, K_3, K_4, G_1, G_2, G_3, and G_4; the load F; and the ordinates y_1, y_2, and y_3. Force F is included in the B-terms, and the ordinates are included in the C-terms, as may be noted in Eqs. 9. They can all be found by applying the following known conditions (the number after the dash in the last column indicates the value of a for the particular panel in question):

Condition No.	Statement of condition		Applies to Eq.:
I	When $x = 0$,	$y = e$	(8–1)
II	When $x = x_1$,	$y = y_1$	(8–1)
III	When $x = x_2$,	$y = y_2$	(8–2)
IV	When $x = x_3$,	$y = y_3$	(8–3)
V	When $x = h$,	$y = e + \Delta$	(8–4)
VI	When $x = h$,	$\frac{dy}{dx} = 0$	(10–4)
VII	When $x = x_1$,	$M_{(0-1)} = M_{(1-2)}$	(8–1) and (8–2)
VIII	When $x = x_2$,	$M_{(1-2)} = M_{(2-3)}$	(8–2) and (8–3)
IX	When $x = x_3$,	$M_{(3-4)} = M_{(2-3)} - R_s e_s + R_m e_m$	(8–3) and (8–4)
X	When $x = x_1$,	$\frac{dy}{dx}(0\text{–}1) = \frac{dy}{dx}(1\text{–}2)$	(10–1) and (10–2)
XI	When $x = x_2$,	$\frac{dy}{dx}(1\text{–}2) = \frac{dy}{dx}(2\text{–}3)$	(10–2) and (10–3)
XII	When $x = x_3$,	$\frac{dy}{dx}(2\text{–}3) = \frac{dy}{dx}(3\text{–}4)$	(10–3) and (10–4)

(It should be noted that Eq. 8 must be multiplied by the proper $E\,I$-values in order to obtain the bending moment when applying Conditions VII, VIII, and IX.)

The twelve equations obtained by applying these conditions may be solved to obtain the twelve unknowns.

Special Case I; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight Considered, but Not Eccentric Roadway Loads.—Use the same procedure as for the "General" Case. Referring to Fig. 1, R_s and R_m now disappear and the problem reduces to that for a tower with three panels. There are nine unknowns: The six constants of integration K_1, K_2, K_3, G_1, G_2, and G_3; the load F; and the ordinates y_1 and y_2. They may all be found by applying nine known conditions which are identical with Conditions I to III, V to VIII, X, and XI of the "General" Case. (In any case that excludes eccentric roadway loads, a concentric load placed at the roadway level may be handled as if it were a tower weight concentration.)

Special Case II; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight and Roadway Loads Not Considered.—Eqs. 9 are now greatly modified, inasmuch as R_s, R_m, and all values of W disappear. Since the shear is now continuous through the panel points, shears instead of slope may be equated at these points. The basic equations then become—

For moment:

$$\frac{d^2y}{dx^2} = -n_a^2\,y - B_a\,x = K_a \sin n_a\,x + G_a \cos n_a\,x \,\ldots\ldots\ldots (11)$$

for shear:

$$\frac{d^3y}{dx^3} = -n_a^2\frac{dy}{dx} - B_a = K_a\, n_a \cos n_a\, x - G_a\, n_a \sin n_a\, x \;\ldots\ldots (12)$$

The solution is obtained by finding the seven unknowns F, K_1, G_1, K_2, G_2, K_3, G_3. Four equations are obtained by equating moments and shears at the panel points. (Eqs. 11 and 12 must be multiplied by the proper $E\,I$-value to obtain moment and shear.) The other three equations are obtained by applying the following known conditions: In Eq. 11, for $a = 1$, $y = e$ when $x = 0$. In Eq. 11, for $a = 3$, $y = e + \Delta$ when $x = h$. In Eq. 12, for $a = 3$, $\frac{dy}{dx} = 0$ when $x = h$.

Special Case III; Variable Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Considered, but Not Eccentric Roadway Loads.—This case is identical with Special Case I, except that $e = 0$.

Special Case IV; Variable Moment of Inertia, No Eccentricity at Tower Top, No Roadway Loads, and Tower Weight Neglected.—This case is identical with Special Case II, except that $e = 0$. In both cases, the labor required may be reduced somewhat by following the procedure outlined in a standard textbook.[5] The value of A is removed from the denominator of the first two terms of the general solution, Eq. 4, and incorporated in the constants K and G. By following through the development of the problem from this point, it may be seen that the resulting simultaneous equations which express equality of moments and shears at the panel points do not require the $E\,I$ coefficients.

Special Case V; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Considered, but No Eccentric Roadway Loads.—The equations required for solving this case are obtained from those of Special Case I by substituting the constant value of I for I_1, I_2, and I_3. Note that this substitution must also be made to obtain the values of n_1, n_2, and n_3. (It is obvious that the same substitution in the equations for the General Case will take the roadway loads into account.)

Special Case VI; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads.—Since the tower consists of only one panel, the problem reduces to the following two equations—

Moment:

$$E\,I\frac{d^2y}{dx^2} = -R\,y - F\,x = E\,I\,(K \sin n\,x + G \cos n\,x) \;\ldots\ldots (13)$$

shear:

$$E\,I\frac{d^3y}{dx^3} = -R\frac{dy}{dx} - F = E\,I\,(K\,n \cos n\,x - G\,n \sin n\,x) \;\ldots\ldots (14)$$

In Eq. 13, when $x = 0$, $y = e$:

$$-R\,e = E\,I\,G \;\ldots\ldots (15)$$

In Eq. 13, when $x = h$ and $y = e + \Delta$:

$$-R\,(e + \Delta) - F\,h = E\,I\,(K \sin n\,h + G \cos n\,h) \;\ldots\ldots (16)$$

In Eq. 14, when $x = h$ and $\frac{dy}{dx} = 0$:

$$-F = E\,I\,(K\,n \cos n\,h - G\,n \sin n\,h) \ldots\ldots\ldots\ldots (17)$$

Solving Eqs. 15, 16, and 17 for F:

$$F = \frac{R\,n\,[e\,(1 - \sec n\,h) + \Delta]}{\tan n\,h - n\,h} \ldots\ldots\ldots\ldots (18)$$

Special Case VII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Considered, No Eccentric Roadway Loads.—Equations may be found for this case by substituting the constant value of I in the equations found for Special Case III.

Special Case VIII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads.—This case is identical with Special Case VI when $e = 0$. Thus, Eq. 18 becomes:[6]

$$F = \frac{R\,n\,\Delta}{\tan n\,h - n\,h} \ldots\ldots\ldots\ldots (19)$$

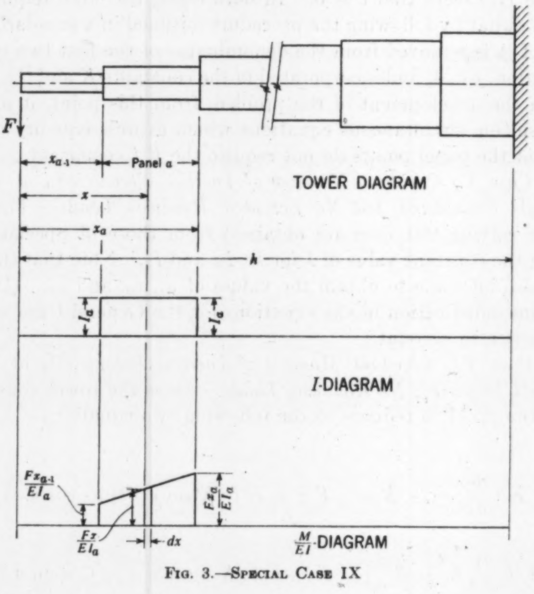

FIG. 3.—SPECIAL CASE IX

Special Case IX; Variable Moment of Inertia, Tower Weight Neglected, No Roadway Loads, and $R = 0$.—The problem (see Fig. 3) is now reduced to that of a simple cantilever of variable cross section. To find the deflection Δ by the moment-area method, find the moment of the $\frac{M}{E\,I}$-diagram about point 0. The moment of the small part of the diagram included in dx, in panel a, is: $dM_a = \frac{F\,x^2}{E\,I_a}\,dx$; integrating for panel a: $M_a = \frac{F}{E\,I_a}\int_{x_{a-1}}^{x_a} x^2\,dx$ $= \frac{F}{E\,I_a}\left(\frac{x_a{}^3 - x_{a-1}{}^3}{3}\right)$. Summing up for all panels:

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[6] This case is also outlined in Pt. II of "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 10th Ed., pp. 278 and 279.

$$\Delta = \sum M = \frac{F}{E}\sum_0^h \frac{x_a{}^3 - x_{a-1}{}^3}{3\,I_a};$$

or

$$F = \frac{3\,\Delta\,E}{\displaystyle\sum_0^h \frac{x_a{}^3 - x_{a-1}{}^3}{I_a}} \quad \text{.....................(20)}$$

Special Case IX, Applied to the Condition in Which the Moment of Inertia Cannot Be Considered Constant Over Any Part of the Tower Leg.—If I can be expressed as a continuous function of x, the deflection may be found as the

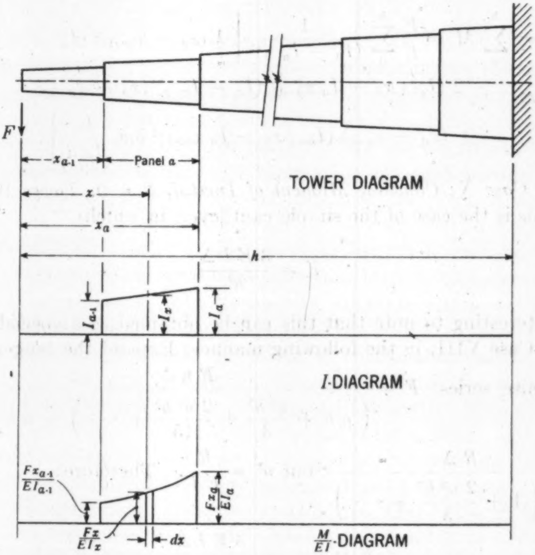


FIG. 4.—LAYOUT FOR FINDING Δ WHEN $R = 0$ AND I VARIES AS LINEAR FUNCTION OF x IN EACH PANEL

moment of the $\frac{M}{E\,I}$-diagram in the following manner:

$$\Delta = \int_0^h \frac{F\,x^2\,dx}{E\,I_x} = \frac{F}{E}\int_0^h \frac{x^2\,dx}{I_x} \quad \text{...............(21)}$$

More often the moment of inertia is not a continuous function of x, but can be expressed as a series of linear functions of x. From Fig. 4, the moment of the diagram for panel a is:

$$M = \frac{F}{E} \int_{x_{a-1}}^{x_a} \frac{x^2}{I_x}\, dx \qquad (22)$$

From Fig. 4, the equation of the I-diagram for panel a is:

$$\frac{I_x - I_{a-1}}{x - x_{a-1}} = \frac{I_a - I_{a-1}}{x_a - x_{a-1}} \qquad (23)$$

Solving Eq. 23 for I_x, substituting in Eq. 22 and integrating:

$$\begin{aligned} M = \frac{F}{E\,(I_a - I_{a-1})^3} \Big[& \tfrac{1}{2}\,(x_a - x_{a-1})^3\,(I_a{}^2 - I_{a-1}{}^2) \\ & - 2\,(I_{a-1}\,x_a - I_a\,x_{a-1})\,(I_a - I_{a-1})\,(x_a - x_{a-1})^2 \\ & + (x_a - x_{a-1})\,(I_{a-1}\,x_a - I_a\,x_{a-1})^2 \log_e \frac{I_a}{I_{a-1}} \Big] \qquad (24) \end{aligned}$$

Summing all segments:

$$\begin{aligned} \Delta = \sum_0^h M = \frac{F}{E} \sum_0^h \frac{1}{(I_a - I_{a-1})^3} \Big[& \tfrac{1}{2}\,(x_a - x_{a-1})^3\,(I_a{}^2 - I_{a-1}{}^2) \\ & - 2\,(I_{a-1}\,x_a - I_a\,x_{a-1})\,(I_a - I_{a-1})\,(x_a - x_{a-1})^2 \\ & + (x_a - x_{a-1})\,(I_{a-1}\,x_a - I_a\,x_{a-1})^2 \log_e \frac{I_a}{I_{a-1}} \Big] \qquad (25) \end{aligned}$$

Special Case X; Constant Moment of Inertia, $R = 0$, Tower Weights Neglected.—This is the case of the simple cantilever, in which

$$F = \frac{3\,E\,I\,\Delta}{h^3} \qquad (26)$$

It is interesting to note that this can be obtained as a special application of Special Case VIII, in the following manner: Expand the tangent in Eq. 19 as an infinite series: $F = \dfrac{R\,n\,\Delta}{\left(n\,h + \dfrac{n^3\,h^3}{3} + \dfrac{2\,n^5\,h^5}{15} + \cdots\right) - n\,h}$. Simplify: $F = \dfrac{R\,\Delta}{\dfrac{n^2\,h^3}{3}\left(1 + \dfrac{2\,n^2\,h^2}{5} + \cdots\right)}$; but $n^2 = \dfrac{R}{E\,I}$. Therefore:

$$F = \frac{3\,E\,I\,\Delta}{h^3\left(1 + \dfrac{2\,R\,h^2}{5\,E\,I} + \cdots\right)} \qquad (27)$$

Then, when $R = 0$, Eq. 27 becomes Eq. 26.

Special Case XI; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Neglected, $F = 0$.—In Eq. 18 of Special Case VI, let $F = 0$:

$$R = \frac{E\,I}{h^2}\left(\sec^{-1} \frac{e + \Delta}{e}\right)^2 \qquad (28)$$

Special Case XII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads, $F = 0$.—In Eq. 28, let $e = 0$. Then, since $\sec^{-1} \infty = \frac{\pi}{2}$:

$$R = \frac{E\, I\, \pi^2}{4\, h^2} \qquad (29)$$

Thus, in this case, the vertical load R is independent of the deflection Δ. This is Euler's formula for a column with fixed base and top free to move laterally. It yields the value of R for which the tower will be in unstable equilibrium. If, for any reason, the vertical load becomes slightly eccentric with respect to the center line of tower at its base, the tower will bend continuously until failure occurs.

For values of R greater than $\frac{E\, I\, \pi^2}{4\, h^2}$ or values of $n\, h$ greater than $\frac{\pi}{2}$ a negative F is required to hold the tower in equilibrium for any desired deflection. This may be verified by inspection of Eq. 19.

GENERAL OBSERVATIONS ON THEORY

Although not more than two changes in moment of inertia have been used in any case, and only four tower panels were used in the so-called "General Case," the same method may be applied for any number of changes in moment of inertia, and any number of tower panels. A more complex case simply increases the labor required. No matter what the conditions may be, this analysis requires two constants of integration for each section of tower in which there is no change in moment of inertia and no point at which a load is assumed to act. If the concentrated loads along the shaft are considered, the ordinate to each load point is another unknown that must be found. The remaining unknown is the horizontal tower-top load F. As soon as the setup of the problem is known, the number of simultaneous equations required may thus be determined by inspection.

If the effect of wind loads is desired in combination with any or all of the other loads, the wind can be considered as a series of concentrated horizontal loads acting at convenient points along the shaft. This will add to the complexity of the function of x in the expression for moment and may or may not introduce additional constants of integration, depending on whether or not the tower is thereby divided into a larger number of panels.

Since the lateral bracing of the tower adds somewhat to the effective moment of inertia, some designers find it advisable to add from 3% to 5% to the calculated moments of inertia of the tower leg to take account of this effect.

All of the simultaneous equations for solving any case are linear. Therefore, it is evident that, for a given value of R, F will be a linear function of Δ. For the same reason, the ordinate to any point on the elastic curve is a linear function of Δ.

As will be shown in the numerical examples, when R is plotted against F for a given Δ, the resulting curve is so nearly a straight line that it may be considered so for all practical purposes. This approximate linear relationship is demonstrated mathematically in the complete unabridged paper.[2]

These characteristics are a great aid in determining the flexure curve for a tower. For a given Δ, which may conveniently be taken as unity, the curve of applied vertical load plotted against F may be determined by finding only two points. Furthermore, for any given value of vertical load R along the curve, the F corresponding to a different Δ may be found by direct proportion if R is concentric. If R is eccentric, the value of F for various values of Δ may be found by a simple linear relationship only at the points where F has been calculated.

Numerical Applications

The results of a few numerical applications are given herein to support some of the foregoing general observations and to indicate the error to be expected if certain approximate short cuts are used. The dimensions and physical properties of the tower shafts are given in the unabridged paper.[2]

Tower A (Height = 188 Ft, One Change in Moment of Inertia, Equivalent I = 1,061.7 In²-Ft²).—The results for this tower are shown in Fig. 5(*a*), where values of F in terms of Δ are plotted against values of R. Curve 1 is based on Special Case III.

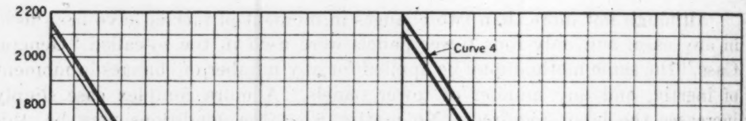


FIG. 5.—FLEXURE LINES

In finding curve 2, the weight of the tower was neglected. The F-intercept was found by using Special Case IX. The other two points were found by means of Special Case IV.

In finding curve 3, the weight of the tower was neglected and a constant moment of inertia was used. Curves that were established by finding more than two points indicated that, for all practical purposes, F is a linear function of R. This is also confirmed by the discussion in Appendix II of the unabridged paper.[2] For these reasons, only two points were found on curve 3. The F-intercept was found by using Special Case X, and the R-intercept by using Special Case XII. The constant I was found by equating the value of F on curve 2 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was then solved for I to obtain an equivalent moment of inertia.

Tower B (Height = 188 Ft, Two Changes in Moment of Inertia, Equivalent $I = 1{,}078.2$ In²-Ft²).—The results for this tower are shown in Fig. 5(*b*). Curve 1 is based on Special Case III and curve 2 is based on a weightless tower. The F-intercept for curve 2 was found by using Special Case IX. The other three points were found by means of Special Case IV.

Curve 3 is also based on Special Case IV, but the following correction was applied to take account of the weight of the tower. For each of two of the points found on curve 2, the ordinates to the elastic curve were found at the points at which tower-weight loads were applied in finding curve 1. The tower-weight loads were then placed at these points. The sum of their moments about the center line of tower at the tower base, divided by the total height of tower, represents the approximate relief of F as found for curve 2, caused by the presence of the tower-weight loads.

Curve 4 is based on a weightless tower with constant I. The F-intercept was found by using Special Case X, and the R-intercept by using Special Case XII. The constant moment of inertia for this curve was found by equating the value of F on curve 2 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was then solved for I to obtain an equivalent I.

Curve 5 is also based on a weightless tower with constant moment of a inertia equal to that used for curve 4. In this case, however, the vertical load R was given an eccentricity of 2 ft at the tower top, and all points were found for a tower-top deflection of 1 ft. Special Case XI was required to find the R-intercept, and Special Case VI to find the other points.

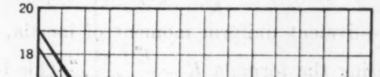


FIG. 6.—FLEXURE LINES FOR TOWER C

Tower C (Height = 702 Ft, Thirteen Changes in Moment of Inertia, Equivalent $I = 906{,}740$ In²-Ft²).—The results for this tower are shown in Fig. 6. In this case the ordinates represent values of F for a deflection of 1 in. Curve 1 is based on a weightless tower and required the use of Special Cases IV and IX.

Curve 2 is also based on Special Case IV, but the following correction was applied to take account of the effect of the tower weight. Using the tower elastic curve for the point on curve 1 where $R = 52{,}000{,}000$ lb, a reduction in F was found by the method used in finding the points of curve 3 for tower B. For the point where $R = 0$, the same numerical reduction in F was used.

Curve 3 is based on a weightless tower with a constant moment of inertia, and required the use of Special Cases VIII, X, and XII. The constant I was found by equating the value of F on curve 1 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was solved for I to obtain an equivalent moment of inertia.

Discussion of Numerical Applications

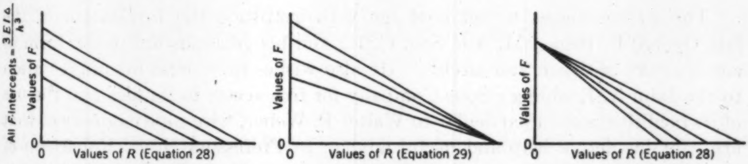
The results of this numerical work clearly indicate that, as nearly as it can be plotted, the horizontal load required to produce a given deflection is a linear function of the vertical load. This is confirmed by the discussion in Appendix II of the unabridged paper,[2] wherein the deviation from a straight line is discovered algebraically. It appears that for many purposes of design and estimate, it would be sufficiently accurate to use the following method of obtaining the flexure characteristics of a tower:

Using Special Case IX, find F for the weightless tower when $R = 0$. Equate this value to $\frac{3\,E\,I\,\Delta}{h^3}$ (Special Case X) and solve for I. Using this as an equivalent uniform moment of inertia, find the value of R when $F = 0$ by using the formula $R = \frac{E\,I\,\pi^2}{4\,h^2}$. This is from Special Case XII, and is also Euler's formula. The line joining these two intercepts is a close approximation of the true flexure line for the weightless tower. The worst case arising in the foregoing numerical applications occurs for tower C. Curve 3 is found as in the foregoing, and curve 1 is the true flexure line for the weightless tower. In the vicinity of the dead-load tower-top reaction (52,000,000 lb), there is an error in F of 650 lb in 11,000 lb, or approximately 6%. This can be brought to within about 6% of the true value for the weighted tower by the following method: Divide the tower into convenient panels. Find the weight of each section and apply it at the proper panel point as a concentrated load. Find the ordinates to these points on the elastic curve of the weightless tower by using Special Case IX, or simply by assuming the tower to take the shape of some well-known curve such as the parabola. Find the sum of the moments of the weight loads about the center line of the tower at its base. Divide this by the total height of the tower and reduce all values of F found on the approximate line for the weightless tower by the amount of this quotient.

Finally, when it is desired to obtain the greatest accuracy which is possible without following the rigorous solution for the weighted tower, the following method may be used: Depending on whether or not the tower-top reaction is eccentric, use Special Cases II and IX, or Special Cases IV and IX, to find two points, the F-intercept and one other, on the flexure line for the weightless tower. For one of these conditions, find the ordinates to the elastic curve, and proceed as before to reduce F by taking moments of the tower weights about the tower base. The accuracy of this method for tower B may be seen by comparing curve 3, Fig. 5, which was found by this method, with curve 1, found by the exact method. It is doubtful, however, that this approximate method saves enough time, in comparison with the exact method, to make it worth while. In the exact method it is necessary to find only two points of the flexure line. It is the opinion of the writer that, if the degree of accuracy desired is such as to require the use of either the latter approximate method or the exact method, the exact method may as well be used unless the tower-weight concentrations cannot reasonably be placed at the points of change in moment of inertia. If they must be placed at intermediate points, additional integration constants will be required, and the last approximation described in the foregoing would be far less tedious than the rigorous solution.

All of this reasoning is based on the assumption that the principal objective of the solution is the deflecting force F required at the tower top. However, if the problem is to be solved in order to review the design of the tower shaft, and the principal objectives are the true elastic curve and the stresses in the tower sections, it appears to the writer that the only safe procedure is to use the rigorous solution.

In some design or erection problems, it may be desirable to set down a series of flexure lines for various values of deflection or eccentricity. The general disposition of various sets of curves for a few simplified cases is shown in Fig. 7. The following methods of attack are suggested by the arrangement of these lines.


All F-Intercepts $= \frac{3EI\Delta}{h^3}$
Values of F
0
0
Values of R (Equation 28)
Values of F
0
0
Values of R (Equation 29)
Values of F
0
0
Values of R (Equation 28)

(*a*) SPECIAL CASES VI, X, AND XI' (*b*) SPECIAL CASES VIII, X, AND XII (*c*) SPECIAL CASES VI, X, AND XI

FIG. 7.—ARRANGEMENT OF FLEXURE LINES FOR VARIOUS CASES

Special Cases VI, X, and XI.—In Fig. 7(*a*), $e > 0$ = constant, and each curve represents one value of Δ. The procedure is to find the F-intercept and one other value of F in terms of Δ. Any desired number of lines may then be found by applying various values of Δ to these expressions.

Special Cases VIII, X, and XII.—In Fig. 7(*b*), $e = 0$, and each curve represents one value of Δ. The procedure is the same as in Fig. 7(*a*) to find the F-intercept for each line. For the simple case of a constant moment of inertia and a weightless tower, the common R-intercept is found by Special Case XII. For other cases: Find a point on one line; produce this line from its F-intercept through the point just found to the R-axis; and draw all other lines through this common R-intercept.

Special Cases VI, X, and XI.—In Fig. 7(*c*), $e > 0$; Δ = constant; and each curve represents one value of e. The procedure is to find the common F-intercept and solve for one other point on each line.

CONCLUSION

The relation between horizontal and vertical tower-top loads for any tower-top deflection can be found by rigorous mathematical analysis, which includes the effects of the following influences:

(1) An eccentricity of the vertical tower-top load;

(2) The eccentricity of the weight of the tower (considered as a series of concentrated loads) caused by deflection;

(3) Reactions at the roadway level; and

(4) Wind loads (considered as a series of concentrated horizontal loads).

The analyses of this case and of several simpler cases have been developed in this paper. Many of the latter have been published elsewhere, but the writer feels that there is a definite advantage in having all these cases assembled for ready reference.

It is possible to obtain approximate values which are very close to the true values by means of short-cut methods, several of which are described herein.

It is mathematically correct to state that, for any given combination of other loads, the horizontal tower-top load is a linear function of the tower-top deflection. For all practical purposes, it is correct to state that, for a given deflection, the horizontal tower-top load is a linear function of the vertical tower-top load.

ACKNOWLEDGMENT

The writer wishes to acknowledge with gratitude the inspiration of the late George E. Beggs, M. Am. Soc. C. E., and his generous aid in the form of constructive criticism and advice. He also wishes to express his appreciation to the John A. Roebling's Sons Company for free access to its files and the use of its facilities, and in particular to Walter F. Weber, wire rope engineer of that firm, for his active help and useful advice; to Professors H. P. Robertson of Princeton University, Princeton, N. J., and M. S. Knebelman of Washington State College, Pullman, Wash., for their helpful advice in developing the mathematical theory which is included in the paper and the appendixes of the unabridged paper.[2]

DISCUSSION

Francis P. Witmer,[7] **M. Am. Soc. C. E.**—A complete method for analyzing the stresses in a suspension bridge tower under a variety of imposed conditions is presented in this paper. It is particularly interesting that the author has based his development upon one of the almost forgotten members[3] of the series that were classics in their field fifty years ago. It is unfortunate that the more detailed study upon which this paper was based[2] could not have been presented in full. The derivation of Eq. 4 would then have been clear to one who happens not to be acquainted with Professor Robinson's valuable little treatise published in 1882, and who is not in a position readily to consult the complete paper in the Engineering Societies Library in New York City.

For the usual case of a tower without eccentricity at the top and with a variable moment of inertia, the writer has had satisfactory results by using Eq. 19 of Case VIII, first determining an equivalent moment of inertia I by equating Δ in Eq. 20 of Case IX to the deflection for a cantilever with constant moment of inertia, $\Delta = \frac{F\,h^3}{3\,E\,I}$. The value of I, so found, may then be used in the quantity $n = \sqrt{\frac{R}{E\,I}}$ without serious error. The author has applied a similar procedure to several numerical cases with results that seem to justify this conclusion, particularly when it is remembered that the fiber stress in the tower due to bending is generally small in comparison with the direct unit stress. Incidentally, Eq. 19 may be readily derived by the writer's method, which is cited in the paper.[4]

John I. Copp,[8] **Assoc. M. Am. Soc. C. E.**—The procedure for the exact solution of tower stresses for all loading conditions that ordinarily occur in a plane parallel to the bridge axis is described in this paper. The author has also indicated some approximations that may be used with accuracy for certain loading conditions in evaluating the tower stresses and deflections. His proposed method of finding the effect of the tower dead load on the horizontal force F at the tower top by "* * * assuming the tower to take the shape of some well-known curve such as the parabola" (see heading "Discussion of Numerical Applications") has been used by the writer and found to give a very close approximation to the true value.

When preliminary investigations are being made for planning cable-spinning operations or tower erection, it is of value to have some rapid means of approximating the tower deflections and stresses prior to the final analysis by the exact method. By applying the foregoing parabolic assumption, it is comparatively simple to derive the approximate equations for any and all loading conditions that are indicated by the author in his various cases. The assumption of a parabolic curve for the tower eliminates the solution of all differential equations and the determination of the constants of integration when deriving the equations.

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The writer believes that as a supplement to the author's formal solution of the various cases the method of deriving the approximate equations would be of value. The derivation of the equations for a few typical loading conditions is given herein with some numerical applications by the exact and approximate methods for comparison.

In the derivation of the approximate equations the author's notation will be used in so far as it is applicable with additions as required:

h = tower height, in feet;

Δ' = initial tower-top deflection for a free standing tower, in feet;

Δ = final tower-top deflection, in feet;

δ_n = deflection increment of tower top (caused by tower moments that are functions of the tower curve), in feet;

$\Sigma\delta$ = total tower-top deflection (caused by tower moments that are functions of the tower curve), in feet;

R = known vertical load at tower top, in pounds;

F_Δ = horizontal force at tower top to maintain a known horizontal deflection Δ, in pounds;

F = a horizontal force at tower top, in pounds;

E = modulus of elasticity for the tower, in pounds per square inch;

e = eccentricity of the vertical load R with respect to the center line at tower top, in feet;

M_x = bending moment in tower at a distance x from the tower top, in foot-pounds; and

w = dead load of tower per linear foot of height (assumed uniform).

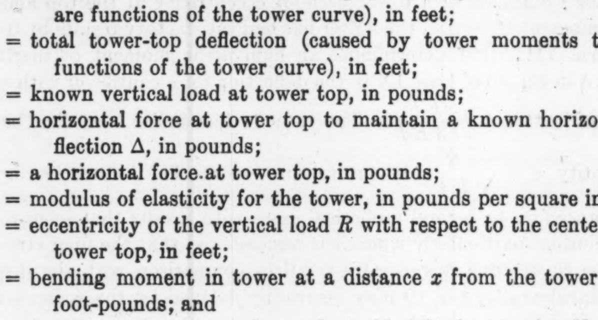

FIG. 8

Free Standing Tower (Total Deflection of Tower Top Due to the Dead Load of Tower).—Assume the tower curve $T\ A\ B$ (Fig. 8(a)) to be a parabola with its vertex at point B and with an initial deflection Δ' (Fig. 8(*b*)) resulting from any tower moments that are not functions of the tower curve.

The total deflection $\Sigma\delta$ (Fig. 8(*b*)) due to the dead load of the tower is determined by taking the sum of a geometrical series, since the deflection δ' will induce an additional deflection δ'' and an infinite number of deflection increments will result with a constant ratio between successive increments—that is, $\frac{\delta'}{\Delta'} = \frac{\delta''}{\delta'} =$ etc. The first term of this series, δ', is determined by taking moments of the $\frac{M}{E\,I}$-diagram, for the tower dead load, about the tower top.

The formula for the sum of a geometrical series is

$$s = \frac{a\,(r^n - 1)}{r - 1} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

which, applied to the tower deflection increments, gives $a =$ first term $= \delta'$; $r =$ ratio for the series $= \frac{\delta'}{\Delta'}$; $n =$ number of terms $= \infty$; $s =$ sum of the series $= \Sigma\delta$; and the sum of the series then will be

$$\Sigma\delta = \frac{\delta'\left[\left(\frac{\delta'}{\Delta'}\right)^{\infty} - 1\right]}{\frac{\delta'}{\Delta'} - 1} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

In Eq. 31 the ratio $\frac{\delta'}{\Delta'}$ must be less than unity (if the critical loading for the tower has not been reached); hence

$$\lim_{n \doteq \infty}\left(\frac{\delta'}{\Delta'}\right)^n = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

and the equation for deflection will be

$$\Sigma\delta = \Delta'\left(\frac{\delta'}{\Delta' - \delta'}\right) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

In Eq. 33 the value of δ' is equal to the moment of the $\frac{M}{E\,I}$-diagram, about the tower top—

$$\delta' = \int_{x=0}^{x=h} \frac{M_x\,x}{E\,I}\,dx \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which the value of M_x, the moment at the point x, due to the dead load of the tower above point x, for a parabolic curve, can be shown to be

$$M_x = \frac{\Delta'\,w}{h}\,x^2 - \frac{2}{3}\,\frac{\Delta'\,w}{h^2}\,x^3 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

Substituting this value of M_x in Eq. 34 and evaluating the integral—

$$\delta' = \frac{1}{E\,I}\int_{x=0}^{x=h}\left(\frac{\Delta'\,w}{h}\,x^2 - \frac{2}{3}\,\frac{\Delta'\,w}{h^2}\,x^3\right) x\,dx = \Delta'\left(\frac{7\,w\,h^3}{60\,E\,I}\right) \ldots (36)$$

The deflection for the free standing tower due to its own dead load is then found by substituting the value of δ' from Eq. 36 in Eq. 33

$$\Sigma\delta = \Delta'\left(\frac{7\,w\,h^3}{60\,E\,I - 7\,w\,h^3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The value of Δ (Fig. 8(*b*)) then will be

$$\Delta = \Delta'\left(1 + \frac{7\,w\,h^3}{60\,E\,I - 7\,w\,h^3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Tower with Controlled Deflection.—When the tower-top deflection is controlled by a known amount Δ', as in cable-spinning operations, the reduction in the horizontal force F (say F') due to the dead load is found by equating the deflection caused by a single horizontal force F' to the deflection δ' and solving for F'; that is,

$$\frac{F'\,h^3}{3\,E\,I} = \Delta'\left(\frac{7\,w\,h^3}{60\,E\,I}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

and

$$F' = \Delta'\left(\frac{7}{20}\,w\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Applying this reduction, F', to a tower with a controlled deflection Δ', the horizontal force F is determined by deducting, from the force required to deflect the weightless tower, an amount Δ', the force F'.

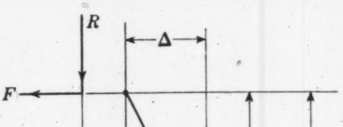

FIG. 9

The force required to deflect the weightless tower an amount Δ' is

$$F_\Delta = \frac{\Delta'\,3\,E\,I}{h^3} \ldots\ldots\ldots (41)$$

Hence, the net force will be

$$F = \frac{\Delta'\,3\,E\,I}{h^3} - \Delta'\,\frac{7}{20}\,w \ldots . (42)$$

in which, as previously stated, Δ' is an initial deflection of the tower top resulting from moments that are not functions of the tower curve.

Free Standing Tower with Horizontal and Eccentric Vertical Loads at Tower Top.—In Fig. 9 the moment at any point x, y will be

$$M_x = F\,x + R\,(e + y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

which may be considered as the sum of three moments, $F x$, $R e$, and $R y$. The moments $F x$ and $R e$ are independent of the tower curve, whereas the moment $R y$ is a function of the tower curve. The deflection resulting from the moment $R y$ must be treated as the sum of a geometrical series, for a free standing tower, as was done in the case for the dead load of the tower. The deflection due to the horizontal force F will be

$$\Delta_F = \frac{F h^3}{3 E I} \quad \text{(44a)}$$

The deflection due to the moment $R e$ will be

$$\Delta_{Re} = \int_{x=0}^{x=h} \frac{R e x}{E I} dx = \frac{R e h^2}{2 E I} \quad \text{(44b)}$$

The tower-top deflection due to these two moments then will be

$$\Delta' = (\Delta_F + \Delta_{Re}) = \frac{F h^3}{3 E I} + \frac{R e h^2}{2 E I} \quad \text{(45)}$$

which is the initial deflection for determining the deflection caused by the moment $R y$.

The deflection due to the moment $R y$ is determined by the moment of the $\frac{M}{E I}$-diagram about the tower top:

$$\delta' = \int_{x=0}^{x=h} \frac{R y x}{E I} dx \quad \text{(46)}$$

in which

$$y = \frac{\Delta'}{h^2}(2 h x - x^2) \quad \text{(47)}$$

Then:

$$\delta' = \int_{x=0}^{x=h} \frac{R \Delta'}{E I h^2}(2 h x - x^2) x \, dx = \Delta' \frac{5 R h^2}{12 E I} \quad \text{(48)}$$

The total deflection due to the moment $R y$ will be equal to the sum of a geometrical series with an infinite number of terms as indicated by Eq. 33. Substituting for Δ' and δ' their values as given by Eqs. 45 and 48, respectively, in the general Eq. 33, and adding the initial deflection Δ':

$$\Delta = \left(\frac{F h^3}{3 E I} + \frac{R e h^2}{2 E I}\right)\left(1 + \frac{5 R h^2}{12 E I - 5 R h^2}\right) \quad \text{(49)}$$

which is the total deflection for a free standing tower with a horizontal and eccentric vertical load at the tower top.

For a free standing tower with a horizontal force F and a concentric vertical load R, the value of e in Eq. 49 becomes zero and the equation for deflection will be

$$\Delta = \frac{F\,h^3}{3\,E\,I}\left(1 + \frac{5\,R\,h^2}{12\,E\,I - 5\,R\,h^2}\right) \dots\dots\dots\dots (50)$$

It is of interest to note that an approximate value of the critical concentric load R, for a free standing tower, may be evaluated from the general Eq. 31. It can be shown that Eq. 31 approaches infinity as the ratio $\frac{\delta'}{\Delta'}$ approaches unity. Equating the ratio in Eq. 48 to unity and solving for R,

$$R = \frac{12\,E\,I}{5\,h^2} \dots\dots\dots\dots (51)$$

A comparison of this approximate critical load with Euler's theoretical load shows an error of less than 3% in the approximate value.

When the tower is loaded at the top with an eccentric vertical load and with the horizontal deflection controlled, the horizontal force for equilibrium is determined by equating (for a weightless tower) the deflection resulting from a horizontal force F' to the sum of the first increments of deflection caused by the moments $R\,e$ and the vertical load R and solving for F':

$$\frac{F'\,h^3}{3\,E\,I} = \frac{R\,e\,h^2}{2\,E\,I} + \Delta'\,\frac{5\,R\,h^2}{12\,E\,I} \dots\dots\dots\dots (52)$$

or,

$$F' = \frac{3}{2}\,\frac{R\,e}{h} + \Delta'\,\frac{5\,R}{4\,h} \dots\dots\dots\dots (53)$$

which gives the reduction in the horizontal force required to deflect the tower an amount Δ' with no other loads. The reduced force F resulting from the eccentric vertical load will then be

$$F = \frac{3\,\Delta'\,E\,I}{h^3} - \frac{3}{2}\,\frac{R\,e}{h} - \Delta'\,\frac{5\,R}{4\,h} \dots\dots\dots\dots (54)$$

In Eq. 53 the value of e may be either negative or positive, depending on whether the moment $R\,e$ decreases or increases, respectively, the moment $F\,x$ at any point in the tower. The value of e will be zero for a concentric load R. When R is concentric, the resulting horizontal force F will be

$$F = \Delta'\,\frac{3\,E\,I}{h^3} - \Delta'\,\frac{5\,R}{4\,h} \dots\dots\dots\dots (55)$$

Numerical Applications of the Approximate Solutions.—Refer to curve 3, Fig. 5(*a*) of the paper, as a comparable example and apply the corresponding approximate Eq. 55 with the following values: R (assumed) = 1,200,000 lb; h = 188 ft; I = 1,061.7 in^2 ft^2; Δ' (assumed) = 1.0 ft; E (assumed) = 29,000,000 lb per sq in.; and F = 5,922 lb.

Entering Fig. 5(*a*) with R = 1,200,000 lb, the value of $\frac{F}{1{,}000\,\Delta} = 6$, which gives a value of F = 6,000 lb and shows an error, in the approximate solution, of about 1.5% when compared to the author's comparable case.

Using the same tower properties and assuming it as free standing, the total deflection by the approximate solution (Eq. 50) is $\Delta = 1.013$ ft, in which R, E, I, and h are as stated previously and F (from Fig. 5(a)) is 6,000 lb. From curve 3, Fig. 5(a): $\Delta = \frac{F}{6{,}000} = 1.00$ ft, which shows an error of about 1.5% in the approximate solution.

It appears from the foregoing derivations and numerical examples that the approximate solutions may be used to advantage as an approach to the formal solutions as outlined by the author. They also might be used for the preliminary design of towers or as the first approximation if the tower stresses were to be determined by successive approximations.

BLAIR BIRDSALL,[9] ASSOC. M. AM. SOC. C. E.—The writer appreciates the discussion presented by Professor Witmer. The reference in the paper to Professor Robinson's booklet[3] was intended purely as an historical note.

To obtain the derivation of the general solution (Eq. 4) of the differential Eq. 3, it is not necessary to refer to Professor Robinson's booklet or to the writer's complete paper which is on file in the Engineering Societies Library in New York City. This solution may be obtained by applying the procedure recommended in textbooks on differential equations for linear differential equations with constant coefficients.

Mr. Copp has presented an interesting method of obtaining closely approximate results without the use of trigonometric functions. For this reason, many engineers may find Eq. 54 a more handy tool than Eq. 18 for finding approximate results under Special Case VI. This may also be true of a combination of Eqs. 42 and 54 or Eqs. 42 and 55 when the effect of tower weight is considered. However, the writer believes that if tower weight is to be considered at all, it should be handled as a series of voussoirs of weight increasing from top to bottom of tower.

The writer does not believe that, for a weightless tower with uniform moment of inertia, Eq. 55 has any advantage in ease of application over Eq. 26 (Special Case X) and Eq. 29 (Special Case XII), which were used to obtain curve 3 of Fig. 5(a). Since Eqs. 26 and 29 are statements of its two intercepts, the equation of curve 3 is—

$$F = \frac{3\,E\,I\,\Delta}{h^3} - \frac{12\,\Delta\,R}{\pi^2\,h} \quad \text{.................... (56)}$$

The similarity of Eqs. 55 and 56 and the fact that they are equally easy to apply may be seen at a glance. They are both based on the assumption that F is a linear function of R, and both assume a weightless tower and a constant moment of inertia. However, both intercepts of Eq. 56 are true values for this type of tower, whereas that is true of only the F-intercept of Eq. 55. For that reason, the writer is inclined to favor Eq. 56.

In his numerical application, Mr. Copp has compared a value of F (5,922 lb) computed from Eq. 55 with a corresponding value (6,000 lb) taken from a graph of Eq. 56. It appears to the writer that, for a true comparison, computed values should be used in both cases. The corresponding computed value of F from Eq. 56 is 6,140 lb. When this is compared with 5,922 lb, the resultant discrepancy is 3.6%. This is a small error but one that can be avoided without increase in labor.

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[9] Asst. Chf. Engr., Bridge Dept., John A. Roebling's Sons Co., Trenton, N. J.

As stated in the "Introduction," one of the writer's principle aims in presenting the paper was to make available to the profession certain handy tools for the closely approximate solution of an otherwise tedious problem. Mr. Copp's discussion is a valuable addition to this "tool kit."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2149

EVALUATION OF FLOOD LOSSES AND BENEFITS

By Edgar E. Foster,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. E. L. Chandler, E. F. Chandler, Charles B. Burdick, H. K. Barrows, Roger E. Amidon, Hyman J. Fine, Otto F. Buzhardt, Malcolm Elliott, John B. McAleer and John B. Drisko, and Edgar E. Foster.

Synopsis

The analysis of the damages, including the annual flood loss, constitutes the outstanding problem of the economics of flood control in which benefits must be weighed against costs. It is the purpose of this paper to present some methods that have been used by the writer in the U. S. Engineer Department. The primary condition imposed is that the method must be based on sound principles of mathematics, hydrology, and economics. In order to demonstrate that this condition was met, the paper contains a brief description of the various types of damage, some fundamental concepts of economics, a discussion of frequency and damage curves, and an example of the computation of the annual loss.

Introduction

Economic studies form an essential part of the investigation of practically every constructive enterprise, and works for flood control or flood protection are no exception, particularly when the loss of human life is not a factor, Indeed, lacking (as they usually do) any direct esthetic or intellectual appeal, their justification must be made primarily upon an economic basis. Although it is true that in great catastrophic floods the loss of life is appalling and the prevention of this loss is also a justification, nevertheless the costs of flood control are necessarily measured in economic terms and, to make adequate comparison of cost against benefit, the latter must be gaged on the same basis. Moreover, aside from the loss of life, flood control or protection is entirely an economic matter.

The losses from which the benefits are computed may be found by investigation or appraisal, and the costs can be estimated in the same manner as

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Note.—Published in May, 1941, *Proceedings*.

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costs of other structures. When the losses are known, the annual benefits can be computed and compared with annual costs. This is the object of the economic study. On this comparison depends: (1) Whether or not any protective measures are justified; (2) to what extent protection may be given; and (3) in part, what kind of works may be provided.

The analysis of flood losses and computation of annual loss must proceed on a rational basis as far as such bases can be found in the relevant fields of economics, mathematics, and hydrology. It is the purpose of this paper to develop some methods applicable to such analysis.

DESCRIPTION OF FLOOD DAMAGE

Flood losses arise from the occupation of the fluvial plains that are sufficiently high to escape the frequent and smaller freshets but are still low enough to be inundated by the greater and rarer floods. From the beginning of civilization, the flood plains have been favored lands for settlement. Prior to the development of modern railways, rivers formed the most important means of commercial transportation and commerce, which required a settlement where navigation could be had if such a site were at all possible. This factor is probably the most potent of all that promoted riverside settlement and, moreover, is a world-wide factor in city growth. Another reason for such settlement is the ease of construction on the flat terrane. This means cheaper construction for other and more modern forms of transportation, as railways and highways. In some cases, notably in New England, the existence of potential water power induced the development of industries on the banks of rivers. Then, regardless of the original reason for settlement, once a city is established on a flood plain, it will stay. People are so loath to leave their property that only the worst catastrophes will force them to leave any except the smallest towns. Therefore, as long as cities and towns occupy flood plains, and as long as rivers have floods, there will be flood losses from which protection will be sought.

The diversity of losses is as great as the economic interests of society itself, since floods inundate everything on the plain as, perhaps, may be the right of floods. However, since society has found the land of flood plains most desirable for nearly every human purpose, it will continue to occupy that land and consequently continue to suffer losses in every line of activity. Industrial, commercial, utility, municipal, recreational, social, and agricultural interests all pay toll for trespassing on a flood plain, and others, throughout the present-day closely knit social and economic organization, contribute in a diminishing degree, although far from the scene of disaster.

Industrial losses include those sustained by establishments engaged in manufacturing activities in which raw materials are processed to form a finished product for use by a consumer or in another manufactory. Usually the structures used for manufacturing purposes are sufficiently strong to withstand the onslaught of a flood and will survive unless they are located so close to the riverbank that they become undermined. Small auxiliary buildings may be washed away or wrecked by other means. Wooden floors will swell and bulge so that extensive repairs or replacements are required. Floor covering and wall finishing will be damaged and windows will be broken.

Machinery is injured to a varying degree that depends upon the type of machine. Engines and pumps and similar heavy machines will not suffer any great damage from simple immersion but will require dismantling and thorough cleansing. Electrical machinery, particularly motors and generators, requires thorough cleansing and even more thorough and careful drying. Even then, such machinery is subject to rapid deterioration and early replacement. The more fragile machinery, such as is used in the textile industry, is subject to breakage by floating debris as well as the foregoing damage. Many classes of raw materials and finished products, such as textiles and clothing, are liable to total destruction. Others (some heavy rubber goods, for example) may be reclaimed by thorough cleaning. Still others may be damaged in quality so that their value is reduced. Some industries (rayon, for example) that operate in a continuous process suffer a loss simply because of the untimely interruption which necessitates clearing the plant of material in process.

Industrial losses also arise from the destruction or damage to records if the administrative offices are so located that they too are flooded. Some papers may be recovered by careful drying but others may be a total loss. A case of the latter kind occurred in the Merrimack River Valley in the flood of March, 1936. The owner of a chemical manufacturing plant had some chemical formulas that had been obtained after several years of study and experimentation. The formulas, having been written out and left in the office, were completely destroyed by the flood and could be reproduced only by repeating the investigations at a heavy cost. Furthermore, all factories suffer heavy indirect losses because of the interruption of the usual business with a continuation of fixed costs. This loss may be incurred whether or not the plants are located in the flood zone, but, of course, not to the same extent.

Under commercial losses may be included all those sustained by establishments engaged in trade, wholesale and retail; banks and other financial institutions; professional and other offices; service concerns such as laundries, plumbers, barbers, beauty shops, restaurants and hotels, theaters, and similar lines of business that are too numerous and diversified to list in detail.

Retail and wholesale shops lose heavily in merchandise that has been submerged. Goods that have been soaked by dirty and polluted flood water are usually a total loss, although there may be some kinds that may be reclaimed to sell at much reduced prices. Damage to accounts and records results in a loss, the minimum value of which is the cost of restoring them by cleansing and drying into some recognizable condition. Buildings and fixtures are always damaged at least to the extent of requiring cleaning and redecorating. Plate-glass fronts, which are a conspicuous feature of retail shops, are heavy sufferers if the flood rises high enough to carry debris against them. Likewise, all affected concerns sustain an indirect loss because of suspension of business.

Under the term "utilities" may be grouped all agencies or private and public organizations that provide or sell some form of service to the consumer or patron. These agencies include electric, gas, telephone and telegraph, and railroad companies, and water works, among private organizations; under public ownership may be included water works, sewerage systems, and streets and highways.

Since utilities deal in service rather than goods, there is relatively little to be lost in stocks of goods, either in raw materials or finished products of their principal business. Gas companies, however, may lose gas from their containers or pipe lines, and both gas and electric utilities may suffer damage to merchandise which is sold for the utilization of their services. Both also may lose stocks of operating supplies, and both may sustain losses in business due to the inundation of their patrons and the suspension of other business. Hydroelectric power houses and dams and river crossings of pipe lines and transmission lines are danger points and frequently are damaged severely.

Railroads and highways suffer because grades are damaged by slides or are washed out entirely, taking pavements or rails with them as the case may be. Railroads sustain damage to rolling stock, which is lost not infrequently when freight cars are loaded with rock or other ballast and placed on bridges that are in danger of going out. Both railroads and highways suffer loss of bridges. In many cases the entire bridge is destroyed; in others, the piers may be damaged or the approaches eroded. Occasionally the bridge superstructure may be damaged, although left in place. The submergence of both highways and railways causes a temporary rerouting of traffic with consequent greater cost to the public in the case of the former and to the railway company in the latter case.

Dams, like bridges and all other structures in rivers, are particularly vulnerable to floods. Flashboards may be washed away by any freshet, gates may be damaged, or the abutments washed out by heavy and high water; or the entire dam may fail and release the water in storage to augment the flood below. Because a dam has successfully passed one flood is no reason to be certain that it will pass all others. All structures in the channels are subject to damage and may be weakened by successive floods until one finally washes them out.

Under the term "municipal" losses may be grouped all damage to property and all increased cost of municipal operation during the flood: These include all losses sustained by city, county, state, or other governmental agencies, including the federal government.

Streets and roads suffer heavy damage in flood from washing and erosion, and heavy deposits of debris must be removed after the water has subsided. Sewer outlets in streams and water-main crossings are subject to damage. Municipal buildings are liable to the same kind of damage as other structures, because floods are no respecters of authority. In addition to the physical loss, the government must spend large sums of money for relief of those who are driven from their homes. The police duties are increased by the emergency, and frequently other forces are required to maintain civic order, the cost of which is a true flood loss.

Farm and rural losses include all damage sustained by agricultural lands and forests, together with all appurtenant improvements. This includes the loss of livestock that may be trapped and drowned; damage or destruction of buildings and farm equipment; loss of crops that may be in the field, growing or harvested; and loss due to prevention of planting, erosion of valuable land, or deposits of debris that may render the soil untillable. The flood may wash out fertilizer that has been placed on the land in preparation for crop, or it may necessitate repeating the work of preparing the field for planting. It may erode the topsoil so that replacement may be necessary or one or more years of fruitless cultivation may be required to restore the fertility.

Residential losses include all damage to dwellings, both the buildings and the grounds, and the furnishings and other personal property commonly used or kept around homes. As in other types of damage, the extent varies greatly as the desires and means of the occupant vary. Entire houses may be carried away if they are located in areas submerged by swift, deep water or they may be undermined by erosion of the banks of the stream. Buildings, household goods, and generally the land may be charged off as a total loss in cases where the entire house is taken away. In other cases the personal property is damaged by immersion; furniture and other household furnishings are badly damaged and many pieces (pianos, for example) must be replaced. The dwelling must be thoroughly cleansed and redecorated and may require more or less extensive repair work. Moreover, there is usually some cost of moving out because of rising water and providing other habitation for the duration of the flood. This cost is properly chargeable to flood losses.

The term "social losses" may apply to all losses arising from the interruption of normal, individual, or collective social activity and to the personal risk of life and health endangered by floods. The term is necessarily rather vague and very broad. The losses are indefinite in most cases and may be practically nothing, or they may be large. For example, an interruption of an afternoon bridge game is an inconvenience but the loss caused by such an interruption is negligible or nonexistent. On the other hand, the personal risk may be very real. The inundation may result in sickness and death, a loss that cannot be measured in terms of money. These social losses may be considered as indirect losses accompanying the direct residential damage.

SOME DEFINITIONS OF WEALTH AND VALUE

The considerations of flood damages and benefits involve the most basic and fundamental concepts of the science of economics. It is desirable to define these concepts herein before proceeding to the analysis of losses.

The first concept is that of wealth. It is necessary to know what can be classed logically as the real wealth of a community that has suffered flood damage in order to determine the true loss. By the term "wealth" is meant the sum of all goods, fixed or movable, which have value to individuals or organizations of individuals who have a commonly recognized title to ownership of the goods or property in question. Wealth may be owned by private or public agencies. "Wealth can be described by means of a stock-taking or inventory, but it can be summed or measured as a whole only in terms of its money value. * * * Its value is the sum of the values of existing property rights."[1]

Economic goods may be divided into two general classes—productive and consumptive goods. These two classes include all forms of property that constitute the real wealth of the community or the country. They include both real and personal property. They do not, however, include evidences of indebtedness, such as bonds, because such papers are essentially claims to other property, the title of which rests in other owners.

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[1] "Wealth," *The Encyclopædia Britannica*, 14th Ed., Vol. 23, p. 448.

Productive property is taken to be that which is devoted to the use of producing other goods or service in commercial operations. It includes both real and personal property—that is, land and improvements and all equipment devoted to such production. It consists of the property used in all commerce, industry, utilities, and agriculture; it is fundamentally all property used to produce an income.

Consumptive or nonproductive property, on the other hand, includes all property, both real and personal, put to uses that do not produce other goods or commercial services. Nonproductive fixed property is essentially a kind of "consumptive good" because it is used directly by the consumer or patron. Its utility is obtained directly by the user. Nonproductive property includes residences, religious and educational institutions (both the land and buildings), libraries and art galleries, parks and similar recreational areas for which admission is not charged, streets, highways, sewers (generally), and governmental and municipal structures. This differentiation between productive and consumptive property is essential to the proper comprehension of the effect of flood losses on depreciation.

Another concept for which a definition must be made is value. For the purposes of this paper, two definitions are given, interrelated but viewing the concept differently. First, the value of any particular item of property may be defined or measured as the quantity of other goods that may be given or received in exchange; under existing economic conditions the "other goods" is nearly always money. The value arrived at will depend upon two or more parties involved in a transaction. This is the exchange or objective value and is the definition usually in mind when the worth of goods is discussed.

In contrast with the exchange value is the subjective value, which is the worth of an item of property in the consideration of the holder in comparison with other goods. This value depends upon an individual's sentiments and preferences, without effective regard to the utility of the property in question. It cannot be measured directly in money since the holder usually places a greater consideration on the property than its utility to others is worth.

Actually, the subjective value enters the appraisal of a personal need and the purchase of every article of consumptive goods obtained for the usefulness to the purchaser. The particular item that is bought will be what the buyer believes will give him greater utility than some other "good" that may be had for the same money. This displacement value of consumptive goods is entirely subjective, but in the case of production goods it will depend upon the relative costs of production attainable and may be measured or estimated in terms of money. An item, therefore, is worth to the consumer what he will pay for it and to the producer what he can earn with it.

If the item of property is offered for sale at a price within the replacement value, the objective or exchange value is reached. On the other hand, if the individual refuses to pay the exchange price or cost, it should be safely inferred that the subjective value is not worth to him as much as is asked. Questions of indirect losses, business losses, depreciation of property, risk of health and life, and evaluation of indirect benefits must be answered by reference to the basic concepts of value.

Definition of Flood Losses and Benefits

It becomes necessary now to make some definitions of what constitute valid flood losses and benefits. The economic effect of floods touches such a multitude of diverse phases of human activity that, unless some guiding principles are established for analysis, confusion and, consequently, erroneous conclusions will be deduced. That these principles must be basically sound and in accordance with well-established economic doctrines that are recognized to be fundamentally correct goes as a matter of course. Because of the aforementioned diversity of flood losses, the principles and definitions can be only the broadest.

Flood losses may be defined as those losses arising from the following exigencies:

(a) *Any Destruction or Damage Caused by a Flood to the Existing Real or Personal Property of a Community.*—This includes all physical and such intangible property as may constitute a part of the total wealth of the community. As a rule, it will not include the loss of the evidence of indebtedness; only where the debtor is outside the damaged community and when the debt cannot be proved except by the papers lost can the value of the debt be allowed as a valid loss, and in that case it is a loss only in that community and not in an area including both the debtor and the creditor.

The operation of this rule can be shown clearly by an illustrative property account, which may be called the Community Property Account (See Fig. 1).

Community Property Account

(Debits)			(Credits)		
Date	Item	Amount	Date	Item	Amount
	Property of A Property of B Property of C	X_1 X_2 X_3		Community wealth	X_n
	Total	X_n		Total	X_n

Fig. 1

The account must balance. Hence, any loss that affects the property of *A* (who, it is assumed, has no connection with the others) will result in a decrease of the debits and necessarily will cause a reduction on the credit side of the account, which represents the total community wealth. Suppose again that *B* sustains a loss which is a payment that goes entirely to *C*. There is no change in the debits, and consequently there will be no change in the community wealth.

(b) *Loss of Earnings or Loss of Services.*—Earnings, whether they are wages, salaries, or professional fees, are compensation for services and are an economic commodity, as are material goods. It follows, therefore, that loss of services constitutes an economic loss to the community, but that both loss of services and loss of compensation should not be counted is brought out by considering a Community Income Account similar to the Community Property Account in Fig. 1. Assume that *D* works for *E* and that *E*'s plant is closed because of the flood. In this case *E* loses the services but saves the payment of the wages, and *D* loses his compensation; there are two individual losses and one individual gain, which leaves one net loss to the community—the loss of *D*'s services, which have the value of the compensation. This rule will be applied later in the consideration of business losses.

(*c*) *Increased Expense of Usual Operations.*—Floods interfere with normal operations and cause increased expense either without, or in addition to, other damage. This situation is likely to be encountered in the utility field, which is obliged to maintain service at all times regardless of the emergency. Certain functions of the municipal governments are likewise greatly increased in periods of flood. A somewhat similar type of loss exists in all commercial or industrial firms. They have certain overhead expenses, such as taxes, rent, insurance, and so forth, that must be paid regardless of interruption of income. When the interruption is a flood, the expenditures for fixed charges or overhead may be fairly charged to flood losses.

(*d*) *Diminution of Capital Value of Real Estate.*—This type of loss is emphasized in the case of residential property and is applicable to nearly all nonproductive or consumptive property. The repairs of flood damage, which constitute an added expense, and the personal risk involved, make this property less desirable for use. The subjective evaluation of this lessened desirability is reflected in the capital value by diminishing it. Productive property is subject to the same capital loss as a result of a flood not previously experienced or expected. If such property is located on a site known to be subject to floods, that added expense should be foreseen and the capital value adjusted or fixed on the basis of such anticipated expense, which becomes a charge against income. For all property, however, this diminution of property value should be considered as a nonrecurring loss if it is allowed in one flood. This type of loss will be considered again in connection with indirect losses and one method of evaluation of flood damages.

(*e*) *Expenditures Caused by the Flood Emergency or a Threat of Flood.*—These expenditures may be roughly grouped into two classes: (1) Emergency flood protection, and (2) cost of relief to flood sufferers.

Whenever a sufficient warning is had, occupants of the flood plain take steps to protect their property. Dikes may be constructed to protect low areas; machinery may be moved to high ground or to upper stories or may be protected by other means; and stocks of merchandise are raised to elevations above the expected flood. This work is undertaken before it can be known to what stage the water will rise, and entails expense—frequently heavy expense—which must be met from some source. These expenditures are properly chargeable as flood losses even though it is proved later to have been needless.

Floods invariably drive large numbers of people from their homes and cause the loss of their usual source of subsistence and shelter, which must be furnished from other sources. The refugees gain what is expended by the municipality and relief agencies, but the loss of their usual sources of subsistence and shelter is probably worth considerably more to them on account of the subjective value, if for no other reason. The full amount of relief expenditures may reasonably be taken as a flood loss.

TWO GENERAL CLASSES OF FLOOD LOSSES

For the purpose of evaluating annual flood losses and benefits, it is convenient to divide the damages into two general classes—direct and indirect. The broad basis of division is the immediate origin of the damage; but since the object of the classification is convenience rather than inescapable necessity, other factors may be given consideration that may entail variations in treatment. In the category of direct losses may be placed all damages to physical property (Class (*a*)), and the direct expenditures (Class (*e*)) for flood emergency.

In addition to the direct losses sustained by the physical property, other losses (Classes (*b*) and (*c*)) arise indirectly from many sources because of the flood. These constitute the "indirect losses." The sources of the indirect losses are as diverse as the direct. They arise in the flood zone and outside, and may occur far from the locality of the flood.

Perhaps first in importance among the sources of indirect flood losses is the interruption of commercial activity. In this respect floods are similar to fires; they stop the normal earning capacity of all commerce and industry for the duration of the high water and for whatever length of time is needed to clean the premises and repair the damage. This business loss consists of the gross business profit of the operations. It includes loss of profit, wages and salaries, all expense for overhead, and all indirect operating costs. Where no goods are manufactured or sold, no raw material or stock of goods will be used, and consequently the difference between the cost of raw materials or cost of stock of merchandise and the normal value of the goods sold will form the indirect flood loss caused by the interruption of business.

The data obtainable for computing this kind of indirect loss consist of the value of sales for the period of suspended activity, and they should be examined critically. The amount of sales should be normal for the season. Items of cost that are suspended or reduced should be omitted from the losses. It commonly happens that, after the flood has subsided, labor normally used in the business is assigned to cleaning and repairing the plant. In this case, labor costs should be charged to direct loss and not the indirect loss. For computing the indirect loss from the normal sales, rates of gross profit, less labor costs (usually), may be estimated for each line of business and applied to the data of loss of sales.

Another source of indirect loss arises in the transportation systems—the railroads and the highways. When the highways and railroad tracks are submerged, the grades eroded, or bridges damaged or destroyed, the traffic must use other, and in practically all cases, longer routes. This necessarily results in greater expense of transportation. In the case of the railroads, the cost is borne by the companies and could probably be estimated from their records, although a search would be long and tedious. Furthermore, there is likely to be some loss or at least inconvenience to the consignees or shippers and the passengers because of the longer route.

In contrast with rail traffic, the losses caused by floods to highway transportation are borne by the public. The individual losses may be little or they may be considerable, for they will depend upon how much use is made of the roads by the individual. With rail traffic, these losses arise because of rerouting and consequent longer haul. For example, it may cost 1.5 cents per mile to operate an ordinary passenger car. If, then, that car is forced to use a route 5 miles longer, and is forced to traverse it four times daily, it will sustain an additional expense of 30 cents a day for the duration of the interruption. One or several cars will not produce a great indirect loss; but the operation of many vehicles, including commercial trucks with much higher operating cost, can amount to a considerable sum in a short time.

The interruption of the services of utilities can cause the suspension of operations of business outside of the flood zone. Electric power is particularly vulnerable in this respect, because hydroelectric plants (which usually generate a part of the supply) and transmission lines at river crossings form prospective sources of interruptions.

There are other sources of indirect losses, most of which are even more difficult to express in terms of money. There is the loss of the use of playgrounds, parks, schools, churches, and similar property. Each flood brings with it a definite risk of loss of health and frequently life that can be evaluated only subjectively. No attempt can be made herein to enumerate all possible forms of indirect losses, because there are manifold sources, but the foregoing are the principal ones. Others should be searched for in each flood.

Flood Risk

Before proceeding to describe the methods of evaluating the annual flood loss, it is desirable to discuss further the economic risk of such damage. Although floods are usually more likely to occur in some seasons than in others, the time of occurrence is unknown until the rising stages may give warning of impending danger. Even that foreknowledge is available only on the larger rivers, and disastrous floods may occur on smaller streams without any indications of their approach. The magnitude, perhaps, is more uncertain than the time of occurrence. There are many areas, moreover, which may be inundated but yet have not in recorded history experienced a freshet great enough to cover them. The occupants are subject to flood damage, therefore, even though it is unknown to themselves. These people who dwell or work on a flood plain are thus liable to an unknown and uncertain expense that can be designated the "flood risk." Economically, this flood risk is in no way different from that of other uncertain catastrophic events such as fire or earthquake.

From an economic standpoint,

> "* * * risk may be defined as uncertainty in regard to cost, loss, or damage. In this definition, emphasis is on the word 'uncertainty.' Where destruction of capital is certain in connection with a business process, it can be charged up in advance as a cost. It is not a risk. When the destruction or loss is uncertain, it may be dealt with in accordance with the judgment of probabilities and presents a problem of risk."[3]

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[3] "Risk and Risk-Bearing," by Charles O. Hardy, Inst. of Economics, Washington, D. C.

This definition fits the flood situation perfectly. Moreover, the definition may be extended a step further by defining the annual flood loss as the cost of the flood risk.

Flood risk, as well as other kinds, may be conveniently divided into two classes—property or capital, and personal risks. The first kind pertains to the loss of property or goods. It may be satisfied with the money value of goods or damage sustained. The personal risk is due to the subjective value that an individual puts on an object. It is usually greater than the exchange value. It is a value that arises from long association with the object in question, such as, for example, an old chair, store, house, or the building of a life business. The same category will cover the loss of means, employment, health, and life itself.

The capital loss can be evaluated satisfactorily on an annual basis when the property damage is known; but the personal risk can be evaluated only indirectly, either by determining the value from the adverse effects of the risk or by determining what the individual will pay to ameliorate his situation. This paper is concerned primarily with the property risk.

Since flood risk depends upon uncertain events, its evaluation must be based upon some expression of the probabilities of occurrence. These estimates of probability fall into three classes,[3] namely: (1) Mathematical *a priori*, (2) statistical, and (3) judgment. In a further statement, Charles O. Hardy states that judgment is only a crude application of the statistical method. In determining the probability of flooding, unaided judgment should be used only when there are no other means possible.

On the other hand, mathematical *a priori* methods cannot be used to determine the probability of floods, and hence that method is eliminated from further consideration. There remains, therefore, only the methods of statistics, which can be based on the records of flood data that have been compiled. It is on the principles of statistics that the methods of evaluating the annual flood losses, discussed in this paper, are based.

THE COMPUTATION OF ANNUAL FLOOD LOSSES

The annual flood loss has been defined as the cost of the flood risk, and it may be considered as an expense to be met in the same manner as any other similar cost, such as fire losses. The cost is not uniform in the absence of insurance but varies as time and chance provide the floods. The risk of floods is continuous, however, the same as that of any other catastrophe; when it occurs the damage is done and the expense must be met.

The intermittent and sometimes heavy cost of floods may be reduced to an annual basis in accordance with the mathematical expectancy as expressed by the formula

$$A = p\,D \text{.........................} (1)$$

in which A is the annual flood loss, p the probable number of floods per annum, and D the total damage caused by the floods. Eq. 1 is well known and is the mathematical expectancy of payment or cost of a chance event. The number of floods per annum can be taken from a frequency curve which must be computed from a record of flood discharge. The damage, D, must be taken from a stage-damage or similar curve that has been constructed from data obtained from a field survey.

Frequency Curves

It can be seen from Eq. 1 that the probability of a flood is one of the two essential factors entering into the computation of the expected annual flood loss. Therefore, it is advisable that some consideration be given to this matter. Although data of floods will be approximations at best, they should nevertheless be analyzed carefully, and the frequency should be computed by the best possible means.

Many methods of computing flood frequencies have been proposed, some purely empirical and others partly based on the mathematical theory of probability. It is plain, however, that the importance of accurate evaluation of the annual flood losses is such that the frequencies should be based on a sound mathematical and hydrological theory. Although it is not the purpose of this paper to enter into a long discussion of the various methods of computing frequencies of floods, it seems, nevertheless, desirable to describe briefly the applicability of the theoretical methods.

Floods are the result of extraordinary rainfall, or rapid melting of snow, or a combination of both. Behind each of these two primary causes there must function a long chain of preliminary events, each of which has its own peculiar variations and may operate in various degrees in the combination of circumstances that produce the flood. In fact, a peak is a result of fortuitous combination of favorable chance variations of the usual meteorological events. For that reason, data of floods, both of magnitude and of number, are amenable to treatment by the mathematical theory of probability.

Since the magnitude and the number of floods are the result of chance circumstances, one cannot expect a given series to be duplicated. The engineer must anticipate that there will be floods in the future as in the past; but when, or how big, a given flood will be he does not know; and he has no way of learning. On the other hand, he may expect a similar distribution of floods (that is, a similar proportion of high, moderate, and low peaks in a given period), and from the distribution he may expect approximately the same average and the same mean square variation (or coefficient of variation). These two quantities constitute the first and second statistical moments. If the record is long enough, it is also reasonable to anticipate similarity in the higher statistical moments, and from a record of moderate length the moment may be obtained with suitable accuracy. The frequency of floods then may be computed from the moments by the mathematical theory of probability.

Several sound methods exist that may be used for the computation of frequency curves. First in point of publication is the method proposed by H. Alden Foster,[4] M. Am. Soc. C. E., for which the probability functions of Karl Pearson formed the basis. Secondly, J. J. Slade, Jr.,[5] presented an asymmetrical probability function that can be used appropriately for computing frequencies of peak discharge or volume. If the data of floods consist of stages only, a frequency curve can be computed readily by means of a Gram-Charlier probability series.[6] In using the same data these functions will yield closely comparable results. They agree very closely for the range of data which the former will cover. Likewise Professor Slade's function gives results practically identical with those obtained by means of a Gram-Charlier series when the data consist of stages.

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[4] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[5] *Loc. cit.*, Vol. 101 (1936), p. 35.

Before attempting to construct any frequency curve, the records of discharge should be examined carefully to determine whether or not the data of crests, especially in the earlier part of the period, are comparable to those of current observation. The regimen of streams, even in the natural state, changes more or less as time passes. The greatest danger is that conditions affecting stream flow may have been altered by human agencies in the development along the bank in a way that changes the stage-discharge relationship or even modifies the discharge. In such an examination, each record must be treated on its merits and given individual attention. Nevertheless, there are some sources of modification for which a special search should be made. Some of these factors modify discharge and the resulting stage, and others change the stage-discharge relationship so that only the stage is affected.

First, there may be diversion from one watershed to another. This diversion is most likely to occur in projects for municipal water supply but may arise in development for irrigation or water power. All discharges will be reduced, but the low-water flow will be affected most appreciably. If the diversion works provide a spillway into the old stream, the effect on floods may be relatively small or even negligible.

Second, there may be storage reservoirs upstream from the gaging station. These reservoirs will modify all floods to an extent that will depend upon the capacity of the reservoir and the drainage area above. Small reservoirs well up in the headwaters and controlling only a small part of the watershed may be neglected. The effect of the larger ones, especially those closely above the gaging point, must be considered. The best procedure, perhaps, is to compute the probable effect the reservoir would have had on the crests that occurred previous to its construction. At best, however, a new uncertainty has been introduced in the data by the vagaries of the human operation of the reservoir.

Third, the construction of levees for local flood protection will modify greatly the stage-discharge relationship, although it will not affect the rate of discharge appreciably unless much valley storage is lost.

Fourth, encroachment on the channel by bridges, highway and railroad grades, and the construction of buildings on the flood plain and channel modify the crests chiefly by causing backwater which changes the stage-discharge relationship.

Fifth, the construction of navigation dams and run-of-river power plants that have little or no storage affects the crests to some extent. It is probable that works of this type will have little or no effect on floods downstream other than perhaps to shift the time of crest slightly. The effect on the reduction of flood crests of power dams with pondage only is likely to be overestimated, but it should be investigated.

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[6] "Probability and Its Engineering Uses," by Thomas C. Fry, D. Van Nostrand Co., Inc., N. Y., 1928.

Sixth, there is usually some intermittent erosion and shifting of the channel. This situation is likely to occur when the stream flows through a loose geological formation such as alluvial silt. The rate of flow will not be affected, but the stage-discharge relationship is very uncertain. Unless the entire record can be worked over with reliable discharge rating curves or the entire discharge can be based on frequent—almost daily—measurements, it is better to compute a frequency curve of stage only.

Seventh, there are some other events that must be looked for, such as ice jams and dam failures. The effect of the former should be included in frequency curves to be used with damages possible during the spring or winter seasons. The latter are so rare that the effect should be eliminated from the flood record.

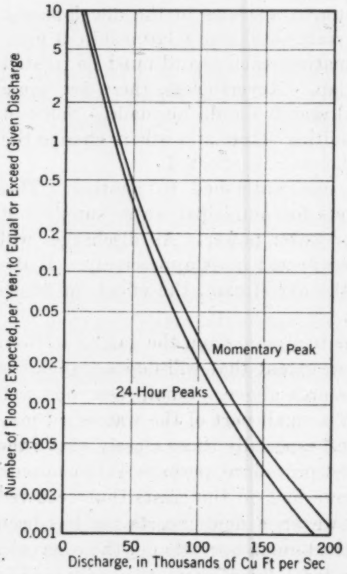

Fig. 2.—Flood Frequency; Merrimack River, Lawrence, Mass.

The preferable form of frequency curve is obtained by plotting the number of peaks or floods expected per year against magnitude, either of discharge or stage. Quantities needed for the computation of the annual loss may then be read directly from the curve. Fig. 2 shows a sample curve computed from the records of the Merrimack River at Lawrence, Mass.

Because the potential damage causable by floods is not uniformly distributed through the year, it is sometimes necessary to make frequency curves for floods experienced in different seasons. The most satisfactory manner of accomplishing this is to use the data of floods to compute separate curves for the different seasons. This may be done where a long record with ample data exists. For each given frequency the sum of the number of floods expected during each season or fraction of the year should equal the total number for the year.

It may be necessary to make seasonal frequency curves from data of those periods during which a single cause of floods, such as melting snow, operates exclusively of others. Such a condition prevails in regions where early spring floods arise from melting snow prior to a season when intense rainstorms can occur. In other sections, such as the southern parts of the United States where heavy rainstorms are the sole primary cause of floods, the annual frequency curves may be divided into seasonal curves on the basis of the yearly distribution of the peak data.

STAGE-DAMAGE CURVES

A stage-damage curve represents the losses caused by floods at various stages. An example, for Lawrence, is shown in Fig. 3. It is assumed in this example, and in all similar curves, that the total direct loss caused by a flood is given at any stage, regardless of how, or from what data, the curve was constructed. It may be constructed from data obtained by a survey of actual damage caused by one high flood, or from the damage data of a number of floods that have occurred at different times and to diverse stages, or it may be computed from the estimated potential damage. In any case, a point on the curve should give the total direct loss from a flood to the corresponding stage.

A variant of the stage-damage curve is one which shows the loss plotted against peak discharge. The discharge-damage curve will frequently facilitate computation of the annual loss.

The stage-damage curve links the damage to the frequency of floods through the elevation of the property to the stages of the stream. The essential data of each item of damage required for its construction are the location and the elevation of the damage above a datum plane which can be correlated with stages of the stream. The elevation of damage above ground or below a visible high-water mark can be obtained by measurement. All variations of elevation, whether caused by difference in topography or by construction of the property, are taken into account by the stage-damage curve.

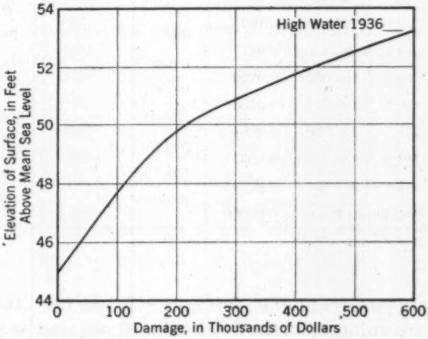

FIG. 3.—STAGE-DAMAGE CURVE FOR LAWRENCE, MASS.

An example of the computation of the annual loss of an urban area is shown in Table 1(*a*). The frequencies are taken from Fig. 2 and the damage from Fig. 3. The stage, 53.1 ft (Table 1), is the maximum that has been experienced. The damage of greater floods has been limited to the same amount in this computation, as a matter of conservatism.

ESTIMATING FUTURE FLOOD LOSSES

Of course, data of flood damage can be obtained only for floods that have been experienced; there is no other basis on which data of actual, experienced losses can be had. Yet, since comparison must be made with future costs, it becomes necessary to use the data at hand to estimate future losses and benefits also. The compiled data should be examined carefully, therefore, to eliminate nonrecurring losses and sometimes to make adjustments for past economic growth or expected future trend of development.

Damage that manifestly will not occur again should be eliminated from the computation of annual losses. In this category may be included (1) bridges that have been washed out and will not be replaced, or new structures that have sufficient elevation or waterway capacity to obviate all future danger; (2) dams that will not be reconstructed after failure; (3) other buildings that have been destroyed and will not be replaced; and (4) areas that have been evacuated and cannot be reoccupied because of physical conditions or legal restrictions. Items of damage that are eliminated should be scrutinized carefully and cast out only where it is reasonably certain that other developments, subject to similar damage, will not take their places. If replaced structures are vulnerable, the flood risk will remain the same as before the flood. Furthermore, unless some restrictions are made by conversion of land into parks or other legal enactment, evacuated areas may be reoccupied and the flood risk re-established. Moreover, there are unusual losses in nearly every flood so that an item should not be thrown out simply because it is unique.

TABLE 1.—ANNUAL LOSS FROM FLOODS, LAWRENCE, MASS.

Stage, in ft	Discharge, in cu ft per sec	Frequency (No. per Yr)		(a) Natural Conditions			(b) Reduced 1 Ft		
				Damage (Thousand Dollars)		Annual loss (dollars)	Damage (Thousand Dollars)		Annual loss (dollars)
		For discharge	For interval	For discharge	For interval		For discharge	For interval	
45.1	47,700	0.530		0			0		
			0.300		20.5	6,150		0	0
46.1	62,000	0.230		41			0		
			0.130		59.5	7,735		20.5	2,665
47.1	77,000	0.100		78			41		
			0.053		96.0	5,078		59.5	3,153
48.1	92,100	0.047		114			68		
			0.025		136.0	3,400		96.0	2,400
49.1	108,000	0.0220		158			114		
			0.0100		192.5	1,925		136.0	1,360
50.1	124,900	0.0120		227			158		
			0.0056		275.0	1,540		192.5	1,078
51.1	141,700	0.0064		323			227		
			0.0027		384.5	1,038		275.0	742
52.1	158,700	0.0037		446			323		
			0.00125		518.0	648		384.5	481
53.1	174,000	0.00245		590			446		
			0.00245		590.0	1,446		446.0	1,093
Maximum Flood		0.00000		590			446		
Total annual loss		....	....	....	....	28,960	....	....	12,972

It is impossible, of course, to predict definitely what economic development will take place in the future, but at least the trend must be examined in extending flood losses to compare with future costs. In this respect, flood-control works are on an equal footing with other structural developments, such as hydroelectric plants, railroads, large office buildings, and industrial plants. All must be constructed in the light of present economic conditions plus what trend may be predicted safely.

A key to economic development may be found in the growth of population, since all such activity arises from satisfying human needs. The growth of population in the United States apparently has become stabilized following the full settlement of the country, and relatively small changes may be expected in the future except in localities where a new industry has settled or some other attraction to development exists. These factors may be determined by investigation. Except in special circumstances, it will be in the interest of conservatism to assume no increasing losses in the future.

Some difficulty will be encountered in correcting or adjusting for seasonal changes in the potential flood damage. Usually the seasonal fluctuations of the retail trade are too small to be considered in view of the probable error of the flood-damage data. There are some industrial activities, however, which have a seasonal trend that may require some special consideration. Activities such as fairs, which commonly last only a week or ten days, will subject much perishable property to loss in event of a flood, if the grounds are in an area liable to inundation. Plainly, such potential loss cannot be computed with an annual frequency curve but should be used with the number of floods to be expected during the period of such activity.

Agricultural Losses

Other than crops, rural and agricultural losses can well be treated in the same manner as urban damage. The reaches may be longer and the losses will be more widely scattered, but otherwise there is no essential difference from urban damage, and there is no material seasonal fluctuation of risk. The outstanding source of such seasonal changes, of course, is agricultural crop losses, which must be given special consideration.

In contrast with urban and other rural damage, potential crop losses resulting from floods vary greatly from season to season; and they vary with the area flooded in a manner distinct from other damage. Beyond a relatively shallow depth, they do not increase with added inundation, for, if a crop is flooded sufficiently to be a total loss, it suffers no further damage by deeper water. These factors necessitate different treatment and entail the use of two other curves instead of the stage-damage curve. The first of these is the "area-flooded" curve, which represents the area of land flooded at various stages. The second is the "crop-loss" curve, which shows the potential loss of crops for the various seasons of the year.

The area-flooded curve may show the cultivated area inundated in a given reach or (preferably) the total area. The latter is the more constant value. The area flooded depends upon the stage and the topography of the valley or flood plain. Accurate topographic maps are necessary for its construction.

The risk of crop loss varies from nothing in the winter season in the northern latitudes to full, or nearly full, value of the harvested crop in summer. In southern or warm regions there may be some loss throughout the year because the work of preparation, such as plowing or the application of fertilizer, may be nullified by floods. The maximum monetary loss, of course, will depend also upon the value of the crop.

The "crop-loss" curve, which shows this seasonal flood risk, should preferably give the loss per acre, for clarity and convenience in computation. A curve may be used for each individual crop, or one curve may be a combination of several crops weighted by the average plantings of each. The example in Fig. 4 illustrates a curve of the latter type. Data needed for the construction of these two curves include the following:

(*a*) Total area within the reach or area in which crop losses are to be computed;

(*b*) The average proportion of the total area that is cultivated;

(*c*) The average acreage devoted to various crops raised in the area;

(*d*) The average cost of preparation of land and drop at various seasons;

(*e*) The average prices of the harvested crops;

(*f*) The dates or seasons of various farm processes performed in the raising of crops (that is, planting, cultivation, harvesting, and plowing); and

(*g*) Latest date on which a crop may be replanted so as to produce a harvest.

Items (*a*), (*b*), and (*c*) are used in the construction of the stage versus area-flooded curve, and items (*d*) to (*g*) are required for the crop-loss curve.

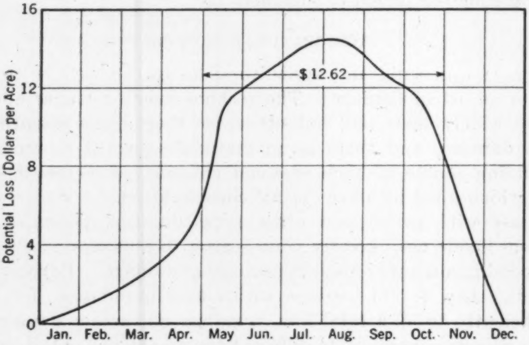

FIG. 4.—POTENTIAL CROP LOSS; REACH ON THE LITTLE RIVER, ARK.

The area-flooded curve is based primarily on the total area of the reach, since the land uses are more easily checked on that basis. Aside from that reason, it could be made to show cultivated area only. A computation of annual loss is given in Table 2.

COMPUTATION OF INDIRECT LOSSES

Indirect annual losses cannot be computed on the foregoing basis, primarily because the relation between the stage and loss is not constant. Other factors enter, which differ with the source of the indirect loss. Business losses, for example, depend also upon the time during which the plant is closed for repairs, which period will vary also with the nature of the business as well as the extent of the physical damage. The cost of rerouting of traffic will depend upon the length of available detours and the time needed to make repairs. Each source of indirect loss contains some other factor to affect the relationship.

There appears, however, a somewhat more constant, closer relationship between total direct and indirect losses of individual business concerns, since greater indirect losses will be sustained when the more extensive damage must be repaired. On the basis of this relation, a ratio of indirect to direct damage may be computed from the field data, adjusted if necessary, and applied to the annual direct losses. For example, the indirect loss in a particular flood may amount to 40% of the direct damage; then the total annual loss will equal 140% of the direct annual loss.

TABLE 2.—Crop Loss for Period from May to October, Reach on Little River, Ark.

Stage, in ft	Frequency, No. of Floods per Yr		Land Flooded, in Acres		Crop Loss		Annual loss
	For stage	For interval	For stage	Mean for interval	Per acre	Total	
26.0	0.608		0				
		0.257		270	$12.62	$ 3,407	$ 876
27.0	0.351		540				
		0.1847		760		9,591	1,771
28.0	0.1663		980				
		0.0992		1,155		14,576	1,446
29.0	0.0671		1,330				
		0.0431		1,465		18,488	797
30.0	0.0240		1,600				
		0.01615		1,710		21,580	348
31.0	0.00785		1,820				
		0.00545		1,895		23,915	130
32.0	0.00240		1,970				
		0.00220		2,055		25,934	57
34.0	0.00020		2,140				
		0.00018		2,160	12.62	27,259	5
36.0	0.00020		2,180				
Total annual crop loss for the period							$5,430

Benefits from Flood Control

The primary source of benefits from protective works is the elimination of flood losses. The benefits are of the same diverse nature as the flood losses, and various means must be used to evaluate all types. Floods are recurrent phenomena, and therefore the risks are continual although the damages are intermittent. The losses, or cost of the risk, are a charge against income; that is, they are an expense which, without protection, must be paid, although it is irregular, and for which nothing is received in return. The benefits are equal to the losses eliminated. In accordance with this principle, the direct annual flood benefits from works giving complete protection will be identical with the total annual losses.

The annual benefits obtaining from works (reservoirs, for example) which may give only partial protection may be computed to the extent that floods are reduced. For example, suppose that a reservoir had been constructed which would reduce all floods at a given point by 1 ft. The benefits result from reducing each flood that may occur and thereby save a part of the possible loss. For a given flood the same causes, or substantially the same in effect, will operate to produce the given peak after the construction of the reservoir, as previously. Therefore, the frequency, expressed as the expected number of floods per year, will remain the same; since the flood is reduced 1 ft, however, the damage will be taken from the stage-damage curve at a point 1 ft lower than the natural stage. The stage at which damage begins will remain the same so that all floods above that point to the limit of the reduction will be reduced below the danger point. The computation of the annual loss is repeated with control (see Table 1(*b*)), and the difference in annual loss under the two conditions is the annual benefit:

Annual loss under natural conditions (Table 1(*a*))	$28,960
Deduct annual loss, with reduction of 1 ft (Table 1(*b*))	12,970
Annual benefits with 1-ft reduction	$15,990

The damage center is the same as that used for natural conditions (Table 1(*a*)).

Reservoirs will not reduce all floods equally. Small floods below the danger point generally will be passed with no reduction. A reservoir usually will be designed to control a specified "design flood," the reduction for which will probably be the maximum since larger floods will pass over the spillway with a smaller relative reduction. A curve will be required to show the reduction obtainable over the full range of floods. The computation of annual loss is substantially the same except that the variable reduction obtained from the curve will replace the uniform reduction. As before, the annual benefit will be the difference in annual losses obtained before and after control.

Levees will eliminate all direct losses caused by inundation up to the stage at which they are overtopped. Beyond that stage the loss is as great as if there were no protection. The annual benefits will equal the annual losses below the elevation of the top of the levee or wall.

Losses and benefits can be computed graphically from a diagram such as is shown in Fig. 5. Values on the ordinate scale are the numbers of floods per year, and on the abscissa, the damage caused by a peak at the corresponding stage. The area under the curve (area *O A B C O*) then represents the total annual loss from uncontrolled floods. Next, a similar curve is constructed for losses after control (area *O D E F O*, Fig. 5). The area between the two curves (area *A B C F E D A*) represents the average annual benefits.

Accurate distribution of benefits is possible by the foregoing analysis. It may be desirable in some cases to determine the benefits accruing from two or more means of protection. It is only necessary to fix the range of stage through which each method of protection and the benefits obtained may be readily computed. The benefits derived by each type of protection will be the reduction of losses through the range of stage in which it operates. It should be remembered that the benefits per uniform increment of stage vary in accordance with the frequency of the floods expected.

In some cases, the benefits from protection of agricultural land are obtained as average value per acre. No objection can be raised against this, provided that it is used only where the total for the given area can serve equally well. It is an erroneous step, however, to assume that the higher areas covered by the greater floods will have the same average annual benefits as the lands at lower elevations. For example, levees may protect to elevation *a*; land above stage *a* will not yield the same average annual benefits with protection as those below for the reason that the latter are more frequently flooded.

EVALUATION BY ESTIMATING DEPRECIATED PROPERTY VALUES

In the foregoing evaluation of flood losses and benefits, the entire process was predicated on the basis of an annual loss or expense of risk, and therefore a charge against revenue or income. In contrast with this method, evaluation of flood losses has been made on a capital basis, which consisted essentially of an estimate of the decrease of capital value of property caused by one large flood. This process has been used in computing the value of flood protection and hence must be given some consideration.

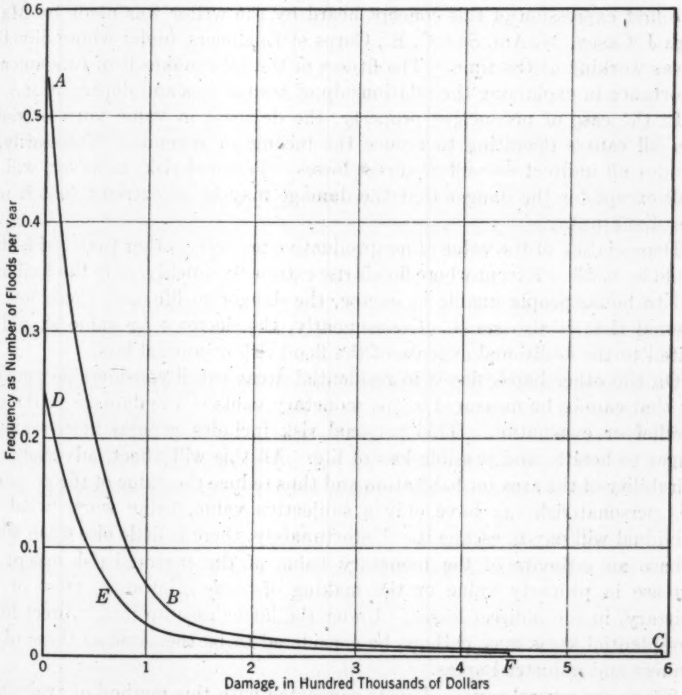

FIG. 5.—GRAPHIC COMPUTATION OF FLOOD LOSSES AND BENEFITS, LAWRENCE, MASS.

An appraisal is made of the affected property after the flood to estimate its value then and the value as before the inundation. The difference between the two values is taken as the flood loss and, if damage is fully eliminated by protective works, the benefits to be obtained. These appraisals must be considered primarily as a sale value or equivalent since construction costs will not be materially affected by the flood risk of the completed structures. For present purposes it will be assumed that the difference in appraisals includes all the costs.

Except for the effect of the construction costs, which constitutes a factor tending to resist change, the value of property will depend upon the income that can be earned by productive property or upon the desirability for use intended in the case of nonproductive property. This value will be manifest in the sale price of property, which should correspond to the appraised values. If a piece of property becomes subject to a flood loss, its desirability either for profit-making, habitation, or other use becomes less, and its capital value falls. Therefore, the appraised value or sale price will be less. The decrease in capital value, or depreciation of value, is seen to be simply a subjective evaluation of the total annual flood loss in terms of the capital value of the property. (The first expression of this concept heard by the writer was made by Major Hugh J. Casey, M. Am. Soc. C. E., Corps of Engineers, under whose direction he was working at the time. The fitness of the idea makes it of fundamental importance in explaining the relationship of annual loss and depreciation.)

In the case of productive property, the decrease in value must originate from all causes operating to reduce the income or revenue. Necessarily, it includes all indirect as well as direct losses. Personal risk, however, will be small except for the danger that the damage may be so extreme that it may cause bankruptcy.

Depreciation of the value of nonproductive property, other than residential, should be small. Except where floods rise extremely quickly, or in the buildings used to house people unable to escape, the danger to life and, therefore, the personal risk, is also small. Consequently, the decrease in value should be limited to the additional expense of the flood risk or annual loss.

On the other hand, floods in residential areas entail considerable personal risk that cannot be measured in the monetary value of the damage or the cost of relief or evacuation. This personal risk includes general inconvenience, danger to health, and possible loss of life. All this will affect, adversely, the desirability of the area for habitation and thus reduce the value of the property. This personal risk can have only a subjective value, being worth what the individual will pay to escape it. Unfortunately, there is little else upon which to base an estimate of the monetary value of the personal risk except the decrease in property value or the making of some allowance, more or less arbitrary, in the indirect losses. Under the latter assumption, indirect losses of residential areas may perhaps be considered to be the same as those of the business and industrial areas.

There are several serious defects connected with this method of evaluation, however, which preclude its acceptance for general use. Depreciation of value will be quite high immediately after a flood, especially if it is the first experience in the area; but, as the hysteria subsides, property will regain a part of the loss in value, and, as time erases the memory of the flood and new people come into the area, values will probably reach practically their former levels. The decrease in value will depend upon the time elapsed since the last great flood, but the real flood risk will continue as before unless some means of protection have been provided.

The capital-loss method bases the estimated damage on a single flood. As is well known, there are other floods of lesser magnitudes and possibly others of greater, and therefore there is only the faintest chance that the total damage of a single flood can equal the capitalized value of the annual flood loss. The annual flood loss or risk sustained is the total of all floods, great and small—not from just one large inundation. Furthermore, the damage caused by one flood cannot be compared properly with the capital cost of protection because of the element of time, which enters through the frequency of floods and the interest on the investment.

In order to determine the capitalized value of the annual flood loss, the latter must first be obtained by some means, after which there would be little reason to capitalize it. It may be said that the annual cost can be obtained from the total flood loss by applying a suitable rate of interest. What rate? This question must be answered whether the annual loss is desired from the total damage or the latter is obtained by capitalizing the annual loss. In either case the rate must be selected arbitrarily or on a basis not relevant to the flood risk, which action will yield a result little better than a guess.

Other Benefits from Flood Control

Flood-protection works frequently result in benefits other than the elimination of the losses discussed herein. A source of benefit may be found in some localities in the possible appreciation of land values when an area is adequately protected from a recognized flood menace. The lands subject to such appreciation must be situated adjacent to, or in, a highly developed locality where other land is scarce so that there can be little or no doubt about the utilization of the land once it is protected. Appreciation of land values opens a way to assume highly speculative returns as benefits for protective works, and for this reason such possibilities should be examined with great care. Conservative estimates of benefits will eliminate the assumption of increased land values except in the most favorable cases. Before it can be allowed, there should be in view a need for the land and a manifestly ready market, two conditions which practically limit appreciation to favorably located urban property. At least for the present the alleged overproduction of farm products should be a sufficient objection to allowing appreciation of rural lands as a benefit of flood protection.

Occasionally a vast, ill-defined, and unevaluated benefit (designated "social benefit") is advanced as a reason for flood protection. Social benefits are essentially the total elimination of all the foregoing losses, including the personal risk. Except the latter, all such benefits can be evaluated by some means. The value of the personal risk can be evaluated subjectively only, and will depend upon what the community will pay to eliminate it. Certainly a community that has voted against contributing as little as the right of ways has stated in no unequivocal terms that the flood risk, personal or other, has no value for the people living there.

When flood protection is sought by means of reservoirs, other benefits may (in favorable cases) be obtained. In large reservoirs there may be storage for power, or sanitary, or navigational, uses by augmenting the stream flow in low-water season. Pools may be maintained at fixed elevations so that more or less permanent lakes are created, which may be utilized for summer resorts or other forms of recreation. All these, however, are by-products peculiar to one type of protective works, and to certain favorable sites, and will assist in reducing the cost chargeable to flood protection if they have a recognizable value. They are not obtainable by the reduction of flood losses, however, and for that reason are not considered further in this paper.

Collection of Damage Data

The collection of data of flood damage is a task that requires care, experience, and a broad understanding of economic values in all phases of human activity. Much of the data must necessarily be estimated, particularly the indirect losses; and, although there are many cases of actual costs obtainable shortly after a flood, considerable judgment necessarily must still be exercised. Reliable judgment can be derived only from experience. The compiled data should be as full and complete as can be secured with reasonable effort. Information of certain kinds is necessary, and other kinds are desirable for guidance of judgment or basis of estimate.

The essential data required for computing the value of total damage and the annual loss fall into four distinct classes: (*a*) Monetary value of the damage sustained; (*b*) elevation of the property above a fixed datum which is, or can be, correlated with the available data of flood crests; (*c*) location of the damage; and (*d*) stage or discharge of the flood. In addition to the foregoing data, more information is needed to support the primary data of damage. This information is extremely varied, since it pertains to every variety of economic activity, and can be classified as follows—

For buildings: Approximate dimensions, condition, type, and material of construction.

Bridges: Purpose, type, dimensions, material, and approximate age and condition.

Highways and railroads: Type of surface, length affected by the flood, and amount of traffic.

Business losses: Nature of business, approximate payrolls, annual sales or production, and gross profits.

Acknowledgment

The ideas and opinions expressed herein have been developed by the writer while engaged in work on flood-control studies in the various district offices of the U. S. Engineer Department. They have grown with the duties undertaken and have been shaped by innumerable conferences with many colleagues. The opinions expressed herein, however, are those of the author and do not necessarily reflect the official view of the U. S. Engineer Department.

DISCUSSION

E. L. Chandler,[7] **M. Am. Soc. C. E.**—Probably there will be no dissent from a statement that the construction of a flood-protection project should not be considered justifiable unless the resulting benefits will exceed the cost. Benefits include the elimination of prospective losses directly attributable to floods, the expansion of existing facilities, and the development of new areas, which will result from improved conditions.

The author refers to "The losses from which the benefits are computed * * *" (see "Introduction"), and predicates his study on the thought that "When the losses are known, the annual benefits can be computed and compared with annual costs." This seems to be a limited and ultraconservative point of view. Any full measure of benefits must include not only computations covering tangible losses but an estimate of future expansion and development of potential resources that will be made available by reason of flood protection. Mr. Foster has not ignored this phase of the subject, but the writer is of the opinion that strict adherence to the methods advocated is likely to result in overemphasis of the specific "losses and damages" due to floods under existing conditions, at the expense of fuller consideration of the many indirect benefits and of benefits to be anticipated from potential developments that will be encouraged by flood protection. Any public development should be built with an eye to the future, and this is especially true of flood-protection works.

The writer realizes that an estimate of future benefits is a matter to be approached with caution. It is entirely possible to rise to flights of fancy that may seem to warrant expenditures for flood-protection works which should not be built. Conservatism must be exercised in making computations contingent on prophecy.

It happens that there is no question as to the serious need for flood protection for Chattanooga, Tenn. Careful estimate indicates that, with the existing development of the community, measurable damages that would result from the maximum flood to be expected would amount to between two and three times the cost of protective works. On the other hand, there are thousands of acres of land within the city limits that are unusually well adapted for industry and business developments except for the flood menace. These areas, which are well located for service by railroad, highway, navigation, and utilities, now stand idle. No surrounding territory is available that will compare favorably with these areas after they have been guaranteed freedom from flood. No measure of the benefits to be gained from flood protection would be complete without an estimate of development to be confidently expected as the result of protection. Although such estimate is not required to prove the necessity for protection at Chattanooga, it may easily happen that, in other instances, economical justification for worthy projects might fail of recognition unless future benefits were given due weight.

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[7] Care, Chas. T. Main, Inc., Wilmington, Del.

The writer's thought may be summed up by the statement that, although it is poor economics to build works that cannot be justified by the benefits to be received, it is also poor judgment to refuse to make expenditures because of failure to recognize a full measure of benefits that would be gained.

The author states (see heading, "Some Definitions of Wealth and Value") that "value * * * may be defined or measured as the quantity of other goods that may be given or received in exchange * * *," the "other goods" being, in general, money. Again, in referring to the appraisal of property values, he states that "These appraisals must be considered primarily as a sale value * * *" (see heading, "Evaluation by Estimating Depreciated Property Values"). The true value of a piece of property is not necessarily its sale value. Mr. Foster recognizes this elsewhere in his paper, but the writer wishes to place particular emphasis on the fact. It is a common saying that "a thing is worth what you can get for it." Probably such a statement is satisfactory for the purposes of the real-estate dealer or merchant, or the casual buyer or seller, but it should not serve as a criterion for an engineer studying the economics of a project for flood protection; nor will it serve for appraisals of properties to be used as a basis for special benefit assessments in the case of assessment districts. Many factors may tend to render sale price inaccurate as a measure of value for the purpose at hand. Sale prices during boom times or during a period of depression would scarcely be proper. An influence which, perhaps, is more pertinent is the lapse of time since the occurrence of a flood of considerable magnitude. The author correctly states that a marked depreciation of value will follow the occurrence of a serious flood, and that, on the other hand, if a considerable period of time elapses without disastrous flooding, the sale value of properties will reach heights greater than the true values, due to the ignorance of property owners as to potential danger.

Mr. Foster mentions these circumstances under his discussion of evaluation of losses on a "capital basis" and concludes that they "preclude its acceptance for general use" (see heading, "Evaluation by Estimating Depreciated Property Values"). The writer does not agree with this statement. The "capital basis" method affords a feasible and valuable approach to the problem in many instances, particularly in the case of urban communities where complete protection is to be provided. Mr. Foster states that "The capital-loss method bases the estimated damage on a single flood." This is a limited approach. A study based on the effects of a single flood could scarcely be considered adequate.

In this connection the author states that "An appraisal is made of the affected property after the flood to estimate its value then and the value as before the inundation. The difference between the two values is taken as the flood loss * * *." Why "after the flood"? Probably Mr. Foster has in mind the historic apathy of flood-menaced communities. Generally, this has been such that no flood-protection works have been built except in the wake of serious floods, with the result that appraisals of property values and estimates of benefits have been made while the memory of flood disaster was fresh in mind. It is hoped that Chattanooga will prove to be one exception to this distressing rule. In such event, the situation will be the reverse of that suggested by the author. There has been no serious flood at Chattanooga for a long time and none of consequence since the community reached its present state of development. Flood-protection works have been planned and the Chattanooga Flood Protection District appraisers are faced with the problem of determining values now when the gravity of the flood menace is not generally recognized. Such values must be compared with an estimate of the enhanced values that will result from flood protection. Regardless of which conditions prevail, it is very evident that sound appraisal judgment is necessary.

The author states (see heading, "Benefits from Flood Control") that

> "Levees will eliminate all direct losses caused by inundation up to the stage at which they are overtopped. Beyond that stage the loss is as great as if there were no protection. The annual benefits will equal the annual losses below the elevation of the top of the levee or wall."

This thought should be accepted only with qualification. It appears to be substantially correct in the case of protection for agricultural areas although, even there, the overtopping of a levee, with consequent sudden flooding, is likely to result in losses far greater than those that would result from a flood of corresponding magnitude if there were no protection at all. In the case of protective works for urban communities, levees that would give anything less than complete protection should seldom be considered. The presence of an inadequate levee will furnish a false sense of security that will encourage residents to remain, with their belongings, in low areas in the face of approaching disaster. The sudden overtopping of a levee surrounding a thickly populated district is almost certain to result in loss of life and property far in excess of that which would have occurred if there were no protective works at all.

E. F. CHANDLER,[8] M. AM. SOC. C. E.—As viewed by the writer, this paper seems to be the best discussion, well rounded, with brief mention of each of the complete list of possible contributing factors or modifying phases that he has ever seen. It is especially interesting as showing the advance in accepted ideas on this subject during the twenty years, since 1920, and in particular the progress in methods used by the author himself since he first occasionally undertook such investigations.

However voluminous the base data, however elaborate and reasonable the chain of reasoning, there is no absolute certainty concerning the amount and intensity of the rainfall in any assigned future period; there is still less concerning flood flows, and less yet as to the total resulting damages in the future. Rules and formulas assist, but good judgment on the part of the engineer designing flood-protection works is also a necessity.

It can be argued plausibly that the problem of the evaluation of the benefits from flood-protection works, to determine the magnitude and expense of works advisable that will be equal to the cost of the future probable flood losses thereby eliminated, is, for two reasons, the most complicated, difficult, most nearly insoluble, of all engineering investigations. The problem confronts the engineer, however, because he will certainly have occasional floods with their ensuing losses in future years; so he is obliged to try to do the best that he can with it.

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[8] Prof., Civ. Eng. and Dean Emeritus, College of Eng., Univ. of North Dakota, Grand Forks, N. Dak.

The first of these reasons is the uncertainty as to the actual cash loss resulting from any future flood of any assigned height; in business districts, it will vary greatly with business season, with the length of the advance warning, etc., and in agricultural regions even more with the time of year and advancement of the crops. This is not markedly more difficult, however, than the uncertainties and indefinite risks of any business venture.

The second, almost overwhelmingly large, difficulty in making advance estimates of the benefits accruing hereafter from any type or magnitude of flood-protection works lies in the extreme uncertainty as to the probable magnitude of future floods. Rainfall itself is very uncertain in its future times, amounts, and intensities. Even the annual total, in North Dakota, varies in different years from a maximum of about 150% of the normal annual to a minimum of about 50% of the normal; and short storms, of a day, few days, or few weeks, which are usually the cause of the floods, of course, are far more irregular. The portion of the rainfall that becomes floodwater is not logically a fraction of the rainfall, but a remainder after soil absorption, evaporation, small-basin detention en route, etc., have been subtracted. Therefore, doubling the rainfall in any period does not merely double the flood flow but usually multiplies it many times.

For example, in the prairie regions of North Dakota, with a many-year average total annual rainfall of about 18 in., the average total annual runoff in the streams of the region is slightly less than 1 in., although in any single year the total can easily be double the normal, or even much more than that, and in a dry year perhaps be only a quarter, or merely a tenth, of the normal annual total. In some Pennsylvania areas of 40-in. total normal annual rainfall, however, the average annual runoff is about 20 in.; and furthermore, because of the steeper sloping topography, the Pennsylvania flood crests are likely to be more accentuated.

Thus it is self-evident that in any region a slight increase in the total rainfall will presumably cause a disproportionately large increase in the stream flow. It is also self-evident that the peak height of flood flow depends on the intensity and abruptness of the rainfall (especially on small streams) far more than on the total of rainfall, and is very greatly affected by soil conditions, surface vegetation, frozen condition of soil, and a vast number of other factors, all of which the author seems to have included in his lists.

Charles B. Burdick,[9] M. Am. Soc. C. E.—The Society and the author of this paper are to be congratulated upon this concise statement and scholarly discussion of the methods applicable to the computation of benefits from flood-relief projects. Mr. Foster discusses a subject almost as important as the design and construction of the projects for flood relief. The paper deals with a subject too often neglected, partly on account of the difficulties and uncertainties that are usually involved. There may be cases in which the need for flood-relief projects is sufficiently evident to justify the cost involved in the proper remedy without computation of the benefits. Such cases, however, are very few and must necessarily involve only a few people well conversant with the facts. Even in such cases, if the interested people are called upon to "foot the bills," a very convincing case must be presented if the necessary money is forthcoming, and there must be a marked preponderance of benefits if the project is to be financed.

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[9] Cons. Engr. (Alvord, Burdick & Howson), Chicago, Ill.

Every one reasonably familiar with stream flow realizes that a bankfull capacity of creeks and rivers is but a small percentage of the flows that the streams are required to carry as a result of unusual rainfall conditions. It is the rule that at greater or less intervals the adjoining river bottoms must be flooded under exceptional flow conditions. The fact that these occurrences are comparatively rare has led to the occupancy of the bottom lands, in many cases including costly improvements. Thus, there is a flood problem of more or less urgency upon almost every mile of the many thousands of streams that drain the continental United States.

In the days when the doctrine prevailed that works for flood relief must be built at the expense of the benefited property, such works were more or less automatically confined to the most worthy projects. In recent years, a number of exceptionally disastrous floods, which directly affected many people and of which nearly every one became cognizant through the ready means of communication today, have been instrumental in effecting state and federal legislation designed to take the burden largely from the shoulders of the chief beneficiaries of relief. It becomes especially important, therefore, to scrutinize carefully the benefits arising from such works. The public dearly loves to "get something for nothing." Immediately following a great flood, it is good politics to build works for flood relief.

Although there is no doubt that expenditures for flood relief can be justified only by prospective losses without relief, the argument sometimes has been advanced that losses have nothing to do with prospective benefits. This position has been taken by courts in some cases. The attitude probably arose from legal procedures in ordinary special assessment cases of which many thousands have been tried in which there was no way to determine benefits except through opinion testimony. Immediately following a very disastrous local flood detrimentally affecting property values, it is often practicable to secure well-informed layman testimony that certain benefits to property would arise from adequate flood protection. However, such testimony is unprocurable and would be well-nigh useless in the case of an extensive flood problem involving a large river system. There is nothing more practical that can be done than to determine as accurately as possible the probable losses of the past and to project them into the future as a guide to justify expenditures. The difficulty and the uncertainty of such a task cannot be urged as reasons to omit such an investigation. Unless a preponderance of benefits can be reasonably and clearly shown, measured in dollars, the project should not be built because there are far too many other good uses for money.

Many public expenditures are made in these days that necessarily are only subject to justification on the basis of a "hunch" that they are worth while. These often prove to be valuable improvements, although the reverse is also true. An alleged "social benefit" is a general term frequently heard and usually impossible of measurement. Flood-relief projects can be built only with dollars. Many social benefits can be accomplished only by the expenditure of dollars. Human life, sickness, and death are subject to measurement in dollars at least so far as the general public is concerned. The courts have placed values on such things in many cases, and this type of loss and prospective benefit may very properly be considered where it is an important element.

There are sometimes cases in which it is comparatively simple to show a tangible, strong preponderance of benefits based on indisputable evidence. There are many other borderline cases in which the accuracy of benefit computations may be seriously questioned. The public would do well to confine its flood-relief expenditures to cases in which the preponderance of benefits can be clearly shown.

H. K. BARROWS,[10] M. AM. SOC. C. E.—In his discussion of "Evaluation by Estimating Depreciated Property Values" the author has indicated clearly the objections to using, and difficulties encountered in attempting to use, this method of estimating flood losses and benefits, which he calls the capital-loss method. The writer is quite in accord with his views on this matter and desires to accentuate them.

If the estimates of annual flood losses substantially cover all direct and indirect losses, including proper allowance for any intangible losses, the annual depreciation losses of a property or plant, if correctly determined, will be a reflection of the sum of all these determined losses and hence another determination of the same quantity. In other words, the capital-loss method gives another, less perfect, determination of what has already been obtained by determining average annual losses, based upon the sum of direct, indirect, and intangible losses, and expressed as a capitalized amount for comparison.

In the case of the Connecticut River Flood Control project,[11] in which depreciation was included in estimating flood benefits, the following yearly figures resulted:

Class	Yearly benefits	Ratio to direct benefits
Direct	$ 853,700	1.0
Indirect	808,400	0.94
Depreciation	3,597,000	4.20
Total	$5,259,100	6.15

In this case depreciation was based upon estimates made for the 1936 flood—the largest on record—whereas the direct and indirect benefits were based upon the average yearly flood over a long period of years. Direct plus indirect yearly benefits total $1,662,100. Allowance for intangible losses may raise this total to, say, $2,000,000. The depreciation method, which is (or should be) another determination of the same quantity, gives $3,597,000. This is obviously not based upon a long-time average flood, but the greatest flood of record, of very recent occurrence, and hence inflated in amount.

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[10] Cons. Engr., Boston, Mass.

[11] H. R. Doc. No. 455, 75th Cong., 2d Session, pp. 108–109.

The fallacy of adding together two determinations of the same quantity instead of averaging them, if of equal weight in accuracy, is obvious. Considering the uncertainty of the estimated yearly depreciation, it appears that yearly benefits for the Connecticut River project should not rationally be greater than about $2,000,000. This warrants an expenditure for flood relief of about $40,000,000, instead of about $100,000,000, which resulted when depreciation benefits were added in (as was done in the report previously cited, upon which the approval of this flood-control project was based).

The comprehensive flood-relief project for the Connecticut River is now estimated to cost about $75,000,000. It is clearly of doubtful economy and indicates that the project should not have been approved in the form presented.

The author is to be commended for his excellent presentation of a timely subject.

Roger E. Amidon,[12] **Assoc. M. Am. Soc. C. E.**—The evaluation of flood losses and flood-control benefits is an intricate and involved science. The paper by Mr. Foster is comprehensive and as complete a treatise on the subject as the writer has seen. It certainly will be widely used as reference. It approaches the category of a handbook. The author has stated with clarity the enormous number of details involved in an analysis, as well as the indefiniteness of the data upon which every study must be made. Judgment based on experience is rightly given important emphasis.

It is the object of the writer to add a word of caution to the use of the generalized formula in specific cases. The experienced analyst will not be likely to use this information in a manner that was not intended, but the inexperienced analyst may take it too literally and end up with results that are extremely erroneous or, what is more likely, he may become so "bogged down" with inconsistent results that he will throw up his hands in despair.

The watersheds of Southern California are a case in point. The methods of analysis presented by the author could be applied to a study of the Los Angeles River, or any of the larger watersheds with probably fair results. These same methods cannot be applied to the smaller tributaries originating in the steep mountains. Stage-discharge relationships are not applicable to these mountain streams because of the character of the torrential flows of muddy and debris-laden waters that debauch from the canyon mouths on to the alluvial fans below. A comparatively small flow of water may cause damage to areas that have not been damaged by previous major floods. The lodging of a boulder or several logs may cause a stream to leave an apparently well-defined channel and cut through areas that have not been touched by floods since the coming of the white man. Denudation by fire may cause a small tributary to go on a rampage from a comparatively small rainfall. With present fire detection and fighting facilities, it is improbable that a sufficient part of a large watershed would be burned at any one time to cause a flood that would be extremely abnormal for the amount of rainfall over the entire area. Even within canyons, above their mouths, the flood flows will meander over bench lands and cause damage to recreation and other developments entirely out of proportion to the stage-discharge relationship. The high percentage of solids in the flows from these canyons causes the fluid to act considerably different from that which normally could be expected from a flow of flood water. Henry B. Lynch,[13] M. Am. Soc. C. E., stated this fact clearly in 1939. In analyzing the economics of an area where such irregular conditions exist, it is necessary to give consideration to these conditions if good results are to be obtained. In areas that are bordered by mountains where denudation by fire is possible and probable, it becomes necessary to correlate the occurrence of fire with the occurrence of flood. It is possible to do this and obtain reasonable results from a study of past occurrences within the area or within adjacent watersheds. In such a comparison it is necessary to make adjustments for differences in values, types of vegetative cover, topography, etc., between the area under analysis and the one with which it is being compared. Values may be adjusted by comparing population or assessed valuation. Areas subject to this type of damage in Southern California are frequently desirable for urban development and consequently may change from an agricultural or waste area to a highly valuable residential section during the period for which the study is being made. Broad experience, judgment, and understanding are essential in making reasonable adjustments to take care of such conditions.

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[12] Lt., 2d Air Force, U. S. Army, Fort George Wright, Wash.

The author's definition of "social benefits" does not describe what the writer prefers to term "Benefits for which it has been impossible to assign monetary values." In special cases these benefits may be so extremely important as to overshadow other benefits. Mountain canyons adjacent to highly developed residential and industrial centers may lose high recreation value as a result of torrential floods. The transformation of such a canyon bottom from a wooded, shady, vegetation-covered glen, to a barren waste of rocks and boulders, is a definite loss to society. Hikers, picnickers, and others seeking outdoor recreation do not gain as much pleasure from the use of an area so ravished. Although the actual number of people using the canyon may not be materially reduced, the pleasure and value to the user are less. If a correct analysis is to be made such flood losses must be given consideration.

The loss of a home and land by people who have given years of their life to their building is usually much greater than any factor commonly applied to the assessor's figures. The writer is familiar with cases in which people have spent several times the market value of their property repairing damage done by a single flood. In one case, about $40,000 was spent on replacing soil, trees, roads, and stream channel on a piece of property that could not be sold on the open market for $15,000. The writer is familiar with many other cases in which the individual cannot replace damage that was done by floods, but goes on year after year feeling a hurt in his soul each time he looks at the spot where the great oak once stood. This is a value that is not customarily included in flood loss. Sentimental values such as this are essential to human happiness and well being.

Other items exist for which it has been impossible to assign values. Loss of human life as well as loss of wild life is not usually considered in evaluating flood-control measures. Many streams in Southern California have been made useless for fishing as a result of floods.

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[13] "Transient Flood Peaks," by Henry B. Lynch, *Transactions*, Am. Soc. C. E, Vol. 106 (1941), p. 199 (published in *Proceedings* in November, 1939).

In the economic analysis of a proposed flood-control program all benefits that will accrue to the proposed measures must be considered and evaluated if the results are to show correct justification. In light of the many indefinite items that combine to make up most studies, such as hydrology, damage relationships, etc., it does not appear to the writer to be such a far step to the monetary evaluation of intangible items such as the aforementioned. Widespread questionnaires, designed and analyzed by competent psychologists, could probably answer, with reasonable accuracy, the question regarding the loss of recreation value. A factor relationship might be developed between measurable and immeasurable benefits by a detailed study by competent technicians in various types of areas. Where measurable values are large, immeasurable values will usually also be large.

It appears to the writer that if flood-control expenditures are to be based on economic justification it will be necessary to set up "yardsticks" of evaluation for all items involved. When this is done in a consistent and uniform manner it will be a step toward the elimination of pressure-group flood-control expenditures. The "cards will be face up on the table" where every one can see the "winning hand." This will be particularly important during the coming years when flood-control projects will be in the foreground for taking up the slack of unemployment resulting from the reduction of the war industry.

Hyman J. Fine,[14] Assoc. M. Am. Soc. C. E.—The importance of the paper by Mr. Foster is readily realized by the fact that the justification for the millions of dollars spent annually on flood-protection projects must be based on a study of the seriousness and extent of the flood problem and on a comparison of the annual charges of the proposed projects with the annual benefits to be derived from them.

As stated in the paper, the average annual flood loss for a particular reach of river is based on: (*a*) The frequency curve for the control gage in the vicinity of the area under study; and (*b*) the stage-damage curve.

Frequency Curves.—On certain streams the available record of flood heights is not sufficiently complete for the preparation of reasonably accurate frequency curves. In many instances, recourse must be made to the flood data available for adjoining watersheds. Furthermore, frequency curves derived by equally acceptable methods of procedure may indicate a variation in the probability of flood stages.

The modified frequency graph for a certain section of a stream, wherein the floods are reduced by tributary storage reservoirs, depends on the flood routing studies which indicate the effect of such tributary regulation on the main stream flows. Such studies are usually complicated by the lack of adequate hydrological data of past storms for floods of varying magnitude.

The aforementioned factors indicate the importance of extensive hydrological research prior to the preparation of the required frequency curves.

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[14] Associate Engr., U. S. Engr. Office, Norfolk, Va.

Stage-Damage Curve.—Mr. Foster properly emphasizes the necessity for exercising considerable judgment in the compilation of the data required for the stage-damage curve. In general, reliable information relative to the monetary losses from past floods is not readily obtained unless the floods are of recent occurrence. Furthermore, the establishment of a stage-damage curve, which is accurate throughout the range of probable flood heights, usually requires estimates of the losses for floods higher and lower than those that have occurred recently.

The three classes of property that generally constitute the major part of the damage appraisals to be made in urban areas are large industrial developments, small commercial establishments, and residential property. Large industrial establishments must be covered by separate interviews, and the plant engineer and manager are usually able to furnish the damage data required. In the case of the small commercial establishments or residences, it may be difficult to ascertain the future flood damages to the structures or improvements therein from the owners or representatives inasmuch as these people usually do not have sufficient knowledge of the cost of repairs. Such information can be obtained more readily from other sources.

As an example, Table 3 illustrates the collection of flood-damage data for a small commercial concern. The damage to the stock has been obtained from the proprietor who is thoroughly capable of furnishing such information. One important factor to take into consideration in estimating the stock damage is the time of flood warning available to the community, wherein this concern is located, so that the owner may raise his stock to a height higher than the crest of the anticipated flood. Damages to structures and improvements have been obtained from a curve indicating the cost of repairing floors, walls, plaster, fixtures, heating system, refrigerators, windows, doors, etc., for various heights of flooding. This latter curve is based on data furnished by contractors who perform such work. The repair costs should be reduced by a depreciation factor to determine the actual flood loss to the business establishment.

TABLE 3.—Appraisal of Flood Damages in Zone 1, Roanoke, Va., for the October, 1937, Flood and for Probable Floods Above and Below the 1937 Stage

(Mr. John R. Scott, Owner, Gille Clothing Co., 601 Chestnut Street, Roanoke, Va. [Left Bank of Stream])

Item of damage	Feet Above or Below the 1937 Stage:[a]									
	−8[b]	−6	−4[c]	−2	0	+2	+4	+6[d]	+8	+10
Direct Damage										
Structure and improvements			0	100	300	500	600	700	1,000	1,100
Machinery			0	50	200	350	350	350	600	700
Stock				0	400	600	600	600	1,200	1,500
Miscellaneous										
Total			0	150	900	1,450	1,550	1,650	2,800	3,300
Total indirect damage			0	100	300	440	500	580	1,120	1,320

[a] Elevation of 1937 stage in vicinity of establishment is 862.5 ft above mean sea level. [b] Ground. [c] First floor. [d] Second floor.

The indirect loss is expressed by the following formula:

$$\text{Indirect loss} = N\,S\,P \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which: N = number of days the establishment must be closed due to evacuation and rehabilitation following the flood; S = average daily sales lost during the flood period; and P = average gross markup per dollar of sale (including overhead, labor, and profit). A curve indicating the number of days of shutdown and rehabilitation for small business concerns can usually be established from actual flood data and from information furnished by contractors relative to time involved for repairs, etc. The average daily sales and the average gross markup can be obtained readily from the various business establishments affected by floods.

Agricultural Losses.—With reference to the method of deriving agricultural losses explained in the paper, it is believed that one additional item, which may be referred to as the inundation factor, should be taken into consideration and listed as item (*h*). This factor is based on the best available damage data for past floods of varying stages and may be expressed as follows: Inundation factor (for a flood of certain magnitude occurring in a particular month) $= \dfrac{\text{actual crop loss per acre}}{\text{maximum possible crop loss per acre for that month}}$. The total crop loss resulting from a flood of certain magnitude can now be expressed by

$$\text{Crop loss} = I\,(A\,C_1\,S_1\,L_1 + A\,C_2\,S_2\,L_2 + A\,C_3\,S_3\,L_3 + \cdots) \ldots\ldots (3)$$

in which: A = total cultivated area inundated; C = percentage of cultivated area in a certain crop; S = sales value of that crop per acre; L = maximum possible crop loss in the month that the flood occurred, in percentage of the sales value; and I = inundation factor. The $(S\,L)$-value would be obtained from a graph similar to that in Fig. 4.

If the inundation factor is taken into consideration, the crop loss value of $12.62 shown in Table 2 would not remain a constant for all stages. Depending on the topography of the area inundated, the crop loss would probably be a minimum at the lower stages, would gradually increase to a maximum at the medium stages, and then would remain at a maximum or possibly reduce slightly at the higher stages.

Annual Benefits Attributable to Local Protection.—Local protection improvements will usually eliminate all the losses caused by inundation up to the stage at which the flood-prevention structures are overtopped. Floods higher than those against which protection have heen provided have a certain probability of occurrence within the life of the protection works and may destroy or impair the usefulness of these structures. Therefore, it is logical to deduct, from the annual benefits, the incremental annual damages caused by such floods over what they would normally have caused without the construction of the flood prevention works.

Standardization of Data.—It is believed desirable to standardize on certain of the data required for a flood study. Stage-damage curves above first-floor levels for certain types of property, monthly crop-loss graphs, and other similar data, if available in standard form, would obviate the necessity of extensive studies for the preparation of such graphs.

Mr. Foster's paper is a distinct contribution toward the determination of the average annual losses resulting from floods and the benefits to be derived from flood-prevention structures.

Otto F. Buzhardt,[15] **Esq.**—A clear picture of the economics of flood control is presented in this paper. One point of special interest is the emphasis placed on indirect and unique losses and damages. It is usually a relatively simple matter to estimate physical flood losses or damages, but to evaluate indirect losses properly is often more difficult. These losses often exceed direct damages.

As Mr. Foster stated, stage-area and stage-damage curves may be derived through the use of information obtained through flood damage surveys. As a matter of fact, all estimations of flood damages and flood-control benefits depend primarily on field surveys of the flooded areas. When used in conjunction with the record of a conveniently located gage, areas and damages other than to crops may be determined readily for any stage. Typical examples of these curves are given in the following:

Fig. 6(*a*) shows the areas flooded, subdivided into cleared and total areas for respective gage heights. It will be noted that the cleared area usually begins well above bankfull stage because of the high frequency of overflow at the lower elevations. Naturally, under varying conditions, the areas that were inundated for a certain gage height may not be precisely the same but in general the difference will compensate.

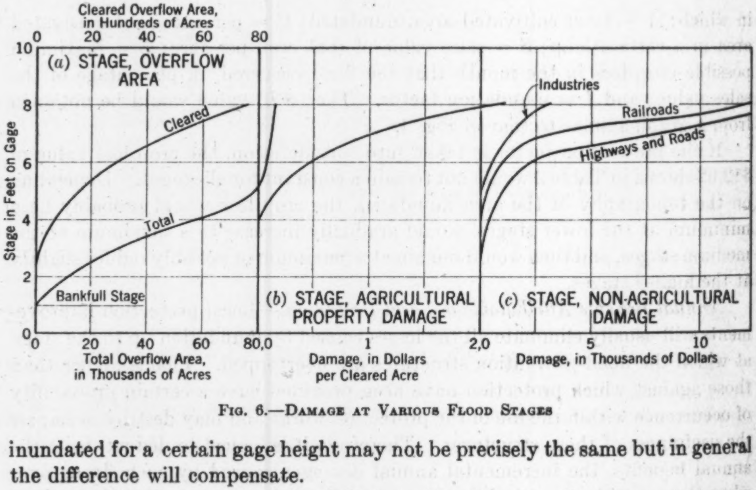

Fig. 6.—Damage at Various Flood Stages

The author asserts correctly that agricultural losses, other than crops, may be treated as urban losses, but it is logical to place damages of this kind on a per-cleared-acre basis (see Fig. 6(*b*)) since the types and extent of improvements primarily depend upon the cleared farm area. Usually these curves have a tendency to flatten out for higher stages due to the fact that improvements are located on higher parts of the flood plains. Mr. Foster has a good point in that, as a matter of conservatism, the maximum estimated damage for any property-value curve should be limited to that for which an actual computation has been made.

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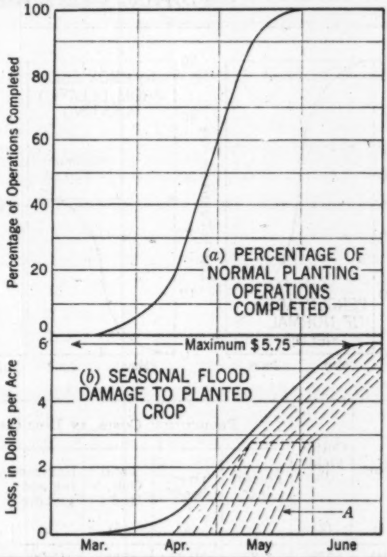
[15] U. S. Engr. Office, Vicksburg, Miss.

Stage-Nonagricultural Property Damages.—In order to construct a curve such as the stage-damage curve for Lawrence, Mass., damages such as those of Fig. 6(*c*) must first be ascertained. It is important to note Mr. Foster's explanation that the composite curve is of direct damages. Since indirect damages depend on factors other than stage, their values may not be a part of this or similar curves.

TABLE 4.—Derivation of Factors Used in Plotting Curve for Damages to Prepared Ground and Planted Crop

First date of flood	Preparing and Planting		Other Operations		Total damages
	% completed	Loss per acre	% completed	Loss per acre	
March 15	0	0.00	0	0.00	0.00
April 1	6	0.17	0	0.00	0.17
April 15	20	0.55	0	0.00	0.55
May 1	63	1.73	6	0.18	1.91
May 15	94	2.58	20	0.60	3.18
June 1	100	2.75	63	1.80	4.55
June 15	100	2.75	94	2.82	5.57
July 1	100	2.75	100	3.00	5.75

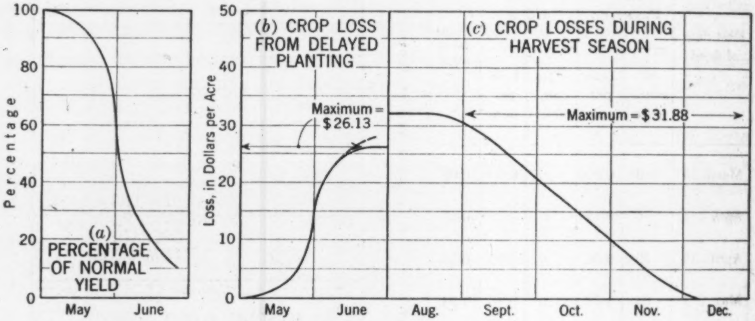


The author has summed up very neatly the data needed for the construction of crop curves, but since agricultural damages in some parts of the United States constitute the greater part of losses, it is believed that a more detailed explanation of the derivation of these curves is needed. Accurate estimates of crop losses are difficult to compute because of varying types of farming and crop distribution. Topography, as well as the sectional locality, determines the type of equipment used on farms, with a resulting difference in production costs. For this reason the basic information used in the following curves must apply to a specific area, but it is believed that the principle may be applied to a crop loss in any locality. Although the areas encountered support a variety of crops with a wide range in gross value of yields, curves relating to cotton crop damages are used for illustrative purposes.

Curve (*a*) in Table 4 shows the average first date (March 15) that cotton will be planted, with normal planting variations between these dates. This curve is considered with normal conditions only and is an integral part of curve (*b*), Table 4.

As Mr. Foster states, it is assumed that crops are destroyed when lands are flooded, regardless of the depth of water; therefore, the season of flooding is the governing factor in estimating crop losses. Curve (*b*), Table 4, the planted crop damage curve, is designed to show the accumulated expense incurred in planting and cultivating one acre of cotton. These values are referred to days of the year in order that planted crop damages may be obtained for any day on which an overflow may occur. Should a flood destroy a crop between March 15 and July 1, the resulting damage would be the amount corresponding to

TABLE 5.—Derivation of Factors Used in Plotting Curve for Crop Losses



% yield[a]	Cotton yield[b] (lb)	Production Costs, in Dollars					Value of yield	Col. 8 minus Col. 7	Profit[e]	Date
		Preparing[c]	Overhead[d]	Total, Cols. 3 and 4	Harvesting and ginning	Total				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
100	375	5.75	20.25	26.00	15.00	41.00	46.88	5.88	0	April 20
90	340	5.75	20.25	26.00	13.50	39.50	42.20	2.70	− 3.18	May 20
80	300	5.75	20.25	26.00	12.00	38.00	37.50	− 0.50	− 6.38	May 26
70	260	5.75	20.25	26.00	10.50	36.50	32.80	− 3.70	− 9.58	May 30
60	225	5.75	20.25	26.00	9.00	35.00	28.10	− 6.90	−12.78	May 31
50	190	5.75	20.25	26.00	7.50	33.50	23.40	−10.10	−15.98	June 1
40	150	5.75	20.25	26.00	6.00	32.00	18.75	−13.25	−19.13	June 3
30	110	5.75	20.25	26.00	4.50	30.50	14.05	−16.45	−22.33	June 8
20	75	5.75	20.25	26.00	3.00	29.00	9.40	−19.60	−25.48	June 15
10	40	5.75	20.25	26.00	1.50	27.50	4.70	−22.80	−28.68	June 25
0	0	5.75	20.25	26.00	0	26.00	0	−26.00	−31.88	....

[a] From curve (*a*). [b] Lint. [c] Includes planting, cultivating, and poisoning. [d] Includes work stock care. [e] Col. 9 minus 100% potential loss ($5.88), to be used in plotting curve (*b*).

the date of flooding. If this flood occurred at such a date that reasonable return could not be expected from a replanted crop, then floods subsequent to that date would naturally not damage crops below the elevation of the previous flood. In general, season permitting, crops are replanted within approximately ten days after lands are free from flood waters, the cost being almost equal to that of the first planting. As the season progresses, the speed of replanting operations is increased as shown by the broken lines "*A*" of curve (*b*)(Table 4). In estimations for a series of floods, the damages to a replanted crop are indicated by lines "*A*," the initial point falling ten days after the preceding flood. Consideration has been taken of the fact that flood water will gradually recede to bankfull stage, and it is assumed that a farmer will plant from the higher to lower elevations.

Curve (*a*), Table 5, shows the decline of production with respect to dates of late crop planting. This curve is used in deriving factors for curve (*b*), Table 5, "seasonal value of crop loss from delayed planting."

Crops destroyed by floods and planted after the average date of April 30 are usually reduced in production value. It is logical that a farmer will not continue replanting to a date beyond which his return is less than his cost of overhead and work stock care plus his normal net gain. Curve (*b*), Table 5, shows the rate of decline in production value. Curve (*c*), Table 5, "crop losses during harvest season," shows values of the crop from the time the crop is "laid by" through the harvest season. The maximum value shown in the curve is the sum of maximum values found in curves (*b*), Tables 4 and 5. When these curves, ((*b*), Table 4; and (*b*) and (*c*), Table 5) are combined, a curve such as Fig. 4 of the paper will be the result.

Under the heading "Benefits from Flood Control" it should be stated that channel improvement is used for stage reduction, as are reservoirs. The stages are reduced through better drainage facilities and as a rule the duration of flooding will be shorter. The entire flood will be moved forward—the first day of flooding occurring possibly a day or more earlier than it would have in the unimproved channel and the last day of flooding will probably be several days earlier, depending, of course, on the individual flood.

Contrary to Mr. Foster's view, the writer believes that the total benefits derived from a flood-control project should be: (1) Flood damages prevented, through protection, plus an increase to allow for future development of the area considered, without protection; (2) enhancement to lands already cleared; and (3) enhancement to woodland to be cleared. Some areas are subject to frequent flooding and, as a result, rich alluvial deposits have been built up. As a matter of fact, some farmers find this land so fertile that crops are planted in spots that are not above a three-year flood frequency. These are unusual cases, of course, and should the land be placed on the market for sale it would be found to have little value. Land values in these areas increase, however, as the elevation increases and the flood frequency decreases. It has been found through field investigation of protected areas, similar to those mentioned herein, that land values will be increased by improved flood conditions. This is due to the probability of increase in the productivity of the land through the use of more economical or intensive cultivation methods and rotation of crops.

In estimating the increased value of cleared land on the basis of increased productivity, the lands, under existing conditions, are analyzed to determine the average annual acre cost and income within the area considered. The comparison of these estimates with corresponding values representative of adjacent areas, not subject to frequent overflows, indicates the annual cleared land value to be expected by reason of flood control. If possible, it is preferable to compare the area under consideration with lands already protected by works similar to those proposed, rather than with lands above flood plains where values are dependent upon conditions other than flooding. Since the value of agricultural land varies, in general, according to its productivity or net income, comparison of values before and after protection is indicative of the annual increased land values. This value is exclusive of flood damages prevented.

Annual increased value of woodland, afforded protection, depends primarily upon its value as potential cultivatable land. Flood damages in wooded areas are usually negligible, and the wooded areas receiving benefits through flood control are restricted largely to those areas which might be suitable or desired for crop lands. Generally, parts of wooded areas to be protected are unsuitable for clearing without considerable expenditures for drainage improvement. Certain areas would be retained as timber or woodland for purposes other than crop production. Therefore, the amount of ultimate clearing to be expected in an area, as a result of flood control, may be estimated through comparison with other similar areas having essentially the same degree of protection as proposed for the area considered.

Remembering Mr. Foster's warning that values vary with elevation, for illustrative purposes it will be assumed that lands, in an area under discussion, average $25 per acre in value. Also, it will be assumed that the average gross value of crops on these cleared lands is equal to approximately 50% of the capital value of land at $12.50 per acre. Computed on a share rental basis, the net rental income will be approximately $2.50 or 10% of the capital value of land, from which interest on investment has not been deducted. To the annual gross value of crops ($12.50 per acre) may be added the annual value of damage to agricultural land that would be prevented by flood protection ($1.50 per acre for this particular area) the sum of which is the average annual gross value of crops given flood protection, or $14 per acre. Since the gross value of crops is equivalent to approximately 50% of the capital value of the land, a crop valued at $14 would be the yield of $28 land, the $3 per acre difference between $25 and $28 representing the capitalized value of flood damage prevented. However, it has been found that lands of the same type, and with the same degree of flood protection, have a value of $35 per acre. From this it is indicated that an average increase in capital value of $7 per acre would be obtained by flood protection beyond the value of flood-damage elimination, and it may be attributed to more favorable conditions. Based on the 10% expected on this cultivated land, an annual average increase in net return of $0.70 per acre on increased capital value of land, aside from that due to flood-damage elimination, would result.

In enhancement to lands to be cleared, it is assumed that the present value of unprotected woodland is $8 per acre, and that the average cost of clearing this land for farming purposes is approximately $15 per acre. From this, the capital value of the land when cleared must be in excess of $23 to give a profitable return. Should investigations prove that the ultimate value of land, when cleared, would be $33 per acre, the indication would be an increased capital value of $10 per acre if afforded protection. Based on an annual net return of 10%, the average annual net return in this case would be $1 per acre for the woodland to be cleared.

It should be remembered that the lands described herein are subject to frequent overflows, and values are not based on hysteria caused by an unusual flood, as was suggested by Mr. Foster in his description of the capital value method of land enhancement.

MALCOLM ELLIOTT,[16] **M. AM. SOC. C. E.**—The author correctly stresses the difficulties attending the use of depreciated property values in the evaluation of flood losses. In first place among these difficulties is the uncertainty as to the rate of interest to use in computing the capitalized value of the annual flood losses. However, lest it be understood that this difficulty is avoided by computing losses on an annual basis, the writer desires to emphasize that, regardless of whether the capital-loss or annual-loss methods are used, the determination of economic desirability of any flood-protection project depends on the adoption of a suitable rate of interest. The question raised by the author—"What rate?"—will inevitably plague the analyst in any case.

The difficulty can be illustrated by an assumed example pertaining to a flood-menaced area of 10,000 acres of corn land:

Benefits:		
Average flood losses, per acre per annum		$ 5
Average annual flood loss (10,000 × 5)		50,000
Costs:		
Cost of protection works		600,000
Annual interest and amortization combined, say 5%	$30,000	
Annual maintenance, operation, and all other annual charges	10,000	
Total annual charges		$ 40,000
Ratio, $\frac{\text{Benefits}}{\text{Costs}} = \frac{50{,}000}{40{,}000} =$		1.25

Can one conclude from this favorable ratio that the project is economically desirable? The capital investment in this instance may be considered the sum of the initial cost, $600,000, and the capitalized cost of future maintenance and operation, say $200,000, or a total of $800,000. This is the equivalent of $80 per acre and if, as often may be the case, other lands, flood-free and of equal fertility, may be bought in the vicinity for from $75 to $100 per acre, an investment of $80 per acre for flood protection of the bottom lands would appear uneconomical.

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The foregoing case is purely imaginary, yet there have been actual cases studied and reported in which, although the comparison of annual benefits and costs appears to justify the project, the capital investment seems out of proportion to the capitalized benefits. Why (as in this case) is it possible to arrive at two different answers according to whether one uses the annual or lump-sum method of analyzing costs and benefits? Obviously the uncertainty is due to the rate of interest adopted. The 5% used in this example, for combined interest and amortization, corresponds reasonably to the rate of interest the federal government pays for borrowed money and an assumed 50-yr life of the project. Manifestly, some other rate of interest must be used if the annual and lump-sum methods of analysis are to be brought into agreement. Again, what rate?

Perhaps there is a fallacy involved in crediting the entire value of flood damages to benefits. By so doing the engineer assumes in effect that, had the floods been prevented, the farmer would have gained his customary return, whereas in fact he might have lost the crop anyway by reason of other causes such as drought, insects, glutted market, etc. It is even conceivable that a flood could benefit him in some years by wiping out early in the year a crop which if saved from floods might later have been destroyed by other causes.

The writer has searched in vain for a formula that will determine whether or not a project is meritorious or in fact complies with the mandate of Congress in the Flood Control Act of 1936 that the "Federal Government should improve or participate in the improvement of navigable waters or their tributaries * * * for flood control purposes if the benefits to whomsoever they may accrue are in excess of the estimated costs * * *." Although the ratio of cost to benefit is ordinarily computed on an annual basis using customary interest rates, it is believed that the final determination cannot depend on this ratio alone. A comparison of the capitalized benefits must also be computed and compared with the total cost of the improvements (including capitalized maintenance and operation) before the engineer can be sure that the investment is a good one.

JOHN B. MCALEER,[17] JUN. AM. SOC. C. E., AND JOHN B. DRISKO,[18] ESQ.—Some of the basic concepts of flood losses are described in this most valuable paper. The author discusses benefits derived from the prevention of direct and indirect flood losses, on the basis of his experience. The method outlined cannot be applied indiscriminately, however, to each and every problem of flood protection. The evaluation of some types of benefit will require other methods. The author mentions some of these (for example, "social losses" and "diminution of capital value of real estate"), but does not include them. In some cases, there are intangible losses that are not included in the category of indirect losses, and there are benefits that do not spring directly from the elimination of a recorded loss. Proper evaluation of flood losses can be realized only by thorough consideration of the entire problem in a river basin coupled with detailed investigations and studies. Superficial studies usually result in partial or inconclusive results.

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The author defines wealth and value as related to flood losses, based upon the *Encyclopædia Britannica*.[2] He makes no practical application of these concepts, however, and under the heading, "Collection of Damage Data," overlooks the determination of the value of the property to be protected. A knowledge of property values is essential to a common-sense view of protection. Determination of property values is required for "evaluation by estimating depreciated property value," a method briefly touched upon by Mr. Foster as an alternative method. Although not suggested by the author, it should be noted that this method ("evaluation by estimating depreciated property value") may also be used to supplement the "annual loss method."

The purpose of this discussion is to bring out the practical application of value and wealth as (1) an approach to a method of rational determination of benefits beyond the method of the author, and (2) a "yardstick" for comparison with the cost of the protective works. The method to be discussed is an application of the approach referred to by Mr. Foster as "capital loss method." The discussion will attempt to clear the basic principles of valuation and increases in property value that result from flood protection—increases that are often quite independent of flood losses. A superficial study of the matter leads one to believe that such benefits are completely evaluated under the annual direct and indirect losses and benefits. The discussion citing the benefits of the Connecticut River Flood Control Plan is an example of this very common error.

Proceeding with the discussion of a rational method of determining losses and benefits, flood losses may be classified in three basic types: "direct," "indirect," and "intangible."

"Direct losses" are the physical damage to property and goods; the usual indexes are the costs of repair, replacement, temporary shifting of goods, and cleanup. Direct losses are caused by each individual flood.

The "average annual direct losses" may be determined by: (*a*) Summing up the total losses over a period of years and dividing by the period of record in years; or, (*b*) determining the average ordinate of a curve showing the relation between direct loss and the frequency. Such a curve is obtained by combining the stage-loss relation and the stage-frequency relation.

"Indirect losses" are the value or cost of the service, or the use lost or made necessary as a result of flood conditions. Losses of business, extra costs of rerouted transport, and other similar losses, both in the flooded area and remote from it, are all indirect losses. Indirect losses are caused by each flood.

The "average annual indirect losses" may be estimated as a proportion of annual direct losses, the proportion being indicated by investigations made for outstanding floods.

"Intangible losses" are the third type of loss. These losses are not susceptible of direct evaluation although they are sometimes of commanding importance. The intangible losses should be given separate statement and consideration. They are partly reflected in changes of property value and may often be evaluated from these changes. The total depreciation of property values as a result of floods reflects not only the capitalized direct and indirect losses, but also the lessened utility and other adverse effects of an intangible nature which determine the use, the value, and the future of property. These intangible losses may be evaluated as real losses of property value, and they are equal to the total depreciation minus the capitalized direct and indirect losses. They may be called "depreciation losses," as distinct from "total depreciation," and classed under a separate category. They are equivalent to a portion of the intangible losses. Recoverable depreciation losses are more practicably evaluated under equivalent benefits—that is, as "increases of property value," because the extent of recovery will vary with time of protection, general economic conditions, prosperity of the region, demand for such properties, and condition of repair and use.

Intangible losses—including life, health, social and economic security, mental and physical distress—are caused by an individual flood but are intensified by a succession of floods—that is, by the flood history of a river. The knowledge that a certain area is subject to flooding casts a blight upon that area; and this causes a decrease in value which is often far greater than represented by the direct and indirect losses. Areas flooded frequently never develop fully, unless they are protected; areas visited for the first time by a record flood may suffer enormous depreciation, especially if the area is flooded or threatened more than once within a brief span of years.

Losses of the third class, intangible losses, are not susceptible of evaluation on the basis of recurring stage, or frequency. These intangible losses are reflected in real losses in the value and utility of property. Protection from floods will restore the value of such property and increase its utility. The benefits to be derived from the restoration of such values may best be classed as increase of property value.

The benefits of flood protection may be classed, then, as (*a*) direct, (*b*) indirect, (*c*) increases of property value, and (*d*) intangible benefits. Benefits (*d*) cannot be measured in dollars but include such items as prevention of loss of life, promotion of general welfare, and social security of the population. The benefits to be derived from a flood-control program are estimated on the basis of recurring losses which will be prevented, and upon the potential increases of value that may result from rendering recognized flood zones free of danger or from restoring highly developed zones that have been severely damaged by one or more unprecedented floods.

Direct and indirect benefits (Benefits (*a*) and (*b*)) are represented by the reduction in recurring losses. Many experienced losses are non-recurring by reason of altered usage or abandonment, or because replacement is combined with a betterment which renders future loss impossible or less likely. Recurring losses become less with each successive major flood, since property gradually becomes adjusted to the greater flood experience. Also, as a zone becomes recognized as a flood-dangerous zone, new capital is no longer introduced and the area may become stunted.

Increases of property value (Benefit (*c*)) are based on the higher utility and value that will result from adequate flood protection, insuring freedom from the disturbing disruption of possible flooding and increasing the net annual value or income of the property. Since many increases of property value that result from flood protection simply reflect the capital value of the direct and indirect losses prevented, benefits in this third class must be limited to a separate increment in value. Where property reflects important intangible losses or where the utility and value of property have been forced recently to a lower level by record flood experiences, increases in property values result from restoration to full utility by flood protection. These are equivalent to the restoration of depreciation losses. Often, benefits will result also from the more productive use of idle or partly developed lands where development has been retarded by floods, depending on future demand for such properties. In evaluating the annual benefits due to increases in property value, each area and class of property presents a problem for individual analysis. The capitalized value of average annual direct and local indirect losses that have been experienced should be deducted where overlapping occurs.

After the benefits have been determined they are compared with the costs of protection. If benefits exceed costs, the project may have merit, but if benefits are less, the project is not economically justified.

A few examples will show the necessity for considering the value of the real estate in relation to the total value of the property to be protected:

Case (*a*).—Consider a lumberyard which is subject to frequent flooding, and suppose the annual cost of protection to be less than the annual value of the losses sustained. If the cost of protection is high compared with the value of land and buildings involved, however, the business should be moved to high ground and flood protection for this property should not be considered.

Case (*b*).—Consider a dwelling that is frequently flooded, and for which annual costs of protection are found to be less than the value of the losses. If the capital cost of flood protection is a large percentage of the value of the house (say, 50%), then protection of the property is not economically sound. Similar reasoning will apply to larger areas also. In certain cases where it is found that the justifiable cost of protection exceeds the value of the property Congress provides in the Flood Control Act of 1938, for evacuation of an area as an alternate to protection.

Case (*c*).—Compare two flood properties—one, a farm that is frequently flooded, and the second a hotel building that is rarely flooded. Although annual direct and indirect losses of the hotel are equal to those of the farm, it is evident that, were the owners of the property to be assessed for the cost of local protective works, the hotel management would be willing to pay more for protection than the farmer, simply because it has a much larger investment or value at stake. In this connection it is of interest to remark that in most cases where the cost of protective works has been borne by the community protected, assessments against the individual properties have been based upon a percentage of valuation or upon an increase in value rather than upon the damage-frequency relation.

Suppose the benefits do exceed costs. Then, is the project economically sound? Although the government finances most flood protection the project should be considered from a strictly economic point of view. Consider a group of business men viewing an investment, with costs to be assessed from the owners of property protected or the owner of an industrial plant considering protection of the plant against floods:

The cost of protection must bear a reasonable relation to (1) the total value or capital value of the property to be protected and (2) the value of real estate or fixed equipment which may not be relocated or moved out of flood danger. This is elementary. If either of these relations is high the project should be abandoned.

Consider the example of the lumberyard described in Case (*a*). Suppose protection is estimated to cost $30,000, whereas capitalized annual benefits amount to $35,000. Protection would appear reasonable because the total worth of the establishment amounts to, say, $200,000; but closer inspection reveals that land and buildings are worth only $33,000, and the remaining worth is represented by stock. It is unreasonable to spend $30,000 to protect a bad investment of $33,000. The business should be moved to a more favorable site.

It is evident that the economic limit of protection should be set at some fraction of the value of the property to be protected, possibly 25%, and some greater portion of the value of real estate. Expenditures for flood protection should be held far below these limits unless benefits greatly exceed the cost of protection.

Consideration should be given to alternates to protective works: (1) Evacuation of the flood area, (2) altered use and zoning for future construction, (3) flood warnings, and (4) insurance.

The method of determining benefits, including increases of value, as outlined in this discussion, has definite advantages. It combines the precision of estimating benefits by the "annual loss" or "summation method" with the completeness of the methods based on "capital loss" or "differential" method. It is possible to determine, rationally, the "intangible" benefits which bear no relation to the direct and indirect losses and sometimes overshadow them. The method is no less accurate than any estimate of the worth of an industrial plant. The method is far more accurate than a computation by the annual-loss method.

"Intangible benefits" based upon "depreciation losses" and representing increases of property value minus capitalized direct and indirect losses will vary in each locality and watershed and are generally found limited to densely populated growing urban and industrial areas or adjacent areas where development has been seriously retarded by floods. In areas that are retrogressing, in which there are idle mills, and in which industry and activity are shrinking rather than growing, properties become greatly depreciated by economic conditions and floods, but there can be little recovery of value. Even direct and indirect benefits in these areas based upon past experience represent an exaggeration. In agricultural areas benefits based upon direct and indirect losses are usually found to equal or exceed the total increase in value so that intangible benefits are zero.

Since September, 1938, flood-control investigations have been made by the Providence (R. I.) Engineer District on five rivers in the Providence District: The Blackstone, Pawtuxet, Thames, Connecticut, and Housatonic.

The principal industrial development along the Blackstone and Pawtuxet rivers has been textile, and these industries have not prospered in late years.

Property values in these valleys have undergone economic depreciation, and recent floods have caused no recoverable depreciation.

The situation in the Connecticut Valley is quite different. The valley as a whole has prospered and activity is expanding. The valley has suffered three major floods in recent years—the 1927 flood, greatest for some seventy years; the 1936 flood, the greatest of record; and the 1938 flood, the second greatest of record on the main river and the greatest of record on the down-river tributaries.

This rapid succession of three great floods, during an otherwise favorable economic period of the valley, has caused substantial depreciation, a major part of which will be restored (and in some cases already has been restored) by proper flood-control measures.

Benefits accruing to the approved Connecticut River comprehensive flood control project have been analyzed by the method outlined herein. Annual flood control benefits are as follows:

Direct	$1,170,800
Indirect	1,029,100
Increases of property value	1,940,200
Total	$4,140,100

Capitalized at 5%, this is about $83,000,000 excluding incidental benefits to existing water power, navigation, recreation and pollution betterment, and to National Defense value, and without allowing for inevitable general growth and development of the area in the next fifty years, the period used for amortization of the works in all economic studies. The cost of the approved project is approximately $71,000,000. The total value of the property protected is approximately $856,000,000, and the real estate totals $596,000,000. Hence, benefits exceed the cost by 17%, and the cost of the project amounts to only 8.3% of the value of the property protected.

The increases of property value occur almost exclusively in the highly developed urban and industrial areas of the lower Connecticut Valley, where complete protection is afforded by the combination of a well-distributed reservoir system and the local protective works—that is, Northampton, Holyoke, Chicopee, Springfield, and West Springfield, in Massachusetts, and Hartford, and East Hartford, Conn. In Hartford, annual flood losses are as follows:

Direct	$ 191,800
Indirect	186,000
Depreciation	1,192,000
Total	$ 1,569,800
Capitalized at 5%	$31,396,000

It will be noted that depreciation losses are large in relation to direct and indirect losses. This is partly a result of the high value of the flood area (nearly $400,000,000) and partly because a dike built before the initiation of the federal project furnished protection against a flood having a probable frequency of occurrence of once in about fifty years and thus eliminated 60% of the annual direct and indirect losses. Had these low dikes not existed, the annual direct and indirect losses would have been about $950,000, and the depreciation would have been correspondingly less. The dikes, which had been constructed by local agencies, were topped in 1927, 1936, and 1938 with disturbing effect on property values in the flood areas. The "intangible" losses as measured by real "depreciation losses" are large and form the major part of economic justification for the dikes and walls which are now under construction. Bearing in mind that the initial benefits of stage reduction are credited to the reservoir system, the remaining benefits based upon the annual loss method would not justify raising the dike in spite of the fact that Hartford was flooded three times in eleven years, and that direct and indirect losses in this period amounted to $19,000,000. By estimating increases in value a more nearly correct measure of the benefits is obtained.

The foregoing study was based on data obtained prior to 1939. Since that time much of the local protection has been completed. Experience has shown that the estimate of benefits was extremely conservative. In one large area in Hartford, land in the flood plane increased 300% in value before the dike protecting it was completed. A large municipal airport which was relocated at a field 5 miles from the center of Hartford after the 1936 flood was returned to its location in the city as soon as dike protection was completed. Expansion of housing and industrial plants within the flood area has progressed rapidly.

Hartford is typical of the other cities of the lower valley, although on a larger scale. Existing dikes at Northampton, Holyoke, Chicopee, Springfield, and West Springfield which afforded partial protection prevented many of the direct and indirect losses, which would otherwise have been experienced frequently.

In these communities, local protection works which could be justified by the annual direct and indirect benefits would be only half-measures, and would neither provide valuable flood protection nor promote any confidence in the value of the works. A true summation of benefits, however, including increases of property value which reflect the "intangible" benefits based principally on "depreciation losses," gives a more nearly complete determination of the real benefits, and justifies the construction of complete protection for these areas. Complete protection, in turn, insures the realization of the manifold benefits.

It is on such a sound basis that local works in the Connecticut Valley have been constructed, and even brief experience shows that anticipated results will be exceeded.

In conclusion, the economics of flood protection requires a thorough consideration of the entire problem in each river basin. A detailed determination of the valuation of the property to be protected is one essential part of such an analysis. In some river basins a thorough consideration of values will show the necessity of supplementing and extending the method outlined by the author to include benefits resulting from increases of property value. With this addition, the suggestions offered by the writer have more general application to river basins of varying character, and make possible a complete and rational evaluation of benefits. The result is a more accurate comparison of benefits with the cost of flood protection.

Edgar E. Foster,[19] **Assoc. M. Am. Soc. C. E.**—The discussion brought forth by the paper has been highly gratifying to the writer. The discussers generally have approached the problem of flood losses and benefits from protection from different angles and have illuminated or elaborated on many points.

The uncertainty of occurrences of floods and extent of damage has been noted by Professor Chandler. Variability and uncertainty attend every factor entering into the formation of a flood and the aggregate of losses caused thereby. It is for this reason that the writer prefers to approach the problem of flood frequency from the statistical standpoint and to consider the prospective losses as a risk or a hazard. The danger in appropriate seasons is always present for the exposed localities so that there is a continual flood risk just as there is a never-ending fire risk.

The case for conservative treatment of flood economics was well stated by Mr. Burdick. Because of the uncertainty and difficulty of estimating flood losses and benefits of protection, more than ordinary precautions should be taken in their calculation. With all due precautions to obtain fully justified benefits, it is still possible to include indirect and intangible benefits where they are plainly suffered.

On the other hand, E. L. Chandler believes that the principles enunciated in the paper are ultra-conservative in that allowance is not made for benefits to be obtained by protecting economic developments likely to be made after flood protection is achieved. All flood-control projects require individual study and treatment, and perhaps there may be individual cases in which benefits to development subsequent to protection may justify a project. Such cases would be recognized in the allowance of enhancement of land benefit of areas to be protected when it is manifest that the flood risk has been the sole or principal factor that has retarded development and where it would be reasonably certain that the area would be developed to the degree attained on adjacent land after the removal of that risk.

To allow the elimination of losses to prospective development would open the way to speculative benefits in an undesirable degree. Certain considerations militate against claiming as benefits for a flood-control project the elimination of losses of economic development resulting from that project. A conservative view of the benefits should be taken because of the uncertainty of the final estimates of eliminating the direct losses; the estimates of the indirect and intangible benefits are even more unreliable. The development of an area given protection usually depends on more factors than merely flood protection. The general economic conditions of the country and the demand for land in the particular area should be considered; these factors may easily prevent development regardless of complete protection. Furthermore, recognition should be made of the probability that damage by future floods may not be as large as in the past because of the development of accurate means of flood prediction. In recent years, the U. S. Weather Bureau has built an extensive organization for the purpose of flood warning.

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[19] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

Professor Barrows agrees that estimates of annual benefits obtained by flood-damage surveys should not be combined with estimates of capital loss based on a single flood. There seems to be no serious question that such procedure is fallacious. It is now prohibited in the U. S. Engineer Department under the existing Orders and Regulations.

Colonel Elliott advocates the use of a capital-loss method as well as the annual-loss; but it should be noted that the capital-loss method he favors is not based on the depreciation of property value caused by one severe flood, but is in reality a capitalization of the estimated annual losses. The question of interest rate arises in such capitalization. It appears to the writer that the rate to be used in this conversion should be comparable to that which may be received or expected by an entrepreneur as a return of a business rather than the interest rate on a good loan or government bond. A return on a business enterprise usually would be higher than a rate of interest paid by the government. It is to be noted, however, that Colonel Elliott does not advocate combining the results of the two methods of computing benefits but only a comparison.

Colonel Elliott states that the problem of interest rate is not avoided by using the annual-loss method. In this he is right, for the interest rate appears in all financial matters. However, when a rate of interest is used in an estimate of cost, it is in the same status as any other item of cost such as steel, concrete, or land: It can be estimated at the current or expected rates for appropriate loans.

The annual cost of a large project consists of diverse items—interest on investment, amortization, repairs and maintenance, and cost of operation. Only the first is a constant function of capital cost. Amortization varies with the assignable life of parts of the project; repairs, maintenance, and operation may vary with little regard to the capital costs, although the relationship may become close with few diverse elements and low-cost operation. It appears, therefore, that a comparison between costs and benefits can be made more satisfactorily on the annual basis.

Flood control is expensive, and protection can be given only where there is a definite well-defined need for it. For a number of reasons it would be advantageous to eliminate losses from forest and recreational areas such as are described by Mr. Amidon, but manifestly it is not economically feasible to eliminate widely scattered losses in mountainous areas. This limitation usually will preclude the necessity for computing benefits. Nevertheless, occasionally there will be a need to estimate benefits on small streams subject to the "cloudburst" type of storm which occurs with such rarity that few statistics are available for the methods outlined. Recourse must then be made to whatever method can be used to determine a reasonable and conservative answer.

Mr. Amidon describes an excellent example to illustrate the subjective value as distinguished from exchange value of property. The person who spent $40,000 on his place must have considered it worth that much to him, although the sale value was only $15,000: The difference, $25,000, must be considered the personal or subjective value. This appears to be an extreme instance, but undoubtedly there are many cases of smaller degree in any large flood. A different type of subjective value is illustrated in the loss of pleasure of picnickers in denuded valleys: If they will pay nothing for flood protection, the value of their pleasure must be considered small.

In homes or places of business there undoubtedly are many items which may have no market value but which still furnish valuable service and have some personal or subjective value to the owners. These goods, if damaged by a flood, must be replaced. Such losses, reported and used in a determination of annual losses, may account for a part of the discrepancy between the capitalized annual losses and estimated property values mentioned by Colonel Elliott.

Since the annual loss depends upon the frequency as well as the loss per flood, it is almost self-evident that careful study must be given to the hydrological aspects of the evaluation of flood losses. The writer is glad that Mr. Fine has emphasized this feature.

Mr. Fine has also given some additional information of the mechanics of the computation of annual losses. Table 3 is a useful step in compiling data for the construction of the stage-damage curves. Turning to Mr. Fine's statement of agricultural losses, however, it appears that the use of the average value of crop loss throughout the season would eliminate the use of the "inundation factor" for purposes of obtaining the average annual loss per acre.

Mr. Buzhardt has given considerably more details of the methods used in the Mississippi Valley for computing the average annual crop losses. Such losses constitute the principal type of losses in that region, and his discussion is of value for that reason. Where any one type of flood becomes predominant, it should be treated in considerable detail. In the broad, fertile, alluvial valleys of the Lower Mississippi River Basin, agriculture is the predominant industry and therefore agricultural flood losses there have been studied probably more extensively than elsewhere in the United States. For this reason, the additional details of the computations of agricultural flood losses are welcome.

In referring to agricultural losses other than of crops, Mr. Buzhardt states that stage-damage curves flatten out. When the stage is plotted on the ordinates and damages on the abscissas, the curve will approach some ordinate as an asymptote as the damages approach the probable total property value. The same situation would occur in any stage-damage curve, urban as well as rural, except that in the former the total property value within a given area may exceed the probable flood loss so greatly that the change in the direction of the curve is not reached by the maximum observed flood.

Mr. Buzhardt believes that the total benefits of protective works would be derived from: (1) Flood damages prevented, plus an allowance for future development without protection; (2) enhancement to lands already cleared; and (3) enhancement to woodland to be cleared.

Item (1), flood damages prevented, is taken to mean average annual flood loss. If the losses are to be estimated on the annual basis, it is agreed that this item is correct. Whether or not an allowance is made for future development should depend entirely upon the local conditions. The writer believes that, as a general rule, in the interest of conservatism, no allowance or very little allowance should be made for future development. Nevertheless, perhaps there are localities to which this general rule could not be applied, such as the existence of particularly important advantages of a site that would induce settlement despite a known flood hazard.

In item (3), there would be little or no annual losses, as Mr. Buzhardt states. The enhancement of value would be made possible because the land is made susceptible of higher development if protected and therefore would return a greater income. As stated by the writer under "Other Benefits from Flood Control," such increased values may be accepted as a benefit for protection if there is a reasonable certainty of utilization. The critical question in this case is whether or not the flood menace was the sole or principal cause of lack of development. If so, there is good reason to believe that the land would be developed if protected, and the increased value may be assumed as a benefit.

On the other hand, item (2), the enhancement of value of cleared land, appears to be an increase in land value brought about by the elimination of the annual flood losses. The correctness of this view is indicated by Mr. Buzhardt's statements: "Land values in these areas increase, however, as the elevation increases and the flood frequency decreases"; and again, "Since the value of agricultural land varies, in general, according to its productivity or net income, comparison of values before and after protection is indicative of the annual increased land values." These land values are taken as capital values as the word "annual" does not appear to change the general meaning. Since there are intermittent floods, the constant flood risk is recognized by buyer and seller alike, and the capital value of the land is decreased by an amount that is substantially a subjective capitalization of the annual flood losses, including the personal risk. The land enhancement of the cleared land, therefore, is simply a restoration of this capitalized loss. The writer firmly believes that this land enhancement should not, therefore, be combined in any manner with the annual losses to obtain a total benefit.

The purpose of including the definition of wealth was to lay the groundwork for the summation of total wealth, the Community Property Account and, in conjunction with the definition of "value," to set a standard by which the personal risk could be measured. By these definitions it was hoped that duplications of losses and benefits could be eliminated.

Since the provision of flood protection is an economic matter and since the costs are measured in dollars, the benefits must be reduced to the same basis in order to make a comparison. The loss of human life may be serious, but it is not possible to evaluate it in terms of money loss to the community without making many more or less arbitrary assumptions. Fundamentally, the value of human life is what the individual makes it in terms of associations with his fellow men rather than in dollars. It will be seen readily that the value will vary greatly among individuals; the possible loss is a personal risk that can be gaged only by that which the individual will pay to prevent it.

The economic losses caused by a flood may be classified in many ways, but the writer believes that, aside from the loss of life, the two classes are sufficient for all purposes, namely, direct and indirect. This indirect category would also include all intangibles listed by Messrs. McAleer and Drisko which have a monetary value.

These so-called intangibles generally constitute the controversial items of flood loss. The fact that they are difficult to define or evaluate should be sufficient reason to regard them with suspicion and treat them conservatively. It is easy to make claims for extensive losses on such grounds but every claim should be examined for a market value or a prospect that the sufferer would pay good money to escape a repetition of the loss. Life and health will have a definite value to the individual, but the average person is not likely to pay much to eliminate danger to either once in 50 or 100 years.

Social and economic security are matters of every-day concern; there is only a relative difference in these items during floods except as the latter affect property adversely. During high water there is undoubtedly physical and mental distress among the flood victims; but what market value can be placed on those factors except that which the sufferers will pay to escape? On the other hand, if one can ascribe a value to such items as mental distress, why not credit floods with an entertainment benefit obtained by the many, untouched by the flood, who spend days in viewing the novel and interesting scenes? The writer has heard of floods in places where the inhabitants got a jug of corn liquor and really enjoyed themselves.

The utility of property is not, in general, intangible in the sense that it has no economic value. All commercial and industrial property exists for one purpose—business—and the object of business is profit. Between floods, business proceeds as usual. There is no reduction of utility because of floods except at the time of occurrence and, therefore, no loss of profit except when flood damage is chargeable against income. Therefore, the loss of utility is measured in loss of income. Noncommercial property such as residences, churches, schools, parks, and libraries present a somewhat, but not fundamentally, different picture. The loss of utility is confined to the period of high water and repairs; the utility is obtained by direct use rather than through income; there is a greater personal risk involved; the losses constitute increased cost of maintenance rather than charges against income. Except for the personal risk, the loss of utility can be measured by the rehabilitation and loss of service or rents.

Regardless of the class or kind of property, the owner can suffer only the losses caused by a flood during his period of ownership, and suffer those as they occur. He will not suffer the loss of capital value until he sells; the known flood risk, of course, will cause a reduction of property value; but after the owner has sold, and accepted his capital loss, he will not be liable for the periodic flood damage. Any given individual cannot suffer both losses and therefore the community cannot suffer the total of both, since community wealth is simply the aggregate of the individuals' wealth.

Messrs. McAleer and Drisko present what they term a more accurate method of determining flood losses. Essentially the method consists of a determination of the capital losses of depreciation of property value which result from floods and a determination of the annual direct and indirect loss. The annual direct and indirect loss is capitalized and the result deducted from the total depreciation loss; some rate of interest is applied to the remainder and the result is called the annual intangible loss.

It can be seen readily that the determination of the total depreciated property loss is the controlling factor; since the total annual loss includes the intangible, the determination of the direct and indirect annual losses is merely a means of allocating portions of the total losses to those categories, the portions being fixed to a large extent by the rate of interest used in capitalization. This method is open to all the objections of basing the economics of flood control on one flood: It does not give adequate consideration to small floods; the frequency of the one flood may be such that the observed damage does not fairly represent the expected average over a period of years; the hysteria following a major disaster is probably a large contributing factor in depreciating property values. Messrs. McAleer and Drisko state that their "method is far more accurate than a computation by the annual-loss method." This statement is questionable in view of the uncertainties introduced and the necessarily arbitrary selection of interest rates.

Occasionally, losses in a community are claimed because industrial or commercial units move from a locality, town, or city that is subject to floods or because a known flood menace causes business organizations to locate elsewhere. In this matter, the interests of the community and state or nation diverge. The loss of a business unit will undoubtedly have an adverse effect on the community because some employees are likely to be left without employment and the community loses a source of tax revenue; but such losses do not accrue to the state or country as long as the business relocates within its boundaries. There will be some expense in moving but this should be chargeable to the savings in flood losses. The total wealth or income of the state or country is the aggregate of the wealth or income of individuals, and it is not affected by the location of the individuals within its boundary. For this reason, an intangible loss should not be claimed because of relocation or lack of industrial development of a flood-menaced area, when the cost of flood protection is to be borne by the United States.

Before closing this discussion, another aspect of the economics of floods must be mentioned. Through correspondence, R. M. McCrone, M. Am. Soc. C. E., raised the question of direct flood benefits. In a humid climate, such benefits are scarcely conceivable; but in the sub-humid and semi-arid regions of the West where dry-farming is practiced, real benefits are obtained from the smaller overflows which provide needed irrigation water. In one instance that came to the writer's attention, a net benefit of $6,000 was reported from one spring flood that covered an extensive area of hay land. This situation is another indication that each flooded locality provides a separate problem and should be studied in the light of sound and conservative basic economic principles.

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WIND STRESS ANALYSIS BY THE *K*-PERCENTAGE METHOD

By F. P. Witmer,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. M. Goodrich, Francis L. Castleman, Jr., Clyde T. Morris, Otto Gottschalk, and F. P. Witmer.

Synopsis

The purpose in preparing this paper was twofold:

(1) To describe in detail a procedure for applying the *K*-percentage method of wind stress analysis which was presented[2] in 1939, in order to facilitate its practical utilization; and

(2) To present a method of design, using the aforementioned principles, but with the assumption that vertical wind reactions and direct wind stresses in columns must be in accordance with the cantilever relation, thus eliminating all secondary moments due to change in length of columns under their direct wind stresses.

Present Methods of Design

In the analysis and design of a building bent of the usual type, having girders framed between columns, with small connections designed to resist pure bending, the engineer is confronted at the outset with two alternative methods of procedure, for the determination of wind stresses:

(1) An "approximate" method, in which the distribution of vertical wind shear across the bent is assumed to occur in some arbitrarily assigned relation, without consideration of the elastic behavior of the various members of the bent; and

(2) One of the so-called "exact" methods, in which the elastic action is considered in a more or less accurate theoretical manner.

The approximate methods are much more readily and quickly performed, and, for buildings of usual proportions, are quite satisfactory, both practically and economically, although their resultant stresses may be considerably in error. However, they ignore the secondary bending moments resulting from

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Note.—Published in June, 1941, *Proceedings*.

[1] Director, Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

[2] "Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings," *Proceedings*, Am. Soc. C. E., June, 1939, p. 975.

change in length of columns under direct wind stresses and, for extreme proportions of height to width of bent, these moments may attain serious magnitude.

The exact methods give correct values for primary wind stresses, but, as usually applied, they also ignore these secondary moments. In one case, however, that of the "Spurr method,"[3] this procedure is entirely justified, because, since the bent is assumed to act as a cantilever fixed at the base, and is carefully designed elastically upon this premise, all joints in each floor will remain in a plane under wind load, and these secondary moments, therefore, will be non-existent.

By any method other than the Spurr method, it is thus seen that these effects of column axial wind distortion are habitually neglected, unless a separate additional analysis of them is made by some more or less laborious process.

The principal approximate methods in use have been two—the "Portal Method" and the "Cantilever Method"—both described[4] by Robins Fleming in 1913. The portal method assumes that vertical wind reactions and direct wind stresses for all interior columns are equal to zero. The cantilever method assumes the direct-wind unit stresses in columns to be proportional to the distance of the column from the neutral axis of the bent. Neither method takes into account the fact that these particular relations of reactions and stresses cannot exist unless the relative sizes of columns and girders are such as to develop them by elastic action. The reactions assumed in the portal method are theoretically correct for primary wind moments if all sizes have been computed from vertical floor loads alone, assuming discontinuous girders, and also assuming: That the floor load is constant throughout any floor; that girders in any floor are of the same depth throughout; and that column areas are always proportional to their moments of inertia.[5] If these assumptions are not made, the portal method is only approximately correct, the degree of accuracy depending upon the degree of departure from these assumptions. The cantilever method can only give correct results if the relation of sizes happens to be such as to develop the assumed reactions and direct column stresses when the bent distorts elastically. This condition will obviously be only of accidental occurrence.

Of the so-called "exact" methods, the "slope deflection method," introduced in 1915 by W. M. Wilson and G. A. Maney,[6] Members, Am. Soc. C. E., and the method of end-moment distribution, presented by Hardy Cross,[7] M. Am. Soc. C. E., are the most outstanding. Each will produce a correct analysis of primary wind moments for a bent whose members are of any assumed size. Both are laborious, that of Professor Cross being far more practical in the case of bents many stories high. Neither method, as before stated, takes account of secondary bendings from column, axial, wind distortion, without an additional analysis made for this purpose.

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[3] "Wind Bracing," by Henry V. Spurr, M. Am. Soc. C. E., McGraw-Hill Book Co., Inc.

[4] *Engineering News-Record*, March 13, 1913.

[5] *Proceedings*, Am. Soc. C. E., November, 1936, p. 1496.

[6] *Bulletin No. 80*, Univ. of Illinois Experiment Station, Urbana, Ill., 1915.

[7] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

PROPOSED METHOD OF DESIGN

If it is considered necessary that these secondary moments be not neglected, the only method heretofore available is the Spurr method which is also very

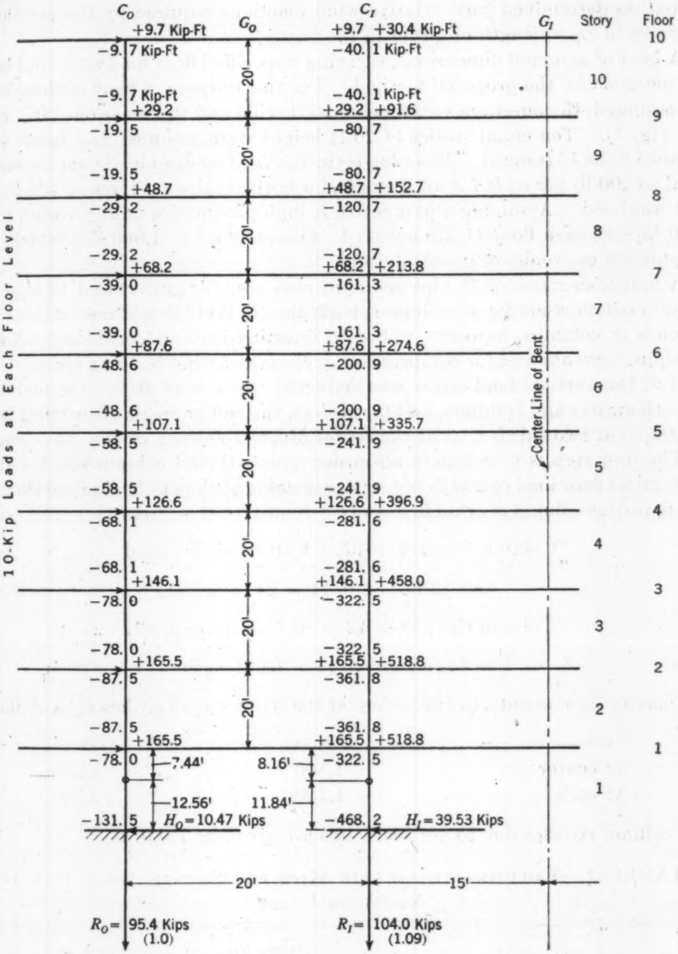
C_O
C_I
G_O
G_I
Story
Floor
+9.7 Kip-Ft
+9.7
+30.4 Kip-Ft
-9.7 Kip-Ft
-40.1 Kip-Ft
10
-9.7 Kip-Ft
-40.1 Kip-Ft
+29.2
+29.2
+91.6
9
-19.5
-80.7
9
-19.5
-80.7
+48.7
+48.7
+152.7
8
-29.2
-120.7
8
-29.2
-120.7
+68.2
+68.2
+213.8
7
-39.0
-161.3
7
-39.0
-161.3
+87.6
+87.6
+274.6
6
-48.6
-200.9
6
-48.6
-200.9
+107.1
+107.1
+335.7
5
-58.5
-241.9
5
-58.5
-241.9
+126.6
+126.6
+396.9
4
-68.1
-281.6
4
-68.1
-281.6
+146.1
+146.1
+458.0
3
-78.0
-322.5
3
-78.0
-322.5
+165.5
+165.5
+518.8
2
-87.5
-361.8
2
-87.5
-361.8
+165.5
+165.5
+518.8
1
-78.0
-322.5
1
7.44'
8.16'
12.56'
11.84'
-131.5
H_O = 10.47 Kips
-468.2
H_I = 39.53 Kips
20'
20'
15'
R_O = 95.4 Kips (1.0)
R_I = 104.0 Kips (1.09)
10-Kip Loads at Each Floor Level
Center Line of Bent

FIG. 1.—WIND MOMENTS, IN KIP-FEET (CANTILEVER REACTIONS)

time-consuming. In the present paper a new method is presented which is both rapid and convenient, at the same time insuring practical agreement with the cantilever relation of column, vertical, wind reactions which is necessary to justify ignoring the secondary wind effects. The proposed method is based upon the "method of K-percentages" first reported in 1939,[2] and particularly utilizes the relation[8] that ΣI varies as $V_g\, L_g^2$, in which ΣI = approximate sum of moments of inertia of all girders in a bay; V_g = relative vertical wind shear in the bay as determined from relative wind reactions required by the cantilever theory; and L_g = length of girder in the bay.

A bent of assumed dimensions, carrying a specified floor load and wind load, was designed by the proposed method. For this purpose, a bent of three bays was assumed, the outer ones each being 20 ft wide and the inner one 30 ft wide (see Fig. 1). Ten equal stories of 20-ft height were assumed and bents were assumed to be 15 ft apart. Floor loads (inclusive of all dead load) were assumed equal to 200 lb per sq ft for all floors and a horizontal wind force of 33⅓ lb per sq ft was used. Assuming a parapet 10 ft high produced a uniform wind force of 10 kips at each floor (1 kip equals 1 "kilo-pound," or 1,000 lb), which was adopted for convenience in calculations.

A unit fiber stress of 18 kips per sq in. was used for girders and 12 kips per sq in. in compression for all columns, regardless of their slenderness ratios. At the ends of columns, however, where no buckling need be considered, 18 kips per sq in. were allowed for compression, inclusive of wind bending stress. One third of the vertical load stress was deducted when wind stress was included, for both girders and columns; and for girders, the end moment for vertical loads was taken at two thirds that at the center of a simple span of the same length.

The first step is to compute all girder moments and column stresses from the vertical floor load of 200 lb per sq ft, assuming girders to be non-continuous. The tentative column areas of first-story columns for this stress are as follows —

Column C_O: $300 \div 12 = 25.0$ sq in. required

Use 12-in., 85-lb H = 24.98 in.^{2}

Column C_I: $750 \div 12 = 62.5$ sq in. required

Use 14-in., 215-lb H = 63.21 sq in.

The maximum moments, in kip-inches, at the centers of all girders are as follows:

Location	Girders G_O	Girders G_I
At center	1,800	4,050
At ends	1,200	2,700

The column stresses due to vertical loads are given in Table 1.

TABLE 1.—COLUMN STRESSES, IN KIPS PER SQUARE INCH, DUE TO VERTICAL LOADS

Column (see Fig. 1)	Floor No.:									
	1	2	3	4	5	6	7	8	9	
C_O	300	270	240	210	180	150	120	90	60	30
C_I	750	675	600	525	450	375	300	225	150	75

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[8] *Proceedings*, Am. Soc. C. E., June, 1939, p. 977.

Assume that the unit stresses in columns are proportional to their distance from the neutral axis of the bent, which, in this case, is at the center. Then, if A_O and A_I are the tentative areas of the outer and inner first-story columns respectively, and the vertical wind reaction R_O at the outer column is taken at unity, the inner-column wind reaction will be (see Fig. 1): $R_I = \frac{15}{35} \times \frac{A_I}{A_O} = \frac{15}{35} \times \frac{63.21}{24.98} = 1.09$. If $R_O = 1.0$, this also equals the vertical wind shear V_O in the outer bay. Then the vertical wind shear in the inner bay will be $V_I = R_O + R_I = 1.0 + 1.09 = 2.09$.

Hence, if I_O equals the moment of inertia of the outer girder in any floor, that of inner girder in the same floor will be $I_I = I_O \times 2.09 \times \frac{30^2}{20^2} = 4.70$.

Thus the moment of inertia of any inner girder should be equal to 4.70 times that of outer girder in the same floor in order to produce the cantilever relation of vertical wind reactions. This ratio will be subject to verification and possible change after the actual design of girders and columns for the inclusion of wind stress.

The next step is to compute wind moments in the girders, assuming vertical, column, wind, reactions and direct stresses in the various stories to be in the proportion of 1.0 and 1.09 for outer and inner columns respectively, and assuming all points of contraflexure at the mid-point of the member except first-story columns. For the latter it is better to assume the point of contraflexure at sixth tenths of the story height above the bottom. Each of the outer girder moments is taken as equal to the product of one half the girder length center to center of columns by the shear received from the column at its outer end. This shear equals the difference between outer column direct wind stresses above and below the floor in question.

The direct wind stress in the outer column of any story will be found by the following moment equation:

$$M = 70\, C_O + (1.09 \times 30)\, C_O = 102.7\, C_O \quad \ldots\ldots\ldots\ldots (1)$$

from which

$$C_O = \frac{M}{102.7} \quad \ldots\ldots\ldots\ldots (2)$$

In Eqs. 1 and 2, M equals the wind moment about the point of contraflexure of the column in the story in question and C_O is the stress in the outer column of this story.

It is not actually necessary to compute these wind moments.[9] The horizontal wind shears in all stories are always readily available and the difference between M_9 and M_{10}, for instance, in stories 9 and 10, will be

$$M_{10} - M_9 = \frac{H_9\, h_9}{2} + \frac{H_{10}\, h_{10}}{2} \quad \ldots\ldots\ldots\ldots (3)$$

in which H_9 and H_{10} are the story, horizontal, wind shears, and h_9 and h_{10} are the story heights. This relation is easily shown to be true. Hence, the wind moment in the outer girder of floor 9 (between stories 9 and 10) is equal to $\frac{1}{102.7}\left(\frac{H_9 h_9}{2} + \frac{H_{10} h_{10}}{2}\right) \times 10$. For the top girder, this moment becomes $\frac{1}{102.7}\left(\frac{H_{10} h_{10}}{2}\right) \times 10 = \frac{1}{102.7}(10 \times 10) \times 10 = +9.7$ kip-ft. For the ninth floor girder, it is $\frac{1}{102.7}(10 \times 10 + 20 \times 10) \times 10 + 29.2$ kip-ft and similarly for the other floors above the first floor. For the first story, the point of contraflexure being at six tenths of the story height above the base, or four tenths of this height below the floor, $M_1 - M_2 = \frac{H_2 h_2}{2} + H_1 \times 8$, from which the moment in the girder is $\frac{1}{102.7}\left(\frac{H_2 h_2}{2} + H_1 \times 8\right) \times 10 = \frac{1}{102.7}\left(90 \times \frac{20}{2} + 100 \times 8\right) \times 10 = +165.5$ kip-ft.

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[9] *Proceedings*, Am. Soc. C. E., June, 1939, p. 980.

For inner girders, the moments will be equal to those for the corresponding outer girders, multiplied by $\frac{V_I}{V_O} \times \frac{15}{10}$, in which V_I and V_O are, respectively, the vertical wind shears in the inner and outer bays. As found before, if $V_O = 1.0$, $V_I = 2.09$. Hence, if M_O and M_I are moments in outer and inner girders, respectively, $M_I = M_O \times \frac{2.09}{1.0} \times \frac{15}{10}$. All inner girder moments are thus quickly found.

By the condition that $\Sigma M = 0$ at each joint, and working from the top of bent downward, column moments are all readily determined from the girder moments, except those at the bottom of the first story. Thus (in kip-ft):

Story		C_O
10		-9.7
9		$-(29.2 - 9.7) = -19.5$
: : :	etc.	: : :

For the bottom of the first-story columns, the total wind moment in the first story (that is, 100 × 20 ft = 2,000 kip-ft) is assumed divided among the four columns in proportion to their *K*-values, or, as all have the same length, in proportion to their moments of inertia. The tentative areas of C_O and C_I are, respectively, 24.98 sq in. and 63.21 sq in. Assuming these columns to be tentatively 12-in., 85-lb H ($I = 723.3$) and 14-in., 215-lb H ($I = 2{,}730.0$) these columns will have total moments of $\frac{2{,}000}{2 \times 3{,}453.3} \times 723.3 = 209.5$ kip-ft and $\frac{2{,}000}{2 \times 3{,}453.3} \times 2{,}730.0 = 790.7$ kip-ft, respectively. Deducting from these moments the moments previously found at the top of columns in first story, the base moments given in Fig. 1 are determined, this figure also giving the wind moments throughout, computed as described herein. Wind reactions in Fig. 1 are found as follows:

By taking moments about the contraflexure point in the first story, assumed to be 8 ft below the first floor,

$$R_O = \frac{(10 \times 10)\ (20\ \text{ft} \times 4.5 + 8\ \text{ft})}{102.7} = 95.42\ \text{kips}$$

and

$$R_I = R_O \times 1.09 = 104.0\ \text{kips}.$$

The total moment in the outer column of the first story was found to be 209.5 kip-ft. Hence, $H_O = \frac{209.5}{20} = 10.47$ kips.

$$\text{Similarly, } H_I = \frac{790.7}{20} = 39.53\ \text{kips}.$$

The corresponding location of points of contraflexure will be at distances above the base of $\frac{131{,}500}{10{,}470} = 12.56$ ft and $\frac{468{,}200}{39{,}530} = 11.84$ ft respectively, instead of sixth tenths of 20 ft = 12 ft as was assumed originally in computing wind moments.

The next step is to proportion columns and outer girders for vertical loads, or for vertical loads combined with wind stresses, whichever will require the greater section. Inner girders are then to be proportioned by making the moment of inertia of any girder, as nearly as practicable, equal to 4.70 times that of the outer girder in the same floor, unless its stress from vertical and wind loads requires a larger section. In the latter case the corresponding outer girder must be increased until its moment of inertia is equal to $\frac{1}{4.70}$ times that of the inner girder. In other words, the relation $I_I = 4.70\ I_O$ must be maintained in order to produce the assumed cantilever relation of wind reactions, while, at the same time, no girder or column may be of less section than is required by its own maximum stresses.

TABLE 2.—SIZES OF H-BEAMS AFTER PROPORTIONING FOR WIND AND VERTICAL LOADS

Floor or story	Columns		Girders	
	Outer (C_O)	Inner (C_I)	Outer (G_O)	Inner (G_I)
10	12 in., 32 lb	14 in., 58 lb	21 in., 59 lb	33 in., 125 lb
9	12 in., 32 lb	14 in., 58 lb	21 in., 59 lb	33 in., 125 lb
8	12 in , 45 lb	14 in., 111 lb	21 in., 59 lb	33 in., 125 lb
7	12 in., 45 lb	14 in., 111 lb	21 in., 59 lb	33 in., 125 lb
6	12 in., 58 lb	14 in., 158 lb	21 in., 59 lb	33 in., 125 lb
5	12 in., 58 lb	14 in., 158 lb	21 in., 59 lb	33 in., 125 lb
4	12 in., 79 lb	14 in., 211 lb	21 in., 68 lb	33 in., 132 lb
3	12 in., 79 lb	14 in., 211 lb	24 in., 74 lb	36 in., 160 lb
2	12 in., 85 lb	14 in., 237 lb	24 in., 74 lb	36 in., 160 lb
1	12 in., 85 lb[a]	14 in., 237 lb[b]	24 in., 74 lb	36 in., 160 lb

[a] With two plates, 9 in. by $\frac{7}{8}$ in. by 3 ft at bottom. [b] With two plates, 11 in. by $\frac{11}{16}$ in. by 4.0 ft at bottom.

In proportioning columns, the ratio of the moments of inertia in inner and outer columns should be maintained as nearly constant as practicable for the same reason as was stated in the case of girders. This condition, however, is not nearly so essential as is the proper relation of girder moments of inertia. The effect upon the wind-reaction ratio of a change in the relative size of girders is much more pronounced than is the effect of a similar change in columns.

Table 2 shows sizes of all members after proportioning them for vertical loads and wind. This operation, in some detail, is as follows:

Design of Outer Girders G_O, First Floor.—

Maximum moment at center = 1,800 kip-in. at 18 kips per sq in. = 100 (minimum section modulus required)

Maximum moment at end = 1,200 kip-in.
Less one third = 400 kip-in.

Net moment from vertical load = 800 kip-in.
Wind moment 165.5 kip-ft × 12 in. = 1,986 kip-in.

Total end moment = 2,786 kip-in.; at 18 kips per sq in. = 154.8 section modulus required

Use 24-in. 74-lb H (170.4 section modulus; $I = 2{,}033.8$)

A similar procedure is followed for each floor to determine the sizes shown in Table 2.

Design of Inner Girders G_I, First Floor.—

Maximum moment at center = 4,050 kip-in., at 18 kips per sq in. = 225 (minimum section modulus required)

Maximum moment at end = 2,700 kip-in.
Less one third = 900 kip-in.

Net moment from vertical load = 1,800 kip-in.
Wind moment 518.8 kip-ft × 12 in. = 6,225 kip-in.

Total end moment = 8,025 kip-in.; at 18 kips per sq in. = 445.8 section modulus required

A 33-in. 141-lb H (446.8 section modulus; $I = 7{,}442.2$) is sufficient for stress. However, the designer must consider the relative moments of inertia of girders G_O and G_I:

$$I \text{ of } G_O = 2{,}033.8\text{; hence, the required—}$$
$$I \text{ of } G_I = 2{,}033.8 \times 4.70 = 9{,}560 \text{ in.}^4$$

Therefore, for G_I, use a 36-in. 160-lb H ($I = 9{,}738.8$).

In a similar manner sizes for all inner girders are found, as given in Table 2.

Design of Outer Columns C_O, First Story.—

Vertical load = 300 kips, at 12 kips per sq in. = 25.0 sq in.
Less one third = 100 kips

Net vertical load = 200 kips

Wind Load (Top).—

78 kip-ft × 12 in. × $\frac{c}{r^2}$ = 200 ($c = 6.25$ in. and $r = 5.38$ in.)

Total at end = 400 kips, at 18 kips per sq in. = 22.2 sq in.
Use a 12-in. 85-lb H (24.98 sq in.; $I = 723.3$).

Wind Load (Bottom).—

131.5 kip-ft × 12 in. × $\frac{c}{r^2}$ = 343 kips
Net vertical load = 200 kips

Total at end = 543 kips, at 18 kips per sq in. = 30.2 sq in.
Use a 12-in. 85-lb H (24.98 sq in.), with two plates
9 in. by ⅜ in. by 3.0 ft = 6.75 (at base)

Total at base = 31.73 sq in.

Design of Outer Columns C_O, Second Story.—

Vertical load = 270 kips, at 12 kips per sq in. = 22.5 sq in.
Less one third = 90 kips

Net vertical load = 180 kips

Wind 87.5 kip-ft × 12 in. × $\frac{c}{r^2}$ = 227 kips ($c = 6.25$ in. and $r = 5.38$ in.)

Total at end = 407 kips, at 18 kips per sq in. = 22.6 sq in.
Use 12-in. 85-lb H (24.98 sq in.) for first and second stories.

Similarly sizes for other stories are obtained as given in Table 2.

Design of Inner Columns C_I, First Story.—

Vertical load = 750 kips, at 12 kips per sq in. = 62.5 sq in.
Less one third = 250 kips

Net vertical load = 500 kips

Wind Load (Top).—

322.5 kip-ft × 12 in. × $\frac{c}{r^2}$ = 709 kips ($c = 8.06$ in. and $r = 6.65$ in.)

Total at end = 1,209 kips, at 18 kips = 67.2 sq in.
Use a 14-in. 237-lb H (69.69 sq in.; $I = 3{,}080.9$ in.^{4}).

Wind Load (Bottom).—

468.2 kip-ft × 12 in. × $\frac{c}{r^2}$ = 1,024 kips
Net vertical load = 500 kips

Total at end = 1,524 kips, at 18 kips per sq in. = 84.8 sq in.
Use a 14-in. 237-lb H = 69.69 sq in.
Plus two plates, 11 by $\frac{11}{16}$ by 4.0 ft = 15.14 sq in. (at base)

Total at base = 84.83 sq in.

Second Story.—

Vertical load = 675 kips, at 12 kips per sq in. = 56.25 sq in.
Less one third = 225 kips

Net vertical load = 450 kips
Wind 361.8 kip-ft × 12 in. × $\frac{c}{r^2}$ = 793 kips

Total at end = 1,243 kips, at 18 kips per sq in. = 69.1 sq in.
Use 14-in. 237-lb H (69.69 sq in.) for the first and second stories.

Similarly, sizes are obtained for other stories, as given in Table 2.

All sizes having been determined, a check for the reaction ratio $\frac{R_I}{R_O}$ will now be made by the *K*-percentage method:

	Moments of inertia, *I*, for:	
Stories	C_O	C_I
1st and 2d	723.3	3,080.9
3d and 4th	663.0	2,671.4
5th and 6th	476.1	1,900.6
7th and 8th	350.8	1,266.5
9th and 10th	246.8	597.9
	2,460.0	9,517.3

Average I = 492.0 = 1,903.5
Average K = 24.6 = 95.17
Ratio $\frac{C_I}{C_O}$ = 3.87

Although this ratio differs from the ratio 2.50 upon which the original computations of column areas were made, the difference will probably have little effect upon resulting wind stresses, these being influenced mainly by the relation between girder moments of inertia in the different bays. This relation is found as follows:

Floor	Moments of inertia, I, for: G_O	G_I
1	2,033.8	9,738.8
2	2,033.8	9,738.8
3	2,033.8	9,738.8
4	1,478.3	6,856.8
5	1,246.8	6,354.7
6	1,246.8	6,354.7
7	1,246.8	6,354.7
8	1,246.8	6,354.7
9	1,246.8	6,354.7
10	1,246.8	6,354.7
	15,060.5	74,201.4

$$\begin{array}{lcrcr} \text{Average } I & = & 1{,}506.05 & = & 7{,}420.14 \\ \text{Average } K & = & 75.30 & & 247.30 \\ \text{Ratio } \dfrac{G_I}{G_O} & = & \dfrac{7{,}420.14}{1{,}506.05} & = & 4.92 \end{array}$$

This value checks reasonably well with 4.70 which was previously determined and used in proportioning the inner girders.

ANALYSIS OF A "COMPROMISE 3-STORY BENT"

The computation of the compromise 3-story bent shown in Fig. 2 is as follows:

Determination of K-Percentages.—From the K-values at the joints:

75.30	99.90	75.30	
24.60	24.60	247.30	417.77
99.90)75.30	124.50)75.30	95.17	95.17
0.754	0.605	417.77)75.30	512.94)75.30
		0.181	0.147
		)247.30	)247.30
		0.592	0.482

Determination of Reaction Ratio R.—

$$\begin{array}{lcl} 0.181 & & 0.754 \\ 0.147 & & 0.605 \\ \underline{0.147} & & 0.605 \\ & & \underline{1.836} \\ 0.475 \times \dfrac{95.17}{24.60} & = & 3.800 \div 20 = 0.1900 = R_O = V_O \end{array}$$

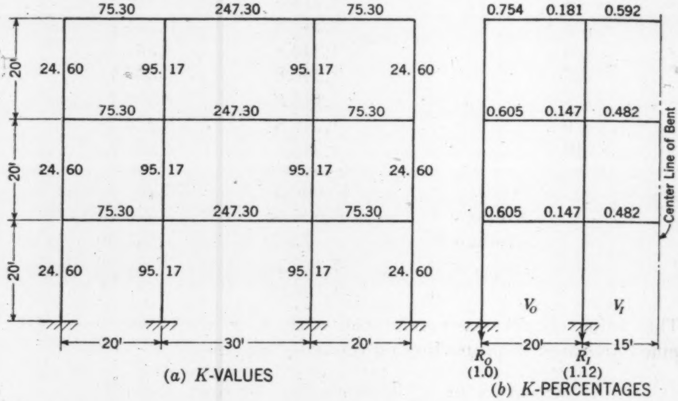
$$\begin{array}{l} 0.592 \\ 0.484 \\ \underline{0.484} \\ 1.560 \times 2 \times \dfrac{95.17}{24.60} = 12.120 \div 30 = 0.4040 = V_I \end{array}$$

$$V_I - V_O = 0.2140 = R_I$$

$R = \dfrac{R_I}{R_O} = 1.12$. Hence, if $R_O = 1.0$, $R_I = 1.12$.

This checks closely with 1.09 in original calculations. The design is therefore consistent with the cantilever reaction relation.

Should a discrepancy be found great enough to necessitate further computation, the wind reactions for the new reaction ratio R may be found readily as was done before, and the new wind moments in girders found by simple proportion from those in Fig. 1, the outer girder moments being obtained by multiplying the former moments by the ratio of the respective outer vertical wind shears V_O $(= R_O)$ and the inner girder moments by multiplying the former moments by the ratio of respective inner vertical wind shears V_I $(= R_O + R_I)$. Column moments result, as before, from the condition that $\Sigma M = 0$ at each joint, with special procedure in the first story.



(a) K-VALUES

(b) K-PERCENTAGES

FIG. 2.—COMPROMISE THREE-STORY BENT

The sizes of girders and columns may then be checked against these new moments, and changes made where required. Generally if the first design was made carefully to accord with the proper ratio of I's for inner and outer girders, found as described, such corrections will be very slight, even if required at all.

The proposed method of designing for cantilever behavior, and the K-percentage method for obtaining wind reaction ratio R for any design, may be applied to bents of any number and spacing of bays. The latter principle is particularly useful in the investigation of wind stresses in existing structures or in designs that have been made without due consideration of their elastic action.

As a final check on the accuracy of the foregoing wind-moment computation, a moment-distribution analysis[7] was made for the design. In Table 3, comparisons are made with the average of the Cross moments at the two ends of any member; in other words, the total moments in the member are compared. These show a very good agreement. All differences are of negligible proportions except in one bottom column (16%) and in certain members in the ninth and tenth floors and stories. The latter show differences as high as 26%; but as the actual moments are very small the sizes used are ample without change. Such a result may generally be relied upon. Ordinarily, large percentage discrepancies in wind moments near the top of the bent are of but little practical importance in the design of members.

TABLE 3.—CHECK COMPUTATIONS OF MOMENTS FOR COMPARISON

(C = Cross Method; W = Witmer Method; L = Left End; R = Right End; D = Percentage Difference; T = Top; B = Bottom)

Floor or story	OUTER GIRDER, G_O				INNER GIRDER, G_I			OUTER COLUMN, C_O				INNER COLUMN, C_I			
	C		W	D	C	W	D	C		W	D	C		W	D
	L	R						T	B			T	B		
10	15.0	11.1	9.7	−26	28.5	30.4	+ 7	14.2	11.4	9.7	−24	38.4	35.9	40.1	+ 8
9	37.0	30.7	29.2	−14	81.8	91.6	+12	26.8	25.7	19.5	−26	74.8	71.8	80.7	+10
8	56.4	49.1	48.7	− 8	147.6	152.7	+ 3	29.8	28.3	29.2	+ 1	124.6	117.2	120.7	0
7	72.4	68.2	68.2	− 3	222.0	213.8	− 4	42.7	41.1	39.0	− 7	173.3	143.1	161.3	+ 2
6	88.0	83.2	87.6	+ 2	270.5	274.6	+ 2	47.3	45.4	48.6	+ 5	210.0	197.2	200.9	− 1
5	104.5	101.3	107.1	+ 4	340.6	335.9	− 1	59.1	59.1	58.5	− 1	245.8	235.9	241.9	0
4	126.3	125.4	126.6	+ 1	385.3	396.9	− 3	66.9	69.3	68.6	+ 1	275.9	287.8	281.6	0
3	147.0	147.9	146.1	− 1	460.7	458.0	− 1	78.5	77.3	78.0	0	321.5	322.6	322.5	0
2	164.7	165.3	165.5	0	521.5	518.8	− 1	87.8	85.8	87.5	+ 1	364.4	361.1	361.8	0
1	171.9	167.0	165.5	− 2	557.7	518.8	− 7	86.4	96.7	106.5	+16	364.2	452.6	393.5	− 4
$\frac{\Sigma M}{20} =$	96.6		95.4	$\frac{\Sigma M}{30} =$	201.4	199.4	....	....	....	....	....	....	....	....	....

The design was made using moments at the center of intersections for all girders and columns. If it is desired to use moments at the face of columns or at the top or bottom of girders, corresponding corrections may always be made, in proportioning the members. The most practicable way to do the original computation is to consider moments at the centers of intersections of members.

If it is found that the horizontal deflection of the building exceeds a proper assigned proportion of its height, a correction may be applied readily by making a proportional reduction in the allowed column unit stress, keeping the stiffness relation unchanged between outer and inner columns. A change in girder stiffness will have little effect upon deflection. The deflection can be computed from column wind moments at either inner or outer column, in the manner described elsewhere.[10]

In general, a design based on the assumption of a cantilever relation between vertical wind reactions may not be the most economical as to weight although it is to be recommended because it eliminates secondary wind bending moments due to axial distortion of columns under direct wind stresses.

For 3-bay bents, the loss in economy will be greater when the width of the inner bay is relatively greater. This is apparent since in such cases the inner girders G_I are longer; and the foregoing computations show that, in order to produce the cantilever relation of wind reactions, the sections of these girders must be increased arbitrarily above the requirements for vertical loads alone.

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[10] *Proceedings*, Am. Soc. C. E., February, 1932, p. 229.

GENERAL APPLICABILITY OF *K*-PERCENTAGE METHOD

It should be remembered that the *K*-percentage method is not limited to the type of case considered herein—namely, one in which vertical wind reactions are made to accord with the cantilever relation—but is applicable to any sizes that may exist in the bent, and to a bent of any number and any relative width of bays. It is particularly useful in the investigation of wind stresses in an existing bent, for which the assumptions as to wind distribution which were made in its design are unknown.

If it is not considered necessary to maintain the cantilever relation of vertical wind reactions, the bent may first be designed throughout for vertical loads alone. Using the sizes thus obtained, wind reaction ratios are found by the *K*-percentage method and from these ratios wind moments are computed. The members are then re-designed to include these wind moments and finally a *K*-percentage check is made on wind-reaction ratios. These operations are similar to those described in the foregoing example. If the reaction ratios last found do not vary too greatly from those used in computing the wind moments, no further change in sizes is required. In this connection it should be noted that a considerable variation may occur in the values of wind reaction ratios without materially changing the wind moments.[11] For this reason a second revision of design will generally be unnecessary.

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[11] *Proceedings*, Am. Soc. C. E., June, 1936, p. 974.

DISCUSSION

C. M. Goodrich,[12] **M. Am. Soc. C. E.**—Mr. Witmer's paper is a corollary to the Witmer Method in the Sixth Progress Report of the Committee on Wind Bracing in Steel Buildings,[2] where it was shown that, for cantilever action, multiplying each bay shear by the bay width gives a result proportional to the *K*-value of the girder treated. It supersedes clumsier methods, and gives a welcome program of calculations for one to follow.

In summing loads on the columns, the axial wind load is apparently omitted. In the design of outer columns C_O, first story, one might consider this computation:

Vertical load	$= 300 @ 12 = 25$ sq in.
Axial wind load	$= 95$
Axial wind bending $78 \times 12 \times \frac{c}{r^2}$	$= \underline{200}$
Top of column	$595 @ 18 \times \frac{4}{3} = 24.8$ sq in.

It is conceivable that in buildings of great height, and of relatively small lower story height, the axial load from wind would be of more importance than the wind bending.

Under the heading "Proposed Method of Design: Second Story," the average *K*-ratio of the column is found to be 3.87; the columns tentatively selected have a *K*-ratio of 3.77. The value of 2.50 given in the text refers to the relative areas of the tentative columns.

A little difference in the sequence of calculations might be considered desirable. If alongside Fig. 1 one places the total moments to points of contraflexure, then the axial wind loads may be calculated from them by one setting of the slide rule for C_O, dividing by 102.7; C_1 loads may be found from these, also by one setting. Moments in G_O may then be found by multiplying successive differences by the half girder length, and moments in G_1 taken in ratio to these.

The moment in G_O and C_O in the tenth floor may be approximated, on the high side, by taking the shear in C_O as $10 \times \frac{1}{6.18} = 1.62$, and multiplying by the half height or half bay 10, giving 16,200 ft-lb. Unfortunately such a procedure gives results increasingly too great as one goes down the column.

Among the "exact" methods is that of Georg Unold,[13] including wind and vertical loads in one equation per joint. It is an end-tangent method, of course, after the Mohr-Greene, elastic-weight, area-moment, graphical integration devices, as used sometime in the last century by the late Prof. C. E. Greene, M. Am. Soc. C. E., in an undated pamphlet entitled "Partially Braced Frames," given to the writer by his son about 1903. A line in this pamphlet indicates that Professor Greene regarded area moments as graphical integration. Professor Mohr introduced angle changes as unknowns in 1892.

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[12] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[13] "Die Praktische Berechnung der Stahlskelettrahmen," by Georg Unold, W. Ernst und Sohn, Berlin, 1933.

Professor Unold suggests that in many cases the stiffness of the end walls may be such as to justify disregarding sway in interior bents, in view of floor stiffness. It is interesting to speculate as to whether it is better to make end walls adequate, or to make the interior bents adequate but capable of acting effectively only after the walls are shattered.

It would be of interest if Mr. Witmer would give an exposition of his favorite method of calculating sway in his closure.

Francis L. Castleman, Jr.,[14] **Assoc. M. Am. Soc. C. E.**—For the mathematically minded, or those who insist on extreme refinement in the accuracy of the assumptions and calculations, the *K*-percentage method of wind stress analysis will offer little appeal. To practicing engineers who, on occasion, must utilize the conventional methods, either theoretical or approximate, the *K*-percentage method and the accompanying new method of cantilever design offer a means whereby wind stresses can be determined with an accuracy approaching the so-called theoretical methods, and with little more time than that required for the simplest of the approximate methods. Verification of the accuracy of these new methods has been backed up by sufficient research, both analytical and experimental, to warrant full confidence in their use as presented.

The studies evolving these new methods, although very circumspect in the cases and conditions considered, lead, upon thoughtful consideration, to questions definitely not considered by them, but to which they are unavoidably joined and for which additional information is needed. The method of *K*-percentages can be used on any tier building, as can the new cantilever theory; but, as inferred in the paper under "Present Methods of Design," the conventional approximate methods are satisfactory for buildings of the usual proportions. Inspection of the building codes of various cities, however, leaves one in a quandary as to what a building of the usual proportions is.[15] Lacking any conclusive information, analytical or experimental, the following three cases are proposed arbitrarily:

$$\text{Case 1;}\quad \frac{\text{Height}}{\text{Least width}} \leqq 2\text{; wind stresses neglected.}$$

$$\text{Case 2;}\quad \frac{\text{Height}}{\text{Least width}} \leqq 5\text{; wind stresses to be considered.}$$

$$\text{Case 3;}\quad \frac{\text{Height}}{\text{Least width}} > 5\text{; tower buildings; wind stresses to be carefully determined.}$$

Of course, it is assumed in cases 1 to 3, inclusive, that the same intensity of wind pressure exists. Modifications should naturally be made for allowable increases in unit stresses (*a*) where wind is involved (an almost universal specification) and (*b*) where wind is omitted over protected heights.

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[14] Prof. and Head, Dept. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

[15] *Carnegie Pocket Companion*, 24th Ed., p. 357.

Cases 1 and 3 offer no difficulty. In case 1, assuming the cases divided correctly, no consideration need be given; in case 3 wind stresses must be determined carefully either by one of the theoretical methods or by the method of K-percentages, preferably based on the new cantilever method. For extreme upper limits a combination of methods may be preferable before final sizes are determined.

Case 2 is the range in which most buildings that require wind stress analysis occur. Offhand, it would be assumed that all buildings in this class can be designed by any one of the approximate conventional methods (probably either the portal or cantilever method). The writer has often questioned this and felt that in the upper limits of this range, say from 4 to 5, more careful consideration should be given, but that the great accuracy (somewhat imaginary) furnished by the theoretical methods was not necessary. The method of K-percentages here offers a method that is precisely suitable and, in this range (or an equivalent one), will find its maximum use.

A series of studies, either analytical, experimental, or both, to determine ranges similar (but not necessarily identical) to those given by cases 1, 2, and 3, would be most helpful to the designer, who could then at once classify his building and the method to be used in its design for wind. It would seem that this series of studies should be made by methods similar to those that evolved the method of K-percentages and was a logical extension of the same.

The author states under "Analysis of a Compromise 3-Story Bent" that "* * * a design based on the assumption of a cantilever relation between vertical wind reactions may not be the most economical as to weight although it is to be recommended * * *." This would appear contradictory and at least only partly true. Obviously, if used, the cantilever method of design will fall under case 3 and then should be used when the secondary moments from column shortening cause the sizes of the members to be in excess of those obtained by the new cantilever method. Secondary moments from column shortening are not objectionable provided they are cared for and that the sum total weight involved is less than that required by cantilever theory. Again it is suggested that a series of supplementary theoretical studies should be made determining an approximate height-to-least-width ratio where this change takes place. Such studies might in themselves lead to shorter methods for determining the secondary moments set up in the beams from column shortening.

The bent as designed in the paper offers several interesting possibilities for variation in design, subject to possible verification by future analytical and experimental investigation. Two cases will be considered, both dealing with the girders of the first floor. (A similar procedure can be followed for girders of the other floors if desired.) All sizes are based on gross section following a similar procedure used in the paper. Inside girder G_1, as designed, is a 36-in., 160-lb H ($I = 9{,}738.8$) and was determined from the fact that its moment of inertia must be at least 4.70 times that of outside girder G_O. For vertical load alone a 27-in., 91-lb H ($I = 3{,}129.2$) is sufficient. Suppose cover plates are added to this at each end to provide enough strength to furnish a moment of inertia of at least 9,560 in.^{4} Calculation shows that a 27-in., 91-lb H gives poor size covers and that the base section is changed to a 30-in., 108-lb H with covers $16 \times \frac{11}{16} \times 6.75$ ft top and bottom at each end. The point of contraflexure for combined loading is taken as three eighths the span of 30 ft, and 1 ft is added to the theoretical length of each cover. For the end portions $I = 9{,}602$ in.^{4} and for the center portion $I = 4{,}461$ in.^{4} The end portion comprises 22.5% of the 30-ft span. It should be noted that this girder should be connected to the column for its moment-resisting capacity rather than the total end moment of 8,025 kip-in. For shear it should be connected for its moment-resisting capacity divided by one half the span. With the foregoing arrangement the ratio of the end moment of inertia of the inside span to that of the outside span is 4.72. A weight saving of 11.4% is obtained. For total economy this saving would have to be balanced against the increased cost of fabrication. Obviously, the *K*-values have been upset and only by future additional studies could such a procedure be justified theoretically. However, it should be noted that the author has used cutoff cover plates on the first-story columns. In all fairness it should be stated that such a procedure has much less influence on the design than that of using cutoff covers on the girders.

A more rational method than the foregoing, and one conducive to greater economy, is as follows: Replace the author's design of the outer first-floor girder G_O by an 18-in., 55-lb H ($I = 889.9$) with covers $10 \times \frac{3}{8} \times 3.75$ ft top and bottom at each end. This gives an *I*-value in the end portions of 1,531 in.^{4} and in the center portion of 889.9 in.^{4} The point of contraflexure for combined loading is taken at three eighths the span of 20 ft, and 1 ft is added to the theoretical length of each cover. Sufficient resisting moment is furnished at all points. The end portion comprises 18.75% of the total span of 20 ft. A weight saving of 12.7% is obtained.

From the foregoing design multiply the *I*-values in the end and center portions of outside girder G_O by 4.70, giving theoretical values of $I = 7{,}200$ in.^{4} and 4,190 in.^{4}, respectively, to be used in the design of the inside girder G_1. Again, for the same reason, a 30-in., 108-lb H ($I = 4{,}461.0$) is used for the base section. Covers 5 ft long, top and bottom, are used at each end with the point of contraflexure for combined loading taken at three eighths the span of 30 ft. One foot is added to the theoretical length of each cover. Sufficient resisting moment is furnished at all points with the added feature that, in the end portions, $I = 7{,}221$ in.^{4} and in the center portion $I = 4{,}461.0$ in.^{4} The ratio of these *I*-values, respectively, to those in the side span is, for the end portion, 4.71; and, for the center portion, 5.01. These do not vary greatly from the theoretical value of 4.70. A weight saving of 24% in the inside bay is effected by this arrangement. The end portion comprises 16.70% of the total span of 30 ft.

The latter case, in which girders in the inside and outside bays are both cover-plated and the ratio of the moment of inertia is held to the value of 4.70 as nearly as possible, is more effective than the case in which cover plates were used only in the inside bay. That this is true is seen by the much greater saving in weight. However, it must again be emphasized that such a saving can be justified only by the verification of more extended investigations both analytical and experimental.

Professor Witmer's studies in wind stress analysis have opened up a new approach to this interesting field. Based upon both analytical and experimental evidence that is incontrovertible, they offer to the profession two new approaches that considerably simplify a somewhat difficult problem. As stated in this discussion the investigations can be extended considerably further and the writer sincerely hopes means can be found whereby this will be done. An interesting possibility would be an adaptation or modification to the Vierendeel truss. A tier building is no more than a cantilever Vierendeel truss.

CLYDE T. MORRIS,[16] M. AM. SOC. C. E.—The design procedure outlined by Professor Witmer gives results closely in accord with the Spurr method of design.[3] The fundamental relationship on which the *K*-percentage method is based ("that I_g varies as $V_g L^2_g$"), so far as the writer is aware, was first called to the attention of the profession and utilized as a basis for design in a paper, read at the meeting of the Structural Division of the Society on October 13, 1927, entitled "The Design of Tall Building Frames to Resist Wind,"[17] by the writer in collaboration with A. Ward Ross, Jr. Instead of the *K*-percentage method as developed by Professor Witmer, the writer's paper presented a chart showing the variations in girder shears due to changing ratios of column-to-girder *K*'s.

In discussing Table 3 Professor Witmer states that when the horizontal deflection of the building exceeds a proper assigned proportion of its height, it may be corrected by making a proportional reduction in the allowed column unit stress, keeping the stiffness relation unchanged between outer and inner columns. According to Professor Witmer: "A change in girder stiffness will have little effect upon deflection." This is in direct contradiction to the statement made earlier, in his discussion of Table 2: "The effect upon the wind-reaction ratio of a change in the relative size of girders is much more pronounced than is the effect of a similar change in columns."

It is also at variance with the facts, as shown by the following computations.[18] Fig. 3 shows the elastic lines of the members at a floor with the deflections greatly exaggerated. The drift due to column bending is ϕh and that due to girder bending is θh. From area moments,

$$\phi = \frac{H_c h^2}{12 E I_c} \quad \text{(4a)}$$

and

$$\theta = \frac{V_g L^2_g}{12 E I_g} \quad \text{(4b)}$$

Using the stresses and member sizes given by the author for the first floor: For column C_O(12-in. 85-lb H), $\phi_O = \frac{10.47\ (2 \times 8 \times 12)^2}{12\, E\, 723.3} = \frac{44.4}{E}$; for column C_I (14-in. 237-lb H), $\phi_I = \frac{39.53\ (2 \times 8 \times 12)^2}{12\, E\, 3{,}080.9} = \frac{39.4}{E}$; for girder G_O

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[16] Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

[17] *Proceedings*, Am. Soc. C. E., May, 1928, p. 1395; see also *Bulletin No. 48*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

[18] *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

(24-in. 74-lb **WF**), $\theta_O = \dfrac{16.55\,(20 \times 12)^2}{12\,E\,2{,}033.8} = \dfrac{39.0}{E}$; and for girder G_I (36-in. 160-lb **WF**), $\theta_I = \dfrac{34.6\,(30 \times 12)^2}{12\,E\,9{,}738.8} = \dfrac{38.4}{E}$.

Thus, for the author's design, the effect of girder flexure on deflection is about equal to the effect of column flexure. If the story height were about 12 ft instead of 20 ft, as is usual in most buildings, the effect of column flexure would be still less.

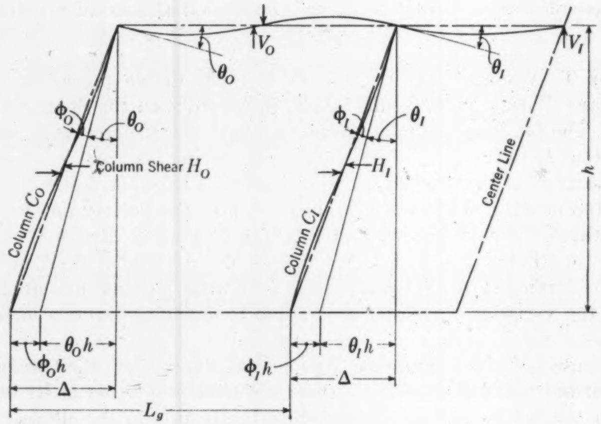

FIG. 3

When it is desirable to reduce the deflection of the bent, the girder stiffnesses should be increased. The column flexure has little effect upon the total deflection. The deflection due to direct lengthening and shortening of the columns (chord deflection) is still smaller.

OTTO GOTTSCHALK,[19] ESQ.—For more than a century structural analysis has been almost entirely characterized by abstract mathematical speculation. This is especially true of the eighties. It seems as if this trend was started by specialists to establish—as a mystery—the art of expressing simple relations in endless formulas and equations. Even today this characteristic of structural analysts is admired and emulated the world over. In the United States, writers of textbooks have mitigated the learned excesses, but fundamentally confusing and complicating expressions such as "redundants," "least work," etc., still abound—concepts that are not essential to the type of structure to be designed. The present trend is toward simplified methods of balancing moments or rotations, such as that developed by Professor Cross.[7] This symbolizes a much needed reaction to a study of how a structure acts in nature and of relative efficiencies. The cycles of computation, back and forth, to balance a disturbed equilibrium in a structure are easily perceptible to the analyst. Even these arithmetical devices are unnecessary, however, if the analyst will study, directly, the geometrical deformations that can be observed by the eye.

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[19] Buenos Aires, Argentine Republic.

Professor Witmer states (see heading "Present Methods of Design") that "Of the so-called 'exact' methods, the 'slope deflection method' * * * and the method of end-moment distribution * * * are the most outstanding." This statement is incomplete. A form of visual experiment has been advocated by the writer for many years[20] as the logical basis of all structural analysis. A model composed of flexible splines, subjected to a unit geometrical alteration at any place, bends into curves which are the parts of the influence line for the stress represented by the flexed part. The resistance to rotation at the end of a member, of specific stiffness $K = \frac{I}{L}$, is the relative stiffness[21]

$$S = K\,(1 - 0.5\,m) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This simple expression defines implicitly and automatically the end value of a balancing series and is obtained in a minimum of time.

One of the principal lessons derived from the use of splines is to keep the analysis of structures entirely separate from load and stress computations in order to avoid relapsing, even partly, into the former state of "learned" chaos. To analyze a structure for wind stress assume the upper floor (Fig. 4) to be pushed to the left by a unit distance 1. The columns then assume a curved form whose end tangents intersect the floor levels at distance *f* and *f'*, as shown at *B*. The resistance of any one column to the unit horizontal sway is:[22]

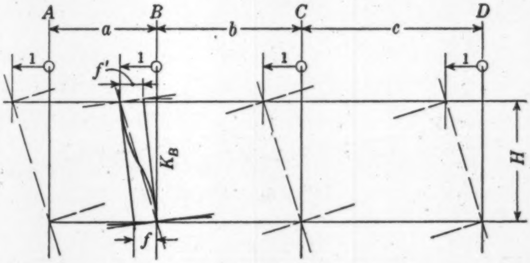

FIG. 4.—DISPLACEMENT "1" OF UPPER FLOOR

$$\Delta P = \frac{6\,E\,K\,(f' - f)}{H^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which, generally, *f* and *f'* have opposite signs and *P* is the force required to produce the unit displacement. Then $\frac{\Delta P}{\Sigma\,\Delta P}$ indicates the share absorbed by that column of the total horizontal thrust *P* acting at the upper floor. The per-

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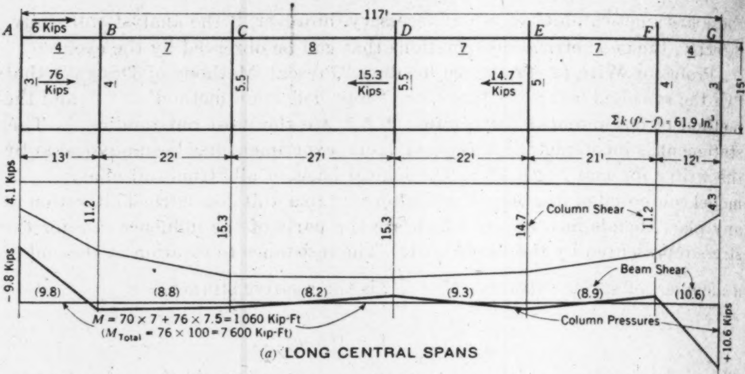
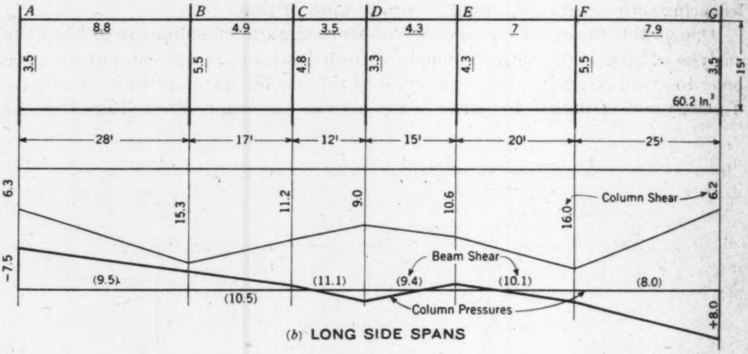
[20] *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1019.

[21] "Structural Analysis Based Upon Principles Pertaining to Unloaded Models," by Otto Gottschalk, *ibid.*, Vol. 103 (1938), p. 1020.

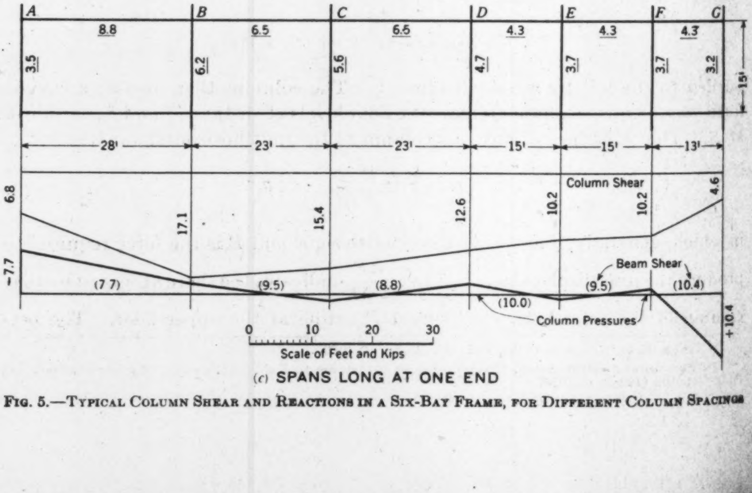
[22] *Ibid.*, Vol. 105 (1940), Eq. 5, p. 1020.


117'
A
6 Kips
B
C
D
E
F
G
4
7
8
7
7
4
3
76
Kips
4
5.5
15.3
Kips
5.5
14.7
Kips
5
4
3
15'
Σk (f' − f) = 61.9 In.³
13'
22'
27'
22'
21'
12'
4.1 Kips
11.2
15.3
15.3
14.7
Column Shear
11.2
4.3
−9.8 Kips
(9.8)
(8.8)
(8.2)
(9.3)
(8.9)
Beam Shear
(10.6)
M = 70 × 7 + 76 × 7.5 = 1 060 Kip-Ft
(M_Total = 76 × 100 = 7 600 Kip-Ft)
Column Pressures
+10.6 Kips

(a) LONG CENTRAL SPANS


A
B
C
D
E
F
G
8.8
4.9
3.5
4.3
7
7.9
3.5
5.5
4.8
3.3
4.3
5.5
3.5
15'
60.2 In.³
28'
17'
12'
15'
20'
25'
6.3
15.3
11.2
9.0
10.6
16.0
Column Shear
6.2
−7.5
Beam Shear
(9.5)
(11.1)
(9.4)
(10.1)
(8.0)
(10.5)
Column Pressures
+8.0

(b) LONG SIDE SPANS


A
B
C
D
E
F
G
8.8
6.5
6.5
4.3
4.3
4.3
3.5
6.2
5.6
4.7
3.7
3.7
3.2
15'
28'
23'
23'
15'
15'
13'
Column Shear
6.8
17.1
15.4
12.6
10.2
10.2
4.6
−7.7
Beam Shear
(7 7)
(9.5)
(8.8)
(9.5)
(10.4)
(10.0)
Column Pressures
+10.4
0
10
20
30
Scale of Feet and Kips

(c) SPANS LONG AT ONE END

FIG. 5.—TYPICAL COLUMN SHEAR AND REACTIONS IN A SIX-BAY FRAME, FOR DIFFERENT COLUMN SPACINGS

centage thus obtained is multiplied by the horizontal thrust P to compute the thrust at each column. The thrust P does not enter and should not possibly be introduced into the geometrical analysis of $(f' - f)$ itself, yet that is what is being done all the time.

The ΔP values in every story are relative. Therefore, it is possible to introduce any simplification that affects all members in approximately equal proportions. Generally, in solving Eq. 6 for any single story $\frac{6\,E}{H^2}$ is canceled and (for example) it will be admissible to substitute S by $K = \frac{I}{L}$ for all the structural members, without affecting the results materially. Similarly, in structures generally the columns joining at the upper and at the lower floor of each story will influence the members of that story in equal proportions, and for the purpose of interior analysis, may be omitted.

Fig. 5 shows clearly[23] how the distribution of all these shears is influenced by the relative density of the columns at the sides, at the center, or at one side of the building, proving again that the load distribution is entirely different from that determined with the assumptions made in some of the older methods, as rightly emphasized by Professor Witmer.

Fig. 6 is the three-bay, 10-story bent presented elsewhere by Professor Witmer[24] which the writer has analyzed according to his views. It will be noted that the flexibility of exterior columns is exceptionally great in relation to the remainder of the structure. The analysis can be demonstrated as applied to the story between the sixth and fifth floors (Fig. 7).

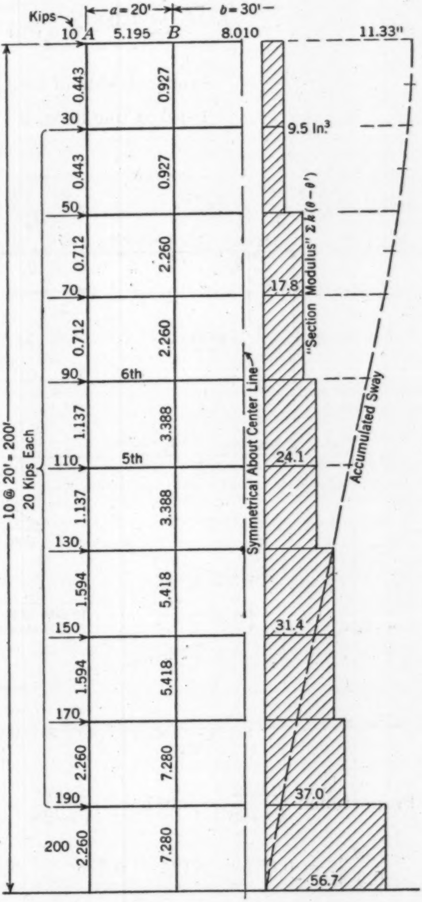

Kips
a = 20'
b = 30'
10
A
5.195
B
8.010
11.33"
0.443
0.927
30
9.5 In.³
0.443
0.927
50
0.712
2.260
70
17.8
0.712
2.260
90
6th
1.137
3.388
110
5th
24.1
1.137
3.388
130
1.594
5.418
150
31.4
1.594
5.418
170
2.260
7.280
190
37.0
200
2.260
7.280
56.7
10 @ 20' = 200'
20 Kips Each
Symmetrical About Center Line
"Section Modulus" Σk(θ − θ')
Accumulated Sway

FIG. 6.—THREE-BAY, 10-STORY BENT

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[23] *Transactions*, Am. Soc. C. E., Vol. 105 (1940), Fig. 7, p. 1038; see also Fig. 10, p. 1043, which shows the possibility of obtaining comparative values for the flexibility and sway of a steel building frame.

[24] "Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings," *Proceedings*, Am. Soc. C. E., June, 1939, p. 969.

The girder shears are obtained as follows:

$$K_A\,\theta_a = 1.137 \times \frac{5.195}{5.195 + 1.137} = 0.932$$

$$K_B\,\theta'_a = 3.388 \times \frac{5.195}{5.195 + 3.388} = 2.050$$

$$K_B\,\theta_b = 3.388 \times \frac{8.010}{8.010 + 3.388} = 2.382$$

Sum each side of center line 5.364

Total at one floor, 2 × 5.364 = 10.728

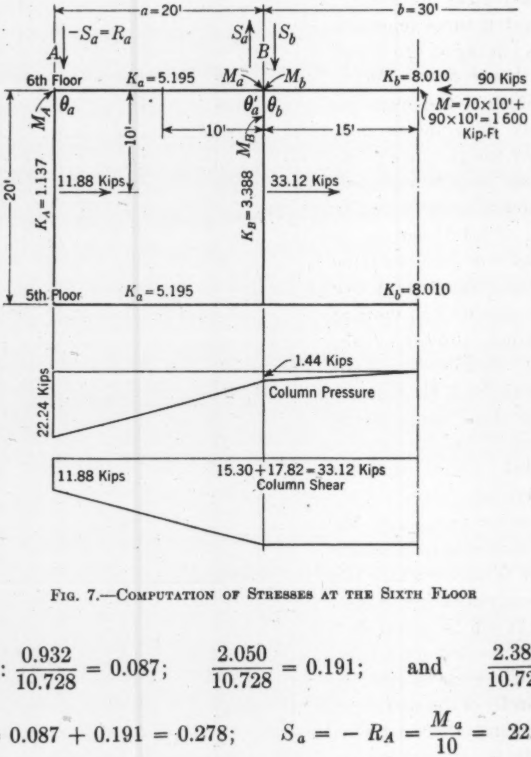
a=20'
b=30'
$-S_a=R_a$
A
S_a
S_b
B
6th Floor
K_a=5.195
M_a
M_b
K_b=8.010
90 Kips
θ_a
θ'_a
θ_b
M=70×10'+90×10'=1 600 Kip-Ft
M_A
10'
10'
15'
M_B
20'
K_A=1.137
11.88 Kips
33.12 Kips
K_B=3.388
5th Floor
K_a=5.195
K_b=8.010
1.44 Kips
Column Pressure
22.24 Kips
11.88 Kips
15.30+17.82=33.12 Kips
Column Shear

FIG. 7.—COMPUTATION OF STRESSES AT THE SIXTH FLOOR

Proportions: $\frac{0.932}{10.728} = 0.087$; $\frac{2.050}{10.728} = 0.191$; and $\frac{2.382}{10.728} = 0.222$.

$$\frac{M_a}{0.5\,M} = 0.087 + 0.191 = 0.278; \qquad S_a = -R_A = \frac{M_a}{10} = 22.24 \text{ kips}$$

$$\frac{M_b}{0.5\,M} = 0.222 + 0.222 = 0.444; \qquad S_b = \frac{M_b}{15} = 23.68$$

Reaction at column B: $R_B = S_b - S_a = +1.44$ kips

The shear at the columns between the fifth and sixth floors may be computed as follows:

$$2\,K_A\,\theta_a = \frac{2 \times 1.137 \times 5.195}{5.195 + 2 \times 1.137} = 1.583$$

$$K_B\,\theta'_a = \frac{3.388 \times 5.195}{5.195 + 3.388} = 2.050$$

$$K_B\,\theta_b = \frac{3.388 \times 8.010}{8.010 + 3.388} = 2.382$$

Sum each side of center line 6.015

Total, 2 × 6.015 each at bottom and top = 12.030

$$\text{Section modulus for this level: } \frac{I}{c} = \Sigma K(\theta - \theta') = 24.060$$

$$\text{Proportions: } \frac{1.583}{12.030} = 0.132; \qquad \frac{2.050}{12.030} = 0.170; \quad \text{and} \quad \frac{2.382}{12.030} = 0.198.$$

$$S_A = 90 \times 0.132 = 11.88\ K; \quad \text{and} \quad S_B = 90\ (0.170 + 0.198) = 33.12\ K.$$

TABLE 4.—COMPARATIVE MOMENTS IN A SYMMETRICAL THREE-BAY TEN-STORY BENT (FIG. 6)

(C = Cross Method (Averages); W = Witmer Method; G = Gottschalk Method; D_c = Percentage Difference Between G and C; and D_w = Percentage Difference Between G and W)

Floor or story	OUTER MEMBERS					INNER MEMBERS				
	C	W	G	D_c	D_w	C	W	G	D_c	D_w
(*a*) GIRDERS										
10	15.6	13.7	14.8	−5	+8	21.7	22.7	20.6	−5	−9
9	60.5	54.7	59.2	−2	+8	79.5	90.6	82.1	+4	−10
8	110.6	109.4	111.2	+1	+2	182.3	181.3	177.6	−3	−2
7	164.3	164.1	166.8	+1	+2	281.0	271.9	266.4	−5	−2
6	217.0	218.7	222.4	+2	+2	368.7	362.5	355.2	−4	−2
5	273.4	273.4	278.0	+2	+2	453.5	453.1	444.0	−2	−2
4	326.6	328.1	326.4	0	−1	551.5	543.8	547.2	−1	+1
3	379.8	382.8	380.8	0	−1	649.7	634.4	638.4	−2	+1
2	431.8	437.5	430.4	0	−2	729.4	725.0	742.3	+2	+2
1	448.8	440.3	433.1	−3	−2	753.5	729.5	747.0	−1	+2
(*b*) COLUMNS										
10	15.6	13.7	15.9	+2	+13	33.4	36.4	34.1	+2	−7
9	48.0	41.0	47.7	−1	+14	103.0	109.0	108.3	+5	−7
8	62.3	68.4	62.5	0	−8	187.6	181.7	187.5	0	+3
7	88.6	95.7	87.5	−1	−8	261.3	254.3	262.5	0	+3
6	120.1	123.0	118.8	−1	−3	328.9	326.9	331.2	+1	+2
5	147.0	150.4	145.2	−1	−3	403.0	399.6	404.8	0	+1
4	166.5	177.7	163.8	−2	−7	485.8	472.3	486.8	0	+3
3	196.7	205.1	189.0	−4	−8	558.5	544.9	561.0	0	+3
2	228.4	232.4	222.7	0	−5	621.6	617.6	627.3	+1	+2
1	208.6	207.9	208.4	0	0	597.6	552.2	631.9	....	....
Base	267.9	251.5	255.2	−5	+2	825.9	884.4	804.5	....	....

Table 4 shows the bending moments obtained by the foregoing analysis in girders and columns respectively, compared with the average moments of the Cross method and the moments of the Witmer methods. It will be seen that the writer's results are close to the average of the Cross moments and generally between Professor Cross and Professor Witmer.

The lower part of Fig. 7 shows the column pressures and column shears produced by the 90 kips of wind pressure acting at the sixth floor, and by the $M = 1{,}600$ kip-ft of moment, quite distinct again from general, arbitrary assumptions.

Knowing the accumulated column pressures, and their shortening or lengthening in relation to the nearest column, the correction is simple enough to determine by Eq. 6, substituting for H the length L of the connecting beam between columns.[20] Additional important data obtained by the writer[20] are the section modulus and "accumulated sway" in Fig. 6, which facilitate a valuable comparison for the flexibility of different column arrangements in a bent.

Altogether the merits of Professor Witmer's paper and the *K*-percentage method cannot be questioned, especially in comparison with the results of the older arbitrary methods; but a method, not contained in the structure itself, is not needed if it is analyzed in the manner suggested herein. The three-bay, 10-story bent is most convenient for training purposes and demonstration; yet results obtained with it do not apply to the irregular bents encountered in daily practice, where exterior columns are less flexible compared with the other members of the structure.

F. P. Witmer,[25] M. Am. Soc. C. E.—It is gratifying that Professor Morris believes that the design procedure outlined in the paper gives results closely in accord with the Spurr method of design. This was a main incentive in preparing the paper.

Professor Morris states that the writer has made contradictory statements regarding the effect upon deflection of a change in girder stiffness. This criticism is not quite warranted. The statement made by the writer in discussing Table 2 was with reference to the effect of such change upon the wind-reaction ratio, and not upon deflection. The effect upon the reaction ratio was amply demonstrated[26] in the model study by the writer and H. H. Bonner published in 1936. The effect upon deflection is less obvious. The writer has made a further study, results of which do not support his original views regarding deflection as expressed in his present paper. This study will be commented upon later.

Professor Morris' Eqs. 4*a* and 4*b* are not properly applicable unless the point of contraflexure lies on the straight-line axis of the member, which can occur only when this point is at midlength. Therefore, it is quite incorrect to apply Eq. 4*a* to the first-story column. To do so results in an error of nearly 150% in the value of ϕ_O.

The discussion of Professor Morris was very stimulating and impelled the writer to make a further effort to satisfy himself as to the facts in the matter. The writer stated that a change in column stiffness would have a greater effect upon deflection than a change in girder stiffness. Professor Morris states that

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[25] Director, Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

[26] "Tall Building Frames Studied by Means of Mechanical Models," by F. P. Witmer and H. H. Bonner, *Transactions*, Am Soc. C. E., Vol. 102 (1937), p. 416.

"When it is desirable to reduce the deflection of the bent, the girder stiffnesses should be increased. The column flexure has little effect upon the total deflection." The writer's investigation leads him to conclude that neither of these statements is correct, but that an increase in either girder size or column size alone will have only a negligible effect upon deflection, and that, to produce an appreciable reduction in deflection, both girders and columns must be increased in approximately equal proportions. In other words, the stiffness of the bent as a whole must be increased. If the bent is made twice as stiff, the deflection will, of course, be reduced by one half.

TABLE 5.—COMPARISON OF MOMENT-DISTRIBUTION RESULTS FOR WIND MOMENTS

Story	OUTER MEMBERS						INNER MEMBERS					
	Case I.-Witmer design		Case II.-Girders doubled		Case III.-Columns doubled		Case I.-Witmer design		Case II.-Girders doubled		Case III.-Columns doubled	
	(*a*) COLUMNS											
	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
10	14.2	11.4	14.7	13.8	13.3	9.7	38.4	35.9	35.7	36.0	41.1	36.0
9	26.8	25.7	28.3	27.7	25.3	24.0	74.8	71.8	74.2	70.1	78.2	72.0
8	29.8	28.3	33.3	29.9	29.0	26.6	124.6	117.2	120.0	116.7	128.5	115.9
7	42.7	41.1	42.5	41.7	42.1	37.8	173.3	143.1	159.5	156.3	166.4	153.6
6	47.3	45.4	49.1	46.9	48.0	42.0	210.0	197.2	204.4	199.7	214.0	195.9
5	59.1	59.1	59.4	59.4	56.4	58.1	245.8	235.9	241.5	239.7	245.1	243.0
4	66.9	69.3	68.6	70.0	67.2	72.1	275.9	287.8	277.5	284.0	271.6	293.0
3	78.5	77.3	79.7	78.9	75.9	74.9	321.5	322.6	323.0	318.4	327.5	321.6
2	87.8	85.8	86.0	85.6	81.7	88.5	364.4	361.1	365.3	363.2	365.6	367.7
1	86.4	96.7	90.7	100.8	75.7	114.8	364.2	452.6	381.6	426.8	325.7	488.3
	(*b*) GIRDERS											
	Left	Right	Left	Right	Left	Right	Left and right		Left and right		Left and right	
10	15.0	11.1	13.4	10.7	13.4	10.7	28.5		28.4		26.7	
9	37.7	30.7	42.4	34.0	34.9	30.2	81.8		89.8		85.1	
8	56.4	49.1	58.1	49.8	53.0	48.6	147.6		143.6		152.0	
7	72.4	68.2	70.5	66.1	68.0	65.3	222.0		213.0		218.7	
6	88.0	83.2	89.1	84.9	85.5	83.8	270.5		277.5		282.2	
5	104.5	101.3	106.6	101.7	97.6	103.7	340.6		340.6		335.6	
4	126.3	125.4	127.1	126.7	124.4	124.8	385.3		390.6		386.3	
3	147.0	147.9	146.3	146.1	147.3	147.1	460.7		462.4		470.8	
2	164.7	165.3	163.0	163.1	156.0	160.3	521.5		521.8		525.0	
1	171.9	167.0	173.7	175.7	164.1	161.0	557.5		569.6		529.9	
ΣM	983.9	949.2	990.2	958.8	944.2	935.5	3,016.0		3,037.3		3,012.3	
V R	V_O = 96.65[a] R_O = 1.080[b]		= 97.45[a] = 1.078[b]		= 93.98[a] = 1.137[b]		V_I = 201.07[c] R_I = 104.42[d]		= 202.49[c] = 105.04[d]		= 200.82[c] = 106.84[d]	

[a] $V_O = R_O = \frac{983.9 + 949.2}{20}$ = 96.65, etc. [b] $R = \frac{R_I}{R_O}$ = 1.080, etc. [c] $V_I = \frac{\Sigma M \times 2}{30}$ = 201.07, etc.
[d] $R_I = V_I - V_O$ = 104.42, etc.

Two new designs were compared with the design in the paper. In the first of these the girders were doubled in stiffness, and in the second the columns were doubled, other parts remaining as in the original design. Both designs were checked by the Cross method. The wind-reaction ratios were found to be practically the same in all three cases, as was to be expected, since the relation between inner and outer columns and between inner and outer girders was unchanged, the assumed cantilever relation of reactions thus being maintained. The surprising fact was noted, however, that the wind moments in girders and columns throughout the bent were also practically the same in all three cases.

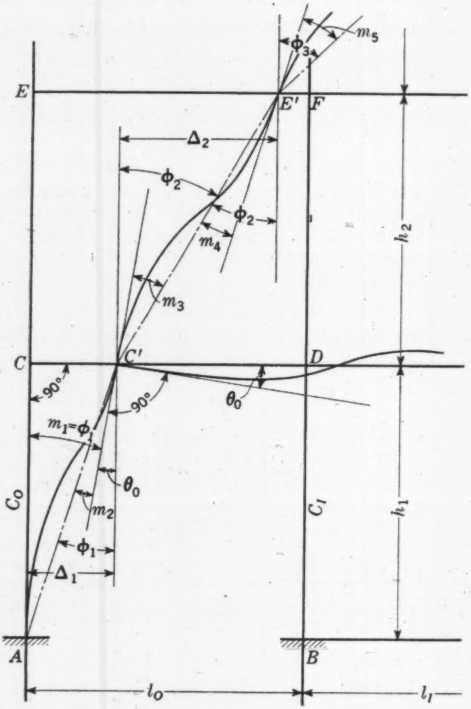

FIG. 8

Table 5 shows the moments computed for the three bents. This result should not be surprising, since it would necessarily follow from the similar wind-reaction ratios if the *K*-percentage method of analysis were used, and in every one of the many cases previously tested by the writer[2] the moments found by this method were seen to be very closely in accord with those obtained by the Cross analysis.

Deflections were next computed for all three cases. Instead of following the method suggested in the Second Progress Report of the Sub-Committee on Wind Bracing,[27] the writer used a much simpler method. End-deflection angles m_1, m_2, m_3, etc., for columns shown in Fig. 8, were computed from the column moments by the method of slope deflection. The formula for this is

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[27] *Proceedings*, Am. Soc. C. E., February, 1932, p. 213.

$$m_1 = \frac{M_2 - 2\,M_1}{6\,E\,K} \qquad (7)$$

From these angles, the angles ϕ_1, etc., being the slopes of the straight-line axes of the columns with reference to the vertical, were found as follows:

$$\left.\begin{aligned} \phi_1 &= m_1 \\ \phi_2 &= \phi_1 - m_2 + m_3 \\ \phi_3 &= \phi_2 - m_4 + m_5, \text{ etc.} \end{aligned}\right\} \qquad (8)$$

It is seen that the terms $(\phi_1 - m_2)$, $(\phi_2 - m_4)$, etc., in Eq. 8 represent angles identical with θ_O, etc., in the discussion of Professor Morris, and therefore correspond to what he terms the "girder drift," whereas m_3, m_5, etc., are comparable to Professor Morris' ϕ_O, etc., and represent "column drift."

From these values of ϕ_1, ϕ_2, etc., the deflections in the several stories will equal, respectively, $\phi_1\,h_1$, $\phi_2\,h_2$, etc.; and since, in the case under consideration, the story height is the same for all floors, the total deflection will be

$$\Delta = (\phi_1 + \phi_2 + \phi_3, \text{ etc.}) \times h \qquad (9)$$

This procedure produced the results that follow:

Case I, Witmer Design—$\Delta = 5.592$ in. (by outer column moments); column drift = 65.7%; and girder drift = 34.3%. (By inner column moments, $\Delta = 5.760$ in.—a good check.)

Case II, Girders Doubled—$\Delta = 4.932$ in. (a reduction of 11.6%); column drift = 74.2%; and girder drift = 25.8%.

Case III, Columns Doubled—$\Delta = 5.136$ in. (a reduction of 8.2%); column drift = 32.0%; and girder drift = 68.0%.

Thus, an increase of 100% in either girders or columns reduced the deflection by only about 10%, which was not enough to be of great practical value, when compared with the increased weight of metal involved. The relative effect upon column and girder drift in each case, however, was of the nature to be expected from the character of change made in sizes.

It thus seems reasonable to conclude that a reduction in deflection of an effective proportion would require an increase in both columns and girders. This result was so surprising and so conclusively established for the particular cases considered that the writer felt justified in presenting it rather fully, particularly as it seems so at variance with the expressed views of Professor Morris.

The discussions of Mr. Goodrich and Professor Castleman are both interesting and gratifying. The foregoing method of computing deflection may be considered by Mr. Goodrich as a response to his query in this regard The omission of direct wind stress in the design of columns, which was mentioned by Mr. Goodrich, was an oversight, pure and simple, and fully deserves his criticism; but it does not seriously affect whatever utility the paper may have in regard to its announced purposes.

Referring to Professor Castleman's discussion, the reason for the suggestion that it is better to use the cantilever relation of reactions in design, even though maximum economy may not result, is not because of any objection to secondary stresses from direct column wind distortion, "provided they are cared for and that the sum total weight involved is less than that required by cantilever theory." The real reason is that without the cantilever relation these stresses must be computed, and their determination is quite laborious, requiring an additional moment-distribution analysis based upon the direct column wind distortions. It was considered that the comparatively small loss in economy to be expected might be properly offset against this increased cost of analysis, although the writer realizes that an increased cost in engineering processes is not generally a valid reason for entailing an additional cost of steel. The matter is largely one of judgment on the part of the designer.

The writer has not had time to study further the effect of a variable moment of inertia in girders upon the availability of the *K*-percentage method, but he believes that the principles underlying this method are so fundamentally applicable that it can be adapted readily to such a modification in detail.

Mr. Gottschalk's discussion very acceptably presents the methods of bending behavior that he has described on many occasions, and had the writer possessed greater familiarity with these methods he would surely have mentioned them in conjunction with those of slope deflection and end-moment distribution, as a practical type of design procedure.

It is interesting to note the close comparison of the moments as computed by Mr. Gottschalk's method with those as computed by the Cross and the *K*-percentage methods, in the case considered by Mr. Gottschalk.

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TRANSACTIONS

Paper No. 2151

SIMPLIFIED THEORY OF THE SELF-ANCHORED SUSPENSION BRIDGE

By C. H. Gronquist,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. A. J. Meehan, C. B. McCullough, Jaroslav J. Polivka, William Bertwell, A. A. Eremin, and C. H. Gronquist.

Synopsis

The theory of the self-anchored suspension bridge is presented in this paper in a simplified form that lends itself to a straight-forward and expeditious analysis of that type of structure.

The self-anchored suspension bridge, unlike the externally-anchored type, may be properly analyzed by the elastic theory, since the effect of distortions under live load is practically eliminated in this "closed" structure. The formulas which follow are therefore based on the elastic theory.

Vertical camber of the stiffening girder is effective in reducing cable and girder stress in the self-anchored bridge by the arch-like action of the cambered girder, and this factor is included throughout. Moreover, as the resultant stress in the girder, like that in an arch rib, is that produced by direct thrust and moment, influence lines for girder flange stress may be constructed as for moment about the kern points of the girder. This method of stress analysis is used in the computations, which illustrate the application of the theory of the self-anchored suspension bridge.

The formulas for the self-anchored suspension bridge have been derived for a three-span symmetrical bridge with continuous girders. By dropping the continuity terms, the formulas for girders hinged at the towers are obtained. It will be observed that these formulas, with girder camber and shortening eliminated, are the same as those for the externally-anchored suspension bridge analyzed by the elastic theory.

Constant girder moment of inertia within each span has been assumed; the effect of this approximation, as well as that of neglect of suspender elongation, is considered, as is the modification of the formulas for unsymmetrical and multiple spans. A method of correcting the value of cable and girder H as

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Note.—Published in February, 1941, *Proceedings*.

[1] Assoc. Engr., Robinson & Steinman, New York, N. Y.

computed by the elastic theory, in order to take into account the movement of the ends of the girder produced by its vertical deflection, is indicated.

INTRODUCTION

In the self-anchored suspension bridge the cable is attached to the stiffening girder at the outer ends of the side spans. Thus the horizontal pull of the cable is resisted by the girder in compression, and the anchorage foundations are subjected only to the upward vertical component of the cable stress, in addition to the girder reactions. This type of construction, consequently, permits the use of a suspension structure where foundation conditions do not justify the ordinary externally-anchored suspension bridge, with anchorages that must resist the horizontal pull of the cable.

There is some doubt as to whether the self-anchored suspension bridge was originated by an Austrian, Joseph Langer, in 1870, or whether that type of structure was covered by an American patent issued to Charles Bender in 1867. Although Bender's design and Langer's Wrsowic Bridge (1870) were of the self-anchored type, the cable was attached to the girder in the main span as well as at the outer end of the side spans of these bridges. The first truly self-anchored suspension bridge of the modern type was built at Lübeck, Germany, in 1899. The self-anchored suspension bridge has since been adopted extensively in Europe and in one instance in Japan. In the United States three almost identical examples of this type of bridge were built from 1926 to 1928 in Pittsburgh, Pa. Two bridges have also been built in the United States in which rolled beam girders and open-strand cables were used—the first in 1933 over the Little Niangua River in Missouri, and the second in 1939 with a 350-ft main span over the Wabash River in Indiana. In Guatemala a small self-anchored bridge was built in 1937 on the Pan-American Highway. (Howard Mullins, in 1936, supplied a list and bibliography of the self-anchored suspension bridges of the world.)[2]

Although several of these bridges are of moderately long span, ranging from 600 ft to 1,000 ft, most of them are approximately 400 ft, or less, in length of main span. The application of this type of structure to spans of less than 400 ft—in which, for light loading, rolled beams can be used as stiffening girders and open-strand construction for the cables, and where the shortness of span will lessen the disadvantage that deflection does not appreciably relieve girder moment—appears to offer distinct possibilities for improving appearance in bridges of this size in locations which would not be considered suitable for the externally-anchored suspension bridge.

So far as is known to the writer, no complete treatise on the theory of the self-anchored suspension bridge is available in English, although some valuable information has been published on the subject. (Computations for several self-anchored suspension bridge designs were presented by G. G. Krivoshein in 1930;[3] and, as stated, a review of the history and analysis of the characteristics of this type of structure was published by Mr. Mullins in 1936.[2]) The following is offered as a simplified statement and extension of the existing treatments of the theory of the self-anchored suspension bridge in the belief that, with a more complete, simplified treatment available, the use of the self-anchored suspension bridge for spans of moderate length will increase.

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[2] *Engineering News-Record*, January 9, 1936, p. 45.

[3] "Simplified Calculation of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, 1930.

NOTATION

The letter symbols in Fig. 1 and elsewhere in the paper are defined in the text where they first appear and, for convenience of reference, are listed alphabetically in the Appendix.

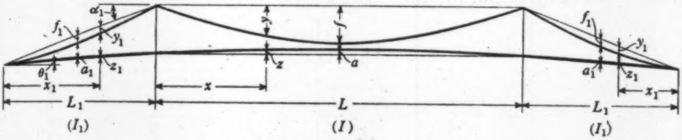

FIG. 1.—NOTATION DIAGRAM

DEAD LOAD

The stiffening girder, of symmetrical cross section and constant depth, of a self-anchored suspension bridge is subjected to no dead-load moment provided it is laid out during fabrication for dead-load position, and that the lengths of the towers, cable, suspenders, and girder are fabricated correctly to bring the girder exactly to this position after the application of the dead load. If the girder is cambered parabolically in any span, it will carry a portion of the uniformly distributed dead load as a parabolic arch, the total stress being axial. If the side-span cable is unloaded, the side-span girder must naturally be self-supporting for dead load, and when, in addition, the girder is continuous through the towers, it will be subjected also to dead-load bending stress in the main span. The latter type of structure is not considered herein; but by including this dead-load moment in the formulas that follow, they become applicable to that type of structure.

To determine the value of the horizontal component of cable and girder stress due to dead load, the expression for moment in the girder at the center of the main span may be written and placed equal to zero. In Fig. 2 let:

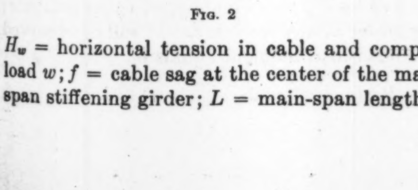

FIG. 2

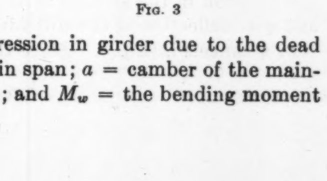

FIG. 3

H_w = horizontal tension in cable and compression in girder due to the dead load w; f = cable sag at the center of the main span; a = camber of the main-span stiffening girder; L = main-span length; and M_w = the bending moment due to dead load. Then,

$$M_w = \frac{w\,L^2}{8} - H_w\,a - H_w\,f = 0 \qquad (1a)$$

and

$$H_w = \frac{w\,L^2}{8\,(f + a)} \qquad (1b)$$

The portion of the dead load carried by the girder as a parabolic arch (w_a) may be evaluated by taking the second derivative of the expression for the negative moment that tends to act on the girder as a result of its parabolic camber. The expression for this moment is:

$$M_w' = -\,H_w\,z = \frac{-4\,a}{L^2}(L\,x - x^2)\,H_w \qquad (2a)$$

and

$$w_a = \frac{d^2}{dx^2}(-\,H_w\,z) = 8\,H_w\,\frac{a}{L^2} = \frac{w\,a}{f + a} \qquad (2b)$$

in which: z = main-span stiffening girder ordinate; and x = abscissas along the center span, measured from the tower. The value of the side-span cable sag f_1 is obtained by writing, as in the main span, the expression for moment in the girder at the center of the span (see Fig. 3):

$$M_{w1} = \frac{w_1\,L_1^2}{8} - H_w\,(f_1 + a_1) = 0 \qquad (3)$$

and therefore,

$$f_1 = \frac{w_1\,L_1^2}{w\,L^2}(f + a) - a_1 \qquad (4a)$$

Again (see Eq. 2*b*):

$$w_{a1} = \frac{w_1\,a_1}{f_1 + a_1} \qquad (4b)$$

LIVE LOAD

The application of live load to the bridge produces deflections of the cables and stiffening girders that change the dimensions of the structure. The changes in cable span lengths due to tower-top movement are negligible since they are small in proportion to the total span lengths. That the proportionately large changes in cable sag and girder camber have no effect on the moments in the girder may be seen by writing the general expression for moment in the girders (see Fig. 4):

$$M = T + M_D + M_0 - (H_w + H)\,(y + \eta) - (H_w + H)\,(z - \eta) \qquad (5)$$

in which: M = the total resultant bending moment at any section x of the stiffening girder; T = bending moment at any section x due to continuity; M_D = simple-beam bending moment for dead load; M_0 = simple-beam bending moment due to a live load p; H = horizontal tension in the cable and compression in the girder due to a live load p; y = main-span cable ordinate; and η = deflection of the stiffening girder at any section x. It will be observed that η cancels in Eq. 5. Since the dead-load moment equals 0,

$$M_D - H_w\,(y + z) = 0 \qquad (6a)$$

and, in any span,

$$M = T + M_0 - H\,(y + z) \qquad \text{(6b)}$$

The load taken by the girder in moment is equal to the negative of the second derivative of that moment.

$$p_0 = \frac{-d^2 M}{dx^2} = p - \frac{8\,(f + a)\,H}{L^2} \qquad \text{(7)}$$

This is the difference between the applied distributed load p and that taken by the cable and by the girder as an "arch."

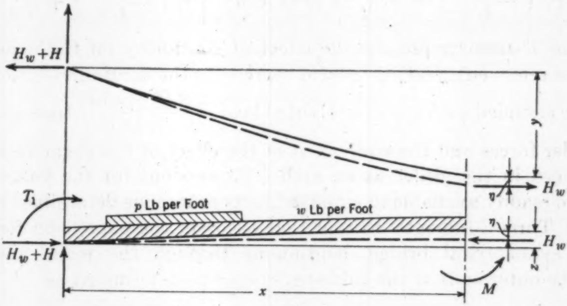

FIG. 4

Thus it may be stated that the self-anchored suspension bridge is an independent, closed structure, in its action under live load, in many ways more nearly resembling an inversion of the tied arch than an externally-anchored suspension bridge. Whereas the stresses in the latter type of structure are greatly affected by the changes in the cable sag produced by deflection, those of the self-anchored bridge are not so affected due to the simultaneous change in girder moment of H multiplied by the deflected z. Because the sum of $(y + z)$ or $(f + a)$ in any span is constant during deflection, neglecting suspender elongation, the value of the dead load H_w remains unchanged during deflection; the dead load carried by the girder-arch is decreased, that by the cable is increased.

Inasmuch, then, as the deflection of the bridge has no appreciable influence over the dead load H_w or the live-load moment in the girders, except over the latter in the side span when the side-span cable is unloaded, the self-anchored suspension bridge may be analyzed by application of the elastic theory. The following assumptions have been made:

(1) The cable curve before deflection is a parabola whose equation, with the closing chord as the x-axis and the origin at one end, is

$$y = \frac{4\,f\,x}{L^2}\,(L - x) \qquad \text{(8)}$$

in all spans.

(2) The girder camber is parabolic. (For a girder that lies in a straight line the a of the formulas (Eqs. 1 to 7) is made equal to 0.) With the closing

chord as the x-axis and the origin at one end:

$$z = \frac{4\,a\,x}{L^2}\,(L - x) \ldots\ldots (9)$$

in all spans.

(3) The average moment of inertia of the stiffening girder throughout each span is used as the constant I of that span.

(4) The work of deflecting the tower top is negligible.

The moment of continuity at any point may be written:

$$T = T_1'\left(\frac{L - x}{L}\right) + T_2'\,\frac{x}{L} + H\,(f + a)_m \left[e_1\left(\frac{L - x}{L}\right) + e_2\,\frac{x}{L}\right] \ldots (10)$$

in which the T'-terms represent the effect of continuity on the independent girder (cable removed), and the e-term represents the continuity effect on the girder of the assumed uniformly distributed load of $\frac{8\,(f + a)\,H}{L^2}$ from the sum of the suspender forces and the equivalent of the effect of the negative moment, $-H\,z$, induced in the girder as an arch. Expressions for the values of the T'-terms are readily available in standard texts or may be determined from the Theorem of Three Moments. From that theorem the expression for e for a three-span symmetrical bridge, continuous through the main towers but hinged at the outer ends of the side spans, may be determined as

$$e = e_1 = e_2 = \frac{2\,(1 + i\,r\,v')}{3 + 2\,i\,r} \ldots\ldots (11)$$

at the main towers and throughout the main span; in the side span the e-term varies from 0 at the free end to $e_{1,2}$ at the tower. In Eq. 11 (see Appendix): i = the ratio of moments of inertia, $\frac{I}{I_1}$, main span to side span; r = ratio of span lengths $\frac{L_1}{L}$, side span to main span; and v' = ratio of total sag and camber $\frac{f_1 + a_1}{f + a}$, side span to main span.

The general moment equation applying to all spans may be written in the following form as a working equation:

$$M = (T' + M_0) + H\,[e'\,(f + a)_m - (y + z)] \ldots\ldots (12)$$

In this equation e is written with a prime to indicate that it varies as a straight-line function within any span, and $(f + a)$ is written with the subscript m to indicate that it takes the main-span value in all spans; T', similarly to e, varies as a straight-line function within any span.

The vertical shear in the girder corresponding to this moment is in general form for all spans:

$$V = \frac{dM}{dx} = \left[\frac{(T_2' - T_1')}{L} + V_0\right]$$
$$- H\left[4\left(\frac{f + a}{L^2}\right)(L - 2\,x) - (f + a)_m\,\frac{(e_2 - e_1)}{L}\right] \ldots\ldots (13)$$

in which V_0 = simple-beam shear due to live load.

DETERMINATION OF H FOR LIVE LOAD

To derive the expression for H by the method of the redundant member, the cable may be conceived as cut, with the horizontal component of cable stress H the redundant. Then:

$$H = -\frac{\sum \frac{S'\,u\,L}{A\,E} + \sum \int_0^L M'\,m\,\frac{dx}{E\,I}}{\sum \frac{u^2\,L}{A\,E} + \sum \int_0^L m^2\,\frac{dx}{E\,I}} = \frac{N}{D} \quad\cdots\cdots\cdots\cdots (14)$$

In Eq. 14 S', the direct stress in the various members of the system with the cable cut, equals zero throughout; M', the moment in the members of the system with the cable cut, is the moment in the independent girder in each span; $M' = T' + M_0$ in all spans. The term u denotes the direct stress in the members of the system corresponding to a horizontal component of stress in the cable and girder of unity. In the cable $u = \frac{ds}{dx}$; and in the girders $u = \frac{ds'}{dx}$. The bending moment in the members of the system due to a unit horizontal component of cable stress is m. Calling $T' + M_0 = 0$ and $H = 1$ in the working equation for moment in the girders,

$$m = e'\,(f + a)_m - (y + z) \quad\cdots\cdots\cdots\cdots (15)$$

in all spans. The first term in the denominator of the H-equation (Eq. 14) is:

$$\sum \frac{u^2\,L}{A\,E} = \sum \int_0^L \left(\frac{ds}{dx}\right)^2 \frac{ds}{E_c\,A_c} + \sum \int_0^L \left(\frac{ds'}{dx}\right)^2 \frac{ds'}{E\,A_g} = \frac{L_s}{E_c\,A_c} + \frac{L_s'}{E\,A_g} \quad\cdots\cdots\cdots\cdots (16)$$

neglecting the effect of change of length of suspenders and tower legs. With a parabolic cable of constant section:

$$L_s = \sum L\,(\sec^3 \alpha + 8\,n^2 \sec \alpha) \text{ (very nearly)} \quad\cdots\cdots\cdots\cdots (17a)$$

For an eyebar cable in which A_c may be assumed to vary with the cable secant:

$$L_s = \sum L \left(\sec^2 \alpha + \frac{16\,n^2}{3}\right) \text{ (very nearly)} \quad\cdots\cdots\cdots\cdots (17b)$$

The values of A_c at the low point of each span are used with the value of L_s given in Eq. 17b.[4] In the same manner:

$$L_s' = \sum L\,(\sec^3 \theta + 8\,c^2 \sec \theta) = \sum L \text{ (very nearly)} \quad\cdots\cdots\cdots\cdots (17c)$$

The second term in the denominator of the H-equation (Eq. 14) is:

$$\sum \int_0^L m^2 \frac{dx}{E\,I} = \sum \int_0^L [e'\,(f + a)_m - (y + z)]^2 \frac{dx}{E\,I}.$$

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[4] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), Eqs. 21*c* and 21*d*, p. 1145.

Thus for a three-span symmetrical bridge,

$$D = \left(\frac{L_s}{E_c\, A_c} + \frac{L_s'}{E\, A_g} \right) + \frac{(f + a)^2\, L}{3\, E\, I} \left(3\, e^2 - 4\, e + \frac{8}{5} \right)$$
$$+ \frac{2\, L_1}{3\, E\, I_1} [e^2\, (f + a)^2 - 2\, e\, (f + a)\, (f_1 + a_1) + \frac{8}{5} (f_1 + a_1)^2] \ldots \text{(18}a\text{)}$$

Abbreviating further, Eq. 18*a* may be written also as:

$$D = \frac{(f + a)^2\, L}{3\, E\, I} \left\{ \left(\frac{8}{5} + 3\, e^2 - 4\, e \right) + 2\, i\, r \left[\frac{8\, (v')^2}{5} + e^2 - 2\, e\, v' \right] \right.$$
$$\left. + \frac{3\, I}{(f + a)^2\, L} \left(\frac{E}{E_c} \times \frac{L_s}{A_c} + \frac{L_s'}{A_g} \right) \right\} \ldots\ldots\ldots\ldots \text{(18}b\text{)}$$

The numerator of Eq. 14 is: $- \sum \int_0^L M'\, m\, \frac{dx}{E\, I} = - \sum \int_0^L (T' + M_0)$ $\times [e'\, (f + a)_m - (y + z)] \frac{dx}{E\, I}$; but $\sum \int_0^L T' \left[e'\, (f + a)_m - (y + z) \right] \frac{dx}{E\, I} = 0$, which can be verified by integration. Therefore

$$N = - \sum \int_0^L M_0\, [e'\, (f + a)_m - (y + z)] \frac{dx}{E\, I} \ldots\ldots\ldots \text{(19)}$$

From the foregoing general expression for *N* (Eq. 19), the value of the numerator *N* of the *H*-equation (Eq. 14) for a specific bridge and various loadings may be derived by performing the indicated integration. A three-span symmetrical structure has been assumed in the cases that follow. For a concentrated load *P*, at a distance *k L* from the left tower in the main span:

$$N = - P\, (f + a) \frac{k\, L^2}{E\, I} \left[\frac{e}{2} (1 - k) - \frac{1}{3} (1 - 2\, k^2 + k^3) \right] \ldots\ldots \text{(20}a\text{)}$$

and

$$H = \frac{1}{D'\, n'} \left[B(k) - \frac{3}{2} e\, (k - k^2) \right] P \ldots\ldots\ldots\ldots \text{(20}b\text{)}$$

in which

$$D' = \frac{3\, E\, I\, D}{(f + a)^2\, L} \ldots\ldots\ldots\ldots \text{(21}a\text{)}$$

and

$$B(k) = k\, (1 - 2\, k^2 + k^3) \ldots\ldots\ldots\ldots \text{(21}b\text{)}$$

For a concentrated load at a distance $k_1\, L_1$ from the free end of the side span:

$$N = - \frac{P\, k_1\, L_1^2}{6\, E\, I_1} [e\, (f + a)\, (1 - k_1^2) - 2\, (f_1 + a_1)\, (1 - 2\, k_1^2 + k_1^3)] \ldots \text{(22}a\text{)}$$

and

$$H = \frac{i\, r^2}{D'\, n'} \left[v'\, B(k_1) - \frac{e}{2} (k_1 - k_1^3) \right] P_1 \ldots\ldots\ldots\ldots \text{(22}b\text{)}$$

in which

$$B(k_1) = k_1\, (1 - 2\, k_1^2 + k_1^3) \ldots\ldots\ldots\ldots \text{(23)}$$

By a process of integration over proper limits the values of N and H can be obtained from these expressions, for various distributed load conditions. For all spans fully covered with distributed loads of p in the main span and p_1 in the side spans:

$$N = \frac{-p\,L^3}{3\,E\,I}(f + a)\left(\frac{e}{4} - \frac{1}{5}\right) - \frac{p_1\,L_1^3}{3\,E\,I_1}\left[\frac{e}{4}(f + a) - \frac{2}{5}(f_1 + a_1)\right] \ldots (24a)$$

and

$$H = \frac{p\,L}{D'\,n'}\left(\frac{1}{5} - \frac{e}{4}\right) + \frac{2\,i\,r^3}{D'\,n'}\left(\frac{v'}{5} - \frac{e}{8}\right)p_1\,L \ldots\ldots\ldots (24b)$$

The value of N for the left or right half of the main span covered with a uniformly distributed load is one half the value for the entire span so loaded. Combined with the value of N for a side span fully loaded, this value is useful in obtaining preliminary values of the maximum moment at the tower and quarter-point of the main span. The value of N for the center 0.4 of the main span covered with a uniformly distributed load, which may be used in obtaining a preliminary value of the maximum moment at the center, is:

$$N = \frac{-p\,L^3}{E\,I}(f + a)\,(0.0473\,e - 0.0390) \ldots\ldots\ldots\ldots (25a)$$

and

$$H = \frac{3\,p\,L}{D'\,n'}(0.0390 - 0.0473\,e) \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

The several formulas given herein, for use in the computation of the live load H of a self-anchored suspension bridge, will all reduce, upon the substitution of f for $(f + a)$, y for $(y + z)$, and $L_s' = 0$, to those of the externally-anchored suspension bridge as derived by others[5,6] for the elastic theory.

TEMPERATURE

As the self-anchored suspension bridge is a "closed" structure, no temperature stresses are produced for conditions of uniform temperature change except those caused by the expansion of towers of unequal length below the girders. For the usual case in which the length of the main tower from pier to girder is greater than that at the free end of the side span, the stress produced, however, is merely that in the independent continuous girder as a result of the relative vertical movement of the main tower supports. The moment over the upward deflected supports for a symmetrical three-span girder is:

$$T_1' = T_2' = -\frac{6\,E\,\Delta y_t}{L_1\left(\frac{3\,L}{I} + \frac{2\,L_1}{I_1}\right)} = -\frac{6\,E\,I\,r\,\Delta y_t}{(3 + 2\,i\,r)\,L_1^2} \ldots\ldots (26)$$

The corresponding value of H is zero, neglecting the slight effect of horizontal movement of the ends of the girder and the rise of the tower saddles.

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[5] "A Practical Treatise on Suspension Bridges, Their Design, Construction and Erection," by D. B. Steinman, M. Am. Soc. C. E., 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1929.

[6] "Modern Framed Structures," by the late J. B. Johnson and C. W. Bryan, Members, Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., 9th Ed., Pt. 2, John Wiley & Sons, Inc., New York, N. Y., 1911.

In the girder hinged at the towers there is no stress produced by the expansion of the towers, since the structure is fully articulated and free to deflect under uniform temperature change without causing stress. Towers fixed at their bases, however, are subject to temperature moments produced by the deflection of the tower tops with the cables.

LIVE-LOAD DEFLECTIONS

The equation of the girder deflection curve may be obtained by performing the double integration of the equation: $\frac{d^2\eta}{dx^2} = -\frac{M}{E\,I}$; from which—

$$\eta = -\int\int\left\{(T' + M_0) + H\left[e'\,(f + a)_m \frac{-4}{L^2}(f + a)\,(L\,x - x^2)\right]\right\}\frac{dx}{E\,I} \quad \text{..(27)}$$

The deflection of the girder at a particular point may be written from this general equation by the substitution of the proper coordinate, or by integration of the familiar equation:

$$\eta' = \int_0^L \frac{M\,m\,dx}{E\,I} \quad \text{.......................(28)}$$

The change of grade or slope of the deflected girder and roadway is found by integrating Eq. 27 once:

$$\tan\Delta\phi' = \frac{d\eta}{dx} = -\int\frac{M\,dx}{E\,I} \quad \text{..................(29)}$$

This slope of the elastic curve of the girder at a particular point is again found by the substitution of the proper coordinate in the general expression. At the towers—$x = 0$, or $x = L$. The formulas that follow have been derived for the three-span symmetrical bridge.

In the main span for a uniformly distributed load extending a distance $k\,L$ from the left tower, the equation of the deflection curve from $x = 0$ to $x = k\,L$ is:

$$\begin{aligned} E\,I\,\eta = &-\left[\frac{T_1'}{2\,L}\left(L\,x^2 - \frac{x^3}{3} - \frac{2\,L^2\,x}{3}\right) + \frac{T_2'}{6\,L}(x^3 - L^2\,x)\right] \\ &+ H\left[\frac{e}{2}(f + a)\,(L\,x - x^2) + \frac{2}{3}\frac{(f + a)}{L^2}\left(L\,x^3 - \frac{x^4}{2} - \frac{L^3\,x}{2}\right)\right] \\ &- p\left\{\frac{1}{6}\left[k\,L\left(1 - \frac{k}{2}\right)x^3 - \frac{x^4}{4}\right] - \frac{x\,L^3}{6}\left(\frac{k^4}{4} - k^3 + k^2\right)\right\} \quad \text{....(30)} \end{aligned}$$

For the deflection curve from $x = k\,L$ to $x = L$, only the p-terms change. These are: $-\,p\left[\frac{k^2}{4}L^2\left(x^2 - \frac{x^3}{3\,L}\right) - \frac{x\,L^3}{6}\left(\frac{k^4}{4} + k^2\right) + \frac{k^4\,L^4}{24}\right]$.

At the left tower the change of grade is, for this loading:

$$\begin{aligned} E\,I\tan\Delta\phi_1' = &\frac{L}{3}\left[T_1' + \frac{T_2'}{2} + H\,(f + a)\left(\frac{3\,e}{2} - 1\right)\right] \\ &+ \frac{1}{6}\,p\,k^2\,L^3\left(1 - k + \frac{k^2}{4}\right) \quad \text{.................(31a)} \end{aligned}$$

At the right tower:

$$E\,I \tan \Delta\phi_2' = -\frac{L}{3}\left[\frac{T'}{2} + T_2' + H\,(f + a)\left(\frac{3\,e}{2} - 1\right)\right]$$
$$+ \frac{p\,k^2}{12} L^3 \left(\frac{k^2}{2} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots (31b)$$

For full main-span loading and without load on the side spans ($k = 1$, and $T_1' = T_2'$), the deflection at the center of span, where $x = \frac{L}{2}$, is found from the general equation (Eq. 30):

$$\frac{8\,E\,I}{L^2}\,\Delta f = T_1' + \frac{5}{48}\,p\,L^2 + H\,(f + a)\left(e - \frac{5}{6}\right) \ldots\ldots (32a)$$

The deflection under a concentrated load at the center of the main span may be written from the previous formula by changing simply the deflection coefficient of the independent simple beam:

$$\frac{8\,E\,I}{L^2}\,\Delta f = T_1' + \frac{P\,L}{6} + H\,(f + a)\left(e - \frac{5}{6}\right) \ldots\ldots\ldots (32b)$$

The formulas for deflection and load in the side span are most easily obtained by substituting for $T_1 = 0$ in those of the main span. For a uniformly distributed load extending a distance $k_1\,L_1$ from the free end of the span:

$$E\,I_1\,\eta_1 = \frac{-T_1'}{6\,L_1}\,(x_1^3 - L_1^2\,x_1)$$
$$+ H\left[\frac{-e}{6\,L_1}\,(f + a)\,(x_1^3 - L_1^2\,x_1) + \frac{2}{3}\left(\frac{f_1 + a_1}{L_1^2}\right)\left(L_1\,x_1^3 - \frac{x_1^4}{2} - \frac{L_1^3\,x_1}{2}\right)\right]$$
$$- p_1\left\{\frac{1}{6}\left[k_1\,L_1\left(1 - \frac{k_1}{2}\right)x_1^3 - \frac{x_1^4}{4}\right] + \frac{x_1\,L_1^3}{6}\left(\frac{k_1^4}{4} - k_1^3 + k_1^2\right)\right\} \ldots (33a)$$

The change of grade at the free end of the side span is:

$$E\,I_1 \tan \Delta\phi_1' = \frac{L_1}{3}\left\{\frac{T_1'}{2} + H\left[\frac{e}{2}\,(f + a) - (f_1 + a_1)\right]\right\}$$
$$+ \frac{p_1\,k_1^2}{6}\,L_1^3\left(1 - k_1 + \frac{k_1^2}{4}\right) \ldots\ldots\ldots\ldots\ldots (33b)$$

At the tower:

$$E\,I_1 \tan \Delta\phi_1' = -\frac{L_1}{3}\left\{T_1' + H\left[e\,(f + a) - (f_1 + a_1)\right]\right\}$$
$$+ \frac{p_1}{12}\,k_1^2\,L_1^3\left(\frac{k_1^2}{2} - 1\right) \ldots\ldots\ldots\ldots\ldots (33c)$$

The deflection at the center of the fully loaded side span is:

$$\frac{16\,E\,I_1}{L_1^2}\,\Delta f_1 = T_1' + \frac{5}{24}\,p_1\,L_1^2 + H\left[e\,(f + a) - \frac{5}{3}\,(f_1 + a_1)\right] \ldots (33d)$$

For a central concentrated load, as in the main span, $\frac{P_1 L_1}{3}$ should be written in place of $\frac{5}{24} p_1 L_1^2$ in Eq. 33*d*. Eqs. 27 to 33, with the p-terms made equal to zero, are those for the deflection of spans unloaded themselves, which are influenced by the loading in other spans through the action of girder continuity and cable effect. For girders hinged at the points of support, the continuity terms are made equal to zero in these and all formulas previously derived for continuous girders. For bridges with unloaded side-span cables, f_1 is made equal to zero in Eqs. 3 to 33.

STRESSES

The formula for cable stress is the same as that for the externally-anchored bridge. Therefore the stress at any point in the cable from the combination of dead and live load is $(H_w + H) \sec \phi$, in which the angle ϕ is defined by $\tan \phi = \tan \alpha + \frac{4f}{L^2}(L - 2x)$. The influence of deflection on the cable angle ϕ, as well as on the girder angle ϕ', is negligible in its effect on both cable and girder stresses.

Since the girder, in helping to carry the dead load of the bridge, is ordinarily subject to no moment or normal shear, the only dead-load stress that it must be designed to sustain is the direct compression that amounts to $H_w \sec \phi'$ at any point. This stress is comparable to the dead-load stress in the cable.

Live-load girder stresses are those of direct compression, moment, and shear. The stresses from direct compression and moment about the axis of the girder may be treated, as in the arch, by finding, at each section examined along the girder, the combined compressive stress and the resultant tensile stress corresponding to the moments alone about the proper kern points, which are distant $\frac{r^2}{c_{U,L}}$ from the neutral axis.[6] The final flange stresses are $f_{U,L} = M_{U,L} \frac{c_{U,L}}{I}$. Therefore, influence lines for flange stress may be obtained by determining the values of the moments about the kern points for unit live-load concentrations along the bridge. This method of obtaining influence lines for girder stress is used by some German writers.[7]

The kern moments may be determined either analytically or graphically. Analytically, they are equal to the moment previously found about the geometrical girder axis (on which lie the cable reaction pins at the span ends) corrected for the effect of the components of H and V acting through the lever arms $\frac{r^2}{c_{U,L}} \mp h$. Thus in Fig. 5:

$$M_{U,L} = M \pm [V \sin \phi' + H \cos \phi'] \left(\frac{r^2}{c_{U,L}} \mp h \right) \ldots\ldots\ldots (34)$$

in which

$$\phi' = \tan^{-1} \left[\tan \theta + \frac{4a}{L^2}(L - 2x) \right] \ldots\ldots\ldots\ldots\ldots (35)$$

The kern points shown in Fig. 5 for the upper and lower flanges are $K_{U,L}$.

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[7] "The Second Fixed Bridge Over the Rhine at Cologne," by W. Dietz, *Zeitschrift des Vereines Deutscher Ingenieure*, 1920.

Eq. 34 might also be written:

$$M_{U,L} = (T' + M_0)_{U,L} + H\,[e'\,(f + a)_m - (y + z)]_{U,L}$$
$$\pm H\left(\frac{r^2}{c_{U,L}} \mp h\right)\cos\phi' \ldots\ldots\ldots\ldots\ldots\ldots\ldots(36)$$

The influence of the component of V in Eq. 34 is included in Eq. 35 by writing the values of $(T' + M_0)$ and $H\,[e'\,(f + a)_m - (y + z)]$ directly with respect to the kern points; or, in other words, as in the definition, by taking moments directly about the kern points. This can be done more easily graphically than analytically, however. Fig. 6 shows the method of determining graphically the value of $\frac{M_{U,L}}{H}$ at any section for a unit load at a given point. The equilibrium polygons for $-(y + z)$, $e'\,(f + a)_m$, $\frac{T'}{H}$, and $\frac{M_0}{H}$ are added graphically, and the $\frac{M_{U,L}}{H}$-intercept can then be scaled from points distant $\pm\left(\frac{r^2}{c} \mp h\right)\cos\phi'$, from points on the $(y + z)$-line directly over or under the kern points of the girder. The case given in Fig. 6 is that of a symmetrical three-span bridge, with a girder (symmetrical top and bottom) that lies on a straight grade line

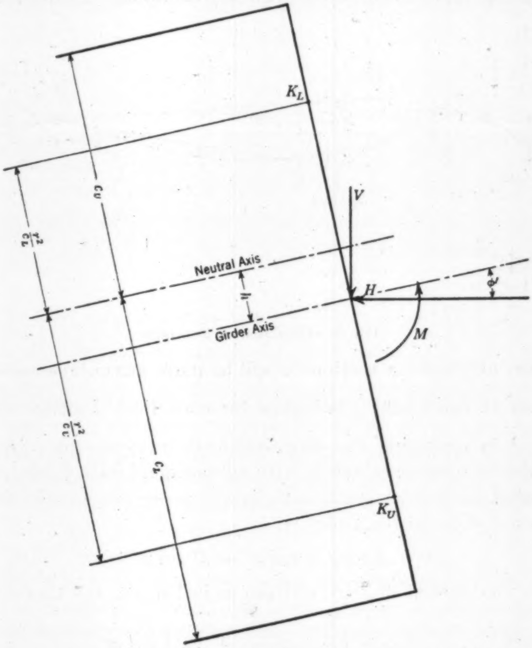

FIG. 5

in the side spans, loaded with concentrated loads in the main span and left side span. It will be noted that it is necessary to redraw only the $\frac{T'}{H}$-polygon and $\frac{M_0}{H}$-polygon for the various loads, since the other polygons remain unchanged.

Ordinarily, the shearing stresses in the girder will be small, and therefore it should be necessary to determine the shear merely at a few critical sections to test the adequacy of the web, to estimate the maximum permissible rivet and stiffener spacing, and to compute the girder reactions. To draw the influence lines for shear at these few sections it will be more advantageous to draw them directly than to construct $\frac{V}{H}$-polygons for each load position similar to the method used in obtaining the stress-moment influence-line ordinates. The latter can also be obtained directly with advantage if only a few sections are to be investigated.

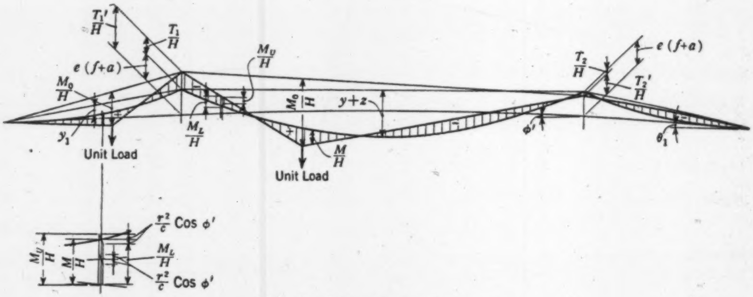
$\frac{T_1'}{H}$
$\frac{T_1}{H}$
$e\,(f+a)$
$\frac{M_0}{H}$
$\frac{M_U}{H}$
y_1
Unit Load
$\frac{M_L}{H}$
$\frac{M_0}{H}$
$y+z$
$\frac{M}{H}$
Unit Load
$\frac{T_2}{H}$
$e\,(f+a)$
$\frac{T_2'}{H}$
ϕ'
θ_1
$\frac{r^2}{c}$ Cos ϕ'
$\frac{M_U}{H}$
$\frac{M}{H}$
$\frac{M_L}{H}$
$\frac{r^2}{c}$ Cos ϕ'

FIG. 6.—EQUILIBRIUM POLYGONS

The normal shear in the girder is:

$$V_N = V \cos \phi' - H \sin \phi' \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

in which V, the vertical shear, is written as in Eq. 13, and therefore:

$$V_N = \left(\frac{T_2' - T_1'}{L} + V_0 \right) \cos \phi'$$

$$- H \left[4 \left(\frac{f + a}{L^2} \right) (L - 2\,x) - (f + a)_m \left(\frac{e_2 - e_1}{L} \right) + \tan \phi' \right] \cos \phi' \quad \ldots (38a)$$

To construct the influence line, this equation may be rewritten in the following form:

$$\frac{V_N \sec \phi'}{\left[4 \left(\frac{f + a}{L^2} \right) (L - 2\,x) - (f + a)_m \left(\frac{e_2 - e_1}{L} \right) + \tan \phi' \right]}$$

$$= \frac{\left(\frac{T_2' - T_1'}{L} + V_0 \right)}{\left[4 \left(\frac{f + a}{L^2} \right) (L - 2\,x) - (f + a)_m \left(\frac{e_2 - e_1}{L} \right) + \tan \phi' \right]} - H \quad \ldots (38b)$$

EFFECT OF MOTION OF ENDS OF GIRDER DUE TO DEFLECTION OF BRIDGE

Due, principally, to vertical deflection in the main spans, the ends of the girder will move horizontally to produce a change in H from that evaluated by the elastic theory. This change in H can be determined best by successive trials from the values of the main-span deflection calculated for the new values of H. The change in H may be written

$$\Delta H = \pm \frac{\Delta x}{D} \qquad (39a)$$

in which $\pm \Delta x$ is the entire change in horizontal length of girder.

$$\Delta H = \pm \frac{3 \, \Delta x \, E \, I}{(f + a)^2 \, D' \, L} \qquad (39b)$$

In some cases this correction in H amounts to several per cent—with corresponding percentages of correction in moment. The moment corrections will be greater for load in the main span and less for load in the side span, but generally are not great enough to affect the design, although they should be considered in analyzing the motion of the structure during erection.

EFFECT OF VARIABLE GIRDER MOMENT OF INERTIA

Although the integrations required for the determination of N and D in the H-formula (Eq. 14) may be performed for varying girder moment of inertia, as in arch analysis, the value of H is usually little affected by variation in I within the span. The values of the moments, of course, are more greatly affected by the variation in I. To account for this effect the values of T' and e' corresponding to such variation can be computed by the moment-area or a comparable method and the results used in combination with the H obtained for constant I. Where constant-depth girders are used in short spans, the moment of inertia will tend to be fairly constant since it is impracticable actually to follow the moment curve closely in selecting the girder sections; and therefore the assumption of a uniform average I within the span will usually yield sufficiently accurate results.

EFFECT OF SUSPENDER ELONGATION

Analysis by the usual procedure to consider the effect of suspender elongation indicates that this action produces a negligible change in the values of moment, shear, and deflection of the girder from those computed neglecting this effect.

UNSYMMETRICAL AND MULTIPLE SPANS

The formulas for unsymmetrical three-span or multiple-span bridges require different values of the continuity terms e' and T' from those of the symmetrical three-span bridge. The changed values of e' and T' result in modification of N and D in the H-equation and of the expressions for shear, moment, and deflection. The formulas for dead load, however, are unchanged. The form of all the equations for bridges with girders hinged at the towers remains unchanged from the form of the equations of the three-span symmetrical bridge, since the girder continuity terms e' and T' become zero for the hinged condition.

COMPUTATIONS FOR A SELF-ANCHORED SUSPENSION BRIDGE WITH A CONTINUOUS STIFFENING GIRDER

A bridge with a 20-ft roadway, two 2-ft walkways, and a lightweight floor designed for H-15 highway loading is examined. The structure is symmetrical, with a 350-ft main span and two 150-ft side spans on 5% grades. The dead-load cable sag is 35 ft, and the parabolic camber of the main-span floor and girder is 4.38 ft. The open cable is made up of nine twisted wire strands, and single 36-in. wide flange beams are used for the stiffening girder.

Span and Load Constants.—Let $L = 350$ ft; $L_1 = 150$ ft; $f = 35$ ft; $n = 0.10$; $a = 4.38$ ft; and $n' = 0.1125$. The dead loads are: $w = 1{,}330$ lb per ft, and $w_1 = 1{,}430$ lb per ft; and the live load is $p = 500$ lb per ft (all loads are for one girder). For the cable, $E_c = 25{,}000{,}000$ lb per sq in.; and, for the girder, $E = 29{,}000{,}000$ lb per sq in. The dead load (see Eq. 1b) is: $H_w = \frac{1{,}330 \times 350^2}{8(35 + 4.38)}$ $= 517$ kips; the side-span cable sag (Eq. 4a) is $f_1 = \frac{1{,}430 \times 150^2}{1{,}330 \times 350^2} \times 39.38$ $= 7.82$ ft; $L_s = 743$ ft; $L_s' = 651$ ft; $I = 15{,}625$ in.^{4}; $I_1 = 21{,}175$ in.^{4}; the area of the cable is $A_c = 12.1$ sq in.; of the main-span girder, $A_g = 70.3$ sq in.; of the side-span girder, $A_{g1} = 86.1$ sq in.; $i = 0.738$; $r = 0.429$; $v' = 0.199$; $e = 0.585$; and $D' = 0.444$.

Live Load—Spans Fully Loaded.—With the main span fully loaded (Eq. 24b) $H = \frac{500 \times 350}{0.444 \times 0.1125}\left(0.20 - \frac{0.585}{4}\right) = 189$ kips; and $T_1' = \frac{-500 \times 350^2}{4 \times 3.632}$ $= -4{,}215$ kip-ft. By Eq. 32a: $\frac{8 \times 29{,}000 \times 15{,}625}{350^2 \times 144} \times \Delta f = -4{,}215 + \frac{5}{48}$ $\times 500\,(350)^2 + 189 \times 39.38\,(0.585 - 0.833)$; or $\Delta f = +1.58$ ft. The live load camber equals $a - \Delta f = 2.80$ ft.

To obtain the correction in H due to the movement of the ends of the girder corresponding to the change in camber: $\Delta L' = \frac{8}{3}(4.38^2 - 2.80^2)\frac{1}{350} = +0.087$ as a parabola; and $\frac{16 \times 29{,}000 \times 21{,}175}{150^2 \times 144}\Delta f_1 = -4{,}215 + 189\,(0.585 \times 39.38$ $- 1.667 \times 7.82)$. That is, $\Delta f_1 = -0.77$ ft; $\Delta L' = \frac{8}{3}(0.77)^2 \times \frac{2}{150} = -0.020$ ft; total $\Delta L' = +0.087 - 0.020 = +0.067 = \Delta L$ (very nearly); and (Eq. 39b) $\Delta H = \frac{+3 \times 0.067 \times 29{,}000 \times 15{,}625}{39.38^2 \times 350 \times 0.444 \times 144} = 2.6$ kips $= 1.4\%$. Furthermore, $H' = 188.5 + 2.5 = 191$ kips; $\Delta f' = +1.48$ ft; $\Delta f_1' = -0.76$ ft; the maximum stress in the side-span cable is $(517 + 191)\,1.147 = 813$ kips; the maximum direct stress in the girder is $(517 + 191)\,1.001 = 709$ kips; and (Eq. 12) the negative moment at the center of side span is: $M = -2{,}108 + 191$ $\times \left(\frac{0.585}{2} \times 39.38 - 7.82\right) = -1{,}400$ kip-ft. At the kern point: $M = -1{,}400$

-191×1.04 ft = $-1{,}600$ kip-ft. It will be noted that the effect, on the moment, of the correction in H is not important.

With the side span fully loaded (Eq. 24*b*): $H = \dfrac{0.738 \times 0.429^3}{0.444 \times 0.109} \times \left(\dfrac{0.199}{5} - \dfrac{0.585}{8}\right) 500 \times 350 = -7.0$ kips; $T_2' = \dfrac{+0.738 \times 0.429^3}{3.632 \times 1.632} \times \dfrac{500 \times 350^2}{4} = +150$ kip-ft; and $T_1' = -395$ kip-ft.

The positive moment at the center of the side span is $M = -\dfrac{395}{2} + \dfrac{500 \times 150^2}{8} - 7.0\left(\dfrac{0.585}{2} \times 39.38 - 7.82 \mp 1.04\right) = +1{,}190$ kip-ft; and, with far side span loaded: $M = \dfrac{+150}{2} - 7.0\left(\dfrac{0.585}{2} \times 39.38 - 7.82 \mp 1.04\right) = +55$ kip-ft.

Influence Line Solution for Live-Load Girder Flange Stress.—The influence-line values for H are as follows: With a load in the main span—

$$H = \frac{1}{0.444 \times 0.1125}\left[B(k) - \frac{3}{2} \times 0.585\, k\,(1 - k)\right] \ldots\ldots (40a)$$

and with a load in the side span—

$$H = \frac{0.738\,(0.429)^2}{0.444 \times 0.1125}\left[0.199\, B(k_1) - \frac{0.585}{2} k_1 (1 - k_1^2)\right] \ldots (40b)$$

TABLE 1.—COMPUTATIONS FOR INFLUENCE LINES

	(a) H				(b) M_0 AND T'					
k and k'	Y	Z	$Y - Z$	H	M_0	$\frac{M_0}{H}$	T_1'	$\frac{T_1'}{H}$	T_2'	$\frac{T_2'}{H}$
MAIN-SPAN COMPUTATIONS										
0.1	0.0981	0.0790	0.0191	0.382	31.5	82.5	−20.7	−54.1	− 5.3	−13.9
0.2	0.1856	0.1404	0.0452	0.905	56.0	61.9	−33.4	−36.9	−12.8	−14.2
0.3	0.2541	0.1843	0.0698	1.397	73.5	52.6	−39.3	−28.2	−21.3	−15.3
0.4	0.2976	0.2106	0.0870	1.742	84.0	48.2	−39.9	−22.9	−29.5	−16.9
0.5	0.3125	0.2194	0.0931	1.864	87.5	47.0	−36.1	−19.4	−36.1	−19.4
SIDE-SPAN COMPUTATIONS										
0.2	0.0369	0.0562	−0.0193	−0.0525	24.0	−457	− 4.03	76.8	+1.53	−29.1
0.4	0.0591	0.0983	−0.0392	−0.1060	36.0	−340	− 7.05	66.5	+2.68	−25.3
0.6	0.0591	0.1123	−0.0532	−0.1440	36.0	−250	− 8.05	55.9	+3.06	−21.2
0.8	0.0369	0.0842	−0.0473	−0.1280	24.0	−187.5	− 6.05	47.2	+2.30	−18.0

The computations are given in Table 1. To simplify typography in Table 1(*a*), Eqs. 40 may be written

$$H = X\,(Y - Z) \ldots\ldots\ldots\ldots (41)$$

the proper quantities being substituted from Eq. 40*a* in the case of main-span computations and Eq. 40*b* in the case of side-span computations.

As previously explained the value of $e' (f + a) m - (y + z)$ is laid out from the center line of the girder on the elevation of the bridge drawn to scale. The values of $\frac{T' + M_0}{H}$ are then plotted to the same scale for each unit load, and the intercepts between these two lines at points along the span are measured from points distant $\pm \frac{r^2}{c} \cos \phi'$ from the $(y + z)$-line. The product of these intercepts by the values of H for the respective loads are flange-stress moments for live load. Adding these moments the maximum total positive and negative flange-stress moments are obtained. The maximum live-load flange-stress moments are:

Side Span		Main Span	
Point	Stress moment (kip-ft)	Point	Stress moment (kip-ft)
0	0	1.0 (tower)	−1,190
0.2	−1,120	0.1	− 680
0.4	−1,560	0.2	+ 930
0.6	−1,525	0.3	+1,045
0.8	−1,040	0.4	+ 910
		0.5	+ 735

The total live-load flange stresses for both direct and bending stress are then obtained by dividing the foregoing moments by the section modulus of the symmetrical girder at the several points. The dead-load stress, which is direct, is added to the live-load stress to obtain the total compressive flange stress for dead and live loads.

Live-Load Shear.—The maximum live-load shears at the ends of the main and side spans may be obtained by constructing influence diagrams that consist of graphically combining the influence lines for: V_0; $\left(\frac{T_2' - T_1'}{L}\right)$; and $-H\left[4\frac{(f + a)}{L^2}(L - 2x) - (f + a)_m\left(\frac{e_2 - e_1}{L}\right) + \tan \phi'\right]$. In obtaining vertical shear, $\tan \phi'$ under the bracket is omitted. Proceeding in this manner the following values are obtained:

Span	Vertical shear (kips)	Normal shear (kips)
End of main span.	+36.2, −28.6	+36.8, −36.0
Tower end of side span.	−44.1, +41.0	−43.8, +31.8
Free end of side span.	+36.6, −39.3	+37.4, −48.1

The end shears were all determined for full-span loading except for the main-span shear from main-span loading and normal shear at the tower end of the side span for main-span loading, which was slightly increased for partial main-span loading. The influence diagram for shear at the end of the main span is shown in Fig. 7.

The lateral wind stresses in the stiffening girders for spans such as these may be neglected since they amount to less than 25% of the total dead-load plus live-load stresses. In general, the wind stresses in self-anchored suspension bridges are relatively unimportant, since span lengths are moderate, elastic theory design of the girders requires that they be comparatively stiff, and the girders must be designed to carry dead and live load thrust in addition to moment.

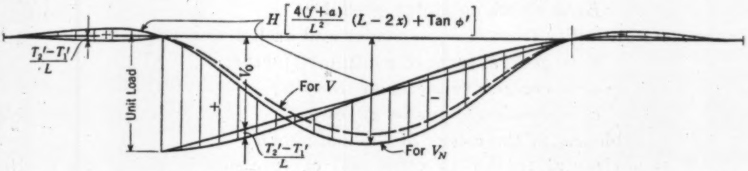

FIG. 7.—INFLUENCE DIAGRAM FOR SHEAR AT END OF MAIN SPAN

ACKNOWLEDGMENT

This paper was submitted by the author, in 1940, as a thesis to the graduate faculty of Rutgers University at New Brunswick, N. J., in partial fulfilment of the requirements for the degree of Civil Engineer.

CONCLUSION

The theory of the self-anchored suspension bridge has been presented in the foregoing in such a manner as to enable the practicing engineer readily to apply the formulas derived to the design of that type of structure. It is evident that the self-anchored suspension bridge has many advantages over other types in certain locations; and, with the theory for its design conveniently at hand, the self-anchored suspension bridge of moderate span should become widely adopted, where conditions justify its use.

APPENDIX

NOTATION

For the most part the notation in this paper (see Fig. 1) is the same as that adopted by D. B. Steinman.[5,8] An effort is made, furthermore, to conform substantially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials compiled by a Committee of the American Standards Association[9] with Society representation, and approved by the Association in 1932.

Throughout, a subscript 1 is used to distinguish side-span symbols from main-span symbols; subscript U = "upper kern point" and L = "lower kern point"; and w = "due to dead load."

A = area of cross section, the subscript c denoting "cable" and g denoting "stiffening girder";

a = center camber of stiffening girder;

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[8] "A Generalized Deflection Theory for Suspension Bridges," by D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1135.

[9] ASA—Z10a—1932.

c = distance from the extreme fiber to the neutral axis;

D = general form of the denominator of the H-equation (Eqs. 14 and 18):

D' = working form (see Eq. 21a);

E = modulus of elasticity of girder:

E_c = elastic modulus of cable;

e = continuity factor for suspender and arch action (Eq. 10):

e' = general form of continuity factor;

e_1 = continuity factor at tower 1;

e_2 = continuity factor at tower 2;

f = cable sag at the center of the main span;

H = horizontal tension in cable and compression in girder due to live load p:

H_w = horizontal tension H due to dead load;

I = moment of inertia of stiffening girder;

i = ratio of moments of inertia, main span to side span, $\frac{I}{I_1}$;

K = a kern point;

k = ratio of span length:

$k\,L$ = a span segment;

L = main-span length:

L_s = cable length function;

L_s' = girder length function—functions defined by Eq. 17;

M = resultant bending moment at any section x of the stiffening girder:

M' = the moment in the members of the system with the cable cut;

M_w' = dead-load moment carried by the girder, as a parabolic arch;

M_0 = simple-beam bending moment due to a live load p;

M_D = simple-beam bending moment due to dead load w;

m = bending moment in the members of the system due to a unit horizontal component of cable stress;

N = numerator of H-equation (Eq. 14);

n = ratio of cable sag to span length, $\frac{f}{L}$:

$n' = \frac{f + a}{L}$;

P = concentrated load;

p = live load per unit length:

p_g = part of the live load taken by the girder in moment;

r = ratio of length of side span to main span, $\frac{L_1}{L}$;

S = direct stress in the various members of the system:

S' = direct stress in the various members of the system with the cable cut;

s = length of cable:

ds = increment of cable length;

T = bending moment at any section x due to continuity:
  T_1 = moment at tower 1;
  T_2 = moment at tower 2;
  T' = moment effect of continuity on the independent girder (cable removed); general form;
  $T_1' = T'$ at tower 1;
  $T_2' = T'$ at tower 2;
t = temperature;
u = direct stress in the members of the system, corresponding to a horizontal component of stress equal to unity in the cable and girder;
V = total vertical shear at any section x of the stiffening girder, due to live load p:
  V_N = normal shear;
  V_0 = simple-beam vertical shear due to live load;
$v = \dfrac{f_1}{f}$:
  $v' = \dfrac{f_1 + a_1}{f + a}$;
w = uniformly distributed dead load:
  w_a = part of the dead load carried by the stiffening girder as an "arch";
X = a substitution factor in Eq. 41;
x = abscissas in main span measured from the tower:
  x_1 = abscissas in the side span measured from the free end of the side span;
Y = a substitution factor in Eq. 41;
y = cable ordinate;
Z = substitution factor in Eq. 41;
z = stiffening girder ordinate;
α = slope of cable chord in any span;
Δ = increment:
  Δf = midspan deflections;
  Δy_t = vertical deflection of the girder at the towers due to temperature change;
  $\Delta\phi$ = angular deflection of cable;
  $\Delta\phi'$ = angular deflection of girder;
  Δx = change in horizontal length of girder;
η = deflection of truss at any section x;
θ = slope of the girder chord in any span;
ϕ = slope of cable at any point:
  ϕ' = slope of the girder at any point.

DISCUSSION

A. J. MEEHAN,[10] M. AM. SOC. C. E. (by letter).—As much as the suspension bridge has occupied the spotlight during recent years, the self-anchored type has not enjoyed great prominence. The complexity of its analysis, which has existed heretofore, has been one of a number of contributory factors responsible for its infrequent selection. The long-awaited "Simplified Theory of the Self-Anchored Suspension Bridge" is now at the disposal of designers, for which warm thanks are due to Mr. Gronquist.

Possibly a discussion on this paper should be confined strictly to theory. However, taking a liberal viewpoint of "theory," it may be regarded as a cog in the production of something concrete or tangible. As such, the writer submits his generalizations, prompted by the experiences of his association in the rather complete preparation of preliminary designs for two separate river crossings by self-anchored suspension bridges. These preliminaries each consisted of a main span of 480 ft. Loaded side spans on one were 165 ft each, and flanking approach spans gave an aggregate length of 1,128 ft for this project. For the other layout, side spans of 150 ft were planned, to which were added a number of approach spans, resulting in a total bridge length of 1,590 ft. Each project provided a 26-ft roadway and two cantilevered sidewalks of 4-ft clear width.

In the self-anchored group of suspension bridges are those having a combination of counterweighted anchorage pier and self-anchored side span. At the sites previously mentioned, high water conditions prohibited this construction. During flood stages, the end piers, acting as gravity anchorages, would have been submerged to such an extent that the resulting buoyancy would have seriously impaired their usefulness.

Column action may require excessive sections in the side-span stiffening girder or truss to resist the horizontal component of the cable pull. In some stiffening trusses, the lower chord takes the horizontal cable reaction. In such cases, the top chord details may become awkward in order that a direct cable connection may be made with the bottom chord.

To transmit end-span loads ideally (including lateral forces, such as wind and earthquake), it is probably more important in this type, than in any other kind of structure, to adopt a compact and effective framing arrangement. Such a requirement gives stiffening girders the advantage of selection over stiffening trusses, due to a more unified action. It is probable that a steel floor might serve as the element to carry transverse shear by web action between stiffening girders acting as flanges.

In the projects with which the writer was connected, although trusses were considered, stiffening girders were given the "decision." The stiffening girders were cambered nearly sufficient to carry the entire dead-load moment. In other words, the stiffening-girder dead-load thrust practically coincided with the neutral axis. An effort was made to carry the remainder of the moment in the floor system. Difficulty was encountered in providing enough rivets and transmitting the loading from one floor panel to the next. Excessive material was required for shear development between the floor system and the concrete slab. However, the studies included steel-floor adaptation and also an attempt to relieve some of the stiffening-girder thrust with the lateral system.

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[10] Senior Bridge Engr., State Dept. of Public Works, Div. of Highways, Bridge Dept., Sacramento, Calif.

Except for shorter spans, the satisfactory solution of the end connection, of either parallel wire cables or wire-rope strands, to the stiffening member can become as involved as any problem in structural steel detailing. Of course, the cables or chains could be connected to points other than at the extreme end of the stiffening girder, but this adds moment and shear due to the upward component of the cable reaction.

This type of bridge is obviously penalized from a cost standpoint by additional dead load due to details whose sole purpose is to counteract eccentricity or whose function is to restrain buckling tendencies of the stiffening girders acting as columns. Of further economic concern is the handicap involved in the provision for materials required for erection purposes only, which may include temporary connections or auxiliary and jacking struts or increases in sections of the permanent members due to the excess of erection stresses over normal. Some savings might be effected by scheduling the erection procedure so that use would be made in a temporary position—during the earlier stages of erection—of some of the materials required for other positions in the finished structure.

Eyebar chains are superior to cables because of easier connections at the stiffening member ends and simpler erection connections. At present (1941) eyebar production has been curtailed due to defense needs, so that all desirable sizes are not available; but at no time have eyebars been competitive bid items since the acceptable kind have been manufactured by only one company.

Falsework costs are greater in proportion for longer spans using trusses than in shorter spans using girders. Falsework for self-anchored suspension spans must remain in place not only during steel erection, but possibly until the concrete slab has been completed. This long period tends to increase the charge for the contractor's plant, which the owner pays indirectly. In streams where floods may develop during steel erection, the hazard is obvious.

Girders are generally more economical than trusses for the stiffening members in cases of reasonable span lengths and in ordinary roadways. To date, self-anchored spans have generally been short enough to permit girders to be used in lieu of stiffening trusses. However, recent tests have revealed that, from the standpoint of aerodynamic stability, holes should be provided in the tops and bottoms of girder webs up to 25% of the web area as one means of effecting such stability. Perhaps if such an area of open space is desirable through the sides of stiffening girders, one's preference may turn to stiffening trusses. It is recognized that the aerodynamic phenomenon does not demand attention when the span-to-width ratios are low as compared to the range of tried and proved structures.

Since simplification of the theory is the author's aim, with the ultimate objective of popularizing the type, it may be well to recognize some of the further obstacles to such progress. At present there is no accepted design specification for self-anchored suspension bridges. As a consequence, such development has stagnated. What, for instance, would be the acceptable live loading for a structure financed under such circumstances? Invariably what one gets when trying to secure an advance opinion (from an agency disbursing public bridge funds—of what, for instance, would be acceptable as to live loading for a structure financed under such circumstances) is some arbitrary answer ranging up to 50% more than factual conditions may permit. More explicitly, the required value might be 60 lb per sq ft live load instead of a submitted 40 lb per sq ft. Glenn B. Woodruff and Norman C. Raab,[11] Members, Am. Soc. C. E., have demonstrated the ultraconservativeness of such a requirement. They indicate that the maximum possible vehicular live load on a suspension bridge will be obtained with traffic at a standstill (hence, without impact), and with cars placed bumper to bumper, which condition produces a live load of 35 lb per sq ft. When the load is moving slowly, it may be only 25 lb per sq ft (impact to be added).

Another case in point concerns the cable. The effect of live loading per square foot may expand the safety factor tremendously. To produce the maximum stress in the main cable, the full length of the bridge is loaded. If a safety factor of three were required for the breaking strength of the cable when stressed under full dead and live load, and if the ratio of dead-to-live load were 3 to 1, then the safety factor for the live load would attain the unreasonable value of 9.

The unsatisfactory situation with respect to secret production details of heat-treated eyebars and of processing cable wire is alike, inasmuch as the designer openly admits his dependence upon the manufacturer. To protect himself, the designer resorts to specifying minimum acceptable materials and conditions. Although secretive, the manufacturers are nevertheless cooperative.

The variations in wire-rope strand construction make it difficult to obtain an accurate forecast of the resulting modulus of elasticity after the pre-stressing operation. However, if pre-stressing is adopted, one should require that the procedure develop a stress at least equal to the maximum design stress. The fullest advantage of pre-stressing will result if, as an added precautionary measure, it is specified that the ropes are to be socketed before pre-stressing.

It seems possible that, for certain locations, skewed suspension bridges are required, although they are never seen. In view of the present National Defense demands and the attendant difficulties in securing adequate quantities of structural steel, it seems likely that the concrete tower might find application in suspension-bridge design, plastic flow of concrete and other difficulties to the contrary notwithstanding.

An effort has been made herein to summarize some of the pitfalls that the enthusiast for self-anchored suspension bridges will encounter before his dream bridge can begin to function. Personal observation of the three self-anchored suspension bridges in Pittsburgh, Pa.—at Sixth, Seventh, and Ninth streets—

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[11] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 614.

has demonstrated amply the practicality of such structures to the writer. It is sincerely hoped that a newly rejuvenated era of self-anchored suspension bridges will be the response to the stimulating influence of Mr. Gronquist's illuminating contribution.

C. B. McCullough,[12] M. Am. Soc. C. E. (by letter).—Little can be added to Mr. Gronquist's most able treatment of the theory of self-anchored suspension spans. His presentation is complete yet concise, and the subject matter should prove of distinct value to every designing bridge engineer.

The self-anchored type presents greater possibilities of economic utilization than are generally realized. This statement finds corroboration in the results of certain studies of short-span highway structures made by the Oregon State Highway Department in 1937 and 1938. The first phase of this latter investigation comprehended the development of designs and estimates for fifteen two-lane highway structures, each having a total length of approximately 750 ft, in groups, as follows:

Group	Type
A	Simple truss spans
B	Cantilever spans
C	Externally-anchored suspension spans (truss stiffened)
D	Self-anchored suspension spans (girder stiffened)
E	Self-anchored suspension spans (truss stiffened)

The various span lengths are given in Table 2. In the self-anchored girder-stiffened suspension spans (Group *D*) the stiffening girders were rigidly anchored to the deck by means of perforated T-bars attached to the girder webs with the deck reinforcing passing through the perforations. The deck was also anchored to the stringers by means of Z-bar connections spaced approximately 4 ft center to center. The girders and stringers, acting as a composite unit, were designed to resist the stiffening frame moment. This arrangement affords considerable economy of material and accounts, in part, for the relatively low cost of this type.

Based upon unit prices prevalent at that time (1937 and 1938), the cost per lineal foot for the various design types and span arrangement was as indicated in Fig. 8. (The general trend beyond data points is shown by broken lines.) The comparisons indicated in Fig. 8 were based upon structures having the same total length but with varying side-span ratios. Since some of the span arrangements selected were obviously uneconomical, the investigation was extended to include a comparison of structures of varying total lengths but constant side-span ratio, as indicated in Fig. 9.

TABLE 2.—Study of Short-Span Highway Bridges in Oregon

(Span Lengths, in Feet)

No.	Group *A*		Group *B*		Groups *C*, *D*, and *E*	
	Center span	2 side spans	Center span	2 side spans	Center span	2 side spans
1	250	250	300	225	300	225
2	300	225	350	200	375	187.5
3	350	200	400	175	400	175

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[12] Asst. Chf. Engr., Oregon State Highway Dept., Salem, Ore.

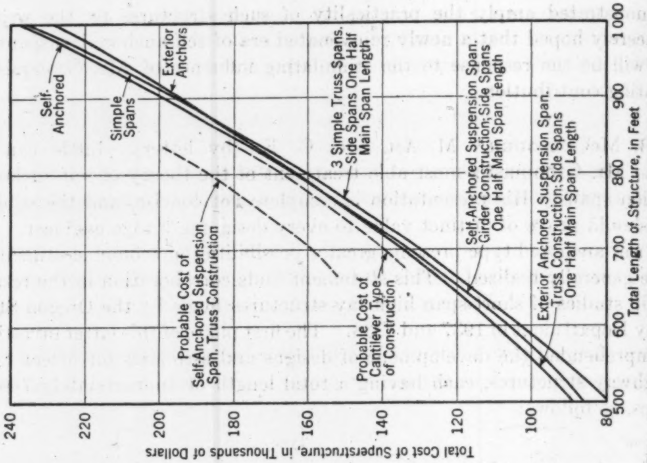

FIG. 9.—TOTAL SUPERSTRUCTURE COST FOR VARIOUS TYPES OF BRIDGES (SIDE-SPAN RATIO = 0.50; 27-FT ROADWAY; AND TWO 5-FT WALKS)

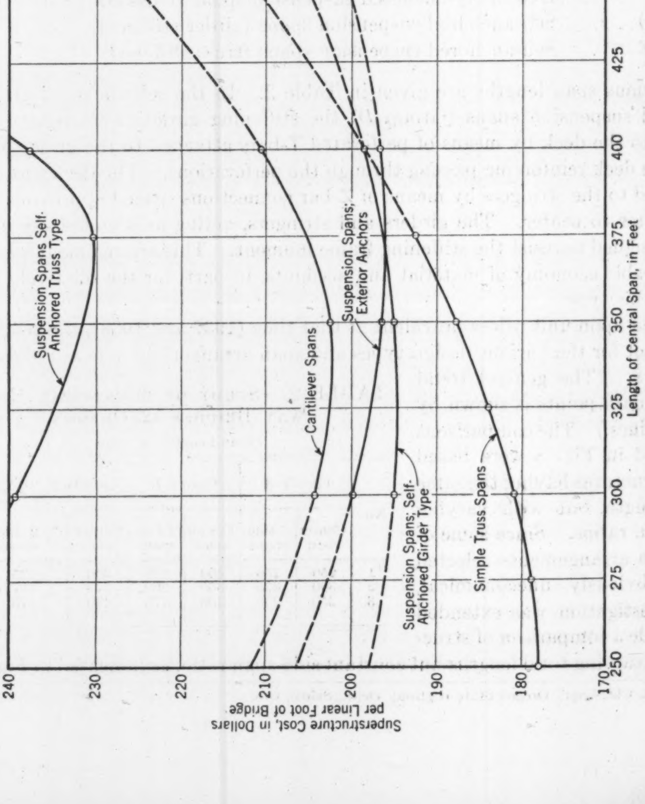

FIG. 8.—COST COMPARISONS PER LINEAL FOOT FOR THE VARIOUS COMPARATIVE TYPES

Since the cantilever type and the self-anchored suspension type using truss construction indicated no economy in the first comparison, they were not included in the second. The probable cost trends for these latter types through the central range of the diagram (Fig. 9) are indicated by dotted lines.

An inspection of Figs. 8 and 9 indicates that the self-anchored suspension design, utilizing the girder system, is quite likely to show economy in first cost for three-span arrangements in which the central span exceeds 400 ft. This is only true when the central span is the controlling factor. If it is possible to cut the central opening, the simple truss type will show economy in first cost, as indicated in Fig. 10.

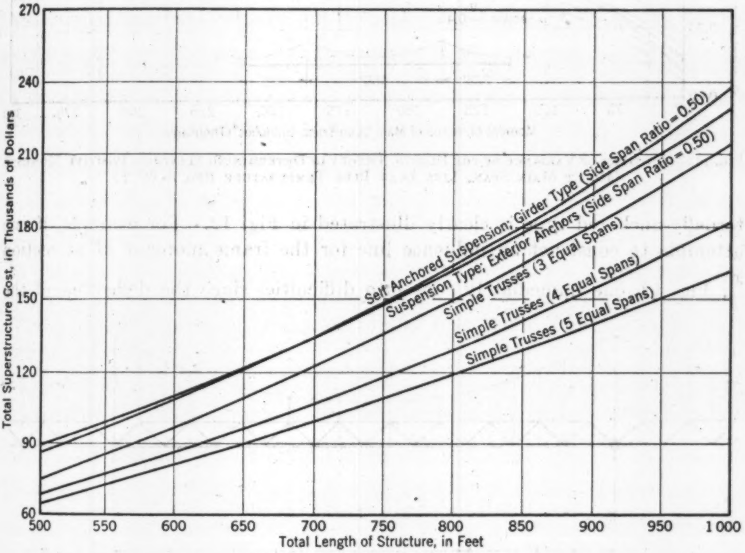

FIG. 10.—COMPARATIVE SUPERSTRUCTURE COST, SUSPENSION TYPES VERSUS SIMPLE TRUSSES

A most important property of the self-anchored suspension type is the fact that the effect of the deflection term is canceled in the moment equation so that a simplified procedure is possible in the analysis. This is nicely developed by the author in his discussion of live loads.

The effect of this deflection term is considerable even for short spans, as will be seen from an inspection of Fig. 11, which indicates the degree of variance between the exact method and the so-called elastic theory when applied to a 225–450–225 ft, externally-anchored, suspension design, for varying values of the moment of inertia of the stiffening frame. For the more flexible frames the divergence is of material magnitude and this fact (with the further fact that the degree of variance is not constant but varies from point to point along the span) renders the approximate or elastic theory of little value for externally-anchored structures.

Another important advantage in the method of analysis developed by the author for self-anchored spans is the fact that it is possible to develop and utilize influence diagrams. That such diagrams cannot be used for the externally-anchored type is clearly illustrated in Fig. 12. For example, if one attempts to construct an influence line for the frame moment M at section $\frac{x}{l}$, Fig. 12, one immediately runs into difficulties since the deflection of the cable fluctuates with the magnitude and position of the load. In other words, as a unit load moves across the span from right to left, the increasing cable deflection shifts the node point n so that the negative-moment area is a continuously fluctuating quantity dependent upon the position of the load.

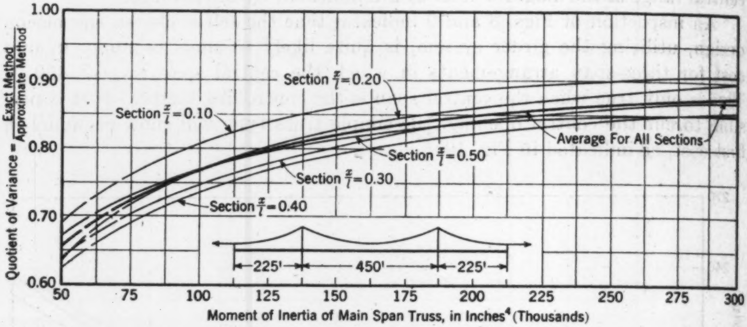

FIG. 11.—QUOTIENT OF VARIANCE OF THE ELASTIC THEORY IN DETERMINING MAXIMUM POSITIVE MOMENTS (450-FT MAIN SPAN; LIVE LOAD PLUS TEMPERATURE RISE = 60° F)

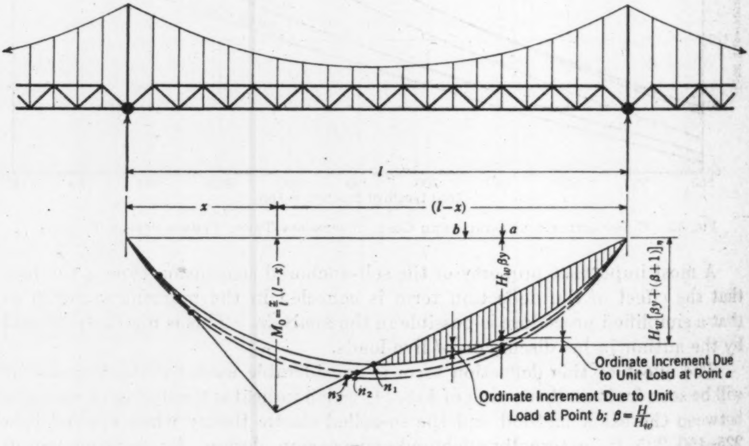
l
x
$(l-x)$
b
a
$H_w\beta y$
$M_0=\frac{x}{l}(l-x)$
$H_w[\beta y+(\beta+1)]_n$
n_3
n_2
n_1
Ordinate Increment Due to Unit Load at Point a
Ordinate Increment Due to Unit Load at Point b; $\beta=\frac{H}{H_w}$

FIG. 12

In summary, therefore, the self-anchored suspension type not only indicates distinct possibilities for economic utilization but also lends itself to certain simplified design procedures which are not applicable to externally-anchored designs. The author is to be congratulated for presenting, in a most readable manner, a method of analysis that should be of great benefit to the designer.

Jaroslav J. Polivka,[13] M. Am. Soc. C. E. (by letter).—A valuable contribution to the analysis of the self-anchored suspension bridge is contained in this paper. The arch-like action of the cambered girder may reduce cable and girder stresses considerably, and the economic advantage of such a structure is evident. The economy increases, naturally, with the amount of the vertical camber. Usually the vertical camber is limited by the required grade of the girder, and is relatively small in such a characteristic type of a suspension bridge as is shown in Fig. 1. In this case assumption (3) is justified—namely, that the average moment of inertia of the stiffening girder throughout each span is constant.

This assumption involves the approximation

$$\frac{\cos^2 \alpha}{\cos^3 \phi} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

in which α is the angle of the tangent at the ends and ϕ is the average angle of the structural sections of the arched girder, both with respect to the chord. The condition expressed by Eq. 42 results from the differential equation of the elastic curve,

$$E\,I \cos^3 \phi \frac{d^2\eta}{dx^2} = -M_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43a)$$

and from equations of continuity having the form

$$\frac{\cos^2 \alpha_1}{\cos^3 \phi_1}\theta_1 + \frac{\cos^2 \alpha}{\cos^3 \phi}\theta = 0 \ldots\ldots\ldots\ldots\ldots\ldots (43b)$$

Eq. 11, representing the continuity of the independent girder, is correct for the varying moment of inertia of the cambered girder following the relationship,

$$I = \frac{I_0}{\cos \phi} \ldots\ldots (44)$$

in which I_0 is the moment of inertia of the cambered girder at the crown.

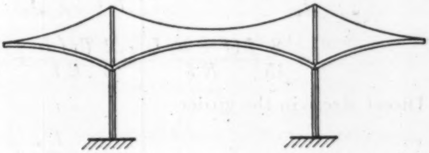

Fig. 13

The theory can be extended and applied for structures as shown in principle by Fig. 13, which represents a new type of structure of great economy, especially for large-span buildings, such as hangars, assembly plants of plane factories, etc.

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[13] Research Associate, Univ. of California, and Cons. Engr., Berkeley, Calif.

WILLIAM BERTWELL,[14] ASSOC. M. AM. SOC. C. E. (by letter).—Although this paper is interesting and valuable, the method of design presented involves the use of fairly complicated formulas. The reduction of computations to substitution in formulas is often useful and desirable, but the writer believes that in this instance a more flexible procedure is preferable.

A clear conception of the action of the structure is obtained through consideration of the geometry of its deflections. If the cable is cut at one connection to the stiffening girder, a unit horizontal tension applied to the cable, and a unit horizontal compression applied to the girder, then the vertical deflections of the girder, divided by the relative horizontal movement of the girder and cable at the cut, yield the influence line for the horizontal component of cable stress. Although this well-known principle is expressed by Eq. 14, the writer believes that if the indicated steps are performed independently the possibility of overlooking a structural peculiarity of a given bridge is minimized, and the work is often simpler. The use of a truss or a girder with varying moment of inertia then results in no more difficulty than in an ordinary continuous structure without a cable.

It also appears that, if the points of actual suspension are chosen for the computation of influence ordinates, the effects of distributed loads might well be determined from those ordinates, rather than by integrations, which assume a continuous supporting medium.

Adopting the author's numerical example, the work would proceed about as follows in determining the influence line for H: The vertical load on the girder is $\frac{8\,(f + a)}{L^2} = -\,0.00278$ lb per ft in the side spans, and $-\,0.002571$ lb per ft in the main span. The moment at the tower may be determined by moment distribution, for example, and is 23.041 lb-ft.

The relative horizontal movement of the cut ends is:
Direct stress in the cable—

$$\frac{L_s}{A_c\,E_c}.$$

Bending in the cable—

$$\int_0^L (y + z + T_1)\,y\,\frac{dx}{E\,I} + 2\int_0^{L_1}\left(y_1 + T_1\,\frac{x_1}{L_1}\right)y_1\,\frac{dx_1}{E\,I_1}$$

$$= \frac{8}{15}\,\frac{f\,(f + a)\,L}{E\,I} + \frac{2}{3}\,\frac{T_1\,f\,L}{E\,I} + \frac{16}{15}\,\frac{f_1^2\,L_1}{E\,I_1} + \frac{2}{3}\,\frac{f_1\,L_1\,T_1}{E\,I_1}.$$

Direct stress in the girder—

$$\frac{L'_s}{A_g\,E}.$$

Bending in the girder—

$$\int_0^L (y + z + T_1)\,z\,\frac{dx}{E\,I} = \frac{8}{15}\,\frac{(f + a)\,a\,L}{E\,I} + \frac{2}{3}\,\frac{T_1\,a\,L}{E\,I}.$$

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[14] Structural Engr., Public Roads Administration, San Francisco, Calif.

Totaling these terms:

$$\frac{L_s}{A_c\,E_c} + \frac{L'_s}{A_g\,E} + \frac{8}{15}\,\frac{(f + a)^2\,L}{E\,I} + \frac{2}{3}\,T_1\,\frac{(f + a)\,L}{E\,I} + \frac{16}{15}\,\frac{f^2_1\,L_1}{E\,I_1} + \frac{2}{3}\,\frac{f_1\,L_1\,T_1}{E\,I_1}$$

$$= \frac{144}{29 \times 10^6}\,(0.49465 + 0.05890 + 18.52200 - 13.54811$$

$$+ 0.46207 - 0.85091) = 5.1386\,\frac{144}{29 \times 10^6}.$$

TABLE 3.—INFLUENCE ORDINATES FOR *H*

Description	Fraction of Side Span				Fraction of Main Span				
	0.2	0.4	0.6	0.8	0.1	0.2	0.3	0.4	0.5
Vertical deflections[a]	−0.2694	−0.5467	−0.7426	−0.6611	1.9656	4.6469	7.1795	8.9460	9.5761
Influence Ordinate for *H*..	−0.0524	−0.1064	−0.1445	−0.1287	0.3825	0.9043	1.3972	1.7410	1.8636

[a] Multiplied by $\frac{29 \times 10^6}{144}$.

The vertical deflections of the girder may be computed by any method the designer prefers. Table 3 shows the vertical deflections $\left(\text{multiplied by } \frac{29 \times 10^6}{144}\right)$ of the girder, and the influence ordinates for *H*, which are obtained by dividing by 5.1386.

It is not clear to the writer what the author includes in his correction of *H*. He states (see heading "Effect of Motion of Ends of Girder Due to Deflection of Bridge") that Δx is the "entire change in horizontal length of girder." The computation he indicates in his numerical example appears to bear out this definition. However, the writer believes that if this simple interpretation is intended the author has overlooked the fact that the elastic theory determines *H* by computing, among other values, the change in horizontal length of girder. The error in the elastic theory arises from the fact that deflections change the arms by which angle changes are multiplied to obtain horizontal movements. This error is greatest for the condition of a heavy load in the main span only, which results in deflections that are large compared to the camber of the girder and the sag of the cable. The error will be the difference between the change in horizontal length calculated by the elastic theory, and that which actually occurs. If this correction is made, it should not be limited to consideration of the girder only, as the cable contributes appreciably, particularly in side spans where the sag is small. Using the author's example, for the case of a 500-lb uniform load in the main span only, the value of *H* was found to be

TABLE 4.—CHANGES IN HORIZONTAL LENGTH CAUSED BY BENDING

Member	Elastic Theory		Actual	
	Main span	Side spans	Main span	Side spans
Cable...	−0.8240	+0.4096	−0.8288	+0.3237
Girder..	+0.1030	0	+0.0865	−0.0210

188,210 lb; the deflection at the center line of the main span was 1.5847 ft; the deflection at the center line of the side spans was − 0.769 ft; and the changes in horizontal length were as shown in Table 4.

Both the cable and girder are actually shorter than is indicated by the elastic theory. (The changes in horizontal length caused by direct stress, which are in addition to those shown in Table 4, are a lengthening of 0.4623 ft for the cable and a shortening of 0.0551 ft for the girder.) The correction in H will be: $\frac{0.0907 - 0.0375}{5.1386} \times \frac{29 \times 10^6}{144} = 2{,}080$ lb. Although this value is not much different from that determined by the author, it would appear that this is a coincidence, and not a result to be expected generally.

A. A. Eremin,[15] Assoc. M. Am. Soc. C. E. (by letter).—A simplified method of analyzing stresses in a self-anchored suspension bridge is presented in this paper. The computations serve to illustrate its simplicity.

It may be of interest to determine the error that may be introduced due to the simplifying assumptions adopted. The author assumed that the moment of inertia of a stiffening girder is constant for the entire length of the girder in each span. Prof. G. G. Krivoshein computed the stresses in a self-anchored suspension bridge with a continuous girder, assuming a constant moment of inertia in the girders and a varying moment of inertia increasing toward the supports. He found that the error due to neglecting the variation of moment of inertia may be more than 15%, and he stated that the method with the constant moment of inertia cannot generally yield suitable results.[16]

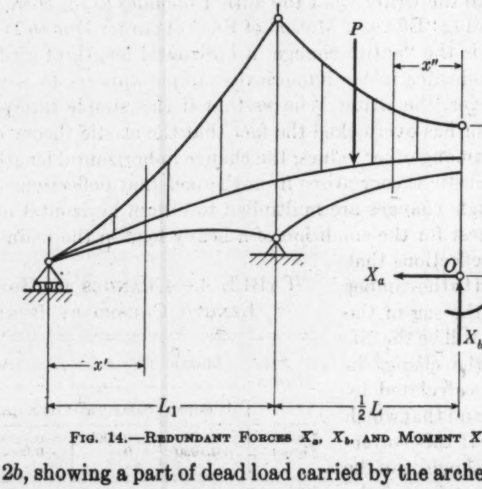

Fig. 14.—Redundant Forces X_a, X_b, and Moment X_c

In Eq. 2b, showing a part of dead load carried by the arched stiffening girder, the author has neglected the effect of bending stresses on the amount of dead or live load carried by the arched girder. Evidently, this effect varies in direct proportion to the arch rise.

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[15] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[16] "Simplified Calculation of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, 1930, p. 204.

The author has shown that the elastic theory applied to the analysis of stresses in a self-anchored suspension bridge gives satisfactory results. In his analysis of stresses in a self-anchored bridge, Professor Krivoshein applied the theory of elasticity,[16] with similar results, using the same method as that adopted for the analysis of stresses in an arch bridge. He severed the suspension bridge at the center of the middle span and replaced the removed half bridge by three redundant stresses: X_a, a horizontal force; X_b, a vertical shear; and X_c, a bending moment (see Fig. 14), as follows:

$$X_a = -\frac{\int M_q\, M_a \frac{1}{I}\, dx}{\int M^2{}_a \frac{1}{I}\, dx + \int S^2{}_o \frac{1}{A_o}\, dx + \sum S^2{}_c\, s \frac{1}{A_c}} \ldots\ldots\ldots (45a)$$

$$X_b = -\frac{\int M_q\, M_b \frac{1}{I}\, dx}{\int M^2{}_b \frac{1}{I}\, dx} \ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

$$X_c = -\frac{\int M_q\, M_c \frac{1}{I}\, dx}{\int M^2{}_c \frac{1}{I}\, dx} \ldots\ldots\ldots\ldots\ldots\ldots (45c)$$

and

$$d = \frac{\frac{1}{L_1}\int_0^{L_1} z\, x' \frac{1}{I}\, dx' + \int_0^{\frac{L}{2}} z \frac{1}{I}\, dx'}{\frac{1}{L^2{}_1}\int_0^{L_1} (x')^2 \frac{1}{I}\, dx' + \int_0^{\frac{L}{2}} \frac{1}{I}\, dx''} \ldots\ldots\ldots\ldots (46)$$

in which (in addition to the notation of the paper): M_q = moment at any section of the main statically determinate system (see Fig. 14), resulting from the given external loading P only (X_a, X_b, and X_c being removed); M_a = moment at any section of the main statically determinate system due to loading $X_a = 1$ (all other loads being removed); M_b = moment at any section of the main statically determinate system due to loading $X_b = 1$ (all other loads being removed); M_c = moment at any section of the main statically determinate system due to loading $X_c = 1$ (all other loads being removed); and d = length of stiff arm to which forces X_a, X_b, and X_c applied.

Evidently each redundant force for a given loading may be computed directly, and the influence lines for stresses may be constructed by Eq. 45.

Unlike the formulas in the paper, Eq. 45 cannot be reduced for application to an externally-anchored suspension bridge. This agrees with the author's statement that the self-anchored bridge acts more like a tied arch than an externally-anchored suspension bridge. It is to be noted that the elastic and geometrical qualities of the self-anchored bridge are considered more directly in Eq. 45. However, the author's method of analysis is more simple and requires less time for computation.

C. H. GRONQUIST,[17] ASSOC. M. AM. SOC. C. E.—The discussions have added greatly to the value of the paper. The presentation of alternative methods of analysis, although leading to the same results, serves to indicate the advantages and disadvantages of the simplified method, and gives the reader a proper perspective of the problem of analyzing the structure. Study of the practical phases of the design of the self-anchored suspension bridge, and economic comparisons with other types of structures, serve to complete the picture.

That the self-anchored suspension bridge is a type of structure which ordinarily will not be used except where conditions are particularly suitable is excellently stated by Mr. Meehan in his thorough discussion of the practical aspects of the subject. There are many locations, however, where the type is eminently suitable, as shown in the paper, and recognition of the fact will undoubtedly lead to the wider adoption of the self-anchored suspension bridge as the proper solution of many moderate-span crossing problems.

The difficulties attendant on the design and construction of the self-anchored suspension bridge appear in several instances to have been overemphasized by Mr. Meehan. Design of the stiffening girders to take compression usually does not require an excessive amount of material, considering that the foundations are lightened, since vertically the live-load bending moments require a stiff section, and lateral support can be obtained at close intervals either from the floor or from an adequate system of lateral bracing. While the details of the cable connection to the girder are somewhat troublesome for longer spans, they can be handled without much difficulty for spans up to 600 ft. Probably for the longer spans it is well to avoid great eccentricity in this connection, although for shorter spans the additional material required to connect the cable and girder above the latter will not be sufficient to offset some practical advantages in so doing. Connection of a wire-rope-strand cable to the girder is made with little more difficulty than for an eyebar cable, as it merely involves the transition from the wire cable to the anchor plates that connect directly to the girder in the same manner as for eyebars.

Erection of the self-anchored suspension bridge is usually done by means of falsework. The eyebar cable spans in Pittsburgh were erected by the cantilever method, however, and erection of wire-cable spans by use of temporary external anchorage to eliminate the falsework has been proposed also. Mr. Meehan states that erection stresses necessitate increase in the section of the stiffening girder or truss members, but this need not be so if the falsework bents are moderately spaced. Moreover, it is not necessary to keep the bridge on falsework after the girders, floor beams, and laterals have been erected, as it usually will be easier to make the cable-to-girder connection at that time rather than later when all dead load is on the structure. This procedure was used on the recently completed Wabash River Bridge (Illinois-Indiana), as well as on many others.

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[17] Assoc. Engr., Robinson & Steinman, New York, N. Y.

As stated by Mr. Meehan, girders, in a great many respects, have an advantage over trusses for use as the stiffening member of the self-anchored suspension bridge. The girder has exhibited undesirable aerodynamic characteristics in light externally-anchored suspension bridges, but this objection to the girder as the stiffening member does not appear to be valid in the case of the self-anchored suspension bridge. Inasmuch as the girder in the latter case must be designed in accordance with the requirements of the elastic theory, which was likewise pointed out by Mr. McCullough, and since it must be stiff enough also to carry the dead-load and live-load direct compression, the self-anchored suspension bridge invariably will be a relatively stiff structure.

As a means of comparing the vertical stiffness of suspension bridges, a constant may be derived from the deflection formula for simple beams under fully distributed loads. Assuming half-span loading, which would produce contraflexure near the center of the main span, such a constant might be written:

$$C = \frac{1}{L^2}\left(H_w + 4 \times \frac{9.6\,E\,I}{L^2}\right) \dots\dots\dots\dots\dots\dots (47)$$

For the self-anchored suspension bridge, the term H_w will not appear, as the cable stress does not contribute to stiffness in the "closed" structure that is analyzed properly by the elastic theory. Even with the girder producing the entire stiffness of the system, however, the value of this constant for self-anchored suspension bridges proves to be relatively high and, where in addition the stiffening girders are made continuous, further stiffening is produced.

Mr. Meehan mentions the possibility of using concrete towers as a substitute for the steel towers of suspension bridges. Although it is not applicable necessarily to the self-anchored type, it is of interest to note the use of fixed-base concrete towers on short suspension spans with unloaded side-span cables in Northern Sweden in recent years, apparently with successful results.

In making preliminary designs for self-anchored suspension bridges for which he made studies, Mr. Meehan states that it was not possible to camber the girders sufficiently to eliminate all dead-load moment. Unless eccentric details are used, there will be no dead-load moment in the stiffening girders of the self-anchored as well as the externally-anchored suspension bridge, with suspended side spans, so long as the cables, girders, and suspenders are cut, fabricated, and erected to proper length and elevation for the stress condition existing at the time of the various operations computed back from the dead-load or normal position.

The discussion by Mr. McCullough makes a distinct contribution to the paper in presenting in graphic form economic comparisons between the self-anchored suspension bridge and the simple span and cantilever, as well as the externally-anchored suspension bridge, for varying lengths of span and total length of structure. His conclusions confirm the more general observations of the writer. Structural connection of the steel floor system and concrete roadway slab to the stiffening girder to take advantage of the natural tendency of the former two to participate with the girder in reacting against the pull of the cable and stiffening the structure, used in the studies by Mr. McCullough, accounts partly for the economy secured with the self-anchored suspension type. Some of the details required to obtain complete interaction of the floor and girder are likely to be rather complicated; but this interaction is a definite characteristic of the self-anchored suspension bridge, as of the tied arch, and the savings to be made by recognition and proper exploitation of the fact should not be lost sight of by the designer.

The remarks of Messrs. Polivka and Eremin in regard to the effect of variable moment of inertia of the stiffening girder are of interest. Where girders of varying depth, which increases toward the towers, are used, it is to be expected that moments obtained on the assumption of constant moment of inertia will be considerably in error, and the simplified method of analysis should be modified in such cases, as outlined in the paper. Where the stiffening girders are of constant depth, even though the moment of inertia varies, the simplified method should prove sufficiently accurate. While the formulas presented would not apply directly to the structure described by Mr. Polivka and shown in Fig. 13, the same general method of approach undoubtedly could be used to advantage in its analysis.

The writer agrees with Mr. Bertwell that it is well not to rely entirely on formulas for the analysis of indeterminate structures. His discussion explains clearly the geometrical basis for the solution of the self-anchored suspension bridge, and constitutes a valuable addition to the paper.

The formulas for D, the denominator of the H-equation, can be simplified somewhat, as was the formula for N, the numerator of the H-equation, by recognition of the fact that, in computing the deflection of an indeterminate structure by the method of the dummy unit load, the value of m, or the moment produced by the unit load, may be taken for a statically determinate condition, and that therefore in this case

$$\int_0^L \left[e'(f + a)_m - (y + z) \right] e'(f + a)_m \frac{dx}{E\,I} = 0 \ldots\ldots\ldots (48)$$

The value of D then may be written (compare Eq. 18b):

$$D = \frac{(f + a)^2 L}{3\,E\,I} \left[\left(\frac{8}{5} - 2\,e \right) - 2\,i\,r\,v' \left(\frac{8}{5} v' - e \right) + \frac{3\,I}{(f + a)^2 L} \left(\frac{E}{E_c} \times \frac{L_s}{A_c} + \frac{L'_s}{A_g} \right) \right] \ldots\ldots\ldots\ldots (49)$$

The formula for the denominator of the H-equation when stated in this form agrees with that of Mr. Bertwell.

As stated by Mr. Bertwell, the method of correcting the value of H for the effect of deflection requires modification. The change in horizontal span of cable or girder may be written:

$$\Delta L = \frac{8}{3 \sec^4 \alpha} \left[\frac{f^2}{L} - \frac{(f + \Delta f)^2}{(L + \Delta L)} \right] + \Delta L_d \text{ (very nearly)} \ldots\ldots (50a)$$

in which ΔL_d is the change in length, not span, due to direct stress, and equals:

$$\Delta L_d = \frac{H\,L}{E\,A}\left(\sec\alpha + \frac{8\,n^2}{3\sec^3\alpha}\right)^2 \quad \text{(50b)}$$

The sum of the changes of span of the cable must equal the sum of the changes of span of the girder, obtained by substitution in the foregoing formulas of the deflections Δf and the value of H given by the elastic theory. The correction in H therefore may be computed by substitution in Eq. 39*b* of the amount by which these changes in span fail to agree. The amount of the correction of both H and moment is negligible, although that of deflection is not, as already indicated in the computations of the paper. The use of the factor 2.6 in place of $\frac{8}{3}$ in Eq. 50*a* will give more accurate results for change of span of main-span cable. The correction in the H of the numerical example of the paper, obtained in this manner, would be little changed from that originally given. The writer believes that the 2.08-kip change in H computed by Mr. Bertwell is not correct, since the total actual changes in span of cable and girder, as given, do not agree.

Mr. Eremin states that the effect of bending stresses on the amount of dead or live load carried by the arched girder has been neglected in deriving Eq. 2*b*. For the type of self-anchored bridge considered in this paper—that is, the bridge with a suspended side span—there will be no dead-load moment if the various parts of the structure are fabricated and erected to be cambered for the dead-load condition. Consequently, for dead load the stiffening girder takes only direct stress but, due to its parabolic arching, it carries a part of the dead load as stated by Eq. 2*b*. The girder, of course, is subjected to bending as well direct stresses due to live load, and the load taken in bending is expressed in Eq. 7.

Eqs. 45 for the independent values of the redundants that are assumed acting at the "elastic center" of the self-anchored suspension bridge are given by Mr. Eremin following the method suggested by Professor Krivoshein.[16] These or similar equations were first written by H. Müller-Breslau,[18] and also have been used by others. The lever arm d, or the vertical distance from the cable to the elastic center, is equal to $(f + a)(1 - e)$, in the notation of the paper, and also represents the moment at the center of the main span in the independent girder produced by the suspender forces for a unit value of H. It should be observed that in Eq. 46 the variable z has a different meaning from the z of the paper. The z of Eq. 46 is the vertical distance between the cable and the girder.

It may be of interest to elaborate somewhat on the type of self-anchored suspension bridge with unloaded side spans mentioned in the paper. Two bridges of this type have been built, both with the side-span girders hinged a short distance from the towers. The main-span girders were also hinged at the center for the dead-load condition. Neglecting the effect of side-span girder deflection, which is not neutralized as in the case of the main span by equal cable deflection, the formulas of the paper for the live-load condition will apply throughout when f_1 is placed equal to zero.

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[18] "Die graphische Statik der Baukonstruktionen," by H. F. B. Müller-Breslau, Stuttgart, Alfred Kroner, 1920–1927.

In the self-anchored suspension bridge with unloaded side-span cables, the main-span girders will not be subjected to bending due to dead load when the stiffening girders are hinged at the towers. When the girders are continuous, however, the main-span girder must resist a constant negative dead-load moment that is the tower moment produced in the independent girder by a side-span loading equal to the difference between the side-span dead load and that portion of it taken by the side-span girder as a parabolic arch, provided the side-span girder is constructed to such a roadway camber. Both the side-span and main-span girders must be cambered vertically, for this side-span loading. If it is assumed that the entire side-span dead load is applied to the girders before the cables are connected to them, the girders then will deflect to a position with respect to their proper dead-load vertical position that will be below in the side span and above in the main span. The distortion required to make the cable-to-girder connection, due to the no-load shortness of the cable with respect to the girder, will be neutralized on application of the entire dead load in the main span. The parabolic arching of the side-span girders, where those girders do not follow a straight roadway grade, will relieve, in addition, the side-span girders of a part of the side-span dead load. This relief will have the effect of raising the side-span girders and lowering the main-span girders to their correct dead-load position.

The equation for dead-load bending at the center of the main span is:

$$M_w = T_D + \frac{w\,L^2}{8} - H_w\,a - H_w\,f = T_D \ldots\ldots\ldots\ldots (51)$$

in which T_D is the value of the bending moment at the tower, and therefore $H_w = \frac{w\,L^2}{8(f + a)}$, as for the bridge with loaded side spans. In the side span the equation for dead-load bending moment is:

$$M_{w1} = T_D\,\frac{x}{L_1} + \frac{w_1}{2}\,(L_1\,x - x^2) - H_w\,z_1 \ldots\ldots\ldots\ldots (52)$$

The second derivative of this expression gives the load taken by the side-span girder in bending:

$$w'_1 = w_1 - 8\,H_w\,\frac{a_1}{L^2_1} = w_1 - \frac{w\,L^2}{L^2_1} \times \frac{a_1}{(f + a)} \ldots\ldots\ldots (53)$$

The tower moment produced by this loading in the independent girder is the dead-load moment in the main span of the self-anchored suspension bridge with unloaded side-span cables.

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ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

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With Discussion by Messrs. Maurice P. van Buren, Wayne W. Smith, Leonard P. Zick, Jr., and Conrad C. Wan, S. D. Lash, Dean F. Peterson, Jr., R. W. Stewart, Jaroslav J. Polivka, and Bruce Johnston and Edward H. Mount.

Synopsis

Methods applicable to the analysis of building frames with semi-rigid riveted or welded connections between the beams and columns are presented in this paper. The methods of analysis are too complex for ordinary design use, but the writers have presented simple design procedures, based on these methods of analysis, elsewhere (2) (3),[3] and have made them expeditious by the use of charts and diagrams. Such design methods effect permissible economy in the required beam sizes, made possible by considering the partial restraint afforded by standard or near standard connections, particularly riveted or welded connections of the top and seat angle type.

This paper also presents test results of a welded building frame that corroborate the methods of analysis. A study of the effect of neglecting the width of members in the analysis is presented. ("Member width" is used in this paper to indicate column width or beam depth, as the frame is viewed in elevation.) The essential features of the methods of analysis have been presented elsewhere (1) (13) (14), and it is the intention of the writers to modify them slightly so as to simplify the technique by conforming in every way to the usual slope-deflection and moment-distribution procedures.

Introduction

The design of the steel frames that form the skeleton of multiple-story steel buildings is usually based upon certain simplifying assumptions, chief of which are: (*a*) For the purpose of beam design the beam-column connections

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[3] Numerals in parentheses, thus: (2) (3), refer to corresponding numbers in the Bibliography; see Appendix.

are assumed to be pin connections, or simple supports; (*b*) columns are designed without attempting to evaluate the moments introduced by frame action; and (*c*) the beam-column connections are assumed to be rigid in calculating stresses due to lateral or wind loads.

These assumptions have afforded a means of rapid design calculation. Riveted building frames constructed on the basis of these design assumptions have proved to be safe and reliable, but there remains the possibility of achieving greater economy through the use of more nearly correct design assumptions. According to the British investigations (1), an average saving in the weight of beams of as much as 20% may be expected by taking advantage of the partial end restraint of riveted beam-column connections. Welded construction, with its inherent continuity, also makes similar saving in weight possible.

The basis for the application of more accurate design methods to frames with semi-rigid connections has already been laid in Great Britain (1) and in work of a parallel nature in the United States (2) (3) (4). The comparison of analytical and experimental results presented in this paper was made possible by the construction and test of a welded building frame with three bays and two stories. The beam-column connections used in this frame were of a semi-rigid type previously studied at the Fritz Engineering Laboratory (2) (5) in connection with research programs sponsored by the American Welding Society. Similar tests on riveted building frames have been made in Great Britain (1).

The slope-deflection and moment-distribution methods have as a common purpose the determination of the bending moments at the ends of the individual members of a statically indeterminate frame. Both methods in their usual form are based on the assumption that the deformations of the frames are caused entirely by bending of the members and that the relation between bending moment and distortion is given by the beam formula:

$$\frac{M}{E\,I} = \frac{d^2y}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The derivation of the beam theory and the assumptions on which it is based may be found in any text on the strength of materials.

In 1915 the slope-deflection method was applied in the United States (by Wilbur M. Wilson and George A. Maney, Members, Am. Soc. C. E.) to the analysis of wind stresses in tall buildings (6). A more complete treatment followed in 1918 (by Professor Wilson, with F. E. Richart and Camillo Weiss, Members, Am. Soc. C. E.) (7), and in 1931 a modification (8) was introduced by L. T. Evans, Assoc. M. Am. Soc. C. E., to take care of members with varying moments of inertia. The method of moment distribution was first presented in mimeographed form by Hardy Cross, M. Am. Soc. C. E., in 1926 (9) (10) (12). Innumerable variations and short cuts have been applied to the moment-distribution method and, although some of these have merit, the original method remains the outstanding development in recent times for rapid and effective analysis of continuous frames. The application of both the slope-deflection and moment-distribution methods to the analysis of frames with semi-rigid connections was made by John F. Baker, Assoc. M.

Am. Soc. C. E. (13). The width of the members and the semi-rigid nature of the connections are both taken into account, as it is found that the neglect of member width gives rise to considerable error, particularly in the case of analyses of frames with semi-rigid connections.

ANALYSIS OF FRAMES WITH SEMI-RIGID JOINTS

It will be assumed that the reader is already familiar with the usual application of the slope-deflection and moment-distribution methods, for which references are readily available (12) (15) (16) (17). The methods herein presented are identical in mode of application to the usual simple form, with the exception that special coefficients must be used in the slope-deflection equations and for the carry-over and distribution factors in the moment-distribution method. The following method is identical with that previously presented by Professor Baker (13) (14) when the width of member is neglected. When the width of member is considered, the following method differs in two respects from that of Professor Baker: (*a*) The interior of the joint between connections is assumed infinitely rigid, whereas Professor Baker assumes it to have the same stiffness as the beam; and (*b*) hypothetical moments are computed at the joint centers by the usual slope-deflection and moment-distribution procedures, whereas in Professor Baker's method separate expressions are given for the moment and shear at the connection, or column face, and are dealt with separately.

The following assumptions apply both to the slope-deflection and to the moment-distribution relations as herein presented: (*a*) Members are of uniform cross section between their end connections; (*b*) the semi-rigid connection at the end of a member behaves elastically as defined by the connection constant γ; and (*c*) the interior of the joint between connections is assumed to be infinitely rigid, although free to rotate as a rigid body.

The notation used is shown subsequently in Figs. 5 and 6. The hypothetical moments and shears at the joint centers, when width of column is considered, are designated by the bar above the letter M or V, thus; $\bar{M}$ and $\bar{V}$.

The "Semi-Rigid" Connection.—The semi-rigid connection may be thought of as a locally weakened section between the end of the beam and the face of the column to which the connection is made. The effect on analysis is the inverse of the effect produced by end haunches or added cover plates. The typical test behavior of a riveted or welded connection of this type is shown in Fig. 1, which presents the plot of the relationship between moment transmitted through the connection and the angle change between the joint center and the end of the beam. In the design range the relationship is assumed linear and the inverse slope is termed the connection constant, γ, thus:

$$\gamma = \frac{\phi}{M} = \frac{\text{Angle change}}{\text{Moment}} \qquad \text{(2)}$$

The connection constant γ may be defined as "angle change for unit moment" and may be determined experimentally by testing typical joints. The vertical line through the origin in Fig. 1 would indicate the behavior of a perfectly fixed connection with $\gamma = 0$, and the horizontal line would represent the behavior of a frictionless pin connection, in which case $\gamma = \infty$. Fig. 2(*a*)

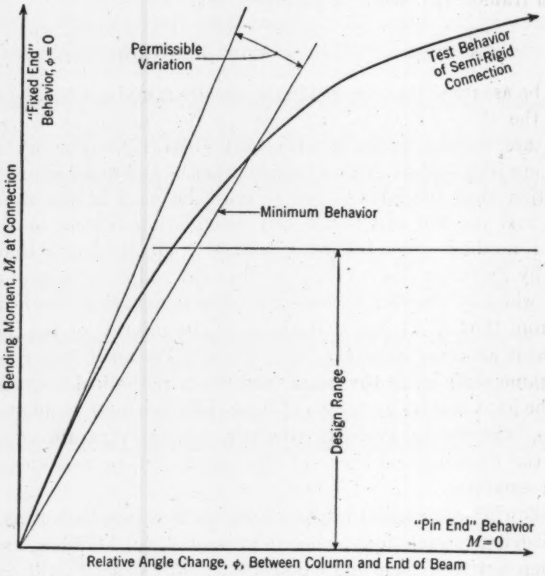
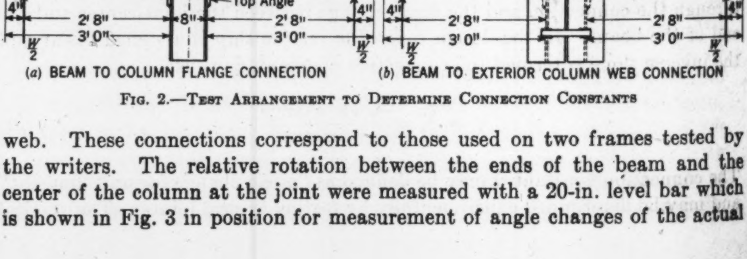


FIG. 1.—TYPICAL M-ϕ RELATION IN THE TEST OF A WELDED OR RIVETED BEAM-COLUMN CONNECTION

shows the test setup for determining the connection constant at an interior joint of a frame with beam-to-column-flange connections, and Fig. 2(*b*) shows a similar setup to test the connection between a beam and an exterior column



(*a*) BEAM TO COLUMN FLANGE CONNECTION (*b*) BEAM TO EXTERIOR COLUMN WEB CONNECTION

FIG. 2.—TEST ARRANGEMENT TO DETERMINE CONNECTION CONSTANTS

web. These connections correspond to those used on two frames tested by the writers. The relative rotation between the ends of the beam and the center of the column at the joint were measured with a 20-in. level bar which is shown in Fig. 3 in position for measurement of angle changes of the actual test frame. Fig. 4 is a graph within the test-design range of measured angle change plotted against moment in typical joint tests corresponding to the actual test frame.

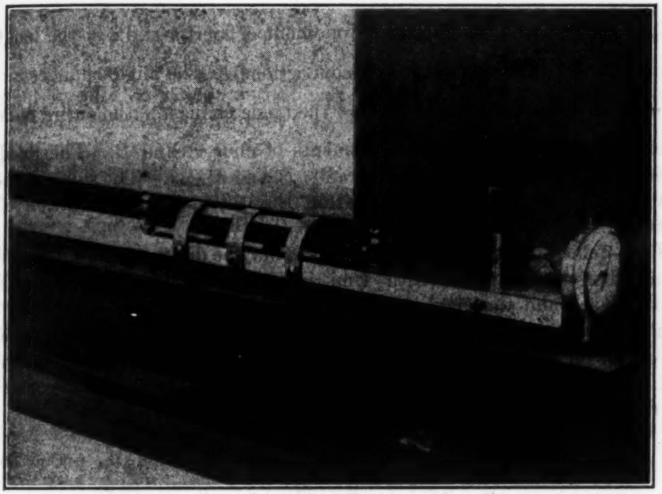


FIG. 3.—LEVEL BAR USED TO MEASURE ROTATIONS

The Slope-Deflection Equations.—For any individual member of a frame, the relation between its end moments, the angle changes at each end, and the angle change of the member as a whole may be expressed by a pair of slope-deflection equations. For the usually assumed case of uniform beam cross

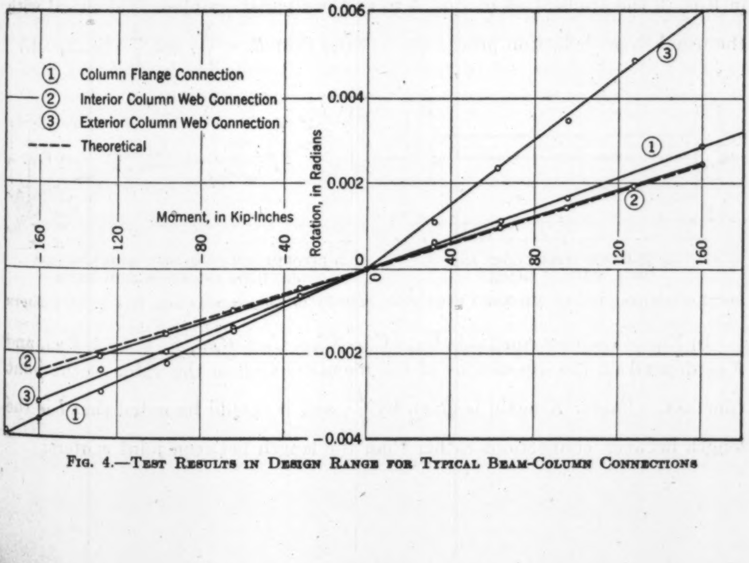


FIG. 4.—TEST RESULTS IN DESIGN RANGE FOR TYPICAL BEAM-COLUMN CONNECTIONS

section, and with rigid end connections, these equations are written:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) - M_{RAB} \qquad (3a)$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) + M_{RBA} \qquad (3b)$$

In Eqs. 3, $K = \frac{I}{l}$, in which I = the moment of inertia and l = the length of the member (distance between joint connections). The angle changes at ends A and B are θ_A and θ_B; and $R = \frac{\Delta}{l}$ = the angle change of the entire member, Δ being the relative lateral displacements of ends A and B. The fixed-end moments for the lateral loads alone are M_{RAB} and M_{RBA}. The slope-deflection equations may be derived directly from Eq. 1 or by an application of the moment-area principles.

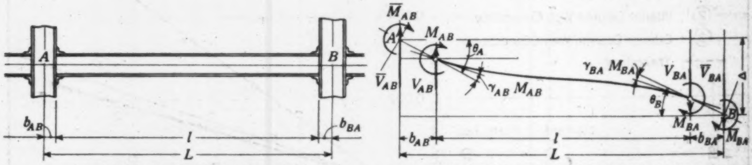
Slope-deflection equations such as Eqs. 3 may be derived by similar methods for members which frame with semi-rigid connections. Fig. 5 shows the notation used and the geometry of the general deflected curve of any such member. Note especially Fig. 5(*b*), which shows the hypothetical moments at the center of the joint. The slope-deflection equations corresponding to Eqs. 3 for the hypothetical moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ at the joint centers for any member $A\,B$, as shown in Fig. 5, may be written:

$$\bar{M}_{AB} = \frac{1}{1 + 2\,\alpha + 2\,\beta + 3\,\alpha\,\beta}\,[2\,E\,K(C_{AA}\,\theta_A + C_{AB}\,\theta_B - C_{AC}\,R) - F_{AA}\,M_{RAB} - F_{AB}\,M_{RBA}] - V_{AB}'\,b_{AB} \qquad (4a)$$

and

$$\bar{M}_{BA} = \frac{1}{1 + 2\,\alpha + 2\,\beta + 3\,\alpha\,\beta}\,[2\,E\,K(C_{BB}\,\theta_B + C_{BA}\,\theta_A - C_{BC}\,R) + F_{BB}\,M_{RBA} + F_{BA}\,M_{RAB}] + V_{BA}'\,b_{BA} \qquad (4b)$$

Except for the fact that new coefficients replace the even integer coefficients in Eqs. 3, the application of Eqs. 4 to any particular problem is identical with the usual slope-deflection procedure. (Note that $R = \frac{\Delta}{l}$, not $\frac{\Delta}{L}$, in Eqs. 4.)



(*a*) SEMI-RIGID CONNECTIONS AND FINITE WIDTH OF MEMBER

(*b*) DEFORMATION OF MEMBER WITH SEMI-RIGID CONNECTIONS AND FINITE JOINT WIDTH

FIG. 5.—DEFORMATION OF MEMBER CONSIDERING SEMI-RIGID CONNECTIONS AND FINITE JOINT WIDTH

In Eqs. 4 the new constants C_{AA}, C_{AB}, C_{AC}, C_{BB}, C_{BA}, F_{AA}, F_{AB}, F_{BA}, and F_{BB} depend on the dimensions of the members and on the value of the joint constant. Factor K again is given by $\frac{I}{l}$, and it should be noted that l is the length between connections rather than the length between joint centers.

The connection constants γ_A and γ_B for the connections at the two ends of the beam are introduced into new constants α and β by the relations (1):

$$\alpha = 2\,E\,K\,\gamma_A \tag{5a}$$

and

$$\beta = 2\,E\,K\,\gamma_B \tag{5b}$$

The fixed-end moments for a member with fixed, rigidly connected, ends of span length l are M_{RAB} and M_{RBA}; and V_{AB}' and V_{BA}' are the shears or reactions at the ends of a member with freely supported ends and span length l.

The constants C_{AA}, C_{AB}, C_{AC}, F_{AA}, and F_{AB} in Eq. 4*a* are given in Table 1 for four different cases. All four cases are for unsymmetrical conditions, the first and the third considering semi-rigid connections, and the second and fourth considering rigid connections. In cases I and II, a finite width of member is considered; and, in cases III and IV, width is ignored. The values in case IV are those commonly assumed—that is, frames with rigid joints and with width of member neglected. In Eq. 4*b* the constants C_{BB}, C_{BA}, C_{BC}, F_{BB}, and F_{BA} for $\bar{M}_{BA}$ are obtained from C_{AA}, C_{AB}, C_{AC}, F_{AA}, and F_{AB}, respectively, by interchanging α and β and the subscripts A and B. It is noted that C_{BA} is equal to C_{AB}.

Eqs. 4 with the preceding coefficients are for moments at the joint centers and therefore are used with exactly the same equilibrium conditions as in the simpler form (2); namely,

For joint equilibrium,

$$\Sigma \bar{M} = 0 \tag{6a}$$

and, for story equilibrium,

$$\Sigma \bar{M} + V\,h = 0 \tag{6b}$$

In Eq. 6*b*, h is the story height between neutral axes of two beams.

With Eqs. 4 it is now possible to determine the hypothetical end moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ at the joint centers. Moments and shears are assumed as positive when they act on the ends of the beam with a clockwise sense, or act on the joint with a counterclockwise sense. The hypothetical shears in the joint are constant and equal to the actual shear at the connection. The shears $\bar{V}_{AB}$ and $\bar{V}_{BA}$ may be calculated from the moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ by the following:

$$V_{AB} = \bar{V}_{AB} = -\left(\frac{\bar{M}_{AB} + \bar{M}_{BA}}{L}\right) + \bar{V}_{AB}' \tag{7a}$$

and

$$V_{BA} = \bar{V}_{BA} = -\left(\frac{\bar{M}_{AB} + \bar{M}_{BA}}{L}\right) - \bar{V}_{BA}' \tag{7b}$$

in which $\bar{V}_{AB}'$ and $\bar{V}_{BA}'$ are the end shears in a member having span length L with simply or freely supported ends. The moments at the connections, M_{AB} and M_{BA}, may now be calculated from the following:

$$M_{AB} = \bar{M}_{AB} + \bar{V}_{AB}\,b_{AB} \tag{8a}$$

and

$$M_{BA} = \bar{M}_{BA} + \bar{V}_{BA}\,b_{BA} \tag{8b}$$

TABLE 1.—SLOPE-DEFLECTION AND MOMENT-DISTRIBUTION CONSTANTS—NONSYMMETRICAL CASES

Constant	Case I—Nonsymmetrical, semi-rigid connections, finite width of members	Case II—Nonsymmetrical, rigid connections, finite widths	Case III—Nonsymmetrical, semi-rigid connections, zero width	Case IV—Rigid connections, 0 width
	(a) SLOPE DEFLECTION			
C_{AA}	$2 + 3\beta + 6(1+\beta)\frac{b_{AB}}{l} + 3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}$	$2 + \frac{6\,b_{AB}}{l} + \frac{6\,b_{AB}^2}{l^2}$	$2 + 3\beta$	2
$C_{AB} = C_{BA}$	$1 + 3(1+\alpha)\frac{b_{AB}}{l} + 3(1+\beta)\frac{b_{BA}}{l} + 3(2+\alpha+\beta)\frac{b_{AB}\,b_{BA}}{l^2}$	$1 + \frac{3\,b_{AB}}{l} + \frac{3\,b_{BA}}{l} + \frac{6\,b_{AB}\,b_{BA}}{l^2}$	1	1
C_{BB}	$2 + 3\alpha + 6(1+\alpha)\frac{b_{BA}}{l} + 3(2+\alpha+\beta)\frac{b_{BA}^2}{l^2}$	$2 + \frac{6\,b_{BA}}{l} + \frac{6\,b_{BA}^2}{l^2}$	$2 + 3\alpha$	2
C_{AC}	$3(1+\beta) + 3(2+\alpha+\beta)\frac{b_{AB}}{l}$	$3 + \frac{6\,b_{AB}}{l}$	$3(1+\beta)$	3
C_{BC}	$3(1+\alpha) + 3(2+\alpha+\beta)\frac{b_{BA}}{l}$	$3 + \frac{6\,b_{BA}}{l}$	$3(1+\alpha)$	3
F_{AA}	$1 + 2\beta + (1-\alpha+2\beta)\frac{b_{AB}}{l}$	$1 + \frac{b_{AB}}{l}$	$1 + 2\beta$	1
F_{AB}	$\beta + (\beta - 2\alpha - 1)\frac{b_{AB}}{l}$	$-\frac{b_{AB}}{l}$	β	0
F_{BB}	$1 + 2\alpha + (1-\beta+2\alpha)\frac{b_{BA}}{l}$	$1 + \frac{b_{BA}}{l}$	$1 + 2\alpha$	1
F_{BA}	$\alpha + (\alpha - 2\beta - 1)\frac{b_{BA}}{l}$	$-\frac{b_{BA}}{l}$	α	0
	(b) MOMENT DISTRIBUTION			
$\bar{M}_{SAB}$ semi-fixed end moment at joint center	$\frac{\left\{\left[1+2\beta+\frac{b_{AB}}{l}(1+2\beta-\alpha)\right]M_{RAB}+\left[\beta-\frac{b_{AB}}{l}(1+2\alpha-\beta)\right]M_{RBA}\right\}}{1+2\alpha+2\beta+3\alpha\beta}+V_{AB}'\,b_{AB}$	$\left(1+\frac{b_{AB}}{l}\right)M_{RAB} - \frac{b_{AB}}{l}M_{RBA} + V_{AB}'\,b_{AB}$	$\frac{[(1+2\beta)\,M_{RAB} + \beta\,M_{RBA}]}{1+2\alpha+2\beta+3\alpha\beta}$	M_{RAB}
r_{AB} carry-over factor between joint centers	$\frac{1 + 3(1+\alpha)\frac{b_{AB}}{l} + 3(1+\beta)\frac{b_{BA}}{l} + 3(2+\alpha+\beta)\frac{b_{AB}\,b_{BA}}{l^2}}{2 + 3\beta + 6(1+\beta)\frac{b_{AB}}{l} + 3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}}$	$\frac{1 + \frac{3\,b_{AB}}{l} + \frac{3\,b_{BA}}{l} + \frac{6\,b_{AB}\,b_{BA}}{l^2}}{2 + \frac{6\,b_{AB}}{l} + \frac{6\,b_{AB}^2}{l^2}}$	$\frac{1}{2+3\beta}$	$\frac{1}{2}$
$\bar{S}_{MAB}$ end rotation stiffness	$\frac{2\,E\,K\left[2 + 3\beta + 6(1+\beta)\frac{b_{AB}}{l} + 3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}\right]}{1+2\alpha+2\beta+3\alpha\beta}$	$2\,E\,K\left(2 + \frac{6\,b_{AB}}{l} + \frac{6\,b_{AB}^2}{l^2}\right)$	$\frac{2\,E\,K\,(2+3\beta)}{1+2\alpha+2\beta+3\alpha\beta}$	$4\,E\,K$
$\bar{S}_{VAB}$ sidesway stiffness	$\frac{6\,E\,K}{l^2}\left(\frac{2+\alpha+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$	$\frac{12\,E\,K}{l^3}$	$\frac{6\,E\,K}{l^2}\left(\frac{2+\alpha+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$	$\frac{12\,E\,K}{l^2}$
$\bar{M}_{VAB}$ sidesway end moment at A	$-\frac{6\,E\,K}{l}\left[\frac{1+\beta+\frac{b_{AB}}{l}(2+\alpha+\beta)}{1+2\alpha+2\beta+3\alpha\beta}\right]$	$-\frac{6\,E\,K}{l}\left(1+\frac{2\,b_{AB}}{l}\right)$	$-\frac{6\,E\,K}{l}\left(\frac{1+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$	$-\frac{6\,E\,K}{l}$

The slope-deflection equations are simplified when symmetrical conditions of loading and structure exist with respect to any particular member. In such a case $\alpha = \beta$. Furthermore, $b_{AB} = b_{BA} = b$; $M_{RAB} = M_{RBA} = M_R$; $\bar{V}_{AB}' = \bar{V}_{BA}' = V_{AB}' = V_{BA}' = V'$; and the slope-deflection equations corresponding to case I of Table 1 may be reduced to:

$$\bar{M}_{AB} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A + \left(\frac{1}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B - \left(3 + \frac{6\,b}{l}\right)R\right] - \left(\frac{M_R}{1 + \alpha} + V'\,b\right) \qquad (9a)$$

and

$$\bar{M}_{BA} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B + \left(\frac{1}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A - \left(3 + \frac{6\,b}{l}\right)R\right] + \left(\frac{M_R}{1 + \alpha} + V'\,b\right) \qquad (9b)$$

For case II of Table 1, considering the semi-rigidity of the joints but neglecting the width of the members, the equations for symmetrical conditions may be reduced to a simple form by letting $b = 0$ in Eqs. 9; thus:

$$\bar{M}_{AB} = M_{AB} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha}\right)\theta_A + \left(\frac{1}{1 + \alpha}\right)\theta_B - 3\,R\right] - \frac{M_R}{1 + \alpha} \qquad (10a)$$

and

$$\bar{M}_{BA} = M_{BA} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha}\right)\theta_B + \left(\frac{1}{1 + \alpha}\right)\theta_A - 3\,R\right] + \frac{M_R}{1 + \alpha} \qquad (10b)$$

A similar simplification may be made for case III of Table 1 by letting $\alpha = 0$ in Eqs. 9, in which case the following equations result:

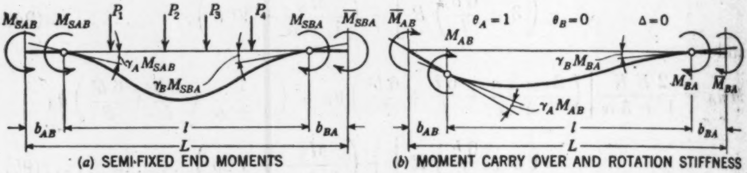
$$\bar{M}_{AB} = 2\,E\,K\left[\left(2 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A + \left(1 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B - \left(3 + \frac{6\,b}{l}\right)R\right] - (M_R + V_b') \qquad (11a)$$

and

$$\bar{M}_{BA} = 2\,E\,K\left[\left(2 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B + \left(1 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A - \left(3 + \frac{6\,b}{l}\right)R\right] + (M_R + V_b') \qquad (11b)$$

Moment Distribution Applied to Frames with Semi-Rigid Connections.—The moment-distribution method serves identically the same purpose as the slope-deflection method; that is, the moments at the ends of the individual members of a frame or continuous beam are determined. It will be assumed, as in the case of the slope-deflection method, that the reader is familiar with the usual moment distribution procedure. The procedure as applied to frames with semi-rigid connections is identical with the usual method, although there are differences in the numerical value of the carry-over, stiffness, and other factors.

The factors used in the moment-distribution procedure may be derived from the slope-deflection equations or by use of the column analogy (11), as will be described herein. Fig. 6 shows the deformation conditions for determining semi-rigid end moments and the carry-over factor for moment distribution. Table 1 gives the "semi-rigid" end moments, "carry-over" factor, rotation-stiffness factors, sidesway-stiffness factors, and sidesway end moments for the nonsymmetrical cases I, II, III, and IV. Table 2 presents the same factors for the symmetrical cases I, II, and III.



(a) SEMI-FIXED END MOMENTS (b) MOMENT CARRY OVER AND ROTATION STIFFNESS

FIG. 6.—DEFORMATION CONDITIONS USED IN MOMENT DISTRIBUTION FOR MEMBERS WITH SEMI-RIGID CONNECTIONS AND FINITE JOINT WIDTH

TABLE 2.—MOMENT DISTRIBUTION FOR SYMMETRICAL CASES
$\alpha = \beta$, $b_{AB} = b_{BA} = b$, AND $M_{RA} = M_{RB} = M_R$

Constant	Symmetrical case I, semi-rigid connections, finite width of members	Symmetrical case II, rigid connections, finite member width	Symmetrical case III, semi-rigid connections, zero member width
$\bar{M}_{SAB}$ semi-fixed-end moment at joint center	$\frac{M_R}{1+\alpha} + V' b$	$M_R + V' b$	$\frac{M_R}{1+\alpha}$
$\bar{r}_{AB}$ carry-over factor between joint centers	$\frac{1+6(1+\alpha)\frac{b}{l}+6(1+\alpha)\frac{b^2}{l^2}}{2+3\alpha+6(1+\alpha)\frac{b}{l}+6(1+\alpha)\frac{b^2}{l^2}}$	$\frac{1+\frac{6b}{l}+\frac{6b^2}{l^2}}{2+\frac{6b}{l}+\frac{6b^2}{l^2}}$	$\frac{1}{2+3\alpha}$
$\bar{S}_{MAB}$ end rotation stiffness	$\frac{2EK}{1+3\alpha}\left(\frac{2+3\alpha}{1+\alpha}+\frac{6b}{l}+\frac{6b^2}{l^2}\right)$	$2EK\left(2+\frac{6b}{l}+\frac{6b^2}{l^2}\right)$	$2EK\left(\frac{2+3\alpha}{1+4\alpha+3\alpha^2}\right)$
$\bar{S}_{VAB}$ sidesway stiffness	$\frac{12EK}{l^2(1+3\alpha)}$	$\frac{12EK}{l^2}$	$\frac{12EK}{l^2(1+3\alpha)}$
$\bar{M}_{VAB}$ sidesway end moment at A	$-\frac{6EK}{l}\left(\frac{1+\frac{2b}{l}}{1+3\alpha}\right)$	$-\frac{6EK}{l}\left(1+\frac{2b}{l}\right)$	$-\frac{6EK}{l(1+3\alpha)}$

All of the moment-distribution factors relate to hypothetical moments at the centers of the joints, and the general procedure is therefore identical with that used in the simple case. After the hypothetical moments at the joint centers are obtained, the moments and shears at the connections result as before from Eqs. 7 and 8.

In applying the moment-distribution method to frame problems, sidesway may be induced either by lateral loads or by unsymmetrical conditions of loading or frame arrangement. The simplest form of sidesway problem is one involving only lateral loads applied to a one-story frame. The lateral load is distributed to the columns in proportion to their "sidesway stiffness" $\bar{S}_V$, and semi-fixed or fixed-end moments are distributed to the ends of each column in proportion to the $\bar{M}_V$ moments for unit sidesway. The next step is the usual moment distribution, but at the conclusion the summation of column shears will not account for the total lateral force. All of the end moments are then multiplied by a constant ratio sufficient to bring the shears into equilibrium with the external lateral force. If the structure is more than one story in height, the procedure is progressively more complicated. Shears are distributed in any one story in proportion to their lateral or sidesway stiffness; but in special cases in which two-story sections adjoin open auditoriums or halls, the combined rigidities of the two stories of columns "in series" must be determined. The stiffness of a two-story group of columns "in series" is given by:

$$\bar{S}_{VABC} = \frac{\bar{S}_{VAB}\,\bar{S}_{VBC}}{\bar{S}_{VAB} + \bar{S}_{VBC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

in which $\bar{S}_{VABC}$ = sidesway stiffness of two columns or groups of columns "in series." The combined rigidity of three tiers of columns "in series" is given by:

$$\bar{S}_{VABCD} = \frac{\bar{S}_{VAB}\,\bar{S}_{VBC}\,\bar{S}_{VCD}}{\bar{S}_{VAB}\,\bar{S}_{VBC} + \bar{S}_{VBC}\,\bar{S}_{VCD} + \bar{S}_{VCD}\,\bar{S}_{VAB}} \quad \ldots\ldots (12b)$$

The analysis of two-story and three-story problems of the foregoing type is taken up in texts (12) (16) (17) and the procedure for frames with semi-rigid joints follows exactly the same course. In applying these analytical methods to the development of design methods for multi-storied buildings, under the action of vertical loads alone, it is reasonable to neglect the effect of sidesway.

Certain short cuts for special conditions may be used, provided that their use in simpler forms of moment distribution is already familiar. For example, if the end B of member $A\,B$ is pin-connected, or freely supported, β becomes equal to ∞. The "semi-fixed" end moment at A then becomes:

$$\bar{M}_{SAB} = \left(\frac{2 + \frac{2\,b_{AB}}{l}}{2 + 3\,\alpha}\right) M_{RAB} + V_A'\,b_{AB} \quad \ldots\ldots\ldots\ldots (13)$$

In Eq. 13 M_{RAB} is the fixed-end moment at A due to lateral loads in a beam freely supported at B. The rotation stiffness, or distribution factor, for end A of member $A\,B$ with B freely supported is:

$$\bar{S}_{MAB} = \frac{2\,E\,K}{2 + 3\,\alpha}\left(3 + \frac{6\,b_{AB}}{l} + \frac{3\,b_{AB}{}^2}{l^2}\right) \quad \ldots\ldots\ldots\ldots (14a)$$

The sidesway stiffness factor for the same case, with one end freely supported, is as follows:

$$\bar{S}_{VAB} = \frac{6\,E\,K}{l^2\,(2 + 3\,\alpha)} \quad \text{(14b)}$$

Cases II and III in Table 2 may be obtained from Eqs. 13, 14*a*, and 14*b* by letting α and b_{AB}, respectively, be equal to zero.

Another type of special case occurs when the entire frame and loading upon it are symmetrical. If the center line of the frame is on line with a column, there will be no rotation of the column joints and the center line of the center column may be assumed equivalent to a fixed wall. If there are an odd number of panels, the center line of the frame will cut through the center of the beams in the center bay. The rotation of the two ends of each beam in the center panel will be equal in magnitude and opposite in sign. From this condition it follows that the modified moment stiffness or "distribution factor" may be taken as:

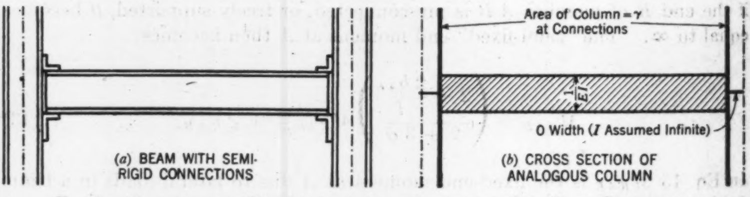
$$\bar{S}_{MAB} = \frac{2\,E\,K}{1 + \alpha} \quad \text{(14c)}$$

for the ends of beams in the center panel. No carry-over is used in the center panel when the modified stiffness factor is used.

Application of the Column Analogy to Members with Semi-Rigid Connections.—It will be assumed that the reader is familiar with the application of the column analogy, originally developed by Professor Cross (11), to the determination of moment-distribution factors for beams with variable moments of inertia. The width of the "analogous column" is equal to $\frac{1}{E\,I}$, and the area of any elemental length ds of the analogous column is therefore equal to $\frac{ds}{E\,I}$. From the fundamental relations of the bent beam,

$$\frac{d\phi}{M} = \frac{ds}{E\,I} \quad \text{(15)}$$

in which $d\phi$ = the angle change in any elemental length of beam. At the


Area of Column = γ at Connections
$\frac{1}{EI}$
0 Width (*I* Assumed Infinite)

(*a*) BEAM WITH SEMI-RIGID CONNECTIONS

(*b*) CROSS SECTION OF ANALOGOUS COLUMN

FIG. 7.—APPLICATION OF COLUMN ANALOGY TO BEAMS WITH SEMI-RIGID CONNECTIONS

particular location of the semi-rigid joint, from the definition of the "connection constant,"

$$\gamma = \frac{d\phi}{M} = \frac{ds}{E\,I} \quad \text{(16)}$$

Hence, the localized area of the analogous column at the semi-rigid connection is equal to the connection constant γ. Professor Cross (11) has shown that the area of the analogous column at a pin connection is infinite, and in the region of a completely rigid zone it is equal to zero. The semi-rigid connection obviously is a case somewhere between these two extremes, and the column analogy may readily be used to obtain the moment-distribution factors for a member so connected. Fig. 7 illustrates a cross section through the analogous column of a member with semi-rigid end connections.

ANALYSIS BY SLOPE-DEFLECTION METHOD

An illustrative example will be presented in detail to demonstrate the application of both the slope-deflection and moment-distribution methods to the analysis of a frame with semi-rigid joints, taking into account the width of the members.

The frame shown in Fig. 8 corresponds to one of the frames actually tested (see Fig. 9), and the connection constant used in the analysis was obtained experimentally from tests of a sample joint. All of the connections were identically alike, and each beam, therefore, was individually symmetrical. The results of the connection test gave an experimental value of $\gamma = 0.01775 \times 10^{-3}$ in inch-kip units. The stiffness of the frame members was measured by bending tests preliminary to fabrication of the frame, and the quantity $E\,I$ was thus found to be $3{,}550 \times 10^3$ and $3{,}321 \times 10^3$ for the beams and columns, respectively, in inch-kip units. The net length l of the beams between connections was 168 in. − 8 in. = 160 in. The columns are continuous, and the beam connections were of the welded seat and top angle type. An approximate correction for column length may be shown to be one third of the beam depth at each end that frames with a beam (see heading "Effect of Width of Member Upon Analysis"). Hence, for the second-story columns, $l = 120 - 6.67 = 113.33$ and, for the first-story columns, $l = 120 - 3.33 = 116.67$. This correction could well be omitted with but little error.

The constant α for the beams was

$$\alpha = \beta = \frac{2\,E\,I\,\gamma}{l} = \frac{2 \times 3{,}550 \times 10^3 \times 0.01775 \times 10^{-3}}{160} = 0.7877.$$

The fixed-end moment for the loading shown is 221.0 in-kips.

Because of the individual symmetry of the beams, the slope-deflection equations in the form of Eqs. 9 were applicable. The typical equation for any beam is written by substituting the values of α, $E\,K$, b, l, etc., in Eqs. 9, which for any loaded beam results in the following:

$$\bar{M}_{AB} = \frac{2 \times 3{,}550}{160(1 + 3 \times 0.7877)}\left\{\left[\left(\frac{2 + 3 \times 0.7877}{1 + 0.7877}\right) + \frac{6 \times 4}{160} + \frac{6 \times 4^2}{160^2}\right]\theta_A\right.$$

$$\left. + \cdots + \left[\left(\frac{1}{1 + 0.7877}\right) + \frac{6 \times 4}{160} + \frac{6 \times 4^2}{160^2}\right]\theta_B\right\} - \left(\frac{221.0}{1.7877} + 6.5 \times 4\right) \quad \text{..(17)}$$

The right-hand side of this and the following equations has been divided by 1,000 to give more convenient values of θ. The moment $\bar{M}_{AB} = 34.233\,\theta_A$

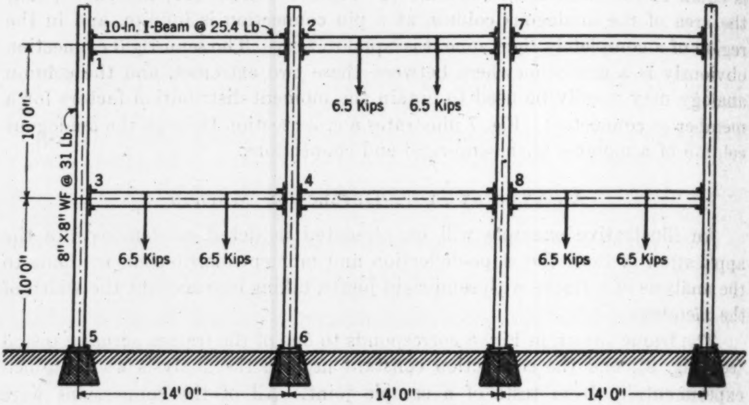

10-In. I-Beam @ 25.4 Lb
1
2
7
3
4
8
5
6
10' 0"
10' 0"
8"×8" WF @ 31 Lb.
6.5 Kips
6.5 Kips
6.5 Kips
6.5 Kips
6.5 Kips
6.5 Kips
14' 0"
14' 0"
14' 0"

FIG. 8.—TEST FRAME AND ILLUSTRATIVE ANALYSIS (LOAD SPACING: 4 FT 5 IN., 5 FT 2 IN., AND 4 FT 5 IN., EQUALS 14 FT 0 IN.)

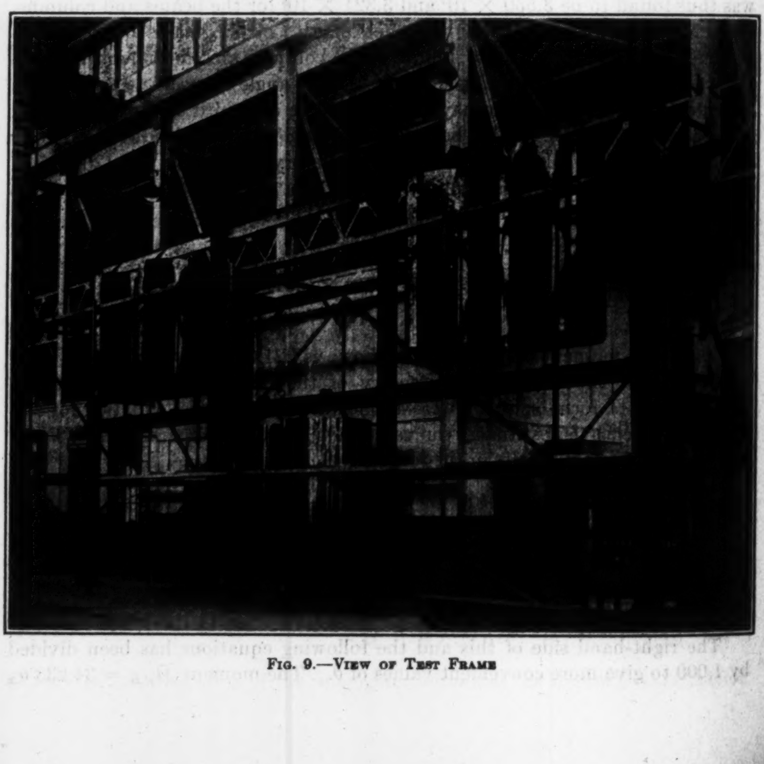


FIG. 9.—VIEW OF TEST FRAME

$+ 9.411\,\theta_B - 149.62$; and similarly:

$$\bar{M}_{BA} = 34.233\,\theta_B + 9.411\,\theta_A + 149.62 \qquad (18)$$

Equations of this type are written for all of the beams in a symmetrical half of the frame, as follows:

$$\left.\begin{aligned}
\bar{M}_{12} &= 34.233\,\theta_1 + 9.411\,\theta_2 \\
\bar{M}_{21} &= 34.233\,\theta_2 + 9.411\,\theta_1 \\
\bar{M}_{27} &= 34.233\,\theta_2 + 9.411\,\theta_7 - 149.62 \qquad (-\theta_2 = +\theta_1) \\
&= 24.822\,\theta_2 - 149.62 \\
\bar{M}_{34} &= 34.233\,\theta_3 + 9.411\,\theta_4 - 149.62 \\
\bar{M}_{43} &= 34.233\,\theta_4 + 9.411\,\theta_3 + 149.62 \\
\text{and} \quad \bar{M}_{48} &= 24.822\,\theta_4 \qquad (-\theta_4 = +\theta_8)
\end{aligned}\right\} \quad (19)$$

Similar equations for the column moments are written by making the proper substitutions in slope-deflection Eqs. 11, as follows:

$$\left.\begin{aligned}
\bar{M}_{13} &= 127.865\,\theta_1 + 69.257\,\theta_3 \\
\bar{M}_{24} &= 127.865\,\theta_2 + 69.257\,\theta_4 \\
\bar{M}_{31} &= 127.865\,\theta_3 + 69.257\,\theta_1 \\
\bar{M}_{42} &= 127.865\,\theta_4 + 69.257\,\theta_2 \\
\bar{M}_{35} &= 123.910\,\theta_3 \qquad (\theta_5 = 0) \\
\bar{M}_{46} &= 123.910\,\theta_4 \qquad (\theta_6 = 0) \\
\bar{M}_{53} &= 61.810\,\theta_3 \qquad (\theta_5 = 0) \\
\text{and} \quad \bar{M}_{64} &= 61.810\,\theta_4 \qquad (\theta_6 = 0)
\end{aligned}\right\} \quad (20)$$

The sidesway is obviously zero because of symmetry, and the only unknowns are the four angle changes θ_1, θ_2, θ_3, and θ_4. The necessary and sufficient conditions for the solution are obtained by applying the joint equilibrium equation $\Sigma M = 0$ to the four joints 1, 2, 3, and 4:

$$\left.\begin{aligned}
\bar{M}_{12} + \bar{M}_{13} &= 0 \\
\bar{M}_{21} + \bar{M}_{24} + \bar{M}_{27} &= 0 \\
\bar{M}_{31} + \bar{M}_{34} + \bar{M}_{35} &= 0 \\
\text{and} \quad \bar{M}_{43} + \bar{M}_{42} + \bar{M}_{48} + \bar{M}_{46} &= 0
\end{aligned}\right\} \quad (21)$$

Rewriting these equations in terms of the unknown θ's:

$$\left.\begin{aligned}
+ 162.098\,\theta_1 + 9.411\,\theta_2 + 69.257\,\theta_3 &= 0 \\
9.411\,\theta_1 + 186.920\,\theta_2 + 69.257\,\theta_4 &= + 149.620 \\
+ 69.257\,\theta_1 + 286.008\,\theta_3 + 9.411\,\theta_4 &= + 149.620 \\
+ 69.257\,\theta_2 + 9.411\,\theta_3 + 310.830\,\theta_4 &= - 149.620
\end{aligned}\right\} \quad (22)$$

The solution of these four simultaneous equations may be made by systematic elimination of unknowns (15) (16) or by a method of successive approximations (15). The following solution was obtained by the first method: $\theta_1 = -0.33174$;

$\theta_2 = +1.09276$; $\theta_3 = +0.62795$; and $\theta_4 = -0.74385$. These values of θ are 1,000 times the actual values but will give the correct moments when substituted in the moment equations which previously had been divided by 1,000. The actual moments at the connections may be found from the hypothetical joint center moments by computing the shears with Eqs. 7 and the connection moments with Eqs. 8. An alternate method would be to construct, graphically, the simple beam moment diagram for the full lengths L upon the joint moment base line. The connection moments then could be scaled off as the ordinate to the moment diagram at the face of the connecting member. Table 3 gives the results by the analytical method.

TABLE 3.—COMPUTATION BY SLOPE DEFLECTION

Location, joint and member	Joint moment $\overline{M}$ from slope-deflection equation	$\overline{V}$ by Eqs. 7	Connection moment M by Eqs. 8
1–2	− 1.07	−0.198	− 1.86
2–1	+ 34.29	−0.198	+ 33.50
2–7	−122.49	+6.500	− 96.49
3–4	−135.12	+6.531	−109.00
4–3	+130.06	−6.469	+104.18
4–8	− 18.46	0	− 18.46
1–3	+ 1.07	−0.515	− 0.65
3–1	+ 57.31	−0.515	+ 55.59
3–5	+ 77.79	−1.000	+ 74.46
5–3	+ 38.82	−1.000	+ 38.82
2–4	+ 88.20	−0.607	+ 86.18
4–2	− 19.44	−0.607	− 21.46
4–6	− 92.15	+1.186	− 88.20
6–4	− 45.99	+1.186	− 45.99

ANALYSIS BY MOMENT DISTRIBUTION

The factors required in the moment-distribution procedure have been presented in Tables 1 and 2. Some of the necessary computations in the following have already been made under the heading "Analysis by Slope-Deflection Method":

Semi-Fixed End Moment at Joint Center.—

$$\bar{M}_{SAB} = \frac{221.0}{1.7877} + 6.5 \times 4 = 149.62 \text{ in-kips.}$$

Carry-Over Factors Between Joint Centers.—

Beams (Both Directions).—

$$\bar{r}_{ab} = \bar{r}_{ba} = \frac{1 + 6\,(1.7877)\,\frac{4}{160} + 6\,(1.7877)\,\frac{4^2}{160^2}}{2 + 3\,(0.7877) + 6\,(1.7877)\,\frac{4}{160} + 6\,(1.7877)\,\frac{4^2}{160^2}} = 0.275.$$

Second-Story Columns (Both Directions).—

$$r = \frac{1 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)}{2 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)} = 0.542.$$

First-Story Columns (Top to Bottom) (See Table 1).—

$$r = \frac{1 + 3\left(\frac{3.33}{116.67}\right)}{2 + 6\left(\frac{3.33}{116.67}\right) + 6\left(\frac{3.33}{116.67}\right)^2} = 0.499.$$

End Rotation Stiffness at Joint Centers.—

Beams (End Bay).—

$$\bar{S}_M = \frac{2\ (3{,}550) \times 10^6}{160\ [1 + 3\ (0.7877)]} \left[\frac{2 + 3\ (0.7877)}{1.7877} + 6\left(\frac{4}{160}\right) + 6\left(\frac{4^2}{160^2}\right)\right] = 34.2 \times 10^6.$$

Beams (Modified Stiffness in Center Bay Due to Symmetry Requiring Analysis of Only One Half of Frame—Eq. 14*c*).—

$$\bar{S}_M = \frac{2\ (3{,}550) \times 10^6}{160\ (1 + 0.7877)} = 24.8 \times 10^6.$$

Second-Story Columns.—

$$\bar{S}_M = \frac{2\ (3{,}321) \times 10^6}{113.33}\left[2 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)\right] = 127.9 \times 10^6.$$

First-Story Columns (Upper End).—

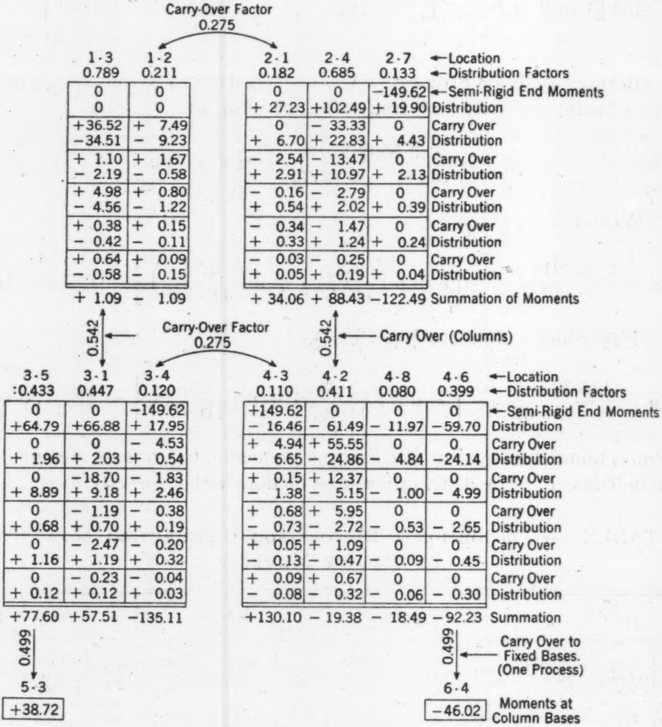
$$\bar{S}_M = \frac{2\ (3{,}321) \times 10^6}{116.67}\left[2 + 6\left(\frac{3.33}{116.67}\right) + 6\left(\frac{3.33}{116.67}\right)^2\right] = 123.9 \times 10^6.$$

Proportional factors for distributing moments to ends of members are given in Table 4. The distribution may be done either directly on a diagram of the frame in the manner frequently followed or in tabular form. The solution is herein presented in tabular form (see Fig. 10), through five cycles after the initial distribution. Each cycle consists successively of: (*a*) The carry-over of moments from the previously distributed moments; and (*b*) the distribution of the new unbalanced moment at each joint to the ends of the members. The final summation of moments may be compared with the results of the solution by the slope-deflection method, and the results are seen to check with a maximum error of two in the third significant figure, or a fraction of 1%, except in the case of the smallest moment of 1.07 in-kips,

TABLE 4.—PROPORTIONAL FACTORS FOR DISTRIBUTING MOMENTS TO ENDS OF MEMBERS

JOINT 1			JOINT 2			JOINT 3			JOINT 4		
Member	End rotation stiffness	Distribution factor	Member	End rotation stiffness	Distribution factor	Member	End rotation stiffness	Distribution factor	Member	End rotation stiffness	Distribution factor
1–3	127.9	0.789	2–1	34.2	0.182	3–5	123.9	0.433	4–3	34.2	0.110
1–2	34.2	0.211	2–4	127.9	0.685	3–1	127.9	0.447	4–2	127.9	0.411
...	.....	.....	2–7	24.8	0.133	3–4	34.2	0.120	4–8	24.8	0.080
...	.....	.....	...	.....	.....	...	.....	.....	4–6	123.9	0.399
...	162.1	1.000	...	186.9	1.000	...	286.0	1.000	...	310.8	1.000

when the error is about 2%. The moments resulting from the distribution procedure are hypothetical moments at the joint center, and the actual moments at the connection may be found by the method previously described.


Carry-Over Factor
0.275
0.542
Carry-Over Factor
0.275
0.542
Carry Over (Columns)
0.499
0.499
Carry Over to Fixed Bases. (One Process)

1·3	1·2	2·1	2·4	2·7	←Location
0.789	0.211	0.182	0.685	0.133	←Distribution Factors
0	0	0	0	−149.62	←Semi-Rigid End Moments
0	0	+ 27.23	+102.49	+ 19.90	Distribution
+36.52	+ 7.49	0	− 33.33	0	Carry Over
−34.51	− 9.23	+ 6.70	+ 22.83	+ 4.43	Distribution
+ 1.10	+ 1.67	− 2.54	− 13.47	0	Carry Over
− 2.19	− 0.58	+ 2.91	+ 10.97	+ 2.13	Distribution
+ 4.98	+ 0.80	− 0.16	− 2.79	0	Carry Over
− 4.56	− 1.22	+ 0.54	+ 2.02	+ 0.39	Distribution
+ 0.38	+ 0.15	− 0.34	− 1.47	0	Carry Over
− 0.42	− 0.11	+ 0.33	+ 1.24	+ 0.24	Distribution
+ 0.64	+ 0.09	− 0.03	− 0.25	0	Carry Over
− 0.58	− 0.15	+ 0.05	+ 0.19	+ 0.04	Distribution
+ 1.09	− 1.09	+ 34.06	+ 88.43	−122.49	Summation of Moments

3·5	3·1	3·4	4·3	4·2	4·8	4·6	←Location
:0.433	0.447	0.120	0.110	0.411	0.080	0.399	←Distribution Factors
0	0	−149.62	+149.62	0	0	0	←Semi-Rigid End Moments
+64.79	+66.88	+ 17.95	− 16.46	− 61.49	− 11.97	−59.70	Distribution
0	0	− 4.53	+ 4.94	+ 55.55	0	0	Carry Over
+ 1.96	+ 2.03	+ 0.54	− 6.65	− 24.86	− 4.84	−24.14	Distribution
0	−18.70	− 1.83	+ 0.15	+ 12.37	0	0	Carry Over
+ 8.89	+ 9.18	+ 2.46	− 1.38	− 5.15	− 1.00	− 4.99	Distribution
0	− 1.19	− 0.38	+ 0.68	+ 5.95	0	0	Carry Over
+ 0.68	+ 0.70	+ 0.19	− 0.73	− 2.72	− 0.53	− 2.65	Distribution
0	− 2.47	− 0.20	+ 0.05	+ 1.09	0	0	Carry Over
+ 1.16	+ 1.19	+ 0.32	− 0.13	− 0.47	− 0.09	− 0.45	Distribution
0	− 0.23	− 0.04	+ 0.09	+ 0.67	0	0	Carry Over
+ 0.12	+ 0.12	+ 0.03	− 0.08	− 0.32	− 0.06	− 0.30	Distribution
+77.60	+57.51	−135.11	+130.10	− 19.38	− 18.49	− 92.23	Summation

5·3	6·4	
+38.72	−46.02	Moments at Column Bases

Fig. 10.—Solution by Moment Distribution

Sidesway Induced by Unsymmetrical Vertical Loads

The writers have analyzed the frame shown in Fig. 11 by both slope deflection and moment distribution in order to study the effect of neglecting sidesway as induced by unsymmetrical vertical loads. Space does not permit the details of the analysis, which follows usual procedures, however. The dimensions of the frame and size of members are shown in Fig. 11 and the beam *a b* is assumed to carry a uniformly distributed load of one kip per foot. As in the previous case, it was assumed that the columns were fixed at the base, but the beam-column connections were assumed to have "50% rigidity," which corresponds to $\alpha = 1$. The results are presented in Table 5.

Although no general conclusions should be drawn from this single case, it is seen that in Table 5 sidesway could have been neglected without great

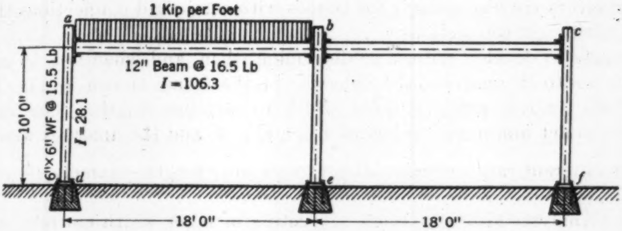


FIG. 11.—UNSYMMETRICALLY LOADED FRAME

error in the end moments. Sidesway due to vertical loads usually will be less in frames with semi-rigid connections as compared with the same frames rigidly connected, and will be further decreased in the actual structure by walls and concrete incasement.

TABLE 5.—SIDESWAY INDUCED BY UNSYMMETRICAL VERTICAL LOADS

Joint and member	By Moment Distribution			By slope deflection: Moment at joint center by slope-deflection method	Connection Moments		
	Moments at joint centers, neglecting sidesway	Sidesway moment to balance shear	Moment at joint center, corrected for sidesway		b, in in.	Shear at end of member by Eqs. 7	Moment at face of connecting member
a d	+119.02	− 6.74	+112.28	+112.32	4	−1.34	+106.96
d a	+ 59.32	−10.53	+ 48.79	+ 48.80	0	−1.34	+ 48.80
a b	−119.02	+ 6.74	−112.28	−112.32	3	+8.59	− 86.52
b a	+141.71	+ 5.10	+146.81	+146.85	3	−8.91	+120.12
b e	− 88.61	−10.20	− 98.81	− 98.88	4	+1.29	− 93.72
e b	− 44.16	−12.25	− 56.41	− 56.42	0	+1.29	− 51.26
b c	− 53.10	+ 5.10	− 48.00	− 47.97	3	+0.23	− 47.28
c b	− 7.63	+ 6.74	− 0.89	− 0.89	3	+0.23	− 0.20
c f	+ 7.63	− 6.74	+ 0.89	+ 0.89	4	+0.05	+ 1.09
f c	+ 3,80	−10.53	− 6.73	− 6.72	0	+0.05	− 6.52

EFFECT OF WIDTH OF MEMBER UPON ANALYSIS

In the analysis of frames, the length of each member is often assumed to be equal to the distance center-to-center of joints. The moments thus computed at the joint centers will usually be higher than the actual moment at the connection at the end of the member. This method of computation is usually on the safe side in determining end-connection moments but generally will be on the unsafe side in determining the maximum positive moment near the center of the beam.

An approximate correction is sometimes made for the effect of member width. The end moments computed in the foregoing manner are used to construct the moment diagram. The actual end-connection moment to be used in design is then taken as the ordinate to the moment diagram at the face of the column or connecting member. This method usually gives values of end-connection moments that are too low.

The error by either of the foregoing methods becomes greater as the ratio between the width of the joint and the length of the member increases. The average errors are also greater for frames with semi-rigid connections than for frames with rigid connections.

In order to develop criteria to determine when, and when not, to consider member width in analysis, the behavior of the frame shown in Fig. 12 was studied for various ratios of joint width to member length. The load was assumed to act uniformly on beams 1–2 and 3–4, and the analyses were made for four different ratios of joint width to member length—namely, $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{9}$, and $\frac{1}{6}$. Analyses also were made neglecting member width entirely, and the method of arbitrary correction previously outlined was tried also. The analyses were made both for a frame with rigid connections and for a frame with continuous columns but with semi-rigid beam-to-column connections. All of the analyses were made by the method of moment distribution.

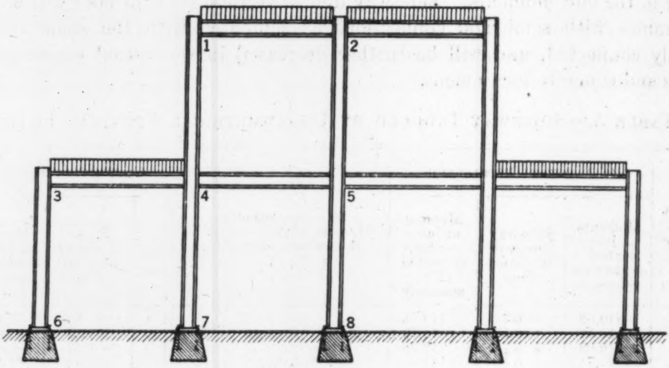


FIG. 12

Special note should be made of the length of the columns in relation to beam depth. The methods herein presented to take account of width of joint are based on the assumption that the interior of the joint may be considered infinitely rigid in comparison with the bending stiffness of the member. In the case of a beam framing into a column, this assumption seems reasonable, particularly if the column runs through the joint without a splice. In the case of the continuous column, however, the connection moments are introduced by concentrated lateral forces acting at the top and bottom of the beam in the type of connection shown in Fig. 2(*b*). In such a case it may be shown that nearly correct results may be obtained for the moments in the column by assuming a length correction for the column of one third the beam depth at each end instead of one half the beam depth. This correction was made in the analyses under consideration and was found to give good results in actual frame tests.

The results of these studies are shown in Fig. 13 for the frames with rigid and semi-rigid connections, respectively. The solid lines give the percentage of error of moments determined with a neglect of joint width as compared with corresponding moments correctly computed at the face of the joint. The broken lines give the percentage of error resulting from the arbitrary correction for joint width by neglecting it in the analysis but using the ordinate of the moment diagram at the face of the joint.

It is noted in Fig. 13 that the maximum percentage of error occurs in the case of the large end moments in the loaded beams at joints 1 and 4. It also may be seen that the errors are usually larger in the frame with semi-rigid connections than in the frame with rigid connections. The errors are appreciable even for low ratios of joint width to beam length. In the case of a one-to-twenty ratio, for example, the error may be as high as 20%, with the average error about 5% for the rigid frame and as high as 25%, and with an average error close to 10% for the frame with semi-rigid connections. As the ratio of joint width to beam length increases, the errors become increasingly larger.

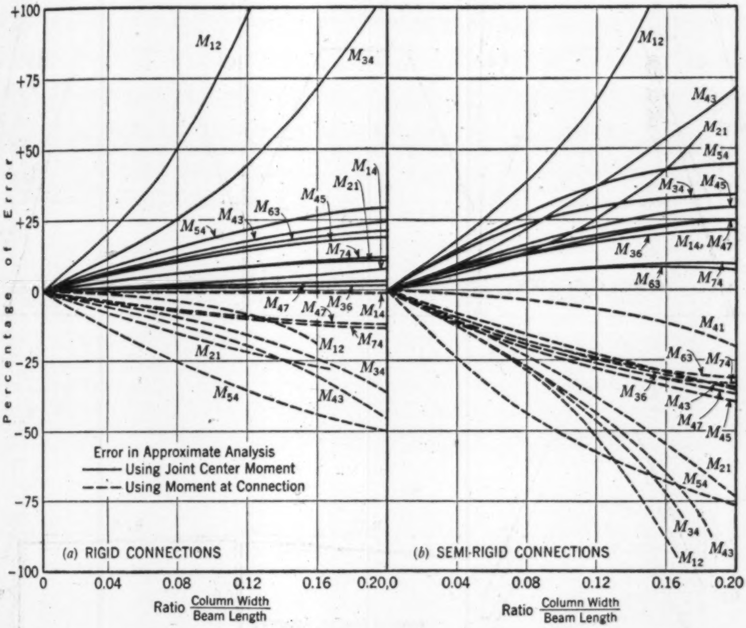


FIG. 13.—PERCENTAGE OF ERROR IN CONNECTION MOMENT, NEGLECTING MEMBER WIDTH

A fairly close approximation for the moment at the connection is obtained by neglecting joint width in the analysis and using, as the connection moment, the moment halfway between the connection and joint center.

COMPARISON BETWEEN THEORETICAL ANALYSES AND TEST RESULTS

In order to compare the results of analyses with the actual behavior of building frames, two full-size, all-welded, model building frames were con-

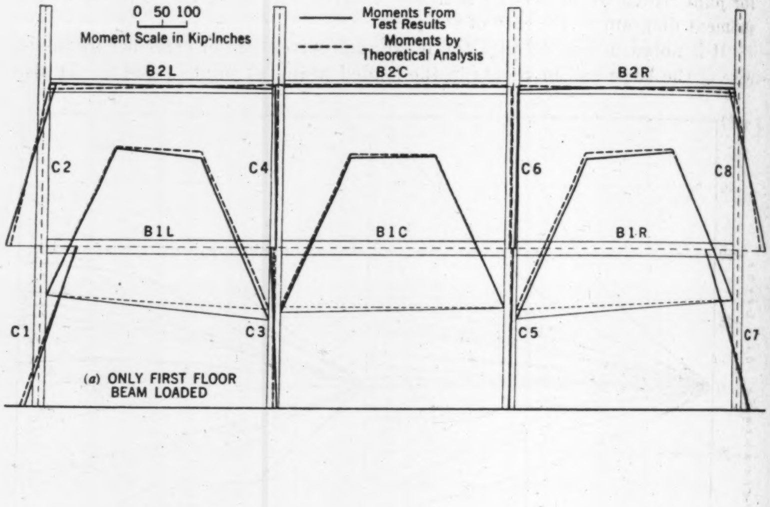
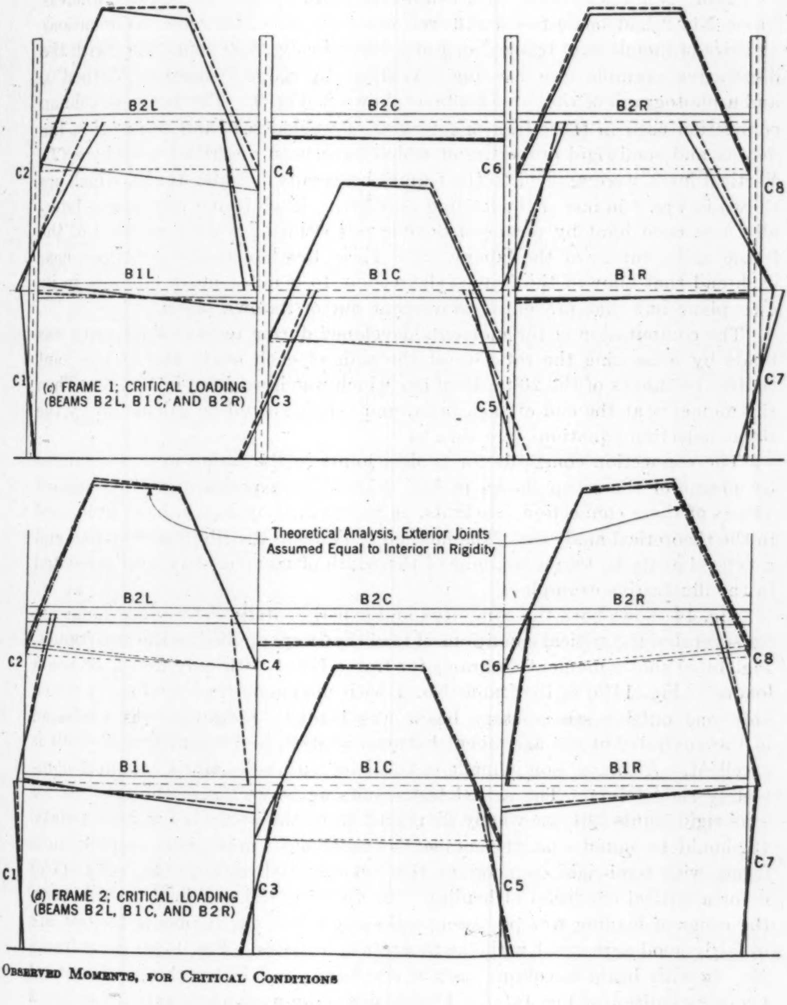

0 50 100
Moment Scale in Kip-Inches
Moments From Test Results
Moments by Theoretical Analysis
B2L
B2C
B2R
C2
C4
C6
C8
B1L
B1C
B1R
C1
C3
C5
C7
(a) ONLY FIRST FLOOR BEAM LOADED
B2L
B2C
B2R
C2
C4
C6
C8
Theoretical Analysis, Assuming 100% Connection Rigidity
B1L
B1C
B1R
C1
C3
C5
C7
(b) FRAME 1; UNSYMMETRICAL LOADING; ONLY ONE OUTSIDE SECOND STORY LOADED

FIG. 14.—COMPARISON OF COMPUTED AND


B2L
B2C
B2R
C2
C4
C6
C8
B1L
B1C
B1R
C1
C3
C5
C7
Theoretical Analysis, Exterior Joints Assumed Equal to Interior in Rigidity
B2L
B2C
B2R
C2
C4
C6
C8
B1L
B1C
B1R
C1
C3
C5
C7

(*c*) FRAME 1; CRITICAL LOADING (BEAMS B2L, B1C, AND B2R)

(*d*) FRAME 2; CRITICAL LOADING (BEAMS B2L, B1C, AND B2R)

OBSERVED MOMENTS, FOR CRITICAL CONDITIONS structed. Details of these tests have already been presented in another paper by the writers (2).

Frame No. 1 was made with beam-to-column flange connections, whereas frame No. 2 had beam-to-column web connections. The general dimensions and size of members of frame No. 1 are shown in Fig. 8 in connection with the illustrative example (see heading "Analysis by Slope-Deflection Method"), and a photograph of the same frame is shown in Fig. 9. The beam-to-column connection used in these frames consisted of welded seat and top angles, the details and semi-rigid properties of which have been described elsewhere (2). Vertical loads were applied to the frames by means of water tanks, which are shown in Fig. 9 in one of the loading positions. Each frame was braced laterally near each joint by means of flexible ties welded between columns of the frame and columns of the laboratory. These ties had reduced sections near each end that allowed the frame full freedom to bend or move laterally in its own plane but that prevented movement out of its own plane.

The computation of the moments developed during tests of the frames was made by measuring the rotation at the ends of each beam and at the joint centers by means of the 20-in. level bar which was illustrated in Fig. 3. Then the moments at the end of each beam and column could be calculated by the slope-deflection equations (see Eqs. 3).

The connection constants for typical joints in the frame were determined by means of the setup shown in Fig. 2(*b*). The experimentally determined values of these connection constants, as determined by Fig. 4, have been used in the theoretical analyses. The method of moment distribution was used and a typical analysis, taking account of the width of member, has been presented in the illustrative example.

Fig. 14 shows both the computed and experimentally determined moments for several of the critical conditions of load that were applied to the two frames. Fig. 14(*a*) shows moment diagrams for frame No. 1 with only first-floor beam loaded. Fig. 14(*b*) is for frame No. 1 with unsymmetrical loading in which only one outside second-story beam was loaded. Sidesway was neglected in the analysis but the agreement between analysis and experimental result is excellent. A comparison is made in this case with an analysis assuming completely rigid joints. The actual test results agree well with the analysis for semi-rigid joints but are widely divergent from the analysis for rigid points. It should be noted that the moments "taper out" much more rapidly in a frame with semi-rigid connections than in one with rigid joints. Fig. 14(*c*) is for a critical condition of loading. In applying the test load for this case, the order of loading was purposely unbalanced but the moments by test are in fairly good agreement with the theoretical analysis. Fig. 14(*d*) is for frame No. 2, with beam-to-column web connections, and is for the same critical loading condition as Fig. 14(*c*). The outside column connections in frame No. 2 has less rigidity than the inside, and this was taken into account in the analysis. The analysis based on the assumption that the outside joints are as rigid as the interior joints is also given, and it is seen that the test results usually fall between the two different analyses.

In general, the test results agree well with the methods of analysis which have been presented. The results also show that the test of a single joint to determine the connection constant gives a satisfactory measure of the behavior of the same type of joint used in an actual frame.

THE DESIGN OF FRAMES FOR PARTIAL RIGIDITY

The methods of analysis which have been presented in this paper obviously are not directly applicable to design. Any method of statically indeterminate analysis requires an assumed structure as a preliminary to design. To assume a building design, and then to analyze such a highly redundant structure by the methods that have been presented, would be an impractical design procedure, warranted only for very special problems.

For routine building design, a suitable method must be direct and simple in application. Such design methods have been presented by the writers in conjunction with a particular type of all-welded, beam-to-column connection (2), and in a more general article covering the application to any semi-rigidly connected structure (3). The British Steel Structures Research Committee (1) has developed design procedures for frames with semi-rigid riveted connections. In a letter dated December 26, 1940, S. D. Lash, secretary of the Subcommittee on Steel Construction, National Building Code, National Research Council of Canada, stated that simplifications in the original design method have been made in Great Britain and that similar steps are in progress in Canada.

The design procedure for the beams in welded building frames with semi-rigid connections that has been developed by the writers (2) may be outlined as follows:

1. The beams are designed by the usual procedure of computing the required section modulus for maximum simple beam moment.
2. The section modulus for maximum simple beam moment is multiplied by a reduction factor that depends on the distribution of load and relative stiffness of the simple beam and adjacent column sections. This reduction factor is obtained from a graph or simple formula and is based on the most critical combination of load possible.
3. The final beam selection is determined by the reduced section modulus found by step 2.

Although the method was developed for designing beams with welded connections, it is applicable to any frame having connections with the desired semi-rigid properties.

In computing the reduction factor, the stiffening effect of adjacent beams was neglected, and the same formula applies to exterior and interior bays. By this procedure, with end connections designed for 50% end restraint, an average saving in the weight of beams of between 15% and 20% was found possible. If greater refinement and complexity are introduced into the design procedure, the average saving in weight of beams might be raised to more than 20%.

CONCLUSION

The methods presented and corroborated by test in this paper represent a refinement in the analysis of building frames. It may be questioned whether such refinement is warranted. The concrete encasement of beams, columns, walls, and partitions, and the uncertainties of applied load, all represent indeterminate quantities which, undoubtedly, may have as great, or greater, effect upon frame behavior as does the semi-rigidity of the bare steel connection. Nevertheless, discounting these uncertainties as assets that cannot be counted upon definitely, there remains the certain dependable bare connection end-restraint. This influence can be determined and applied to the development of improved and more economical methods of design.

ACKNOWLEDGMENT

The tests of the all-welded steel building frame were part of an investigation conducted at the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., in cooperation with the Structural Steel Welding Committee of the American Welding Society. Leon S. Moisseiff, M. Am. Soc. C. E., was chairman of the committee. The writers received the cooperation of Prof. Hale Sutherland, M. Am. Soc. C. E., director of the Fritz Engineering Laboratory, and Howard J. Godfrey, engineer of tests.

APPENDIX

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DISCUSSION

Maurice P. van Buren,[4] **Assoc. M. Am. Soc. C. E.** (by letter).—A basis for effecting valuable economies in certain types of structures is afforded by this interesting paper. However, one important correction to the analysis should be made. In comparing the moments at the face and at the center of a joint, consideration must be given to the reaction as a load on the structure. It does not appear that this has been included in the examples given. It can be provided for easily by the addition of a term to both Eq. 4*a* and Eq. 4*b*. At the point of inflection of the column, the reaction is uniformly distributed, and will cause a moment about the joint center of $V\,g\,b$, in which $g\,b$ is the distance from the joint center to the centroid of the half column. This moment is of opposite sense to the moments $V\,b$, and the last terms of Eqs. 4*a* and 4*b* will then read: $-\,V_{AB}'\,(1-g)\,b_{AB}$ and $V_{BA}'\,(1-g)\,b_{BA}$, respectively. The significance of this effect is apparent in the case of the flanged column illustrated in Fig. 5(*a*), for which g approaches 1, and the term involving V practically vanishes. The authors' comments on the extent to which final results will be affected is awaited with interest.

Wayne W. Smith,[5] **Leonard P. Zick, Jr.**,[6] **Juniors, Am. Soc. C. E., and Conrad C. Wan**,[7] **Esq.** (by letter).—As the authors have shown, there are reasonable possibilities for a more economical design of frame construction by considering the semi-rigidity of connections. As they have stated, their method, in practice, is too cumbersome to be used directly. Their suggested procedure using simple beam moments and reduction constants thus appears to be the most practical method of design. However, it should be emphasized that every continuous structure or frame is an individual problem and should be dealt with as such. Graphs, charts, and simple formulas may be used to simplify certain designs, but a thorough knowledge of their limitations must guide their use.

According to the authors' test results, the maximum moment developed was about 274 kip-in. (Fig. 14(*d*)). For the 10-in., 25.4-lb, I-beam used, this meant that a stress of 11 kips per sq in. was developed, a stress considerably below the elastic limit of steel. The use of the connection constant γ seems quite proper within the "design range," but as seen in Fig. 15 the constant changes rapidly beyond this range. This design range is not clearly specified in the paper. Making the logical assumption that the design range is one half the elastic limit, there is an "overload range" for which γ is no longer constant. Failure should not occur until the elastic limit is reached, which limit is the farther boundary of this overload range. Realizing that the maximum moment does not necessarily occur at the joint, the question arises as to how important is the discrepancy due to assuming γ constant when the member is stressed nearly to the elastic limit. To ignore this point may reduce the factor of safety for the member.

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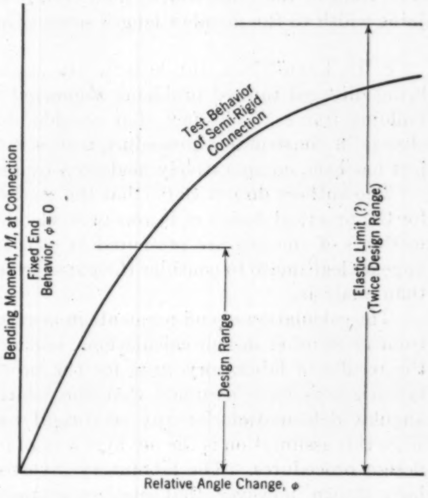

FIG. 15

It seems that the authors' suggestion of neglecting the sidesway moments due to vertical unsymmetrical loadings is contradictory to the degree of refinement in their analysis. As shown in Table 5, the sidesway moments range from 6.74 to −12.25 kip-in., which, in the latter case, amounts to a decrease of about 29%.

In the comparison by the method of moment distribution of a rigid structure and a semi-rigid structure, it is very interesting to notice the changes in the carry-over factors and the end rotation stiffness (distribution factor).

The solutions in Table 6 are for the test frame shown in Fig. 8 assuming rigid connections and neglecting joint width. The authors' results are taken from Fig. 10. As shown, the carry-over factor for the beams was decreased from the usual value of 0.5 to 0.275. Since the case of the semi-rigid beam is the opposite of the haunched beam, the results are as expected. The changes in the carry-over factors for the columns are due to the consideration of the joint width only. The results given in Table 6 for the analysis with joints 100% rigid is merely additional information. The authors have already shown the large

TABLE 6.—COMPARISON OF MOMENTS AND STIFFNESS FACTORS FOR RIGID AND SEMI-RIGID CONDITIONS

Member	JOINT MOMENT		CARRY-OVER FACTOR		DISTRIBUTION FACTOR	
	Authors	100% fixed	Authors	100% fixed	Authors	100% fixed
1–2	−1.09	7	0.275	0.500	0.211	0.443
1–3	1.09	−7	0.542	0.500	0.789	0.557
2–1	34.06	93	0.275	0.500	0.182	0.362
2–4	88.43	91	0.542	0.500	0.685	0.456
2–7	−122.49	−184	.....	.....	0.133	0.182
3–1	57.51	83	0.542	0.500	0.447	0.358
3–4	−135.11	−187	0.275	0.500	0.120	0.284
3–5	77.60	104	0.449	0.500	0.433	0.358
4–2	−19.38	−40	0.542	0.500	0.411	0.313
4–3	130.10	198	0.275	0.500	0.110	0.249
4–6	−92.23	−113	0.449	0.500	0.399	0.313
4–8	−18.49	−45	.....	.....	0.080	0.125
5–3	38.72	52	0.449	0.500	.....	.....
6–4	−46.02	−56	0.449	0.500	.....	.....

disagreement considering 100% rigid connections (see heading "Comparison Between Theoretical Analysis and Test Results").

The authors are to be commended for the close agreement between the theoretical and experimental results that they obtained. Also, in their consideration of the joint width, their results showed that for large ratios of the joint width to the member length some adjustment should be made.

S. D. LASH,[8] ESQ. (by letter).—It is gratifying to find that attention is being directed toward problems connected with the design of beams in steel building frames. The fact that considerable economies are possible without change in construction procedure makes it the more surprising that this subject has been comparatively neglected by practising engineers.

The authors do not claim that the methods presented by them are suitable for the practical design of beams in steel building frames. However, since the methods of analysis are presented as a basis for simpler design procedures, it appears legitimate to consider the paper from the point of view of design rather than analysis.

The calculation of end moments in beams with semi-rigid connections differs from most other design calculations, inasmuch as it is necessary to rely upon the results of laboratory tests for the properties of the connections. Many investigators have assumed that the relation between applied moment and angular deformation for any semi-rigid connection is approximately linear, since this assumption is the obvious way of introducing modifications of regular design procedures. The laboratory investigations referred to by the authors have shown, however, that such an assumption is incorrect for most types of semi-rigid connections. The relation between moment and angular deformation is not linear, and the behavior of a connection on first loading differs considerably from its behavior on subsequent reloadings. In an experiment on a steel frame, if the loads are applied, removed, and reapplied, measurements of strains being taken on the reloading, these strains will depend upon the reloading curves for the connections, but the total strains will be those corresponding to the initial loading curves.

Since moment-angle curves for semi-rigid connections are not linear, it would appear to be necessary to introduce some form of "limit design" when determining allowable restraining moments. Thus, for example, if a factor of safety, or load factor, of n is desired, the allowable restraining moment at working load should be $\frac{1}{n}$ times the computed restraining moment at n times the working load.

It should also be pointed out that the "connection constant γ," referred to by the authors, depends not only upon the type of connection, but also, in the case of flange-angle connections, at least, upon the depth of the beam. The constant as determined experimentally cannot be used for beams of a depth other than that used in the original tests unless suitable corrections are made.

In order to make the foregoing points more specific, Fig. 16 has been prepared using information previously published by the Steel Structures Research

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[8] Acting Secretary, National Building Code, National Research Council of Canada, Ottawa, Canada.

Committee in Great Britain.[9] Curve (1) represents the moment-angle relationship for 4-in. by 4-in. by ½-in. flange-angle connections on a beam of 12-in. depth. Actually, this curve defines a lower limit obtained by combining the curves resulting from tests on seven riveted specimens submitted by different fabricators.

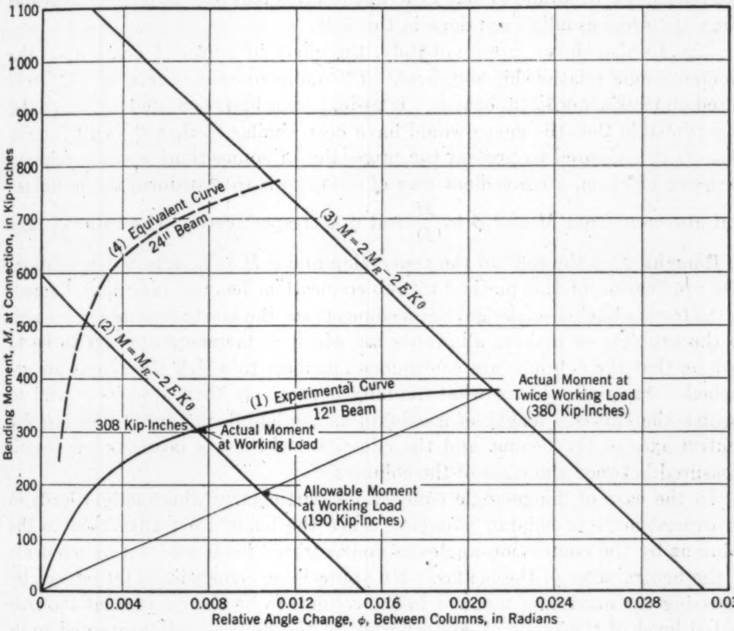

FIG. 16

For simplicity it will be assumed that a beam is attached to completely rigid columns. In such a case for symmetrical loading, the end moment may be conveniently determined by solving the equations graphically:

$$M - f(\phi) \qquad (23)$$

representing the curve of the connection; and

$$M = M_R - 2\,E\,K\,\phi \qquad (24)$$

representing the slope-deflection equation.

In Fig. 16 two lines, (2) and (3), are drawn corresponding to slope-deflection equations for two different magnitudes of load applied to a 12-in. beam 20 ft long. Line (2) corresponds approximately to a working load and line (3) to a load twice as great. In the first instance the end moment is 308 kip-in. and in the second, 380 kip-in. Thus, for a load factor of two, the allowable moment at working load should be 190 as indicated by line (5) instead of 308 kip-in. This diagram will also serve to indicate that the working range of the curve for a connection is actually quite extensive. For case shown it will extend at working loads from angular deformations of about 0.002 radians up to 0.12 radians and, if overload is being considered, up to twice these values. For this reason it is desirable to take experimental readings over a somewhat greater range than has usually been done in the past.

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[9] Final Report, Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, p. 282.

Fig. 16 also shows, approximately, the effect of depth of beam upon the moment-angle relationship obtained. The moment-angle curve (1) shown is based on tests using 12-in. beams. If 24-in. beams had been used in these tests, it is probable that the curve would have been similar to that shown by curve (4). If it is desired to present the properties of connections without relation to depth of beam, a convenient way of doing so is to transform the ordinates and abscissas from M and ϕ to $\frac{M}{D}$ and ϕD, respectively. The ratio $\frac{M}{D}$ may be thought of as the pull on the connection and ϕD as its linear deformation. The application of this method to web connections has not been investigated.

In frames having semi-rigid beam connections, the most reasonable approach to the problem of making allowance for width of members appears to be to assume that the columns are continuous members to which the beams are attached. After all, this is what actually occurs. It then appears logical to assume the effective length of a column as the length measured between the neutral axes of the beams, and the effective length of a beam as the length measured between the faces of the columns.

In the case of flange-angle connections particularly, this method leads to an overestimate of column moments, since the loads are transmitted to the columns by the connection angles as concentrated loads and not as moments at the neutral axes of the beams. No appreciable error will be introduced by assuming the maximum moment in the column to be the moment at the estimated level of the point of contraflexure of the flange angle connected to it. Although this correction may be worth making from the point of view of the design of the column, it is doubtful if it is worth making when considering the distribution of moments throughout a framework, since an appreciable rotation of the end of the beam may result from local shear deformation of the column in the vicinity of the connection. Results published elsewhere[10] have shown that this deformation may increase the total rotation of the column at the level of the beam by 30% or 40%.

The National Research Council of Canada has recently published Part 3 (Engineering Requirements) of the National Building Code (a model code for the use of Canadian municipalities). In this Code an attempt has been made to give some weight to the considerations mentioned by the authors at the beginning of their paper. In particular, it is required that bending moments in columns other than those supporting a symmetrical arrangement of beams of approximately equal span shall be investigated, and the stresses resulting from them shall be provided for. With this mandatory requirement is a permissive requirement stating that "where a beam is restrained at either end due allowance may be made for such restraint." The Code does not restrict the designer to any particular method for estimating moments in columns or restraining moments in beams, but acceptable methods of doing these things are given as Appendixes. A description of the method given for computing allowable end moments was published in 1941.[11]

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[10] Final Report, Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, p. 357.

Dean F. Peterson, Jr.,[12] Assoc. M. Am. Soc. C. E. (by letter).—The methods presented by the authors have made an excellent rational start toward a better basic understanding of what actually occurs in a loaded frame in which the members are connected in the customary way. Expressing the laboratory-determined "stiffness" of the connection in terms of the "stiffness" of the

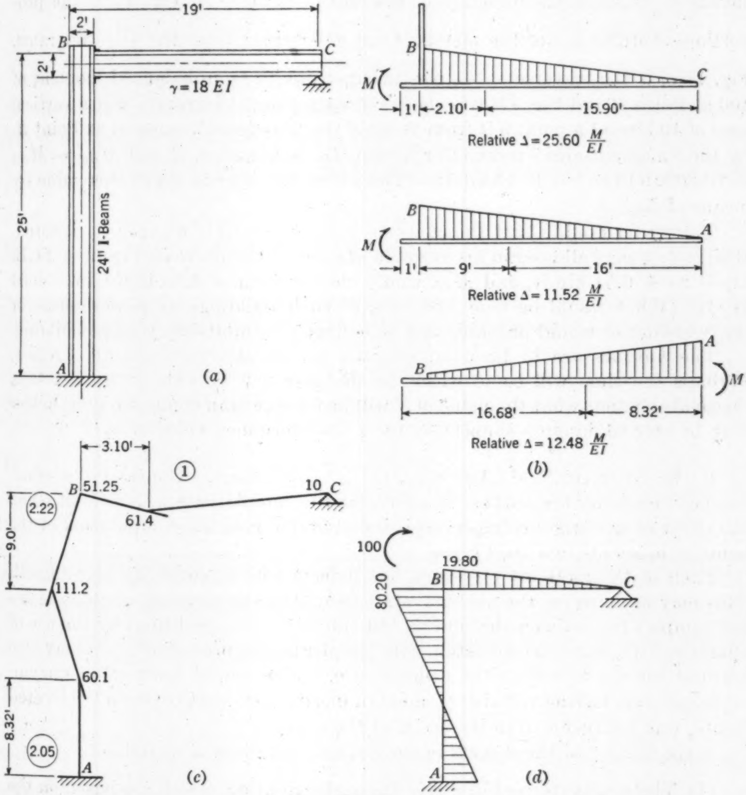
19'
2'
B
2'
C
γ = 18 EI
25'
24" I-Beams
A
(a)
B
M
C
1'
2.10'
15.90'
Relative Δ = 25.60 M/EI
B
M
A
1'
9'
16'
Relative Δ = 11.52 M/EI
A
B
M
16.68'
8.32'
Relative Δ = 12.48 M/EI
(b)
3.10'
①
B
51.25
10
C
2.22
61.4
9.0'
111.2
60.1
8.32'
2.05
A
(c)
100
19.80
B
C
80.20
A
(d)

Fig. 17

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[11] "The Design of Beams in Steel Frame Buildings," N. R. C. 992, National Research Council of Canada, 1941.

[12] Asst. Engr., Sanderson Porter, Pine Bluff, Ark.

connecting member "opens the door" for the application of the "moment-distribution" and "slope-deflection" methods. The writer believes, however, that the most direct and simple method for solving such an elastically complex structure as a building frame with semi-rigid connections is the "flexure factor" method.[13] Using this method it is unnecessary to solve either simultaneous equations or to make the rather involved adjustments of the "moment-distribution" method. Since no mention is made of the application of the "flexure factor" method, either in the text or in the references given, it may add to the completeness of the discussion to call attention to it.

To illustrate the application of this method, the writer has chosen the simple frame of Fig. 17(*a*). The "stiffness" of any member is the moment required at one end, with the other end freely supported, to produce an angle change of unity in the direction of the end tangents. In Fig. 17(*b*) the proportionate stiffnesses of the members are determined from the $\frac{M}{E\,I}$-diagrams. Fig. 17(*c*) is the traverse of the elastic curve due to an unbalanced moment of 100 at Point *B* and Fig. 17(*d*) is the resulting moment diagram. For a vertical load of 10 kips at a point 9 ft from Point *B* the "fixed-end" moment at Point *B*, by the "moment-area" method or by Eq. 13, is 13.86 kip-ft and $M_{BC} = M_{BA} = (0.8020)(13.86) = 11.15$ kip-ft. The writer was able to check this value by means of Eqs. 4.

The writer doubts that the effect of sidesway should be generally ignored. If sidesway were allowed in his example M_{BA} would be decreased from $+ 11.15$ kip-ft to $+ 6.54$ kip-ft, and M_{AB} would change from $+ 5.55$ kip-ft to $- 6.54$ kip-ft. What would be true, however, of such buildings as powerhouses or low warehouses would probably not be so true of a multi-story office building.

The authors are to be complimented on an excellent research project. Perhaps the time will come when the designer will be able to predict to a reasonable extent what the value of γ will be for a certain connection, or, better still, be able to design a connection for a predetermined value of α.

R. W. Stewart,[14] M. Am. Soc. C. E. (by letter).—Considerable effort has been made by the authors to overcome the complexities which result when the effect of semi-rigid connections is incorporated into the computation of the bending moments in a steel frame.

Each of the methods presented has defects which render its use difficult. This may account for the authors' statement that the methods of analysis are too complex for ordinary design use, but can be made expeditious by the use of charts and diagrams in connection with simpler design procedures. It may also account for the fact that the general case, which would occur if an unsymmetrical beam having variable moment of inertia were used between the riveted joints, was not included in the scope of the paper.

A specific list of the defects in the authors' solutions is as follows:

(1) The constants used in end-moment distribution, which are based on the

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[13] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

[14] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

moment required to produce unit rotation at one end of a beam when the other end is fixed, require long series of algebraic terms to express their values if a member is affected by asymmetry or other special conditions like the yielding joints treated by the authors. A glance at the right-hand column and then at the left column (Case I) of Table 1 will disclose how the expressions for moment-distribution constants expand when conditions other than the simplest are introduced. If these constants were further encumbered by unsymmetrical tapering members between the riveted joints, the computation of α and β would become complex, making the use of the formula very tedious and difficult.

(2) The slope deflection equations (4a and 4b) are based on $K = \frac{I}{l}$-values which are applicable only to members of uniform section between joints. The introduction of unsymmetrical tapering members would add substantially to the difficulty of computing the necessary constants.

(3) The use of the authors' methods independently of a set of charts pertaining thereto would require having at hand, for reference, formulas that are too complicated to remember.

(4) The authors' methods offer no easy facility for detecting errors which may occur in computations.

The following solution of Fig. 11 entirely eliminates defect 3 attributed to the authors' methods, as any one familiar with this method of attack would not use any references to solve any of the authors' problems, except a steel handbook giving the moments of inertia of rolled beams. It greatly alleviates defect 4 in that an automatic check involving a single setting of a slide rule will verify a large portion of the solution. It alleviates to a considerable degree defects 1 and 2 by using "illustrated" member constants which are simpler than moment-distribution constants and which can be altered without difficulty to include the general case of unsymmetrical members between joints. Fig. 18(a) represents an $\frac{M}{I}$-diagram due to a moment at one end of a beam shown in Fig. 11, the other end being hinged. It requires no explanation; Fig. 18(b) is the appurtenant graph of the tangents to the elastic curves in the beam, known as a traverse of the elastic curves, in which Δ represents the curvature between joints. To evaluate ϕ, which represents the angle of yield at a joint, Eqs. 2 and 5 yield

$$\alpha = 2\,E\,K\,\frac{\phi}{M} \qquad \text{(25)}$$

For the flexure of a beam of constant section, hinged at one end, $\Delta = \frac{M}{2\,E\,K}$. For $\alpha = 1$ (which is the condition for Fig. 11), by solving each of these equations for M, it is found that $\phi = \Delta$ for the case in which Δ is the curvature represented by a triangular $\frac{M}{I}$-diagram. For Fig. 18(b), ϕ will equal the area of the $\frac{M}{I}$-diagram under the dotted line in Fig. 18(a). With this explanation and the traverse principle[15] that in traverse triangles the lengths of the sides (considered as their horizontal projections since altitudes are negligible) are proportional to the opposite angles, Fig. 18(*b*) can be readily sketched and evaluated. The difference between Figs. 18(*a*) and 18(*b*) is of interest. Fig. 18(*a*) is a moment-area diagram that cannot show joint yield angles or joint rotation angles. Fig. 18(*b*) is a traverse diagram that shows all the elements of the flexure. The moment-distribution constants in Table 1 are necessarily based on flexure due to the existence of moments at both ends of a beam which is why they become more complicated for unsymmetrical conditions than Fig. 18(*b*) which deals with a moment at one end only. The stiffness factor computed from Fig. 18(*b*) is first computed as the moment necessary to give the value of unity to the angle shown as $\frac{432}{216}\Delta$, which represents the over-all curvature in the beam. Top and bottom stiffnesses for the columns are computed by a procedure that is similar but is simplified because there are no ϕ-angles in the columns. After all stiffness factors are computed, they are multiplied by a factor which will reduce the deck stiffness to unity in order to simplify subsequent arithmetical work.

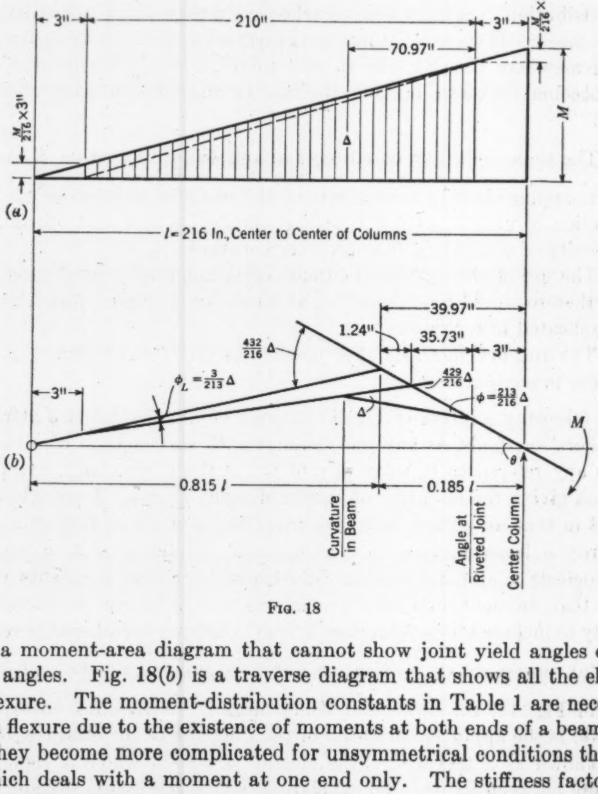
3"
210"
3"
$\frac{M}{216}\times 3$
70.97"
$\frac{M}{216}\times 3"$
Δ
M
(a)
l = 216 In., Center to Center of Columns
39.97"
1.24"
35.73"
3"
$\frac{432}{216}\Delta$
$\phi_L = \frac{3}{213}\Delta$
$\frac{429}{216}\Delta$
$\phi = \frac{213}{216}\Delta$
3"
Δ
M
θ
(b)
0.815 l
0.185 l
Curvature in Beam
Angle at Riveted Joint
Center Column

FIG. 18

The computation for Fig. 19(*a*) is a much faster procedure than the uninitiated would suspect. When the traverse computation arrives at the top of a column both column moments are obtained by one setting of the slide rule as they bear the same ratio to the moments written on column *cf* that the angle at the top of the column bears to the angle at the top of column *cf*.

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[15] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

Fig. 19(*b*) is nearly all copied from Fig. 19(*a*). The computations for Fig. 19(*b*) take less than one minute. As soon as they are complete a single slide-rule setting will disclose whether either Fig. 19(*a*) or 19(*b*) contains an error.

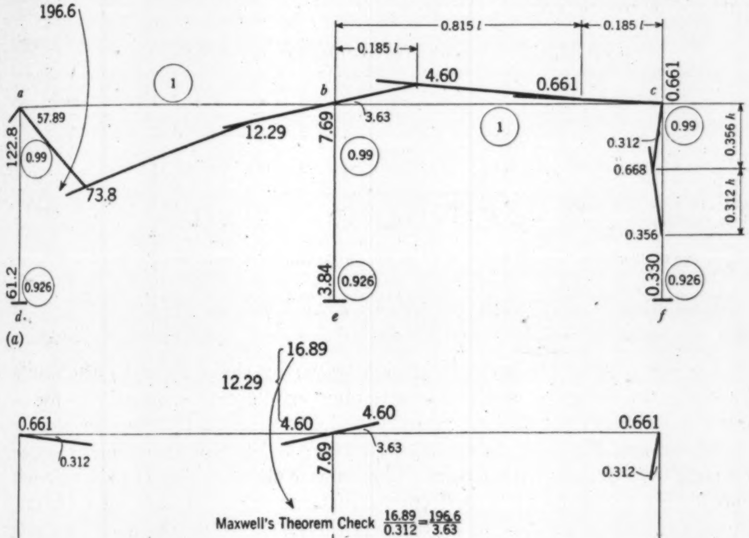
196.6
0.815 *l*
0.185 *l*
0.185 *l*
4.60
0.661
0.661
a
b
c
1
1
57.89
122.8
0.99
12.29
7.69
3.63
0.99
0.99
0.312
0.356 *h*
0.668
73.8
0.312 *h*
0.356
61.2
0.926
3.84
0.926
0.330
0.926
d
e
f
(*a*)
16.89
12.29
0.661
4.60
4.60
0.661
0.312
7.69
3.63
0.312
Maxwell's Theorem Check $\frac{16.89}{0.312}=\frac{196.6}{3.63}$
0.330
3.84
0.330
(*b*)
$1.293\,\theta_1+0.293\,\theta_2$
$1.293\,\theta_2+0.293\,\theta_1$
0.815 *l*
0.185 *l*
0.185 *l*
θ_1
1
0.305
0.305
1
0.408
0.610
0.408
0.99
0.99
θ_2
0.99
0.356 *h*
$1-2.135\,\theta_1$
$1-2.135\,\theta_2$
0.312 *h*
0.312
$1-1.135\,\theta_1$
$1-1.135\,\theta_2$
0.636
0.741
Shear $=\frac{3.45}{h}$
0.636
0.926
0.926
0.926
(*c*)

FIG. 19

There is more than one way to compute sidesway moments using the traverse method. Fig. 19(*c*) represents an elastic curve traverse converted into an amplified form of slope deflection which can be applied to the two-story structures treated in the authors' paper. The key to this method is to express the values of the angles which govern the stiffnesses in terms of the θ-angles and then proceed as in slope deflection.

Table 7 shows the moments computed from Fig. 2. The check with the authors' moments is exact.

TABLE 7.—MOMENTS COMPUTED FROM FIG. 19

Description	*d*	*a*	*ba*	*be*	*e*	*bc*	*c*	*f*	Shear
$\frac{179.38}{196.6}$ × Fig. 19(*a*)	55.8	112.0	11.2	7.0	3.5	4.2	0.6	0.3	158.2 + *h*
$\frac{179.38}{16.89}$ × Fig. 19(*b*)	3.5	7.0	130.5	81.6	40.7	48.9	7.0	3.5	101.3 ÷ *h*
Moments without sidesway	59.3	119.0	141.7	88.6	44.2	53.1	7.6	3.8	56.9 ÷ *h* (check)
Johnston-Mount	59.3	119.0	141.7	88.6	44.2	53.1	7.6	3.8	....
Sidesway Correction:									
$\frac{56.9}{3.45}$ × Fig. 19(*c*)	10.5	6.7	5.1	10.2	12.2	5.1	6.7	10.5	56.9 ÷ *h*
Moments with sidesway	48.8	112.3	146.8	98.8	56.4	48.0	0.9	6.7	Zero
Johnston-Mount	48.8	112.3	146.8	98.8	56.4	48.0	0.9	6.7	....

To summarize: The methods demonstrated by the authors to establish a basis for the design of structures with semi-rigid joints required the use of derived constants and derived formulas which are inherently very complex.

The use of the basic constants of flexure[16] in an orderly manner assisted by a pictorial graph of the flexure will eliminate the necessity of making reference either to special slope deflection equations or to formula for constants in an independent solution of problems of this class. ("Independent" solution means one not dependent on a set of charts and diagrams prepared by some one else. In court testimony in a building failure case the solution should be independent.)

It will also eliminate most of the anxiety regarding the possibility of errors.

JAROSLAV J. POLIVKA,[17] M. AM. SOC. C. E. (by letter).—A complete analysis of building frames with semi-rigid connections is presented in this interesting and valuable paper. The authors have corroborated their analytical methods by tests. Both methods applied (slope deflection and moment distribution) have the same disadvantage—namely, that the computations must be made separately for each type of loading, and the exact and refined method, demonstrated by the authors, becomes complex and tedious, even for such a simple case as a two-story building with three equal bays (Fig. 8). The authors agree that the method is not applicable to ordinary design. Three types of loading (Fig. 14) require the determination of sixteen unknown slopes and this number increases to forty eight for an unsymmetrical four-story building.

The writer has attempted to find the answer to three questions relating to the analysis of building frames with semi-rigid connections:

(1) How are the results affected by certain assumptions that would simplify the analysis considerably?

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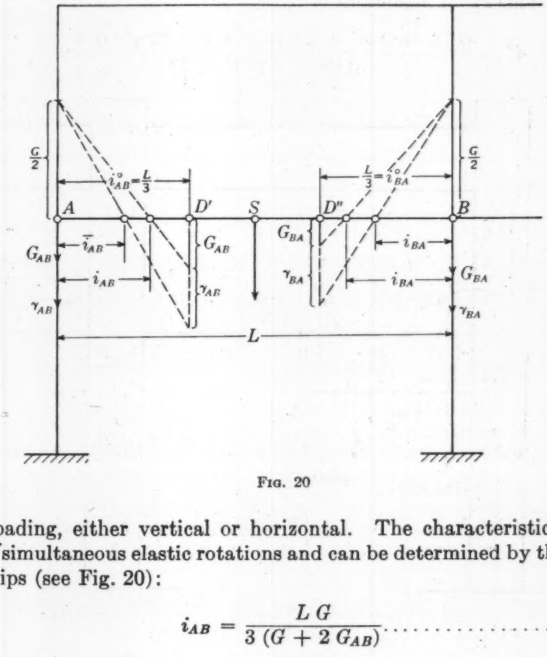
[16] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 41–44.

[17] Research Associate, Univ. of California, and Cons. Engr., Berkeley, Calif.

(2) Is it possible to determine certain constants of a given building frame that would be applicable for any type of loading?

(3) What are the short cuts of a general exact method permitting sufficient accuracy in the analysis?

Using the method of elastic weights,[18] two characteristic points for each structural member can be determined. The characteristic points are constants of the building frame and represent sufficient data to analyze the frame for any type of loading, either vertical or horizontal. The characteristic points are centers of simultaneous elastic rotations and can be determined by the following relationships (see Fig. 20):



$\frac{G}{2}$
$\overset{\circ}{i}_{AB} = \frac{L}{3}$
A
D'
S
D''
B
$\frac{L}{3} = \overset{\circ}{i}_{BA}$
$\frac{G}{2}$
G_{AB}
$\bar{i}_{AB}$
G_{AB}
i_{AB}
γ_{AB}
γ_{AB}
G_{BA}
γ_{BA}
i_{BA}
i_{BA}
G_{BA}
γ_{BA}
L

FIG. 20

$$i_{AB} = \frac{L\,G}{3\,(G + 2\,G_{AB})} \qquad (26a)$$

and

$$i_{BA} = \frac{L\,G}{3\,(G + 2\,G_{BA})} \qquad (26b)$$

in which, in addition to the notation of the paper, G is the elastic weight of the beam $\left(= \frac{L}{E\,I} \right)$; and G_A and G_B are the elastic weights of the supports A and B. Angular rotations at A and B due to a unit moment are G_{AB} and G_{BA}.

For semi-rigid connections Eqs. 26 become

$$i_{AB} = \frac{L\,G}{3\,[G + 2\,(G_{AB} + \gamma)]} \qquad (27a)$$

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[18] "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by J. J. Polivka, Berkeley, Calif., 1940 and 1941.

and

$$i_{BA} = \frac{L\,G}{3\,[G + 2\,(G_{BA} + \gamma)]} \quad \text{(27b)}$$

In the simple case in which the width of a member is assumed equal to zero (Fig. 20), the centers of rotation D' and D'' relating to rigidly restrained supports at A and B are distant $\frac{L}{3}$ from the supports.

For a finite member width, this distance can be determined graphically, as shown in Fig. 21, or algebraically by

$$i^\circ = \frac{G\,[(l + 2\,b)^2 + 2\,b\,(l + b)] + 12\,\gamma\,b\,(l + b)}{3\,(G + 2\,\gamma)\,(l + 2\,b)} \quad \text{(28)}$$

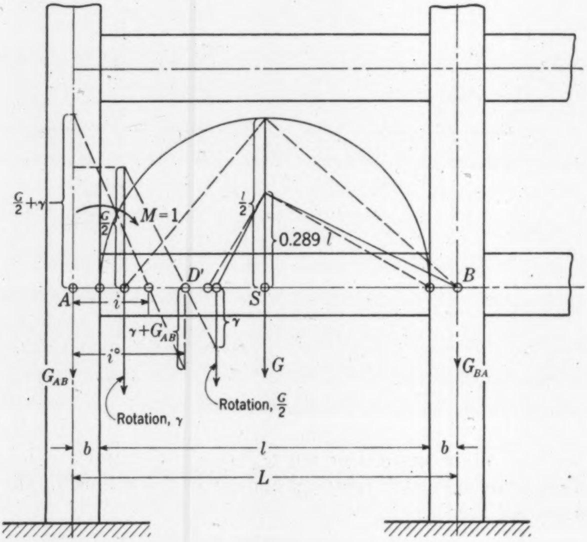
$\frac{G}{2}+\gamma$
$\frac{G}{2}$
$M=1$
$\frac{l}{2}$
$0.289\,l$
D'
B
A
i
S
$\gamma+G_{AB}$
γ
i°
G_{AB}
G
G_{BA}
Rotation, γ
Rotation, $\frac{G}{2}$
b
l
b
L

Fig. 21

Using the values in the authors' example—$G = 0.04507$, $l = 160$ in., $b = 4$ in., and $\gamma = 0.01775$—the distance $i^\circ = 36.223$ in. Since the carry-over factor is

$$r = \frac{i^\circ}{L - i^\circ} \quad \text{(29)}$$

the accuracy of this method may be checked with the results obtained by the authors: $i^\circ = \frac{36.223}{131.777} = 0.27488$ (compared with 0.275 found by the authors).

In the example discussed (Fig. 8), the elastic weight of support of the beam 3-4 results in

$$G_{3-4} = \frac{G_{3-5}\,G_{3-1}}{G_{3-5} + G_{3-1}} \quad \text{(30)}$$

That is, $G_{3\text{-}4} = \dfrac{0.00878 \times 0.01008}{0.00878 \times 0.01008} = 0.00469$ and the center of resultant rotation is determined by $i = i^{\circ}\,\dfrac{G + 2\,\gamma}{G + 2\,(G_{3\text{-}4} + \gamma)} = 32.444$ in. Practically the same value is obtained using the simple term $i = \dfrac{L\,G}{3\,[G + 2\,(G_{3\text{-}4} + \gamma)]}$

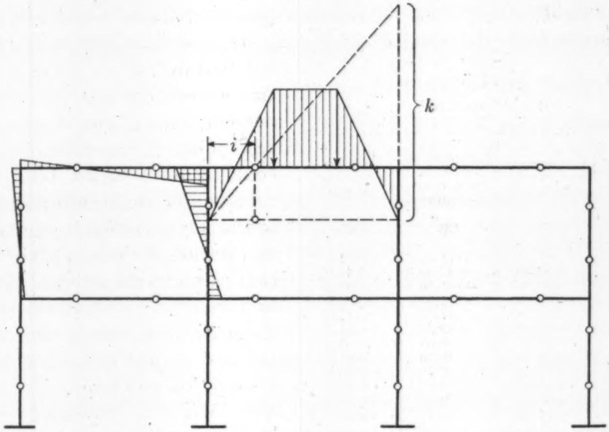

Fig. 22

$+ b = 28.058 + 4 = 32.058$ in., the difference for this short cut being only 1.2% affecting the center moment of the beam on the side of safety. This simplification eliminates entirely the complex term in Eq. 30.

Knowing the characteristic points of each structural member determining all carry-over factors, the moment diagrams for any type of loading can readily be plotted, as shown in Fig. 22.

Bruce Johnston,[19] Assoc. M. Am. Soc. C. E., and Edward H. Mount,[20] Esq.—The worth-while discussions contributed to this paper are appreciated.

Mr. van Buren assumes uniform distribution of the beam reaction in the column at a point of inflection and then concludes that the "reaction as a load on the structure" should be considered to act at the centroid of the half-column width adjacent to the beam. This would produce a moment at the joint center equal to "$V\,g\,b$, in which $g\,b$ is the distance from the joint center to the centroid of the half column." If a complete free-body diagram is drawn of the end of the beam, extended just to the column center line, it will be obvious that the moment $V\,g\,b$ is balanced by an equal and opposite moment resulting from normal stresses acting on the vertically cut section of the column web. These stresses are in equilibrium with the section of the column cut away from the imaginary free body.

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[19] Engr., Bureau of Yards and Docks, U. S. Navy, Washington, D. C.

[20] Formerly Am. Welding Soc. Research Fellow at Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

It should be emphasized that the "hypothetical" moment at the joint center is a pure fiction, created simply for convenience of analysis, as a result of which moments are balanced at the joint center without separate consideration of the shears. The actual distribution of stresses acting in and around the joint is very complex. Exact stress analysis is out of the question and the general behavior of the connection in so far as it affects frame behavior is of more importance than a complete knowledge of stresses.

In a "rigid" frame joint, the reaction stress introduced into a column by a beam (apart from the moment producing stresses) would be uniformly distributed over the column cross section a short distance below the connection; but, in the case of a semi-rigid joint, the reaction of the beam could cause a slight tilting of the column and would shift the resultant of the column stress (again assuming no rotation of the end of the beam—that is, no frame action). The reaction would not shift all the way from the center to the column face, but this amount of shift makes a convenient assumption and happens to coincide with Mr. van Buren's suggestion. Table 8, Col. 2, presents the results of an analysis of the frame as loaded in Fig. 8, giving the results of the writers' original analysis taken from Table 3. Col. 3 is for the analysis with column reactions assumed at the connection face (as suggested by Mr. van Buren for flanged columns), and Col. 4 gives moments actually determined by test. The results are not very consistent, but may be compared on the basis of the following percentage of summed differences:

TABLE 8.—COMPARISON OF MOMENTS

Location, joint and member (1)	Connection moment, by Table 3 (2)	Connection moments, revised assumptions (3)	Average moments, by test (4)
1-2	−1.86	−1.36	−1.9
2-1	+33.50	+27.83	+41.9
2-7	−96.49	−101.10	−103.5
3-4	−109.00	−111.57	−102.9
4-3	+104.18	+107.50	+117.0
4-8	−18.46	−15.28	−21.3
1-3	+0.65	+0.68	+1.6
3-1	+55.59	+46.79	+61.5
3-5	+74.46	+62.76	+70.9
5-3	+38.82	+31.97	+28.7
2-4	+86.18	+72.47	+89.1
4-2	−21.46	−16.74	−32.8
4-6	−88.20	−74.69	−88.5
6-4	−45.99	−38.05	−37.2

$$\frac{\Sigma(\text{Difference between Col. 4 and Col. 3 or Col. 2})}{\Sigma\ \text{Col. 4}} \times 100$$

This percentage of error in the summed differences gives most weight to the moments of largest magnitude. The percentage of the summed differences is 9.95% for the writer's original analysis (Col. 2), and 15.05% for the assumption of column reaction at column face (Col. 3).

Although the analysis assuming column reaction at column face appears to be definitely less in agreement with test results, it is perhaps more important that the difference between the two is less than the percentage of error in the summed differences by the writers' original analysis. It should be added that the percentage of error in the summed differences of a "rigid frame analysis" is 58.61% (see analysis in discussion by Messrs. Smith, Zick, and Wan). Obviously, no analysis, however complicated, can ever account for variations in fabrication, and all of the other factors that affect the behavior of the actual bare frame, to say nothing of what finally happens in the actual building when cross framing, fireproofing, walls, and partitions are added. Nevertheless, the behavior of the bare frame represents a dependable minimum on which to base a more rational design procedure than that commonly used at present.

Messrs. Smith, Zick, and Wan, in their discussion, state: "It seems that the authors' suggestion of neglecting the sidesway moments due to vertical unsymmetrical loadings is contradictory to the degree of refinement in their analysis." The writers' statement was not intended to apply to analysis but rather to design, and particularly to design of sections of multi-story tier buildings where enough duplication and regularity make semi-rigid design a practicable possibility. In such cases local tendencies toward sidesway will be absorbed largely by adjacent parts of the building frame. For example, the agreement between test results and theoretical analysis for the unsymmetrical loading in Fig. 14(*b*) should be noted. Sidesway was neglected in this analysis. Hence, refined methods of analysis may be used to show that sidesway in some cases may be neglected in simplified and necessarily less refined procedures for design. In cases where sidesway affects analysis to an appreciable degree, as in the illustrative example, Fig. 11, there probably would not be enough duplication of structure to make actual design for semi-rigid behavior an economical procedure.

Mr. Lash provides a stimulating discussion on the subject of design. Messrs. Smith, Zick, and Wan also discussed the design aspects of the nonlinear relation between moment and angle change in a semi-rigid connection. The general subject of design is one that requires much more study before complete, simple, and practical rules may be specified for all load conditions, types of structure, and connection. The writers avoided this subject, their primary object being to set up methods of analysis that might be used as a basis from which to build rational procedures of design. One of the writers, in collaboration with Robert A. Hechtman, Jun. Am. Soc. C. E., has presented elsewhere a simple approach to the design problem (3).

Mr. Hechtman, in collaboration with one of the writers, recently completed a program of tests on forty-seven different riveted beam-column connections with beam depths ranging from 12 in. to 24 in. These tests have been sponsored at Lehigh University by the American Institute of Steel Construction. Preliminary studies of these test results indicate that the procedure of estimating the behavior of a 24-in. beam connection from test results obtained with a 12-in. beam connection is misleading, if not entirely unsatisfactory. This procedure was used in the reports of the Steel Structures Research Committee of Great Britain, as illustrated by Mr. Lash in Fig. 16. The M-ϕ relationship of a top and seat angle beam connection is influenced by size and arrangement of rivets, thickness of seat and top angles, thickness of beam and column flanges, and other factors, many of which cannot be similar in the 12-in. and 24-in. beam connection because of practical design requirements.

The beam lines for symmetrical loads, as shown by Mr. Lash in Fig. 16, originally suggested as a criterion of design requirement by Cyril Batho,[21] have been used to advantage in reporting recent tests of welded beam connections.[22] These tests and later tests now in progress indicate the possibility of contriving welded semi-rigid connections that will develop nearly linear behavior up to a moment which gives an adequate factor of safety.

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[21] "Investigations on Beam and Stanchion Connections," by C. Batho and H. C. Rowan, Second Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, pp. 61–137.

It should be noted that the "beam line" shown by Mr. Lash in Fig. 16 is for "constant beam load." If the semi-rigid connections are actually considered in design, the load capacity of the beam will increase as the connection stiffness increases. If the columns are assumed not to rotate, and the semi-rigid connections are counted upon to their full capacity, the design will be for "constant maximum beam stress," and the design requirements are then more severe than for "constant beam load."

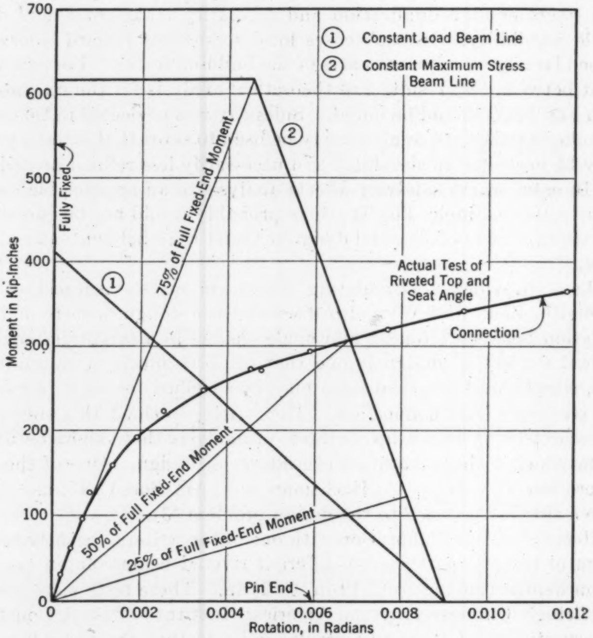

FIG. 23.—DESIGN REQUIREMENTS FOR 12 WF 25 BEAM

The beam-line equation for "constant beam load" has been noted by Mr. Lash in Eq. 24 ($M_R = M + 2\,E\,K\,\phi$). Eq. 24 is plotted as curve (1) in Fig. 23 for a 12 WF 25 beam (12-in., 25-lb wide flange I-beam) for $\frac{l}{d} = 20$. For the condition of constant maximum beam stress, M_R is not a constant. If the maximum design stress of 20 kips per sq in. is developed, the following equation gives M_R for the uniformly loaded beam:

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[22] "Tests of Miscellaneous Welded Building Connections," by Bruce Johnston and Gordon R. Deits, *The Welding Journal*, Vol. 21, No. 1, January, 1942, Research Supplement, pp. 5-s to 27-s.

$$M_R = \tfrac{2}{3}(20\,S + M) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

in which S is the section modulus of beam. Substituting Eq. 31 in Eq. 24, the beam-line equation for "constant maximum beam stress" is obtained:

$$M + 6\,E\,K\,\phi = 40\,S \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Eq. 32 is plotted as curve (2) in Fig. 23. The beam line, curve (2), is cut off by a horizontal line corresponding to joint rigidities equal to or greater than 75%, in which case the end moment governs the design. The experimental curve in Fig. 23 represents results of a typical connection test of a riveted seat and top angle connection, using a 12 WF 25 beam, as tested at the Fritz Laboratory.

In the foregoing, the columns are assumed not to bend or rotate at the joint. When column bending due to unbalanced beam loads is allowed for, the design loads will be less than assumed in Eq. 32. In such cases the design requirements usually will be somewhere between beam curves (1) and (2), Fig. 23. The beam line for uniform load and "constant maximum stress," therefore, seems to be a good criterion of connection performance for actual semi-rigid design.

When column bending is allowed for, there may be some question as to whether or not it is necessary to penalize the design load by the straight-line construction shown in Fig. 16, drawn from the "twice working load" beam line, because the design requirements will be modified by column rotation. However, in order to maintain a safe structural load factor, some reduction in design value must be made if the M-ϕ curve is decidedly nonlinear up to a beam line constructed on the basis of (working load) × (factor of safety). Mr. Lash has suggested a factor of safety of 2, in Fig. 16, and this is a conservative value.

The discussions by Messrs. Peterson and Stewart both claim advantages for the "flexure factor" method of analysis as compared with moment distribution or slope deflection. In Table 7, Mr. Stewart[13] shows an exact check by the flexure factor method with the writers' illustrative example, Fig. 11.

Any designer who is actively engaged in analyzing continuous structures will be well repaid by a thorough study of the flexure factor method, as is attested by the discussions of Mr. Stewart's paper[13] by a number of designing engineers who have used it in their work. The method is particularly advantageous if variable load conditions are to be considered on any one structure.

In spite of the advantages of the flexure factor method as an analytical tool, the ultimate application to semi-rigid design probably will be made only in cases of multi-story buildings with regular repetition of bays, or in the case of a large factory with similar repetition of design in roof construction. In the case of the multi-story building, the design probably will be made by means of standardized connections and simplified design rules, and the designer will not make a complete analysis at all, either by the writers' method or by Mr. Stewart's method. It appears to the writers that the slope-deflection method will be the most suitable in establishing general design formulas for typical and limiting conditions, thereby leading to simplified design rules or charts.

Mr. Polivka has shown, in his well-written discussion, how readily the graphical method of "characteristic points" lends itself to the semi-rigid analysis problem. His discussion, like Mr. Stewart's, is a valuable addition to what might be termed a symposium of different methods of semi-rigid frame analysis. Perhaps it would be better to say "techniques" than "methods," since all are merely special ways of determining terminal moments in framed members, with the fundamental basis in each case being expressed by

$$\frac{M}{E\,I} = \frac{d^2y}{dx^2} \quad \text{(33)}$$

which is the well-known equation of the bent beam.

In conclusion, the writers wish to emphasize that the important problem is not analysis, but design. More rational methods of analysis are significant only to the extent that they result in more rational methods of design, thereby reducing the "factor of ignorance" and effecting greater economy in the use of materials.

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TRANSACTIONS

Paper No. 2153

SURFACE RUNOFF DETERMINATION FROM RAINFALL WITHOUT USING COEFFICIENTS

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Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. L. L. Harrold, C. E. Ramser, LeRoy K. Sherman, A. J. Schafmayer, C. S. Jarvis, G. W. Musgrave, F. L. Flynt, W. I. Hicks, Albert S. Fry, and W. W. Horner and S. W. Jens.

Synopsis

In hydraulic engineering practice, the relation between rainfall and runoff has generally been represented as a ratio or coefficient. It has been recognized that the form of this relationship should be "rainfall minus losses equals runoff." Heretofore the inadequacy of hydrologic data has discouraged attempts to evaluate losses as they occur during a storm period. In this paper the writers call attention to the recent improvement in hydrologic data with respect to precipitation and stream flow, and to the information with respect to infiltration that has developed from the research program of the U. S. Department of Agriculture; and they outline a method of applying this information to the evaluation of surface runoff from precipitation data without the use of a coefficient. The method is presented as being generally applicable to all drainage basins, and is described in detail as it would be used in urban storm drainage.

PART I.—GENERAL PRINCIPLES

Introduction

Fig. 1 shows two patterns of one-day storms, each producing a total precipitation of 6 in. On these storm patterns have been indorsed the values of infiltration capacity that might prevail in a midwestern drainage basin, under average agricultural land use, in midsummer. The two infiltration capacity curves are basically the same, but appear differently on the two diagrams as a result of the difference in precipitation pattern.

Note.—Published in April, 1941, *Proceedings*.

[1] Cons. Engr. (Horner & Shifrin), St. Louis, Mo.

[2] Associate Consultant, W. W. Horner & H. Shifrin, St. Louis, Mo.

The shaded area represents, by hourly mean values, the excess of precipitation rate over infiltration capacity rate, and therefore the hourly production of excess rainfall. With some allowance for interception and depression storage, this will become an evaluation of surface runoff, as it is produced.

The mass values of surface runoff are materially different on the two diagrams. Obviously these two storms on the same area and in the same season will produce materially different values of flood flow. Engineers have been accustomed to attempt to express the volume of surface runoff from a storm of particular duration as a percentage of the total rainfall. Fig. 1 indicates why no single percentage or coefficient can be highly significant. It indicates further why these coefficients might be expected to have a wide range of values, even in a single season of the year.

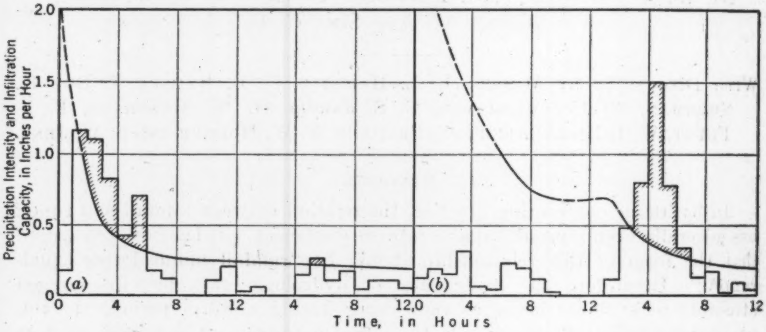

Fig. 1

The hourly variation in surface runoff production is quite different in the two cases; it is readily imaginable that the peak rates of surface runoff which these storms would produce in any stream system would be quite different, and it would follow that any attempt to express peak or crest runoff rate as a percentage or coefficient of the rainfall rate, for some critical time, would also result in a wide range of coefficients even in a single season of the year.

Hydrologic Data

The efforts of the engineering profession to develop, from available data, coefficients of runoff, either by volume or rate, have been due largely to the fact that, until recently, insufficient hydrologic data have been available to justify an attempt at more refined procedure.

Precipitation information was not formerly collected or published with any great consideration for the needs of engineering practice. For large areas in the United States, the only rainfall data available have consisted of the published records of daily totals. Recording rain gages existed at the first-order stations of the U. S. Weather Bureau, but the records were not published in detail.

Where excess rainfall is to be determined by subtracting infiltration rates from precipitation rates, the engineer must know the pattern of precipitation occurrence; if he is to utilize hourly rainfall rates, he must do it with an understanding of the manner in which precipitation actually occurs, on the average within the hour. The result of the study by Erwin R. Breihan,[3] Jun. Am. Soc. C. E., reproduced as Fig. 2, may be startling to the engineer who has been accustomed to using hourly mean rates. Such an engineer will have some difficulty in assimilating the idea that, on the average, half of the rainfall occurring within an hour actually falls at intensities equal to, or exceeding, twice the mean hourly rate.

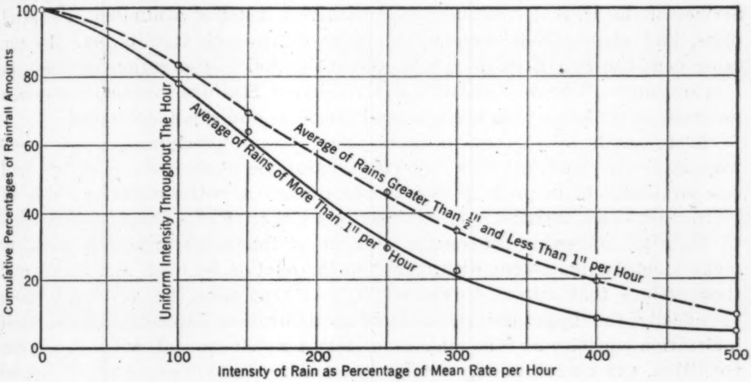

FIG. 2.—RELATION OF PRECIPITATION INTENSITY WITHIN THE HOUR, TO MEAN HOURLY RATE

The tremendous expansion of both public and private work in the hydraulic field, and the corresponding expansion of the engineering practice in this field, have produced a continuing pressure for better information, which has finally resulted in an extension of the basic data-collection services and installations. The expansion of the network of recording rain gages by the Weather Bureau and other agencies, and of the stream gaging work of the U. S. Geological Survey and of the states, has multiplied hydrologic stations and has resulted in a great increase in the number with recording equipment. With the wealth of accurate information at hand, it is to be expected that engineering practice itself will be reorganized, and that hereafter the design of important hydraulic structures should not, and need not, be based on crude over-all general relationships in the hydrologic field.

INFILTRATION

Hydraulic practice has recognized that flood flows represent a less volume of water than the related precipitation, because a considerable part of the precipitated water infiltrates into the soil. Prior to 1920 there was no appreciable body of derived data that gave quantitative values of infiltration "loss" in detail.

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[3] *Civil Engineering*, May, 1940, p. 303.

The research of the Miami Valley Conservancy District, particularly that related to the small control plots, revealed the possibilities of securing new information and better ideas of rainfall-runoff relationships through studies on such small segregated areas.

Under the inspiration of the Miami example, one of the writers, in 1921, arranged for a series of small plot studies on plastic clay soils on the Washington University campus at St. Louis,[4] Mo., and about 1930 conducted similar studies on the Texas black soils, at Dallas.[5] As early as 1922, the Forest Service of the U. S. Department of Agriculture installed a number of control plots, and about 1933 extended this type of research installation. In the same year the Soil Erosion Service (later the Soil Conservation Service), in cooperation with State Agricultural Experiment Stations, began its research program on a nation-wide basis, using both plots and small watersheds.

As to nearly all of these projects, the values of precipitation and runoff were originally analyzed in an effort to determine coefficients of runoff, and comparatively little progress was made toward a better understanding of infiltration characteristics. It remained for Robert E. Horton,[6] M. Am. Soc. C. E., after reviewing the results of some of this earlier research work, to make a definite statement with regard to the relation between infiltration and these values that appear to control it. At that time, he developed more specifically the hypothesis of a limiting infiltration capacity, and defined infiltration capacity as "the maximum rate at which the soil, when in a given condition, can absorb falling rain." Later, Mr. Horton[7] presented his initial development of the theory of overland flow as related to surface detention. It is interesting that as late as 1935 he assumed that infiltration capacity might be satisfactorily approximated as having a constant uniform value during the first hour or two of a precipitation period.

During the four years (1937–1941), the basic information needed for a better understanding of infiltration capacity became available out of the extended research program of the Department of Agriculture, and values of infiltration capacity are derivable as secondary data from possibly as many as 100,000 controlled experiments. Although a tremendous amount of analytical work must yet be done before this mass of secondary data can be produced and published so as to cover conditions, reasonably, in the United States, it is possible at this time to secure and analyze such parts of this information as may be pertinent to a particular problem.

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[4] Under agreement between the Engineering Dept. of the City of St. Louis and the Engineering School of Washington Univ. Original in the thesis file, Washington Univ. Published in part in *Municipal and County Engineering*, December, 1922.

[5] Rept. to City Council, Dallas, Tex. (unpublished); also "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935, pp. 17–18.

[6] "Role of Infiltration in the Hydrologic Cycle," by Robert E. Horton, *Transactions*, Am. Geophysical Union, 1933, p. 446.

[7] "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935.

Character of Infiltration-Capacity Curves

Characteristic curves of infiltration capacity, such as have been found to be derivable from different types of basic data,[8] are shown in Fig. 1. Fig. 3(*a*) shows a graph of infiltration capacity under artificially applied precipitation on grass, on an old hydraulic fill area near Washington, D. C., one day after this plot received a rain of 3.50 in. per hr. Fig. 3(*b*), similarly, was prepared from one of the plots tested at Washington University in 1923.

Fig. 4 also shows infiltration capacities at Edwardsville, Ill., for the storms of March 30, 1938. Antecedent rainfall consisted of 0.66 in. which fell on March 28, with a gross duration of 11.75 hr (6.5 hr net). The 49.95-acre watershed was a poor pasture with grass and weeds 6 in. to 8 in. high; 61% of the soil is Bogota silt loam; surface slopes vary between 0.8% and 30%.

The efforts of the engineering profession to express the relationship between rainfall and surface runoff in terms of coefficients of runoff, involving either volumes or rates, have been due to the fact that until recently sufficient hydrologic data have not been available to permit visualization of their relationships in more detail.

It will be noted that these curves all have characteristically similar shapes, the value of infiltration capacity being relatively high in the beginning of precipitation, decreasing rapidly as precipitation continues, and tending to reach rather definite minimum values, for a particular precipitation period, within a time of 2 to 20 hr. The shape of the early segment of these curves is particularly important when this information is to be applied to the calculation of runoff from small areas under intense precipitation. Where the problem involves high intensity storms of several hours' duration, relatively high initial values may not greatly affect the volume of total runoff. Nevertheless, in the Edwardsville storm of July 17, 1938, the first 8 hr of rainfall, amounting to 1.5 in., produced no runoff.

The production of such characteristic curves as those in Figs. 3 and 4 represents a long advance over the earlier conception of uniform infiltration capacity. Hydrologists are not as yet fully agreed as to the mechanics of infiltration that result in the production of high initial values and the steady reduction toward a constant minimum, and this matter is being investigated intensively.

The results of research indicate that: (1) Infiltration capacity varies little with surface slope; (2) it probably varies materially with soil porosity and with soil moisture, possibly with soil moisture deficiency below field capacity; (3) it may change rapidly with an alteration of soil surface condition such as may occur under the puddling action of rain impact, or under erosion and inworking of fines where the soil is not protected by good vegetal cover;

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[8] For methodology see:

For Plats—"Analysis of Runoff Plat Experiments With Varying Infiltration Capacity," by R. E. Horton, *Transactions*, Am. Geophysical Union, 1939, p. 693.

"Sprinkled Plat—Infiltration and Runoff Experiments on Arizona Desert Soils," by E. L. Beutner, R. R. Gaebe, Jun. Am. Soc. C. E., and R. E. Horton, *loc. cit.*, 1940, Pt. I, p. 550.

"A Graphical Method of Analysis of Sprinkled-Plot Hydrographs," by A. L. Sharp and H. N. Holtan, *loc. cit.*, p. 558.

For Small Watersheds—"Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois," by W. W. Horner and C. L. Lloyd, *loc. cit.*, p. 522.

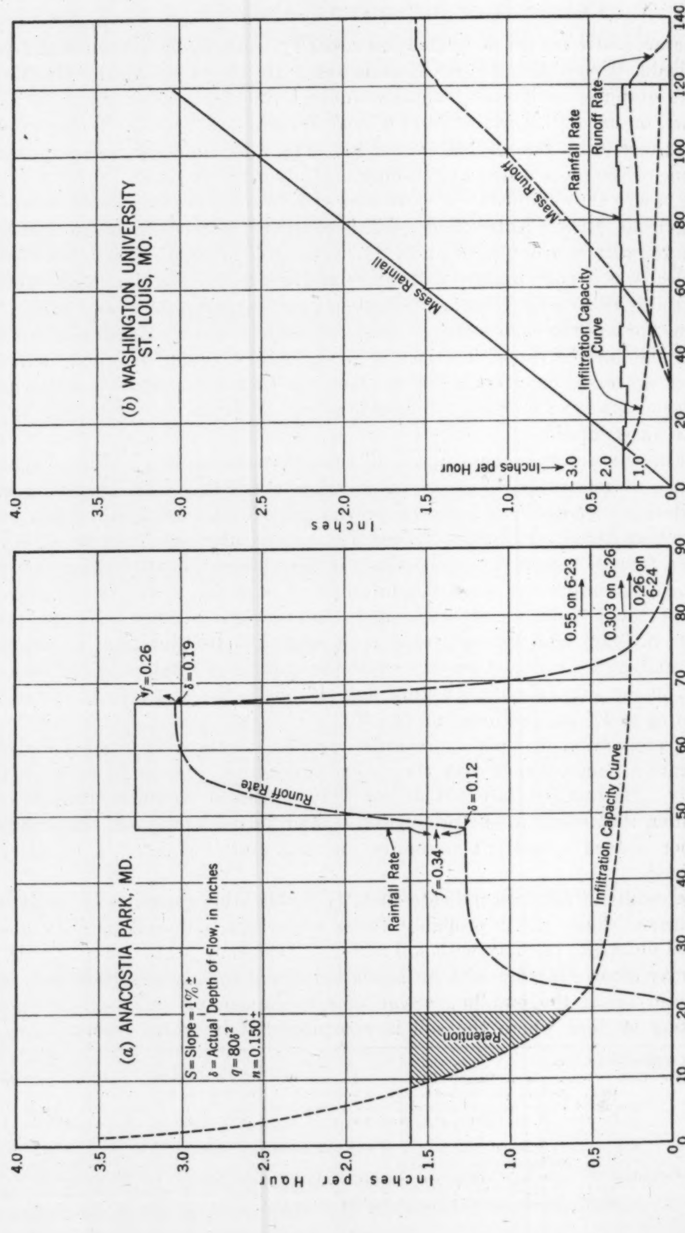

FIG. 3.—SPRINKLING TESTS FOR INFILTRATION CAPACITY

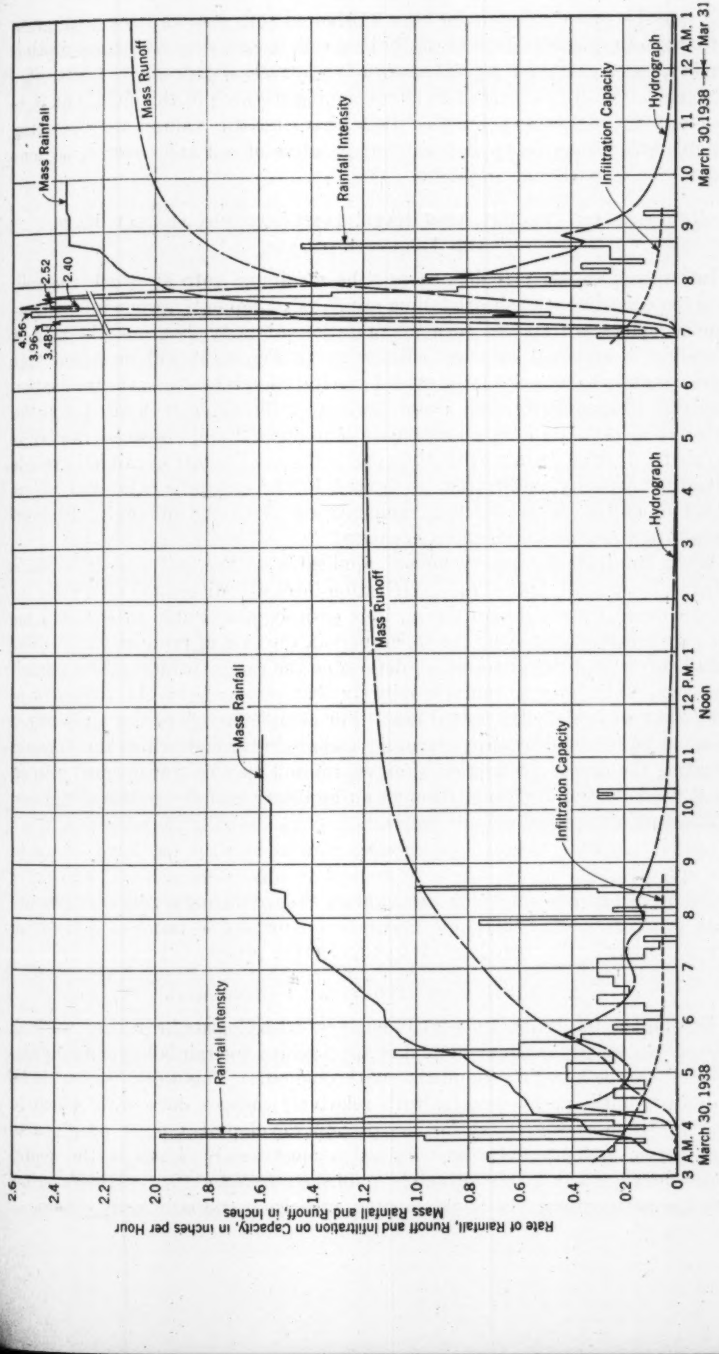

FIG. 4.—STORMS OF MARCH 30 AND 31, 1938, AT EDWARDSVILLE, ILL.

(4) it may be quite different for bare cultivated soils as compared with grass or other good vegetal cover; and (5) for bare soils it may vary with precipitation intensity, but under good vegetal cover it is relatively independent of intensity.

Much is yet to be learned from continuing research in this field, but it is important to engineering practice that characteristic values are becoming available which may be applied as representative of soil and cover conditions encountered in particular problems.

RELATION OF PRECIPITATION RATE, INFILTRATION-CAPACITY RATE, AND EXCESS RAINFALL

Infiltration capacity is defined as "the maximum rate at which the soil, in a given condition, can absorb falling rain." Consequently, when infiltration-capacity values are applied to a precipitation-intensity diagram, the rate of infiltration is apparent at once. Referring to Fig. 4, it will be noted that between certain time periods, the rate of precipitation is less than the infiltration capacity. Consequently, for those periods, infiltration is limited by the precipitation rate. All the precipitation is infiltrated and no excess rainfall is produced. During certain other time periods where the rate of rainfall exceeds infiltration capacity, infiltration is limited to the capacity rate, and excess rainfall or supply is produced in amounts equal to the difference between precipitation rate and infiltration capacity.

When infiltration-capacity curves applicable to the soil and cover in a drainage basin are applied to precipitation-intensity diagrams, either for an actual storm or for a design storm, it is entirely practicable to compute for time increments, throughout the storm period, the rate of production of excess rainfall; or, in equivalent terms, to determine the rate at which excess rainfall will occur. This process removes entirely that phase of the old coefficient of runoff that was related to actual loss. For complete engineering application, this must be followed by supplementary technique through which the diagram indicating the rate of production of excess rainfall may be transformed into an actual flood flow hydrograph, through an application of the unit hydrograph, pluviograph, or channel storage methods.

Infiltration has ceased to be a vague phenomenon; it has been shown to have quantitative rate values with respect to any particular soil and cover condition; the manner in which these values change during a period of precipitation is determinable; therefore, infiltration expressed in terms of infiltration capacity may now be introduced into engineering practice.

APPLICATION TO HYDRAULIC ENGINEERING

During the ten years, 1931–1940, engineer-hydrologists have been working to develop a strictly rational method of approaching the rainfall-runoff relationship that can be based on sound theory throughout. This work has had to be conducted, until quite recently, with relatively meager data with which to test it. Much of the credit for the present development must be given to Mr. Horton, and probably the publication most nearly classic to the course of this development is his privately published *Bulletin 101.*[7] However, his work has been extensively complemented, criticized, and refined by others.

The results of research in this field have been presented in publications of the Department of Agriculture, or cooperating State Experiment Stations, of the U. S. Geological Survey, and in the *Transactions* of the American Geophysical Union, Section of Hydrology. A reading of selected papers from the latter for the period 1934 to 1940 gives an adequate perspective of the investigational work that has been done. As yet there has been no presentation to engineering practice of a definite technique through which the new principles and newly derived data may be applied to the evaluation of surface runoff under specific conditions.

The writers have recently had the opportunity to study and develop a methodology for such an application, first for small areas, conventional in character, at the Washington (D. C.) National Airport, and second for a natural drainage basin of about 800 sq miles. Out of the first experience there has been evolved a methodology presented in this paper as applicable to the determination of surface runoff from urban areas. Out of the second experience has come a clearer concept of a satisfactory procedure for large natural drainage basins.

The essential stages involved in this procedure are as follows:

I—Delineation of the precipitation pattern from which surface runoff is to be evaluated;
II—Choice of basic curve of infiltration capacity;
III—Adjustment of infiltration capacity values to antecedent conditions and precipitation pattern;
IV—Determination of the rate of production of excess rainfall;
V—Interception, depression storage, and infiltration out of surface detention; and
VI—Translation of mass surface runoff to hydrograph form.

I—Delineation of the Precipitation Pattern from Which Surface Runoff Is To Be Evaluated.—This pattern may be set up either in the form of a rainfall intensity diagram or of the mass curve. In the latter form the relations of precipitation and infiltration capacity are not so readily visible from hour to hour. The writers much prefer to show precipitation as an intensity diagram. For small drainage areas it is essential that this diagram be closely representative of the rainfall intensity as it actually occurs. In the illustration in Part II, a time unit of 10 min has been used and this appears to be sufficiently refined and generally satisfactory. For such small areas, retention and infiltration, out of surface detention reached significant amounts and must be evaluated separately.

For large natural drainage basins, the appropriate time unit will depend somewhat on the character of the available rainfall data. For such large areas precipitation pattern should be set up at normal Thiessen centers. At the points usually chosen for such centers generally only daily rainfall amounts are available, although often with observers' notes as to the beginning and end of heavy precipitation. It is necessary to transform this information into the most probable intensity pattern, and this must be done through comparison with the nearest available recording gage records. Obviously, if there are no records of hourly rainfall available within a reasonable distance from such Thiessen centers, it would be useless to attempt preparation of a detailed pattern of precipitation intensity at those points, and time units as great as 3 to 6 hr may be the smallest that are justified. Where the data permit, however, the pattern should be set up in at least hourly amounts.

II—Choice of Basic Curve of Infiltration Capacity.—Information with respect to land use is now quite readily available for most parts of the United States in the form of aerial photographs. Using such photographic information, the percentage of the land under any particular type of vegetal cover, such as cultivated land, pasture land, woodland, etc., may be readily determined for any drainage basin or any part of such area. Soils maps are also available for a large part of the United States. Using both types of information, it is not a difficult matter, by well-devised sampling, to take an inventory of land use and soil for a complete drainage basin of considerable size. From the results of the research programs, basic infiltration capacity curves may be selected that will be satisfactorily representative of any particular combination of soil and cover under specific seasonal conditions.

III—Adjustment of Infiltration-Capacity Values to Antecedent Conditions and Precipitation Pattern.—Although, for any soil, cover, and seasonal condition the graph representing the march of infiltration-capacity values appears to have a quite definite form under continuous excess rainfall, nevertheless, the appearance of these values during a particular precipitation period will vary with the following:

(*a*) The initial soil moisture and soil condition will vary from storm to storm, and a wide variation of initial values of infiltration capacity may be expected. For very dry, open, or cracked condition of the soil, the initial values may exceed 4 in. per hr; for wet or plastic soils, it may be as low as 0.2 in. to 0.3 in. per hr. The writers have found that for any one soil these values are responsive to antecedent precipitation, and show a fair correlation with antecedent infiltration when expressed in amount and elapsed time since occurrence. A study of a number of infiltration capacity curves produced under different antecedent conditions makes possible a reasonable choice of the initial value to be used.

(*b*) Referring to Fig. 3 it will be noted that, except for the first ten minutes, precipitation exceeded infiltration capacity. The graphs of infiltration capacity are typical of those derived from infiltrometer test runs and are representative of the march of values, throughout time, in which infiltration is continually occurring at capacity rates. Very nearly the same curves occur under natural precipitation during compact storms starting at high intensity such as the first storm shown in Fig. 4. For storms in which the precipitation rate in the early part is less than the infiltration-capacity rate, the same values of infiltration capacity will occur, but at some later time in the course of the storm.

To permit a satisfactory relation of infiltration-capacity values to precipitation rates for any particular storm pattern requires a knowledge not only of the normal capacity curve such as that in Fig. 3(*a*), but also as to where, in time, these values will appear under intermittent or varying precipitation. The required adjustment can be made from an examination of a good series of such curves derived from small watershed data under widely different rain patterns; or it may be approximated from infiltrometer tests designed for that purpose. For the latter tests, precipitation would not be applied continuously at excess rates; the tests would include several series having intervals at lower rates and also varied intervals during which no precipitation is applied.

For light open soils, the writers have found that an approximate adjustment of infiltration capacity rates to the time of occurrence may be made on the basis of mass infiltration as illustrated in Part II (see heading "Infiltration Capacity"). For soils of high clay content where the decrease in infiltration capacity rate is not so directly responsive to soil moisture change (but may be due in part to change in soil structure such as that resulting from colloidal swell) a different type of adjustment may be required.

The type of adjustment, both as to initial and continuing infiltration capacities suggested previously, is at this time the most difficult phase of the application of the proposed methodology. With the present knowledge of the mechanics of infiltration there is no definite rule that can be formulated in allocating these values. They must be applied to each storm pattern on something of a case system in which the judgment is guided by what actually happened under conditions for which infiltration-capacity curves have been derived. In addition to study of small watershed data, the guide to judgment can also be obtained from a well-devised series of infiltrometer tests.

For large drainage basins and long storms a considerable error in the choice of initial values of infiltration capacity will have little significance. For small areas, as illustrated in Part II, this item may be of major importance. This stage of the procedure requires a thorough knowledge of infiltration-capacity values for the soils and covers involved, and a complete study of all available data is a pre-essential to the undertaking of an evaluation of surface runoff.

The procedure outlined is usable at this time for any project in which the probable expenditures involved appear to justify an extended engineering analysis. Under these conditions it can be applied with a much greater assurance of accuracy, and with the use of much lower factors of safety than one involving a choice of a coefficient of runoff. Undoubtedly out of the continuing research programs, and out of the experience resulting from applications to large drainage basins, information will develop that may be expected to simplify and clarify this type of adjustment greatly. In that the method attempts to recognize the more important controlling variables and to allocate logical values to them, it will probably never be possible to reduce it to a simple routine.

IV—Determination of Rate of Production of Excess Rainfall.—With the precipitation diagram and the infiltration capacity values adjusted as outlined, a determination of the rate of production of excess rainfall is purely a matter of the subtraction of coincident values for each of the time units utilized. These values represent the gross quantity of excess rainfall produced at the ground surface during each time unit. They are subject to some further modification as outlined in Section V.

V—Interception, Depression Storage, and Infiltration Out of Surface Detention.—Surface detention is the term now generally applied to water in transit on the ground surface. In its broadest use it includes also depression storage—that is, water required to fill depressions in the ground surface below their overflow levels. Expressed in another way, it is the depth of water that must exist on the ground surface to permit flow over the surface, toward the stream margin, inlet or other approach to the main drainage system. It is not intended to cover water actually in stream channels of appreciable size.

The effect of surface detention has long been recognized in the discussion of engineering practice, but has generally been assumed to be so highly variable and so intangible in its dimensional aspects that it was not capable of direct engineering evaluation. It remained again for Mr. Horton to develop, specifically, the hydraulics of overland flow, and to illustrate it with the meager data which he found available in 1935.[7] An evaluation of surface detention has now been found to be of extraordinary engineering importance in studies of rainfall-runoff relationships from small areas. As shown in detail in Part II, it is fundamental that at the end of a period of excess rainfall a part of the "supply" will be in the form of a layer of water in transit over the surface. So long as this persists on any part of the surface, infiltration will occur out of it at capacity rate, and the surface runoff will be less than the supply by the volume so extracted. The depth of surface detention will vary with slope, character of surface and cover, and with the rate of production of supply. For surfaces such as pasture or forest litter on flat slopes at the end of periods of high excess rainfall, surface detention will be very great; the time required to dispose of it will be considerable, and the abstraction through infiltration a significant quantity. For smooth surfaces, steep slopes, and small rates of excess rainfall, the quantity may be negligible.

The term "retention" is used herein to define that part of the supply which is abstracted permanently during the period of excess rainfall. It includes interception and depression storage. Its requirement must be satisfied out of the early precipitation; interception out of the first period of precipitation; and depression storage out of the first period of excess. The water quantities so abstracted will remain on the foliage or in depression storage at the end of the period.

For smooth, cultivated land, the value of retention appears to be on the order of 0.05 to 0.10. For contour cultivation it is considerably greater under good management. For good pasture it may be as high as 0.2, and for good forest litter up to 0.3. Abstractions in such amounts will occur during the early part of the storm. These quantities will be removed by evaporation and infiltration after the end of precipitation, or as to depression storage, after the end of excess rainfall. In large storms, if there are several hours of little or no precipitation, retention may be wholly or partly disposed of, and the requirements must again be filled during the subsequent part of the storm. In one long storm studied for a forested area, it appeared that retention actually exceeded one inch of rainfall. For compact storms, except where the cover is good grass or forest litter, the values of retention may be negligible.

In the procedure outlined herein, the foregoing factors should be recognized and applied in the following manner:

Characteristic values of interception and of depression storage may be determined from an analysis of the small watershed hydrographs. After excess rainfall has been computed for each time unit, values of interception and depression storage should be deducted from such excess in the time units where either interception or the filling of depression storage may obviously occur.

For small drainage areas where the infiltration pattern is set up in 10-min units and where it reflects precipitation intensity accurately, an allowance for infiltration out of surface detention must be determined in the manner illustrated in Part II. For large drainage basins where precipitation is set up in hourly mean values, and where no recognition is given to the rates which may occur within the hour, the resulting net rainfall computed will probably be less than the true value. Under these conditions, infiltration out of surface detention may be neglected and the two errors will be considered as compensating.

VI—Translation of Mass Surface Runoff to Hydrograph Form.—The final stage of this application involves the translation of the excess rainfall diagram into stream flow, generally through unit hydrographs of excess rainfall, or through surface detention and channel storage computations. It is assumed herein that, for most basins, net excess rainfall produced on the land represents surface runoff, and that this in general will be substantially equivalent to stream flow volume for a storm period and related rise of the hydrograph. A reduction will occur between surface runoff and stream flow where a part of the runoff is lost as seepage into stream bed or bank or on flood plains. An increase will occur where a part of the infiltered water returns to the stream system, during the rise of the hydrograph. There are exceptional conditions, as for some basins, where the return flow through the ground is so rapid as to constitute a considerable part of the flood flow. For such basins, infiltration capacity may be used to evaluate the hydrograph of surface runoff, and a separate procedure is developed to determine the hydrograph of ground water return flow. For many basins the latter is an insignificant part of the flood hydrograph and may be neglected.

PART II.—APPLICATION OF DESIGN METHODS

INTRODUCTION

It is intended in this part of the paper to accomplish two objectives:

(*a*) To permit a better visualization of the various factors and processes discussed in Part I by actually applying them to a small restricted area of conventional type, such as a developed city block; and

(*b*) To show the character of the runoff that will occur from such a conventional area, since the hydrograph will differ because of differences in certain controlling variables. After producing the actual synthetic hydrograph for

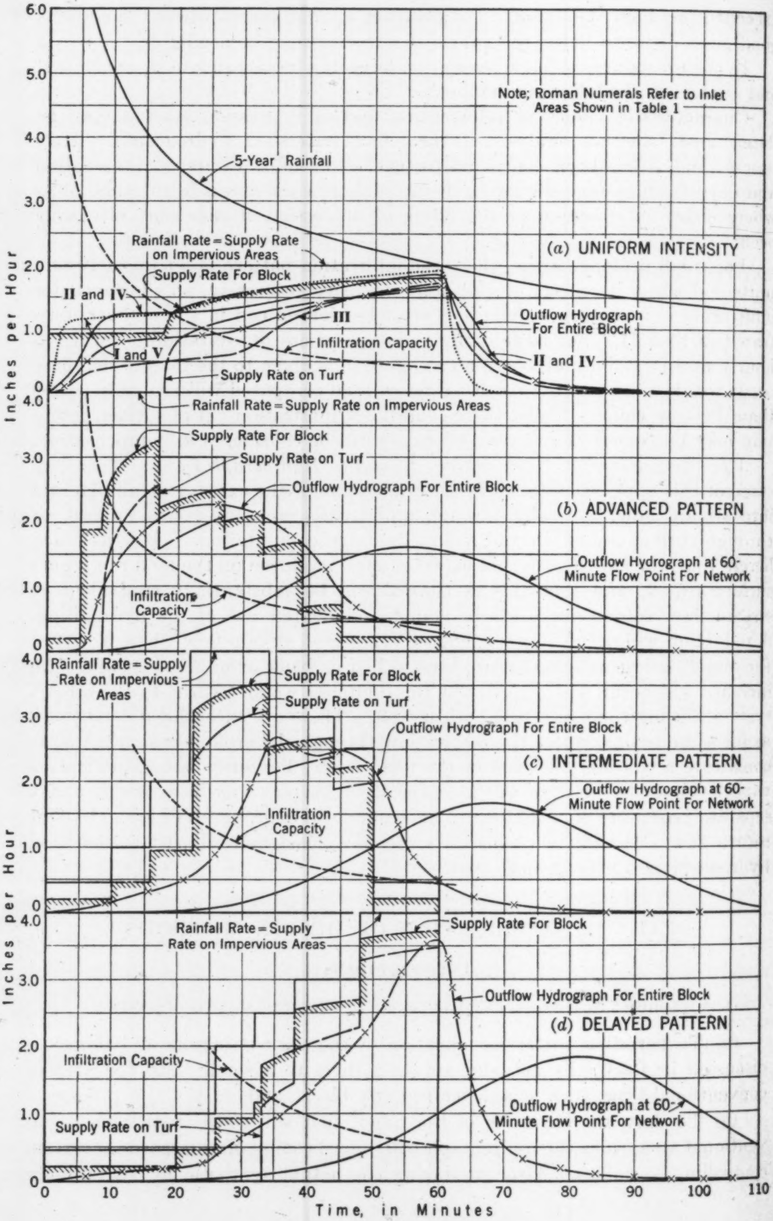

FIG. 5.—RAINFALL CURVES; 60-MIN RAIN

TABLE 1.—TRIBUTARY AREAS

No.	Description	CURVES (SEE FIGS. 5 AND 14)					
		I	II	III	IV	V	Total
1	Tributary area, in acres......	1.3509	0.1222	2.4325	0.1223	1.3509	5.3788
	Percentage of Area:						
2	Pervious...............	38.9	18.9	75.5	18.9	38.9	53.6
3	Impervious............	61.1	81.1	24.5	81.1	61.1	46.4

each combination of conditions, it is proposed to discuss the relation of the peak flow to the mean rainfall in terms of the accepted definition of the coefficient of runoff.

For the purpose of this demonstration, three precipitation periods have been chosen, having respective durations of 20, 40, and 60 min. The mean intensity for each of these durations is that taken from the rainfall curve shown in Fig. 5(*a*), which is substantially a 5-yr rainfall curve for St. Louis (see Table 1 for inlet areas corresponding to curves I to V). In the ordinary application of the rational method, precipitation intensity for specific duration periods is customarily considered as if it were uniform for the period. This is a condition that practically never occurs in nature. The probable occurrence of intensities for parts of the duration period has been investigated (see Fig. 2). For the purpose of this demonstration, the mean intensity for each period has been broken up into probable intensities and durations in approximate accord with the results of the analysis by Mr. Breihan.[3] For the 60-min precipitation period, these sub-intensities and duration units are shown on each of the diagrams, Figs. 5(*b*), 5(*c*), and 5(*d*). Mr. Breihan's investigation did not extend to the order in which these sub-intensity blocks might be expected to appear, and no other good investigation of this kind has been reported. The nearest approach to such a study is that described by D. I. Blumenstock,[9] but this relates only to variations in mean intensity, from hour to hour, within storms of considerable duration.

Experience has shown that each of the intensity patterns shown in Figs. 5(*b*), 5(*c*), and 5(*d*) may be expected to occur at some time, pattern 5(*b*) being the most common. The three patterns shown have been chosen arbitrarily for the purpose of this investigation and are referred to hereafter as the "advanced," "intermediate," and "delayed" patterns.

Mr. Breihan's investigations did not extend to a similar variation of intensities within precipitation periods of less than one hour. For the purpose of this investigation, the variations during the periods of 40 min and 20 min have been proportioned in a manner similar to that which was found for the one-hour period (see Figs. 6 and 7).

PHYSICAL CHARACTERISTICS OF THE AREA TO WHICH RAINFALL IS APPLIED

For the purpose of this investigation, Fig. 8 shows a conventional city block bisected by a paved alley. There are twelve 50-ft lots on each street frontage. The actual areas devoted to roofs, sidewalks, driveways, street and alley

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[9] *Technical Bulletin No. 698*, U. S. Dept. of Agriculture.

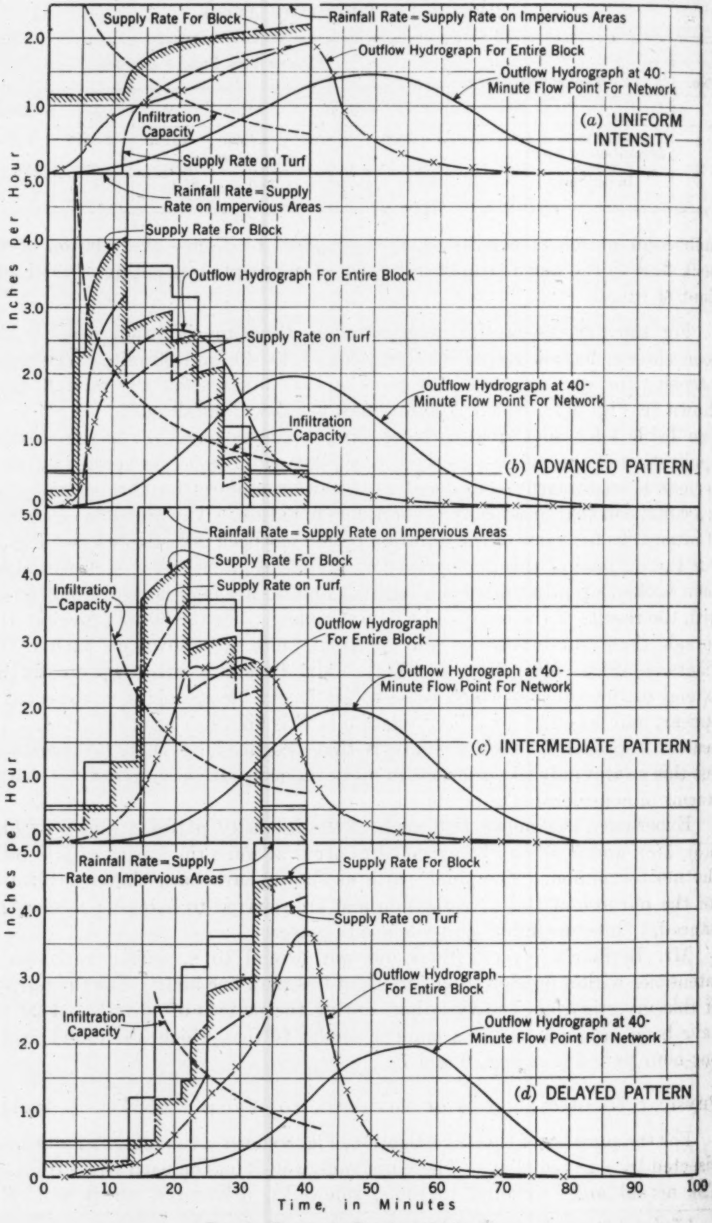


FIG. 6.—RAINFALL CURVES; 40-MIN RAIN

pavements, and lawns are shown in detail in Fig. 8(*b*) and Table 2. The general conditions correspond rather closely to one of the city blocks heretofore gaged in St. Louis. Not only is the percentage of the area of each of these classes important (see Table 2), but the position of the area within the block significantly affects the character of its runoff hydrograph. Equally important are the surface slopes of each component part of the area. Figs. 5, 6, and 7 are based on overland flow slopes of 1% and alley gutter slopes of 0.5%.

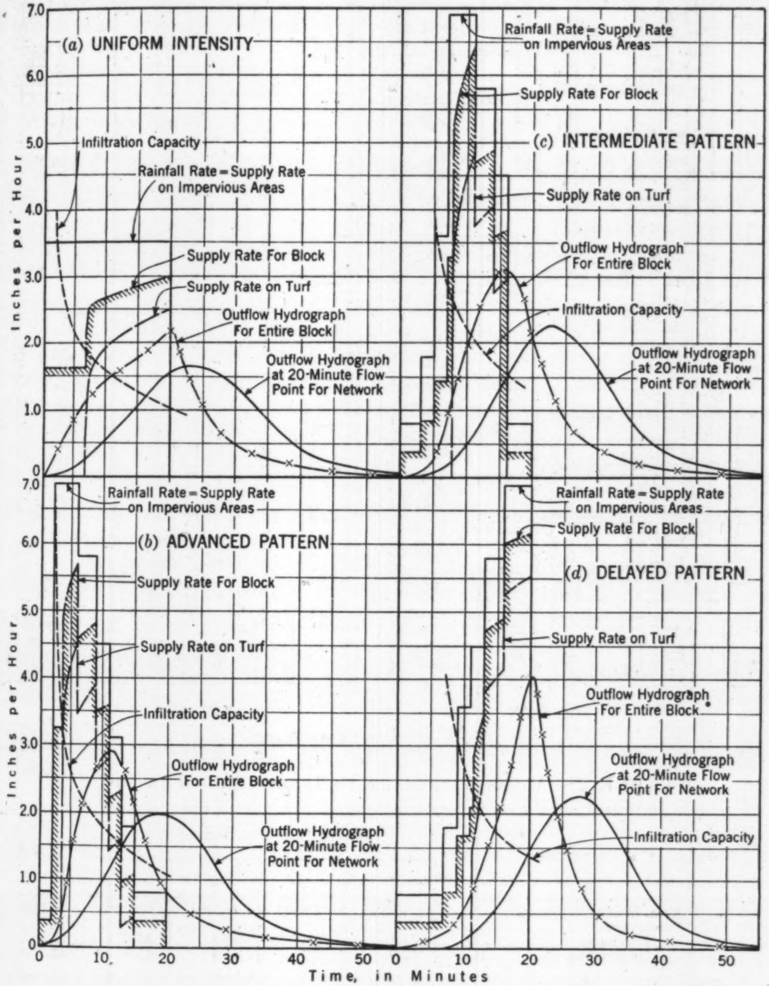
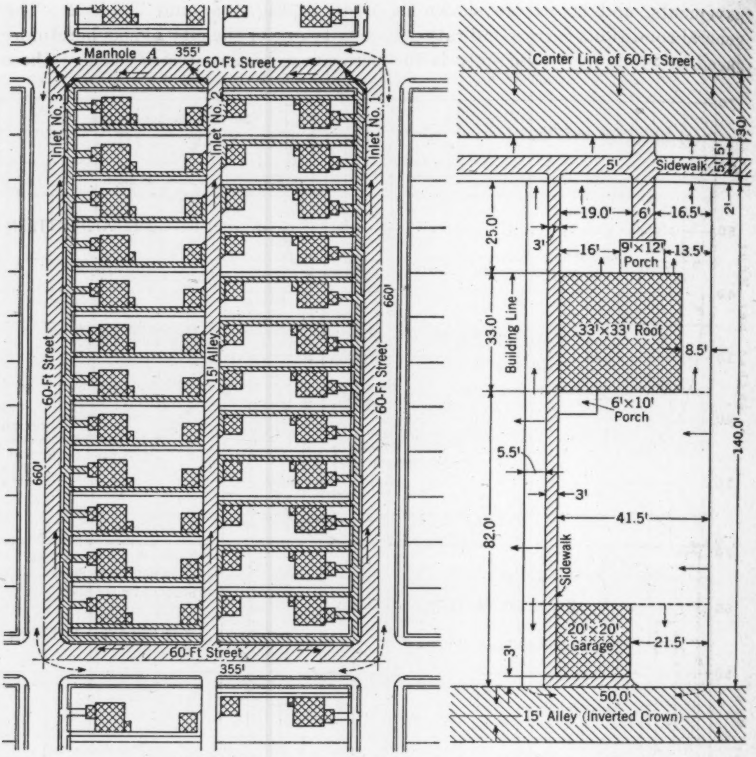


FIG. 7.—RAINFALL CURVES; 20-MIN RAIN


Manhole A
355'
60-Ft Street
Inlet No. 3
Inlet No. 2
Inlet No. 1
660'
60-Ft Street
15' Alley
60-Ft Street
660'
60-Ft Street
355'
Center Line of 60-Ft Street
30'
5'
Sidewalk
5'
5'
2'
19.0'
6'
16.5'
25.0'
3'
16'
9'×12' Porch
13.5'
33'×33' Roof
8.5'
33.0'
Building Line
6'×10' Porch
140.0'
5.5'
3'
41.5'
82.0'
Sidewalk
20'×20' Garage
21.5'
3'
50.0'
15' Alley (Inverted Crown)

(*a*) PLAN (*b*) DETAILS OF LOT

FIG. 8.—TYPICAL URBAN BLOCK

TABLE 2.—AREAS TRIBUTARY TO INLETS SHOWN IN FIG. 8

No.	Description	Inlets Nos. (See Fig. 8(*a*)):				Fig. 8(*b*)
		1	2	3	Manhole *A*	
1	Area, in acres	1.3509	2.5547	1.4731	5.3787	0.2038
	Percentage:					
2	Pervious	37.1	72.8	35.6	53.6	57.2
3	Impervious	62.9	27.2	64.4	46.4	42.8

It is assumed herein that the sewers to be designed are separate storm sewers, that there is no storm sewer abutting the lot frontage, and that therefore all drainage is routed through either the street gutters or the alley pavement. Roof drainage is presumed to be collected in house gutter systems and carried through downspouts and short sewers to outlets through the curbing into the gutter. In this respect the conditions are different from those heretofore described for the St. Louis blocks gaged, where the sewers were of the combined type, and the roof drainage was connected directly to the sewers.

Routing roof water through the street gutters will increase, slightly, the time of its arrival at the collecting point manhole (manhole A, Fig. 8(a)) and give a slightly delayed runoff peak for the city block as a whole.

INFILTRATION CAPACITY

Fig. 5(a) shows a curve of infiltration capacities throughout the precipitation period. This curve is not intended to represent a specific condition accurately, but follows somewhat the curve of infiltration capacity under 2-in. rainfall rates, as computed from the small plots tested at Washington University in 1923 (see Fig. 3(b)). It may be characterized as representing, rather closely, the march of infiltration capacities that would exist for a turfed area; a soil condition consisting of about 4 in. of loam underlain with yellow clay, and a moisture condition that would exist in St. Louis during a midsummer period in which heavy rains occurred about twice a month, the last one approximately two weeks in advance of the conditions that this curve represents.

As far as is known, no sprinkling plot experiments have been made on well-developed city lawn grass. The nearest approach to such a situation, other than the Washington University tests, is in the runs made on the Anacostia Parkway, for which one graph has been shown (Fig. 3(a)). For that area, however, the soil is a plastic mud fill and not comparable to the ones referred to herein.

A preliminary study of rainfall-runoff relationships reported in 1934[10] indicates that both infiltration capacity and retention characteristics are higher for such lawns than for the sprinkling plots otherwise reported. This is undoubtedly due to the fact that such lawns are frequently mowed, are undisturbed by cultivation, and that it is possible, therefore, for extensive porosity to develop because of dead root growth and absence of compaction. Retention values are also increased because of the mat of grass-cutting litter that accumulates at the ground surface, and infiltration opportunity is extended for the same reason.

The calculations herein will involve somewhat lower infiltration capacities than may be expected under the same summer conditions on the city lawns, and somewhat less retention than will exist on such lawns; and therefore the resulting rates of runoff will be higher than would be expected for such conditions. This fact, however, does not affect the validity of the calculations with respect to the assumptions on which they are based.

In Fig. 5(a), the infiltration-capacity curve is shown in the position it would have held had it been actually derived from such a plot under a uniform 2-in. rainfall rate. For application to the precipitation patterns shown in Figs. 5(b), 5(c), and 5(d), the position of the curve has been adjusted slightly. For the purpose of this adjustment, it was assumed, the initial condition being the same for each diagram, the infiltration capacity would arrive at any specific rate when the mass infiltration was the same as that which preceded the occurrence of such a rate in Fig. 5(*a*). Specifically, in the adjusted positions, the 2-in. infiltration-capacity rate is so placed in time that the mass infiltration occurring prior to that time will be the same for each of the diagrams; and a similar adjustment was made with respect to other critical rate points on the curve.

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[10] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140 (first published in *Proceedings*, Am. Soc. C. E., October, 1934).

For the rates of 40 min and 20 min duration (Figs. 6 and 7) the infiltration capacity was adjusted in a similar manner. It is recognized that present knowledge of the mechanics of infiltration is not such as to validate this adjustment procedure fully, but a study of the St. Louis infiltration-capacity curves appears to indicate that for any initial condition, the march of infiltration-capacity rates is closely correlated with mass infiltration. There is good evidence of a similar correlation at the Edwardsville station of the Soil Conservation Service.

PRECIPITATION, INFILTRATION CAPACITY, AND NET RAINFALL

The elemental surfaces shown on the city block fall into either the pervious or impervious class. The position, size of these surfaces, and the direction and length of overland flow are shown subsequently in Figs. 10(*b*) and 11(*b*).

For simplicity, it is assumed for the impervious surfaces that there is no permanent retention and no infiltration, although in an actual case a very small amount of each may exist. Therefore, net rainfall is equal to the precipitation, and the rate of production of net rainfall or "supply" (hereafter given a symbol σ) is represented by a precipitation intensity diagram.

For the pervious surfaces, it is assumed in all cases that the permanent retention is 0.1 in. It appears to be reasonably representative of the turfed areas that have been studied under sprinkling plot experiments. One half of this retention, or 0.05 in., is assumed to be of the character of interception, and is taken out of the first 0.05 in. of rainfall that occurs. The other 0.05 in. is treated as depression storage and is taken out as the first 0.05 in. of net rainfall after infiltration capacity drops below the precipitation rate. As stated, these values are undoubtedly somewhat lower than will actually exist on well-developed city lawns.

With this allowance for retention, the rate of production of net rainfall or "supply" is determined by subtracting the infiltration capacity rates from the precipitation intensity rates, and the result is shown in Figs. 5, 6, and 7; and, as so shown, represents the diagram of net rainfall or rate of production of surface runoff for the pervious area.

These two "supply" diagrams (that is, the precipitation intensity diagram as to impervious surfaces and the net rainfall diagram as to pervious surfaces) determine the mass values of surface runoff that are thereafter unchanged (except for a small amount of infiltration during the recession period), and appear in the same values under the varying rate diagrams for overland flow for each particular surface, for inflow into the street and alley gutters and for the final runoff hydrograph for the city block as a whole, at manhole *A* (Fig. 8(*a*)).

OVERLAND FLOW AND SURFACE DETENTION

The characteristics of overland flow have been the subject of extensive investigation through the analysis of rainfall plots. It has been extensively discussed by Mr. Horton.[7, 11] The field investigations have shown that overland flow in thin sheets may be entirely turbulent or partly turbulent and partly laminar. For smooth surfaces, it appears that fully turbulent flow will be developed normally, and that the Manning formula is applicable. For subdivided flow through grass, most of the experimental data indicate that the flow is laminar to some extent and may be reasonably represented by a condition of 75% turbulent flow, in which case the profile of overland flow may be assumed to be parabolic and the flow to take place in accordance with the relation,

$$Q = k\,\delta^2 \qquad (1)$$

in which: δ is the depth of flow at any point; k is a constant; and Q is the rate of flow. Since the profile of the water surface is parabolic, in this case, the mean depth of surface detention may be taken as two thirds of the depth at the outflow margin of any strip, and the rate of outflow may be expressed either in terms of mean depth or of outflow depth. For the purpose of this demonstration, it is assumed that the flow from paved impervious surfaces will take place in accordance with the Manning formula, using an n of 0.015, and may be represented by the equation

$$V = 18.9\,\delta^{0.67}\,S^{0.5} \qquad (2a)$$

in which V is in feet per second and δ is in inches. It is assumed that the flow over turfed surfaces will be 75% turbulent. Coefficients have been adopted from a study of several sets of experimental data, and the flow formula used is

$$V = 0.96\,\delta\,S^{0.5} \qquad (2b)$$

The values of n derived from experimental studies of turfed plots and used in Eq. 2*b* are not of the same order as the Manning n, because the condition is actually one of subdivided flow. For evaluation of overland flow through turf, it seems advisable to avoid the use of values comparable to the Manning n and to discuss rather the velocity coefficients in Eq. 2*b*. The Anacostia plot (Fig. 3(*a*)) gives a coefficient of slightly more than 0.6. The aforementioned value of 0.96 was derived by the writers from experiments reported in 1938.[12] Unpublished values for flows 1 to 2 in. deep through Bermuda grass[13] indicate that these can be expressed by a coefficient of about 1.2, although the flow was turbulent. In general, these values over a considerable range of turf conditions seem to have less variation than the n-values for natural stream channels.

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[11] *Transactions*, Am. Geophysical Union, 1939, p. 693.

[12] Unpublished manuscript of the SCS entitled "Results of Studies Involving the Application of Rainfall at Uniform Rates to Control Plot Conditions," by C. M. Woodruff, D. D. Smith, and Darnell M. Whitt, June, 1938.

[13] "Some Experiments on Shallow Flows over Grass Slope," by W. O. Ree, Spartansburg Lab., SCS, *Transactions*, Am. Geophysical Union, 1939, Pt. IV, p. 653.

If any specific project is undertaken for the development of truly representative hydrographs of flow involving the turf on city lawns, the value of the coefficient in Eq. 2*b* should be given some further study. For the reasons heretofore discussed, it appears that on well-developed city lawns, this coefficient may be as low as that found for the Anacostia Parkway—that is, on the order of 0.6. This assumption is borne out in part from an examination of an actual hydrograph for one of the city blocks studied previously,[10] where the delayed peak and the considerable bulk of flow occurring after the end of rainfall indicate in part relatively low velocities of overland flow.

On the basis of the flow-depth relationships of the foregoing type, Mr. Horton has developed, for 75% turbulent flow, the equations for the rising side of the hydrograph, and for the depth of detention corresponding to any rate of flow.[14] These equations, when developed from Eqs. 2, become:

For turf—

$$q_s = \sigma \tanh^2 \left[\frac{1.5 \sqrt{3{,}520}\; \sigma^{0.5}\; S^{0.25}\, t}{60\; l^{0.5}} \right] \ldots\ldots\ldots\ldots (3a)$$

and

$$\delta = \frac{l^{0.5}\, \sigma^{0.5}}{S^{0.25} \sqrt{3{,}520}} \tanh \left[\frac{1.5 \sqrt{3{,}520}\; \sigma^{0.5}\, S^{0.25}\, t}{60\; l^{0.5}} \right] \ldots\ldots\ldots (3b)$$

Figs. 9(*b*) and 9(*c*) show families of curves for runoff across turf strips for different slopes and lengths.

For pavement, the writers have developed the following relationships:

$$q = \sigma \tanh^{1.67} \left[\frac{1.60\; (1{,}020)^{0.60}\, S^{0.30}\, \sigma^{0.60}\, t}{n^{0.60}\, l^{0.60}\; 60} \right] \ldots\ldots\ldots\ldots (4a)$$

and

$$\delta = \frac{\sigma^{0.60}\, n^{0.60}\, l^{0.60}}{(1{,}020)^{0.60}\, S^{0.30}} \tanh \left[\frac{1.60\; (1{,}020)^{0.60}\, S^{0.30}\, \sigma^{0.60}\, t}{n^{0.60}\, l^{0.60}\; 60} \right] \ldots\ldots (4b)$$

In Eqs. 4, q is expressed in inches per hour, σ = the rate of supply of excess rainfall in inches per hour, S = the absolute slope, l = the length of overland flow in feet, t = time in minutes, n = the coefficient of roughness, and δ = the depth of detention, in inches, at the outflow margin at any time t. The mean depth of detention is for turf $\frac{2}{3}\,\delta$ and for pavement approximately $\frac{5}{8}\,\delta$.

For any specific area of surface, such as those shown in Fig. 8(*a*), S, l, and n are inserted and Eqs. 4 then show the relation between the rate of overland flow or the marginal depth of overland flow and the rate of supply at any time t. For each condition of flow, Eqs. 4 have been graphed for overland flow over 10.5-ft and 7.5-ft lengths of pavement, for various rates of supply. (See Fig. 9(*a*).)

In determining the hydrograph of overland flow, the q equations, Eqs. 3*a* and 4*a*, are applied to the rate of supply shown on the diagram, as for example, in Fig. 5(*a*). Where the rate of supply is varying, the procedure is simplified by reducing the rates to uniform rates for short periods. Where the rate of supply is uniform for a considerable time, the rising side of the hydrograph is that represented by Eqs. 3*a* and 4*a* for any single rate. If the uniform rate of supply continues for a considerable time, equilibrium flow is established and the rate of overland runoff becomes equal to the rate of supply.

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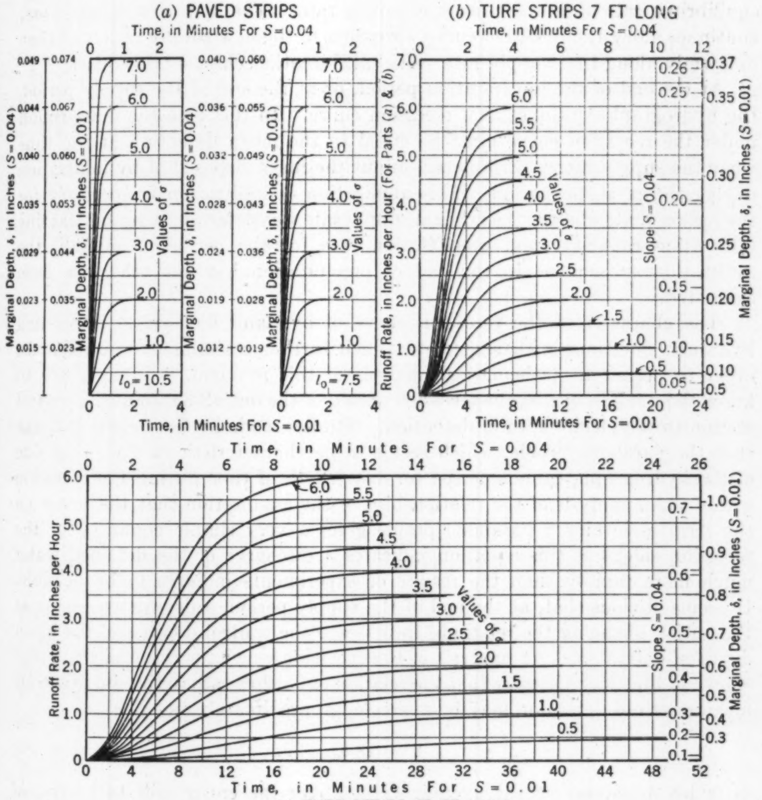
[14] "The Interpretation and Application of Runoff Plat Experiments," by Robert E. Horton, *Proceedings*, Soils Science Soc. of America, Vol. 3, 1938.

If the supply rate increases either before or after equilibrium flow has been established, the rising side of the hydrograph is continued as follows: The surface detention at a time when the supply rate increases is related to the value of Q by Eq. 1. This same value of detention would have been produced under the second supply rate at a different time, and the Q corresponding to such δ at that time has the same value as the Q resulting from the first supply rate. Accordingly, it is possible to join two different curves representative of the rising side of the hydrograph for different supply rates. For example, in Fig. 9(*c*), if the initial supply rate was 1.5 in. per hr and it lasted for 16 min,



(*a*) PAVED STRIPS

(*b*) TURF STRIPS 7 FT LONG

(*c*) TURF STRIPS 65 FT LONG

FIG. 9.—RUNOFF RATES, OVERLAND FLOW

the rate of runoff at that time would be 1 in. per hr. If at that time the supply rate changed to 2.5 in. per hr, the hydrograph can be continued by using the curve for the 2.5-in. supply rate at the point of equal flow which, for that curve, is found at 8 min. The rising side of the hydrograph will then develop along this curve until the supply rate is again changed.

When the supply rate is reduced from one value to another, the hydrograph takes the form of a recession curve, the rate at the lower end of which is the equilibrium rate of flow for the new supply rate. The hydrograph, therefore, continues along the recession curve, dropping to the new supply rate, and then uniformly along that supply rate value until another change takes place.

At the end of the precipitation period, or at the end of the supply period, the hydrograph continues as a recession curve. In this case the mass runoff under the recession curve must be equal to the mean detention at the time when the supply rate is terminated, minus the mass disposed of by infiltration during the recession period. This application of infiltration is necessary for the reason that, whereas the original supply rate was determined by subtracting infiltration capacity from precipitation rate for the period of rainfall, the infiltration subsequent to the end of precipitation has not otherwise been evaluated.

The character of the recession curve of overland flow raises interesting hydraulic questions with respect to which further research is desirable. In order to permit a mathematical analysis of this problem, it is necessary to know the relation during the recession period of the outfall depth at the control section to the mean depth of detention. Studies made by the writers indicate that the parabolic profile, which seems to be characteristic of the rising side of the hydrograph, cannot persist for any length of time during the recession period. An analysis of the relationships of the assumption that the water on the strip maintains a parabolic profile gives a very simple equation for the recession side, but this equation indicates a discharge of the detained water much more rapidly than the results of experiments indicate to be possible. It seems obvious that, at the end of the supply period, a drawdown occurs at the outlet, changing the relationship between the control depth and the mean depth of detention. An analysis of the recession curve for the Anacostia plot shown in Fig. 3(*a*) indicates that the relation of outflow rate to the simultaneous mean depth of detention may be expressed approximately as

$$q = 18\,\delta_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

in which δ_a represents the average depth, over the entire tributary area, of the residual detention at that time. Further study of a large number of such recession curves may give a better indication of how the water surface profile is modified during this period. For the purpose of the demonstration of this paper, the recession curves have been adjusted to conform generally to those of experimental plots that have been examined.[15]

The effect of this procedure in transforming the supply rate shown in Fig. 5(*a*) to a hydrograph of surface runoff, for each of the elemental areas

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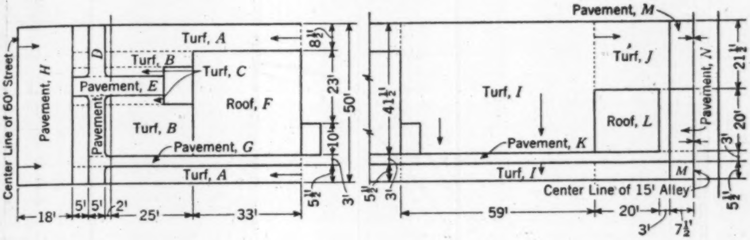
[15] See "A Graphical Analysis of Sprinkled Plat Hydrographs," by A. L. Sharp and H. N. Holtan, *Transactions*, Am. Geophysical Union, 1940, p. 558.

TABLE 3.—AREAS INVOLVED IN HYDROGRAPHS OF FIGS. 10 AND 11



(a) LOT TO STREET CORNER

Unit (See Fig. 10)	Area, in acres	Percentage of total	Length of strip, l_0, in ft
A[a]	0.0209	20.40	65
B[a]	0.0158	15.40	32
C[a]	0.0032	3.10	23
D[b]	0.0047	4.58	5
E[b]	0.0038	3.81	28
F[c]	0.0288	28.10	....
G[b]	0.0045	4.39	65
H[b]	0.0207	20.22	18
Total	0.1024	100.00	....
Pervious	0.0399	39.00	....
Impervious	0.0625	61.00	....

(b) LOT TO ALLEY

Unit (See Fig. 11)	Area, in acres	Percentage of total	Length of strip, l_0, in ft
I[a]	0.0023	61.30	47
J[a]	0.0143	14.03	23
K[b]	0.0062	6.09	89.5
L[c]	0.0092	9.05	....
M[b]	0.0046	4.57	7.5
N[b]	0.0048	4.76	10.5
Total	0.1014	100.00	....
Pervious	0.0766	75.5	....
Impervious	0.0248	24.5	....

[a] Turf. [b] Pavement. [c] Roofs.

shown in Table 3, is illustrated in detail in Figs. 10 to 14 which follow. In all of these computations, the runoff rates are maintained in inches per hour.

The rainfall chosen to illustrate the application of this method is the simple one presented in Fig. 5(*a*). This rainfall is reproduced in Fig. 10(*a*), showing the respective outflow graphs for each of the elemental areas draining to the street. It is noticeable that for the smoothly paved areas equilibrium flow is quickly established, and the outflow graph departs comparatively little from the precipitation or "supply" diagram. The hydrographs for the turfed areas reflect both the effect of infiltration and surface detention. It will be noted that runoff becomes perceptible at about 18 min, which is approximately the beginning of the supply rate diagram as shown in Fig. 5(*a*). The difference between the hydrographs for areas *A*, *B*, and *C* reflects the difference in the length of overland flow, all of them having the same supply rate and the same surface slope, for this illustration, of 1%. The effect of the longer overland flow for unit *A* is also apparent in the increased depth of surface detention which results in the greater mass under the recession curve *A*.

In Table 3(*a*), each of these unit areas is shown in its actual acreage and also as a percentage of the area of the lot draining to the street.

In Fig. 10(*b*), the ordinates of the hydrographs in Fig. 10(*a*) have been multiplied by their respective areal percentages, and therefore appear in terms of inches per hour for the entire tributary area shown on the small plat in

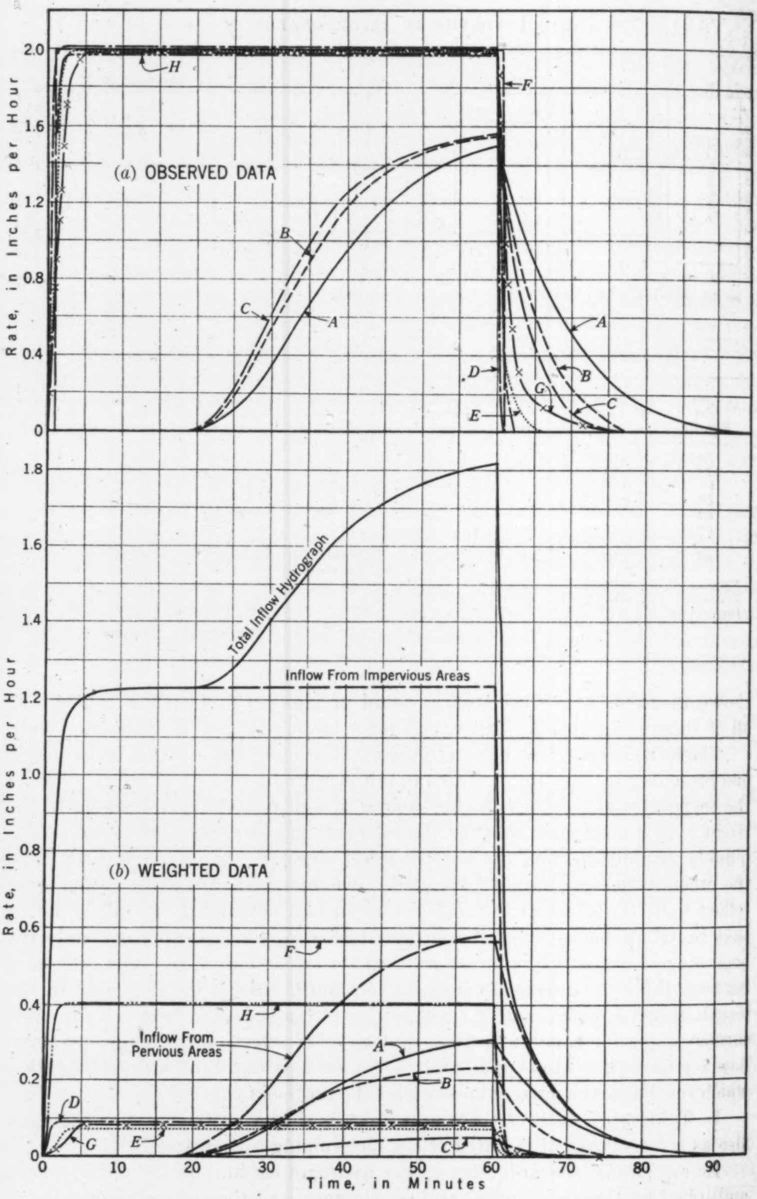


FIG. 10.—HYDROGRAPHS OF FLOW FROM A TYPICAL LOT TO A STREET GUTTER (RAIN, 2 IN. PER HR FOR 60 MIN)

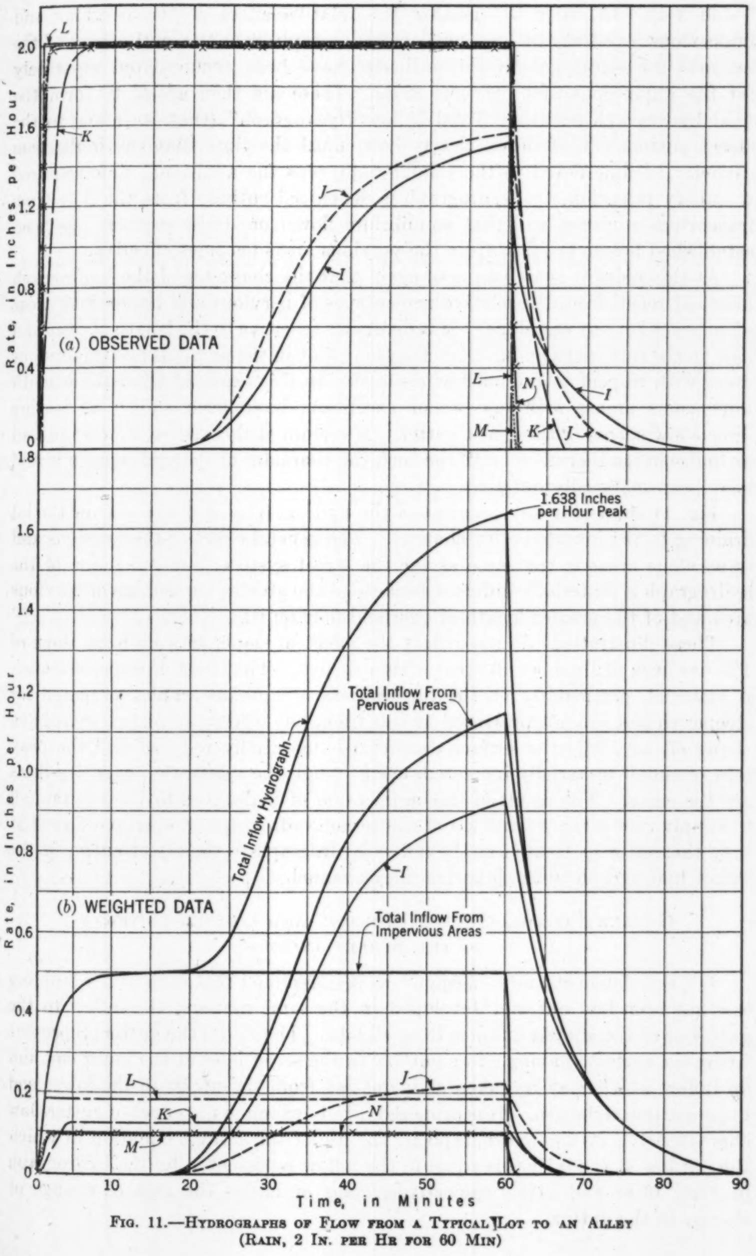


FIG. 11.—HYDROGRAPHS OF FLOW FROM A TYPICAL LOT TO AN ALLEY (RAIN, 2 IN. PER HR FOR 60 MIN)

Table 3(*a*). In order to visualize the relative effect of the pervious and impervious areas on the final outflow hydrograph from this entire part of the lot into the street gutter, the ordinates have been accumulated separately for the impervious and pervious units. These are then added to form the final hydrograph entitled "Total Inflow Hydrograph" (that is, inflow to the street gutter). This shows clearly how, until the time that the infiltration capacity became equal to the rainfall rate, plus the additional time required to satisfy retention, the hydrograph is developed entirely from the flow from impervious surfaces, and that equilibrium flow from these surfaces has been established before the flow from the pervious areas becomes effective.

At this point it is well to note again that the character of the hydrograph does not result from the relative percentages of pervious and impervious areas alone, even for any one slope. It is distinctly sensitive to the length of overland flow on the turf strips and to the relative position of the pervious and impervious areas with respect to the outflow channel. In this example all of the smooth impervious areas, with one minor exception, have been shown as having direct-flow access to the street gutter. If certain of these areas were so placed as to discharge their flow on to the turf, the character of the hydrograph would have been materially different.

Fig. 11 shows the development of the hydrographs from the rear of the lot draining to the inverted alley channel. The general effect of the pervious and impervious areas is the same as for the street system, but the shape of the hydrograph is materially different because of the greater percentage of pervious area and of the greater length of overland flow on it.

These illustrations do not reflect the effect of slope, as a uniform slope of 1% has been utilized for all areas within the lot. The effect of slope, however, is material, particularly on the turfed areas. Separate graphs, prepared for steeper slopes, are not presented at this time, but where the turfed areas have slopes of, say, 4%, the surface detention is materially reduced and the peak rate of runoff materially increased, even though the infiltration capacity rates are the same. The result of this application of hydraulics to the net rainfall or supply rate is the production of the actual hydrograph of overland flow that may thereafter be treated as the inflow hydrograph to the street gutter or the inflow hydrograph to the inverted alley channel.

GUTTER INFLOW, GUTTER STORAGE, AND GUTTER OUTFLOW TO THE STREET INLET

For this demonstration, inasmuch as it is assumed that each of the building lots has been laid out and developed in the same manner, the inflow to the gutter or to the alley is uniform from all lots. Therefore, the gutters represent hydraulic systems having a free outflow to the sewer inlet at the lower end and an inflow which may be taken as increasing from the upper to the lower end in proportion to length. It is proposed here to examine the effect of gutter-flow characteristics on the hydrograph by the use of the storage equation in which the outflow at the inlet Q is equal to the inflow represented by the hydrograph in Figs. 10(*b*) and 11(*b*), respectively, plus or minus the rate of change of storage in the gutter.

To permit this application to be made, it is necessary to develop, for any condition of equilibrium flow, the relation of the storage volume in the gutter to the discharge Q at the inlet. This storage volume is readily determinable when the character of the water surface profile is known. In this case, the relation between rate of flow to length of flow along the gutter is a linear one, in which

$$\frac{Q_x}{x} = \frac{Q_l}{l} \qquad \text{(6)}$$

being the total length of the gutter, and x the distance along the gutter measured from the upper end. For the alley-channel gutter, the width of the water surface is great compared to the depth, and the hydraulic radius may be closely approximated as being equal to the mean depth.

In ordinary municipal practice, it has been the custom to assume that the water surface is parallel to the flow-line gradient, in which case the depth h_x at any point x becomes a direct exponential function of the rate of flow Q_x at that point. For the conditions: $A = 15\,h^2$; $v = \frac{1.486}{0.015} R^{0.67} S^{0.5}$; $R = \frac{h}{2}$; and $v = 62.4\,h^{0.67}\,S^{0.5}$

$$Q = A\,v = 936.1\,h^{2.67}\,S^{0.50} \qquad \text{(7a)}$$

and, assuming a gutter 600 ft long,

$$V = 5{,}142.9\,h^2 \qquad \text{(7b)}$$

For this condition of flow, in the alley gutter shown in Fig. 12(*b*), the equation between h_x and Q, the outfall discharge becomes

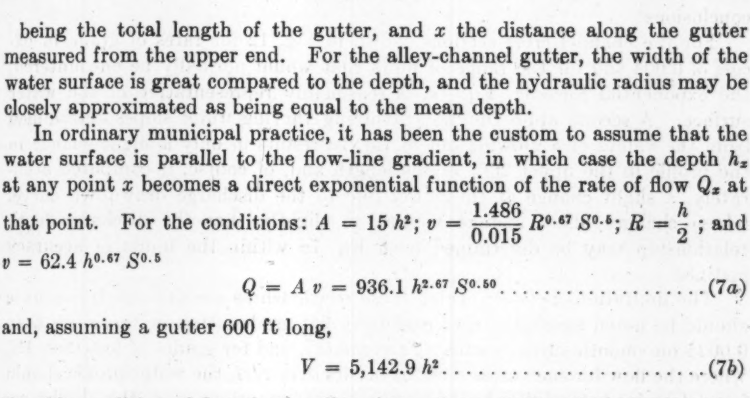


FIG. 12.—HYDRAULIC CHARACTERISTICS

$$Q = \frac{936\,l}{x} S^{0.5}\,h_x^{2.67} \qquad \text{(8)}$$

The assumption involved in Eq. 7*a*, of course, is physically inconsistent, as the values of h cannot vary along the gutter without producing varying values of the water surface slope i.

In connection with this demonstration, considerable study was made with respect to the character of the genuine water surface profile for the alley gutter. It was found that a rigid analysis of this hydraulic problem is quite involved, and that the matter could be approached best by successive approximations. The water surface profile has been investigated in this manner with the following conclusions:

For the channel cross sections shown in Fig. 12 for rates of grade in excess of 0.004 and for the range of flows that would normally be encountered, the exponential formula, Eq. 7*a*, is reasonably representative of the water surface. A second approximation involving varying water slopes determined from the values of h growing out of Eq. 7*a* results in only a slight change in the profile in the upper 20% of the length and, of course, if computed accurately, a slight change at the outlet due to the discharge drawdown curve. These changes affect storage volume so slightly that the discharge-depth relationship may be determined from Eq. 7*a* within the limits of accuracy justified.

The limitations between which these relationships are sufficiently accurate should be noted carefully. For extremely flat grades (that is, those less than 0.0025 on smooth surfaces such as pavement), and for grades of less than 1% where the flow for such cross sections occurs over turf, the water profile should be determined carefully by successive approximations and the discharge-storage equation developed by summation. This has been found to be particularly important in studies made for the drainage of the National Airport at Washington, D. C., where the channel flow is entirely through flat turfed gutters. Under these conditions, storage volume reaches large quantitative values with respect to discharge Q, must be rather accurately determined, and affects, importantly, the relation between channel inflow and outflow.

The relationship between storage volume and depth at the outlet has been evaluated and is shown for the alley on the control graph, Fig. 12(*b*). It can also be expressed directly as a relation between discharge rate and storage volume.

This can be done by eliminating h between Eqs. 7*a* and 7*b* for the alley, which results in the following relationship:

$$V = k\,Q^{0.75} \qquad \text{(9a)}$$

n which V is in cubic feet; and Q in inches per hour for tributary area A in acres and, for this block,

$$k = \frac{60\,l}{7}\left(\frac{A}{936\,S^{0.5}}\right)^{0.75} \qquad \text{(9b)}$$

With Q expressed in cubic feet per second, Eq. 9*b* becomes

$$k = \frac{60\,l}{7}\left(\frac{1}{936\,S^{0.5}}\right)^{0.75} \qquad \text{(9c)}$$

This is the value of k used in Eqs. 12. For a short finite Δt in which the hydrograph may be taken as a straight line, the insertion of these values in the storage equation, with inflow I in cubic feet per second, gives

$$60\,I\,\Delta t - \frac{Q_1 + Q_2}{2}\,60\,\Delta t - (k\,Q_2^{0.75} - k\,Q_1^{0.75}) = 0 \quad \text{(10)}$$

Rearranging the terms, this can be expressed as

$$fQ_2 = I + fQ_1 \quad \text{(11)}$$

in which (expressing I, Q_1 and Q_2 in inches per hour)

$$fQ_2 = \frac{2\,k\,Q_2^{0.75}\,A^{-1/4} + 60\,Q_2\,\Delta t}{120\,\Delta t} \quad \text{(12a)}$$

and

$$fQ_1 = \frac{2\,k\,Q_1^{0.75}\,A^{-1/4} - 60\,Q_1\,\Delta t}{120\,\Delta t} \quad \text{(12b)}$$

This relationship is shown graphically in Fig. 13. Similar diagrams were prepared for the street gutter and were used to translate the gutter inflow hydrograph to the gutter outflow hydrograph. Because of the character of the street gutter cross sections, the equations are quite involved, and the work was done graphically.

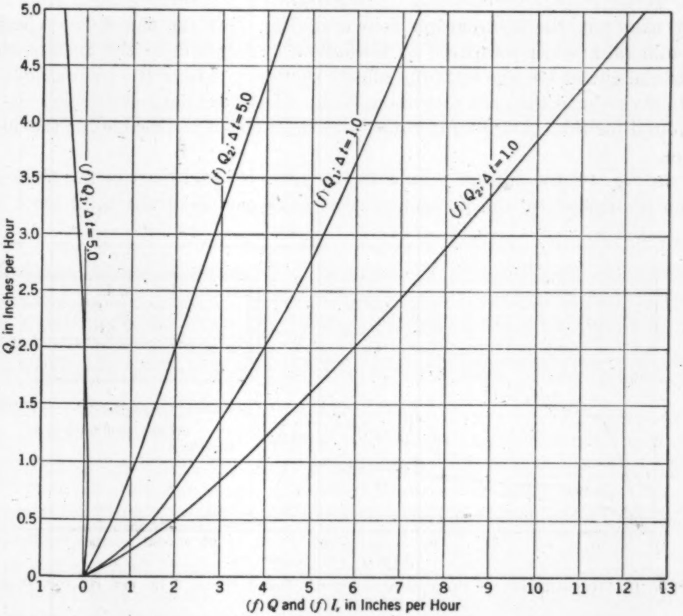


FIG. 13.—GRAPHS FOR THE SOLUTION OF THE STORAGE EQUATION FOR THE ALLEY

In this application, the value of I is the mean value for the period Δt; Q_1 is the value at the beginning of Δt; and Q_2 is the value at the end of period Δt in min; A is in acres. The graph in Fig. 13 has been prepared by inserting the value of k representative of the total tributary area to the alley for a length of 600 ft, and a slope of 0.005. The method of determining Q_2 from Q_1 is shown in an example by the dotted lines in this diagram.

For the purpose of this application, the inflow hydrographs of the type shown in Fig. 11(*b*) were modified by evaluating the rate of change of storage and adjusting the ordinates accordingly. The resulting outflow graph to the inlet from the alley channel is shown in Fig. 5(*a*).

It will be noted that for this case the storage volume is so small with respect to the mass of the flow quantity that the channel storage has no effect on the peak rate. However, it does modify the earlier part of the hydrograph somewhat. The earlier part of the hydrograph is important in translating these values to flow at a critical point on the sewer system, and therefore this modification also becomes important to the ultimate peak flow in the sewer.

The effect of gutter channel storage for the shorter and more intense rains is not apparent from the graphs. Actually, however, the reducing value of this gutter storage becomes highly important for the rains of 20-min duration. For example, the hydrograph shown in Fig. 7(*d*) for the delayed-pattern, 20-min rain, when compared to the inflow hydrograph to the gutters, shows that the gutter storage not only affects the rising side of the hydrograph, but actually reduces the peak rate about 30%. A similar but less drastic reduction is found for the other 20-min rains, and the effect is noticeable for the 40-min rains.

In Fig. 14, the outflow graphs to the inlets for each part of the block have been multiplied by the percentage that each part is of the total block area, and therefore transformed to inches per hour for the block as a whole (see Table 1 for areas). It has been assumed that the inlets are discharging into a sewer on the cross street; for this purpose the velocity in the sewer is taken as 5 ft per sec, the time of flow between the inlet connections has been evaluated, the hydrographs have been offset accordingly, and the ordinates have been

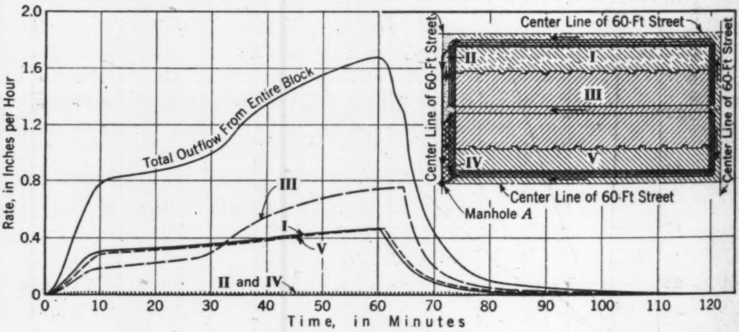


FIG. 14.—HYDROGRAPHS OF FLOW FROM A TYPICAL BLOCK (RAIN, 2 IN. PER HR FOR 60 MIN)

summed up to produce the total outflow graph from the entire block. (In Fig. 14, curves I, II, III, and IV are the weighted hydrographs of flow into inlets from areas bearing the same numbers.)

This graph, and the companion graphs in Figs. 5, 6, and 7, can be accepted as representing, rather accurately, the manner in which the flood flow from this block will occur at the concentrating manhole *A*. The flow so represented may be accepted as that which would result from the conditions assumed. Among these conditions, the character of the infiltration capacity curve and the amount of retention are important. As has been previously discussed, such graphs, if they are to be used quantitatively in sewer design, should be based on infiltration capacities and on retention values that have been developed out of actual flow gagings of such city lots. Undoubtedly the infiltration capacity for the turf on such lots and the characteristic retention on such turf will be found to be higher than those determined from the analysis of sprinkling plats. In this connection, the writers have made a quick preliminary review of the results of the gagings from station *B* (city block 4841 in St. Louis).[10] The development of this block as to character of surface cover and as to slopes is rather similar to the assumed city block outlined in Fig. 8(*a*). The principal difference is that the actual St. Louis block is nearly twice as long as that shown in Fig. 8(*a*). An analysis of the actual losses that occurred from the 70% pervious area in city block 4841 for a number of rains shows the results presented in Table 4. In this form, the values of losses have no specific significance. However, they can be broken down by analysis, between retention and total infiltration. The amount of infiltration, by comparison with the precipitation diagram, can be checked against infiltration opportunity. In this way, not only can mean infiltration capacity be determined, but by analogy to sprinkling plot results, an approximation can be made of the trend of infiltration capacity curve.

TABLE 4.—CITY BLOCK 4841, ST. LOUIS, MO.; 70% PERVIOUS

Date	Duration of rain, in min	Precipitation, in inches	Infiltration plus retention, in inches	Peak rate of runoff from area tributary to alley, in inches per hr	Date	Duration of rain, in min	Precipitation, in inches	Infiltration plus retention, in inches	Peak rate of runoff from area tributary to alley, in inches per hr
8- 2-15	60+	1.29	1.22	1.45	10-27-18	15	0.63	0.42	1.00
5-28-16	30	0.64	0.24	1.50	4-19-20	60+	1.30	0.68	1.95
6- 2-16	20	0.91	0.55	1.55	8-27-21	40	1.77	1.27	1.60
8-11-16	20	0.77	0.60	1.55	8- 8-23	50	3.51	3.00	2.45
8-12-16	60+	1.14	0.65	1.35	6-23-24	50	1.36	0.92	1.50
9- 7-16	10	0.78	0.54	1.65	8-24-24	15	1.13	0.89	1.80

Of course, the peak rate of block flow such as the 1.68 in. per hr of Fig. 5(*a*) for manhole *A* is not the critical rate for which the sewer would be designed at the outlet of this block. The importance of this hydrograph relates to the design of the sewer at some point where the so-called "critical time" is approximately 60 min. At that point on the sewer, if all the blocks in the drainage basin had the same typical development as shown herein, the hydrograph of discharge may be determined by offsetting the hydrographs from the various blocks, adding the ordinates, and making some further modification in the shape of the hydrograph that would result from the translation of the flood wave under this condition. This process has been applied to each of the hydrographs, the routing being traced through a typical rectangular sewer system.

The outflow graph at manhole *A*, representing the hydrograph of outflow from the entire city block, has been calculated for each of the three duration periods and each of the four rain patterns, and the full hydrographs are shown in Figs. 5, 6, and 7. In these figures there has also been indicated a value of the net rainfall or supply graph for the block as a whole. This is a weighting of the supply graphs for the pervious and impervious area with respect to the proportionate amount of these two types in the block.

For ease in inspection there has also been noted on each of the graphs the character of the hydrograph as it would appear after routing to the flow time point on the sewer system equivalent to the duration represented by the particular rain. By inspection, therefore, it is possible to note the following:

Comparison between the precipitation diagram and the weighted supply line shows the effect of the infiltration and the permanent retention that was introduced into the computations. A comparison between the supply line and the hydrograph at manhole *A* (Fig. 14) shows the combined effect of surface detention during overland flow and of gutter storage. The comparison between the hydrograph at manhole *A* and the hydrograph at the design point shows the equalizing effect resulting from the non-synchronous concentration of the flow from the various city blocks at this point. These last hydrographs, as to their peak values, are representative of the quantities such as have heretofore been commonly used in sewer design under the rational method. For those

TABLE 5.—SUMMARY OF VALUES FOR ASSUMED CITY BLOCK, FIG. 8(*a*) (46% Impervious, No Impervious Surfaces Draining Over Turf)

No.	Unit Rates	20-Min Duration Precipitation Pattern				40-Min Duration Precipitation Pattern				60-Min Duration Precipitation Pattern			
		Uniform	Adv.[b]	Int.[b]	Del.[b]	Uniform	Adv.[b]	Int.[b]	Del.[b]	Uniform	Adv.[b]	Int.[b]	Del.[b]
	In Inches Per Hour:												
	Precipitation Rate—												
1	Mean	3.5	3.5	3.5	3.5	2.5	2.5	2.5	2.5	2.0	2.0	2.0	2.0
2	Maximum	3.5	6.9	6.9	6.9	2.5	5.0	5.0	5.0	2.0	4.0	4.0	4.0
3	Maximum supply rate	3.0	5.64	5.80	6.15	2.20	4.01	4.25	4.50	1.80	3.22	3.52	3.75
	Peak Flow Rate at:												
4	Manhole *A*	2.15	2.91	3.09	4.00	1.95	2.65	2.76	3.65	1.68	2.27	2.68	3.60
5	20-min point[a]	1.63	1.98	2.24	2.26	1.70	2.48	2.49	2.58	1.58	2.17	2.43	2.88
6	40-min point[a]	1.27	1.37	1.55	1.56	1.47	1.96	1.98	1.95	1.44	1.92	2.03	2.28
7	60-min point[a]	0.99	1.00	1.17	1.17	1.25	1.55	1.57	1.57	1.30	1.60	1.66	1.84
	In Inches of Depth:												
8	Mass precipitation	1.17	1.17	1.17	1.17	1.67	1.67	1.67	1.67	2.0	2.0	2.0	2.0
9	Mass infiltration	0.33	0.31	0.26	0.24	0.45	0.46	0.39	0.38	0.52	0.56	0.42	0.41
10	Mass supply	0.84	0.86	0.91	0.93	1.22	1.21	1.28	1.29	1.48	1.44	1.58	1.59

[a] Rectangular drainage network (see text). [b] Advanced, intermediate, and delayed, respectively.

who are accustomed to thinking in terms of the coefficient of runoff, these peak rates may be compared with the mean precipitation intensity for the original duration period. These particular peak rates, of course, result from the particular network routing system used. This rectangular system with equally spaced branches, and with the flow time in the branch equal to the flow time in the main, is only one of a number of patterns that might have been applied. Higher peak rates would have resulted if the network system had been chosen of the sector type with the branches converging toward the time design point.

In order to give a general perspective view of the results of the calculations here presented, Table 5 has been prepared. All of the values given in this table are expressed in inches per hour, or the approximate equivalent cubic feet per second per acre, except the mass values of supply Q and infiltration F, which are in inches in depth distributed over the entire tributary area. It should be noted again that, quantitatively, the values are responsive directly to the rainfall patterns and to the infiltration capacity values used, and as to the latter, result in runoff rates greater than would be expected with the frequency involved. The values given for peak flow at the various time-design points reflect the particular network sewer pattern previously described and would be materially different for a different type of collecting system. However, they all have been derived on the same basic assumption and through the same application of hydraulics, and therefore are definitely comparable.

Summary and Conclusions

Conventional Urban Areas.—The conception of a uniform rainfall rate for a specific duration period is an unsatisfactory basis with which to compare runoff rates. In the opinion of the writers a fully rational basis of design for urban storm drainage involves the following steps:

(*a*) A study of rainfall intensity distribution and of the sequence of intensities within certain arbitrary duration periods, such as those chosen in this paper.

(*b*) For a particular area, some further research is needed with respect to infiltration capacity and overland flow for the type of turf normally utilized for city lawns.

(*c*) With the necessary basic data, it is entirely practicable and not an unreasonably involved or expensive procedure to develop, through hydraulic processes, the type of hydrographs shown in this paper.

(*d*) The rates so determined, on the basis of the conditions for which they are derived, should be subjected to a further frequency investigation, and from these studies a series of design curves might be prepared that would give the peak rate of runoff expected from each of the typical blocks, for specific slope conditions, and at any particular frequency.

(*e*) No attempt should be made to reduce these basic design values to generally applicable flow rates at any "time point," but in the design of any particular sewer system they would form the basis of flow routing through the particular drainage network.

(*f*) The procedure in steps (*a*) to (*e*) is subject to organization and tabulation in a manner similar to that under which the rational method has been commonly applied. It removes, entirely, the completely irrational coefficient of runoff; it involves a further study and revision of the criteria of critical time; it implies that the critical runoff rates at the sewer inlets will be routed through the sewer network in such a manner that the maximum rate of flow which can occur at any design point will have been determined; its application initially will require the extensive preparation of essential basic data, and extensive studies as to the physical characteristics of typical city blocks. Its application in general, after basic hydrographs of determined frequency have been prepared, will be little more difficult than the methods now used.

PART III.—CONCLUSION

The demonstration presented herein for small conventional areas shows, in principle and outline, the manner in which hydraulic methods may be applied to the organization of basic data and the derivation of secondary data essential to the computation of flood flow rates. The writers believe that basic hydrologic data are now becoming available, of a quality and in an extent that justify the more detailed type of hydrologic analysis set out, and that hereafter the basis of design of important hydraulic structures may be developed in a more logical and dependable manner than has heretofore been considered feasible. That the computations involved will be tedious in some cases is recognized; but they need be no more extensive, nor complex, than the technical computations commonly underlying the design of an important bridge.

The procedure outlined in detail for small areas of conventional type has been applied with some modifications, but upon the same basic principles, to larger drainage basins and to natural channels.

When applied to larger areas, all matters may obviously be on a somewhat different scale, and some phases of the computation are properly of a more approximate character. However, even as to such large areas:

(*a*) Precipitation patterns can be prepared for each important sub-basin, preferably in units not exceeding 1,000 sq miles;

(*b*) Infiltration capacity values reasonably well related to actual soil and cover conditions can be accumulated separately for each sub-basin, and can be matched with the pertinent precipitation pattern in a logical manner;

(*c*) From such a combination, the rate of production of surface runoff from each sub-basin will be developed in the order and manner in which it would actually occur;

(*d*) The diagram of surface runoff can then be translated into the hydrograph of stream flow at any particular point in each sub-basin through the unit graph or a proper evaluation of surface detention and channel storage. The hydrographs for the sub-basins can then be synchronized and combined for the main stream. This application unquestionably requires more detailed physical data as to stream channels and valley storage areas, and involves a material increase in the cost of the design phase of the engineering of hydraulic

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structures; but it permits a marked reduction in the factors of ignorance that need to be applied thereto.

This procedure is restricted to the determination of flood flow from surface runoff. For many basins, ground water or base flow during the rise of the hydrograph will have a small value and may be neglected. For some conditions it may be large and may add materially to flood flow. Any detailed discussion of this question is beyond the scope of this paper.

DISCUSSION

L. L. HARROLD,[16] ASSOC. M. AM. SOC. C. E.—To obtain data for the design of urban storm sewers, the authors have undertaken to utilize both hydrologic and physical data to develop "in place" surface runoff, and then have shown how the "in place" surface runoff can be routed through gutter systems by established hydraulic principles.

The paper implies that these hypotheses can be adapted to natural watersheds. It appears to the writer that there is no reason why they are not applicable as long as the natural watersheds produce only surface runoff. Many drainage areas are of this class, whereas the hydrology of other areas is complicated by the presence of ground-water flow and subsurface storm flow. By "subsurface storm flow" is meant that water which filters through the soil surface, forming temporary perched water, and flows laterally and appears at the ground surface lower down the slope. This subsurface storm flow, which joins the stream while surface flow is still in the drainage system, has not the characteristic of true ground-water flow because of its quick return to the surface and its short duration.

Even though the application of these hypotheses may be limited to those drainages where surface runoff constitutes the entire storm flow, there is still a wide field of possible use.

For example, if the hydraulic engineer in charge of designing the size of highway bridge openings knows the hydrologic and physical data of the natural drainage above the site (as did the authors for the urban block), the corresponding hydraulic design data can be derived. To the extent that certain watershed changes, such as the introduction of different land use practices or the construction of conservation structures, increase or decrease the surface retention and infiltration capacity of the basin, they will affect the rates and amounts of runoff from the area. The hypotheses presented furnish a vehicle for computing this effect at various points in the drainage system.

The problem can be related more closely to the example of the urban block described by the authors. If the 46.4% area of impervious surface were increased to 70%, could the sewer system handle the runoff? By applying the method to obtain the revised hydraulic design data, the engineer could tell the zoning commission that, if the district is to become "commercial," additional impervious surfaces are possible, and a certain sum of money will be necessary to enlarge the storm sewer system.

The writer has not overlooked the fact that these hypotheses must be checked in the field. It soon will be possible to perform the check over widespread areas. The Geological Survey, U. S. Department of the Interior, is collecting topographic and runoff data. The Weather Bureau, U. S. Department of Agriculture, is collecting rainfall data. The Department of Agriculture also is engaged in obtaining data on soils, land use, and infiltration. Several federal agencies are engaged in computing infiltration-capacity values for large areas by correcting the runoff record for channel storage and by relating the resulting direct runoff value to the precipitation data. At this point the writer would like to emphasize the importance of using the precipitation data separately for each gage, rather than combining the records for several gages. This is especially important when some of the rain falls at rates less than the infiltration capacity. When the rainfall rate exceeds the infiltration rate at all points, average basin rainfall rates can be used. The following example explains the reason for this caution.

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[16] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

TABLE 6.—RAINFALL INFILTRATION AND RUNOFF—A HYPOTHETICAL EXAMPLE

Area[a]	INCHES PER HOUR			15-min runoff (inches)
	Rainfall intensity	Infiltration	Excess rain	
(1)	(2)	(3)	(4)	(5)
A	8.0	2.00	6.00	1.50
B	4.0	1.00	3.00	0.75
C	0.40	0.80	0.00	0.00
D	0.04	0.30	0.00	0.00
Aver.[b] . .	3.11	1.02	2.25	. . . .
15 min[c] .	0.78	0.26	0.56	0.56

Average rain intensity on watershed, in in. per hr....	3.11
Average infiltration on watershed, in in. per hr......	1.02
Average rain excess on watershed, in in. per hr......	2.09
Total runoff for 15 min, in in......................	0.52

[a] Equal size. [b] Weighted average. [c] Amounts for 15 min.

Assume that in a given watershed or drainage area (see Table 6) there are four rain gages serving equal areas of the basin. Each area served by a rain gage has similar physiographic characteristics. The infiltration rates for each area in Col. 3, Table 6, give an average rate on the entire watershed of 1.02 in. per hr. The corresponding rainfall intensity for each area is shown in Col. 2, Table 6, from which the rain excess over infiltration, in a 15-min period, is found to be $(3.11 - 1.02)\,\frac{15}{60} = 0.52$ in., the average for the watershed. On the other hand, the 15-min runoff values in Col. 5, Table 6, indicate a total excess of 0.64 in.

Assuming that the true runoff was 0.52 in., the computed infiltration is $(0.78 - 0.56) = 0.22$ in., or 0.88 in. per hr instead of the weighted average of 1.02 in. per hr. For storms having rainfall rates over parts of the area less than the corresponding infiltration capacity, the computed value of infiltration capacity for the drainage basin will always be less than the actual.

From plats and small watersheds the infiltration capacities for various surface conditions are obtained. If engineers are to use the methods set forth in the paper for applying these infiltration-capacity values to larger drainage systems, it seems imperative that the rainfall records for each gage be applied, through the use of isohyetals for short-time intervals, to the separate drainage units having uniform infiltration capacities.

C. E. RAMSER,[17] M. AM. SOC. C. E.—The science of hydrology as applied to practical engineering problems has been advanced notably by this paper. The method presented is especially appropriate for use in storm-sewer design where large expenditures are involved. With an increase in rainfall and runoff data made available through hydrologic research, no doubt the method can be simplified greatly. This is true particularly in connection with the estimation of runoff from agricultural areas where, quite often, the cost of improvements would not justify a large expenditure for a detailed analysis of the various hydrologic factors.

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[17] Chf., Hydrologic Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

LeRoy K. Sherman,[18] M. Am. Soc. C. E.—A large amount of valuable scientific research and study has been made on the phenomena of infiltration since 1930. The use of the coefficient of runoff has been condemned as irrational and erroneous. Little has been presented in applied hydrology that would induce the engineer to discard the coefficient and make use of the infiltration theory. In this connection the authors' paper is timely.

Some of their remarks on variable conditions—"not yet fully agreed on the mechanics of infiltration," "probably never be possible to reduce it to a simple routine," etc.—may lead the reader to the conclusion that the application of the infiltration theory is still too complex to serve as a working tool. It will be constructive, therefore, to emphasize the major elements of the infiltration theory that are now accepted as facts. In the writer's opinion, they include the following points noted by the authors:

(*a*) "* * * curves [of infiltration capacity] all have characteristically similar shapes, * * * relatively high in the beginning of precipitation, decreasing rapidly * * *, and tending to reach * * * definite minimum values * * *" (see heading "Character of Infiltration-Capacity Curves").

(*b*) "* * * the march of infiltration-capacity rates is closely correlated with mass infiltration" (see heading "Infiltration Capacity"). The writer would supplement this item with the following: The curve of infiltration capacity is readily converted into a curve of mass infiltration. In this form it furnishes a simple device for estimating the volumes of runoff due to real or hypothetical storms.[19]

(*c*) Infiltration capacity varies: (4) With vegetal cover (and hence with season); (2) with soil moisture and soil porosity; and (1) little with surface slope (see heading "Character of Infiltration-Capacity Curves").

(*d*) "Infiltration has ceased to be a vague phenomenon; * * * infiltration expressed in terms of infiltration capacity may now be introduced into engineering practice" (see heading "Relation of Precipitation Rate, Infiltration-Capacity Rate, and Excess Rainfall").

The writer concurs with the foregoing statements of the authors. The scientist here can select many exceptional cases, flaws, variations, and inaccuracies in his search for perfection. The engineer, however, will find that these statements of facts, combined with allowable approximations, can furnish reliable working procedures for estimating infiltration and runoff.

The authors' procedure for correlating the periods of observed rain intensities with the curve of infiltration capacity is a practical requirement in the application of the infiltration theory. Their presentation of this detail appears a little involved. Since infiltration capacity decreases with an increase of soil

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[18] Cons. Engr., Chicago, Ill.

[19] "The Unit Hydrograph and Its Application," by L. K. Sherman, *Bulletin*, Associated State Eng. Societies, April, 1941.

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moisture in the manner shown by the capacity curve, it follows that the equivalent time T, referred to the capacity curve, is equal to the volume of infiltration divided by the average infiltration capacity. Thus, if rain fell for two hours at an intensity of 0.2 in. per hr, and the infiltration capacity was 0.4 at the beginning of rain, then T, due to the first hour of rain, is $\frac{0.2}{0.4} = 0.5$ hr. At the beginning of the second hour of rain, the volume of infiltration had been increased by 0.2 in. and, by the mass curve of infiltration, this lowered the infiltration capacity to 0.35. For the second hour of rain T is $\frac{0.2}{0.35} = 0.57$ hr. The equivalent value of T for the two hours of this rain is 1.07 hr on the capacity curve. Also, if the rain excess (observed surface runoff) for a storm was 1.5 in., and the derived value of average areal infiltration capacity was 0.3 in. per hr, then the equivalent duration time of the storm $T = \frac{1.5}{0.3} = 5$ hr.

In Part II the authors have described in detail the phenomena and procedure of computing surface runoff from small areas. This is predicated upon the use of curves of infiltration capacity. They have given little space in this paper to the methodology for deriving this curve. They do, however, refer the reader to several papers[8] on the subject.

One of these papers,[20] in the writer's opinion, is a valuable and essential prelude to the authors' paper. The methodology in this article for deriving the curve of infiltration from observed rainfall and runoff applies to small basins in which the downpours of rain are readily correlated with the corresponding volumes or hydrographs of runoff. For small areas, this methodology of derivation is ideal in both simplicity of procedure and accuracy in results. For larger areas, or for small plots on which runoff is measured volumetrically without the hydrograph, the procedure is not directly applicable.

No mention is made, in the paper, of the derivation or use of average areal infiltration capacity. This procedure is in general use. It was first presented by Mr. Horton in 1937.[21] The methodology is simple and applicable to all basins, large or small. On small areas, the initial losses (interception and pocket pondage) may be segregated to give the volume of infiltration and the average areal infiltration capacity (f_a). On large areas, this segregation is generally impracticable. The derived value in this case may be properly termed the "average loss rate" (f_{av}). Studies by the staff of the Illinois State Planning Commission[22] show that reliable capacity or loss-rate curves, and their corresponding mass curves, may be derived from these capacity or loss-rate values.

Attempts have been made by the writer and others to make direct use of these f_a or f_{av} values without subsequent derivation of the curves of infiltration capacity or loss rates. Such attempts, based on averages from unrelated storms, have been futile and misleading. The results are no better than the similar use of a batch of runoff coefficients. Possibly this situation accounts for the authors' silence on the subject of average infiltration capacity.

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[20] "Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois," by W. W. Horner and C. L. Lloyd, *Transactions*, Am. Geophysical Union, 1939, p. 522.

[21] "Determination of Infiltration Capacity for Large Drainage Basins," by R. E. Horton, *loc. cit.*, 1937, Pt. II, pp. 371–385; also, "Determination of Areal Average Infiltration-Capacity from Rainfall and Runoff Data," by R. E. Horton and R. Van Vliet, Office of Land Use Coordination, Flood Control, U. S. Dept. of Agriculture, November, 1940 (mimeographed).

[22] "Application of the Infiltration Theory to Engineering Practice," *Transactions*, Am. Geophysical Union, 1941 (publication pending).

The authors have made a useful contribution to engineering hydrology. Like all pioneering steps, it can and will be improved and simplified.

A. J. SCHAFMAYER,[23] M. AM. SOC. C. E.—The method of arriving at the rate of runoff from a given area by deducting the rate of infiltration into the soil plus other losses from the rainfall intensity rate is an interesting departure from former methods. The thorough analysis of the large number of variables entering into runoff provides a vivid explanation of the wide variations found in actual runoff for various types of storm. The fact that different agencies, notably the Weather Bureau and Soil Conservation Division of the Department of Agriculture, are gathering more extensive data on rainfall and runoff than heretofore should tend to reduce these errors. Also, the increasing mass of data on stream gaging being secured by the U. S. Geological Survey, in cooperation with various states, should effect further reductions. The present studies by the authors, when compared with these data, will undoubtedly provide further valuable information as to infiltration and the consequent runoff.

The development of the unit hydrograph by LeRoy K. Sherman,[24] M. Am. Soc. C. E., together with the distribution graph by Merrill Bernard,[25] M. Am. Soc. C. E., and the studies in infiltration by the authors and others mentioned in their paper, have added methods by which the designer may make independent checks of his studies of probable runoff.

Also, the authors' careful consideration of the effects on resultant runoff of the varying conditions of surface as to slope, vegetation, method of cultivation, type of soil and its moisture content, and the proportion and disposition of impervious surfaces in any given area, emphasizes the bearing of chance events on any estimate of runoff rates or of the probable frequency of any particular rate.

For the engineer designing storm conduits to carry peak runoffs, a mass of data on simultaneous rain gaging, infiltration, and stream gaging, and simple formulas for its use, should ultimately prove a valuable aid. The material submitted by the authors is a contribution in this direction, since it tends toward removal of one of the unknowns (rate of infiltration) from the coefficient of runoff in the so-called rational formula.

City of Chicago design in recent years has dealt mostly with storm sewers to serve urban areas up to several thousand acres, and the so-called rational formula has been used. The engineer designing sewers or conduits for storm flow is concerned with maximum or peak rates, and, although it is evident that the contribution of the authors may also be of value to engineers concerned with average and minimum rates of flow, the work in Chicago has not been along

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[23] Asst. Chf. Engr. of Sewers, Bureau of Sewers, Dept. of Public Works, City of Chicago, Chicago, Ill.

[24] "Stream Flow from Rainfall by Unit Graph Method," by LeRoy K. Sherman, *Engineering News-Record*, April 7, 1932.

[25] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100, 1935, p. 347.

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these lines so that those aspects of the study of runoff are not considered in this discussion.

For peak flows, the commonly accepted formula has been

$$Q = C\,I\,A_w \qquad (13)$$

in which, in addition to the notation of the paper, C is the coefficient, I the intensity rate, A_w the watershed area in acres, and Q the peak runoff. After making determinations required by the method proposed by the authors, the equation could be stated

$$Q = (I - L)\,A_w \qquad (14)$$

in which L would represent the rate of loss through infiltration and pondage. This equation might be considered an intermediate step between the so-called rational formula and the equation

$$Q = 10{,}000\,P\,\sqrt{M} \qquad (15)$$

by C. S. Jarvis,[26] M. Am. Soc C. E., or

$$Q = R\,M \qquad (16)$$

as suggested by George W. Pickels,[27] M. Am. Soc. C. E., in which R represents the expected rate of runoff per square mile, M is the area in square miles, and P is a runoff percentage coefficient. With the increasing mass of available data, it may be that, before many years, this last form of equation (Eq. 16) can be used by selecting a runoff factor directly from observed data. This would enable one to avoid both the wide range of probable errors in intensity and frequency involved in applying the rational formula and the extended analytical work involved in the method proposed by the authors. Until such time, however, it will be preferable to use one of the indirect methods.

The application of the infiltration capacity method to a specific problem, as indicated in Part II, will require considerable adaptation and judgment. The type of city block shown in Fig. 8 and used in the hydrographs in Figs. 5, 6, and 7 differs materially from the normal city block in Chicago. The area, shape, and arrangement of the buildings differ. Also, in Chicago the sewers are usually located in the streets in front of the buildings and the roof drainage connects directly to the sewers. This would advance the crest of the hydrograph somewhat in time. However, since the objective of the designer is to determine the peak storm flow, the time factor, which is important in the rational method, loses its importance in the method proposed by the authors. In Figs. 5, 6, and 7, the outflow hydrographs show a notable uniformity of peak flow for the 60-min, 40-min, and 20-min points whether the storm is of the advanced, intermediate, or delayed pattern. The studies of the flow in gutters for streets and alleys are based upon conditions practically identical with the ordinary conditions in Chicago. The equations and curves (Figs. 12 and 13), therefore, could be applied directly to Chicago calculations under similar conditions without changes in factors.

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[26] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 364.

[27] "Run-Off Investigations in Central Illinois," by George W. Pickels, *Bulletin No. 232*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., September, 1931, p. 130.

To apply the authors' method, as they have stated, it is necessary to determine infiltration capacity and overland flow for the particular areas entering into the specific problem. This involves determinations of both initial and continuing infiltration capacities and of the precipitation diagram, and it requires considerable additional investigation for many tributary areas with extensive and tedious computations. Therefore, its immediate application for local studies will probably consist of calculations for runoff, using the rational formula with a coefficient of runoff, and checking and comparing the results with results obtained by other methods, including that suggested by the authors, with estimated values of infiltration capacity, overland flow, and for the precipitation diagram adapted from the data and diagrams submitted by the authors.

C. S. Jarvis,[28] M. Am. Soc. C. E.—No matter how commendable the author's purpose may be in attempting to determine surface runoff from rainfall without using coefficients, both the procedure and the objective obtained are subject to challenge—in popular parlance it might be said that the remedy is at least equal to the ills it is intended to cure. Specifically, the obvious implication is that ample data from properly distributed stations are available wherewith to account for the changing intensities and migrations of turbulent centers during the storm period. The painstaking work done in the Soil Conservation Service has demonstrated the wide digressions from point observations ordinarily experienced between stations much more closely spaced than has yet been found practicable on extensive drainage basins. With such spacings of rainfall stations as have been provided over the United States at large, or are now in process, it appears that the authors, in the matter of detail and apparent refinement, may go far beyond the limits of accuracy and reasonable interpretation of basic data, even with such detailed records as they had assembled for a metropolitan center that is especially favored in both length and quality of hydrologic records. It is conceivable that the assumptions involved in the application of this method to less-favored extensive areas by less skilled technicians may include greater errors and uncertainties than some of the more familiar devices.

It is readily conceded that a large army of explorers examining each unit of area in a strange country may ultimately acquire greater volumes and details of information than would a reconnaissance or scouting party; yet comprehensive reconnaissance and rapid scouting with bird's eye views from points of vantage may more nearly satisfy the immediate requirements, and at the same time determine whether the detailed study should be undertaken. Candidly, the writer has deplored the general neglect of voluminous basic data which have been shunted aside during recent years in favor of the microscopic approach, the most satisfactory test and confirmation of which would be the derivation of those same basic data by integration of discharges from elementary areas. Results from individual efforts as well as those of small groups in research projects have brought such returns from a few months' effort as could not be encompassed in years or decades by such procedures as would ban the normal use of coefficients, formulas, and other means of summarizing available observations and experience.

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[28] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

The data in Table 7, representative samples of extensive tabulations ready for publication, illustrate the advantageous use of coefficients, in this instance expressing percentages of the 5-yr (1934–1938) mean discharges of the Mississippi River and some of its principal tributaries, as well as of some minor tributaries with notable records. The discharges for individual years of the 5-yr period, together with the recorded or estimated means for each period progressively increasing by 5-yr steps up to 30 yr, by 10-yr steps up to 60 yr, thence by 20-yr steps up to 80, 100, or 120 yr according to available data, have provided some fundamentals which could not conceivably be attained except by use of coefficients showing relationships that may be used to bridge gaps in the record. Would it be venturing too far to suggest that the consistent trends and variations from the latest 5-yr means reflected in the percentage coefficients for yearly or longer-period means should provide some guidance for such detailed analyses as the authors have so ably undertaken? Instead of building up exclusively from small increments, using the synthetic method, they would be employing derivations from larger units by analysis of basic data. It would be comparable to making use of the broad perspective views from reconnaissance scouting in order to locate and orient the small plot of strategic importance, and would in no way restrict the detailed studies within the particular plot.

Granting that there is a fundamental difference between the authors' use of instantaneous hydrologic phenomena and the writer's approach through yearly or longer period means, it should be realized that the latter represents the total natural integration of the former, and is fully as susceptible to forecasting, if not more dependable than are the instantaneous or short-period phenomena.

The assertions, such as the authors', that empirical formulas and devices are unscientific and outmoded, including the use of coefficients, do not seem to eliminate them from practice nor from the preferred equipment where basic data are either meager or broken. Surely the results of procedures outlined in this paper may be expressed in the form of coefficients showing what portion of the rainfall appeared as runoff. Whether this percentage is identical with that of excess rainfall or divergent from it, such expression should not be ruled out of reports dealing with hydrologic data. For example, the Ohio River flood volume of March, 1936, at Pittsburgh, Pa., represented about 95% of the rainfall and snow mantle on the tributary watersheds, and about 40% of the total flood discharge occurred on the day of maximum flood. Likewise the Ohio River flood volume of January, 1937, at Metropolis, Ill., represented 69% of the storm precipitation plus the water content of snow mantle, of which not more than 4% was discharged on the day of maximum flood.

Fortunately, the 11-yr record period from 1928 to 1938, inclusive, for Spur, Tex., afforded opportunities for practical tests of runoff determinations by coefficients and by other means. The total precipitation of twenty-eight recorded, runoff-producing storms for this period amounted to 40.89 in., and the observed runoff was 16.26 in. or 40% for watershed No. 1, with 53, 38, 57, and 21%, respectively, for watersheds Nos. 2, 3, 5, and 6. Applying these

TABLE 7.—Record of Mean Discharges at Stream-Gaging Stations of Method," or Estimated from all Available Data, for Various September 30, 1938; the Latest

Item No.	Stream and station	Drainage area, in sq miles	Years of Record[b]			1937–1938	
			Observed		Gaged and estimated	cfs[c]	Percentage of 5-yr mean
			Gagings	Gage heights			
1	Mississippi River, St. Paul, Minn.	36,800	46	61	100	8,218	167.4
2	Mississippi River, LaCrosse, Wis.	62,800	9	64	100	30,390	143.7
3	Mississippi River, Le Claire, Iowa	88,600	65	65	100	49,770	137.5
4	Mississippi River, Davenport, Iowa	88,800		79			
5	Mississippi River, Keokuk, Iowa	119,000	60	71	100	65,520	135.4
6	Mississippi River, Grafton, Ill.	170,000	39[d]	59	100	(99,000)	123.8
7	Mississippi River, Alton, Ill.	171,500	11	49			
8	Illinois River, Peoria, Ill.	13,480	28	67	80	17,410	114.2
9	Illinois River, Beardstown, Ill.	(22,500)	18	60	80	22,260	112.0
10	Missouri River, Fort Benton, Mont.	24,600	57	57	100	6,010	130.4
11	Missouri River, Fort Benton, Mont. (including consumptive use)	24,600	57	57	100		
12	Missouri River, Boonville, Mo.	505,700	13	65	100	40,190	103.4
13	Missouri River, Hermann, Mo.	528,200	39[d]	65	100	56,640	106.3
14	Mississippi River, St. Louis, Mo.	701,000	10	81	100	156,800	116.6
15	Mississippi River, Cape Girardeau, Mo.	716,000	5	47	100	(173,000)	118.9
16	Greenbrier River, Alderson, W. Va.	1,357	41	41	100	2,061	94.8
17	Kanawha River, Kanawha Falls, W. Va.	8,367	61	61	100	13,170	105.4
18	Wabash River, Mt. Carmel, Ill.	28,600	11	54	100	35,020	146.5
19	Cumberland River, Nashville, Tenn.	12,860	50	65	100	(18,000)	90.8
20	Tennessee River, Chattanooga, Tenn.	21,400	64	65	100	35,720	103.3
21	Tennessee River, Florence, Ala.	30,800	44	67	100	49,600	100.8
22	Tennessee River, Johnsonville, Tenn.	38,520	49	62	100	58,330	98.4
23	Ohio River, Pittsburgh, Pa.	19,100		67	100	(32,600)	106.5
24	Ohio River, Sewickley, Pa.	19,500	5	5	100	33,120	106.5
25	Ohio River, Huntington, W. Va.	55,200	4	25	100	85,240	109.7
26	Ohio River, Cincinnati, Ohio	76,580		68	100	(108,715)	107.8
27	Ohio River, Louisville, Ky.	91,200	4	67	100	124,000	106.1
28	Ohio River, Metropolis, Ill.	203,000	12	65	100	269,800	105.4
29	Ohio River, Cairo, Ill.	203,900	39[d]	80	100	(270,700)	105.4
30	White River, De Valls Bluff, Ark.	23,800	11	11	100	30,140	131.4
31	White River, Clarendon, Ark.	25,750	11	54	100	(37,000)	129.0
32	Arkansas River, Little Rock, Ark.	157,900	39[d]	67	120	46,590	129.1
33	Yazoo River, Greenwood City, Miss.	7,450	11	35	120	(11,240)	114.5
34	Yazoo River, Yazoo City, Miss.	(8,500)		54	120	(12,600)	114.5
35	Red River, Garland, Ark.	51,500	9	9	120	27,380	150.8
36	Red River, Alexandria, La.	65,850	39[d]	67	120	(50,000)	166.3
37	Mississippi River, Memphis, Tenn.	932,800	9	67	120	445,500	107.7
38	Mississippi River, Arkansas City, Ark.	1,130,700	39[d]	59	120	550,700	110.1
39	Mississippi River, Vicksburg, Miss.	1,144,500	56	68	120[e]	567,300	109.4
40	Mississippi River, Vicksburg, Miss., by "sampling method"		56	68	120	((583,814))[f]	112.7
41	Mississippi River, Natchez, Miss.	1,149,400	35	105	122[e]	(573,200)	109.5
42	Miss.-Atchafalaya, Red River Landing and Simmesport, La.	1,242,700	65[d]	65	120	(668,000)	114.8
42*a*	Added items from Mississippi commission reports						
	Test of Official Measurements						
43	Totals of 13 tributaries, Meramec River, Mo., to White River, Ark., inclusive	43,488	9–17	17	120	50,570	125.8
44	Ungaged intervening areas	27,212			120	(30,700)	108.9
6	Mississippi River, Grafton, Ill.	170,000	39[d]	59	120	(99,000)	123.8
13	Missouri River, Hermann, Mo.	528,200	39[d]	65	120	56,640	106.3
29	Ohio River, Cairo, Ill.	203,900	39[d]	80	120	(270,700)	105.4
32	Arkansas River, Little Rock, Ark.	157,900	39[d]	67	120	46,590	129.1
45	Totals, items 6, 13, 29, 32, 43, and 44	1,130,700				(554,200)	112.0
46	Departures of derived data					+3,500	
38	Mississippi River, Arkansas City, Ark.	1,130,700	39[d]	59	120	550,700	110.1
47	Departures in percentage of the latest 5-yr mean					+0.7	

THE MISSISSIPPI RIVER SYSTEM, EITHER OBSERVED, DERIVED BY THE "SAMPLING
PERIODS OF RECORD TERMINATING WITH THE WATER-YEAR,
5-YR MEAN BEING TAKEN AS 100%

Item No.	1936–1937		1935–1936		1934–1935		1933–1934		5-YR MEAN 100%	
	cfs[a]	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	Percentage of 5-yr mean	cfs	csm[b]
1	5,379	109.6	5,330	108.6	3,688	75.1	1,935	39.4	4,910	0.133
2	17,580	83.1	23,570	111.4	22,930	108.4	11,300	53.4	21,154	0.337
3	33,160	91.6	37,390	103.3	41,820	115.5	18,870	52.1	36,200	0.409
4										
5	49,900	103.1	47,680	98.6	57,260	118.4	21,540	44.5	48,380	0.407
6	86,000	107.5	72,200	90.3	106,000	132.5	36,800	46.0	80,000	0.471
7									80,600	0.470
8	16,120	105.7	13,320	87.3	18,590	121.9	10,810	70.9	15,250	1.131
9	21,710	109.2	16,540	83.2	26,540	133.5	12,360	62.2	19,882	0.884
10	3,618	78.5	4,559	98.9	4,307	93.5	4,952	107.5	4,608	0.187
11									(6,000)	0.244
12	38,900	100.1	33,860	87.2	53,560	137.9	27,740	71.4	38,850	0.077
13	59,000	110.7	41,090	77.1	80,010	150.1	29,750	55.8	53,300	0.101
14	146,800	109.1	113,700	84.5	187,600	139.5	67,700	50.3	134,520	0.192
15	162,000	111.4	117,000	80.4	203,600	140.0	71,730	49.3	145,466	0.203
16	2,078	95.6	2,435	112.0	3,007	138.3	1,291	59.4	2,174	1.602
17	11,480	91.8	13,840	110.7	16,180	129.5	7,824	62.6	12,499	1.494
18	39,250	164.2	14,510	60.7	22,710	95.0	8,043	33.6	23,907	0.836
19	(24,600)	124.1	(16,300)	82.2	(23,500)	118.6	(16,700)	84.3	(19,820)	1.541
20	36,810	106.5	38,600	111.7	35,700	103.3	(26,000)	75.2	34,566	1.615
21	53,570	108.9	55,740	113.3	49,650	100.9	37,400	76.0	49,192	1.597
22	67,700	114.2	61,740	104.1	62,540	105.5	46,140	77.8	59,290	1.539
23	(38,900)	127.1	(31,500)	102.9	(29,400)	96.1	(20,600)	67.3	(30,600)	1.601
24	39,520	127.1	32,030	103.0	29,840	96.0	20,960	67.4	31,094	1.595
25	95,110	122.4	79,370	102.1	82,920	106.6	(46,030)	59.2	77,734	1.408
26	(125,971)	124.9	(100,609)	99.7	(111,897)	110.9	(59,009)	58.5	(100,863)	1.317
27	149,000	127.4	111,900	95.7	(131,586)	112.5	(68,195)	58.3	116,919	1.282
28	337,100	131.7	226,600	88.5	285,200	111.4	(161,000)	62.9	255,940	1.261
29	(338,500)	131.7	(227,500)	88.5	(286,400)	111.5	(161,600)	62.9	(256,940)	1.260
30	26,600	116.0	10,110	44.1	32,190	140.4	15,630	68.2	22,934	0.964
31	(35,163)	122.6	10,246	35.7	43,252	150.8	17,709	61.8	(28,674)	1.112
32	35,700	98.9	14,010	38.8	65,400	181.2	18,740	51.9	36,088	0.229
33	(14,900)	151.8	(7,000)	71.3	10,230	104.2	5,720	58.3	(9,818)	1.318
34	(16,700)	151.8	(7,900)	71.8	(11,400)	103.6	(6,400)	58.2	(11,000)	1.294
35	16,726	92.1	7,298	40.2	28,350	156.2	(11,000)	60.6	18,151	0.352
36	(27,000)	89.8	10,885	36.2	42,447	141.2	(20,085)	66.8	30,060	0.456
37	516,700	124.9	349,900	84.6	511,400	123.7	244,285	59.1	413,551	0.444
38	623,600	124.7	387,900	77.6	651,500	130.3	286,900	57.4	500,140	0.442
39	641,200	123.7	401,600	77.5	676,800	130.6	305,000	58.8	518,380	0.453
40	((632,589))	122.1	((405,886))	78.3	((660,815))	127.5	((318,929))	61.6	((518,100))	0.453
41	(647,800)	123.7	(405,740)	77.5	(683,660)	130.6	(308,150)	58.8	(523,667)	0.456
42	(701,080)	120.5	(424,730)	73.0	(772,870)	132.8	(342,665)	58.9	(581,870)	0.468
42a			421,610	72.5	750,235	128.9	545,545	59.4		

AND ESTIMATED DISCHARGES

Item No.	1936–1937		1935–1936		1934–1935		1933–1934		5-YR MEAN 100%	
43	49,288	122.6	18,325	45.6	58,247	144.8	25,112	62.4	40,214	0.925
44	(40,000)	141.9	(14,000)	49.7	(44,000)	156.1	(15,000)	53.2	(28,196)	1.036
6	86,000	107.5	72,200	90.3	106,000	132.5	36,800	46.0	80,000	0.471
13	59,000	110.7	41,090	77.1	80,010	150.1	29,750	55.8	53,300	0.101
29	(340,500)	131.7	(227,500)	88.5	(288,400)	111.5	(161,600)	62.9	(256,940)	1.260
32	35,700	98.9	14,010	38.8	65,400	181.2	18,740	51.9	36,088	0.229
45	(610,488)	123.4	(387,125)	78.3	(642,057)	129.8	(287,002)	58.0	(494,738)	0.438
46	−13,112		−775		−9,443		+102		−5,402	
38	623,600	124.7	387,900	77.6	651,500	130.3	286,900	57.4	500,140	0.442
47	−2.6		−0.16		−1.9		+0.02		−1.1	

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TABLE 7.—

Item No.	10-YR MEAN		15-YR MEAN		20-YR MEAN		25-YR MEAN		30-YR MEAN		40-YR MEAN	
	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%
1	4,906	99.9	5,346	108.9	6,492	132.2	7,771	158.3	7,723	157.3	8,810	179.4
2	19,947	94.3	(21,900)	103.5	(23,100)	109.7	(24,250)	114.6	(24,100)	113.9	(27,500)	130.0
3	35,845	99.0	37,734	104.2	39,566	109.3	41,456	114.5	41,250	114.0	45,000	124.3
4												
5	48,218	99.7	50,352	104.1	51,880	107.2	54,340	112.3	54,586	112.8	58,320	120.5
6	84,844	106.1	93,130	116.4	93,900	117.4	96,700	120.9	96,700	120.9	101,240	126.5
7												
8	16,405	107.6	17,617	115.5	17,158	112.5	16,700	109.5	16,532	108.4	(17,173)	112.6
9	21,581	108.5	24,354	122.5	23,490	118.1	(22,500)	113.2	(22,500)	113.2	(23,000)	115.7
10	5,122	111.2	6,232	135.2	6,496	141.0	7,181	155.8	7,615	165.3	8,005	173.7
11	(6,500)	108.3	(7,300)	121.7	(7,400)	123.3	(8,000)	133.3	(8,300)	138.3	(8,550)	142.5
12	49,775	128.1	52,955	136.3	(55,000)	141.6	(58,000)	149.3	(58,500)	150.6	(60,000)	154.4
13	59,520	111.7	71,257	133.7	73,165	137.3	75,664	142.0	74,027	138.9	74,900	140.5
14	145,670	108.3	(165,500)	123.0	(168,200)	125.0	(173,500)	129.0	(171,900)	127.8	(177,400)	131.9
15	(154,500)	106.2	(178,400)	122.6	(181,400)	124.7	(187,000)	128.6	(185,200)	127.3	(190,800)	131.2
16	2,038	93.7	2,054	94.5	2,067	95.1	2,086	96.0	2,080	95.7	2,111	97.1
17	11,561	92.5	11,656	93.3	11,821	94.6	11,910	95.3	11,884	95.1	12,510	100.1
18	25,929	108.5	(27,000)	112.9	(27,000)	112.9	(26,800)	112.1	(26,800)	112.1	(26,000)	108.8
19	(20,128)	101.6	20,579	103.8	21,584	108.9	21,483	108.4	21,559	108.8	20,992	105.9
20	35,283	102.1	34,762	100.6	36,702	106.2	36,357	105.2	36,604	105.9	37,131	107.4
21	51,506	104.7	50,604	102.9	53,443	108.6	52,534	106.8	52,960	107.7	52,760	107.3
22	63,205	106.6	62,597	105.6	66,258	111.8	64,538	108.9	65,000	109.6	64,030	108.0
23	(29,500)	96.4	(31,100)	101.6	(31,000)	101.3	(31,100)	101.6	(31,000)	101.3	(30,700)	100.3
24	(30,000)	96.5	(31,600)	101.6	(31,500)	101.3	(31,600)	101.6	(31,500)	101.3	(31,200)	100.3
25	(71,680)	92.2	(73,000)	93.9	(74,000)	95.2	(75,244)	96.8	(75,244)	96.8	(77,600)	99.8
26	(94,696)	93.9	(96,000)	95.2	(98,000)	97.2	(99,814)	99.0	(99,700)	98.8	(102,000)	101.1
27	(111,219)	95.1	(114,000)	97.5	(116,000)	99.2	(117,537)	100.5	(116,000)	99.2	(118,700)	101.5
28	253,970	99.2	(266,500)	104.1	(274,300)	107.2	(273,000)	106.7	(275,500)	107.6	(270,000)	105.5
29	255,060	99.3	267,600	104.1	275,300	107.1	273,900	106.6	276,600	107.6	271,500	105.7
30	24,200	105.5	27,000	117.7	(25,500)	111.2	(27,000)	117.7	(26,300)	114.7	(26,600)	116.0
31	30,250	105.5	(32,000)	111.6	(30,000)	104.6	(30,500)	106.4	(30,000)	104.6	(30,000)	104.6
32	36,200	100.3	40,260	111.6	41,240	114.3	40,000	110.8	37,200	103.1	37,600	104.2
33	10,293	104.8	(10,500)	106.9	(9,500)	96.8	(9,000)	91.7	(8,200)	83.5	(8,800)	89.6
34	(11,500)	104.5	(11,800)	107.3	(10,700)	97.3	(10,100)	91.8	(9,200)	83.6	(9,900)	90.0
35	16,920	93.2	(17,060)	94.0	(17,420)	96.0	(17,060)	94.0	(16,340)	90.0	17,060	94.0
36	30,120	100.2	29,090	96.8	30,630	101.9	29,150	97.0	27,525	91.6	28,870	96.0
37	426,000	103.0	(460,000)	111.2	(472,000)	114.1	(475,000)	114.9	(477,000)	115.3	(480,000)	116.1
38	514,400	102.9	555,800	111.1	574,870	114.9	582,900	116.5	586,900	117.3	588,000	117.6
39	534,240	103.1	(561,500)	108.3	(573,400)	110.6	(574,500)	110.8	(573,800)	110.7	576 300	111.2
40	((533,033))	102.9	((567,500))	109.5	((580,000))	111.1	((588,975))	113.7	((592,000))	114.3	((593,000))	114.5
41	(539,580)	103.0	(567,000)	108.3	(579,000)	110.6	(580,000)	110.8	(579,500)	110.7	(582,000)	111.1
42	596,085	102.4	629,200	108.1	649,800	111.7	644,200	110.7	645,400	110.9	646,300	111.0
TEST OF OFFICIAL MEASUREMENTS												
43	42,010	104.5	46,407	115.4	(54,400)	135.3	(58,000)	144.2	(57,200)	142.2	(58,200)	144.7
44	(28,526)	101.7	(31,000)	109.9	(34,000)	120.6	(38,000)	134.8	(39,000)	138.3	(36,000)	127.7
6	84,844	106.1	93,130	116.4	93,900	117.4	96,700	120.9	96,700	120.9	101,240	126.6
13	59,520	111.7	71,257	133.7	73,165	137.3	75,664	142.0	74,027	138.9	74,900	140.5
29	(255,060)	99.3	(267,600)	104.1	(275,300)	107.1	273,900	106.6	276,600	107.7	271,500	105.7
32	36,200	100.3	40,260	111.6	41,240	114.3	40,000	110.8	37,200	103.1	37,600	104.2
45	(506,160)	102.3	(549,654)	111.1	(572,005)	115.6	(582,264)	117.7	(580,727)	117.4	(579,440)	117.1
46	−8,240		−6,146		−2,865		−636		−6,173		−8,560	
38	514,400	102.9	555,800	111.1	574,870	114.9	582,900	116.5	586,900	117.3	588,000	117.6
47	−1.6		−1.2		−0.6		−0.13		−1.2		−1.7	

[a] Cubic feet per second.

[b] Cubic feet per second per square mile.

[c] Unofficial, derived, or estimated quantities are enclosed in parentheses; thus: (8,901).

[d] Based on discharge data for the years 1900 to 1931 in Tables 2 and 13, House Document 259, 74th Congress, 1st Session, "Comprehensive Report on Reservoirs in Mississippi River Basin"; together with later official yearly records.

Continued

Item No.	50-Yr Mean		60-Yr Mean		80-Yr Mean		100-Yr Mean		120-Yr Mean	
	cfs	%	cfs	%	cfs	%	cfs	%	cfs	%
1	(8,901)[e]	181.3	(9,330)	190.0	(9,000)	183.3	(8,700)	177.2	....	....
2	(27,300)	129.1	(29,000)	137.1	(29,400)	139.0	(29,000)	137.1	....	....
3	44,690	123.5	47,350	130.8	(48,000)	132.6	(47,000)	129.8	....	....
4										
5	57,120	118.1	60,770	125.6	(62,000)	128.2	(61,000)	126.1	....	....
6	(99,000)	122.6	(104,000)	130.0	(106,000)	132.5	(104,000)	130.0	....	....
7										
8	(16,900)	110.8	(17,400)	114.1	(18,000)	118.0	....	....	....	....
9	(22,700)	114.2	(23,300)	117.2	(24,000)	120.7	....	....	....	....
10	8,281	179.7	(8,413)	182.6	(8,100)	175.8	(7,800)	169.3	....	....
11	(8,740)	145.7	(8,820)	147.0	(8,410)	141.7	(8,050)	138.2	....	....
12	(60,000)	154.4	(64,000)	164.7	(61,000)	157.0	(60,000)	154.4	....	....
13	(72,000)	135.1	(74,000)	138.8	(70,000)	131.3	(68,000)	127.6	....	....
14	(172,200)	128.0	(179,250)	133.3	(176,250)	131.0	(173,200)	128.8	....	....
15	(185,000)	127.2	(192,600)	132.4	(189,000)	129.9	(185,800)	127.7	....	....
16	(2,150)	98.9	(2,250)	103.5	(2,250)	103.5	(2,200)	101.2	....	....
17	12,960	103.7	13,020	104.2	(12,900)	103.2	(12,800)	102.4	....	....
18	(26,500)	110.8	(27,000)	112.9	(27,000)	112.9	(28,000)	117.1	....	....
19	20,806	105.0	(20,725)	104.6	(20,600)	103.9	(21,000)	106.0	....	....
20	37,307	107.9	38,100	110.2	(38,000)	109.9	(39,000)	112.8	....	....
21	(53,000)	107.7	(54,000)	109.8	(54,000)	109.8	(56,000)	113.8	....	....
22	63,620	107.3	(65,000)	109.6	(65,000)	109.6	(67,000)	113.0	....	....
23	(30,800)	100.7	(31,100)	101.6	(30,600)	100.0	(30,100)	98.4	....	....
24	(31,300)	100.7	(31,600)	101.6	(31,100)	100.0	(30,600)	98.4	....	....
25	(80,500)	103.6	(81,000)	104.2	(80,000)	102.9	(79,000)	101.6	....	....
26	(105,000)	104.1	(105,800)	104.9	(104,000)	103.1	(103,000)	102.1	....	....
27	(122,000)	104.3	(123,000)	105.2	(121,000)	103.5	(120,000)	102.6	....	....
28	(266,000)	103.9	(275,000)	107.4	(274,000)	107.1	(284,000)	111.0	....	....
29	(267,000)	103.9	(276,000)	107.4	(275,000)	107.0	(280,000)	108.9	....	....
30	(27,600)	120.3	(26,000)	113.4	(26,300)	114.7	(26,000)	113.4	....	....
31	(31,000)	108.1	(29,000)	101.1	(29,000)	101.1	(29,000)	101.1	....	....
32	(40,000)	110.8	(36,000)	99.8	(37,600)	104.2	(37,000)	102.5	(36,000)	99.8
33	(9,200)	93.7	(9,500)	96.8	(9,800)	99.8	(9,700)	98.8	(9,600)	97.8
34	(10,300)	93.6	(10,600)	96.4	(11,000)	100.0	(10,800)	98.2	(10,700)	97.3
35	(18,000)	99.2	(17,500)	96.4	(18,000)	99.2	(18,500)	101.9	(18,000)	99.2
36	(30,000)	99.8	(29,000)	96.5	(30,000)	99.8	(31,000)	103.1	(30,000)	99.8
37	(468,000)	113.2	(484,000)	117.0	(479,000)	115.8	(482,000)	116.6	(480,000)	116.1
38	(575,000)	115.0	(576,000)	115.2	(570,000)	114.0	(585,000)	117.0	(580,000)	116.0
39	(570,900)	110.1	(572,500)	110.4	(580,000)	111.9	(599,000)	115.5	(593,000)	114.4
40	((579,000))	111.8	((580,000))	111.9	((584,000))	112.7	((603,000))	116.4	((595,000))	114.8
41	(576,500)	110.1	(578,000)	110.4	(586,000)	111.9	(605,000)	115.5	(601,000)	114.8
42	(638,000)	109.6	(640,000)	110.0	(648,000)	111.4	(668,000)	114.6	(660,000)	113.4

AND ESTIMATED DISCHARGES

43	(55,600)	138.3	(49,000)	121.8	(47,000)	116.9	(48,000)	119.4	(50,000)	124.3
44	(35,000)	124.1	(33,000)	117.0	(32,000)	113.5	(33,000)	117.0	(34,000)	120.6
6	(99,000)	123.8	(104,000)	130.0	(106,000)	132.5	(104,000)	130.0	(102,000)	127.5
13	(72,000)	135.1	(74,000)	138.8	(70,000)	131.3	(68,000)	127.6	(66,000)	123.8
29	(267,000)	103.9	(276,000)	107.4	(275,000)	107.0	(285,000)	110.9	(282,000)	109.8
32	(40,000)	110.8	(36,000)	99.8	(37,600)	104.2	(37,000)	102.5	(37,000)	102.5
45	(568,600)	114.9	(572,000)	115.6	(567,600)	114.7	(575,000)	116.2	(571,000)	115.0
46	−6,400		−4,000		−2,400		−10,000		−9,000	
38	(575,000)	115.0	(576,000)	115.2	(570,000)	114.0	(585,000)	117.0	(580,000)	116.0
47	−1.3		−0.8		−0.5		−2.0		−1.8	

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[e] Based on all available hydrographs, rainfall records, and other hydrologic data, 1817 to 1938, together with most authoritative evaluations of yearly discharge from Humphreys and Abbot's report and later analyses, supplemented by official discharge measurements for recent years.

[f] Uncorrected quantities derived from the gagings of headwaters and principal tributaries together with estimates for the intervening ungaged areas are enclosed in double parentheses; thus: ((583,814)).

percentages, as runoff coefficients for the respective watersheds for the entire list of twenty-eight storms that produced runoff within the record period, provides a total of 5 × 28 = 140 separate determinations. Comparison of the estimated runoff as derived from those average coefficients with observed runoff resulted in some 41.5% of the number falling within a ± 20% tolerance zone. Variations of the average runoff coefficient by some arbitrary factors designed to take account of the antecedent rainfall, whether copious, moderate, or meager, would increase the items within the 20% tolerance to nearly 70%, or a result comparable to that achieved with the most rigorous, painstaking, and time-consuming analyses of which the writer has knowledge. However, in view of the time, labor, and specialized technical guidance required for the rigorous and complicated analyses, and because of their susceptibility to gross errors not readily detected or controlled, it appears questionable whether the more tedious and detailed methods are justified when comparable results may be had by recourse to runoff coefficients, where in practice the consistency or inconsistency is made plainly apparent along with results.

Perhaps the main divergence of opinion herein expressed has to do with the statement in the final sentence of the "Synopsis" of the paper to the effect that the method is generally applicable to all drainage basins. The writer takes no exception to the use of the most detailed and exacting procedures that can be utilized to advantage in urban or other important or strategic areas where both damages and benefits may be of great magnitude and the necessary basic data from many stations are available. However, to attempt the expansion strictly along those same lines of procedure to cover all drainage basins seems to be an attempt at covering targets entirely beyond effective range of small-caliber weapons. If resort must be had to other devices, there can be no question that these will include coefficients expressing the percentage of rainfall appearing as runoff, either total volume, peak discharge, or average discharge for the critical periods of both storm and flood.

TABLE 8.—EXTENSION OF TABLE 4, CITY BLOCK 4841, ST. LOUIS, MO.

Storm No.	Duration (min)	Mean rainfall intensity (in. per hr)	Total Depth: Rainfall (in.)	Total Depth: Runoff (in.)	Runoff: Total (%)	Runoff: Maximum: %[a]	Runoff: Maximum: In. per hr	Storm No.	Duration (min)	Mean rainfall intensity (in. per hr)	Total Depth: Rainfall (in.)	Total Depth: Runoff (in.)	Runoff: Total (%)	Runoff: Maximum: %[a]	Runoff: Maximum: In. per hr
1	60+	1.29−	1.29	0.07	5	112	1.45	7	15	2.52	0.63	0.21	33	40	1.00
2	30	1.28	0.64	0.40	63	117	1.50	8	60+	1.30−	1.30	0.62	48	150	1.95
3	20	2.73	0.91	0.36	40	57	1.55	9	40	2.65	1.77	0.50	28	60	1.60
4	20	2.31	0.77	0.17	22	67	1.55	10	50	4.21	3.51	0.51	14	58	2.45
5	60+	1.14−	1.14	0.49	43	118	1.35	11	50	1.63	1.36	0.44	33	92	1.50
6	10	4.68	0.78	0.24	31	35	1.65	12	15	4.52	1.13	0.24	21	40	1.80

[a] Percentage of mean rainfall rate.

Even where plentiful records are available for rainfall, temperature, soil moisture, surface and cover conditions, infiltration, and all other essential factors capable of influencing runoff, it is well to keep in mind a timely caution from one of the most authoritative technical reports yet published.[29]

> "* * * Because of the existence of impervious or thinly mantled areas most rains, however light, produce some surface runoff, and to this extent the usefulness of the average infiltration capacity as a relative measure of the soil of a given basin to dispose of rainfall is diminished."

Table 4, presenting the results of gagings on a city block in St. Louis, together with the authors' analysis of the actual water losses that occurred for a dozen separate rains, may be extended to show the derived and associated data given in Table 8.

It is significant that the records show variations in mean rainfall intensity for the separate storm periods from 1.14 to more than 4 in. per hr, with storm duration ranging from 10 to more than 60 min, with total depths ranging from 0.63 in. to 3.51 in., and runoff yields varying from 5% to 63% of the rainfall, whereas the "peak rate of runoff from area tributary to alley, in inches per hour," according to the final column of Table 4, averaged 1.62, with extreme variations from 1.00 to 2.45 in. Furthermore, from casual examination, the basic data presented for storms Nos. 1 and 10, Table 8, seem to be so irregular as to warrant review. It is difficult to accept such high infiltration, retention, and peak discharge rates for the same events, depending upon either 5% or 14% runoff coefficient, to produce the momentary peak discharges representing 1.45 and 2.45 in. per hr, respectively. Comparison with the adjacent storms Nos. 2 and 9, Table 8, nearly equal in mean rainfall intensity or in volume of storm runoff, as shown diagrammatically in Fig. 15, indicates the improbability

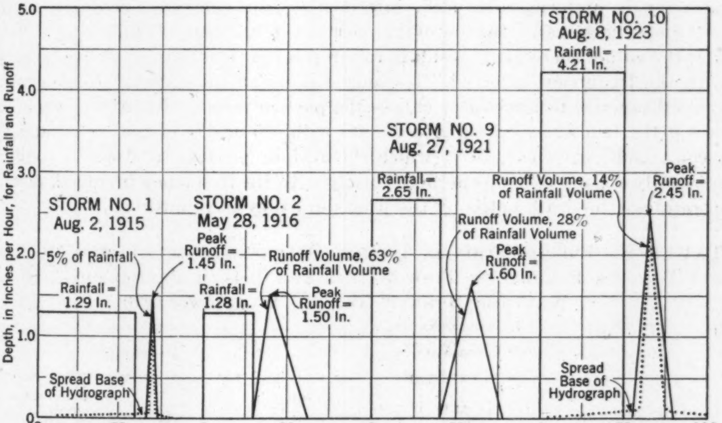

FIG. 15.—DIAGRAMMATIC REPRESENTATION OF RAINFALL AND RUNOFF VOLUMES AND RATES FOR SELECTED STORMS AT ST. LOUIS, MO., DATA FOR STORMS NOS. 1 AND 10 APPEARING TO BE QUESTIONABLE

of such runoff phenomena from the given rainfall. If the time bases of storms Nos. 1 and 10 were extended to cover the rainfall duration or double that period, then (maintaining the same volumes and maxima in accordance with the dotted hydrographs) the runoff peak would necessarily become more needle-like, and would challenge the reliability of the basic data, provided the underlying concepts and procedure of the unit hydrograph are valid. Conceding the fact that the maximum intensity may be much greater than the mean for a storm period, the writer is still unable to accept the 5% and 14% runoff yields of storms Nos. 1 and 10, in association with the peak rates of Table 4.

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[29] "Floods of Ohio and Mississippi Rivers, Jan.–Feb. 1937," *Water Supply Paper No. 838*, U. S. Geological Survey, pp. 491–492.

It is suggested that the only items inclining to show stability or a definite trend among the quantities of Table 4 are in the final column, and that for those twelve notable storms the average runoff rate might be assumed at its average value with more assurance than may be accorded many of the assumptions underlying the procedure developed by the authors, especially if widespread use is contemplated, apart from their special guidance.

Examination of Table 5, summarizing the results of painstaking studies, drawings shown as Figs. 5, 6, and 7, and detailed analysis by the authors disclose that the mass supply, or rainfall minus infiltration of item 10, Table 5, persistently represents nearly 76% of the total rainfall, varying from 72% to 80% for both the 20-min and 60-min durations, and from 72% to 77% for the 40-min duration, all depending upon the rainfall pattern (that is, whether the intensity is uniform, advanced, intermediate, or delayed, according to the authors' definitions). In view of their admission immediately following Table 5, that the values there used "result in runoff rates greater than would be expected with the frequency involved," it seems almost an unnecessary refinement in general practice to forecast or choose the particular rainfall pattern, when any one of the four types would give results within 5 or 6% of the mean, and the mean runoff coefficient, 76% (which the authors propose to discard), could be so readily altered to conform more closely with the frequency involved, instead of retracing intricate paths by the procedure they have outlined.

TABLE 9.—PRECIPITATION IN INCHES, OBSERVED AT ST. LOUIS, MO., FOR VARIOUS PERIODS OF TIME WITHIN THE 104-YR PERIOD OF U. S. WEATHER BUREAU RECORD, INCLUDING 1940

MAXIMUM AND NEXT THREE VALUES						YEARLY		
3-min	5-min	15-min	60-min	24-hr	Month	Minimum and next three values	Mean	Maximum and next three values
(0.36)	0.59	1.39	3.51	7.02	17.07	23.23	39.23	68.83
(0.34)	0.56	1.19	3.47	5.08	11.43	23.38	....	65.36
(0.33)	0.55	1.03	3.36	4.80	11.26	24.80	....	61.40
(0.31)	0.51	0.92	2.42	4.19	10.90	25.00	....	52.72

The writer has repeatedly gone on record to the effect that the microscopic approach, unduly emphasized, has contributed to the general neglect of the great mass of hydrologic data, and of their readily recoverable values. For example, it has been shown that, in general, the average daily precipitation on days of rainfall (the average annual rainfall divided by the average number of rainy days per year) is a fair approximation to the 3-min maximum rainfall depth at many stations with reasonably long records.[30] Nearly a straight-line relationship on logarithmic paper seems to connect the 5-min, 15-min, 60-min, and daily maximum precipitation, whereas the maximum monthly depth naturally exceeds the maximum daily, and in humid areas often closely approaches the minimum yearly precipitation. Armed with those facts, it is possible to make fair evaluations of maximum hourly or other short-period rainfall intensities to be expected for given frequencies.

Thus the readily available records for St. Louis, Mo., yield the items listed in Table 9. Now, assume that the maximum storm intensities for 24 hr, 60 min, and lesser periods are desired, corresponding to frequencies of 104 yr or other period, or for one half, one third, or one fourth of such periods. They are found in Table 9 in the same order, beginning with the top items.

Rainfall depths for intermediate periods may be derived from their approximately straight-line relationship on logarithmic paper. For any assumed storm pattern with intensities high enough to produce notable runoff, it would seem evident that a fair approximation to the resulting hydrograph could be obtained for city block 4841 in St. Louis, Mo., as follows: By using a time base for the flood rise equal to two or three times the rainfall duration, up to an hour or perhaps two hours of storm; and, by adopting as a first approximation to the maximum ordinate, from 1 to 2 in. per hr (depending on antecedent conditions), subject to such adjustment as shall accommodate from 40 to 70% of the rainfall volume under the outflow hydrograph, and in general accordance with the observed data or derivations from Table 4. A satisfactory approach to estimated runoff may thus be made by the use of coefficients with a minimum of assumption and computations and the maximum support from characteristic behavior of the drainage area, as well as neighboring basins.

It seems to be more reasonable, if not more scientific, to accord due consideration to observation, experience, and the readily understandable empirical expressions that summarize such experience, to serve as controls and safeguards in general on all new methods and technical procedures designed to advance the frontiers of scientific knowledge and achievement.

G. W. MUSGRAVE,[31] ESQ.—If the authors of this interesting paper are successful in doing no more than convincing the great body of technical workers in the field of hydrology of the first point that they make—namely, the fallacy of using runoff coefficients—it is believed by this observer that a genuine service will be rendered. That runoff coefficients are a misconception has been shown by several workers, including LeRoy K. Sherman, M. Am. Soc. C. E., who has noted that runoff is a difference, not a ratio. Despite the evidence that has been presented, this writer feels that the use of this misconception will not immediately be discontinued. For this reason it is desired to present, by way of emphasis, a little additional data, which may be regarded as a more or less random sample of much similar evidence from other locations.

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[30] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 398; also Fig. 4.

[31] Head, Infiltration Studies, Office of Research, SCS, U. S. Dept. of Agriculture, Washington, D. C.

At the Conservation Experiment Station at Clarinda, Iowa, runoff records have been made for each storm of record since 1932, and for a large number of areas differing in treatments, sizes, slopes, or vegetal covers. In the data cited the runoff measurements are accurate to within 0.01 in. (more probably ± 0.001 in.) and rainfall is recorded by a number of Freiz, Weather Bureau, and special types of rain gages. Table 10 gives the rainfall and runoff for a 3-yr sample period from two areas having similar soil and slope, but differing in vegetal cover.[32] On one of these plots corn is grown annually and on the other bluegrass. It will be noted from the table that the average runoff from the corn for this period was 17.45% and from the bluegrass 2.03%. However, the range in magnitudes is not greatly different, being from 0 to 62.45% for the area growing corn and from 0 to 61.45% for the area in bluegrass. For the area growing corn, fifteen storms produced runoff in excess of 14% of the rainfall. For the area in bluegrass only two storms produced as much runoff as 14% of the rainfall. If one were using runoff coefficients, what value could be selected that would represent each of these two areas satisfactorily? It is obvious that the use of any single percentage to represent the runoff that might result from a given storm on either of these areas would be entirely fallacious.

TABLE 10.—Percentage Runoff from Plots at Soil Conservation Experiment Station, Clarinda, Iowa

Date	Rainfall (inches)	Annual Runoff, Plot 3; Corn		Runoff, Plot 8; Bluegrass	
		Inches	%	Inches	%
(1933)					
Mar. 31	0.52	0.034	6.54	0.000	0.00
May 12	0.82	0.030	3.66	0.000	0.00
May 22	0.29	0.000	0.00	0.000	0.00
June 27	2.88	0.646	58.73	0.091	3.16
June 29	1.10	0.234	8.13	0.000	0.00
Aug. 21	3.70	0.088	2.38	0.097	2.62
Sept. 25	1.76	0.107	6.08	0.000	0.00
(1934)					
May 12	1.30	0.740	56.92	0.000	0.00
Aug. 31	1.15	0.116	10.09	0.000	0.00
Sept. 3	1.22	0.063	5.16	0.000	0.00
Sept. 15	0.84	0.160	19.05	0.000	0.00
Sept. 26	1.73	0.493	28.50	0.000	0.00
Oct. 19	2.37	0.824	29.19	0.106	4.47
Oct. 20	0.37	0.108	34.77	0.000	0.00
Nov. 3	1.39	0.214	15.39	0.000	0.00
Nov. 22	0.47	0.043	9.15	0.000	0.00
(1935)					
Jan. 19	0.53	0.331	62.45	0.326	61.51
May 21	2.22	0.000	00.00	0.000	0.00
May 23	0.73	0.109	14.93	0.000	0.00
May 28	1.55	0.285	18.39	0.000	0.00
June 1	1.57	0.667	42.48	0.000	0.00
June 3	0.97	0.377	38.87	0.000	0.00
June 4	0.47	0.138	29.36	0.065	13.83
June 10	0.30	0.037	12.33	0.000	0.00
June 24	0.79	0.228	57.54	0.000	0.00
June 26	0.61	0.351	28.86	0.000	0.00
Sept. 2	1.84	0.000	00.00	0.000	0.00
Sept. 16	0.42	0.058	13.81	0.093	22.14
Sept. 26	1.98	0.100	5.05	0.000	0.00
Oct. 17	1.28	0.000	0.00	0.000	0.00
Oct. 31	1.10	0.098	8.91	0.000	0.00
Total	38.27	6.679	....	0.778	....
Average	....	....	17.45	....	2.03

The authors have indicated a number of the factors that affect infiltration capacity. In connection with the reference to soil porosity, it may be well to mention that it is probable that the total volume of porosity does not affect the rate of infiltration so much as does the pore-size distribution. This is obvious from a consideration of the soil porosity of a fine-grain soil such as clay, which in total volume of porosity usually is relatively large, but in which the infiltration rate is usually low because of the small average size of the individual pores.

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[32] "Soil and Water Conservation Investigations," by G. W. Musgrave and R. A. Norton, *Technical Bulletin No. 558*, U. S. Dept. of Agriculture, February, 1937.

In reference to surface slope, also, it may be well to indicate that, although existing evidence indicates little effect of degree of slope upon infiltration, there is evidence indicating that the effect of length of slope commonly is appreciable.

TABLE 11.—COMPARISON OF INFILTRATION (f_c VALUES), WITH ANTECEDENT SOIL MOISTURE FOR THREE DEPTHS, AND SOIL TEMPERATURE DURING REPEATED INFILTROMETER WET RUNS AT THE EDWARDSVILLE, ILL., WATERSHED

Plot	Date (1940)	Infiltration, f_c, in in. per hr	SOIL MOISTURE (%)			Soil temperature (degrees F)
			0 in. to 7 in.	7 in. to 14 in.	14 in. to 21 in.	
7	May 7	0.25	27.3	28.7	25.4	64
7	June 25	0.41	22.9	21.1	17.4	68
7	July 29	0.45	24.8	23.2	20.7	78
7	Sept. 18	0.25	24.4	23.2	21.8	71
8	May 9	0.23	26.4	25.8	24.4	59
8	June 26	0.39	22.3	19.1	20.1	74
8	Aug. 1	0.42	22.3	22.2	20.0	78
8	Nov. 7	0.24	23.0	21.3	19.8	49
9	May 23	0.06	25.4	24.2	23.3	62
9	June 27	0.18	25.5	16.9	21.0	73
9	Aug. 2	0.31	26.4	22.6	22.0	77
9	Sept. 23	0.34	25.8	22.0	22.5	70
10	May 25	0.04	29.0	25.5	26.2	62
10	June 29	0.04	27.1	27.1	27.7	65
10	Aug. 5	0.07	29.0	23.5	26.2	72
10	Nov. 29	0.18	28.2	26.5	26.5	46
11	June 6	0.02	30.6	28.7	27.5	74
11	July 1	0.17	30.5	32.3	27.2	68
11	Aug. 7	0.07	28.7	29.2	25.0	76
11	Sept. 30	0.08	28.6	29.8	26.0	60
12	June 5	0.04	20.2	26.9	26.0	71
12	July 2	0.12	29.6	28.5	26.6	64
12	Aug. 9	0.00	30.8	30.2	27.6	76
12	Nov. 20	0.05	31.9	29.0	25.6	52
13	May 27	0.00	26.2	25.9	25.4	60
13	July 8	0.25	27.2	26.0	25.2	74
13	Aug. 13	0.22	27.3	26.0	25.4	76
13	Oct. 7	0.20	26.8	26.3	25.6	57
14	May 28	0.34	28.0	24.4	28.5	60
14	July 9	0.50	25.5	21.8	18.6	76
14	Aug. 15	0.28	25.3	25.7	21.9	78
14	Dec. 9	0.13	26.1	27.4	23.3	40
15	June 3	0.27	24.4	24.8	25.7	69
15	July 10	0.24	18.0	16.6	19.0	78
15	Aug. 19	0.185	23.1	18.9	18.5	62
15	Dec. 20	0.05	25.4	27.7	25.2	38
16	June 4	0.40	28.9	27.9	27.3	69
16	July 12	0.35	26.2	26.2	25.1	63
16	Aug. 20	0.10	20.5	22.7	21.6	64
16	Oct. 11	0.12	22.0	19.0	19.4	60
Mean		0.20	26.0	24.9	23.7	65.8

In referring to the variation in initial rates, the authors do not attribute the cause solely to soil-moisture variation, but properly take the broader view and consider "soil condition." To this also might be added the temperature of soil and of water. The effect of antecedent rain upon infiltration is a matter that is of great interest to hydrologists at the present time. Although its effect has been considered by some workers solely as that of filling the voids of a soil profile with water, the authors avoid this misconception. Clearly, the antecedent rain does much more to soil and soil structure than fill the voids with water. The impact of rain drops is such as to apply a large amount of energy to the surface of the soil. During 1941, J. O. Laws[33] provided the necessary basic data upon which the amount of energy of impact may be calculated, and it is apparent that much of the destruction of aggregated soil structure at the surface can be accounted for readily by this ample source of energy.

In addition to these mechanical effects, of course, there are many other effects of antecedent rains. There may be an inwashing of fine particles that lodge in constricted portions of the pores and channels of the upper part of the profile. There is likely to be swelling not proportional to the quantity of water. Also, with the withdrawal of moisture upon drying, the soil does not necessarily return at once to its former structural or permeability status. Actually, direct relations between the content of soil moisture and the amount of subsequent infiltration may be expected only under certain unusual conditions, such, for example, as unigranular sands lying upon impermeable substrata—conditions in which the only effect of water is that of filling voids.

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[33] "Measurements of the Fall-Velocity of Water-drops and Raindrops," by J. Otis Laws, *Transactions*, Am. Geophysical Union, 1941, p. 709.

At the Edwardsville project of the Soil Conservation Service, which has been mentioned by the authors (Fig. 4), data are available that include repeated infiltrometer runs on a series of ten plots distributed over the watershed. These data include soil moisture at three different depths; and the temperature of soil, air, and water; as well as other environmental conditions. The data on soil moisture and soil temperature for 1940 are given in Table 11.

It will be seen that repeated runs on the same plot do not give infiltration f_c values (the minimum rate of infiltration occurring during the last part of the wet runs) that invariably correlate with soil moisture. More detailed analyses of the data, however, throw a little more light upon the subject. Four successive rounds of determinations were made on each of the ten plots, and the means for each round are presented in Table 12. Here it is seen that infiltration for rounds 2 and 3, which were made during the summer period, is higher than for rounds 1 and 4, during the spring and fall, and that the average soil-moisture content at the 14-in. to 21-in. depth was lower for rounds 2 and 3 than for rounds 1 and 4. However, it is seen also that the temperature of the soil was higher during the summer period, as would be expected. This quadratic or second-degree curve of seasonal trend of infiltration with changing moisture and temperature is statistically significant.

TABLE 12.—COMPARISON OF MEANS OF SOIL MOISTURE, SOIL TEMPERATURE, AND INFILTRATION FOR EACH OF FOUR ROUNDS OF DETERMINATIONS ON TEN PLOTS AT EDWARDSVILLE, ILL. (WET RUNS)

Round	Period (1940)	Infiltration, f_c, in in. per hr	Soil Moisture (%)			Soil temperature (degrees F)
			0 in. to 7 in.	7 in. to 14 in.	14 in. to 21 in.	
1	May 5 to June 6	0.165	26.64	26.28	25.47	65.0
2	June 25 to July 12	0.265	25.48	23.56	22.79	70.3
3	July 29 to Aug. 20	0.210	25.82	24.42	22.89	73.7
4	Sept. 18 to Dec. 20	0.164	26.22	25.22	23.57	54.3

An examination of the results in Table 12, without due allowance for known differences such as time, might readily lead to erroneous conclusions since they appear to show a very strong inverse correlation between moisture and temperature, as well as between moisture and infiltration, and also a strong positive correlation between temperature and infiltration. More detailed analyses of the individual determinations, however, show that the apparent relationship between moisture and temperature is not significant, that the inverse relation of moisture and infiltration is fairly marked for rounds 2 and 3 but not for rounds 1 and 4, and that for the entire series of forty determinations moisture and temperature in combination are strongly associated with infiltration.

One must recognize, however, the strong probability that other variables are involved and the possibility that the rate of infiltration is actually dependent primarily upon such factors. For example, it is known that the pore-size distribution of a soil varies and that it well may be least in the spring and fall and highest in the summer—in other words has the same trend as the seasonal infiltration curve. In these data there are indications not only that changes in both temperature and moisture are associated with changing infiltration, and that under certain conditions one of these variables may appear more important than the other, but also that at the present time the hydrologist may be measuring merely "shadows" of the true causal factor—pore-size distribution.

This example is fairly typical of the effect of soil moisture as it has been found in many studies covering the period 1929 to date (1941). Briefly, it may be stated that ordinarily:

(*a*) Seasonal trends of soil moisture and soil temperature tend to be correlated inversely;

(*b*) Infiltration is lower when soil moisture is higher, but the magnitude of reduction is not proportional to the increase in moisture;

(*c*) Subsoil moisture often shows a more marked effect upon infiltration than does soil moisture in the surface;

(*d*) Seasonal trends (similar to those of Table 12) ordinarily are statistically significant; and

(*e*) There are continuing evidences that other factors in addition to that of soil moisture are exerting their effects upon the rate of infiltration.

Looking at the matter from a purely physical basis, there seems to be no reason why one should commonly find mathematical proportionality relations between soil moisture and infiltration, since obviously soil moisture alone is not an adequate basis for expressing the size of soil pores. Perhaps the additional variables in the complex may be identified and measured during the course of some of the many investigations now under way.

Messrs. Horner and Jens have been making studies of rainfall and runoff for some time and have had opportunity to test, under many conditions, the analytical procedures that they have presented. In their example of a rational procedure without the use of runoff coefficients, they have presented material that should be distinctly useful. The writer agrees with them wholly in principle, and merely suggests caution against oversimplification in the handling of some of the variables, the effects of which are undoubtedly highly complex, and the magnitude and effect of which are only now being investigated.

F. L. FLYNT,[34] ASSOC. M. AM. SOC. C. E.—In their detailed analysis of storm-water runoff from typical urban areas, the authors have demonstrated beyond reasonable doubt that it is more rational to consider runoff as rainfall minus losses than it is to assume that runoff may be expressed as a constant percentage of the rainfall for a given storm. They have also demonstrated that it is practicable to use this principle in the design of drainage systems. The application of the equations for overland flow and the routing of the resulting hydrographs through alley and gutter storage introduce a new technique in the analysis and synthesis of runoff from urban areas.

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[34] Associate Engr., U. S. Engr. Dept., Rock Island, Ill.

It is evident that the refinements introduced into the design procedure, through a more complete understanding of the fundamental principles involved, tend to increase the work required to design a given drainage system, but this is not a valid argument against the use of the improved method. It would be just as logical to ignore secondary stresses in the design of large framed structures or to provide for them by an arbitrary increase in the factor of safety. The old saying "Anything worth doing at all is worth doing well" applies to the design of engineering structures as well as to lay activities.

Perhaps the weakest link in the chain of the demonstration is the first of the six essential stages of design as listed in Part I under "Application to Hydraulic Engineering"—namely, the delineation of the precipitation pattern. This weakness is not inherent in the method but is due to the present lack of information that further research may some day supply. So far as the writer is aware, no determinations have been made of the relative frequency of various types of storm patterns, such as "advanced," "intermediate," and "delayed," which would enable a satisfactory choice to be made, appropriate to a given design frequency. Fortunately, the effect of rainfall pattern upon the maximum rate of runoff is less apparent in the hydrograph of outflow from a drainage network serving a large area, where the time of concentration is comparatively long, than it is for a single block. This is made clear from a study of Figs. 5, 6, and 7.

The detailed method of analyzing the hydrograph of runoff from urban areas, as demonstrated in the paper, is equally applicable to the problem of designing drainage structures to carry surface runoff from airport areas. Such an analysis, when the grading layout is closely coordinated with it, enables a balanced design to be made that not only assures an adequate drainage system, but also one which is the most economical for the given conditions.

In the analysis of stream-flow hydrographs, the fundamental principle of considering runoff as rainfall minus losses rather than as a ratio or coefficient is rapidly coming into general use. The methods of analysis in the case of drainage areas, involving many square miles, of course, are quite different in detail from the examples given in the paper. The reasons for this difference may be outlined as follows:

(*a*) In the case of streams the volume of water in storage as overland flow or surface detention is usually quite small when compared with water in channel storage.

(*b*) Time periods of hours, rather than minutes, must be used, which means that loss rates must be averaged not only over large areas but also over comparatively long periods of time. This makes it more difficult to relate the observed loss rates with the infiltration capacities of the various soils in the area.

(*c*) The infiltration rate for a stream basin varies over a wider range than it does for urban or airport areas. This is due to the fact that in agricultural areas the condition of the top layer of soil is modified from season to season and from year to year through cultivation and changes in vegetal cover.

(*d*) Because of the present lack of an adequate method of segregating the various kinds of water loss, the experimental values of loss rate as determined from a comparison of rainfall and runoff volumes do not, necessarily, reflect the true value of infiltration loss but the aggregate of all losses, whatever their nature.

(*e*) Another disturbing factor, common to stream basins but insignificant in urban areas, is ground-water return to the flood hydrograph. The problem of segregating the ground-water and surface-water hydrographs has not yet been completely solved. This is partly because there is often no sharp line of demarcation between the two. Some of the ground water travels underground only a few feet before appearing in stream flow, whereas some of it travels through subsurface strata for great distances.

(*f*) Uniform rainfall over small urban areas may be assumed, but for large areas uniform rainfall of significant intensity is practically unknown. The areal distribution of rainfall becomes increasingly important as the size of the watershed increases. Cases have been observed in which appreciable abnormalities in the shape of runoff hydrographs have been traced to heavy rainfall which had been unofficially observed over limited areas, but which had not been recorded at any Weather Bureau Station.

The effects of all the aforementioned conditions, which tend to complicate the rainfall-runoff relation, become more pronounced with the increase in size of the drainage area. It follows that, for the purpose of analyzing hydrographs of record and for synthesizing design floods, it is advisable to subdivide the drainage area into sub-basins and to subdivide the rainfall into periods as short as practicable, noting the areal distribution of rainfall for each period over each sub-basin. By the use of synthetic unit graphs where no hydrographs are available, and by routing and combining the hydrographs of the various sub-basins, hydrographs of past floods may be reconstituted with reasonable accuracy from the rainfall data, and design floods may be synthesized from assumed or transposed rainfalls with more confidence than if large basins were considered as a unit. The amount of work involved in this procedure is considerably greater, of course, than when the older methods are used; but the more reliable results justify the increased work.

Thus it is seen that the principle of breaking down the area tributary to the point of measurement into simpler components for the purpose of analysis and synthesis of runoff hydrographs is of general application and, no doubt, will eventually become the standard practice as will the evaluation of runoff as rainfall minus losses instead of in terms of the percentage factors formerly used.

W. I. HICKS,[35] ESQ.—A comprehensive outline of a method of computation, based on the analysis and synthesis of the factors governing runoff from a city block, is presented in this paper, which points the way to an extension of the computation to a complete drainage system. On account of the wide scope of the subject and the limitations of space, the text must be read carefully to enable one to grasp the entire subject matter.

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[35] Asst. Civ. Engr., Storm Drain Div., Bureau of Eng., Los Angeles, Calif.

The writer has been investigating rainfall-runoff phenomena in Los Angeles, Calif., for a number of years, with the result that a schedule of procedure identical with the six stages listed by the authors (see heading "Application to Hydraulic Engineering") was evolved:

I—A precipitation pattern between "intermediate" and "delayed" was selected;

II—The infiltration curve was based on small test plots and the analysis of rainfall-runoff records gathered from local drainage areas (see Fig. 16);

III—Antecedent precipitation was determined by probability studies and used to weight the basic infiltration curves;

IV—Net rainfall curves were derived for different classes of development after consideration of—

V—Infiltration curves, depression storage varying from 0.10 in. for heavy soils to 0.20 in. for sand; and

VI—The net rainfall curves were translated into hydrographs after consideration of the various forms of detention.

The summation-of-hydrograph method of computation is well suited to Los Angeles terrain where the intensity of a storm of given frequency varies as much as 2 to 1 in a single drainage area; the reason for the variation is the mountainous character of a part of the area.

Apart from the agreement in the general principles, there were some variations and differences on various details of more or less importance.

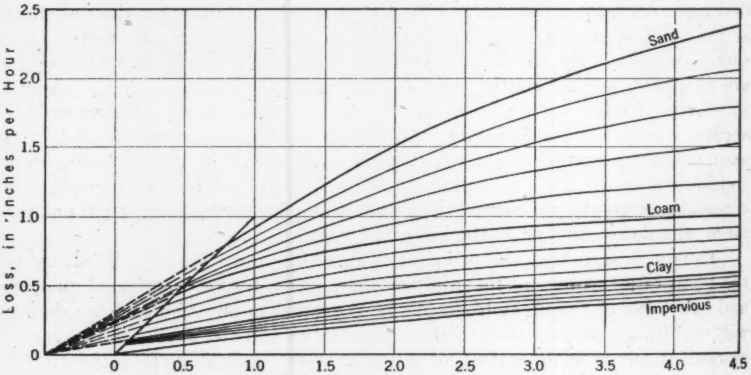

FIG. 16.—RAINFALL RATE—LOSS RATE CURVES

In delineating the precipitation curve, the mass curve was used in preference to the intensity diagram because it was believed that the volume of runoff was of major importance. Likewise, the influence of detention was introduced by suspending the varying depth of storage under the net rainfall curve; and the time functions of the hydrograph were approximated by analyses of hydrographs from drainage areas of various sizes and stages of development.

The general shape of the authors' infiltration-capacity curve is in agreement with local data. Analyses of rainfall-runoff records for a four-day storm culminating on March 2, 1938, show a downward march in capacity to practically zero (the zero capacity was a combination of low infiltration and seepage flow). Because the local rainy season comes in the winter when evaporation is low, because of the delayed position of the peak rainfall, and because of relatively high antecedent precipitation, the flat part of the curve is used, whereas the authors use the sharply descending part of the curve for the "thunderstorm" which breaks a hot, dry period.

The curves in Fig. 16 differ from the authors' assumption of comparative independence of rainfall intensity under turf cover in that the higher intensities of rainfall produce a more complete coverage of the imperfect plane surface of urban yards and so increase the infiltration capacity.

In the authors' typical block, they assume that drainage from impervious surfaces does not flow over turf on its way to the gutter or alley. Analysis of the various classes of urban residential districts indicates that walks, driveway strips, and small porches usually discharge on to turf and that many roof downspouts discharge into flower beds; the total of these areas average from 15% to 20% of the entire area. If this assumption is granted, the gross rainfall for the pervious area is increased by the runoff from these areas, and the impervious supply volume is diminished correspondingly.

In the matter of overland flow and surface detention, limited experimental data were taken locally. The results of the experiments are included for the purpose of discussing the paper and to make them available for use.

The apparatus consisted of:

(1) A rectangular wooden trough 100 ft long by 0.96 ft wide mounted on a series of wooden standards and capable of being adjusted to different slopes.

(2) A 2-in. galvanized pipe, capped at both ends and tapped at intervals to receive nipples, mounted in the wooden standards above the trough. Below this pipe and connected to the nipples were suspended brass sprinkler pipes with close-spaced sprinkling orifices.

(3) A hose connection made from the 2-in. pipe to the valve of a 100-gal tank on a platform set uphill from the apparatus.

The types of surfaces tested were:

(*a*) Tar and sand corresponding to tar-paper roofing, sidewalk, concrete paving, and similar surfaces;

(*b*) Tar and gravel corresponding to the roofing of that type and rough-finish asphaltic concrete paving; and

(*c*) Clipped sod selected as being representative of the average city lawn. Tar-paper baffles were set flush with the ground surface to prevent subsurface flow.

Slopes used varied from 0 to 7%. Lengths of flow were 100, 50, 25, and 10 ft. (The 10-ft runs were on a 10-ft by 10-ft adjustable platform.) Uniform rates of simulated rainfall varied from 0.5 to 7.0 in. per hr.

Data from these tests have been filed in the Engineering Societies Library.[36]

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[36] 29 West 39th Street, New York, N. Y.

They are quite erratic for the tar and sand surface on account of inability to maintain a true plane for so smooth a surface; the data for the other surfaces are more rational but require judgment for interpreting. The data were plotted on logarithmic paper and the following exponential formulas were derived:

Tar and Sand Surface.—

$$\delta_a = \frac{0.0136\, l^{0.318}\, \sigma^{0.535}}{S_0^{0.4615}} \qquad \text{(17a)}$$

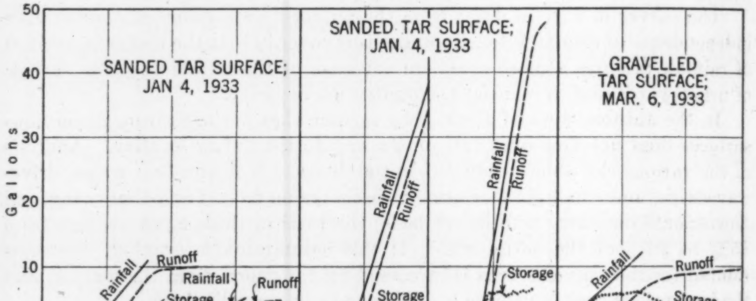

FIG. 17.—SAMPLE RUNS (TROUGHS, 100 FT BY 1 FT; AND S_0 = 0.03)

and

$$t_c = \frac{1.3\, l^{0.323}}{S_0^{0.448}\, \sigma^{0.64}} \qquad \text{(17b)}$$

Tar and Gravel Surface.—

$$\delta_a = \frac{0.0257\, l^{0.384}\, \sigma^{0.351}}{S_0^{0.367}} \qquad \text{(18a)}$$

and

$$t_c = \frac{2.23\, l^{0.373}}{S_0^{0.366}\, \sigma^{0.684}} \qquad \text{(18b)}$$

Clipped Sod Surface.—

$$\delta_a = \frac{0.078\, l^{0.322}\, \sigma^{0.325}}{S_0^{0.281}} \quad \text{(19a)}$$

and

$$t_c = \frac{9.34\, l^{0.298}}{S_0^{0.302}\, \sigma^{0.785}} \quad \text{(19b)}$$

Except that S_0 denotes slope, as percentages in Eqs. 17, 18, and 19 (and in Eqs. 20*a* and 20*b*), the symbols used are those of the paper, t_c being measured from the beginning of the supply rate to the point of full runoff and S_0 = slope expressed as a percentage. It is to be noted that not the marginal but the average depth is used. Nomographs were designed for the convenient solution of Eqs. 17 to 19. Sample runs are shown in Fig. 17.

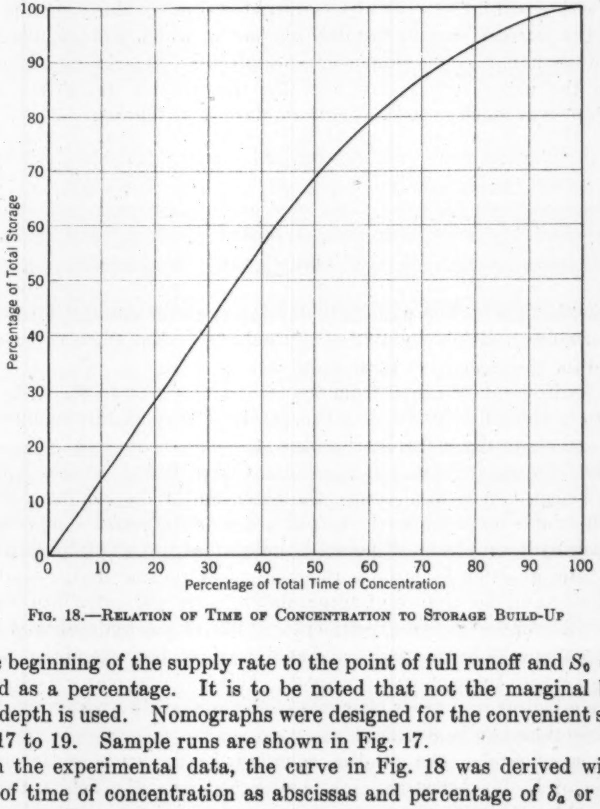

FIG. 18.—RELATION OF TIME OF CONCENTRATION TO STORAGE BUILD-UP

From the experimental data, the curve in Fig. 18 was derived with percentage of time of concentration as abscissas and percentage of δ_a or storage as ordinates. It was based on data from a variable-flow trough, a clipped sod plane surface, a tar and gravel plane surface, and a tar and sand plane surface. With these formulas and their corresponding charts, the alteration of the hydrograph of supply after flow over a plane surface can be computed as outlined in the paper (between Eqs. 4 and 5) except that a mass curve of supply and varying depths of storage are used instead of a supply-intensity curve and the variation of intensity of runoff from the plane surface.

Results obtained by the two methods of treatment are consistent for all practical purposes. When one considers the unavoidable variations in roughness of surface of roofs, pavement, and turf, and the necessary assumption of average conditions, precise mathematical treatment is not to be expected.

In determining the effect of gutter storage on the hydrograph, the writer has followed the general principles stated by the authors. Two formulas similar to those for plane surfaces were computed on the basis of a 30-ft roadway ($n = 0.012$) with a 4.5-in. crown and no crossfall, which was selected as representing average conditions. A series of tests was performed on a V-shaped trough with equal increments of flow introduced at regular intervals along its length; the purpose was to test the manner in which storage was built up. The run on a flat grade supported the authors' remarks on the difference between the water-surface slope and the construction grade; the upstream water slope was much flatter. The formulas are as follows:

$$\delta_a = \frac{0.00044\, l^{0.697}\, \sigma^{0.794}}{{S^{0.417}}_0} \quad \text{(20a)}$$

and

$$t = \frac{0.0443\, l^{0.732}}{{S^{0.39}}_0\, \sigma^{0.252}} \quad \text{(20b)}$$

in which t is time of flow with fully developed gutter storage; for t_c as in the other formulas multiply by 1.33 ±. Similarly to Eq. 19, a nomograph was designed for the solution of these formulas.

The hydrograph of runoff from the lot is transposed to the hydrograph of inflow to gutter inlet by varying the gutter storage in the manner outlined by the authors in the text following Eqs. 4.

Under "Summary and Conclusions: Conventional Urban Areas," the authors suggest, in general terms, the extension of the computation beyond the hydrograph for a single block; paragraph (*e*) indicates a process of summing such hydrographs at all points of inflow (see also text following Table 4). If this interpretation is correct, the volume of routine design work will be considerable and the chance of accumulative error will enter into the computations. A comprehensive experimental study of the hydraulics of the flood wave, confluence of hydrographs, and analytical comparisons of the hydrograph of the block with those of drainage areas of different sizes may yield information as to time factors and other elements of the composite hydrograph by which these objections can be avoided.

Acknowledgment.—The studies reported herein were made under the direction of L. W. Armstrong, M. Am. Soc. C. E., the field activity being supervised by H. W. Fraim.

ALBERT S. FRY,[37] M. AM. SOC. C. E.—The authors have made a valuable contribution to a field of hydraulic engineering which, in many respects, is in a pioneer phase of development, with arguments pro and con as to the practical usefulness of infiltration theory and procedure. Progress in this field will be made as basic knowledge accumulates and as engineers become better acquainted with infiltration and its advantages and limitations.

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It is to be regretted that the paper deprecates past work in hydrology and glorifies infiltration as a new method that puts hydraulic design on a parity with structural steel design in exactness. It is true that the network of stream gages and rain gages has expanded in certain areas of the United States in recent years, providing valuable information of particular use in those areas. The authors expect this to revolutionize engineering practice but overlook the fact that much of the new data are of limited local application and that overenthusiasm in some places has produced data of dubious value. To imply (as the paper does) that, in the past, design of important hydraulic structures has been "based on crude over-all general relationships in the hydrologic field," shows a lack of appreciation and recognition of the invaluable contributions made by men such as C. S. Jarvis, M. Am. Soc. C. E., and others working in this field. Hydrologic data—both past and present—are seldom available on any project to the extent that the engineer desires, but this has not prevented him from understanding hydrologic fundamentals. He has not only understood them but has recognized their limitations in practical application probably better than the disciples of the new order of infiltrationists. Real progress is not achieved in any field by exorbitant and unwarranted claims.

The paper presents the method for surface runoff determination as it relates specifically to urban storm drainage but also recommends it as generally applicable to all drainage basins, and states that it provides a more logical and dependable basis of design for important hydraulic structures than has heretofore been considered feasible. This is more likely to be true of urban storm drainage than of large-scale uni-purpose or multi-purpose projects. The former deals with relatively small areas of physical characteristics susceptible of fairly close determination under detailed study, and design is predicated upon relatively short-time intense rainfall. The latter embraces large watersheds of complex physical and hydrological characteristics difficult of determination, and design is based upon rainfall over a longer period of time.

These large water projects involve extensive works such as reservoirs, channel improvements, and levees, and it is doubted that a more logical and dependable design for these works can be made by substituting infiltration for previously used methods. Such projects must be designed for maximum flood conditions, allowing a factor of safety that might erroneously be considered to be a factor of ignorance, whereas actually too much engineering judgment and experience are involved to apply the term "ignorance." During storm periods that cause maximum floods, the rate of runoff is high—in many cases 90% or more of the rainfall. Infiltration is a relatively minor part of the total rainfall during such great storms and hence is not a primary factor in determining the design basis for a uni-purpose project such as flood control or a multi-purpose project involving flood control, hydroelectric power, navigation, and other purposes.

There is as yet no sound method of estimating accurately the size of the maximum possible storm. Attempts to do this have been made in recent years and represent a forward step in the right direction. However, meteorological scientists, such as J. Bjerknes and C. G. Rossby, do not consider that the science has yet advanced to the point where the factors which produce maximum storms can be closely evaluated so that the maximum storm for a given region can be determined with any degree of reliability. If it were possible to know the maximum storm for which to design, then there would be more justification for the application of infiltration methods. Even in that case, for large areas approaching or exceeding 1,000 sq miles, for proper application of infiltration methods detailed information would be needed for the entire watershed regarding hydrologic factors, physiography, geology, and land use and management, including both agriculture and forestry. Approximations and estimates would have to be made at various steps, and the end result would not be any more sound than it would be by the methods heretofore used by competent hydraulists.

The authors' statement (see heading "Application to Hydraulic Engineering: III—Adjustment of Infiltration-Capacity Values to Antecedent Conditions and Precipitation Pattern") that infiltration procedure allows "the use of much lower factors of safety than one involving a choice of a coefficient of runoff" is not in accord with established facts. Until the engineer can know with certainty the maximum storm that may be expected, there is no basis for expecting infiltration analysis to justify lower safety factors.

The files of the Miami Valley Conservancy District, and its technical publications, will show that the hydraulic engineers, headed by Sherman Woodward, M. Am. Soc. C. E., in charge of the design of the noteworthy flood-control reservoirs and channel improvements in the Miami Valley in Ohio were fully aware of infiltration and other hydrologic factors that are woven into the fabric of the infiltration theory advanced during the past few years. The paper refers to the Miami District research on small plots. Such experiments were made and reported upon by Ivan E. Houk, M. Am. Soc. C. E. They served to evaluate the characteristics of the particular plots, and were interesting within themselves, but furnished no basic information used in the project design. In that District, as in any other large water-control project, the design must provide for storms the size of which, in the final analysis, must be based on the best engineering judgment of a competent engineering staff, supplemented by qualified consultants. Such a determination is a practical integration of hydrologic relationships based on mature consideration of all factors involved.

In the design of flood-control works in the Tennessee Valley to provide protection to the City of Chattanooga, and in determining flood-control capacity in the Kentucky Reservoir at the mouth of the Tennessee River as a link in the flood-protection works for the Mississippi Valley below Cairo, Ill., a board of consulting engineers was called in to work with the Authority's engineers in determining the basis for design. This Board consisted of Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E., the late Harrison P. Eddy, Past-President, Am. Soc. C. E., and Gerard H. Matthes and Ivan E. Houk, Members, Am. Soc. C. E. The judgment of this Board (plus that of the Authority's own engineers, including Joseph H. Kimball, James S. Bowman, and Mr. Woodward, Members, Am. Soc. C. E.) developed design criteria supported by long and successful engineering experience and practice. It is scarcely conceivable that any infiltration derivative could be substituted for the considered decisions of such an engineering group.

The Tennessee Valley Authority has under preparation a plan for flood control for the French Broad River Basin in western North Carolina, an area of about 1,800 sq miles, including the City of Asheville, industrial properties of great value, and a rich agricultural area. A study is being made of the effects of improved land use and better farming methods on flood flows and water conservation in this region. Since 1935 the Authority has conducted an agricultural test program in this watershed involving more than 100 test demonstration farms. It has been clearly demonstrated that, by better land management and the proper application of lime and phosphate fertilizer, definite improvement in infiltration rates and capacity has been brought about that can be evaluated quantitatively in its effect on flood peaks and volumes. However, it is extremely difficult to make practical use of this fact in flood-control design, although supported by concrete information, because the engineers themselves cannot say with positive knowledge just how big a storm should be the basis for flood-control design. Whatever the basis finally determined upon, it must be the best judgment of the engineers and must be large enough to be amply safe to protect the valuable city and industrial properties involved. It would be foolish to apply a method to the extreme flood—the design flood—which might result in lowering the peak a matter of inches when the size of that flood must be estimated with the probability of considerably more error than the total beneficial effect of improved cover under good land management.

The authors recommend aerial photographs for land-use information to determine percentages of land within a watershed having particular types of vegetal cover, such as forests, pastures, and cultivated land. The Tennessee Valley Authority has made extensive use of aerial photographs in its mapping and other programs and has available excellent aerial photographs of the entire 41,000 sq miles in the Tennessee Valley. It is possible from these photographs to determine the percentage of open and forested lands, but for infiltration purposes much more than this is necessary. However, there are great differences in lands with respect to infiltration capacities that cannot be discerned from aerial pictures no matter how good these may be. For example, it is impossible to determine, from the aerial photographs, the extent to which a forest canopy is complete or the amount and character of litter under the trees. Woodlands used for pasture have quite different infiltration characteristics than woodlands not so used, yet both would appear the same in an aerial photograph. In studying the effects of land use on flood control in the French Broad watershed by infiltration methods, it has been necessary for foresters and agriculturalists to go into the field and classify the various types of land use into a number of different classifications for each major usage. Even this work has had to be generalized because of the large watershed involved.

The paper also states that soil maps are available for a large part of the

United States. This is literally true, but it fails to tell the real status of the soil surveys. These are of three general types. There are the reconnaissance maps, covering large areas, which are so generalized that they do not provide the detailed information desirable for infiltration procedure. Detail surveys by counties are available for only 35% of the land area of the United States. There are the much improved detail surveys issued only in the last few years for a small number of counties. The last two types are valuable in providing useful information for infiltration studies, but these are not available for about two thirds of the country.

The unwary and unknowing would be led to believe that with aerial photographs and soil maps it is a simple matter to classify the land-use and soil characteristics of a drainage basin sufficiently for a satisfactory application of infiltration. Experience in application belies this impression. It is this kind of statement that leads thinking engineers to discount infiltration as a useful engineering tool, and it is unfortunate that the infiltrationists have adopted this technique.

From this discussion so far, it may appear that the writer is an "anti-infiltrationist" but, as a matter of fact, he is enthusiastic about the possibilities of this method when used in its proper place. Infiltration has a definite and most useful application in a field not mentioned by Messrs. Horner and Jens. This field is exemplified by the water-control operations of the large system of multi-purpose reservoirs in the Tennessee Valley. The system now (1942) numbers sixteen reservoirs and nine more are under construction so that the dispatch of water through these reservoirs, operated as one integrated system, is quite complex. A group of especially trained hydraulic engineers make detailed determinations of stream flow each day, including both surface and subsurface runoff, from ungaged areas. The efficient operation of the system requires continuous close knowledge of water available throughout the Tennessee Valley in numerous headwater streams and in local areas tributary to the reservoirs. In this work of daily river-flow predictions and water-control operations, infiltration has large possibilities. The condition of sub-basins with respect to physical and geological characteristics, ground-water levels, field moisture, cover, and land use can be known in advance of rain so that, when the rain comes, these facts can be considered and infiltration methods used to estimate storm runoff. The ordinary storm is not the great storm for which the dams and other works are safely designed, but rather the one that is likely to occur at any time, depending upon the season of the year. In this class of rains, where infiltration may account for a considerable part of the rainfall, the method is of real importance and affords a logical approach to the problem of estimating runoff.

W. W. Horner,[38] M. Am. Soc. C. E., and S. W. Jens,[39] Assoc. M. Am. Soc. C. E.—The discussion of this paper is particularly interesting with respect to the wide range in the items that apparently attracted the attention of the different discussers, and in the variety of viewpoints expressed.

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[39] Associate Consultant, W. W. Horner & H. Shifrin, St. Louis, Mo.

The discussions of Messrs. Jarvis and Fry are from the viewpoint of the application of the method to large drainage basins. Since this matter was not presented in detail in the paper, these are primarily discussions of the writers' statement that "The method is presented as being generally applicable to all drainage basins, and is described in detail as it would be used in urban storm drainage" (see "Synopsis"). The writers had no intention of suggesting that the method described in detail for urban storm drainage would be duplicated in its details in an application to a large drainage basin, but rather that the principles involved and the general type of methodology, if adjusted to a different scale, were so applicable. The matter of a similar, but somewhat differently adjusted application to a large drainage basin, has been prepared.[40]

Mr. Fry's discussion appears to be largely a defense of hydraulic practice of the past. He apparently reads into the paper a condemnation of the persons who have been involved in this practice, which was not expressed or intended, or even inadvertently implied. The writers wish to make it quite clear that they have the greatest respect for the men who have been, and who still are, providing the best possible designs for important hydraulic projects on the basis of data that are far from adequate for the purposes.

One of the writers had a responsible part in the development of the so-called "rational method," a method which they have stated that they expect to see supplanted by some procedure similar to that presented in the paper. The rational method represented an important improvement over prior practice, but they would feel derelict to the profession and to themselves if they did not make use of any new data as they became available, and of improvement in technique recently found to be possible. The so-called "rational method" is a good example of the use of "crude over-all general relationships," in that it attempts to relate peak surface runoff rate to mean rainfall rate through a judgment coefficient.

Mr. Fry's primary objections to the method outlined in the paper appear to relate to its possible use as a basis of spillway design, and are on the ground that little is known about maximum storm potentiality and therefore that hydrologists are not justified in attempting a detailed evaluation of the possible maximum flood.

The writers regret that the recent and very proper concentration on the problem of maximum flood flows has relegated to a secondary status the problem of evaluation of flood flows in terms of specific frequency of occurrence, as this phase is a very important determinant in the greater number of hydraulic projects and problems that have a hydraulic background. Some of these are mentioned in the last paragraph of Mr. Fry's discussion.

Mr. Fry also apparently read into the paper a suggestion that the better present coverage of the United States by airplane photographs and by maps of various types would make it possible to evaluate flood flow without going into the field. No such suggestion was intended, and his further elaboration of the field procedure that would be required is a valuable contribution to this phase of the discussion.

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[40] "Rôle of the Land During Flood Periods—Relations Between Rainfall and Stream Channel Inflow," by W. W. Horner (publication in *Proceedings* pending).

Mr. Jarvis breaks a gallant lance in defense of the castigated runoff coefficient, particularly with reference to the writers' statement that the method is generally applicable to all drainage basins. Admittedly, if the available and reliable statistical data for a particular large drainage basin reveal a consistent relationship as to the particular quantities under study, any hydraulic engineer would lean heavily upon such data; but if the magnitude and economic importance of the proposed project are great and there exist sufficient data on rainfall, cover, soils, infiltration capacities, and channel storage discharge characteristics, or if adequate unit graphs can be developed, an application of the principles suggested in the paper can be made to units of, say, less than 1,000 sq miles. The resulting hydrographs can be integrated by ordinary methods to produce a reasonably accurate representation of the flow at the point under analysis, not only as to total runoff or yield but as to momentary peak and durations of rates of various magnitudes. For basins of several hundred thousand square miles, it can be understood that the detailed calculations would be unwieldy; but it is submitted that, for basins with a tributary area of as much as 20,000 sq miles, it is worth while to attempt to evaluate the production of net rainfall as it most probably occurs in nature, hour by hour or in lesser or greater time units as the facts may justify. Such an approach would certainly result in greater reliability than a purely statistical approach.

In the problem used to illustrate the principles, as in most urban storm drainage, peak values are of greatest importance, and no single coefficient would reflect properly the varying runoffs resulting from the different precipitation patterns. The peak resulting from the uniform pattern, it should be noted, differs by much more from any one of the peaks due to the other three patterns (all more likely of occurrence in nature) than any one of the other three peaks differs from either of its two fellows.

Mr. Jarvis presents, in Table 7, some extremely interesting and important data with respect to the variation in annual yield of the Mississippi River and its principal tributaries. The writers agree with the importance of statistical analysis and the use of correlations and factors in organizing data of this class, as such factors are not of the same order as the "coefficients" referred to in the paper. The data presented in Table 7, however, seem otherwise to be scarcely pertinent to the subject of surface runoff in its flood phase as presented in the paper.

Mr. Jarvis sets up runoff coefficients (volumetric) for the Spur, Tex., data, and shows that by varying these coefficients through arbitrary factors their application will reproduce runoff volume for 70% of the storms within a 20% tolerance, a degree of significance that is certainly not impressive. If rate coefficients had been used, the results would have been equally poor. It is interesting that Mr. Musgrave presents similar data from Clarinda, Iowa, to show that "It is obvious that the use of any single percentage to represent the runoff that might result from a given storm * * * would be entirely fallacious." Mr. Jarvis' quotation from *Water Supply Paper No. 838* may serve to clear up a matter that Mr. Sherman also discusses. When losses occurring on only part of a basin are divided by the entire basin area, there results an arithmetical quotient that has no relation to infiltration capacity, areal or otherwise. This situation has been covered by Messrs. Horton and Sherman in presentations before the American Geophysical Union.

Mr. Jarvis' difficulty in accepting the data in Table 4 apparently results from a misunderstanding of what the figures mean. The losses shown as infiltration plus retention practically all occur on the 70%-of-pervious area, and have been reduced to inches of depth in that area, whereas the peak rates include the flow from the 30%-of-impervious area. A study of the relation of these two would require a much more complex approach than that which Mr. Jarvis attempts to use. The writers do not quite understand the reason for the presentation with respect to St. Louis precipitation. There is a long and accurate record of precipitation at St. Louis that has been analyzed many times for the determination of rainfall frequency. It is not presumed that Mr. Jarvis is suggesting that hydrologists approach this matter through a general correlation, when the detailed physical data are available at the site.

Mr. Musgrave's opening paragraphs and his presentation of the Clarinda data support the writers' thesis with respect to the desirability of substituting more accurate methods for coefficients of runoff. His further discussion on the present knowledge of infiltration and infiltration capacity, and on the factors controlling them, is extremely important. The research he mentions is particularly interesting with respect to the efforts to compare infiltration values with antecedent soil moisture and soil temperature. The practical need is very great for some easily applied relationship between infiltration values and one or more controlling variables, readily measured or obtained from normally available data. Another important part of the mechanics of applying infiltration-capacity curves involves a determination of rate of recovery of infiltration capacity for each soil and cover under varying conditions of antecedent infiltration and varying intervals of time between rainfall blocks within a storm itself. The sort of research that Mr. Musgrave indicates is under way will undoubtedly throw helpful light on these and other problems which require satisfactory and practical solutions. However, the writers believe that present knowledge of infiltration capacity and the ability to produce average curves for various complexes and for various seasons, and to recognize variations of infiltration capacity concurrently with, even if not controlled by, changes in soil moisture, justify hydrologists in using the established curves in engineering practice with some confidence.

Mr. Ramser holds out the encouraging expectation that the methodology of applying the latest hydrologic knowledge to engineering problems will undoubtedly be "simplified greatly."

Mr. Harrold's discussion is helpful, both as to his general comments on the methodology proposed and for his emphasis on the necessity of applying it to basins separately by rainfall centers, and separately by soil and cover complexes. The writers agree with him that the use of average areal values applied to average or mean rainfall rates for large basins may yield results that are quite inadequate. He has given one of the limitations of the "average areal infiltration capacity"—namely, when a storm has rainfall rates over parts of the area less than the corresponding infiltration capacity, the computed value of average infiltration capacity for the entire area will always be less than the actual.

As suggested by Mr. Harrold in the example of Table 6, this can be avoided by applying infiltration-capacity curves to each sub-area precipitation diagram separately, thus obtaining Thiessen area-excess rainfall diagrams that can then be weighted and combined.

In a personal letter to the writers Mr. Horton has called attention to the fact that he has given[14] a general equation for the rising side of the hydrograph which is applicable to various types of flow:

$$q = \sigma \tanh^{M} \left[\frac{M + 1}{M} (\sigma K_s)^{(M-1)} t \right] \quad \ldots\ldots\ldots\ldots\ldots (21)$$

Eq. 4*a* follows directly by substitution of the value of $M = \frac{5}{3}$, which is that for fully turbulent flow in Eq. 21. The reader is referred to Mr. Horton's paper[14] for a development of the proper value of K_s for fully turbulent flow.

Mr. Sherman's comments supplement the paper and emphasize certain facts that were not sufficiently covered therein. His discussion of average infiltration capacity may not be clear to those readers who are not familiar with the *Proceedings* of the American Geophysical Union.

Mr. Sherman's calculation of "equivalent time T" for correlating the periods of observed rain intensities with the infiltration-capacity curves is based upon the same fundamental assumption as the method used by the writers—namely, that infiltration decreases with an increase in soil moisture. It may well develop out of further research and analysis that some simple method such as Mr. Sherman's will establish, adequately and practically, the initial value of the infiltration curve that is to be used at the beginning of each storm or of each of several separated parts of one great storm. As indicated by Mr. Musgrave, however, the variation in initial infiltration rates is not solely attributable to variation in soil moisture. It is to be hoped that out of the kind of research exemplified by Table 11 a clear enough picture of the mechanics of infiltration will evolve so that hydrologists may know what the dominant controls of initial capacities are, and accordingly can, with practical accuracy and economy of effort, quickly "position" the applicable infiltration-capacity curve at its correct initial point.

All this is not to deny that present methods are within the range of accuracy attainable by the older methods. The brief experience of several workers in this field indicates, for example, that Mr. Sherman's method of correlation is satisfactory.

It is to be regretted that, in the paper, space limitations caused the omission of some discussion of "average areal infiltration capacity," because the determination of "'average loss rate' (f_{av})" can be a useful approach. The value of "average areal infiltration capacity" does not, of course, represent the actual infiltration capacity of any part of the basin at any time. It is not related to any specific soil and cover combination, and therefore has no physical significance. It seems preferable not to refer to it as infiltration capacity at all, although it is an integration, through area and time, of real infiltration capacities during the storm period from which it is derived. Mr. Sherman's term "average loss rate" seems more truly descriptive.

A series of such values derived from storms occurring over the full range of seasonal variation in vegetal cover is usable in estimating flood flow from any pattern storm likely to occur in a particular season. The relationship developed is much superior to any involving a coefficient of runoff, or one based on total losses. Its major weakness lies in the assumptions involved in setting up rainfall patterns for non-recording stations, and in the common expression of such patterns in hourly values of uniform mean intensity. As shown elsewhere, such hourly mean values do not generally represent precipitation in the manner in which it actually occurs. For short storms, on small basins, the assumption of constant infiltration capacity throughout the storm period would introduce material errors, unless it is also assumed that the storm is immediately preceded by other heavy rainfall and that the infiltration-capacity value is that for the latter part of a total precipitation period.

The method has been presented by Mr. Horton and R. Van Vliet,[21] by Mr. Sherman,[22] and by Walter B. Langbein,[41] and it is to be hoped that a complete illustrative application will be published in the near future for the benefit of the profession.

It is the belief of the writers that where applicable values of actual infiltration capacity are available (as, for example, from small watersheds representative of the principal soil and cover combinations in the basin) the first method of determining excess rainfall using actual infiltration capacity will give more accurate results than one involving average areal values.

As pointed out by Mr. Flynt, the method is particularly applicable to airport and urban drainage—for example, in any instance where close control can be held over the physical factors, such as slope and nature of surfaces, and where the values involved justify a thorough analysis.

The most difficult step in the analysis of large areas can be the groundwater return to the flood hydrograph, but where this is not a problem the steps that Mr. Flynt has outlined for handling large areas are patently more realistic and representative of actual occurrence than any all-embracing coefficient, the choice of which involves an application of judgment possessed only by a relatively small group of highly specialized hydrologists; and, more importantly, the use of coefficients appears easy and hence can be dangerously abused.

Mr. Schafmayer's statement that the many variables involved in runoff determination emphasize "the bearing of chance events on any estimate of runoff rates or of the probable frequency of any particular rate" gives a pertinent reason why the deprecated "microscopic approach" is not only justified but virtually mandatory if those important variables capable of reasonably accurate determination are to accorded due emphasis.

Eq. 14 is not complete without the inclusion of a constant to express the ratio of the peak flow to the mass net rainfall; and, of course, such a "coefficient" must reflect, properly, the particular pattern of the collecting system.

Mr. Schafmayer's hope that Eqs. 15 and 16 will ultimately prevail is an understandable wish; but it is not readily apparent that experience, and analysis of the data coming out of that experience, will easily yield a variety of coefficients, runoff rates per square mile, or the like, which will adequately cover the multitude of varying combinations of surface, slope, vegetation or cover, soil, antecedent rainfall, stream pattern, and soil moisture—to mention only a few of the major variables. It is clear also that, unless the coefficient to be used does reflect the specific conditions being considered, its use is no more accurate than the statement that a certain person will die at the age of 65 because the average span of life, as accurately determined from a great mass of reliable data, is 65 yr.

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[41] "Some Channel Storage Studies and Their Application to the Determination of Infiltration," by Walter B. Langbein, *Transactions*, Am. Geophysical Union, 1938, Pt. I, p. 435.

The writers cannot agree with Mr. Schafmayer in his statement that "since the objective of the designer is to determine the peak storm flow, the time factor, which is important in the rational method, loses its importance in the method proposed by the authors." The time of overland flow, the time for storage to be built up in the particular channel or pipe system, the time for disposition of the detention and channel storage through the recession side of the hydrograph, as well as the time for additional infiltration to take place during such recession, and the time of transit in various branch and trunk elements of the drainage system above the point being investigated are all of utmost importance.

The technique and mechanics involved in the paper seem to offer a tool of especial promise for a thorough reexamination of the current conception of critical time. As commonly stated under the rational theory, the maximum rate of runoff will be produced by a rain having a duration equal to the time of flow from the most remote point of the drainage area to the point under design. From a consideration of the hydrology of the rainfall curve, this hypothesis appears reasonable. From a consideration of the hydraulics of overland flow, as revealed by this demonstration, the conception of the time of transit needs complete reconsideration. For example, in the case of the 40-min uniform rain illustrated in Fig. 6(*a*), it is noticeable that, after the beginning of this "critical" rainfall, there is a period of time during which no surface runoff occurs from the pervious area; call this value t_0. Without, in the first instance, disturbing the theory of the critical time further, it would appear that this time should be at least t_0, plus the time of flow from the remote point of the area.

Further inspection of the hydrographs shows that at the end of this rainfall the rate of runoff from the pervious areas is still increasing rapidly. If the time of transit were developed from a consideration of velocities and lengths, and the velocities were those related to the very small depths existing at the beginning of overland flow, then the time of transit of the first overland flow produced at the extreme limit of the watershed would be quite long. If velocities were considered that were related to larger depths of detention at some indeterminate time after the beginning of overland flow, the time of transit would be less, but the point chosen as the beginning of the time of transit would be later on the hydrograph. From a consideration of velocity and length alone, it would appear that the beginning of overland flow should be taken at some time after the actual beginning and at a point on the rising side of the hydrograph of flow where the velocities would have material values.

When these hydraulic considerations are "brought into the picture," it would appear that the maximum rate of flow would result from a rain having a duration equal to t_0, plus a time t_1 necessary for the rate of overland flow to attain material values, plus a final item t_2 equivalent to the time of transit under these conditions. An actual application of these considerations to any particular rainfall curve and hydrograph would develop a critical time much greater than the current interpretations would indicate.

Finally, however, there is another important item to be considered. It is quite clear that the maximum rate of flow at the design point is not a matter of velocities and distances only, but results from the establishment of equilibrium out of the storage equation. For example, there is a storage lag involved that the foregoing procedure does not evaluate satisfactorily. It is entirely possible that a re-study along the foregoing lines will make it possible to retain the conception of critical time as a valuable index.

It should be further emphasized that the particular curves in the paper (Figs. 5, 6, 7, 12, 13, etc.) are valid only for the conditions from which they have been derived and in no sense should they be used directly for any area or block differing materially from the physical conditions and dimensions assumed.

Mr. Schafmayer comments on the "notable uniformity of peak flow for the 60-min, 40-min, and 20-min points whether the storm is of the advanced, intermediate, or delayed pattern." In studying the peak runoff rates displayed in Table 5, there are certain essential facts to be kept in mind. The first of these is that the results shown by Breihan's curve indicate, purely from a consideration of pattern occurrence, that within a rain of any particular duration there will be included; for periods of less duration, mean intensity precipitation approximately of the order of that which would exist for less duration under the normal rainfall curve. For example, within any one of these 60-min precipitation patterns there will be found a 40-min period for which the mean precipitation will be approximately the value that would be indicated for the 40-min period on the normal 5-yr rainfall curve; and similarly, within either the 60-min or 40-min patterns there will be found a 20-min period having a mean intensity approximately equal to that shown for 20 min on the 5-yr rainfall curve.

In general, the peak rates of flow that are shown in the table, as at manhole *A*, are responsive directly to the periods of high precipitation within the rainfall pattern. In other words, the peak occurring at this point as the result of the 60-min rain is largely influenced by the heaviest 10-min to 20-min precipitation within the 60-min rain.

From this background, it is not difficult to understand why the peak rates at the 20-min point should be higher for the longer storms. In other words, it is not surprising that the values shown in Table 5 appear to violate the criteria of the rational method. When it is remembered, however, that these longer rains also contain the critical shorter rains within them, and that, as a part of the longer rains, the critical shorter rains occur after infiltration rates are reduced and surface detention built up, the result appears to be entirely reasonable. Possibly the aforementioned violation of criteria is only a violation of a common misinterpretation of the criteria, in that hydrologists have been somewhat accustomed to think of these critical rains as isolated periods of precipitation not joined to other precipitation before or after the critical period. An inspection of the records on which any of the rainfall curves are based will show that such a case is the exception rather than the rule, and that the background of the average rainfall curve consists of sections taken out of longer rains and subjected to statistical analysis.

The second matter of importance in studying the values in Table 5 is a recognition that the introduction of patterns introduces a new frequency item. For example, it is imaginable that, of all the patterns in which a 60-min rain might occur, the uniform pattern is the one that practically never occurs; and that possibly one half of the occurrences are of the advanced type, and one quarter each are of the intermediate and delayed types. From this background, the peak runoff rate resulting from the advanced pattern based on a 5-yr rainfall curve would have a real frequency of the order of 10 yr, and the runoff rates resulting from the intermediate and delayed patterns of a similar 5-yr curve would have a real frequency of the order of 20 yr.

Such an analysis approaches the quantitative values in which the engineer is actually interested—the peak rates of flow that will have a specific frequency designation. The foregoing values are only a partial approach to a determination of the flow rates that may be expected with a particular frequency. A complete study of this matter would require a determination of flows under different rain patterns for mean rates taken from rainfall curves of several frequencies, and the undertaking of a new statistical analysis of all of the results to determine the true probability of a specific peak rate of flow for a particular physical situation.

One idea back of the preparation of this paper was that the engineers of one or more of the large cities might be stimulated to test out the suggested method in storm-drainage design. The writers are agreeably surprised to find that such work is already being done at Los Angeles, Calif. It is encouraging to them to learn that Mr. Hicks has arrived at much the same general principles for surface runoff determination through wholly independent studies. It is particularly significant that he too has discarded the unrealistic uniform rainfall pattern.

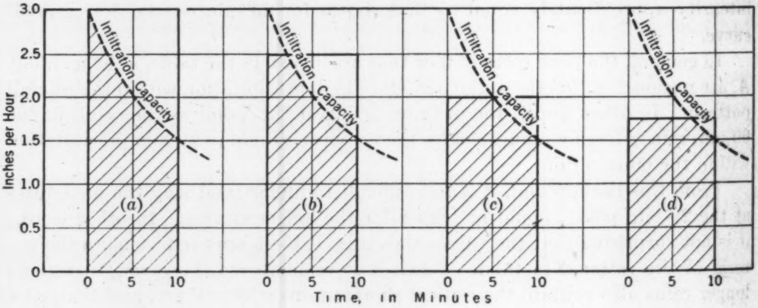

FIG. 19

Fig. 16 is the first experimental evidence that infiltration-capacity values are dependent upon rainfall intensity, and it is not clear from Mr. Hicks' explanation why this is so. That higher rainfall intensities will satisfy more quickly the interception requirements of the turf cover is plain, but that the infiltration capacity is increased because of "more complete coverage of the imperfect plane surface of urban yards" is not readily visualized. It is difficult to understand why higher intensities of precipitation should result in a more complete coverage; the prevailing concepts of surface detention and depression storage postulate runoff from an entire area only after complete coverage has been effected. In the absence of a clear knowledge of the derivation of the curves of Fig. 16, the writers nevertheless have speculated upon whether or not "loss rate," as given in Fig. 16, is identical in definition with infiltration capacity as used in the paper.

If the same infiltration-capacity curve is applied to each of four uniform rains of differing intensities, as shown in Figs. 19(*a*), 19(*b*), 19(*c*), and 19(*d*), it is plain that "loss rates" can be computed which substantiate Fig. 16. By dividing the shaded area in each instance (which represents the mass loss) by the elapsed time—in other words, obtaining the average ordinate for each shaded area—the following values (in inches per hour) result:

Case (Fig. 19)	Rainfall rate	Loss rate
a	3.1	2.10
b	2.5	2.02
c	2.0	1.87
d	1.75	1.69

For the sake of simplicity, all of these hypothetical rains are assumed to occur under identical antecedent conditions, thus placing the assumed infiltration-capacity curve at the same initial values. Fig. 20 is strikingly similar to the curves of Fig. 16; and, for any series of rainfalls each involving blocks at less than infiltration-capacity rates (as in the simplified example), it is obvious that the "loss rate" must vary with the intensity of rainfall.

Mr. Hicks' comment concerning the position of the impervious areas in the typical lot is correct: In the actual urban lot some appreciable part of the impervious area drains on to turf before its flow reaches the street or alley. In the writers' example, the primary purpose was to illustrate the principles involved, and the disposition of the component areas of the typical lot was chosen in part to illustrate the conditions that would contribute the greatest runoff and in part to facilitate the mechanics of evaluating the various contributions.

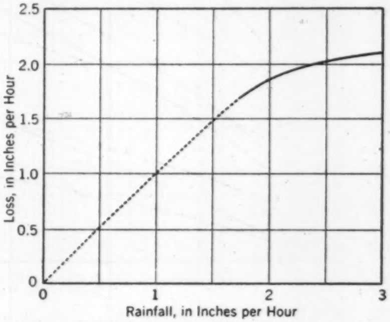

FIG. 20.—RAINFALL RATE-LOSS RATE CURVE

The exponential formulas derived from the experiments reported by Mr. Hicks are strikingly similar in form to Mr. Horton's equations for depth of detention. If Eq. 3*b* is assumed to have the hyperbolic tangent equivalent to unity (which is the case for equilibrium flow), the slope is expressed in percentage, and the result multiplied by $\frac{2}{3}$ to express mean depth of detention:

$$\delta_a = \frac{0.036\, l^{0.5}\, \sigma^{0.5}}{S^{0.25}{}_0} \qquad (22)$$

The constant in Eq. 22 is based upon experimental values reported in 1938,[12] and it is approximately half that of Eq. 19*a*. The larger constant, however, was obtained from clipped sod, whereas the lesser figure resulted from tests on uncut pasture grass of undoubtedly lesser density. The exponents also reflect a difference in the character of flow over the two surfaces. For example, the exponents of Eq. 3*b* are assumptions of 75% turbulent flow, whereas Eq. 19*a* indicates a lesser percentage of turbulence. It seems probable that the exponents and constant of Eq. 19*a* reflect more accurately city lawn conditions than do those used in the paper. Fig. 21 illustrates the influence of the constants and exponents over a range of supply rates for two lengths and slopes.

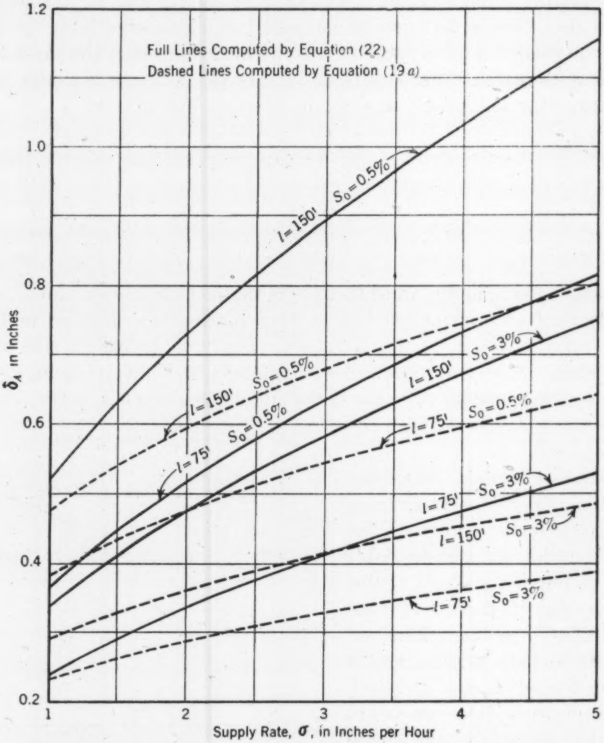

FIG. 21.—COMPUTATION OF δ_a-CURVES BY EQS. 19*a* AND 22

It is clear that the greater exponents in the denominator of Eq. 22 overshadow the lesser constant. The clipped sod appears to develop less detention storage than does the uncut pasture grass. Fig. 21 also shows the considerably greater detention storage built up for the flatter slopes and longer overland flow lengths.

Fig. 18 seems to postulate the theory that time of concentration is that time at which all the surface detention and all the channel storage above the point being studied have been developed or are being utilized; and, from Eq. 20*a*, it appears that Mr. Hicks has evaluated gutter storage in terms of equivalent average depth of flow in the gutter. The symbol "*l*" in Eq. 20*a* is length of gutter.

With reference to Mr. Hicks' comments concerning the summation of the block hydrographs, admittedly there is a danger of accumulative error, but the basic procedure is believed to have the decided advantage of simulating realistically the actual building up of flow as it progresses downstream. It should be noted that for closed systems (such as pipes or large conduits), the hydraulics of the flood wave will not enter the problem, if as is usual the analysis is confined to examination of peak flows. A properly proportioned closed system will flow full or under head when carrying the design flow, and the flood wave obviously cannot exist.

Mr. Hicks has stated in a letter to the writers that the fifth paragraph following Eqs. 12 "seems to indicate that a hydrograph of short duration is not reduced as much by gutter storage as the broader hydrographs for a 40-minute rain." It is regretted that any ambiguity should have arisen, because the work graphs (which could not be reproduced due to space limitations) and tables do show that the gutter storage does have a considerably greater reducing effect for the shorter, more intense rains. The relative effects of surface detention and of gutter channel storage are not evident in the graphs of Figs. 5, 6, and 7; their combined effect is indicated by the difference between the "Supply Rate for Block" and "Outflow Hydrograph for Entire Block."

It is to be earnestly hoped that Mr. Hicks will publish a full report on the commendable hydrologic research at Los Angeles so that all of the data and his interesting analysis and methodology may be available to engineers and hydrologists.

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MOMENTS IN CONTINUOUS RECTANGULAR SLABS ON RIGID SUPPORTS

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WITH DISCUSSION BY MESSRS. MAURICE P. VAN BUREN, RALPH E. BYRNE, JR., PIERCE P. FURBER, AND L. C. MAUGH.

SYNOPSIS

A method of analysis for rectangular slabs, supported on four sides and continuous over the supports, is described in this paper. The solution is so arranged that the equations providing continuity between the panels can be expressed in terms of the restraining moment at the middle of each support. The computations are simplified by using a sine distribution for the edge moments and by establishing continuity between the panels only at the middle of the supports. These approximations enable the solution to be based on assumptions that more nearly represent the fundamental structural action than does the use of hypothetical beam strips. Diagrams are provided from which the equations can be expressed easily. These equations are solved most conveniently by successive approximations.

INTRODUCTION

The moments, shears, and deflections in a single rectangular panel, either simply supported or fixed at the edges, have been computed for many types of loading. When a floor slab is continuous over the supports in all directions, an exact mathematical solution is possible but time-consuming, as the behavior in any panel then varies with the loading and physical characteristics of contiguous panels. Along with this action, the vertical deflection and torsional stiffness of the supporting beams provide additional disturbing factors. Confronted with these difficulties, the engineers who have developed the various building codes have naturally emphasized the most obvious action of the slab—that is, the flexure of the member in its two principal directions.[3] When the slab is simply supported, this flexural action is apparent, and the selection of

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NOTE.—Published in May, 1941, *Proceedings*.

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[2] Jung Yen, Kwangsi, Hongkong.

[3] "Building Regulations for Reinforced Concrete," *Proceedings*, A. C. I., 1936, p. 181.

two orthogonal strips, such as *AB* and *CD*, Fig. 1(*a*), that are considered as simply supported beams is a natural approximation. The distribution of the applied loading to these strips is an indeterminate problem that is frequently solved by making the deflections of the strips equal at their intersection. Ordinarily such assumptions give safe results for the maximum bending moments at the center of the slab, as the torsional resistance of the slab is entirely neglected. By referring to Fig. 1(*b*), it can be seen that the twisting moments T_3 and T_4 on an element of the slab can assist in the transfer of the vertical load to the support along the strip *CD* and thereby reduce the bending moments M_1 and M_2. In the same manner, the twisting moments T_1 and T_2, Fig. 1(*c*), can reduce the bending moments M_3 and M_4 in any strip *AB*.

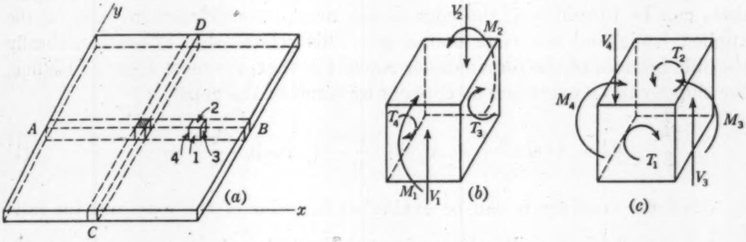

FIG. 1.—COUPLES ACTING ON AN ELEMENT OF A SLAB

In some building codes, such as the German Building Regulations, this reduction in the bending moments due to the torsional stiffness of the slab is effected by multiplying the moments that are obtained from an analysis of the individual strips by a reduction factor (Marcus approximation) that has been obtained from a slab analysis.[4] The present American Concrete Institute (A. C. I.) building regulations[3] in the United States also adhere to the use of individual strips that are considered as continuous beams in the case of continuous floor slabs. In this case, the only correction for the slab action is made by an empirical distribution of the load between the two hypothetical beams. In such analyses, continuity of strain throughout the slab is provided as a secondary factor, if at all, rather than as a fundamental part of the solution. The only alternative to the strip method appears to be in the direction of using the entire panel as a structural unit.

The use of the entire panel was adopted by Nathan M. Newmark, Assoc. M. Am. Soc. C. E., in his analysis of slabs that are continuous in one direction and simply supported on the edges parallel to this direction.[5] In that problem, where only two edges of any panel are subjected to bending moments, numerical tabulation of edge moments and of the corrections for edge rotations is feasible and can be applied directly. On the other hand, when slabs are continuous in all directions, there is considerable advantage in an algebraic arrangement that enables the conditions of continuity to be expressed by equations of standard form. Successive approximations then can be used in the numerical solution of these equations. This procedure was used by Prof. S. Timoshenko in his solution of a rectangular panel with fixed edges.[6]

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[4] "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by H. Marcus, Berlin, Springer, 1932, p. 89.

[5] "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by Nathan M. Newmark, *Bulletin No. 304*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

Regardless of whether an algebraic or direct arithmetical form of arrangement is used, the solution requires that the edge slopes of the elastic curve of the slab be expressed as the sum of the slope in a simply supported slab due to the applied loading, plus the slopes due to the restraining moments at the four edges. In the case of fixed edges, the resultant slope must be zero, but for continuous slabs the resultant slope in one panel must be equal and opposite to the resultant slope of the same edge in the adjacent panel. It should be noted that the foregoing procedure calls for satisfying the continuity conditions entirely around the boundaries of the various panels. Also, before the equations can be formulated, the edge slopes must be expressed in terms of the applied loads and the edge moments. This relationship is mathematically possible because of the fortunate circumstance that, for most types of loading, the slopes can be expressed as converging series of the type[7]

$$\theta_y = A_1 \sin \frac{\pi x}{a} + A_2 \sin \frac{2 \pi x}{a} + A_3 \sin \frac{3 \pi x}{a} + \cdots \qquad (1)$$

in which the coefficients can be evaluated in terms of the load and the ratio $\frac{b}{a}$, or $\frac{a}{b}$, which involves the dimensions of the slab. An exact solution is obtained if the first terms of each series are made to satisfy the required slope condition, then the second terms, then the third terms, etc. The actual edge moments will be the algebraic sum of the moments that satisfy the separate terms of the series. For instance, if the slopes along the boundary *AD* of the simply supported panel *ABCD*, Fig. 2(*a*), are considered, the rotation θ_y' at any distance x from the origin A, for a uniform load p per unit area over the entire panel, when $\frac{b}{a} = 1.0$, is equal to

$$\theta_y' = \frac{p a^3}{D}\left(0.0137 \sin \frac{\pi x}{a} + 0.00025 \sin \frac{3 \pi x}{a} + 0.000033 \sin \frac{5 \pi x}{a} + \cdots \right) \qquad (2a)$$

and, when $\frac{b}{a} = 1.5$:

$$\theta_y' = \frac{p a^3}{D}\left(0.0184 \sin \frac{\pi x}{a} + 0.00025 \sin \frac{3 \pi x}{a} + 0.000033 \sin \frac{5 \pi x}{a} + \cdots \right) \qquad (2b)$$

In all such equations the term D is used to represent the flexural resistance of the slab and is equal to

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[6] "Bending of Rectangular Plates with Clamped Edges," by S. Timoshenko, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938.

[7] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1940, Chapters 5 and 6.

$$D = \frac{E\,h^3}{12\,(1 - \mu^2)} \ldots\ldots.(3)$$

in which h is the thickness of the slab and μ equals Poisson's ratio. The restraining moments along any boundary, such as AD, Fig. 2(b), can be represented by a corresponding sine series, as

$$M_y = M_1' \sin\frac{\pi\,x}{a} + M''' \sin\frac{3\,\pi\,x}{a} + M''''' \sin\frac{5\,\pi\,x}{a} + \cdots\ldots(4)$$

For this distribution of moments, the slopes along any edge also can be expressed as a sine series, thus, when $\frac{b}{a} = 1.0$:

$$\theta_y'' = \frac{a}{D}\left(0.156\,M' \sin\frac{\pi\,x}{a} + 0.053\,M''' \sin\frac{3\,\pi\,x}{a} + 0.0318\,M''''' \sin\frac{5\,\pi\,x}{a} + \cdots\right).(5a)$$

and, when $\frac{b}{a} = 1.5$:

$$\theta_y'' = \frac{a}{D}\left(0.159 \sin\frac{\pi\,x}{a} + 0.053\,M''' \sin\frac{3\,\pi\,x}{a} + 0.0318\,M''''' \sin\frac{5\,\pi\,x}{a} + \cdots\right).(5b)$$

If the edge AD is fixed while the others are simply supported, $\theta_y' + \theta_y''$ must be equal to zero for all points along the boundary AD. This condition can be realized if each term of the series for θ_y' is made equal but with opposite sign to the corresponding term of θ_y''. Thus, using Eqs. 2a and 5a, for θ_y' and θ_y'', when $\frac{b}{a} = 1.0$: For $\sin\frac{\pi\,x}{a}$,

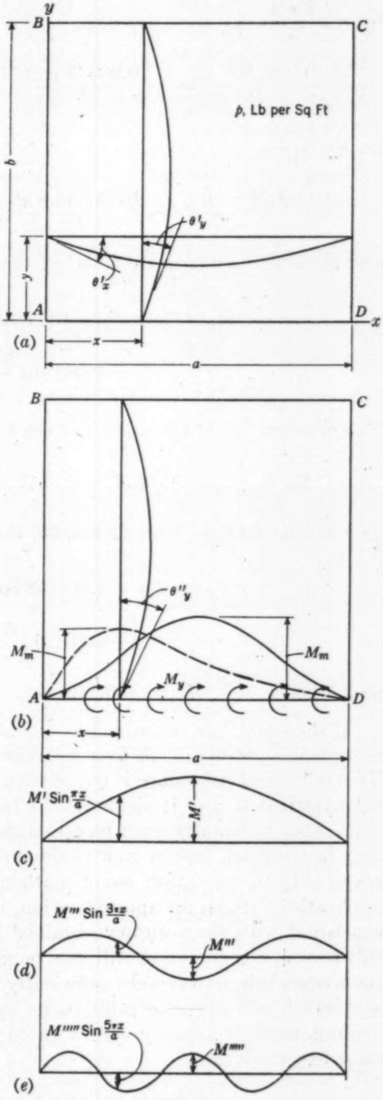


FIG. 2.

$$\frac{0.156\,M'\,a}{D} = -\,\frac{0.0137\,p\,a^3}{D}; \quad \text{or,} \quad M' = -\,0.0877\,p\,a^2$$

for $\sin \frac{3\pi x}{a}$,

$$\frac{0.053\, M'''\, a}{D} = -\frac{0.00025\, p\, a^3}{D}; \qquad \text{or,} \qquad M''' = -0.0047\, p\, a^2$$

for $\sin \frac{5\pi x}{a}$,

$$\frac{0.0318\, M'''''\, a}{D} = -\frac{0.000033\, p\, a^3}{D}; \qquad \text{or,} \qquad M''''' = -0.0010\, p\, a^2$$

and the edge moments are given by the series:

$$M_y = -p\, a^2 \left(0.0877 \sin \frac{\pi x}{a} + 0.0047 \sin \frac{3\pi x}{a} + 0.001 \sin \frac{5\pi x}{a} + \cdots \right) \ldots\ldots (6a)$$

The moment M_a at the middle of the edge, when $x = \frac{a}{2}$, will be

$$M_a = -p\, a^2 (0.0877 - 0.0047 + 0.001 + \cdots) = -0.084\, p\, a^2 \ldots (6b)$$

In a similar manner, Eqs. 2*b* and 5*b*, the corresponding series for $\frac{b}{a} = 1.5$, give

$$M_y = -p\, a^2 \left(0.1158 \sin \frac{\pi x}{a} + 0.0047 \sin \frac{3\pi x}{a} + 0.001 \sin \frac{5\pi x}{a} + \cdots \right) \ldots\ldots (7)$$

and, when $x = \frac{a}{2}$, $M_a = -0.112\, p\, a^2$.

If the panels are restrained on all sides, the slopes on the four sides must be expressed in terms of all four restraining moments and the applied loading. This numerical procedure is not particularly difficult when only one panel is to be investigated, and it yields results that are mathematically correct. However, when continuity must be established between several panels, all of which may be irregular, and in view of the uncertainties of the assumptions, it seems reasonable to relinquish some mathematical exactness for more facility of application. Such an approximation, of course, should give results that are consistent with the accuracy obtained in the average design procedure. For this reason, consideration will now be given to an approximate solution which the writers believe will yield sufficiently accurate results for average conditions and which will be practicable in its application. Any unusual condition of loading, such as a heavy concentrated load near one corner, will require the more exact solution.

APPROXIMATE METHOD OF SOLUTION

The distribution of the restraining moments along any support of a continuous floor slab may be either asymmetrical, as shown by the dotted line for side AD, Fig. 2(b), or symmetrical, as shown by the solid line. For many practical conditions of loading, the maximum value M_m will not vary greatly from the value M' (Fig. 2(c)) for a distribution of $M' \sin \frac{\pi x}{a}$, although the actual distribution is somewhat different. The type of loading best suited for such an approximation will naturally be one that is fairly symmetrical about the center of the panel, such as a uniform load over the entire panel or concentrated loads near the center. Unusual conditions, such as heavy concentrated loads near a support, will ordinarily be critical with respect to shear and, therefore, will require special consideration.

To simplify the numerical solution further, continuity between the various panels will be maintained only at the middle of each edge instead of entirely around the boundaries of each panel, as for the exact method. For any continuous slab, such as that in Fig. 3, the edge moments can be expressed in terms of the center moments $M_{12} = M_{21}$, $M_{15} = M_{51}$, etc., and continuity between the various panels is obtained by equating the slopes at the middle of each edge, as $\theta_{12} + \theta_{21} = 0$, $\theta_{15} + \theta_{51} = 0$, etc. Because of the fact that the distribution of moments along all edges is regarded as a sine curve, the slopes can be expressed directly in terms of the edge moments and the applied loading, as for a continuous beam on rigid supports. Thus, in Fig. 4, the angles α, β, and γ are given in terms of the moment $M' \sin \frac{\pi x}{a}$ for various values of $\frac{b}{a}$. Therefore, any edge slope such as θ_{12}, Fig. 3, can be expressed in the form

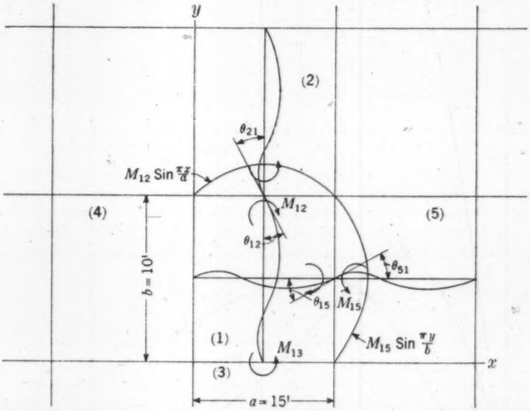


FIG. 3.—MOMENTS AND SLOPES USED IN THE APPROXIMATE METHOD

$$\theta_{12} = \alpha \left(\frac{M_{12}\, a}{D} \right) + \beta \left(\frac{M_{13}\, a}{D} \right) + \gamma \left[\frac{(M_{14} + M_{15})\, b}{D} \right] + \phi_s \ldots\ldots (8)$$

in which ϕ_s is the slope in a simply supported slab for the applied loading. Values of ϕ_s are given in Fig. 5(a) for a uniform load p per unit area over the entire panel, for a concentrated load P at the center of the panel, and, in Fig. 5(b), for a line load L across the center of the panel. It should be noted that, to determine the value of θ for a particular edge, the angles α and β are obtained by taking a as the length of that side, whereas for selecting γ the values of b and a are interchanged.

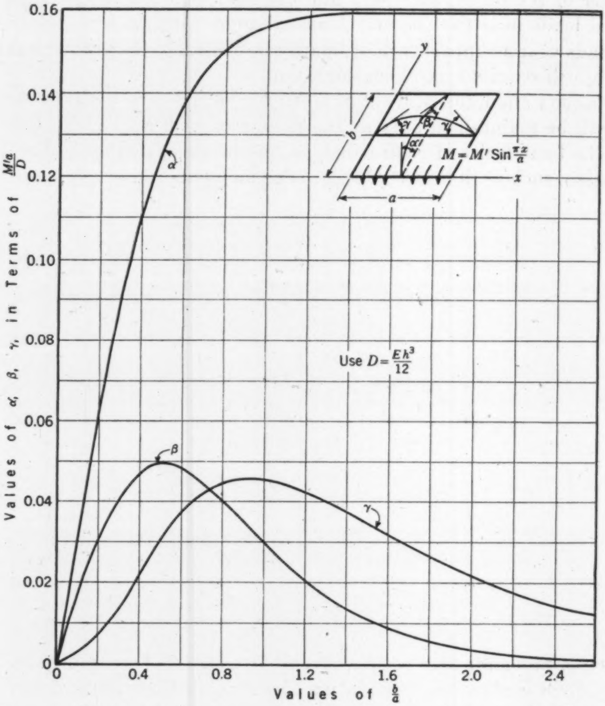


FIG. 4.—EDGE SLOPES α, β, AND γ IN A SIMPLY SUPPORTED SLAB DUE TO MOMENTS APPLIED ALONG ONE SIDE

To illustrate the use of the foregoing values, the equation for θ_{12} in panel (1), Fig. 3, will be given for a uniform load p over the entire area. For selecting α, β, and ϕ_s, $\frac{b}{a} = \frac{10}{15} = 0.67$; and, for selecting γ, $\frac{b}{a} = \frac{15}{10} = 1.5$. By Figs. 4 and 5($a$):

$$\theta_{12} = 0.143 \left(\frac{15\, M_{12}}{D} \right) + 0.046 \left(\frac{15\, M_{13}}{D} \right) + 0.035 \times 10 \left(\frac{M_{14} + M_{15}}{D} \right) + 0.0077 \times 15^3 \frac{p}{D} \quad \ldots\ldots\ldots (9)$$

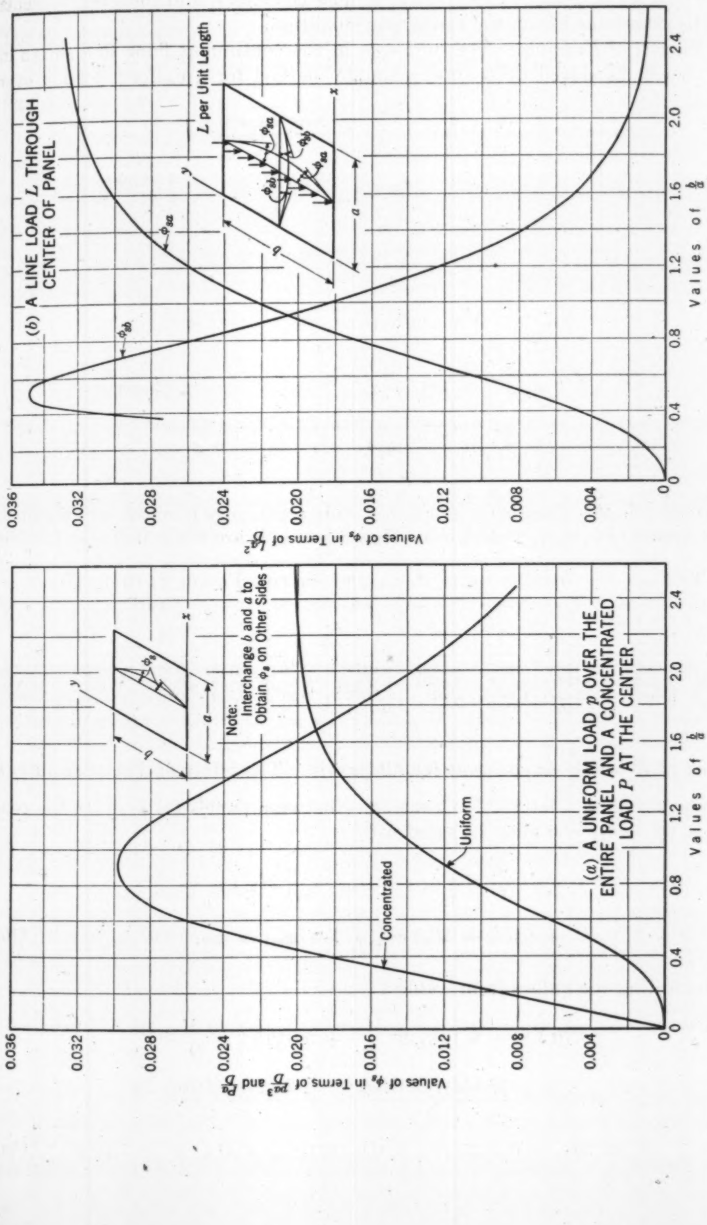


FIG. 5.—EDGE SLOPES ϕ_s

Other values of θ can be expressed by similar equations, and the edge moments can be determined from the continuity condition.

Numerical Example.—The moments in the continuous floor slab shown in Fig. 6 will be solved by the approximate method for a uniform load p over the entire floor. Because of symmetry, only seven edge moments are required. The values of α, β, γ, and ϕ_s from Figs. 4 and 5(a) are listed in Table 1. The value of $D = \frac{E\,h^3}{12}$ is constant for all panels. The effect of Poisson's ratio n will be considered later. For continuity between panels (1) and (2), the condition $\theta_{12} + \theta_{21} = 0$ must be satisfied:

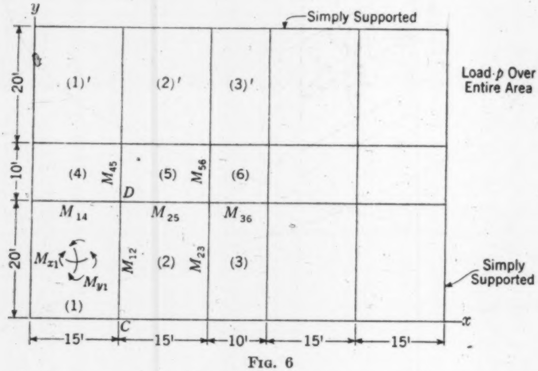


FIG. 6

TABLE 1.—VALUES OF α, β, γ, AND ϕ_s FOR FLOOR SLAB IN FIG. 6

Panel No.	$\frac{b}{a}$	α	β	γ	ϕ_s	$\frac{b}{a}$	α	β	γ	ϕ_s
1, 2	1.33	0.1583	0.0153	0.0395	0.0171	0.75	0.148	0.0425	0.0435	0.00925
3	0.5	0.126	0.049	0.0306	0.00425	2.0	0.159	0.003	0.0215	0.0198
4, 5	0.67	0.143	0.046	0.041	0.0077	1.50	0.159	0.011	0.035	0.0182
6	1.0	0.156	0.030	0.045	0.0134	....	....	....	....	....

$$\frac{a}{D} M_{12} (\alpha_{12} + \alpha_{21}) + \frac{a}{D} M_{23} \beta_{21}$$

$$+ (M_{14} + M_{25}) \gamma \left(\frac{b}{D}\right) + \phi_{s12} + \phi_{s21} = 0 \ldots\ldots\ldots\ldots (10)$$

Substituting the values from Table 1:

$$(0.148 + 0.148)\, 20 \frac{M_{12}}{D} + 0.0425 \left(\frac{20\, M_{23}}{D}\right)$$

$$+ 0.0395 \frac{15\,(M_{14} + M_{25})}{D} = - \frac{2 \times 0.00925\, p\, 20^3}{D};$$

or,

$$0.296\, M_{12} + 0.0425\, M_{23} + 0.0295\, (M_{14} + M_{25}) = - 7.40\, p \ldots (11a)$$

In a similar manner, the following equations can be obtained:

For $\theta_{23} + \theta_{32} = 0$—

$$0.0425\ M_{12} + 0.3239\ M_{23} + 0.0295\ M_{25} + 0.0107\ M_{36} = -\ 5.324\ p \qquad (11b)$$

for $\theta_{45} + \theta_{54} = 0$—

$$0.318\ M_{45} + 0.0107\ M_{56} + 0.1213\ (M_{14} + M_{25}) = -\ 3.64\ p \qquad (11c)$$

for $\theta_{56} + \theta_{65} = 0$—

$$0.0107\ M_{45} + 0.3448\ M_{56} + 0.1213\ M_{25} + 0.0914\ M_{36} = -\ 3.16\ p \qquad (11d)$$

for $\theta_{14} + \theta_{41} = 0$—

$$0.3478\ M_{14} + 0.0580\ M_{12} + 0.0230\ M_{45} = -\ 5.55\ p \qquad (11e)$$

for $\theta_{25} + \theta_{52} = 0$—

$$0.3478\ M_{25} + 0.0580\ (M_{12} + M_{23}) + 0.0230\ (M_{45} + M_{56}) = -\ 5.55\ p \qquad (11f)$$

and for $\theta_{36} + \theta_{63} = 0$—

$$0.3449\ M_{36} + 0.1238\ M_{23} + 0.0914\ M_{56} = -\ 3.321\ p \qquad (11g)$$

Eqs. 11 can be solved conveniently with a slide rule by the method of iteration. If approximate values of the larger moments are first obtained, the entire set of moments can be tabulated quickly. The numerical values of these moments, together with the corresponding values of the same moments that are given by the exact method and by the A. C. I. Code, are given in Table 2.

TABLE 2.—EDGE MOMENTS AT MIDDLE OF SUPPORTS, IN TERMS OF p, FOR FLOOR SLAB IN FIG. 6

Description	M_{12}	M_{23}	M_{45}	M_{56}	M_{14}	M_{25}	M_{36}
Approximate.....	−20.98	−12.64	−2.90	−4.64	−12.25	−9.83	−3.87
Exact...........	−20.56	−12.14	−3.95	−5.40	−12.17	−10.05	−4.26
A. C. I...........	−13.18	−9.20	−3.54	−2.94	−8.18	−5.84	−2.80

The distribution of edge moments along the support CD between panels (1) and (2) (Fig. 6) is shown in Fig. 7 for the exact method, together with the distribution that is obtained by the approximate method. It can be seen that the maximum ordinate for the exact distribution is near the center and is practically equal to the maximum ordinate of the assumed sine curve.

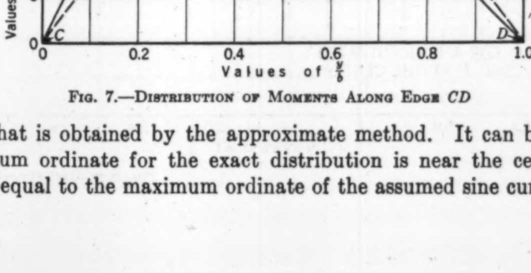


FIG. 7.—DISTRIBUTION OF MOMENTS ALONG EDGE CD

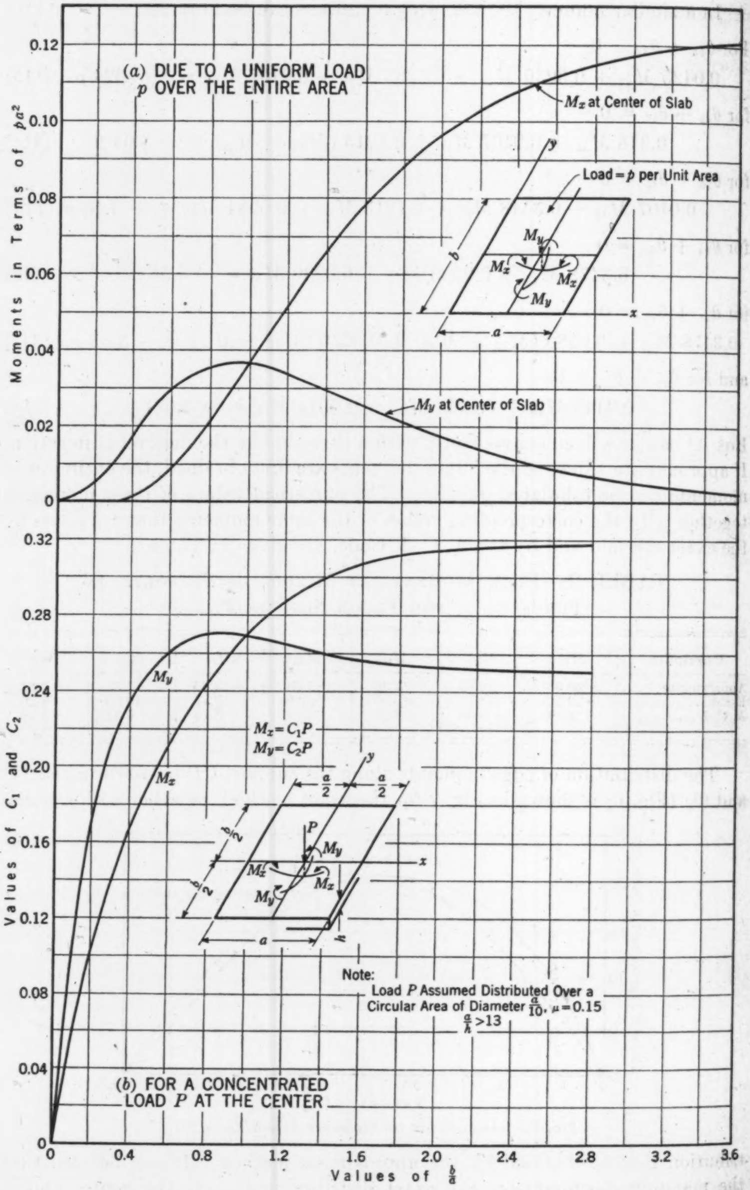
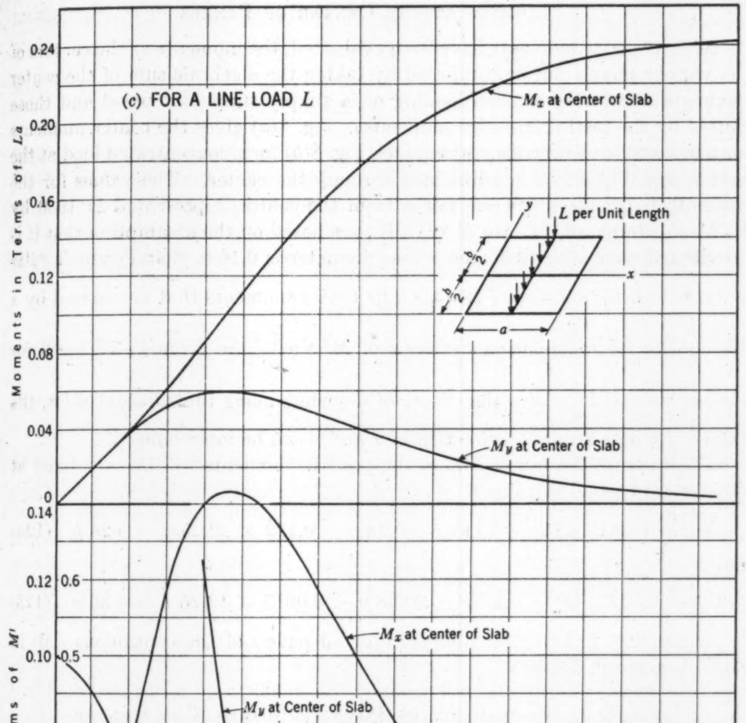


FIG. 8.—MOMENTS AT THE CENTER



OF A SIMPLY SUPPORTED SLAB

Moments at the Center of Panels

After the edge moments have been evaluated, the moments at the centers of the various panels can be computed by taking the algebraic sum of the center moments due to the applied loading on a simply supported panel and those caused by the restraining edge moments. Fig. 8(*a*) gives the center moments for a uniform load over the entire panel, Fig. 8(*b*) for a concentrated load at the center, and Fig. 8(*c*) for a line load through the center. The values for the concentrated load, which are taken from the solution[8] presented in 1930 by H. M. Westergaard, M. Am. Soc. C. E., are based on the assumption that it is distributed over a circular area whose diameter is 0.10 a, that Poisson's ratio is equal to 0.15, and that $\frac{a}{h} > 13$. The center moments that are caused by a distribution of moments alongside a of $M' \sin \frac{\pi x}{a}$ are given in Fig. 8(*d*) for various values of $\frac{b}{a}$. For the effects of moments along the adjacent sides, the values of a and b and the direction of x and y can be interchanged.

To illustrate the use of Fig. 8, the positive moments will be calculated at the center of panel (1), Fig. 6:

$$M_{x1} = 0.0618\, p\, 15^2 - 0.144 \times 20.98\, p - 0.132 \times 12.25\, p = 9.28\, p \quad \text{(12a)}$$

and

$$M_{y1} = 0.032\, p\, 15^2 - 0.1365 \times 20.98\, p - 0.0015 \times 12.25\, p = 4.36\, p \quad \text{(12b)}$$

A comparison of the center moments for all panels of the continuous slab in Fig. 6 is given in Table 3.

TABLE 3.—Comparison of Positive Moments at Center of Panels for Slab in Fig. 6, in Terms of p

Description	M_{x1}	M_{y1}	M_{x2}	M_{y2}	M_{x3}	M_{y3}	M_{x4}	M_{y4}	M_{x5}	M_{y5}	M_{x6}	M_{y6}
Approximate Method:												
$\mu = 0$	9.28	4.36	7.96	2.60	2.08	−0.98	−0.26	2.48	0.42	2.87	2.07	1.93
$\mu = 0.2$	10.15	6.22	8.48	4.19	1.88	−0.56	+0.24	2.43	0.99	2.95	2.46	2.34
Exact Method; $\mu = 0$	9.34	4.27	7.88	2.72	1.98	−0.76	−0.27	2.38	0.35	2.84	2.06	1.83
A. C. I.	7.21	6.16	5.94	4.13	2.30	1.55	1.72	−0.06	1.81	0.84	1.28	1.10

In the foregoing analysis, Poisson's ratio μ has been taken equal to zero. This assumption has little effect upon the edge moments, but the positive moments may be affected considerably. However, when the positive moments are known for $\mu = 0$, they can be computed for any other value of μ from the relations:

$$M_x = M_x' + \mu\, M_y' \quad \text{(13a)}$$

and

$$M_y = \mu\, M_x' + M_y' \quad \text{(13b)}$$

in which M_x' and M_y' are the values for $\mu = 0$. For instance, the positive moments in panel (1), Fig. 6, for $\mu = 0.2$, are:

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[8] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, Bureau of Public Roads, U. S. Dept. of Agriculture, Vol. II, No. 1, 1930.

$$M_{x1} = 9.28\,p + 0.2 \times 4.36\,p = 10.15\,p \qquad (14a)$$

and

$$M_{y1} = 0.2 \times 9.28\,p + 4.36\,p = 6.22\,p \qquad (14b)$$

This change in the value of μ produces an increase in the center moments of 9.4% and 42.6%, which indicates that some consideration should be given to this factor. All diagrams in this paper are for $\mu = 0$, except Fig. 8(*b*), which is based on $\mu = 0.15$. If the moments M_x and M_y are known for any value of μ, the values of M_x' and M_y' for $\mu = 0$ can also be determined by Eqs. 13.

CONCLUSION

From the numerical examples that are given in the paper, together with many others that are not recorded herein, the following conclusions appear warranted:

(*a*) The solution of continuous rectangular slabs can be made more accurately by treating the entire panel as a structural unit than by analyzing individual strips as continuous beams.

(*b*) The use of an approximate method which assumes that the edge moments are distributed according to a sine curve, and which provides continuity at the middle of each edge, is feasible and gives maximum positive negative moments with sufficient accuracy for average loading conditions on any panel. Unusual loading conditions have not been considered.

(*c*) The equations that are used can be solved quickly by the method of iteration with the aid of an ordinary slide rule. Various loading conditions can be investigated by changing the constant term in each equation.

(*d*) The only apparent alternative to the foregoing approximate method appears to be in the development of tables of values for positive and negative moments for various panel arrangements and various ratios of live to dead load. Such a compilation of values by an exact method would be a long and laborious procedure; but it is possible with the aid of the curves and equations now available.

ACKNOWLEDGMENT

Most of the numerical data and diagrams in this paper have been taken from a dissertation by C. W. Pan on the "Analysis of Continuous Slabs," submitted in April, 1939, in partial fulfilment of the requirements for the Degree of Doctor of Science at the University of Michigan, Ann Arbor, Mich.

DISCUSSION

Maurice P. van Buren,[9] Assoc. M. Am. Soc. C. E.—A simplified method of analysis for slabs supported on four sides is contained in this interesting paper. By the assumption that edge slopes and support moments are distributed according to a sine curve, formulas for moments at any point are obtained. Unfortunately, the derivations of Eqs. 2 and 5 are not given, and it is not clear what fundamental relationships connecting slope, load, and moment were used by the authors in their analysis, or whether the effect of torsional rotation has been included. It is to be hoped that this omission will be remedied in the closing discussion.

Values given in the text for support moments in slabs fixed at one edge, and the curves in Fig. 8(*a*) for positive moments in simply supported slabs, agree closely with the theoretical values obtained from elastic analysis using Fourier series by H. M. Westergaard,[10] M. Am. Soc. C. E., in 1921; but it should be pointed out that, in the long direction of oblong panels, the moment M_y (Fig. 8(*a*)) at the center of span may be less than the theoretical maximum. For example, when the ratio of spans is 2, maximum moments occur at about the 0.3 point of the span from each end with a depression of the moment curve at midspan. However, in 1926, Dean Westergaard further modified his formulas[11] to agree with tests by taking into account redistribution of stress. As the load on a panel is gradually increased from zero, bending moments will increase more rapidly in the stiffer parts of the slab than in the more flexible parts. This involves a transfer of bending moment from the center strips to the side strips with a consequent flattening out of the curve of moment distribution across the panel width. During this action there also may be a change in the proportion of load carried by each direction of the slab, and also to some extent a transfer of moment from the support to the center of span. Dean Westergaard's conclusions indicated a reduction in maximum bending moments due to redistribution of 28%. From these considerations it follows that, although a sine-curve distribution may portray conditions satisfactorily at the beginning of the loading, it does not represent the true conditions in the later stages and yields moments considerably too high for purposes of design.

As one of the authors of the A. C. I. regulations for slabs supported on four sides,[12] the writer wishes to correct the impression that these provisions are merely a beam strip method in which "the only correction for the slab action is made by an empirical distribution of the load between the two hypothetical beams" (see "Introduction"). A brief description of the basis of the A. C. I. formulas will be given. This method of analysis is devised for practical use, and simplifying approximations and empirical constants are introduced within the limits of required accuracy without sacrificing the rational form of the equations. Under the A. C. I. regulations, consideration is given to the action of the entire panel as a unit, and the proportion of the total panel load to be carried in each direction is determined by the relative stiffness of the two spans. The distances between lines of inflection when uniform load is applied separately to each span form convenient measures of the relative stiffness of the two directions. For a panel $A \times B$, this results in a load distribution formula:

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[9] Cons. Engr. (J. Di Stasio & Co.), New York, N. Y.

[10] "Moments and Stresses in Slabs," by H. M. Westergaard and W. A. Slater, *Proceedings*, A. C. I., 1921, p. 415.

[11] "Formulas for the Design of Rectangular Floor Slabs and the Supporting Girders," by H. M. Westergaard, *loc. cit.*, 1926, p. 26.

[12] "Slabs Supported on Four Sides," by J. Di Stasio and M. P. van Buren, *loc. cit.*, 1936, p. 350.

$$r_A = \frac{1}{1 + \left(\frac{F_A\,A}{F_B\,B}\right)^3} = 1 - r_B \quad\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which r_A and r_B are the proportions of total panel load carried in each direction, and $F_A\,A$ and $F_B\,B$ are distances between lines of inflection as defined above. The factors $F_A\,A$ and $F_B\,B$ have small variation for usual conditions, and the effect of loads on adjacent panels should be neglected in their determination. It is to be noted that Eq. 15 is not the fourth power distribution, based on the deflections of two intersecting center strips commonly used in the "beam strip" methods, but is a third power distribution that implies the equality of deflections in each span at all corresponding points in the panel.

In addition to distributing the total load between the two directions of the panel, the variation in intensity of loading along each span is also provided for. For instance, at the center point of a square symmetrical panel, one half the unit load is carried in each direction, but, at a point near a support, practically all of the unit load goes directly to the support. Load distribution coefficients are therefore modified by equivalent uniform load factors:

$$e_A = \frac{2}{4 - \frac{F_B\,B}{F_A\,A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

and

$$e_B = \frac{2}{4 - \frac{F_A\,A}{F_B\,B}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

These factors vary from $\frac{2}{3}$ for a symmetrical square panel to 1 for a panel in which the rectangularity is 2, indicating a uniformly distributed load that approaches the condition of one-way construction.

All required values for F_A, r_A, e_A, etc, are tabulated for ready reference in the A. C. I. Code, from which equivalent uniform loads $e_A\,r_A\,w$ and $e_B\,r_B\,w$, to be carried in each direction, can be obtained directly. Bending moments are then computed exactly as for one-way construction by loading successive spans in each direction with their equivalent uniform loads. This procedure is theoretically feasible because of the phenomenon of redistribution of stress that results in the curve of bending moments across the width of the panel being so flattened out that designs may be based on the average moment that corresponds to about 72% of the maximum theoretical moment without this provision.

The A. C. I. Code also covers the design of the supporting beams on the statical principle that beams and parallel slab together carry the total load in each direction. Thus, corresponding equivalent uniform unit loads for beam design are $(1 - e_A\, r_A)w$ and $(1 - e_B\, r_B)w$.

An interesting comparison results from an extension of Table 2 to include the authors' values reduced for redistribution of stress (see Table 4). It is to be noted that the average ordinate of a sine curve is about 64% of the maximum. Therefore, a 36% reduction of moments derived by the sine-curve method may be more nearly correct. Both values are shown. The reasonably close agreement between either of these values and the A. C. I. Code is apparent, and affords definite evidence of its accuracy and reliability in the practical design of unequal panels.

TABLE 4.—Edge Moments Redistributed for Stress

Description	M_{12}	M_{23}	M_{45}	M_{56}	M_{14}	M_{25}	M_{36}
Authors' "exact" formulas	20.56	12.14	3.95	5.40	12.17	10.05	4.26
Authors' "exact" formulas; 28% reduction	14.80	8.74	2.84	3.89	8.76	7.23	3.06
Authors' "exact" formulas; 36% reduction	13.16	7.77	2.53	3.46	7.79	6.43	2.73
A. C. I.	13.18	9.20	3.54	2.94	8.18	5.84	2.80

Sine-curve distribution as developed by the authors is a noteworthy contribution to the analysis of continuous slabs supported on four sides. It provides a method by which the moments in a series of unequal panels of varying degrees of rectangularity may be solved with relative simplicity. The discussion of the effect of Poisson's ratio on positive moments is of technical interest, but has limited scope in reinforced-concrete design where the slabs are subject to tension cracking under working loads. If the formulas are modified to include the effect of redistribution of stress, a useful tool will result for checking practical design constants and methods.

Ralph E. Byrne, Jr.,[13] **Jun. Am. Soc. C. E.**—A comparatively simple method of analyzing a very complex problem is presented in this paper. The method is admittedly approximate, and yet, in the particular case presented in the paper, it produces results that are quite satisfactory for design purposes.

Its most striking feature is that the complicated problem of a continuous slab is reduced to the much simpler problem of a continuous beam. Consequently, the various methods of solving continuous beam problems are made available for the solution of continuous slab problems. In the theory of continuous beams, methods of distributing moments,[14] and of distributing angle changes,[15] have been developed primarily to avoid the necessity of solving a large number of simultaneous algebraic equations. The latter method (balancing angle changes) is well adapted to the solution of the slab problem as presented by the authors.

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[13] Mathematical Physicist, Barber-Coleman Co., Rockford, Ill.

[14] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[15] "Analysis of Continuous Frames by Balancing Angle Changes," by L. E. Grinter, *loc. cit.*, Vol. 102 (1937), p. 1020.

Each slab is considered as simply supported and the angles ϕ_s are determined from Fig. 5, or otherwise. At each support in turn, continuity is restored by applying a moment (the same moment to each slab meeting at the support); and the change in angle, θ, for each slab, caused by this moment, is directly proportional to the factor α (Fig. 4) for the corresponding slab. It is assumed that during this process of restoring continuity on one side, the other sides remain simply supported. Consequently, the three remaining sides will be rotated also, the amount of this rotation being equal to θ (the rotation of the side on which continuity has been restored temporarily) multiplied by a carry-over factor. These carry-over factors are seen to be $C_\beta = \frac{\beta}{\alpha}$ and $C_\gamma = \frac{\gamma}{\alpha}$ for the opposite and adjacent sides, respectively, β and γ being given in Fig. 4.

The process to be followed is thus a well-known one. Continuity is first restored at each side of the various slabs. The angles which, as a consequence, must be carried to the other sides of each slab are determined, and again continuity is restored by the balancing process. This sequence of operations is repeated until the results have converged to within the required limits of accuracy. The only difference between the foregoing process and the well-known one for continuous beams is that it is necessary to consider three carry-over factors, instead of one, for each member at each support.

By this process, the angles θ_{ij} (as defined by the authors) are determined for each support of each slab; and now, to determine the moments, four equations of the type of Eq. 8 must be written, one equation for the angle at each side of a slab panel. These equations are then to be inverted to give the moments as functions of the angles. Let the subscripts a and b refer to quantities relating to the sides of the slab panel of lengths a and b respectively; and let subscripts 1 and 2 further distinguish quantities relating to the sides of the same length. Write

$$\psi = \theta - \phi_s \tag{17}$$

With this notation, the aforementioned equations of the type of Eq. 8 become

$$\left.\begin{aligned}
a\,\alpha_a\,M_{a1} + a\,\beta_a\,M_{a2} + b\,\gamma_b\,(M_{b1} + M_{b2}) &= D\,\psi_{a1}\\
a\,\beta_a\,M_{a1} + a\,\alpha_a\,M_{a2} + b\,\gamma_b\,(M_{b1} + M_{b2}) &= D\,\psi_{a2}\\
a\,\gamma_a\,(M_{a1} + M_{a2}) + b\,\alpha_b\,M_{b1} + b\,\beta_b\,M_{b2} &= D\,\psi_{b1}\\
a\,\gamma_a\,(M_{a1} + M_{a2}) + b\,\beta_b\,M_{b1} + b\,\alpha_b\,M_{b2} &= D\,\psi_{b2}
\end{aligned}\right\} \tag{18}$$

Eqs. 18 are easily inverted to give the moments, as follows:

$$\left.\begin{aligned}
M_{a1} &= \frac{D\,(\alpha_b - \beta_b)}{a\,\Delta}\,[A_a\,\psi_{a1} + B_a\,\psi_{a2} + C_a\,(\psi_{b1} + \psi_{b2})]\\
M_{a2} &= \frac{D\,(\alpha_b - \beta_b)}{a\,\Delta}\,[B_a\,\psi_{a1} + A_a\,\psi_{a2} + C_a\,(\psi_{b1} + \psi_{b2})]\\
M_{b1} &= \frac{D\,(\alpha_a - \beta_a)}{b\,\Delta}\,[C_b\,(\psi_{a1} + \psi_{a2}) + A_b\,\psi_{b1} + B_b\,\psi_{b2}]\\
M_{b2} &= \frac{D\,(\alpha_a - \beta_a)}{b\,\Delta}\,[C_b\,(\psi_{a1} + \psi_{a2}) + B_b\,\psi_{b1} + A_b\,\psi_{b2}]
\end{aligned}\right\} \tag{19}$$

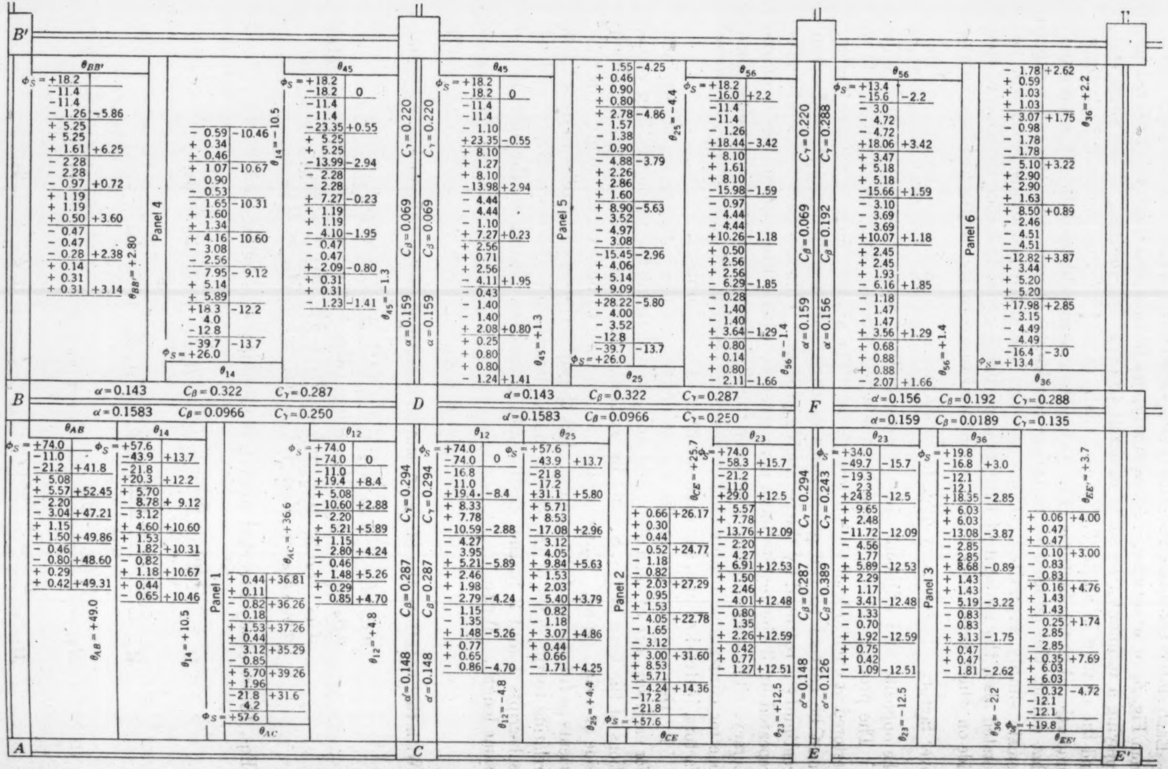
B'
B
A
C
D
E
F
E'
$\theta_{BB'}$
ϕ_S = +18.2
−11.4
−11.4
− 1.26 −5.86
+ 5.25
+ 5.25
+ 1.61 +6.25
− 2.28
− 2.28
− 0.97 +0.72
+ 1.19
+ 1.19
+ 0.50 +3.60
− 0.47
− 0.47
− 0.28 +2.38
+ 0.14
+ 0.31
+ 0.31 +3.14
$\theta_{BB'}$ = +2.80
Panel 4
− 0.59 −10.46
+ 0.34
+ 0.46
+ 1.07 −10.67
− 0.90
− 0.53
− 1.65 −10.31
+ 1.60
+ 1.34
+ 4.16 −10.60
− 3.08
− 2.56
− 7.95 − 9.12
+ 5.14
+ 5.89
+18.3 −12.2
− 4.0
−12.8
−39.7 −13.7
ϕ_S = +26.0
θ_{14}
θ_{14} = −10.5
θ_{45}
ϕ_S = +18.2
−18.2 0
−11.4
−11.4
−23.35 +0.55
+ 5.25
+ 5.25
−13.99 −2.94
− 2.28
− 2.28
+ 7.27 −0.23
+ 1.19
+ 1.19
− 4.10 −1.95
− 0.47
− 0.47
+ 2.09 −0.80
+ 0.31
+ 0.31
− 1.23 −1.41
θ_{45} = −1.3
C_γ = 0.220
C_β = 0.069
α = 0.159
C_γ = 0.220
C_β = 0.069
α = 0.159
θ_{45}
ϕ_S = +18.2
−18.2 0
−11.4
−11.4
− 1.10
+23.35 −0.55
+ 8.10
+ 1.27
+ 8.10
−13.98 +2.94
− 4.44
− 4.44
− 1.10
+ 7.27 +0.23
+ 2.56
+ 0.71
+ 2.56
− 4.11 +1.95
− 0.43
− 1.40
− 1.40
+ 2.08 +0.80
+ 0.25
+ 0.80
+ 0.80
− 1.24 +1.41
θ_{45} = +1.3
Panel 5
− 1.55 −4.25
+ 0.46
+ 0.90
+ 0.80
+ 2.78 −4.86
− 1.57
− 1.38
− 0.90
− 4.88 −3.79
+ 2.26
+ 2.86
+ 1.60
+ 8.90 −5.63
− 3.52
− 4.97
− 3.08
−15.45 −2.96
+ 4.06
+ 5.14
+ 9.09
+28.22 −5.80
− 4.00
− 3.52
−12.8
−39.7 −13.7
ϕ_S = +26.0
θ_{25}
θ_{25} = −4.4
θ_{56}
ϕ_S = +18.2
−16.0 +2.2
−11.4
−11.4
− 1.26
+18.44 −3.42
+ 8.10
+ 1.61
+ 8.10
−15.98 −1.59
− 0.97
− 4.44
− 4.44
+10.26 −1.18
+ 0.50
+ 2.56
+ 2.56
− 6.29 −1.85
− 0.28
− 1.40
− 1.40
+ 3.64 −1.29
+ 0.80
+ 0.14
+ 0.80
− 2.11 −1.66
θ_{56} = −1.4
C_γ = 0.220
C_β = 0.069
α = 0.159
C_γ = 0.288
C_β = 0.192
α = 0.156
θ_{56}
ϕ_S = +13.4
−15.6 −2.2
− 3.0
− 4.72
− 4.72
+18.06 +3.42
+ 3.47
+ 5.18
+ 5.18
−15.66 +1.59
− 3.10
− 3.69
− 3.69
+10.07 +1.18
+ 2.45
+ 2.45
+ 1.93
− 6.16 +1.85
− 1.18
− 1.47
− 1.47
+ 3.56 +1.29
+ 0.68
+ 0.88
+ 0.88
− 2.07 +1.66
θ_{56} = +1.4
Panel 6
− 1.78 +2.62
+ 0.59
+ 1.03
+ 1.03
+ 3.07 +1.75
− 0.98
− 1.78
− 1.78
− 5.10 +3.22
+ 2.90
+ 2.90
+ 1.63
+ 8.50 +0.89
− 2.46
− 4.51
− 4.51
−12.82 +3.87
+ 3.44
+ 5.20
+ 5.20
+17.98 +2.85
− 3.15
− 4.49
− 4.49
−16.4 −3.0
ϕ_S = +13.4
θ_{36}
θ_{36} = +2.2
α = 0.143 C_β = 0.322 C_γ = 0.287
α = 0.1583 C_β = 0.0966 C_γ = 0.250
α = 0.143 C_β = 0.322 C_γ = 0.287
α = 0.1583 C_β = 0.0966 C_γ = 0.250
α = 0.156 C_β = 0.192 C_γ = 0.288
α = 0.159 C_β = 0.0189 C_γ = 0.135
θ_{AB}
ϕ_S = +74.0
−11.0
−21.2 +41.8
+ 5.08
+ 5.57 +52.45
− 2.20
− 3.04 +47.21
+ 1.15
+ 1.50 +49.86
− 0.46
− 0.80 +48.60
+ 0.29
+ 0.42 +49.31
θ_{AB} = +49.0
θ_{14}
ϕ_S = +57.6
−43.9 +13.7
−21.8
+20.3 +12.2
+ 5.70
− 8.78 + 9.12
− 3.12
+ 4.60 +10.60
+ 1.53
− 1.82 +10.31
− 0.82
+ 1.18 +10.67
+ 0.44
− 0.65 +10.46
θ_{14} = +10.5
Panel 1
+ 0.44 +36.81
+ 0.11
− 0.82 +36.26
− 0.18
+ 1.53 +37.26
+ 0.44
− 3.12 +35.29
− 0.85
+ 5.70 +39.26
+ 1.96
−21.8 +31.6
− 4.2
ϕ_S = +57.6
θ_{AC}
θ_{AC} = +36.6
θ_{12}
ϕ_S = +74.0
−74.0 0
−11.0
+19.4 +8.4
+ 5.08
−10.60 +2.88
− 2.20
+ 5.21 +5.89
+ 1.15
− 2.80 +4.24
− 0.46
+ 1.48 +5.26
+ 0.29
− 0.85 +4.70
θ_{12} = +4.8
C_γ = 0.294
C_β = 0.287
α = 0.148
C_γ = 0.294
C_β = 0.287
α = 0.148
θ_{12}
ϕ_S = +74.0
−74.0 0
−16.8
−11.0
+19.4. −8.4
+ 8.33
+ 7.78
−10.59 −2.88
− 4.27
− 3.95
+ 5.21 −5.89
+ 2.46
+ 1.98
− 2.79 −4.24
− 1.15
− 1.35
+ 1.48 −5.26
+ 0.77
+ 0.65
− 0.86 −4.70
θ_{12} = −4.8
θ_{25}
ϕ_S = +57.6
−43.9 +13.7
−21.8
−17.2
+31.1 +5.80
+ 5.71
+ 8.53
−17.08 +2.96
− 3.12
− 4.05
+ 9.84 +5.63
+ 1.53
+ 2.03
− 5.40 +3.79
− 0.82
− 1.18
+ 3.07 +4.86
+ 0.44
+ 0.66
− 1.71 +4.25
θ_{25} = +4.4
Panel 2
+ 0.66 +26.17
+ 0.30
+ 0.44
− 0.52 +24.77
− 1.18
− 0.82
+ 2.03 +27.29
+ 0.95
+ 1.53
− 4.05 +22.78
− 1.65
− 3.12
+ 3.00 +31.60
+ 8.53
+ 5.71
− 4.24 +14.36
−17.2
−21.8
ϕ_S = +57.6
θ_{CE}
θ_{CE} = +25.7
θ_{23}
ϕ_S = +74.0
−58.3 +15.7
−21.2
−11.0
+29.0 +12.5
+ 5.57
+ 7.78
−13.76 +12.09
− 2.20
− 4.27
+ 6.91 +12.53
+ 1.50
+ 2.46
− 4.01 +12.48
− 0.80
− 1.35
+ 2.26 +12.59
+ 0.42
+ 0.77
− 1.27 +12.51
θ_{23} = +12.5
C_γ = 0.294
C_β = 0.287
α = 0.148
C_γ = 0.243
C_β = 0.389
α = 0.126
θ_{23}
ϕ_S = +34.0
−49.7 −15.7
−19.3
− 2.3
+24.8 −12.5
+ 9.65
+ 2.48
−11.72 −12.09
− 4.56
− 1.77
+ 5.89 −12.53
+ 2.29
+ 1.17
− 3.41 −12.48
− 1.33
− 0.70
+ 1.92 −12.59
+ 0.75
+ 0.42
− 1.09 −12.51
θ_{23} = −12.5
Panel 3
θ_{36}
ϕ_S = +19.8
−16.8 +3.0
−12.1
−12.1
+18.35 −2.85
+ 6.03
+ 6.03
−13.08 −3.87
− 2.85
− 2.85
+ 8.68 −0.89
+ 1.43
+ 1.43
− 5.19 −3.22
− 0.83
− 0.83
+ 3.13 −1.75
+ 0.47
+ 0.47
− 1.81 −2.62
θ_{36} = −2.2
+ 0.06 +4.00
+ 0.47
+ 0.47
− 0.10 +3.00
− 0.83
− 0.83
+ 0.16 +4.76
+ 1.43
+ 1.43
− 0.25 +1.74
− 2.85
− 2.85
+ 0.35 +7.69
+ 6.03
+ 6.03
− 0.32 −4.72
−12.1
−12.1
ϕ_S = +19.8
$\theta_{EE'}$
$\theta_{EE'}$ = +3.7

FIG. 9.—DETERMINATION OF THE ANGLES θ BY DISTRIBUTING ANGLE CHANGES

The several quantities used in writing Eqs. 19 are defined as follows:

$$\left.\begin{aligned}
\Delta &= (\alpha_a - \beta_a)(\alpha_b - \beta_b)[(\alpha_a + \beta_a)(\alpha_b + \beta_b) - 4\,\gamma_a\,\gamma_b] \\
A_a &= \alpha_a\,(\alpha_b + \beta_b) - 2\,\gamma_a\,\gamma_b \\
B_a &= -\,[\beta_a\,(\alpha_b + \beta_b) - 2\,\gamma_a\,\gamma_b] \\
C_a &= -\,\gamma_b\,(\alpha_a - \beta_a) \\
A_b &= \alpha_b\,(\alpha_a + \beta_a) - 2\,\gamma_a\,\gamma_b \\
B_b &= -\,[\beta_b\,(\alpha_a + \beta_a) - 2\,\gamma_a\,\gamma_b] \\
C_b &= -\,\gamma_a\,(\alpha_b - \beta_b)
\end{aligned}\right\}\dots\dots(20)$$

It is to be noted that Eqs. 19 are exactly analogous to the slope-deflection equations of beam theory. Having determined the moments from these equations, the accuracy of the entire process may be judged by the agreement of moments at the supports as computed from the angles of the two adjoining slab panels, and by the computed magnitude of moments over supports that have been assumed to be simply supported.

By this method the writer has recomputed the example given by the authors, and the results obtained are in substantial agreement with the authors' results as given in Table 2. The computation of the angles θ by the balancing process is shown in Fig. 9. The computations are so arranged that, for each support, the left-hand column shows the angles for simple support, ϕ_s, each carry-over angle, and each balancing angle; the right-hand column shows the cumulative total after each balancing. The convergence in this example is not rapid; and such cumulative totals not only give a means of judging the degree of convergence, but also, when the balancing process is terminated, they give a means of estimating a better value of the angle θ than the final number in the cumulative total column. Thus, the final angles θ to be used in computing moments were estimated from consideration of the sequence of numbers appearing in the cumulative-totals column.

The computation of moments from the angles thus obtained is a matter of straightforward substitution in Eqs. 19; a convenient tabular form will immediately suggest itself to the computer. The results obtained in this manner are shown in Table 5. The last five moments of this table are computed moments over supports that were assumed to be hinged, and are seen to be small compared to the other moments of the corresponding panels. The remaining moments agree substantially with the authors' results (see Table 2).

TABLE 5.—COMPUTATION OF MOMENTS

Moments	Panels						Mean
	1	2	3	4	5	6	
M_{12}	−20.5	−20.45	....	....	....	....	−20.5
M_{23}	....	−12.42	−13.02	....	....	....	−12.7
M_{45}	....	....	....	− 2.55	−2.72	....	− 2.64
M_{56}	....	....	....	....	−4.53	−4.61	− 4.57
M_{14}	−12.0	....	....	−12.56	....	....	−12.3
M_{25}	....	− 9.94	....	....	−9.80	....	− 9.87
M_{36}	....	....	− 3.78	....	....	−3.79	− 3.79
M_{AB}	+ 0.018	....	....	....	....	....	....
M_{AC}	− 0.017	....	....	....	....	....	....
M_{CE}	....	− 0.175	....	....	....	....	....
$M_{EE'}$	....	....	− 0.01	....	....	....	....
$M_{BB'}$	....	....	....	+ 0.21	....	....	....

Theoretically, it would be possible to apply the method of distribution of moments in a similar manner. However, there are several practical difficulties. The starting point of this process is the determination of "fixed-end moments"—that is, the moment at the center of each edge of each slab when the conditions of fixity are satisfied along these edges. Likewise, "stiffness" and "carry-over" factors must be computed, one edge being rotated through a unit angle while the other edges are held fixed, and the corresponding moments at the centers of the four edges determined. The computation of these quantities for the fixed slab is essentially more difficult than the computation of the corresponding quantities for the simply supported slab. Furthermore, the assumption, in the former case, of a sine distribution of moment along the edges is not as natural an assumption to make as in the latter case. Nevertheless, should these quantities be available for the fixed slabs, the method of distributing moments would be immediately applicable to the problem.

In conclusion, it should be stated that no superiority is claimed for the methods herein suggested over that used by the authors; they are essentially the same. Rather, it has been the purpose of this discussion to point out the analogy between the theory presented in this paper and beam theory, and to suggest other possible methods of attack on the problems that present themselves as a natural consequence of this analogy.

PIERCE P. FURBER,[16] M. AM. SOC. C. E.—Eqs. 13 have been written incorrectly, thus leading to an erroneous conclusion, which is not a little surprising.

Lateral deformation in a body has long been recognized as of opposite sign to the direct strain. Works on elasticity treat the subject in general. Experimental determinations of the ratio of lateral deformation have been made for many materials, by many men. Poisson was among the pioneers; hence, his name has been connected with it.

The theory of elasticity shows that like strains or deformations at right angles are accompanied by lateral deformations of the opposite sign, in each direction. Consequently, the primary strains are reduced by some amount. In a plate or slab under load, therefore, the extensions and compressions are both reduced, to some extent, by the lateral deformations. The true stresses are less, and hence the true moments of resistance in each direction are less than the apparent moments. Eqs. 13 should be written: $M_x = M'_x - \mu M'_y$, etc.

Engineers of varying opinions have engaged in spirited controversy concerning not only the value of Poisson's ratio for reinforced-concrete slabs, but whether any value at all existed for it. Some seemed to believe that the composite plate of concrete and steel constituted an exception to the general theory of elasticity in that respect. However, scientific explanations for that view have been entirely lacking.

The paper contains the first statement the writer has noticed that the effect of lateral deformations is actually to increase the moments (and consequently the stresses) in a slab or plate supported on four sides. As the paper seems not to limit the equations to reinforced concrete, these formulas must be intended to apply as well to other materials.

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L. C. MAUGH,[17] ASSOC. M. AM. SOC. C. E.—In Mr. Van Buren's discussion he mentioned that the fundamental relationships used in deriving the various algebraic expressions, and the numerical values of the slopes and moments, given in the various diagrams, were not stated. These factors will now be considered and the basic principles of the solution given.

The basis of all values is the solution of Lagrange's equation for the flexure of plates, namely

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Any displacement surface, whose values are designated by w, which satisfies this equation will also satisfy the equilibrium and continuity conditions for every element of the slab. It is also assumed that the principle of superposition applies and therefore the displacements can be calculated separately for the applied loads and for the edge moments and then combined to give the total value. The solution of Eq. 21 for the types of loading considered in the paper was made by Levy's method which represents the deflection surface by a Fourier series in the form

$$w = \sum_{n=1}^{\infty} Y(m) \sin \frac{m\,\pi\,x}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which $Y(m)$ is a function of y only. As the application of Levy's method to the solution of slab problems can be found in various books and technical periodicals,[7] it will not be repeated. As stated in the paper this solution enables the slopes and moments to be represented by a sine series which, in most cases, is rapidly convergent. If the edge moments are also represented by a similar sine series as Eq. 4, then an exact solution for the value of these moments can be made as explained in the "Introduction." However, in the approximate solution, the edge moments were represented by the first term of the series so as to simplify the equations. Nevertheless, all displacements, slopes, and moments due to the edge moments were determined by the solution of Eq. 21 and therefore satisfy the shearing, flexural, and torsional resistances of the slab.

After the deflection surface was obtained from Eq. 21, the slopes and moments were obtained from the following equations:

$$\theta_x = \frac{\partial w}{\partial x} \quad \text{and} \quad \theta_y = \frac{\partial w}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

and

$$\left.\begin{aligned} M_x &= -\,D\left(\frac{\partial^2 w}{\partial x^2} + \mu\,\frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\,D\left(\frac{\partial^2 w}{\partial y^2} + \mu\,\frac{\partial^2 w}{\partial x^2}\right) \\ M_{xy} &= -\,D\,(1-\mu)\,\frac{\partial^2 w}{\partial x\,\partial y} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots (24)$$

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[17] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

When Poisson's ratio μ is assumed equal to zero, the value of the moments becomes

$$\left.\begin{aligned} M'_x &= -D\frac{\partial^2 w}{\partial x^2} \\ M'_y &= -D\frac{\partial^2 w}{\partial y^2} \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

which are in the same form as for a beam. By comparing Eqs. 24 and 25, it can be seen that the relations given by Eq. 13 are correct.

In making the calculations, the bending moments are computed at the center of the slab and the middle of the supports although it is known that the absolute maximum values do not necessarily occur at these points. No redistribution of the edge moments was made for the reason that the writer is not at all certain just how this yielding at one point affects the entire slab. When the end connection of a beam yields one knows that the moment at the center increases. In a slab, however, it is doubtful if the moment at the center increases correspondingly until yielding has progressed some distance along the support. As the rotation at the middle of the support increases, there will be some increase in the torsional moment in that vicinity and therefore some increase in the edge moments contiguous to that zone. However, the determination of the amount of this redistribution and its effect upon the ultimate strength of the slab appear to be a matter of judgment and consequently should be made after the theoretical values are obtained. Nevertheless, the writers are indebted to Mr. Van Buren for his discussion of this subject. His recalculation of the edge moments on the basis of some redistribution as tabulated in Table 4 give an interesting comparison with the corresponding values that were computed by the A. C. I. Code. The values computed by the exact formulas would have to be reduced by percentages varying from about 10% to 42% to equal the values obtained by the A. C. I. Code. If a reduction is to be made it should be a constant ratio. The results indicate that the values given by the A. C. I. Code, considering an allowance of about 32% for redistribution, are fairly accurate when the adjacent panels are approximately of the same dimensions.

The use of the slopes as unknowns rather than the moments, as illustrated by the solution submitted by Mr. Byrne, may be advantageous for certain types of problems. For example, if the torsional resistance of the beam is to be included, this factor could be introduced in terms of the rotation angle. For the type of problems considered in the paper, however, the algebraic arrangement that is used was chosen after an investigation of both the moment distribution and slope methods. If Eqs. 11 are written in the following form (p equal to unity)—

$$\left.\begin{aligned} M_{12} &= -25.0 - 0.144\,M_{23} - 0.10\,(M_{14} + M_{25}) \\ M_{23} &= -16.4 - 0.131\,M_{12} - 0.091\,M_{25} - 0.033\,M_{36} \\ M_{45} &= -11.45 - 0.034\,M_{56} - 0.382\,(M_{14} + M_{25}) \\ M_{56} &= -9.15 - 0.031\,M_{45} - 0.351\,M_{25} - 0.265\,M_{36} \\ M_{14} &= -16.0 - 0.167\,M_{12} - 0.066\,M_{45} \\ M_{25} &= -16.0 - 0.167\,(M_{12} + M_{23}) - 0.066\,(M_{45} + M_{56}) \\ M_{36} &= -9.65 - 0.358\,M_{23} - 0.264\,M_{56} \end{aligned}\right\}\ldots(26)$$

—the solution can be made with a slide rule by means of the method of iteration. The rate of convergence is shown by the values for successive cycles as recorded in Table 6(*a*).

TABLE 6.—SOLUTION BY THE METHOD OF ITERATION

Cycle	M_{12}	M_{23}	M_{45}	M_{56}	M_{14}	M_{25}	M_{36}
(*a*) GENERAL CASE							
0	−25.0	−16.4	−11.45	−9.15	−16.0	−16.0	−9.65
1	−19.44	−12.09	+ 3.89	−1.10	−13.02	−10.93	−2.40
2	−20.86	−12.58	− 2.26	−4.61	−12.61	− 9.97	−3.93
3	−20.93	−12.62	− 2.66	−4.53	−12.34	− 9.93	−3.94
4	......	......	− 2.80	−4.52	......	......	......
(*b*) SPECIAL CASE							
0	−12.50	0	0	0	−11.10	0	0
1	−11.39	+ 1.49	+ 4.24	−0.13	− 9.50	+ 1.38	−0.50
2	−11.90	+ 1.44	+ 3.10	−0.45	− 9.30	+ 1.38	−0.40

The values that are given in the second cycle of Table 6(*a*) are sufficiently accurate for all practical purposes, except M_{45} which should be taken to at least three cycles. In making the calculations in this manner, it is often convenient to combine certain equations so as to reduce the number. When different loading conditions are used in the various spans, only the constant term in the equations need be changed. Thus, if only panels 1 and 1′, Fig. 6, are loaded, the values in Table 6(*a*) would be changed to the values given in Table 6(*b*).

For these edge moments, the moments at the center of panels 1 and 4 are

$$\left.\begin{aligned} M_{x1} &= (0.0618)(15)^2\,p - (0.144)(11.9)\,p - (0.123)(9.3)\,p = +\,11.0\,p \\ M_{y1} &= (0.032)(15)^2\,p - (0.1365)(11.9\,p) - (0.0015)(9.3\,p) = +\,5.6\,p \\ M_{x4} &= (2)(0.13)(-\,9.30\,p) - (0.017)(3.10\,p) = -\,2.47\,p \\ M_{y4} &= (2)(0.18)(-\,9.30\,p) + (0.108)(3.10\,p) = -\,3.0\,p \end{aligned}\right\}\;..(27)$$

Combinations of different panel loading and different types of loading can be used in the equations without much difficulty. The most common source of error is likely to be the interchanging of sides *a* and *b* when calculating the moments at the center of the panel.

Mr. Furber has apparently confused action and reaction when considering the sign of Eq. 13. This equation is no different from the usual expression for the two-dimensional state of stress in terms of the corresponding strains. Thus, if f_x, f_y, e_x, and e_y represent the unit stresses and strains acting upon any element in the x and y directions, respectively, the relationship between them is given by the following equations:

$$\left.\begin{aligned} e_x &= \frac{f_x}{E} - \mu\,\frac{f_y}{E} \\ e_y &= -\,\mu\,\frac{f_x}{E} + \frac{f_y}{E} \end{aligned}\right\}\;.......................(28)$$

Solving these equations for f_x and f_y gives

$$\left.\begin{aligned} f_x &= \left(\frac{E}{1-\mu^2}\right)(e_x + \mu\, e_y) \\ f_y &= \left(\frac{E}{1-\mu^2}\right)(e_y + \mu\, e_x) \end{aligned}\right\} \quad \cdots\cdots (29)$$

Therefore, if $\left(\frac{E}{1-\mu^2}\right) e_x$ is represented by f'_x, and $\left(\frac{E}{1-\mu^2}\right) e_y$ is represented by f'_y, the corresponding stresses for a one-dimensional stress system, the equations become

$$\left.\begin{aligned} f_x &= f'_x + \mu\, f'_y \\ f_y &= f'_y + \mu\, f'_x \end{aligned}\right\} \quad \cdots\cdots (30)$$

which can be translated into moments in the usual manner.

In preparing this final discussion the writer deeply regrets that the co-author, C. W. Pan, is unable to participate because of his duties with the Chinese National Government. Present conditions, however, render communication practically impossible.

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TRAFFIC ENGINEERING AS APPLIED TO RURAL HIGHWAYS

By Milton Harris,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Charles E. Conover, Stephen E. Butterfield, Nathan Cherniack, Park H. Martin, and Charles M. Noble.

Synopsis

Highways are built for one purpose only—to carry traffic. In the operation of American rural highway systems two problems have to be faced:

1. To modernize existing roads so that they may meet the demands of efficient transportation.

2. To incorporate in new design those elements that study of existing traffic has taught are efficient factors.

This paper presents various techniques for accomplishing these ends in part.

Introduction

The practice and literature of traffic engineering heretofore have been applied principally to city streets and the environs of urban areas. As a result, investigations have been narrowed to those items that directly affect this type of traffic native to these localities. Corrective measures in general have been confined to erecting signs, parking, placing, and operating traffic signals, channelizing and alleviating conditions that are conducive to high accident frequency at intersections.

The problem of moving rural traffic safely and speedily now demands attention and, in fact, has come to be regarded as of major importance. Its insistence lies in the fact that although the actual number of accidents inside American urban areas is greater, nevertheless by comparison those that occur outside are the more severe and cause a greater number of fatalities.

To fully appreciate the problem that lies before the highway engineer, it is necessary to give some thought to the fundamentals of traffic that produce these troubles. Of two schools of thought, one considers traffic as a homogeneous stream and the other, as made up of individual units to be dealt with singly and individually. Education and enforcement take the latter view,

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but engineering has to consider the former. Both activities may function in unison.

Taken as an individual, the motorist indulges in a variety of antics that express his own personality and training. Even he himself cannot realize why he does certain things in a particular manner. Taken as mass behavior, however, such actions are reasonably capable of being measured and valued. Experience shows that this mass action follows certain definite patterns, as affected by each motorist's individual life and surroundings. In certain localities vast numbers drive to work at 7:45 a.m. Each has to be at work at eight, and each leaves his home at a different minute, yet the mass moves down the street at a particular time of day.

This stream of cars proceeds at a certain speed, as regulated by the individual's sense of caution and by his reaction to the surrounding physical features. Most drivers follow a definite pattern that is as fixed as the time at which this mass movement takes place. Because traffic follows such a pattern to a great extent the traffic engineer can define the pattern's extent and may cause it to change by introducing human and physical factors. The human factors are the police with their regulatory functions and the educators with their teaching propaganda. The physical factors are the surrounding terrain, signs, signals, stripes, islands, and kindred design of the highway structure. Whatever the origin, the least change affects the traffic stream.

Traffic psychology is complex and as yet unsolved; although it may be expected that certain obvious changes will affect the stream flow in certain ways. No one knows what effect certain types of buildings, certain visibility, or certain other contiguous traffic streams will have on a particular flow of traffic. This type of research should be undertaken in the near future. It seems strange that so much accuracy in detail of design of highways should obtain, whereas so little is known about the element that will use the road.

By analyzing traffic action, traffic behavior can be determined and evaluated. Roads should be built accordingly; hence, it seems reasonable to expect a road design that will accommodate that behavior which is normal to those who drive with prudence.

If all these deductions are valid, they serve as a basis for further development of the thought that engineering principles may be applied to traffic flow and traffic trouble as well as to the analysis of liquid flow or liquid disturbances. Underlying all, there seems to be a pattern. It is the engineer's problem to identify this pattern and, if an adverse one, to effect a change by altering either the physical structure or the human regulation and education of the traffic stream. Doubtless, driving habits can be changed by education and by enforcement, but more probably can be accomplished by natural regulation resulting from properly designed highways which accommodate themselves to reasonable existing traffic patterns, aided at proper points by efficient regulation rather than by severe enforcement.

In applying traffic engineering to rural highways the most outstanding feature that needs correcting is the present structure, which must be made safe for existing traffic. The crying need is to reduce the large mortality on streets and highways, yet provide the maximum mobility possible for users.

Next in order, it is essential to provide designers with traffic data so that future roads will be the most efficient obtainable with the money available. These two projects should occupy the immediate attention of traffic engineers until the hazards have been brought under control. After that, other needs of traffic should be subjected to engineering examination in the order of their importance. Among the problems requiring consideration are congestion, pedestrian control, freeways, allocation of tax moneys, roadside development, integration of transportation facilities, and many other worth-while projects.

This paper will be limited to two phases of the traffic engineering problem: (1) Modernization of present facilities by investigating hazardous locations; and (2) utilization of traffic data for design.

POINTS OF HAZARD

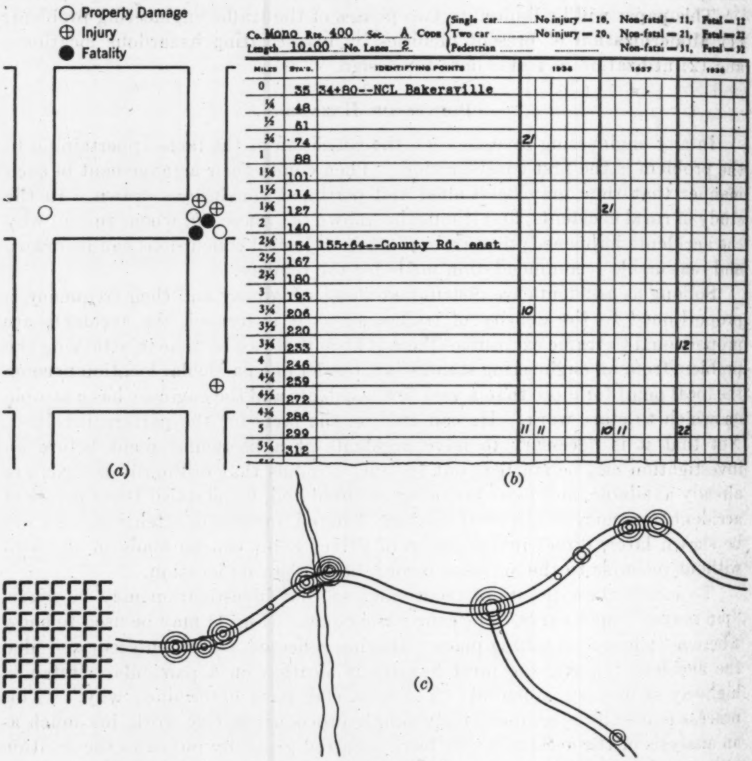
In any engineering investigation the collection of the facts appertaining to the problem is the first consideration. Then comes their arrangement in such manner that they may be studied and pertinent conclusions drawn. In the study of rural accidents, first it must be known (*a*) where, (*b*) when, and (*c*) why the accidents happened; then from these facts logical conclusions can be drawn and reasonable recommendation made for the remedy.

So long as accidents are distributed along a highway and their frequency is proportioned to the density of traffic—or as it is termed, the accidents are proportional to traffic exposure—there is little that can be done in attacking the problem from an engineering standpoint; but let one particular location become the focal point for more than a very few accidents and the engineer has a sample on which to start work. He can analyze the facts for the pattern involved. Not that it is necessary to have accidents at a particular point before an investigation can be started; but the fact remains that ordinarily records are already available, and more are being accumulated, to establish these points of accident frequency as the localities that demand immediate attention. As will be shown later, direct investigation of driver habit can be made in the field without recourse to the accident record, other than its location.

To locate these points of recurrence, so that investigation may be made, "pin maps," "spot cards," or "concentric circles" (Fig. 1) may be used to show where a "pile up" is taking place. Having collected, coded, entered, and filed the accident reports, the most hazardous location on a particular stretch of highway is at once apparent. The next step is to determine "why." This process is usually a comparatively simple procedure in city work, inasmuch as an analysis of the accidents that have occurred generally indicates the solution at once. In rural investigations the causes are more obscure; they are numerous and often are not mentioned in the reports because they were not apparent to the patrol officer at the time. For this reason a different technique is suggested to overcome any lack of information.

This technique is based on the premise that the accident reports reflect certain bad driving habits on the part of some motorists using that particular highway. Due to the changeability of such habits as affected by external influences, it necessarily follows that no two points on a highway will have the same effect. Hence, the investigation must be confined to a relatively small portion or area if the reason is to be determined for a series of accidents at that point.

To indicate those bad driving habits, the accident reports themselves are analyzed for all their elements. Usually one or more outstanding contributory causes motivate the underlying pattern. These are used as indicators with which the traffic engineer arms himself before going out into the field to ascertain the proportion of the traffic that indulges in questionable practices.


○ Property Damage
⊕ Injury
● Fatality
(a)
Co. Mono Rte. 400 Sec. A Code
Length 10.00 No. Lanes 2
Single car .... No injury — 10, Non-fatal — 11, Fatal — 12
Two car .... No injury — 20, Non-fatal — 21, Fatal — 22
Pedestrian .... Non-fatal — 31, Fatal — 32
MILES
STA'S.
IDENTIFYING POINTS
1936
1937
1938
34+80--NCL Bakersville
155+64--County Rd. east
21
21
10
12
11
11
10
11
22
(b)
(c)

(a) Pin Map (b) Spot Card (c) Concentric Circles

FIG. 1.—ACCIDENT INVENTORY

He must use his ingenuity to devise tests to determine these causes. His tests must be extensive and accurate enough to give him results that are trustworthy, because from his results the engineer must formulate a remedy that will exclude habits conducive to accidents, without at the same time introducing other and more severe hazards. It is true that any change may introduce an element giving rise to another habit which in turn may produce accidents, but these accidents should be of a different type. Sometimes it may even be advisable to deliberately introduce into the highway structure a physical change that will convert one type of accident into another less dangerous. For example, a division strip built into a four-lane highway should eliminate head-on accidents, or medial friction, which in rural sections ordinarily results in many fatalities because of the high speeds involved; but a preponderance of rear-end accidents may then occur. However, these are not usually so severe as the head-on type; and by further application of investigative technique, remedies for the rear-end type will be found.

For ease in analyzing accident report data it is customary to list on a summary form the various factors that have been reported for each accident, arranged across the page, one accident to the line, as shown in Fig. 2. (The notation for the driver in the third item as "HBD" is code for "Had been drinking".) By adding up all factors and expressing them as percentages, their relation to each other is noted as one element of the problem, and their relation to state-wide statistics gives immediate indication as to what factors of the problem are worthy of investigation as indicated in Table 1, which relates to the same location as Fig. 2. In fact, comparison of factors, either within the problem or externally with the state-wide data, indicates the accident pattern in many cases. In the example cited, speed is the outstanding contributory cause; hence this can be taken as the indicator for further study in the field.

TABLE 1.—PERCENTAGE ANALYSIS OF ACCIDENT DATA
Route Mono-400-A; Curve 5.2 Miles North of Bakersville

Item	For This Location		State-wide percentage
	No. of Accidents	%	
Daylight	2	40.0	43.63
Darkness	3	60.0	55.85
	5		
Clear	2	40.0	....
Cloudy	2	40.0	....
Rain	1	20.0	....
	5		
Fatal	1	20.0	....
Non-fatal	3	60.0	....
Property damage	1	20.0	....
	5		
Single car	4	80.0	29.61
Two or more cars	1	20.0	70.39
Pedestrian	0	0.0	....
	5		
Dry pavement	3	60.0	....
Wet pavement	2	40.0	....
	5		
Speed	5	50.0	16.43
Wrong lane, not passing	1	10.0	11.75
Soft shoulders	1	10.0	0.17
Out of control	1	10.0	....
Had been drinking	1	10.0	13.26
Blinded by lights	1	10.0	2.14
	10[a]		
Two Lanes:			
Passenger cars (single-car accident)	4	80.0	26.77
Passenger car vs. light truck	1	20.0	4.84
	5		

[a] More than one contributing factor may be assigned to accidents.

Checking the field habits against the accident summary analysis is an activity of rural traffic engineering that is in its infancy. It is comparable to devising surveying methods to obtain geographical data for use in design. In

This Summary Covers — Mono-400-A
Curve 5.2 miles North of Bakersville

Motor Vehicl

					TYPES						PHYSICAL CONDITION OF			PHYSICAL FEATURES OF HIGHWAY				TWO-CAR ACCIDENTS COURSE OF VEHICLES				
					INJURY			ELEMENTS														
DATE	LOCATION OF ACCIDENT	DAYLIGHT	DARKNESS	WEATHER	FATAL	NON-FATAL	PROPERTY DAMAGE ONLY	SINGLE CAR	TWO OR MORE CARS	PEDESTRIAN	DRIVER	VEHICLE	PEDESTRIAN	NO. OF LANES	ALIGNMENT	GRADE	CONDITION	APPROACHING	OVERTAKING	INTERSECTION	SPEED	FOLLOWING TOO CLOSELY
3/20/36	Station 308+40	X		Clear		X		X						2	Curve	Level	Dry				X	
9/ 2/36	Station 311+00	X		Cloudy		X		X						2	Curve	Level	Dry				X	
4/ 4/37	Station 307+90		X	Cloudy			X	X			A HBD			2	Curve	Level	Wet				X	
10/28/37	Station 306+50		X	Clear		X		X						2	Curve	Level	Dry				X	
2/22/38	Station 309+85		X	Rain	X				X		B=Blinded by lights			2	Curve	Level	Wet				X	

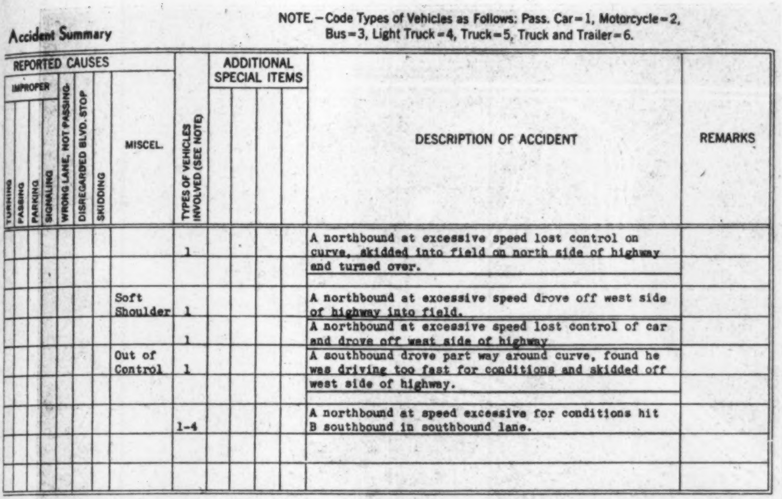
FIG. 2.—TABULATION

fact, it is actually making a traffic survey for use in designing a change in traffic behavior. Of many methods, one is shown in Fig. 3, by photography.

In many cases, a field check of those driver habits, suspected of contributing to the hazard of an accident-prone location, will reveal that the "indicators" are erroneous. In this event, a careful study must be made of the traffic flow and every phase analyzed by partial counts until the crux of the situation is found. Many hours of careful scrutiny may be required. It may also be necessary to return many times to observe traffic at various stages of density as well as under varying conditions of light and weather. It is entirely possible to ascertain and isolate those malign traffic habits from observation alone without prior recourse to an accident summary analysis, though, of course, the analysis sheet is a great help as a starter. Sometimes information secured from local residents or from a study of marks caused by accidents is useful in picking out leads that might indicate the patterns sought. Local police or highway officers often are familiar with the data sought and can be of great help if approached in a diplomatic manner.

The next step is to assemble and analyze the field data, and from them draw whatever conclusion is justified. Most of them may be shown in comprehensive form by means of charts or curves, which are valuable in studying the significance of the facts recorded and also useful for inclusion in a report on the problem (Fig. 4). It is advisable to completely analyze all data taken, because in them may lie the answer to a question that is formed in the mind of a person reading the report, which question eventually would have to be answered.

Applying a physical remedy to a traffic danger demands wide knowledge of highway construction procedure and costs. Obviously, it also requires an expert knowledge of the effect that the new construction will have on the traffic



Accident Summary

NOTE.—Code Types of Vehicles as Follows: Pass. Car=1, Motorcycle=2, Bus=3, Light Truck=4, Truck=5, Truck and Trailer=6.

REPORTED CAUSES								TYPES OF VEHICLES INVOLVED (SEE NOTE)	ADDITIONAL SPECIAL ITEMS				DESCRIPTION OF ACCIDENT	REMARKS
IMPROPER														
TURNING	PASSING	PARKING	SIGNALING	WRONG LANE, NOT PASSING	DISREGARDED BLVD. STOP	SKIDDING	MISCEL.							
								1					A northbound at excessive speed lost control on curve, skidded into field on north side of highway and turned over.	
							Soft Shoulder	1					A northbound at excessive speed drove off west side of highway into field.	
								1					A northbound at excessive speed lost control of car and drove off west side of highway	
							Out of Control	1					A southbound drove part way around curve, found he was driving too fast for conditions and skidded off west side of highway.	
								1-4					A northbound at speed excessive for conditions hit B southbound in southbound lane.	

of Accident Data

stream. Even solutions that seem obvious may produce changes in traffic habits that react unfavorably. An example is the use of three-lane pavements over vertical curves with insufficient sight distance.

The physical remedies devised so far might be classified as changes in the geometric design to accommodate prevailing traffic habits, as follows:

(1) Separation of parallel lanes of traffic;
(2) Separation of intersecting lanes of traffic;
(3) Separation of traffic into lanes of varying speed;
(4) Isolation of various turning movements by means of channelizing islands;
(5) Control of movements by means of signs, signals, lighting, and stripes;
(6) Parking provisions.

Other remedies made by the application of human forces may be effected by means of supervision, enforcement, and education. Application of these remedies is within the realm of traffic engineering, but presupposes a knowledge of traffic police methods, modern educational methods, and public relationship. Their use must be predicated on a complete analysis that indicates a stronger probability of success by such measures than by others within the means of the traffic engineer. He is not limited to physical changes. Some have gone so far as to suggest the combining of the administration of all remedial functions into a highway "operating" department.

In applying remedies to traffic conditions, the item of economics is of direct concern to the traffic engineer. Remedial measures may well be promulgated by any layman but their justification often will not "hold water" from the standpoint of money required for their accomplishment. Much research must be conducted before the traffic engineer can evaluate such factors as fatalities

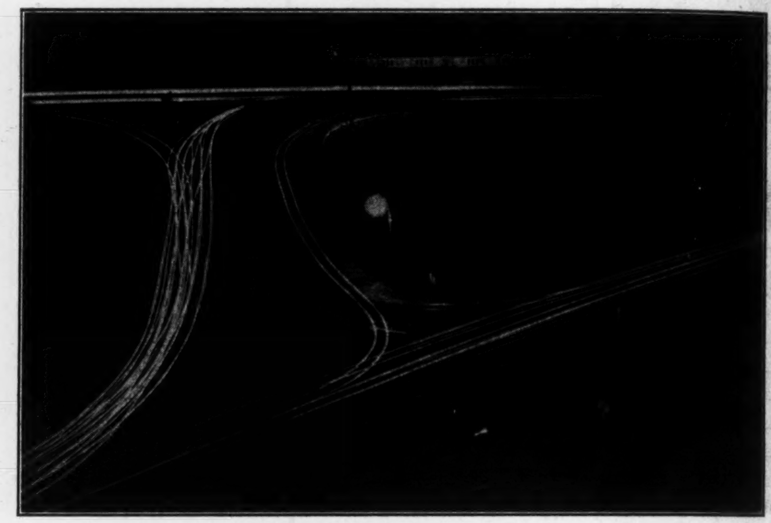


FIG. 3.—TRAFFIC MOVEMENTS TRACED BY PHOTOGRAPHING MOVING HEADLIGHTS. NOTE ONE WRONG ROUTE NEAR CENTRAL LIGHT

and personal injuries, loss of time caused by congestion, cost of car operation when used for pleasure, and others. The answer may lie equally in either engineering or policing, and the final solution may be determined by the relative economies possible. The evaluation of death from the standpoint of insurance risk does not seem to cover the situation except when used on a comparative basis, but the mere totals of deaths or permanent disabilities act as a most potent weapon when waved before the noses of those who must approve the appropriation of funds to correct a hazardous condition.

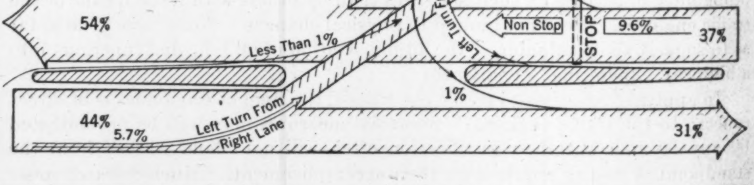

19%
Less Than 1%
15%
From Right Lane
Left Turn
1%
Non Stop
STOP
9.6%
37%
54%
Less Than 1%
1%
44%
5.7%
Left Turn From Right Lane
31%

FIG. 4.—GRAPHIC TRAFFIC DATA FOR ONE INTERSECTION

To show the application to practical problems, a few cases will be analyzed in detail.

CASE NO. 1.—PROBLEM OF DANGEROUS CURVE

It will be presumed that a spot card such as Fig. 1(*b*) has been examined, and the location at mile 5 appears to present a hazard that demands attention in relation to other points of accident occurrence on this highway. A perusal of the individual accident reports is made and an accident summary sheet prepared, as shown in Fig. 2. The contributing factors have been summed up and their percentages calculated for purposes of comparison (Table 1) including a partial comparison with state-wide totals.

Those items that exhibit a tendency to depart from the normal expectancy are immediately used as possible indicators—in this case "single cars", and "speed." From knowledge gained through experience and a study of statistics, it is known that in general a preponderance of single-car accidents points either to a road fault or to drunken driving. Speed and road conditions can be checked in the field.

A collision diagram or pictorial representation of the course of vehicles prior to collision (Fig. 5) is a great help in analysis, especially when used with the corresponding summary sheet (Fig. 2). Notations are the same in both. It will be noted that each accident has happened on the outside of the curve, which important fact will find its place in the complete analysis as shown subsequently.

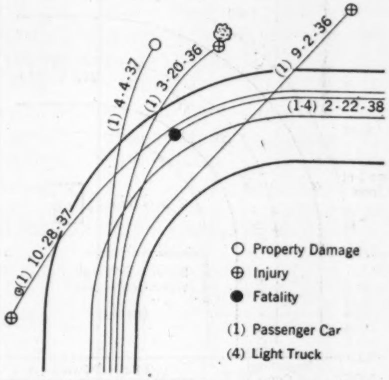


FIG. 5.—COLLISION DIAGRAM, FOR DATA OF FIG. 2 (ILLUSTRATIVE CASE 1)

In this problem the field work consisted of making a site plan of the area (sometimes known as a "condition diagram"); then a survey of the highway itself by means of a five-line profile; and finally a series of speed checks to determine the prevailing speeds. The field results are shown in Fig. 6. In the results of one of the speed checks, plotted as a mass diagram (Fig. 6(*c*)), the curve breaks sharply at approximately 85%, indicating that the increments of speeds up to that point have been used by fairly uniform numbers of drivers; but that above this point, there is a rapid falling off. The "break" of the curve has been designated as the "critical speed" and indicates the top speed that the majority of motorists consider safe for that particular spot under the conditions prevailing at the time of test. The entire curve represents the pattern of drivers' habits, whereas the critical speed represents their habit of top safe speed. Obviously, those who exceed the critical speed are the minority, and they are always present no matter how high the critical speed of the sane drivers. A highway capable of accommodating 100% of the driving range could not be financed practicably even if it were otherwise attainable.

Having determined the prevailing speed, the next logical step is to ascertain if the conditions obtaining are safe to accommodate these speeds. It is noted that the speed of approach to the curve is approximately 50 miles per hr. From Fig. 6(*b*) the superelevation is shown to be only 0.1 ft. One formula for centrifugal motion gives the relationship

$$S + f = \frac{0.067\, V^2}{r} \qquad (1)$$

in which: S = rate of superelevation, as a decimal; f = coefficient of friction (0.16); V = velocity in miles per hour (50 miles per hr); and r = radius of curve in feet (500 ft).

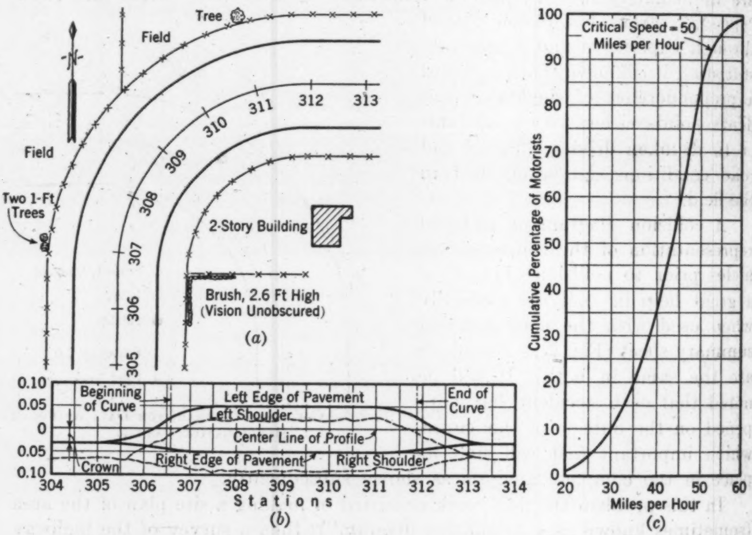


FIG. 6.—SUPPLEMENTARY DATA, INCLUDING (*a*) SITE PLAN, (*b*) FIVE-LINE PROFILE, (*c*) SPEED CHECKS

Applying Eq. 1, it is found that S should be 0.175 as compared with 0.005 obtaining on the ground. This verifies the suspicion that speed too great for conditions prevails at this curve and accounts for the large number of accidents that took a course to the outside of the curve. Even the two-car accident occurred in the outside lane and was evidently caused by the northbound car traveling at too great a speed to keep within its own lane. The single-car accidents occurred on the outside part for the same reason, even though one accident showed drinking as a contributing cause.

Taking the actual $S = 0.005$ and solving Eq. 1 gives a safe $V = 35$ miles per hr satisfied by only 19% of the drivers. One may well wonder what happens to the 81% traveling at a speed in excess of that which is theoretically safe. Examination of the traffic habits on the ground showed that a considerable number of drivers applied their brakes upon reaching this curve and in some cases the rear wheels chattered and skidded sideways as the driver attempted to bring the vehicle under control. A test car was able to negotiate the curve at the top speed of 37 miles per hr without undue sway, which checks very closely with the speed by Eq. 1.

The apparent remedy in this case is to change the superelevation. Obviously, a superelevation of 0.175 is too great to accommodate the slow traffic. In addition, weather conditions, types of vehicles, and their loads may be so affected by such a superelevation as to become a hazard in turn. In practice a superelevation of 0.1 ft per ft of width ($S = 0.1$) is about the maximum that will safely provide for all types of traffic. Using this value and solving for R, the problem resolves itself into relocating the curve to a radius of 644 ft with a maximum superelevation of 0.1 ft per ft of width.

As the design speed for the new curve, the critical speed that obtains at the approach was selected; but there is likely to be an increase in this after the new construction is put into operation. If the curve were to be much flattened and the approach environment changed so that traffic probably would increase its speed at this point, then it would be in order to select a higher design speed than is here indicated. Speed checks taken at comparable locations that have been improved to the proposed standards will often furnish reliable data upon which to base judgment as to proper design speed.

TABLE 2.—SUMMARY OF ACCIDENT DATA FOR CASE 2

Description	For This Location		State-wide per-cent-age	Description	For This Location		State-wide per-cent-age
	No. of Accidents	%			No. of Accidents	%	
			(a) General Data				
Single car	29	19	32.27	Non-fatal	77	44	....
Two or more cars	121	81	67.73	Property damage	86	49	....
Pedestrian	25	17	6.34	Daylight	70	40	45.27
Fatal	12	7	....	Darkness	105	60	54.56
			(b) Contributing Causes[a]				
Driver Condition:				Roadway Condition:			
Had been drinking or drunk	42	17	11.40	Slippery, wet	15	6	3.56
Defective eyes	3	1	....	Impaired sight distance	2	1	4.63
Physical deformity	3	1	....	Obstruction in roadway	1	1	
Asleep	11	4	2.31	Construction under way	3	1	
Fainted	1	1	....				
Blinded by lights	3	1	....	Legal Violations:			
Mental case	2	1	....	Speed too great for conditions	26	11	16.41
				Following too closely	21	8	3.23
Vehicle Condition:				Improper turning	23	9	5.00
Faulty brakes	5	2	1.61	Improper passing	15	6	6.91
Faulty tires	1	1	2.12	Improper parking	5	2	....
Broken wheel	1	1	....	Wrong lane, not passing	35	14	10.90
Faulty windshield	1	1	....	Disregarded boulevard stop	6	2	....
Broken trailer hook	2	1	....	Skidding	11	4	....
Faulty steering mechanism	1	1	....	Violated right of way	6	2	....

[a] In terms of number of times reported; possibly several different causes for one accident.

CASE NO. 2.—ZONE INVOLVING ACCIDENTS

For a section several miles long, the spot cards indicated a series of accidents greater than adjacent areas on the same route, yet with no particular concentration. The summary analysis sheet (Table 2) indicated that "Had Been Drinking" was the cause that appeared most frequently. Drinking habits are scarcely capable of being analyzed by means of a traffic survey, hence recourse was made to another angle of approach. In Fig. 7 the accidents are plotted according to the hour of occurrence. For comparison, the traffic flow is also shown.

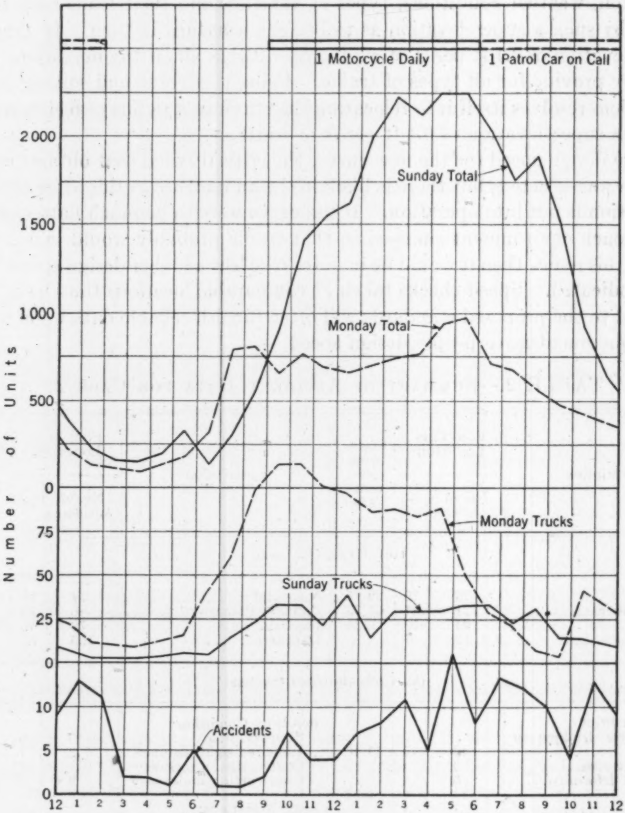


FIG. 7.—GRAPHIC DISTRIBUTION OF TRAFFIC AND ACCIDENTS, CASE 2

From the law of averages, it is reasonable to predict that more accidents will occur during the peak hours of traffic than at other hours, because of the exposure of a greater number of vehicles. The curve shows this to be the case, as between 4 p.m. and 6 p.m. there was a peak in the hourly density curve and a peak in the accident curve. However, it is to be noted that there was another peak in the accident curve at 1 a.m., or when the traffic curve was descending into a trough of minimum density. Obviously a preponderance of drunken-driving accidents occurred at this time in the morning. This was proved further by segregating those accidents that involved drinking.

The remedy in this case lies with the enforcement arm. Further inquiry brought to light the fact that the motor patrol coverage during the peak hour of accidents was only partial (Fig. 7), particularly during the secondary peak at 1 a.m. The coverage on this section of highway therefore should be changed or increased so that adequate patrol would be assured during peak accident hours.

CASE NO. 3.—PRESCRIBING FOR A DANGEROUS ROUTE

Still another case that often arises is partially summarized in Table 3. It will be noted here that 64% of the accidents are rear-end type and 57% of the total number are between passenger cars and trucks. These are the outstanding indicators of this case.

TABLE 3.—SUMMARY OF ACCIDENT DATA FOR CASE 3

Description	For This Location		State-wide percentage	Description	For This Location		State-wide percentage
	No. of accidents	%			No. of accidents	%	
Two or More Cars:				Type:			
Passenger car vs. passenger car	31	41	64.87	Intersection	0	0	29.56
Passenger car vs. truck	43	57	15.18	Approaching	25	33	42.88
Truck vs. truck	2	2	1.23	Overtaking	51	67	27.56
Single Car:				Overtaking Breakdown:			
Passenger	14	70	83.80	Miscellaneous (parked car, etc.)	5	9	29.61
Truck	6	30	4.26	Rear end	32	64	44.87
				Sideswipe	14	27	25.52

A short field observation established the fact that this section of highway is a two-lane road with sustained 6% grades, which slow truck traffic down to a comparatively low speed. The passenger traffic is forced to adopt this low speed for long distances because of the lack of safe passing sight distance. The layout of this section is shown in Fig. 1(*c*) with the position of accidents spotted for reference. It is noted at once that the accidents are "piled up" in those sections that are on curves, with a much smaller number on the tangents.

These data suggest that possibly a group of passenger cars are forced to stay behind a slow-moving truck until a driver becomes impatient enough to attempt to pass even without sufficient sight distance. Proceeding on this theory, it appears that possibly the differential in speed between passenger cars and trucks may account for the high percentage of rear-end accidents. This is borne out further by a study of the relative hourly density of trucks and passenger cars (Fig. 8). Therefore, the main line of investigation might well proceed along the line of ascertaining the relation of speeds between trucks and passenger cars and the effect of congestion on this factor.

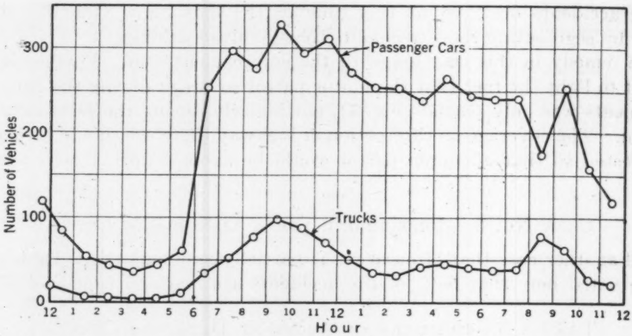


FIG. 8.—PLOT OF TRAFFIC COUNT, CASE 3

To determine the effect of the highway structure and surrounding terrain on the speed of traffic, it is first in order to collect speed data throughout the section, taken close enough together to show the variation in speed, but not taken where congestion exists. The speed, as shown by free-flowing traffic, is the criterion sought. Thus the critical speeds may be plotted on a profile (Fig. 9) and a "virtual speed profile" constructed.

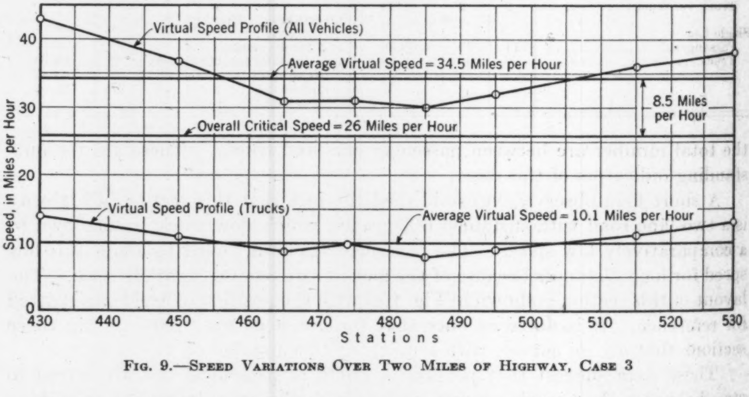


FIG. 9.—SPEED VARIATIONS OVER TWO MILES OF HIGHWAY, CASE 3

The next step is to ascertain the average "over-all" speed, which is the normal speed over this piece of road including time lost by congestion. By plotting a sufficient number of points the usual speed curve is obtained from which a critical speed of 26 miles per hr is selected as the representative safe speed, including time lost by congestion for this section. This has been plotted on the virtual speed profile and indicates that there is a loss of 8.5 miles per hr from the average virtual speed.

A study of the profile indicates those points that will bear further investigation in order that methods may be devised to raise the over-all speed to approach that of the virtual speed. This part of the study consists in the main of separating the slow-speed vehicles from the fast by either additional lanes or a separate truck road. The preponderance of rear-end accidents is added incentive toward separation based on speed; hence, in this case uniformity of speed and safety go "hand in hand."

The average loss of time over this two-mile section was calculated from the data to be 1.08 min or a total over a year's time of 2,222,000 min. This fact has an economic use in selecting the type of design used in the remedy. Various values per minute of lost time have been computed; for example, $0.0075 for passenger cars and $0.0147 for an average truck, in one instance,[2] and $0.01 for passenger cars and $0.0245 for heavy trucks, in another.[3] Using for this example the value of one cent per lost minute, the total loss capitalized at 5% would possibly warrant the expenditure of $444,400 for improvement. The proper value to be assigned to the saving of a minute of time is a matter to be decided on in each particular case.

TRAFFIC DATA FOR DESIGN

Design might be briefly described as the accommodation of structure to facts. Therefore, it is logical to presume that all facts are at hand prior to design. Heretofore only scant attention has been paid to traffic facts, as the highway engineer has been engrossed in the immediate problem of providing smooth, dustless roads at a rate consistent with the availability of money. The demand has been ever greater than the supply of funds. Only in late years has he paused for an instant to realize that the very element of his transportation problem—traffic—for which he has been building roads has not only kept up with the rate of production but has so far exceeded it as to create a problem of its own. Fundamental to design is knowledge of the traffic that will use the structure; therefore all traffic facts possible should be gathered for use in design.

Geographical facts are provided by the survey parties, giving the contour of the ground over which the line is laid, the drainage scheme, topographical data, and soil conditions. The laboratory provides facts concerning the materials—their strength, life, and other physical properties. Legal advisers and right-of-way agents determine facts affecting property rights. All these data combined with the engineer's experience form the foundation for his design of a structure to carry the vehicles of motor transportation; but what of the facts surrounding the use of those vehicles on the completed highway?

Securing and analyzing these facts are within the province of the traffic engineer. Inasmuch as this phase of highway engineering has but recently received recognition, traffic data and their relation to design are still in the first stages of evolution. As more thought is given to this subject, so the field will expand and more and better information will be demanded and obtained; and its analysis will be more definite. Those facts that are readily available at present and have importance in design will now be discussed.

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[2] "Highway Economics," by Sigvald Johannesson, M. Am. Soc. C. E., 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1931.

[3] "The Economics of Highway Planning," by C. B. McCullough, M. Am. Soc. C. E., and John Beakey, *Technical Bulletin No. 7*, Oregon State Highway Dept.

A volume count probably stands first in importance, as the number of vehicles that pass a given point in a certain length of time determines not only the economics of the improvement but the width of lanes and shoulders, the number of lanes and the character of the intersections. A volume count also covers the type of vehicle, the number of passengers—in short, the numerical flow with or without the relative percentages of predetermined subdivisions of type. It is important when making a volume count to have definite knowledge of the use to which that count is to be put in order to gather all necessary data and at the same time to avoid the collection of a mass of information that may be of no value.

The classification of vehicles is of vital importance to the designer if he is to correlate his design to the traffic to be borne. The major classes are usually passenger vehicles and trucks. Each has different elements to be taken into consideration. The ranges of speed are usually different. Their radii of turning are different as well as their wheel loads. Each element has a bearing on design features and the proportion of each classification will affect the design as a whole. Thus, a highway designed to carry trucks alone might conceivably be entirely different from one designed for passenger automobiles. Combine the two classes of vehicles in varying proportions and the design must accommodate the salient features of each classification.

Other types of vehicles would be handled in a similar manner, although their number is usually so small as to be of minor importance in design. The pedestrian, however, although not a vehicle, is a component part of the traffic stream whenever his actions or presence results in conflict. Rural areas are generally free of pedestrian problems except on the outskirts of cities; there it is often necessary to make a pedestrian, as well as a vehicle, count.

For the purpose of design, two elements of a traffic volume count are important. The first is total volume and the second is the peak-hour volume. Total volume for definite periods, such as 24 hr, per month, or yearly, enters into economic calculations. Fluctuations due to season activities sometimes may be handled best on a total monthly basis, especially for comparisons, whereas daily variations have assumed a variety of comparative volumes, based on 24 hr, 16 hr, daily average, daily weekday average, 6 a.m. to 6 p.m. average, and so on, according to the dictates of various engineers' policies. For mutual comparison of these various designated periods it is necessary to know the key hourly percentages to be applied.

The peak-hour traffic is usually the criterion for which the designer plans. By "peak hour" is meant that hour in the daily traffic pattern which most frequently carries the maximum volume during the day. It is the maximum density to be normally expected, yet does not represent unusual conditions prevalent only a few days out of the year—such as traffic for a football game or traffic temporarily detoured over the road in question. Peak-hour density should hold somewhat the same position in traffic engineering when applied to design as maximum stress does in structural engineering.

Successful design must take care of future as well as present conditions. Volumes and densities will change and the designer is obliged to forecast these changes as best he can. Projecting past experience ahead from present knowledge is well established procedure but needs careful manipulation.

In forecasting traffic trends, a straight projection often will lead to fantastic figures. An analysis of past trends will disclose many factors, apparently foreign to traffic, exerting an influence that causes the traffic curve to bend into a form not consistent with past records. Population trends, economic conditions, migration, and even weather conditions have an effect on traffic. In particular locations, public pressure for new roads, improvement of old roads, new industries, and better living facilities, as well as many other causes, are reflected in the traffic curve. For design purposes, the future peak-hour load should be given a great deal of thought and its selection should reflect not only the past trend but also all possible factors that can be estimated as having any effect in the future.

A corollary to volume count, and of importance to the designer, is a record of turning movement. Naturally applicable only at grade intersections, the percentage of total volume that leaves, enters, or crosses the main traffic stream is a definite index of turbulence within the stream. The larger the turning movement, for example, the greater must be the provisions of design to accommodate this factor with as little disturbance of the other traffic units as possible.

Many devices for accomplishing this have been designed and constructed. They range from simple traffic stripe markings to complicated "clover-leaf" intersections. The underlying factors that determine the degree to which an intersection should be improved rest on the total yearly volume for economic reasons; and on the turning and crossing movements, particularly during the peak hour, for traffic reasons. Delays also enter the study of intersection design and are capable of being determined and evaluated.

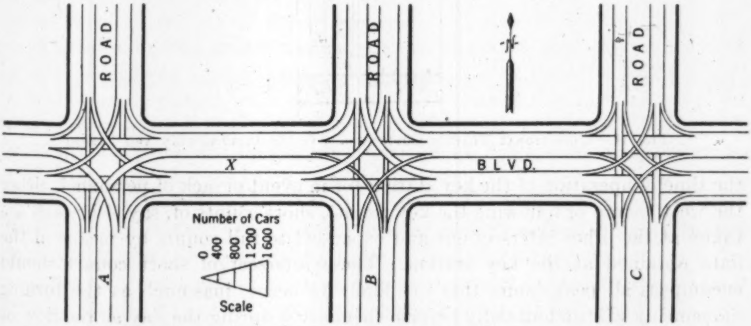
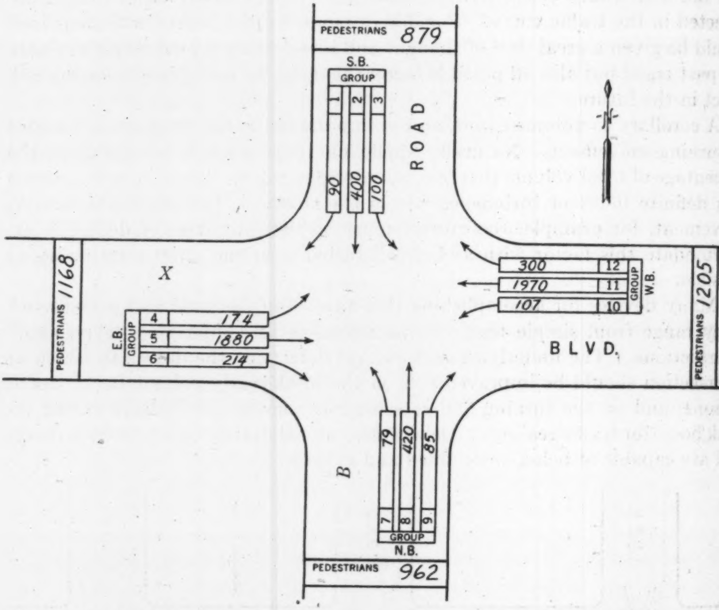


FIG. 10.—GRAPHICAL VOLUME CHART SHOWING PEAK-HOUR TURNING MOVEMENTS

Turning movements may be recorded in the same manner and at the same time that a volume count is made. The hourly percentage of turning movements follows a traffic pattern just as volume passing a point does. Hence, short counts may be expanded by use of key-station counts as a basis. A typical example of the application of volume and turning-movement counts may be seen best by reference to Fig. 10. Boulevard *X* is to be improved and the intersections of *A*, *B*, and *C* roads are also to be improved. It is necessary to ascertain the traffic flow throughout the proposed project and provide the engineer with a flow sheet for his design.

From observation and previous knowledge of the traffic in this vicinity it is decided to set up a key station at the intersection of *B* road and boulevard *X*. The counts consist of volume divided into their respective movements for each hour of the day. The volumes are further classified as to type of vehicles if necessary. The field sheets are then summarized as shown in Fig. 11. During



PEDESTRIANS 879
S.B.
GROUP
1
2
3
90
400
100
ROAD
N
X
PEDESTRIANS 1168
E.B. GROUP
4 174
5 1880
6 214
300 12
1970 11
107 10
W.B. GROUP
PEDESTRIANS 1205
BLVD.
B
79
420
85
7
8
9
GROUP
N.B.
PEDESTRIANS 962

FIG. 11.—DIRECTIONAL TRAFFIC COUNT FOR A GIVEN INTERSECTION AND PERIOD

the time of operation of the key station (or in event of lack of personnel, either the week before or following the key count), short counts of, say, 4 hr each are taken at the other intersections and expanded to full counts by means of the data obtained at the key station. These intersection short counts should encompass all peak hours that are likely to occur, inasmuch as the turning movements will undoubtedly reverse themselves during the day as the flow of traffic is seldom continuous in one direction.

From these data it is easy to place the facts in graphic form by means of a "flow sheet" (Fig. 10) showing peak hourly flow. Average 24-hr daily flow, Sunday 24-hr flow, daily truck flow, or other relationships may be plotted in like manner.

Speed has lately become the common denominator for design, to which it appears reasonable and logical to correlate most other features. Combined with peak-hour volume and classification of vehicles, these three elements have been adopted by the American Association of State Highway Officials to designate the design limits of any road. Their use is not a matter of guesswork on the part of the engineer. Data to determine their value are collectable and a reasonable figure may be assigned, to which the various elements of a project may be so related that the whole will carry the designated volume at the design speed.

To obtain actual data for design speed, it is necessary to know what speeds are prevalent over the present route. In the case of new construction it is necessary to assume a design speed that will carry the expected traffic expeditiously over the proposed route. Both cases require knowledge of speeds that prudent drivers will use when the highways are completed.

Over longer projects and where special conditions may affect the speed, such additional speed checks should be taken to cover the project thoroughly or at least to permit the plotting of an accurate speed profile. Critical speed is usually accepted as a basis for design speed inasmuch as it represents the maximum speed which prudent drivers, usually 85% to 90%, consider safe. It seems reasonable to design for that percentage of traffic inasmuch as, no matter what design speed is used, a few will drive beyond that limit.

In lieu of information as to what might be expected in speed after the project is completed, speed checks can be taken on comparable roads and the information so obtained may be correlated to design conditions. What the future trend of automobile speeds will disclose is problematical, but past trends may be projected ahead and tempered by judgment. Speed checks taken in flat desert country with the latest type of improved surface, and very light traffic, indicate that the critical speed for automobile traffic will remain under 60 miles per hr. In such localities design speeds of 80 miles per hr and more are obtainable without much additional cost, but it is unlikely that the prudent motorist will exceed or even closely approach such speed as a common behavior.

The selection of a proper design speed contemplates that the engineer will provide a highway over which a goodly proportion of the total traffic will be able to pass safely at, or below such speed. This means that each feature of the road, such as width and number of lanes, shoulders, sight distance, intersections, and surface texture, will allow the same safe and orderly passage. Conversely, any section that does not come up to this criterion is a hazard. To select those factors that affect safe speed, it is only necessary to look at almost any neighboring road built some years ago and analyze each element of design. It is on this basis that speed serves as the common denominator for design.

Intimately connected with speed is the element of sight distance. This is usually divided into two parts—safe passing sight distance and safe non-passing sight distance, the latter being synonymous with safe stopping distance.

The many variables that must be assumed in applying any theoretical formula for obtaining passing sight distance leave the designer with no option but to use whatever values are offered until research has provided more data on which to base values. At present, the tentative values presented by the Design

Committee of the American Association of State Highway Officials are given in Table 4. The approximate formulas have been derived by the writer for "rule-of-thumb" use.

TABLE 4.—MINIMUM SIGHT DISTANCES FOR PASSING AND NON-PASSING

Design speed, in miles per hour (M.P.H.)	PASSING SIGHT DISTANCE, IN FT		NON-PASSING SIGHT DISTANCE, IN FT	
	By empiricism	By approximate formula, $(\text{M.P.H.})^2/1.6$	By empiricism	By approximate formula, $10\,(\text{M.P.H.}) - 100$
30	600	563	200	200
40	1,100	1,000	275	300
50	1,600	1,563	350	400
60	2,300	2,250	475	500
70	3,200	3,063	600	600

The application of sight distance to design may be divided into two phases, both interrelated: First, horizontal sight distance; and second, vertical sight distance. Horizontal sight distance is affected by such road elements as cut banks, trees, and structures, whereas vertical sight distance is affected by change in gradient which prevents full vision. The ultimate in design is unlimited sight distance, which obviously cannot always be attained. The minimum obtainable bears a direct relationship to safe speed, hence the use of data such as in Table 4. The two types of sight distance are often found at the same location and the minimum is isolated for governing use.

One factor that seems to have escaped the attention of designers is that traffic itself may introduce a blind area. This is particularly true where traffic volume rises to such a peak that an almost continuous stream of cars occupies a lane. In three-lane construction that embraces curvature, sufficient radius must be used to insure minimum sight distance when the outer lanes are fully occupied (Fig. 12).

In the use of sight distance, the designer must cope with an additional factor. Vertical curvature must permit a certain minimum sight distance; and the horizontal curvature, likewise, must be checked so that the driver's line of sight across an inside chord will not be obstructed by a cut bank, a structure, or traffic. To accomplish this calls for ingenuity.

In some cases it may be economy to widen from two or three lanes to a four-lane highway in order to take advantage of a non-passing sight distance inherent in such design as opposed to the longer passing sight distance necessary with fewer lanes. In two-lane construction in mountainous country, the designer is often confronted with a lack of safe passing distance commensurate with his design speed. In such locality advantage should be taken of every possibility to provide at reasonable cost areas of safe passing sight distance spaced at minimum distances apart. The non-passing areas should be "bunched" or localized, yet not be of such a length that an impatient driver will attempt an unsafe passing movement. A fine point in design is presented here, yet from an economic point of view an advantage may accrue in that lesser radii of curvature may be safely introduced in such non-passing areas.

To further protect the motorist, such non-passing areas should be double-striped, signed, or otherwise brought to his attention as areas throughout which it is dangerous to attempt to pass. In general, traffic should be fully warned as to what the road hazards are and should never be allowed to gain a false sense of security. It may seem paradoxical to build a new highway that contains a known hazard, yet no highway can ever be 100% safe nor is it always economically justifiable to build to an excessively high standard for limited traffic. A study of traffic conditions prior to relocation should dictate the amount of the expenditure. It is a sad commentary that highway engineers often allow minority pressure groups to do the dictating.

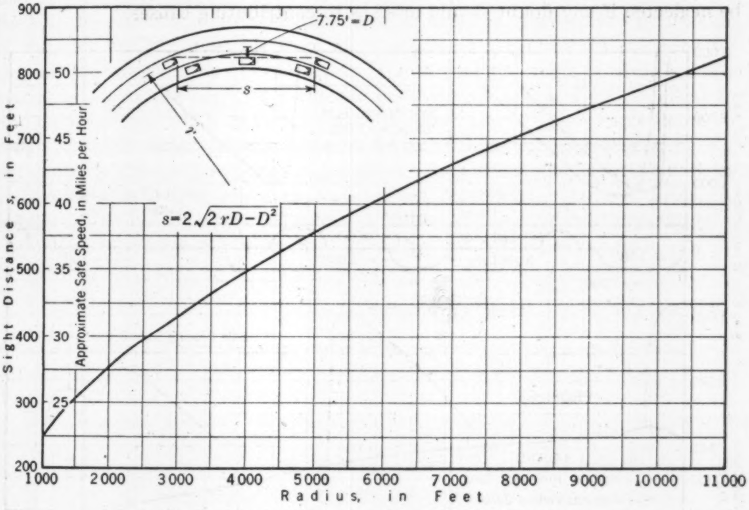


FIG. 12.—SIGHT DISTANCE ON 3-LANE CURVED HIGHWAY

To exemplify the question of sight distance and its relation to design, Fig. 13 presents a study made of a section of proposed relocation designed to a standard speed to be expected for that locality. It will be noted that both horizontal and vertical sight distance have been studied and safe passing sections indicated. Non-passing distance has been isolated and shown in cross hatching. Proper double stripes and signs would safeguard traffic throughout these areas.

Intersection design has received much comment in the technical press because of the innumerable methods proposed to minimize this most hazardous part of a highway. The underlying element is the proper appreciation of the behavior of traffic that will use such construction, as presented in five ways:

(1) Accident history of present intersection;
(2) Traffic flow sheet;

(3) Accident history of proposed design;

(4) Behavior study of present intersection so that proposed design will eliminate hazardous habits; and

(5) Comparative economy in time of passage through intersection using various traffic control devices.

The accident record and collision diagram of a grade intersection should receive careful consideration, as they indicate the intensity of need for change as well as the possible cause of hazard. Accident reports at intersections are much more likely to give a reliable cause than analysis of accidents occurring on the open road. However, the field investigation of driver habit should not be neglected if any doubt should arise as to contributing causes.

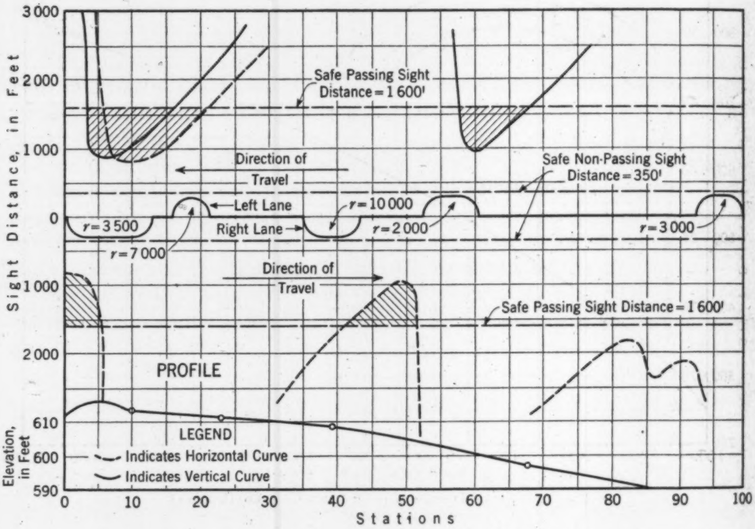


Fig. 13.—Study of Sight Distances on New Layout—Non-Passing Areas Hatched

The traffic flow sheet gives the fundamental factor for design requirements and should contain the maximum of information. For instance, if a secondary peak load with heavy turning movement obtains after dark, that fact may have a direct bearing on whether or not the crossing should be illuminated. If the accident history indicates an unusual number of accidents after dark on this movement, the designer would seldom hesitate to specify lighting in addition to special provision to otherwise safeguard the turning movement.

On the other hand, the expenditure of funds to provide a separate right turn when the flow sheet indicates practically none, is over-design. To fail to provide storage space for cars making left turns across heavy traffic is under-design that will lead to criticism as well as to a necessity to provide relief in some other manner.

The accident history of a constructed design, similar to that proposed and under comparable traffic, is invaluable. A very small change in detail may so affect the flow as to eliminate any danger prevailing in the constructed design. It may be that a certain turning movement or a peak volume may be handled in a slightly different manner or that surrounding conditions may be changed to eliminate potential hazards.

The angle of intersection has a direct bearing on design in that the angle of visibility of the driver is confined within relatively narrow limits from within his compartment. It is true that he can probably scan at least 270° of circumference by moving his head and shoulders, but experience indicates that he will barely move his eyes. For this reason, design should accommodate itself to the driver's habits and consideration must be given to the total angle of visibility easily obtainable from an average car. From data provided by General Motors Proving Ground, it was found that the angle of visibility is a maximum when the angle of incidence lies between 65° and 90° (normal to road approached) when approaching from the right.[4] These facts point to design that will cause one stream of traffic to impinge upon another at nearly right angles. The same is true where one traffic stream joins another flowing in the same direction, unless provision is made for acceleration lanes of sufficient length to allow both streams to become equalized as to speed and to commingle. Fig. 14 is an example of design that illustrates this problem in part.

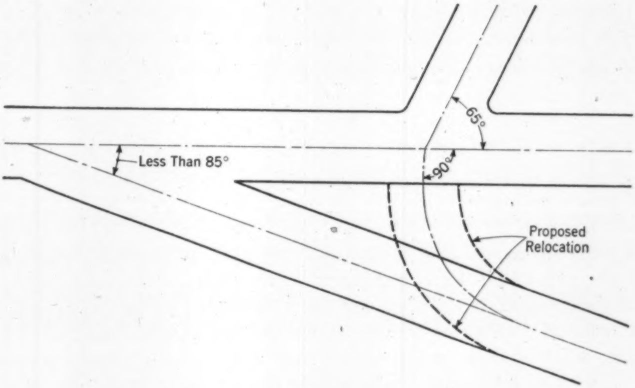


FIG. 14.—REVISION TO SOLVE VISIBILITY PROBLEM

The "loss-of-time" element in intersection design is of vital importance, for the efficiency of movement may be measured in terms of volume and time. The efficient utilization of an intersection is generally much lower than the intervening roadway, hence a proportionately larger amount of time and energy should be devoted to intersection design in order to decrease the over-all time loss and consequent discomfort to the motoring public. That the average motorist is willing to pay for his comfort is axiomatic.

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[4] *Civil Engineering*, August, 1939, p. 494.

Accident records and their analysis should become a part of the fundamental knowledge of a modern highway designer. This is as necessary as knowing the rudiments of a mass diagram. The result of his work should be judged in the light of its ability to carry traffic safely and efficiently. Safety can only be judged by a man experienced in visualizing the effect of all attendant conditions upon the expected traffic stream and asking himself, "What type of accident will this condition impose?" Efficiency of flow must be judged in the same manner, yet both elements depend upon the designer's knowledge of the effect of his labors upon the major fundamental highway factor—traffic.

DISCUSSION

Charles E. Conover,[5] **M. Am. Soc. C. E.**—It is obviously impossible to bank the pavement adequately on curves of less than 500-ft or 600-ft radius, as the author so clearly indicates. Red flash signals on the right side of the pavement approaching the curve are of real value to the careful driver. The difficulty of providing signals that will curb the reckless driver, unless he sees clearly that his life is greatly in peril, is one that all highway engineers clearly recognize. Signs that have been placed along highways, where fatal accidents have occurred, stating the number killed at the location in question, seem to have a sobering effect on such drivers.

Transition curves, not as yet used to any great extent even on main highways, could be used to advantage on curves of less than 1,500-ft radius. The difficulty of securing the additional property in the shape of long, relatively narrow grooves is clearly recognized by the writer. The cost of such acquisition is always far greater than its real value even in undeveloped areas.

Highway pavement on curves should be wider than on tangents. So far as the writer has observed in traveling through many states east of the Mississippi River, it is not present practice to provide this necessary extra width.

The author refers to the accidents that have occurred on a particular section of two-lane highway with sustained 6% grades, which slow the truck traffic down to a comparatively low speed. He shows that accidents are "piled up" in sections that are on curves, with lack of safe passing distances, due largely to impatient drivers of light cars trying to pass the heavy slow trucks. The remedy is not to widen the pavement, but rather to make it obligatory for trucks to have engines powerful enough and so geared that they can maintain the speed of a passenger car. Rear-end collisions with trucks would lose much of their danger if the bumpers were arranged to be at the same elevation as the bumpers of pleasure cars. In other words, car design and truck design are not of less importance in the solution of the accident problem than highway design. Much has been done along this line since 1930, at least within and adjacent to the so-called metropolitan (New York) area.

By Fig. 10, the author portrays vividly the elements of danger always present at crossings, especially in the case of cars making left turns. In urban areas, left turns are often prohibited, and cars are forced, by right turns, to traverse an entire city block in order to accomplish the purpose of a left turn. This affords considerable relief to traffic at very busy intersections. On rural highways, however, the distances usually to be traversed by making right turns around a block, as a substitute for direct left turns, are so great that they compel the making of the left turn as a practical necessity.

At crossings where the traffic on one of the highways is heavy and that on the other is light, a very considerable reduction in the hazards due to such crossings has been effected in many locations adjacent to New York City by installing signal lights so arranged that the red light is set to be normally against the highway with light traffic. A bar is placed transversely in the pavement of the light traffic highway a short distance back of the intersection, electrically connected with the signal light in such a manner that a vehicle passing over it changes the light for that highway for a limited period, the duration of which depends on usual traffic conditions at the intersection.

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[5] Cons. Engr., Pearl River, N. Y.

STEPHEN E. BUTTERFIELD,[6] ASSOC. M. AM. SOC. C. E.—The use of certain corrective engineering techniques to aid in the modernization of existing highways is a practice which to date has been used by a limited number of the more progressive state highway departments. From the point of view of more efficient highway usage, either in terms of accident reduction or increased roadway and intersection capacity, this practice is one that might be used to advantage to a far greater extent. Because many state highway departments are now (1941) faced with curtailed operating revenues, due either to the diversion of gasoline tax moneys or to national defense restrictions in the sale of gasoline, many thousands of miles of antiquated highways must be made to serve as best they may for some years to come. With this the case, it will be necessary for many states to revamp certain of their older highways by the use of any or all of the corrective techniques known to traffic engineers.

As clearly outlined by Mr. Harris, one of the first steps in such a program is the detection of the points of greatest hazard by means of accident records. With the application of the correct treatment to each point of hazard, a sound approach is afforded for a program of selective remedial measures. In applying this technique to several specific examples, the author has indicated how gratifying may be the results of such a selective procedure.

Closely akin to those points of hazard that are revealed by accident records are other points of secondary hazard which are not as easily recognized. They result in congestion and frequently cause accidents, but they do not always appear in the form of such familiar bottlenecks as narrow pavements, bridges, or underpasses. However, under conditions of fairly heavy volumes of traffic, they soon become evident. Frequently these bottlenecks may be intersections of types which are unable to adequately handle even the average volumes of traffic which pour into them for interchange. Thus it is that the capacity (and, therefore, the efficient use) of a roadway may depend almost exclusively upon the frequency and capacity of its interchanges. If this is the case, the solution obviously lies in remodeling the offending intersections. Progressive remedial steps are: The construction of additional turning lanes, channelizing islands, traffic signals, or clover-leaf grade separations.

An invaluable tool in gaging the efficiency of a highway system is the "speed and delay study." Mr. Harris has touched on this subject in his consideration of over-all speeds in Case No. 3. From a study of the speed profiles the points of congestion can be located readily and accurately. By the use of these data the proper corrective treatment can be applied to those specific locations, and only those locations, which are at fault.

The author states that in applying traffic engineering to rural highways the most outstanding feature that needs correction is the present structure, which must be made safe for existing traffic. Unfortunately, some state highway engineers, lacking the essential basic facts concerning points of congestion and locations of accidents, have felt that there is little or no salvage in an older roadway structure. Therefore, depending upon the funds available, a particular stretch of highway that is known to be unsatisfactory for economic transportation is either completely rebuilt or postponed for possible inclusion in the reconstruction program at some later date. Would it not be a wiser procedure, if only limited funds are available, to modernize the old highway in such a way that it may serve fairly efficiently for the additional period? This cost might be almost insignificant compared to the cost of complete reconstruction.

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[6] Traffic Engr., State Highway Dept., Hartford, Conn.

As stated by Mr. Harris, the full use of all available traffic data is essential in order to design a highway that functionally fulfils its sole purpose of providing efficient transportation. The successful highway designer must not only interpret traffic data in terms of present conditions, but he must look into the future. He must boldly appraise and carefully estimate the demands that traffic will make on the proposed highway twenty or even thirty years hence. The design of a highway for current operating highway speeds may result in the construction of a highway already antiquated at the time of its completion.

NATHAN CHERNIACK,[7] **ASSOC. M. AM. SOC. C. E.**—Mr. Harris' paper serves to re-emphasize this point: That in the modernization of existing highway facilities, and in the design of new ones, there is a need for the engineer to adopt a "demand" point of view. This means that the engineer should regard a highway, first, as a facility for regular use by human beings operating machines over its surface for the purpose of traveling between given origins and destinations safely, expeditiously, comfortably, and conveniently; and secondly, as an engineering structure designed to carry specified loads and constructed with definite structural materials. A highway that cannot, at once, satisfy the "demand" specifications is a poor engineering structure, and serves constantly to reveal to the traveling public a disparaging reflection of engineering ability, irrespective of the fact that the engineer may not be to blame for conditions encountered.

Mr. Harris touches only lightly on the subject of economic justification of highway facilities. Where he does, however, he reflects the orthodox theory of economic justification—namely, that reduced highway and vehicle operating costs justify the magnitude of a highway improvement. This theory can stand drastic revision. For example, under "Case No. 3.—Prescription for a Dangerous Route" he states:

> "The average loss of time over this two-mile section was calculated from the data to be 1.08 min or a total over a year's time of 2,222,000 min. * * * Using for this example the value of one cent per lost minute, the total loss capitalized at 5% would possibly warrant the expenditure of $444,400 for improvement."

There seems to be something basically wrong with a statement that a saving of a minute by each vehicle might support a $445,000 improvement.

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[7] Economist, The Port of New York Authority, New York, N. Y.

In the case of a truck, a minute saved does not necessarily mean a penny or two "saved." There is no saving, for example, in the portion representing fixed costs (taxes, insurance, etc.). These items have been paid for, in advance, and there are no rebates. Truck time saved does have value, but only if such time is substantial and can be utilized effectively. What could a truck operator do with a minute saved?

On the other hand, passenger-car time costs much less than truck time. In fact, in most cases it has no commercial cost whatever. Then again, car operating costs may sometimes prove to be higher, because of greater road distances and higher speeds on improved or new highways than on alternate older routes. All of these facts would tend to reduce the expenditures that could be justified for improved or new highways. Despite these facts, however, the experiences of toll highway facilities indicate that, as between a modern vehicular tunnel with a higher toll and alternate antiquated ferries, for example, or as between a modern toll bridge or toll highway and a congested "free" bridge or highway, a substantial group of passenger-car motorists are paying something extra, in tolls, for the privelege of using superior facilities.

A realistic approach to highway economics would therefore recognize these facts: (1) That operators of trucks and buses are more interested in highway economies that would permit them to make more revenue trips (and incidentally increase operating costs) rather than in those that promise unrealistic "paper" reductions in operating costs; (2) that passenger-car motorists (especially around urban areas) are more interested in reducing the "wear and tear" on their nerves than on their cars; and (3) that well-designed highway facilities can offer travelers positive "new" values, such as speed (consistent with safety), comfort, convenience, recreation, pleasure, and (again, especially in urban areas) freedom from the irritations and annoyances of the older hazardous and congested alternate highways controlled by traffic lights. In studies of the economic justification of modern highways, anticipated time savings may be used as indexes of these "new" values. Usually, however, they are largely neglected because they are not readily measurable.

To determine these "new" values, which, in many cases, overshadow doubtful anticipated operating cost reductions, requires the measurement of differences in the nervous energy used in traversing existing traffic-light-controlled streets and highways, on the one hand, and parkways, expressways, and freeways, on the other. Possibly this may be accomplished by means of portable instruments indicating the differences in the blood pressure under which drivers operate under different highway conditions. As Mr. Harris states in his "Introduction," "Traffic psychology is complex and as yet unsolved. * * * This type of research should be undertaken." Such research apparently may not come within the field of engineering or economics, but it would be highly fruitful and would throw a flood of light on the economic aspects of highway design.

Park H. Martin,[8] M. Am. Soc. C. E.—Engineering terminology should be specific, and not subject to several interpretations. The use of the word

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[8] Planning Engr., Allegheny County Planning Commission, Pittsburgh, Pa.

"rural" in the title and body of this paper has caused the writer to speculate at some length as to what the author intended to imply. After careful study, the writer concluded that the term "rural" as used embraced any and all types of highways lying outside of, or beyond, urban centers.

Expression of opinion was sought by the writer from half a dozen engineers of experience in highway work, as to their interpretation of the term "rural highway." Without exception the opinion expressed was that rural highways meant highways extending through rural areas, with very limited vehicular travel.

It seems ridiculous that a term used so much in highway engineering should have so indefinite a meaning. In general, "rural" has been used as defining a certain class of highways. Highway classification terms are not uniform; nor do the words used by one department mean the same when used by another. Several illustrations picked at random follow: From the State of Oregon System—

System	Types
R-1	Rural arterials
R-2	Rural general use laterals
R-3	Rural land service ways
U-1	Urban arterials
U-2	Urban general use laterals
U-3	Urban land service ways

The 1940 report of the State Wide Planning Survey of the Maryland State Road Commission states on page XVII, "This plan for centralization of authority and responsibility contemplates the grouping of all public rural roads into three systems for administration purposes: (*a*) State Primary Road System, (*b*) State Secondary Road System, (*c*) Local or Land Service Roads."

In the State of Pennsylvania a somewhat different classification exists. In 1911, by an Act of the Legislature, a State System of Highways was created. Main roads connecting county seats, cities, and boroughs were taken over as state highways. About 12,000 miles of highway were involved. In 1931, by an additional Legislature enactment, 20,000 miles of highway were added to the system. The 20,000 miles of new highway was designated as the "Rural Route System," the original 12,000 miles being commonly called the "Primary System." The justification given for taking over 20,000 miles of new highway was "To take the farmer out of the mud." Since farmers live in rural areas the term "rural" was given to the addition. Many of the roads taken over were not rural in traffic or service and are less so now, but the term still applies. Conversely many were rural in character and service, and still remain so.

Traffic data made public by the Pennsylvania Department of Highways from an eighteen-county traffic study involving 8,584 miles of road are as follows:

Vehicles per day	Miles of road
<50	1,447
50–99	1,247
100–149	837
150–199	564

The foregoing shows that 4,095 miles of the 8,584 miles investigated carry less than 200 vehicles per day, and the remainder, of course, greater traffic.

The writer is of the opinion that the term "rural highway" is not a proper engineering designation and that highway engineers should have a uniform classification nomenclature to designate the several different classes of highways. The basis for determining the terminology to be applied is debatable. The writer ventures to recommend that traffic volumes and traffic use be the foundation for the classification and terminology. This recommendation is made notwithstanding the opinion of some highway engineers that traffic records cannot be used to divide or classify highway systems. For example, M. R. Keefe, M. Am. Soc. C. E., has written:[9]

> "On studying our traffic records it soon becomes apparent that one cannot divide the existing system into existing primary and secondary roads on the basis of traffic. On many roads one section will be carrying light traffic, 100 to 150 vehicles per day, while on another adjoining section the count may run 750 to 1,000 per day."

The author implies that rural traffic is different from city or urban traffic. The large majority of drivers on rural highways (the term hereinafter used in the largest sense) are urban residents and drivers. The average driving speed is undoubtedly higher, but otherwise the driving habits are not much different on rural highways than upon urban highways and streets.

The author seems a bit inconsistent in his discussion of traffic behavior. Quoting from the "Introduction":

> "By analyzing traffic action, traffic behavior can be determined and evaluated. Roads should be built accordingly; hence, it seems reasonable to expect a road design that will accommodate that behavior which is normal to those who drive with prudence."

In the next paragraph, however, he states:

> "Doubtless, driving habits can be changed by education and by enforcement, but more probably can be accomplished by natural regulation resulting from properly designed highways which accommodate themselves to reasonable existing traffic patterns * * *."

It is true that drivers as a class do have certain uniform habits. For instance, they will not drive close to a curb on an open highway, very many spiralize curves of their own accord, but the engineer must plan from sound engineering principles and not from drivers' changing whims or habits.

The most outstanding project of modern highway engineering with which the writer is familiar and in which has been incorporated every known principle of approved highway design, is the Pennsylvania Turnpike. Yet, notwithstanding the care in planning and expenditures made, drivers will "crack up." It is evident that a wholly foolproof road can never be designed and built.

Four-lane divided highways are given as an example of engineering that will eliminate head-on collision. Several years ago the Pennsylvania State Department of Highways rebuilt a part of the Bigelow Boulevard in the City of Pittsburgh as a freeway with 12-ft lanes, a division strip being protected by curbs with a 10-in. reveal. Experience has shown that it is a rather common occurrence for cars to crash over the curb and wreck themselves on light standards set in the dividing space. The irony of one of the wrecks was that it was a police squad car. This is not to recommend that the divider should be eliminated, but rather to re-emphasize the personal element in driving.

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[9] "The Indiana Secondary Road Program," by M. R. Keefe, *American Highway*, October, 1936.

The author illustrates and mentions the use of photography as an aid in traffic studies. This suggestion is excellent and its use is desirable in various ways, one of which could be the study, by means of moving pictures, of merging traffic behavior.

Another commendable point made by the author is the observation of traffic under varying conditions of light and weather. The writer has long been impressed by the failure of traffic engineers and designing engineers to give more consideration to night visibility on a highway. Traffic islands in many instances serve a useful purpose, particularly when lighted. On rural highways, they may become points of hazard, due to poor visibility at night. Attempts to direct traffic by channelization which involve complicated openings and confusion points should be avoided, particularly when such points are not lighted.

At another place the traffic engineer, or some maintenance crew, may have failed in the placing of directional and speed signs. This criticism refers particularly to older rural highways of poor alinement. The writer has observed many signs that are visible in daylight driving, but are either invisible or only partly visible at night. This is generally the result of improper placing.

The writer is of the opinion that the placement of traffic signs should be studied in the field at night, as well as by day. The proper "signing" of highways, particularly those routes which have poor alinement, and for which funds are not available for reconstruction, is highly important.

The night accident record on rural highways is very bad, as testified[10] by Kirk M. Reid, of the General Electric Company:

> "What are the facts as to the relative day and night hazards on our rural highways? The statistics gathered by your departments show that from three-quarters to four-fifths of the total 24 hour traffic is by day, and yet almost half the non-fatal accidents and over half the fatal accidents occur during the period of dusk and darkness. Hence the average accident hazard per vehicle mile at night is three to four times that by day. And the fatality rate per accident is 64 percent greater at night."

A study of the proper placing of signs that are visible for night driving seems to be a field which needs more attention from traffic engineers. The author has discussed this subject, somewhat, in his example of study involving a curve reduction illustrated in Fig. 6. The result is the recommendation that the curve be reduced with a radius of 644 ft for a speed of 50 miles per hr. This is all very good, but it should be remembered that there are thousands of such curves which probably cannot be corrected because of lack of funds. Why not place a directional curve sign at the proper place at the curve approach and mark on the sign the safe speed for the curve, such as "Speed, 35 Miles"? The distance that the sign should be placed from the point of curvature should not be less than the distance required to brake the speed of the approaching car from what would be the safe speed of the car in the tangent down to the safe speed recommended for the curve. There is also one additional factor to be considered in setting the safe speed to negotiate a curve: On a two-lane highway the speed should not exceed the distance required to bring a car to a stop if an obstruction, such as a stalled car, is on the road in the path of the moving car.

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[10] "The Night Accident Hazard," by Kirk M. Reid, *American Highways*, January, 1936, p. 16.

In the case of the example cited in Fig. 6, the safe sight distance will be less than 250 ft and the safe speed should be less than 50 miles per hr for the corrected curve. The critical speed is sometimes assumed to be about 50% greater than the safe operating speed.

The importance of traffic data for design cannot be stressed too much. Many designers give no thought to this important relationship, if left to themselves. The writer cannot see how there can be a rational design of a highway without all the known facts of traffic being considered. The writer has in mind one project in which the designers proceeded without traffic figures. When those in charge of design were asked for their traffic data, it was found that they had not considered any. A complete origin and destination survey was made and the original preliminary plans were discarded entirely.

The peak-hour traffic is undoubtedly the criterion for which the designer plans. Daily volumes are interesting and of value in presenting the economics of a problem, but the peak-hour load is the controlling factor in design.

Forecasting is interesting but sometimes rather hazardous. Notwithstanding the uncertainties sometimes involved, it must be done. Forecasting the future of a highway may sometimes result in recommendations for a new highway rather than heavy expenditures for the reconstruction of an existing highway. For example, state Route No. 19 is the main route leading north from Pittsburgh. Outside the city limits the road passes, for four miles, through rather populated suburban areas. On one particular section the alinement is poor. For some time there has been considerable agitation to correct this condition by the construction of a bridge. This would entail an expenditure of several million dollars resulting in a considerably depressing effect upon adjacent property values. The construction of a new four-lane divided highway through undeveloped farming land about one mile east was decided to be the better plan. The construction of the new highway (at a cost only slightly more than the cost of the proposed bridge) affords two routes for some distance north of the city, with considerable relief to the existing traffic load on Route No. 19.

The lack of adequate parallel roads, particularly detour routes, in times of construction and repairs, is becoming more apparent and more serious as traffic volumes increase. Minor roads in the vicinity of main routes should always be considered in the light of available detour routes.

The writer cannot agree with the author when he states (see "Traffic Data for Design")—"To obtain actual data for design speed, it is necessary to know what speeds are prevalent over the present route." Maximum speed limits on rural highways are fixed by law. They may vary from 45 to 55 miles per hr. The vast majority of drivers, even under the best conditions, cannot drive safely at a speed greater than 60 miles per hr. It is assumed that the case the author has in mind involves the reconstruction of the highway. Since the highway is to be modernized, it would seem to the writer that the design should provide for the safe legal speed, which in turn would permit a higher critical speed. Unless the highway is a freeway, the design speed should not exceed 60 miles per hr. Since internal-stream friction (the conflict of faster and slower traffic in the same direction) is the cause of the greatest percentage of accidents, it does not seem wise that greater speeds than legal limits should be encouraged.

Minimum sight distances for passing and non-passing are now well determined and accepted in standard practice. The writer, personally, will remain a critic of three-lane highways. Economically, as well as from the standpoint of safety, they do not appear sound. The only exception that may be noted is in the case of truck lanes in mountainous areas. Joseph Barnett, M. Am. Soc. C. E., has presented[11] an interesting and instructive treatment of highway intersections. When funds are not available for separation devices this phase of rural highways is highly important.

Too much stress cannot be placed on the angle of visibility, not only at intersections but at accelerating traffic lanes. The writer has not been convinced of the desirability of expecting a driver to attempt to look over his left shoulder at a very flat angle in merging into high-speed traffic from an accelerating lane.

In 1941 the City of Pittsburgh, County of Allegheny, and Pennsylvania State Department of Highways completed a grade separation in the city of a street and Bigelow Boulevard. A merging traffic lane is brought into the Boulevard by a curve, the deflection angle of the access lane with the Boulevard being 25°. The radius of the curve between the access lane and the Boulevard is 588 ft. Speed on the Boulevard is about 45 miles per hr. Those who have observed its operation have been very much impressed by the speed and ease with which the traffic merges.

The writer agrees with the author (see "Traffic Data for Design") in that: "The angle of intersection has a direct bearing on design in that the angle of visibility of the driver is confined within relatively narrow limits from within his compartment."

In presenting the subject of relationship of traffic engineering to highway design, the author has performed a service for highway engineers. The writer is hopeful that his discussion of highway classification terminology may result in some terminology more acceptable than the word "rural."

Charles M. Noble,[12] M. Am. Soc. C. E.—The modern attitude of the more progressive group of highway engineers is reflected in this paper in that the functional character of the highway in relation to the operation of the vehicle and the characteristics of the driver is stressed. The writer is strongly sympathetic to this view and therefore wishes to strengthen that type of thinking and indorse the paper.

It is encouraging to note the reference to research in traffic (driver) psychology, for it is believed[13] that such research will yield substantial dividends in transportation efficiency and accident reduction by pointing the way to correct design.

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[11] "Highway Intersections, Design and Application," by Joseph Barnett, *American Highways*, January, 1940, p. 18.

[12] Chf. Highway Engr., War Dept., Arlington, Va.

[13] *Proceedings*, Highway Research Board, 1937, p. 246.

In addition to the principle enunciated by the author (that it may be advisable to introduce into the highway structure a physical change that will convert one type of accident into another less dangerous), it is also quite possible for the designer of a new facility to visualize many accident possibilities in advance, during the design process, and to arrange the design in anticipation of such possibilities. This is illustrated in the case of the Pennsylvania Turnpike tunnels, where it was necessary to converge from two lanes into one lane on entering, and the possibility of a skid or other mishap occurring in the confined roadway in front of the portals was anticipated. With a conventionally designed portal such a mishap, in many cases, would result in a direct collision with the immovable portal structure, generally with serious consequences. In the Turnpike design, a wall of the stepped type was projected out from the portal face, curving away from the roadway gradually until it disappeared into the bank of the approach cut. This wall was designed for the purpose of converting a direct impact accident into one of less serious character of the sliding type. Actual operation experience indicates that the walls are functioning with complete success, and many serious accidents have been averted by their use.

From the standpoint of transportation efficiency, the analysis by the author, utilizing a virtual speed profile to determine time losses, is valuable; for, although the average driver would be convinced immediately that an improvement should be made, it is often difficult to convince highway authorities, struggling with budgetary problems, of the necessity for improving part of an existing highway. The method illustrates the critical locations graphically and provides an economic argument that may be used to convince the authorities that funds should be authorized for remedial measures.

The author states (see heading "Traffic Data for Design") that "Design might be briefly described as the accommodation of structure to facts"; and

> "Only in late years has he [the highway engineer] paused for an instant to realize that the very element of his transportation problem—traffic—for which he has been building roads has not only kept up with the rate of production but has so far exceeded it as to create a problem of its own. Fundamental to design is knowledge of the traffic that will use the structure; therefore all traffic facts possible should be gathered for use in design."

He also states that "As more thought is given to this subject, so the field will expand and more and better information will be demanded and obtained; and its analysis will be more definite." All these statements should be applauded. Again the author goes to the heart of the modern traffic problem in the paragraph beginning "The peak-hour traffic is usually the criterion for which the designer plans."

With respect to the author's comments relative to design speed, it is pertinent to remark that, if too low a value for design speed is selected for the main routes, such highways may again become obsolete within a relatively short period (10 to 15 years), resulting in an economic loss, and the engineer may suffer embarrassment and possible loss in prestige in having to acknowledge the adoption of a shortsighted policy.

In closing, a word of caution should be interjected, for there is a tendency in some quarters to feel that engineering can solve all traffic problems, human and otherwise, and that enforcement can be relaxed when modern design is provided. Such is far from the case, as Mr. Harris recognizes clearly, and the writer wishes to emphasize the point that modern highway engineering and enforcement should function in unison.

Mr. Harris has covered his subject quite thoroughly as far as general principles are concerned, and very little can be added except as a matter of indorsement.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2156

MISSOURI RIVER SLOPE AND SEDIMENT

By William Whipple, Jr.,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Samuel Shulits, Leo M. Odom, E. W. Lane, Elliott J. Dent, and William Whipple, Jr.

Synopsis

The project for the improvement of the Missouri River consists primarily of open-channel regulation, which contracts the natural channel in addition to materially changing its shape. A general description of the methods adopted is given, together with a quantitative summary of the effects of the improvement upon the length, slope, width, shape, discharge, velocity, and roughness coefficient of the natural stream between Rulo, Nebr., and Sioux City, Iowa. Data are supplied as to the bed and suspended sediment characteristics of the river, in both improved and unimproved sections. An analysis is presented of the applicability of various bed-load formulas, involving both competence and capacity, to the prediction of the future slope of the river; and results are compared with observations to date (1940) on completed sections of the river. It is generally concluded that: (1) Formulas involving competence will not give the answer to this particular problem; (2) the mean slope of the Missouri River eventually will decrease through the operation of the contraction works; and (3) the bed of the river will scour out progressively for some time to come.

Introduction

The improvement of the Missouri River consists of open-channel regulation of approximately the lower 760 miles of the river, with regulation of flows through the Fort Peck Reservoir (see Fig. 1). It has been prosecuted by the Corps of Engineers primarily in the interest of navigation since authorization by Congress in 1927 and 1935, but some work was done under earlier authorities dating back to 1876. Despite the magnitude of this project and the length of time since it was begun, comparatively little has been published regarding the hydraulic characteristics of the stream. As far as the writer is aware, this paper represents the first published compilation of the basic data from which

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Note.—Published in March, 1941, *Proceedings*.

[1] Major, Corps of Engineers, U. S. Army, Fort Belvoir, Va.

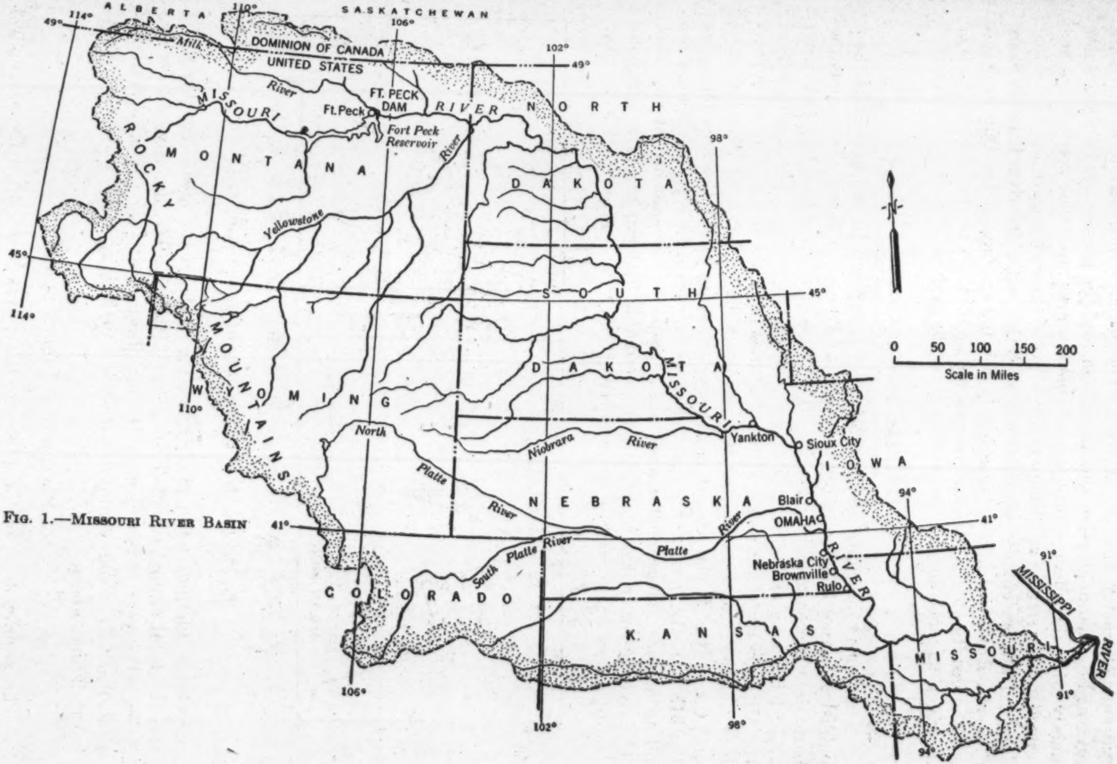
ALBERTA
SASKATCHEWAN
DOMINION OF CANADA
UNITED STATES
Milk
River
MISSOURI
FT. PECK DAM
Ft. Peck
Fort Peck Reservoir
RIVER
NORTH
DAKOTA
SOUTH
DAKOTA
MONTANA
ROCKY
MOUNTAINS
WYOMING
Yellowstone
River
MISSOURI
Yankton
Sioux City
IOWA
Niobrara
River
North
Platte
River
NEBRASKA
Blair
OMAHA
Platte
River
South Platte River
Nebraska City
Brownville
Rulo
RIVER
COLORADO
KANSAS
MISSOURI
MISSISSIPPI
RIVER
0 50 100 150 200
Scale in Miles
N
49°
114°
110°
106°
102°
98°
94°
91°
45°
41°

FIG. 1.—MISSOURI RIVER BASIN

a prediction can be made of the ultimate effect of this improvement upon the hydraulic characteristics of the river. The principal phenomena involved are governed by channel erosion and deposition, and the present study is concerned mainly with the effect of the improvement works upon the competence and capacity of the stream and the probable effect of these changes upon its eventual slope. Data presented deal principally with the part of the river between Sioux City and Rulo, which is approximately the upper third of the improvement, but the results obtained should have some application to the lower sections of the river, as methods used have been generally similar.

GENERAL DESCRIPTION

A complete description of the Missouri River and its tributaries has been published in the "308 report" on the Missouri River.[2] Certain of the applicable basic data are summarized herein. The Missouri River (see Fig. 1) is about 2,470 miles long and drains an area of 529,000 sq miles. The headwaters of the Missouri River and some of its tributaries lie in the Rocky Mountain province, but the greater part of the drainage basin lies in the Great Plains province and the central lowlands. It is noteworthy that, within the area improved, the Missouri River runs through and over deposits of its own sediment, which fill an ancient valley much deeper than the present stream bed at all points. The river never strikes rock or other unerodible material except at occasional points where it approaches the adjoining bluff. The Missouri River experiences wide fluctuations in discharge. Fig. 2 shows the mean monthly runoff and the total annual runoff for the years 1929 to 1937, inclusive, at Omaha, Nebr. The year 1938 was excluded because in that year partial operation of Fort Peck Reservoir changed conditions materially from what they would naturally have been.

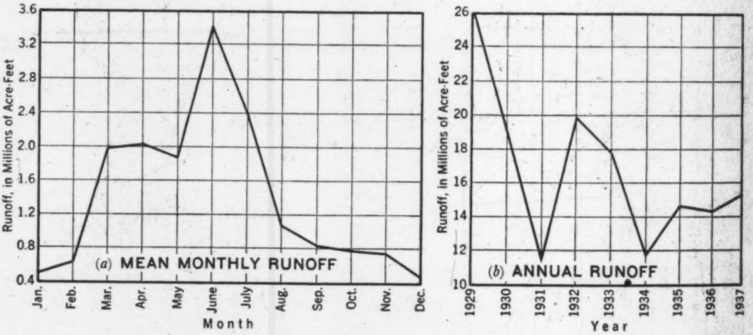

FIG. 2.—RUNOFF DATA, MISSOURI RIVER AT OMAHA, NEBR.

NATURE OF IMPROVEMENT

The improvement of the Missouri River was based upon the assumption that its average slope represented approximately a state of equilibrium under conditions existing at that time. The sediment content of the Missouri River is enormous, and if the existing slope had been insufficient to carry the average load for a long period of time the valley would have aggraded until an equilibrium slope was reached. On the other hand, it was obvious, from the many meandering oxbow lakes and the low banks, that below Sioux City the Missouri River was not in process of eroding a deeper channel. Although the river in its natural state is constantly varying its length and slope locally, these changes tend to counterbalance one another. It was known that during periods of prolonged drought the river generally silted up its bottom at all points, but these effects were not permanent. The net result of changes for several decades prior to the improvement program was an increase in slope; but insufficient information is available to determine whether, under conditions of nature, this increase would have continued, remained static, or reverted to its former level.

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[2] *H. R. Doc. No. 238*, 73d Cong., 2d Session, February 5, 1934.

Method of Improvement.—The plan for improving the river includes a generally contracted channel in a series of easy curves rather than the succession of straight, wide reaches and involved meanders or abrupt changes in direction that had existed previously. The channel decided upon lay as far as possible within the existing channels of the river. Figs. 3 and 4 illustrate the improvement of one of the worst bends in the river.

When it is determined that the river is to be driven away from a given bank a system of permeable pile dikes is constructed, leading away from this bank. Such a system reduces the velocities in the areas selected for deposit and increases the velocities in the areas selected for the regulated channel. Consequently the water passing through the dikes deposits its heaviest sediment, while the water passing around the dikes, with the increased velocity, scours the bed and bank and erodes that part of the channel deeper and wider. As the erosion and deposition process continues, the dikes and areas between the dikes are silted in and the channel is secured.

As the channel, deflected by the completed dike system, erodes back to its designed alinement on the concave side, this bank is revetted and permanently stabilized, progressively as it reaches the designed alinement. The new bars formed on and over the dike system quickly grow up in willows, which further aid the silting process, so that, a few years after construction is initiated, the new channel is entirely within the designed alinement, with no obstruction to flow except the ends of a few dikes that have not yet completely silted in. Regular snagging operations keep the channels free from obstruction. In addition to the dikes and revetment, dredging is used to cut away deposits of clay or gumbo, to cut pilot channels through bars and islands, and to make cutoffs. The first such cutoff on the Missouri River was described in 1938 by Lt.-Col. W. M. Hoge.[3]

This river improvement, although constructed primarily in the interests of navigation, will operate to stabilize the banks of the Missouri River, permanently, thus protecting adjacent farm lands and other improvements that were formerly in continuous danger from the ever shifting channel. Since the Missouri River is a navigable waterway, land ownership extends only to the low-water line; and when all of an original holding has once been taken by

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[3] *Engineering News-Record*, December 15, 1938, p. 755.

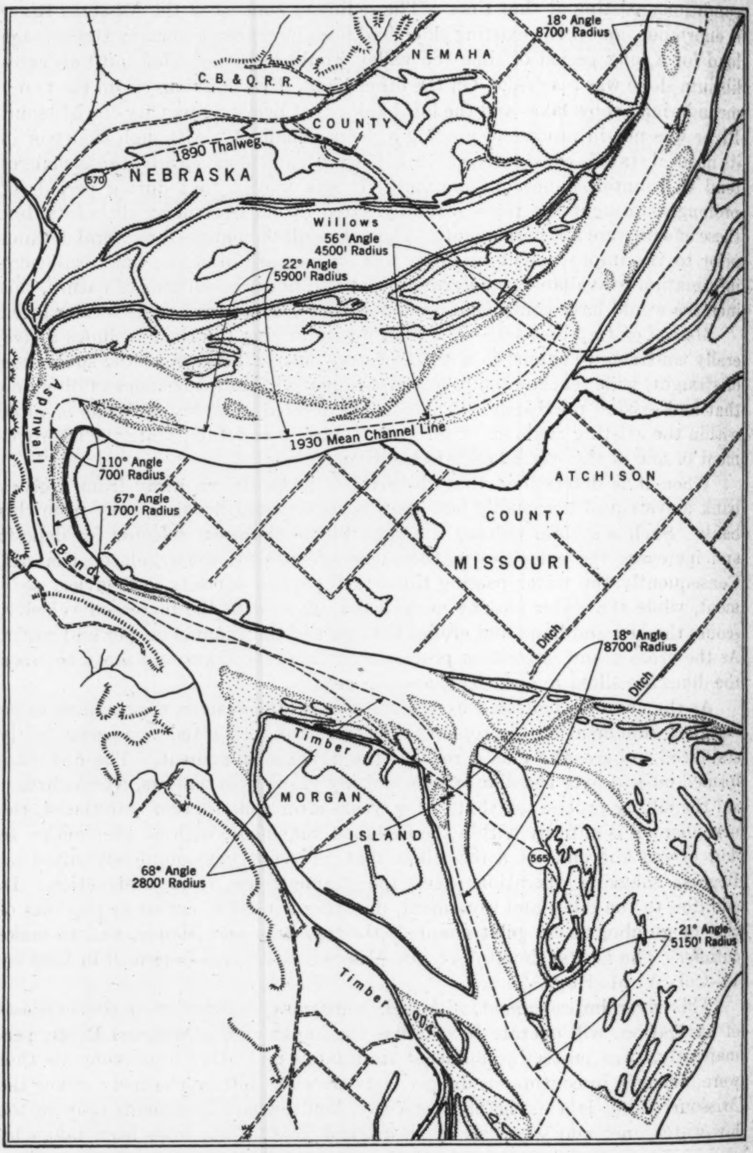
18° Angle
8700' Radius
NEMAHA
C. B. & Q. R. R.
COUNTY
1890 Thalweg
570
NEBRASKA
Willows
56° Angle
4500' Radius
22° Angle
5900' Radius
Aspinwall
1930 Mean Channel Line
110° Angle
700' Radius
67° Angle
1700' Radius
Bend
ATCHISON
COUNTY
MISSOURI
Ditch
Ditch
18° Angle
8700' Radius
Timber
MORGAN
ISLAND
565
68° Angle
2800' Radius
21° Angle
5150' Radius
Timber

Fig. 3.—Aspinwall Bend, 1930

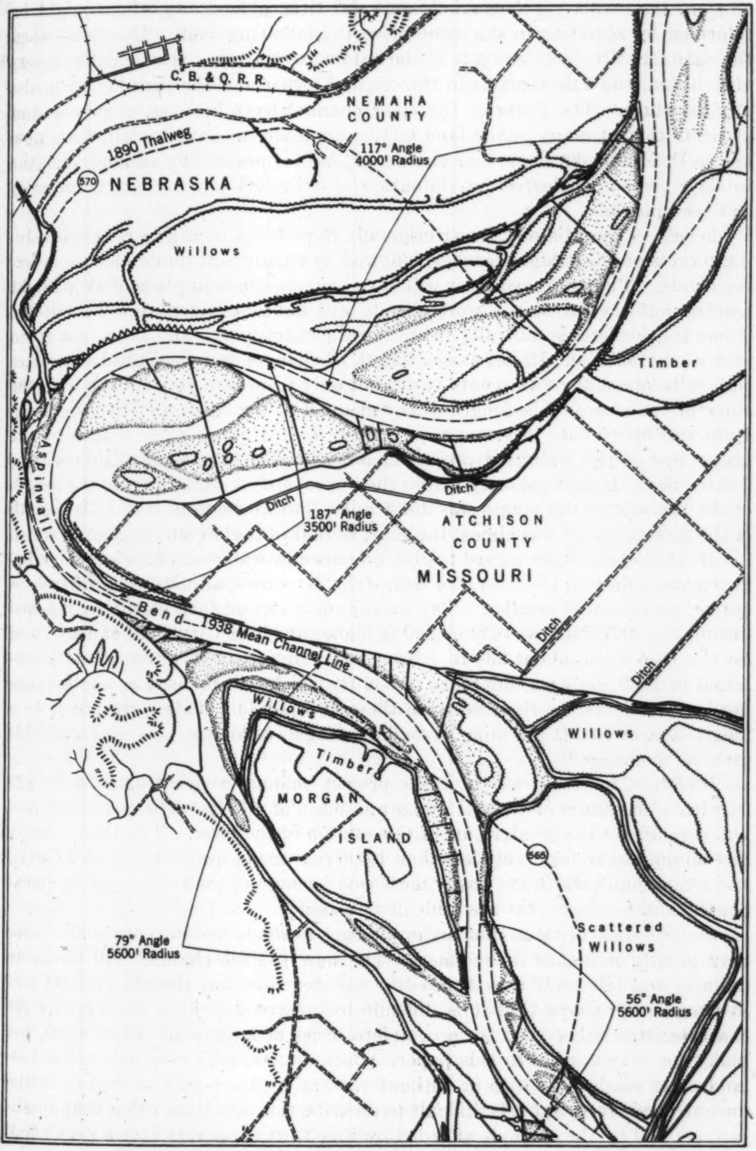
C. B. & Q. R. R.
NEMAHA COUNTY
1890 Thalweg
570
NEBRASKA
117° Angle
4000' Radius
Willows
Timber
Aspinwall
Bend
1938 Mean Channel Line
Ditch
Ditch
187° Angle
3500' Radius
ATCHISON COUNTY
MISSOURI
Ditch
Ditch
Willows
Willows
Timber
MORGAN
ISLAND
565
Scattered
Willows
79° Angle
5600' Radius
56° Angle
5600' Radius

FIG. 4.—ASPINWALL BEND, 1938

erosion, the owner no longer holds a valid title, even though land might be reformed by accretion in the same place the following year. However, when the channel shifts from one side of the holding to the other through the process of avulsion, the title remains in the original owner and the jurisdiction in the original state. The shifts of the river channel have been so numerous and intricate that at many points land known originally to have been in Iowa now lies on the Nebraska bank, and vice versa; and for practically all land adjacent to the river no conclusive determination of either state or private boundaries has been possible.

Length.—The Missouri River originally flowed in a changing course involving successions of straight reaches, followed by abrupt and sometimes meandering bends. The improved river will have sinuous bends in place of all straight reaches but several of the worst bends will have been removed by cutoffs. There is considerable difficulty in stating the exact length and slope of a given part of the Missouri River, as the length of channel and the fall in the water surface between any two points varies not only from year to year but with the stage of the river. The length of the channel of the Missouri River between Rulo and Sioux City was measured (1890) as 270 miles. The length of the center line of the stabilized project channel between the same points will be 245.6 miles. In 1930, shortly before the improvement works began, the length of the thalweg at low stages was 251.8 miles, but at medium stages the length of the mean channel was almost the same as that of the present project channel.

Mean Slope.—With regard to the difference in water-surface elevations between two points on the river, the same difficulty exists, although the differences are proportionately smaller. The maximum water-surface elevation reached during the 1927 flood was about 240 ft higher at Sioux City than at Rulo, and for the 1938 flood, about 234 ft. However, a uniform low-water plane, established in 1938, shows a difference of 236 ft, which will be used as the average head available to the river between these two points. This corresponds to a mean slope of 0.96 ft per mile, at medium stages, as long as the head available remains unchanged.

Width.—The mean width of the project channel above Rulo will be 725 ft, with a maximum of 1,000 ft and a minimum of 700 ft. The latter will prevail throughout the greater part of this section of the river. The 1930 channel had minimums considerably less than 1,000 ft, a mean usable width of 3,650 ft, and a maximum width not easily measured because of the many islands, back-chutes, and low bars that are difficult to classify.

Shape.—It is obvious that major changes in shape and curvature of a river may greatly influence its regimen. The new project channel will be fairly uniform and relatively easy to classify and describe, but the old channel was so irregular in shape that it is difficult to present any idea of its hydraulic characteristics other than by a complete series of large-scale maps much too bulky for presentation in this paper. Back-chutes, split channels around islands, and small bars, with or without vegetation, were very numerous in the uncontrolled river, but it is difficult to describe this condition other than qualitatively and by the example afforded by Fig. 3, which covers only a very small part of the river.

Although the eventual slope of the Missouri River depends primarily upon its sediment-carrying characteristics, certain hydraulic conditions must be evaluated as a basis for the direct approach to the problem. When the improvement of this section of the Missouri River is completed, it will have contracted the channel to a uniform width, left the length unchanged, removed snags, eliminated split channels, generally changed the shape of the channel, substituted dikes and revetment for natural banks, and regulated natural flows through operation of Fort Peck Reservoir. These factors, and other factors, not part of the improvement, all have their effect upon the general hydraulic characteristics of the river, which in turn determine its sediment-carrying capacity; and these relationships are evaluated in this paper to the extent that the available data will permit.

GENERAL HYDRAULICS

Local Slope Variations.—The existence of a substantial uniformity in mean slope taken at various times must not be understood as implying a uniformity of slope over different parts of the river at any one time. Gages set at 5-mile intervals show very considerable variation. Studies of water-surface elevations at gages established at half-mile intervals or less, with precautions to avoid discrepancies due to superelevation, show very great local variation in slope. Even for fairly well stabilized parts of the river, local slopes still vary from less than 0.5 to more than 2.5 ft per mile. Moreover, a new profile taken at the same point a few weeks later may show considerable change.

A study of all available data sheds light on certain slope irregularities and leaves others not yet explainable. For example, two persistent steep slopes were identified as due to the presence of protruding ledge rock into the channel from the adjacent bluff. Some longer reaches, the slopes of which are greater than average at high water only, unquestionably demonstrate disturbing effects of excessively severe bends. In incompleted parts of the river certain slope deviations in both directions from the mean are known to be incidental to the improvement works, and therefore of temporary nature. Sufficient data have not been developed to show the extent of local steepening to be expected at crossings, partly because studies of this nature have not yet been made at extremely low stages and partly because of the presence of so many other disturbing influences of greater magnitude. The effects upon the slope of the Missouri River of the heavy bed load carried by the Platte River have previously been described in detail elsewhere.[4] These effects are of major importance and extend many miles down the river. As measured during the low water of 1930, the mean slope for 31 miles above the mouth of the Platte River was 0.74 ft per mile, and for 44 miles below that point it was 1.24 ft per mile. Locally, steep slopes are found at many points where back-chutes reenter the main stream at an angle, whether or not this occurs at a crossing. This is probably due to bar deposition in the main stream by the chute during times of high water. These half-mile gages have been set only on improved and partly improved reaches of the stream, but similar or greater variations are believed to exist in unimproved sections of the river.

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[4] *H. R. Doc. No. 238*, 73d Cong., 2d Session, February 5, 1934, Appendix XV.

It is probable that the completed channel will always have certain slope irregularities, due to tributary influences, variations in shape, crossings, and irregularities in the transportation of bed load. However, disturbances at reentrant back-chutes will be eliminated. The extent of excessive slopes at crossings and bends will vary according to the number and severity of bends.

Comparison of Shape.—This study was based upon complete large-scale maps of 1930 and 1939, which show, respectively, conditions prior to any improvement in the section between Sioux City and Rulo and conditions of the present, including the adopted channel line. As a first step in classifying bends, lines were drawn on each map to show, as accurately as possible, the mean channel line of the river at medium stages throughout its length. For the improved channel this was taken to be the center of the project channel. For the maps of the unimproved river it was necessary to disregard small irregularities, and in the case of a split channel either to adopt a line intermediate between the two parts or to follow either one of the channels that was considered to be representative, in both length and curvature, of the river as a whole. Figs. 2 to 4 illustrate these lines for the reaches of the river concerned.

Although excess loss of energy in a bend is occasioned primarily by sharpness of curvature (measured by length of radius in the bend) it is also affected (particularly during overbank flow) by the total angle through which the river is turned. In the many bends of varying radii only the minimum radius in each bend was considered. The total angle through which the river is turned in a bend will be known herein (for lack of better nomenclature) as a "central angle." For convenience, all bends of less than 45° central angle were considered separately, as experience shows that, even during overbank flow, relatively little disturbance is caused by flow through bends of small central angle, despite radii less than those ordinarily desirable.

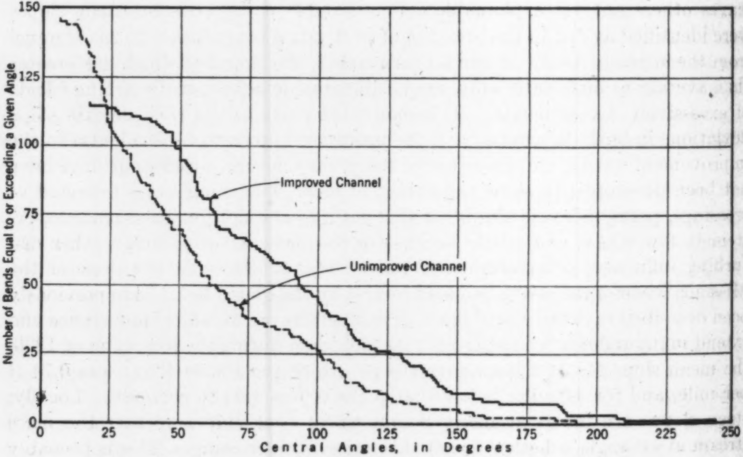

FIG. 5.—NUMBER OF RIVER BENDS CLASSIFIED BY THEIR CENTRAL ANGLES

Fig. 5 shows, for both the improved and the unimproved channel, the number of bends with central angles equal to or greater than any given curvature. These curves indicate that, apparently, the channel stabilization program has increased the number of bends of a central angle of 100° or more, but has made little change in the number of those between 45° and 100°. Actually the number of the longest meanders was reduced by cutoffs, but this fact is not apparent from Fig. 5 because many meanders in the unimproved river had to be classed as two or more successive bends in the same direction, separated by short reaches. Of bends of less than 45° central angle, the improved channel has only about one sixth as many as the unimproved channel. A summary of all data shows that in 1930 the medium-stage, unimproved channel had 147 bends, with a total angular curvature of 8,340°; whereas the future project channel is planned with 116 bends and a total angular curvature of 10,640°. At low stages in 1930, the thalweg had 269 bends with a total angular curvature of 14,286°.

The radii of curvature of the bends and general character of the meanderings of a river depend upon many factors, including the size of the stream, the slope of the valley, and the character of the sediment. The Mississippi River sweeps through its great curves in meanders of many thousands of feet radius, whereas some small stream flowing in curves of similar shape may have prevailing radii of only a few feet. On the section of the Missouri River considered in this study, experience has shown that, if otherwise properly designed, bends of 4,000 to 8,000 ft minimum radius are generally satisfactory and create no unusual hydraulic disturbance at any stages experienced in recent years. Straight reaches and bends with radii much greater than 8,000 ft have undesirable characteristics during low stages because of the excess number of crossings that may develop. These extra crossings introduce an extra sinuosity and impede the flow to that extent, besides being a hazard to navigation. On the other hand, bends of less than 4,000-ft radius, and particularly those of less than 3,000-ft radius, create excessive disturbance and turbulence in the river at bankful and overbank stages, and for this reason are no longer constructed. Fig. 6(*a*) shows that, of bends with central angles greater than 45°, the number with excessively sharp curvature has been greatly reduced by the channel improvement project. A separate study of bends of less than 45° central angle shows a similar result to have taken place, the bends of undesirable curvature in this class having been entirely eliminated by the improvement (see Fig 6(*b*)). If results for all bends in the river are combined, the number with less than any given radius is shown to have been decreased by the channel improvement.

It is believed that, considering solely the planimetric shape of the river, the final results of the improvement program will tend to increase the hydraulic efficiency of the river at all stages of bankful or less, due to reduction in sharp bends, irregular banks, excess crossings, and split channels. During overbank stages efficiency will tend to be somewhat less due to the greater curvature of the improved river, although promoted by the more regular shape, the reduction in the number of sharp bends, and other favorable factors. The relative importance of overbank stages as compared to lesser stages may be roughly estimated from the fact that from 1929 through 1937 (a subnormal period)

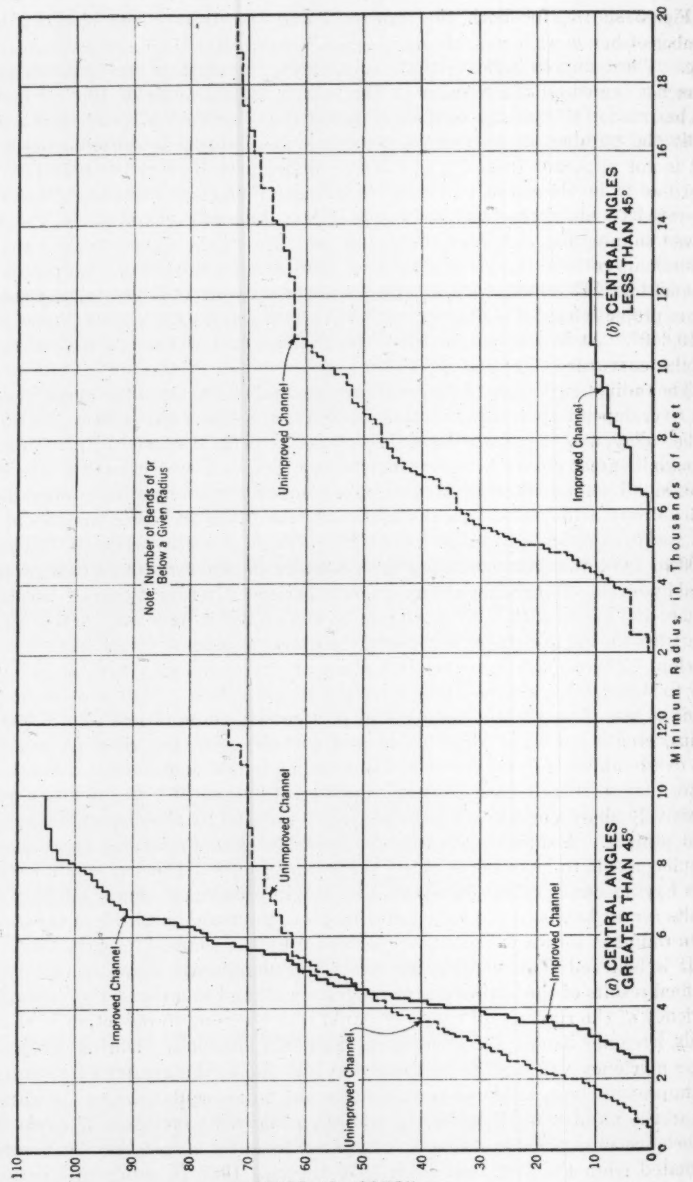

Fig. 6.—Number of River Bends Classified by Their Minimum Radii

the mean flow was about 23,000 cu ft per sec, whereas overbank flow does not start until flows exceed about 75,000 cu ft per sec, and is unimportant except near the crest of unusual floods.

Fort Peck Reservoir.—The regimen of the Missouri River will unquestionably be greatly influenced in some respects by the operation of the Fort Peck Reservoir. This reservoir, with a gross capacity of 19,400,000 acre-ft, controls the entire runoff of the Missouri River above the mouth of the Milk River. Its primary purpose is to regulate the flow of the Missouri River so as to insure required minimum flows during the navigation season at the head of the navigation improvement. Other benefits will result, due to partial flood control of the Missouri River, the production of incidental hydroelectric power, and possible utilization for irrigation.

In general, the effects of this reservoir will be as follows: During all years, flood peaks and overbank stages will be decreased; and during dry years, medium to overbank stages will probably also be reduced. A considerable part of the winter flow will also be stored. During August, September, October, and early November, there will be considerable releases, in practically all years, in order to bring the navigation project up to the desired minimum flow, and during wet years there may be additional excess releases during October in order to draw the reservoir down for flood-control storage. It seems probable that these changes will tend to prevent the aggradation of the channel which has been observed during dry cycles of the past, but in the long run this factor may be balanced by reduction in frequency of high discharges in wetter years. Although detailed studies have not been made for this paper, it is not believed that the operation of this reservoir alone will have an important effect, between Sioux City and Rulo, upon the average ratio between the quantity of sediment brought into the section and its sediment-carrying capacity, for a given slope.

Irrigation Influences.—The extensive past and projected future development of irrigation on a large scale in the Missouri Basin must be a factor tending toward aggradation of the stream bottom. Irrigation projects in the Missouri Basin are ordinarily installed where the water supply is comparatively free from sediment. In this manner a reduced supply of water is available downstream to carry the net sedimentary load of the stream, which is not decreased proportionately. No detailed study has been made as to the extent to which this effect would influence the slope of the Missouri River below Sioux City, but it is probably of minor importance.

Velocity and Slope.—Consideration has been given to approaching the problem of changes in velocity of the Missouri River due to the channel improvements by a comparison of velocities measured by the U. S. Geological Survey in connection with discharge readings over the period of record, from 1929 to the present. However, interpretation would present major difficulties, since the discharge readings were taken at points of the river contracted by bridges to approximately 1,000 ft, which widths have not materially changed since the general reduction in width of the river as a whole. Prior to the improvement work, the contracted sections near the gaging stations would naturally tend to have a slope and mean velocity lower than the average at low stages and higher than the average at high stages of the river. The extent of these departures from the mean would be dependent upon the rate at which the river bed would scour out or silt in following changes in discharge. Data sufficient to determine the slope at these periods are almost entirely lacking. For these reasons, determinations of velocity changes, in order to be applicable to the river as a whole, can be computed by dividing the discharge at the nearest gaging station by the average area of the cross section as obtained by survey.

During 1938 a hydrographic survey and hydraulic study were made of a 25-mile section of river that was almost completely improved. Data were obtained at a discharge of 55,900 cu ft per sec with gages set at approximately 5-mile intervals. The part of the river considered included one long bend, of minimum radius 3,700 ft and central angle 161°; a sharp bend, of minimum radius 3,000 ft and central angle 104°; and three other bends of lesser severity. Results showed considerable variation in roughness coefficient. The mean slope of the entire section was 0.69 ft per mile; the average velocity 4.73 ft per sec; and the mean value of Manning's *n*, 0.0194.

As a check, a similar hydrographic survey and study were made for a 10-mile reach of almost wholly unimproved channel for the same year. This survey was taken at 95,000 cu ft per sec, which is slightly greater than overbank stage in this vicinity, and includes one fairly severe and one very abrupt bend. Only a very few improvement works had been constructed in the part of river under consideration. Values of roughness coefficient obtained varied in about the same range as the values obtained for the improved section of river. The mean slope was 0.91 ft per mile, the average velocity 4.80 ft per sec, and the mean value of Manning's *n*, 0.0197.

Overbank flow at these stages is of minor importance at this point. Any allowance for overbank flow would raise the roughness coefficient and lower the aforementioned average velocity, since these values were computed from the total discharge of the river at this time, as obtained from the nearest gaging station.

TABLE 1.—MISSOURI RIVER SLOPES

Section (see Fig. 1)	1890 milage reference	1930		1938		1939	1940
		Length, in miles	Slope at low water, in feet per mile	Length, in miles	SLOPE, IN FEET PER MIL		
					At low water	At flood	At low water
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sioux City, Iowa	804.2 to 799.2	43	1.06	5.3	0.87	0.99[a]	0.93
Blair, Nebr	694.7 to 683.6	29	0.74	13.6	0.71	0.70	0.77
Omaha, Nebr	668.9 to 644.7	23	0.76	24.1	0.73	0.86[b]	0.75
Nebraska City, Nebr	607.8 to 601.9	20	1.25	5.7	0.83	1.26	0.88
Brownville, Nebr	586.0 to 570.8	14	0.90	12.3	0.77	0.88	0.93

[a] A 10-mile section. [b] A 17-mile section.

As of 1938, although construction was underway throughout most of the river between Sioux City and Rulo, there were only five parts of the river in which a single completed channel was established over a sufficient length for comparative slopes to be available. These were, besides the section near Omaha already discussed, sections near Sioux City, and Blair, Nebraska City, and Brownville, Nebr. Recent slopes of these sections are given in Table 1.

It is noted in Table 1, Cols. 3 and 4, that the initial improvement by contraction of five stretches of the river resulted in all cases in a reduction of slope and a resultant slope below the average for the section between Sioux City and Rulo. The reduction was much greater for short sections and particularly the one section with a wide, divided, natural channel. Flood slopes in 1939 were greater on four of the five sections. Low-water slopes in 1940, although still below the general average, were all increased over 1938, coincidentally with the extension of contraction works to other parts of the river.

Values of roughness coefficient are apparently about the same for the improved and the unimproved section of the river. A computation by means of the Manning formula of the velocity that would obtain at each reach at a slope of one foot per mile shows that the unimproved section would have a velocity of 5.04 ft per sec, whereas the improved section would have a velocity of 5.71 ft per sec, although the data for the unimproved channel were taken at a 70% greater discharge. Similar comparisons made between present improved channels and unimproved sections of river in 1932 also show that the mean velocity obtaining, in terms of equal slope and discharge, is considerably higher for the improved river.

SEDIMENT TRANSPORTING CHARACTERISTICS

Data.—In introducing this single problem the writer must acknowledge his indebtedness to comprehensive sediment studies made in this basin for the U. S. Engineer Department by L. G. Straub,[4] M. Am. Soc. C. E. For a general description of the sedimentary characteristics of this basin, as well as for a large amount of basic data, reference must be made to this work. For the section of river under consideration no data were taken between those of the 308 report[2] and a program that was undertaken early in 1939. In the 1939 program, bed-load and suspended sediment samples were taken at Omaha, near the center of the largest section of the improved river above Rulo, and at Yankton, S. Dak., which is many miles above the head of the navigation improvement and represents the unregulated river. The mean width of the river in the vicinity of Omaha is about 725 ft; at Yankton it is about 2,000 ft. The slope at Omaha is 0.71 ft per mile, and at Yankton it is 1.13 ft per mile, both slopes taken during October, 1938, over a stretch of river more than 8 miles long. For the period of sampling the discharge at Yankton was more than nine tenths of the Omaha discharge.

Nature of Problem.—The Missouri River carries a heavy bed load, which has been sampled and analyzed, but which has not been measured as to quantity. In order to establish a future equilibrium, the competence of the improved channel must be sufficient to move the largest particles brought down to it by the unimproved channel above; and, similarly, the capacity of the improved channel must be sufficient to move the total bed load of the unimproved channel. The Missouri River also carries a heavy suspended load, which has been both analyzed and measured. A third condition for equilibrium is that the downstream part of the river must carry the entire suspended load brought to it by the upstream part.

If the river develops transporting characteristics over and above those sufficient to carry its total sediment load in a reach where its banks are stabilized, it will scour deeper, provided that the competence is sufficient to move the material of which the bed is composed.

Suspended Sediment.—Curves (1) to (3), Fig. 7, represent the average of mechanical analyses of suspended sediment at Yankton and Omaha. For 1939 a very close correspondence is shown between the sediment at Yankton and Omaha, with the latter very slightly coarser. In these cases about 20% of the materials shown is very fine sand or coarser, as compared to about 32% as shown on the 1930 curve for Omaha. Fig. 8 shows the rating curves for the relationship between the discharge and the total suspended load at Yankton for 1939, and at Omaha for 1930 and 1939. The wide divergence between observed results at high discharges is noteworthy. In the case of the Missouri River the quantity of finer sediments carried is probably always dependent

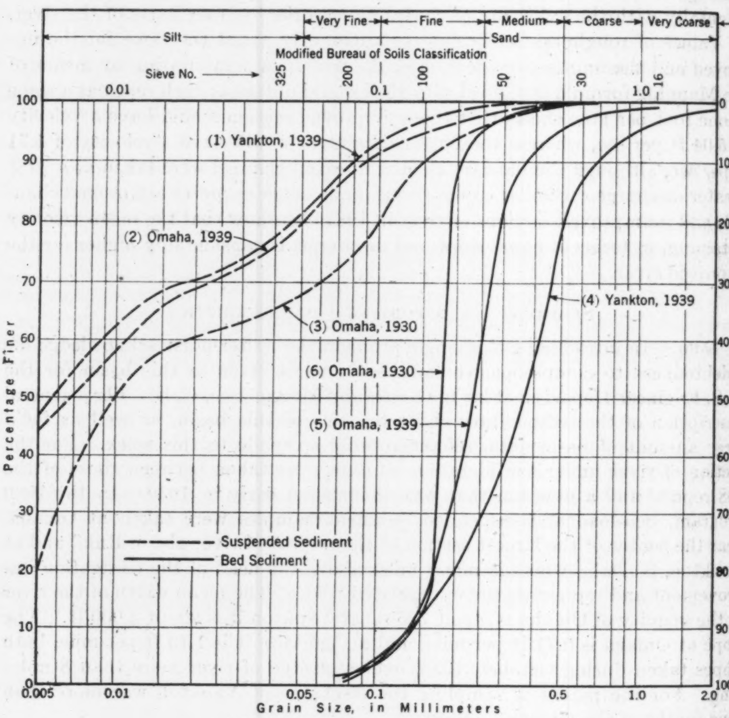

FIG. 7.—AVERAGE OF MECHANICAL ANALYSES

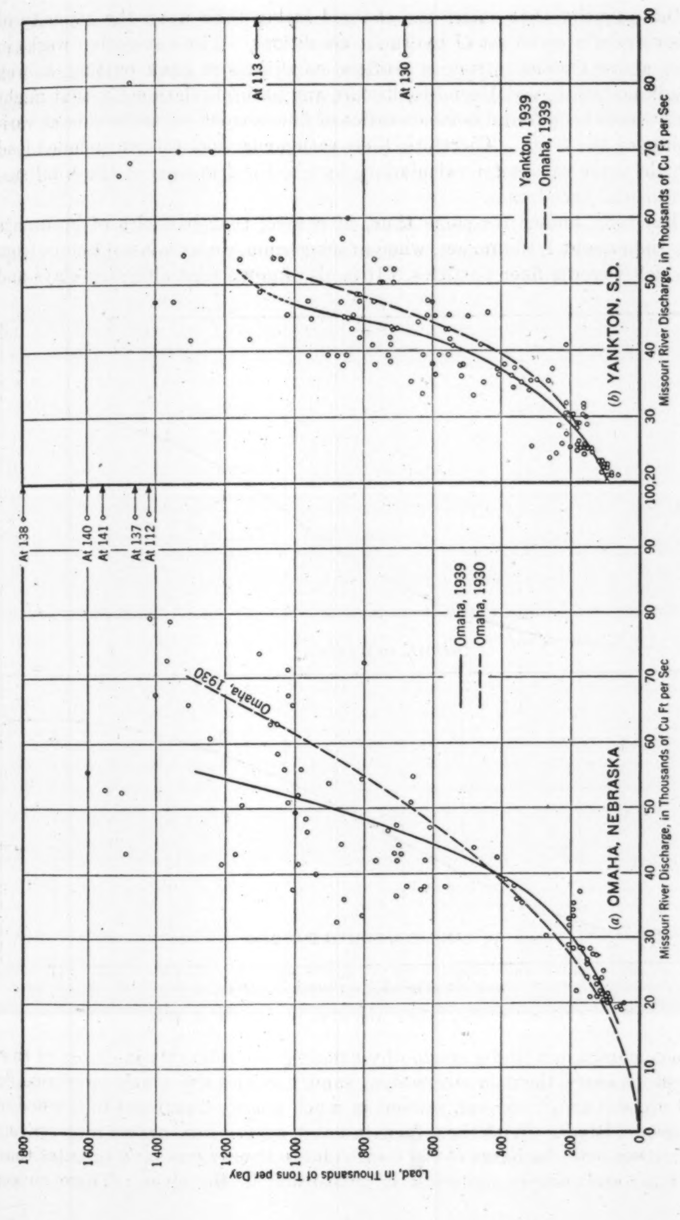

FIG. 8.—RATING CURVES FOR TOTAL SUSPENDED LOAD, 1939

upon the quantity that enters the channel rather than upon the capacity of the river under a given set of hydraulic conditions. The navigation works in progress above Omaha introduce artificial conditions of bank cutting, as well as of sedimentation, which tend to obscure any natural relationship that might otherwise have been found between ratios of finer suspended sediments at various points on the river. Therefore, these rating curves of total suspended load have little value except for calculation, by empirical means, of the total suspended load of the stream.

It has been known for some time, of course, that particles of grain size greater than about 1/16 mm act, when in suspension, under a law different from that which governs finer particles. It is also known that although clays and silts, once compacted, resist erosion by velocities considerably in excess of that sufficient to carry them in suspension, sand particles are much more readily picked up, and are, moreover, present in much greater quantities in the bed of the Missouri River. With these facts in mind, curves were drawn representing the variation with discharge of the total volume of very fine sand (greater than 0.074 mm) and coarser materials in suspension in the river. These curves

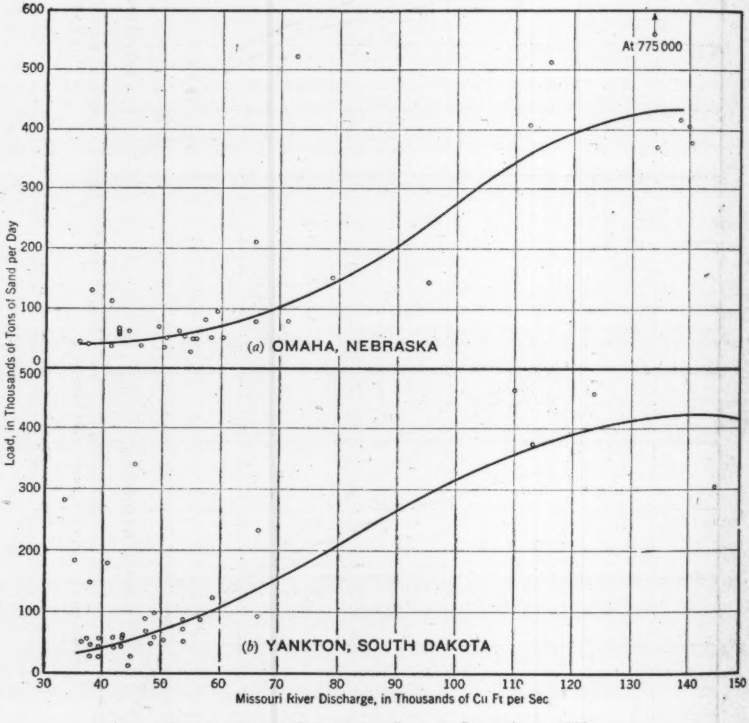

FIG. 9.—RATING CURVES FOR SAND IN SUSPENSION, 1939

are reproduced in Fig. 9. Unfortunately, they show that the suspended sand passing a given point, as well as the total suspended content, varies with parameters other than discharge. Variation in slope and velocity, due to the passage of sand waves on the bottom and to rising or falling stages, are probably among the influences to be considered. However, these local variations should balance when the average is taken of the considerable number of variations, and the mean quantity of suspended sand, rather than total suspended sediment, should be more nearly proportional to the capacity of the stream.

In order to make a more direct comparison of quantities of coarse suspended sediment (material coarser than 0.074 mm) at Omaha and Yankton, respectively, data were taken for a corresponding though non-consecutive period covering 29 days, from April through July, 1939, with allowance for a 2-day time of travel between Yankton and Omaha, in order that the data would be comparable. These are the only corresponding days on which composite samples were taken large enough to contain sufficient quantities of suspended sediment to permit mechanical analyses to be made. Results are given in Table 2.

TABLE 2.—COMPARISON OF DATA, APRIL THROUGH JULY, 1939

No.	Description	Omaha	Yankton	Ratio
1	Average water discharge, in cu ft per sec	70,000	61,800	1.13
2	Average coarse sediment discharge, in tons per day	174,000	134,800	1.29
3	Average total sediment discharge, in tons per day	1,240,000	1,293,000	0.96

Results of Item 3, Table 2, are checked by the fact that for a period of 84 corresponding days (with allowance for travel time) the average total sedimentary discharge was exactly the same for the two stations.

From the foregoing it appears that the Missouri River at Omaha carried a 14% higher concentration of the coarser suspended sediments than it did at Yankton during the same period, the excess having been picked up by scour from the bed or banks or having been brought in by the tributaries. On the other hand, the content of finer sediments was reduced absolutely as well as relatively. Since silts and clays in any appreciable quantity do not settle in the channel of the river, there must have been a large deposit of the finer sediments on bars and behind dike systems between Yankton and Omaha. For lack of an available supply, the river is unable to replace this finer material by scour, although the capacity of the stream for the finer sediments is probably seldom if ever reached at any point, as shown by previous investigations by Professor Straub.

As far as suspended load is concerned, therefore, it appears that the improved section of the river has a tendency, at present, to pick up and carry a net addition to its suspended sediment content from its own bed and banks, even at slopes less than the average now prevailing. The conclusion appears justified that, for a given slope, the improved channel has a greater capacity than the natural river for the transportation of suspended sediment.

Bed-Load Competence.—Fig. 7(*b*) shows graphically the average mechanical analyses of bed sediment samples taken. The results show a predominance of fine and medium sand, with 1939 results for Omaha coarser than in 1930, but not as coarse as those of Yankton in 1939. The Yankton results are probably influenced by its location below the mouth of the Niebrara River, which carries a heavy bed load, and the natural sorting processes present tend to create finer deposits in the lower reaches of a river than are found in the upper sections.

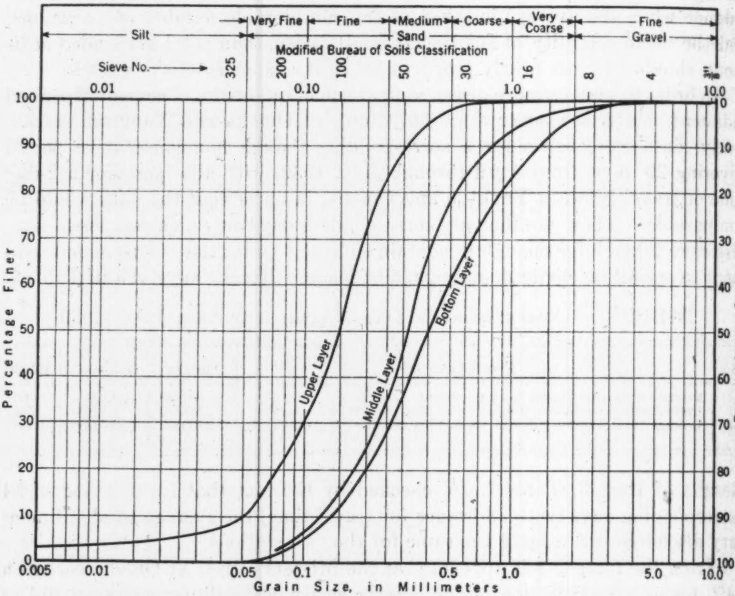

FIG. 10.—SOIL CLASSIFICATION, 10-FT LAYERS OF A BAR ISLAND, GAVINS POINT, YANKTON, S. DAK.

Fig. 10 shows results of analyses of approximately fifty samples taken from a bar island in the river a few miles above Yankton, separated into curves representing successive 10-ft layers from the surface. The lower surface of the bottom layer extends 5-ft below the deepest point of the stream bed at this locality. Results show that nearly all of this sediment is sand, with only 4% of the bottom layer exceeding 2 mm in diameter. Other analyses and field inspections at hundreds of other points throughout the length of the river confirm this general result: There is little gravel in the bed of the Missouri River between Yankton and Rulo except near the mouth of the Platte River.

W. W. Rubey[5] indicates that competence is governed by tractive force formulas rather than the sixth power law when the particles are equal to or finer than a mean diameter of 0.688 mm. Fig. 7 (curves (4), (5), and (6)) and Fig. 10 indicate that Missouri River bed loads may be expected to fall within this range, and therefore should be governed by tractive force formulas.

As has been shown previously, the available slope in the Missouri River in the part being considered is 0.96 ft per mile. It is necessarily assumed that this was sufficient to carry the average bed load with the prevailing discharges prior to the improvement. Roughness coefficients and discharge schedules have been little changed, but mean width is being reduced from 3,650 ft to 725 ft.

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[5] "The Force Required to Move Particles on a Stream Bed," by W. W. Rubey, *Professional Paper 189-E*, U. S. Geological Survey, 1937.

According to the tractive force criterion, competence equal to that which prevailed previously will be retained if the depth-slope product remains constant. Applying this and other conditions, given previously, to the Manning formula, a slope of 0.24 ft per mile would be sufficient to retain competence unchanged.

A similar result is obtained by applying an equation derived from the Schoklitsch formula[6] by Samuel Shulits, Assoc. M. Am. Soc. C. E., and W. E. Corfitzen, Jun. Am. Soc. C. E.:

$$D_{\max} = 4{,}770 \frac{Q\, S^{4/3}}{B} \qquad (1)$$

in which $D_{\max}$ is the diameter of the largest grain size moved in millimeters; Q is the discharge in cubic feet per second; S is the energy-gradient; and B is the width of the river in feet. According to Eq. 1 a slope of 0.285 ft per mile would be sufficient to maintain competence unchanged.

This result is particularly interesting inasmuch as Messrs. Shulits and Corfitzen were attempting to define not only competence but also, apparently, conditions of a non-silting, non-scouring channel at an actual canal location; and they considered the results from this formula to fall within the approved band of results. To quote, "Hence, the maximum particle-size that can be transported as part of the bed load represents the size of material which would create a non-silting, non-scouring channel." According to this interpretation a slope of 0.285 ft per mile would not only provide competence equal to that existing previously, but would also serve, apparently without further analysis, as the equilibrium slope of the contracted channel.

Messrs. Shulits and Corfitzen also quote a table of critical tractive forces observed by the Nuernberg Kulturamt.[7] For ordinary quartz sand from 0.4 to 1.0 mm in diameter the critical tractive force required was 0.25 to 0.30 kg per sq m. Checking these data by ordinary measurement is complicated by the fact that, when taking a bed-load sample, it is impossible to determine whether or not this sample is in process of movement at the time. Therefore, of the various bed sediment samples for Omaha in 1939 (which were taken at discharges averaging 46,000 cu ft per sec) it is not known whether all samples were taken from moving deposits. However, the assumption will be made that the average sample would move at the average discharge. Substituting the known hydraulic characteristics of the river in the vicinity of Omaha into the Manning formula an average depth of 12.9 ft is obtained for discharges of 46,000 cu ft per sec and the present average slope of 0.71 ft per mile for the nine miles in the vicinity of the gaging station. Substituting, a tractive force of 0.53 kg per sq m is obtained as compared to the 0.25 to 0.30 of the Kulturamt results.

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[6] "Bed-Load Transportation and the Stable-Channel Problem," by S. Shulits and W. E. Corfitzen, *Transactions*, Am. Geophysical Union, 1937, p. 457.

[7] "Der Wasserbau," by A. Schoklitsch, Julius Springer, Vienna, Vol. 1, 1930, p. 127.

Examining the average mechanical analysis of bed-sediment samples, at Omaha (Fig. 7), only 1% of the sample was found to be coarser than 1.0 mm. It is probable that this small percentage of larger particles was either moved, during the discharges, much higher than the average that occurred during the period considered, or consisted of particles of shape and specific gravity considerably different from those of ordinary sand particles. For the average discharge it is believed that 1.0 mm represents, as nearly as can be estimated, the largest particle that was moving. Moreover, when mean grain diameter is plotted against discharge on the day the sample was taken, there is comparatively little variation. Mean grain diameter increased less than 50% between 20,000 and 140,000 cu ft per sec, and the three coarsest samples were actually taken at fairly low discharges. The only reasonable explanation for these results is that the Missouri River does not now find, in its bed, sediments as coarse as those which it is competent to move at fairly high discharges. This being the case, the criteria of competence cannot define the future equilibrium of the channel except possibly below the mouths of gravel bearing tributaries.

Capacity.—The Schoklitsch bed-load formula is:

$$G = \frac{86.7\, S^{1.5}}{\sqrt{D}} (Q - B\, q_0) \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

in which G is the bed load in pounds per second; and D is the grain-diameter in inches. In this formula the factor $B\, q_0$ represents the discharge at which bed-load movement begins, which has been shown in the Missouri River 308 report to be very small in this case, in proportion to the average discharge. Therefore, since the discharge schedule will have no changes of material effect, changes in the expression $Q - B\, q_0$, as between the improved and unimproved channel, can be dismissed as negligible. Surprizingly enough, there are no other factors in this equation that show any variation under the conditions stated. Therefore, Eq. 2 must involve some inherent assumption as to the relationship between width and slope, or width and discharge, which entirely unfits it for present uses. Since this equation was principally based on flume experiments, its empirical form renders its application to natural streams questionable unless verification can be made on a large scale.

Formulas probably more applicable to the conditions in the Missouri River were developed by Professor Straub[8] for the "308" report. His applicable equation is:

$$G_w = \frac{u\, i^{1.4}\, Q^{3/5}}{c^{1.2}} (Q^{3/5} - Q_0^{3/5}) \dots\dots\dots\dots\dots\dots (3)$$

in which G_w is the quantity of sediment transported along the stream bed in pounds per unit width of channel; Q_0 is the discharge per unit width of channel (for a slope i) at which sediment transportation begins, c is the roughness coefficient in an open-channel formula, corresponding to the Manning expression $\frac{1.486}{n}$, and u is the "sediment characteristic," depending upon the physical characteristics of the sediment carried. For present purposes Q_0 may be neglected, since, although it may be varied somewhat by the contraction of the channel, it was very small (even prior to contraction) in proportion to the average discharge. The quantity of sediment, G, must be assumed to be the same for the improved or unimproved channel, which is also true of C and the sediment characteristic u. Substituting known characteristics, the reduction in width of the channel from 3,650 to 725 ft is found to cause a reduction in the equilibrium slope from 0.96 to 0.76 ft per mile, as an equilibrium condition for a bed load for which the steeper slope would otherwise have been an equilibrium condition. This accords quite closely with the slope of 0.69 ft per mile which is actually in existence in the 25-mile improved stretch in the vicinity of Omaha at the present time, and with slopes on other improved parts of the Missouri River.

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[8] "Effects of Channel-Contraction Works Upon Regimen of Movable Bed Streams," by L. G. Straub, *Transactions*, Am. Geophysical Union, 1934.

In fairness to Professor Straub it must be noted that his formula was designed for a uniform discharge, whereas the Missouri River discharge is variable; and that he contemplated measurement of widths at low water, whereas the data used are taken for convenience at medium stages. Greater refinements would probably show slightly different results from the foregoing. Investigations such as those summarized in this paper bring to light great deficiencies in basic data as to the principal hydraulic characteristics of major streams. A few more years of study may provide data which will change, materially, the incomplete and short-period records utilized for this paper.

Using all data available, a very approximate estimate could be made as to the time required for a 250-mile stretch of river with an existing slope of 0.96 ft and an equilibrium slope of 0.76 ft to scour its bed by any given amount. This attempt will not be made herein because, although certain parts of the river are practically completed, there are other parts where the channel is only partly formed, and where contracted sections, split channels, and protruding dikes, not yet silted in, form obstructions to flow that interfere with, and delay, the process and would render any estimate in quantitative terms very approximate. However, recent stage discharge relationships show that the process is already under way.

CONCLUSIONS

It is concluded that locally steep slopes are caused by rock protrusions, sharp bends, crossings, tributary influences, and reentrant chutes, besides temporary factors. Long meanders also increase slope at overbank stages. Since the improvement project will reduce the number of crossings, eliminate chutes, and reduce the number of sharp bends, its general shape will tend to allow the river to reach a flatter slope, even though at overbank stages the additional curvature of the improved river is a counteracting influence.

It is concluded that Fort Peck Reservoir will probably not have any important effect on the mean slope of the stream; and that future irrigation development, although tending toward aggradation, will probably be of slight importance.

The improved and unimproved channels have equal roughness coefficients in the reaches examined; and the improved channel has a greater mean velocity relative to the slope, due to its greater hydraulic radius. Improved sections of the river now maintain themselves at less than the present (or the 1930) average slope.

It is concluded that during 1939 the Missouri River in the improved section at Omaha had a greater capacity for suspended sediment than it had on the unimproved section at Yankton, despite a considerably flatter slope.

An analysis of comparative bed-load competence of the improved and unimproved rivers shows that the tractive force criterion, and an equation derived from the Schoklitsch formula by Messrs. Shulits and Corfitzen, both indicate that a future equilibrium slope of less than 0.3 ft per mile would be sufficient to retain competence unchanged in the contracted channel. Since the Missouri River does not now find in its bed sediments as coarse as those which it is competent to carry at high discharges, it is concluded that bed scour of this stream is limited by capacity, and that formulas for competence will not define an equilibrium condition in this case. The Schoklitsch bed-load formula is inapplicable to the change of regimen of the Missouri River.

Applications of Professor Straub's bed-load formula indicate that, if other things remain equal, contraction of the Missouri River should increase its capacity for bed load by 38%, or reduce from 0.96 to 0.76 ft per mile the average slope at which a section could carry an unchanged bed load. Slopes on parts of the river first contracted correspond fairly well with this analysis. In view of the great length of river involved, no appreciable change in mean slope can be expected; but a progressive lowering of the bed of the stream through scour will probably continue for some time.

DISCUSSION

Samuel Shulits,[9] Assoc. M. Am. Soc. C. E. (by letter).—Captain Whipple's paper merits attention as an analytical treatment of a problem in river regulation. Too frequently river engineers opine qualitatively on the behavior of rivers—one reason why the science of river engineering has moved forward so slowly.

The assumption of an average slope as the basis of the improvement of the Missouri River seems untenable. The word "average" implies a time interval that must affect the magnitude of the "average." It is not easy to accept the tenet of an average slope when the author states that the slope had been increasing for several decades prior to the improvement program and that there have not been enough data to predict the future trend. An average slope does not "jibe" with the recognized fact that the eventual Missouri River slope depends primarily on its sediment-carrying characteristics, which can scarcely be related to an average gradient. If the long view is taken, an average has no significance; and if a short range is taken, the application of the average to a specific problem seems precarious. The writer believes that there is no basis for the rule of thumb that a river has an average slope to which it must return regardless of the regulation measures adopted.

Although detailed studies have not been made for the paper, the effects of Fort Peck Reservoir and of irrigation projects are deemed to be of negligible future influence between Sioux City, Iowa, and Rulo, Nebr. The present state of knowledge is inadequate to permit such conclusions. Is it not possible that a very slow progressive degradation may occur below Fort Peck Reservoir due to a desilting of silt-laden flood flows in the reservoir? The Bureau of Reclamation is measuring, very carefully, the morphologic behavior of the Colorado River below Hoover Dam almost all the way to the Mexican border.

Of particular interest is the application to the Missouri River of some of the findings since 1930 in the field of bed-load formulas. Since these formulas are the keystone of the conclusions on the equilibrium slope of the Missouri River, it is incumbent to subject them to a rigorous critique.

The Schoklitsch bed-load formula (Eq. 2) is imputed to be based principally on flume experiments and it is asserted that its empirical form renders its application to natural streams questionable unless verification can be made on a large scale. Then the belief is expressed that the Straub formula (Eq. 3) is more applicable to the conditions in the Missouri River. There is little evidence, if any at all, for these contentions because: First, as far as the writer is aware, the Schoklitsch formula is the only one for which there is any large-scale verification and it has been confirmed for three rivers;[10] second, the Schoklitsch formula has a good rational basis, whereas the Straub formula is the result of substituting the Manning velocity formula into the du Boys bed-load formula, and the latter is predicated on an assumed manner of sand motion known to be untrue;[10] and third, both formulas found their claims and coefficients on flume investigations—the numerous tests of G. K. Gilbert (mainly) in the case of Professor Schoklitsch and the comparative few of F. Schaffernak in the case of Professor Straub. These three facts render untenable the author's assumption that the Straub formula is more representative of Missouri River conditions; besides there is no physical evidence to substantiate the claim for the Straub formula.

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[9] Hydr. Engr., U. S. Engr. Office, Charleston, S. C.

[10] "The Schoklitsch Bed-Load Formula," by S. Shulits, *Engineering*, June 21 and 28, 1935, pp. 644–646, and 687.

The surmise is incorrect that the Schoklitsch formula contains some inherent assumption of the relation of width to slope or discharge. The formula is based almost entirely on the Gilbert measurements, using the bed load and discharge per unit width of flume, as is the case with all such formulas. The difficulty here, not remedied by any bed-load formula known to the writer, is the fact observed in rivers[11] that movement occurs only over a part of the bed. There is no movement near the sides or banks. The width of the moving strip seems to vary directly with the discharge, but this is not incorporated into the Schoklitsch formula.[12]

The point is made that the Schoklitsch formula does not have more factors to vary for given conditions and yet it is assumed that G, Q, and u in the Straub formula must be the same for the improved or unimproved channel. This leaves the roughness coefficient, $\frac{1.486}{n}$, and the width as the only variables to reflect altered channel conditions. Since the improved and unimproved channels are stated to have equal roughness coefficients, the width remains the only variable factor, and it occurs in the Schoklitsch formula. It may be inferred, therefore, that the Schoklitsch formula is also applicable to the Missouri River problem under discussion. Actually, the magnitude of n, when the bed load is in motion, can be 20% to 35% over its value at the critical discharge of incipient movement,[13] a possibility neglected in this paper.

The computation of the equilibrium slope is based on the valid assumption that the critical discharge is negligible. This changes the Straub formula to:

$$G = \frac{u}{c^{1.2}} S^{1.4} Q^{1.2} \qquad (4)$$

If the Schoklitsch formula is treated in the same way,

$$G = \frac{86.7}{D^{0.5}} S^{1.5} Q \qquad (5)$$

Current knowledge of solids transportation is still too meager to permit a positive decision for 1.4 instead of 1.5 for the exponent of S or 1.2 instead of 1 for the exponent of Q. Consequently little can be said for or against the one equation that cannot be applied to the other—and any choice must be arbitrary to a considerable degree. The situation is similar to the choice exercised in the use of exponential velocity formulas, always a matter of personal familiarity with the formula and its coefficients. With regard to Eqs. 4 and 5, the selection simmers down to whether one prefers to choose a value of u (which is determined by the grain diameter) and estimate c (a roughness coefficient which, for some reason, is made to be $\frac{1.486}{n}$), or to select the effective grain diameter (D) in the Schoklitsch formula.

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[11] "Beobachtungen ueber Geschiebefuehrung," by S. Kurzmann, Munich, 1919; also "Untersuchung ueber den Schwebestoff und Geschiebefuehrung des Inn naechst Kirchbichl (Tirol)," by L. Muehlhofer, *Die Wasserwirtschaft*, 1933, p. 1.

[12] "Der Geschiebetrieb und die Geschiebefracht," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, No. 4, 1934, p. 37.

[13] "Beitraege zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen fuer Stroeme, Kanaele und geschlossene Leitungen," by A. Strickler, *Mitteilungen des Amtes fuer Wasserwirtschaft*, No. 16, Bern, Switzerland, 1923.

The resemblance between the two formulas becomes even greater upon closer analysis of the coefficient u. It can be shown from experimental data that:

$$u = \frac{K}{D^{0.75}} \qquad (6)$$

in which K is a constant, equal to 111,000 if D is expressed in millimeters. Eq. 4 then becomes:

$$G = \frac{K}{c^{1.2} D^{0.75}} S^{1.4} Q^{1.2} \qquad (7)$$

so that there is $D^{0.75}$ to compare with $D^{0.5}$; and the case is just as strong for the one as for the other. Since considerable latitude is possible in practice in the choice of c or n for rivers like the Missouri, close agreement should be possible between Eqs. 5 and 7 by using in the latter a value of the roughness coefficient well within the limits of accuracy of the coefficient. Thus it is not just to condemn the Schoklitsch formula for the Missouri River and to uphold the Straub formula as "probably more applicable."

The writer would be interested to know what value of u was used and how it was obtained in the section, "Capacity"; what c or n was employed; and then how the equilibrium slope of 0.76 ft per mile was computed.

Leo M. Odom,[14] Assoc. M. Am. Soc. C. E. (by letter).—Because of the tremendous expenditures that have been made on the improvement of the Missouri River, it is highly gratifying that the profession has received the benefit of a discussion of the principles upon which the general improvement is based and of the results obtained.

Improvement of the Missouri River for navigation is certainly a task of magnificent proportions. The totals are overwhelmingly immense and the design in practically all of its phases involves forces and actions of Nature, the understanding of which (in spite of all the mathematical gymnastics and lengthy dissertations that an intensely scientific age has produced on the subject) seems almost as remote as it did when General Humphreys commenced his Topographical and Hydrographical Survey in 1850.

In arriving at a solution for river control, as in most of the problems that beset the river engineer, he may utilize any and all of the following approaches: (1) Cut and try; (2) judgment based on observation of similar problems; (3) mathematical formulas; and (4) small-scale models. It seems to the writer that all of the writing in the past few years has been about approaches (3) and (4), and most of the work has been performed by the use of approaches (1) and (2).

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[14] Chf. Engr., Dept. of Public Works, State of Louisiana, Baton Rouge, La.

The author sets forth the general plan of improvement that has been evolved and gives briefly the methods and procedure by which the general plan is to be accomplished.

It was decided at the beginning of the improvement that, although the river meandered widely in Nature, its average slope was still ample to provide self-cleaning velocities and that the course of improvement for low-water navigation should be restricted, in so far as practicable, to confinement of the low-water flow to a well-defined channel, meanwhile revising the alinement so as to obviate direct attack and crossing bars. These considerations led the Engineer Department to propose changing the alinement of the river to a continuous series of sweeping curves. The adoption of this proposal embarked the federal government on a project which, of its kind, is the largest that man has ever attempted. The determination of the controlling radii and central angles of the curves to be used was undoubtedly done in the broad laboratory of Nature itself. All that remained was to fit in these curves as economically as possible, and the general plan was complete. The writer believes that the method of cut and try, combined with judgment, was used practically exclusively in evolving this plan.

The author presents a good summary of the type of material borne by the stream, both as to bed load and suspended sediment. The writer cannot follow his discussion of the mechanics of bed load and suspended load movement. The suspended load to be found in a cross section is dependent on the turbulence to a much greater extent than on the velocity, and, although the velocity may give a rough index of the turbulence, it does not do so necessarily. Furthermore, the movement of the major percentage of the detritus progresses downstream by "trading" and not as a continuous process.[15] With these two facts in mind, the author's discussion and conclusions in some cases seem vague. The detritus content in any cross section would not necessarily indicate that the total quantity is progressing downstream since it is highly probable that a large part of such detritus is in the process of transverse movement.

The author's comparison of the results obtained by the improvement with the answers obtained by formulas is interesting and enlightening. The writer had never seen the first two formulas before and was, therefore, not disturbed to learn that they will not work. As to the Straub formula, it is rather a disappointment to learn that it will not work on the river from observations of which it was derived.

The author's statement that the Straub formula was designed for uniform discharge only is in disagreement with the derivation of the formula given in the "308" report on the Missouri River. It is stated therein that the slope is assumed to remain constant; variation of depth and discharge is not supposed to affect the accuracy of results. It is not exactly clear what effect the stage of the river at which widths were measured could have on results given by the formula as cited, since the formula is intended to give the quantity of sediment transported per second along the stream bed in pounds per unit width of channel.

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[15] "Mineral Composition of Sands from Mississippi River Tributaries," by R. Dana Russell and Leo W. Hough, *Proceedings*, Geological Soc. of America, 1937, p. 109.

Two glaring weaknesses that are apparent in most river work today are revealed by this paper. One is the fact that the location and number of continuous observation stations frequently are not suitable for design requirements. Another weakness is in the practice of constructing improvement works without an adequate knowledge of what was occurring in the section of the river prior to the improvements. This practice is indicated in the paper in that the author is forced to compare the action in a 25-mile section of the improved river with a different 10-mile stretch of unimproved river at a different discharge. A comparison such as that is of doubtful value. The only logical way to find out whether improvements are accomplishing the desired end would be to have sufficient data on the improved section taken prior to the improvement to provide a basis for comparison with its condition after improvement. The adoption of a common-sense theory of improvement such as was followed in this case is good practice; but all theories, of whatever origin, bear careful checking in the field. The only way that engineers can be absolutely sure that their improvement works are accomplishing the desired results is to make a detailed "before and after" comparison, on the section of river improved.

The author has examined the value of the formula approach to design of river-control works. It would be very enlightening to have a similar examination of the value of small-scale models, since, undoubtedly, they have been used in connection with the design of works on the Missouri River.

E. W. Lane,[16] M. Am. Soc. C. E. (by letter).—Considering the magnitude of the Missouri River improvement project, very little has been written about it, and hydraulic engineers will be very much indebted to the author for his interesting exposition of the principles underlying it. This improvement probably constitutes the most extensive application of the contraction of streams as a means of deepening them for improving their navigability, a procedure which is one of the oldest in the science of hydraulic engineering. A search, by a hydraulic engineer, of the ancient writings of Egypt, Mesopotamia, and China would probably show earlier writings on the principles involved, but the earliest known to the writer is by Chi-Husn Pan in the latter part of the sixteenth century.[17] Mr. Pan argues at length in favor of confining the Yellow River between embankments and making it deepen its channel by scouring out its bed.

Although the Chinese apparently knew many of the principles of river control before the Europeans, this knowledge has had no influence on the development of the present-day science. So far as is known, the earliest work leading to modern knowledge was that of Guglielmini in the latter part of the seventeenth century. He stated that the greater the quantity of water that a river carries, the less will be its fall, which statement was used as the basic principle of plans for contracting rivers. The subject seems to have been a matter of frequent discussion in Europe before the latter part of the eighteenth century. At this time Paul Frizi and M. Gennete[18] argued in favor of keeping rivers in a single channel rather than having them branch near the mouth, as the single channel would have less fall and cause less flooding.

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[16] Prof., Hydr. Engr., State Univ. of Iowa, Iowa City, Iowa.

[17] "Review of River Protection," by Chi-Husn Pan (translation by Hsing and Wong).

In the United States the principles involved were the subject of active discussion in connection with the improvement of the Mississippi River nearly a century ago. At that time it was proposed to deepen the Mississippi River for navigation and flood control by preventing the overflow of the banks at high-water time by means of levees. This discussion developed into one of the greatest of engineering controversies in connection with the improvement of the passes of the Mississippi River by the late James B. Eads, F. Am. Soc. C. E., and the confinement principle of improvement was adopted as the basis of the early work of the Mississippi River Commission. Although successfully applied to the improvement of the mouths of the river, it was not a success when applied, as a means of flood control, to the entire river, and was superseded by the "levees only" principle. The success in one case and failure in the other was largely due to the tremendously greater quantities of material involved in the case of the entire river, and the lack of a quantitative analysis of the problem, for, although data for an exact solution are still unavailable, it is possible to show by a computation of the quantities that would be moved and the time required, using data then available, that its application to the entire river would fail. The difference between the quantitative relations given in the paper and the almost entire lack of discussion in quantitative terms during the early Mississippi controversy show that appreciable progress has been made in the science, but the qualitative nature of the author's conclusions indicates that considerable progress is still necessary before a prediction can be made of the time interval required to bring about the desired results.

It is believed that the author would have clarified somewhat the picture of the results of the contraction works if he had amplified the statement in his "Conclusions" that "In view of the great length of river involved, no appreciable change in mean slope can be expected." In the "Synopsis" one reads the statement that the mean slope of the river will eventually decrease through the operation of the contraction works, and later quantitative relations are given tending to show that the equilibrium slope would be reduced from 0.96 to 0.76 ft per mile, a change of 21%, which is in agreement with the 0.69 ft per mile slope observed in an improved stretch. When these statements are followed in the "Conclusions" of the paper, with no further explanation, by the statement that no appreciable change in mean slope can be expected, it seems probable that some readers of the paper may be confused. If this is the case, perhaps the following discussion will aid in clearing up the picture of what will probably take place.

It appears to the writer that there are two more or less distinct effects of the construction of the contraction works: (1) The change of shape of the channel from a wide shallow one to a narrower and deeper one, and (2) a change of the gradient of the stream as it tends to reach equilibrium under the changed conditions. The first of these will tend to take place relatively rapidly, especially the major part of the total change, but the second will be active for a much longer period, and equilibrium will not be reached for many years even from a practical standpoint, for theoretically equilibrium would never be achieved.

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[18] "An Investigation of the Transportation of Bed Load by Flowing Water," by F. T. Mavis; thesis presented to the Univ. of Illinois, Urbana, Ill., in 1935, in partial fulfilment of the requirements for the degree of Doctor of Philosophy (see p. 110).

To carry the flow of the river at a width of 725 ft, which formerly flowed over a width of 3,650 ft, without raising the water surface, will require a deepening of the bed. Fig. 11 shows the depths required for various discharges using the slope of 0.96 ft per mile in both cases. The difference in depth at any given discharge that would give the same water surface elevation for these two conditions is indicated by the vertical distance between the two lines. Thus, to obtain, at bankfull elevation, the same discharge in the contracted as in the uncontracted channel (at which stage the unimproved stream discharged about 80,000 cu ft per sec) would require a deepening of the bottom of about 16.5 − 6.3 = 10.2 ft. Under these conditions the area of the channel at bankfull level would be reduced from 23,000 sq ft for the unimproved channel to 12,000 sq ft for the improved channel, a difference of 11,000 sq ft. The area of excavation would be 725 × 10.2 = 7,400 sq ft and the area of fill (3,650 − 725) 6.3 = 18,400 sq ft. The space that would be filled in with sediment under the improved conditions, if it fills to former bank levels, is therefore 67% more than the volume that it is necessary to excavate in the contracted section. Since the competence of the stream is much greater than required to move the largest bed particles, if the size of the bed material does not increase substantially with depth, it is not unreasonable to expect that considerable of the deepening due to contraction will proceed rapidly, as the material excavated at one point along the channel can be deposited outside the contracted section a short distance downstream. However, the river carries great quantities of material in suspension which will deposit in, and rapidly fill up, the parts of the old channel outside the new channel line. As this filling takes place, the deepening of the contracted channel will slow up, and

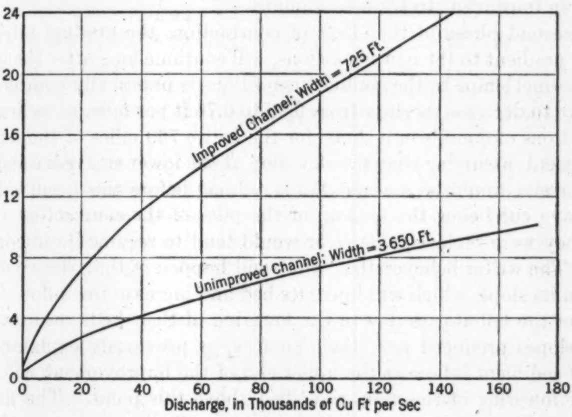

FIG. 11

although the first part of the deepening will probably proceed quite rapidly it may be some time before the final depth is reached.

The deepening of the channel will result in a cutting down of the bottoms of all tributary streams that are not restrained by rock ledges or deposits of coarse material. After the adjustment of channel cross section is substantially completed, at high discharges the water levels in the Missouri River will differ little from those at present; but at lower discharges in the contracted channel they will be considerably lower than in the unimproved conditions, and the bottoms of tributary streams will tend to cut down to lower levels, due to the increased fall that will be caused near their mouths as a result of the lowering of the bed in the main river. This action will bring into the main river an additional load of material that will have to be deposited in the unimproved channel area if the improved channel is to continue to deepen. This action will further slow up the deepening of the bed of the improved channel. There appears to be no way of making a close estimate of the rate at which lowering of the bed of the improved channel can be expected, except by using the experience in improved stretches as a guide.

The second phase of the effect of contraction, the gradual adjustment of the river gradient to the new conditions, will continue long after the adjustment of the channel shape to the confined conditions is practically complete. If the river were to decrease its slope from 0.96 to 0.76 ft per mile, as indicated by the computations of equilibrium slope, for the entire 760 miles of the length of the improvement, assuming that the elevation at the lower end remained the same, the upper end would be lowered 152 ft. Long before this occurred, the river would have cut below the bottom of the piles of the contraction works, and, unless they were replaced, the river would tend to resume its unconfined condition. The writer believes that what will happen is that the river will tend to flatten its slope, which will lower its bed and increase the inflow of sediment load from the tributaries due to the lowering of their beds resulting from the steeper slopes produced near their mouths, as previously explained. An increase of sediment inflow at the upper end of the improvement will also result from the lowering of the main river bed above this point. The flattening of the grade due to confinement will continue until the sediment inflow will increase and become equal to the capacity of the stream to transport material in its confined condition, and when this condition is reached no further decrease in slope will be observed. It is the writer's belief that this condition of substantial equilibrium will occur before much lowering of the stream bed due to changed slope has occurred, although it is probable that the lowering at the upper end will be sufficient to be easily detectable in the stage-discharge relations at that point. It is doubtful if the mean slope of the river will be changed more than 2% or 3% for many years to come, which substantially agrees with the author's statement that the change will not be appreciable.

If this be true, what then is the "equilibrium slope" computed by the author? This is the slope at which the confined river would carry the same load of material as the unconfined river carried at its natural slope. Soon after the works are constructed, however, the load to be carried will be increased and a slope greater than the computed equilibrium slope will be required.

The author states that the Schoklitsch formula could not be used to compute the equilibrium slope, which is true for the form given in Eq. 2. An examination of the paper, however, shows that this form was obtained by Schoklitsch by multiplying the load and discharge per unit width by the width, thus obtaining the equation in terms of the total load and total discharge. It can be expressed as well in terms of the load and discharge per unit width, which gives

$$G_1 = \frac{86.7\, S^{1.5}}{\sqrt{D}} (Q_1 - q_0) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which G_1 and Q_1 are the sediment discharge per unit width and water discharge per unit width, respectively. Using this equation with the formula for q_0 given by Schoklitsch, it is possible to compute the author's equilibrium slope by this formula.

In discussing the formula developed by Professor Straub, the author states that the quantity of sediment must be assumed to be the same for the improved or unimproved channel. As previously shown, the quantities of sediment would not be the same in these two cases, and hence the formula will not give the slope that will actually be obtained in the river; it will give a flatter slope. The author also states that the sediment characteristics must be the same in the contracted as in the uncontracted stream. If the size of material increases with depth as shown in the analysis of samples given in Fig. 9, this condition also is not fullfilled, since coarser material would be encountered as the channel scoured deeper, and a steeper slope than computed would result from this cause.

TABLE 3.—COMPARISON OF BED LOAD AND SUSPENDED SAND DISCHARGE; MISSOURI RIVER, AT OMAHA, NEBR.

(All Sediment Discharges Are in Thousands of Tons per Day)

Size of Sand Particles, in mm		Ten-Foot Depth 29,400[a]			Fifteen-Foot Depth 58,000[a]			Twenty-Foot Depth 93,800[a]			Twenty-Five-Foot Depth 136,000[a]		
From	To	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)
0.05	0.075	33.3	7,750	0.43	67.4	25,350	0.27	110	76,100	0.14	160	161,500	0.10
0.075	0.10	83.1	6,600	1.26	169.5	16,750	1.01	278	41,100	0.68	405	71,600	0.57
0.10	0.15	224.	6,440	3.48	465	21,300	2.18	767	41,800	1.83	1,120	75,300	1.49
0.15	0.25	765	4,120	18.5	1,639	15,950	10.3	2,740	35,700	7.69	4,020	77,800	5.17
0.25	0.30	306	453	67.5	678	1,880	36.0	1,145	4,920	23.3	1,690	10,650	15.85
0.30	0.40	98.2	44	245	227	188	121	390	605	64.4	581	1,468	39.60
0.40	0.60	38.4	2	1,870	99	0	∞	175	20	874	262	962	27.2
0.60	0.80	4.8	0	∞	15.1	0	∞	27.6	0	∞	42.8	0	∞
0.80	1.20	2.2	0	∞	10.8	0	∞	21.6	0	∞	33.7	0	∞
Total computed		1,555	25,409	6.1	3,371	81,418	4.1	5,654	200,245	2.8	8,315	399,280	2.1
Observed			39,000			70,000			230,000			425,000	
Ratio 1[b]			4.0			4.8			2.5			2.0	
Ratio 2[c]			65.1			118.5			85.7			93.8	

[a] Cubic feet per second. [b] Ratio of computed bed load to observed suspended load in per cent. [c] Ratio of computed suspended load to observed suspended load in per cent.

There is considerable reason to question, however, whether accurate results can be obtained in computing the effect of contraction works on the Missouri River by considering only the bed load. The suspended load of sand carried by this river is very large, as shown by Fig. 9. A comparison of this observed sand load at Omaha and Yankton with the bed loads computed according to the Schoklitsch formula for a wide range of discharges, as shown in Tables 3 and 4, indicates that the bed load is probably never as much as 10% of the suspended load of sand in the Missouri River. Of course, it is a much smaller part of the total load, but the sand particles settle rapidly and therefore have a much larger effect on stream formation than the silt and clay sizes. The suspended sand load no doubt consists, on the average, of finer material than the bed load, and some sizes carried by bed load are moved in suspension in negligible quantities, if at all. In Tables 3 and 4 are also given the computed bed loads for various size ranges and the quantities carried in suspension of these same size ranges computed by the method developed by A. A. Kalinske,[19] Assoc. M. Am. Soc. C. E., and the writer. This comparison shows that the suspended loads are greater than the bed loads for all sizes less than about 0.3 mm at Omaha and 0.4 mm at Yankton, and for high discharges, where the most active bed formation exists, for all sizes less than about 0.6 mm at Omaha and 0.5 mm at Yankton. In order to indicate the reliability of the relation used in computing the suspended load, a comparison is also given between the computed total load of sand and the observed load, as shown in Fig. 9. This comparison shows a reasonably close agreement between computed and observed suspended sand loads at Omaha, and somewhat less agreement at

TABLE 4.—COMPARISON OF BED LOAD AND SUSPENDED SAND DISCHARGE; MISSOURI RIVER AT YANKTON, N. DAK.

(All Sediment Discharges Are in Thousands of Tons per Day)

SIZE OF PARTICLES, IN MM		FIVE-FOOT DEPTH 32,700[a]			EIGHT-FOOT DEPTH 71,800[a]			TEN-FOOT DEPTH 104,000[a]			TWELVE-FOOT DEPTH 141,000[a]		
From	To	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)	Bed load (1)	Suspended load (2)	Col. 1 ÷ Col. 2 (%) (3)
0.05	0.1	261	12,880	2.0	598	43,500	1.4	878	73,500	1.2	1,200	115,000	1.0
0.1	0.2	680	5,120	13.3	1,635	16,300	10.0	2,420	39,600	6.1	3,330	62,000	5.4
0.2	0.3	522	440	119	1,350	2,330	57.8	2,040	4,620	44.0	2,830	9,500	29.8
0.3	0.4	405	88	461	1,145	390	294	1,760	730	241	2,460	2,300	107
0.4	0.5	350	0	∞	1,105	0	∞	1,730	240	720	2,450	380	645
0.5	0.7	119	0	∞	478	0	∞	775	0	∞	1,080	0	∞
0.7	1.0	13	0	∞	113	0	∞	195	0	∞	291	0	∞
1.0	2.0	0	0		55	0	∞	133	0	∞	222	0	∞
Total computed		2,350	18,528	12.7	6,479	62,520	10.4	9,931	118,690	8.3	13,863	189,180	7.3
Observed			28,000			165,000			330,000			425,000	
Ratio 1[b]			8.4			3.9			3.0			3.3	
Ratio 2[c]			66.2			37.9			35.9			44.6	

[a] Cubic feet per second. [b] Ratio of computed bed load to observed suspended load in per cent. [c] Ratio of computed suspended load to observed suspended load in per cent.

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[19] "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, 1939, Pt. IV, p. 637; also "Engineering Calculations of Suspended Sediment," *ibid.*, 1941.

Yankton, but amply close to indicate a great preponderance of suspended over bed load. In view of the data shown in Table 3, it does not seem unreasonable to believe that in the Missouri River the suspended load may be as potent as the bed load in the channel-forming process.

There seems to be little doubt that the confinement of the Missouri River will cause an increased flow of heavy sediment in it, and that this increased flow will be carried out into the Mississippi River. What will be the effect of thus adding to the load of that stream? Fortunately, the improvements previously built in the middle Mississippi River have been causing, over a long period, a lowering of the bed of this stream up to Chain of Rocks, a rock ledge across the river just below the mouth of the Missouri River. The extent of this lowering has become such that the water depths over the rock ledge have been greatly decreased and a by-pass canal with locks, or a movable dam with locks, is badly needed. The increase of the sediment load from the Missouri River, therefore, will tend to offset this lowering. Whether or not it will be sufficient to reverse the trend and cause a raising of the water levels downstream is a question; it is possible but unlikely. An increase in deposit, however, must occur at some point below the mouth of the Missouri River. Such effects are usually not considered in making river improvements, and their results usually do not become apparent for many years. Sometimes they require expensive corrective measures, as shown by the proposed improvements at the Chain of Rocks, previously cited.

ELLIOTT J. DENT,[20] M. AM. SOC. C. E. (by letter).—From Sioux City to Rulo the proposed channel will have a length of 245.6 miles, and in 1938 the total fall was 236 ft, or an average of 0.96 ft per mile. In his closing paragraph the author states that the improved channel will be able to carry 38% more bed load than the natural channel, or that it could carry the present bed load with a slope of only 0.76 ft per mile which corresponds to a drop of 186.5 ft in the reach under discussion.

If this section of the Missouri River is contracted and otherwise improved so as to increase its bed-load carrying capacity, it is logical to expect erosion at the upper end of the reach until a capacity load has been picked up. This increased bed load should then be carried through the remainder of the section and dumped upon the next lower reach at Rulo. To increase the bed load of the river channels materially, from Rulo to the Gulf of Mexico, is a matter that deserves very serious consideration.

WILLIAM WHIPPLE, JR.,[21] ASSOC. M. AM. SOC. C. E.—The interest shown by those who contributed discussions is much appreciated. Mr. Odom is entirely correct in assuming that the Missouri River control works were designed largely by "cut and try," combined with judgment, and that even now there is a considerable lack of suitable observation stations along the Missouri River (as well as most other rivers). However, practice in this respect has greatly improved; and long-range engineering investigations of the Missouri River are now as thorough as available funds will permit.

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[20] Col., U. S. Army (*Retired*), Cons. Engr., Washington, D. C.

[21] Major, Corps of Engrs., U. S. Army, Fort Belvoir, Va.

Mr. Odom's point concerning the applicability of the Straub formula to conditions of non-uniform discharge depends upon the rate of change of discharge. Variations in discharge on the Missouri River above Rulo are usually irregular enough to be accompanied by slope changes. Otherwise the formula is applicable.

Professor Lane is to be thanked for his clear explanation of the writer's rather abrupt conclusion that no appreciable change in mean slope for the river as a whole can be expected. He shows that there are difficulties in predicting the time required for the river to scour down to a given slope, by reason of increased sediment inflow from the tributaries, as the main stream lowers its bed. Another reason for caution in this respect lies in the hydrologic irregularity of the Missouri River system. In this basin, sediment conditions differ so widely from year to year that naturally occurring changes may affect the result of artificial works over a period of several years.

Professor Lane questions the assumption made by the writer that the quantity of sediment and the sediment characteristics of the river should be considered as unchanged by the contraction works. This assumption would be unjustified when applied to the river as a whole; but it is a close approximation when applied to the short stretches first contracted. As shown by Table 1, all contracted sections initially assume low slopes, thus verifying the formula applied and the assumptions made. The reach near Omaha had a flat slope before the improvement; but this section was already narrow. The increase in all of these slopes by 1940 was due to the contraction of other portions of the river, with a consequent increase in quantity of sediment. If the quantity of sediment and sediment characteristics of the river as a whole did remain unchanged, the entire river someday would inevitably assume its reduced slope within a deep, canyon-like valley. Such an eventuality is certainly not foreseen by this writer. However, the presence of rock ledges all the way across the channel at some points does raise the possibility that the river may erode a deep channel below one of the ledges, unhindered by increased channel scour from above that point.

The writer agrees with Colonel Dent that it is a serious matter to increase the bed load which must be carried from Rulo to the Gulf of Mexico. However, these channels have been improved concurrently with the Missouri River above Rulo, and should be able to handle the increase.

Mr. Shulits centers his attention on the application of bed-load formulas involving capacity to the change in regimen of the Missouri River brought about by the contraction works. Specifically he objects to the conclusion of the writer that the Straub formula is "probably more applicable" than the Schoklitsch formula to this particular problem. He states that there is no physical evidence to substantiate the Straub formula, and that the Schoklitsch formula "has been confirmed for three rivers," has a good rational basis, and is essentially the same as the Straub formula except for the exponents of two variables.

It is true that the Schoklitsch formula has been applied to three rivers; but such confirmation as was given in the reference cited was partial and approximate. Moreover, the three rivers to which it was applied are the

Danube at Vienna, the Inn, and the Tiroler Ache (all in Austria), which are not similar to the Missouri; and contraction of channel width was not a factor covered by those investigations.

Mr. Shulits demonstrates that the Straub and Schoklitsch formulas can be reduced to forms which are generally similar, and draws the conclusion that the Schoklitsch formula must also be applicable to the problem. However, he did not state any results obtained by applying this formula. In a strict sense, neither formula should be used in the form in which he gives it. The result of channel contraction, as Professor Lane states, must be computed in terms of the unit width of the channel and not of the channel as a whole. However, these equations can be considered to apply to unit widths of a wide channel. If such a channel is contracted, the water discharge per unit width must increase. According to the Straub formula (Eq. 4), the bed-load discharge would increase more than proportionately to the water discharge; but according to the Schoklitsch formula (Eq. 5) the proportion would remain the same. Therefore, considering the total water discharge, the Schoklitsch formula (Eq. 5) would lead to the conclusion that contraction of the channel could not increase the scouring power of a stream or reduce its equilibrium slope in the contracted portions. This conclusion is in violation of the observed facts; and it is for this reason that the Schoklitsch formula was rejected for the purpose. The Straub formula, on the other hand, is well substantiated by the facts.

Mr. Shulits inquires regarding the method of computing the equilibrium slope of 0.76 ft per mile, particularly the values of u and c or n used. In this instance it was not necessary to substitute numerical values for either coefficient, since it had previously been shown that they are unaffected by changes in channel width. Since it is very small in proportion to the average discharge, Q_0 is omitted. This reduces Eq. 3 to:

$$G_w = K\, i^{1.4}\, Q^{1.2} \qquad (9)$$

for bed-load discharge in a unit width of channel. Let W be the channel width; and conditions prior to and after contraction may be denoted by subscripts 1 and 2, respectively. If the slope reduces after contraction so as to carry the same bed load as before, and the water discharge schedule remains unchanged,

$$G_{w1}\, W_1 = G_{w2}\, W_2 \qquad (10)$$

and

$$Q_1\, W_1 = Q_2\, W_2 \qquad (11)$$

Substituting:

$$i^{1.4}{}_2 = \left(\frac{W_2}{W_1}\right)^{0.2} i^{1.4}{}_1 \qquad (12)$$

Substituting 3,650 for W_1, 725 for W_2, and 0.96 for i_1, $i_2 = 0.76$ ft per mile, the new equilibrium slope for the given conditions.

On the subject of average slope, Mr. Shulits has apparently misunderstood the writer. The view, which he condemns, that the river has an average slope to which it must return, regardless of regulation measures adopted, will not be found in this paper. The writer stated that it was assumed in planning the improvement of the Missouri River that its average slope represented approximately a state of equilibrium under conditions then prevailing; and reasons were given for the assumption. As a matter of fact, at the time this improvement was planned, the river was about to reverse its established trend and revert to an aggradation of the channel similar to conditions observed approximately forty years before. An assumption that the existing trend would continue would have been more in error than the actual decision to base plans upon existing conditions. Perhaps the application of this conclusion to a specific problem was precarious, as Mr. Shulits states, but unless a reliable method can be developed to foretell for a decade or more in advance the sedimentary characteristics of a river valley, empirical judgments, however "untenable" theoretically, will continue to be relied upon.

Mr. Shulits points out that a slow degradation of the river bed may tend to occur below Fort Peck Reservoir, due to a desilting of the silt-laden flood flows in the reservoir. There will also be other effects, such as the reduction of the dangerous ice-jam floods on the Missouri River in North Dakota. Data for the solution of the entire problem of the effects of this reservoir have been gathered for about six years, and necessary studies are being made.

However, the Missouri River above Fort Peck carries comparatively little sediment; and Fig. 1 shows the many hundreds of miles of sediment-bearing tributaries that enter between that point and the head of navigation at Sioux City. Long before a given flow reaches the contracted section, any deficiency in the heavier sediments, due to Fort Peck storage, will have been picked up through erosion of banks or bed. Deficiencies in lighter sediments are not an important influence, since these sediments cannot deposit in appreciable quantities in the bed of the contracted channel. It is for these reasons that it is tentatively concluded that Fort Peck Reservoir operation will probably not have an important bearing on the future slope of the Missouri River within the limits considered in this paper. Any influence that does occur will probably be due to alternations in the discharge schedule by Fort Peck storage and not to retention of sediment in the reservoir.

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RIGID FRAMES WITHOUT DIAGONALS (THE VIERENDEEL TRUSS)

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WITH DISCUSSION BY MESSRS. JAROSLAV J. POLIVKA, W. A. MILLER, A. A. EREMIN, YVES NUBAR, AND LÉON G. RUCQUOI.

SYNOPSIS

Attention is called herein to a simple method for the design of Vierendeel trusses with chords of equal moment of inertia, the loads being applied only at the joints. The method is based on the fact that, for these trusses, a point of contraflexure exists in each vertical and that, according to theory checked by photoelasticity, this point is at the midheight of the vertical.

INTRODUCTION

Until recently, the Vierendeel truss has been adapted mostly to designs of highway bridges, railway bridges, steel framed towers, and the structural steel frames of buildings. There is now a tendency to extend this range to the design of cranes, locomotives, steel passenger cars, etc. This type of truss also has possibilities in the design of reinforced concrete. Among the examples of its use in the United States may be cited a group of eight steel bridges, all of the Vierendeel type, constructed in 1936 in Los Angeles, Calif. Since designers in the United States are showing more and more interest in this type, the time seems favorable to open the subject for technical discussion.[2]

HISTORICAL

The Vierendeel Truss Before 1914.—The concept of the simple rigid truss without diagonals (variously termed the "ladder" truss, or the "arcade" truss) is due to Prof. Arthur Vierendeel. In 1893, he adopted this type for the steel steeple of the church of Dadizeele, in Flanders, Belgium. In Tervueren, near Brussels, Belgium, on the occasion of the International Exhibition of Brussels in 1897, Professor Vierendeel designed a riveted steel test bridge, 103 ft long, having two main girders of uniform depth divided into nine panels without

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NOTE.—Published in January, 1941, *Proceedings*.

[1] Prof. of Stability, Univ. of Brussels, Brussels, Belgium.

[2] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 869.

diagonals. The verticals were joined to the chords by large curved gussets. The angles that were used as the outer flanges of the vertical members were bent, as shown in Fig. 1, to become the inner flanges of the chord members. When it was subjected to load tests,[3] rupture occurred in the panels near the very end of the bridge. Even during this early period, Professor Vierendeel was actively promoting the general acceptance of this type of truss.[4,5]

The introduction of this type of truss without diagonals was such a distinct departure from the accepted practice of the time that it aroused considerable controversy in Belgium. Practically all engineers were radically opposed to the system, some on general principles, and others for reasons of economy or efficacy.

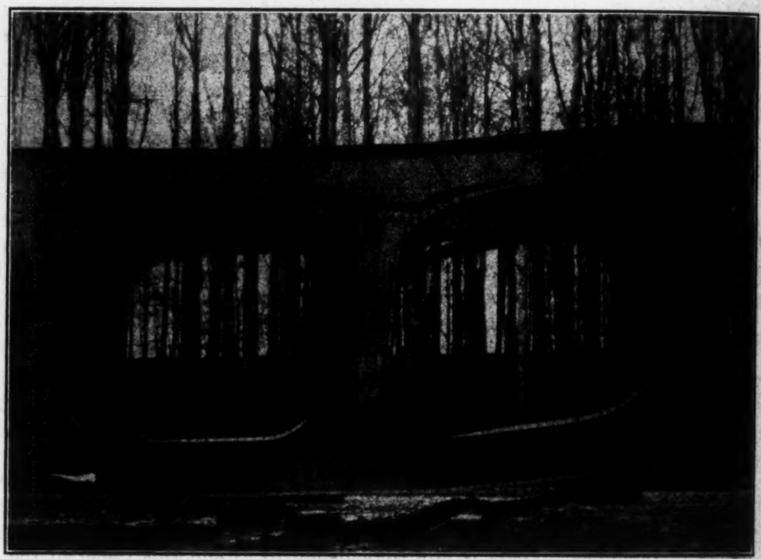

FIG. 1.—END PANELS OF EXPERIMENTAL VIERENDEEL BRIDGE TESTED TO FAILURE AT TERVUEREN (BELGIUM) IN 1897

It is true that, architecturally, these early trusses, for the most part with parallel top and bottom chords, were not attractive. Furthermore, it seems that it might be difficult to rivet a large curved gusset plate at the joint.

Except for the experimental bridge at Tervueren, the first steel highway bridge of this type was built in 1902 at Avelghem, on the Schelde River, with a span of 138 ft. The first steel single-track railway bridge of this type was built, over the Lys River, at Grammene, Belgium, in 1923. The span length was 185 ft and the structure was subjected to systematic load tests recorded in 1925.[6] The first reinforced concrete footbridge was constructed in La Louvière in 1911 with a span of 184 ft. The upper chord was arched and intersected the lower chord at the end of the span.

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[3] "Le Pont système Vierendeel: Expériences de Tervueren," by A. Vierendeel, Bruges, 1898; see, also, *Annales des Travaux Publics de Belgique*, February, 1898.

[4] "Longerons en treillis et longerons à arcades," by A. Vierendeel, Brussels, 1897; and "Théorie Générale des Poutres Vierendeel," by A. Vierendeel, *Mémoires de la Société des Ingénieurs Civils de France*, August, 1900.

[5] "Les ponts architecturaux en métal," by A. Vierendeel, *Annales des Travaux Publics de Belgique*, October, 1896.

Prior to 1914, six steel highway bridges were constructed in Belgium. One drawbridge had been designed for Bruges, but was not constructed until 1920. Each of these steel structures had riveted connections. In the first type (the Tervueren Bridge) many of the members had to be cambered; that is, as previously stated, the flanges of the verticals were bent so as to serve as the inner flanges of the chords. No other bridge of this exact type was ever built in Belgium. In subsequent designs, Professor Vierendeel introduced an important simplification permitting a more rational solution; that is, he made the chords continuous throughout. In general, the verticals were made of constant cross section, the flanges being extended up against the chord flanges, the joints being stiffened by angles or by large gusset plates. In other cases, the verticals were made gradually deeper toward the chords, thus forming the large connecting gussets at the joints.

Development Since 1920.—From 1920 to 1939, and more especially from 1928, there has been a marked increase in the number of Vierendeel trusses built in Belgium and in the Belgian Congo. For example, there are about twenty such structures in the Congo, among which may be cited Bukama Bridge with four 202-ft spans. Belgium has more than seventy Vierendeel bridges.

These structures differed, however, from the unattractive types created before 1914, and, following the concept of the bowstring truss with tie rods, most of the Vierendeel bridges built after 1920 were designed with curved top chords (second-degree parabolas) with the height of the end verticals theoretically reduced to zero.

Sixty such highway bridges were built in Belgium between 1928 and 1937. In the list are included two drawbridges and six steel railway bridges, with a few examples of reinforced concrete structures. Examples are shown in Figs. 2, 3, and 4.

Characteristics of the Vierendeel Truss Designed for Bridges

From experience with Vierendeel bridges in Belgium and in the Belgian Congo, the writer has noted the following characteristics favorable to the adoption of the Vierendeel truss with the curved upper chord:

(1) When they are suitably planned and skilfully designed (especially the gusset plates connecting the verticals to the chord members, and the shape of the ends), this truss may be made very attractive architecturally (Figs. 2 and 3).

(2) Under dead load this truss does not react as a Vierendeel truss, but as an ordinary bowstring arch. Consequently, under dead load, the various members are bent very little, if any, and are only slightly stressed.

(3) It is only under moving live loads that the structure really acts as a Vierendeel truss. The maximum bending moment in this truss will be found to be less than in the tied arch.

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[6] "Essais de réception du pont-rail de Grammene," by A. Vierendeel, *Annales des Travaux Publics de Belgique*, April, 1925.

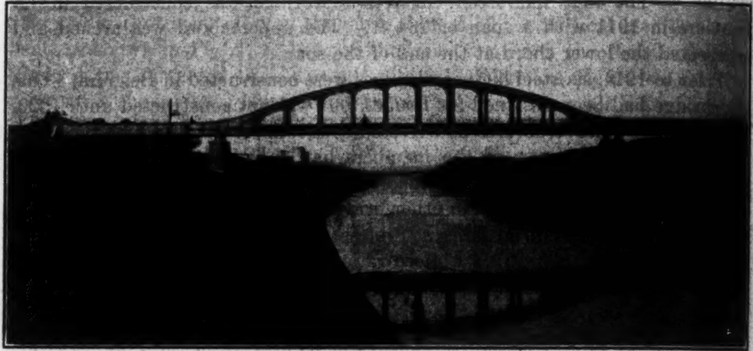

FIG. 2.—THE EYGENBILSEN BRIDGE, BUILT IN 1935; TOTAL LENGTH, 369 FT; AND LENGTH OF MAIN SPAN, 211 FT

FIG. 3.—THE DUDZEELE BRIDGE, BUILT IN 1935; SPAN, 104 FT

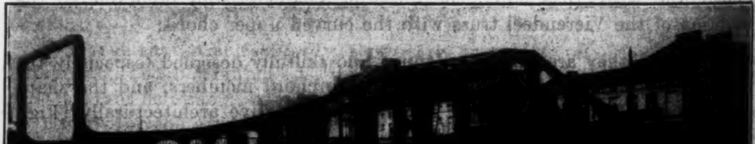

FIG. 4.—DRAWBRIDGE OF MUIDE (GHENT, BELGIUM), BUILT IN 1933; CHANNEL SPAN, 68 FT; AND COUNTERWEIGHT SPAN, 43 FT 6 IN.

(4) Analyzed from the viewpoint of items (1) to (3), the curved Vierendeel truss will often be found more economical than the ordinary tied arch.

(5) Compared with the conventional steel tied arch, the Vierendeel truss inherently has greater stiffness which is a valuable characteristic especially in railroad bridges constructed for heavy and fast traffic.

(6) It is logical and desirable to make both chords continuous. The verticals are simply attached to the chords by means of large connecting plates.

(7) It is logical in principle to design the upper chord with the same section as the lower chord, provided the floor beams are attached only at panel points. Furthermore, it is convenient, in general, to design all the verticals with the same cross-sectional area. These conditions will help to simplify problems of design as well as construction.

(8) As compared with the Vierendeel truss having a curved upper chord, the parallel-chord type is seldom attractive in appearance. The latter should be adopted only in cases where the uniform height is required by the conditions of the design. Furthermore, the stress distribution in the parallel-chord type is much less favorable than in the arch type, particularly in the end panels where the shearing forces are extremely high. Where the parallel chord is necessary, this condition at the end panels can be met by spacing the vertical members closer together near the ends of the truss, as suggested by J. D. Gedo,[7] M. Am. Soc. C. E. The designer, however, might encounter difficulties at least in applying this principle to bridge structures.

(9) With the assistance of arc welding performed with care and skill, the Vierendeel truss becomes a sturdy structure with simple and sober lines, permitting easy maintenance and offering little encouragement to corrosion.

PRINCIPLES OF DESIGN FOR TRUSSES WITHOUT DIAGONALS

Undoubtedly, the greatest obstacle to the general acceptance of the truss without diagonals has been (and still is) its reputation for including certain design assumptions and for being difficult to design.

In the present state, however, the design of trusses without diagonals, having two chords of equal reduced moment of inertia, $I \cos \alpha$, and receiving the loads only at the joints, is as easy as the design of any other type of truss.

Point of Contraflexure in the Verticals.—From the very beginning of his studies, Professor Vierendeel has stated that the design was predicated upon the fact that there must be a point of contraflexure in each vertical of the truss. When the upper and the lower chords are parallel and have the same moment of inertia, Professor Vierendeel assumed this point to be at the midheight, regardless of the loading, with the assumption that the loads were applied only at the joints. For two chords that are not identical he assumed that the bending moment taken by them was proportioned in the ratio of their moments of inertia, at the sections under consideration. For the tied arch type of truss, he sometimes assumed (to simplify the design) that the points of contraflexure were at the top of the verticals.

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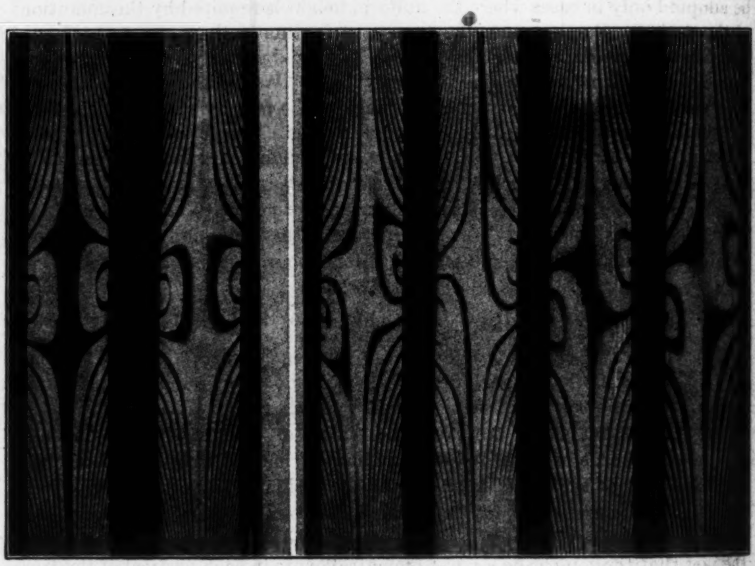
[7] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 933.

Professor Vierendeel and later writers[8] have attempted to demonstrate that when the parallel upper and lower chords are not of equal cross sections, the point of contraflexure in the verticals occurs in the ratio, $\frac{h_u}{h_l} = \frac{I_u}{I_l}$, in which I_u and I_l are the moments of inertia of the upper and the lower chords, respectively, and the height, h, between the chord axes, is divided, by the point of contraflexure, into an upper section, h_u, and a lower section, h_l. This assumption has been disproved, but it continues to mislead many designers. It is important to emphasize this fact because advocates of this theory claim to have checked it and, thereafter, apply it erroneously but confidently. Other writers have claimed that $\frac{h_u}{h_l} = \sqrt{\frac{I_u}{I_l}}$, a relation which is likewise incorrect.[9]

Still other writers, including Professor Vierendeel himself, have tried to locate the points of contraflexure in the chord members; and they advance systems of design based on this principle.[10] It seems clear now, however, that the points of contraflexure that may occur in the chord members are not fixed points. Their location is variable and depends on loading conditions. Moreover, in the arch type of upper chords, there may be no points of contraflexure



(a) $\frac{N}{T} = 0.3$ (b) $\frac{N}{T} = 2.0$

FIG. 5.—PHOTOELASTIC VIEWS SHOWING STRESS PATTERNS IN THE REGION OF THE POINT OF CONTRAFLEXURE OF A STRAIGHT SPECIMEN SUBJECT TO PLANE BENDING AND AXIAL LOAD

in the chord. Consequently, it is impossible to establish generalized reliable methods of design based on locating points of contraflexure in the chord members. It would be necessary to establish a separate design system for each condition of loading or to obtain a rough approximation.

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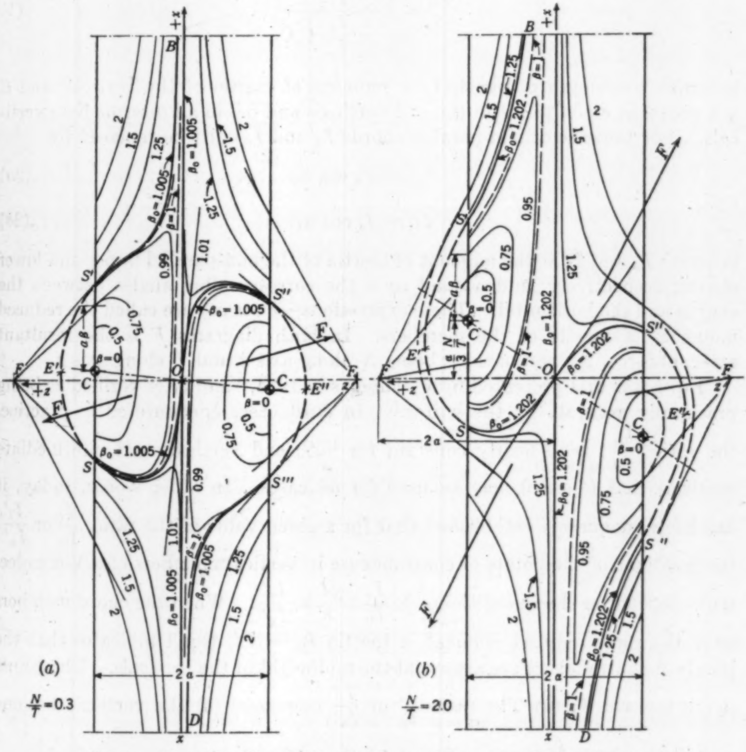
[8] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 908–910.

[9] "Der Eisenbetonbau," by R. Saliger, 6 Aufl., Leipzig, 1933.

[10] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), pp. 901–907.

The controversy has been greatly aided by the introduction of photoelasticity as a laboratory method of analyzing structures. By this means, checked by analytical computations (which agree remarkably well), it has been found that in monochromatic light the lines of extinction in the vicinity of the points of contraflexure are quite characteristic.

Fig. 5 shows two photoelastic stress patterns in the region of a point of contraflexure in a straight specimen subjected to plane bending combined with axial stress. The ratios are $\frac{N}{T} = 0.3$ and 2.0, respectively, in which N = the axial force and T = the shearing force. This photoelastic test was made with monochromatic light. The black curves are extinction lines, or curves of equal


+x
B
2
1.5
1.25
$\beta_0 = 1.005$
$\beta = 1$
0.99
1.01
S'
S''
S'''
S
$\alpha\beta$
0.75
0.5
$\beta = 0$
E'
E''
C
O
F
+z
z
$\beta_0 = 1.005$
0.99
1.01
1.25
1.5
2
2a
D
x
$\frac{N}{T} = 1.25$
$\beta_0 = 1.202$
0.95
$\beta = 0.5$
$\frac{a}{3}\frac{N}{T}$
$\beta = 1$
$\beta_0 = 1.202$
S''
S'
0.75
0.5
$\beta = 0$

(a) $\frac{N}{T} = 0.3$

(b) $\frac{N}{T} = 2.0$

FIG. 6.—GRAPHICAL REPRESENTATION OF THE STRESS PATTERNS IN FIG. 5 AS PLOTTED FROM CORRESPONDING THEORETICAL FORMULAS

tangential maximum stress. Their number depends upon the intensity of the loads applied.

The equations of these curves have been completely derived, and are represented[11] graphically in Fig. 6 for the two cases shown in Fig. 5. It will be noticed how closely these curves check with the experimental photographs.

An extensive program of experiments, covering a range of cases beyond all possible practical requirements, has enabled the writer to confirm the following expression for fixing very approximately the position of the points of contraflexure in vertical members when they all have the same cross section and all panels have the same length:

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_l} + C}{\frac{l}{h} \times \frac{I_v}{I_u} + C} \quad \text{.......................} (1)$$

in which l = length of panel; I_v = moment of inertia of the vertical; and C is a constant equal to 2.5 in the end verticals and 6.0 in all intermediate verticals. For the case of non-parallel chords I_u and I_l must be replaced by

$$I_u' = I_u \cos \alpha_u \text{......................} (2a)$$

and

$$I_l' = I_l \cos \alpha_l \text{......................} (2b)$$

in which I_u and I_l = the moment of inertia of the non-parallel upper and lower chords, respectively; and α_u and α_l = the corresponding angles between the chords and the horizontal. These expressions—$I \cos \alpha$—are called the reduced moments of inertia of the members. In both diagrams F is the resultant external force, the components being N along axis x and T along axis z.

Eq. 1 was first presented by P. Thomas in 1924,[12] and later verified as being practically accurate by the writer.[11] In most cases encountered in practice, the ratio, $\frac{h_u}{h_l}$, is so nearly constant for both end verticals and intermediate verticals that $C = 6.0$ may be used for all cases. In other words, today, it has been adequately established that for a given value of the ratio, $\frac{I_l}{I_u}$ or $\frac{I_l'}{I_u'}$, the positions of the points of contraflexure in vertical members of a Vierendeel truss depend on the quantities, $\frac{l}{h} \times \frac{I_v}{I_u}$ or $\frac{l}{h} \times \frac{I_v'}{I_u'}$. When the chord members have the same reduced moment of inertia $I_u' = I_l'$, Eq. 1 indicates that the points of contraflexure are always at the midheight of the verticals. The points move upward if: (*a*) The ratio $\frac{I_l}{I_u}$ or $\frac{I_l'}{I_u'}$ increases; (*b*) the verticals become stiffer (that is, if $\frac{I_v}{I_u}$ increases); and (*c*) the ratio $\frac{l}{h}$ increases.

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[11] "La poutre Vierendeel," by L. Baes, *L'Ossature Métallique*, October, 1936, Brussels; also, "Étude de la région voisine des points d'inflexion d'une pièce droite prismatique sollicitée par flexion plane composée, avec effort tranchant," by L. Blanjean et F. Temmerman, Prize Paper, *Bulletin de la Société Royale Belge des Ingénieurs et des Industriels*, Brussels, 1937.

[12] "Le calcul des poutres Vierendeel," by P. Thomas, *Revue Universelle des Mines*, Liége, Belgium, November 15, 1924; see, also, issues of August 1, 1926, and November 15, 1926.

Although, theoretically, the position of the point of contraflexure depends on the condition of loading for all ordinary cases, it is practically independent of it if the loads are applied only at the panel points. Moreover, it is only for higher values of the quantity, $\frac{l}{h} \times \frac{I_v'}{I_u'}$, that the points of contraflexure are at an appreciable distance from the center. For all ordinary cases, furthermore, the points of contraflexure in verticals of a Vierendeel truss, carrying no loads except those at the panel points, are at midheight, or very nearly so. The foregoing questions, which have been subjects of discussion since before 1900, have now been solved by analytical methods checked by photoelasticity.

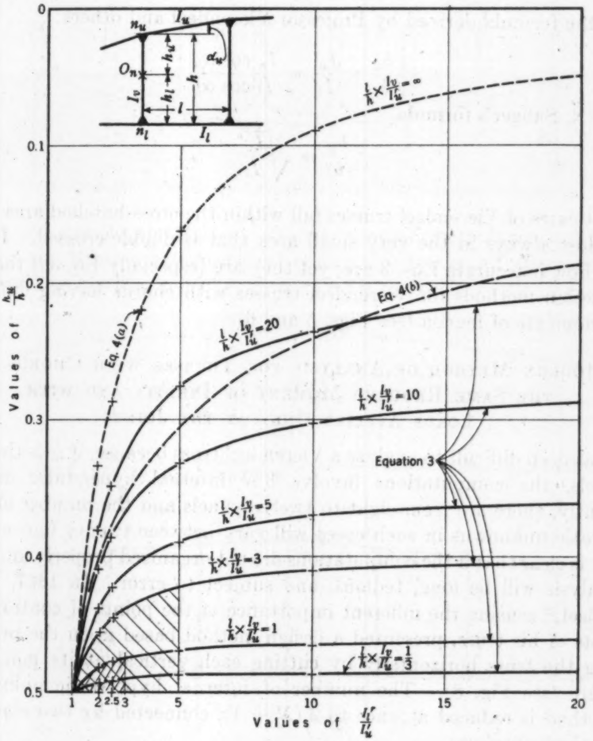

FIG. 7.—COMPARISON OF FORMULAS GIVING POSITION OF POINT OF CONTRAFLEXURE IN THE VERTICALS

This determination of the location of the points of contraflexure in the verticals, or of the distribution of the bending moment between the two chords, is essential. In fact, it makes it possible to determine by experiment whether a design method is accurate or not. Fig. 7 shows the considerable difference between the results given by Eq. 1:

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_n'} + 6}{\frac{l}{h} \times \frac{I_v}{I_l'} + 6} \qquad (3a)$$

or

$$\frac{h_u}{h_l} = \frac{\frac{l}{h} \times \frac{I_v}{I_n'} + 6}{\frac{l}{h}\left(\frac{I}{I_u'} + \frac{I}{I_l'}\right) + 12} \qquad (3b)$$

and by the formula derived by Professor Vierendeel and others:

$$\frac{h_u}{h_l} = \frac{I_u'}{I_l'} = \frac{I_u \cos \alpha_u}{I_l \cos \alpha_l} \qquad (4a)$$

or Prof. R. Saliger's formula

$$\frac{h_u}{h_l} = \sqrt{\frac{I_u'}{I_l'}} \qquad (4b)$$

Most cases of Vierendeel trusses fall within the cross-hatched area of Fig. 7 and almost always in the very small area that is double-crossed. It will be noticed how inaccurate Eqs. 3 are; yet they are (especially Eq. 3*a*) the basis of many design methods for Vierendeel trusses with chords having different reduced moments of inertia (see Figs. 5 and 6).

THE MODERN METHOD OF ANALYSIS FOR TRUSSES WITH CHORDS HAVING THE SAME REDUCED MOMENT OF INERTIA AND WITH LOADS APPLIED ONLY AT THE JOINTS

It has been difficult to analyze a Vierendeel truss because, if N is the number of panels, the computations involve $3N$ internal hyperstatic unknowns. Frequently, there are from eight to twelve panels and the number of internal hyperstatic unknowns in such cases will vary between twenty four and thirty six. It is clear that if the computations are not organized properly and skilfully the analysis will be long, tedious, and subject to error. In 1897, Professor Vierendeel,[13] sensing the inherent importance of the points of contraflexure in the posts of his truss, presented a design method based upon the principle of dividing the truss horizontally by cutting each vertical at its point of contraflexure (see Fig. 8). The number of internal hyperstatic unknowns by this method is reduced at once to $2(N + 1)$, connected by two statical conditions of equilibrium.

In the thirty years since about 1908, however (especially since the introduction of reinforced concrete), many hyperstatic systems with rigid connections have been introduced and designers have developed many ingenious methods of analysis. The Vierendeel truss, of course, has not been ignored in this development. Unfortunately, methods applying to the analysis of the Vierendeel truss in general have been such as to confirm a popular opinion that the computations are long, difficult, and, consequently, subject to inadvertent errors. In the United States, for example, certain methods of analyzing this truss have been proposed which are very remarkable from an analytical point of view but which do not simplify the problem.

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[13] "Longerons en treillis et longerons à arcades," by A. Vierendeel, Brussels, 1897.

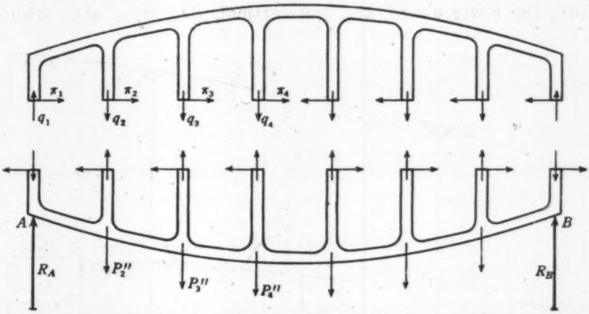
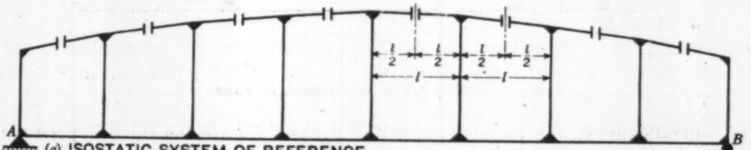
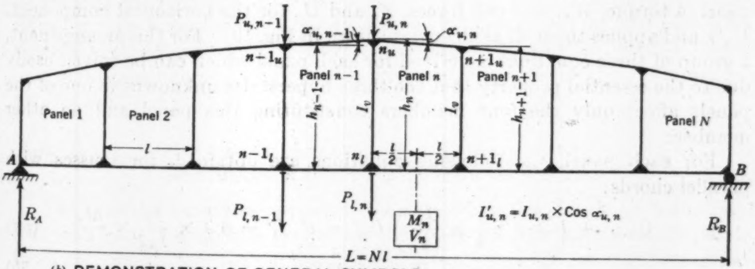

x_1
x_2
x_3
x_4
q_1
q_2
q_3
q_4
A
B
R_A
R_B
P_2''
P_3''
P_4''

FIG. 8.—ISOSTATIC SYSTEM OF REFERENCE USED BY PROFESSOR VIERENDEEL

The writer has shown[14] that the simplest method of analyzing a Vierendeel truss involves choosing as a principal or isostatic system of reference the arrangement shown in Fig. 9. The upper chord is cut at the center of each panel.


$\frac{l}{2}$
$\frac{l}{2}$
$\frac{l}{2}$
$\frac{l}{2}$
l
l
A
B

(a) ISOSTATIC SYSTEM OF REFERENCE


$P_{u,n-1}$
$P_{u,n}$
$\alpha_{u,n-1}$
$\alpha_{u,n}$
$n-1_u$
n_u
$n+1_u$
Panel 1
Panel 2
Panel $n-1$
Panel n
Panel $n+1$
Panel N
h_{n-1}
I_v
h_n
I_v
h_{n+1}
l
$n-1_l$
n_l
$\frac{l}{2}$
$\frac{l}{2}$
$n+1_l$
A
B
R_A
R_B
$P_{l,n-1}$
$P_{l,n}$
M_n
V_n
$I'_{u,n} = I_{u,n} \times \cos \alpha_{u,n}$
$L = Nl$

(b) DEMONSTRATION OF GENERAL SYMBOLS

FIG. 9.—DESIGN SKETCHES

Since the two chords have the same reduced moment of inertia, and since the loads are only applied at the panel points, the points of contraflexure in the verticals are at midheight.

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[14] "La poutre Vierendeel," by L. Baes, *L'Ossature Métallique*, Brussels, October, 1936, and March and September, 1937.

A method may be applied which was developed by K. Kriso[15] and redemonstrated, in another manner, by the writer.[14] An interpretation of general symbols is afforded by Figs. 9(*b*) and 10, in which M_n is the external bending moment; V_n is the external shearing force, at the center of panel *n*; I_{un}, I_{ln}, and I_v are the moments of inertia of the cross sections, respectively, of the upper chord, the lower chord, and the verticals; O_n, O_{n+1}, etc., are the points of contraflexure in the verticals; and W_n, Z_n, and U_n are the three hyperstatic unknowns in panel *n*. The designer selects, as hyperstatical unknowns in each panel, a torque, W_n, and two forces, Z_n and U_n (or the horizontal component, U_n'), and applies them all at the point shown in Fig. 10. For this arrangement, a group of three equations is derived for each panel which can be solved easily due to the essential property that the three hyperstatic unknowns in one of the panels affect only the four members constituting this panel and no other member.

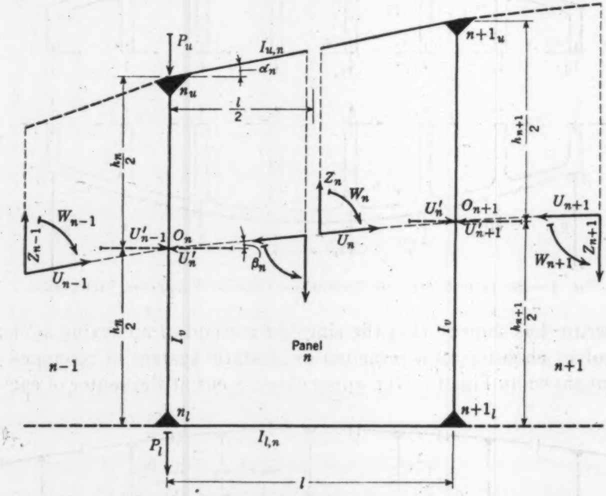

FIG. 10.—EXPLANATION OF SYMBOLS IN PANEL *n*

For each panel, the following equations are obtained, for trusses with parallel chords:

$$-U_{n-1}' + \left(2 + 6\frac{l}{h} \times \frac{I_v}{I_{un}}\right) U_n' - U_{n+1}' = 6\frac{l}{h} \times \frac{I_v}{I_{un}} \times \frac{M_n}{h} \ldots (5a)$$

$$W_n = \tfrac{1}{2} M_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

and

$$Z_n = \tfrac{1}{2} V_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5c)$$

in which M_n and V_n are the external bending moment and shearing force acting on the truss at the midlength of panel *n*.

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[15] "Stabilité des poutres Vierendeel," by K. Kriso; translated from the German by E. Barbieux and E. Dryon, Paris, France, Béranger, 1926.

For trusses with non-parallel chords, W_n and Z_n are expressed by Eqs. 5*b* and 5*c*, but Eq. 5*a* is replaced by equation:

$$- U_{n-1}' + \left\{ 1 + \left(\frac{h_{n+1}}{h_n} \right)^3 + \frac{3}{2} \times \frac{l}{h_n} \times \frac{I_v}{I_{un}'} \left(\frac{h_{n+1}}{h_n} + 1 \right)^2 \times \left[1 + \frac{1}{3} \left(\frac{h_{n+1} - h_n}{h_{n+1} + h_n} \right)^2 \right] \right\} U_n' - \left(\frac{h_{n+1}}{h_n} \right)^3 U_{r+1}' = \frac{3}{2} \times \frac{l}{h_n} \times \frac{I_v}{I_{ln}'} \left(\frac{h_{n+1}}{h_n} + 1 \right)^2 \left(\frac{M_n + \frac{1}{6} \times \frac{h_{n+1} - h_n}{h_{n+1} + h_n} \times l \times V_n}{\frac{h_n + h_{n+1}}{2}} \right) \ldots\ldots (6)$$

In the case under consideration, two of the hyperstatic unknowns in each panel (W_n and Z_n) become known at once, and it is virtually impossible to make an error in sign.

The entire problem is thus greatly simplified, being reduced to the simple task of solving the system of formulas of the same type as Eq. 5*a* or Eq. 6, which may be called the equations for the two or three U'-values of the panels separated by two adjacent verticals. For all cases, therefore, the solution involves a group of successive linear equations, in stages, starting with two variables, and involving as many formulas as there are panels. Such equations may be solved readily by a slide-rule, and they involve no complications due to successive approximations.

Eqs. 5 and 6 characterize the method and are as important in their connection as the three-moment equation (Clapeyron equation) for the moments in a continuous beam. Their solution is direct and simple as the writer has demonstrated in describing the design of a railroad bridge constructed in Belgium in 1937.[16] In designing a tower only one assumption as to loading is required,[17] and, in the case of a bridge, the method makes it easy to draw the influence line for every element of the computation.[16] Kriso[15] had considered it necessary to develop the equation for every single influence line. Although it is analytically quite remarkable, his study is difficult to read, and for that reason has not attracted attention. The direct solution of Eq. 5 or Eq. 6 for a limited number of positions of a single load is preferable; it demonstrates the simplicity and reliability of that method.

THE MODERN METHOD OF ANALYSIS FOR TRUSSES WITH CHORDS HAVING DISSIMILAR REDUCED MOMENTS OF INERTIA

The preceding method applies in the case when the chords have the same reduced moment of inertia and when the loads act only at the panel points. In all other cases, the method must be completed using the remarkable separating properties of the elastic centers of the panels. This new step was the subject of a thesis presented in 1939 by Emile Robert to the Faculty of Applied Sciences, University of Brussels. The problem is beyond the scope of the present paper; but the method can be used to solve all cases, without making any special assumption, and calling only on very simple first-degree equations, separating the three hyperstatic unknowns of each panel.

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[16] "La poutre Vierendeel," by L. Baes (second paper), *L'Ossature Métallique*, Brussels, March, 1937.

[17] "La poutre Vierendeel," by L. Baes (third paper), *loc. cit.*, September, 1937.

CONCLUSION

Personal experience with this subject leads the writer to conclude that:

(1) Regardless of the type of Vierendeel truss, and regardless of the use to which it is put, its analysis is simplest when a section is cut through one of the chord members between two verticals. This method is simple and altogether reliable.

(2) The method described in this paper is based essentially upon the assumption that the points of contraflexure in the vertical members may be determined precisely. Analytical investigation as well as experimental investigation reveal important characteristics pertaining to the location of these points. With the photoelastic method available, an exact theoretical solution can be checked.

(3) The method involves an equation applied to two or three panels which are separated by two adjacent verticals. A given case can be solved readily without successive approximations or other "trick" methods of computation. Such solution involves a complete analysis of the effects of live loads by means of influence lines.

(4) A preliminary design, which in itself is quite accurate, involves the analysis of only one or two conditions of loading.

(5) Thereafter, the computation of a Vierendeel truss, with chords having equal reduced moments of inertia and in which the loads are applied only at the panel points, is shown to be quite simple and free from too broad assumptions. This encourages the writer to believe that this type of truss, without diagonal members, will receive more attention on the part of structural designers in the future.

The writer desires, in this paper, merely to emphasize the existence of an accurate method of designing Vierendeel structures which is simpler, for example, than those currently in use in the United States.

ACKNOWLEDGMENT

The paper was translated into English by L. G. Rucquoi, M. Am. Soc. C. E., to whom the writer wishes to make sincere acknowledgment.

DISCUSSION

Jaroslav J. Polivka,[18] M. Am. Soc. C. E. (by letter).—In the "Synopsis" of his paper Professor Baes states that the points of contraflexure in each vertical of a Vierendeel truss, with chords of equal moments of inertia, are at the midheights of the verticals. This is true only for open-web trusses with inclined chords when the "reduced" moments of inertia of the chords in each panel are equal, as mentioned later in the paper. In other words, when the elastic weights of the chords in each panel are equal:

$$G_u = \frac{l_u}{E\,I_u} = G_l = \frac{l_l}{E\,I} \,..\,(7)$$

In such a case the reactions due to a vertical displacement (Δ, Fig. 11) of the panel intersect at the midheight of the vertical member. (In Fig. 11, C_v, C_u, and C_l are the points of contraflexure in vertical, upper chord, and lower chord members, respectively.)

The occurrence of equal "reduced" moments of inertia in the chords and verticals is quite exceptional and will not be true in most cases encountered in practice. However, except for this restriction, the assumption of fixed points of contraflexure in the verticals will shorten the computations considerably in many cases.

Eq. 5*a* does not involve the influence of shear and direct stress on deformation. This equation can be derived in a general form by introducing the elastic weights of the structural members, and their relationships with the deformations.[19] This further step enables the designer to analyze open-web trusses, com-

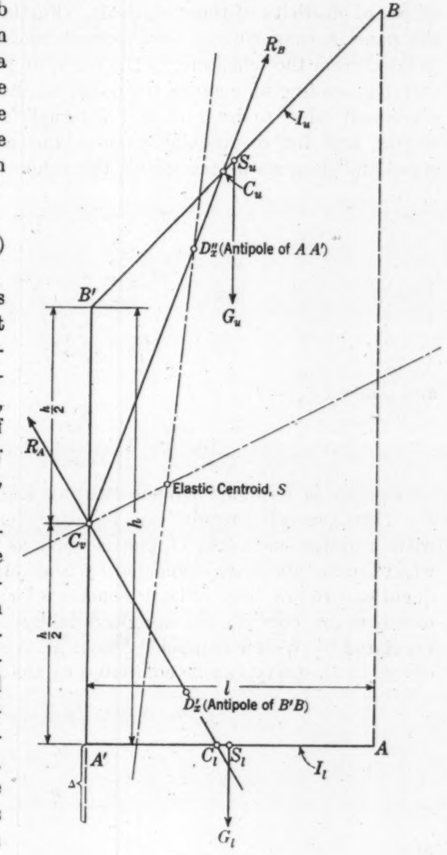

Fig. 11

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[18] Research Associate, Univ. of California, Cons. Engr., Berkeley, Calif.

[19] "Graphical Analysis of Framed Structures on the Basis of Ellipse of Elasticity," by Jaroslav Polivka, Soc. of Eng. Students, Inst. of Technology, Prague, 1919; also, "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by Jaroslav Polivka, Berkeley, Calif., 1940 and 1941.

posed of members of any shape, and to consider the effect of direct stress and shear.

The general equation is

$$- U'_n \, G_{v,n} \, r^2_{v,n} + U'_n \, G_n \, r^2_n - U'_{n+1} \, G_{v,n+1} \, r^2_{v,n+1} = \Delta_n \ldots\ldots (8)$$

in which $G_{v,n}$ and $G_{v,n+1}$ are the elastic weights of the vertical members at the joints n and $n + 1$ (Fig. 10) and $r_{v,n}$ and $r_{v,n+1}$ are the respective semi-axes of ellipse of elasticity of these verticals. Furthermore, G_n is the elastic weight of the panel n consisting of two verticals and two chords, considered as being isolated from the remainder of the truss; and Δ_n is the displacement due to the external bending moment of the panel n. Eq. 8 expresses the equality of displacements due to internal and external forces. For constant moments of inertia, and for rectangular panels (and neglecting the influence of direct stress and shear on deformation), the values of Eq. 8 become:

$$G_{v,n} = G_{v,n+1} = \frac{h}{E\,I_v} \ldots\ldots (9a)$$

$$r^2_{v,n} = r^2_{v,n+1} = \frac{h^2}{12} \ldots\ldots (9b)$$

$$G_n = \frac{2}{E}\left(\frac{h}{I_v} + \frac{l}{I}\right) \ldots\ldots (9c)$$

and (since $I_{v,n} = I_{l,n} = I$):

$$G_n \, r^2_n = 2\left[\frac{h^3}{12\,E\,I_v} + \frac{l}{E\,I}\left(\frac{h}{2}\right)^2\right] \ldots\ldots (9d)$$

Eq. 5*a* is obtained by substituting Eqs. 9 in the general formula, Eq. 8. This general formula can be used for trusses composed of members with variable moments of inertia such as those that result, for example, when the members are connected by brackets or fillets. This is the most frequent case in practice. Elastic constants for several regular shapes of haunched members are compiled in standard tables. From the values u, v, p, and q tabulated by Walter Ruppel,[20] Assoc. M. Am. Soc. C. E., the constants of the ellipse of elasticity can be computed by the following relationships:[21]

$$G = (p + q)\, G_B \ldots\ldots (10a)$$

$$r^2 = s\,(d - s) \ldots\ldots (10b)$$

$$s = \frac{p}{p + q}\, l \ldots\ldots (10c)$$

and

$$d = (1 - u)\, l \ldots\ldots (10d)$$

The relative displacement (Δ) of the end faces of a structural member, perpendicular to its axis, is caused by a force R passing through the elastic centroid S and having the magnitude

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[20] Discussion by Walter Ruppel of "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

[21] *Proceedings*, Am. Soc. C. E., Vol. 67, April, 1941, p. 710.

$$R = \frac{\Delta}{G\,r^2} \quad \text{(11)}$$

A general equation similar to Eq. 8 can be derived for panels having members with different moments of inertia.

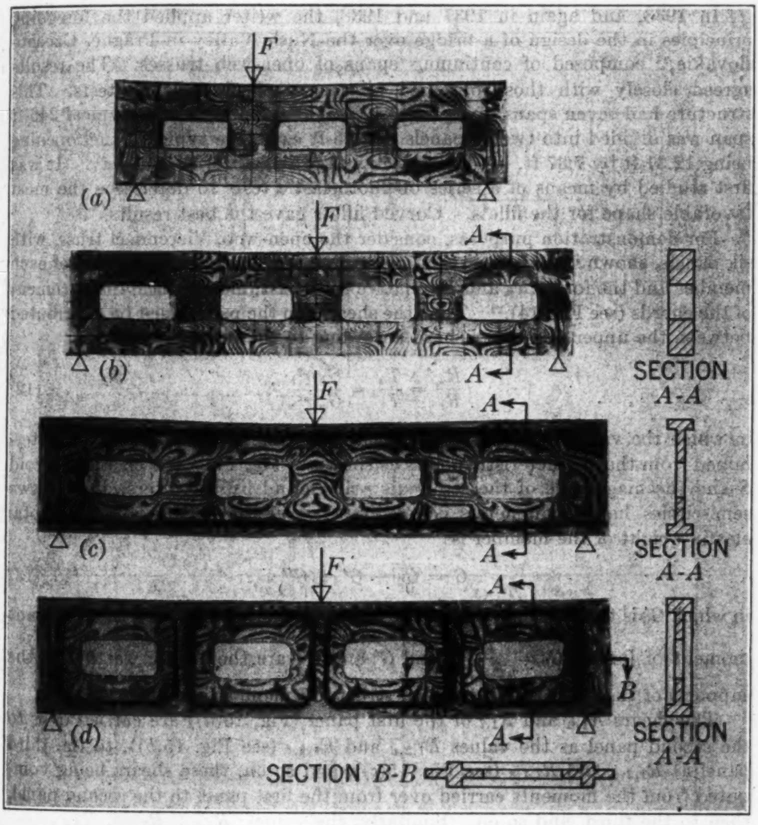

FIG. 12

When the members are relatively slender and the effect of bending is predominant, the points of contraflexure in all members may be obtained by photoelastic investigation and the truss may be considered as statically determinate, having imaginary hinges in all members.[22] The check of the static solution consists in determining the pattern (and even the magnitude, if necessary) of the internal forces by photoelastic investigation.[23] The photoelastic method, however, becomes more complicated whenever the truss consists of deep members. In this case the effect of shear and of stress concentrations, which the fillets cause near the joints, must be considered. The points of contraflexure in the chords approach the joints and in some cases disappear (Fig. 12).

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[22] "Analysis of Statically Indeterminate Structures on the Basis of Photoelastic Data," by J. J. Polivka and H. D. Eberhart, Assoc. M. Am. Soc. C. E., paper presented before the Structural Engineers Assoc. of Northern California, San Francisco, August 6, 1940 (mimeographed).

In 1933, and again in 1937 and 1938, the writer applied the foregoing principles in the design of a bridge over the Nusle Valley in Prague, Czechoslovakia,[24] composed of continuous spans of open-web trusses. The results agreed closely with those obtained by means of photoelastic tests. This structure had seven spans, five of which were 246 ft long. The typical 246-ft span was divided into twelve panels of 20.5 ft each, the typical panel opening being 12.31 ft by 7.37 ft. A lower deck was provided for the subway. It was first studied by means of a series of photoelastic tests to determine the most favorable shape for the fillets. Curved fillets gave the best results.

For demonstration purposes, consider the open-web, Vierendeel truss, with six panels, shown in Fig. 13. First determine the characteristic points of each member and the forces R_u and R_l relative to the assumed vertical displacement of the chords (see Fig. 14).[19] Then the shear T in the panel must be distributed between the upper and lower chord according to the ratio

$$\frac{R_u}{R_l} = \frac{T_u}{T_l} = \frac{G_l\, r^2_l}{G_u\, r^2_u} \qquad (12)$$

in which the values G and r, of the elastically restrained members, are determined from the characteristic points as shown in Fig. 14. The elastic centroid S and the magnitude of the semi-axis r are found by the intersection of two semi-circles having diameters equal to $l\,(1 - v)$ and $l\,(1 - u)$. The total elastic weight of the member is

$$G = G_0 + G' + G'' \qquad (13)$$

in which G_0 is the elastic weight of the isolated structural member (for constant moment of inertia $G_0 = \frac{l}{E\,I}$), and G' and G'' are the elastic weights of the supports of the elastically restrained structural member.

The shears $R_{1,u}$ and $R_{1,l}$ of the first panel (Fig. 15(*a*)) are carried over to the second panel as the values $R_{1,2,u}$ and $R_{1,2,l}$ (see Fig. 15(*b*)), to the third panel as $R_{1,3,u}$ and $R_{l,1,3}$ (see Fig. 15(*c*)), and so on, these shears being computed from the moments carried over from the first panel to the second panel, then to the third, and so on. Similarly, the values: $R_{2,1,u}$, $R_{2,3,u}$, $R_{2,4,u}$, $\cdots$ etc., and $R_{2,1,l}$, $R_{2,3,l}$, $R_{2,4,l}$, $\cdots$ etc., are obtained by distributing the shears $R_{2,u}$ and $R_{2,l}$ to the other panels (see Table 1).

It can be seen that the values of the shear distributed to the fourth and following panels are negligible. The total shear in each panel is obtained by

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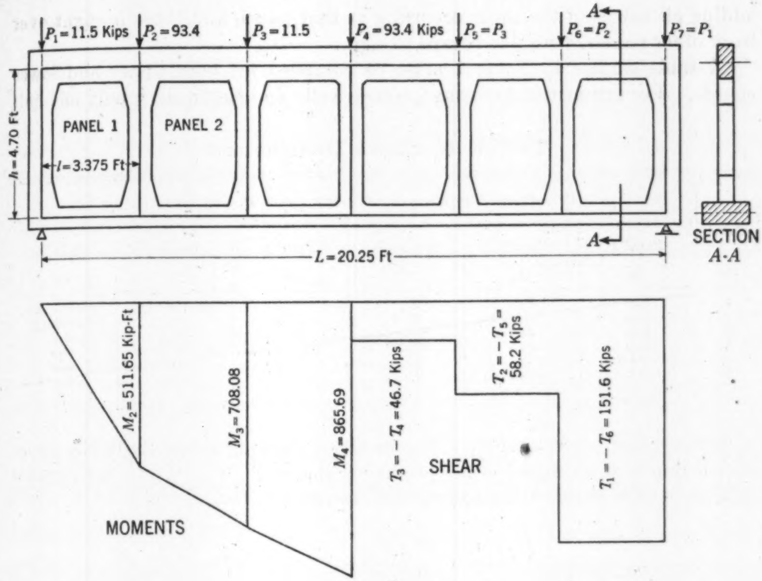
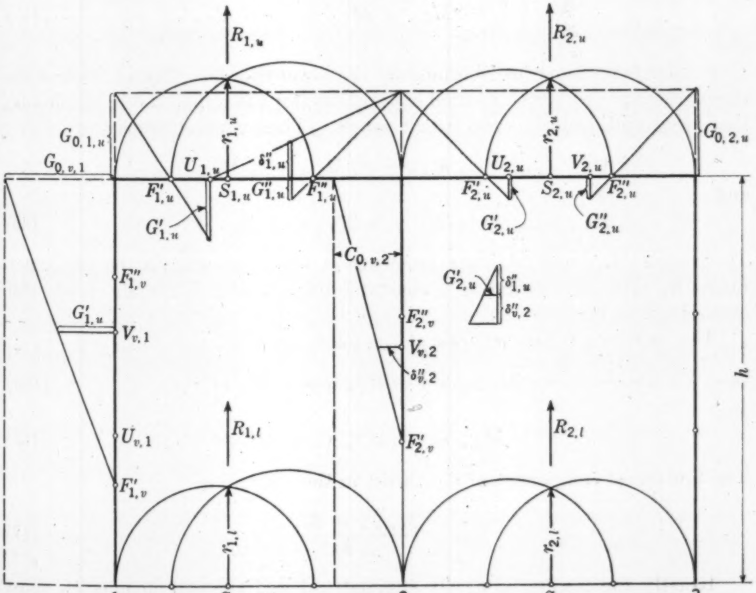
[23] "Some Stress Relationships in Photoelasticity," by J. J. Polivka and H. D. Eberhart, *Proceedings*, Tenth Semi-Annual Eastern Photoelasticity Conference, Cambridge, 1939.

[24] "The Vierendeel Truss," by J. J. Polivka, *Technical Encyclopædia*, Vol. XIV, Prague, 1937, p. 730



$P_1 = 11.5$ Kips
$P_2 = 93.4$
$P_3 = 11.5$
$P_4 = 93.4$ Kips
$P_5 = P_3$
$P_6 = P_2$
$P_7 = P_1$
A
A
PANEL 1
PANEL 2
$h = 4.70$ Ft
$l = 3.375$ Ft
$L = 20.25$ Ft
SECTION A-A
$M_2 = 511.65$ Kip-Ft
$M_3 = 708.08$
$M_4 = 865.69$
MOMENTS
$T_3 = -T_4 = 46.7$ Kips
$T_2 = -T_5 = 58.2$ Kips
$T_1 = -T_6 = 151.6$ Kips
SHEAR

FIG. 13



$R_{1,u}$
$R_{2,u}$
$r_{1,u}$
$r_{2,u}$
$G_{0,1,u}$
$G_{0,2,u}$
$G_{0,v,1}$
$U_{1,u}$
$\delta''_{1,u}$
$U_{2,u}$
$V_{2,u}$
$F'_{1,u}$
$S_{1,u}$
$G''_{1,u}$
$F''_{1,u}$
$F'_{2,u}$
$S_{2,u}$
$F''_{2,u}$
$G'_{1,u}$
$G'_{2,u}$
$G''_{2,u}$
$C_{0,v,2}$
$F''_{1,v}$
$G'_{1,u}$
$V_{v,1}$
$G'_{2,u}$
$\delta''_{1,u}$
$\delta''_{v,2}$
$F''_{2,v}$
$V_{v,2}$
$\delta''_{v,2}$
h
$R_{1,l}$
$R_{2,l}$
$U_{v,1}$
$F'_{2,v}$
$F'_{1,v}$
$r_{1,l}$
$r_{2,l}$
1
$S_{1,l}$
2
$S_{2,l}$
3
l
l

FIG. 14

adding all values of the shear occurring at that panel, and those brought over from other panels, with due regard to sign.

A table similar to Table 1 must be compiled for both upper and lower chords. For symmetrical trusses (geometrically and elastically), only one half of these values are computed. The resulting bending moments in the upper chords due to the actual loading producing shears T_1, T_2, T_3, $\cdots$ in the panel 1, 2, 3, $\cdots$ are obtained by adding the expressions

TABLE 1.—SHEAR DISTRIBUTION

Panel	SHEAR DISTRIBUTED FROM THE PANEL IN COL. 1 TO:					
	Panel 1 (2)	Panel 2 (3)	Panel 3 (4)	Panel 4 (5)	Panel 5 (6)	Panel 6 (7)
(1)						
1	R_1	$R_{1,2}$	$R_{1,3}$	$R_{1,4}$	(vanishing)	(vanishing)
2	$R_{2,1}$	R_2	$R_{2,3}$	$R_{2,4}$	$R_{2,5}$	(vanishing)
3	$R_{3,1}$	$R_{3,2}$	R_3	$R_{3,4}$	$R_{3,5}$	$R_{3,6}$
4	$R_{4,1}$	$R_{4,2}$	$R_{4,3}$	R_4	$R_{4,5}$	$R_{4,6}$
5	(vanishing)	$R_{5,2}$	$R_{5,3}$	$R_{5,4}$	R_5	$R_{5,6}$
6	(vanishing)	(vanishing)	$R_{6,3}$	$R_{6,4}$	$R_{6,5}$	R_6

$$\frac{T_1}{R_{1,u} + R_{1,l}} \left(M_{1,u} + \frac{R_{1,2,u}}{R_{2,u}} M_{2,u} + \frac{R_{1,3,u}}{R_{3,u}} M_{3,4} + \cdots \right) \qquad (14a)$$

$$\frac{T_2}{R_{2,u} + R_{2,l}} \left(\frac{R_{2,1,u}}{R_{1,u}} M_{1,u} + M_{2,u} + \frac{R_{2,3,u}}{R_{3,u}} M_{3,4} + \cdots \right) \qquad (14b)$$

etc.

Similar expressions are obtained for the lower chords. The addition of the six expressions produces two final expressions (for the upper and lower chords) that are simultaneously valid for all panels. Their general forms are:

$$M_{n,u\,(\text{total})} = a_u\, M_{1,u} + b_u\, M_{2,u} + c_u\, M_{3,u} + \cdots \qquad (15a)$$

and

$$M_{n,l\,(\text{total})} = a_l\, M_{1,l} + b_l\, M_{2,l} + c_l\, M_{3,l} + \cdots \qquad (15b)$$

The values $M_{n,u}$ and $M_{n,l}$ are moments at the joints due to the imaginary forces R_n (Eq. 11), assuming a constant displacement Δ for all chords (for example, $\Delta = 1$).

The moments in the verticals at the joints are then

$$M_{v,n,u} = -\,(M''_{n-1,u} + M'_{n,u}) \qquad (16a)$$

and

$$M_{v,n} = -\,(M''_{n-1,l} + M'_{n,l}) \qquad (16b)$$

The horizontal component of the thrust in the vertical is

$$U_n = \frac{M_{v,n,u} + M_{v,n,l}}{h_n} \qquad (17)$$

Usually the loading is nearly symmetrical and the components U_n must satisfy the condition $\Sigma U_n = 0$. For unsymmetrical truss and loading it will

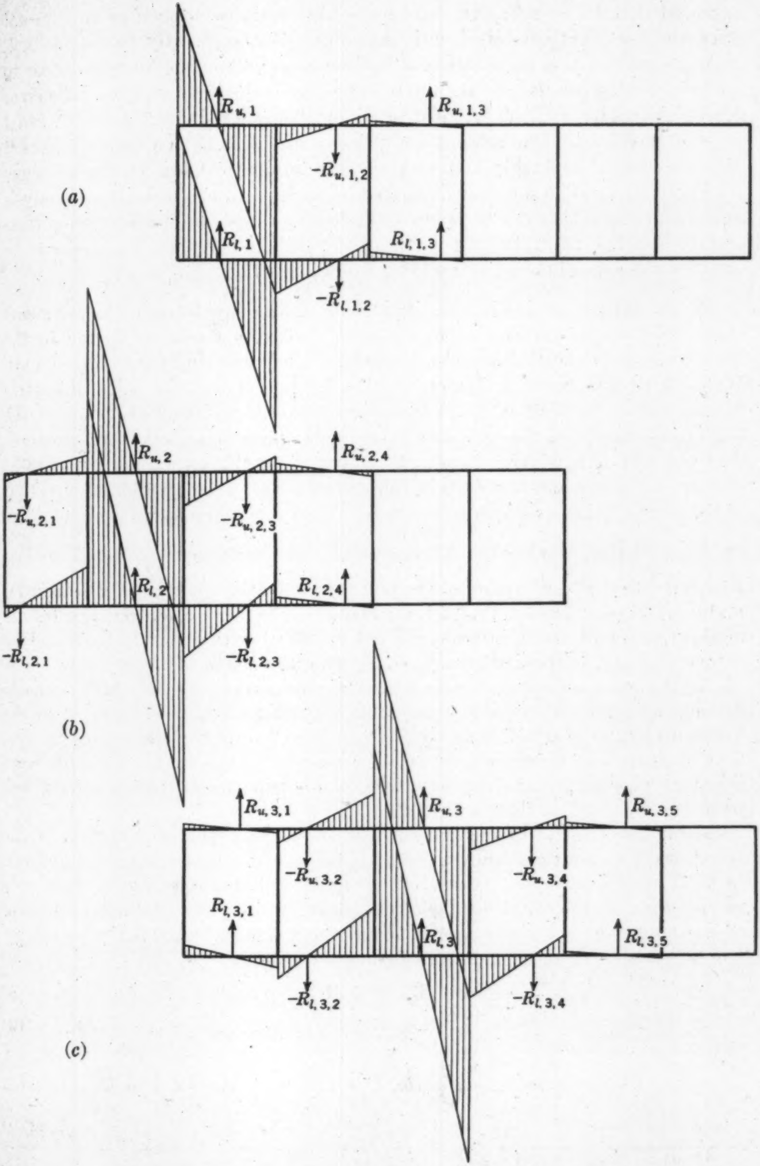

(a)
$R_{u,1}$
$R_{u,1,3}$
$-R_{u,1,2}$
$R_{l,1}$
$R_{l,1,3}$
$-R_{l,1,2}$
(b)
$R_{u,2}$
$R_{u,2,4}$
$-R_{u,2,1}$
$-R_{u,2,3}$
$R_{l,2}$
$R_{l,2,4}$
$-R_{l,2,1}$
$-R_{l,2,3}$
(c)
$R_{u,3,1}$
$R_{u,3}$
$R_{u,3,5}$
$-R_{u,3,2}$
$-R_{u,3,4}$
$R_{l,3,1}$
$R_{l,3}$
$R_{l,3,5}$
$-R_{l,3,2}$
$-R_{l,3,4}$

FIG. 15

be found that $U_n = \Delta U$. In this case $- \Delta U$ must be applied as an external force and its effect combined with the moments due directly to the loading.

If a slight error is permissible, it is not necessary to distribute the shear to the neighboring panels, and the analysis becomes relatively simple. The error decreases rapidly with the number of panels, being less than 5% for the usual type of structure. The adaptation of the method as shown herein makes it possible to analyze highly complicated grid framings of the Vierendeel type.

Conclusion.—The suggestions offered herein extend Professor Baes' analysis to a general form that can be applied to any size, shape, and inclination of truss members and to multiple tiers or stories of panels in the truss, such as may be used for the special design of building frames.

W. A. MILLER,[25] ASSOC. M. AM. SOC. C. E. (by letter).—Experiments along lines somewhat similar to those of Professor Baes, conducted in the Department of Civil Engineering, University of Sydney, in Sydney, New South Wales, Australia, by J. I. Miller,[26] verify the position of the point of contraflexure in the verticals of the Vierendeel truss and substantiate some of the other conclusions of the paper. Using steel spline models and macroscopic deformations, Mr. Miller considered cases with parallel as well as non-parallel chords. The moment of inertia of the verticals was taken about one third that of the upper chord, and that of the lower chord was taken to vary from "equal to" to eight times that of the upper chord. For the range of $\frac{I_u}{I_l}$ from 1 to 4, it was found that the departure of the point of contraflexure in the verticals from midpoint did not exceed 1% of their height. Further examination of frames in sheet celluloid, using microscopic deformations with the aid of a standard deformeter gage, supported the findings using the spline models.

As the main object of the investigation undertaken by Mr. Miller was to develop an approximate and dependable algebraic analysis, applicable to the Vierendeel type of truss likely to be encountered in tower and bridge frames, that analysis was continued, based on the assumptions that the chords were equal (or nearly so) and that the points of contraflexure of the connecting web members occurred at their midpoints.

Using the U-axis (as did the author) and a Z-axis on any vertical in the panel, the hyperstatic unknowns were applied at the intersection and equations in U, W, and Z derived. It was found finally that the selection of the Z-axis to pass through the elastic center of the panel considerably simplified the form of the equations. For the case in which the author's "modified moments of inertia" are equal, these equations reduce to

$$W_n = \tfrac{1}{2} M^0{}_n \qquad \text{(18)}$$

$$Z_n = \tfrac{1}{2} V^0{}_n \qquad \text{(19)}$$

and

$$- h^3{}_{n-1} U'{}_{n-1} + \left[h^3{}_{n-1} + 2 \frac{I_v}{I_n} (h^2{}_{n-1} + h_{n-1} h_n + h^2{}_n) \right] U'{}_n - h^3{}_{n+1} U'{}_{n+1} = 3 l (h_{n-1} + h_n) M^0{}_n \qquad \text{(20)}$$

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[25] Prof., Civ. Engr., Univ. of Sydney, Sydney, New South Wales, Australia.

[26] From a thesis presented to the University of Sydney, in 1939, in partial fulfilment of the requirements for the degree of Bachelor of Engineering in Civil Engineering.

The symbols are those used by the author, except V^0_n and M^0_n, which are the simple beam shear and moment, respectively, on the vertical through the elastic center. Eq. 20 reduces to Eq. 5*a* for a truss with parallel chords.

A. A. EREMIN,[27] ASSOC. M. AM. SOC. C. E. (by letter).—An interesting review of the historical development of a Vierendeel truss is presented in this paper. In a short time the Vierendeel truss has found wide application, and various materials are used in its construction. Analysis of stresses in this type of truss has revealed many interesting problems in the theory of mechanics, and many computation methods have been developed. Such methods are generally based on previously computed or assumed points of contraflexure in the vertical members. The author has given an interesting illustration of the application of the photoelastic method in determining points of contraflexure in Vierendeel trusses.

In conclusion (1), Professor Baes suggests that, for the analysis of stresses, the upper chord can be assumed cut at each panel. Eq. 5 or 6 may also be developed assuming the truss to be cut at the points of contraflexure in the vertical members. Obviously, the simplicity of the computation method depends not upon the development of the basic equations, but upon the type of redundant forces and their application to the structure. Determination of the stresses with Eq. 5 or 6 requires the solution, simultaneously, of a system of equations with three unknown redundant forces. The author's analysis of stresses may be compared with that proposed by Dana Young, Assoc. M. Am. Soc. C. E., in 1936.[2] Professor Young (and the writer in a discussion of this paper) cited equations for redundant forces in a Vierendeel truss with a factor including the summation of the horizontal redundant forces. The simplicity of computation by either of these types of equations varies with the individual ability of the designer. Furthermore, by algebraic summation, the equations with a summation factor may be easily transformed into the author's type of equation with three unknowns. In conclusion (3) Professor Baes disapproves the method of successive approximations in solving the system of equations for stresses in a Vierendeel truss. However, under the heading "The Modern Method of Analysis for Trusses with Chords Having the Same Reduced Moment of Inertia and with Loads Applied Only at the Joints," he shows that Eq. 5 or 6 may be solved by successive approximations.

The method of successive approximations in analysis of stresses in statistically indeterminate structures has been proved to be very important and has found wide application in the design of structures. Therefore, it scarcely could be considered as evidence that this method is complex. This paper is a valuable contribution.

YVES NUBAR,[28] M. AM. SOC. C. E. (by letter).—In illustrating the application of photoelastic analysis to the study of the distribution of bending moments in the verticals of Vierendeel trusses, this paper by Professor Baes is of timely interest. It contains several formulas and a number of references to comparatively recent research in European technical literature.

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[27] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[28] Cons. Engr., New York, N. Y.

The writer would appreciate knowing whether the photoelastic test specimens to which the author refers formed a part of the model of a Vierendeel truss, or whether they were isolated specimens subjected to given external axial loads and moments. It will also be interesting to follow the derivation of Eq. 1, in particular whether it was based on further photoelastic tests or exclusively on analytical investigation. In either case, this formula, which appears to have a rather logical structure for cases within the limitations specified by the author, will facilitate greatly the reduction of the general problem to simpler elements, certainly at least for a first approximation. In this connection, it is difficult to understand why Professor Vierendeel's isostatic system of reference (Fig. 8) should not be simpler and of more immediate application than that of the author (Fig. 9), since the points of inflection in the verticals may be assumed located by means of Eq. 1 or other equivalent formulas.

The statement of the author (see heading "Principles of Design for Trusses Without Diagonals: Point of Contraflexure in the Verticals") that "This determination of the location of the points of contraflexure in the verticals * * * is essential" may lead to misinterpretations; it cannot possibly mean what it would convey to the casual reader. Many analyses of Vierendeel trusses have been presented in the technical literature that seem to attach to this determination an undue importance, ascribing to it, explicitly or tacitly, an essential rôle in the solution of such frames. The position taken by Professor Vierendeel himself, as described by the author (see heading "Principles of Design for Trusses Without Diagonals: Point of Contraflexure in the Verticals") illustrates this attitude to some extent. The fact of the matter is that the problem is determinate, and its solution (theoretically, at least, quite simple) requires no further assumptions beyond those commonly accepted in the general theory of rigid frames. The function of a preliminary assumption of the location of

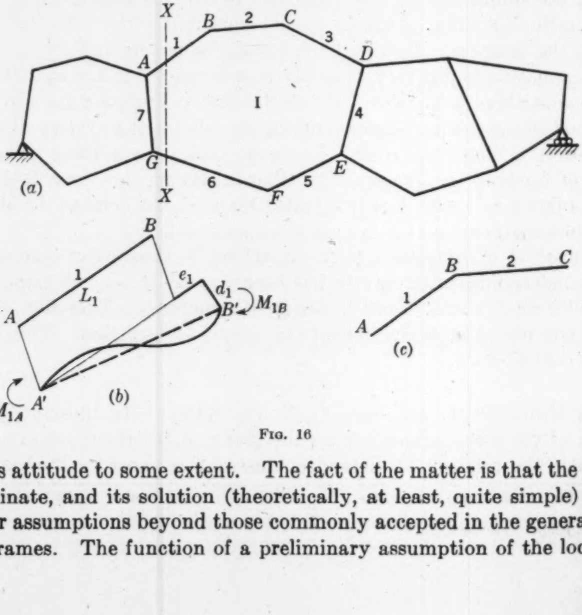

FIG. 16

points of contraflexure is merely to simplify the mathematical analysis involved. More precisely, if N is the number of the panels, it may be shown that the complete solution of a Vierendeel truss of a general type leads to a system of $2(N + 1)$ linear equations of the "six-moment" form (instead of $3(N + 1)$ equations of a more complicated type derived by the great majority of conventional methods of analysis); any assumption as to the location of the points of zero moment in the verticals will simply decompose this system into two other independent systems, each of $N + 1$ equations of the "three-moment" type. This may be brought into formal evidence by means of a different approach to the general problem of rigid frames,[29] summarized partly as follows under four headings, A, B, C, and D:

Consider a Vierendeel truss (or a viaduct tower), of a very general type, composed of single panels of any polygonal shape, joined to each other along a single member (Fig 16(a)) (no claim is here made as to the engineering suitability of such frames, greater generality of results being the only purpose in mind). All symbols and notations entering in the development which follows are condensed in Figs. 16(b) and 16(c). For example, in Fig. 16(b), $A\ B$ is the original position of any member in a given frame; $A'\ B'$ is its position after deformation; M_{1A} and M_{1B} are end moments exerted by the member upon the joints; $R_1 = \frac{d_1}{L_1}$; and $S_1 = \frac{e_1}{L_1}$. Furthermore (see Fig. 16(c)), let $R_{12} = R_1 - R_2$ and $S_{12} = S_1 - S_2$; and in general R_{pq} and S_{pq}, given by similar formulas, refer to any two adjacent members p and q.

A.—Given any panel I, apply the vectors R_{pq} and S_{pq} at each joint as shown in Fig. 17, these vectors being orthogonal but inclined an arbitrary angle γ_1 to the coordinate axes. Three independent relations may be established and stated as follows:

$$\left.\begin{array}{l} \Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad\qquad\qquad \text{inclined an angle } \gamma_1) = 0 \\ \Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad\qquad\qquad \text{inclined an angle } \gamma_2) = 0 \\ \Sigma(\text{the moment about any point, of vectors } R_{pq} \text{ and } S_{pq}, \\ \qquad\qquad\qquad\qquad \text{inclined an angle } \gamma_3) = 0 \end{array}\right\} \quad (21)$$

all summations being taken around the complete boundary of the panel. Vectors S_{pq} in Eqs. 21 and in Fig. 17 will drop out if axial deformations are assumed to be negligible, at least for a first approximation. In addition, vectors R_{pq} may be assumed written in terms of the end moments of the given members p and q. For instance, for members 1 and 2:

$$R_{12} = \frac{1}{6\,E}\left(\frac{2\,M_{1B} - M_{1A}}{K_1} - \frac{2\,M_{2B} - M_{2C}}{K_2}\right) \ldots\ldots\ldots\ldots (22)$$

in which $K = \frac{I}{L}$. Eq. 22 is derived from the slope-deflection equations applied to members 1 and 2, assumed rigidly connected to each other. Any one of Eqs. 21 may be replaced by $\Sigma\ (R_{pq}) = 0$. Three independent equations such as Eqs. 21 may be written for any panel; they involve no unknowns except end moments (and axial loads or axial deformations when these latter are not assumed negligible). These relations have been termed compatibility equations.

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[29] See dissertation entitled "Displacements and Stresses in the General Two-Dimensional Framework," presented by the writer to Columbia University, New York, N. Y., in 1941, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

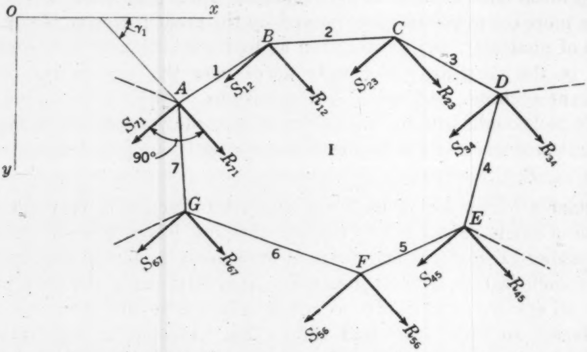

FIG. 17 (THE DIRECTION OF THE ORTHOGONAL VECTORS R_{pq} AND S_{pq} IS THE SAME AT ALL JOINTS)

B.—In addition, three equilibrium equations: $\Sigma X = 0, \Sigma Y = 0$, and $\Sigma M = 0$ may be written at any joint. The joint loading will include:

(*a*) Loads and moments externally applied at the joint;
(*b*) Axial loads in members meeting at the joint;
(*c*) End shears in these members; and
(*d*) End moments in these members.

Since item (*c*) may readily be expressed in terms of (*d*), it is again seen that these joint equations contain no other unknowns but axial loads and end moments.

C.—Returning now to panel *I* of Fig. 16(*a*), the following relations may be written, assuring the equilibrium and continuity of the frame:

(*a*) Three equilibrium equations for the part of the entire structure on one side of line *X X*;

(*b*) Three times three, equals nine, intermediate joint equations for joints *B*, *C*, and *F*; and

(*c*) Three compatibility equations for the panel.

This gives a total of fifteen equations involving the following nineteen unknowns: Five axial loads for members *A B*, *B C*, *C D*, *E F*, and *F G*; and fourteen end moments for all seven members forming the panel. All of these unknowns may then be expressed in terms of $19 - 15 = 4$ of them. This property is general; all unknowns such as the foregoing may be expressed as functions of four of them, regardless of the type of the panel and the number of its sides. The complete solution of the system is not necessary, at this time at least; choose the four end moments of two consecutive web members as parameters and solve only for M_{1A}, M_{3D}, M_{5E}, M_{6G} in terms of them.

D.—Perform a similar operation for each panel; then write equation $\Sigma(M) = 0$ at each joint of the frame at the ends of the web members, such as *A*, *D*, *E*, and *G*. This will yield a system of $2(N + 1)$ linear equations, each involving the six end moments of three consecutive web members. The solution of this system is possible, although often laborious. Any assumption relating the two end moments of each web member (that is, any assumption as to the location of the point of inflection in these members) will break this system of $2(N + 1)$ equations into two other independent systems, each containing $N + 1$ equations between the moments at one end of three successive web members. The similarity of the resulting systems to that of "three-moment" equations is obvious. Such an assumption, if reasonably correct, will lead to quite accurate results which, if desired, may be used to determine a second set of values more closely convergent.

The writer has applied this method to the solution of the symmetrical, five story Kinzua Viaduct.[30] For each panel, the equations listed under heading *C* reduce in this case to only two, as follows:

(*a*) One equilibrium equation;
(*b*) No intermediate joint equation; and
(*c*) One compatibility equation.

These two equations contain only four unknowns: Two end moments for the chord member and one more end moment for each web member in the panel. Proceeding as outlined, a system of five linear equations is obtained between the five end moments of the web members, only two or three of these entering into each equation. The solution of this system is greatly facilitated by the fact that the coefficient of one of the unknowns in each equation is, on the average, twenty times larger than that of the other one or two unknowns. As a result, some appropriate method of solution by successive approximations yields a first set of values, with errors ranging from 1% to a maximum of 5%.

Eq. 6 given by the author is also of the "three-moment" type and will be found very useful in many cases. However, it appears to be predicated upon the following particular assumptions: A straight bottom chord, rectangular or trapezoidal panels of equal spans, equality of reduced moments of inertia for the two chords, equality of moments of inertia for all verticals, and location of the points of inflection at midheight of these verticals. The foregoing analysis is not subject to these restrictions. It is only fair to add, however, that these assumptions will be found almost entirely verified in the greatest majority of actual Vierendeel trusses, and Eq. 6 has a much wider applicability than might appear at first.

Léon G. Rucquoi,[31] M. Am. Soc. C. E.—Professor Baes' candid intent in presenting his paper to American engineers was to obtain the point of view of those in this country who have devoted their attention and studies to the same problem. In this effort the writer was privileged to appear as an intermediary and friend. The fruitful discussion and constructive remarks have certainly fulfilled the author's wish.

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[30] "Modern Framed Structures," by the late J. B. Johnson, the late C. W. Bryan, and F. E. Turneaure, Pt II, John Wiley & Sons, Inc., New York, N. Y., 9th Ed., p. 404.

[31] Care, Import-Export Industries, Inc., New York, N. Y.

Circumstances now prevent Professor Baes from transmitting his concluding comment. When last heard of near the close of 1941, he was in occupied Belgium. It is not known if he is still allowed to continue his work at the University of Brussels in the present circumstances.

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Paper No. 2158

CONSUMPTIVE USE OF WATER FOR AGRICULTURE

By Robert L. Lowry, Jr.,[1] M. Am. Soc. C. E., and Arthur F. Johnson,[2] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. E. B. Debler, Rhodes E. Rule, Edgar E. Foster, Charles H. Lee, Harry F. Blaney, J. L. Burkholder, S. T. Harding, C. S. Jarvis, R. W. Davenport and Gordon R. Williams, and Robert L. Lowry, Jr. and Arthur F. Johnson.

Synopsis

Transpiration and evaporation, together accounting for practically all consumptive use of water, have been shown by experimental investigations to be influenced by climatic factors, of which temperature gives one of the better correlations. Consumptive use in a number of adequately watered irrigated valleys and humid watersheds, representing a wide range in climate, latitude, elevation, and type of crops, is shown in this paper to bear a straight-line relation, within narrow limits, to accumulated daily maximum temperatures above 32° F during the growing season. Factors responsible for deviations from average consumptive use are discussed. The relation of consumptive use to growing-season temperatures offers to the engineer a ready means of estimating probable consumptive use on projects under investigation as an initial step in determining the irrigation requirement at the farm or at the point of diversion. Short descriptions of each area studied, with summaries of annual data, are given in the Appendix.

Introduction

The problems of consumptive use of water have been studied intermittently by engineers of the Bureau of Reclamation since the early years of the organization. The relation of consumptive use to temperatures was first studied in the Bureau in 1920. Other approaches have been made from time to time. Many engineers have collaborated in the studies, and this paper, therefore, is the result of accumulated effort of the organization.

Note.—Published in April, 1941, *Proceedings*.

[1] Engr., National Resources Planning Board, Denver, Colo.; formerly Hydr. Engr., National Resources Planning Board, Pecos River Joint Investigation, Roswell, N. Mex.

[2] Engr., Bureau of Reclamation, Denver, Colo.

The study was made for the purpose of obtaining a working basis for estimating the probable annual consumptive use of water on projects under investigation for conditions of full development with an adequate water supply. With consumptive use known, estimates of irrigation requirement may be made by taking into account the effective precipitation and expected losses incident to the diversion and application of the water consistent with the characteristics of the individual project studied. Efforts were directed toward finding a dependable correlation between consumptive use and some factor that influences consumptive use, or some related factor for which records are readily available in most areas, such correlation to be used in estimating the consumptive use on the areas under investigation. Growing-season temperatures were found to be more satisfactory than other factors studied, and this paper discusses their use for this purpose.

The determination of consumptive use for agriculture, in its full sense, involves allowance for consumptive use by nonagricultural areas also. Where nonagricultural use is limited to precipitation, it need not be considered, but, where it is supplied in part from present or potential irrigation supplies, it cannot be neglected. On many projects, control or elimination of nonproductive water uses may determine the feasibility of additional agricultural development. In this study an attempt has been made to evaluate nonagricultural losses contributing to valley consumptive use in order to arrive at the true value of consumptive use for agriculture.

DEFINITION

In a basic sense, consumptive use[3] is "* * * the quantity of water, in acre-feet per cropped acre per year, absorbed by the crop and transpired or used directly in the building of plant tissue, together with that evaporated from the crop-producing land." The Committee's definition of valley consumptive use includes also the consumptive use on the uncropped area and nonrecoverable deep-soil percolation. Valley consumptive use is represented by the difference between annual inflow to the valley (consisting of surface and subsurface movement of water into the valley and of precipitation) and the total outflow therefrom in the same period (consisting of surface and subsurface movement of water out of the valley), with appropriate correction for changes in surface and subsurface storage. For the purpose of this paper, the Committee's definition of valley consumptive use has been modified by the omission of the term "nonrecoverable deep percolation loss" and the substitution of the term "equivalent valley area" for cropped or irrigated area. "Nonrecoverable deep percolation loss" is a form of outflow independent of the factors influencing transpiration and evaporation, the forms of use primarily considered herein. An effort has been made to limit consideration to valleys where deep percolation losses are believed to be negligible. The equivalent valley area in each case comprises the entire area or valley (except for areas consuming no stream flow), reduced to an area consuming water at a rate equivalent to that by the cropped area. Every area considered contains open water or waterlogged areas from which the loss of water is materially higher than on cropped lands, and the abnormal use on such areas counterbalances the incomplete supply on cropped areas. There are often also extensive areas with partial supplies feeding on ground waters or surface wastes from the irrigated areas.

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[3] "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1349–1377.

Earlier Investigations

Many studies have been made for the purpose of determining consumptive use of water. These earlier studies usually have been undertaken for special purposes. They reflect many different conceptions of the meaning of consumptive use, and therefore are not generally comparable. Their results must be interpreted in terms of their purposes.

Tank Experiments.—Some of the first experiments on consumptive use were aimed at measuring plant transpiration in relation to growth, and many of these were designed to eliminate surface evaporation. The laboratory experiments of T. A. Kiesselbach,[4] L. J. Briggs and H. L. Shantz,[5] and many others, with tanks and potometers, have demonstrated that, with other factors constant, production of dry matter for any one crop varies in direct proportion to the quantity of water transpired, over a wide range, but that plant efficiency is affected by soil fertility, frequency of cutting, and many other factors. The efficiencies of various plant species were also found to vary widely under similar conditions.

Other experimenters have used tank data as a basis for estimating consumptive use. J. C. Stevens,[6] M. Am. Soc. C. E., used the Briggs and Shantz data for various crops in estimating consumptive use by applying the relation between transpiration and dry matter produced to crop yields. W. C. Hammatt[7] and E. B. Debler,[8] Members, Am. Soc. C. E., and others, have made more direct application by endeavoring to simulate natural conditions. The integration method used by H. F. Blaney,[9, 10] M. Am. Soc. C. E., involves the determination of the consumptive use of various types of crops, native vegetation, and soil surfaces by experiments using undisturbed samples in tanks set in the native habitat of the plants. The rates of use so determined are weighted by the area devoted to that type of use within the study area or valley to obtain the total consumption and weighted average use.

Because of the difficulty of simulating natural conditions and eliminating individual differences, tank experiments were declared by the Duty of Water Committee[3] to be of questionable value. The improved equipment and methods now available remove many of the uncertainties characterizing early tank experiments.

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[4] "Transpiration as a Factor in Crop Production," by T. A. Kiesselbach, *Research Bulletin No. 6*, Nebraska Agri. Experiment Station, 1916.

[5] "The Water Requirements of Plants," by L. J. Briggs and H. L. Shantz, *Bulletins Nos. 284 and 285*, Bureau of Plant Industry, 1913; "Relative Water Requirements of Plants," by the same authors, *Journal of Agricultural Research*, Vol. 3, 1914, pp. 1–64; and "The Water Requirements of Plants at Akron, Colorado," by H. L. Shantz and L. N. Piemeisel, *loc. cit.*, Vol. 34, 1927, pp. 1093–1190.

[6] "The Duty of Water in the Pacific Northwest," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 2094.

[7] "Determination of the Duty of Water by Analytical Experiment," by W. C. Hammatt, *loc. cit.*, p. 200.

[8] "Final Report on Middle Rio Grande Investigations," by E. B. Debler, Bureau of Reclamation, May, 1932 (unpublished); see also discussion by Ivan E. Houk, M. Am. Soc. C. E., of "Experiments to Determine Rate of Evaporation from Saturated Soils and River-Bed Sands," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol 94 (1930), pp. 982–995.

[9] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, and Colin A. Taylor and A. A. Young, Assoc. Members, Am. Soc. C. E., *Bulletin No. 33*, California Div. of Water Resources, 1930.

[10] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Part III, Water Utilization," by H. F. Blaney and others, National Resources Committee, 1938, pp. 295–427.

Plot and Farm Experiments.—A large number of plot experiments are constantly in progress at the many federal and state agricultural experiment stations. Although many of these studies are directed toward comparison of plant varieties as to yield, resistance to heat, cold, plant diseases, etc., there have also been many experiments (such as those conducted by J. A. Widtsoe[11, 12] and F. S. Harris[13] in Utah; W. L. Powers[14] in Oregon; M. R. Lewis,[15] M. Am. Soc. C. E., and D. Bark[16] in Idaho; R. G. Hemphill[17] in Colorado; and W. H. Snelson[18] in Alberta, Canada) that have been directed toward use of water. These experiments have shown definite relations between production and application of water under farm conditions and have contributed much to irrigation practice. Generally, they have not attempted to measure deep percolation losses and surface wastes, and so have fallen short of determining consumptive use.

Project Studies.—In some cases, studies of project areas, such as those by W. G. Steward and D. J. Paul[19] and L. Crandall,[20] M. Am. Soc. C. E., in Idaho, have met the requirements for determination of valley consumptive use in method. In general, such studies do not account for uses of water outside the irrigated or cropped area, and the entire use is charged against such area. The increasing necessity for conservation of water resources requires that use of water by nonproductive areas be considered in relation to irrigation efficiency, and in the future most studies must be made valley wide.

Valley Studies.—Many studies of valley consumptive use have been made using inflow-outflow methods and the data submitted as testimony in water-right adjudications and interstate suits. Among these may be mentioned the following: Sevier Valley, Utah[21]; Cache la Poudre Valley, Colorado[17]; Cache la Poudre and South Platte valleys, Colorado, and Little Laramie Valley, Wyoming[22, 23]; Arkansas Valley, Colorado[24]; San Luis Valley, Colorado[25];

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[11] "The Production of Dry Matter with Different Quantities of Irrigation Water," by J. A. Widtsoe, *Bulletin No. 116*, Utah Agri. Experiment Station, 1912.

[12] "The Yield of Crops with Different Quantities of Water," by J. A. Widtsoe, *Bulletin No. 117, loc. cit.*

[13] "The Duty of Water in Cache Valley, Utah," by F. S. Harris, *Bulletin No. 173, loc. cit.*, 1920.

[14] "Irrigation and Soil Moisture Investigations in Western Oregon," by W. L. Powers, *Bulletin No. 140*, Oregon Agri. Experiment Station, 1914.

[15] "Experiments on the Proper Time and Amount of Irrigation, Twin Falls Experiment Station, 1914, 1915, and 1916," by M. R. Lewis (U. S. Dept. of Agriculture cooperating with Twin Falls County Commissioners, Twin Falls Canal Co., and Twin Falls Commercial Club, 1919).

[16] "Experiments on the Economical Use of Irrigation Water in Idaho," by D. Bark, *Bulletin No. 339*, U. S. Dept. of Agriculture, April 21, 1916.

[17] "Irrigation in Northern Colorado," by R. G. Hemphill, *Bulletin No. 1026*, U. S. Dept. of Agriculture, 1922.

[18] "Irrigation Practice and Water Requirements for Crops in Alberta," by W. H. Snelson, Irrigation Series, *Bulletin No. 6*, Dept. of the Interior, Canada, 1922.

[19] "Report on Drainage Investigations of Pioneer and Nampa-Meridian Districts in Boise Valley for the Years 1916 and 1917," by W. G. Steward and D. J. Paul, Bureau of Reclamation, 1919 (unpublished).

[20] "Report of Use of Water on Twin Falls North Side Project, 1918," by L. Crandall (unpublished).

[21] "Hearing of the Colorado River Commission, Salt Lake City, Utah," by C. J. Ulrich, March 27 and 28, 1922.

[22] Various investigations, mostly unpublished, by R. I. Meeker, M. Am. Soc. C. E. Studies in the South Platte Valley were based on data published in Colorado Agricultural Experiment Station *Bulletin No. 279*, 1922.

[23] "Return Flow Water from Irrigation Developments," by R. I. Meeker, *Engineering News-Record*, Vol. 89, July 20, 1922, pp. 105–109.

[24] "Arkansas River Basin Investigations—Consumptive Use of Arkansas River Waters, Pueblo to Holly," by C. L. Patterson, Colorado Water Conservation Board, 1940 (in course of preparation); see also Amsley Reports, 1922–1925, state engineer of Colorado (unpublished).

[25] "San Luis Valley Field Investigations, Consumptive Use Determinations, Evaporation Experiments, Drainage Measurements, 1930, 1931, 1932," by R. J. Tipton and Francis C. Hart, Assoc. Members, Am. Soc. C. E., Colorado State Engr's Office (unpublished).

Truckee River, Nevada[26]; Owens Valley, California[27]; San Gabriel Valley, California[28]; Columbia River and its tributaries, Washington-Oregon[29]; and Rio Grande in Colorado, New Mexico, and Texas.[10] The results from some of these studies have been used in this paper. Others were either incomplete or involve valleys with an inadequate water supply and so have been excluded.

The methods and results of these studies are not discussed herein as many of the reports are readily available. Several of the studies are discussed in Society publications and reviewed in the Committee Report.[3]

INFLUENCE OF METEOROLOGICAL FACTORS

Although small-scale experiments using tanks, potometers, etc., for determining consumptive use are of questionable value in some cases, they have been of great value in pointing out the importance of transpiration and evaporation as the chief processes in consumptive use and the influence on these phenomena of meteorological factors. Considerable research has been directed toward the determination of both transpiration and evaporation.

The potometer experiments on transpiration of plants of various kinds conducted by Messrs. Briggs and Shantz[30, 31] are among the most extensive of their kind. They indicate a very close correlation between transpiration and meteorological phenomena, including evaporation from water surfaces, air temperatures, solar radiation, and wet-bulb depression readings. Experiments on sorghum, wheat, oats, rye, alfalfa, and pigweed, which cover a wide range of rates of water use, all show similar changes in hourly rate of use throughout the daily cycle.[32] Experiments on the rate of transpiration of tules at the Prado Station in California[33] show similar hourly cycles for transpiration, evaporation, and temperature, of air and of water.

The latter experiments showed that 93.5% of the total daily transpiration occurred between 9 a.m. and 9 p.m. Observations of ground-water levels in an alfalfa field in the Escalante Valley, Utah,[34] showed a distinct lowering of the water table during daylight hours and a recharge at night. Similar results were obtained[35] for native vegetation in the Middle Rio Grande Valley. Experiments on evaporation[36] at Fort Collins, Colo., showed that two thirds of the daily evaporation occurs during daylight hours. Obviously, solar radiation or some secondary phenomenon connected with sunlight is an important factor in both transpiration and evaporation. In fact, the Escalante Valley experiments showed a marked decrease in transpiration on cloudy days from that on clear days.

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[26] "Irrigation Studies on the Truckee River, Nevada," by S. T. Harding, M. Am. Soc. C. E.—data from testimony in Truckee River Adjudication, 1918.

[27] "The Determination of Safe Yield of Underground Reservoirs of the Closed-Basin Type," by Charles H. Lee, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 148; also, *loc. cit.*, Vol. 94 (1930), pp. 1382–1397.

[28] Discussion by H. Conkling, M. Am. Soc. C. E., of "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1378.

[29] "Water Powers of the Cascade Range—Pt. III—Yakima River Basin," by G. L. Parker, M. Am. Soc. C. E., and F. B. Store, *Water Supply Paper No. 369*, U. S. Geological Survey, 1916.

[30] "Hourly Transpiration Rate on Clear Days as Determined by Cyclic Environmental Factors," by L. J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, Vol. 5, January 3, 1916, pp. 583–650.

[31] "Daily Transpiration During the Normal Growth Period and Its Correlation with the Weather," by L. J. Briggs and H. L. Shantz, *loc. cit.*, Vol. 7, October 23, 1916, pp. 155–212.

[32] "Hourly Transpiration Rate on Clear Days as Determined by Cyclic Environmental Factors," by L. J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, Vol. 5, January 3, 1916, p. 617.

[33] "Water Losses Under Natural Conditions—Pt. I—South Coastal Basin Investigations," by H. F. Blaney, *Bulletin No. 44*, California Div. of Water Resources, 1933.

[34] "A Method of Estimating Ground Water Supplies Based on Discharge by Plants and Evaporation from Soil," by W. N. White, *Water Supply Paper No. 659–A*, U. S. Geological Survey, 1932, pp. 87 and 88.

[35] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Pt. II—Ground Water Resources," by C. V. Theis, National Resources Committee, 1938, pp. 272–277.

The Briggs and Shantz experiments over long-time periods show the variation in rates from day to day as affected by changes in solar radiation and temperatures. A comparison of daily changes in solar radiation and temperatures indicates that the former is a function of the altitude of the sun and thus varies with the seasons, whereas the latter (temperature), which is dependent upon absorption of heat by air and earth, and the circulation of the air, lags behind the seasons. Although solar radiation gives one of the best correlations with transpiration and evaporation, growing-season temperatures more nearly parallel the cycle of plant growth. Fig. 1 shows clearly the relation of consumptive use to the monthly mean of maximum daily temperatures above 32° F for the Mesilla Valley in New Mexico, one of the few areas for which monthly values of consumptive use are available. Maximum daily temperatures are used as being more representative of daylight hours when most of the transpiration occurs.

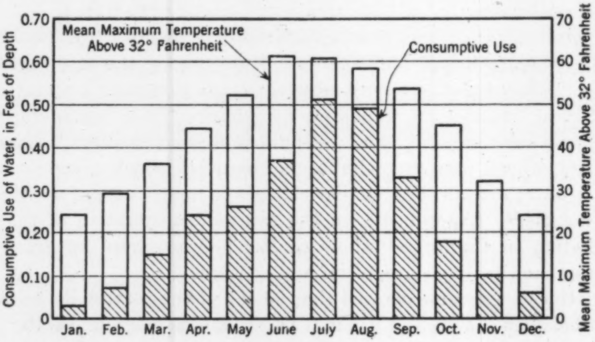

FIG. 1.—COMPARISON OF MONTHLY DISTRIBUTIONS OF CONSUMPTIVE USE AND MEAN TEMPERATURES ABOVE 32° F

Consumptive use (see Fig. 1) is relatively lower with respect to temperature in winter than in summer and has a different distribution in the spring than in the fall. These variations are produced by differences in plant water requirement as related to plant development. In winter, perennial plants are practically dormant and annual plants are entirely so. As spring advances, annual crops are seeded and all plants increase in size. As their leaf area enlarges, their water requirements increase, and reach a maximum soon after maximum temperatures pass the peak of the season. In the fall, water requirements drop rapidly as the function of the plant is transferred from growth to processes of ripening and reproductive development; and growth practically ceases with harvest or with the onset of sub-growing temperatures. The pronounced lag in consumptive use noted in May is due in part to the cutting of alfalfa and the harvest of winter crops and in part to the preparation of seedbeds for planting cotton and other summer crops. In the Mesilla Valley 91% of annual consumptive use and 85% of annual evaporation occur during the eight-month growing season, March to October.

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[36] "Evaporation from Free Water Surfaces," by C. Rohwer, Assoc. M. Am. Soc. C. E., *Technical Bulletin No. 271*, U. S. Dept. of Agriculture, December, 1931, pp. 23–26.

Hedke Method of Estimating Consumptive Use.—The method of estimating consumptive use derived by C. R. Hedke,[37] M. Am. Soc. C. E., is based on the principle that each crop requires a definite quantity of heat above its germinating temperature to bring it to maturity, and that moisture and plant food are required in proportion to the heat utilized. Basing his study on the years 1916 and 1917 and the estimated normal year, in the Cache la Poudre Valley, Mr. Hedke accumulated mean daily temperatures, in degrees Fahrenheit above the germinating temperature, for each crop produced in the valley over its period of growth; and he weighted each crop total by the area devoted to that crop. The weighted average consumptive use of heat per cropped acre was then plotted against consumptive use of water in acre-feet per cropped acre to obtain a correlation curve. In determining consumptive use of water, 50% of the annual precipitation was added to the measured stream depletion. The ultimate object of the method was apparently a guide for estimating stream depletion in other valleys, and it was assumed that effective moisture not obtained from precipitation must be supplied from stream flow. Mr. Hedke assumed that precipitation, falling on the soil surface, evaporates twice as rapidly as normal soil moisture drawn from depth. He did not allow for variations in ground-water storage in computing consumptive use, and he charged the total valley use to the cropped area without regard to relative uses by non-cropped land.

METHOD OF STUDY

The present study, like that of Mr. Hedke, is predicated on correlating consumptive use of water to effective heat during the growing season; but in details it departs widely from the Hedke method. Of necessity the study has been confined to a rather limited number of areas for which essential data are available. These data have been utilized in accordance with a method of procedure found by trial to give the most satisfactory results.

Measurement of Use of Water.—On each area used in the study, adequate records or estimates were obtained of surface inflow to, and outflow from, the area; of changes in surface and ground-water storage, if appreciable; and of precipitation. The study was confined to areas believed to have negligible or compensating underground inflow and outflow. Where ground-water storage was considered to be fairly stable from year to year, no correction for changes in such storage was deemed necessary, as any error due to this factor is practically eliminated from the mean consumptive use over a period of several years. Precipitation was computed by weighting the recorded quantity at available stations by the area nearest each station. The full year, rather than the growing season, was used as a basis of study in order to eliminate the effect of seasonal changes in water table and soil moisture, which are affected by the surplus or deficiency in precipitation compared with consumptive use. For the sake of uniformity, the calendar year was used for all areas, although it is perhaps less satisfactory on some areas than some other operating year for securing accurate measurements because of unfavorable distribution of precipitation, snow storage, high stream discharge, or ground-water changes. As these factors vary with the locality, no one period would be satisfactory for all areas.

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[37] "Consumptive Use of Water by Crops," by C. R. Hedke, New Mexico State Engrs. Office, July, 1924 (unpublished).

Correction for Years of Short Water Supply.—In most of the areas studied, shortages of water supply occurred in one or more years of the period of record. Omitting the records of the dry years in arriving at mean values for each area introduces errors in those cases where no records of ground-water storage are available because it is usually withdrawn in dry years, through consumptive use and natural drainage into streams, and is replaced in wet years. When dry years are omitted, ground-water recharge enters into the mean as consumptive use. It was believed preferable, therefore, to include all years of record in the mean and to add a correction factor to the water supplies in low years sufficient to make them adequate. From an inspection of the records of inflow and consumptive use in dry years, it appears that consumptive use deviates from the mean much less, relatively, than does total inflow. In the humid regions the lessened rainfall is more fully used because the soil capacity of the root zone is overtaxed less frequently with reduced escape of water to the underlying water table. Runoff is also relatively less. In the irrigated areas, the same results are achieved by reduced irrigation applications and reduction in surface waste. Withdrawals from ground water both for plant life and through natural drainage are important in such years. In succeeding wet years there is a recharge of depleted ground water.

Equivalent Area.—Instead of referring use to cropped area or to gross valley area as a basis for computing consumptive use (as has been done frequently), an estimate was made of the area, within the limits defined by points of measured inflow and outflow, consuming water at a rate equivalent to that on the cropped area or land representative of prevailing conditions in the valley. In irrigated valleys, high areas consuming only precipitation and contributing a negligible quantity of runoff were omitted. Corrections were made in the remaining area, as believed advisable, to compensate for unbalanced areas in swamp land consuming water at a higher rate than cultivated land and in natural vegetation adjacent to irrigated areas consuming less water than cultivated land. Although these corrections are a matter of judgment, the net effect is small, since the cropped land in most agricultural valleys amounts to 85% or more of the gross area.

Correction for Variations in Irrigated Area.—In irrigated valleys the acreage actually irrigated and cropped may vary from year to year. After full development has been reached, fluctuations in area are normally so small that they may be neglected in computing consumptive use; but, if economic conditions cause major changes, the variations in area should be considered. Since

1933 the program of the Agricultural Adjustment Administration (AAA) in many localities has effected reductions in the area devoted to certain staple crops and has freed the land for other uses. In some cases land has been removed from cultivation and has received little or no regular irrigation. Where the cropped area in a given valley has been reduced during the period of study, consumptive use values in the affected years have been computed on an equivalent area, resulting from a deduction of one half the area temporarily out of cultivation that year, on the theory that a limited weed growth and soil evaporation would still occur. This is equivalent to fixing the consumptive use on the fallow land at one half that on cropped land.

Effective Heat.—As used in this paper, the term "effective heat" is the accumulation, in day-degrees, of maximum daily temperatures above 32° F during the growing season. This measure of heat was decided upon after comparative studies had been made using mean daily, as well as maximum, temperatures with bases both above and below 32°. The choice of the maximum temperature is in line with experiments showing that transpiration and evaporation take place almost entirely during the daylight hours, when temperatures are normally higher than at night. Although evaporation of moisture from soil and other surfaces continues at a low rate during the winter, transpiration is almost entirely confined to the growing season. During the winter season, maximum temperatures are frequently considerably above the freezing point even when minimum temperatures prevent plant growth. Because of this break in the relation of consumptive use to heat, a much better correlation is obtained between annual consumptive use and accumulated growing-season heat units than with total annual heat units.

Length of Growing Season.—The growing season is frequently considered the time between killing frosts. For annual crops the growing season is necessarily shorter than the frost-free period, in order to permit planting in the spring after danger of frosts is past and to permit maturing and possibly harvesting before the first serious frost in the fall. For most perennial crops, growth starts as soon as the maximum temperature rises far above the freezing point for any extended period of days and continues in spite of later freezes throughout the season—and sometimes for many weeks after the first so-called killing frost. In the spring, and to a less extent in the fall, daily minimum temperatures may fluctuate 1° or 2° above and below 32° for several days before remaining above or below that point, even though the temperature for most of the day is quite warm and the hardier crops are able to grow unharmed by the few hours of subfreezing temperature. In fact, many hardy crops, and especially grasses, mature although temperatures drop below freezing at frequent intervals throughout the summer.

In order to define the limits of the growing season more definitely than the dates of the latest and earliest frosts, minimum temperature data were smoothed by obtaining twice-repeated five-day moving averages, equivalent to applying the expression, $\frac{a + 2b + 3c + 4d + 5e + 4f + 3g + 2h + i}{25}$, to the consecutive daily minimum temperatures (represented in the expression by letters) and offsetting the result to the middle day of the series to maintain proper phase. The ends of the growing season are thus defined by the dates between which the smoothed temperatures remain above 32°.

Valley Areas Studied

The work of the Bureau of Reclamation is concerned primarily with the arid West where agricultural development is dependent upon irrigation. In this paper, available studies on consumptive use were utilized. Some of them were on Bureau of Reclamation projects, constructed or under investigation; others were studies by various agencies in connection with local and interstate litigation. Many other areas were examined but discarded for lack of adequate records.

As a check on the validity of the conclusions reached for irrigated areas, the study was extended to a number of valleys in the humid regions of the United States and to one mountain basin in Colorado, where an essentially adequate water supply is obtained from precipitation. In these cases the entire watersheds are included in the study areas, and consumptive use is computed by deducting outflow from precipitation. Consumptive use in these humid valleys was found to vary with effective heat of the growing season in exactly the same manner as in irrigated arid valleys.

Table 1 lists the various valley areas used, with comparative data on area, elevation, and number of years of study. Fig. 2 shows the average consumptive use on each area for the period of study, plotted against the corresponding average effective heat. A brief description of each area, with annual data on consumptive use, effective heat, and source of basic information, is given in the Appendix.

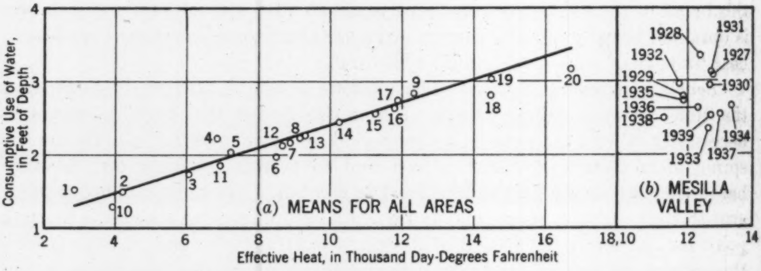
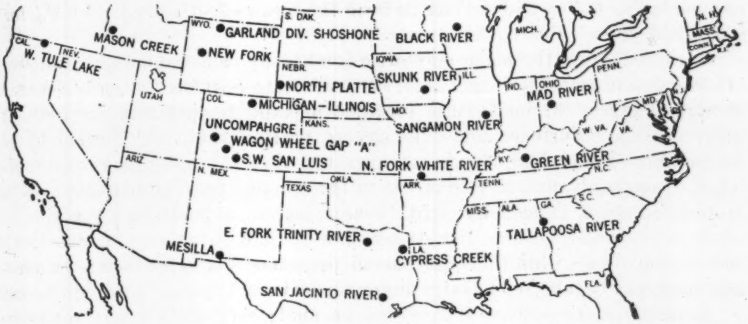


FIG. 2.—RELATION OF CONSUMPTIVE USE TO EFFECTIVE HEAT (NUMBERS IDENTIFIED IN TABLE 1)

The areas included in the study represent practically the entire range of growing conditions and types of agriculture found in the United States. Climatic conditions range from arid to humid and from frigid to subtropical, growing seasons range from a few days to the entire year, latitudes range from near the northern to the southern boundaries of the nation, and altitudes from near sea level to over 10,000 ft; crop diversification varies from the mountain meadow hay and alpine forests to cotton and citrus fruits. Notwithstanding the wide differences in crops, the relation between consumptive use and effective heat is maintained closely.

TABLE 1.—COMPARATIVE DATA ON VALLEY AREAS STUDIED


CAL.
NEV.
W. TULE LAKE
MASON CREEK
WYO.
GARLAND DIV. SHOSHONE
S. DAK.
BLACK RIVER
MICH.
N.Y.
MASS.
CONN.
R.I.
NEW FORK
IOWA
NEBR.
SKUNK RIVER
ILL.
PENN.
UTAH
NORTH PLATTE
IND.
OHIO
MAD RIVER
COL.
MICHIGAN-ILLINOIS
MO.
MD.
UNCOMPAHGRE
KANS.
SANGAMON RIVER
W. VA.
VA.
WAGON WHEEL GAP "A"
S.W. SAN LUIS
ARIZ.
N. MEX.
N. FORK WHITE RIVER
KY.
GREEN RIVER
N.C.
OKLA.
TEXAS
ARK.
TENN.
S.C.
MISS.
ALA.
GA.
TALLAPOOSA RIVER
E. FORK TRINITY RIVER
MESILLA
LA.
CYPRESS CREEK
SAN JACINTO RIVER
FLA.

No.	Valley (see Appendix)	State	Approximate mean elevation, in ft	Equivalent area, in acres	Length of record, in years	Mean consumptive use, in ft	Mean effective heat, in day-degrees (F)
	(a) IRRIGATED VALLEYS						
1	New Fork	Wyoming	7,400	25,000	2	1.59	3,960
2	Michigan and Illinois	Colorado	8,300	43,000	3	1.50	4,210
3	Southwest Area, San Luis	Colorado	7,750	390,000	1	1.74	6,070
4	West Tule Lake	California	4,150	6,300	7	2.22	6,890
5	Garland Division of Shoshone Project	Wyoming	4,350	41,900	4	2.02	7,240
6	North Platte	Wyoming-Nebraska	4,100	462,000	9	1.98	8,560
7	Mason Creek and Boise	Idaho	2,500	13,570	2	2.17	8,940
8	Uncompahgre	Colorado	5,500	137,700	3	2.28	9,550
9	Mesilla	New Mexico-Texas	3,800	109,000	13	2.83	12,370
9*a*	Greenfields Division of Sun River Project	Montana	3,800	37,500	2	1.98	7,240
9*b*	Pecos River near Carlsbad	New Mexico	3,100	37,850	18	2.94	13,600
	(b) NON-IRRIGATED WATERSHEDS						
10	Wagon Wheel Gap "*A*"	Colorado	10,000	222	14	1.30	3,980
11	Black River	Wisconsin	1,200	494,000	13	1.85	6,950
12	Mad River	Ohio	1,100	307,000	13	2.15	8,760
13	Skunk River	Iowa	700	1,850,000	16	2.25	9,340
14	Sangamon River	Illinois	700	1,640,000	16	2.43	10,270
15	North Fork of White River	Missouri	1,200	755,000	14	2.58	11,260
16	Green River	Kentucky	600	5,000,000	6	2.62	11,810
17	Tallapoosa River	Alabama-Georgia	900	1,060,000	13	2.75	11,900
18	East Fork of Trinity River	Texas	600	531,000	14	2.82	14,460
19	Cypress Creek	Texas	300	545,000	13	3.02	14,500
20	San Jacinto River	Texas	200	1,159,000	9	3.19	16,710

ANNUAL VARIATIONS IN CONSUMPTIVE USE

The plotted points of Fig. 2(*a*) show only a small deviation from the curve. When individual years instead of means are plotted, a rather wide variation is noted. As an example, Fig. 2(*b*) shows the relation of consumptive use to effective heat for the Mesilla Valley, New Mexico, for the period of study 1927 to 1937. Measurements on this valley are probably more complete than on other areas studied, and yet deviations from the mean consumptive use amount to as much as 18.7% and deviations from the mean effective heat to 6.4% for individual years.

Variations from the mean may arise from many causes, among which are: (1) Inaccurate records of discharge; (2) inaccurate estimates of ground-water storage changes; (3) inadequate precipitation and temperature records; (4) differences in occurrence and intensity of precipitation, and variability of temperatures, wind, hail, cloudiness, humidity, etc.; (5) inaccuracy in estimating the area equivalent in rate of use to the cropped area; and (6) changes in crop distribution, damage by plant diseases, insects, animals, and weeds.

In many valleys where the stream flow is quite large as compared with stream depletion within the valley, small percentages of error in stream measurement may be reflected by large discrepancies in estimates of consumptive use.

Corrections for surface storage can be made accurately where reservoir-capacity data and storage records are available but, because of the heterogeneous nature of most valley fills, it is difficult to evaluate ground-water storage changes. Water-level measurements from only a few wells may be misleading. Application of a single yield coefficient for converting differences in depth into storage changes gives a rough estimate at best and may not reflect actual conditions, especially where changes in water-table levels are not uniform throughout the valley.

Inadequate coverage by precipitation stations may result in wide discrepancies where rainfall distribution tends to "spottiness." Even with adequate coverage, variations still will prevail where there are marked differences in seasonal distribution. For the same annual precipitation, soil-surface evaporation will be much greater in a year when there are numerous light showers than when there are only a few heavy rains which penetrate deeply and also produce considerable surface runoff. The distribution between summer and winter also affects consumptive use, as evaporation is higher in summer than in winter.

Temperatures do not usually vary greatly over a valley area, and for that reason inadequate coverage by recording stations is not likely to be an important source of error in estimating effective heat. Differences in temperature from year to year will normally cause corresponding differences in consumptive use; but changes in consumptive use may vary widely from the relation exhibited by valley means (see Fig. 2(*a*)) because of changes in temperature distribution and plant efficiency. Abnormally low temperatures may retard plant development, or unusually high temperatures may cause plants to become dormant. The latter effect is especially noticeable in the case of the southern study areas where consumptive use is actually reduced in the hotter years, even when the water supply is adequate. Consumptive use may vary widely in years of equal accumulated temperatures because of deviations from the normal seasonal distribution, as transpiration is influenced by the area of leaf surface and plant functional requirement (both related to stage of maturity), as well as by temperature. The effect of increased wind movement is normally to increase evaporation; and the effect of increased humidity and cloudiness is to reduce it. Hail may damage crops and thus reduce, materially, their rates of transpiration.

The annual consumptive use varies with type of management and land use, which are influenced by economic factors. These factors may change crop distribution and methods of irrigation. Thus, a change from grain to alfalfa will increase transpiration. Consumptive use by native vegetation on noncropped areas may be expected to remain more or less constant from year to year where the water supply is adequate, but surface evaporation will increase in wet years because of the increase in area of water surface and moist soil.

In irrigated valleys there is usually a lavish application of irrigation water when the supply is abundant; and there is a consequently larger amount of waste and return flow available for nonproductive consumption, as well as an increased soil evaporation on the cropped lands. For these reasons the equivalent valley area is seldom a constant; actually, it increases in wet years and decreases in dry years. Plant diseases and pests reduce consumptive use by inhibiting plant growth. The spread of noxious weeds may reduce the area irrigated if crops cannot be grown in infested areas. Indirect results of infestation may appear in widespread changes in crops.

Most of the factors mentioned fluctuate from year to year. Consumptive-use studies continued over a series of years may be expected to eliminate most of the inaccuracies in measurements and smooth out short-time trends due to changes in crop distribution and other factors.

APPLICATION

An essential feature of the water-supply investigation of an irrigation project, whether it is an undeveloped tract needing a full supply or a developed project needing supplemental water, is the determination of its water requirement. Annual consumptive use is a convenient approach in this determination. From the relation developed in this paper it is possible to estimate consumptive use from temperature data, which are usually available. The crop irrigation requirement may then be estimated by deducting from consumptive use that part of the annual precipitation that it is estimated will be consumed. The farm delivery requirement then may be estimated by adding to the crop irrigation requirement an allowance for deep percolation and surface waste incident to application of water to the land, consistent with anticipated irrigation practice under local conditions. The diversion requirement then may be estimated by providing for canal and lateral operation waste and losses in transit from stream or reservoir to the farm. In case the project is to be devoted chiefly to a single crop, differing materially in its water requirements from the average requirements common to the temperature conditions of the project, the consumptive use estimated from the relation curve may be corrected as required. Although the determination of the project-diversion requirement leaves much to judgment after the proper consumptive use has been adopted, a mass of experimental data on the other factors is available to guide the investigating engineer in his determinations.

ACKNOWLEDGMENT

Data for this paper were secured from various sources, and where possible the source has been stated. Data on land use in the humid areas were obtained from U. S. Census reports, or from personal knowledge, and climatic and runoff data used in connection with area studies, not specifically acknowledged, were obtained from publications of the Weather Bureau and Geological Survey, respectively. Appreciation is expressed for the advice and assistance of R. I. Meeker, M. Am. Soc. C. E., and R. J. Tipton, Assoc. M. Am. Soc. C. E., in the conduct of this study.

This paper was prepared in the course of official duties for the Bureau of Reclamation of which John C. Page, M. Am. Soc. C. E., is commissioner, and S. O. Harper, M. Am. Soc. C. E., is chief engineer. The work was done under the immediate supervision of E. B. Debler, M. Am. Soc. C. E., hydraulic engineer. Many of the project superintendents and investigating engineers of the Bureau of Reclamation have assisted in supplying valuable data. Assistance in making studies and computations was rendered by assistant engineers R. R. Reed, Jun. Am. Soc. C. E., and C. L. Myers; and junior engineers I. F. Richards and C. E. Mohler, Juniors, Am. Soc. C. E., and R. I. Brasaemle, H. Y. Hsu, and J. S. Fulton.

APPENDIX

DESCRIPTION OF AREAS STUDIED

New Fork Valley, Wyoming.—The valley is in the upper Green River Basin above Pinedale, Wyo., at an elevation of about 7,400 ft. It has a gross area of 25,400 acres, of which 16,650 acres in meadow hay and pasture are irrigated. A part of the remainder is in sagebrush and a part in willows and seeped alkali land. The equivalent area is estimated at 25,000 acres. The climate is severe in winter. Summer temperatures reach 90°, but frosts frequently occur at night in every month. Precipitation averages 10.2 in., of which 4.3 in. fall in the months of May to August. Irrigation by wild flooding is practiced, and the area always has adequate water. Consumptive-use studies were made by the Bureau of Reclamation in 1939 (Table 2, No. (1)) and include inflow and outflow measurements and precipitation. Ground-water storage, as indicated by test wells, did not show any change for the year, and ground-water movement is believed to be small. Because of the prevalence of summer frosts, the effective growing season is doubtless longer for the type of crop grown than that computed for the study.

Michigan and Illinois Rivers, North Park, Colorado.—The study area includes the narrow valley of the Michigan River between the Haworth School and Cowdrey and the tributary valley of the Illinois River below Rand, in North Park, near Walden, Colo. Grass and hay, covering 35,250 acres, are the only crops. The remainder of the area is in willows and water surface. The equivalent area is estimated at 43,000 acres. The average elevation is about 8,300 ft. The climate is rigorous. Temperatures range from − 49° to 109° with a May-to-September average of 53°. The frost-free period is about

60 days. Annual precipitation averages 10 in., about one half falling during the growing season. The meadows are irrigated by flooding until hay is cut, and the supply obtained from the Michigan and Illinois rivers is adequate. Data (Table 2, No. (2)) were supplied by the Colorado Water Conservation Board.[38] Winter runoff not covered by records was estimated. Ground-water storage as shown by well measurements was essentially the same each year with no movement into, or out of, the basin.

TABLE 2.—CONSUMPTIVE USE SUMMARY, IRRIGABLE VALLEYS ((1) TO (4)) IN WYOMING AND COLORADO[a]

Year	(1) NEW FORK VALLEY		(2) MICHIGAN AND ILLINOIS		(3) SAN LUIS VALLEY		(4) WEST TULE LAKE	
	U	H	U	H	U	H	U	H
1933	....	....	....	....	....	....	1.47	5,520
1934	....	....	....	....	....	....	2.66	6,850
1935	....	....	....	....	....	....	2.31	6,440
1936	....	....	....	....	1.74	6,070	1.82	6,900
1937	....	....	1.70	4,630	....	....	2.19	6,890
1938	....	....	1.27	4,550	....	....	2.24	7,200
1939	1.53	2,860	1.53	3,570	....	....	2.86	7,520
1940	1.65	5,020	....	....	....	....	....	....
Mean	1.59	3,940	1.50	4,250			2.22	6,890

[a] U = consumptive use, in ft; and H = effective heat, in degree-days (F).

Southwest Area, San Luis Valley, Colorado.—The area includes the valley lands tributary to the Rio Grande on the west, amounting to 400,000 acres. The topography is comparatively flat—about 7,750 ft in elevation. About 56% of the area is farmed, principally to hay, grain, and potatoes. The equivalent area is estimated at 390,000 acres. The climate is moderately warm, dry, and windy during the short summers, and cold in winter. Precipitation averages about 7 or 8 in. per yr, about 65% of it in the growing season. Water for irrigation is supplied from the Rio Grande and its tributaries. Subirrigation is practiced widely. Consumptive use has been determined by a number of agencies from inflow-outflow measurements, but the present study (Table 2, No. (3)) includes only the 1936 determination by the Bureau of Agricultural Engineering as a part of the Rio Grande Joint Investigation.[10]

West Tule Lake, Klamath Project, California.—The area includes land under the J-1 lateral of the Tule Lake Division of the Klamath Federal Reclamation Project lying within the dikes west of Lost River. The mean elevation is about 4,150 ft. The effective area is estimated at 6,300 acres, of which 5,700 acres have been irrigated in recent years. The soil is a sandy lake-bed deposit and drains well. Alfalfa, cereals, and potatoes are the principal crops. Temperatures vary from sub-zero to 100°. The growing season is about 140 days. Precipitation averages about 8.5 in., most of which occurs in the fall and winter. Irrigation is necessary for agriculture, and an adequate supply is obtained from the Lost and Klamath rivers. Inflow is measured in the supply lateral and outflow at drainage pumps. Ground-water storage is maintained nearly constant by well-regulated drains; and, because of the small differential between Lost River and drains, little underground flow is believed probable. Data (Table 2, No. (4)) were furnished by the Klamath Project office and from the report by J. R. Iakisch,[39] Assoc. M. Am. Soc. C. E.

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[38] "Jackson County Investigations—Special Studies on Consumptive Use," by C. L. Patterson, Colorado Water Conservation Board, 1940 (in course of preparation).

Garland Division, Shoshone Project, Wyoming.—The area consists of 48,400 acres of fairly level bench land, 4,200 to 4,600 ft in elevation, lying north of the Shoshone River near Powell, Wyo. Although only 30,000 acres were irrigated at the time of the study, 41,900 acres are irrigable and estimated to consume water equivalent to the rate on cropped land. Soils vary from sand to clay. About 50% of the cropped area was in alfalfa, the remainder being mainly grain, sugar beets, and potatoes. Inflow is supplied by the Garland Canal and some storm runoff; outflow is measured in the Frannie Canal, Bitter Creek, and river drains. Complete records of inflow and outflow are not available in any one year, but the missing data (storm inflow, and drains to river and Bitter Creek) have been estimated from measurements available in other years. Deep percolation losses are believed negligible. The water supply was average in two years, abundant in one, and deficient in one. Data in Table 3, No. (5), were supplied by the Bureau of Reclamation project office and from reports by H. H. Johnson.[40]

TABLE 3.—CONSUMPTIVE USE SUMMARY, VALLEYS (5), (7), AND (10), IDAHO, WYOMING, AND COLORADO[a]

Year	(5) GARLAND DIVISION			(7) MASON CREEK		(10) WAGON WHEEL GAP[c]		
	U_p	U_r	H	U	H	U_p	U_r	H
1912	....	....	....	....	....	0.86	1.05[b]	3,990
1913	....	....	....	....	....	1.30	1.30	3,920
1914	....	....	....	....	....	1.27	1.27	4,200
1915	....	....	....	....	....	1.35	1.35	3,600
1916	....	....	....	2.13	8,760	1.48	1.48	3,710
1917	....	....	....	2.20	9,130	0.75	0.91[b]	3,930
1918	....	....	....	....	....	1.65	1.65	4,410
1919	....	....	....	....	....	1.43	1.43	4,060
1920	....	....	....	....	....	1.14	1.14	3,910
1921	....	....	....	....	....	0.97	1.13[b]	4,060
1922	....	....	....	....	....	1.54	1.54	4,520
1923	2.06	2.06	7,480	....	....	1.43	1.43	3,960
1924	1.64	1.84[b]	6,960	....	....	0.75	1.02[b]	3,860
1925	2.16	2.16	7,120	....	....	1.43	1.43	3,540
1926	2.02	2.02	7,390	....	....	....	....	....
Mean	1,97	2.02	7,240	2.17	8,940	1.24	1.30	3,980

[a] U = consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Non-irrigable watersheds; Nos. (5) and (7) are irrigable.

North Platte Valley, Wyoming, and Nebraska.—The study area includes the valley between Whalen, Wyo., and Lisco, Nebr. This valley has an average elevation of 4,100 ft, a length of 120 miles, and a width of 7 miles. It includes an equivalent area estimated at 462,000 acres, which includes the North Platte Federal Reclamation Project and private lands. The bottom soils are black loam, the terrace soils sandy loam. There is a considerable area of river bed, reservoir surface, and seeped bottom land. Part of the area once seeped has been drained. The underlying Brule clay precludes deep percolation, and as the channel is cut into this material there is little underflow at the river gaging stations. The area is in general crops with alfalfa, sugar beets, and cereals prevailing. Temperatures vary from sub-zero to more than 100°. The growing season is about 165 days. Precipitation averages about 17 in., of which 75% falls in the growing season. Additional water is supplied by irrigation from the North Platte River and from reservoir storage. The supply has been adequate except in years such as 1931 and 1934. Records of inflow and outflow (Table 4, No. (6)) were corrected for changes in surface storage, but no correction for ground storage was considered necessary. Records were obtained from the North Platte Project Office.

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[39] "Report on Pumping from Tule Lake and Wild Life Refuge Development," by J. R. Iakisch and others, Bureau of Reclamation Project Investigations Report No. 5, April, 1938.

[40] "Consumptive Use of Water, Garland Division of Shoshone Project," by H. H. Johnson, Reports for years 1923–1926, Bureau of Reclamation (unpublished).

TABLE 4.—CONSUMPTIVE USE SUMMARY, VALLEYS (6), (8), (9), AND (16)[a]

Year	(6) NORTH PLATTE			(8) UNCOMPAHGRE		(9) MESILLA			(16) GREEN RIVER[d]		
	U_p	U_r	H	U	H	U_p	U_r	H	U_p	U_r	H
1927	....	....	....	....	....	3.09	3.14[c]	12,750	....	....	....
1928	....	....	....	....	....	3.36	3.36	12,370	....	....	....
1929	....	....	....	....	....	2.81	2.81	11,840	....	....	....
1930	....	....	....	....	....	2.96	2.96	12,630	....	....	....
1931	1.58	1.71[b]	8,230	....	....	3.13	3.13	12,670	2.77	2.77	11,740
1932	2.31	2.31	7,740	....	....	2.98	2.98	11,750	2.69	2.69	11,630
1933	1.93	1.93	8,930	....	....	2.35	2.43[c]	12,570	2.57	2.57	11,680
1934	0.95	1.82[b]	8,380	....	....	2.67	2.79[c]	13,230	2.41	2.41	11,430
1935	1.69	1.69	7,720	....	....	2.63	2.82[c]	12,240	2.60	2.60	11,740
1936	2.10	2.10	9,380	....	....	2.73	2.76[c]	11,860	2.21	2.66[b]	12,660
1937	2.30	2.30	8,020	....	....	2.54	2.54	12,940	....	....	....
1938	2.09	2.09	9,310	2.20	9,200	2.50	2.54[c]	11,310	....	....	....
1939	1.83	1.83	9,370	2.27	9,190	2.53	2.56[c]	12,630	....	....	....
1940	....	....	....	2.38	10,250	....	....	....	....	....	....
Mean	1.86	1.98	8,560	2.28	9,550	2.79	2.83	12,370	2.54	2.62	11,810

[a] U = consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Years of reduced acreage, U_r based on equivalent area less one half of reduction in area. [d] Non-irrigable watershed; Nos. (6), (8), and (9) are irrigable.

Mason Creek Area—Boise Project, Idaho.—The area includes 13,570 acres of the Pioneer Irrigation District in the Boise Project of the Bureau within the Mason Creek drainage unit near the towns of Caldwell and Nampa. The elevation is about 2,500 ft and the topography fairly flat. It is all irrigated except for right of ways and farmsteads. At the time of the study it was more than 50% in alfalfa and clover hay, and the remainder was largely grain. An adequate water supply is furnished through the Phyllis Canal from Boise River and Arrowrock Reservoir. Precipitation amounts to about 10 in. per yr, little of it occurring in the growing season. .Outflow is measured in the various drains and by observing the ground-water changes in 55 wells. There is apparently little difference in ground-water inflow and outflow as the area is surrounded with uniformly well-drained irrigated land (see Table 3, No. (7)).[19]

Uncompahgre Valley, Colorado.—The valley is at the foot of the San Juan Mountains in the Gunnison Basin and includes Montrose and Delta, Colo., at an average elevation of about 5,500 ft. Most of the area is in the Uncompahgre Federal Reclamation Project. It has a gross area of 169,000 acres, of which 86,500 acres are irrigated and cropped to alfalfa, cereals, sugar beets, and onions. The soil is heavy, much of it has been seeped, and a part is affected by alkali. From a field study of the non-cropped land, the area equivalent in rate of use of water to the cropped land is estimated at 137,700 acres. Temperatures range from −25° to 102° with an average during the growing season of about 67°. The growing season is about 145 days. The average annual precipitation is 9 in., of which about one half falls in the growing season. Abundant water for irrigation is obtained from the Uncompahgre and Gunnison rivers and the Taylor Park Reservoir. Ground-water inflow and outflow are believed to be small. Corrections were made for ground-water storage change effected by drainage operations. Data were secured in a cooperative study by the Bureau of Reclamation and Colorado Water Conservation Board (see Table 4, No. (8)).

Mesilla Valley, New Mexico-Texas.—The Mesilla Valley is one of the many narrow valleys of the Rio Grande. It extends from Fort Seldon, New Mexico, almost to El Paso, Tex., and comprises the Leasburg (N. Mex.) and Mesilla divisions of the Rio Grande Federal Reclamation Project. The valley bottom is flat and varies in elevation from 3,700 to 3,900 ft. Of the gross area of 109,000 acres, about 77,000 acres are cropped, principally to cotton and alfalfa, 13,000 acres are in native vegetation, and the remainder in miscellaneous uses. Studies by the U. S. Bureau of Agricultural Engineering[10] in 1936 indicate the consumptive use by the non-cropped land to be essentially at the same rate as by the cropped land except in the case of areas temporarily out of cultivation, which vary from year to year. An equivalent area of 109,000 acres has been used except for the years noted in Table 4, No. (9). Temperatures range from −8° to 106° with an annual mean of about 60°. Annual precipitation is about 8 in., two thirds falling in the 240-day growing season. Irrigation water is supplied from the Rio Grande with storage in Elephant Butte and Caballo reservoirs. The data in Table 4, No. (9), were supplied by the Rio Grande Project Office, and for the years 1927 to 1936 were included, in uncorrected form, in the study by Mr. Debler.[41] The effect of annual differences in the factors affecting consumptive use of water is well illustrated by the annual variations in consumptive use in the Mesilla Valley.

At Agricultural College, N. Mex., precipitation in 1931 was the highest of the years of study and consumptive use of water second highest. In 1935 precipitation was second highest in total, but approximately one half of the total fell in one day and consumptive use was below the average of the period. In 1928, the year of greatest consumptive use, precipitation occurred in many light rains. The year of least rainfall, 1933, was also the year of least consumptive use.

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[41] "Valley Consumptive Use," by E. B. Debler, *Transactions*, Am. Geophysical Union, Pt. II, National Research Council, 1937, Washington, D. C., pp. 532–536.

The effective heat in 1933 was approximately the same as in 1931 but consumptive use was 0.78 ft less. Lower temperatures in April, May, and June may account for a part of the lower use in 1933, with rainfall and other factors accounting for the remainder.

Land use is one of the most important factors causing variation in consumptive use. The cotton acreage in the Mesilla Valley declined between 1930 and 1935 and has increased since that time. Compensating changes were made in general crops or in the fallow acreage. The alfalfa acreage has remained practically constant for many years. The effect of differences in land use is most apparent in 1933, 1934, and 1935 (Table 4, No. (9)), when the government cotton-reduction program retired large areas not used for other crops. The difference of 0.73 ft in consumptive use for the years 1928 and 1935, years of nearly equal effective heat, is largely due to a reduction of 13,882 acres in irrigated area. It is possible, too, that water-surface evaporation and consumption by native vegetation growing along the Rio Grande was much less in 1935 than in 1928 as the runoff of the river at Leasburg, head of the valley, was only 69% as great as in 1928. The reduction in consumptive use in 1933 as compared with 1931 is explained in part by the fact that 6,870 acres were plowed under in 1933 and no other use made of the land that year. Nonproductive uses of water in these two years were probably about the same.

The low consumptive use in 1937 cannot be explained by any one of the factors mentioned. Unusually favorable circumstances must have prevailed in that year to account for the high crop yields and low consumptive use of water.

The corrected use data in Table 4, No. (9), have been corrected to a full-area basis by reducing the equivalent area by 50% of the area temporarily out of cultivation. This is based on an estimated consumptive use, by the land that is temporarily uncropped, of 50% of the cropped land. In 1927 development was incomplete; in 1933, 6,870 acres of cotton were plowed under but reported in crop; in 1934, 9,220 acres of cotton land were fallow but not reported; and, in other years, the corrected full irrigation development was set at 76,900 acres, which is the mean for the period 1928 to 1934, and the year 1937.

Wagon Wheel Gap Area, Colorado.—The area consists of a 222-acre watershed tributary to Rio Grande at Wagon Wheel Gap, Colo. Elevations range from 9,400 to 11,300 ft. The area is forested with Douglas fir and other growth typical of the central Rocky Mountains, but includes no agricultural land. Its water supply is limited to precipitation, which varies little from the average of 21 in. per yr. Runoff, and hence consumptive use, varies from year to year because of differences in intensity and occurrence of precipitation. There are no known seepage losses from the area. The data in Table 3, No. (10), are for watershed "*A*," used in the experiments of the Forest Service and Weather Bureau described by C. G. Bates and A. J. Henry.[42]

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[42] "Forest and Streamflow Experiment at Wagon Wheel Gap, Colorado," by C. G. Bates and A. J. Henry, *Monthly Weather Review*, Supplement No. 17 (1922) and Supplement No. 30 (1928).

Black River, Wisconsin.—The drainage basin above Neillsville, Wis., comprises 770 sq miles of gently rolling land, 62% of which is cropped, 30% pastured, and the remainder wooded. The average elevation is about 1,200 ft. Temperatures vary from −40° to 101°, although summer temperatures are normally moderate. The growing season is 164 days. Annual precipitation averages only 31 in. but as a large percentage occurs in the growing season, and as winter precipitation is held in snow storage until spring, agriculture is successfully practiced without irrigation. During the period of runoff records, water shortage was severe in only one year. Consumptive use (Table 5, No. (11)) was obtained by deducting runoff from precipitation on the entire watershed.

Mad River, Ohio.—The watershed above Springfield, Ohio, covers an area of 480 sq miles of flat to rolling land with an average elevation of 1,100 ft lying about half way between the Ohio River and Lake Erie. About 33% of the area is cropped, 38% is pastured, and the remainder is woodland.

TABLE 5.—CONSUMPTIVE USE, VALLEYS (11), (12), (13), (14), AND (17); NON-IRRIGABLE WATERSHEDS[a]

Year	(11) BLACK RIVER			(12) MAD RIVER			(13) SKUNK RIVER			(14) SANGAMON RIVER			(17) TALLAPOOSA RIVER		
	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H
1921	....	....	....	....	....	....	2.53	2.53	9,560	2.80	2.80	10,900	....	....	....
1922	....	....	....	....	....	....	2.29	2.29	9,990	1.96	2.06[b]	11,360	....	....	....
1923	....	....	....	....	....	....	2.26	2.26	8,930	2.80	2.80	9,550	....	....	....
1924	1.77	1.77	6,590	1.75	1.75	8,370	2.05	2.05	9,130	2.29	2.29	9,380	2.72	2.72	12,150
1925	2.09	2.09	6,380	2.27	2.27	9,130	2.32	2.32	9,490	2.09	2.09	10,020	2.51	2.51	13,040
1926	1.93	1.93	7,020	2.65	2.65	8,520	2.10	2.10	8,750	2.53	2.53	9,740	3.10	3.10	11,590
1927	1.76	1.76	6,860	1.99	1.99	8,830	1.99	1.99	9,340	3.01	3.01	9,020	2.66	2.66	12,800
1928	1.60	1.60	6,400	1.77	1.77	8,470	2.55	2.55	8,800	2.42	2.42	9,150	3.04	3.04	11,010
1929	1.60	1.60	7,020	2.15	2.15	8,720	1.91	1.91	9,240	2.17	2.17	10,360	2.91	2.91	11,820
1930	1.94	1.94	7,090	1.50	1.79[b]	8,680	1.86	1.86	9,100	1.69	2.19[b]	10,100	2.38	2.38	11,230
1931	2.04	2.04	7,110	2.43	2.43	9,630	2.84	2.84	9,830	2.70	2.70	11,410	2.35	2.35	11,810
1932	1.70	1.70	6,870	2.29	2.29	8,760	2.43	2.43	8,910	2.52	2.52	10,540	3.59	3.59	11,500
1933	1.39	1.80[b]	7,770	1.71	1.83[b]	8,990	1.66	2.04[b]	9,530	2.11	2.11	10,850	1.90	2.25[b]	11,420
1934	2.27	2.27	7,970	1.61	2.28[b]	9,090	2.05	2.05	10,240	2.52	2.52	10,900	2.91	2.91	11,860
1935	1.94	1.94	6,060	2.77	2.77	7,510	2.71	2.71	8,770	2.44	2.44	10,540	2.90	2.90	12,910
1936	1.40	1.68[b]	7,220	1.92	1.92	9,110	1.86	2.12[b]	9,870	2.19	2.19	10,500	2.38	2.38	11,510
Mean	1.80	1.85	6,950	2.06	2.15	8,760	2.21	2.25	9,340	2.39	2.43	10,270	2.72	2.75	11,900

[a] U = Consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Non-irrigable watersheds; Nos. (5) and (7) are irrigable.

Temperature extremes vary from −34° to 108°. The growing season is six months. Precipitation averages 38 in. per yr, of which about 21 in. occur in the growing season. This is adequate in all except a few short years for crop production. Consumptive use (Table 5, No. (12)) was obtained by deducting runoff from precipitation on the watershed.

Skunk River, Iowa.—The drainage area above Coppock, Iowa, is about 145 miles long and 20 miles wide and contains 2,890 sq miles of gently sloping prairie land, lying immediately east of Des Moines. The average elevation is about 700 ft. Crop land occupies 68% of the area, pasture 26%, and woodland 6%. Temperatures vary between extremes of −30° and 110°, and relative humidity is high. The growing season is about 205 days. Precipitation averages 32 in. per yr but has dropped as low as 18 in. Nearly two thirds of the annual precipitation falls in the spring and summer months, and the supply is ordinarily enough to mature crops. In a few recent years the supply has been inadequate. Consumptive use (Table 5, No. (13)) was obtained by deducting runoff from the precipitation falling on the entire watershed.

Sangamon River, Illinois.—The area comprises 2,560 sq miles of Sangamon River watershed above Springfield, Ill. It has an average elevation of 700 ft. The topography is gently rolling; 78% of the watershed is cropped, 18% is pastured, and the remainder is wooded. Temperatures vary from −24° to 107°, and the relative humidity is high. The growing season is about 220 days. Precipitation averages 36 in. per yr and is favorably distributed for crop use. A full water supply is obtained in nearly every year. Consumptive use (Table 5, No. (14)) was obtained by deducting runoff from precipitation over the entire watershed for the period 1921 to 1936.

North Fork of White River, Missouri.—The watershed above Tecumseh, Mo., comprises 1,180 sq miles of rough foothill country, with an average elevation of 1,200 ft, lying along the southern edge of the Ozark Plateau. There is a larger percentage of woodland than in any other area studied except the small Wagon Wheel Gap area. Only about 25% of the area is cropped and 28% pastured. Extremes of temperature vary from −29° to 106°. The growing season is about eight months. Precipitation, which furnishes the sole water supply, averages 42 in., of which 26 in. occurs during the spring and summer. The amount of precipitation fluctuates considerably but is seldom short.

TABLE 6.—CONSUMPTIVE USE, VALLEYS (15), (18), (19), AND (20); NON-IRRIGABLE WATERSHEDS[a]

Year	(15) WHITE RIVER			(18) TRINITY RIVER			(19) CYPRESS CREEK			(20) SAN JACINTO RIVER		
	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H
1924	2.78	2.78	10,630	2.00	2.65[b]	14,330	....	....	....	....	....	....
1925	2.63	2.63	11,180	1.87	2.72[b]	16,070	2.76	3.20[b]	15,470	....	....	....
1926	2.43	2.43	10,880	3.60	3.60	12,480	3.95	3.95	14,950	....	....	....
1927	3.33	3.33	11,520	3.18	3.18	14,640	3.00	3.00	15,650	....	....	....
1928	2.22	2.22	10,440	2.66	2.66	14,120	2.93	2.93	14,270	....	....	....
1929	2.45	2.45	11,150	2.46	2.46	13,990	2.21	2.21	14,170	3.47	3.47	15,920
1930	2.46	2.46	10,950	2.36	2.86[b]	14,890	2.92	2.92	13,360	2.91	2.91	16,340
1931	2.54	2.54	12,020	2.33	2.90[b]	14,800	3.04	3.04	13,430	3.03	3.24[b]	19,480
1932	2.54	2.54	11,490	2.81	2.81	13,420	2.57	2.74[b]	13,280	2.90	2.90	13,630
1933	2.43	2.43	11,700	2.52	2.85[b]	16,450	2.70	2.90[b]	14,980	2.51	3.15[b]	16,840
1934	2.23	2.23	12,350	2.23	2.79[b]	14,870	2.32	2.80[b]	15,290	3.20	3.20	17,880
1935	3.04	3.04	11,210	3.04	3.04	14,240	3.12	3.12	14,110	3.56	3.56	16,110
1936	2.04	2.63[b]	11,160	1.94	2.70[b]	15,240	2.10	2.90[b]	14,700	3.02	3.02	16,410
1937	2.35	2.35	10,970	2.33	2.33	12,880	3.12	3.12	14,850	2.93	3.30[b]	17,820
Mean	2.53	2.58	11,260	2.52	2.82	14,460	2.83	3.02	14,500	3.06	3.19	16,710

[a] U = Consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use.

Consumptive use (Table 6, No. (15)) was computed for years of recorded runoff by deducting runoff from precipitation on the entire watershed.

Green River, Kentucky.—The watershed above Livermore, Ky., has an area of 7,800 sq miles, covering a large part of Kentucky and part of Tennessee, and is the largest area studied. The topography is rolling, with an average elevation of 600 ft. About 40% of the area is cropped, about the same percentage is pastured, and the remainder is wooded. Temperatures vary widely between winter and summer. The growing season is about eight months. Annual precipitation averages about 45 in., of which more than one half occurs in the growing season. This is normally sufficient for full crop requirements, although occasional shortages occur. Consumptive use (Table 4, No. (16)) was obtained by deducting runoff at Livermore from precipitation over the entire watershed in the period of runoff record, 1931 to 1936.

Tallapoosa River, Alabama-Georgia.—The Tallapoosa River above Wadley, Ala., drains an area of 1,660 sq miles ranging from an elevation of 1,200 ft at the southern end of the Blue Ridge Mountains in Georgia to 200 ft on the coastal plain of the Gulf of Mexico. The area is only 42% cropped, with 15% in pasture, the remainder being woodland. Temperatures have an extreme range of −8° to 103° but are normally moderate due to the proximity of the Gulf. The area is occasionally subjected to severe storms. The average annual precipitation is about 52 in., a considerable part of which occurs in winter. Spring and summer rainfall is usually adequate for crop requirements. Consumptive use (Table 5, No. (17)) was obtained by deducting runoff from precipitation on the entire watershed.

East Fork of Trinity River, Texas.—This area of 830 sq miles is upstream from the stream gage near Rockwall in northeastern Texas in the blackland prairie region, one of the richest agricultural sections of the state. The topography is relatively flat, ranging in elevation from 400 to 800 ft. Of the total area, 75% is in crops, 16% is in pasture, and 9% is woodland. The distance from the Gulf of Mexico is sufficient so that temperature variations are wide. The growing season is from April to October. Precipitation furnishes the entire water supply but in many years is inadequate. It averages 21.6 in. in the spring and summer months. Runoff is flashy and dependent on storm rainfall. Consumptive use (Table 6, No. (18)) was determined by deducting runoff from precipitation on the entire watershed.

Cypress Creek, Texas.—The drainage area, containing 848 sq miles upstream from the gaging station at Jefferson, is in eastern Texas. The elevation ranges from 200 to 400 ft and the topography is gently rolling. The area was once heavily timbered but is now (1941) about 50% cropped and 20% pasture. Soils vary from sandy to tight clay. A wide variety of crops, including much truck and fruit, is grown. Temperatures vary widely from below zero to 110°. The growing season is about 280 days. Precipitation, the sole source of water supply, averages 43.9 in. per yr, of which about 70% occurs in the growing season and, except in dry years, is adequate for crop production. Relative humidity is high. Consumptive use (Table 6, No. (19)) was determined by deducting runoff from precipitation on the entire watershed.

San Jacinto River, Texas.—The San Jacinto River Valley is a flat sandy area of 1,811 sq miles, varying from 50 to 400 ft in elevation, on the coastal plain of the Gulf of Mexico a few miles north of Houston, Tex. Of the total area, 29% is pine forest, 38% prairie, and 33% cultivated. The climate is mild and humid. The precipitation is adequate for crop requirements in most years, but is frequently so intense that high runoff and shortages of moisture occur. Consumptive use (Table 6, No. (20)) was estimated as the difference between precipitation and measured outflow. Ground-water movement out of the basin was considered negligible.

Greenfields Division, Sun River Project, Montana.—The area consists of benchlands lying between Sun River and Muddy Creek near Great Falls, Mont., and draining into Muddy Creek. Elevations range from 3,600 to 4,000 ft (see No. 9*a*, Table 1). The topsoils are underlain with terrace gravels over shales so that excess waters are readily drained away. There is negligible runoff from non-irrigated lands tributary to Muddy Creek except in the case of unusual flood conditions. The acreage irrigated in 1937 and 1938 was 34,250. With allowance for 1,000 acres of seeped land and for right of ways and farmsteads, the area equivalent in use to irrigated land is estimated at 37,500 acres. Temperatures range from −46° to 112°. The growing season, based on smoothed minimum temperatures, is about 180 days. An adequate water supply is furnished through the Greenfields Canal from Sun River and from storage in Pishkun and Gibson reservoirs. Alfalfa and small grains are the main crops, although some sugar beets and seed peas are also grown. Ground-water levels at the beginning and end of the year are believed to be essentially the same. The following consumptive-use data were furnished by the Sun River Project office:

Year	Consumptive use (ft)	Effective heat (degree-days, F)
1937	2.02	7,800
1938	1.95	6,670
Mean	1.98	7,240

Pecos River Area near Carlsbad, N. Mex.—This area includes the lands of the Carlsbad Reclamation Project plus adjacent lands east of the Pecos River, all of which are supplied with irrigation water from the Pecos River. The average elevation of the area is approximately 3,100 ft (see No. 9*b*, Table 1). The irrigated soils are fairly tight, but adequate drainage removes any excess water. During the period covered by this study the acreage irrigated has remained about constant. Allowing for water-using areas in addition to those lands actually irrigated, the total water-using area has averaged 37,850 acres, which has been adopted as the equivalent area. Temperatures range from −17° to 112°. The growing season averages about 240 days. An adequate water supply is provided from the Pecos River, which is more or less controlled by storage reservoirs upstream. The main crop for a number of years has been cotton, but a large acreage is also devoted to alfalfa. Ground-water levels were assumed the same at the beginning and end of the period, although variations in the depth to the water table undoubtedly explain some of the wide variations in computed consumptive use for individual years.

Consumptive-use data comparable with Tables 2 to 6, inclusive, for the Pecos River area, are as follows:

Year	Consumptive use (ft)	Effective heat (degree-days, F)
1921	2.38	12,400
1922	2.68	13,000
1923	2.66	12,600
1924	2.74	12,450
1925	2.53	13,640
1926	4.50	12,850
1927	2.60	13,870
1928	2.54	13,740
1929	4.02	13,190
1930	3.54	14,620
1931	2.96	14,110
1932	2.92	13,050
1933	2.83	15,290
1934	2.21	14,700
1935	2.29	14,680
1936	3.25	13,750
1937	3.74	12,640
1938	2.61	14,420
Mean	2.94	13,600

Of the total consumptive-use average of 2.94 ft, 0.94 ft is supplied by precipitation, leaving 2.00 ft to be supplied by irrigation water. The data used consist of runoff records from the New Mexico State Engineer and the Geological Survey with other information furnished by the Carlsbad Project office.

DISCUSSION

E. B. Debler,[43] M. Am. Soc. C. E.—In regard to the orchard areas discussed by the authors, the writer made some study of the Kings River area in California, which is probably the most typical upon which information is available in that region. There seems to be no material departure from the curve of Fig. 2. In other words, crops are a corollary to the number of thousands of heat units. They go with that factor, and, although the authors state that the consumptive use in orchards is low, the writer believes that, in areas where cover crops are maintained to produce fertility, consumptive use will lie on this general graph (Fig. 2). The areas that are getting along with a lower consumptive use are, as a rule, the so-called clean cultivated orchards, and agriculturists now agree that such a condition can be only more or less temporary, and that with the maintenance of fertility those areas, in time, must go to cover crops.

It may appear rather extraordinary that so few areas were selected in what might be called the more highly developed, long-irrigated regions, such as the South Platte and Arkansas River areas in Colorado, and others. The situation is that, almost without exception, the older irrigated sections are subject to chronic shortages of water. The writer doubts very much that the irrigated area of the South Platte River Basin enjoys a water supply that (on the average) is more than two thirds of a full supply. Two difficulties were encountered in that area:

(1) The situation presents an almost insurmountable condition for the correction of data to represent consumptive use with a full supply, as records on water deliveries are lacking; and

(2) Reliable records on irrigated areas are not available, a situation that could not be overcome without considerable expense.

The available information was limited to reports of the water commissioners, and there are too many of those reports made with an eye to future water-rights adjudication suits! For these reasons, in areas where water is most valuable and where there is a fight for water from year to year, engineers are handicapped in arriving at consumptive use.

As to the possibility of using annual temperatures: The classic example encountered by the writer is a comparison between the Klamath Project in Oregon and a project in eastern Montana. They have about the same annual temperature. The Klamath Project has an average temperature even higher than the one in eastern Montana; yet it is subject to frost in every month of the year. Naturally, its consumptive use is very much less than that of the eastern Montana Project, where the temperatures are extremely low in the winter, but the summer temperatures are as high as they are in Texas.

In other words, what the engineers were attempting to arrive at was a correlation applicable in every location rather than an easier correlation that might not have general correlation for the United States as a whole. The

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[43] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

Klamath Project will show a frost-free period on an average of less than 100 days. The project in Montana will show a frost-free period of 150 days. The results just are not comparable. Notwithstanding this difference in the records, the Klamath area grows plenty of potatoes, which cannot be done in some areas showing a much higher average temperature. The writer is only hopeful that more information can be developed. He found an extreme dearth of records, faulty in many particulars—most commonly with reference to the irrigated areas. In some areas the consumptive use is so small in proportion to the inflowing and outflowing waters that no one can be sure of results, as a very minor error in stream flow may equal the entire consumptive use!

RHODES E. RULE,[44] ASSOC. M. AM. SOC. C. E. (by letter).—For many years it has been known to irrigation engineers who have investigated the subject that a close correlation exists between maximum temperatures and consumptive use. The authors' paper is refreshingly practical in its application of this relationship. Too often the results of extensive research are submitted in involved and inconclusive treatises beyond the grasp of the average engineer who has neither the time nor equipment to attempt to translate them into terms of useful application.

Determination of valley consumptive use lends itself to several methods of approach. By pursuing these courses separately and then reconciling the results as the final step it is possible to arrive at a conclusion with a probable error not larger than required by the uses to which the data may be put. The authors have added another avenue of attack which is simple and apparently effective, subject to certain limitations.

The following discussion relates to specific details taken in the order of their presentation in the paper and concludes with data for two Southern California basins which, as will be shown, do not accord with the empirical relationship developed by the authors.

Definitions.—Omission of the term "nonrecoverable deep percolation loss" from the definition of valley consumptive use clears up an inconsistency. This term should never have been introduced by the Committee on Duty of Water[3] since, as the authors correctly state, it "is a form of outflow independent of the factors influencing transpiration and evaporation."

A confusion of terms exists here. "Deep penetration," "deep percolation," or simply "penetration" and "percolation" have been used variously to designate that part of the moisture reaching the earth's surface which passes downward to below the root zone. As such it is not lost to available water supply and is certainly not consumptive use. Under ordinary conditions it ultimately reaches the permanent water table and becomes available for recovery by pumping or for use where it reappears at the surface as effluent seepage. In all but a few exceptional cases this availability for use is within the valley where the deep penetration originally occurred and there is no net loss of supply to the valley. On the other hand, if "nonrecoverable deep percolation" is taken to mean ground water which escapes the valley through deep subterranean channels or simply as underflow at the lower end of the area it should be included in "valley outflow" even though its estimation be a mere guess. Certainly if its quantity is large in a particular case it becomes an important item for investigation and estimate. If carelessly lumped in with consumptive use the latter would be too high and the results misleading.

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[44] Cons. Eng. Geologist, Burbank, Calif.

Lest the foregoing be thought to be mere quibbling over terms of no particular consequence it should be realized that, in spite of its basic simplicity, knowledge of consumptive use has been hampered by lack of agreement and confusion over definitions and, perhaps, a misunderstanding of the physics and geology of subsurface waters. Involving nothing more complicated mathematically than simple arithmetic the inventory of valley-water supplies is one subject concerning which there should be little disagreement as to methods and definitions.

Field-Plot Experiments.—In the discussion of plot experiments the authors have failed to recognize the valuable and determinative work done by H. F. Blaney, M. Am. Soc. C. E., F. J. Veihmeyer, Colin A. Taylor, Assoc. M. Am. Soc. C. E., and others. The thought seems to persist that plot experiments have failed to account for deep percolation and surface losses. This was the conclusion of the Committee on Duty of Water,[3] and was unquestionably true of the earlier investigations; but there are now at hand the results of much definitive work in which these losses have been measured or can be calculated from the experimental data. Some of these data are reported in papers referred to by the authors.[9, 10] The principal deficiency is in variety of crops and climatic conditions. Much of the available information relates to tree crops in California.

By proper use of the field-plot method consumptive use can be measured under actual field conditions with remarkably small error. Essential to success is careful selection of a location having uniform soil and representative crop conditions. Having such a location the soil moisture is determined by sampling before and after measured applications of irrigation or rain water. Sampling is extended to below the root zone. Surface runoff can be trapped and measured or kept to a negligible quantity by suitable control. Evaporation and transpiration are readily computed from the decrease in measured soil moisture between applications of water. Plotted foot by foot, the consumptive use follows smooth curves which permit ready detection and evaluation of deep percolation since any excess of applied water over that accounted for by these curves, plus runoff, may be ascribed with certainty to deep percolation. By careful application of water to suit the crop demands, deep percolation may be practically eliminated. The only qualifying conditions are that the water table must be beyond the reach of plant roots, the soil should be uniform, and the crop representative.

There is no surer method for the determination of consumptive use by individual crops or measurement of evaporation from bare soils than by the use of field plots if suitable locations are available and the work carefully done. It is admittedly expensive but the results justify the cost.

Integration Method.—The authors state that the integration method used by Mr. Blaney depends upon tank experiments. This is only partly true. Long-continued and successful use of the field-plot method provides the background for estimating many of the unit values in the integration method as applied by Mr. Blaney and his associates. Tank experiments become necessary where a high water table exists and the field-plot method cannot be applied.

In the last analysis the integration method relies on the judgment of the person applying it. If this judgment is to be good it must be based on a thorough knowledge of the experimental field including field-plot, tank experiments and all other sources of information.

Quantitative values of annual consumptive use by individual crops are known to cover a wide range. Under fixed climatic conditions consumptive use varies directly with total leaf surface and, with annual crops, the length of the actual growing period. Double cropping results essentially in a doubled consumptive use on an equal area. The data of Fig. 2 must be recognized as being based on a variety of vegetative types and farming practices which, when averaged, produce a high correlation between consumptive use and effective heat as defined by the authors. The same degree of correlation would not exist as between different crops. Grape vines consume far less water per acre than mature citrus trees or alfalfa. This is reasonable from a consideration of the relative leaf surfaces. A valley in which vineyards predominate doubtless would produce a point which would fall well below the curve shown. It follows that successful use of the authors' method in estimating valley consumptive use must be predicated upon a variety of conditions which, in the aggregate, produce "average" results. Specialty crops require special attention and analysis.

Knowledge of consumptive use will be served best by concerting research effort toward accurate determination of use by the more common crops under true field conditions and the relation of the resulting experimental data by some such common denominator as effective heat in order that reasonable estimates may be made for regions where experiments have not been undertaken. It is probable that comparison on the basis of relative pan evaporation would be more satisfactory but temperature records are available nearly everywhere and the effective heat method has a correspondingly wider field of utility.

All this discussion sums up to the writer's opinion that the integration method, based on reliable experimental data and checked by an inflow-outflow computation, provides the best method for estimating existing consumptive use.

The effective heat method appears to offer real promise in two ways:

(1) As a basis for comparison in estimating unit values of consumptive use for regions where experimental data are lacking (these would then be most useful of application in the integration method); and

(2) In estimating average consumptive use for proposed projects where temperature records alone are available and where a preliminary value will suffice.

Effective Heat.—Selection of 32° F as a base for computation of effective heat is subject to question and further analysis. A rigorous study of the subject belongs in the province of pure science and involves many variables. It is doubtful if a better solution could be obtained than by further investigation along empirical lines.

A glance at Fig. 1 suffices to show that, for the particular data from which this block diagram was prepared, mean maximum temperatures above, say, 52° F would produce a better monthly correlation with consumptive use and, roughly, each 10° above 52° would result in monthly consumptive use of 0.1 ft in depth.

The authors state that comparative studies were made on other bases before 32° was selected. Nevertheless, lacking definite evidence to the contrary, the writer inclines to the belief that a somewhat higher base would be more satisfactory.

Length of Growing Season.—The authors' method of calculating the length of growing season is tedious and the results do not appear to warrant the work involved. Further, since it is arbitrary, it fails to take account of farming practice which varies from place to place. Where full information regarding such practice is available or can be readily obtained in the field it offers a much better basis for computation.

Failure of the method is most clearly shown in Southern California for which no data have been given in the paper. For all practical purposes the use of twice-repeated five-day moving averages results in a 365 day growing season for the coastal plain and interior valleys of Southern California. It is obvious that some other criterion must be adopted for this location.

Careful attention to cultural practices will provide the basis for estimating an average growing season for any given area.

In many parts of California annual row crops are planted during March and April and require from 90 to 120 days to mature. After the spring crop is harvested some of the acreage is replanted. On the average, taking double-cropping into consideration, the "growing season" for row crops probably does not exceed 150 days. Following harvest some farmers practice weed control by cultivation; others permit weeds to flourish and later plow them under for green manure. Clearly variations in consumptive use by such crops are dictated more by individual cultural practice than by any arbitrarily defined growing season.

Similarly, tree crops will have widely varying uses depending upon practice with respect to cover crops now being used to an increasing extent by California farmers. Deciduous trees without a cover crop will have a growing season of perhaps 250 days, whereas a cover crop would result in appreciable consumption during every month of the year.

In other words the length of the season during which plants will grow is not controlling unless there are growing plants to utilize the available heat during this season.

The authors have based their studies on the assumption of an adequate water supply. Perhaps this should be further qualified by the assumption of an "adequate plant supply." As pointed out this is not always true, particularly where the frost-free period is long. It is concluded that cultural practice is more important than an inflexible definition of growing season based on minimum temperatures.

Annual Variations in Consumptive Use.—Annual variations in calculated consumptive use by the inflow-outflow method are usually such as to discourage reliance on one-year computations. To the causes mentioned by the authors should be added the impossibility of evaluating, accurately, water in transit to the water table. Where the preceding year has been exceptionally wet and the depth to the water table is large (in Southern California it may be several hundred feet) the quantity of water in transit may be sufficient to result in gross errors in calculated use during a single year. Experiments by the writer demonstrate that a number of years may elapse before influent seepage at the valley margin reaches the main ground-water body by lateral percolation. Vertical seepage to the water table is not usually so delayed but may require a year or more in older alluvium.

As distinguished from these apparent but not true variations there also must be considered the normal variation resulting from rainfall differences. The authors have corrected for dry years by adding a quantity considered adequate to insure "normal consumptive use." Presumably the object was to arrive at a set of figures of consumptive use based on a full evaporation opportunity. The difficulty here lies in the fact that heavy rainfall encourages, and scanty rainfall discourages, all types of vegetative growth, including a variety of weeds and grasses which are heavy consumers of water. This variation in use is independent of effective heat. The net effect is the same as if the equivalent area were increased during wet years and decreased during dry years. "Normal consumptive use" is not a constant but an average. If an addition is made for dry years a subtraction would be equally in order for wet years.

Data for San Jacinto Valley, California.—Studies[45] by Mr. Blaney, A. A. Young, Assoc. M. Am. Soc. C. E., and Paul A. Ewing, reported in 1941, provide the basis for the following analysis of an interior valley in Southern California.

The San Jacinto Basin comprises 162,100 acres, of which (1939) 31,500 are irrigated, 95,500 dry farmed, 28,200 in native vegetation, and 6,900 miscellaneous (water surfaces, towns, etc.).

Irrigated crops of the basin consist of some 10,000 acres of irrigated trees, both citrus and deciduous; 6,000 acres of alfalfa; 12,000 acres of row crops; and 2,500 acres of miscellaneous crops including grain; 1,000 acres were classed as prepared land—idle.

Consumptive use for an 18-yr period, 1922 to 1939, was computed at 198,500 acre-ft by the inflow-outflow method, and for 1938–1939 was estimated as 204,200 acre-ft by the integration method. Consumptive use on irrigated lands approximates 2.3 ft.

The writer has estimated the equivalent area at 90,000 acres with a consumptive use of 2.25 ft.

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[45] "Utilization of the Waters of Beaumont Plains and San Jacinto Basin, California," Progress Report, U. S. Dept. of Agriculture, SCS, 1941.

Mean maximum temperatures are not published for any station in San Jacinto Basin but records are available for two nearby stations having similar climatic conditions and less than a degree difference in mean temperature.

Applying the authors' moving average scheme for determination of the growing season it was found that for all practical purposes the entire year would be so classified. Since it was apparent that this was not in accord with the facts a different method was adopted.

Based on cultural practice as observed in the field the "growing season" ranges from a minimum of about 150 days for row crops to a maximum of 300 days for trees. By an approximate system of weighting according to acreage a mean growing season of 214 days was estimated. In general this covers the period from April through October and coincides with the irrigation season for this location.

Computed average effective heat during the 18-yr period, for the average year and for a 214-day season (April 1 to October 31), was as follows:

Length of season (days)	Day-degrees
365	17,700
214	11,900

Plotted in Fig. 2, on the basis of 11,900 day-degrees and a consumptive use of 2.25 ft, it is seen that the point falls below the defined curve. In terms of consumptive use it falls 0.4 ft short of the amount indicated by the curve, even after adjusting the growing season as explained herein.

Three apparent reasons come to mind to explain the discrepancy. First is the difficulty and expense of water development from which has grown a frugal system of irrigation practices. Much of the supply is pumped by individual farmers or sold on a unit basis, both conducive of careful husbanding of irrigation water. Second, the average crop grown may consume less water than the average in the valleys studied by the authors. Third, throughout most of the basin the water table lies well below the root zone, thus limiting consumptive use to water applied to the surface, either artificially or naturally.

Data for San Gabriel Valley, California.—Mr. Conkling made extensive investigations in the San Gabriel Valley,[46] in the intermediate climatic zone of the South Coastal region of Southern California. Of 133,000 acres in the basin, 85,000 were irrigated in 1926; 33,000 were fallow, and 15,000 classed as waste. The equivalent area, as estimated by the writer, was 103,000 acres. Consumptive use from inflow-outflow computations from 1923–1924 to 1926–1927 averaged 188,000 acre-ft. By integration the use was estimated to be 195,000 acre-ft or 1.9 ft in depth on the equivalent area.

Temperature records are published for the City of Pasadena, Calif., at the westerly edge of the basin. The mean effective heat for three seasons, 1924–1926, was 10,600 day-degrees for a growing season from April 1 to October 31.

Plotted in Fig. 2 the point for San Gabriel Valley falls below the curve, consumptive use being 0.6 ft less than would be inferred from the effective heat. As in the San Jacinto Valley this is in spite of a materially shorter growing season than would have resulted from using the authors' method of determination.

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[46] "San Gabriel Investigation—Analysis and Conclusions," by Harold Conkling, Report, Div. of Water Rights, Dept. of Public Works, State of California. 1929.

Conclusions.—The situation in Southern California is such that modifications of the proposed method must be made before it will yield satisfactory results. As has been shown the growing season must be ascertained by reference to agricultural practice and field observation of the growth habits of the local plant life.

Also, the water supply situation must be considered. Shortage of supply is the rule if viewed in relation to regions such as have been studied by the authors. This does not necessarily imply an actual shortage of water required to produce commercial crops. More nearly the truth is that scarcity and high cost of water have promoted a careful conservation of all available water so that a portion of what would be normal consumptive use in other parts of the country has been salvaged by cultural methods aimed toward the highest possible duty of water.

It is possible, also, that crop types prevailing in Southern California are such as to have a lower average use than in the regions represented by the curve in the paper.

Finally, and perhaps most important of all, is the situation regarding evaporation opportunity by reason of the ground-water level. Apparent from a reading of the area descriptions in the Appendix is the fact that the study areas chosen by the authors are nearly all regions of high ground water. This is emphasized by the repeated references to artificial drains. Under such conditions the evaporation opportunity is high regardless of whether land is in crop or not. This being the case there exists a near maximum utilization of available heat in evapo-transpiration processes. There is little opportunity to conserve moisture during the period when lands are not in crop. On the contrary, an attempt at such conservation might well result in concentrating alkali near the ground surface to the detriment of the lands.

Contrasted with these conditions an entirely different situation exists where depth to ground water is considerably in excess of the depth of the root zone. Here transpiration may be practically stopped during the period between crops, and evaporation is limited to losses following precipitation. Available water supply thus may be utilized more efficiently. Identical crops may be grown with a materially lower consumptive use without affecting yields or depriving plants of water which they are capable of using beneficially.

In this connection it should not go without mention that because of high ground water under cropped lands an enormous quantity of water is unnecessarily consumed each year while on the same stream systems there exist severe downstream shortages. From the comparisons given herein this excess consumption may be in the order of 0.5 ft in depth each year. This constitutes a potential water supply of significant quantity in many regions. Available supply is never exhausted until the water table is kept at sufficient depth below the surface to preclude such losses. In many cases drainage by pumping or substitution of a pumped supply for a portion of the gravity supply would result in benefits such as alkali and weed control which may compensate in large measure for increased costs.

From these general considerations, substantiated in a measure by the data cited herein, it is suggested that in the application of the authors' method to a particular location careful consideration must be given and proper correction made for differences in consumptive use which may arise from: (*a*) Irrigation practice, (*b*) crop types, and (*c*) depth to ground water.

In closing the writer wishes to express again his appreciation of the value and practicability of the authors' work. If most of this discussion has been critical let it be added that it is intended to be constructive. Mere confirmatory discussion is usually little more than sterile.

EDGAR E. FOSTER,[47] ASSOC. M. AM. SOC. C. E.—The consumptive use of water for agricultural purposes provides one of the crucial problems of irrigation engineering, and the authors are to be commended for searching for some logical basis for solution. It is evident that they have made a careful study of the subject.

Since water used in irrigation is consumed chiefly in evaporation and transpiration, it may be reasonably expected that there should be some relationship between heat and the consumptive use of water; yet the graph, Fig. 2(*a*), shows such a remarkably close correlation (in contrast with the authors' statement that a rather wide variation exists in plotting individual years as shown by Fig. 2(*b*)) that it appeared to the writer that the data should be tested for correlation by the methods of statistics.

TABLE 7.—CORRELATION COEFFICIENT FOR WAGON WHEEL GAP

Year	U_r, in acre-ft	H, in day-degrees	X	Y	$X\,Y$	X^2	Y^2
1912	1.05	3,990	−0.25	10	− 2.5	0.0625	100
1913	1.30	3,920	0	− 60	0	0	3,600
1914	1.27	4,200	−0.03	220	− 6.6	0.0009	48,400
1915	1.35	3,600	0.05	−380	− 19.0	0.0025	144,400
1916	1.48	3,710	0.18	−270	− 48.6	0.0324	72,900
1917	0.91	3,930	−0.39	− 50	19.5	0.1521	2,500
1918	1.65	4,410	0.35	430	150.5	0.1225	184,900
1919	1.43	4,060	0.13	80	10.4	0.0169	6,400
1920	1.14	3,910	−0.16	− 70	11.2	0.0256	4,900
1921	1.13	4,060	−0.17	80	− 13.6	0.0289	6,400
1922	1.54	4,520	0.24	540	129.6	0.0576	291,600
1923	1.43	3,960	0.13	− 20	− 2.6	0.0169	400
1924	1.02	3,860	−0.28	−120	33.6	0.0784	14,400
1925	1.43	3,540	0.13	−440	− 57.2	0.0169	193,600
Means	1.30	3,980	....	....	....	....	....
Totals	....	....	....	....	+204.7	0.6141	974,500

The coefficients of correlation between the water used, in acre-feet, and the effective heat, in day-degrees, were calculated for ten stations with records ranging from thirteen to sixteen years, the data for which are given in the Appendix. These coefficients were computed by the formula,[48]

$$r = \frac{X\,Y}{\sqrt{\Sigma X^2 \,\Sigma Y^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

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[47] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[48] "Handbook of Mathematical Statistics," by H. L. Reitz, Houghton Mifflin Co., 1924.

in which: r = the correlation coefficient; X = the deviation from the mean water used for the individual stations; and Y = the deviation from the mean effective heat for the individual stations.

The method of computation is given in Table 7, which contains the entire work of the calculation of the correlation coefficient for the valley of Wagon Wheel Gap. Substituting the appropriate values from this table in Eq. 1, $r = +0.265$, with a probable error, e, of $e = \dfrac{0.6745\,[1 - (0.265)^2]}{\sqrt{14}} = 0.168$.

The characters, U_r and H, have the same significance as in Table 3, Valley (10). It will be noted that the corrected values of the consumptive water are used as given by the authors. In general, the corrected data would give better correlation than the uncorrected values, but because the object is to determine how much water could be used, rather than how much had been used, the corrected values are justifiable.

TABLE 8.—SUMMARY OF THE CORRELATION COEFFICIENT r AND THE PROBABLE ERROR e

Name of stream	MEANS		CORRELATION COEFFICIENTS	
	U_r	H	r	e
Wagon Wheel Gap...	1.30	3,980	+0.265	±0.168
Mesilla............	2.83	12,370	+0.085	0.186
Black River........	1.85	6,950	+0.266	0.174
Mad River..........	2.15	8,760	−0.212	0.179
Skunk River........	2.25	9,340	−0.073	0.168
Sangamon River....	2.43	10,270	−0.254	0.158
Tallapoosa River....	2.75	11,900	−0.089	0.186
White River........	2.58	11,260	+0.010	0.180
Trinity River.......	2.82	14,460	−0.150	0.176
Cypress Creek......	2.99	14,500	+0.282	±0.172
Total............	....	....	+0.120	....
Mean............	....	....	+0.012	....

The computed correlation coefficients of the ten basins, together with their probable errors, are given in Table 8.

As is shown in the theory of statistics, the value of the correlation coefficients varies from − 1.0 for an inverse mathematical relationship, through values approximating 0.0 for purely chance, to 1.0 for a direct mathematical relationship. From Table 8 it will be seen that the correlation coefficients between consumptive water used and the effective heat as defined and calculated by the authors vary from − 0.254 to + 0.282 with a mean of 0.012. There are five positive values and five negative values. The probable errors of the coefficients, being dependent upon the number of observations as well as upon the magnitude of the deviations, are large (in some cases several times as large as the coefficients themselves), but in no case will the combined coefficient and probable error be great enough to show significant correlation. The low values of the coefficients, the diversity of the signs, and the relatively large probable errors show conclusively that there is no correlation between consumptive water and the effective heat for the individual years.

Nine of the ten valleys in Table 8 are not in regions where irrigation is practised. From this it may be inferred reasonably that the correlation is not caused by inaccuracies in estimating the area equivalent in rate of use to the cropped area, or changes in crop distribution or crop damage, but may be attributed to the lack of correlation in the various hydrological data.

The correlation coefficient was not computed for the averages of the effective heat and water in Table 1 because the data are not entirely comparable on account of the variable number of years of record. A high degree of correlation is plainly evident between the average water used and the effective heat as shown by Fig. 2(*a*), and a computed coefficient would probably have little added significance.

There now arises the question of what is the reason for the high correlation between the means as shown by Fig. 2(*a*), whereas there is no correlation between the water used and the effective heat for the individual years. A part of the improved correlation is probably obtained by averaging the inaccuracies noted by the authors. These sources of error are: Inaccurate records of discharge; inaccurate estimates of ground water; inadequate precipitation and temperature records; variability in the climatic elements; inaccurate estimates of the water used; and changes in crop distribution and reduction in the water demand because of crop damage. Since all these factors operate each year to a varying degree, it is likely that the variations caused by these factors offset each other to a considerable extent. However, there will very likely still be deviations that will be reduced further by averaging a series of data of successive years.

Nevertheless, in addition to the valid averaging of deviations, a kind of spurious correlation is obtained in the averages of water used and effective heat, which is the result of dividing the individual yearly values by the same factor—in this case, the number of years of record. This false or exaggerated type of correlation is pointed out by various writers of mathematical statistics.[49,50] Improvement in the correlation between two variables is caused by operating on the concurrent data in a series of observed values with a common divisor. It is not possible to estimate how much apparent improvement in the correlation may be due to this cause but it may be considerable. The uncertainty is enough to vitiate the results observed in Fig. 2.

In view of the lack of correlation for individual years, it is evident that the method proposed could not be used to estimate the year-to-year requirement of consumptive water with any degree of certainty since, for any given value of annual heat, it would be a matter of chance whether or not the computed depth of water would be sufficient for that year. The use of Fig. 2(*a*) would provide a somewhat more reliable value for the average annual consumptive water but, because of the spurious correlation obtained by the use of averages as data, the reliability of the value would not be great.

It is hoped that the authors will continue their search for means of estimating the quantity of consumptive water used in agriculture. A method utilizing more than one climatic element would probably be more successful. However, a high degree of correlation cannot be expected, although a coefficient should be higher than 0.50.

Charles H. Lee,[51] M. Am. Soc. C. E.—This important subject has been presented in an analytical manner from limited, although widely distributed, data, and the authors have developed a simple method of computing con-

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[49] "Mathematical Analysis of Statistics," by C. H. Forsyth, John Wiley & Sons, Inc., 1924.

[50] "Frequency Curves and Correlation," by W. P. Elderton, Cambridge University Press, 1938.

[51] Cons. Engr., San Francisco, Calif.

sumptive use of water that is very promising. From this as a base, they point out that it is an easy step to determine water requirements for crops and from thence to farm delivery and diversion requirements, all of which are essential data for the design of an irrigation project. They are to be commended for the broad view they have taken of the problem and their recognition that many of the earlier studies were of limited scope or objective, had differing concepts of the meaning of consumptive use, and hence have little value in a generalized study. An example of this fact was the frequent use of stream flow depletion as a measure of valley consumptive use. In 1930 the writer pointed this out in commenting upon the Report of the Duty of Water Committee of the Irrigation Division on consumptive use of water in irrigation.[52] At that time he suggested the adoption of the climatic instead of the crop year and the topographic or geologic unit of area instead of the project area. It is noted that the authors have proceeded in accord with these ideas.

The authors state that their basic data of consumptive use have been obtained from stabilized irrigation areas in which general crops are grown and which have adequate water supply and full project development. An added condition has been absence of nonrecoverable loss by deep percolation. These conditions should be considered and, if necessary, adjusted in applying their results to other areas.

The authors have developed a master curve for consumptive use (Fig. 2), well supported by plotted points, which they state (see heading "Valley Areas Studied") "represent practically the entire range of growing conditions and types of agriculture found in the United States." This statement is believed to be rather broad and to overlook entirely the Southwest, the Pacific Coast and other areas where growing conditions and types of irrigated agriculture differ widely from those in the cold mountain valleys that make up most of the irrigated areas selected for study. From the standpoint of production and economic importance, the omitted areas are of greater importance than are those for which representative valleys have been included. The master curve, therefore, should be accepted conditionally and should be considered as applicable only to the Rocky Mountain and humid sections of the United States until proved to have wider scope.

In order to define more clearly the limitations of the master curve, Table 9 has been compiled from data in the Appendix, and gives both average annual precipitation and that during the growing season, the length of growing season, and vegetation. It appears that, in most of the selected valleys, 50% or more of the precipitation falls within the growing season. In contrast to this, in the Southwest, and particularly on the Pacific Coast, practically no precipitation falls within the growing season.

It is further apparent that the principal irrigated crops are limited to alfalfa, native hay, grain, potatoes, and cotton, omitting entirely all the extensive orchard, vineyard, field, and vegetable crops raised in the Southwest and on the Pacific Coast, as well as rice, lettuce, cantaloupes, celery, artichokes, beans, seeds, and many other specialty crops of which large acreages are irrigated. With respect to the length of the growing season, the only irrigated area considered by the authors exceeding 165 days is Mesilla Valley, whereas in most of the irrigated sections of the Southwest and Pacific Coast the growing season exceeds 225 days and in some favored localities 300 days. In addition to differences in crops in the two regions, there are also vast differences in the methods of handling water and in the economics practised in its distribution and application. Some of these differences may well have sufficient influence upon consumptive use to overshadow that of effective heat.

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[52] Discussion by Charles H. Lee of "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1385.

TABLE 9.—PRECIPITATION AND CROP DATA FOR THE VALLEY AREAS STUDIED

No.	Valley (see Table 1)	PRECIPITATION (IN.)		Length of growing season (days)	VEGETATION		
		Average annual	Growing season		Cropped	Pastured	Wooded
(a) IRRIGATED VALLEYS							
1	New Fork	10.2	4.3	120	Meadow, hay, and pasture		
2	Michigan and Illinois	10	5	....	Grass, hay, and willows		
3	Southwest Area, San Luis	7 to 8	5	....	Hay, grain, and potatoes		
4	West Tule Lake	8.5	0+	140	Alfalfa, cereals, and potatoes		
5	Garland Division of Shoshone Project	....	....	....	Alfalfa, grain, sugar beets, and potatoes		
6	North Platte	17	13	165	Alfalfa, sugar beets, and cereals		
7	Mason Creek and Boise	10	Little	....	Alfalfa, clover, hay, and grain		
8	Uncompahgre	9	4.5	145	Alfalfa, cereals, sugar beets, and onions		
9	Mesilla	8	5.3	240	Cotton, alfalfa, and native vegetation		
(b) NON-IRRIGATED WATERSHEDS							
10	Wagon Wheel Gap "A"	21	....	....	....	....	100[d]
11	Black River	31	31 −	164	62	30	28
12	Mad River	38	21	180	33	38	29
13	Skunk River	32	21	205	68	26	6
14	Sangamon River	36	36 −	220	78	18	4
15	North Fork of White River	42	26	245	25	28	47
16	Green River	45	23 +	245	40	40	20
17	Tallapoosa River	52	....[a]	....	42	15	41
18	East Fork of Trinity River	....	21.6	215	75	16	9
19	Cypress Creek	43.9	31	280	50	20	30
20	San Jacinto River	....	....[a]	....	33[b]	38[c]	29[e]

[a] Adequate for crop requirements. [b] Cultivated. [c] Prairie. [d] Forested. [e] Pine forest.

The writer regards the element of precipitation as one most likely to cause confusion and error in the use of the master curve. The mean consumptive-use values that support the curve have apparently been built up with the inclusion of total annual precipitation (see heading "Method of Study: Measurement of Use of Water"). This varies for the selected valleys from 7 in. to 10 in. (with one instance of 17 in.) and 50% or more falls within the growing season (see Table 9). Considering irrigated areas throughout the West, precipitation varies from 3 in. to more than 20 in. and on the Pacific slope occurs mostly during the winter season. Precipitation is ordinarily disposed of by: (1) Interception; (2) immediate evaporation from the soil; (3) runoff; (4)

temporary absorption as soil moisture, returning to the atmosphere later as soil evaporation and transpiration; and (5) deep percolation passing beyond the reach of plant roots and ultimately becoming ground water. The relative distribution of total precipitation among these fractional elements varies greatly, depending upon the season of the year, the frequency of storms, the intensity of precipitation, the type of vegetation, the slope of the ground surface, the character and condition of the soil, etc. With precipitation corresponding to that in the valleys selected by the authors, the deduction of 7 in. to 10 in. in compiling water requirements from the master curve would probably not involve appreciable error; but an attempt to apply this assumption in valleys where the precipitation exceeds 10 in., and occurs during the winter, is likely to cause a very serious error. For this reason, the writer suggests that the master curve be supported by consumptive-use values that include only soil moisture at the beginning of the growing season (or precipitation during the growing season, whichever is more appropriate), rather than total precipitation, thus omitting the features that are foreign to consumptive use. Established methods are now available for measuring available soil moisture within the root zone at the beginning of the irrigation season so that this quantity is no longer indeterminate.[53] Its use in areas with little or no precipitation during the growing season, and the use of precipitation during the growing season in other areas, for the purpose of determining consumptive use, is far more logical than the use of total precipitation. Conversely, when computing water requirement from the master curve, either soil moisture at the beginning of the growing season or precipitation during it should be deducted.

The writer is in agreement with the authors that with present improvements in method and equipment many of the uncertainties of tank data have been eliminated. Careful consideration must still be given to the matter of exposure, however, to insure dependable results from tank observations.

The authors refer to the close correlation between transpiration and meteorological phenomena, listing especially evaporation from water surface, air temperatures, solar radiation, and wet-bulb depression readings. The evaporation from water surface probably represents the integrated effect of all the other factors mentioned. As an illustration of this fact, the writer prepared a graph on which were plotted the transpiration ratio and evaporation from water surface for alfalfa during the growing season as measured in tanks by Messrs. Briggs and Shantz at Akron, Colo., from 1911 to 1917,[54] and by A. C. Dillman at Newell, S. Dak., from 1912 to 1918, and at Mandan, N. Dak., from 1919 to 1922.[55] The observations were made in sunken land pans, 8 ft in diameter, reduced to reservoir surface by the use of a coefficient of 0.94. Although it shows a considerable spread among plotted points, Fig. 3 indicates a definite trend in the form of a straight-line relation.

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[53] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, Colin A. Taylor, and A. A. Young, *Bulletin No. 33*, California Div. of Water Resources, 1930, Pt. II.

[54] "The Water Requirement of Plants at Akron, Colorado," by H. L. Shantz and L. N. Piemeisel, *Journal of Agricultural Research*, Vol. 34, 1927, Table 27, p. 1144.

[55] "The Water Requirements of Certain Crop Plants and Weeds in Northern Great Plains," by A. C. Dillman, *loc. cit.*, Vol. 42, 1931, pp. 187–238.

The authors state (see heading "Influence of Meteorological Factors") that "Although solar radiation gives one of the best correlations with transpiration and evaporation, growing-season temperatures more nearly parallel the cycle of plant growth." The latter statement is not confirmed by the work of R. F. Goudey, M. Am. Soc. C. E., at Los Angeles, Calif. He found that the growth of algae in reservoirs follows the ultraviolet component of solar radiation more closely than it does temperature.[56] The writer believes that there is much yet to be discovered through correlation of evaporation and transpiration with the components of solar energy, and especially with the ultraviolet radiation.

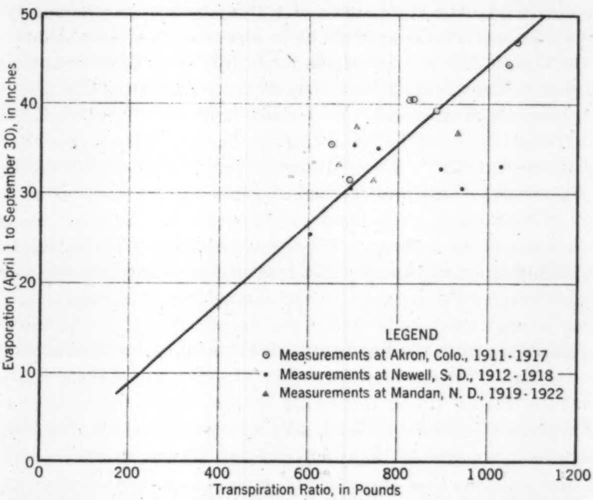

FIG. 3.—RELATION OF EVAPORATION FROM RESERVOIR SURFACE AND TRANSPIRATION RATIO FOR ALFALFA

An especially interesting feature of the authors' paper is the correlation of consumptive-use and watershed losses. Water loss from large land areas in the United States has been the subject of engineering study by two specialized groups—irrigation engineers working in the plains and valleys of the semi-arid and arid West, and water supply engineers working in the humid East and the mountain watersheds of the West. The former have adopted the term "consumptive use of water" to represent the losses in irrigated agriculture by transpiration and soil evaporation. To represent the loss from watersheds by transpiration, soil evaporation, and interception, the latter have used various terms, such as evaporation, evaporation from land areas, evapo-transpiration, total loss, water losses, fly-off, and total evaporation, of which the latter has received much recent use.[57] (The term "total evaporation" has been adopted by the writer as the most expressive short term in current use.)

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[56] "Sun Ray Counts Save Sulphate," by R. F. Goudey, *Engineering News-Record*, April 11, 1940, p. 95.

[57] "Hydrology of the Physics of the Earth," National Research Council (see chapter on "Transpiration and Total Evaporation," by Charles H. Lee).

Although the factors of interception and soil evaporation are of greater importance in total evaporation than in consumptive use, yet otherwise the two quantities (total evaporation and consumptive use) have similarity in that losses under both normally occur with conditions of ample water supply. This fact gives validity to the comparison of water losses from adequately watered irrigated valleys and humid watersheds as attempted by the authors.

The selection of humid watersheds made by the authors is not duplicated in the literature and suggests that they have not followed the line of recent technical papers, beginning with W. G. Hoyt, M. Am. Soc. C. E., in 1936,[58] Joseph Kittredge, Jr., in 1938,[59] and Gordon R. Williams, Assoc. M. Am. Soc. C. E., and others in 1940.[60] These studies have had as their aim the determination of total evaporation from watershed areas by the method of deducting runoff from precipitation. The results of these studies, however, are generally in line with those of the authors. Of especial interest is the correlation made by Mr. Williams of mean annual water loss with mean annual temperature for twenty-eight watersheds in which mean annual precipitation is in excess of 20 in. This relation can be expressed by a straight line, although the departure of individual points from the line is greater than found by the authors for consumptive use and effective heat (see Fig. 2). By eliminating consideration of temperature during the non-growing season, the authors have evidently devised a method of correlation that adheres more closely to the temperature conditions directly related to plant activity and transpiration.

The writer believes that the authors' method will bear further study and has a field of application that will ultimately be useful in water supply studies as well as in irrigation planning.

HARRY F. BLANEY,[61] M. AM. SOC. C. E.—The treatise on consumptive use of water for irrigated agriculture is excellent, and the authors have developed a practical method of estimating evapo-transpiration based on the relation of measured consumptive use to maximum temperatures above 32° F during the growing season for average water supply and crop conditions. The effect of sunshine and heat in stimulating transpiration was studied as early as 1691, according to a review of literature by the U. S. Department of Agriculture in 1905.[62] For many years irrigation and hydraulic engineers have considered the use of temperature data in estimating consumptive use of water. However, the results usually have not been satisfactory for individual years as the annual variations in rainfall, water supply, crop distribution, and methods of irrigation were not taken into account. For an average consumptive use over a series of years the method is also subject to criticism, when projects under consideration do not have similar crop distribution or the quantities of water available for irrigation per acre are materially different. The authors have made allowance for these varying conditions, but caution should be exercised by those not so familiar with the problems involved. For example, two projects having about the same effective heat unit in day-degrees might have considerable difference in average unit consumptive use if one project had 50% alfalfa and 15% cotton and the other had 20% alfalfa and 60% cotton, since alfalfa may consume 50% more water than cotton.

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[58] "Studies of Relations of Rainfall and Run-off in the United States," by W. G. Hoyt and others, *Water Supply Paper No. 772*, U. S. Geological Survey, 1936.

[59] "The Magnitude and Regional Distribution of Water Losses Influenced by Vegetation," by Joseph Kittredge, Jr., *Journal of Forestry*, Vol. 36, 1938, No. 8, p. 775.

[60] "Natural Water Losses in Selected Drainage Basins," by G. R. Williams and others, *Water Supply Paper No. 846*, U. S. Geological Survey, 1940.

[61] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.

[62] "A First Report on the Relations Between Climates and Crops," by Cleveland Abbe, *Weather Bureau Bulletin No. 36*, U. S. Dept. of Agriculture, 1905.

Differences in distribution of crops is illustrated by a crop survey of the districts in Pecos County, Texas, made in 1940 by the Department of Agriculture in connection with the Pecos River Joint Investigation. The distribution of the irrigated acreage in the Fort Stockton area was: Cotton, 18%; alfalfa, 58%; and other crops, 24%; whereas the distribution in the Imperial-Buena Vista area some 50 miles away was: Cotton, 53%; alfalfa, 26%; and other crops, 21%. The writer computed the consumptive use by both the integration and inflow-outflow methods for these areas, finding that the consumptive use in the Fort Stockton area was considerably more than in the Imperial-Buena Vista area, although there was very little difference in the effective heat units. The greater use in the Fort Stockton area was attributed to a larger percentage of alfalfa and a more abundant water supply.

The authors mention the inflow-outflow and integration methods used in estimating consumptive use of water in the Upper Rio Grande Basin[10] in 1936. In connection with this investigation and the Pecos River study in 1940, the writer reached the conclusion that further research into the relation of available heat to consumptive use of water must be conducted to make the effective heat method comparable in accuracy and reliability to the inflow-outflow and integration methods. Tables 10 and 11 show how closely the results of the inflow-outflow and integration methods in Mesilla Valley will check under favorable conditions. However, conditions are not always favorable for use of these methods.

Before the integration method can be successfully used it is necessary to know unit consumptive use and the areas of various classes of agricultural crops, native vegetation, bare land, and water surfaces. The reliability of the method depends primarily upon the selection of proper unit consumptive-use values.

In the consumptive-use studies conducted by the Division of Irrigation in cooperation with the National Resources Planning Board on the Pecos River in 1940, the writer developed the irrigation efficiency method of determining unit consumptive use for various agricultural crops. Briefly, the irrigation efficiency method consists of estimating evaporation and transpiration losses during the growing season and winter period based on data available as to the number of irrigations and depth of water applied, farm duty of water, and efficiency of irrigation water application for various crops and soils; and precipitation and evaporation from free water surface. Precipitation may be a large factor in winter consumptive use.

A survey of irrigation conditions was made throughout the Pecos River Basin. Water superintendents, county agricultural agents, engineers, and farmers were interviewed to ascertain the quantity of irrigation water supplied to various crops, both for the growing season and the winter period. Precipita-

TABLE 10.—CONSUMPTIVE USE OF WATER IN THE MESILLA VALLEY AREA (NEW MEXICO AND TEXAS), 1936, BASED ON INFLOW-OUTFLOW METHOD[a]

Period	ACRE-FT					$\frac{K}{A}$ in acre-ft per acre
	I	P	R	$G_s - G_e$	K	
January	3,800	7,180	8,600	+3,310	5,690	0.05
February	13,540	1,660	10,700	+1,660	6,160	0.06
March	50,840	1,200	29,400	−3,310	19.330	0.18
April	93,550	830	50,800	−9,940	33,640	0.30
May	89,858	4,050	62,500	−3,310	28,098	0.25
June	105.204	2,300	62,800	−1,660	43,044	0.39
July	123.299	9,850	76,500	−1,660	54,989	0.50
August	123,190	14,080	79,700	−3,310	54,260	0.49
September	63,832	22.540	50,500	+4,970	40,842	0.37
October	12,170	2,940	17,800	+4,970	2,280	0.02
November	9,020	10,120	12,500	+6,620	13,260	0.12
December	7,000	5,430	12,000	+1,660	2,090	0.02
Total	695,303	82,180	473,800	0	303,683	2.75

[a] As determined by the equation $K = (I + P) + (G_s - G_e) - R$; from inflow ($I$), precipitation ($P$), difference in ground-water storage ($G_s - G_e$), outflow (R), and valley consumptive use (K). Total area (A) = 110,418 acres; irrigated area = 82,923 acres; $\frac{K}{A}$ = consumptive use per acre.

TABLE 11.—CONSUMPTIVE USE OF WATER IN MESILLA VALLEY AREA (NEW MEXICO AND TEXAS), AS ESTIMATED BY THE INTEGRATION METHOD, 1936

Classification	Area, in acres	CONSUMPTIVE USE	
		Acre-ft per acre	Acre-ft
(a) IRRIGATED LAND			
Alfalfa and clover	17,077	4.0	68,308
Cotton	54,513	2.5	136,282
Native hay and irrigated pasture	216	2.3	497
Miscellaneous crops	11,117	2.0	22,234
Total or weighted average	82,923	2.74	227,321
(b) NATIVE VEGETATION			
Grass	2,733	2.3	6,286
Brush	6,933	2.5	17,332
Trees—bosque	3,532	5.0	17,660
Total or weighted average	13,198	3.13	41,278
(c) MISCELLANEOUS			
Temporarily out of cropping	5,569	1.5	8,354
Towns	1,523	2.0	3,046
Water surfaces: Pooled, river, and canals	4,081	4.5	18,364
Bare land, roads, etc.	3,124	0.7	2,187
Total ($a + b + c$)	110,417	2.72	300,550

tion records were collected and estimates made of the efficiency of irrigation for various crops in the different areas. The latter data were supplemented by water-application efficiency studies made on typical farms by measuring the quantity of water applied and taking soil samples before and after irrigation to determine the proportion of soil moisture available for crop transpiration use within the root zone. All this information was used to determine the unit consumptive use of water for various crops in each area. A summary of the results is shown in Table 12.

For many years the Division of Irrigation, U. S. Department of Agriculture, has been correlating observed evaporation and evapo-transpiration data as a means of estimating consumptive use of water. The writer made some studies of this character on field crops at the Denver (Colo.) Irrigation Field Laboratory of the Division in 1919. The adaptability of the evaporation pan as an index for estimating evapo-transpiration losses from moist areas was demonstrated by studies in 1931.[63] For tules growing in a large tank within the confines of a swamp area at Victorville, Calif., the percentage of consumptive use with reference to evaporation from a near-by exposed Weather Bureau pan was 95%. Evaporation from pans was used as an index of consumptive use of water on the Rio Grande in 1936,[10] and on the Pecos River in 1940,[64] when evaporation stations were established in valleys extending several hundred miles along those rivers.

TABLE 12.—NORMAL UNIT CONSUMPTIVE-USE VALUES FOR PRINCIPAL IRRIGATED CROPS, PECOS RIVER VALLEY AREAS (NEW MEXICO AND TEXAS) IN ACRE-FEET PER ACRE (ESTIMATED BY IRRIGATION EFFICIENCY METHOD)

Location	ESTIMATED UNIT CONSUMPTIVE USE				
	Alfalfa	Cotton	Annuals	Other crops	Weighted average for area
			(a) NEW MEXICO		
Las Vegas	2.4	...	1.8	1.7	1.9
Fort Sumner	3.1	...	2.3	2.0	2.5
Roswell	3.3	2.6	...	2.1	2.6
Carlsbad	3.4	2.8	...	2.2	2.8
			(b) TEXAS		
Barstow	3.5	2.2	...	1.9	2.6
Grandfalls	3.6	2.4	...	2.2	2.7
Imperial	3.3	2.6	...	2.2	2.8
Fort Stockton	4.2	2.6	...	2.2	3.0

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[63] "Water Losses Under National Conditions from Wet Areas in Southern California," by Harry F. Blaney, Colin A. Taylor, Harry G. Nickle, and Arthur A. Young, *Bulletin No. 44*, California Div. of Water Resources, 1933.

[64] "Consumptive Water Use and Requirements," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Pecos River Joint Investigation (Pt. IV-B) Section 2 (mimeographed report of Division of Irrigation, SCS to National Resources Planning Board, 1941).

In the Pecos River investigation an empirical formula was developed by the writer and Karl V. Morin[65] for use in computing evaporation from a lake surface and unit consumptive use of water by vegetation having access to ground water, where temperature and humidity records are available. Both evaporation and transpiration respond freely to temperature and humidity. Records of temperature, humidity, and evaporation in New Mexico and Texas were used. The first step in the process was to introduce a factor of evaporative force which is the average temperature times the percentage of (yearly) daytime hours for each month, or other time unit considered. A chart using evaporative force as ordinates and use of water in inches as abscissas was developed, on which it was found that plotted observed evaporation pan records, reduced to lake surface by the factor 0.70, appeared thereon as a fairly straight line.

From this the following general formula was developed for estimating annual unit values of evaporation from free water surface and evapo-transpiration by native vegetation for use in the integration method:

$$E = F\,K\,(114 - H) \ldots\ldots\ldots\ldots (2)$$

in which E = annual evaporation in inches; F = average temperature multiplied by percentage of yearly daytime hours = evaporative force (varying with latitude); K = constant; and H = annual average relative humidity (a percentage).

It is desirable to modify Eq. 2 when computing consumptive use of water by vegetation by substituting U for E, as follows: The annual consumptive use, U, in inches, equals

$$U = F\,K\,(114 - H) = u_s + u_w \ldots\ldots\ldots\ldots (3)$$

in which: u_s = growing-season consumptive use; and u_w = winter consumptive use. Then

$$U = f_s\,k_s\,(114 - h_s) + f_w\,k_w\,(114 - h_w) \ldots\ldots\ldots\ldots (4)$$

in which the subscript s denotes the growing (summer) season and w the winter period. The symbol f varies with temperature and latitude whereas k will vary with type of vegetation.

The advantage of the evaporative force method over the effective heat method is that it includes the factors of humidity and latitude as well as temperature and length of growing season.

The authors state (see heading "Method of Study: Equivalent Area") that "the cropped land in most agricultural valleys amounts to 85% or more of the gross area." This may be true for some federal projects but the percentage of cropped land is probably considerably less in most agricultural valleys unless areas within the general irrigated boundaries are eliminated.

In the Upper Rio Grande Basin[10] in 1936, the proportion of cropped area to gross area was 56% in the Southwest Area, San Luis Valley; 46% in the Isleta-Belen Area; and 75% in the Mesilla Valley. The author's data (heading "Description of Areas Studied: Mesilla Valley, New Mexico-Texas" (Appendix)) show only 71% in Mesilla Valley, which is more than the average valley percentage.

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[65] "Consumptive Water Use and Requirements," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Pecos River Joint Investigation (Pt. IV-B) Section 2 (mimeographed report of Division of Irrigation, SCS to National Resources Planning Board, 1941), p. 104.

The Division of Irrigation has computed the annual consumptive use of water by the inflow-outflow method in Mesilla Valley for the period 1921 to 1939, inclusive, and the results check closely with the data given in Table 4, varying from 1.86 acre-ft per acre in 1921 to 3.53 in 1926, with an average of 2.78 acre-ft per acre for the 19-yr period.

Conditions are not always favorable or the expense may be too great to use some of the foregoing methods.

The authors have developed a simplified and unique method for estimating consumptive use under average conditions when a limited time is available. The paper is a valuable contribution to the literature on the subject.

J. L. BURKHOLDER,[66] M. AM. SOC. C. E.—The method originated by the authors provides for estimating the consumptive use of water in agriculture by correlating mean effective heat, and such consumptive use on extensive areas of cropped land having a range of climatic conditions. The adaptation of mean effective heat as the independent variable for estimating consumptive use is based on the fact that this variable is readily determinable for any locality and on the theory that it is the most important factor influencing the quantity of water consumed by transpiration and evaporation from cropped areas.

A number of other variables affect the consumptive use of water in agriculture. Some of the more important are: Soil types; alkali and drainage conditions; types of crops and their yields; and water application with respect to deficiency or surplus.

On irrigated areas, soil types, and alkali and drainage conditions constitute a complex network of factors that influence the consumptive use. If drainage is quite effective, the net influence of these factors is likely to result in a higher or above normal consumptive use due to increased evaporation losses from heavy and frequent irrigation. In the reverse case of tight soils and restricted drainage, the "rise of salt" will prove an effective brake to over-irrigation, which would increase evaporation losses. Thus consumptive use might be lowered.

The yield of most common crops under given soil and climatic conditions will vary directly with the quantity of water beneficially applied. For instance, sugar cane will normally increase its yield about 20% for each additional acre-foot of water so applied up to a total application of about 5 acre-ft. Heavy crop yields transpire more water; hence the consumptive use is increased.

On most irrigated projects water is provided to the farms in normal years practically on a demand basis. Only in years of deficient supply is the use curtailed by rationing. The authors have attempted to correct their data for any deficiency in water supply. However, without detailed information on the type of crops produced on the various farms, and on the quantity of water required for each crop, it would be very difficult to establish the amount of the deficiency. Also, under the authors' method of estimating consumptive use, it may be equally important to correct for excessive evaporation due to an overabundance of applied water.

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[66] Senior Engr., International Boundary Comm., El Paso, Tex.

A consideration of the complex nature of the variables affecting the consumptive use of water in agriculture leads to the belief that the annual consumptive use for a given area may be expected to vary considerably from the mean for that area. The correlation between consumptive use and effective heat, therefore, should reflect the true value of the annual consumptive use so that the range of such use can be established.

The consumptive-use data presented in the paper include a total length of record of 42 years for nine irrigated valleys, of from 6,000 acres to 462,000 acres in area, and a total length of record of 141 years for eleven non-irrigated watersheds of from 200 acres to 5,000,000 acres in area. Of the eleven plotted points (Fig. 2) representing consumptive use on non-irrigated areas, nine fall below the line. It is thus apparent that the authors' study is based primarily on records of consumptive use on non-irrigated watersheds. Also, it is apparent that the mean consumptive use on these areas is somewhat lower than mean consumptive use on irrigated areas. This may be accounted for by the fact that crop yields are usually less on non-irrigated areas than on irrigated areas, or the lower consumptive use may be due to deficient or poorly distributed rainfall.

On the irrigated areas the length of the consumptive-use record ranges from as low as one year to as many as 13 years. For non-irrigated watersheds this record has a much narrower range of from 6 years to 14 years. By reducing the annual consumptive-use record of each area to a mean for the entire period of record, the authors seem to have given the same weight to records of one year as to records of 14 years.

The data presented for irrigated valleys do not include areas having high effective heat such as are found in the subtropical climates of southern California and southern Texas. The writer has attempted to extend the range of the authors' data by the inclusion of two such projects. This study has led to the preparation of a new chart (Fig. 4) on which consumptive-use data for the Barahona Project, Dominican Republic, and the Lower Rio Grande Valley, Texas, have been plotted, together with the data presented by the authors for individual years. It has thus been possible to define roughly the upper and lower limit of consumptive use. The writer is of the opinion that there is an additional advantage in expressing consumptive use as a zone rather than as a mean, since in investigating new projects the upper limit of consumptive use would then provide a safer basis for estimating the required maximum water supply and the capacity of canals and structures. Also, the lower limit may provide a useful gage of minimum water requirements when it becomes necessary to ration the supply.

It will be noted that line 2, Fig. 4, representing the mean of both the authors' and the writer's data, is almost identical in its relative position with line 1 based exclusively on the authors' data. However, if the position of the line representing the mean of the authors' data is fixed by the method of least squares, the authors' line is shifted to position 3.

The data for the Lower Rio Grande Valley include the consumptive use for the years 1922 to 1939, inclusive. The average cropped area for this project is 326,000 acres. Crop distributions are not known for each year, but, in

1929, 53% of the irrigated area was planted to cotton, 19% to orchards, 17% to vegetables, and 11% to grain and sorghum. The growing season includes the entire year. The valley has made rapid strides toward improvement of irrigation facilities and preparation and care of the irrigated lands in recent years, especially since 1928. In this connection it is interesting to note that consumptive use for the years 1922 to 1925, inclusive, is greater than for any of the following years.

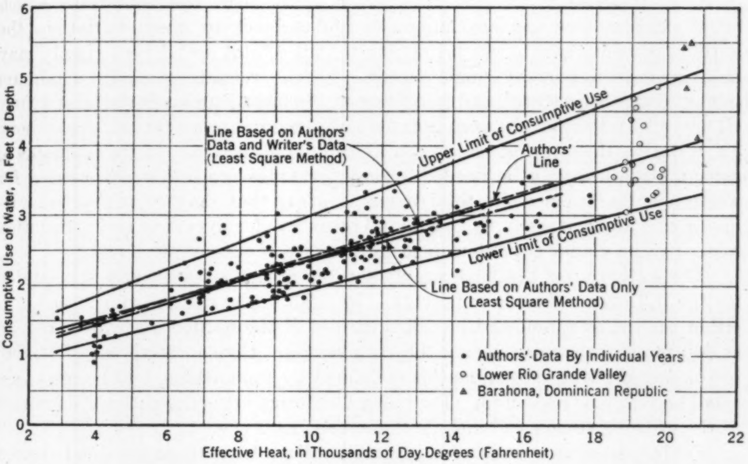

Fig. 4.—Relation of Consumptive Use to Effective Heat by Individual Years

Located on latitude 16° North, the Barahona Project is devoted exclusively to the growing of sugar cane, with the exception of a small area of pasture. Irrigation was started on this area in 1920, and the average gross cropped acreage increased from 6,332 in 1921 to 19,354 in 1924. The normal rainfall is about 12 in. and the growing season extends over the entire year. An extensive system of deep open drains was under construction during the years of record. The deltaic soils are relatively tight, and the salt content is high. These conditions forced a reduction in the quantity of water used for irrigation in 1923 and 1924.

The consumptive use for the Barahona area indicates that sugar cane uses more water than the mean requirement for similar climatic conditions as reflected by Fig. 4. The project produced an average of about 30 tons of cut cane per acre in addition to the heavy foliage that was stripped from the cane as it was harvested. It is the sole example of a single-crop area included in the study, and its inclusion results in a steepened line of mean consumptive use.

The authors have made a very able presentation of the subject, and their effort should be commended by those interested in irrigation development. Additional records of consumptive use on irrigated areas are needed to amplify and extend the study presented by them. It is hoped that this paper will stimulate engineers in charge of the many irrigation projects to undertake the required measurements and calculations in order that the range of consumptive use can be more definitely established.

Acknowledgment.—Assistance was rendered by Victor L. Streeter, Assoc. M., Am. Soc. of C. E., in preparing Fig. 4 and the data on the Barahona Project and Lower Rio Grande Valley.

S. T. HARDING,[67] M. AM. SOC. C. E.—Although the conditions which affect consumptive use are numerous and subject to much variation, the authors present records which indicate that the total useful heat supply may be the most important single factor. As the records of daily maximum temperatures are secured and published at Weather Bureau Stations in nearly all agricultural areas, the accumulated daily maximum air temperature above 32° F during the growing season can be computed for areas where estimates of consumptive use may be needed. Such estimates are frequently needed for western areas because the total irrigated areas that may be supported adequately by any water supply (where full recovery of conveyance and percolation losses can be made) are determined by the rate of consumptive use.

The authors use the heat supply for the growing season and the rainfall for the full year. This is consistent. Rainfall in the non-growing season will either appear as runoff and be taken care of in the outflow records or remain as soil moisture for use during the crop season. Consumptive use of rainfall in the non-growing season, as this is defined by the authors, will be relatively small and may be neglected. For many California areas the authors' definition of the growing season will include the entire year.

Table 1 separates results for irrigated valleys from those for non-irrigated watersheds. The latter are similar to the watersheds for which other computations of consumptive use have been published in *Water Supply Paper No. 846*.[68] The water losses (as this term is used in *Water Supply Paper No. 846*) are equivalent to the authors' definition of consumptive use, since adjustments were made for the loss from water areas. The values of the resulting mean annual water loss in *Water Supply Paper No. 846* for eighteen different parts of the drainage area of the Delaware River vary from 16.9 in. to 32.8 in. although the precipitation varied only from 92% to 110% of the mean of 44 in. These results are secured from more general records than those used by the authors and would be expected to vary more widely. The authors limited the records they used to areas which they considered to be fully supplied with moisture; the rainfall in the Delaware Basin should be sufficient to approach or exceed this standard. An application of the authors' method of comparison of effective heat and consumptive use to the results shown in *Water Supply Paper No. 846* would indicate the range of variation to be expected where only such general records are available.

The authors make a distinction between the consumptive use on agricultural and nonagricultural areas and reduce their results to the rate of consumptive use per acre of cropped area. Allowance when needed, both for areas using only rainfall and for areas of swamp or other excess rates of use, is essential if results applicable to crop areas are to be secured. The authors' procedure for such areas and the amount of the differences made for these lands are not consistent, however, with their treatment of all crops as having the same rate of consumptive use. They state that, notwithstanding the wide differences in crops in the areas used (mountain meadow hay and alpine forests to cotton and citrus fruits), the relation between consumptive use and effective heat is maintained closely. This should not be accepted too completely. The close relationship shown in Fig. 2 appears to be the result of balancing factors in the different areas rather than the uniformity of use by different crops. In several of the areas listed grain can be grown without irrigation with rainfalls less than the amount of the consumptive use shown. The differences in the moisture requirements of forage and orchard crops are also recognized in irrigation practice. The relationship shown in Fig. 2 would need to be modified when applied to local areas to allow for the types of crops to be grown.

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[67] Prof., Irrig., Univ. of California, Berkeley, Calif.

[68] "Natural Water Loss in Selected Drainage Basins," by G. R. Williams and others, *Water Supply Paper No. 846*, U. S. Geological Survey, 1941.

The points plotted in Fig. 2 fall as consistently along the straight line used as can be expected in results of this type. This does not mean that this line can be extended outside of the range of these records, however. Its extension downward would indicate a consumptive use of about 1 ft in depth, per year, in an area having no effective heat. This is a probable value as yearly moisture losses on an area where the climate permits any growth of vegetation or evaporation from the soil would be expected to approach this amount. The authors' points 18 to 20 indicate that the extension of the curve upward should be on a reduced slope reaching a maximum consumptive use somewhat above 3 ft per yr.

Such a maximum consumptive use under large values for effective heat is also indicated by other records. The maximum values for effective heat in the United States are about 20,000 day-degrees F per yr. This value occurs on the lower Colorado River areas in Arizona and California. For such amounts of effective heat the extension of the line in Fig. 2 gives a consumptive use of about 4 ft per yr. This is about the average rate of delivery to the land of irrigation water plus rainfall in the Imperial Valley and exceeds the similar figure for the Yuma Project. The delivery to the land in both of these areas includes some deep percolation losses not consumed by the crops so that the actual consumptive use will be less than that indicated by Fig. 2. This does not mean that the relationship in Fig. 2, within the limits of its supporting data, is incorrect; but it indicates that, in areas of greater effective heat, consumptive use should reach a maximum at or greater than 3 ft per yr. In such areas it is not customary for land to be fully cropped for the entire year and this difference is the result of not utilizing the full available heat supply.

The authors have included rainfall in the water supply from which the consumptive use is determined. This is desirable in results that may be used in both irrigated and nonirrigated areas. In some parts of California the rainy and irrigation season are sufficiently separate that values of the consumptive use may be derived which are applicable to the irrigation supply only. This practice is useful in these areas in determining the irrigation supply that needs to be provided but such results should not be applied to areas having different amounts or distribution of rainfall without making allowances for this difference in the basis of the results. The consumptive use from irrigation separately from the rainfall has been determined for some areas served by Kings River,[69] in California, the values derived being about 1.7 ft for orchards and vines and 2.0 to 2.25 for areas mainly in alfalfa. The mean rainfall is about 0.75 ft occurring during the winter months and should be added to these figures to give results in the terms used by the authors. Effective heat is available in all months of the year and some growth occurs throughout the year. Consumptive use by the trees and vines during their dormant period is replaced by the consumptive use of volunteer or planted cover crops, which receive their moisture supply mainly from the rainfall. The total annual effective heat for Fresno, Calif., is about 16,000 day-degrees for which Fig. 2 indicates a general consumptive use of 3.4 ft. This is a larger rate of consumptive use than the local records support and agrees with the indications of the authors' points 18 to 20 that consumptive use by crops will seldom exceed 3 ft.

C. S. JARVIS,[70] M. AM. SOC. C. E.—An unusual, although apparently logical, approach to a difficult problem has been used by the authors, and they have brought forth interesting as well as concordant data concerning water use on the various projects and regions dealt with. These data agree so closely with the fragmentary information heretofore available, and supplement the previous authoritative statements so satisfactorily as to call for a continuation of the research to cover many more river systems.

The writer's recent approach to consumptive use of water within a number of river basins was incidental to tracing the variations of annual or other periodic river discharge as related to recorded precipitation. It became apparent, for example, in dealing with the upper Missouri River and the entire Platte River basins, that the yield from the headwaters and the precipitation were closely related for separate years, pairs or groups of years, such as decades, and up to the total periods of record, whereas the recorded discharges along the middle valleys showed more and more marked deficiency as the irrigated area, and the storage facilities serving such regions, became further developed. So persistently were these deficiencies displayed within the main irrigated regions that it became advisable to make an evaluation of such "increased consumptive use due to irrigation and storage projects." It thus appears that the quantities derived by the writer are not strictly comparable to those published by the authors, although there is some fairly close relationship. Any river basin in the arid region on which considerable regulation and utilization of its water resources have been achieved should show a marked reduction of outflow as a result of such development. For example, the Platte River discharging at Ashland, Nebr., can be reconciled with the summation of inflows from its four main tributaries only by allowing for some 1,600 cu ft per sec as representing consumptive use due to irrigation developments during recent years in the lower valley, as shown in Table 13.

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[69] "Ground Water Resources of Southern San Joaquin Valley," by S. T. Harding, *Bulletin No. 11*, California Dept. of Public Works, Div. of Eng. and Irrig., 1927, and later records.

[70] Hydr. Engr., SCS, Dept. of Agriculture, Washington, D. C.

Similarly, the Missouri River discharge at Fort Benton, Mont., averaging 4,608 cu ft per sec for the 5-yr period from 1934 to 1938, apparently should be increased by about 1,400 cu ft per sec to account for the increased consumptive use due to irrigation in upstream areas.

TABLE 13.—Mean Discharge Compared with Increased Consumptive Use for Irrigation Along the Platte River System, Colorado and Nebraska

Item No.	Stream and station	Drainage area, in sq miles	Length of Record, in Years, to September 30, 1938	
			Observed	Estimated
1	South Platte River, South Platte, Colo.	2,050	36	....
2	South Platte River, Julesburg, Colo.	20,600	34	....
3	North Platte River above Pathfinder Dam, Wyoming	7,410	22	....
4	North Platte River below Pathfinder Dam, Wyoming	10,700	31	....
5	North Platte River, North Platte, Nebr.	32,000	43	....
6	Loup River, Columbus, Nebr.	14,200	24	....
7	Elkhorn River, Waterloo, Nebr.	6,390	12	24
8	Totals, items 2. 5, 6, and 7	73,190		
9	Ungaged intervening areas	10,610		
10	Consumptive use, storage, and losses in transit chargeable to irrigation	....	....	24
11	Platte River, Ashland, Nebr.	83,800	10	24

TABLE 13.—(*Continued*)

Item No.	Record-Period Average Discharge			Latest 10-Yr Mean			Latest 5-Yr Mean		
	Cu ft per sec	Cu ft per sec per sq mile	Depth per yr on drainage area, in inches	Cu ft per sec	Cu ft per sec per sq mile	Depth, in inches	Cu ft per sec	Cu ft per sec per sq mile	Depth, in inches
1	391	0.1907	2.59	323	0.1575	2.14	306	0.1494	2.03
2	485	0.0235	0.32	(300)	0.0146	0.20	201	0.0098	0.13
3	1,600	0.2159	2.93	1,303	0.1759	2.39	1,113	0.1502	2.04
4	1,784	0.1667	2.27	1,488	0.1391	1.89	1,197	0.1119	1.52
5	2,540	0.0794	1.08	(1,700)	0.0531	0.72	1,212	0.0379	0.47
6	2,984	0.2101	2.85	2,750	0.1937	2.63	2,574	0.1813	2.46
7	(776)	0.1214	1.65	711	0.1113	1.51	549	0.0859	1.17
8	6,785	0.0927	1.26	5,461	0.0746	1.01	4,536	0.0620	0.84
9	(984)	0.0927	1.26	(1,119)	0.1046	1.42	(1,082)	0.1920	1.39
10	(−1,469)	....	....	(−1,583)	....	....	(−1,679)	....	....
11	(6,300)	0.0752	1.02	4,988	0.0595	0.81	3,939	0.0470	0.64

Utilizing the 1930 census data on "irrigation of agricultural lands," as shown in Table 14, and calculating the consumptive use for irrigated acreage in accordance with the quantities given in Tables 1 to 6, produce the results in the final columns of Table 14, some of which are but estimates, while others are official record data. It is obvious that the most nearly complete utilization of water resources among the larger drainage basins as of 1929 was along the

Platte River, in Colorado and Nebraska, where increased consumptive use due to irrigation in the entire valley nearly equals the average outflow discharge at Ashland, within 25 miles of the Missouri River. However, if the Platte River discharge is measured near Duncan, Nebr., instead of at Ashland, the 10-yr average is 1,668 instead of 4,968 cu ft per sec, thus indicating that more nearly $\frac{4{,}780}{4{,}780 + 1{,}668} = \frac{4{,}780}{6{,}448} = 74.1\%$ of the available discharge is utilized within the region where irrigation is required, instead of approximately 50% of the total above Ashland including some areas where only supplemental if any irrigation is needed. Both the Great Basin and the Rio Grande drainage areas show utilization of water representing slightly more than 40%, computed as in the foregoing, for the year 1929, whereas the San Joaquin and Colorado River basins show 29.6% and 24.1%, respectively.

It is known that in many small drainage basins, components of the larger ones listed in Table 14, the utilization has closely approached 100% in recent years. However, where open storage over a long period is required to accomplish such development, considerable actual loss is necessarily entailed through evaporation, year after year, in addition to the inevitable deep percolation and seepage, small or otherwise in amount, which deplete the outflow without

TABLE 14.—IRRIGATED ACREAGE AND ESTIMATED CONSUMPTIVE USE OF WATER FOR IRRIGATION IN 1929

Drainage system	1930 census irrigated acreage of 1929	Estimated average consumptive use, in cu ft per sec	Record-period average discharge, in cu ft per sec [a]	Estimated utilization for irrigation; 1929 (%)
Missouri River	4,185,180	(9,650)	(75,000)	11.4
Jefferson River	363,380	(750)	1,541	32.7
Milk River	88,218	(182)	(500)	26.7
Big Horn River	361,926	(750)	2,211	25.3
Yellowstone River	861.145	(1,760)	9,365	15.8
Cheyenne and Belle Fourche rivers	71,550	(148)	(1,200)	11.0
Platte River	2,315,297	(4,780)	4,988	48.9
Kansas River	26.139	(640)	4,881	11.6
Republican River	23,387	(580)	777	42.7
Mississippi River, exclusive of Missouri River	902,560	(3,050)	530,000	0.6
Arkansas River	753,533	(2,560)	37,750	6.4
St. Francis River	13,282	(27)	1,424	1.9
White River	85,884	(176)	24,570	0.7
Red River	39,298	(130)	30,000	0.4
Gulf Streams other than Mississippi and Rio Grande	662,958	(2,370)	....	....
Rio Grande	1,468,913	(5,040)	7,350	40.7
Independent streams in Rio Grande Basin	95,812	(400)	....	....
Colorado River	2,537,124	(7,250)	22,800	24.1
Great Basin drainage	2,069.986	(5,280)	(7,000)	43.0
Columbia River	3,393,640	(7,020)	(300,000)	2.3
Snake River	2,339,264	(4,840)	48,860	9.0
Independent streams of Snake River Basin	82,074	(170)	....	....
Pacific Ocean streams other than Columbia	4,225,971	(10,750)	(75,000)	12.5
San Joaquin	2,405,380	(6,970)	16,550	29.6
Western States	19,547,544	(56,000)	(1,050,000)	5.1

[a] Unofficial estimates or derivations are in parentheses.

adding materially to crop growth of such basins, thus reducing the net amount available for use.

Now consider the fact that the Mississippi River discharge at its mouth represents approximately 25% of the precipitation within the entire basin, with the Ohio River and smaller tributaries from the eastern portion yielding more nearly 50%, while the river systems draining the Great Plains area yield more nearly 5% to measurable channel discharge and undetermined amounts to subsurface flow. Then the high utilization percentage of the Platte River (74.1 at Duncan or 48.9 at Ashland) applied to 5% runoff yield, results in only 3.7% or 2.5% of the precipitation which the consumptive use for irrigation diversions may represent, even though it largely depletes the Platte River discharge. The consumptive use due to both natural agencies and artificial devices or practices represents more than 96% of the total rainfall, with less than half the optimum moisture requirements maintained throughout more than 90% of the drainage basin, and only about 10% of the area adequately supplied, the irrigated and high mountainous areas with plentiful moisture being of nearly equal extent, or about 5% of the basin area in each classification. Furthermore, it is evident that the limit of practicable irrigation development within the Platte River basin is within view, unless importations of additional volumes are to be accomplished.

R. W. DAVENPORT,[71] M. AM. SOC. C. E., AND GORDON R. WILLIAMS,[72] ASSOC. M. AM. SOC. C. E.—In the period from 1935 to 1939, the writers participated in studies,[73] made by the U. S. Geological Survey, which produced results that may be compared with those presented in the paper. The studies consisted of the determination of natural water loss (or consumptive use) for about 200 non-irrigated drainage areas in humid and sub-humid regions east of the Rocky Mountains. The areas studied are similar to areas 10 to 20, in Table 1.

The relation between temperature and consumptive use was determined for only twenty-eight areas and in the published report[73] mean annual temperatures were used instead of day-degrees. It was realized that day-degrees of mean daily temperature above 32° might give a better correlation than mean temperatures. In order to determine the relation between mean temperature and day-degrees above 32° for yearly periods, the records for twenty-three representative stations, totaling sixty-nine record years, were studied. The results of the study were plotted and a mean curve drawn.[74] This curve showed that, for mean annual temperatures above 48° F, the mean temperature should give as good a correlation as total day-degrees.

Beyond showing the relation between the two methods of indicating temperature no use was made of this curve but it could be used to change the mean annual temperature to equivalent total day-degrees above 32° F. This has been done for the purposes of this discussion and the results are shown in Fig. 5. The plotted points in Fig. 5 are identified by their numbers as follows:

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[71] Principal Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[72] Hydr. Engr., U. S. Engr. Dept., Wilkes Barre, Pa.

[73] "Natural Water Loss in Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940.

[74] "Natural Water Loss in Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940, Fig. 4.

Point No. (Fig. 5)	Station
1	South Branch Nashua River at Clinton, Mass.
2	Sudbury River at Framingham Center, Mass.
3	Lake Cochituate Outlet at Cochituate, Mass.
4	West River at Newfane, Vt.
5	Swift River at West Ware, Mass.
6	Middle Branch of Westfield River at Goss Heights, Mass.
7	Clearfield Creek at Dimeling, Pa.
8	Swatara Creek at Harper Tavern, Pa.
9	Upper Little Swatara Creek at Pine Grove, Pa.
10	Oconee River near Greensboro, Ga.
11	Chattahoochee River near Norcross, Ga.
12	Conecuh River near Andalusia, Ala.
13	East Fork of Tombigbee River near Fulton, Miss.
14	Pearl River at Edinburg, Miss.
15	Red Bank Creek at St. Charles, Pa.
16	Miami River at Dayton, Ohio
17	West Fork of White River at Noblesville, Ind.
18	Tittabawassee River at Freeland, Mich.
19	Red River at Fargo, N. Dak.
20	Red River at Grand Forks, N. Dak.
21	La Crosse River near West Salem, Wis.
22	Kickapoo River at Gays Mills, Wis.
23	Blackwater River at Blue Lick, Mo.
24	South Grand River near Brownington, Mo.
25	Little Arkansas River at Valley Center, Kans.
26	Walnut River at Winfield, Kans.
27	Neches River near Rockland, Tex.
28	Angelina River near Lufkin, Tex.

The consumptive use was originally computed in inches but has been changed to feet to be comparable with the authors' study.

The differences in procedure between the two studies are listed in Table 15. The greatest difference lies in the method of determining effective heat. If the authors' curve from Fig. 2 is drawn on Fig. 5, a close similarity is shown in the slopes and positions of the curves even though the bases for computing the total day-degrees are different. The conclusion to be drawn is that the two methods of computing effective heat have compensating influences, that they lead to similar totals in day-degrees and are perhaps equally effective methods of correlating consumptive use to climatic conditions.

There are notable differences between the irrigated valleys and non-irrigated watersheds studied, shown in respect to altitude and various climatic features. Moreover, the way in which the water supply comes to the land is different, that for the irrigated valleys coming to a greater degree in a time when the capacity for use is greater. In view of these differences the correspondence of the relation between consumptive use and effective heat for the two kinds of areas is especially interesting and suggests that here also compensating influences are in operation.

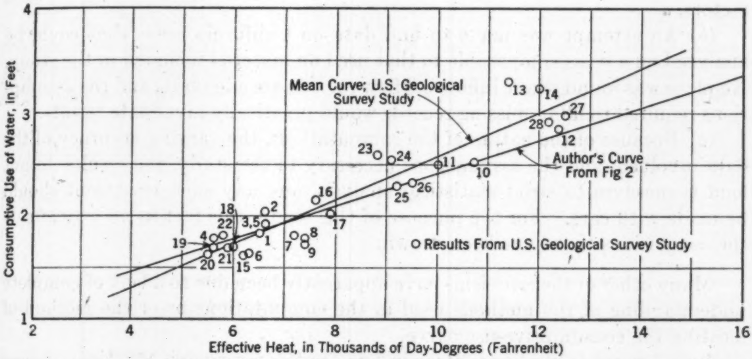

FIG. 5.—COMPARISON OF RELATIONS BETWEEN CONSUMPTIVE USE AND EFFECTIVE HEAT, USING DIFFERENT METHODS OF COMPUTING EFFECTIVE HEAT

TABLE 15.—COMPARISON OF METHODS

Item	Authors' study	Geological Survey study[a]
Period used in computing consumptive use..	Calendar-year	Water year ending September 30
Correction for years of short water supply..	Corrections made	No corrections
Period used in computing effective heat....	Growing season	Water year ending September 30
Temperature ranges used in computing effective heat..........................	Degrees by which maximum temperature for the day exceeds 32° F	Degrees by which mean temperature for the day exceeds 32° F

[a] "Natural Water Loss in Selected Drainage Basins," by Gordon R. Williams and others, *Water-Supply Paper No. 846*, U. S. Geological Survey, 1940.

ROBERT L. LOWRY, JR.,[75] M. AM. SOC. C. E., AND ARTHUR F. JOHNSON,[76] ASSOC. M. AM. SOC. C. E.—The constructive criticism evoked by this paper is gratifying to the writers. The paper as submitted was recognized as only a step toward the solution of the problem of using water for irrigation. It is hoped that as new studies are completed the results may be made available to the profession.

Criticism of the paper has centered largely around the following items:

(*a*) Failure to use plot and tank experimental data;
(*b*) Lack of California projects in the study; and
(*c*) Poor correlation when analyzed by statistical methods.

When reviewed carefully in relation to the purpose underlying the study, none of these criticisms afford a basis for casting doubt on the results as reported.

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The answers to the foregoing criticisms may be stated briefly thus:

(*a*) Field-plot experimental data are not applicable to valley consumptive use. They are undoubtedly valuable for indicating specific losses by certain crops, but their application to field conditions depends upon too many unknown factors.

(*b*) An attempt was made to find data on California areas that could be analyzed on a basis comparable to that used on material included in the study. No place was found where inflow-outflow records are adequate, and the assumptions required to fill in missing records would practically invalidate results.

(*c*) Because of the nature of the computations, the varying accuracy of the data involved, and the assumptions necessary to the study, the results do not lend themselves to strict statistical analysis, and any such treatment should be made with care. For the purpose of this study, this fact in no way affects the conclusions that have been drawn.

Many other of the criticisms have apparently been due to a lack of complete understanding of the methods used in the computations or of the method of locating the consumptive-use curve.

By reference to studies of algae in Los Angeles reservoirs, Mr. Lee criticizes the writer's statement that growing-season temperatures more nearly parallel the cycle of plant growth than does solar radiation. The life cycles of most varieties of algae are not similar to those of plants normally grown in agricultural practice. The observed increase in algae concentrations in the spring, when solar-radiation intensities are high (and incidentally, when food supplies are abundant because of the spring turnover of the water), may mean only that there are more individuals at that time rather than that each individual is making the greatest growth in its life cycle. Further, the reference loses significance when it is considered that algae do not consume water in the same sense that higher types of plants do. From a practical standpoint solar radiation cannot well be used as a measure of consumptive use because of the dearth of records. Only twenty-one stations measuring solar radiation are listed by the Weather Bureau for the United States, of which only two (Lincoln, Nebr., and Albuquerque, N. Mex.) are between the Mississippi River and the Pacific Coast.

Mr. Foster has attempted to show that there is no correlation between effective heat and consumptive use. The writers have no criticism of Mr. Foster's mathematics but point out that by confining his computations of correlation coefficients to individual areas, all but one of which are nonirrigated, and averaging the individual coefficients, he has ignored the writers' premise that over a wide range of conditions a very high correlation exists between effective heat in the growing season and consumptive use. For any one area, effective heat varies relatively little from year to year as compared with consumptive use because of the effect on the latter of variations in the accuracy of records, distribution of rainfall, changes in crops, inroads by pests, and effect of management. Over the entire range of climatic conditions the variation in effective heat, a universal factor, is large as compared with the variation caused by more or less local factors. The correlation coefficient for a series consisting of the mean values of effective heat and consumptive use for each study area was found to be 0.974 by Eq. 1 quoted by Mr. Foster. Similarly, a sample consisting of the effective heat and consumptive use values of that year which varied most from the curve of Fig. 2 from each study area gives a correlation coefficient of 0.70.

Messrs. Davenport and Williams have raised the question of correlating consumptive use with effective heat as estimated from daily temperatures. Although mean daily temperatures show an excellent correlation, maximum daily temperatures were found by the writers to show a better one, primarily, it is believed, because maximum temperatures reflect better the effect of radiant solar energy available only during daylight hours. Fig. 5, presented by Messrs. Davenport and Williams, does not give a true comparison of methods because effective heat for the study areas was computed by only one method.

In discussing the paper at the time it was presented before the Irrigation Division, Royce J. Tipton, M. Am. Soc. C. E., suggested that consumptive use be correlated with mean annual temperatures. This gives a very good correlation and offers a somewhat simpler computation than the writers' method. It is also of somewhat wider application because mean annual temperatures are frequently published for stations for which daily maximum and minimum temperatures, though available at central offices, are not published. Mean annual temperatures, however, do not reflect differences in growing-season temperatures, as was stated by Mr. Debler in comparing the Klamath Project in Oregon and California with an eastern Montana area such as the Lower Yellowstone Project. The mean annual temperatures at Tulelake, Calif., and Glendive, Mont., for the period 1933 to 1939 were 46.7° F and 45.9° F, respectively, but growing-season effective heats were 6,890 degree-days and 8,760 degree-days, respectively.

Mr. Rule criticizes the writers' method of determining effective heat, first for using a base of 32° F, and second, for the tedious work involved in computing the length of the growing season. He states that a base of 52° F would probably fit the Mesilla Valley better than 32° F. This may be true, but the same cannot be said for the cold New Fork and Michigan-Illinois areas where a base as low as 25° F would probably be better. The writers chose a base of 32° F after much study, as it appeared to give the best results over the entire range of climatic conditions represented by the study areas. Computation of the end points of the growing season is seldom tedious because these points can be picked by inspection in most cases, with recourse to the weighting process only in questionable cases.

Mr. Blaney suggests the irrigation efficiency method of determining consumptive use as an improvement over the effective heat method. As he states, that method

> "* * * consists of estimating evaporation and transpiration losses during the growing season and winter period based on data available as to the number of irrigations and depth of water applied, farm duty of water, and efficiency of irrigation water application for various crops and soils; and precipitation and evaporation from free water surface."

Such data would not be available for an undeveloped irrigation project. The purpose of the method proposed by the writers is to furnish estimates of consumptive use in areas where no past records are available. Mr. Blaney compares the method directly with the inflow-outflow and integration methods. The inflow-outflow method is basic for the determination of consumptive use under existing conditions where all factors can be measured. The integration method is one of approximation where inflow-outflow methods cannot be applied readily but where data on the relative areas in various classes of vegetation can be secured. It can be applied to a new area only if the consumptive-use requirements of the crops to be grown on the area are known. The effective heat method serves the latter purpose by referring to average requirements as indicated by effective heat. As Mr. Jarvis points out, the method also may be used in connection with estimates of runoff from precipitation, but it was not developed with this in mind. Because of errors in annual estimates of consumptive use due to local influences other than temperature, this application should be confined to averages unless more dependable methods for estimating annual runoff values are lacking.

Mr. Lee suggests that the master curve be supported by points based on eliminating non-growing-season precipitation, except as such precipitation can be stored in the soil at the beginning of the growing season, or that growing-season precipitation alone be included. In so doing, he does not distinguish between the development of the curve and its application. In developing the curve from inflow-outflow data, precipitation is a part of inflow and runoff from precipitation not consumed—a part of outflow. Therefore, precipitation must be included in the total. In application, only that part of precipitation adjudged available for plant use or soil storage for plant use is considered effective, and the remainder is discounted as runoff or deep percolation. The effectiveness of precipitation varies widely, depending upon its intensity and amount, its distribution through the year, the character of the soil, the surface slope, and the vegetative cover. Under normal conditions it is probably 75% to 100% effective during the growing season according to the writers' definition. Under conditions of low winter precipitation, characteristic of the Great Plains, rainfall may be insufficient to supply winter consumptive-use requirements, and consequently the soil dries out. Irrigation then becomes necessary in the spring, before planting. On the West Coast winter precipitation normally exceeds the winter consumptive use, and the excess over that used or stored in the soil is lost.

There is little information on which to base the monthly distribution of consumptive use for estimate purposes. The writers frequently have based estimates on the monthly determinations for the Mesilla Valley and the Boise Project, and on various tank and plot experiments. Fortunately, the soil-moisture reservoir is usually sufficient to provide for most errors of estimating the monthly distribution of water for irrigation.

Mr. Burkholder suggests that because of the greater number of years of record for the humid areas included in the writers' study it is apparent that the latter is based primarily on consumptive use by non-irrigated areas. This is not the case because, as Mr. Burkholder himself states, nine of the eleven points representing non-irrigated areas fall below the curve. The curve was based on results from irrigated areas, and the non-irrigated areas were added merely as a general check on the validity of the conclusions. Least-square methods were not used as the data represents studies of varying length and dependability. The curve was established on judgment, with the greater weight given to the records believed to be the more dependable. Fig. 6 shows the curve in relation to irrigated areas alone.

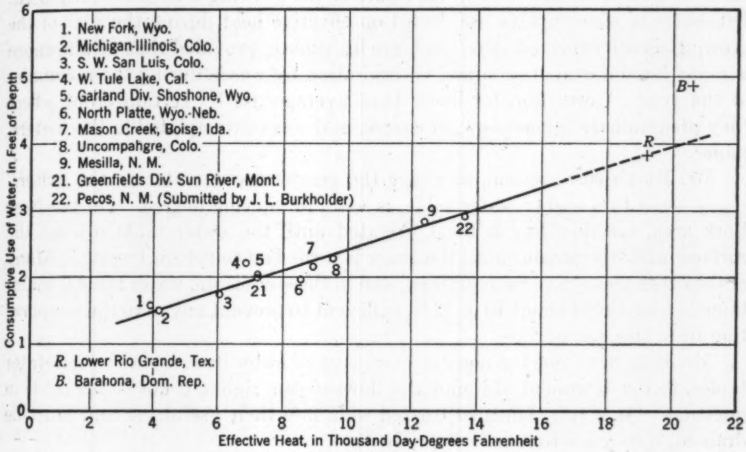

FIG. 6.—RELATION OF CONSUMPTIVE USE TO EFFECTIVE HEAT-MEANS FOR IRRIGATED AREAS

Mr. Burkholder also suggests using curves for upper and lower limits of consumptive use rather than the average value indicated by the curve presented. This suggestion appears useful for investigation studies. The upper limit could be used in determining system capacity and the lower limit as a criterion of minimum requirements in lieu of the usual allowance of a permissible shortage for the driest years.

Professor Harding and Mr. Rule criticize the writers' curve as not applicable to California and the Southwest. Mr. Rule's three suggested reasons for the apparent discrepancy, especially economic conditions and type of crops, are believed a reasonable explanation of a part of the difficulty. The writers have noted a tendency among California investigators to discount precipitation and ground-water movement toward heavily pumped areas, and a part of the difficulty may be attributed to these factors. The curve was designed to represent conditions of liberal use of water on general crops suited to the climatic conditions indicated by the growing-season temperatures and with full utilization of the growing season. Normal evaporation of moisture from precipitation, even though that precipitation is of no direct benefit to plant growth, is included as a part of consumptive use. It is well known that, through a wide range, crop production increases with an increase in water applied, but the most economical application depends upon the relative cost of the increased application to the value of the incremental gain in production. The relation varies, too, with soil fertility and character of the crop. Doubtless, because of the limited surface supplies, the high cost of pumping, and the favorable climatic conditions for growing fruits, vines, and certain row crops in addition to winter grain, which can utilize the winter precipitation with little need for irrigation, Central and Southern California consumptive uses appear low in comparison with the curve values, even when a correction is made to limit effective heat to that part of the growing season actually used. Estimates of consumptive use, based on effective heat during the part of the growing season expected to be used, are legitimate, provided that an allowance is made for soil evaporation and transpiration by weeds during the remainder of the year. Correction for lower than average use by certain crops where they predominate is necessary, of course, and was contemplated in the writers' paper.

Mr. Rule's third reason, ascribing the greater use of water on the writers' study areas to a higher water table, is valid in varying degree. On the New Fork area, meadow hay is flood irrigated until the water table reaches the surface, where it remains until the water is turned off to permit haying. Many of the other areas have been drained, and in these cases the water table is maintained at depths of about 10 or 12 ft, sufficient to prevent any material evaporation from this source.

Mr. Rule mentions the possible excess use of water by areas with high water tables, to the detriment of junior and downstream rights, if not to the lands in question. As a rule, lands so treated soon lose their usefulness and must be drained, if they are to remain in production.

By way of recapitulation, Fig. 6 is presented to show the writers' curve of consumptive use in relation to effective heat supported by data from irrigated areas only. Mean values for the Lower Rio Grande Valley and the Barahona Project, obtained from annual values given by Mr. Burkholder in Fig. 4, are shown on the graph for comparative purposes, but the writers have not had the opportunity to examine the basic data supporting them.

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TRANSACTIONS

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AN INVESTIGATION OF PLATE GIRDER WEB SPLICES

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With Discussion by Messrs. C. H. Gronquist, Charles Stratton Davis, Bruce Johnston, Francis L. Castleman, Jr., and J. M. Garrelts and I. E. Madsen.

Synopsis

In the design of plate girders several types of web splices are in current use. Probably the most common is the type with one plate on either side of the web extending the full depth between the toes of the flange angles. This splice was investigated by the writers both for the condition of uniform rivet spacing and for that of nonuniform rivet spacing. The six-plate splice, with three plates on each side of the web between the toes of the flange angles, two for moment and one for shear, was also investigated. The other type considered was that in which the web is spliced for its full depth between the toes of the flange angles for shear and strap plates over the flange angles to splice for moment. In many cases fill plates are required under the strap plates. In most cases the cost of the splice increases with the number of pieces to be handled.

Apparently the design of such splices has been based largely on assumptions which have never been checked by tests, and even the relative merit of these various designs has been unknown. This paper reports a series of tests designed to secure information concerning the behavior of plate girder web splices under load and to determine the relative values of four types of web splice design commonly used in modern practice.

Introduction

Four plate girders, each having a depth of 24¼ in. and a length of 24 ft and identical except for the types of web splices used, were subjected to test. The procedure was divided into two main parts: (1) All girders were first tested at working stresses (that is, at loads producing stresses below the elastic limit, thus

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Note.—Published in June, 1941, *Proceedings*.

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affording a basis for the comparison of their behavior in the elastic range); and (2) the tests of each girder were continued to failure, in order to reveal possible variations under increased loading.

In the first phase, or elastic range tests, strain rosette readings were taken along the center line of the splice and on two parallel lines, one on each side of the splice except for G_4 where two parallel lines were used on each side of the splice. The strains were measured on three intersecting gage lines at each strain rosette with tensometers using 1-in. gage lengths. Normal and shear stresses were computed for a vertical cross section through each rosette point using the measured strains. The longitudinal stresses in the flange angles and cover plates were determined from additional strain gage measurements on these elements using tensometers with 0.5-in. gage lengths. The variations in direct and shear stress intensities over the full depth cross section of each girder are given by curves drawn to represent the intensity of stress at any point. The variations along the girder of the longitudinal stresses in the toes of the vertical legs of the flange angles are also given by curves drawn to represent those stresses.

TABLE 1.— TOTAL MEASURED MOMENT AND SHEAR ON A CROSS SECTION

Location[a]	BENDING MOMENT, IN KIP-IN.[b]				SHEAR,[c] IN KIPS[b]	
	Applied	Measured	In the 20-In. Depth Between Flanges			
			Measured	Theoretical	Measured	Theoretical
GIRDER G_1						
Line *A*	1,110	1,059	148.66	125.4	12.20	13.84
Line *B*[d]	1,190	1,135	144.74	......	12.45	......
Line *C*	1,273	1,280	160.0	144.0	13.40	13.84
GIRDER G_2						
Line *A*	1,110	1,076	152	125.4	11.3	13.84
Line *B*[d]	1,190	1,138	140	......	12.0	......
Line *C*	1,273	1,289	171	144.0	12.9	13.84
GIRDER G_3						
Line *A*	1,080	1,067	173.3	122.0	13.0	13.84
Line *B*[d]	1,190	1,120	174.3	......	14.5	......
Line *C*	1,300	1,284	167.7	147.0	13.8	13.84
GIRDER G_4						
Line *A*	1,070	1,031	184.51	121.0	12.6	13.84
Line *B*	1,140	1,066	145.51	129.0	10.3	......
Line *C*[d]	1,192	1,121	162.67	......	10.38	......
Line *D*	1,243	1,192	152.48	140.4	11.0	......
Line *E*	1,320	1,312	172.60	149.1	11.45	13.84

[a] See Fig. 1. [b] 1 kip = 1 "kilo-pound" = 1,000 lb. [c] Applied shear = 16 kips in each case. [d] Splice.

During the second phase, or ultimate load tests, 10-in. strain gage measurements were taken, from which the relative rotation and the relative vertical displacement at the splice were computed. The stresses at the top and bottom of the splice plates and in the adjacent toes of the flange angles were determined from 2-in. strain gage measurements.

Although the results are given in terms of stresses, these have been computed on the basis of a constant proportionality factor of stress to strain. The values given for stresses above the yield point represent strain rather than stress.

Deflections were measured by the stretched wire and scale method. A load-deflection curve for one girder is included in this report. The curves for the other girders were similar.

The total measured moment and shear on a cross section were computed by a process of summation, using the stress curves drawn to represent the measured values. The results for these values are given in Table 1 for the gage lines shown in Fig. 1.

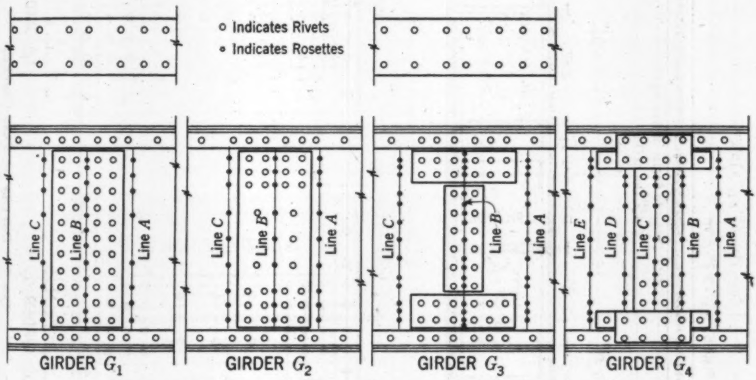


FIG. 1.—GAGE LINES FOR LOCATION OF STRAIN ROSETTES

DESCRIPTION OF TEST SPECIMENS

The dimensions of the test girders were selected by making them proportional to those of girders used in practice and as large as could be handled in the laboratory.

The four test girders were alike, except for the material and arrangement at the splices. A typical test girder, with the splices omitted, is shown in Figs. 2(*a*) and 2(*b*). The main material consisted of the 24-in. by $\frac{3}{16}$-in. web plate, four angles 3 in. by 2 in. by $\frac{3}{16}$ in., two cover plates $6\frac{1}{4}$ in. by $\frac{3}{16}$ in., and two cover plates at $6\frac{1}{4}$ in. by $\frac{1}{8}$ in. Rivets 0.5 in. in diameter were used. The girders were fabricated with a depth of $24\frac{1}{4}$ in. back to back of flange angles and a span of 24 ft center to center of bearings. The relative position of each of the splices in a girder is specified by referring to them as 1, 2, or 3 as shown in Fig. 2(*b*).

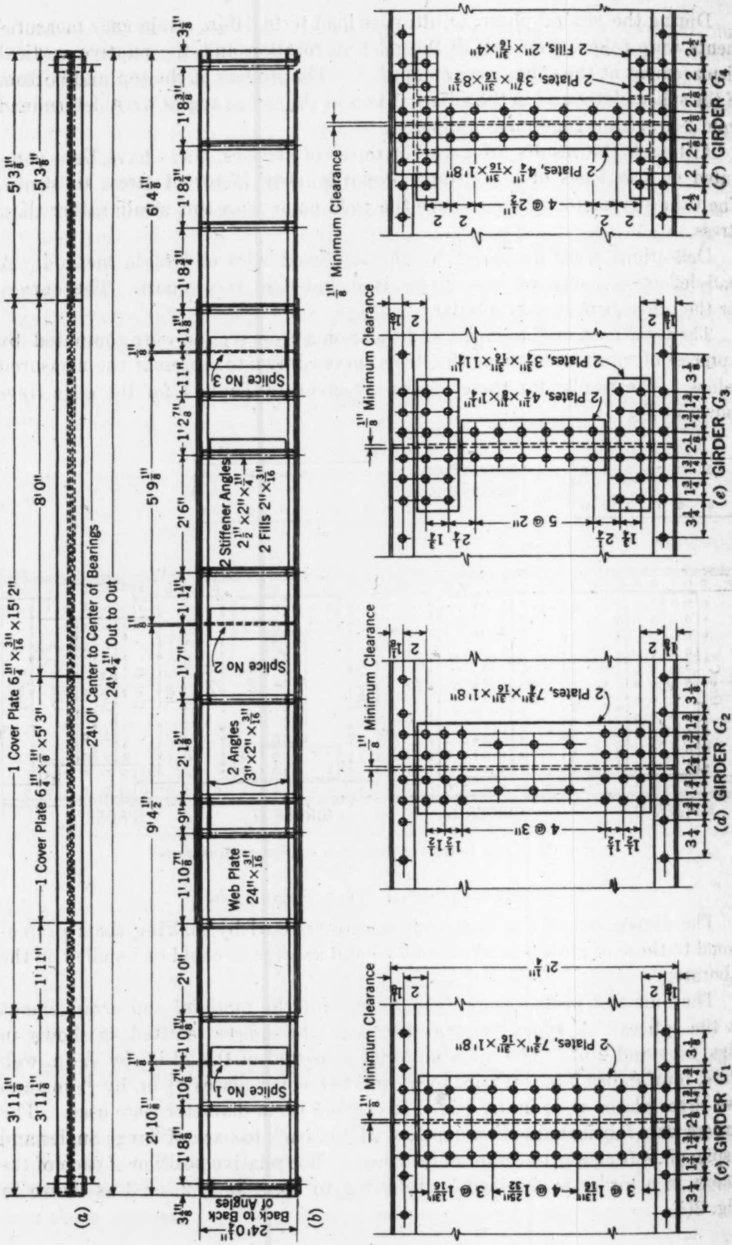

5'3 1/8"
5'3 1/8"
8'0"
1 Cover Plate 6 1/4"×3/16"×15'2"
1 Cover Plate 6 1/4"×1/8"×5'3"
24'0" Center to Center of Bearings
24'4 1/4" Out to Out
1'11"
3'11 1/8"
3'11 1/8"
(a)
3 3/8"
1'8 3/8"
6'4 1/8"
1'8 1/4"
1'8 1/4"
1'1 1/8"
1/8"
Splice No 3
1'1"
5'9 1/8"
1'2 7/8"
2'6"
2 Stiffener Angles 2 1/2"×2"×1/4"
2 Fills 2"×3/16"
1'1 1/4"
1/8"
Splice No 2
1'7"
2'1 1/8"
2 Angles 3"×2"×3/16"
9'4 1/2"
9"
1'10 7/8"
Web Plate 24"×3/16"
2'0"
1'0 7/8"
1/8"
Splice No 1
10 7/8"
2'10 1/8"
1'8 1/8"
3 3/8"
2'0 1/4" Back to Back of Angles
(b)
2 Fills, 2"×3/16"×4"
2 Plates, 3 7/8"×3/16"×8 1/2"
2 Plates, 4 1/4"×3/16"×1'8"
1/8" Minimum Clearance
4 @ 2 1/2"
2
2
2
2
2 1/4
2 1/8
2
2 1/8
2 1/8
2 1/8
2
2 1/4
(f) GIRDER G4
1/8" Minimum Clearance
2 1/8
2
1 1/8
2 Plates, 3 3/4"×3/16"×11 3/4"
2 Plates, 4 1/4"×3/8"×1'1"
5 @ 2"
1 3/4
2 1/4
2 1/4
1 3/4
3 1/4
1 3/4
1 3/4
2 1/8
1 3/4
1 3/4
3 1/4
(e) GIRDER G3
1/8" Minimum Clearance
2 1/8
2
2 Plates, 7 3/4"×3/16"×1'8"
4 @ 3"
1 1/2
1 1/2
1 1/2
1 1/2
3 1/4
1 3/4
1 3/4
2 1/8
1 3/4
1 3/4
3 1/4
(d) GIRDER G2
1/8" Minimum Clearance
2'1"
2 1/8
2
2 Plates, 7 3/4"×3/16"×1'8"
3 @ 1 13/16"
4 @ 1 25/32"
3 @ 1 13/16"
3 1/4
1 3/4
1 3/4
2 1/8
1 3/4
1 3/4
3 1/4
(c) GIRDER G1

FIG. 2.—DETAILS OF TEST GIRDERS

The splice materials and the rivet arrangement for each type of splice are shown in Figs. 2(*c*), 2(*d*), 2(*e*), and 2(*f*). Each of the four different types of splice arrangement is identified by the same notation as that which is used to designate the girders; that is, G_1, G_2, G_3, and G_4.

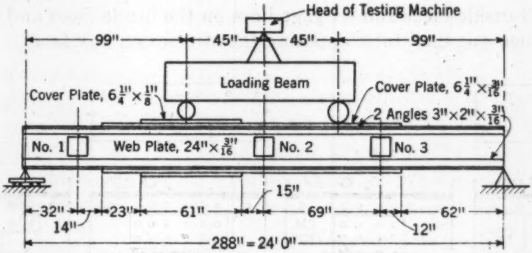


FIG. 3.—LOADING ARRANGEMENT

The preliminary test program at the time the girders were fabricated called for the application of a single concentrated load at a third point of each girder. The cover plates were designed to suit this loading condition, although the method of loading the girders was later changed to a symmetrical arrangement.

TABLE 2.—PHYSICAL TESTS ON STEEL IN GIRDER

Description	Flange angles	Web plates	3/16-in. cover plates	1/8-in. cover plates
Yield point (lb per sq in.)	43,225	32,950	31,300	33,825
Ultimate strength (lb per sq in.)	61,300	46,300	44,575	48,700
Modulus of elasticity (kips per sq in.)	29,400	29,100	29,300	29,700
Percentage elongation in two inches	46	51	55	49
Percentage reduction of area	69	71	71	69

Tensile flats were made of coupons taken from the angles, cover plates, and web plates of each of the test girders. These specimens were tested in tension to determine the yield point, ultimate strength, ductility and modulus of elasticity. The yield point stress was determined by the "drop of the beam" method. The results of the tests are summarized in Table 2.

DESCRIPTION OF TESTS

The loading arrangement used for each girder is shown in Fig. 3. The loads were applied to the girders by a 600-kip, hydraulically-operated, universal testing machine, through rollers located near the third points (1 kip = 1 "kilo-pound" = 1,000 lb). End reactions were provided by a knife-edge support at one end of the girder and a roller nest at the other end.

In the first phase of the program, strain readings were taken at 1-kip and 33-kip loads. The difference between these strain readings corresponds to a load of 32 kips which is necessary to produce a theoretical maximum extreme fiber stress of 17.6 kips per sq in. In this elastic range, a line of strain rosettes was used along the center line of splice 3, and in addition parallel lines of rosettes were taken on the web plate adjacent to the splice plates (see Fig. 1).

Longitudinal strains were measured on vertical legs of the flange angles and on both the inside and outside faces of horizontal elements of the flanges. The strains on the horizontal parts of the flanges were taken along seven gage lines on the outside faces and six gage lines on the inside faces and the average of these values was used for computing the stresses on any face.

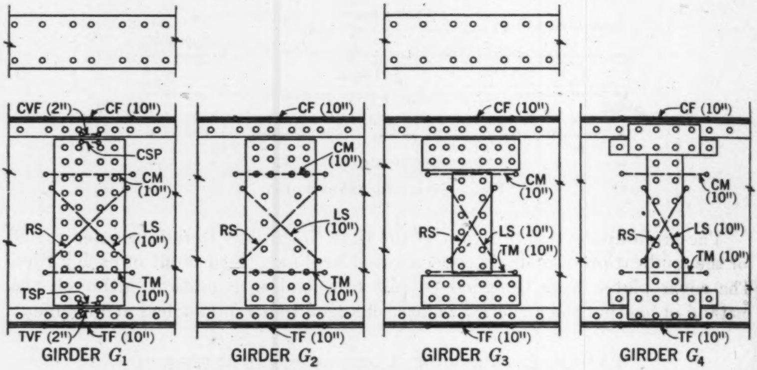


FIG. 4.—GAGE STATIONS FOR STRAIN READINGS

In the second phase or ultimate-load tests, strain-gage readings were taken at the following loads:

Girder	Initial test loads (kips)	Final increments of increase to failure (kips)
G_4	0, 1, 33, 36, 39, etc.	3
G_3	0, 11, 22, 33, 37, 41, 45, etc.	4
G_1 and G_2	0, 33, 37, 41, 45, etc.	4

Strain readings were taken at the stations shown in Fig. 4, in which:

CF = Compression flange—outstanding leg............10-in. gage length
CVF = Compression flange—vertical leg................ 2-in. gage length
CSP = Compression edge—splice plate................ 2-in. gage length
CM = Compression rotation..........................10-in. gage length
RS = Right shear strain............................10-in. gage length
TSP = Tension edge—splice plate.................... 2-in. gage length
LS = Left shear strain.............................10-in. gage length
TM = Tension rotation..............................10-in. gage length
TVF = Tension flange—vertical leg.................. 2-in. gage length
TF = Tension flange—outstanding leg................10-in. gage length

RESULTS OF TESTS

First Phase (Elastic Range Tests).—Normal and shear stresses along lines *A*, *B*, and *C* for splice 3 of girder G_1, computed from strain rosette readings, are shown in Fig. 5. Longitudinal stresses for the toes of the vertical legs of the flange angles are also shown. The curves for both measured and theoretical stresses represent the values for a 32-kip load on the girder. The gross moment of inertia of the girder section was used for calculating theoretical stresses.

Curves representing the stresses for girder G_2 are shown in Fig. 6. Stresses for girder G_3 are shown by the curves of Fig. 7. The strain readings for the flange-stress curves were taken at cross sections through the rivets and at cross sections half-way between the rivets. Although these stresses are computed from average strains they show a definite zigzag effect due to the rivets.

The stresses for girder G_4 are shown by the curves of Fig. 8 for five cross sections *A*, *B*, *C*, *D*, and *E*. Longitudinal stresses in the toes of the flange angles, where exposed, and along a corresponding line in the plates covering the angles, are also shown.

TABLE 3.—RESULTS OF BREAKING TESTS (MAXIMUM LOADS AND MOMENTS)

GIRDER G_1			GIRDER G_2			GIRDER G_3			GIRDER G_4		
Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)
Load	Shear		Load	Shear		Load	Shear		Load	Shear	
63.5	31.75	3,140	63.5	31.75	3,140	65.3	32.65	3,230	63.0	31.5	3,120

The values of the applied moments and shears, and the measured moments and shears for each of the lines of rosettes at and near splice 3, are given in Table 1. The values of the measured moments and shears were computed by numerical summation using the stresses determined from the strain readings.

Second Phase (*Ultimate Load Tests*).—From strain readings on gage lines RS and LS, splice 1 (Fig. 4), the relative vertical or shear displacements were computed and the values are given for all four girders by the curves of Fig. 9.

The relative angular displacements at splice 2, for the four girders, are given by the curves of Fig. 10. This displacement is obtained by dividing the sum of the strains on gage lines TM and CM by the 12-in. distance between these lines.

TABLE 4.—SUMMARY OF DESIGN COMPUTATIONS

Description	Web plate	G_1	G_2	G_3	G_4
For rivet group Σd^2 (in.^{2})................	....	727	719	895	960
Plate moment, in kip-in....................	322.0	366.0	366.0	332.0	797.0
Plate shear, in kips....................	62.4	97.5	97.5	76.3	135.3
Rivet moment, in kip-in. (no shear)........	....	322.0	319.0	391.0	353.0
Rivet shear, in kips (no moment)..........	....	88.0	60.0	72.0	60.0

The average tension and compression flange stresses computed from strains measured on 10-in. gage lengths along lines TF and CF on both sides of the girder at splice 2 are given by the curves of Fig. 11.

Deflection curves for girder G_1 at each of the test loads are shown in Fig. 12. The corresponding curves for the other girders were similar and have not been

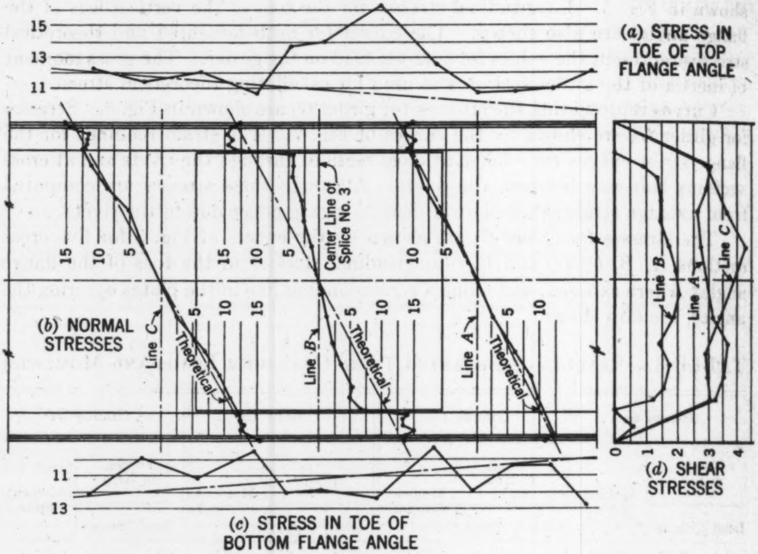


FIG. 5.—STRESSES AT SPLICE 3, GIRDER G_1, IN KIPS PER SQUARE INCH

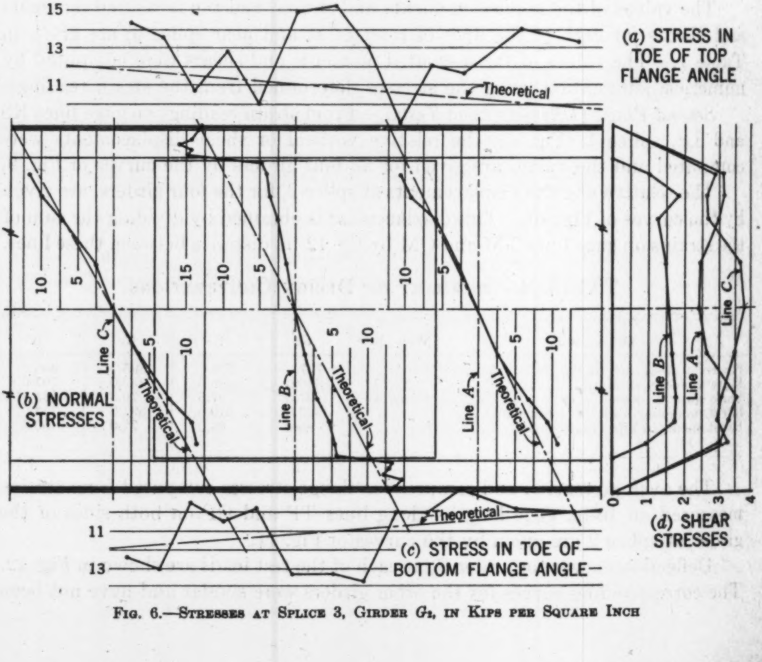


FIG. 6.—STRESSES AT SPLICE 3, GIRDER G_2, IN KIPS PER SQUARE INCH

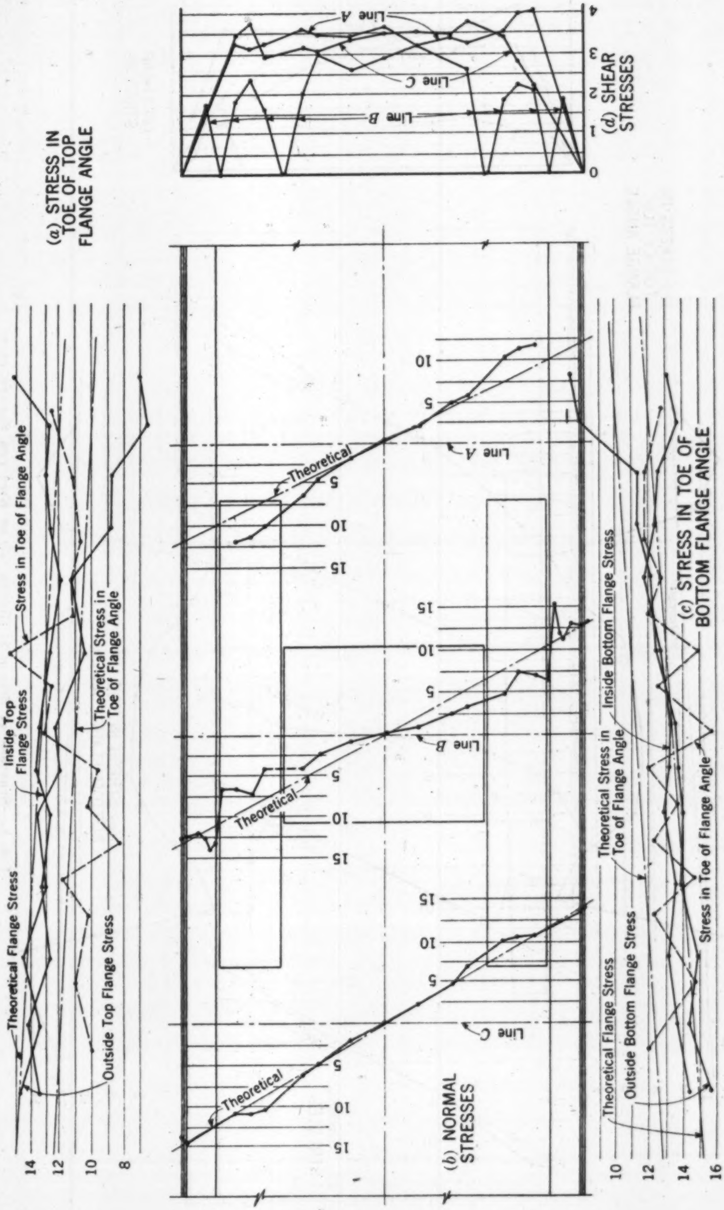


FIG. 7.—STRESSES AT SPLICE 3, GIRDER G_3, IN KIPS PER SQUARE INCH

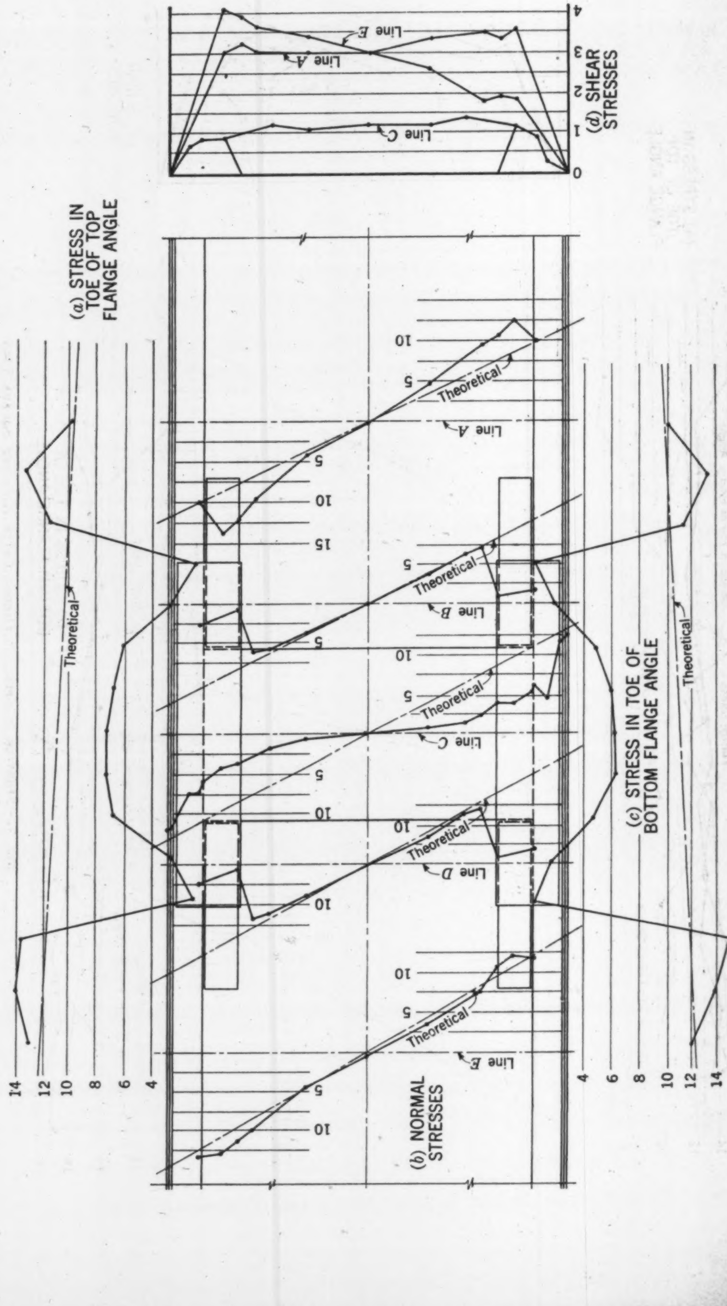


FIG. 8.—STRESSES AT SPLICE 3, GIRDER G_4, IN KIPS PER SQUARE INCH

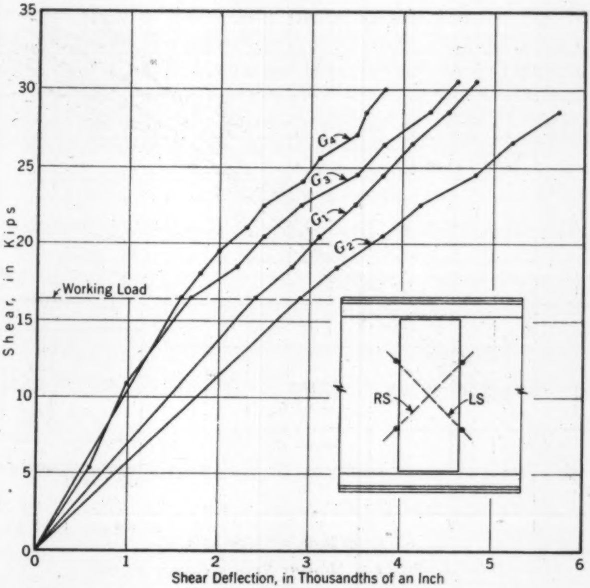


FIG. 9.—SPLICE 1; SHEAR VERSUS RELATIVE SHEAR DISPLACEMENT

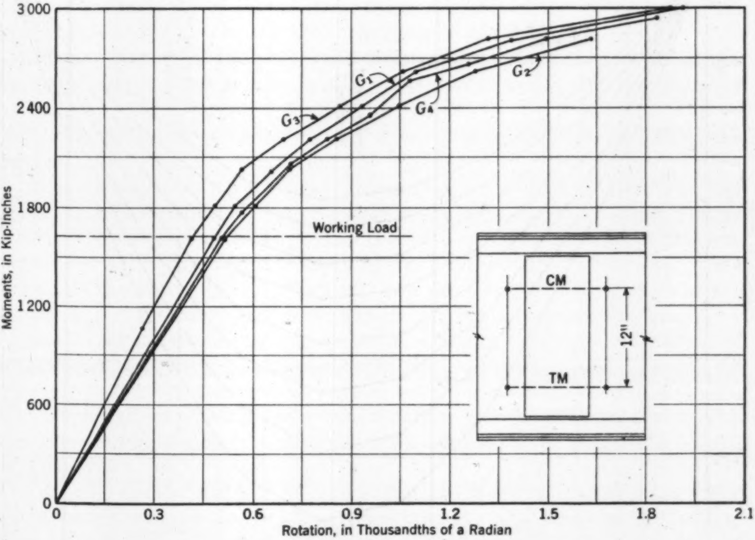


FIG. 10.—SPLICE 3; MOMENT VERSUS RELATIVE ROTATIONS

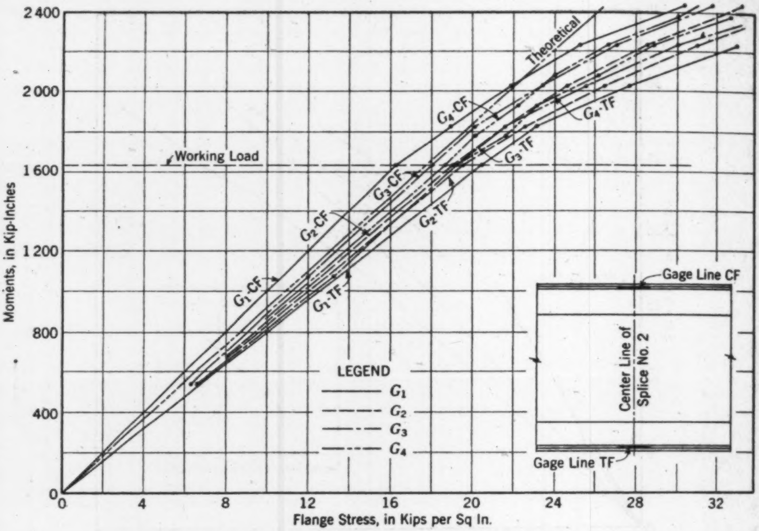


FIG. 11.—SPLICE 2; MOMENT VERSUS FLANGE STRESS

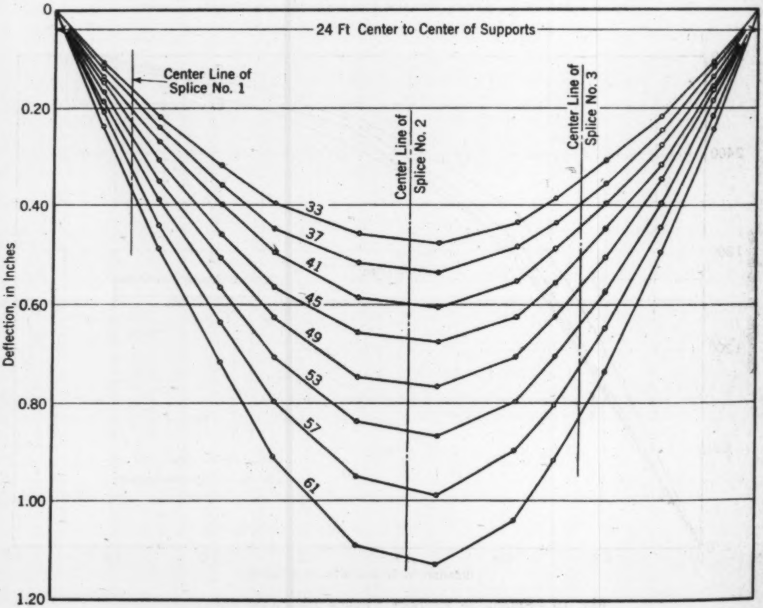


FIG. 12.—GIRDER G_1; DEFLECTION CURVES

included. The curves of load versus maximum deflection in Fig. 13 show a similar behavior for all four girders.

Stresses in the top and bottom edges of the splice plates of girder G_1 and in the edges of the adjacent flange angles are shown, for splice 2, in Fig. 14.

The ultimate loads for the girders are given in Table 3. In each case, failure was by buckling in the top flange, at splice 2 (see Fig. 15).

Discussion of Results

A summary of the design values for the four types of splices tested is given in Table 4. The design stresses and loads are as follows:

Unit stress	Kips per square inch
Bending in extreme fiber	18
Shear on gross section	13
Allowable stresses for rivets—	
Shear	15
Bearing (double shear rivets)	40
Maximum stress in plates at a distance d from the neutral axis	$18 \times \frac{d}{12.3}$
Working load, P, to produce a maximum extreme fiber stress of 18 kips per sq in	32.8
Maximum moment at working load in kip-in	(1,620)

The maximum stress in the rivet of any group due to bending is taken at the maximum bearing value. The theoretical moments and shears in the 20-in. depth of the web plates between the flange angles and the values computed from the measured strains are also included in Table 1.

An examination of the curves for direct stress reveals that at the center line of splice the stress is not directly proportional to the distance from the neutral axis. This variation is partly dependent on the relative thicknesses of the web plate and splice plates, on the length of splice plates parallel to the center line of girders and on the number of rivets in the splice plates.

Computations show that the moments carried by the splice plates on girders G_1, G_2, and G_3 are equal to the moments in the 20-in. depth of the web plate between the flange angles (Table 1). The stress due to moment in the web plate under the flange angles is taken by the flanges at the cut section. This increase in the flange stress is effective over a short length and is greater in the vertical legs of the flange angles than in the horizontal parts of the flanges.

The direct stresses along lines A and C adjacent to the spliced section of girder G_1 are essentially in agreement with the theoretical values. The curves representing the direct stresses on lines A and C for girders G_2 and G_3 and lines A and E for girder G_4 show some variation from the theoretical values. The lines representing the theoretical stresses are assumed to be unaffected by the adjacent splice.

From the data of Table 1, it is seen that the measured shear in the splice plates along the center line of splice is less than the total shear on the section. This suggests that the flange angles carry a greater part of the shear than is

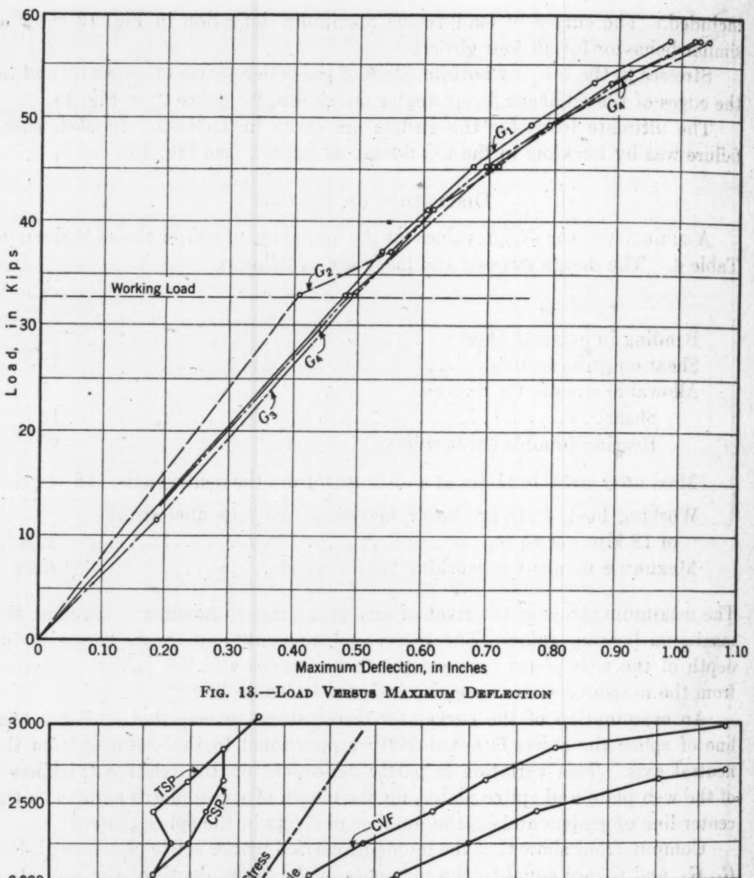


FIG. 13.—LOAD VERSUS MAXIMUM DEFLECTION

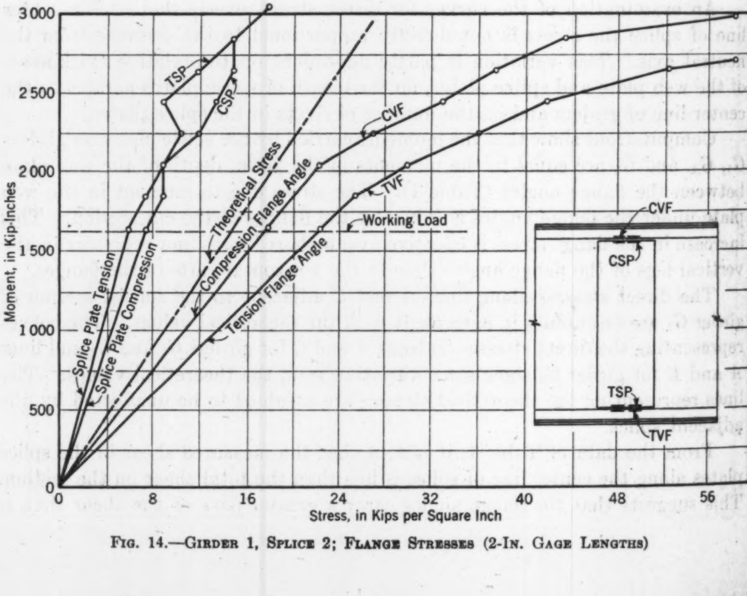


FIG. 14.—GIRDER 1, SPLICE 2; FLANGE STRESSES (2-IN. GAGE LENGTHS)

usually assigned to them. The stress curves for the toes of the flange angles show that the angles are subjected to a secondary bending stress at the cut section due to this shear. The well-known parabolic curve of shear distribution results when the loads producing the shear are farther apart than the depth of the beam. Since the loads taken by the splice plates are applied through the rivets, the shear distribution in the splice plates is affected by this method of loading the plates. There is also some disturbance in the shear curves for the lines outside the splice plates due to the adjacent splices.

The relative shear-displacement curves of Fig. 9 show a slightly different behavior for the girders. The curve for G_1 is essentially a straight line. The girders had all been subjected to the working load several times so that only the part of the curves above that load represents the action under the first application of load. The displacement at the splice on girder G_2 is about 80% greater than that of G_4 at the working load and that of G_1 is about 50% greater at that load.

The relative rotation curves of Fig. 10 show that the four splices act in a similar manner in the region of pure moment. At the working load the variation between the degree of rotation for G_3 which was minimum and G_2, the maximum, was about 25%.

The flange-stress curves of Fig. 11 show a similar behavior for the flanges of all girders. At the working load and above, the flange stresses are higher than the theoretical value based on the gross section.

The deflection curves of Fig. 12 do not indicate a noticeable weakness in any of the splices. The curves of Fig. 13 break away from the straight line at a load of about 40 kips, which is consistent with the action shown by the curves of Figs. 10 and 11.

The curves of Fig. 14 show that the stresses developed at the inside edges of the flange angles are larger than the theoretical, and the stresses in the adjacent splice plates are less than the theoretical, although both elements are essentially the same distance from the neutral axis. The curves indicate that there is a difference in these stresses for all loads and that the angles and the splice plates are acting independently. This difference is affected by the local bending in the angles and by the stress transfer through the rivets.

Behavior of the different splices was similar in character but different in degree. The action of splice G_4 was somewhat involved, but the values obtained indicate that the flange splice plates did not have enough rivets to develop the full action of the plates.

Distribution of stresses recorded during the breaking tests is consistent with that obtained for the elastic range. The buckling of the compression flanges occurred near the splice and in the region carrying an overstress due to the splicing action of the flange. Although failure did not occur in the splices, the relative actions of the four types of splices can be observed. For a girder of the type tested, there can be no failure of the splice due to moment until after one of the flanges has yielded.

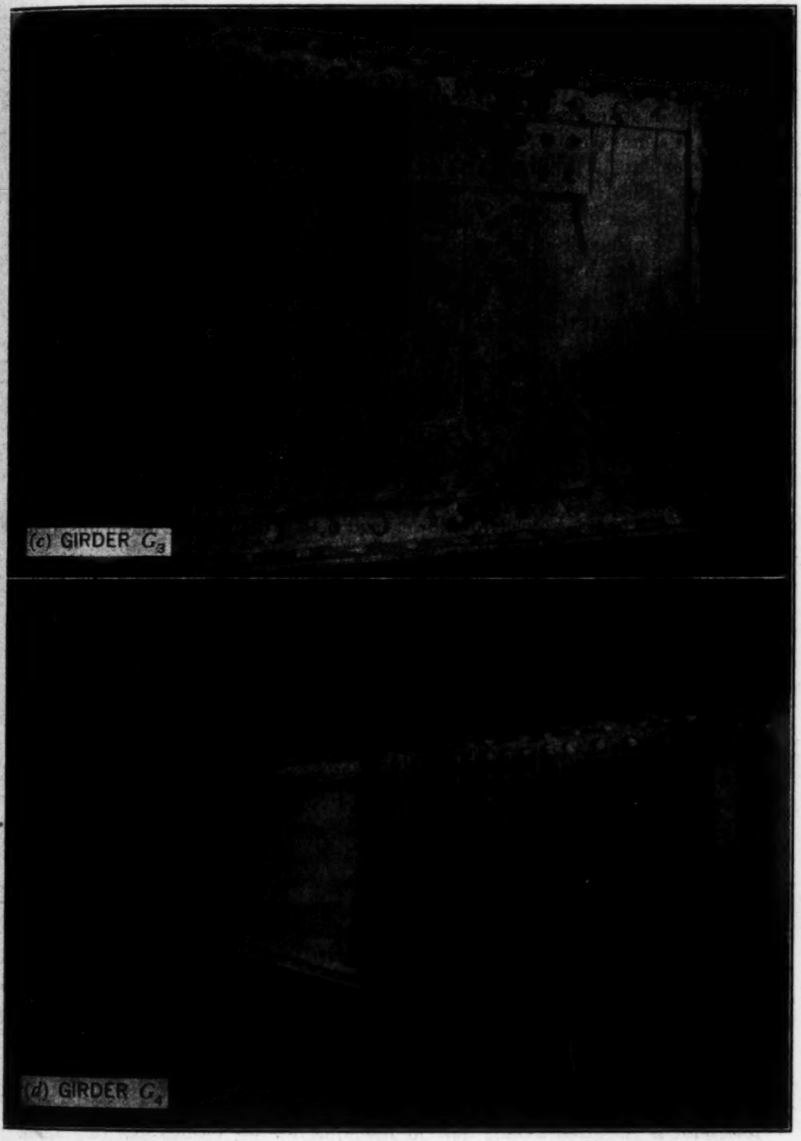
The load tests on the girders indicate the following conclusions:

(1) For all of the splices tested, each component part of any splice carried only the stress in that part of the web plate beneath that element of the splice.

(*a*) GIRDER G_1

(*b*) GIRDER G_2

FIG. 15.—VIEWS OF TEST


(c) GIRDER G_3
(d) GIRDER G_4

SPECIMENS AT FAILURE

(2) That part of the web-plate stress, not carried by the splice plates, produces an increased stress in the flanges.

(3) The splices on girders G_1, G_2, and G_3 carried only the moment in the 20-in. depth of web plate between the flange angles.

(4) At a spliced section, the component parts of the splice do not act together as a unit with the girder unless the splice is designed so that the strains in the elements of the splice are consistent with those of the girder.

(5) A splice that is weak in shear-carrying capacity will cause secondary bending stresses in the flange angles due to the shear carried by the angles.

(6) The behavior of all splices within the working load was satisfactory.

(7) The length of splice material along the girder apparently affected the action of the splices. The longer splice material of girder G_3 restrained the angular rotation at the splice. It is believed that splices of G_1 or G_2 type would have been at least as effective as G_3 in this respect if they had been extended one line of rivets on either side.

(8) A variation in the type of web splice used, between those tested, does not appreciably affect the ultimate strength of the girder if failure does not occur in the elements of the splice itself.

(9) Excess stress in the flanges, produced by the splicing action of the flanges, probably influenced the buckling of the compression flange and caused that flange to buckle at a slightly lower load than it would have for an unspliced girder.

(10) The lack of shear resistance at horizontal planes where the splice plates were discontinuous did not measurably affect the behavior of the splice.

ACKNOWLEDGMENT

The tests reported herein were planned by the American Institute of Steel Construction in cooperation with the Research Laboratories of the Department of Civil Engineering, Columbia University, New York, N. Y. Preliminary studies were made by Bruce Johnston, Assoc. M. Am. Soc. C. E., in 1938 while he was at Columbia University. The girders were designed and fabricated by the Bethlehem Steel Company.

F. H. Frankland, M. Am. Soc. C. E., and other members of the Committee on Technical Research of the Institute cooperated in the development of the program and offered valuable suggestions and criticisms as the work progressed.

The conduct of the tests was under the supervision of Prof. W. J. Krefeld, M. Am. Soc. C. E., director of the Engineering Materials' Laboratories, Columbia University, who also gave freely of his time and advice in the analysis and interpretation of results.

The authors also desire to express their appreciation for the encouragement and cooperation of J. K. Finch, M. Am. Soc. C. E., head of the Civil Engineering Department.

DISCUSSION'

C. H. GRONQUIST,[3] ASSOC. M. AM. SOC. C. E.—The investigation of stresses in plate girder web splices reported in this paper is most timely and valuable. With the increasing use of plate girders for bridges of comparatively long spans ranging up to 300 ft in length, it is proper that the theoretical bases for the design of these girders should be reviewed and checked by experimental investigation.

A type of plate girder web splice that is very frequently used was not included in this investigation. It is similar to the splices of girders G_1 and G_2, with the exception that the web splice plates are supplemented by vertical flange plates which are placed only over the vertical legs of the flange angles, and are not extended to take a row of rivets in the web splice plates as in the splice of girder G_4. This splice has the merit of providing for the cross-sectional area of the part of the web plate under the flange angles, although it does not provide shear resistance at the horizontal plane at the toe of the flange angles. According to conclusion (10) of the paper, this lack of horizontal shear resistance should not be objectionable, but it would be helpful to have the opinion of the authors in regard to this type of splice.

The flange plates, since they splice indirectly, should be extended, at least on the low-moment side of the splice, a sufficient distance to provide rivets for the transfer of the flange-stress increment in addition to those required for the splice of the web under the flange angles. This was revealed in the authors' original notes and reports, although it does not seem to have been emphasized in the paper.

CHARLES STRATTON DAVIS,[4] M. AM. SOC. C. E.—The experiments conducted by the authors to secure information concerning the behavior of plate girder web splices under load and to determine the relative values of four types of web-splice design commonly used in modern practice are most interesting and instructive.

The experiments were conducted on girders proportional to those used in practice; it might be desirable also to have some tests made on girders with excess flange strength in order to cause failure in the splices. It appears from Fig. 15 that ultimate failure occurred in the outstanding legs of the flange angles and the cover plates of the compression flanges of all girders tested. Fig. 15 further indicates that the rivet pitch in the flange plates might have been too great in proportion to the thickness of the flange angles and plates. The use of thicker material in the flanges, creating excess flange strength, might demonstrate additional information in regard to the various types of splices.

The desire to secure economical construction is an incentive that has led designers to try to splice webs of plate girders, fully, for both shear and moment and has caused the development of web splices requiring a relatively large amount of material and extra labor. Greater economy may be obtained through the use of simple splices placed at favorable locations. Flange plates are stressed to their full value only at or near their centers. They have excess flange value near their ends. It would therefore be wise to take advantage of this excess flange strength when designing web splices and place splices near the ends of flange plates. In case it should become necessary to splice the web at the center of a girder, short flange plates should be added to equal the value of the web plate in bending. It is believed that the advantages gained in simplicity of construction and resultant saving labor will more than offset all extra flange material required.

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[3] Associate Engr., Robinson & Steinman, New York, N. Y.

[4] Cons. Engr., Pittsburgh, Pa. (Mr. Davis died on January 2, 1942.)

Bruce Johnston,[5] **Assoc. M. Am. Soc. C. E.**—Ever since the need for these tests was first suggested in 1936 by engineers of the Bethelehem Steel Company, the writer has followed the progress of this program with interest. The authors have made a concise and able presentation of their results, and deserve much credit for their perseverance and patience in making the many strain measurements and rosette calculations.

The fact that all four splices showed nearly the same behavior constitutes an important finding applying to design economy. Splice G_1 or G_2 would obviously be the most economical, and it is interesting to tabulate the following data pertaining to the relative economy of all four splices.

Splice	No. of rivets	No. of plates
G_1	44	2
G_2	30	2
G_3	36	6
G_4	26	14

(The 26 rivets in splice G_4 do not include eight rivets in the flange angle.)

A number of years ago the writer was prejudiced against splices of the G_3 type by statements in textbooks, as well as by engineers, to the effect that this type of splice is poor design. One argument states that the splice has "four planes of shear weakness." It is interesting to have these prejudices dispelled by the fact that splice G_3 is slightly superior to the other splices on the following evidence taken from the authors' report:

(1) According to Table 1, splice G_3 carried in the working range a greater proportion of both bending moment and shear than any of the other splices;
(2) Table 3 shows that the girder with splice G_3 carried a greater maximum load than any other girder; and
(3) Fig. 10 shows that web-plate rotation at working load across the spliced part was about 14% less for splice G_3 than for any other splice.

On other counts the difference in splice behavior is extremely small, and the very slight superiority of splice G_3 is more than offset by the greater economy of splice G_1 or G_2. It is nevertheless an important fact that splice G_3 is proved to be not as bad as sometimes imagined.

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[5] Engr., Bureau of Yards and Docks, U. S. Navy, Washington, D. C.

On the basis of Table 2 the writer has calculated the approximate weighted yield-point stress of the flange section, assuming one sixth of the area of the web to be acting therein. Assuming a linear stress-strain relationship, and using the gross moment of inertia, the maximum calculated stress at failure is only about 80% of the weighted yield-point average. The calculated stress at maximum load is very close to the weighted yield-point average if the net moment of inertia is used. It does not necessarily follow that the net moment of inertia should govern the design because there are several non-separable variables involved—for example, local bending in the flange angles at the splice, the lower-than-average yield point of the cover plates which apparently buckled first, and the unsupported length of the compression flange. However, it is true, apparently, that in the case of spliced girders of the type tested, the use of the net moment of inertia as a design basis would not appear to be overconservative. In the light of current design practice, this is a subject that warrants further study and test.

Reserve strengths more than that calculated by substituting the yield point in a maximum-stress formula are sometimes present in compact steel members, according to the principles of "limit design." Such reserve strength is conspicuously "more than absent" in the case of the spliced plate girders tested by the authors.

In their discussion of results, the authors present design data that are evidently based on the use of net moment of inertia and a maximum fiber stress of 18 kips per sq in. On such a basis the load-factor ratio, or real factor of safety for girders G_1 and G_2, adjusted from a weighted average flange yield point of 36.7 kips per sq in. to the 33 kips per sq in. minimum specified for structural steel, is calculated by the writer to be 1.73. This factor of safety is fairly well in line with the ratio of tensile yield point to working stress—that is, $\frac{33}{18} = 1.83$. If the design had been based on gross moment of inertia and a maximum allowable stress of 20 kips per sq in., the adjusted load factor of safety would be 1.32, which provides a margin of safety (between 1.0 and the tensile stress ratio of $\frac{33}{20} = 1.65$) of only 50%.

In the foregoing calculations the writer used 90% of the reported girder strength to correct the weighted average of the material yield strength of 36.7 kips per sq in. down to the specified minimum of 33.0 kips per sq in. for structural steel. Tests[6] made in 1941 on a number of different representative heats of structural steel, from different manufacturing plants, indicate that structural steel frequently has a yield point very near or even below the specification minimum of 33 kips per sq in. Such steels may show satisfactory yield points when tested at standard test speeds allowed by the American Society for Testing Materials (A.S.T.M.). Such tests are "material control" or "acceptance" tests and do not represent the minimum behavior to be expected at the slow speed of the test. The standard speed of test affects the reported yield point in the laboratory test and has little relation to the speed of loading a structure. The writer raises these questions because they are of considerable importance if the excellent service record made by plate girders in the past is to be maintained. Data available in the authors' tests have a direct relation to these questions. The subject is one that deserves more attention than has been given to it by the authors.

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[6] "Compression and Tension Tests of Structural Alloys," by Bruce Johnston and Francis A. Opila, 1941 Meeting, A. S. T. M.

As indicated in conclusion (9), it is possible that the lower-than-yield-point buckling stresses in the flange angles may have been caused partly by local bending stresses in the flange at the spliced section. The writer suggests that the only way to make a spliced girder equally as effective as an unspliced girder may be by butt-welding the web plates before riveting the flange angles, thus eliminating riveted web-splice plates altogether.

It is obviously impossible on the basis of these tests for the authors to answer all of the questions that could be raised regarding plate girder splices. They have thrown new light on a subject that has been largely a field of speculation heretofore. Tests of this type are shown to have practical value for the engineering profession, and the authors deserve much credit for doing a difficult job in such a satisfactory manner.

FRANCIS L. CASTLEMAN, JR.,[7] ASSOC. M. AM. SOC. C. E.—The designing engineer will benefit greatly from this series of investigations. As stated in the "Synopsis," the theory of the design of plate girder web splices has been based largely on assumptions that have never been checked experimentally.

Considered broadly, these investigations give confidence in the four types of splices investigated, and designers need feel no hesitation in using any one of them. A theory does not necessarily have to be correct; all it has to do is "work"! Even casual inspection of the results of these tests shows great departure from theory, but such results that the conventional methods are quite safe and will suffice until better and more rational ones are established and accepted. Certain conclusions can be drawn from these investigations that possibly can be interpreted as suggesting minor modifications in the present theory as used.

It is difficult to choose the optimum of the four splices tested, taking into account both theoretical and practical considerations. From the latter standpoint splices G_1 and G_2 are undoubtedly the best, having fewer pieces and being more simply fabricated. Their variation in strength from the other types is negligible when considered from a practical viewpoint. They also possess a certain theoretical merit in that on lines *A* and *C*, Figs. 5 and 6, a nearer approach to straight-line theory is shown.

From theoretical considerations alone, splice G_4 probably would be preferred by most designers. However, Table 3 shows this to be located on a girder that failed under the smallest loading. In other words, assuming that the splices influenced the failure of the compression flanges, this type of splice appears to have exerted the greatest damaging influence. To the writer this is one of the most interesting features of this series of investigations. It is true that certain modifications could perhaps be made in this splice that would alter these results, but these modifications are by no means apparent upon starting the initial design. The designer has little time to ponder over such modifications even if they were finally forthcoming.

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[7] Prof. and Head, Dept. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

Splice G_3, which is the least rational of the four types considered, shows the greatest breaking strength. This is interesting, particularly to the writer, in that he has seen this type frequently criticized (as well as most other types) as lacking certain theoretical qualifications. It would appear that the ordinary methods used in calculating the strength of such a splice are of such a fortuitous nature that they inadvertently lead to great strength.

In designs of type G_1 and G_2 it is often common to reduce the working value of the extreme rivet in the splice plate by the ratio of its distance from the neutral axis to the distance of the extreme rivet in the web (through the flange angle) from the neutral axis. Since the tests under consideration show that each component part of such a splice carries only the stress in that part of the web plate beneath it, this would seem unnecessary. Whether desirable or not, the flange acts as a separate unit in splicing the cut web at the top and bottom, and its action is entirely independent of the splice plates on the web. It is interesting to observe here that in addition to the increase in flange stress set up by the splicing action this increase also throws an extra increment on the flange rivets in the vicinity of the splice.

From a standpoint of stiffness, which is often an important consideration, most splices are usually too short. Splices of the type of G_3 will overcome this as the tests show, but the same results can be obtained by increasing the length of splices of the type G_1 or G_2. In most cases this would mean extra material, and rivets would have to be added in the interest of increased stiffness. There is no valid objection to this, if it is desirable, as the same problem frequently arises in the design of main members.

In addition to the action of the splices these investigations give a very good picture of the buckling action of the compression flanges of the girders. Using G_3 as a basis, its theoretical modulus of rupture, by proportion, would be $\frac{17.6}{32} \times 65.3 = 36$ kips per sq in. This, in itself, is not sufficient to cause failure of the compression flange, but, when the increase in stress due to sidewise buckling, plus a small allowance produced by the splicing action, is added, it is easy to understand how failure occurred. It should be noted that the yield point of the flange angles is approximately 43 kips per sq in. and that of the $\frac{3}{16}$-in. cover 31 kips per sq in. Both of these values would have to be considered, as well as the restraining influence of the web, in determining the elastic instability of the flange.

It should be borne in mind that the four girders tested to failure had top flanges that were not supported laterally, or, if so, only that small amount furnished by the loading apparatus. Furthermore, the top ends were not restrained against sidewise rotation. Such an arrangement would scarcely be met in problems of a practical nature and therefore the results can be looked upon as extreme.

With regard to shear it is unfortunate that other loading arrangements were not investigated in addition to the symmetrical arrangement used. In the elastic range considered, shearing values obtained were so small that it is questionable whether a conclusive statement could be made safely concerning the lack of shear resistance at the horizontal planes where the splice plates were discontinuous. It is often necessary to locate a splice at a point of relatively high shear, and it is in such cases that these points of discontinuity become critical. In the cases under consideration, taking G_3 as a basis and using the breaking load, the theoretical horizontal shear at the vertical toe of the flange angle in the web, using the well-known relation $\frac{V\,Q}{I\,t}$, would be 6.4 kips per sq in. at splice 3. It is also interesting to observe that, at this same point and for the same load, the theoretical stresses on a cross section due to moment fall within the elastic range (but, of course, greater than the allowable).

It would have been instructive if at least one of the girders could have been loaded in such a manner that splice 2 would have been subjected to a high shear as well as moment. As given, no shear occurs at this splice, and the loading producing such a condition is generally only of academic interest, rarely, if ever, occurring in practice. In fact, the authors state under "Discussion of Results": "* * * there can be no failure of the splice due to moment until after one of the flanges has yielded." Of course, when this has happened the girder has failed structurally.

The ideal web splice would be welded, without the use of side plates, and with the webs butted together for a V-weld or a modification of it. With such an arrangement, and using coated electrodes, stress conditions would be practically the same as in the adjacent unspliced parts of the web. As yet there is no ideal riveted splice in the sense that a stress condition is set up similar to that in the unspliced web. The writer questions seriously if such a riveted splice, or a rational solution for it, can be devised. The rivets create so many individual load points that the consequent complicating ramifications become enormous and defy rigid theoretical analysis. However, experience, and the investigations under discussion, have shown ably that the present methods are adequate, even if they are not based on entirely sound theory.

J. M. GARRELTS,[8] ASSOC. M. AM. SOC. C. E., AND I. E. MADSEN,[9] JUN. AM. SOC. C. E.—The criticisms and suggestions offered by discussers have added to the value of the paper as a record of tests made to increase the present knowledge of the behavior of plate girder splices.

The writers are of the opinion that a splice like either G_1 or G_2, with the addition of vertical flange plates as suggested by Mr. Gronquist, would provide ample splicing action for the cut web, but the distribution of stresses in the splice material will be somewhat complicated, as for example, in splices G_3 and G_4.

Mr. Davis has suggested that it might be desirable to have some tests made on girders with excess flange strength in order to cause failure in the splices, but the writers believe that the flanges in a plate girder will fail, at least as soon as the web splice. In other words, before the splice can yield in shear or bending, the flange must also yield. The location of a web splice near the end of a flange plate would provide reinforcement for the direct stress in the flange and, of course, would be preferable to splicing at a point where the flange stresses are high, unless the flange is reinforced.

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[8] Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y.

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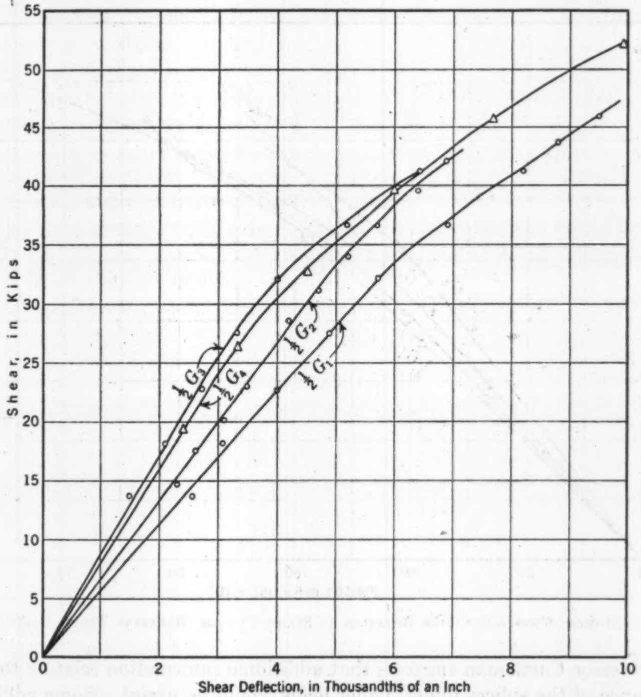

FIG. 16.—SHEAR-VERSUS-RELATIVE-SHEAR DISPLACEMENT CURVES (BREAKING TESTS, HALF-GIRDERS)

Professor Johnston has stated that the calculated stress at maximum load is very close to the weighted yield-point average if the net moment of inertia is used. However, it seems to the writers that, since failure of all of the girders was due to buckling of the compression flange, no particular relation exists between net moment of inertia and the failure load. The fact that very little is known about the structural action involved when a plate girder fails is ample proof that the problem warrants further study and tests. Professor Johnston is quite right in emphasizing the uncertainty as to the margin of safety in a plate girder, but the data reported in this paper were intended primarily to give information relating to the behavior of the splices and their effect on the structural behavior of the girder near the splices.

Professor Castleman has assumed that the top flanges of the girders were not braced laterally, since the writers did not mention the method of supporting the girders. Actually, the top flanges were supported laterally at a point near the end of each girder and the third-points of the span by steel angle frames attached to the supporting beams. With these supports, the flange appeared to remain straight until failure had occurred due to the local buckling action.

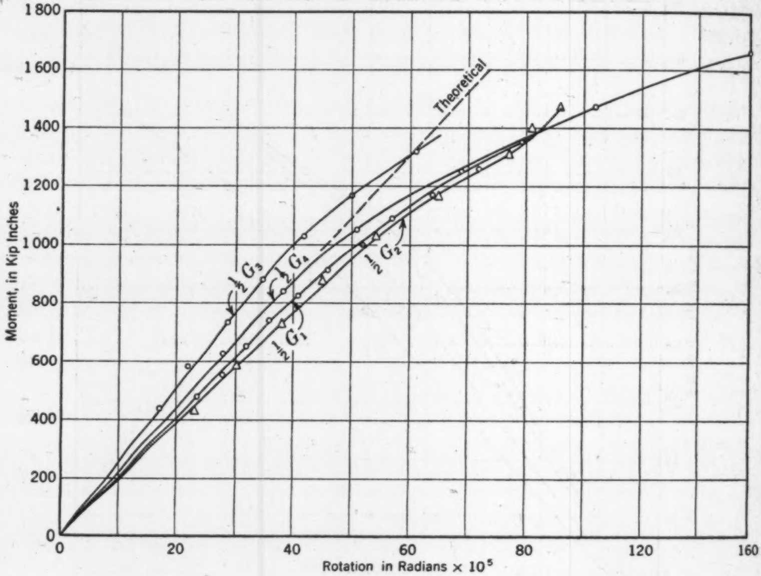

FIG. 17.—MOMENT-VERSUS-RELATIVE ROTATION AT SPLICE CURVES (BREAKING TESTS, HALF-GIRDERS)

Professor Castleman suggests that additional information relating to shear resistance of the splices under higher loads would be useful. Some additional data were obtained that may be helpful in this respect. Following the tests on the full-length girders, a 10-ft 5-in. length of girder containing splice 1 was cut from the end of each girder, and will be referred to as $\frac{1}{2}\,G_1$, $\frac{1}{2}\,G_2$, etc. The girders $\frac{1}{2}\,G_1$, $\frac{1}{2}\,G_2$, and $\frac{1}{2}\,G_3$ were tested to failure by the application of a single load at a point 69 in. from the end of the girder and 37 in. from the center line of splice 1. Girder $\frac{1}{2}\,G_4$ was loaded at a point 46 in. from the end of the girder and 14 in. from the center line of splice 1.

The girders failed by buckling of the compression flange at the load point. This failure, like that in the full-length girders, was caused by local buckling of the flange, due to the stresses caused by the moment, and by local stresses under the load. Although the splice was not placed at the point of maximum moment, it was subjected to higher shear stresses than in the tests of the full-length girders.

Strain measurements were taken to determine the shear-versus-relative-shear displacement and moment-versus-relative rotation. These values are shown in Figs. 16 and 17. The maximum loads and moments for these half-girders are shown in Table 5. The section modulus of $\frac{1}{2}\,G_4$ at the load point was only two thirds of that for the other girders, since the cover plate is not effective on $\frac{1}{2}\,G_4$. The action indicated in Figs. 16 and 17 is consistent with that shown for the full-length girders in Figs. 9 and 10.

TABLE 5.—MAXIMUM LOADS AND MOMENTS

Girder	FORCES, IN KIPS		Moments at load point, in kip-in.	Moment at splice, in kip-in.
	Load	Shear		
$\frac{1}{2}\,G_1$	104	47.7	3,210	1,530
$\frac{1}{2}\,G_2$	112	51.4	3,460	1,640
$\frac{1}{2}\,G_3$	105	48.2	3,260	1,540
$\frac{1}{2}\,G_4$	92	60.1	2,610	1,920

These tests have demonstrated that plate girder splices do not act in accordance with the assumptions made in designing them. The problem is complicated by the fact that the lengths of the splice plates have an effect which cannot be fully taken into account and by the fact that the rivets act in a manner which does not admit of exact mathematical analysis.

The writers are grateful to those who discussed this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2160

COMPACTION OF COHESIONLESS FOUNDATION SOILS BY EXPLOSIVES

BY A. K. B. LYMAN,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. REUBEN M. HAINES, R. G. HENNES, JOHN D. WATSON, PAUL BAUMANN, AND A. K. B. LYMAN.

SYNOPSIS

The method of compacting loose, cohesionless foundation soils in their natural state by detonating buried charges of explosives has been used successfully at Franklin Falls Dam, Denison Dam, and Almond Dam. The use of this method, as reported herein, is intended to draw attention to a new economical and efficient method of obtaining a satisfactory degree of compaction in cohesionless foundation materials. To employ this method successfully, the materials must be in a condition approaching complete saturation, and under such conditions, this method is widely applicable to all loose, cohesionless structural foundations. The results obtained in investigations to date (1941) indicate that in addition to increasing the degree of compaction of the materials, the horizontal permeability of stratified deposits is greatly reduced. Further investigations and tests are needed to determine the limits of compaction for specific materials and whether optimum results can be obtained in fully or partly saturated material.

INTRODUCTION

Franklin Falls Dam on the Pemigewasset River near Franklin, N. H., was designed for the control of floods on the Merrimack River. In their natural state at this site, there are deposits of loose, fine alluvial sand, especially in the river terraces of the foundation area of the embankment that constitutes a part of the project development. Extensive investigations disclosed that the stability of the embankment would be considerably increased if these loose sand deposits could be consolidated to reduce the possibility of their failure by liquefaction due to disturbance caused by earthquakes, demolitions, or blasting in the vicinity.[2] Estimates of cost were prepared for various compaction

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NOTE.—Published in May, 1941, *Proceedings*.

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[2] *Civil Engineering*, April, 1940, p. 205.

methods, including that of driving piles and the use of large vibrators and tamping equipment to reduce the voids ratios of the loose terrace materials below critical values, as defined by Arthur Casagrande, Assoc. M. Am. Soc. C. E. Consideration was also given to the excavation of the loose materials and replacement by the same or other materials in a dense state by accepted compaction methods. The uniformly high cost of the treatments considered led the writer to investigate the feasibility of compacting the loose deposits by use of buried charges of explosives.

As a necessary prerequisite to the application of this method to compaction of loose soils it is necessary to consider: (*a*) Type of soil; (*b*) condition or degree of saturation of soil; and (*c*) depth of deposit. Although compaction by blasting was initially developed for the treatment of loose deposits of uniformly graded, slightly silty, fine to medium sands at Franklin Falls, subsequently sands containing a substantial percentage of silt have been successfully compacted at one other locality.

Saturation of the soil to be compacted, at least in the region of the explosive charges, is regarded as conducive to the best results. However, fairly effective compaction has been obtained in tests at the Denison (Tex.) Dam under conditions of partial saturation. The pore water probably acts initially as a tamping medium, thus facilitating the progression of the vibratory waves or shocks emanating from the charges. The explosion of buried charges induces a "liquefaction" of the soil mass, followed by an escape to the ground surface of excess pore water which acts as a lubricant to facilitate a rearrangement, in a more compact state, of the soil grains in the vicinity of, and overlying, the charge. Extensive explorations after blasting disclosed the absence of cavities which may have been created momentarily by blasting in the soil mass.

There are no apparent limitations to the depths of deposits that may be consolidated by means of explosives. On the contrary, deposits of sand at considerable depth may be compacted in this fashion, whereas they may not be readily accessible to other forms of treatment.

It is thought that where loose strata of sand greater than 30 ft thick are to be compacted, two or more tiers of small charges are to be preferred to one tier of large charges. For deposits less than 30 ft thick charges at depths, below the top surface of the mass to be compacted, of approximately two thirds of the thickness of the loose soil mass, will generally suffice.

The size and distribution of charges should be such as to shatter the soil mass thoroughly but not to the extent of creating permanent surface craters. A surface crust of impervious material inhibits the emission of the surplus pore water from the ground and must be disturbed, fractured, or removed. Following the dull thudding sound of the blast there is an upheaval of the ground surface in the range of influence, a fracturing of the surface, and the escape of gas and water for periods of from minutes to hours from crater-like openings similar to sand boils which appear on the surface. The primary surface settlement immediately follows the initial upheaval, and continued settlement is apparent for a few minutes. No doubt settlement continues for some time as part of the adjustment of the soil mass accompanying the continuing expulsion of surplus pore water.

The lateral distribution of charges should be based on results obtained from a series of single shots. A horizontal grid spacing of charges, 20 ft on centers, for the initial coverage, with an orderly interspersion of four subsequent coverages in the space intervals between prior coverages, proved satisfactory at Franklin Falls.

The degree of compaction obtained by blasting can be demonstrated by tests of undisturbed samples taken before and after the compaction operation. Undisturbed sampling below the ground-water table, although requiring expensive test pits, is necessary if the results are to be evaluated accurately. Observation of ground-surface settlement by means of stakes or anchored plates permits a generalized interpretation of the effectiveness of the blasting method. Moreover, such observations indicate the settlement for each blasting coverage and, hence, are a key to the number of coverages required.

The method of compacting loose, cohesionless foundation materials by means of buried explosive charges has been applied successfully to the river terraces which form a large part of the embankment foundation area at Franklin Falls Dam. Tests have also been conducted at Denison Dam in Texas and at the site of Almond Dam in western New York State.

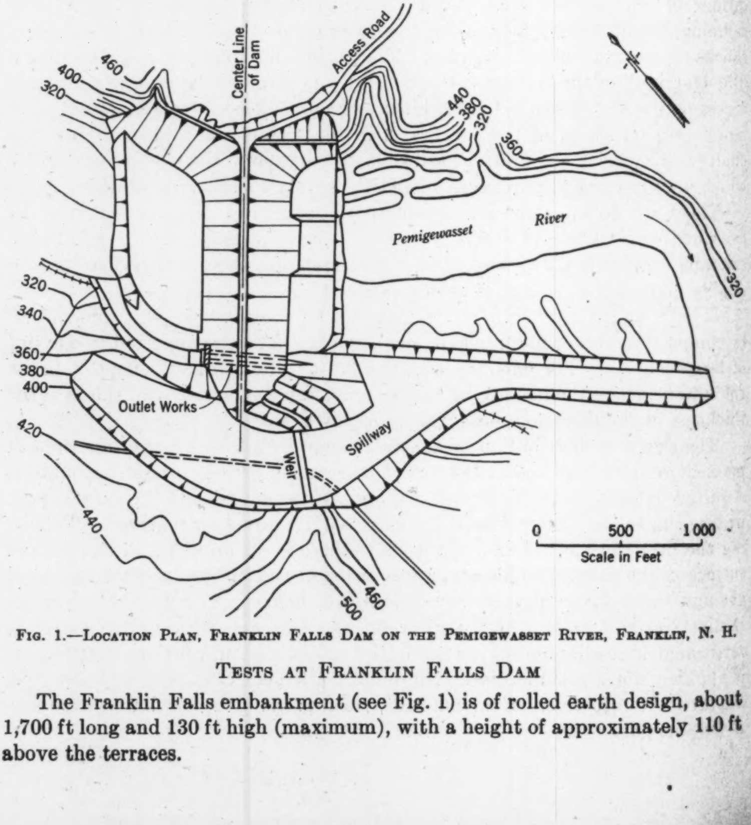
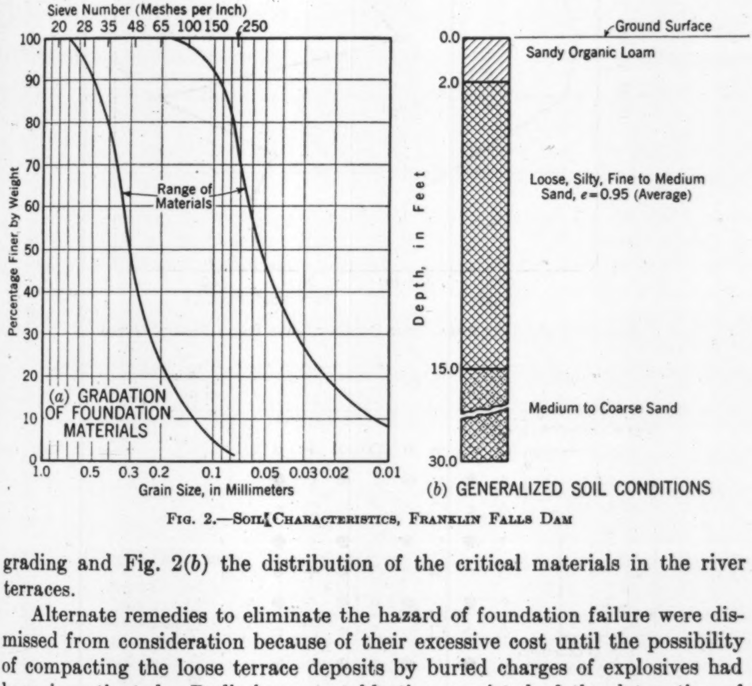


Fig. 1.—Location Plan, Franklin Falls Dam on the Pemigewasset River, Franklin, N. H.

Tests at Franklin Falls Dam

The Franklin Falls embankment (see Fig. 1) is of rolled earth design, about 1,700 ft long and 130 ft high (maximum), with a height of approximately 110 ft above the terraces.

The embankment foundation area consists in part of two overbank flood plains or terraces, each about 400 ft wide, of loose, fine-to-medium, alluvially-deposited sands extending to depths of from 15 to 30 ft. Below this stratum are stable coarse sands and gravels. As construction of the dam proceeds the loose, cohesionless materials will consolidate slightly, but not sufficiently to reduce the fine materials below their critical voids ratios for the loads to be imposed. Apprehension was felt that these deposits might suddenly "liquefy" and flow from beneath the embankment due to disturbance caused by earthquakes, slides, settlements, or vibrations. Fig. 2(*a*) indicates the range of grading and Fig. 2(*b*) the distribution of the critical materials in the river terraces.



(*b*) GENERALIZED SOIL CONDITIONS

FIG. 2.—SOIL CHARACTERISTICS, FRANKLIN FALLS DAM

Alternate remedies to eliminate the hazard of foundation failure were dismissed from consideration because of their excessive cost until the possibility of compacting the loose terrace deposits by buried charges of explosives had been investigated. Preliminary test blasting consisted of the detonation of individual charges of dynamite ranging from 2 lb to 8 lb at depths of 10 to 16 ft below the ground surface; optimum consolidating effect was produced by charges of 8 lb of 60% dynamite at a depth of 15 ft below the ground surface.

A test area was next blasted with twenty-one charges at depths of 15 ft distributed into four coverages with final grid intervals of 10 ft. An average ground-surface settlement of the test area of 1.25 ft resulted. Tests of undisturbed soil samples taken from a test pit excavated to a depth of 8 ft in the center of the blasted area yielded the results indicated in Fig. 3. It is estimated that the average voids ratio before blasting was 1.0 and the degree of compaction 36%.

To confirm these results and further develop the method, a second test area, with five blasting coverages (see Fig. 4), resulted in an average ground-surface settlement of 2.0 ft, the compaction being effective over an estimated depth of 20 ft. Pits were excavated before (Pit 610) and after (Pit 611) blasting. Tests of undisturbed soil samples indicate changes in voids ratio and degree of compaction as plotted in Fig. 5. No evidence of cavities was revealed by the test pits excavated after the blasting operations. A total of six soil samples taken at each elevation indicated a fairly uniform degree of compaction laterally at each depth.

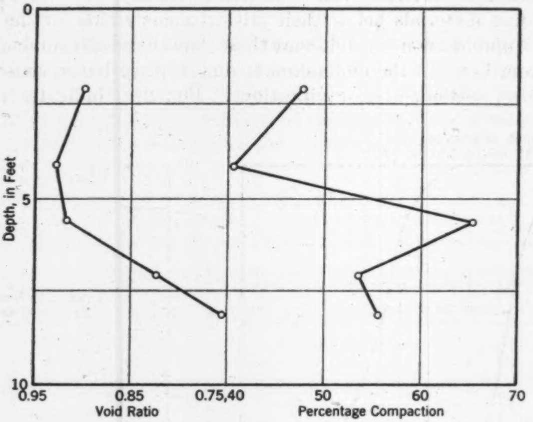


Fig. 3.—First Test Area, Test Pit 609; Voids Ratio Versus Depth

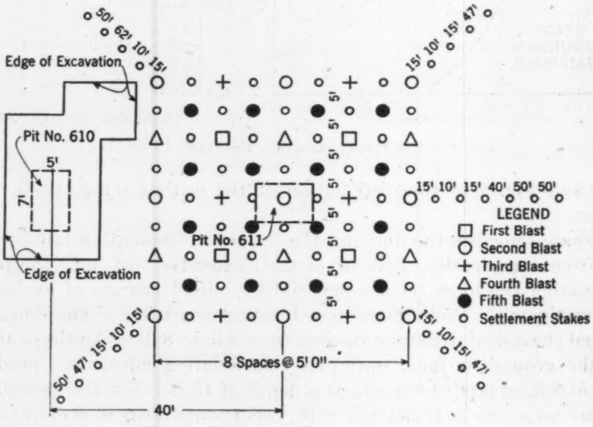

50'
62'
10'
15'
15'
10'
15'
47'
Edge of Excavation
Pit No. 610
5'
7'
Edge of Excavation
Pit No. 611
5'
5'
5'
5'
5'
5'
5'
5'
15' 10' 15' 40' 50' 50'
15'
10'
15'
47'
50'
47'
15'
10'
15'
8 Spaces @ 5' 0"
40'
LEGEND
□ First Blast
○ Second Blast
+ Third Blast
△ Fourth Blast
● Fifth Blast
∘ Settlement Stakes

Fig. 4.—Layout of Second Test Area

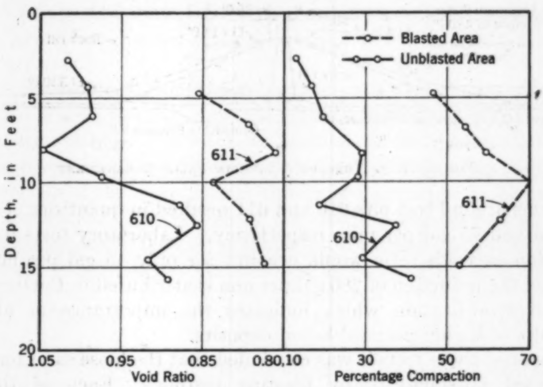


FIG. 5.—SECOND TEST AREA, TEST PITS 610 AND 611; VOIDS RATIO AND DEGREE OF COMPACTION VERSUS DEPTH

Permeability is reduced substantially by the compaction operations and is attributable to two causes: First, the reduction in void spaces between soil grains; and second, the breaking up of lensing and stratification of the natural deposits. The relative magnitude of these two effects is shown in Fig. 6. Field pumping tests before and after compaction operations from identical well

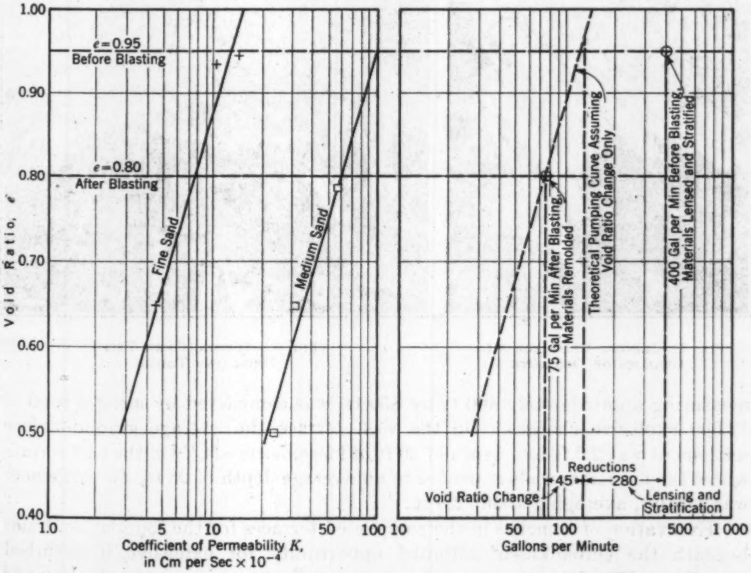


FIG. 6.—LABORATORY PERMEABILITY CURVES ON REMOLDED SAMPLES; FIELD PUMPING TEST

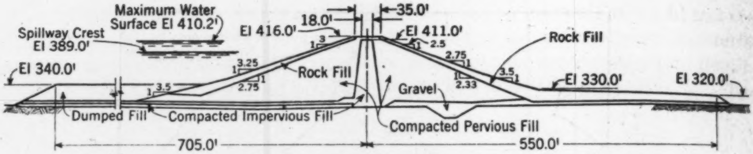

Maximum Water Surface El 410.2'
Spillway Crest El 389.0'
El 340.0'
El 416.0'
18.0'
35.0'
El 411.0'
Rock Fill
3
1
3.25
1
1
2.75
Rock Fill
1
3.5
2.5
1
2.75
1
1
2.33
3.5
1
El 330.0'
El 320.0'
Gravel
Dumped Fill
Compacted Impervious Fill
Compacted Pervious Fill
705.0'
550.0'

Fig. 7.—East Terrace, Franklin Falls Embankment

point systems around test pits 610 and 611 resulted in quantities of flow of 400 gal per min and 75 gal per min, respectively. Laboratory tests indicate that the reduction in voids ratio would account for only 45 gal per min, and the remainder of the reduction of 280 gal per min is attributed to the breaking up of lensing and stratification which indicates the importance of altering this characteristic of highly permeable soil deposits.

As a result of these tests it was concluded that the loose sand terraces could be compacted satisfactorily by blasting methods. Each of the terraces, measuring approximately 400 ft by 850 ft, was compacted by using a total of 12,000 explosive charges. On the west terrace the average ground-surface settlement was 2.0 ft in a depth of 20 ft of loose materials. On the east terrace where the loose materials extended to an average depth of 25 ft, the settlement was greater, averaging about 2.5 ft.

Fig. 8.—Jetting Drill for 8-Lb Charges of Dynamite

Fig. 9.—Jetting Crew Using Drill and Tripod

Excavation of trenches in the compacted terraces for the foundation drains beneath the embankment afforded opportunity for extensive undisturbed sampling. The fifty-seven samples tested indicate an average voids ratio of

0.780 for the compacted materials in the west terrace and 0.736 in the east terrace, with respective degrees of compaction of 45.7% and 56% for the west and east terraces.

The breaking up of lenses and stratifications in the Franklin Falls terraces by blasting results in a slight mixture of the material and is important as a means of reducing permeability. Undisturbed samples (after blasting) of sufficient size for tri-axial testing present such a wide variation of grading from point to point that isolation of the more critical uniformly fine materials for testing was found to be infeasible. The results of the blasting compaction at Franklin Falls must be evaluated in part by judgment and experience in critical density testing of fine cohesionless materials similar to those which are of concern in the terrace foundations. The uniform, fine, cohesionless embankment materials at Franklin Falls (which are similar in character and grading to the critical, fine alluvial sands in the terraces) have been tested extensively; and, for the design loadings, a critical voids ratio of 0.67 has been determined. Although not subject to strict correlation, the voids ratios in the terraces, after blasting (with values as low as 0.656 but generally ranging from 0.72 to 0.80), may be favorably compared with those of the mechanically compacted embankment. It is manifest that a comparison of these voids ratios provides considerable assurance that dangers of liquefaction have been reduced greatly.

Decreases in thickness of the loose sand deposits, from 2 to 2.5 ft in a depth of 20 ft, and reduction of the average voids ratio in the natural state of approximately 0.95 to the average values after blasting, previously cited, are indicative that the major settlement hazard has been eliminated. The berms at the toe of the embankment slopes, shown in Fig. 7, serve to contain the foundation; and the deep downstream foundation drain trench, which is backfilled with gravel, provides a ready path for the escape of surplus pore water should the terraces be subjected to vibratory stress. The section of the embankment in the river channel has its foundation approximately 20 ft below the embankment on the terraces, thus virtually enclosing the critical terrace foundations. These facts all provide assurance that adjustments in the terrace foundations, under stress conditions of reasonable expectancy, will be of a purely local nature and that the possibility of "flow" of the terrace foundations is quite remote.

The cost of compacting the Franklin Falls terrace foundation is approximately \$35,000, or \$0.09 per yd, for the material compacted. This compares with a minimum estimated cost of \$125,000 for treatment by other methods. The equipment used at Franklin Falls to jet the blasting charges in place is shown in Figs. 8 and 9.

Tests at Denison Dam, Texas

At the Denison Dam Site, on the Red River near Denison (see Fig. 10), the river bed is about 900 ft wide and consists of loose deposits of sand and silt, with some intermixed clay extending to depths of 30 ft. During low-water periods the river occupies only a small part of the river bed. Preliminary compaction tests in the South River area, consisting of single charges of dynamite, showed that charges of 16 lb at depths of 16 ft would yield the best results.

A test area 250 ft by 180 ft (North River test area, Fig. 10) was then blasted using seventy-three charges exploded in five individual blasts. An average surface settlement of 0.68 ft was obtained for the area. However, some of this settlement represents a migration of material toward the unconfined river bank.

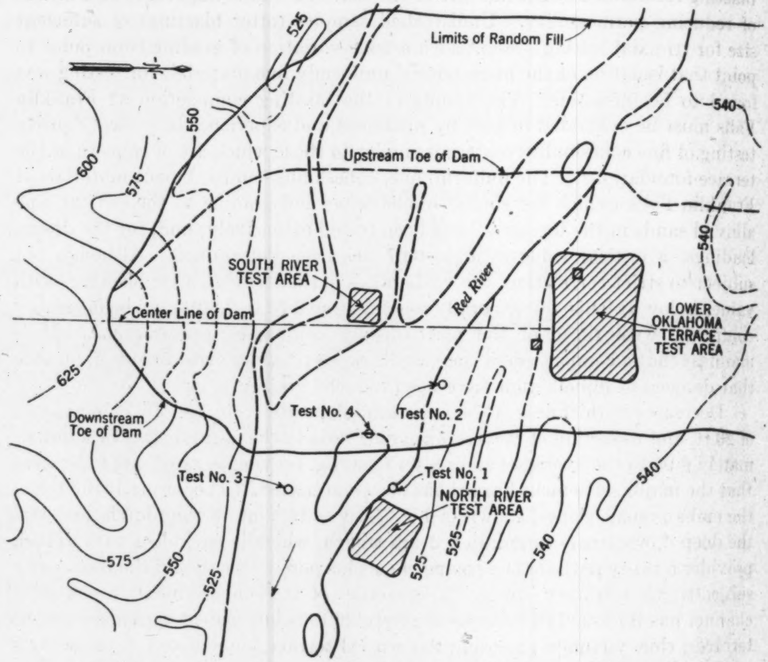

525
Limits of Random Fill
540
550
600
575
Upstream Toe of Dam
540
SOUTH RIVER
TEST AREA
Red River
Center Line of Dam
LOWER
OKLAHOMA
TERRACE
TEST AREA
625
Downstream
Toe of Dam
Test No. 4
Test No. 2
Test No. 3
Test No. 1
NORTH RIVER
TEST AREA
540
575
550
525
525
525
540

FIG. 10.—RIVER AND LOWER TERRACE AREAS, SITE OF DENISON DAM, IN TEXAS

Tests of undisturbed samples before and after blasting gave the voids-ratio decreases indicated in Fig. 11(*a*).

A second test area on the Oklahoma Terrace (left bank) of the dam where loose, cohesionless materials existed in the embankment foundation was then blasted. The ground-water table in this vicinity is approximately 22 ft below the surface, and pumping and ponding of water were necessary to produce the saturated condition requisite for consolidation blasting. An area 145 ft by 200 ft was enclosed in a small dike and pumping operations were continued with few interruptions for eighteen days with the result that the ground-water table was raised to within 10 ft of the surface; the material between the ground surface and the artificial water table was well moistened. Of the two blasts in the Oklahoma Terrace the first consisted of 312 lb of dynamite split into fifty-one charges placed in three tiers at depths of 5, 10, and 15 ft. The charges in each tier were placed in a pattern of squares 40 ft on a side. Each tier was oriented so that the charges were spaced midway between the charges of the other tiers in plan. The lower tier of sixteen charges of 12 lb of 60% dynamite was set off with instantaneous caps; the middle tier of fifteen charges of 6 lb of 40% dynamite was set off with first-delay caps; and the upper tier of twenty charges of 1.5 lb of 40% dynamite was set off with second-delay caps. The second blast consisted of 414 lb of dynamite divided into fifty-one charges placed in two tiers at depths of 10 and 15 ft.

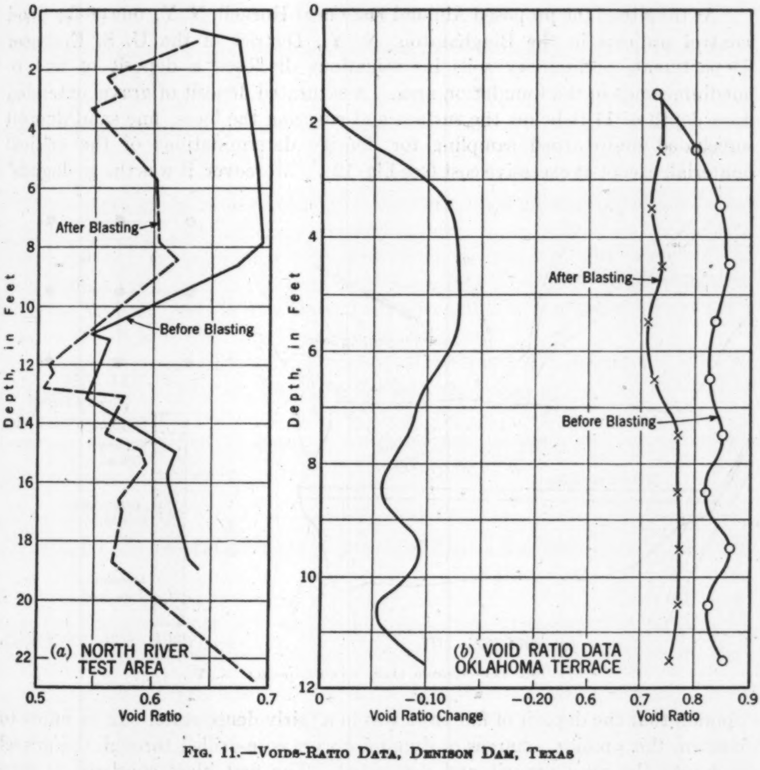


FIG. 11.—VOIDS-RATIO DATA, DENISON DAM, TEXAS

Average settlements of 0.27 ft for the first blast and 0.22 ft for the second blast, or a total of 0.49 ft, were observed. Two test pits, one before blasting and one after blasting, were excavated to a depth of 11.5 ft for obtaining laboratory samples for voids-ratio determinations. A voids-ratio decrease of approximately 0.09 resulted from the two blasts, the major compaction being between the depths of 2.5 ft and 7 ft in the zone where the material was well moistened rather than fully saturated (see Fig. 11(*b*)). The report[3] of the tests at Denison states:

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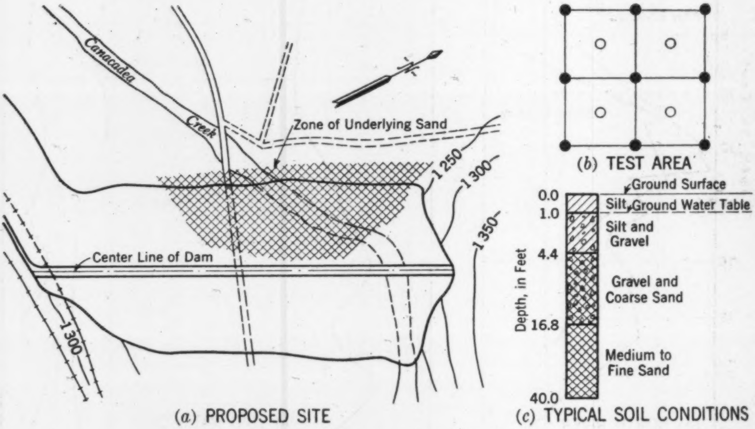
[3] "Inspection Report—Dynamite Compaction Tests, Denison Dam and Reservoir," U. S. Engr. Office Denison, Tex., dated December 7, 1939 (unpublished).

> "Compaction can be obtained by blasting. Results in the River Area show that the critical voids ratio of 0.6 can be very closely approached and that with an average surface settlement of less than one foot. On the Oklahoma Terrace experiments were not carried far enough to learn if the critical voids ratio could be attained, but it is evident that the voids ratio can be decreased considerably even without complete saturation."

Tests at Almond Dam, Hornell, N. Y.

At the site of the proposed Almond Dam near Hornell, N. Y., one of the flood control projects in the Binghamton, N. Y., District of the U. S. Engineer Department, preliminary soils investigations disclosed a deposit of fine to medium sands in the foundation area. A saturated deposit of gravel extended to a depth of 17 ft below the surface and covering the loose, fine sand deposit precluded undisturbed sampling for density determinations of the critical material, except at excessive cost (see Fig. 12). Moreover, it was the geologists' opinion that the deposit of fine sand was in a fairly dense state. In an effort to confirm this opinion, charges of dynamite were core-drilled through the gravel and into the sand deposit and detonated. The first blast consisted of nine charges of 50 lb each of 60% dynamite. These were placed in a pattern of squares 50 ft on a side, all charges being at a depth of 40 ft below the surface. The second blast consisted of four charges, also 50 lb each in size and at a depth of 40 ft below the surface. The lateral spacing of these four holes was intermediate to the initial nine holes. The third and fourth blasts consisted of reloading and firing the preceding nine-hole and four-hole blasts, respectively. However, the size of each charge was decreased to 20 lb of explosive and the depths were reduced to 30 ft below the surface.



(*b*) TEST AREA

(*a*) PROPOSED SITE

(*c*) TYPICAL SOIL CONDITIONS

Fig. 12.—Almond Dam, Near Hornell, N. Y.

As a result of the four blasts the ground surface of the test area settled an average of 0.1 ft. This was such a slight settlement that it was concluded that the fine sand deposit was initially in a fairly dense state of deposition and no compaction or other treatment would be necessary. It must be emphasized, however, that until more investigations have been conducted in connection with this new method, the failure of an area to settle upon blasting cannot be regarded as conclusive evidence that the material is below the critical voids ratio or that it is stable under all possible stress conditions.

SUMMARY

The method of compacting cohesionless foundation soils by means of explosives was developed and used in the embankment foundation at Franklin Falls Dam. Considerable economy was effected over any previously available methods of compaction, such as vibrators or pile driving, and also over the process of excavating the unsuitable material and recompacting with suitable materials.

The method, in brief, consists of the detonation of charges of explosives spaced laterally and vertically throughout the loose soil mass. Successive coverages of the same area give additional compaction, but the limits of compaction for a specific material by this method have not been investigated. Neither have the tests to date been extensive enough to show whether optimum results are obtained in fully or partly saturated soil.

It can be stated definitely, however, that the present indications are that the method is widely applicable to loose, cohesionless structural foundations provided the materials are substantially saturated prior to blasting. In addition to increasing the degree of compaction of the materials, the horizontal permeability of stratified deposits is greatly reduced.

ACKNOWLEDGMENT

The investigations and tests at Franklin Falls Dam were conducted under the supervision of the writer. Investigations at Denison Dam were conducted under the direction of Maj. Lucius D. Clay, M. Am. Soc. C. E., district engineer, Dension District of the U. S. Engineer Department. Investigations at Almond Dam were conducted under the direction of Lt.-Col. George J. Nold, M. Am. Soc. C. E., district engineer, Binghamton District of the U. S. Engineer Department.

DISCUSSION

Reuben M. Haines,[4] **Assoc. M. Am. Soc. C. E.**—The initial tests and first actual application of the method of compacting loose, cohesionless foundation soils by detonation of buried charges of explosives are described in this paper. Colonel Lyman is to be highly commended for conducting the investigations, and for his presentation of data, since this method of compaction may provide an economical and practical solution to produce a suitable foundation for many structures. Before the limitations of the method are known, additional information and data will need to be obtained by future field and laboratory investigations. Since the author has proved that the method is practicable for certain conditions, it is believed that additional information will become available in the near future. The costs of making preliminary investigations where the method might be used would appear to be inexpensive. Such investigations should be made to determine if settlement can be obtained for the particular soil deposit and to determine the spacing, order, and size of charges that would give the most compaction. It appears that the further development of the method is highly desirable. For certain soil conditions, it may be possible that the use of well points to remove excess water after detonation, or the loading of the area, will produce greater compaction.

This method of compaction can be used only for soil deposits in which a new and denser soil structure will be formed if the material is completely disturbed. The compaction that can be obtained is limited and depends upon the type of soil particles, type of soil structure, the initial density, the permeability, the drainage conditions, the stress conditions, and the degree of saturation. The detonation of the buried charges in a saturated loose deposit produces a liquid or partly liquid mass by destroying the soil structure. The particles of soil settle to form a new structure as the excess moisture seeps from the mass. The rate and quantity of seepage probably affect the degree of compaction obtained after the detonation of a charge. After the excess moisture has escaped and the new soil structure formed, the detonation of another charge produces additional compaction; however, the data of the tests at Denison Dam show that there will be less change in density than by the detonation of the previous charge. The number of coverages to produce optimum compaction can be determined only by field tests. It would be interesting if Colonel Lyman, in the closing discussion, would present any additional available data that may indicate the change in density by the detonation of each successive charge. The writer believes that the inertia and tamping actions produced at the time of detonation have little effect upon the compaction of saturated materials; however, if the soil deposit is only partly saturated, these actions may be primary causes of the compaction of the soil particles as rearranged by resettlement. The writer believes that only a minor degree of compaction can be obtained by this method unless there is sufficient moisture to produce partial liquefaction when the soil structure is shattered.

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The development of the method may lead to many practicable uses. During the construction of many earth dams, the current procedure is to dewater the channel areas and place rolled-fill materials. Investigations may develop this method so that cohesionless material may be placed and satisfactorily compacted in the channel areas of some projects without the construction of cofferdams and dewatering of the area. Until more is known about the stress-strain relations of cohesionless materials when subjected to disturbances, and until the critical void ratios for various stress conditions can be determined, it will not be known if this method can be used to produce a density below the critical void ratio for any imposed loads of appreciable magnitude. Any foundation, however, would have a satisfactory density to insure against liquefaction if this method is used after the stresses of the imposed load are produced in the deposit; but this procedure may be very expensive for large earth-dam projects. The method may be economical and practicable for many earth-dam projects, as in the case of the Franklin Falls Dam where only a moderate degree of compaction of the foundation deposits on the terraces was required due to foundation and design features.

R. G. HENNES,[5] ASSOC. M. AM. SOC. C. E.—Many engineers have emphasized the importance of making generally available the results of experience with foundations and earthwork if the principles of soil mechanics are to be developed and utilized to best advantage. As a significant contribution to this field, the present paper should be most welcome. The value of such contributions is not fully exploited except by discussion, which preferably should spring from a background of comparable experience in the field. Description of new construction methods is unavoidably handicapped by the absence of parallel experience upon which adequate discussion could be based. It is to be hoped, therefore, that the author's closure of the present paper will be made the vehicle for presenting further information on this new method of compaction for the use of the profession.

Especially appreciated would be any additional test data that may be available which would assist the engineer in applying this method to other projects without unnecessary duplication of effort. In any such application the main thing to be determined is the size and distribution of charges. The variables upon which the layout depends are stated to be the type of soil, its degree of saturation, and the depth of the deposit. Fortunately, the problem is somewhat limited by the practical consideration that only in loose sandy deposits does the critical void ratio tend to be a controlling factor. Theory would seem to approve the author's conclusion that best results are to be expected when the soil is saturated; for the very danger associated with the concept of a critical void ratio is the "liquefaction" produced by the excess of water in the voids. The Denison tests suggest that the effect of saturation is less simple, and point to the need for further research in this field. The effect of depth is something that can be disclosed only by experiment, and it is here that the paper comes closest to permitting at least tentative general conclusions. It would be helpful if the author's closure considered this matter at greater length. Both at Franklin Falls and at Denison best results appear to have been obtained by placing charges at a depth of 15 ft, but in neither case is it clear how deep the effect was felt, and it would be of interest to learn what evidence led to the conclusion "that where loose strata of sand greater than 30 ft thick are to be compacted, two or more tiers of small charges are to be preferred to one tier of large charges" (see "Introduction").

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[5] Associate Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

It is not the thought of the writer that any amount of discussion can obviate the need for preliminary tests on each individual project, but rather that each new investigation should be shaped by the fullest comprehension of what has already been accomplished.

John D. Watson,[6] **Assoc. M. Am. Soc. C. E.**—The results presented in this paper show positively that the method is economical and that loose cohesionless soils can be made much denser, and also considerably less permeable. Colonel Lyman is to be commended most heartily for originating and developing a method of foundation improvement that will be of great value to foundation engineers throughout the world.

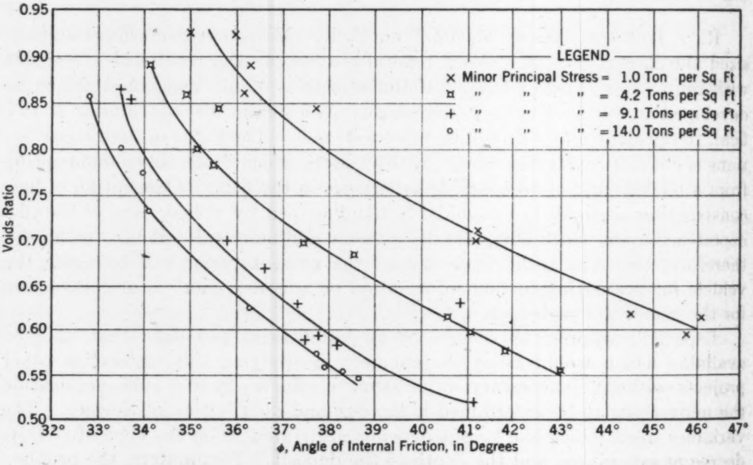

Fig. 13.—Empirical Relationship Between Voids Ratio and Angle of Internal Friction for Franklin Falls Sand

Although the author states (see "Introduction") "that the stability of the embankment would be considerably increased if these loose sand deposits could be consolidated to reduce the possibility of their failure by liquefaction * * *," he did not take the opportunity to point out the concomitant advantage of the greater resistance to rupture that these soils would have by reason of the higher density. In 1938, when the writer was serving as research fellow in soil mechanics for the Graduate School of Engineering at Harvard University, in Cambridge, Mass., he made many tri-axial compression tests on samples of a fine cohesionless sand from the Franklin Falls Dam site.

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[6] Asst. Prof., Civ. Eng., College of Eng., Duke Univ., Durham, N. C.

The samples for these tests were prepared with a wide range of voids ratio (0.925 to 0.52, comparable to a unit dry weight of 88 lb per cu ft to 112 lb per cu ft), and the tests were made with a surrounding hydrostatic pressure ranging from 1.0 to 14.0 tons per sq ft. The results of these tests are shown in Fig. 13. For the same minor principal stress (hydrostatic pressure), the angle of internal friction is inversely proportional to the voids ratio. However, the amount of the variation changed with the intensity of the minor principal stress. When the minor principal stress was low, the increase in the angle of internal friction with a decrease in the voids ratio was greater than when the minor principal stress was high. The following explanation for this fact has been offered tentatively: When the minor principal stress is low, rupture occurs by one grain of sand sliding over another, and the interlocking effect of the grains is considerable. When the minor principal stress (and the major principal stress as well) is high, then many grains in the plane of failure break apart and the interlocking effect is less pronounced. Similar variations in the angle of internal friction with changes in density and minor principal stress were found for other sands.

Karl Terzaghi, M. Am. Soc. C. E., has pointed out many times that the strength of a cohesionless soil foundation does not depend upon the first, but rather upon the fourth, power of tan $\left(45 + \frac{\phi}{2}\right)$, in which ϕ is the angle of internal friction. Hence, when the angle of internal friction of any sand is raised by compaction from 34° to 38°, the resistance to rupture of this soil beneath a foundation load increases not just 9%, but rather it increases 42%; and, if the angle of internal friction could be raised from 34° to 46°, the strength of the material against rupture by a foundation load would increase 200%. Although the possibility of loading a foundation of cohesionless soil to the point of rupture by means of an embankment is admittedly remote, nevertheless the large increase in the strength of such soils through compaction is quite pertinent for other types of foundation loading and for the stability of embankments as well.

PAUL BAUMANN,[7] M. AM. SOC. C. E.—Presented with military brevity and precision, this paper is very timely indeed and should prove to be of distinct significance in view of many floating dams that have been built or are proposed to be built in connection with the national flood control program.

The consolidation of granular foundation materials by means of shock or vibration, as a result of blasting, stands in contradistinction to the chemical methods of solidification of similar materials. In principle, the first method consists of rocking the particles into closer contact, whereas in the second case the particles remain undisturbed and the voids between them are filled partly or completely by a suitable combination of liquid chemicals. Both methods have at least one feature in common—namely, the difficulty of access and inspection as well as sampling to ascertain the results of the treatment. Hence, the control of the stabilization of foundations of this character is definitely at variance with the control of the stabilization by means of compaction of embankments above the ground surface. The latter are ordinarily built in relatively thin layers, offering ready access to testing at all times.

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[7] Senior Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

The blasting technique, as described by the author, resembles certain methods of demolishing mine tunnels or underground communications of various types well known in military engineering, particularly in connection with field fortifications. One of the many tasks of the sapper is the demolition of such enemy underground passages without appreciable disturbance of the ground surface—that is, especially without the formation of a crater or a "mole hill." This is accomplished by the so-called squash mines that have a striking resemblance to the type of mines described in the paper.

Contrary to the normal crater charge with a sphere of influence touching the ground surface as shown in Fig. 14 by the solid circle, the sphere of influence of the squash mine is much smaller, possibly as indicated by the dotted circle. Expressed in kilograms and meters for the weight W of the dynamite charge and the radius R of the sphere of influence, respectively, the simple relation

$$W = c \times R^3 \qquad (1)$$

exists, in which c is a coefficient that approaches unity for a 60% dynamite placed in fully tamped blast holes in medium-fine sand.

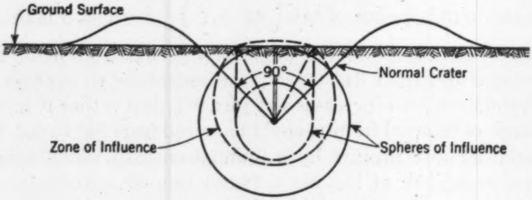

FIG. 14

According to the author's information, best results were obtained, at least in one location, if a charge of 8 lb of dynamite was used 15 ft below the ground surface. For the normal crater charge that would result in a 90° cone, more or less, the formula would call for approximately 200 lb of dynamite. Hence, the charge actually used was one twenty-fifth of the normal crater charge. Ordinary squash mines would probably range between one tenth and one fifteenth of the normal crater charge, depending on the character of works subject to demolition. Therefore, they would receive roughly double the charge as used for best compaction. This means that test pits within the blast area, if adequately timbered, would not be subject to destruction from compaction blasts unless they were located within the zone of influence—that is, in this case, within about 5 ft of the charge. Hence, there appears to be no reason why, with a relatively small additional expense, a more rigid control of the process could not be attained.

It is to be expected that compaction by blasting would result in somewhat erratic degrees of compaction. This is particularly notable in the data shown for the Denison Dam where the foundation material within the range of blasting was not saturated. A saturated condition is undoubtedly very desirable for more uniform propagation of the pressure waves, as the author clearly states. In studying Fig. 11, and particularly the part applying to the North River test area, it is significant that, at about 13 ft of depth, the density of the material was greater before blasting and that below some 18 ft of depth the trend of the curve showing the conditions after blasting is in the direction of diminishing density with increasing depth. In this connection, the author's emphasis of the need for further investigation before definite conclusions can be drawn from results so far obtained is believed to be well supported.

The writer had occasion to make studies and to direct tests on the solidification or petrifaction of granular material by means of chemical methods. Although this was a number of years ago and prices of chemicals may have changed considerably in the meantime, it is the writer's conviction that any one of the feasible chemical treatments to accomplish what apparently was accomplished by means of blasting would have been far more expensive than the latter method.

A. K. B. LYMAN,[8] M. AM. SOC. C. E.—The tests for developing the method of compaction by detonating buried charges of explosives were unique and, therefore, no additional data have been presented in the discussions. They have been mainly limited to descriptions of the compaction phenomenon, the limitations and possible applications of the method, and a request for additional information. The purpose of the tests was to prove that, for certain conditions, foundation soils could be compacted economically by detonating buried charges of explosives. As Mr. Haines states, the limitations of the method will not be known until additional information and data are obtained by future field and laboratory tests. The writer believes that the tests conducted to date must be considered as preliminary and, in the future, the procedures for various conditions may be radically different from the one used at the Franklin Falls Dam site. Mr. Haines suggests that "For certain soil conditions, it may be possible that the use of well points to remove excess water after detonation, or the loading of the area, will produce greater compaction." A preliminary test was made at the Franklin Falls Dam site, using well points. The results of the test indicated that no appreciable increase in compaction could be obtained by using well points at this particular site. However, the writer fully agrees that various ideas should be used in tests for various conditions. It is quite probable that greater compaction could be obtained by using well points, particularly if compaction is desired at a greater depth, or the soil is less permeable, than at the Franklin Falls Dam site.

Mr. Haines believes that data which indicated the settlement due to each blast may be of interest. Such data were obtained during the compaction in the test areas at the Franklin Falls site, but the writer regrets that they are not available for publication. In general, at the Franklin Falls site, approximately two thirds of the settlement was obtained during the first two blasts.

The writer is unable to present, as suggested by Professor Hennes, any additional data that would assist the engineer in applying the compaction method to other projects without making complete tests. Professor Hennes has questioned the writer's thought "that where loose strata of sand greater than 30 ft thick are to be compacted, two or more tiers of small charges are to be preferred to one tier of large charges." Professor Hennes has failed to note that this is not a definite conclusion made by the writer. Based on a study of the phenomenon that occurs and the results of the tests, the writer believes that more uniform and satisfactory results can be obtained with dispersed small charges of explosives in place of one large charge. However, future investigations may show that the present belief of the writer is not valid.

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[8] Col., Corps of Engrs., U. S. Army, 3d Engrs., Schofield Barracks, Hawaii.

Professor Watson has discussed the effect of the increase of density upon the angle of internal friction of fine cohesionless sand. Although the initial tests were made with the view in mind to reduce the density to below the critical voids ratio, the compaction method is equally applicable for increasing the density of foundation soils for the purpose of increasing the angle of internal friction, which is the critical factor in design for some structures.

Mr. Baumann emphasizes that the control of compaction by the method of detonating buried explosives cannot be as rigid as the method used to compact embankments. The writer believes that as careful a control as possible should be applied when a new method is being developed for the purpose of collecting data. However, after the method is developed beyond the research stage, the amount of control required depends upon the aspects of design and the effect that variations have upon the stability of the structure. In the case of the Franklin Falls Dam foundation, it is believed that ample investigations were conducted prior to and after compaction to insure that the method produced satisfactory results.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2161

OPERATION EXPERIENCES, TYGART RESERVOIR

BY ROBERT M. MORRIS,[1] ESQ., AND THOMAS L. REILLY,[2] ESQ.

WITH DISCUSSION BY MESSRS. NICHOLLS W. BOWDEN, AND EDGAR E. FOSTER.

SYNOPSIS

Some of the problems involved in the operation of a large flood-control and water-supply project are reported in this paper. The writers demonstrate how effectively the Tygart Dam can perform the functions for which it was planned. For example, the effect of the reservoir in reducing flood crests at downstream points is demonstrated. The accuracy of these estimates is dependent on the method of flood routing and the thoroughness with which it is consummated. The elements involved in routing these floods are described only briefly because a complex problem of this type would require a separate paper for complete presentation.

INTRODUCTION

At the junction of the Monongahela and Allegheny rivers, in Pennsylvania, lies Pittsburgh, the steel city, favored by nature with bounteous coal supplies and three broad rivers for its transportation. Nature can be exacting for its favors, however. Records indicate that almost annually since the eighteenth century flood waters rolling down from the western slopes of the Appalachian Mountains have inundated some parts of the expanding city. At the time of the 1907 flood, when Pittsburgh had grown to be the steel center of the world, the valleys of the Allegheny and Monongahela rivers were lined with industrial developments. The low-lying "Triangle" at the junction of the two rivers was the crowded business center of this great industrial district. The flood of 1907, reaching then unprecedented heights, took a terrible toll in lives and property damage and awakened the citizenry to the realization that, since their city had been built practically in the flood plain of the rivers, some means must be found to check the annual flood menace.

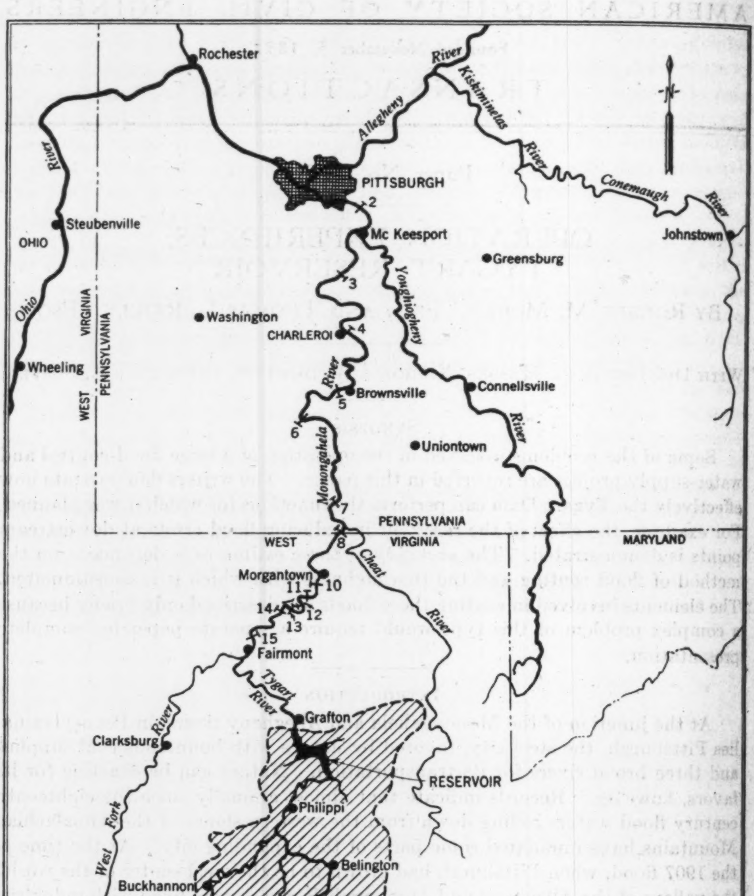
The Pittsburgh Flood Commission, formed after the 1907 flood, made a detailed study of methods of flood control and published a complete report.[3]

NOTE.—Published in April, 1941, *Proceedings*.

[1] Assoc. Engr., U. S. Engr. Dept., Pittsburgh, Pa.

[2] Asst. Engr., U. S. Engr. Dept., Pittsburgh, Pa.

[3] Report of Flood Commission of Pittsburgh, Penna., April 16, 1912.


Rochester
River
Allegheny
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River
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PITTSBURGH
2
Conemaugh
River
Johnstown
Steubenville
OHIO
Mc Keesport
Greensburg
3
Youghiogheny
Ohio
Washington
CHARLEROI
4
VIRGINIA
PENNSYLVANIA
WEST
Wheeling
River
Brownsville
5
Connellsville
6
River
Uniontown
Monongahela
7
PENNSYLVANIA
WEST
VIRGINIA
MARYLAND
8
Cheat
Morgantown
11
10
14
12
13
River
15
Fairmont
Tygart
River
Grafton
Clarksburg
River
RESERVOIR
Philippi
Fork
West
Belington
Buckhannon
Elkins
LEGEND
Lock and Dam
DRAINAGE AREA
River
Tygart
0
10
20
30
40
50
Scale in Miles

Fig. 1.—Location Map, Tygart Dam

Later surveys and studies were made by the United States Army Engineers. A complete plan for a ten-reservoir system for flood control in the Upper Ohio Basin was developed by the Army Engineers; and finally, in 1934, funds were appropriated by the federal government for the first unit in this system, a dam on the Tygart River, one of the main tributaries to the Monongahela. The latter river is one of the world's greatest industrial waterways, linking the bituminous coal fields of Pennsylvania and West Virginia with the steel mills and other industries of the Pittsburgh District. Since navigation on this stream was hampered periodically by low water in the dry summer and autumn months, this dam on the Tygart River was designed for the dual purpose of flood control and water supply.

The Tygart Dam and Reservoir, constructed and operated by the U. S. Army Engineers, have been in service since January, 1938. Following the great record-breaking flood of March, 1936, which caused an estimated monetary loss of $178,674,000 and heavy loss of life in the Ohio River basin above Wheeling, W. Va., money was appropriated by Congress for the extension of the reservoir system and four other units are now (1941) under construction. This paper purports to discuss some of the problems involved in the operation of a large flood-control and water-supply project and to demonstrate how effectively the Tygart Dam can perform the functions for which it was planned.

The dam is on the Tygart River in Taylor County, West Virginia. It lies just two miles by stream above the City of Grafton and 27 miles above Fairmont, W. Va., where the Tygart and the West Fork rivers join to form the Monongahela River. This latter river is canalized over its entire length of 128 miles, and from its mouth at Pittsburgh (where with the Allegheny River

FIG. 2.—VIEW OF TYGART DAM

it unites to form the Ohio River) to its source at Fairmont there are thirteen navigation locks and dams. The tributary drainage area of the Tygart Dam is 1,183 sq miles. This is 16.1% of the total Monongahela Basin (drainage area, 7,384 sq miles) and 6.2% of the combined drainage area of 19,117 sq miles of the Monongahela and Allegheny rivers at Pittsburgh.

This paper deals primarily with the operation of the reservoir rather than design or construction of the dam and only a brief description of the structure is presented herein for the purpose of clearness in explaining methods of operation.

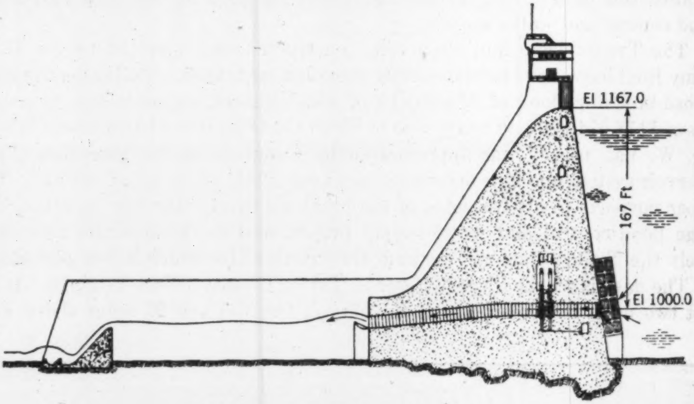

FIG. 3.—SPILLWAY SECTION, TYGART DAM

The dam is of the concrete gravity type, 1,880 ft long, and rises about 230 ft above bedrock. The spillway section, at about the middle of the dam, is 489 ft long and is flanked by bulkhead sections 23 ft higher than the spillway crest. The outlet works consist of eight conduits 5 ft 8 in. by 10 ft, each controlled by two, vertical, hydraulically operated, steel slide gates and two, 48-in., internal, differential, needle valves for low-water regulations. A cushion pool is provided below the main dam for the dissipation of the kinetic energy of discharge through the outlet works and over the spillway. Fig. 1 shows the location of the dam; a view of the completed structure is shown in Fig. 2; and a cross-sectional drawing is given in Fig. 3. The cost of the completed project, including railroad relocation, was about $18,000,000.

GENERAL FEATURES OF OPERATION

Reservoir Capacity.—The size of a flood-control reservoir is governed both by the economic feasibility and the hydrologic potentialities of the project. The dam should be built to a height necessary to produce a reservoir of sufficient capacity to provide a reasonable degree of control over floods similar to those of past record and over theoretical floods that may possibly occur in the future. Where storage for low-water control must also be considered, as in the case of the Tygart Reservoir, this feature becomes doubly important. The height of the dam and attendant reservoir capacity are also limited by the point at which the cost of the completed project, converted to annual charges, approaches the monetary benefits that will result from its operation. Exhaustive hydrologic and economic studies made by the Army Engineers, prior to the construction of the dam, limited the capacity of the reservoir to only 290,000 acre-ft at the spillway crest, which is equivalent to 4.5 in. of runoff from the tributary drainage area of 1,183 sq miles. Between the spillway crest and the top of the bulkhead sections an additional quantity of 88,000 acre-ft as surcharge storage is available. Fig. 4 shows the capacity of the reservoir in terms of elevation.

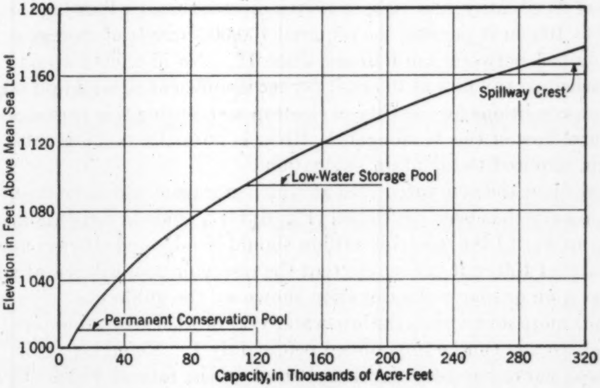

FIG. 4.—CURVE OF RESERVOIR CAPACITY

Seasonal Storage.—A schedule of dual-purpose operation is required for a flood-control reservoir of limited capacity if it must augment the seasonal low-water flow in the streams below the dam. The rate and quantity of low-water storage must be coordinated with flood-control operation to provide satisfactory results for both purposes. In allocating reservoir storage capacity to low-water regulation and flood control, during the months of low flow, a study was made to determine the water required to maintain satisfactory navigation conditions on the Monongahela River, under the most adverse conditions that might reasonably be expected. Using the historic 1930 drought as a criterion, it was determined that sometime it might be necessary to maintain a constant discharge of 340 cu ft per sec from the Tygart Reservoir for the 5½-month period from July 1 to December 15. Translated into volume, this flow would be 675 acre-ft per day, or 113,400 acre-ft for the 168-day period. The record of the Tygart River during the 1930 drought reveals that, although on occasion the discharge was as low as 1 cu ft per sec, the total volume of discharge from July 1 to December 15 was approximately 13,000 acre-ft. Since the difference between the total volume of required discharge and the minimum expected inflow determines the volume of storage that must be in the reservoir prior to the low-water period the use of 1930 as a criterion would necessitate 100,000 acre-ft of storage by July 1.

In planning the operation of the reservoir, it was necessary to determine when, and at what rate, storage should be impounded in order to have the required storage by this time. It is obviously desirable to reserve the entire reservoir capacity for flood storage during the periods when floods are most prevalent. A study of Fig. 5 shows the distribution by months of floods at Pittsburgh, and indicates that flood frequencies increase from November to March and decrease from April through October. Therefore, maximum storage capacity is maintained in the period from November to March and, since the probabilities of damaging floods begin to decrease in April, at this time a gradual increase in storage may be started. An analysis of stream discharge during the months of April, May, and June in 1930 indicates that, by limiting the reservoir outflow to 100 cu ft per sec, the required 100,000 acre-ft of storage could have been obtained between April 10 and June 15. No ill effects would be caused by this schedule as a flow of 100 cu ft per sec is sufficient to maintain satisfactory sanitation conditions for the City of Grafton and, during this period of the year, the natural flow of the Monongahela River is normally much greater than the minimum required to maintain navigation.

Based upon the computed rate at which storage could have been obtained in 1930, a curve has been developed (Fig. 6) for a guide in impounding storage. Starting on April 1 the reservoir outflow should be adjusted (always maintaining a minimum of 100 cu ft per sec) so that the reservoir pool will rise at a rate that will keep it on or above the elevation shown on the guide line. Thus, even if conditions more severe than the low-water year of 1930 are encountered, 100,000 acre-ft of storage can be impounded before July 1. Referring again to Fig. 6, a depletion curve has been developed showing the rate at which the reservoir pool would have receded in 1930, under a constant outflow of 340 cu ft per sec, assuming the total inflow for the period to have been equally distributed. If the rate of drawdown throughout the low-water season keeps the pool elevation on or above the guide-line requirement, it is reasonably certain that the guaranteed minimum discharge of 340 cu ft per sec can be supplied continuously from July 1 to December 15.

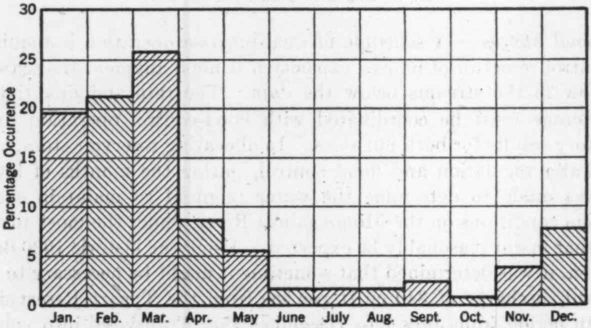

FIG. 5.—MONTHLY DISTRIBUTION OF PAST FLOODS AT PITTSBURGH, PA.

General Features of Flood-Control Operation.—As previously mentioned, the gross capacity of the Tygart Reservoir is equivalent to 4.5 in. of runoff from the tributary drainage area. However, when the low-water storage of 100,000 acre-ft is in the reservoir, the remaining flood-control capacity is equivalent to only 2.9 in. of runoff. Obviously a reservoir of this capacity cannot completely contain the entire runoff from a great storm of the type that occurs frequently over similar drainage basins in Eastern United States. This limited storage capacity, therefore, presents a problem of operation since the outlet works cannot be closed arbitrarily at the beginning of a flood and opened when the flood is past. A proper balance must be maintained between outflow and inflow so that the maximum benefits will be obtained from the available storage capacity. Flood storage in the reservoir should not begin until it will aid in preventing or reducing damage at some downstream point. As the flood passes and forecasts of stages indicate that an increased outflow from the reservoir will not arrive at the critical downstream points until after the rivers at those points have receded below damage stage, the rate of storage in the reservoir may be reduced by a gradual increase in reservoir outflow.

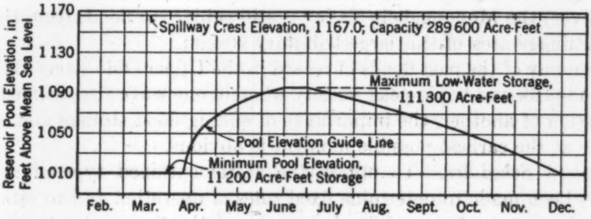

FIG. 6.—ANNUAL STORAGE SCHEDULE

The true functions of a reservoir of this type during a general flood are to delay the rise of its tributary flow only until the streams to which it is affluent are falling below damage stage, and to keep the magnitude of their ultimate rise below this critical point. This principle is shown diagrammatically in Fig. 7, which illustrates the necessity to store, not the entire flood flow, but only that part represented by the single cross-hatched area of the hydrograph. The double cross-hatched area represents the discharge of storage; the two areas,

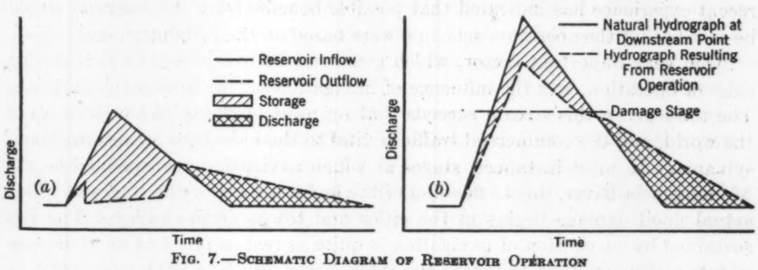

FIG. 7.—SCHEMATIC DIAGRAM OF RESERVOIR OPERATION

representing reservoir storage and draft, would, on an actual flood, be equal. The ideal result obtainable from this type of operation is shown diagrammatically in Fig. 7(*b*), which represents the desired effect at some distant downstream flood-damage point over which complete control cannot be procured. This condition obtains on the Monongahela River below Lock 7. Above this point the Tygart Reservoir can exercise almost continuous control over Monongahela River stages but beyond Lock 7 (which is immediately below the mouth of the Cheat River) the Monongahela Valley is still subject to floods from the uncontrolled drainage area of this large tributary stream.

Since many of the past floods of record in the Upper Ohio River Basin are of the double-peak and triple-peak variety wherein one wave rises immediately on the recession of another, the importance of ending flood storage and beginning discharge at the earliest possible moment is obvious.

Operation Schedules.—To obtain the aforementioned type of results, an effort has been made to determine fixed rules of operation and to establish criteria for the time and rate of flood storage and the time and rate of discharge of storage. The first attempt at establishing a fixed operation schedule was based upon stages at Pittsburgh, since it is in this district that the great monetary flood losses occur. From a study of past floods, a certain stage combined with a definite anticipated rate of rise on the Pittsburgh "Point" gage located at the confluence of the Monongahela and Allegheny rivers was set as the starting limit for storage in the Tygart Reservoir, and likewise a certain stage and rate of fall at Pittsburgh indicated the time for release of storage. Because of the complexities of the drainage basin above Pittsburgh it was soon evident that an operation schedule based entirely on stage conditions at this point would be subject to wide variation in results. The floods that occur at Pittsburgh are not always caused by simultaneous flood flows of the Allegheny and Monongahela rivers as either stream is sufficiently large, individually, to cause or contribute principally to flood stages in the Upper Ohio River. Therefore, although it is possible to set criteria at Pittsburgh that would have given excellent results from the Tygart Reservoir on some of the past great floods of record, recent experience has indicated that possible benefits from the reservoir would be sacrificed if the operation schedule were based on the Pittsburgh stage alone.

One very important factor, which could not be overlooked in establishing rules of operation, was the influence of navigation on the Monongahela River. The tonnage on this stream exceeds that on most of the inland waterways of the world, and the commercial traffic is vital to the industries of western Pennsylvania. In most instances, stages at which navigation is suspended on the Monongahela River, due to flooding of the locks, are below the stages at which actual flood damage begins in the cities and towns of the valley. The loss sustained by suspension of navigation is quite as real as physical flood damage in urban districts, and therefore the river stages affecting navigation must be taken into consideration in the regulation of the reservoir.

Because of its comparatively short dam section, Lock 5 at Brownsville, Pa., is usually the first lock on the Monongahela River to be inundated by a rising river and the last lock to return to normal service. This point, therefore, forms a control index for the entire river. If the Tygart Reservoir is operated to delay the inundation of this lock as long as possible or prevent it altogether, and if, during reservoir discharge, the outflow is regulated to keep the stage at Lock 5 below the top of the lock walls, then the maximum benefits should normally be obtained throughout the length of the Monongahela River Valley. In the current operation studies Lock 5 has been used to a large extent as the point on which operating criteria must be based, and in a later paragraph the results obtained at Lock 5 on several actual floods will be demonstrated. The experience gained thus far strongly indicates that predetermined rules will serve only as a partial guide for successful reservoir operation and maximum efficiency can only be accomplished from judicious use of accurate forecasted stages at the critical points. The following example will tend to illustrate this point.

The average flood-wave translation time from Tygart Dam to Lock 5 is approximately 14 hr. When the rivers are rising the Tygart Reservoir, to be most effective, must begin storage at least 14 hr before the predicted time for Lock 5 to be overtopped by an uncontrolled flood wave. Storage must continue at the maximum rate in the reservoir until it is predicted that 14 hr later the stage at Lock 5 will have receded below the top of the walls. The rate at which the reservoir outflow can be increased must be equal to, or somewhat less than, the predicted rate of recession at Lock 5. If all these conditions are forecast accurately the reservoir can be operated so that the loss of lock service will be reduced to a minimum, and the maximum reduction of the flood crest will also be obtained at Pittsburgh and other critical points throughout the length of the Monongahela River.

One possible objection to the use of Lock 5 for operation control is that positive storage in the reservoir may be used for the benefit of the Monongahela River when the river at Pittsburgh is below flood stage. If a great general flood should occur in the Tygart Basin immediately following this period, sufficient in magnitude to overtop the spillway of the dam, the efficiency of the reservoir might be reduced, as part of the storage capacity would be occupied at the beginning of the flood. This possibility can be greatly lessened by accurate meteorologic forecasts and intelligent use of hydrologic data.

Flood Prediction.—In the preceding section the importance of predicted river stages below a flood-control dam has been emphasized. It is evident, of course, that the forecast of inflow into the reservoir itself is of great importance, particularly where the storage capacity is limited and where there is a possibility that the reservoir will be completely filled by a large flood or succession of smaller floods. In this section some of the factors affecting flood prediction will be discussed briefly, and the sources of river and rainfall data will be enumerated.

The first factor to be considered in making river-stage forecasts is the condition of the streams at the beginning of the storm period. This is very important as a flood of some definite magnitude may be comparatively small if isolated. If it has occurred closely after another flood, however, which has utilized all the natural storage in the stream valleys, it may result in extremely harmful stages. The floods of January, 1937, in the Ohio Valley were of this type.

Another factor affecting flood flows is the percentage of runoff to be expected from a given rain. This is largely a function of the absorption capacity of the soil and condition of vegetation. A constant check on the rainfall-runoff relation must be maintained as it varies through the year. When they are unaffected by surface runoff, river stages represent outflow of ground water, or subsurface flow, giving an indication of soil conditions and the rate of infiltration to be expected with rainfall. Studies are being conducted along these lines to correlate these factors more accurately for use in determining runoff.

The most important factors in flood prediction, of course, are the rate and distribution of rainfall, combined when necessary with the water content of fallen snow. A network of rain gages and river-gaging stations is maintained over the Upper Ohio Drainage Basin by the U. S. Weather Bureau, U. S. Army Engineers, and U. S. Geological Survey. Daily reports from these stations are telegraphed to the Pittsburgh Office of the Weather Bureau at 7:00 a.m. and during periods of heavy rainfall at 6-hr intervals. The Weather Bureau, in conjunction with the Federal-State Forecasting Service, makes river-stage predictions for principal points in the Pittsburgh District of the U. S. Engineer Department.

In the tributary area of the Tygart Reservoir a network of ten rain gages and six river gages is maintained. From the reports of these stations, forecasts are made of reservoir inflow using runoff distribution percentages developed from a unit hydrograph for this basin. Fairly good results have been obtained and investigations are being made to define a small drainage basin above the dam to be used as an index of the runoff to be expected from the entire basin.

Accurate prediction of rainfall over an area before the rain has actually fallen represents the ultimate hope of those engaged in flood-protection work. Rapid strides in the science of air-mass analysis indicate that such predictions from data obtained by the Weather Bureau are a distinct possibility.

Flood Routing.—In simple terms, flood routing is an analytical redevelopment and transition of a flood wave—its inception from surface runoff in the headwater areas of one basin, its subsequent movement downstream augmented by inflow from successive tributary streams and diminished by the flattening of the wave as it progresses downstream, and the effect of natural valley storage. If the natural inflow from one of the tributary streams is withheld by a flood-control dam, the net effect at some downstream point will be a function of the valley storage between the dam and the point at which the reduction is computed. This amount of valley storage will itself be a function of the rate of discharge from the uncontrolled parts of the drainage basin.

The Army Engineers of the Pittsburgh District have been conducting extensive investigations into various analytical methods of flood routing and have adopted the so-called "coefficient method," first used in the Muskingum Watershed Conservancy Project studies, as the mathematical type most applicable to their region. In this method of routing, the stream is divided into routing reaches. The ends of these reaches are marked by stream-flow stations, so selected that the time of flow translation between the reach limits is as nearly equal as possible and corresponds somewhat to the interval of time used in routing computations. The flood wave is then routed analytically through each reach. The inflow at the head of the reach is added to the computed tributary inflow throughout the reach; and the inflows are modified for time, and by valley and wave slope storage. The outflow at the reach termination is thus determined.[4] The outflow so determined at the end of the reach is then checked against the actual flood hydrograph. A representative group of floods in the Upper Ohio Basin have been routed by this method and very close agreements between the reproduced flood hydrographs and the actual flood hydrographs have been obtained. The flood routing coefficients determined by this method of routing have been used in computing the reductions effected by the Tygart Reservoir at Lock 5 and at Pittsburgh.

Results from Flood-Control Operation

Floods Encountered.—It is a curious fact that, since the beginning of construction of the Tygart Dam, four floods have occurred that exceeded in crest magnitude all but one of the previous floods of a thirty-year period of record at this site. In descending order of peak discharge the first five floods of record on the Tygart River are those of July, 1912, October, 1937, April, 1939, February, 1939, and March, 1936. Since the dam was not fully completed until January, 1938, it was not possible to control all of the four recent large floods. Construction was not far enough advanced to have any appreciable effect on the flood of March, 1936; some control was exercised over the flood of October, 1937; and, during the floods of February and April, 1939, the dam was in full operation. One other flood occurred in December, 1937, which, although of minor proportions on the Tygart River, occurred coincidentally with flood stage at Pittsburgh.

October, 1937.—This flood was the second greatest of record on the Tygart River, reaching a peak reservoir inflow of about 52,000 cu ft per sec. The installation of the slide-gate machinery in the dam was not yet completed and one of the slide-gate conduits was closed by a bulkhead at this time. The upper reaches of the reservoir had not been completely cleared and the available storage capacity, therefore, was limited. During this flood the dam was operated as a detention reservoir. All seven available slide gates and both needle valves were open from the beginning of the rise until the reservoir was again empty. The inflow into the reservoir so far exceeded the capacity of the outlet works at a low storage head that some 104,000 acre-ft of storage were impounded in the reservoir. The rainfall causing this flood on the Tygart River was general over the entire Monongahela Basin with the result that navigation locks on the Monongahela River were flooded out and flood stage at Pittsburgh was exceeded. Fig. 8(*a*) shows the hydrographs of the Tygart Reservoir inflow and outflow, and Figs. 8(*b*) and 8(*c*) demonstrate the effect of the reservoir on the flood hydrographs at Lock 5 and at Pittsburgh.

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[4] "Muskingum River, Ohio (308), Vol. IV," report of U. S. Engr. Office, Zanesville, Ohio, December 1, 1934.

December, 1937.—This flood was the first on which the Tygart Dam could actually be operated. The flood on the Tygart River itself was not particularly severe, the peak reservoir inflow being 22,000 cu ft per sec, but the flood stage at Pittsburgh was exceeded and the reservoir effected some reductions at this point. Fig. 9(*a*) shows the reservoir inflow and outflow, and Figs. 9(*b*) and 9(*c*) show the effect of the reservoir on the hydrographs at Lock 5 and at Pittsburgh.

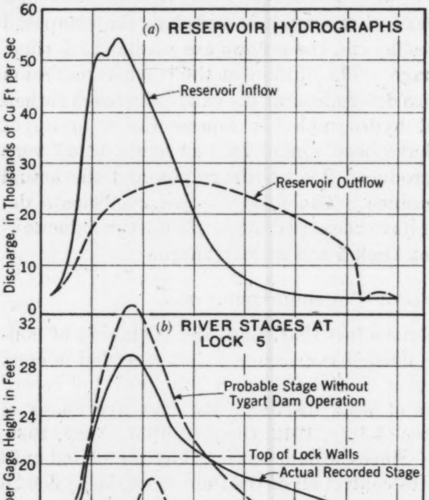

FIG. 8.—FLOOD OF OCTOBER, 1937

February, 1939.—This was a double-peak flood, the crests occurring on January 31 and February 4. The peak reservoir inflows were 28,000 cu ft per sec and 43,500 cu ft per sec, respectively, the latter being the fourth greatest of record in the Tygart River. Precipitation causing these floods was general over the Monongahela Basin and this river would have experienced a flood of major proportions had it not been for the Tygart Reservoir. Coming almost entirely from the Monongahela River and reaching flood stage at Pittsburgh, this flood furnished an example of the effectiveness of the Tygart Reservoir. The reservoir inflow and outflow are shown in Fig. 10(*a*), and the reductions of the flood hydrographs at Lock 5 and at Pittsburgh are shown in Figs. 10(*b*) and 10(*c*).

A double-peak flood of the type occurring on January 31 and February 4 furnishes a severe test of operation technique. Some of the storage impounded during the first peak was discharged as rapidly as possible when meteorological forecasts indicated the approach of additional rain. Referring to Fig. 10(*b*), it can be seen that the rate of discharge was so calculated that the Monongahela River was held stationary about 2 ft below the lock walls at Lock 5, thus allowing navigation to proceed on the entire river. Near the beginning of the second flood wave, storage was again begun in an effort to reduce this crest at downstream points. The total volume of reservoir inflow from January 30 to February 5, inclusive, was about 264,000 acre-ft. The maximum storage in the reservoir at any one time was only 128,700 acre-ft, so that it is evident that, by this system of operation, a possible third and greater peak could have been effectively reduced.

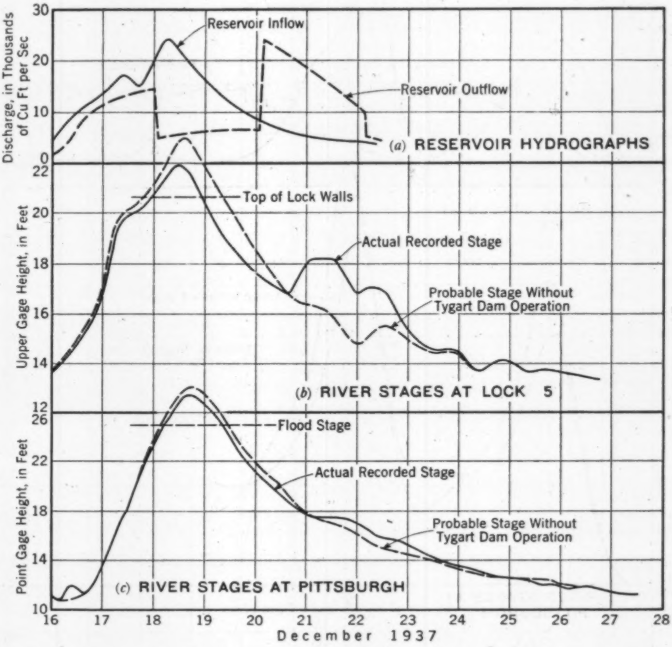

FIG. 9.—FLOOD OF DECEMBER, 1937

April, 1939.—At the beginning of April, 1939, storage was started in the reservoir for the purpose of impounding 100,000 acre-ft for use during the low-water season. The natural discharge of the Tygart River during the first two weeks of the month was such that about 90,000 acre-ft had already been impounded by April 14. With this volume of storage already in the reservoir another major flood occurred on the Tygart and Monongahela rivers. A peak reservoir inflow of 44,000 cu ft per sec on April 16 places this flood third in order of crest magnitude in the history of the Tygart River. A large flood of this type, occurring with the low-water storage already in the reservoir, furnished a thorough test of the capability of the Tygart project to perform its dual function of flood control and water supply. The results, as shown in Fig. 11, were satisfactory. During this flood the reservoir pool reached the

highest level yet attained at El. 1,144.75, representing a gross storage of 220,975 acre-ft. Although the net storage during the flood period was about 131,000 acre-ft there was still about 68,000 acre-ft of storage capacity available.

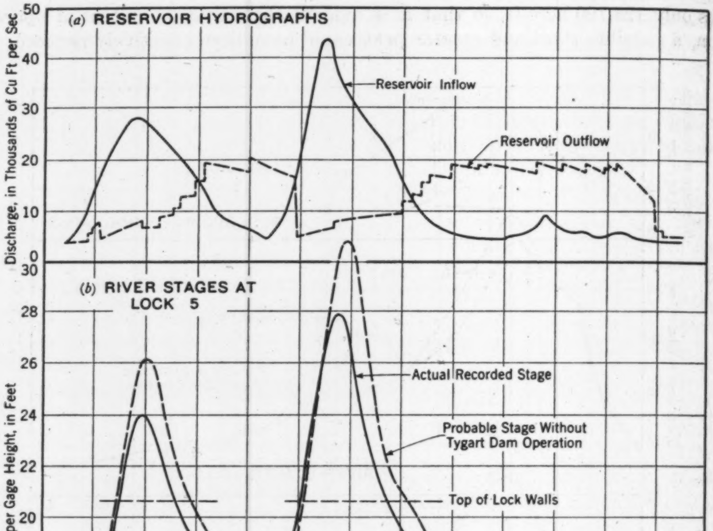

FIG. 10.—FLOOD OF JANUARY AND FEBRUARY, 1939

Minor Rises on Monongahela River.—There are frequent occasions throughout the year when a sharp rise in the rivers will threaten the suspension of Monongahela River navigation without approaching actual flood damage stages. On these occasions the Tygart Reservoir is utilized to equalize, as far

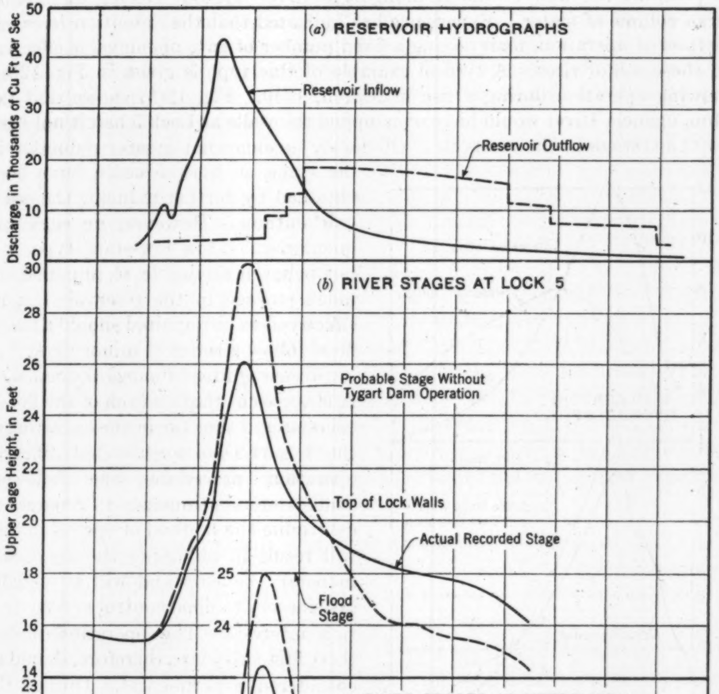

Fig. 11.—Flood of April, 1939

as possible, the flow of the Monongahela River without storing an unduly large volume of water. Experience has indicated that the detention reservoir method of operation, maintaining a fixed number of gate openings, is effective on these minor rises. A typical example of this type is given in Fig. 12(*a*), showing operation during a rise in March, 1940. Fig. 12(*b*) shows that the Monongahela River would have overtopped the walls at Lock 5 had it not been for the storage in the reservoir. Obviously, a somewhat greater reduction in the stage at Lock 5 could have been obtained by further reducing the reservoir outflow. However, on such frequent rises of this low-stage type, it is not believed advisable to impound too much storage in the reservoir lest its effectiveness be impaired should a major flood follow a series of minor rises.

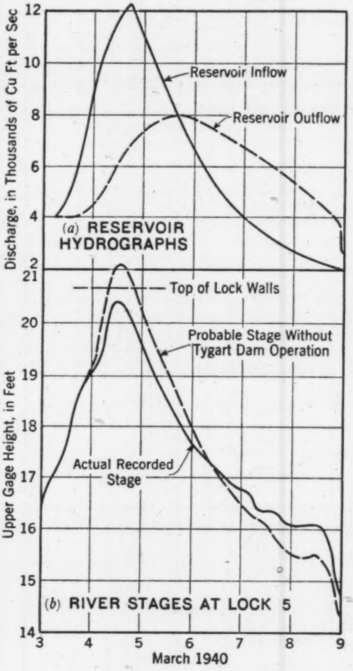

FIG. 12.—FLOOD OF MARCH, 1940

Review of Flood-Control Operation.—It is apparent that on each of the floods encountered thus far in the operation of the Tygart Dam a somewhat different operation procedure was followed. Studies are continuously in progress to determine the method of operation that will result in obtaining the maximum monetary benefits and will be equally satisfactory to flood control and navigation interests. The operation during these first two years, therefore, should be considered experimental. During the flood of December, 1937, an effort was made to correlate the time of reservoir storage with the river stage at Pittsburgh. As a consequence, the maximum possible rate of storage was delayed until nearly the time of the peak reservoir inflow. The rate of discharge, also based on the Pittsburgh gage, produced somewhat undesirable conditions on the Monongahela River. In the flood of January–February, 1939, it was found that an attempt to maintain a fixed passing flow during the storage period, and an increasing discharge rate after the flood had passed, resulted in excessive manipulation of the gates and valves and an extremely irregular hydrograph immediately below the dam. This flood also emphasized the fact that, to be most effective, storage must be started in the reservoir at the beginning of the rise in inflow without waiting for a predetermined stage at some downstream point.

Somewhat better results might have been obtained had storage been started about six hours earlier on the second peak. During the flood of April, 1939, the operation was predicated largely upon forecasted stages at Lock 5. The passing flow during the storage period was maintained at the capacity of one slide gate, thus increasing the outflow as the available storage capacity was decreased. The rate of discharge after the flood peak had passed permitted the Monongahela River to maintain a steady recession at the navigation locks and was satisfactory to navigation interests. The operation during this April flood is believed to be the most successful yet attained and will be used as a guide on future floods of a similar nature.

Monetary Benefits from Flood Control.—In planning the system of flood-control reservoirs a very comprehensive study was made of flood damages in the Upper Ohio River Basin to determine the benefits that would result from reductions of flood peaks. The details of this damage study are quite extensive and only the results are presented herein. The Monongahela Valley has been divided into three damage districts as follows: Pittsburgh District, from the junction of the Ohio and Beaver rivers to Lock 2, Monongahela River; McKeesport District extending from Lock 2, Monongahela River to Lock 4, Monongahela River; Upper Monongahela District from Lock 4 to the head of the river. Damage curves for these three reaches of the river, as developed from the comprehensive study of flood damages, are shown in Fig. 13. The reductions of the crests effected by the Tygart Reservoir on the floods shown in Figs. 8 to 12 were applied to the damage curves to determine the flood-control benefits attributable to the Tygart project. The results in detail are presented in Table 1, the total sum being $2,818,000.

TABLE 1.—ECONOMIC BENEFITS, OPERATION OF TYGART DAM

Flood	LOCK AND DAM 5			LOCK AND DAM 3			PITTSBURGH, PA.			Total benefits
	Actual gage height[a]	Reduction[a]	Benefits[b]	Actual gage height[a]	Reduction[a]	Benefits[b]	Actual gage height[a]	Reduction[a]	Benefits[b]	
Oct. 29, 1937	28.9	3.8	180	28.9	3.7	980	27.8	1.6	520	1,680
Dec. 18 and 19, 1937	21.9	1.1	5	20.9	0.8	0	27.4	0.5	100	105
Jan. 31, 1939	24.0	2.2	23	22.9	1.9	160	23.3	0.8	0	183
Feb. 4, 1939	27.9	3.0	70	27.5	2.3	240	25.4	0.6	70	380
Apr. 17, 1939	26.1	3.8	85	25.3	3.0	385	23.2	1.8	0	470
Total			363			1,765			690	2,818

[a] Feet. [b] Thousands of dollars.

LOW-WATER REGULATION; NAVIGATION

One of the primary purposes of the construction of the Tygart Dam was to provide a storage reservoir to supply an adequate flow for navigation during the low-water season. Prior to the construction of the Tygart Dam, there were many occasions when the natural flow in the Monongahela River was not sufficient to maintain full pools behind the navigation dams. When the volume of water required for lockages, plus the normal leakage past the dams, exceeded the natural flow of the river, the pools would recede below the dam crests and the navigable depths in the pools would be reduced, thus necessitating lighter loading of barges and an increased number of towboat trips to haul the same tonnage. The historic example is the drought of 1930 at which

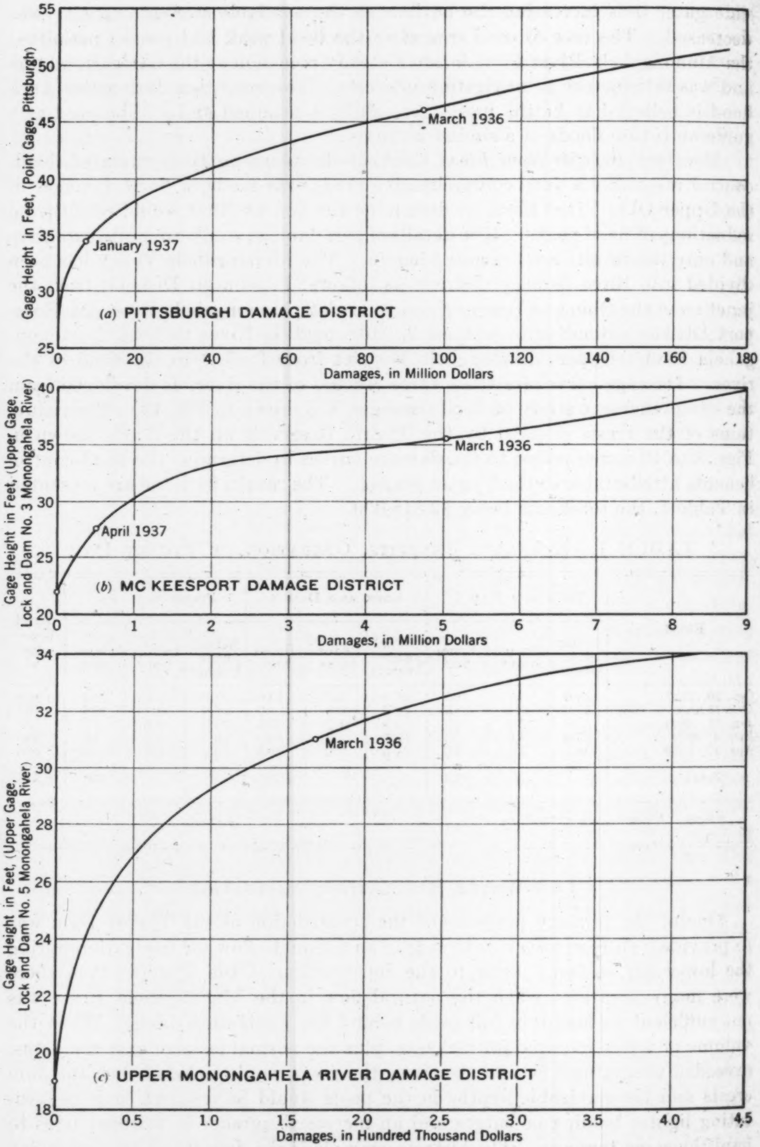

FIG. 13.—APPROXIMATE FLOOD DAMAGE (1936 DOLLAR STANDARD)

time navigation was completely suspended above the pool of Lock and Dam 7, and was maintained at a restricted rate below Lock 7 only by virtue of water obtained from a hydroelectric development on the Cheat River. The action of the Tygart Reservoir in alleviating this condition during the two years 1939 and 1940 has been effectively demonstrated, especially during the late summer and early fall of 1939 when the most severe low-water conditions since 1930 were encountered. Fig. 14 shows the daily reservoir pool elevations for 1938 and 1939, demonstrating the time and rate of storage and discharge.

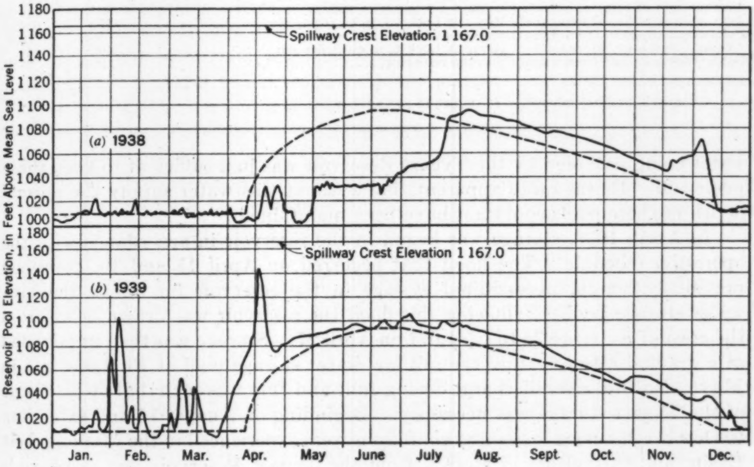

FIG. 14.—DAILY POOL ELEVATIONS

In 1938 it was not possible to begin storage in the reservoir in April (as called for on the operation schedule) because of complications in land acquisition in the reservoir area. Storage was finally begun late in June, and by July 20 about 40,000 acre-ft had been impounded by limiting the outflow to 100 cu ft per sec. Fortunately the natural flow of the Monongahela River was sufficient to permit this procedure. On July 20 and 21 a storm occurred over the Tygart Basin and, by storing almost the entire runoff, the full storage pool of 100,000 acre-ft was obtained by August 1.

Release of this storage was begun on August 6 when the Monongahela River began to recede to low stages, and proceeded continually to November 18. The data in Table 2, Col. 2, demonstrate the increase in the low-water flow provided by the reservoir storage during this period.

The low-water season of 1938 was not particularly severe except for about two weeks in late August and early September. However, the effect of the release of storage from the Tygart Reservoir produced conditions so far superior to those experienced in normal years that this feature of the reservoir operation resulted in widespread public approval. A head of 0.5 ft was maintained on the crests of the navigation dams on the Monongahela River from Dam 15 to Dam 7, inclusive, throughout the low-water period. Below Dam 7 the head on the dam crests was less than 0.5 ft due in part to the greater number of lockages in the lower reaches of the river. In late August and early September the pools behind Dams 5 and 6 receded to the dam crests and the additional water being provided by the Tygart Reservoir was just sufficient to keep these pools full. It was quite apparent that without this water supply the normal depth in these pools could not have been maintained.

TABLE 2.—LOW-WATER REGULATION

Description (1)	1938[a] (2)	1939[b] (3)
Total volume of storage released, in acre-ft	69,800	60,950
Average rate of release, in cu ft per sec	334	433
Average discharge, Monongahela River at Lock 15, in cu ft per sec	570	624
Percentage supplied from Tygart storage reservoir	58.7	69.5
Average discharge, Monongahela River at Lock 5, in cu ft per sec	1,404	1,135
Percentage supplied from Tygart Storage reservoir	23.8	38.2

[a] August 6 to November 18. [b] August 15 to October 25.

In April, 1939, storage was begun in the reservoir in accordance with the operation schedule. The flood that occurred on April 15 and 16, described previously herein, necessitated storage in the reservoir far above the low-water storage pool. Following this flood the reservoir was drawn down from the crest of El. 1,144.75 to El. 1,077 on April 27. Storage was then impounded at a gradual rate to attain the full low-water storage pool at El. 1,094 late in May. Natural river discharge during June and July was such that no discharge of the Tygart storage was necessary. Beginning on August 15 and extending to October 25 severe low-water conditions were experienced in the Monongahela Basin, and the release of storage from the Tygart Reservoir was responsible for maintaining unrestricted navigation on the Monongahela River. The data in Table 2, Col. 3, illustrate the effectiveness of the reservoir on the Monongahela River discharge.

As a result of this regulation by the Tygart Reservoir, conditions on the Monongahela River were maintained much the same as during the previous year. A 0.5-ft head was held on the dams from Lock 15 to Lock 7, whereas the pools at Locks 6 and 5 were barely full. The marked deficiency of rainfall and the acute low-water conditions on other streams of the Pittsburgh District made the effect of the Tygart storage on the Monongahela River quite apparent to the general public and many favorable comments were expressed by those interested in river transportation.

Evaluation of the monetary benefits attributable to the Tygart Reservoir during this period were made through recourse to the records of the 1930 drought. In that year the low water began about July 1 and by July 20 the pool at Lock 5 had receded 4 ft below the crest of the dam, and navigation was seriously impaired. Several slight rises in discharge increased the pool depth about 2 ft, but in mid-September a deficiency of slightly more than 4 ft existed and navigation authorities declared that, should conditions become any worse, it would be impossible to navigate towboats on the Monongahela River. Subsequently, arrangements were made to obtain the storage impounded in the hydroelectric development on the Cheat River for low-water control. This additional flow was just sufficient to maintain navigation in the lower part of the Monongahela River, but in the upper reaches of the Monongahela River, above Lock 7, the pools with the exception of Lock 10 were completely empty and navigation had ceased entirely.

A comparison of the stream-flow records of 1930 and 1939 reveals, of course, that the 1930 drought was more prolonged and severe than that of 1939. However, the conditions experienced from mid-August until late October in 1939 are comparable with the early period of the 1930 drought.

Since the increase in flow (amounting to a total of 60,950 acre-ft) provided by the Tygart Reservoir was barely sufficient to keep the low Monongahela navigation pools full, it is certainly evident that under natural conditions these pools would have been well below the dam crests.

In 1930 information was obtained from the larger companies engaged in Monongahela River traffic to the effect that complete suspension of navigation would result in increased direct costs to these companies of about $1,230,000 per month. It was also estimated at that time that restricted navigation with the pools below the dam crests resulted in increased costs of approximately $139,000 per month. The average monthly Monongahela River tonnage for July, August, and September of 1930 was 2,130,930 tons. In August, September, and October of 1939 the average monthly tonnage was 2,160,900 tons.

The navigation pools of the river would have unquestionably fallen below the critical dam crests for at least two months in 1939 and, since the volume of traffic was comparable to that of 1930, a direct saving of at least $140,000 per month, or $280,000 for this low-water period, was effected. A determination of the exact amount which the navigation pools would have receded under natural conditions was considered inadvisable, as the quantity of water lost by leakage can only be approximated. It is problematical, therefore, whether or not navigation could have been maintained even constantly in a restricted manner throughout the dry period without additional water supply. The cost figures obtained in 1930 pertain only to the additional cost of providing coal by means other than river shipment to the industries normally served by water transportation. The far-reaching indirect effects of disrupted transportation and delayed shipments upon the plants immediately concerned and the general effect upon other related industries cannot be readily reduced to a money basis. Therefore, the estimated saving of $280,000, representing only direct additional costs of coal shipments and based on the assumption that navigation could have continued in a restricted manner without the Tygart Reservoir, is believed to be extremely conservative.

LOW-WATER REGULATION; WATER SUPPLY AND SANITATION

General.—The seasonal storage in the Tygart Reservoir and the regulation during the low-water periods is based primarily on the demands of Monongahela River navigation. However, since this river is extensively used as a source of domestic and industrial water supply and at the same time as a disposal channel for sanitary and industrial wastes, any increase in the seasonal low-water flow constitutes a major benefit to industry and to the general public in the Monongahela Valley. Deplorable conditions existed in the Monongahela River during the 1930 drought, constituting a serious menace to the public health. The improved condition of the river water since the Tygart Dam has been placed in operation has been quite apparent at the water works and industrial plants on the river.

Quality of Tygart River Water.—In the drainage area tributary to the Tygart Reservoir there is a sewered population of from 15,000 to 20,000 persons discharging domestic wastes directly into the river. Industrial development above the dam is sparse, there being but two plants of any size using the river for industrial disposal. During periods of low-water flow the West Virginia State Department of Health has recognized major sewage nuisance conditions in the vicinities of Philippi, Buckhannon, Belington, and Elkins and sewage treatment plants are contemplated for the latter two towns. There has been rather extensive coal-mining activity in the area above the reservoir, and the drainage from abandoned mines imposed an acid load upon the streams. Activities of the federal government in the mine-sealing field have resulted in the sealing of 124 out of 187 abandoned mines above the Tygart Reservoir.

The storage period of the Tygart Dam begins early in April (see Fig. 6). Ordinarily the major part of the 100,000 acre-ft of storage will be impounded during the spring freshets of April and May. During these periods of high river stages the water is turbid but relatively pure, and this vast supply in the reservoir serves as an effective dilution basin for the contaminated low-water flow in the dry summer months. The City of Grafton, immediately below the dam, obtains its raw water supply directly from the reservoir and considers this source far superior to the run-of-river supply previously used. The West Virginia Department of Health has classified the water in the reservoir as "safe."

TABLE 3.—INCREASE IN RIVER DISCHARGE

Period ending	No. of days	Tygart River[a]		Monongahela River[c]	
		Average discharge[b]	Percentage from storage	Average discharge[b]	Percentage from storage
Aug. 31[d] .	26	637	52	2,181	15
Sept. 15. .	15	485	89	1,186	36
Sept. 30. .	12	460	50	1,325	17
Oct. 31. . .	31	447	86	866	45
Nov. 18. .	18	546	58	1,444	22
Average. .	(102)[e]	520	67	1,404	25
June 15[f] .	9	785	56	3,053	15
June 27. .	4	2,430	51	8,288	15
July 19. . .	13	1,714	48	5,312	15
July 29. . .	6	2,355	28	6,987	9
Aug. 31. .	31	830	45	1,985	19
Sept. 30. .	30	558	87	1,054	46
Oct. 26. . .	26	464	93	1,126	38
Nov. 30. .	27	651	57	1,562	24
Average. .	(146)[e]	858	57	2,303	21
Average[g] .	(248)	719	60	1,933	22

[a] At the dam. [b] In cubic feet per second. [c] At Charleroi, Pa. [d] Beginning August 6, 1938. [e] Total. [f] Beginning June 7, 1939. [g] Average, both periods.

Chemical analyses of the water discharged from the reservoir during the past summers have been made and the average values are as follows: Alkalinity, 5 ppm; soap hardness, 29 ppm; turbidity, 7 ppm; free carbon dioxide (CO_2), 3 ppm; potential of hydrogen (pH), 6.1; temperature, 68° F; and indicated number of B. coli (24 hr presumptive), 1 per 10 million. Turbulent discharge through the needle valves insures a dam-site flow that is high in dissolved oxygen, thus aiding in the assimilation and purification of domestic and other wastes that have a high oxygen demand.

Increase in Flow of Monongahela River.—Disposal by dilution is an accepted method of handling almost all of the common wastes incident to human and industrial activities. The success of the method is dependent upon the quantity and quality of diluting water available. The quality of the Tygart Reservoir water has been discussed, and Table 3 gives the increase in low-water flow supplied to the river during various critical periods of 1938 and 1939. Under the heading: "Low-Water Regulation; Navigation," average values were given for the increase in the low-water flow for a continuous period in 1938 and 1939. The values contained in Table 3 are for intermittent critical periods and are based at different points.

Chemical Changes in Monongahela River.—An estimate has been made of the probable effect of the Tygart Reservoir on the chemical characteristics of the Monongahela River during the low-water seasons of 1938 and 1939. Chemical analyses of the reservoir water and of the river as recorded by the various water works and industrial plants along its course were obtained continuously throughout these periods. The approximate natural discharge of the river, as it would have been without the additional discharge from Tygart Reservoir, was determined. Stream-flow records were investigated to find years prior to the construction of Tygart Dam when the Monongahela River discharge compared closely with the natural discharge during 1938 and 1939.

TABLE 4.—TYGART DAM OPERATION EFFECT ON ACIDITY AND HARDNESS IN MONONGAHELA RIVER

(Reductions in Parts per Million; Averages Are Weighted for Time)

Date, 1938 and 1939	CHARLEROI, PA.		MCKEESPORT, PA. (ABOVE YOUGHIOGHENY RIVER)		MCKEESPORT, PA. (BELOW YOUGHIOGHENY RIVER)		PITTSBURGH, PA.	
	Hardness	Acidity	Hardness	Acidity	Hardness	Acidity	Hardness	Acidity
Aug. 6 to 31	21	4	25	6	19	5	19	2
Sept. 1 to 15	60	11	73	13	49	12	65	7
Sept. 19 to 30	45	8	46	9	27	6	31	4
Oct. 1 to 31	78	13	131	31	83	19	97	8
Nov. 1 to 18	44	4	61	12	46	10	53	4
Average (1938)	51	8	73	16	49	11	57	5
June 7 to 15	22	4	28	4	16	2	18	1
June 24 to 27	23	2	18	2	13	1	9	0
July 7 to 19	13	1	11	1	9	1	10	0
July 24 to 29	8	1	4	1	5	1	6	0
Aug. 1 to 31	30	3	25	4	25	4	28	2
Sept 1 to 30	129	14	121	14	105	11	111	5
Oct. 1 to 26	133	16	144	15	114	11	112	7
Nov. 4 to 30	62	8	28	4	30	4	32	1
Average (1939)	72	8	64	8	55	6	57	3
Average (1938 and 1939)	63	8	68	11	52	8	57	4

From the past records of water works and industrial plants the chemical characteristics of the water during these years of similar flow were obtained. After a thorough study of all the factors involved a computation was made of the chemical effect of the Tygart Reservoir water and the results are shown in Table 4.

Monetary Benefits.—The total quantity of water consumed for various purposes throughout the length of the Monongahela River was determined by a field investigation of the records of the municipal water works and industrial plants. These data are shown in Table 5*A*. At the same time time the method and cost of water treatment were ascertained to be as follows:

TABLE 5*A*.—WATER CONSUMPTION, MONONGAHELA RIVER, IN MGD

TABLE 5*B*.—MONETARY BENEFITS (ADD 000)

River reach (see Fig. 1)	Year	Domestic	INDUSTRIAL		DOMESTIC		INDUSTRIAL		Total annual values
			Cooling water	Boiler feed	Neutralization	Softening	Cooling water treatment	Boiler water treatment	
Fairmont, W. Va., to Charleroi, Pa.	1938	10	95	2	0.4	8.5	3.9	2.6	15.4
	1939	10	175	3	0.6	17.1	10.2	7.9	35.8
Charleroi, Pa., to McKeesport, Pa.	1938	4	120	5	0.2	6.3	7.3	7.9	21.7
	1939	4	220	9	0.2	9.9	12.8	22.3	45.2
McKeesport, Pa., to Pittsburgh, Pa.	1938	18	415	12	0.7	24.3	16.9	16.2	58.1
	1939	18	750	21	0.6	36.8	24.6	42.9	104.9
Totals: 1938	....	....	....	....	1.3	39.1	28.1	26.7	95.2
1939	....	....	....	....	1.4	63.8	47.6	73.1	185.9
1938 and 1939	....	....	....	....	2.7	102.9	75.7	99.8	281.1

Neutralization (lime at $12.00 per ton), in cents per million gal, per ppm of acidity neutralized..........................	5
Softening (lime at $12.00 per ton and soda ash at $1.25 per 100 lb), in cents per million gal, per ppm of hardness removed......	25

A combined adjustment of water consumption with the reduction in acidity and hardness effected by the Tygart Reservoir and the unit costs of water treatment (in thousands of dollars) produce total monetary benefits as shown in Table 5*B*.

Review of Water-Supply Benefits.—The monetary benefits in Table 5*B* include only the estimated savings in actual water treatment. No attempt has been made to assign monetary values to various factors, such as the increased oxygen-carrying capacity that aids in the assimilation of domestic sewage and other organic and inorganic wastes. The possible effect on the Ohio River below Pittsburgh has also been omitted. The estimates of monetary benefits, therefore, are believed to be conservative.

Although the operation of the Tygart Dam for low-water supply is only one step forward in the desired purification of the rivers of the Upper Ohio Basin, the presence of this reservoir does insure the population of the Monongahela Valley against repetition of the conditions of 1930. This feature of the project is an important by-product of the primary functions of navigation and flood control.

CONCLUSION

The total monetary benefits attributable to the Tygart Dam for the first two years of operation are as follows: Flood control, $2,818,000; navigation, $280,000; and water supply, $281,100—a total of $3,379,100. Monetary benefits have been determined only where direct savings to the public can be shown. The direct benefit for the maintenance of navigation is most misleading because, as explained previously, many intangible factors, including delay and disruption of industrial production, are too complicated to be evaluated fully in a paper of this nature. The effect of the Tygart Reservoir in maintaining unrestricted navigation on the Monongahela River throughout the summer and fall of 1939 is probably the greatest single benefit attributable to this project.

The moderate reductions effected on flood crests at Pittsburgh by this one reservoir, controlling only about 6% of the total drainage area of the combined Allegheny and Monongahela rivers, serve as an indication of results that may be obtained from the proposed ten-reservoir system. It is also worthy of mention that, although the net reduction in the flood peak is the most important factor in evaluating flood damages, there is also the factor of time that the river remains above damage stage. On some of the floods discussed herein the degree of synchronization of the peak discharges of the Tygart River with those at Pittsburgh was such that only a small reduction in crest stage was effected, but it was accompanied by a much greater reduction during the recession of the flood, with a corresponding reduction in the time that the river remained above flood stage. This time element is particularly important at the locks of the Monongahela River, and the reduction in the time that these locks would have been out of service is one of the important intangible benefits to which no monetary values have been ascribed.

These first two years of operation most probably do not represent average annual conditions since three of the five largest floods of record on the Tygart Basin occurred during this time and the Monongahela Basin experienced one of the worst droughts in its history. On the other hand, only small floods with low damages have occurred at Pittsburgh during this period. A small reduction on a larger flood would result in greater flood-control benefits than have thus far been obtained on any individual flood.

In conclusion, it is believed that the results achieved thus far by the Tygart Dam, even in the experimental stage, amply justify the funds expended for its construction and operation and are the most conclusive proof that can be offered for the extension of the flood-control system of the Upper Ohio River Basin.

ACKNOWLEDGMENTS

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DISCUSSION

NICHOLLS W. BOWDEN,[5] M. AM. SOC. C. E.—The engineering profession in general, and hydraulic engineers in particular, who are interested or engaged in the operation of large reservoirs, or problems connected with them, should feel indebted to the authors for their clear, detailed, and comprehensive treatment of the Tygart Reservoir. There is a dearth of such material both in technical journals and in textbooks. Because of his connection with this and the other proposed storage projects above Pittsburgh while employed by the Corps of Engineers during several years of investigation and planning of these projects and until the construction of the Tygart Dam was well under way, the writer was greatly interested in this timely paper.

The dire need for this reservoir cannot be questioned when one considers the situations brought about by the severe drought in 1930 and the destructive flood in 1936. At times during the drought the natural flow in the Monongahela River was so small as to be negligible—far from enough to provide for lockage of boats and leakage through the locks and dams—and some of the pools in that river above the mouth of the Cheat River became practically empty basins. Below the confluence of the Cheat River, where the bulk of the commerce on this heavy-traffic waterway moves, water released from a privately owned power reservoir on the Cheat River was sufficient in the emergency to supply the pools and maintain navigation. The ravages of the 1936 flood, particularly in the "Golden Triangle" of the City of Pittsburgh, are too well known to need comment here.

As to the efficacy of the Tygart Reservoir in regulating flood flows and affording an adequate, dependable flow during the season of low natural discharge, the authors show quite conclusively that operations during the first two years have been successful. Very properly they state that: The reservoir controls only about 6% of the total drainage area of the Allegheny and Monongahela rivers above Pittsburgh; that the flood storage capacity equals only 4.5 in. of runoff over the contributing area above the dam; and that this capacity is insufficient to store the entire runoff from great storms such as frequently occur in the eastern part of the United States. The relatively small part of the runoff from the drainage area above Pittsburgh that this reservoir regulates is a potent argument for constructing additional reservoirs—and this is now being done—to regulate runoff from a larger part of the area. The relatively small flood-storage capacity of the reservoir seems to merit further discussion, as it is believed that this feature is of the utmost importance in the successful operation of this and other reservoirs. Also, the writer believes that there may be some who do not think that the degree of control which this reservoir affords is adequate.

Various criteria have been advanced for planning and designing reservoirs for flood control in this section of the United States. Some of these may be stated as follows: (1) That the flood-storage capacity should be equivalent to 8 in. or more of runoff over the controlled area; (2) that the flood-storage capacity plus a constant release from the reservoir at bankfull stage downstream, over a given flood period of, say, three or four days, should equal 8 in. or more of runoff; or (3) that the flood-storage capacity should be equivalent to at least one half the average annual runoff. All of these are desirable requirements, if attainable. The capacity of Sacandaga Reservoir in the Hudson River Basin is equivalent to almost 14 in. of runoff; that of the flood-storage space in Norris Reservoir in the Tennessee River Basin, about 12.5 in.; and that of the flood storage space in Lake Mead in the Colorado River Basin, more than three fourths of the average annual runoff. These multiple-purpose reservoirs easily meet the foregoing specifications for flood storage, as no doubt other reservoirs do, and there should be no question as to their suitability for flood control.

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[5] Prin. Hydr. Engr., and Head, River Control Section, TVA (Formerly Senior Engr., and Chf., Projects Div., U. S. Engr. Office, Pittsburgh, Pa.).

Some years ago a prominent engineer remarked to the writer, in a facetious manner, that the planning of large reservoir projects in the Ohio River Basin came about 100 years too late; otherwise, a dam might have been built at or near the mouth of each tributary stream of sufficient height to control flood flows from above and then one on the lower reaches of the Ohio River itself, behind which all of the flood flow from the main-stream drainage could be controlled. This was just a trite way of saying that human occupancy of the valleys with cities, towns, industries, and rail and highway transportation lines had progressed so far that very large reservoirs were no longer feasible. He might have added with equal force that this human occupancy, which had much too often invaded the flood plain, made flood control increasingly important.

Thus, the situation in the Pittsburgh District that confronted engineers who planned the reservoir systems on the Allegheny and Monongahela rivers was one of very great need for flood regulation, as well as for other purposes, and usually of very definite limitations in storage possibilities except at high cost. If the engineers had adhered rigidly to conservative requirements for runoff-storage relationships, a reservoir plan suitable for the protection of the Pittsburgh area might not have been developed. Instead, the studies proceeded on the basis of securing as high a degree of runoff control as possible, giving due regard to the ratio of estimated annual costs to estimated annual benefits.

In order that the writer may not be misunderstood, it should be stated that he is a firm believer in providing ample storage space in reservoirs for complete control of flood runoff wherever feasible, but he does not believe that this important factor should be allowed to control to the exclusion of all other considerations. As the technique for operating large reservoirs improves—and it is improving rapidly—many concepts of yesterday and even of today will undergo marked change. The importance of collecting adequate precipitation data, and translating them quickly and accurately into stream flow, is now recognized as never before; the technique of routing flood flows in open channels and through reservoirs has advanced to a marked degree in recent years; and last, though probably far from least in possibilities of influencing and perhaps revolutionizing the method of operating reservoirs, is the effort being made by the U. S. Weather Bureau to forecast the location, amount, and intensity of rainfall one or more days in advance. Their initial cooperation in work of this character with the Tennessee Valley Authority in the Tennessee Basin, which was begun in 1939, is being followed closely to see what degree of dependability and accuracy can be attained.

Since the space available in the Tygart Reservoir for storing flood water is insufficient for complete control of the contributory area, it is essential to operate the reservoir in such a manner that the space is used when it will be most effective in accomplishing as much flood-stage reduction as practicable at the critical point downstream, and so that the space again will be made available in the reservoir as promptly as possible for subsequent use. The authors show that this principle has been applied with success in the regulation of floods thus far. Obviously, such operations require very careful attention and the exercise of good judgment in the use of the storage space, lest a storm should exceed expectations and produce a volume of runoff beyond estimates either in the form of a higher or broader inflow hydrograph, or possibly one with a double peak like the flood of January and February, 1939. A full reservoir at the wrong time might conceivably result in a high discharge rate from the reservoir at such time as to synchronize with peak flows at one or more critical points downstream. Involved in this problem is the matter of determining whether to attempt the maximum stage reduction, through the use of all available flood storage space to regulate a current flood, or whether to be satisfied with a smaller stage reduction and thus retain storage space as a factor of safety for use in case subsequent flood flows should develop.

The existence of operating problems of this nature, although difficult of solution, should not of itself deter engineers from constructing large multiple-use reservoirs that do not afford as much storage space for the several purposes as might be desired. The importance of the need and the value of such a project to society should have great influence. The writer has complete confidence in the ability of engineers to develop operating techniques, where necessary (such as the authors indicate is being done in the case of the Tygart Reservoir), which will depend for success on the proper handling of water rather than on storing all flood runoff in a catchment basin so large as to be almost foolproof in operation. Likewise, he believes that engineers are resourceful enough to perfect means for the efficient operation of large multiple-purpose reservoir systems.

EDGAR E. FOSTER,[6] **ASSOC. M. AM. SOC. C. E.**—The authors are to be commended for presenting a study of the results of the operation of one of the recently constructed multiple-purpose flood-control projects that the federal government has undertaken in the past decade. Few such projects have been completed long enough to observe the results of operation for a significant period of time. In the pioneer flood-control works constructed by the Miami (Miami Valley, Ohio) Conservancy District,[7] each dam was a single-purpose project only, since it used uncontrolled outlets for emptying the reservoir, and hence its operation could not constitute a complete model for the multiple-purpose projects.

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[6] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[7] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1503.

The purpose of the Tygart Reservoir as stated by the authors is twofold—flood control and water conservation. The latter purpose provides water obtained from floods or high discharge in spring for the improvement of navigation, incidental benefits to water supply, and reduction of pollution during late summer. The utilization of the conservation water requires only releasing during periods of low stages at the controlling points along the stream below. The operation for flood control, on the other hand, must be planned carefully to obtain the desired result or even to prevent a worse flood. The seasonal basis for operating the Tygart Reservoir is depicted in Fig. 5. The plan of operation for both flood control and utilization of the conservation water has been skilfully developed.

Table 6 summarizes the reduction in flood peaks obtained at the site, at Lock and Dam 5, and at Pittsburgh, Pa., by operation of Tygart Reservoir. Generally, the reductions obtained at Tygart are much greater than at either Lock 5 or Pittsburgh, especially in the larger floods. Flood crests from storms centering over the headwaters of the Monongahela River, of course, will be reduced more effectively than storms centering elsewhere on the watershed. Although the reductions at Pittsburgh appear relatively small, they do not seem out of proportion to the drainage area controlled by the Tygart Reservoir as compared with the area above Pittsburgh.

TABLE 6.—SUMMARY OF FLOOD PEAK REDUCTIONS

Date	REDUCTIONS, IN FEET, AT:		
	Tygart	Lock 5	Pittsburgh
Oct., 1937	28.0	3.8	1.6
Dec., 1937	0.1 ±	1.1	0.5
Jan., 1939	8.0	2.2	0.8
Feb., 1939	24.0	3.0	0.6
April, 1939	26.0	3.8	1.8
March, 1940	4.2	0.6	....

Although the general principles of operating a flood-control reservoir are applicable to any watershed, the details of the operating plan must usually depend upon the characteristic features and requirements of each individual basin. Among the peculiarities to be considered in relation to the Tygart Reservoir are its location in the headwaters, the areas susceptible to flood damage at Pittsburgh and in the Monongahela Valley, the existing navigation on the river, the low spillway capacity, and the central location of Dam 5. The stage at Dam 5 is utilized as a control for the release of flood storage. These factors are used in developing the plan of operation of Tygart Reservoir but of course would not be available on any other watershed. On other streams and for other reservoirs a different set of characteristics necessarily would be considered.

Usually the large industrial centers like Pittsburgh that are subject to heavy flood loss would be the controlling point for reservoir operation, but the authors have shown that navigation is of such importance on the Monongahela River that a control point near the center of the basin must be selected. It appears to the writer, however, that Pittsburgh should be considered as a secondary control point for operations in the event of a general flood covering the Allegheny Basin.

A storage reservoir can reduce downstream flood crests by two means: (1) Storage until the flood has subsided throughout the basin, or (2) by delaying the peak at the site until the flood crest of the valley below has moved downstream. The latter plan is uniquely available to a reservoir in the headwaters. It has been used effectively in the operation of Tygart Reservoir as may be seen in the hydrograph of the flood of December, 1937; practically no reduction of peak was obtained at the site, where it was merely delayed, but a reduction of 1.1 ft was achieved at Lock 5. The floods of 1939 likewise show the effect of regulation to delay the peak as well as to reduce it by temporary storage.

The reduction of peaks on the main stem, by a delaying operation at the reservoir, is dependent to a considerable extent upon the location of the reservoir with respect to the main stem. Tygart Reservoir, being situated in the headwaters of the main stem, is favorably located for such operation. In order to illustrate a contrary condition, assume that floods on the Youghiogheny River under natural conditions discharge into the Monongahela River slightly ahead of the crest on the main stem. Under this condition a small reservoir located near the mouth of the Youghiogheny River, unable to hold a substantial part of the flood runoff, would succeed only in delaying the tributary crest, cause it to coincide more closely with the peak on the main stem, and thereby produce a greater flood below the mouth of the tributary. The location of a reservoir near the mouth of the Youghiogheny River and the relative time of the tributary flood is suppositive as far as the writer is aware, but it serves to illustrate the fact that merely delaying a flood crest on a tributary is not always advantageous, although it operates favorably for flood control by Tygart Reservoir.

A flood on a river is not a simple summation of excessive areal runoff as some theoretical studies may lead one to believe, but it is a compound product of the various peaks of its tributaries that combine at different times and places. The "flood-wave" analogy should not be taken too literally.

As another aspect of reservoir operation, the authors emphasize the element of flood prediction both above and below the reservoir. This emphasis is rightly placed, for some measure of discharge prediction at the reservoir and stage prediction at the control point is essential for efficient operation. Such flood prediction should be obtained with the closest possible cooperation with the Weather Bureau and the Water Resources Branch of the U. S. Geological Survey, such as the authors indicate is accomplished. The writer believes that snow surveys should also be utilized to secure data of the moisture content of snow cover if there is any possibility that such cover can contribute an appreciable proportion of the flood runoff.

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THE SURVEYOR AND THE LAW

By A. H. Holt,[1] M. Am. Soc. C. E.

With Discussion by Messrs. E. F. Chandler, Harry Rubey, William H. Richards, Jr., C. B. Humphrey, and D. D. Haines.

Synopsis

It is not intended that this paper shall discuss, primarily, the law of boundaries. That is a very broad subject, capably treated in at least three books which, in whole or in part, are directly concerned with the subject.[2] The subject is also included, as incidental material to even broader subjects, in many treatises and compilations to which one commonly turns in efforts to learn the law. It is desired to present herein a discussion of those aspects of law which tend to indicate and to define the duties and the responsibilities of surveyors in their share of the work of establishing, describing, and recovering the boundaries of land. This is work in which there must be combined (with mutual understanding if best results are to be secured) the training, the experience, and the well-counseled efforts of practitioners of law and of engineering. This paper is a humble attempt to contribute a little to this end by discussion of some impacts of the law on the work of the engineer.

Introduction

Clearly, such a treatment as this cannot be exhaustive. Even though very limited in what it attempts, it is necessarily general in treatment. It is hoped that it will not be misleading, but it is too much to hope that none of its statements will be at variance from, or even contradictory to, that which is good law in some of the forty-nine different jurisdictions within the continental United States. It is assumed that, charitably and with good common sense, the reader will remember this fact; and also that, to be on safe ground, he must be ably advised on any given point concerning the law in the jurisdiction in which he is concerned. That the law of all other jurisdictions on that point might be otherwise would probably not then be material.

Note.—Published in May, 1941, *Proceedings*.

[1] Maj., Corps of Engrs., U. S. Army; Worcester, Mass.

[2] "Law of Operations Preliminary to Construction in Engineering and Architecture" (especially Part III), by John C. Wait.
"The Legal Elements of Boundaries and Adjacent Properties," by R. H. Skelton.
"A Treatise on the Law of Surveying and Boundaries," by F. E. Clark.

QUALIFICATIONS OF A SURVEYOR

Let the reader consider a hypothetical surveyor, Mr. S, as he makes his preparation for his tasks, and go with him through some of the situations that he may be required to meet.

In most of the states of the Union there are now statutes providing that any person who is to do boundary surveying (or, as it is more commonly but less definitely called, "land surveying"), in the sense that he is to be in responsible charge of a boundary survey, must be registered or licensed by an examining board. These statutory requirements vary considerably from state to state, but they are easily discovered by referring to the code or the general laws of the state in question. Let it be assumed that if Mr. S is to practice in a state that has registration laws he has convinced the Board of Examiners that he is qualified to the extent that the statute and the Board require, and that he has been duly registered or licensed to practice.

Thus, the representatives of the state have in effect pronounced Mr. S to have at least the minimum personal and professional qualifications that should permit him to practice. What degree of care, skill, and judgment may a prospective client properly expect of him; and what may be the consequences if he fail to measure up in these respects to the requirements of some job that he may undertake? The courts answer this question by quoting the phrases that have long been used in the law to define that standard of conduct which separates negligence from due care: He must "exercise that degree of care which a skilled civil engineer of ordinary prudence would have exercised under similar circumstances."[3] The engineer will probably wish for a more specific, more tangible criterion that could be generally applied; but he will seek in vain. This standard must be used and interpreted in the light of the circumstances of each situation. In Ferrie vs. Sperry[3] the court says,

> "* * * it was important that the jury should know what such an ordinarily prudent engineer would do under the circumstances of this case. Might he simply examine the land records and muniments of title, and observe the fixed monuments and evidences of present occupancy and ownership; or was he bound to scour the neighborhood to learn whether there had been adverse occupancy and claims of ownership of any part of the premises covered by his client's deed by others than the latter's predecessors in title appearing of record? * * * The jury would not know, unless informed by evidence, and such evidence was admissible."

Thus must the question be resolved if it finally come to a court—by expert testimony of those who profess to know what should be done.

DUTIES AND LIABILITIES OF THE SURVEYOR

The present interest, however, is in having Mr. S do whatever he should do, and in keeping him out of court. Mr. S, then, should make as thorough an examination of all reasonably available sources of information as circumstances seem to permit—and err a little on the side of safety, in diligence of search. This applies to matters of record, to evidences of boundary or of occupancy to be found on the ground, and to any other items of information that his eyes or his ears can discern. He must then pursue his field work and computations by such methods as will secure a precision appropriate to the job, and will eliminate the possibility of there remaining that bugbear of all engineers, a mistake. He should make very clear to his client the situation that he finds, in order that, if necessary, the client may use his own judgment in seeking legal advice as to the significance of conditions of occupancy, conflicting claims, and other matters that may override metes and bounds in fixing the lines.

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[3] Ferrie vs. Sperry, 85 Conn. 337, 82 A. 577.

True it is, the owner wants and expects to have the proper boundaries marked and reported to him, and does not want a list of conflicting items of evidence; and to the credit of the judgment of surveyors it may be said that that is what he usually gets. It is this "judicial function" of the surveyor (thus so aptly termed by Mr. Justice Cooley, of the Supreme Court of Michigan) that calls for experience and sound legal judgment, as well as engineering ability, on the part of a surveyor.

Suppose Mr. S fails to measure up to this required standard of care and skill. It has been generally held that he is answerable in damages to his client for the loss or injury that results from such failure—at least for any loss coming from a use of the land in a manner known to the surveyor to be contemplated at the time of the survey. A surveyor has been required to pay the cost of moving an apartment building, built to the line which he had marked, back some five feet to the true line.[4] The difficulty in that case was that of a mistake in measurement,—which is undoubtedly a clear failure to use due care. The surveyor in this case was not relieved of his burden (to a cost of some $1,267) because he had charged only $12 for the survey—a rather impressive commentary on the undesirability of cheap surveys, from the standpoint either of the surveyor or of the client. The custom of giving a certificate of accuracy, for which a higher fee was charged, was found "wholly immaterial" in this case.

It has been held that if a person is employed as a municipal official (for example, a city engineer) and does work for private persons as a function of his office, he is liable to them only in case of loss from his negligence or fraud; but that if a person works in the capacity of a private practitioner he is liable as well for loss resulting from want of skill.[5] Cases in other jurisdictions seem to indicate that the more general rule is that a surveyor undertakes to bring reasonable skill and knowledge to his work, be it done in either official or private capacity, and that he will be liable for loss resulting to his client because of his failure to do so.[6] The municipality, however, is not liable if its engineer does work erroneously at the request and for the benefit of a private person (in the case cited, setting stakes erroneously to show street grades in front of a house lot); even though he be bound, in his official capacity, to do such work upon request.[7]

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[4] Taft vs. Rutherford, 66 Wash. 256, 119 Pac. 740, 38 L.R.A. n.s. 1043.

[5] McCarty vs. Bauer, 3 Kan. 237.

[6] Highway Commissioners vs. Beebe, 55 Mich. 137, 20 N.W. 826.

[7] Waller vs. City of Dubuque, 69 Iowa 541, 29 N.W. 456.

It is believed that the duties and liabilities to his client of a surveyor who has been registered or licensed to practice do not differ from those of an unlicensed practitioner in a jurisdiction where license is not required. Of course, one who is licensed but is subsequently found incompetent may be prevented (by such process as is provided in the statutes of the state in question) from continuing to practice, or he may have his license revoked.

SURVEYOR'S RIGHT OF ENTRY

Now, suppose that Mr. S, duly advised as to the care to be used, is proceeding with his survey. It may be practically necessary for him, either in actually running out the lines or for the purpose of reaching control monuments to which to refer his survey, to go on other land than that of his client. Surveyors regularly do this, but have they any right to do so?

If the surveyor is conducting in a reasonable manner a survey for an undertaking directed or authorized by act of the legislature, and if "the entry is reasonably necessary, and * * * is but a temporary one, and accompanied with no unnecessary damage," there is no trespass.[8] It is generally held that a body having the right of eminent domain in the furtherance of some project may, without payment of compensation, enter upon private land to make preliminary surveys therefor, and that such entry is not a trespass; and the statutes of some states so provide. Under such rulings, surveys for highways, for railroads, and for other improvements have been made; and the officials of New England towns have performed their duties of periodically "perambulating" their boundaries.[9] Of course, compensation must be made for any actual damage done, and any excessive or unnecessary damage will be enjoined.[10] In at least one case it has been held that where the state's officers (the state engineer and those working under his direction), in making a survey directed by the legislature of a disputed boundary line of certain counties, exceeded the amount of damage that was reasonably necessary, the state was not liable for the excess damage but the state's officers were.[11] In this case the damage complained of consisted in cutting through plaintiff's woods a "slash" 3½ miles long and from 5 to 25 ft wide, for the purpose of establishing a base line. A referee in the case found that the method used was proper, and the best for the purpose, and that the work was skilfully and carefully done. The damage amounted to about $500. It should be mentioned that in this case both the referee and the lower court had found for the defendants (the surveyors) and that in the Appellate Division there was a strong dissenting opinion.

In the absence of any official mission or the possession of any official character, a surveyor who enters the land of another without permission is as much a trespasser as any other intruder would be. The action of trespass will lie even though no actual damage be done to the land or to any of its appurtenances.[12] Perhaps it may be apt to remark that, again, it is much to the credit of the ability of surveyors to "get along" amicably with those on whose land they almost necessarily trespass, that there are in the books very few cases of actions of trespass against surveyors, or because of the mere act of surveying.

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[8] Winslow vs. Gifford, 6 Cush. (Mass.) 327.
Dill. Mun. Corp. 4th Ed. Sec. 614.
Edwards vs. Law, 71 N.Y.S. 1097, 63 App. Div. 451.

[9] Fox vs. Western P. R. Co., 31 Cal. 538.
Burrow vs. Terre Haute & L. R. Co., 107 Ind. 432, 8 N.E. 167.
Brigham vs. Edmands, 7 Gray (Mass.) 359.

[10] Dancy vs. Alabama Power Co., 198 Ala. 504, 73 So. 901.

[11] Litchfield vs. Bond, 186 N.Y. 66, 78 N.E. 719.

This is the situation under the common law, but in some states legislative enactments have brought a modification. As an example, Section 2586 of the Public Laws of Vermont (1933) is quoted:

> "In cases wherein the title to lands, tenements or hereditaments may come in question, the practical surveyor with the necessary assistants, employed by any of the parties to such disputed title, may enter upon such lands or real estate, or other lands, for the purpose of running doubtful or disputed lines and locating or searching for monuments and ascertaining and deciding the lines and monuments of a survey, doing as little damage as possible to the owners of such lands."

Laws are now appearing on the statute books of a few states giving to engineers or surveyors the right to enter upon private land, whereon is a control survey station, for the purpose of using the station for engineering or surveying purposes. The laws usually require that no unnecessary damage be done, and that any anticipated damage be paid for in advance.[13]

A conclusion, drawn from all the foregoing and from the common experiences of surveyors, for the guidance of Mr. S, may well be summarized somewhat as follows:

The survey of a boundary line will almost inevitably require going on other lands than those of the client. Ordinarily, if care is used to cause no appreciable damage, no objection will be raised by the owner or occupant of such land; and if and when such objection is raised a frank explanation of the surveyor's need of permission to enter, a sincere promise of due care, and an explanation of the disinterested and non-partisan character of his work and conclusions will often secure the needed permission. Such entry, however, cannot properly be forced unless the surveyor is acting in an official capacity or for a legally-authorized undertaking; or unless it is under such a statute as that just mentioned. The invasion should be as brief and unobjectionable as practicable, limited strictly to the purposes of the survey. No reasonably avoidable damage should be done, and any unavoidable damage should be explained and compensation made.

FINALITY OF A SURVEY

As Mr. S proceeds with his survey he may reach very firm and, in his opinion at least, very well-founded opinions as to where the lines with which he is concerned are, or should be. It is usually expected by his client that such will be the case, and that monuments of some kind will be set to represent such conclusions. However, Mr. S may find that his conclusions disagree from those of some other survey or from evidences of occupancy and use. May he, if he is "sure" he is right, "establish" the boundary? The same question is sometimes put differently by a client: "Is your survey 'legal'? Will it 'hold'?" The answer probably should be that no survey, unless it be of an original and official character such as were the original surveys of the United States Public Lands, is "legal" in the sense in which people use that term in this connection; nor will it necessarily "hold" without the occurrence of one of two things: (1) The acceptance of the result by those whose interests are affected, or (2) a decree of court that the survey represents the proper solution of the boundary problem involved.

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[12] Dougherty vs. Stepp, 18 No. Car. (1 Dev. & Bat. Law) 371.

[13] An excellent example of such a law is included in Chapter 628 of Laws of Maryland 1939. The part of that chapter to which reference is here made will become Sections 32 to 35, inclusive, of Article 91 of the Annotated Code of Maryland (1924 Ed.).

No modification of this statement is to be made because the surveyor may possess some official character, such as that of a city or county surveyor, nor because that which such a surveyor has concluded is sanctioned by his superiors (the county commissioners, for example). He has no more authority to conclude a matter of boundary than has a private surveyor,—or a blacksmith. Property rights must be settled by those who own them or by the courts. The Michigan court has well said,[14]

> "It is quite manifest that in this case there seems to have been an idea that the ex parte action of surveyors and commissioners was entitled to credence and authority upon the true lines and the fact of encroachment [of a fence and buildings alleged to be within a highway]. Neither of them can affect vested rights or settle controversies. They may be useful witnesses, when they speak of matters with which they are familiar, but they have no greater right than anyone else to determine starting points or boundaries. * * * The rash and vexatious acts of local officers not under the process of law are fruitful sources of bad blood and litigation."

(Since the boundary of a highway has been mentioned let the caution be noted that cases in which boundaries of highways are concerned are ordinarily excluded from the effect of the doctrine of acquiescence or that of adverse possession.[15])

Now that the legal situation has been outlined, let the statement be made —once more to the credit of surveyors in general—that, as a practical matter, many a boundary dispute has been settled by surveyors, many boundaries have been "established" by them; and may such continue to be done. In honest, competent hands, this is one of the most practical, most useful results of the exercise by the surveyor of his unofficial "judicial function."

ARBITRATION

Now let it be supposed that, at the end of his work over some boundary, Mr. S does reach a conclusion which differs from that of some one else, or to which not all items of evidence—or "calls"—agree; or perhaps he is forced to the conclusion that it would be disproportionately expensive, or even practically impossible, to execute surveys that would enable any honest surveyor to say, "There are the stakes which with reasonable certainty and precision mark the line." It may often be true, in such a situation, that it will be possible to say, with relatively small limits of variation, approximately where the line is. Evidences of occupancy might accomplish that, or the differing conclusion from another survey may not represent a large difference. Mr. S's best service, in the interest not only of his client but of all who are concerned, may now lead him to suggest that the best results, both from the standpoint of property rights and also with reference to the amicable relations of all concerned, will be reached by compromise or by arbitration. If interested persons will either come to an understanding of all the conflicting elements involved and then hit upon and accept a solution, or if they will let the surveyor propose a compromise solution, with the reasons therefor, and then accept it, the Gordian knot will be cut,—and the chances are good that all will be happier than at the reaching of a conclusion in a court of law; for, after all, the decision of a court would have to be based on evidence, and it is unlikely that the court would have any more evidence, or have it presented with less bias, than that now in the possession of the surveyor. No suggestion is here made that any substantial, well-founded rights of a client, properly enforceable in a court, should be recommended for surrender; it is believed that the proper setting for the procedure recommended has been made clear. Obviously, to function acceptably in the rôle suggested, Mr. S can use all the legal and engineering training and experience and judgment at his command. To carry the matter to its best conclusion, as a competent and fully-informed court would do, will be another instance of his exercise of his judicial function.

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[14] Gregory vs. Knight, 50 Mich. 61.

[15] Langle vs. Branch, 193 Iowa 140, 185 N.W. 28. Subject is discussed in 6 A.L.R. 1210.

What must be done to make such an acceptance of a compromise solution binding? How should it be perpetuated?

With reference to the first of these two points, it seems clear that nothing further than the parol—oral—agreement of the owners concerned is needed to conclude the matter and to make their agreement binding upon them and upon their subsequent grantees. This is not a situation in which, under the statute of frauds, a writing is necessary to make the agreement effective. In a brief, but clearly-expressed, case[16] the Supreme Court of Arkansas has said,

> "This court has held in accord with the weight of authority that, where there is doubt or uncertainty or a dispute has arisen as to the true location of a boundary line, the owners of the adjoining lands may, by parol agreement, fix a line that will be binding upon them, although their possession under such agreement may not continue for the full statutory period."

The Supreme Court of the United States has held[17] that such an agreement is "not a contract for the sale or conveyance of lands. It has no ingredient of such a contract." Adjoining owners hold up to a boundary thus fixed by consent by virtue of their title deeds, not by virtue of a parol transfer between themselves.[16,18]

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[16] Robinson vs. Gaylord, 182 Ark. 624, 33 S.W. 2d. 710.

[17] Boyd vs. Graves, 4 Wheat. 513, 4 L.Ed. 628.

[18] Cutler vs. Callison, 72 Ill. 113.

Such an agreement to fix a boundary line is void if the owners know or if one of them knows that the agreed line is not the true line.[19] It is then in effect a transfer of land, or perhaps even a fraud on the part of the one who knows. However, that the line could be ascertained by an expensive survey, the cost of which the owners thus avoid, does not prevent the uncertainty that must exist to make the agreement valid—and valid "even though the true line should afterward be ascertained."[20,21]

It may be that the parties will prefer, rather than to try to agree upon a location themselves, to submit the question of boundary to arbitration. Still assuming that the arbitrator is an honest and a competent surveyor, that may be the very best solution. That it is a valid one is amply supported by decisions.[22] It is not necessary that the agreement to do so be in writing[23]; although, to prevent misunderstanding, a writing may sometimes be desirable. When a writing is used, it should be carefully drawn, and it must be taken to be the entire basis of the submission to arbitration. No parol modification would be effective, under the statute of frauds,[24] or the parol evidence rule.

The award, resulting from an arbitration conducted in accordance with the terms of the submission, is valid and conclusive.[25] It cannot be impeached because of an honest mistake of judgment on the part of the arbitrator.[26]

This fact, and the aforementioned fact that a location agreed upon will stand even though subsequent developments show that it was not the true line, are only corollaries of the elementary and fundamental principle of boundary law that a boundary once authoritatively established and adopted remains in that place, regardless of mistakes of measurement or of judgment made in arriving at that location. This principle is too thoroughly established and too well known to require citation of authority or discussion here. It is comparable in importance to, and is directly connected with, the fundamental rule for interpretation or construction of land descriptions and for search for land boundaries—namely, that which is to be accomplished if possible is to ascertain and to carry into effect the intent of the parties. To effectuate these fundamental principles, all rules and all "order of calls" are enunciated, and to them all such rules must yield.

All that is absolutely required to be done to make a location by agreement or by arbitration effective,—to perpetuate it,—is to describe the boundary with certainty.[27] However, as a surveyor, Mr. S should know that to provide a permanent remedy, to prevent a recurrence of the trouble, the line should be adequately and "permanently" monumented and platted, and the plat should be recorded with written, signed, and acknowledged acceptance of that which is shown thereon, or accompanied by a written submission to arbitration and a certificate by the arbitrator that the plat shows the result of the award.

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[19] Moran vs. Choate, 69 S.W. 2d. 994, 253 Ky. 470.
Johnson vs. Buck, 46 P. 2d. 771, 7 Cal. App. 2d. 197.

[20] Lynch vs. Egan, 67 Neb. 541, 93 N.W. 775, 777.
Sobol vs. Gulinson, 94 Colo. 1, 28 P. 2d. 810.
Blanford vs. Biven, 254 P. 1030, 123 Kan. 269.
Ferrill vs. Bryson, 37 S.W. 2d. 841 (Tex. Civ. App.).

[21] The matter is treated, and many citations are given, in 69 A.L.R. 1430, 1433, and in 11 C.J.S., Boundaries Sec. 67.

[22] Turner vs. Spicer, 249 S.W. 1038, 198 Ky. 739.
Robbs vs. Woolfolk, Tex. Civ. App., 224 S.W. 232.

[23] Hill vs. Walker, Tex. Civ. App., 140 S.W. 1159.
Smith vs. Seitz, 89 A. 257, 87 Conn. 678.

[24] Tabor vs. Craft, 116 So. 132, 217 Ala. 276.

[25] Turner vs. Spicer, 249 S.W. 1038, 198 Ky. 739.

[26] Rottman vs. Taft, 204, N.W. 585, 187 Wis. 558.

[27] Giddings vs. Hadaway, 28 Vt. 342.

Now, lest Mr. S might (from the emphasis thus far laid on agreement or acquiescence) think that the author is saying that such acquiescence concerning another boundary than the original surveyed line destroys all significance of the latter, two points should be kept in mind:

The first is merely a matter of interest to be noted in passing; namely, it has been held[28] that the conveyance of land by government subdivisions, without any representation of quantity, conveys only that portion included within boundaries acquiesced in for more than the period of the statute of limitations, and visibly marked, even though it is less than the full area of such subdivisions.

The second point, of more general consequence, is that although such acquiescence or agreement in a boundary line is binding on the parties thereto and on those claiming under them[29] (grantees, heirs, etc.) only such persons are bound by the acquiescence or agreement.[30]

This conclusion has some implications that would lead out of the immediate field of the present discussion; but the point to be noted here is that, although agreement, acquiescence, or adverse possession may have taken away from the original location of a section line, for example, its character as a boundary line between the contiguous parcels, it may still be essential to locate the original position of the section line and of the section corners thereon as sources from which to work in locating other lines or corners. Even though the significance of the first lines as immediate boundaries may have disappeared, they are just as effective (if they can be found) as they ever were in every other respect.

THE SURVEYOR AS AN EXPERT WITNESS

It is to be hoped that Mr. S can conclude for his client the questions of boundary with which he may be concerned without the necessity of the latter becoming involved in an action in court. There are some situations, however, in which only a court can settle the matter properly. In such a case Mr. S should be able to be of material assistance to his client's attorney; and this he can do better if he has some knowledge of the law concerned. Attorneys, especially those in general practice, are not always entirely familiar with matters of boundaries, with the engineering terms involved, with the practicability or the difficulty of accomplishing, in special situations, things which seem theoretically possible. It should not disturb the professional pride of a lawyer to review his trial brief with the surveyor; and the writer has known of cases in which, as he believes, the procedure and possibly the result could have been improved had that been done. Ordinarily the surveyor will be used as an expert witness. That this may be done effectively requires that the examining attorney and the prospective witness go carefully over the matter beforehand. The witness may well explain, first (if it is not already known to the attorney), just what the witness' testimony should show. Questions can then be framed which will clearly and expeditiously bring these facts before the court. If the surveyor listens carefully during the trial, especially as other surveyors give their testimony, he can frequently be of material help in catching points or suggesting questions of a technical nature. None of this savors in any way of shady practice or of any variation from the truth. No reputable attorney will ask or expect a reputable engineer to become involved in any "sharp" practice to gain a technical advantage, nor to vary from the truth as he sees it in giving his testimony.

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[28] Johnson vs. Trump, 161 Iowa 512, 143 N.W. 510.

[29] Amber vs. Cain, 110 N.W. 1053 (Iowa, 1907).
Rydalch vs. Anderson, 37 Utah 99, 107 P. 25.

[30] Ivey vs. Cowart, 124 Ga. 159, 52 S.E. 436, 110 Am. St. R. 160.
Sawyer vs. Coolidge, 34 Vt. 303.
Kyte vs. Chessmore, 94 Kan. 611, 146 P. 1152.

It may be worth while to inject here the statement that even in states which have statutes providing that no surveys shall be legal evidence unless made by a county surveyor or his deputy, or by the United States or by the state, it has been held that such statutes do not preclude the admissibility of testimony of a competent private surveyor concerning a survey made by himself, or concerning a plat of it that he has made.[31]

It is clear that, in giving testimony, a surveyor may refresh his memory by referring to his field notes or to a plat made from his notes. This is true even if the survey or the plat were not made in all respects in accordance with a statute providing methods therefor.[32]

Admissibility of Plats and Field Notes

The following excerpt from Corpus Juris Secundum[33] states concisely the general situation concerning the admissibility as evidence of maps and plats. The citations here given are only a few of those available:

> "As a general rule, maps or plats of surveys are admissible in evidence.[34] These include maps or plats made by the county surveyor or other official surveyors[35]; plats of unofficial surveys when made by a qualified surveyor[36]; maps from the land office,[37] although made subsequent to the beginning of the action[38]; and maps made by order of the state land board. Likewise, plats and maps attached to and made a part of deeds or grants or referred to therein, either expressly or by clear implication,[39] maps which have been recognized as correct by a former owner of the land,[40] and ancient

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[31] Von Eime vs. Fuchs, 8 S.W. 2d 824, 320 Mo. 746.
Cody vs. Black, 192 P. 282, 97 Ore. 343.

[32] Krider vs. Milner, 99 Mo. 145.
Gee vs. Sherman, 221 Mo. App. 121, 126.

[33] 11 C.J.S., Boundaries Sec. 113, p. 709.

[34] Martin vs. Hughes (Pa.) 98 Fed. 556, 39 C.C.A. 160.

[35] Price vs. DeReyes, 119 P. 893, 161 Cal. 484.

[36] Heinrichs vs. Terrell, 21 N.W. 171, 65 Iowa 25.
Nickel vs. Chapman, 158 N.W. 90, 163 Wis. 348.

[37] Giraud vs. Huffman, Tex. Civ. App., 46 S.W. 2d 367.

[38] Thatcher vs. Matthews, Tex. Civ. App., 183 S.W. 810.

[39] Daniel vs. Finn et al., 119 S.E. 307, 156 Ga. 310.
McDaniel vs. Leuer, 230 S.W. 633.

[40] Nichols vs. Turney, 15 Conn. 101.
Gates vs. McCormick, 97 S.E. 626, 176 N.C. 640.

maps and plats when duly authenticated,[41] are admissible to prove a disputed boundary. A map or plat referred to in the testimony is admissible as explanatory of it.

"In all cases the map or plat must be verified or authenticated,[42] and in the case of a surveyor's plat proof offered that the surveyor making the same actually traced the lines on the ground as depicted."[43]

However, in one case[44] it was held that

> "* * * plats and surveys attached to certain deeds and made by reference a part thereof were properly admitted in evidence with the deeds, notwithstanding the attached documents were not proved by the surveyor nor their correctness otherwise established. * * * Aliter [It would be otherwise] if the plats and surveys had not been made a part of the deeds themselves."

With reference to the plats or field notes of surveyors now deceased, one court[45] has held that:

> "* * * ancient plans of lots, showing the original lotting and boundaries assigned, also ancient minutes of surveyors, which were made by capable independent surveyors, having no interest to misrepresent, and which have been preserved, and are produced from reliable sources, may likewise be used on trial, when the boundaries of lots to which said surveys refer come in question."

To the same effect are cases from other jurisdictions.[46]

Some states have by statute made the matter relatively clear. An example is the following section from the Iowa Code:

> "11295. *Field notes and plats.* A copy of the field notes of any surveyor, or a plat made by him and certified under oath as correct, may be received as evidence to show the shape or dimensions of a tract of land, or any other fact the ascertainment of which requires the exercise of scientific skill or calculation only."

In situations quite analogous to those affecting the introduction in evidence of plats, the field notes of an authorized survey, when duly authenticated, are at common law admissible.[47] Also, at common law, the field notes of a deceased surveyor are admissible[48]; but at least one court has held[49] that a statutory procedure, recently enacted, under which county surveyors' field notes may be admitted, abrogates the common law which admitted the notes of surveyors, either public or private, and that therefore all other field notes than those of county surveyors are excluded. This would seem to be an undesirable result.

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[41] Brown vs. Metcalf, 215 Mass. 289, 102 N.E. 413.
Donohue vs. Whitney, 133 N.Y. 178, 30 N.E. 848.
Burgan vs. Siegman, 9 Ohio App. 84.

[42] Kearce vs. Maloy, 142 S.E. 271, 166 Ga. 89.
Cashion vs. Meredith, 64 S.W. 2d 670, 333 Mo. 970.

[43] Marcone vs. Dowell, 173 P. 465, 178 Cal. 396.
Maxcy vs. Norsworthy, Tex. Civ. App., 19 S.W. 2d 926.

[44] Hill vs. Snellings, 154 S.E. 156, 41 Ga. App. 585.

[45] Smith vs. Forrest, 49 N.H. 230, 239.

[46] Hamilton vs. Smith, 74 Conn. 374, 50 Atl. 884.
Prouty vs. Tilden, 164 Ill. 163, 45 N.E. 445.

[47] Giraud vs. Huffman, Tex. Civ. App., 46 S.W. 2d 367.
Neill vs. Ward, 153 A. 219, 103 Vt. 117.

[48] Stewart vs. Stephenson, 89 S.E. 1060, 172 N.C. 81.

[49] Peck vs. Molhoek, 228 N.W. 721, 249 Mich. 360

The older practice has been to take field notes in a bound book, and it is undoubtedly that form of field book which has usually been introduced in evidence. Mr. S may prefer a loose-leaf field book, and a question may arise in his mind as to the admissibility of notes taken on such loose sheets. Whatever may be one's opinion of the judgment of Mr. S if he decides to use loose sheets, even in some form of "binder," for field notes, it is still probable that if the sheets are properly identified and authenticated they would be admissible under the same circumstances that would admit a bound book. The writer knows of no case directly in point. The practical difficulty of authenticating and handling half a dozen separate sheets bearing the notes of a survey, as compared with the corresponding use of a book, would seem likely to produce some annoyances and danger of loss. The foregoing opinion concerning admissibility is based, by analogy, on modern usage concerning office records. By statute in some states, and by court decision in others, loose leaves[50] or cards[51] bearing office records are admissible as evidence when properly identified.

It is true that loose, miscellaneous memoranda have generally been excluded, but "the admissibility or inadmissibility of such entries would appear to rest more on their form than on the nature of the book in which they are made."[52]

DEFINITION AND OWNERSHIP OF FIELD NOTES

It has seemed that not all usages are quite consistent as to what is meant when the term "field notes" is used. Field notes are defined in Corpus Juris as "Notes made by the surveyor in the field while making the survey, describing by course and distance, and by natural or artificial marks found or made by him where he ran the lines and made the corners."[53] "Words and Phrases" uses the same definition, referring to the same case therefor; and with this definition a surveyor should have no quarrel. However, in spite of this plain statement in this case, apparently taken by both authorities quoted as the best available definition of field notes, some usages seem to refer to a description subsequently "written up" from the notes which were actually put down at the time measurements were made or objects were observed. Some substance is lent to this view by an inspection of the books said to contain the "original field notes" of original government surveys of the public lands. To make a very conservative statement, these books, and the neatly ink-written narratives that they contain, do not seem likely to have been with the surveyors in the field under some of the conditions of weather and work described therein. The terms "field book"[54] and "field notes" are sometimes applied to books and contents that are obviously office (usually a public office) records.

These observations are made in reference to suggestions sometimes heard that the field notes of a survey should be turned over to the client upon completion of the work as part of that for which he presumably has paid. It is the opinion of the writer, and of others with whom the matter has been discussed, that this would not be the best arrangement, either from the viewpoint of the surveyor or from that of his client. It is the opinion of the writer that Mr. S should turn over to his client, in the most useful form practicable, all the essential information that has been accumulated at the expense of the client. The most useful form (in addition to whatever physical evidences of the work are left on the site) will probably be a plat. A written description of the boundary may sometimes be useful, and to this some persons might give the name "field notes." A more probable label would be a "survey," which name seems to be used indiscriminately to refer either to the work done or to the report of such work in the form of a description.

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[50] Queen City Sav. Bank vs. Rayburn, 163 Fed. 597 (aff. 171 Fed. 609, 96 C.C.A. 373).
Wylie vs. Bushnell, 277 Ill. 484, 115 N.E. 618.
United Groc. Co. vs. Dannelly, 93 S.C. 580, 77 S.E. 706, Ann. Cas. 1914D, 489.

[51] Haley, etc. Co. vs. Del Vecchio, 36 S.D. 64, 153 N.W. 898, L.R.A. 1916B, 631.

[52] 22 C.J. 871, Sec. 1046.

[53] State vs. Palacios, Tex. Civ. App., 150 S.W. 229, 236.

[54] Neill vs. Ward, 153 A. 219, 103 Vt. 117.

This plat or other report should be as complete as circumstances will permit. It should be sufficient to enable any qualified person to identify the land or to recover an obliterated bound at any time in the future. The withholding of any essential data for the purpose of insuring reemployment on any future work in the vicinity cannot be too strongly condemned; but the field book—the book used in the field at the time the work is done, and in which the data are set down, usually in pencil, as they are observed—may better stay in the hands of Mr. S. It is quite probable that these notes will be in a tabulated form, in which they would be of little use to a client who was not himself familiar with survey work and notes. They probably contain data concerning random traverses, and the like, which would only be confusing to him. Moreover, cases which have been cited herein indicate, incidentally, that field notebooks are frequently found in the possession or among the effects of those who took the notes. The writer has neither read nor observed in his own practice anything that would lead him to believe that there has been, or is, any custom of turning field notes of private surveyors over to their clients; nor has he seen anything in statute or reported decision to cause him to think that it is expected—let alone required.

Of course, the terms or circumstances of the engagement of an engineer, either as a public official, such as a county surveyor, city engineer, etc., or as an employee in a private office, often make it clear that the notes he takes are not his private property but belong to the office. The remarks that have been made concern the engagement of a private surveyor or engineering firm on private work for a client.

SURVEYOR'S REPORT TO HIS CLIENT

One of the final matters with which Mr. S will be concerned in a survey is the furnishing of an adequate description of the land surveyed as one of the chief elements of his report to his client. The two purposes which this description must serve should be constantly kept in mind:

(1) It must make convenient and certain the identification of the tract of land for the purpose of tracing title; and

(2) It must be sufficient to enable any competent engineer to restore conveniently at any future time a bound (or all the bounds) which may have been obliterated.

For the first purpose, in order that it may be incorporated in a deed, a verbal description, giving general location, and perhaps adjoiners, is essential. It should be as brief as practicable, should be such as may conveniently be used over and over in successive instruments, each of which should refer to a preceding one; and, as an item of paramount importance, it should incorporate a plat by referring to it so as to make the plat a part of the deed description. That the complete description given on the plat will then determine the boundaries of the land conveyed is now too well settled to require authority for the statement here.[55] For the most part the cases cited in discussions of this matter are situations in which some special point had caused the raising of a question; for example, it is immaterial by whom the map was made[56]; or the plat was not acknowledged or recorded[57]; or it was invalid.[58]

Needed for the first purpose mentioned, but absolutely essential in many cases for the second, is the incorporated plat. It is a matter of common knowledge and observation that graphical descriptions are often not only much more convenient than verbal ones, but much less likely to be misunderstood. To this rule land descriptions are no exception.

Mr. S has been assumed to be a competent engineer; therefore, one need not here review the specifications for a satisfactory plat,—other than the fundamental requirement expressed under Purpose (2). It should be so numbered or titled as to be referred to conveniently in the verbal part of the description; and it should be certified or acknowledged by the surveyor, and of course made a part of the record.

SURVEYOR'S FINAL OBLIGATION

Finally, with reference to the service which Mr. S should render his client, it may be stated that he should do his work so that it is unlikely that that particular job will ever have to be done again; and, paradoxically, it should be so done that it could be retraced conveniently by any competent engineer —stranger to the locality though he may be—at any time in the future. The first of these two requirements can best be attained by the use of distinctive and enduring monuments. To economize by setting wood stakes or other cheap monuments to perpetuate a survey is indeed being "penny wise and pound foolish." From a legal, engineering, or just good common sense standpoint, the best advice that can be given to a client in this respect (and all too often the client must be "shown") is to add to the cost of the survey enough to monument it well.

Concerning the second requirement—that of adequate description—beyond that which has been said, it is desired to emphasize but one point: It is sometimes true that plats which are complete in themselves are only inadequately located with respect to adjoining land. Until very recently no one satisfactory means of stating or showing such relative locations has been generally available. Now, with state plane coordinate systems devised for every state and permanently connected with the national control system of triangulation, such a means is at hand. As time and surveys go on, this means is becoming accessible to more and more land. Wherever this control is available, practically and economically, it should be used. Whenever practicable the plat should show in terms of coordinates in the state system the positions of one or more monuments on the boundary, and of one or more monuments on a control survey. Directions should then be stated in azimuths, referred, of course, to the same system. A statement on the plat should make it clear that this has been done. Statutes have been enacted in some states making definite the fact that such is an adequate description; but it is believed that, in the absence of any statute, the common law which would be applied in any state is clear that such a description is good. There are many decisions to the effect that a "deed must contain such a description * * * as will enable the property to be readily located by reference to the description"[59]; and also to the effect that "if the description of the land conveyed in a deed is such that a surveyor, by applying the rules of surveying, can locate the same, such description is sufficient * * *."[60]

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[55] See 9 C.J. 180, sec. 50.

[56] Finelite vs. Sinnott, 125 N.Y. 683, 25 N.E. 1089.

[57] Johnstone vs. Scott, 11 Mich. 232.

[58] Young vs. Cosgrove, 49 N.W. 1040, 83 Iowa 682.

It is for the quality of description specified by these quotations that the writer pleads: Such a description as "a surveyor"—any surveyor—can effectively use "by applying the rules of surveying"; not something that merely furnishes an opportunity for the exercise of ingenuity in the solving of a puzzle. The proper use of the state plane coordinate systems, now officially approved for this purpose by both the Society and the American Bar Association, is, in the opinion of the writer, the best means thus far made available for assuring the certainty and the permanence that are required.

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[59] Saterstrom vs. Glick Bros. * * * Co., 5 P. 2d 21, 118 Cal. App. 379.

[60] Neves vs. Flannery, 149 So. 618, 111 Fla. 608.
Brooks vs. Pryor, 189 So. 675.

DISCUSSION

E. F. CHANDLER,[61] M. AM. SOC. C. E.—Special commendation is due this paper because it brings so definitely to attention one very important feature of the work of a land surveyor making field measurements to determine the proper location of doubtful or disputed boundaries. This is the fact that if the surveyor is not merely careful in his surveys but also fairly informed on a certian line of legal questions he is often able (unofficially, but by willing consent of both parties) really to fulfil a judicial function and bring about fairly harmonious agreement between parties who might instead have resorted to costly hearings in court.

The nontechnical purchaser is quite likely to think, and can easily be misled to believe, that his deed gives absolute and complete legal title to the lengths and acreages specified by it, which of course is not invariably true.

For example, careless work in the early surveys many years ago may have vitiated all the data recorded in them. A striking case of this came before the writer in his own early experience, in south central Wisconsin, where there was a dispute concerning boundaries between adjoining land owners, each of whom had a deed of very convincing prima facie appearance. In both cases, dimensions were taken directly from the plats of the original government contract surveyor, who had staked off the townships and sections there in some year of the late 1840's. About fifty years later, the writer retraced a series of the lines, and was fortunate enough still to find indisputable evidence (witness trees, bearing trees, etc.) at almost every adjoining section corner and quarter-section corner. On one line between corners (east and west in Section 6, and therefore not assumed or reported as a precise mile) a so-called mile was found to be actually about 900 ft less than the figures certified for it by the original contract surveyor. No surplus was in any adjoining section that could by any possibility be obtained on the plea of an alleged mistake or blunder by the original surveyor in accidentally witnessing some trees at an unintended point. In this case, however, the land had been so long occupied without objection that it was easy to make the most injured claimant realize that, although his quarter section was really 900 ft narrower than the distances alleged in his deed, it would be a useless expense to bring any suit or to attempt to reclaim something that actually did not exist.

To avoid stirring up needless claims or disputes based upon present-day failures of tracts to fit precisely the reported metes and bounds certified by the surveyors of earlier days (some of whom were quite hurried, careless, and inaccurate, and some even worse in framing up semi-fraudulent description), the land surveyor now should keep clearly in mind the usual rule for accepting priority of calls in the early surveys, which holds (except in special or unusual cases where there is extra evidence supporting one or another of the conflicting items)—namely, the general rule that definite unchanged landmarks, monuments, or permanent stakes and fences outweigh all other evidence; lengths and distances have less weight, but outweigh directions or bearing; bearings are good evidence only as compared with areas; and areas have little or no value in fixing boundary-line locations, being merely corroborative unless the other calls are lacking.

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[61] Prof., Civ. Eng. and Dean Emeritus, College of Eng., Univ. of North Dakota, Grand Forks, N. Dak.

The author refers briefly to another principle that the surveyor must constantly remember if he does not wish to encourage his client in useless disputes—the doctrine of "adverse occupancy." Briefly worded, in most states undisputed continuous possession "under color of title" through ten years gives the occupant indisputable ownership; or even without color of title he gains ownership in twenty years. In different states there are variations, so that in some localities even half this time (five years, or ten years) is sufficient; there are different technical exceptions in different places (such, perhaps, as the law not beginning to hold against minor owners until they reach the age of twenty-one, and never holding against the general public, but merely against individual owners, etc.); and, of course, the occupancy must have been absolutely undisputed, without legal objection at any time in the entire period. The surveyor must, perforce, keep in mind the effect of that statute of limitations, whatever its precise form may be in that jurisdiction, and must not unduly encourage his client to hope to gain or regain, merely by mathematical measurements, the possession of real estate, the ownership of which has really been lost by years of neglect.

If the surveyor is careful, however, is fairly accurate in the engineering work of his surveys, has a well-founded general idea of the legal principles attaching to them, and has good judgment in avoiding the appearance of prejudice or bias, he will usually find his recommendations accepted (perhaps rather unwillingly, but because seeming the most reasonable to both sides) without any need for final resort to courts of law, or to the vote of a jury of twelve men, none of whom is likely to have more than the merest superficial knowledge of such matters.

If a case does actually come to court trial, the evidence of a surveyor as an expert witness on matters to which he can properly testify should be practically incontrovertible. Furthermore, as stated by the author, if he does not claim greater proficiency in legal knowledge than the lawyer himself, the lawyer probably will be glad of his advice or assistance on the technical sides of the question. If the case is quite important, the lawyer may even (as has repeatedly befallen the writer) ask the surveyor-engineer to make for him the preliminary draft of the engineering part of his brief.

HARRY RUBEY,[62] M. AM. SOC. C. E.—This is a valuable paper by the author on the professional aspects of a subject too little understood by engineers. The Society is indeed indebted for the many references that accompany the paper—references that most engineers would otherwise overlook. From the wide range of possible discussion of the paper, the writer will attempt only to emphasize the professional nature of the work, as distinguished from the subprofessional status ordinarily assigned to it.

Too often the subject of boundary surveys is discussed and practiced as a distinctly subprofessional activity. The author is entirely correct in implying that a highly professional engineering judgment, often involving a considerable legal knowledge, is required for boundary surveys that fall outside of a simple routine.

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[62] Chairman, Civ. Eng. Dept., Univ. of Missouri, Columbia, Mo.

The actual surveying—that is, the making of measurements and calculations—frequently might be classified as subprofessional; but when the engineer so engaged is called upon to exercise legal knowledge and judgment in connection with surveys, the work takes on a professional character, and in many cases may become highly professional.

Although the private engineer possesses no formal legal status other than that of expert witness, actually and informally he must advise his client and conduct his surveys in accordance with accepted legal principles. In many cases, the engineer must cooperate on a definitely professional basis with an attorney, both in the preparation of a case and in the negotiations or court proceedings connected therewith. From the moment the engineer is approached by a client regarding a boundary survey, his advice and action must be based on sound legal knowledge and professional judgment.

When it is realized that the interests of the client, the engineer, and the community are best served by securing the facts, by studying the case, by definite solution from a survey, by settling the controversy through agreement or by arbitration, and perhaps finally by court action, the professional nature of the engagement becomes apparent.

Another definitely professional phase of this subject in which the Society cooperates with the American Bar Association was stressed in 1941.[63]

It would be most valuable if the author could give in some detail the engineer's ethics, techniques, and procedures in the matters of arbitration and expert testimony.

All too few qualified engineers and attorneys are available for professional boundary survey assignments, and the published information and discussion of the matter are entirely too limited.

WILLIAM H. RICHARDS, JR.,[64] M. AM. SOC. C. E.—Professor Holt renders a service to those interested in boundary surveys in his discussion of the duties and responsibilities of surveyors when engaged in recovering and establishing boundaries of land. His paper indicates the combined technical and legal character of the land surveyor's work and demonstrates the necessity for some knowledge of the law on this subject and of legal procedure by the civil engineer who engages in this field of activity.

Under the heading "Definition and Ownership of Field Notes" the author observes that field notes, although defined in Corpus Juris as "Notes made by the surveyor in the field while making the survey," also, apparently, under some usages, refer to a description subsequently written up from the notes actually taken in the field. He mentions in this connection the "original field notes" of the public land surveys.

There is a very definite distinction between the notes and data taken by the private surveyor during the progress of his field work and the official field notes of the public land surveys. The current edition of "The Manual of

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[63] "Land Surveys and Titles," Second Progress Rept. of the Joint Committee of the Real Property Div., American Bar Association, and the Surveying and Mapping Div., Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., June, 1941, p. 1065.

[64] Cadastral Engr., Gen. Land Office, Div. of Surveys, Washington, D. C.

Instructions for the Survey of the Public Lands of the United States" contains the following statement in this connection: "Section 542 The initial notes are kept in pocket field tablets. The final field notes for filing are transcribed from the field tablets, and are typewritten upon regulation field note paper."

The 1881 edition of the manual of instructions contained the following statement relative to field notes:

> "From the data thus recorded [in field tablets] at the time when the work is done on the ground, the deputy [surveyor] must prepare true field notes of the surveys executed by him, in the manner hereinafter prescribed, and return same to the surveyor general, together with the required sketches, at the earliest practicable date after completion of his work in the field."

Such final or true field notes, when duly certified by the surveyor, approved by the supervisor of surveys (formerly by surveyor general), and accepted by the commissioner of the General Land Office, are the official field notes.

The actual running of the lines of a survey in the field and the recording of the notes are not alone sufficient to constitute an official survey. Until all the requirements as to approval are complied with, the survey and its record are not regarded as complete, and the lands are still to be classed as unsurveyed. In other words, to justify the application of the term "surveyed" to a body of public land, something is required beyond the completion of the field work—namely, approval by the authorized officials of the federal government.[65]

C. B. HUMPHREY,[66] ESQ.—It is well that, from time to time, there appear, in the technical publications that reach the land surveyor, papers dealing with the problems of the surveyor in establishing and re-establishing boundary lines, and to remind him that, although the true lines of ownership are matters for a court, he may report the lines to the best of his knowledge.

Professor Holt's paper dealing with this subject is timely, when communities in the United States, with Federal Aid, are establishing coordinate systems to which may be referred all public and private surveys. His remarks upon the duties and limitations of a surveyor are most instructive, especially to young surveyors who, at times, seem to overlook the fact that most of their problems, pertaining to the interpretation of a boundary line, already have been determined by a court and the data acquired by a little research work.

The writer recalls reading "The Judicial Functions of Surveyors," by Mr. Justice Cooley of the Michigan Supreme Court and joins with Professor Holt in recommending that it be read by all interested in survey work. It appeared as "Appendix A" in "Theory and Practice of Surveying"[67] in 1892 and the writer, in his official duties, often quotes parts of this paper.

With reference to the establishment of a disputed boundary line by parol evidence or by agreement of interested parties, the surveyor may perform his functions as a surveyor by making a survey, setting bounds and preparing a plan for recording. As Professor Holt states, however, it may not be a judicial determination (which is a matter for a court) but it may be adjusted by direct grants between the respective owners.

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[65] See U. S. Supreme Court decision in the case of Cox vs. Hart, 260 U. S. 427.

[66] Chf. Engr., Massachusetts Land Court, Boston, Mass.

[67] "Theory and Practice of Surveying," by J. B. Johnson, John Wiley & Sons, New York, N. Y., 1892.

In a Massachusetts case[68] that was an appeal from a Land Court decision, there was evidence that the disputed line had been fixed by a parol agreement by predecessors in title and the court ruled that although parties could agree orally, no title would pass by force of such agreement unless followed by occupation.

In proceedings in the Massachusetts Land Court there are many cases in which, after all the evidence is presented, disputed boundary lines are settled by written agreement or stipulation between the parties. The line thus agreed upon becomes a part of the final decree of the court confirming the title to the land and fixing all boundary lines.

The author has carefully emphasized that the determination of a disputed boundary line by either of the foregoing methods does not necessarily mean that it is the exact location as laid down by the original surveyor and shown on some recorded plat referred to, and from which one could, with safety, measure to reproduce all other lines.

In a Massachusetts case[69] the court held that the decision in a previous case whereby the line of a street was determined, although subsequently it was found that it was not determined in accordance with the plan from which the petitioners obtained title, did not prevent the petitioner in an adjacent case from insisting upon having the line of the street located in its proper position where it bounded upon his land.

To whom belong the field notes for a private survey? The writer contends that they belong to the professional surveyor, but that the surveyor should deliver to his client a plan and report that would be of value to a subsequent owner, and that he should show what an expert surveyor would call proper engineering data, and not a plat with plotting lines drawn to scale with distances only.

He should further recommend, as Professor Holt has so well stated, that if no monuments exist, sufficient monuments should be set to perpetuate and mark the lines surveyed and established. It is not always the fault of the surveyor that monuments are not set, but due to a short-sighted policy on the part of his client, which, in many cases, is the cause of the plan and survey being practically worthless when retraced after many years.

The author has presented an unusual paper, with helpful references to court cases, and it is hoped that it will be read by all those engaged in surveying practise. The writer also wishes to call attention to the coordinate systems now established by many states, a part of the expense of which is being borne by the federal government. The use of these systems by the engineering departments of various political subdivisions will enable more points to be established and should to a certain extent eliminate the duplication of survey work.

Many years have passed since G. Washington, Surveyor, placed the "corner stone" for the plantation under the Natural Bridge in Virginia and wrote his initials on the side of the cliff. The poplar trees, black oaks, white oaks and hickory corners mentioned in the "King's Grant" have long disappeared. Directions are no longer given in degrees only and distances in rods. The present-day surveyor, with modern instruments, must regard such common sources of errors as plumbing, alinement, temperature, wind, etc., and must think in terms of coordinates when locating a point on the earth's surface. He must also have in mind the standard used by previous engineers in laying down the Geodetic Control Stations if he is to agree with subdivisions between control lines. With these precautions, and coordinate values of record on recorded plats, there should be fewer disputes in the future as to the location of boundary lines.

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[68] Crawford vs. Roloson, 256 Mass. 331.

[69] Pollard vs. Burchard, 199 Mass. 376.

D. D. HAINES,[70] ASSOC. M. AM. SOC. C. E.—In his "Introduction" Professor Holt makes a very good point when he states that a surveyor "must be ably advised on any given point concerning the law in the jurisdiction in which he is concerned." The mere fact that the law of one state provides that certain procedures must be followed in that state under certain conditions does not necessarily mean that the same procedure should be followed in another state. Statements of good law in one state must never be made to apply in another. A certain amount of local research on the subject must always be made. In laws relating to boundaries each state is an entity unto itself.

It is a well-known fact that not all states require that a surveyor be licensed, or registered, before he can do boundary surveying. However, if a surveyor is registered in a particular state, it is assumed that he is a "qualified" land surveyor; such qualification means that the surveyor has at least the minimum ability to exercise the necessary "ordinary prudence." Students invariably ask, "What is ordinary prudence?" This is a very natural question and one that defies a definite answer. Professor Holt has given the basis for the answer to such a question. Records must be pursued, boundaries examined, occupancy determined, and any other information carefully weighed; and furthermore the surveyor must tell his client just what the facts are as he sees them. The responsibility of a surveyor to his client is the same whether he is a registered or an unregistered practitioner. Should a surveyor fail to exercise "ordinary prudence," it is well known that he can be held liable for damages. To eliminate errors and secure the necessary precision in boundary surveying is not peculiar to that branch of surveying, nor to surveying in general, but to engineering as a whole. Professor Holt mentions the custom of giving a "certificate of accuracy" as a usual way of emphasizing accuracy. The writer would like to have Professor Holt enlarge on this subject.

The writer is frequently asked by students if a surveyor is actually a trespasser. The reply always has been that he is unless he has permission, or is exercising an official duty, and even then the surveyor is liable for actual damages as well as for any unnecessary damage he does. The author has made this point very clear. In the majority of cases, a surveyor can secure the permission of the owner to enter upon his land merely by explaining the need for such entry and promising to exercise due care. The surveyor should always keep in mind that he cannot force his way on to private property unless he has official authority, or unless a state statute definitely provides for such entry, and that in the majority of cases it is far better not to use authority unless absolutely necessary.

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[70] Asst. Prof. of Civ. Eng., Univ. of Kansas, Lawrence, Kans.

It is well that this paper states that any survey will "hold" and become a "legal" survey when: (1) The results are accepted by those affected; or (2) the court rules that the survey properly settles the problem. Therefore, private surveyors and official surveyors have equal authority to "establish boundaries." Not enough laymen understand this fact.

Boundary disputes can be settled without court action. The realization of this fact is frequently startling to the young engineer or surveyor. Agreement may be reached between adjacent landowners when not in the presence of a surveyor, but since a surveyor is generally a part of such discussions he frequently can serve as an arbitrator. The surveyor is a better arbitrator if he understands the law thoroughly and if he has collected and weighed all the facts. Any award he makes will be considered valid, and any boundary fixed by agreement remains fixed regardless of mistakes of judgment or measurements. Although agreements to fix boundaries need not be in writing, it is better practice to make them of permanent record. One point cannot be overemphasized with respect to boundaries established by agreement and that is that the correct boundary line still has its place and importance, in that it (and not the agreed line) must be used as a starting point in the location of other lines and monuments.

The writer would like to have Professor Holt enlarge on the statement: "Such an agreement to fix a boundary line is void if the owners know or if one of them knows that the agreed line is not the true line" (see heading "Arbitration").

Professor Holt is quite right when he states that attorneys of general practice are not always entirely familiar with matters of boundaries. In fact, he could go further than that and state that they are seldom more than moderately familiar with such matters. Along this same line it should be noted that surveyors should not hesitate to consult lawyers as to the correct procedure to follow under certain circumstances. There must be a certain amount of "give-and-take" among the two professions to solve problems to their mutual advantage.

The discussion on the admissibility of plats and field notes is very interesting. The author has given a good bibliography for reference. It seems that field notes and plats from any source, so long as they are properly identified and authenticated, may, under the usual circumstance, be admitted as testimony.

The writer is glad that Professor Holt has seen fit to express his opinion to the effect that the most useful form in which a survey can be submitted to a client is a plat, thus leaving the survey notes in their original form with the original surveyor. Of course, the plat must be adequate and must give all the final data pertinent to the survey that the surveyor's client requested. The author's point (see heading "Surveyor's Report to His Client") that "graphical descriptions are often not only much more convenient than verbal ones, but much less likely to be misunderstood," is a good one and should cause more and more plats to be made and appended to legal description.

The entire paper is very interesting and worth while. Although most of the points discussed are problems that have been present for some time, they are constantly recurring, and are of sufficient importance to make it worth while to have them so ably reviewed.

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TRANSACTIONS

Paper No. 2163

COST OF PUBLIC SERVICES IN RESIDENTIAL AREAS

By F. Dodd McHugh,[1] Esq.

With Discussion by Messrs. Donald M. Baker, Howard Whipple Green, T. T. McCrosky, and F. Dodd McHugh.

The Problem

Material available, although not conclusive, shows that the United States is faced with a slowing up in the rate of growth. The greatest intensity of this change is felt in cities. During the decade 1910–1920 the population of New York, N. Y., increased 17.8% and that of the nation 14.9%. The increase was 23.3% for the city and 6.1% for the nation during the period 1920–1930. The U. S. Census Bureau reports an increase (in 1940) of only 6.5% for New York City and an increase of 7% for the continental United States since 1930.

Modern trends reveal a shifting of people from the older sections of the city to newer residential developments; but the older sections are only partly vacated and are rarely reoccupied by an influx of people or by new commercial activities. The preliminary report of the Census Bureau shows that Manhattan's population remained practically unchanged between 1930 and 1940, whereas the population of the four other boroughs of New York City increased from 4% to 20%. It now appears that, since the 1934 Real Property Inventory of New York City, some of the older sections of Manhattan have continued to lose population. The new developments in the northern part of the borough have apparently attracted a sufficient number of residents to offset the loss in older sections. The exact extent of this population shift cannot be determined until detailed information by census tracts for 1940 has been analyzed.

In 1934 the Real Property Inventory revealed that about 12% of all dwelling units in New York City were vacant, but that about 5% of Manhattan's dwellings were unoccupied. This inventory also shows that only four fifths of the dwelling units were occupied in the older apartment sections

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Note.—Published in June, 1941, *Proceedings*.

[1] Director of Planning, Dept. of City Planning, New York, N. Y.

of Manhattan covered by this study. The Census Bureau's tentative report shows that 7.6% of all dwelling units were vacant in New York City as of April, 1940, and that about 11% were unoccupied in Manhattan. The census shows a gain in the occupancy ratio for the entire city as compared with 1934, but a decline for Manhattan. The meager data now at hand do not suggest that the older apartment districts of this borough have made any considerable gains in population subsequent to 1934. In fact, the preliminary census figures along with building permit, foreclosure, tax delinquent, and rental reports indicate that since 1934 the occupancy of dwellings in the older districts of Manhattan has probably not improved in relation to the city-wide occupancy ratio. Lacking any definite information to the contrary, it may be expected, therefore, that people are still leaving these older apartment districts to live in newer developments.

Why do people move from the older sections of cities? One explanation, given by Clarence A. Perry,[3] is applicable to practically all of the residential sections of New York City and other cities:

> "A prominent New York real estate man recently expressed the opinion that the 'normal life expectancy' of an ordinary dwelling was two or three times that of its neighborhood. In other words, the quality or desirability of a particular district tends to decline long before its component houses begin to wear out. Even so, this observation acquires a general significance only if it is discovered that the neighborhood deterioration affects the value of its individual houses. When the owner of a dwelling finds he can keep it occupied only by progressively lowering its rent, then its actual value is declining and the matter is of importance to the public, since that property is no longer able to bear its original share of the tax burden.
>
> "If this view is correct then the unsatisfactory conditions that lead families to give up their domiciles and move to new quarters become of private and public concern. If the dissatisfaction has arisen from something in the environment, it should be of special interest to the community because the matter is then beyond the power of the individual to correct and must receive attention from the public. That neighborhood conditions do cause people to move is a common view, but there is no unanimity as to just what these conditions are."

When old sections lose population, public services in these sections must be maintained and are usually operated below actual capacity. Meanwhile, additional public services must be supplied to the newer residential developments that are drawing population from the older sections of the city.

A survey made for the (New York City) Department of City Planning shows that one third of the city's gross area was vacant and unimproved in 1938. The 1940 federal census reports that in most of the larger cities of the United States, as well as in New York City, the rate of growth is slowing down. It is apparent from these facts and other considerations that the city has more than enough land available to house its present residential population and commercial activities, as well as any increase to be reasonably expected in the future. This outlook does not evidence a positive need to continue the extension of the city's physical growth to outlying undeveloped sections. The present economic and social conditions found in New York City obviously exist in many other cities. The prevalence of such unstable conditions would seem to require careful re-examination of the urban structure with particular reference to the possibilities of revitalizing the older sections of American cities.

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[3] "Housing for the Machine Age," by Clarence A. Perry, Russell Sage Foundation, New York, N. Y., 1939, p. 15.

Constitutional and statutory provisions usually limit the debt-incurring and real-estate taxing powers of municipalities to a fixed percentage of the assessed valuation of real estate. In general, assessed values are not rising, and the increase of urban government income from all sources, including real-estate taxes, has failed to keep pace with the growing demand for expansion of public services. Consequently the cities' financial capacity to meet changing urban conditions is threatened seriously.

Some of the conditions underlying this study are peculiar to New York City. To the extent that such factors as high density of buildings and population reflect New York characteristics, the findings are not directly applicable to other municipalities. It seems, however, that the methods employed may be useful in measuring the magnitude of a problem that is common to most American cities, and that any differences between one city and another as may be revealed by similar studies will be mainly a matter of degree.

PURPOSE OF STUDY

An attempt is made in this paper to examine the problem in terms of city expenditures for permanent improvements and public services in old and new residential areas. Two types of residential areas are investigated: (1) A specimen community that is typical of the centrally located, existing residential development; and (2) a newer type of "neighborhood unit" development that is applicable to the rebuilding of old sections or in developing outlying vacant areas.

Costs are estimated for the public services found in the specimen community that represents existing conditions and for replacing the obsolete part of existing public facilities in this old area. The study also explores the public costs involved in rebuilding the old area on a neighborhood unit pattern, and in an identical new development to house the same population on raw land.

The sums that the private developer must spend for land and construction and carrying charges are not considered. Extensive studies by Messrs. Thomas Adams, M. Am. Soc. C. E., and the late Robert Whitten,[3] and Clarence Perry,[4] the President's Conference on Home Building and House Ownership,[5] the National Resources Committee, and others,[6] have approached the problem from planning and housing viewpoints with particular reference to the charges borne by the individual home owner and tenant.

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[3] "Neighborhoods of Small Homes," by Robert Whitten and Thomas Adams, Harvard Univ. Press, 1931; and "Design of Residential Areas," by Thomas Adams, Harvard Univ. Press, 1934.

[4] "Housing for the Machine Age," by Clarence Arthur Perry, Russell Sage Foundation, New York, N. Y., 1939; and "Neighborhood and Community Planning," Regional Survey, Vol. VII, Regional Plan of New York and Its Environs, 1929.

[5] "Planning for Residential Districts," Vol. I, and "Slums, Large-Scale Housing and Decentralization," Vol. III, the President's Conference on Home Building and Home Ownership, Washington, D. C., 1932.

[6] "Land, Materials and Labor Costs," *Housing Monograph Series, No. 3*, National Resources Planning Board, Washington, D. C., 1939; see also "Rehousing Urban America," by Henry Wright, Columbia Univ. Press, 1935.

PROCEDURE

A study of this nature is necessarily developed from, and limited by, hypotheses that may not be accepted in their entirety. It is believed, however, that the principles underlying this study furnish a sound approach to a complex problem.

The procedures used in this study involve the selection of the types and geographic location of specimen residential communities; the determination of land uses and building bulk in a specimen community representing old sections, and the allocation of land uses and fixing of building bulk in the "neighborhood units" as the basis for estimating the population that can be accommodated in each type of specimen community; the determination of the character and extent of public services existing in old sections and the needs for public services required to serve the residents of neighborhood units; and estimating the costs to the city for permanent improvements and annual operating expenses in each community.

CHARACTER OF DEVELOPMENT

The several community studies undertaken by the Mayor's Committee on City Planning prior to 1938 supply the most recent information on the character of development existing in the fully developed, high-density apartment districts located in the old central parts of the city. It should be noted that these communities are predominantly residential districts. In addition to dwellings, they have parks, schools, churches, local business, and other community facilities that serve residential needs. There are, however, cemeteries and industries in some of these communities that are not considered desirable within residential districts.

Existing Multi-Family Areas.—The Mayor's Committee Studies designated as East Harlem, East Side, and Yorkville are typical of existing conditions in the older apartment districts of Manhattan. Of their aggregate land area, 35.55% is used for streets on the gridiron system; only 2.7% is not built upon; 3.6% is parks; about 38.5% is residential; and the remainder is made up of business, industry, public buildings, and private institutions (see Table 1).[7] About 95% of the population lives in apartments and more than 87% of the residential land is used for multi-family housing.

The population was reported as 678,446 by the 1934 Real Property Inventory, but the total available family quarters were only 79.6% occupied. With full occupancy of all family quarters, 851,900 persons could be accommodated in the housing existing in these communities in 1934. The land areas expressed as acres per 1,000 persons in Table 1 are based on this estimate of "population provided for in existing housing."

If all apartments were occupied, the 851,900 persons housed on the 1,254.4 residential acres would produce a density of 678 persons per net acre! Allowing 70% coverage of residential land and 242 sq ft of building area per person, apartments would average 5.3 stories in height to house this population. As a rough check on this value for average height, it is only necessary to inspect these communities. One finds that most of the buildings are old "walk-up" flats or brownstone houses converted into apartments. Relatively few "new law" multi-family buildings of more than six stories will be seen and most of them are found on main thoroughfares. In Yorkville, for example, such apartments are situated along the north-and-south avenues and the major east-and-west streets.

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[7] Data from East Harlem, East Side, and Yorkville Community Studies of the Mayor's Committee on City Planning (1935).

TABLE 1.—"BUILT-UP EXPERIENCE" IN CENTRAL APARTMENT COMMUNITIES OF NEW YORK CITY

Land Use	Area		Percentage of:	
	In acres	In acres per 1,000 persons	Gross area	Developed area
Developed Area:				
(a) Net Usable—				
Residential	1,254.4	1.47	38.49	39.56
Business	276.8	0.32	8.50	8.73
Industry	149.8	0.17	4.60	4.72
Institutions	209.4	0.28	6.42	6.61
Parks and Playgrounds	119.9	0.14	3.67	3.78
Cemeteries	1.5	......	0.04	0.05
Total net area	2,011.8	2.36	61.72	63.45
(b) Streets in use	1,158.8	1.36	35.55	36.55
Total developed	3,170.6	3.72	97.27	100.00
Vacant land	89.1	0.10	2.73	......
Gross area	3,259.7	3.82	100.00	......

The 1934 Real Property Inventory shows that about 85% of the buildings in East Harlem were erected prior to 1900, and further, that about 14% of these structures are between twenty and thirty-five years old. More than one fifth of the residential buildings need major repairs and two thirds require minor repairs; only 9% are in first-class condition. The 1935 survey of the Mayor's Committee on City Planning discloses that (1) nearly 23% of the properties in East Harlem were tax delinquent, and (2) that of a total assessed valuation of $263,000,000, land accounted for $125,000,000 or 47% of the total.

New Residential Units.—The diverse and sporadic development of individual properties under built-up experience usually produces the situation reflected in the foregoing examination of existing multi-family areas.

It is believed that urban blight may be combated successfully through large-scale planning and development, both in older, central, deteriorated districts and in new outlying districts. If such large-scale developments are not to become obsolete faster than the individual buildings therein, they must be of a size sufficient to permit the establishment of a self-contained neighborhood that can create a desirable residential environment and can maintain this character by preventing the occurrence of conditions that have caused people to move away from existing residential communities, leaving vacant dwellings and partly used public improvements that must be duplicated elsewhere. These new neighborhoods must also offer the amenities for urban living that will enable them to compete successfully with the newer developments that attract people from older sections of the city to suburban districts.

The Neighborhood Unit.—It is not the purpose of this study to formulate means nor to evaluate procedures suggested by others to promote the development of the newer type of residential neighborhood. Mr. Perry has presented[8] possible approaches to the problems inherent in development of neighborhoods. The neighborhood unit concept, as presented by Mr. Perry, sets the pattern for self-contained communities, and was adopted with minor modifications as the basis for ascertaining the costs of public improvements and services in newer types of residential developments. "The neighborhood unit—a scheme for arrangement for the family-life community" consists of six principles:[8]

(1) *Size.*—A neighborhood unit should provide housing for that population for which one elementary school is ordinarily required, its actual area depending upon its population density.

(2) *Boundaries.*—The unit should be bounded on all sides by arterial streets, sufficiently wide to facilitate its by-passing, instead of penetration, by through traffic.

(3) *Open Spaces.*—A system of small parks and recreation spaces, planned to meet the needs of the particular neighborhood, should be provided.

(4) *Institution Sites.*—Sites for the school and other institutions having service spheres coinciding with the limits of the unit should be suitably grouped about a central point.

(5) *Local Shops.*—One or more shopping districts, adequate for the population to be served, should be laid out in the circumference of the unit, preferably at traffic junctions and adjacent to similar districts of adjoining neighborhoods.

(6) *Internal Street System.*—The unit should be provided with a special street system, each highway being proportioned to its probable traffic load, and the street net as a whole being designed to facilitate circulation within the unit and to discourage its use by through traffic.

In addition to the foregoing principles the neighborhood unit formula as used in this study implies that: (1) Building density should be limited to insure light and air to residents and to prevent unnecessary overcrowding of the land; (2) the unit should be fully developed in order to utilize land, buildings, and public services efficiently; and (3) public and private facilities should be adequate to meet the needs of the people living in such self-contained residential neighborhoods. These principles of the unit scheme can ordinarily be fully applied to new developments only. Consequently, the neighborhood unit principle may be limited to outlying unbuilt areas and to the rebuilding of centrally located deteriorated districts.

Types of Development Studied

This neighborhood unit formula seems to present a sound technique for the development of communities that will avoid the incipient blight and early deterioration experienced in existing residential sections. If this new type of development can establish and maintain a desirable environment for residential neighborhoods, stabilize development, and conserve values by helping to prevent unnecessary shifting of population, then the relative costs of public improvements in such neighborhoods are of considerable interest. This paper, therefore, undertakes to examine two types of development—(1) an area characteristic of existing development, and (2) a new neighborhood unit that could be used in rebuilding the old area or for a new development on vacant land.

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[8] "Neighborhood and Community Planning," Regional Survey, Vol. VII, Regional Plan of New York and Its Environs, 1929.

Existing Area Selected for Study.—The character and extent of development in specimen Area *A*, Fig. 1, is determined by the "built-up experience" exhibited in the three multi-family communities surveyed by the Mayor's Committee on City Planning. (The locations shown in Fig. 1 are not intended to designate specific areas but merely to show the approximate locations.) Area *A* is a fully developed apartment district situated in the central city; it is typical of the "built-up experience" in existing multi-family areas that are developed on a gridiron street plan. Such areas are usually found in the older sections of most large cities.

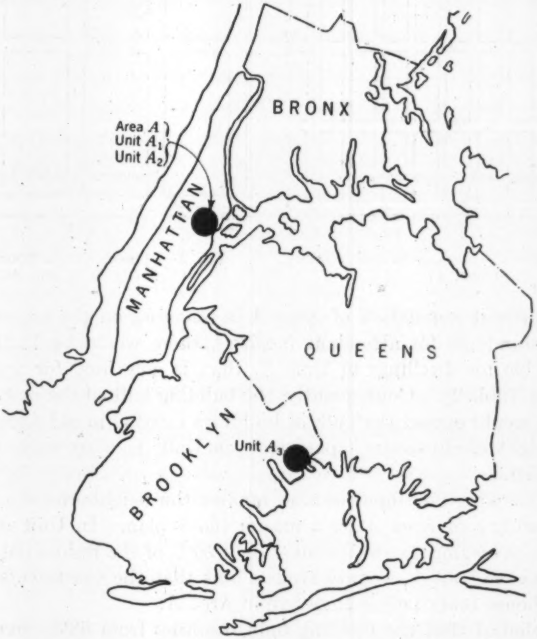


FIG. 1.—APPROXIMATE LOCATION OF THE 160-ACRE COMMUNITIES, NEW YORK, N. Y.

New Units Selected for Study.—Each of the neighborhood unit types of development selected for study is predicated upon the principles underlying the unit formula and upon the following minimum "standards":

Unit A_1.—This development represents an attempt to apply the neighborhood unit scheme in the rebuilding of Area A on a super block plan, which involves the conversion of the "normal" street system shown in Fig. 2 to the master block plan in Fig. 3. It provides sufficient new apartments to rehouse the present population of Area A, and the necessary private facilities and public services required to serve this population.

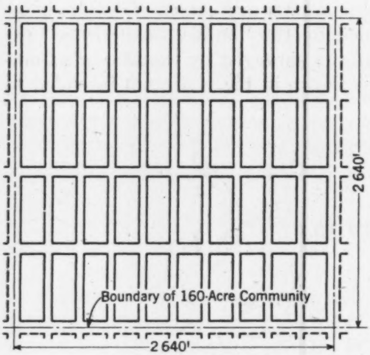


FIG. 2.—"NORMAL" GRIDIRON STREET SYSTEMS (AREA A)

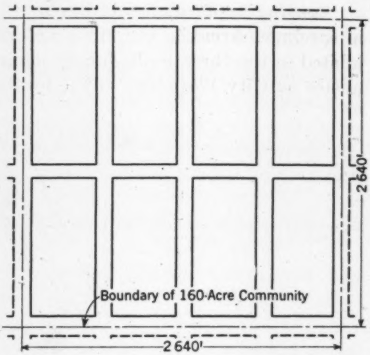


FIG. 3.—MASTER BLOCK STREET SYSTEMS (UNITS A_1 AND A_2)

If the present population of Area A is rehoused on the same site, under minimum standards for attendant facilities, there would be 15.11 acres less land available for dwellings in Unit A_1 than is now used for apartments in Area A (see Table 2). Consequently, the building bulk of the new apartments in Unit A_1 would exceed the bulk of buildings existing in old Area A. At an average height of six stories, apartments in Unit A_1 must cover 68% of the residential land.

Unit A_2.—This development also applies the neighborhood unit formula in the rebuilding of Area A on a master block plan. In Unit A_2, however, apartments averaging six stories cover only 50% of the residential land. The new buildings in Unit A_2 would have less bulk than the apartments and would, therefore, house fewer people than live in Area A.

It is believed that the building bulk resulting from 68% cover, six story apartments, which is required in Unit A_1 to rehouse the present population of Area A on the same site, would make that new unit an obsolete development. The apartments in Unit A_2, covering only 50% of the residential land and averaging six stories in height, provide more open space and permit a better layout which would probably be subject to a much lower rate of obsolescence than the bulkier buildings of Unit A_1.

Unit A_3.—Unit A_3 is a new apartment development on raw land; it is identical to Unit A_2 and houses the same population; but entirely new public facilities are needed in the undeveloped outlying section selected as a site for Unit A_3. This development is predicated on six-story buildings covering 50%

of the residential land. Such a building bulk is not desirable in outlying districts; nor should it be implied that this study so recommends. The purpose of this theoretical development, the same as the other units, is to afford a direct comparison of the results obtained.

Land Area of Speciman Communities.—The specimen area and units studied are considered predominantly residential in character with only such local business, recreational, and cultural facilities within their boundaries as are required for the convenient service of the persons housed in each community.

TABLE 2.—LAND USE ALLOCATION ON EACH 160 ACRES IN RESIDENTIAL NEIGHBORHOODS

(*a*) GENERAL DATA AND POPULATION

Description	Area *A*	Unit A_1	Unit A_2	Unit A_3
Types of residence	Old multi-family	Rebuild Area *A*	Rebuild Area *A*	New multi-family
Percentage covered[a]	....	68	50	50
Borough	Manhattan	Manhattan	Manhattan	Brooklyn
Street system	Fig. 2	Fig. 3	Fig. 3	Fig. 3
Population provided for	46,000	38,550	33,270	33,270
Persons per:				
Residential acre	678	731	540	540
Gross acre	287	241	208	208
Persons actually housed[b]	36,620	36,620	31,600	31,600

(*b*) AREA ALLOCATION

Description	Acres	% of gross	Acres	% of gross	Acres	% of gross	Acres	% of gross
Streets in use	56.88	35.55	42.08	26.30	42.08	26.30	42.08	26.30
Parks and playgrounds	4.79	2.99	22.05	13.78	19.02	11.88	19.02	11.88
Institutions	11.25	7.03	21.00	13.12	18.14	11.32	18.14	11.32
Business	14.88	9.30	22.15	13.86	19.15	11.96	19.15	11.96
Residence	67.83	42.40	52.72	32.94	61.61	38.54	61.61	38.54
Vacant	4.37	2.73	....	....	....	....	....	....
Total	160.00	100.00	160.00	100.00	160.00	100.00	160.00	100.00

[a] Percentage of residential land covered by apartments. [b] Persons housed after allowing for vacancies.

In order to measure public services in the existing area and to ascertain needs in the new units that could be reduced to comparable terms, it was necessary to find a common denominator. Accordingly, each of the communities studied is a type or specimen development predicated on a square containing 160 acres, measured to the center line of boundary streets, in order to secure comparability in one respect—namely, land area. It should be noted that the area of 160 acres, developed with apartments of the character assumed in this study, would house more people than could be efficiently served by one elementary school. Under the foregoing principles of the neighborhood unit, such a development might be served by several elementary schools.

Location of Communities Studied.—There is no particular advantage, however, in examining an area of 160 acres within designated boundary streets. The value of the findings could be easily prejudiced by conditions peculiar to a specific location. It is true that a number of unit costs vary, as between the boroughs of New York City, due to differences in construction practices and in the extent to which certain city services are normally provided, but there is no significant variation within a borough. Approximate locations, within a well-defined section of a borough, furnish ample bases for determining the extent of public development existing in an area typical of that section and the costs of public facilities prevailing in the vicinity. The resulting cost estimates are more representative of average conditions for large sections of the city and, therefore, have broader implications.

Accordingly, Area A (see Fig. 1) is located within the East Harlem section of Manhattan; Units A_1 and A_2, the two schemes for rebuilding multi-family Area A, of course, are on the same site as Area A in East Harlem; and Unit A_3 is located on vacant land, adjacent to Jamaica Bay in Brooklyn, N. Y.

LAND USE, BUILDING DEVELOPMENT, AND POPULATION

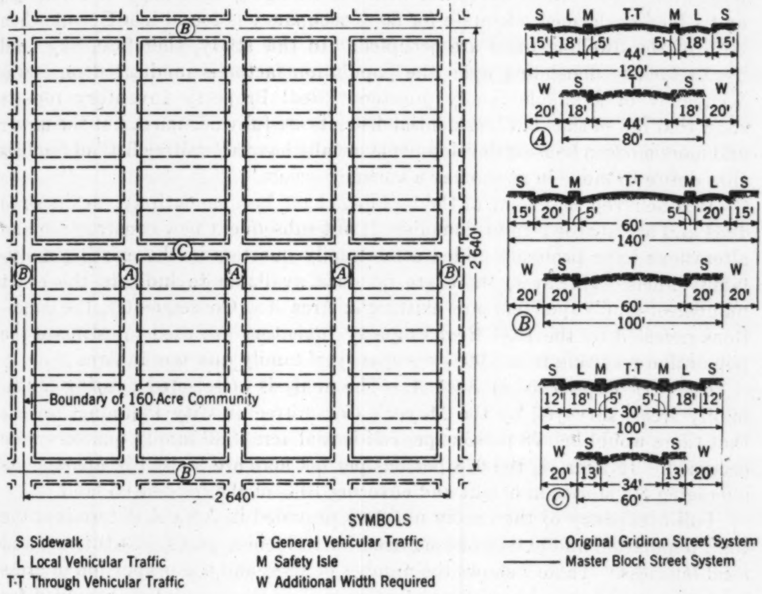
The area of land used for various purposes and the bulk of buildings determine the number of persons that can be accommodated in these communities. The population of the specimen communities must be ascertained before the extent and cost of public facilities can be investigated.

Land Use.—Two major factors are involved in allocating the 160 acres of these typical communities to street, residence, institution, park, and business uses—namely, area and population. The land required for street purposes is determined by the street plan, and in the old area both vacant land and streets are based on the extent of existing development. The land area needed for residence, parks, institutions, and business, however, is dependent upon the number of persons to be housed in a given community; hence, the extent of these land uses is predicated upon the service requirements of the community's population.

The "Area" Factor.—As shown in Figs. 2 and 3, the communities studied are developed on the two types of street systems: (1) The "normal" gridiron street plan; and (2) a master block arrangement.

Gridiron Street Plan.—The usual gridiron street plan existing in New York City is not directly adaptable to a theoretical community of 160 acres, in the form of a square measuring one-half mile on a side. Minor adjustments in the occurrence of the wider streets and avenues in relation to the local streets of standard width are, however, satisfactory for the purposes of this study. The gridiron street system was standardized in a "normal" pattern to permit direct comparisons between communities (see Fig. 2). The amount of land in Area A that is allocated to the gridiron street plan on this "normal" pattern is based upon the built-up experience of existing multi-family areas (see Tables 1 and 2). The boundary streets of Area A average 100 ft in width, and the "normal" gridiron system requires nearly 57 acres, or about 35% of the 160-acre gross area. Each block measures 575 ft by 200 ft.

Master Block Plan.—The master block arrangement indicated by Fig. 3 is used for Unit A_1, Unit A_2, and Unit A_3. This development plan of "super" blocks is designed to limit "through" traffic to boundary and major thoroughfares. Boundary streets of the 160-acre tract are each 140 ft wide, including a local service road. Each master block (1,200 ft by 535 ft) is surrounded by a service roadway separated by a safety isle or planting strip from the wider through-traffic streets bounding the block. A single master block is equivalent to five ordinary blocks in the usual gridiron system, including the area devoted to four minor cross streets in the gridiron system but excluding the area needed for service roadways. The theoretical 160-acre community has eight such master blocks in two rows of four each. They are separated by three north-and-south streets, each 120 ft wide with service road, and by one east-and-west street, 100 ft wide, running through the center (see Fig. 4).


B
C
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2 640'
2 640'
Boundary of 160-Acre Community
S L M T·T M L S
15' 18' 5' 44' 5' 18' 15'
120'
W S T S W
20' 18' 44' 18' 20'
80'
A
S L M T·T M L S
15' 20' 5' 60' 5' 20' 15'
140'
W S T S W
20' 20' 60' 20' 20'
100'
B
S L M T·T M L S
12' 18' 5' 30' 5' 18' 12'
100'
W S T S W
20' 13' 34' 13' 20'
60'
C
SYMBOLS
S Sidewalk
L Local Vehicular Traffic
T·T Through Vehicular Traffic
T General Vehicular Traffic
M Safety Isle
W Additional Width Required
Original Gridiron Street System
Master Block Street System

FIG. 4.—CONVERSION OF "NORMAL" GRIDIRON STREET SYSTEM OF AREA *A* TO MASTER BLOCK LAYOUT FOR UNITS A_1 AND A_2

This master block plan is applicable in the rebuilding of most of the old sections that are now developed on a "normal" gridiron street system and can be easily used for any neighborhood to be built on a vacant and relatively flat site. In this design, streets account for about 42 acres, or 26.3% of the 160-acre gross area.

"Population" Factors and Standards.—With the street pattern fixed, the next step in the determination of land use involved the allocation of the remaining acreage to the parks, institutions, business, and residential uses needed by the community's inhabitants.

Reference to Table 1 will recall that there are cemeteries and industries in the large communities surveyed by the Mayor's Committee on City Planning. Within these larger communities, however, there are many sections of 160 acres that do not contain either a cemetery or any significant industrial uses. For the purpose of this study, therefore, such uses are excluded from all of the communities.

Uses in Terms of Population.—The extent of public services and private facilities needed in the neighborhood units is necessarily determined by the total number of people that can be housed in each community. In the case of an "old" development, it is reasonable to express land use and service needs in terms of the population that could live in the area if all available dwellings were occupied. Otherwise, any measurement of adequacy is based on a population figure which fluctuates with the changing occupancy of dwellings, and a facility deemed adequate for 80% occupancy is found insufficient when 90% of the dwelling units are occupied. In this study, therefore, any land use or facility depending upon the population factor is predicated on 100% occupancy of available family quarters. Real Property Inventory reports show that few of the "old" residential districts are fully occupied, but the newer and more modern housing developments usually have a "waiting list" of families that desire to move in as soon as a vacancy occurs.

It is believed that some of the buildings have become entirely vacant since 1934 and are boarded up or demolished; but subsequent new construction and alterations have probably added some family quarters in the existing multi-family areas. However, there are no data available to indicate the exact number of family quarters now existing in Area *A*. Consequently, the conditions revealed by the 1934 Real Property Inventory are used to estimate the population resulting from 100% occupancy of family quarters in Area *A*.

Built-Up Experience in Area A.—The analysis of selected existing multi-family areas, surveyed by the Mayor's Committee on City Planning, reveals that there would be 678 persons per residential acre if all family quarters were occupied. In Area *A*, the 678 persons per net acre are housed in apartments averaging 5.3 stories in height and covering 70% of the residential land.

Full occupancy of the family quarters provided in Area *A* determines the total population and extent of land used for residence, parks, institutions, and local business. Table 2 shows the number of acres and the proportion of gross land area used for each major purpose as well as the population provided for under built-up experience in the 160-acre specimen community, Area *A*. The acres per 1,000 persons of park, institution, and local business found in Area *A* are given in Table 1.

"Minimum Standards" in Units.—The extent of land required for each major purpose in the neighborhood units is predicated on the unit formula and the minimum standards for parks, institutions, local business, and housing, as follows:

Park and Playground.—Standards believed feasible in New York City, which were developed by the Mayor's Committee on City Planning, call for a minimum of one acre of park and playground per 1,750 persons. This allows for a playground, a small quiet park, and active recreation areas but excludes large parks.

Institutions.—Schools, fire stations, libraries, and other public and private buildings of a local character, as well as churches, are included under "Institutions." Because of the varying spheres of influence of such services, some of which do not coincide with the 160-acre tract under consideration, the land allocated to institutions in each unit is based upon the average built-up experience of eleven residential communities studied by the Mayor's Committee on City Planning. This average is just more than one half of an acre per 1,000 persons, and is believed to be generally adequate.

Business.—Local business includes retail shops, garages, gas stations, laundries, cleaners, barbers, theaters, banks, amusements, local offices, and similar services that would be needed by the people of a self-contained residential community.

In this study the business standard is fixed at 50 ft per 100 people. With lots averaging 100 ft deep this is equivalent to one acre of business per 870 population in each neighborhood unit. Table 1 shows that about 0.3 acre per 1,000 people is used for commercial structures in the existing multi-family areas of the city. The remaining business frontage usually required to serve such communities is found on the ground floor of apartment buildings. Consequently, in Units A_1 and A_2 (the alternative schemes for rebuilding Area A), and in Unit A_3, it is assumed that one half of the required business frontage is on the ground floor of multi-family buildings and the remainder in commercial structures.

Residential Requirements.—In rebuilding Area A to rehouse the present population of 36,620 persons, provision is made in Unit A_1 for a normal vacancy of 5%. On this basis the new apartments must accommodate 38,550 persons. Allowing 242 sq ft of gross floor area per person, and buildings averaging six stories high, these apartments would cover approximately 68% of the net residential land. If land coverage is held to 50%, the buildings would average 8.15 stories in height. In either case there would be 731 persons per net residential acre compared with a potential density of 678 in Area A. This greater density in Unit A_1 results from the relatively larger land areas that are devoted to park, institutional, and business purposes under minimum standards, which reduce the area of land available for residence (compare Area A and Unit A_1 in Table 2). Although these other facilities may be considered adequate, the building bulk of apartments in Unit A_1 is believed to exceed a reasonable standard in overcrowding the land and is therefore considered obsolete.

It is assumed that the housing bulk resulting from 50% land coverage and six-story apartment buildings is the maximum development desirable under reasonable standards. At 242 sq ft of gross floor area per person, these apartments would accommodate 540 people on each residential acre. When properly designed, it is believed that such a development could offer sufficient amenities to maintain a desirable residential character despite the high density. Accordingly, this average building bulk is the housing "standard" in rebuilt Unit A_2 and in new Unit A_3.

Application of Factors and Standards.—Using the aforementioned factors and "standards," the requirements of the population for residence, park, institution, and business were determined for each of the neighborhood units. The resulting population and the land area allocated to these major uses are given in Table 2. Inasmuch as the neighborhood units are fully developed, there is no vacant land.

PUBLIC IMPROVEMENTS AND SERVICES

This investigation attempts to ascertain the nature and extent of public improvements and services now provided in Area *A*. The land uses and degree of development in Area *A* are derived from the built-up experience of the larger communities of which this theoretical 160-acre area is a representative sample. Likewise, the character and extent of public improvements and services in Area *A* are determined from the built-up experience in the larger residential community of East Harlem. The average conditions revealed by public records to exist in East Harlem, when applied to Area *A*, indicate the public facilities now serving this theoretical 160-acre community.

The public services required to serve the neighborhood units adequately are determined on the basis of "minimum service standards," which are discussed herein. Practically all of the public facilities required to serve Unit A_3, which is situated on vacant land, must be entirely new improvements.

In the rebuilding of Area *A* on a master block plan, it is also necessary for the city to provide the adequate and up-to-date facilities needed under "minimum service standards" by the population rehoused in each of the alternative neighborhood schemes, Units A_1 and A_2. Therefore, it is assumed that: (1) Any public facility existing in Area *A* that is not adequate as measured by minimum service standards must be augmented in Units A_1 and A_2 to the full extent of the indicated deficiency; and (2) all existing public services in Area *A* are utilized in so far as needed, but those retained must be replaced in Units A_1 and A_2 to the full extent of the indicated obsolescence.

Improvements and Services Studied.—Only those permanent improvements and facilities supplied by the city that may be directly charged to a local community on the basis of services provided are included in this study. The needs for water supply reservoirs, courts, jails, hospitals, museum buildings, and other general services are not determined by the type of residential development, and they are influenced only indirectly by the geographic location of residential communities. In comparison with local neighborhood facilities, such improvements are city-wide services for which the needs are determined by the entire population and area of the city or a borough. In this study the public services discussed are considered essential to the proper functioning of self-contained residential communities.

Street Utilities.—Sidewalks, curbs, street paving, sewers, water mains, and street lighting are termed "street utilities" in this study. The installation of these utilities in New York City depends almost entirely upon the type of street plan and the various construction practices in each borough of the city rather than upon the number of people to be served.

The borough standards, quantities, and unit costs of street utilities are given in the Appendix. The "minimum service standards" for street utilities in the neighborhood units coincide with current practice in the boroughs, but the quantities needed, or course, are determined by the master block layout.

Population Facilities.—Population facilities include parks and playgrounds, elementary and high schools, branch libraries, district health buildings, fire and police stations, sanitation services, sewage and refuse disposal, and rapid transit. With the exception of rapid transit, which is located in public streets, population facilities are built upon a part of the "developed area" that is classified as "Institutions" in Tables 1 and 2.

The extent to which these public improvements and services are provided in Area *A* and are needed in the neighborhood units is determined by the population to be served. This does not imply that all such services existing in Area *A* are adequate to meet the needs of modern urban living, but rather that the basic requirements for these facilities are not dependent upon the land or "area factor" alone. For example, schools are provided to accommodate a given number of children, and the "area served" depends upon the size of the school and the density of population in the surrounding development. Other parallels can be drawn but the relationship between population and schools illustrates the general nature of "population facilities."

Practically all of these population facilities have been provided in the larger residential communities surveyed by the Mayor's Committee on City Planning. In general, small parks, schools, and sanitation services are entirely local in character, but the other population facilities may have sufficient capacity to take care of a greater number of people than are accommodated in a typical 160-acre community. Existing facilities with an "area of service" greater than 160 acres may not be located within the boundaries of a 160-acre district, and the extent of service available to each community must be based, therefore, on borough or city-wide measurements in relation to the needs of the population of each of the theoretical communities.

The public improvements and services in the neighborhood units that are dependent upon the population factor are identical to the population facilities supplied to Area *A*, and the same procedures are used to determine the extent of each service needed. The "minimum service standards" for parks and playgrounds, elementary schools, libraries, refuse disposal, and rapid transit service in the neighborhood units differ from current borough practices. City standards and departmental programs for the remaining population facilities are considered adequate for the units. The methods of determining needs, and the "standards" and charges against each community for population facilities, are outlined in the Appendix.

OBSOLESCENCE IN AREA *A*

In rebuilding Area *A* under the "standards" adopted for this study, it would be necessary to expand existing public services that are inadequate and to replace any depreciated facility to the full extent of indicated obsolescence.

Street Utilities.—Fig. 4 suggests the extent of alteration and rearrangement of "street utilities" involved in the change from the normal gridiron street system of Area *A* to the master block plan of Units A_1 and A_2. The changes, quantities, and costs involved are given in the Appendix.

Population Facilities.—Many of the existing population facilities serving Area *A* must be replaced in whole or in part when this old district is rebuilt on the master block plan of Units A_1 and A_2. The obsolescence of population facilities serving Area *A* generally is predicated on the present age and useful life of these improvements.

The present age of facilities is obtained from public records, with the exception of libraries, fire stations, police precincts, and sanitation garages and section houses. It was assumed that the age of these facilities is equal to the average age of existing school buildings in East Harlem. The first school in this section, still standing in 1941, was built in 1867, but the average age of all schools in East Harlem is forty-three years. It seems reasonable to suppose that the average age of the aforementioned facilities in East Harlem will approximate the average age of schools.

The useful life of most population facilities is derived from the replacement experience of the appropriate public agencies. Whenever a facility reaches the limit of its useful life, replacement is assumed necessary for the satisfactory continuance of such service. The Appendix contains detailed information on the present obsolescence of public services in Area *A*.

IMPROVEMENT COSTS AND ANNUAL EXPENSES

The reported costs of existing improvements in old Area *A* are used when available. In other instances the estimates are derived from current contract prices reported by city agencies. The cost estimates for improvements in neighborhood units are based upon current practice and contract costs in the appropriate borough, or upon the estimated costs of proposed improvements. Land acquisition is estimated at 1.3 times the average assessed value per acre in each community, but all street land is assumed to be dedicated without cost to the city. The cost estimates for each kind of improvement or service are summarized in Table 3(*a*) by community.

The unit costs used in the estimates of annual expenses are derived from the most recent information available from the operating agencies. The per capita expenses, where used in this investigation, are generally obtained from reported total annual expense and the estimated population actually living in the city. The "actual" population living in each community, rather than the "demand" or total population that could be accommodated with 100% occupancy of dwelling units, is used, therefore, in determining annual expenses that are based on per capita costs.

The estimates of annual expenses indicate the expenditures of the city for operation and maintenance of public services provided in each community. No attempt is made to set up a "balance sheet" of annual expenses and of city revenue derived from a community. The estimates do not in any sense represent the amounts of taxes levied or collected in each community. Annual expenses in any of these residential areas may exceed the taxes paid to the city by the particular community, but the expenses must be met from general city revenues in any event. The estimated yearly expenses charged to each community are summarized in Table 3(*b*), and the procedures in this work are outlined in the Appendix.

Meaning of Estimates.—The estimated costs of permanent improvements represent the present investment of the city in the public facilities now found in Area *A* without any allowance for depreciation or for replacement of the original improvements. The estimates of annual expenses represent the costs to the city for operation and maintenance of existing public services, some at less than capacity because of the relatively low occupancy ratio in dwellings of Area *A*.

TABLE 3.—ESTIMATED TOTAL COSTS TO THE CITY

No.	Description	Area *A*	Unit A_1	Unit A_2	Unit A_3
	(*a*) FOR PERMANENT IMPROVEMENTS				
1	Sidewalk and curb	$ 341,730	$ 145,960	$ 145,960	$ 72,872
2	Street paving	467,626	334,740	334,740	373,146
3	Sewers, etc.	1,070,700	324,450	324,450	310,220
4	Water mains, etc.	151,800	43,660	43,660	133,420
5	Street lighting	....	23,410	23,410	23,410
6	Parks and playgrounds	1,767,550	1,045,850	133,140	383,140
7	Elementary schools	1,740,000	1,672,000	1,365,000	1,936,000
8	High school	2,210,000	450,000	388,500	1,597,500
9	Public library	150,900	69,000	59,500	84,250
10	Health building	50,400	....	....	76,600
11	Fire station, etc.	196,600	75,200	65,900	84,100
12	Police precinct	104,250	69,100	39,400	41,600
13	Sanitation	101,890	61,200	52,800	47,676
14	Sewage disposal	987,000	....	....	431,000
15	Refuse disposal	70,000	28,650	21,650	65,000
16	Rapid transit	1,655,000	1,626,800	783,400	7,080,600
17	Total	$11,065,446	$5,970,020	$3,781,510	$12,740,534
	(*b*) FOR ANNUAL OPERATION AND MAINTENANCE EXPENSES				
18	Street cleaning	$ 96,430	$ 77,063	$ 77,063	$ 56,826
19	Sewer maintenance	2,063	905	905	874
20	Water mains	6,188	2,652	2,652	2,694
21	Street lighting	6,200	1,468	1,468	1,468
22	Parks and playgrounds	2,520	11,575	9,986	9,986
23	Elementary schools	559,000	470,000	405,700	405,700
24	High school	292,560	245,100	211,630	211,630
25	Public library	31,750	26,985	23,275	23,275
26	Health building	2,820	2,150	1,860	2,160
27	Fire protection	153,500	153,500	132,400	99,860
28	Police protection	252,750	252,680	218,000	101,760
29	Refuse removal	79,758	79,758	68,825	63,895
30	Sewage disposal	23,550	19,720	17,050	17,730
31	Refuse disposal	19,987	18,392	15,872	13,248
32	Debt service on rapid transit	80,900	160,400	119,200	346,000
33	Total annual expenses	$ 1,609,976	$1,522,348	$1,305,886	$ 1,357,106

The costs of public improvements in the neighborhood Unit A_3 represent the investment that the city must make to provide new public services that are adequate under minimum service standards. The estimated costs of permanent improvements in the rebuilt communities, Units A_1 and A_2, represent the city expenditures necessary to make existing deficient services adequate and to replace the obsolete part of the present plant that is retained in rebuilding the old apartment Area *A*. The estimates of annual expenses indicate the costs to the city for operation and maintenance of public services in the neighborhood units with 95% of the available housing occupied.

Total and Per Capita Costs.—Rapid transit is not the same kind of local improvement as elementary schools and similar neighborhood facilities. However, transit has exerted a tremendous influence upon the direction and extent of residential development. Since 1900 about 85% of New York's population has lived within one-half mile of rapid transit lines. Urban life depends upon mobility of people as well as goods. Rapid transit, therefore, is considered essential to the life of residential communities in New York City. Rather than permit the transit factor to complicate these findings, the estimated costs will be examined, first with transit charges and second excluding transit charges.

The findings will be considered on the basis of "costs per person housed" in each area. The "costs per person housed" are based upon the actual population living in each area after allowing for vacancies.

With Rapid Transit Charges.—Total improvement costs in new Unit A_3 are 1.15 times the costs in old Area A, and the costs per person housed in Unit A_3 are 1.33 times the per capita costs in Area A (see Table 4(*a*)).

TABLE 4.—ESTIMATED TOTAL AND PER CAPITA COSTS

Area and units	(*a*) WITH RAPID TRANSIT				(*b*) WITHOUT RAPID TRANSIT			
	Improvement Costs		Annual Expenses		Improvement Costs		Annual Expenses	
	Total	Per person	Total	Per person	Total	Per person	Total	Per person
A	$11,065,446	$302.16	$1,609,976	$43.96	$9,410,446	$256.97	$1,529,076	$41.75
A_1	5,970,020	163.02	1,522,348	41.57	4,343,220	118.60	1,361,948	37.19
A_2	3,781,510	119.66	1,305,886	41.32	2,998,110	94.87	1,186,686	37.55
A_3	12,740,534	403.18	1,357,106	42.94	5,659,934	179.11	1,011,106	31.99

Per capita improvement costs for new Unit A_3 are much larger than the costs involved in rebuilding old Area A; costs per person housed in Unit A_3 are 3.3 times the cost in rebuilt Unit A_2 and 2.4 times the per capita costs in rebuilt Unit A_1. This is significant because the rebuilt Units A_1 and A_2 provide additional services to meet present deficiencies of old Area A, and because the deteriorated parts of existing facilities are replaced to the full extent of the indicated obsolescence. It is also interesting to note that the city needs to spend less than $120 per person to house a population of 31,600 in Unit A_2, compared to $163 per person to house 36,620 people in Unit A_1.

Annual expenses per person housed do not vary as much between the several communities as improvement costs. The range is from $41.32 in Unit A_2 to nearly $44 in Area A. The higher total and per capita expenses in Area A suggest that public services in deteriorated old sections are not utilized efficiently. The costs per person housed in new Unit A_3 are $42.94, as against $43.96 in Area A and $41.57 in rebuilt Unit A_1 (see Table 4(*a*)).

Excluding Transit Charges.—If rapid transit should become an entirely self-sustaining service, the costs to the city for improvements in residential developments would be reduced accordingly. The exclusion of transit charges reduces the remaining improvement costs in new Unit A_3 to about 60% of those in old Area A; but the costs in Unit A_3 are 1.3 times the costs in rebuilt Unit A_1 and nearly 1.9 times the costs for improvements in Unit A_2 (see Table 4(*b*)).

In terms of persons housed, the costs are lower in the rebuilt units than in the new development. In new Unit A_3 the improvement costs per person housed are 1.5 times the costs in rebuilt Unit A_1, and about 1.9 times more per person than need be spent in rehousing an equivalent population in rebuilt Unit A_2.

Annual operating expenses are lower in new Unit A_3 than in other communities when transit charges are deducted. Per capita service expenses are generally higher in Manhattan, and in many instances a larger measure of service is provided than in Brooklyn. The expenses per person housed in new Unit A_3 are about 75% of the expenses in Area A, and approximately 83% of expenses in Units A_1 and A_2.

Rehabilitation Versus Decentralization

The surprisingly low per capita cost for the rebuilt Unit A_2 is particularly interesting. The conditions existing in Area A are indicative of the deficiencies of older residential districts in central parts of the city. The average age of this development exceeds forty years; some of the public facilities and private structures are more than seventy years old.

Certain public services are inadequate as measured by the standards adopted for this study, and many existing public improvements are nearing a point of deterioration that will shortly necessitate their replacement. A variety of circumstances have contributed to the high vacancy in apartments, among others being: (1) The heavy vehicular traffic passing through the district on local streets; (2) an influx of commercial and industrial activities into residential blocks; (3) too great a density of buildings and people on the land; and (4) a hesitancy of owners to modernize old housing. Private owners do not wish to increase their investment in old buildings because of the uncertain outlook for individual properties and the hope of putting the land to a more intensive use. This hope has not materialized, but modernization is difficult because many properties have not been amortized over the years of productive use. These and other circumstances make Area A an unstable district, and the lack of bold correctives further contributes to this unhealthy condition. It is only natural that people should move from districts like Area A whenever they can afford to locate in newer, outlying sections that offer a more desirable residential environment.

When population shifts from older districts to newer developments, the city must provide additional street utilities and population facilities in the new sections to meet the resulting demands. There are enough people left in the older districts, however, to require the maintenance of public services. Consequently, the city is in reality forced to duplicate existing facilities in the newer developments and at the same time to operate the old plant below capacity.

The older residential districts cannot be abandoned entirely and left to stagnate; nor are the people willing to remain in these undesirable neighborhoods so long as modern private developments and cheap transportation enable them to live more pleasantly in outlying communities. What can be done to remedy this apparent impasse? Under present tax structures the city cannot continue this inefficient and expensive process unless it is to face bankruptcy or to experience a far greater population growth than the more optimistic prognosticators now foresee.

This investigation suggests that, in so far as city expenditures are concerned, it would be economical and desirable to rehabilitate deteriorated residential districts completely on the neighborhood unit scheme. A brief examination of the current obsolescence of public services will reveal the extent of investments to be salvaged in this process and the cost of rehabilitation as against decentralization.

Public Facilities in Area A.—Table 3 shows that the costs to the city, for modernization of existing public facilities in Area A on the master block scheme of the neighborhood Unit A_1, amount to about $6,000,000, or $163 per person housed. This value does not indicate the exact obsolescence of existing public facilities because of the changes in street utilities necessitated by the conversion of the present gridiron street system of Area A into the master block plan. The extension of population facilities required by minimum service standards also calls for a greater expenditure than is necessary merely to replace the obsolete part of the existing plant. A calculation of the costs of replacing obsolete public facilities in Area A, without changing the present gridiron street plan, shows that a total outlay of about $4,900,000 would be needed. Of this total, replacing obsolete street utilities would require $850,000 and merely replacing obsolete population facilities would involve expenditures of more than $4,000,000. The replacing of obsolete facilities without altering the gridiron street plan of Area A, or supplying adequate parks, is herein termed operation "Re-Area A."

Assuming that the $11,065,446 cost to the city for improvements now existing in Area A is the original public investment, the estimated expense of replacing deteriorated parts is 44.3% of this original cost. In other words, it appears that public facilities in Area A are already 44% obsolete and must be replaced shortly in order to maintain city services for the 36,620 persons living in this community. This of course excludes replacing such new improvements as the sewage disposal plant and a local health building.

Replacing Facilities with Adequate Parks.—When the city does replace obsolete facilities in old Area A, it would be preferable to provide the additional park acreage required under minimum service standards to serve the present population. The Appendix shows that the transformation of the gridiron street system into the master block plan makes it possible to obtain new park acreage by utilizing the land available from interior streets, which are discontinued. In the case of Unit A_1, the required total park acreage exceeds the sum of existing park area plus leftover street area by only 2.5 acres. On this basis the city needs to purchase 2.5 acres of additional land to meet the park requirements of Unit A_1.

By rehabilitating public improvements in old Area A without changing the existing gridiron street system to the master block plan, the city would not be able to discontinue any local streets. As a result, it would be necessary to purchase more than 17 acres of land which, with the 4.8 acres now in parks, would provide the 22 acres of parks and playgrounds needed to serve, adequately, the present population of old Area *A*. This operation can be designated as "New Area *A*."

With land acquisition at 1.3 times the average assessed value and development costs at $7,000 per acre, parks cost more than $6,400,000 in this rehabilitation operation for New Area *A* under minimum standards. The replacement of obsolete street utilities and population facilities, other than parks, requires the same expenditure as in Re-Area *A*—namely, about $4,900,000. The total cost of up-to-date public facilities in New Area *A* on the old gridiron street plan exceeds $11,300,000, or $308 per person housed.

Cost Ratios Per Person.—It is significant to find that the rebuilt Unit A_2 is less costly in respect to public services than "Re-Area *A*," "New Area *A*," or the new Unit A_3. Because of this finding the per person cost ratios in Table 5 are based on the cost in Unit A_2 as 100.

TABLE 5.—CITY EXPENDITURES PER PERSON

(With Transit Charges)

Community	IMPROVEMENT COSTS		ANNUAL EXPENSES	
	Per person	Ratio	Per person	Ratio
Old Area *A*[a]	$ 302	252	$43.96	106
Re-Area *A*[a]	134	112	42.15	102
New Area *A*[a]	308	258	42.30	102
Unit A_1[a]	163	136	41.57	101
Unit A_2[b]	119	100	41.32	100
Unit A_3[b]	403	337	42.94	104

[a] Population housed is 36,620 persons.
[b] Population housed is 31,600 persons.

The replacement of obsolete public facilities, without altering the gridiron street plan of Area *A*, would cost the city 12% more with present park acreage and nearly 2.6 times more with adequate park area than the rebuilding of old Area *A* on the master block scheme of Unit A_2. The rebuilding of Area *A* on the master block plan of Unit A_1 would cost only slightly more than merely replacing obsolete facilities for Re-Area *A*, but less than for New Area *A*.

The replacing of obsolete facilities for Re-Area *A* and New Area *A* would not, however, produce the social and economic results believed possible with the neighborhood unit communities on a master block plan. Such replacement of public facilities would tend to "freeze" the present character of private development on the unsatisfactory gridiron street plan, and to postpone the necessarily bold public and private corrective measures for at least another fifty or seventy-five years. By that time it is probable that the social and economic conditions of this community will become quite chaotic, unless present trends could somehow be arrested.

The advantages of the neighborhood unit schemes, having already been enumerated, will not be repeated. It is surprising to find, however, that such sound types of redevelopment as the neighborhood Units A_1 and A_2 also have cost advantages over the replacement of obsolete facilities, both for permanent improvements and annual operation. The entirely new neighborhood Unit A_3 is much more expensive than the several types of rehabilitated communities. When one recalls that, to a considerable extent, the public facilities in this new unit would duplicate city services now provided in other sections of the city that are already built up, the apparently favorable ratio for operation in Unit A_3 is reduced accordingly.

These findings are not entirely conclusive, but they indicate that large-scale rehabilitation of old central apartment districts is economically feasible and preferable to new developments in outlying sections in so far as city expenditures are concerned. When one considers that all residents pay local taxes, which are obviously increased through duplication of city services, the case of rehabilitation versus decentralization becomes a question of public concern. It would seem, therefore, that essential studies of the private costs involved, and of the procedures to be used in large-scale rehabilitation of entire neighborhoods, should be undertaken immediately under a joint private and public sponsorship.

ACKNOWLEDGMENTS

The original investigations, from which this paper is drawn, were made during 1939 for the Department of City Planning of the City of New York to ascertain the relative costs of public services in various types of residential developments. Commissioner R. G. Tugwell (who heads the Department and is Chairman of the City Planning Commission) has been kind enough to release the material for this paper prior to publication of the full report on the original study.

APPENDIX

The extent of public services existing in Area *A*, the standards used to determine the needs for improvements in the neighborhood units, the obsolescence of facilities now provided in Area *A*, and the cost estimates are outlined herein.

The reported costs of existing improvements in old Area *A* are used when available. In other instances the estimates are derived from current contract prices reported by city agencies. The cost estimates for improvements in neighborhood units are based upon current practice and contract costs in the appropriate borough, or upon the estimated costs of proposed improvements. Land acquisition is estimated at 1.3 times the average assessed value per acre in each community, but all street land is assumed to be dedicated without cost to the city.

Street Utilities.—The quality of street utilities is fixed by construction standards in the various boroughs, and the quantity is determined by the street plan and character of development. The quantities and unit costs (based on recently reported contract prices) are summarized in Table 6.

Sidewalks and Curbs (Item No. 1, Table 6).—Sidewalk standards in Manhattan call for 7 in. of cinders, 4 in. of concrete, and 1 in. of cement with steel-faced concrete curbs. The Brooklyn standard is 6 in. of cinders and 5 in. of concrete. Straight curbs are concrete in Brooklyn, but granite is used for corners and 3 ft beyond the corner curve.

Street Paving (Item No. 2, Table 6).—One half the cost of paving the through traffic roadway of boundary streets is charged to each community; but the service roadway adjacent to boundary streets in the master block plan, as well as other paving within the community, is charged in full to the neighborhood units.

TABLE 6.—STREET UTILITIES

No.	Item	AREA A		UNITS A_1 AND A_2		UNIT A_3	
		Quantity	Unit cost	Quantity	Unit cost	Quantity	Unit cost
1	Sidewalk (sq ft)	962,192	$ 0.20	409,040	$ 0.20	282,040	$ 0.20
	Curb (lin ft)	66,352	2.25	28,512	2.25	27,712	0.50
	Curb (lin ft)	......	......	......	......	192	2.50
	Curb (lin ft)	......	......	......	......	608	3.50
2	Street paving (sq yd)	34,800	3.50	8,081	3.50	80,800	2.55
	Street paving (sq yd)	122,200	2.83	61,524	2.83	61,478	2.20
	Street paving (lin ft)[a]	......	......	58,592[a]	2.25	57,696[a]	0.50
	Street paving (lin ft)[a]	......	......	......	......	384[a]	2.50
	Street paving (lin ft)[a]	......	......	......	......	512[a]	4.00
3	Sewers (lin ft)	35,690	30.00	15,450[b]	30.00	29,110	5.50
4	Water mains (lin ft)—20 in.	2,640	9.00	2,640[c]	9.00	2,640	9.00
	Water mains (lin ft)—12 in.	5,280	5.00	5,280[c]	5.00	5,280	5.00
	Water mains (lin ft)— 8 in.	29,040	3.50	7,920[d]	3.50	23,760	3.50
5	Street lighting—units	......	......	176	132.50	176	132.50

[a] Curbs for safety islands in master block layout. [b] 70% of length shown obsolete and replaced.
[c] 50% of length shown obsolete and replaced. [d] 66% of length shown obsolete and replaced.

Paving standards are as follows: In Manhattan heavily traveled streets have 3 in. of asphalt on a 9-in. concrete base; other streets are paved with 3 in. of asphalt on a 6-in. concrete base. In Brooklyn heavily traveled streets have an 8-in. concrete base; otherwise the practice is the same as that for Manhattan.

In changing Area A to the master block layout, new sidewalks and curbs are required for all of the local traffic streets and new curbs for one side of the safety islands. Service roadways will have to be newly paved to the full widths and lengths indicated in Fig. 4. It is assumed that the surfaces of existing roadways retained in the master block scheme are in good condition but that curb replacement in connection with safety islands requires repaving of 10% of the road surface retained in the master block plan. Any other incidental repaving is considered a maintenance item rather than a permanent improvement.

Sewers (Item No. 3, Table 6).—There are no trunk sewers in the Manhattan communities, Area A and Units A_1 and A_2; but trunk sewer costs for Unit A_3 are estimated on a flat rate of 3 cents per sq ft of "buildable" area. Catch basins, inlets, and manholes are included in the costs since sewers serve as combination storm-water and sanitary drains. One half of the cost of sewers in boundary streets is charged against each community.

Standards in Manhattan call for sewers averaging 4 ft by 2.33 ft. In Brooklyn the average is 18 in., but, due to the long blocks, sewers in Unit A_3 are assumed to average 24 in. and are placed in service roadways.

The average age of sewers in Area *A* is seventy years and their useful life one hundred years, so that 70% of the sewers in existing streets that are retained in the public streets of the master block plan require replacement in either Unit A_1 or A_2. New surface drains are also required for the service roadways in the master block schemes.

Water Mains (Item No. 4, Table 6).—The costs of water mains include valves, fire hydrants, and manholes but exclude all house connections, which are made at the expense of the property owner. One half the cost of water pipes in the boundary streets is charged to each community, except for Unit A_3, where current practice would require the original installation of water mains in each of the service roadways of the master block plan, rather than a single pipe in the bed of boundary and interior streets.

According to records of the New York Department of Water Supply, Gas and Electricity, the average age of water mains in East Harlem is fifty years. The useful life of 12-in. and 20-in. lines is about one hundred years and for 8-in. pipe, seventy-five years. On this basis, one half the 12-in. and 20-in. mains and two thirds of the 8-in. pipes in Area *A* are obsolete and that portion retained in either Unit A_1 or A_2 would have to be replaced. Obsolete fire hydrants must also be replaced, and new and old hydrants relocated to conform to the master block plan.

Street Lighting (Item No. 5, Table 6).—Under current standards and prices lighting equipment for the neighborhood units costs $93.36 per light. The installation is estimated at $39.15 per light.

The city formerly rented street lighting equipment from utility companies; hence there is no city outlay for this equipment in Area *A*. Under present policy the city purchases and installs street lighting equipment. Both the new and rebuilt communities are therefore charged in full for this equipment as noted in Table 6.

Population Facilities.—The character of development and the size of the population that can be accommodated in a community create well-defined demands for parks and playgrounds, elementary and high schools, branch libraries, local health buildings, fire stations and police precincts, sanitation garages and section houses, sewage and refuse disposal plants, and rapid transit. These public services are termed "population facilities."

The obsolescence of existing facilities in Area *A* is determined by the present age and useful life of these improvements. The present age of most facilities is obtained from public records, but the age of libraries, fire stations, police precincts, and sanitation garages and section houses is assumed equal to the average age of existing schools in East Harlem, which is forty-three years. The useful life of these facilities is considered to be sixty years; hence they are 71.6% obsolete.

Parks and Playgrounds.—Table 2 shows that public parks and playgrounds account for 4.79 acres, which is equivalent to one acre for each 7,100 persons that can be housed in the community. The park land required in the neighborhood units is based on standards used by the staff of the Mayor's Committee on City Planning, which call for one acre for each 1,750 persons.

Closing certain interior streets in converting the gridiron street plan of Area A into the master block layout of Units A_1 and A_2 makes about 14.8 acres of land available for other purposes (see Fig. 4). Inasmuch as this land is now in the bed of publicly owned streets, it is assumed to be a city asset, which could be sold or traded for building purposes in exchange for equivalent acreage in appropriate locations for additional parks at no cost to the city.

In Unit A_1 the 4.79 acres (see Table 7) of existing parks plus 14.8 acres from discontinued streets would produce 19.59 acres of parks. The purchase of 2.46 additional acres would bring park acreage to the 22.05 acres required in Unit A_1. For Unit A_2 the existing park acreage plus the area of discontinued streets would provide the 19.02 acres needed for parks and would leave about 0.5 acre that could be used for other public purposes.

TABLE 7.—PARK NEEDS

Unit	Area, in acres	Land cost	Development ($7,000 per acre)
A	4.79	$1,734,000	$ 33,550
A_1	22.05	891,500	154,350
A_2	19.02	....	133,140
A_3	19.02	250,000	133,140

The extensive reconstruction involved in transforming Area A to a master block plan would necessitate the redevelopment of parks and the provision of new park equipment in either Unit A_1 or A_2. The existing park equipment in Area A, therefore, is assumed unusable in Unit A_1 or A_2.

Elementary Schools (Table 8(*a*)).—The number of children from each unit that may be expected to register in public elementary schools is estimated at 9.7% of the total population. The elementary school standard limits classrooms to thirty-five students, and the school needs of each unit are predicated on this average value.

TABLE 8.—EDUCATIONAL NEEDS

Area and units	(*a*) ELEMENTARY SCHOOLS			(*b*) HIGH SCHOOL			(*c*) PUBLIC LIBRARY			
	Student registration	Needed classrooms	Rooms replaced	Registration (seats)	Percentage of seats replaced	Unit cost (dollars)	Persons per branch	Ratio of use	Replacement ratio	Cost of branch (dollars)
A	4,462	127.4	....	1,840	....	1,200	50,750	90.6	....	166,600
A_1	3,740	106.8	79.6	1,542	24.3	1,200	50,000	77.1	71.6	89,600
A_2	3,227	92.2	65.0	1,331	24.3	1,200	50,000	66.5	71.6	89,600
A_3	3,227	92.2	....	1,331	....	1,200	50,000	66.5	....	126,520

Records of the Board of Education of New York City show that 79% of the classrooms available in East Harlem are in buildings that were erected prior to 1901. These buildings generally cover more than three quarters of plots averaging only 31,200 sq ft in area. It is assumed, therefore, that about 80% of the 127 classrooms available to Area A are obsolete so that only 27 classrooms in buildings less than forty years old are considered usable in Unit A_1 or A_2.

Unit A_1 needs about 107 classrooms, of which 27 are supplied by existing schools, and nearly 80 must be provided in new buildings to replace obsolete classrooms. Unit A_2 requires about 93 classrooms, and, by utilizing the 27 now usable in Area A, new schools containing only 65 classrooms need be built to meet the requirements of Unit A_2.

Since fewer classrooms are needed for Units A_1 and A_2, it will not be necessary to acquire more land for schools. If the new playgrounds required under minimum standards were located adjacent to school sites, both old and new buildings in Unit A_1 or A_2 would have more light, air, and play space than is now available in Area A.

Reports of the Board of Education indicate that the accumulated cost of land, equipment, and building for existing elementary schools in East Harlem average $13,650 per classroom. Elementary school costs in all of the neighborhood units are based on $21,000 per classroom, a city-wide average for schools constructed during recent years.

High Schools.—Approximately 4% of the population of each unit is expected to register in public high school. The standard requires the provision of one seat per student in an up-to-date school building.

On the premise that buildings more than forty years old are obsolete, the Manhattan high school plant measured in classrooms is 24.3% depreciated. This obsolescence factor is applied against the number of old seats needed in Units A_1 and A_2 to determine necessary replacement. The cost per seat for land, building, and equipment (see Table 8(*b*)) is a general average for high schools recently constructed.

Public Library.—The standard used by the Mayor's Committee on City Planning forms the basis for determining public library needs in the units.

The obsolescence of the branch library serving Area A is estimated at 71.6%. In either Unit A_1 or A_2 the need for library service is less than that required by Area A. It is assumed that the obsolete part of this existing branch must be replaced to the full extent of the indicated depreciation.

As evidenced by capital budget and program requests, branch libraries cost about $125,000 each for construction and equipment. This sum forms the basis for cost estimates in all communities, exclusive of plots 50 by 100 ft, which are assumed to cost 1.3 times the average assessed value. The charge to each community is determined by the "ratio of use" (see Table 8(*c*)), which, of course, is population expressed as a percentage of the capacity of a branch library.

Health Building.—The Health Department program for health education, preventive medicine, and local health service aims at the eventual provision of health center buildings throughout the city. The capacity of each health building under this program forms the "standard," and costs are obtained from the same source. The "ratio of use" (Table 9(*a*)) determines the charge against each community (see also Table 3(*a*)). The district health center now serving Area A is adequate for Units A_1 and A_2, and this building is so new that obsolescence is negligible.

Sanitation.—The present distribution of sanitation garages and section houses in each borough is assumed a satisfactory standard for all communities.

In Manhattan there is one sanitation truck for each 1,540 persons and one section house for each 32,300 residents. In Brooklyn the averages are 2,540 persons and 44,350 persons respectively. Data from the Department of Sanitation indicate that trucks require about 6,300 cu ft of garage space, which, at 35 cents per cu ft, amounts to $2,205 per truck. Section houses for the storage of street cleaning and sanitation equipment average $25,000 each.

TABLE 9.—HEALTH AND SANITATION

Area and units	(*a*) HEALTH BUILDING			(*b*) SANITATION BUILDINGS			(*c*) SEWAGE DISPOSAL			(*d*) INCINERATORS		
	Persons per building (thousands)	Ratio of use	Cost of building (thousand dollars)	Garages (dollars)	Section houses (dollars)	Replacement percentages	Persons per plant (thousands)	Ratio of use	Plant cost (thousand dollars)	Daily collection (cu yd)	Ratio burned (%)	Capacity (cu yd)
A	227.4	20.2	50.4	1.44	0.76	....	1,270	3.62	27,255	471.5	44.4	209.5
A_1	227.4	....	....	1.44	0.76	71.6	1,270	....	....	395.3	57	225.5
A_2	227.4	....	....	1.44	0.76	71.6	1,270	....	....	341.2	57	195
A_3	250.0	13.3	76.6	0.87	0.56	....	400	8.32	5,176	341.2	57	195

The per capita costs, estimated on the basis of current distribution of these facilities in each borough, are shown in Table 9(*b*). The total charges to each community are derived from the population and appropriate per capita value. In Area *A* depreciation is estimated at 71.6% and the charges in Table 3(*a*) for Units A_1 and A_2 are to cover replacements of obsolete plant only.

Sewage Disposal.—Sewage disposal is assumed essential in all of the communities, and the program for plants and treatment methods proposed by the Department of Public Works is taken as the standard. Costs are also derived from this source. Units A_1 and A_2 are served by the new Wards Island plant; Unit A_3 would be served by the proposed 26th Ward plant in Brooklyn.

The Wards Island disposal plant is practically new, and its capacity is adequate to serve either Unit A_1 or A_2. Hence, no replacement nor enlargement of existing sewage disposal facilities is involved in rebuilding Area *A* on the master block plan. Area *A* and Unit A_3 are charged according to the ratio of use (see Table 9(*c*)) of the rated capacity of the plants serving their population.

Refuse Disposal.—It is assumed desirable to incinerate all combustible material and to use only ashes and other inorganic refuse for land fill. Estimates made for the Mayor's Committee on City Planning show that 57% of the refuse is combustible material and the remainder is inorganic matter that can be disposed of by land fill. About 45% of the refuse from Area *A* is now incinerated (see Table 9(*d*)). It is assumed that land acquisition is not involved in disposing of the remaining refuse by the land-fill method.

The charges for refuse disposal are based on the use of incinerators of 750-ton daily capacity costing $1,500,000. Combustible material from each community is figured at approximately 600 cu yd per ton.

Estimated daily collection of all refuse is based on 3.2 cu yd per capita annually at 313 working days. The existing plant serving Area *A* handles 209.5

cu yd and is assumed 33% obsolete. An additional capacity of 16 cu yd is needed for the 225.5 cu yd to be incinerated in Unit A_1. New plant costs chargeable to Unit A_1 are $5,350 and replacement of obsolete part, $23,300. The 195-cu-yd capacity needed in Unit A_2 is 15 cu yd less than the existing plant serving Area A; this excess can be used for another community, and Unit A_2 is charged only for replacement of the obsolete part of the capacity of the old incinerator needed to serve this rebuilt community.

Fire Station.—The present distribution of fire stations in each borough is assumed adequate for the protection of residential neighborhoods. The existing facilities serving Area A are assumed 71.6% obsolete (see Table 10(*a*)) and to require replacement to this extent.

TABLE 10.—FIRE AND POLICE BUILDINGS

Area and units	(*a*) FIRE STATION NEEDS				(*b*) POLICE PRECINCT BUILDINGS			
	Persons per station	Ratio of use	Replacement ratio (%)	Cost per station (dollars)	Persons per building	Ratio of use	Replacement ratio (%)	Cost per building (dollars)
A	30,400	151.0	....	116,600	68,000	67.6	....	154,100
A_1	30,400	126.3	71.6	53,600	68,000	56.7	71.6	122,200[a]
A_2	30,400	109.3	71.6	53,600	68,000	48.9	71.6	80,600
A_3	37,400	89.0	....	76,520	92,400	36.0	....	115,540

[a] Additional land included in total replacement cost.

The total cost of stations is estimated at $75,000 per building plus 5,000 sq ft of land at 1.3 times the average assessed value of each community. The charge to each community, of course, is based upon the "ratio of use" shown in Table 10(*a*).

The fire alarm telegraph system now uses underground conduits and alarm boxes at approximately every alternate street intersection. On the normal gridiron layout, this calls for twenty alarm boxes per community. The master block plan, however, needs only sixteen alarm boxes per unit. Installation costs, at $1,000 per box, are $20,000 for the gridiron street plan and $16,000 for the master block plan. One half of the twenty existing boxes in Area A are assumed obsolete, but four of the boxes not needed in Unit A_1 or A_2 have a salvage value of $200 each, which reduces the replacement cost to $7,200 in each of these units.

Police Precinct.—The standards for police protection for all communities are also based upon the present distribution of precinct buildings in each borough. The obsolescence of police stations serving Area A is the same as fire stations—namely, 71.6% (see Table 10(*b*)). Replacement to this extent is necessary for Units A_1 or A_2. Most of the old police precincts occupy plots 50 ft wide, but garages for motor equipment adjacent to new stations require lots 100 ft wide. In Unit A_1 the additional land must be acquired. After providing for park needs in Unit A_2, however, sufficient land would be left over from discontinued interior streets to allow for the larger police station lot.

The costs charged to each community (see also Table 3(*a*)) are based on the respective ratios of use, with buildings at $112,500 each and land at 1.3 times the average assessed value.

Rapid Transit.—This study assumes that adequate transit service is essential to the proper functioning of the neighborhood unit communities. The measure of adequate transit service during the morning rush hours (see Table 11) is predicated on "reasonable" carloadings, as defined by the Transit Commission, and sufficient capacity on lines to transport the passengers contributed by each unit with out exceeding this "reasonable" load.

TABLE 11.—MORNING RUSH-HOUR PASSENGERS

Area and units	Percentage of population	Total load	I.R.T. subway	Independent subway	Elevated lines
A	14.4[a]	5,178	3,791	.....	1,387
A_1	14.4[b]	5,450	4,140	1,310	.....
A_2	14.4[b]	4,705	4,140	565	.....
A_3	18.8[b]	6,255	.....	6,255	.....

[a] Ratio of actual population. [b] Ratio of population at full occupancy of housing.

The "reasonable" carload (about 3 sq ft per standing passenger with all seats filled) does not represent the most desirable or convenient transportation, but it is a practical basis for measuring line capacity. Besides carloading, two other factors are involved in determining the capacity of transit lines serving each unit: (1) The number of cars per train, and (2) the headway between trains during the morning rush hours. Adequate transit service, therefore, must provide sufficient capacity to carry the passengers contributed by each unit, as well as by the remaining half-mile zone served by each line, without exceeding "reasonable" carloadings during the morning rush hours on any line into central business districts. In 1935 the Interborough Rapid Transit Company (I.R.T.) was carrying loads during the morning rush hours that are estimated to be 1.2 times greater than the reasonable capacity of the line serving East Harlem. As a result, the actual number of persons living in Area *A* is used to determine rush-hour loads. In the neighborhood units, the population that could be housed at full occupancy of dwellings determines the loads and the capacity required.

Table 11 shows the estimated rush-hour passengers contributed by each community and the number using each rapid transit line. Of the total passengers contributed by Area *A*, only 83% of those using the I.R.T. subway are assumed within the capacity of this line because of the present overcrowding. With certain remedial measures it is believed possible to increase the capacity of this line by 1.3 times during the morning rush hours. This would increase the present 3,165-passenger reasonable capacity of the I.R.T. subway available to Area *A* to 4,140 rush-hour passengers.

In rebuilding Area *A* it is supposed that the obsolete elevated lines serving East Harlem will be demolished and that remedial measures will be undertaken on the I.R.T. subway. Thus, all traffic from Units A_1 and A_2 must use subway lines, including a new subway to replace the Elevated Railway.

Unit A_1 will contribute 5,450 rush-hour transit passengers, of which 4,140 are expected to use the I.R.T. subway, and 1,310 passengers will have to be accommodated on the new subway line that will be needed after demolition of the Elevated Railway. Unit A_2 is expected to contribute 4,700 passengers during the morning rush hours. With the reasonable capacity available after remedial measures, the I.R.T. can serve 4,140 of these people, and the new subway line will accommodate the remaining 565 passengers from Unit A_2.

The Elevated line in East Harlem was built at private expense and cannot be charged against Area A. On the basis of total city contributions to the I.R.T. subway construction, it is estimated that the city's capital outlay averages $437 per passenger carried during rush hours in 1935. According to studies of the Mayor's Committee on City Planning, remedial measures would increase the capacity of the I.R.T. by 1.3 times at a cost of $147.50 for each passenger that could be carried over and above the present reasonable capacity of this line. Assuming a total of $815,000,000 as the cost of the Independent (including sections under construction), and the rated capacity used in the Mayor's Committee Rapid Transit Study, the city will have spent $1,132 for each passenger that this system can carry into work centers during the morning rush hours. These estimates of capital costs to the city per passenger carried during the morning rush hours on each line serving the several communities determine rapid transit charges summarized in Table 3(*a*).

ESTIMATES OF ANNUAL EXPENSES

This section of the Appendix outlines the methods used in estimating the annual expenses for operating and maintaining the public services provided in each community.

Except as otherwise noted, unit costs are based upon the most recent information available from the operating agencies. The per capita costs are generally derived from the total annual expense and the estimated population actually living in the city. Therefore, the "actual" population living in the several communities, rather than the "demand" or potential population that would result from 100% occupancy of residential quarters, is used in determining annual expenses that are based on per capita costs. The estimated yearly expenses charged to each community are summarized in Table 3(*b*) by item.

Street Cleaning.—Annual costs of street cleaning average 27.3 cents per sq yd in Manhattan and 13.1 cents in Brooklyn. Snow removal averaged 20.3 cents per sq yd over a five-year period. Computed for the roadway area, there are 157,000 sq yd of roadway to be cleaned in Area A and 142,520 sq yd in each unit. On the basis of full street width there are 263,884 sq yd to be cleaned of snow in Area A and 187,969 sq yd in each unit.

Sewer Maintenance.—The unit cost of cleaning sewers in Manhattan during 1938 was $203.50 per mile and $8.60 per catch basin. Since data were not available for Brooklyn, the unit cost is the same as Queens—namely, $102.50 per mile of sewer and $8.60 per catch basin.

There are 6.76 miles of sewers and 80 basin manholes in Area A, but only 2.92 miles of sewers and 36 basin manholes in Units A_1 and A_2. Because pipe is placed in service roadways, Unit A_3 has 5.52 miles of sewers and there are 36 basin manholes.

Water Mains.—The annual expenses per mile of pipe for water-main maintenance include 70% of the motor vehicle operations cost for mains in existing streets, regardless of pipe size. The age of mains is not considered a factor because unconsolidated fill tends to cause new mains to break more frequently than old pipes. The annual maintenance expense in Area *A*, and Units A_1 and A_2, is computed at the Manhattan average of $884 per mile of pipe. The cost in Unit A_3 in Brooklyn is based on $449 per mile of pipe per year. Pipe length is given in Table 6.

Street Lighting.—In Area *A* the annual rental of street lighting equipment from the utility companies is estimated to average $25 per light. In the neighborhood units the city would install and maintain its own equipment. Recent operating experience indicates that the annual expense would be $8.34 per light.

Parks and Playgrounds.—An examination of the Comptroller's Report indicates that appropriations for the operation and maintenance of parks and playgrounds amounted to more than $8,800,000 during 1938. This appropriation represents an average cost of about $525 per acre of park, including all facilities under the jurisdiction of the Park Department. This unit cost is used in estimates of annual expenses in all communities.

Elementary Schools.—The average cost per classroom for physical operation and maintenance of elementary and junior high schools as reported for 1937 is $387 in East Harlem (Area *A*); but the city average approximates $400 per classroom, which value forms the basis for estimates in all neighborhood units.

The cost of instruction, excluding general administration, averages about $4,000 per classroom for the entire city. Similar data are not available by school district and, therefore, instruction costs for all communities are based on the city average.

High School.—The annual expenses for instruction and high school building maintenance, shown by reports to approximate $159 per student registered, are the basis for the estimates given in Table 3(*b*).

Public Library.—Requests for public libraries submitted in connection with the capital budget indicate that the annual operation and maintenance costs average $35,000 for the type of branch that serves residential districts. With this value as a base, costs are allocated to each community on its "ratio of use" of library facilities.

Health Building.—Each community is charged with its proportionate share of the annual expense of maintaining and operating health center buildings, as given in the Health Department program submitted to the City Planning Commission. The expense in the East Harlem district (Area *A*, and Units A_1 and A_2) is given as $13,970 per yr, and in the district serving Unit A_3 the total cost is $16,210.

Fire Protection.—Estimates of yearly expenses for fire protection are based on the following per capita charges:

Borough	Dollars per year
Manhattan	4.19
Brooklyn	3.16

These values are derived from reported costs for operation, including equipment, telegraph, buildings, and personnel in the smaller stations, which are assumed typical for residential districts.

Police Protection.—The annual charges for police personnel, motor and other equipment, and building maintenance average $3.22 per person living in Brooklyn and $6.90 per person living north of 80th Street in Manhattan.

Refuse Removal.—The annual maintenance of sanitation garages and section houses, as well as the costs of collecting refuse, are included under "refuse removal" expenses. These expenses average $2.17 per person in Manhattan and $2.02 in Brooklyn. Application of these figures to the population of each community produces the estimates summarized in Table 3(*b*).

Sewage Disposal.—The costs of sewage treatment given for the various plants in the "Tentative Plan for Sewage Disposal" prepared by the Department of Public Works and submitted to the City Planning Commission are used to determine the total operating expenses for each plant. The estimated operating expenses for the Wards Island plant are $650,400 per yr, and for the 26th Ward plant, $213,100 per yr. Each community is charged with its share of the annual expenses in proportion to the "use ratio" of plant capacity.

Refuse Disposal.—The cost per cubic yard of material incinerated is about 20 cents in Manhattan and 18 cents in Brooklyn (see Table 12). The disposal of ashes and other non-combustible material is assumed at about 10 cents per cu yd in Manhattan and approximately 6 cents in Brooklyn. Table 12 shows the total annual collection of refuse in square yards, the disposition by community, and the unit cost (see Table 3(*b*) for the total annual expenses).

TABLE 12.—REFUSE DISPOSAL EXPENSES

Area and units	Annual (cu yd)	INCINERATED		LAND FILL		DUMPED	
		%	Cost[a]	%	Cost[a]	%	Cost[a]
A	117,477	44.4	20	4	10	51.6	15
A_1	117,477	57	20	43	10	....	..
A_2	101,373	57	20	43	10	....	..
A_3	101,373	57	18.1	43	6.4	....	..

[a] Cents per cubic yard.

Debt Service on Rapid Transit.—This study considers the capital costs of rapid transit a proper charge against communities in proportion to the line capacity used. The annual costs of operation and maintenance of the lines, however, are paid out of operating revenues. Under existing operating conditions these expenses are not met by the city and are not charged against the communities. The city does contribute substantial sums to service its outstanding rapid transit debt. Accordingly, debt service on that part of the subway improvement costs allocated to each community is charged as an annual expense in Table 3(*b*).

The capital cost shown in Table 3(*a*) is assumed to be financed by fifty-year corporate stock requiring annual payments of 4% interest and instalments to a sinking fund earning 3% for amortization of principle.

DISCUSSION

Donald M. Baker,[9] M. Am. Soc. C. E.—The problem of rehabilitating residential areas where housing has fallen below the standards for which it was originally developed is one that has attracted considerable attention during the two decades since 1920, and will achieve much greater importance in the future.

During past periods of relatively high rates of population increase in metropolitan areas, land area devoted to residential housing likewise increased comparably. New housing was provided primarily for residents in the high-income and upper middle-class income brackets, whose vacated dwellings were then occupied by those with lesser incomes. With continued growth, these in turn moved on, to be replaced by a succession of residents in progressively lowering economic levels. Ultimately, an area was occupied by those in the lowest income brackets unless, as sometimes happened, such housing was located adjacent to an expanding commercial district. Then commercial or light industrial uses would invade the area, occupying the former residential buildings, or replacing them with newer buildings specifically constructed for particular uses. Until about 1920, little attention was given to the progressive "blighting" of older residential districts. Such phenomena were accepted as a normal result of metropolitan growth. Owners of property in such areas maintained hopes that expansion of commercial or industrial districts would in time reach their property, and give it values based upon such uses.

Community planning, as it is now beginning to be practiced, based upon the scientific method of collecting, analyzing, and interpreting factual data relative to growth and decay, developed a realization that the progressive decay of such areas was an effect, the causes of which were subject to determination, and that in many instances remedial measures were possible. Without question, economic losses due to "blighting" are very great, not only to owners of property within "blighted" areas, but likewise to entire communities where such property exists, and to residents of such communities.

Owners of such property experience not only reduction of value of land, because of lowered rentals upon improvements, but likewise a loss of adequate return upon that part of the improvement—the foundation and the shell of the structure—whose physical life, if it is well arranged and soundly constructed, may range as high as one hundred or more years. Reserves for depreciation and obsolescence are seldom if ever set up in individually owned properties, and are more or less impracticable. Community losses include a reduction of taxable values and loss of a part of the public's investment in improvements and services. These usually have a useful physical life in excess of the life of the area served for any but the lowest type of residential housing or, in time, they develop excess capacity due to population loss in such area.

Loss to individuals includes an increase in taxes to support cost of improvements and services originally provided for the "blighted" area, which fails to carry its share of such cost because of lowered taxable value, and loss to merchants and property owners in business districts serving the "blighted" areas. There is also the expense and loss of time caused to those who, daily or frequently, must travel between outlying residential areas and the central business district a greater distance than they were formerly required to travel.

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[9] Cons. Engr., Los Angeles, Calif.

Movement into new housing is caused by both "pulling" and "pushing" forces. Improvement in financial circumstances, attraction of new surroundings and environment, and more modern conveniences all initiate the movement away from established housing for those who can afford it. A "vacuum," slight in amount, is created, which tends to "pull" into the area residents in lower economic levels who can afford to move up the scale somewhat. Soon environment and neighbors have changed to a point where residents who were not "pulled" out of the area are "pushed" out. The cycle is continuous, and may require many years to change the character of a district from a high to a low level. In general, however, the rate of obsolescence and blighting of a residential district varies directly with the rate of population increase in the community as a whole.

With a reasonably high percentage rate of population increase, the supply of persons required to fill up the vacuum created by the outward flow of residents from older districts usually exists all through the scale. The 1940 federal census brought home to many people, however, the fact that most metropolitan areas, as well as the nation as a whole, are fast approaching a static condition in regard to population, and that former rates of increase will not be maintained.

This condition will undoubtedly bring the problem of the "blighted" area into a far clearer focus. Unless American society becomes far more static than it has in the past, with barriers between those in various economic strata becoming far more inflexible, there will always be a large segment of the population who will yearn for, and also be able to achieve, better housing, surroundings and environment, modern conveniences, and everything else that goes with improved circumstances. The demand for new housing will continue; and the "blighted" areas will increase also, but the supply of residents for these latter areas will not be forthcoming as it has in the past, and decay and loss of values therein will be accelerated.

New housing, as now produced, is usually supplied around the periphery of present developed areas. It entails the expense of the many facilities indicated by the author. These expenses are usually a community obligation. Arresting decentralization of population through rehabilitation of areas now in this condition, or approaching it, means large annual savings in dollars and cents to the community, and therefore to its taxpayers.

There is another side of this problem of rehabilitation, however, which pertains to the owner of property in a "blighted" area, who might desire to undertake rehabilitation, or to the investor who might desire to finance it. In many instances, rise in price levels over a long period, and likewise rise in values due to general increase in population of most communities, have reduced losses due to depreciation or obsolescence when property has changed hands. The temptation to spend all revenues received from rentals, except that immediately necessary for routine expenses, maintenance, taxes, interest, and amortization of loans, is increased by the feeling of rising valuations offsetting, at least to a degree, depreciation and obsolescence.

An owner, or an agency contemplating development of new housing to replace old existing housing in a blighted area, is faced with the following problems:

(1) A large area must be acquired if a proper environment is to be created, such area ranging from at least 40 acres to possibly several times this extent. Demolition of a few existing structures, and their replacement by new and modern buildings, will not create environment.

(2) To secure control of an adequate area, many individual properties must be assembled and purchased, or condemned, entailing large outlay of capital for land alone.

(3) When the cost of buildings is added to the capital investment required for land alone (which in a project of this nature would include land and obsolete improvements), any such project takes on very large proportions.

(4) When such investments are constructed upon unimproved land in peripheral localities, the owner's investment usually includes the cost of an unimproved site, of buildings, a certain amount of landscaping, and those public improvements and services not usually provided by the community or existing public utilities. It is questionable in many cases if the increased rentals received from new developments in a rehabilitated area will support the added site cost in such an area.

For years sociologists have stressed the economic losses resulting from substandard housing—losses resulting from sickness, from crime developed in such housing, and from increased costs of fire and police protection. These losses have even been evaluated in terms of dollars and cents. So far, research has failed to develop a type of low-cost housing within the reach of those in the lowest economic circumstances, without some form of subsidization. The propriety of public subsidization of housing for this part of the population has recently been accepted, and many low-cost housing projects have been constructed or are contemplated.

With the definite community savings as shown by the author, which are affected by rehabilitation of "blighted" housing through site planning, and construction in accordance with new and modern standards, the writer poses this question: "Would it not be a sound investment if such projects likewise were to receive public subsidization from the community, at least to an extent which allowed the increased burden of added site cost to be overcome?"

The paper is a distinct contribution to the literature of this subject, made in a language that businessmen understand—that of dollars and cents.

Howard Whipple Green,[10] **M. Am. Soc. C. E.**—With the United States approaching a stable population and its large cities beginning to realize that their population increases will be much more moderate in future decades, with several large cities for the first time showing actual decreases in population while their suburbs still continued to increase, with fewer and fewer persons left in the central city to bear larger and larger tax burdens, with industry and commerce showing indications of decentralization, and with slum and blighted areas increasing in extent and severity, the problem of rebuilding cities in the United States ranks near the top of the list of important things to be given attention following the present national emergency.

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[10] Secy., Cleveland Health Council; Director, Real Property Inventory of Met. Cleveland, Cleveland, Ohio.

A possible solution to this serious problem of the depopulation of the center of most large American cities with increases in population around the periphery is indicated by the detailed calculations presented in this paper. Although all computations presented are naturally based upon New York City, standards of land use, and unit costs of public improvements and services, similar computations made in smaller cities with different standards of land use and different unit costs may demonstrate even more striking differences between the rehabilitation of existing blighted areas, the entire rebuilding of these areas, and the relocation of their populations on vacant land in outlying sections within or beyond their corporation limits.

The basic facts made available in Cleveland, Ohio, by the continuous Real Property Inventory started in October, 1932, augmented by unit costs of public improvements and services, make such a study possible with a minimum amount of effort.

Cleveland being built horizontally, contrasted with New York's vertical pattern (especially the Borough of Manhattan), doubtless will show quite different factors. However, it is as true in Cleveland as in New York that the public utilities that were complete in the section designated as census tract J-1 (with a population of 6,114 in 1910), which decreased to 493 in 1920, to 49 in 1930, and to 3 in 1940, were of no assistance to census tract T-8 with a population of 59 in 1910, which increased to 1,557 in 1920 and to 10,577 in 1930, decreasing to 9,986 in 1940; nor were the schools, hospitals, and churches, or the retail stores, water mains, sewers, paving, gas lines, and electric and telephone networks serving the 6,114 inhabitants of census tract J-1 in 1910 and quite useless to the 3 persons living in this area in 1940 of any aid to the suburb of University Heights, which increased from a population of 156 in 1910 to 5,981 in 1940. Such a shifting of population is one problem during periods of rapid population expansion accompanied by proportional expansions of industry and commerce, but it is quite another as a stable population is approached.

T. T. McCrosky,[11] M. Am. Soc. C. E.—Many large-scale real-estate developers have made detailed cost studies of the relative economics of private building operations on high-priced improved slum land versus low-priced raw land; but their studies do not include consideration of the costs to the public that enter into their contemplated developments. To the writer's knowledge, Mr. McHugh's paper is the pioneer research in the field of cost of public services made necessary by large private residential projects. As a former colleague of the author, it was the writer's privilege to follow this study closely during the months that it was being conducted without, however, participating directly in its progress.

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[11] Executive Director, Chicago Plan Comm., Chicago, Ill.; formerly Director of Planning, Dept. of City Planning, New York, N. Y.

Similarly thorough analyses could be made for other cities, with different unit costs and appropriate modifications in the assumptions; but the resulting quantitative comparisons would assuredly be of the same general order of magnitude, differing only in degree. The case is proved by this study. It costs the public very much more to provide services for outlying developments than for equivalent redevelopment projects in "near-in" locations, where many satisfactory public facilities already exist. Whether it be 50% to 90% more per person housed, as computed by the writer (for Units A_1 and A_2, where rapid transit charges are excluded), or some moderately lesser or greater figure, is of secondary importance to the fundamental conclusion that it is manifestly uneconomical from the public standpoint to encourage the decentralization of cities.

The great difference in public costs between development of areas with existing transit facilities and areas where new transit lines must be provided is clearly revealed by the author. The moral is inescapable that the least a city should do is to discourage new large-scale building on outlying vacant land that does not have, but will require, transit service.

It would be very valuable to have an extension of this study, in order to develop consolidated comparisons of capital and operating figures for the combination of public services plus private project costs. The differentials of public outlay are both capital and current account. They result from the cost of duplicating facilities and services already existing at the central location. These have been very thoroughly and ably analyzed by the author. The principal differentials of private outlay are capital items. They are the price of land, the expense of assembling it, the cost of the improvements, and the cost of the demolition of slum buildings to make way for the new project. Unfortunately, slum buildings do not tumble down like the walls of Jericho at the summons of a Joshua.

With a consolidated cost comparison, it would then be fruitful to study the feasibility of offering some partial public subsidy to the private builder to encourage him to redevelop slum areas. This could be scaled so that the costs to the taxpaying public, including subsidy, still would be well below what they would aggregate if the builder developed outlying raw acreage. In practice, the subsidy might often take the form of giving title deeds to tax-foreclosed properties, which were foreclosed because they yielded no taxes and thus made no return to the public, and which can never make a return unless transferred to a private owner for sound redevelopment. Another form of subsidy that costs the public nothing is to deed unnecessary street land to building corporations. Partial or limited tax exemption is also "cheap at the price," inasmuch as slum land is notoriously delinquent in most cities. If the public actually collects moderately more taxes from the new project than it did from the substandard properties that are replaced, it is "ahead of the game."

When peace comes, urban America will need a coordinated program for city rebuilding, embracing all price and rental brackets and clearly determining the extent and limits of public and private participation in each bracket. Preparation of this program is the responsibility of the technical professions in the building field. Mr. McHugh's paper is a valuable contribution to the economics of this broad-scale approach to urban redevelopment.

F. DODD MCHUGH,[12] ESQ.—There have been vast changes in the world since the paper on "Cost of Public Services in Residential Areas" was first published. The United States is at war, and the Nation has passed from a defensive phase of preparing for some vague eventuality into a period of vigorous war activity. Its main efforts are directed toward winning a most difficult war to secure freedom. Many individuals are engaged directly or indirectly in the war effort or in civilian defense activities.

Under the circumstances, it may seem like "fiddling while Rome burns" to discuss such matters as public outlays for municipal services. Upon many recent occasions it has been said, however, that one must not only win the war but the peace. To win the peace the people must define their goals and do an extraordinary amount of enlightened planning while the war is being fought.

When the phenomenal efforts of the United States and its Allies have brought peace to the world, most of the old problems will remain and a host of new ones will have appeared. What will the Nation do about some of the now familiar domestic problems? Plans are already in the making for the readjustment of private business after the war, for public works programs, and, among other things, for urban redevelopment. It would seem the logical time to discuss public outlays for municipal services, if such investigations can aid in the clarification of policy and in the formation of postwar action programs.

Planning for the Postwar Era.—There is already wide recognition of the necessity for careful planning for the postwar period, and many public and private agencies are doing research in a variety of fields.[13] The executive branch of the federal government has stated broad objectives and initiated investigations and long-range programs directed toward meeting the future social and economic needs of the Nation. In transmitting to the Congress the "Development of National Resources—Report for 1942," prepared by the National Resources Planning Board, the President stated in part,

> "Plans and programs to win the war and to win the peace must grow out of our common national purpose and with democratic participation in planning by all of us. Through efforts to state our objectives and public discussion of their merits, we play our parts as free citizens."

This report suggests for discussion the following principal objectives of postwar planning: Full employment and maintenance of the national income at not less than $100,000,000,000 a year, without lowering working standards; decentralization of postwar activities and full use of free enterprise in cooperation with national and government leadership; and individual realization of security, opportunity, and the basic freedoms. It also suggests upbuilding America as the keynote of a postwar program,

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[12] Director of Planning, Dept. of City Planning, New York, N. Y.

[13] See the survey on "Post-War Planning in the United States," by George B. Galloway, The Twentieth Century Fund, New York, N. Y., 1942.

> "* * * including both development of our national resources adding to the National Estate, and service activities, which will increase the vitality, health, skill, productivity, knowledge, and happiness of the American people, and thus together end unemployment and add to our wealth and well-being."

This is not the place to enlarge upon these objectives, but an outline of them indicates the breadth of thinking in the federal approach to winning the peace on the domestic scale. The National Resources Planning Board is correlating plans and programs under consideration in many federal, state, and private organizations for postwar full employment, security, and building America. Among other things, it not only furnishes aid in various phases of the war effort, but it also brings together 6-yr programs of federal agencies and aids in developing programs of useful public improvements and services planned by state and local governments. With the necessary detailed plans completed, this great reservoir of public works can help to absorb the men and industry that will be demobilized after the war.

The New York City Planning Commission is proposing that the 1942 Capital Budget be amended to provide funds for the planning of a postwar program of public works. Some funds for general planning studies, sites, preliminary plans, working drawings, and specifications are provided in the 1942 Budget. The proposed Post-War Works Program would expand the reserve of projects that can be undertaken, whenever men, materials, and funds are available, by completing plans for a considerably enlarged number of desirable municipal improvements.

Regardless of the as yet uncrystallized results of current discussion and planning for postwar activity, one can readily agree that any action programs that may be developed cannot do less than promote the conservation of human and natural resources in a society of free men. It is perhaps a poor commentary, but human society seems able to achieve its greatest organization and productivity in periods of national and international stress. In the intervening peaceful times, the extraordinary cooperation attained in emergencies fades. There is no actual reason, however, why a democratic system cannot function equally as well for peace if it can agree upon broad objectives and mobilize the creative, organizing, and productive abilities of its people to promote the general welfare.

In the discussion, planning, and execution of any postwar program, the individual as well as the system of private enterprise must make substantial contributions. Government may have to give new forms of assistance and some direction in coordinating private action with public programs, but enlightened free enterprise must play a major part in attaining anything like full employment, security, and the upbuilding of America in the postwar era.

Urban Rebuilding.—There will be many possibilities for new cooperative activities after the war, including urban redevelopment. More than one half the population of the United States lives in urban areas, and about 48% of it in some 140 metropolitan districts. The 1940 census data clearly reveal that the rate of population growth in the United States is slowing down to the point where it will probably approach stabilization during the life of the present generation. Urban and metropolitan areas are following this general trend, with a few notable exceptions. Consequently, there is a definite need for ordering the towns and cities of the Nation on a more up-to-date and efficient basis for working and living if they are to serve their functions properly in an era of population stability. The social and economic losses due to overexpansion, blight, and obsolescence must be arrested. Existing transportation facilities should be improved and integrated with the transportation advances that undoubtedly will follow the war; obsolete housing should be replaced by large-scale residential neighborhoods designed to improve living conditions and to stabilize the residential character of communities on a high social level.

These notions are not new; they are being discussed throughout the Nation. The importance of coordinating public and private plans and actions in urban rebuilding is obvious. Similar cooperative efforts are already under way in England. Such a program requires the definition of national and local objectives, and the development of new approaches and techniques to draw the broad outlines and fix the tempo within which the gigantic task of city rebuilding might proceed.

Public Costs and Other Factors.—The "Cost of Public Services in Residential Areas" is one of many kinds of investigations needed, and in fact it was undertaken as part of a larger scheme of approach to municipal problems in New York City. As mentioned in the discussions, these urban problems are much greater than the relative costs of public facilities in certain types of residential development.

Mr. Baker has noted the economic losses of individuals, property owners, and the public at large due to urban blight and decentralization. Incidentally, one phase of the original cost study indicated that overexpansion and duplication of public services in residential areas are costing the City of New York at least $40,000,000 annually for operation and maintenance. This is a rough estimate, to be sure, but it suggests the extent of economic loss suffered by the public as a result of blight and decentralization within the city's boundaries. Mr. Baker also mentioned the problems of land assembly and high site cost in connection with large-scale rebuilding. He asks if rents in a rebuilt area can support the high site cost and suggests that it might be a sound investment to subsidize such residential projects, at least to the extent of overcoming the burden of high land acquisition costs. Here is one of the greatest obstacles to city rebuilding. It would appear questionable that even one community could be rebuilt privately on a large scale and become self-supporting without public financial assistance because the rentals that its tenants could afford are almost certain to be insufficient to cover the high site cost plus the expense of new dwelling construction. The ease of land assemblage and its relatively low cost in suburban and outlying urban areas permit profitable private developments, and account in some measure for the continued physical expansion of cities even after population growth is drastically reduced.

Mr. Green ranks rebuilding problems near the top of the list of important things to be given attention following the present national emergency. The writer agrees with the ranking and with the absolute need for concentration on the war effort, but he is convinced that means must be devised and plans developed for urban rebuilding before the war is over. Otherwise, the Nation will be in a less favorable position in winning the peace. Had Mr. Green presented such computations of public costs in Cleveland, they would have been a welcome addition to the paper. Its continuing Real Property Inventory puts Cleveland in the enviable position of knowing of current changes, and the data thereby available make such investigations as a cost study relatively easy.

Mr. McCrosky suggests that the cost study should be extended so as "to develop consolidated comparisons of capital and operating figures for the combination of public services plus private project costs." He also mentions the problems of site acquisition and suggests an investigation of the feasibility of offering some kind of partial public subsidy to the private builder as encouragement in the redevelopment of slum areas. He makes an excellent point in stating that, since tax delinquency is prevalent in slum areas, limited tax exemption would be justified if the actual tax yield on rebuilt projects exceeded the taxes collected from the replaced substandard properties.

In the section of the city represented by old Area *A* of the cost study, it was found in 1938 that some 23% of the properties were tax delinquent. On the basis of this delinquency ratio and a basic levy of 2.75 on an assessed value of 29.8 million dollars for residential property in old Area *A*, the city collected about $630,000 in taxes from housing in this area. Assuming for the moment that new housing in rebuilt Unit A_2 is assessed at the same value as the existing residential properties (29.8 million) and the tax levy remains at 2.75, collections should average 98% of levy. On this basis the tax yield would approximate $820,000 per yr, which amount is 1.29 times current collections in old Area *A*.

The total costs of land and new housing in rebuilt Unit A_2 may be expected to exceed the present assessed value of existing dwellings in old Area *A* by an amount which would be determined by site and construction costs. However, if the assessed value of new housing in Unit A_2 were held at the same level as existing dwellings in old Area *A*, the private developer thus would receive a form of public subsidy; but the city's debt incurring and taxing powers would not be curtailed by a reduction in assessed value and, as Mr. McCrosky states, the city would be "ahead of the game," in so far as taxes collected on the new housing exceeded the present yield of taxes levied.

The discussions have been most profitable. The writer was disappointed, however, that the assumptions and techniques of the cost study were not subjected to a more critical evaluation. The setting given to the study and the questions raised in the discussions lead to further comment on investigations undertaken since completion of the original cost study in 1939. In this connection it should be made clear that the material contained in this closing discussion, as well as in the paper, does not in any respect represent the opinion nor the policy of the New York City Planning Commission or of the Department of City Planning.

Current Studies.—There is considerable evidence of a widespread approach to the solution of urban problems. The following is by no means a complete summary, but it is illustrative of current trends. In its "Urban Progressive Planning Project" the National Resources Planning Board is attempting to establish procedures for the progressive planning of required urban facilities. The Federal Housing Administration (FHA) has prepared *A Handbook on Urban Redevelopment for Cities in the United States.* It contains timely suggestions for procedures in an integrated long-term program for dealing with slums and blighted urban areas. The Illinois and New York State legislatures have enacted urban redevelopment laws that attempt to attract private capital into the rehabilitation of cities.

The findings of the cost study indicate that large-scale rehabilitation of old central apartment districts is economically feasible and preferable to new developments in outlying sections in so far as city expenditures are concerned. The paper concluded with a suggestion that essential studies of the private costs involved, and of the procedures to be used in large-scale redevelopment of entire neighborhoods, be undertaken immediately under a joint private and public sponsorship. The study showed that, if all the persons living in old Area *A* were rehoused on the same site, the new dwellings in Unit A_1, at an average height of six stories, would cover 68% of the residential land and the net density would be 731 persons. In setting up Unit A_2, it was assumed that apartments averaging six stories and covering 50% of the residential land, with a net density of 540 persons, would produce a better living arrangement and that thus Unit A_2 would be subject to a much lower rate of obsolescence than Unit A_1. The costs per person housed were also found to be lower in Unit A_2 than in Unit A_1. This finding suggests that there may be an optimum density, at which public costs per capita would reach a low point, in the rebuilding of old Area *A*.

A study was made to ascertain this population density or range of densities. Regardless of other considerations, it was assumed that old Area *A* might be rebuilt to house a varying number of people, and the calculations were based on the same suppositions and technique used in the original study. (The computations were made under the author's direction by Roger W. Loewi, "Interne" in the Department of City Planning during the summer of 1940.) A part of the results are given in Table 13, which shows that the lowest per capita city expenditures are attained in rebuilding old Area *A* at a density of 324 persons per residential acre. This particular 160-acre community, identified as Unit *C* in Table 13, could house more than 24,500 people in buildings covering 30% of the residential land at an average height of six stories. The per capita cost varies within narrow limits, however, as between the communities designated as Units *B*, *C*, and *D*. Although not shown in Table 13, it was found that per capita costs for developments with densities greater than

731, and less than 108 persons per residential acre, increase at a more rapid rate than the change in density. This indicates that, other things being equal, it would be better from the point of view of public outlays required to rebuild old Area *A* as a modern community within a density range of about 250 to 430 persons per residential acre. This possibility raises an entire series of questions concerning the community's place in the scheme of things, such as the desirable urban pattern for the entire city and its parts, as well as the objectives, techniques, and standards for planning the redevelopment of American cities—in this case the residential sections of Manhattan in New York City.

TABLE 13.—CITY EXPENDITURES PER PERSON, BASED ON APARTMENTS AVERAGING SIX STORIES IN HEIGHT
(With Transit Charges)

Rebuilt community	Proportion of land covered (%)	Persons per residential acre	Improvement (dollars per person)
Unit A_1	68	731	163
Unit A_2	50	540	119
Unit *B*	40	432	96
Unit *C*	30	324	95
Unit *D*	20	216	98
Unit *E*	10	108	110

Private Costs in Rebuilding.—Mr McCrosky's suggestion that the cost study be extended to develop consolidated comparisons of capital and operating figures for the combination of public services plus private project costs is difficult to carry out, due to a multiplicity of variable factors. Several attempts, made along these lines, have so far failed to produce satisfactory results. Complex problems are involved and available data are insufficient. One angle of approach might be somewhat as follows:

The people now living in the specimen community, old Area *A*, have different incomes—not all of them are in the lower income category. In rebuilding such obsolete sections on a large scale, it will be necessary to make provision for families of varying income. Assume that in the present case the engineer is concerned with an average community that will accommodate all groups. With this assumption it is possible to deal with the average rental that people can afford to pay. In New York City this sum is approximately $520 annually per family, which represents the average "demand." What can the private developer supply to meet this demand?

The answer to this question is conditioned in large measure by the capital outlay required of the private developer and the annual carrying charges on the completed project. For purposes of discussion, assume that the operation and maintenance, including insurance and a vacancy allowance, can be held to an average of $150 per dwelling unit. On the basis of the $520 average annual income received for each dwelling unit, and an annual cost of $150 for operation, etc., the private developer would have $370 left to cover taxes, capital charges, and interest on his equity.

Since the problem concerns a large-scale development that offers unusual stability of investment, it is possible that the private developer can obtain capital at 4% and, because the investor will want his capital repaid, the loan might be amortized at, say, 1.5% annually. With annual interest payments fixed at 4% and amortization at 1.5%, the borrower would repay the entire loan in about thirty-four years.' This setup is similar to recent financing under the FHA mortgage insurance plan.

The private developer also must receive a return on his equity or investment. In these days of low capital earning, one may assume a 6% return to be quite attractive. Inasmuch as the rebuilt community is a large-scale affair, requiring adequate public facilities on modern standards, the city could not afford to supply improvements and services without making some charge for them. Therefore, real estate taxes should be included in considering private costs. These private costs may be expressed as:

Item	Percentage of total capital cost
Interest	4% on 80%, or 3.2%
Amortization	1.5% on 80%, or 1.2%
Equity earning	6.0% on 20%, or 1.2%
Taxes	2.75% on 92%, or 2.53%
Total	8.13%

The private developer receives on the average only $370 per dwelling unit to cover these changes. This amount capitalized at 8.13% is $4,550, which represents the total private investment per dwelling unit that can be made economically in this particular case. In a large-scale operation it may be possible to effect such economies that existing structures on the site could be demolished and the new dwellings erected at an average cost of $3,640 per family unit. For this "average" case this development cost is roughly 40¢ per cu ft. At such a favorable rate for construction costs the private developer could afford to pay $910 per dwelling unit for land. If the community is built to house 250 persons per residential acre, land could not cost more than $1.42 per sq ft; at a density of 430 persons per residential acre, the developer could pay $2.45 per sq ft for land. At a density of 540 (provided in rebuilt Unit A_2 of the cost study), the economic value of land is slightly more than $3 per sq ft under these assumptions.

There is not much land in urban slum districts that can be purchased at these figures. At rentals below the afore-stated $520 average, the situation is obviously more difficult, and, inversely, building for rentals greater than $520 is relatively easier. (This average rental does not include rents from stores, parking garages that could be located in the basements of residential structures, nor other miscellaneous income.) Of course, a higher average income might permit higher rents and greater land values, but the resulting increase in living standards could not be counted upon to reduce building costs.

Public and Private Costs.—The sums used in this illustration of an average condition may or may not be valid in so far as the exact private costs of development are concerned. One of the objectives in urban redevelopment is to rehouse people at rents they can afford to pay. With an average rental income of $520 per yr, the private developer can spend only $4,550 per dwelling unit, or approximately $1,240 per person, under the conditions assumed. If the capital charges and operating expenses are larger than those assumed, the total private capital cost would be reduced. The point the writer wishes to make, however, is that, whatever the exact figure, private costs per dwelling unit or per person, based on a given rental income, remain at the same average amount regardless of any change in population density. On the other hand, the cost study shows that public improvement costs per person vary with the density of population. Consequently, a combination of public and private capital costs does not appear to throw as much light on the problem as Mr. McCrosky anticipated.

Mr. Baker emphasizes these economic difficulties in his discussion and asks if it would not be a sound investment to subsidize such residential projects, at least to the extent of overcoming the burden of high land acquisition costs. If it were possible to exempt the project from real estate taxes, the situation could be eased. On the basis of the foregoing illustration, the private developer could spend a total of $6,600 per dwelling unit if he did not have to pay taxes ($370 capitalized at 5.6%). Allowing the same figure for construction ($3,640 per dwelling unit), he could afford to pay about $3 per sq ft for land at a density of 250 persons per residential acre; about $5.25 per sq ft for land at a density of 430 persons per residential acre; and about $6.60 per sq ft for land at a density of 540 persons per residential acre.

What Are the Possibilities?—Large-scale tax exemption would make it nearly impossible for most cities to remain solvent under current means of raising municipal revenue. Apparently, engineers must look elsewhere for a solution. It may be possible to devise ways of reducing private operating and maintenance expenses, and to cut building costs with mass construction methods; or the cost of capital may also be cut in proportion to the elimination of risk in investment. If all of these economies could be made, the high cost of land would still remain an obstacle to urban rebuilding.

Limited tax exemption would be helpful, but it is well known that revenue received from taxes levied on residential property does not cover the actual cost of public services provided in residential sections. For example, total annual expenses for public services in old Area A are estimated at more than $1,600,000 and the tax yield at some $630,000 per yr. If partial exemption, on the basis heretofore mentioned, were granted to the rebuilt Unit A_2, a tax yield of about $820,000 could be expected; but annual expenses for public services would exceed $1,300,000 per yr. Thus, the city would forego a higher tax return, not for the purpose of reducing rents nor to give greater service for the same rents, but to underwrite site costs that are greater than the actual and potential use value of the land.

Perhaps the creation of public land corporations or authorities, with the power of eminent domain and of issuing bonds secured by the land, should be considered. If the land and improvements were subject to real estate taxes, the city's taxing and debt incurring power would not be impaired. This is not an original suggestion (see the FHA *Handbook on Urban Redevelopment for Cities in the United States*). With this type of public ownership, the land could be leased for appropriate private development, and the purchase price of the land could be amortized over a sufficiently long period so that the ground rent charged the private developer would coincide with the economic use value of the land.

Even if land "costs" can be reduced in this manner, the engineer must still find the means to effect economies in construction costs and operating expenses, as well as in charges for the use of capital. Otherwise, large-scale urban redevelopment cannot be attained at economic rental levels. If these or similar devices can be developed and put into operation in the near future, engineers can also perfect their procedures and techniques, and commence the planning and timing of urban redevelopment as an integrated part of postwar activity.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2164

EXTENSOMETER STRESS MEASUREMENTS, NORTH AVENUE BRIDGE, CHICAGO, ILL.

BY LAWRENCE T. SMITH,[1] M. AM. SOC. C. E., AND
PAUL LILLARD,[2] ESQ.

WITH DISCUSSION BY MESSRS. ARTHUR R. LORD, J. CHARLES RATHBUN, AND THEODORE BELZNER.

SYNOPSIS

The design of the North Avenue Bridge (Chicago, Ill.) presented several major structural engineering problems. In developing a rigid-frame design, certain very critical structural assumptions were made which were believed to be quite valid but which lacked definite quantitative substantiating evidence. The importance of the structure justified a systematic extensometer investigation into the actual field-stress condition and structural behavior of the bridge in order to establish the relative validity of these basic assumptions.

As a result of this investigation, the true functional capabilities of this bridge are now known, definitely and accurately, free from the shadow of doubt that often clouds the reflections of the structural designer in his application of pure mathematical logic to the complex factors affecting the higher type of structure.

FIELD MEASUREMENT OF STRESSES

The first recorded instance of the actual field measurement of stress in an existing structure by means of the extensometer was in 1883 when J. J. Frankel described his investigations (12)[3] upon the primary stresses in a Pratt truss bridge. In 1899 M. Mesnager published (27) an account of extensometer stress measurements made upon a railroad bridge. In 1901 M. Rabut described (34) stress measurements on girder and truss railroad bridges. In 1907 the American Railway Engineering Association (A.R.E.A.) conducted stress measurements of limited scope on girder and truss spans to determine relative impact allowances for design purposes (21).

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[3] Numerals in parentheses, thus: (12), refer to corresponding items in the Bibliography (Appendix).

In all of these investigations, which constitute probably the entire scope of the work done in this field until about 1917, no stress measurements were made to compare computed and measured stress or to determine dead-load stress; and no measurements were made on statically indeterminate structures. Such work as had been done was greatly restricted to the determination of live-load stresses and to the evaluation of relative impact. Many of the results contained serious instrumental errors. The use of the extensometer in the systematic investigation of true field stress and the verification of structural design theory applied to an existing structure were of no consequence until 1917 when D. B. Steinman, M. Am. Soc. C. E., conducted his extensive systematic stress measurements on the Hell Gate Bridge (39). This work was done on a monumental scale and remains today probably the most comprehensive stress survey ever made upon a bridge in service.

In the interval from 1917 to date (1941) there has developed an increasing interest in the extensometer as a precision instrument of value to the structural engineer; but the period has been characterized by few stress measurements on structures under construction or in service. The use of the instrument has been confined mainly to investigations upon small models; upon structures erected for specific test purposes, and subject to structural control; and upon structures or members observed under laboratory conditions. In view of the means at hand it is remarkable that the phenomenal progress made in structural design theory in the two decades 1920–1940 has not been accompanied by a wider utilization of the extensometer by structural engineers.

Field stress measurements are expensive, and they are time-consuming and laborious. They require a high order of instrumental and technical skill in order to be of value. Few organizations have the personnel equipped for this work and few can afford the heavy financial outlay for the apparently intangible results achieved. However, there are undoubtedly many situations that arise in practice where the use of this valuable tool would result in a great economy of effort and easily would secure for the engineer the answer to many otherwise impossible stress problems.

Description of Bridge

The Project.—The North Avenue grade-separation project of the Chicago Park District was initiated to provide adequate pedestrian and vehicular grade-separation and traffic-interchange facilities for the introduction of vehicular traffic from the improved LaSalle Street into the Outer Drive of the Chicago Park District, in the southern area of Lincoln Park (see Fig. 1). The project required a complete redesign and reconstruction of most existing facilities over a large area of the park, involving landscaping and the relocation of roadways and walks, sewers and electric lines, and the construction of several large pedestrian subways and a large, skewed, rigid-frame type of vehicular bridge constituting the main unit of the project.

This bridge is designed to accommodate the extremely heavy Outer Drive traffic passing over the likewise heavy northbound LaSalle Street flow. Both streams are interchanged at a junction immediately north of the bridge. Because of space limitations, it was necessary to cross the two streams at a sharp

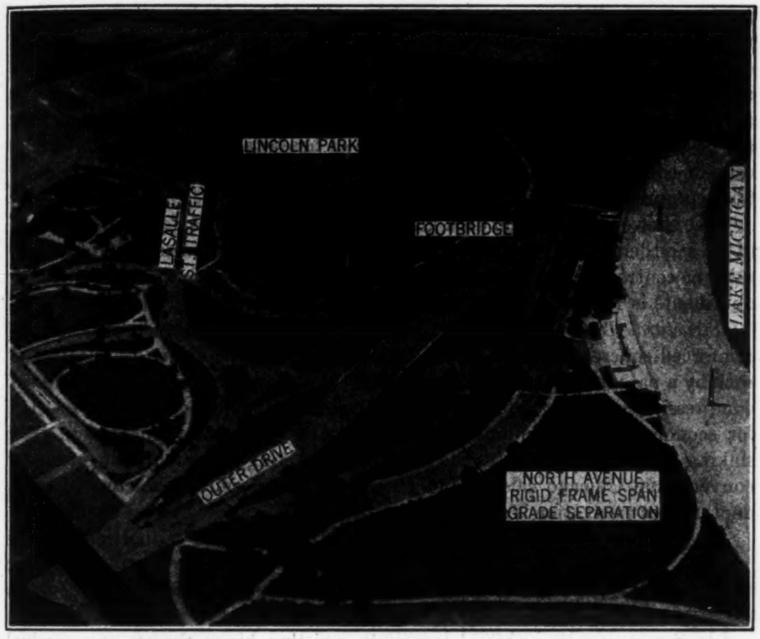


FIG. 1.—MODEL SHOWING LAYOUT OF PROJECT

skew. A high water table prevented an adequate depression of the lower roadway, and the resultant gradient problem placed a definite limitation upon the allowable deck thickness, indicating the use of a structure of the rigid-frame type.

The North Avenue Rigid-Frame Bridge.—The immediate surface stratum at the site consists of a fine lake sand of variable thickness resting upon several strata of glacial till composed of blue-brown clay. Core borings taken at the site showed the upper stratum of this till to be a very wet, soft, plastic, colloidal clay apparently unreliable for major foundation purposes. Load tests made upon this clay showed it to be of little value in supporting a rigid-frame structure. Mantle rock, a reliable Niagara dolomitic limestone of great depth, lies at about 90 ft below Chicago Datum at the bridge site. After considerable study, in which the use of piling was considered, it was decided to rest the structure on cylindrical concrete subpiers driven to mantle rock or to the cemented gravelly claypan which lies immediately above rock. Studies having indicated the existence of much heavier hinge thrusts than could be resisted alone by these subpiers, the lower roadway slab was reinforced and designed to function as a tie in restraining the abutment hinges.

The superstructure is a skewed, two-hinged, reinforced-concrete, rigid frame with deck and abutments of slab-and-girder design and with hinges tied by the reinforcement of the lower roadway slab. The bridge loads are transmitted to rock through spirally reinforced concrete subpiers. The deck is made of slab-and-girder design primarily for economy, but also because of the practical impossibility of predicting stresses in a skewed, rigid-frame bridge of the slab or barrel type. The deck at the crown is of conventional T-beam construction; at the haunches, because of the existence of heavy negative moments, it is of cellular construction with a lower flange slab for the resistance of the bottom fiber compressive stresses.

The skew angle between deck girders and abutments is 55°. The clear span of deck girders is 101 ft and the centroidal span of the frames is 105 ft. The ratio of centroidal rise to centroidal span is 1 : 7. The width of bridge along abutments is 130 ft, and the curb width of the upper roadway is 102 ft.

Structural Design and Stress Analysis.—Supporting subpiers are spirally reinforced-concrete columns of the Chicago caisson type driven to hardpan or rock by a patented rotary process. These subpiers were designed only for axial compression as short reinforced-concrete columns, the diameters being selected for economy and a minimum inspection space. The rotary process used in sinking these piers eliminates the usual large working space requirement of the conventional Chicago caisson and allows the use of small diameters to great depths since excavation is almost completely mechanized. Lateral bending was not provided for in proportioning these subpiers beyond that available in the axially designed section. The tie-slab (Fig. 2) was considered the active agent in resisting hinge thrust, any residual bending being taken by the passive resistance of the substrata. On this basis the tie slab was proportioned to accommodate the entire hinge thrust, unrelieved by active or passive backfill pressures and subpier restraint. In making the design it was not believed that the effect of active or passive backfill forces would be large, either upon hinge thrusts or upon the stress functions of the superstructure. Sleeves for mud-jacking were provided in the tie-slab for the readjustment of the slab in the event of subsequent deleterious settlement.

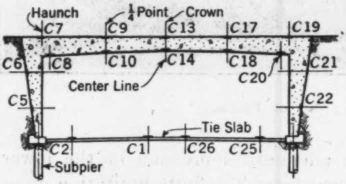


FIG. 2 .—TYPICAL SECTION, SHOWING LOCATION OF STATIONS ALONG CENTER GIRDER

The bridge was designed to carry the Park District standard truck live load, consisting of a file of two 24-ton trucks surrounded by a uniform load of 100 lb per sq ft of deck. Impact was taken at 22%. The maximum temperature differential between superstructure and tie-slab was taken at 25° F, and the effect of hydration shrinkage was based upon a coefficient of 0.0002. No beneficial effect was considered to be derived from active or passive backfill pressures.

Preliminary proportioning was made by means of the moment-distribution method (7), assuming a two-hinged frame of variable moment of inertia. Final analysis of this preliminary proportioned structure, after complete structural detailing, was made by means of the elastic theory, using two separate methods of attack by two computers: (1) Constant $\frac{ds}{I}$, and (2) constant ds. Each analysis was thorough, and the use of the two methods furnished an excellent check. Completion of the analysis by elastic theory showed a very close agreement with the preliminary functions obtained by the moment-distribution method. The bridge was treated for analysis as a series of interior girders or frames, and fascia girders or frames. Each was analyzed separately. As actually analyzed, the frame was assumed to be indeterminate only in the first degree, since the only decisive unknown is the horizontal component of the hinge thrust. The effect of subpier restraint and backfill effort was considered negligible in influencing the hinge thrust and superstructure functions to any significant degree. The horizontal component of the hinge thrust is a function of the properties and loads of the superstructure and the deformations occurring in the tie slab. The equation for hinge thrust under a vertical loading upon the deck is

$$H_0 = \frac{\int_L^R \frac{M'\,y\,ds}{I}}{\int_L^R \frac{y^2\,ds}{I} + \frac{L}{A_s}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which $\frac{L}{A_s}$ represents the restraint of the tie slab. Design studies showed that, theoretically, the effect of the tie slab upon the computed stress functions of the deck is practically negligible, and that small horizontal movements in the hinges produce negligible moment variations in the superstructure. The truth of this statement, in practice, has been established definitely by the results of the extensometer stress study. As measurements during the test showed that there was no movement of the hinges due to applied loads, no elongation of the tie rods was taken into account in evaluating the results. However, the actual design proportioning of the frames was based upon consideration of the tie-slab restraint.

Early stress studies showed the negative moment compression at the haunches to be very high, necessitating the use of a lower flange slab for inverted T-beam action and resulting in the cellular construction of the deck from quarter point to abutment. This cellular part (Fig. 3) was detailed with a taper in the lower flange slab starting at the quarter point and widening out to full width at the abutment. As finally treated, it produced a very satisfactory architectural motif strongly suggestive of greater span and strength.

Shears in the deck girders are high and are accommodated by bent bars and by vertical stirrups. For the accommodation of leg shears, which (according to recent laboratory investigations) are uncertain in actual intensity and are probably much higher than the computed values, more shear reinforcement was used than was indicated by conventional theory.

The abutment hinges were detailed to allow a fairly free rotation with a minimum of moment transmission into the subpiers. The subpiers, however, move horizontally in conjunction with the tie-slab and the abutment hinges. The tie-slab is anchored positively to the lower legs and the hinges. A heavy subpier cap girder was provided to effect a lateral distribution of unequal local loads to the subpiers, and a like provision was made for the distribution of local loads to the lower part of the abutment legs by the thrust girder.

The effect of skew was neglected in analyzing and proportioning the interior frames, although it is obviously a factor in the action of the fascia and adjacent outer girders under certain asymmetrical loadings. However, it was deemed satisfactory to proportion the frames by conventional methods. The wisdom of this course has since been proved by the results of the stress measurements made upon the completed structure.

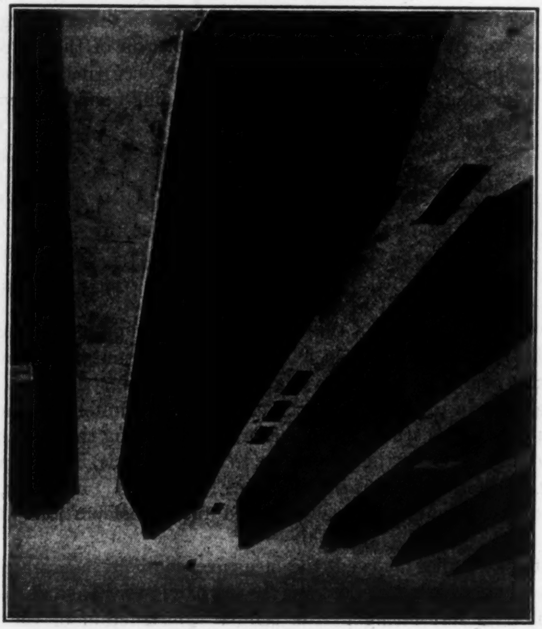


FIG. 3.—MAIN-FRAME DECK GIRDERS BEFORE CEMENT FINISHING, SHOWING CELLULAR HAUNCHES AND EXTENSOMETER STATIONS

The total weight of reinforcing steel used in the superstructure (Fig. 4) and tie slab was 374 tons, and the total volume of concrete was 2,650 cu yd, a volumetric proportion of 2.1%. The setting of steel presents a difficult problem because of the weights and lengths involved. However, this percentage of steel (2.1%) is very reasonable, compares very favorably with that of other structures, and indicates economy in the use of this type of bridge.

Construction.—In sinking the subpiers it was necessary to develop a technique especially adapted to the peculiar subsurface conditions and the rotary process used. A primary requirement in building a structure of this type is the proper design of an unyielding centering and its skilful erection. In this kind of structure, in which few lines are straight, a high quality of workmanship on the part of the carpenter is of vital importance.

Although the structure was built with Park District forces, a close field engineering control was established very early for purposes of securing a uniform production of concrete, satisfactory control-test cylinders for use in the stress measurements, and a qualified inspection service during the construction period.

Control-test cylinders were made from each pour and were subjected to breaking tests to establish the quality of the concrete. A number of these cylinders were given stress-deformation tests to secure the stress-deformation

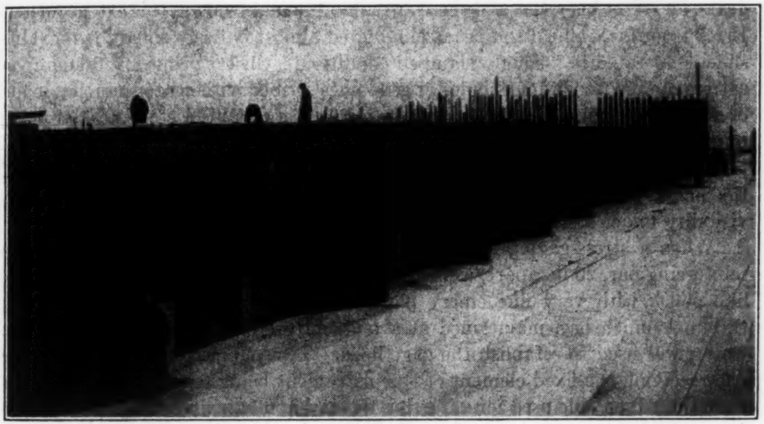


FIG. 4.—VIEW SHOWING REINFORCEMENT AT THE HAUNCH

diagram below the failure point. This was likewise done for specimens of reinforcement cut from the bridge. The results were used in selecting the value of $n = \frac{E_s}{E_c}$, which was taken as 10.

EXTENSOMETER STRESS MEASUREMENTS

Object.—The massive character and the importance of the structure were two of the several reasons why it was considered advantageous to conduct an extensive series of strain-gage extensometer measurements: (1) To ascertain the true stress condition as a matter of safety, (2) to make a comparison of true and theoretical stresses as a matter of structural engineering research, (3) to study the elastic action of the system and the behavior of the bridge under service conditions, and (4) to secure data of value in the future design of similar bridges.

Preparation of Stress-Gaging Stations.—For the measurement of stresses twenty-eight strain-gage stations were provided for in the design and were built into the deck girders, tie-slab and abutment legs of the center-line interior frame, and the east fascia frame (see Fig. 3). At each station the outer face of a length of reinforcing bar carrying computed stress was formed, partly exposed for observation; a thermometer pipe was embedded to the center of the concrete mass; and a pair of thermocouple leads were embedded for the potentiometer determination of internal concrete temperature.

An extensometer designed especially for this study (see Fig. 5) was used most of the time. The instrument consists of a channeled brass bar in which two snugly fitted brass slides move, each holding a sharply pointed center punch. The one slide is clamped securely in position and remains fixed during any given series of measurements. The other slide, being adjustable and actuated by a knurled nut and threaded shaft, is the movable element of the instrument; and its extreme end bears against, and actuates, a strain-gage dial. There are no gears, levers, or bearings, and there are a minimum of parts in this extensometer. Error from mechanical sources is eliminated almost entirely since the movable slide is operated in a snugly clamped condition. Observations are taken upon extremely fine punch-mark holes or gage lines scribed with center punches made of hard alloy drill steel rigidly attached, 20 in. apart, to a steel standardizing punch bar or trammel. The gage points or lines are scribed upon highly polished areas of the reinforcing bar; they are made by a very light pressure of the hand, producing a line as small and as fine as possible consistent with visibility under a strong magnifying glass.

When a stress-gaging station was originally established, two areas of the reinforcing bar, 20 in. apart, were ground smooth and given a very high mirror-like polish with very fine emery paper and jewelers' rouge. The standard, trammel punch bar, maintained at a temperature of 32° by immersion in ice water, was used to establish the gage lines. Two punch holes were made, the one receiving the fixed element of the instrument being made somewhat larger than the punch hole used for adjusting the dial in making readings. The latter hole was made extremely small, and its use was primarily as a reference point, although accurate observations could be made upon it. Actual readings were taken upon the extremely fine gage lines scratched across the polished face of the steel reinforcement.

Stress measurements were made by a team of two structural engineer observers assisted by two or three nontechnical helpers who prepared stations, held flashlights, and carried on the skilled mechanical work required in cutting bars, arc welding, polishing, grinding, building scaffolding, and handling test loadings. These helpers were usually skilled carpenters or ironworkers. In Fig. 5, the strain-gage dial, the reading end of the instrument, and the gage line being observed are at the observer's right hand. The alternate observer is holding the fixed point in its position. The standard punch bar for scribing lines rests on the tool box.

Instrumental work of this nature, conducted under widely varying temperature conditions, requires a base for length comparison that is practically unchangeable under ordinary temperature variations. For this purpose a square, forged bar was used, which was made of a high nickel-iron alloy with a coefficient of temperature expansion of 0.0000004 to 0.0000005, which is about one sixteenth that of structural bridge steel. One face of this bar was brought to a mirror-like polish. Gage points and lines were established on this polished face for each series of stress measurements. This was done with the reference bar and the trammel punch bar maintained at 32°. The function of these gage lines on the reference bar was to provide a base of unchangeable length to which the extensometer could be applied in each reading and given its initial zero setting. By application of the extensometer to similar lines established on the reinforcing bar under observation, a direct comparison of length is made and the true difference or deformation is indicated on the gage dial. For record and identification each set of gage lines was circled with colored paint, different colors being used for the various loadings and stress series. The reference bar, as well as the standard trammel punch bar, was kept in ice water throughout any series of stress measurements. It was found that, with a little care, the temperature changes in the brass extensometer bar could be minimized. As a matter of fact, the short time that ordinarily elapsed between index reading and index check reading upon the reference bar furnished an excellent check on any temperature errors originating in the extensometer. The use of the thermocouple installation was found to be time-consuming, and the main reliance was placed upon the use of a large number of accurate mercury thermometers. All stress readings were corrected for temperature variation by reduction to a common station temperature, assuming a free adjustment of the reinforcing to thermal change without the development of temperature stress. This is not

FIG. 5.—MAKING STRAIN-GAGE MEASUREMENTS WITH SPECIAL EXTENSOMETER

strictly true, however, and does not entirely eliminate the temperature factor from measured stresses since the corrected deformations contain a temperature increment that it is quite impracticable to evaluate. This increment for temperature varies to a maximum of about 15° and is very small and negligible in amount, so that assuming it to be zero produces deformation readings within a very satisfactory range.

Scope of Extensometer Stress Measurements.—Extensometer stress measurements were made upon the structure under the following conditions of loading:

Load	Description
Dead Load	Structure under its own dead load, backfill pressures, and restraint from subpiers.
Test Live Load I	A transverse test load of 65 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 6.6 tons at the crown of each deck girder.
Test Live Load II	A transverse test load of 105 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 8.1 tons at the crown of each deck girder.
Test Live Load III	A longitudinal test load of 79 tons placed directly on the center-line interior girder and uniformly distributed to simulate the conventional truck live load used in design.
Test Live Load IV	A transverse test load of 160 tons uniformly distributed along the crown of the bridge on the skew parallel to abutments, producing a concentrated load of 13.8 tons at the crown of each deck girder.

BEHAVIOR OF STRUCTURE AFTER REMOVAL OF CENTERING AND UNDER TEMPERATURE CHANGES

Temperature Changes in the Structure.—A systematic record was made of hinge movements, deck deflections, and temperature changes in air and bridge concrete during the period of the investigation. The results of these observations furnish an interesting portrayal of elastic condition and present much valuable evidence for certain conclusions concerning the actual physical behavior of the structure.

Temperature observations on air and concrete were made in great detail over a period of about four months, embracing a mean high concrete temperature of 78° and a mean low concrete temperature of 16°, a mean high air temperature of 96° and a mean low air temperature of 15° below zero. When extensometer observations were actively under way, periodic readings were taken at all stress-gaging stations repeatedly during the day in order to furnish a reliable basis for the temperature adjustment of stress measurements.

The result of the observations on air and concrete temperatures is plotted as shown in Fig. 6. No attempt has been made to smooth the curves. The points as plotted in Fig. 6(*a*) represent the mean temperature observed during daylight hours. No temperature observations were made at night so that the physical effect of nocturnal changes is not shown in the temperature curves. Because of this fact, and because the value plotted is actually the daily mean temperature, the lag of concrete temperature behind air temperature is not clearly shown in some instances.

An examination of the air and concrete temperature curves shows the very quick response of the structure to air-temperature changes. This response is very much quicker than is customarily assumed. With air temperatures ranging from plus 32° to plus 95°, the largest observed difference between concrete and air was only 20° and only for a short period of time (six hours). Except for occasions of sudden change, the temperature of the bridge concrete follows very closely that of the adjacent air. Subsidiary local effects were observed—the greater warming of fasciæ under summer sunlight, their greater cooling under strong lake gales, and the quicker response of crown and the slower response of abutments. The mean temperature of the tie-slab coincided closely with that of the superstructure, the greatest daily difference being only 3°. With air temperatures below the freezing point and as low as 15° below zero, however, the thermal effect of the heat retained in the larger backfill mass produced a response in the bridge different from that observed for air temperatures above the freezing point. Thus, when the air temperature dropped to 15° below zero and continued at that low level for two days, remaining below

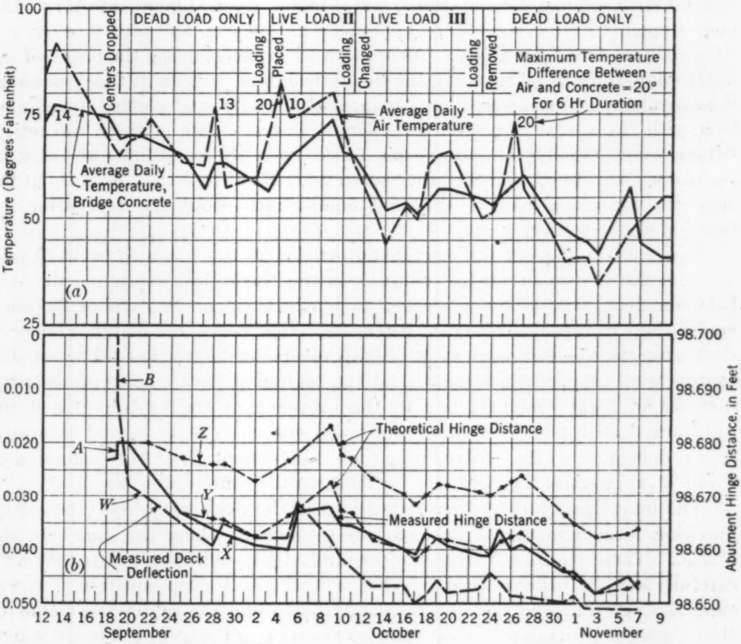


FIG. 6.—DAILY TEMPERATURE VARIATION, MOVEMENT OF HINGES, AND DEFLECTION OF DECK

10° below zero for about seven days, the temperature of the deck dropped only to 15° above zero and remained at that level throughout the low period until the air temperature had risen to above freezing. This action is to be expected because of the slow penetration of frost into the backfill mass. Until this soil is generally frozen, there will be a definite heat transfer from the relatively warm backfill to abutment legs and deck, resulting in the dampening action that is observed in the structure in sub-freezing temperatures.

Hinge Movements and Deck Deflections.—The investigation of the hinge movements was based primarily upon daily precision measurements of the longitudinal horizontal distance between the abutment hinges of the center-line interior frame. No observations were made upon hinge rotations. All measurements were corrected for temperature, tape sag, and tape tension, and were reduced to 32° for study. As an adjunct in ascertaining the origin of the day-to-day hinge displacements, the theoretical hinge lengths of the deck under the same temperatures were computed, reduced to 32°, and plotted in conjunction with the actual measured hinge-length curve. Two of these theoretical curves were actually computed—one based upon the conditions at the time centers were dropped, and the other based upon conditions that obtained at the time the structure reached apparent equilibrium about six days after the removal of the centering.

In Fig. 6(*b*): Curve *X* is the actual adjusted measured distance between gage lines on the abutment hinges; curve *Y* is the theoretical computed distance between deck gage points corrected to temperature on September 25 when equilibrium was reached; curve *Z* is the theoretical computed distance between deck gage points corrected to the temperature on September 19, when the centers were dropped (increment *A* at the beginning of curve *Z* represents the increase in hinge distance, due to the dropping of the centering, and the resultant dead load on the structure); and curve *W* is the actual measured crown deflection of the center-line girder. Increment *B*, at the beginning of curve *W*, represents the deflection at the time the centers were dropped.

The investigation of vertical deflections in the deck was based upon daily precision measurements at five stations established along the crown of the bridge. These stations consisted of carefully built deflection gages that were carefully scribed prior to dropping the centering. A deep concrete post was sunk through a hole in the tie-slab at midspan under the center-line frame for the purpose of providing precision points for checking the results of observations on the deflection gages. A brass plug in the girder overhead and a similar plug in the independently founded post provided these precision points. These deflection observations were likewise reduced to 32° for study.

Readjustment Period Following Removal of Centers.—A study of the curves of observed hinge distance, observed deck deflection, and observed temperature variations indicates the existence of a distinct period of structural readjustment of about six days duration following the removal of the deck centering before the structure reached equilibrium under its own dead load.

At the time centers were struck, this heavy timber framework supporting the deck had been in place eighty-one days under the full load of the deck. It transmitted this load, in turn, to the tie slab upon which it rested. The superstructure, tie slab, and subsoil supporting the tie slab had been in equilibrium under this load condition for many weeks. Striking the deck centering removed the heavy vertical loads which had rested upon the tie slab and which had produced a compression in the subsoil. This subsoil is of poor structural character. Bearing tests made prior to design showed it to be very plastic and low in restoration properties. It is probable that the arched crown of the tie slab facilitated the development of appreciable outward thrusts upon the abutment hinges under the action induced by the plastic condition of the subsoil under the centering load. These thrusts apparently caused an outward movement of the hinges prior to striking of the deck centers, and the system had stabilized in this condition.

Striking of the deck centers removed the heavy centering load from the tie slab, but the plastic nature of the subsoil precluded anything other than a slow recovery of subsoil and tie slab to the unloaded condition. The tie slab recovered slowly, attaining its dead-load equilibrium only after about six days of movement. During this period the hinges drew inwardly at a much greater rate than was indicated theoretically by temperature change. After equilibrium was reached, the horizontal movements in the hinges were almost entirely of temperature origin and were caused by variations in the length of the bridge deck under changes in temperature. The observed day-to-day movements in the hinges are in very close agreement with the theoretical values computed for temperature changes only.

Conclusions Concerning the Behavior of the Bridge.—The following conclusions seem warranted as a result of the study made upon the behavior of the bridge under temperature change and at the striking of centers:

(1) There was a definite readjustment period of about six days, which followed the removal of the deck centering, before the structure attained static equilibrium. During this period the hinges moved inwardly at a greater rate than theoretically computed for the temperatures observed during the period. This phenomenon is traceable to the retarded recovery of the subsoil supporting the tie slab, following the removal of the deck centering load which it formerly sustained. When final equilibrium was reached, at the end of this readjustment period of six days, the structure behaved elastically and geometrically as predicted by theory.

(2) After structural equilibrium is reached following the period of readjustment, the abutment hinges move horizontally in very close response to temperature changes in the superstructure. There is a very close agreement in the curves of theoretical and observed hinge distance, indicating that nearly all of this movement originates in length changes in the deck under temperature variations.

(3) After structural equilibrium is reached, the effect of the restraint of the subpiers and the passive resistance of the approach backfill is apparently quite negligible in modifying the action of the hinges.

(4) The very close agreement of the curves for theoretical and measured hinge distance indicates a probable low magnitude for temperature stresses in the structure. This is further established by the results of the extensometer measurements.

(5) Above the freezing point the internal temperatures of the bridge concrete respond very quickly to variations in air temperature. Below the freezing point the retained heat in the unfrozen backfill soil mass exerts a distinct dampening action on changes in temperature in the superstructure, retarding very greatly the response of concrete temperatures to sub-freezing air temperatures.

Comparative Study of Stress Measurements

Extensometer Measurements Under Dead Load.—The design provided for a crown key to reduce the accumulated shrinkage stresses. This key was poured thirty days after the first deck pour was made. Abutment backfilling started several weeks later, and upon its completion the deck centering was struck—eighty-one days after the pouring of the crown key. The behavior of the deck centering at the time it was struck, the circumstances under which the dead-load stresses were measured, and the close agreement of dead-load stresses measured by standard extensometer procedure and by the cut-bar technique indicated strongly that the centering was still carrying a very considerable part of the weight of the deck.

The measured and theoretical computed stresses under dead load show surprisingly close agreement. For purposes of comparison between measured and theoretical conditions, the measured stresses were used in computing equivalent resisting moments. Compression stress readings in most instances ran excessively high, a condition directly traceable to flexure in the station reinforcing bar in the exposed length that was partly or entirely freed of bonding concrete. The results given in this paper are based accordingly upon the observed tensile stress in the steel and the theoretical linear location of the centroidal axis, assuming no concrete in tension. There is a strong probability of the concrete taking tensile stress, which might tend to modify the measured moments; but it was deemed more practicable to assume the tensile-free condition commonly used in design. The standard linear flexure equation for compression-reinforced T-beams was used in deriving the equivalent measured moment from the measured stress. The moment of inertia of transformed sections was used. The use of a parabolic flexure equation in computing measured resisting moments was given some study but was not deemed practicable.

In order to determine the total combined stress from dead load, temperature change, hydration shrinkage, backfill effort, and subpier restraint (which existed in combination after striking of the deck centering), and to secure a check upon the dead-load stresses that were measured at the time the centering was struck, stress measurements were made upon a number of bars, using the cut-bar technique which the writers developed to facilitate the solution of this special problem. This cut-bar technique involved:

(*a*) Careful removal of the concrete cover from a reinforcing bar carrying computed stress;

(*b*) Preparation of gage lines on the reinforcing bar and on the reference bar;

(*c*) Readings for index reading before any further bonding concrete is removed;

(*d*) Careful removal by hand chipping of all bonding concrete between gage lines;

(*e*) Intermediate reading for deformation;

(*f*) Careful cutting of the bar outside the gage length in a manner causing no temperature changes in the bar;

(*g*) Final reading for deformation in the no-stress condition; and

(*h*) Arc-welding the severed ends of the bar to restore some part of the structural value.

Stress measurement with the cut-bar technique was done in such a manner as to show the true, total, unmodified stress of all origins existing in the bar at the time of measurement. Stresses measured by this technique are entirely free from the error due to imperfect temperature corrections. The series of measurements made by this method furnished an excellent check on the measurements made by the standard dead-load procedure. It is believed that the cut-bar technique is essentially a new departure and certain to be of value in future studies involving extensometer stress measurements.

A study of the data secured in the stress measurements upon the center-line interior frame is summarized in Table 1. The significance of the various column headings in this table may be clarified further, as follows:

Col. No.	Description
2	Extreme temperature range at the station during stress observations
3	Arithmetical mean of observed dead-load deformations at the station, corrected for temperature
	Stress in the Steel Reinforcement, Computed by Means of:
4	Young's modulus
5	Elastic theory
	Bending Moments at the Station:
6	Theoretical
7	Measured resisting moment, computed from the measured tensile stress and an assumed centroid

A graphical comparison of computed and measured stresses and moments is shown in Figs. 7 and 8. On these curves:

C denotes computed theoretical stress or bending moment for the loading indicated;

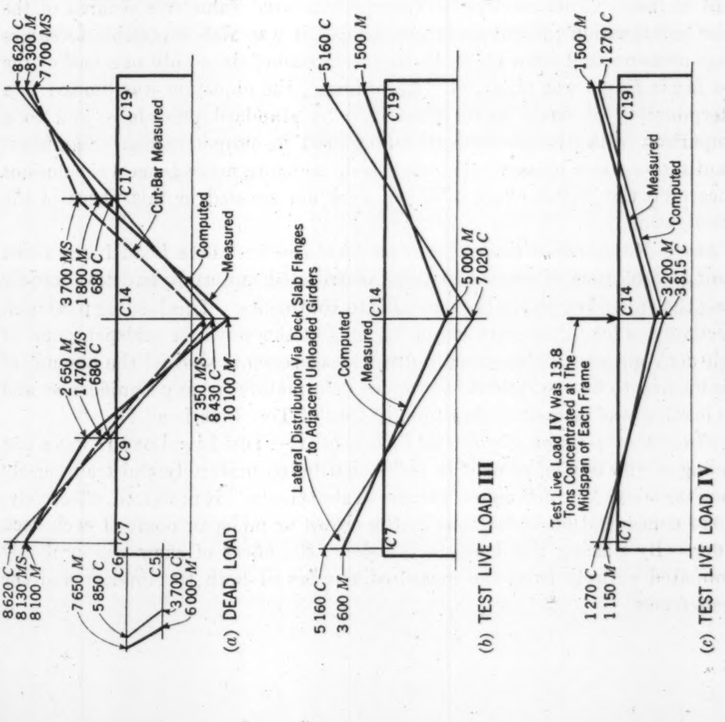
M denotes the measured stress (Fig. 7) secured by means of a standard extensometer, or the equivalent measured resisting moment (Fig. 8), using the measured tensile stress and a theoretical centroid; and

MS denotes the measured combined stress (Fig. 7) secured by the cut-bar technique, or the equivalent measured resisting moment (Fig. 8), using the tensile stress and theoretical centroid in the cut-bar technique.

TABLE 1.—COMPARISON OF MEASURED AND THEORETICAL STRESS FUNCTIONS

Stress gaging station (see Fig. 2) (1)	Temperature range (degrees F) (2)	Mean measured corrected deformation (3)	STRESS IN REINFORCEMENT (LB PER SQ IN.)		BENDING MOMENTS IN HINGED FRAME (FT-LB)		Ratio, $\frac{\text{Col. 7}}{\text{Col. 6}}$ (8)
			Equivalent measured (4)	Theoretical (5)	Theoretical (6)	Equivalent measured (7)	
(*a*) DEAD LOAD; INTERIOR FRAME							
C 7	78–78	0.0054	8,130[b]	8,620	−2,528,000	−2,388,000	0.95
C 7[a]	26–26	0.0054	8,130[b]	8,620	−2,528,000	−2,380,000	0.94
C 8	78–78	−0.0049	−7,350[c]	−3,790	−2,528,000	....	....
C 9	55–68	0.0017	2,650	680	−130,000	−342,000	2.65
C 9[a]	30–30	0.0009	1,470[b]	680	−130,000	−216,000	1.66
C10	71–71	−0.0009	−1,360[c]	−1,745	−130,000	....	....
C13	55–78	−0.0034	−5,100[c]	−2,145	557,000	....	....
C14	71–76	0.0067	10,100	8,430	557,000	665,000	1.19
C14[a]	26–26	0.0049	7,350[b]	8,430	557,000	490,000	0.88
C17	71–76	0.0010	1,500	680	−130,000	−220,000	1.69
C17[a]	30–30	0.0025	3,750	680	−130,000	−247,000	2.50
C18	71–78	−0.0034	−5,100[c]	−1,745	−130,000	....	....
C19	60–71	0.0055	8,300[b]	8,620	−2,528,000	−2,438,000	0.96
C19[a]	71–78	0.0051	7,700[b]	8,620	−2,528,000	−2,268,000	0.90
C20	27–27	−0.0046	−6,900[c]	−3,790	−2,528,000	....	....
(*b*) TEST LIVE LOAD III; INTERIOR FRAME[d]							
C 7	42–56	0.0024	3,600	5,160	−1,515,000	−1,074,000	0.71
C 8	40–57	−0.0025[c]	−3,750[c]	−2,230[c]	−1,515,000	....	....
C13	43–56	−0.0036[c]	−5,500[c]	−1,800[c]	−459,000	....	....
C14	40–56	0.0033	5,000	7,020	−459,000	335,000	0.73
C19	41–58	0.0010	1,500	5,160	−1,515,000	−478,000	0.32
C20	40–50	−0.0016[c]	−2,350[c]	−2,230[c]	−1,515,000	....	....
(*c*) LIVE LOAD IV; INTERIOR FRAME[e]							
C 7	17–24	0.0008	1,150	1,270	−373,000	−328,000	0.88
C 8	17–23	−0.0029[c]	−4,360[c]	−550	....	....	....
C14	16–27	0.0022	3,220	3,815	241,000	204,000	0.92
C19	18–25	0.0010	1,500	1,270	−373,000	−437,000	1.17
C20	17–24	−0.0029[c]	−4,360[c]	−550	....	....	....
(*d*) DEAD LOAD; FASCIA FRAME[f]							
F14[g]	55–60	0.0040	6,000	9,700	....	....	0.62[h]
F17[g]	54–61	−0.0005	−750	1,000	....	....	0.75[h]
F19[g]	55–62	0.0050	7,500	10,700	....	....	0.70[h]

[a] Cut. [b] Denotes that the value of the tensile stress is checked very closely by cutting the station bar. [c] Denotes that the value given is probably in error due to compression flexure in station bar. [d] Test Live Load III was a test loading of 79 tons of sacked cement placed directly on the center-line interior frame to simulate the conventional Park District truck live loading. [e] Test Live Load IV was a test live loading of 160 tons of sand uniformly distributed along the crown of the bridge on the skew parallel to abutments producing a concentrated load of 13.8 tons at the crown of each deck girder. [f] Cut-bar technique was not applied to measurements on fascia frame. [g] Stations located on south half of fascia frame only; north half not established; station numbers correspond to those of similarly located stations in interior frame. [h] Ratio, $\frac{\text{Col. 5}}{\text{Col. 4}}$.

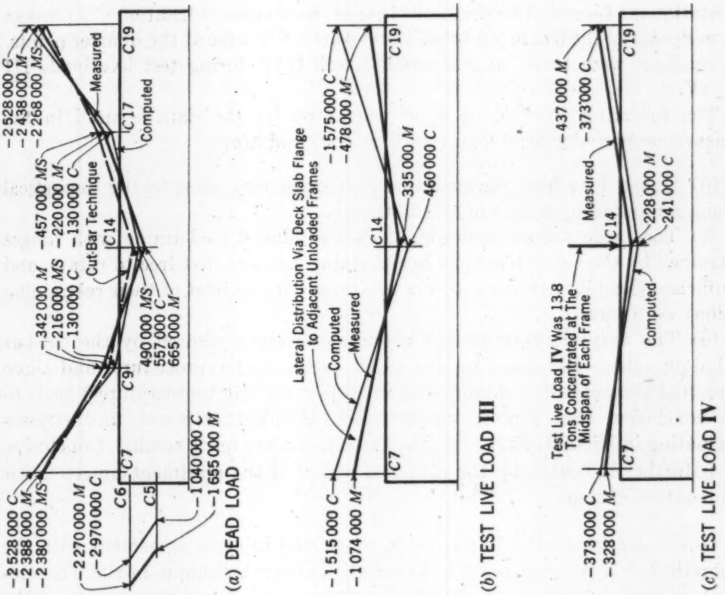


(*a*) DEAD LOAD

(*b*) TEST LIVE LOAD III

(*c*) TEST LIVE LOAD IV

FIG. 7.—COMPARISON OF THEORETICAL AND MEASURED STRESSES (INTERIOR FRAME)



(*a*) DEAD LOAD

(*b*) TEST LIVE LOAD III

(*c*) TEST LIVE LOAD IV

FIG. 8.—COMPARISON OF THEORETICAL AND MEASURED MOMENTS (INTERIOR FRAME)

Stations C 7 and C 19 were at the face of the abutment; Station C 14 was at the midpoint of the frame; and stations C 9 and C 17 were at the quarter points. No readings were taken at stations C 9 and C 17 during test live loads III and IV.

The following conclusions seem warranted by the data secured in the measurements under dead load (see Figs. 7(*a*) and 8(*a*)):

(6) Under dead load the structure is stressed very close to the theoretical values computed by means of elastic theory.

(7) The basic design assumptions (a two-hinged tied frame with hinges restrained by the tie slab against horizontal movement, but free to rotate, and a sufficient flexibility in the subpiers to warrant the neglect of their restraining action) were correct.

(8) The very close agreement between stresses measured by the cut-bar technique, those measured by the usual extensometer procedure, and those computed theoretically (obtained under widely varying temperatures) leads to the conclusion that temperature stresses, shrinkage stresses, and stresses originating in backfill effort and subpier restraint are quite small. Conclusion (8) is further indicated by the observed action of the abutment hinges under temperature change.

Under dead load the fascia frame is stressed in close agreement with the theoretical. It was not possible to use the cut-bar technique in the measurement of fascia girder stresses, so that no measured value was secured of the stress combination present under dead load; it was also impossible to secure stress-measurement data on an entire fascia frame, since only one half of the east fascia frame was observed. Accordingly, the objection was limited to a determination of stress under dead load by standard procedure, and to a comparison with the theoretical values used in proportioning. Significant quantitative stress-measurement data, from which to make general conclusions concerning the elastic effect of skew, were not secured in this phase of the investigation.

Stress Measurement Under Test Live Load I.—Test Live Load I was a test loading of 65 tons of cement in sacks distributed uniformly and transversely along the crown key on the skew parallel to abutments. This loading produced, effectively, a load concentration of 5.5 tons at the crown or midspan point of each deck girder. Extensometer stress measurements showed the amount of this loading to be insufficient to produce deformations of any significance, and the loading was increased, therefore, to that of Test Live Load II.

Stress Measurements Under Test Live Load II.—Test Live Load II was a test loading of 105 tons of cement in sacks distributed uniformly and transversely along the crown key on the skew parallel to abutments. It produced, effectively, a load concentration of 8.1 tons at the crown or midspan point of each deck girder. By placing this load on the skew, the effect of skew was probably eliminated entirely from the measured stresses of both the interior and the fascia frame.

Extensometer measurements showed that this loading was likewise insufficient in amount to produce deformations of significance, and the loading was therefore increased to that of Test Live Load IV.

Stress Measurements Under Test Live Load III.—Test Live Load III was a test loading of 78.7 tons of cement in sacks placed directly on the center-line interior girder between abutment lines. It very closely simulates the live load of conventional design, composed of two 24-ton trucks surrounded by a uniform live load of 100 lb per sq ft. This test loading was applied in such a manner that the entire weight rested upon the girder proper and no weight rested upon the flange slab. Thus placed, the true lateral distributive value of the flange slab is more easily studied. The characteristics of Test Live Load III are shown in Fig. 9.

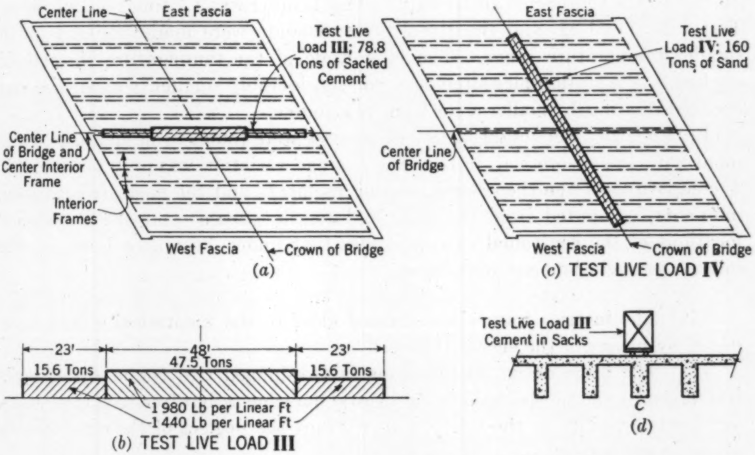

Center Line
East Fascia
Test Live Load III; 78.8 Tons of Sacked Cement
Center Line of Bridge and Center Interior Frame
Interior Frames
West Fascia
Crown of Bridge
(a)
East Fascia
Test Live Load IV; 160 Tons of Sand
Center Line of Bridge
West Fascia
Crown of Bridge
(c) TEST LIVE LOAD IV
23'
48'
23'
15.6 Tons
47.5 Tons
15.6 Tons
1 980 Lb per Linear Ft
1 440 Lb per Linear Ft
(b) TEST LIVE LOAD III
Test Live Load III Cement in Sacks
C
(d)

FIG. 9.—CHARACTERISTICS OF TEST LIVE LOADS III AND IV

Stress measurements on the interior frame under Test Live Load III are summarized in Table 1(*b*). A graphical comparison of computed and measured stresses and moments is shown in Figs. 7(*b*) and 8(*b*). The following conclusions seem warranted by the data secured in the stress measurements under this load:

(9) The structure functions as an elastic two-hinged tied frame with true measured stress functions very close to those indicated by the elastic theory.

(10) The measured crown moment, which was 71% of the theoretical value, indicates that there is apparently some lateral distribution of the load to the adjacent unloaded deck girders via the deck-slab flange.

(11) The measured haunch moments, which average 50% of the value obtained by conventional assumptions as to load distribution, indicate that

there is evidently an appreciable lateral distribution to the adjacent unloaded deck girders via the deck-slab flange and the cellular haunch slabs.

Stress measurements on the fascia frame under Test Live Load III indicate the existence of what is probably a complex torsional condition resulting from the skew of the structure. The objectives of this investigation did not embrace a study of this phase in the fascia and outer frames. The results secured by the stress measurements on the fascia frame were satisfactory and indicate that the stress under Test Live Load III is within safe limits.

Stress Measurements Under Test Live Load IV.—Test Live Load IV (Table 1(*c*)) was a test loading of 159.7 tons of bank sand retained in a timber crib, distributed uniformly and transversely along the crown key on the skew parallel to abutments. It produced, effectively, a load concentration of 13.8 tons at the crown or midspan point of each deck girder. Stress measurements were limited to the center-line girder only. This loading was 1.7 times heavier than Test Live Load II, and the stress measurements were made under a much smaller temperature range. The magnitude of the temperature corrections applied to the station measurements was considerably smaller in each case and resulted in a much smaller error from this source.

In this series the actual measured stresses were found to be in close agreement with the theoretical; in fact, the average variation was only about 12%. A comparison of the theoretical bending moments and the measured resisting moments, computed from the measured tensile stresses and the theoretical locations of the centroidal axis, indicates that under Test Live Load IV the following conclusions are warranted:

(12) The interior frames are stressed close to the theoretical values computed by means of the elastic theory.

(13) The basic design assumptions (a two-hinged tied frame with hinges restrained by the tie-slab against horizontal movement, but free to rotate, and a sufficient flexibility in the subpiers to warrant the neglect of their restraining action) were correct.

(14) The close agreement of measured and theoretical stress functions indicates that subpier restraint and backfill effort are quite negligible in modifying appreciably the stress condition of the superstructure.

Service Load Test.—It was considered desirable to make a service load test upon the completed bridge for assurance of safety under severe conditions. To effect this, the loading of Test Live Load IV was increased to 180 tons and an additional 60 tons of loaded trucks and tractors were arranged along the crown in midspan, bringing the total loading to 240 tons. To augment this static load, and to impose a still more severe condition, a 15-ton tractor was maneuvered rapidly about the deck in a manner to produce heavy impact shocks. Although this service load did not even begin to approximate the capacity of the structure, it was a severe load and is probably the largest the structure as a whole will ever be called upon to accommodate.

CONCLUSIONS

In addition to those presented in the body of the paper, the following general conclusions are offered:

(15) The basic structural assumptions that were made in the original design analysis and in the detailed proportioning of the components of the bridge were found to be valid and correct. They were well sustained by each phase of the investigation.

(16) The effect of active or passive backfill pressures is negligible in modifying appreciably the stress condition of the superstructure. Likewise negligible is the effect of the restraint developed by the subpiers upon the stress condition of the superstructure.

(17) The interior frames function, structurally, very closely as predicted by the elastic theory. In these frames the effect of skew is apparently very small for skewed symmetrical loadings similar to Test Live Load IV, and it may be neglected in proportioning the components in design. Under Test Live Load III there was a lateral distribution of load to the adjacent unloaded frames via the flange slabs. Under dead load and under Test Live Load IV the interior frames are stressed very closely to the theoretical values.

(18) The fascia frames under dead load function similarly to the interior frames. Under a symmetrical loading they show the effect of skew by a departure from the pattern common to the interior frames. However, proportioning the fascia frames by conventional methods is commensurate with safety.

(19) The assumption made in design that the hinges move freely in response to temperature variations in the superstructure, relatively unimpeded by the backfill and subpier resistance, was found to be valid. This assumption is particularly sustained by the study made of the hinge movements where the day-to-day variation in true hinge distance was found to be caused entirely by changes in length of the deck under temperature variations.

(20) By means of the cut-bar technique, the true, total, unmodified stress of all origins existing in the reinforcing at the time of measurement under the dead-load condition may be determined accurately. Use of this technique furnishes a close check on the measurements of dead-load stress secured by the standard procedure.

ACKNOWLEDGMENT

The original thesis from which this paper was prepared was presented to the Armour Institute of Technology (later part of the Illinois Institute of Technology), in Chicago, in 1940, in partial fulfilment of the requirements for the professional degree of Civil Engineer. The extensometer used was devised by Prof. Phillip C. Huntly of Armour College of Engineering, Illinois Institute of Technology, Chicago. It was made in the Institute shop under Prof. Huntly's direction. The instrument is not patented. In addition to Professor Huntly, the writers wish to acknowledge, especially, the valuable advice given them in this investigation by C. Thomas Kelly, and Edward Smulski.

APPENDIX

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DISCUSSION

Arthur R. Lord,[4] M. Am. Soc. C. E.—There is a strange lack of information, in this paper, as to the early state of the art of stress measurement. The statement is made under the heading "Field Measurement of Stress" that:

> "In all of these investigations, which constitute probably the entire scope of the work done in this field until about 1917, no stress measurements were made to compare computed and measured stress, or to determine dead-load stress; and no measurements were made on statically indeterminate structures. * * * The use of the extensometer in the systematic investigation of true field stress and the verification of structural design theory applied to an existing structure were of no consequence until 1917 when D. B. Steinman, M. Am. Soc. C. E., conducted his extensive systematic stress measurements on the Hell Gate Bridge."

The Bibliography given in the Appendix shows a serious lack of important references. To correct this inaccurate record it should be stated that:

(1) The first test for actual stresses in a structure was made on the Deere-Webber Building in Minneapolis, Minn., in the fall of 1910 and described by the writer in a paper published that year.[5] This paper involved extensometer measurements in a statically indeterminate structure and comparison of actual with computed stresses, and it laid the foundation for the design of the reinforced concrete flat-slab construction in the United States and other countries.

(2) Many other tests, each involving thousands of extensometer readings under heavy loads in actual service structures, were made in the years between 1910 and 1917, by the writer[6] and by many others, and were published and discussed throughout the United States. The bulletins of the Engineering Experiment Station at the University of Illinois, Urbana, Ill., give the full record of some of these tests. The *Proceedings* of the American Concrete Institute give others. The *Engineering News-Record* contained abstracts of several. In other words, there is a mass of scientific testing, of precisely the type reported by the authors, that preceded the test in 1917. Dead-load stresses were measured, as well as live-load stresses. Stresses (deformations) developing over periods as long as a year, in structures subject to load and temperature effects, were determined. Effects of "plastic flow" of concrete were noted and evaluated. In fact, the entire scientific design base for hundreds of millions of dollars worth of concrete buildings was developed from such tests. The authors may be justly criticized for ignoring this prior record when essaying to write an historical summary such as is presented in the opening paragraphs and in the bibliography of this paper.

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[4] Consultant, Reports Section, Progress Division, Bureau of Yards and Docks, U. S. Navy, Washington, D. C.

[5] "Measurement of Stresses in Flat Slab Building Under Actual Load," by Arthur R. Lord, *Proceedings*, National Assn. of Cement Users (later Am. Concrete Inst.), 1911.

[6] "Measurement of Actual Stresses in a Cantilever Flat Slab Reinforced Concrete Floor Having Rectangular Panels," by Arthur R. Lord, *Proceedings*, National Assn. of Cement Users, Vol. IX, 1913, p. 127.

It is sincerely hoped that the authors will correct these shortcomings in their closing discussion. For example, the works[7] of Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and the late Willis A. Slater, M. Am. Soc. C. E., as well as that of the writer and many others, antedate any references that appear in the paper.

J. Charles Rathbun,[8] M. Am. Soc. C. E.—The title of this paper does not indicate that it is an attempt at verification of one method of analyzing the stresses in a skew arch; nor does the "Synopsis" so indicate. It is only at the bottom of the third page that the most important phase of the paper appears (see heading "Description of Bridge: The North Avenue Rigid-Frame Bridge").

The "Synopsis" of this paper indicates that field tests were made in order to verify certain very critical structural assumptions "which were believed to be quite valid but which lacked definite quantitative substantiating evidence." The writer takes issue with this statement as well as with the second paragraph of the "Synopsis."

The type of design and method used were fully covered from a theoretical standpoint in a paper[9] by the writer published in 1924; and the results of the authors' test could all have been anticipated in terms of that theory. The general case of the skew arch ring was analyzed, as well as the special case considered by the authors. All test loads used by Messrs. Smith and Lillard were included in this special case. Therefore, it seems fair to state that the action of the North Avenue Bridge under a more general loading condition has not been shown experimentally.

Although the authors may have been justified in using it for other reasons, the slab-and-girder type of construction has two objectionable features in the case of this bridge. The deep girders cut down the headroom materially compared to that of a simple barrel arch; and this type of construction increases the difficulty of analysis and the uncertainty of stress conditions materially. The writer knows of no formula for torsional rigidity or stress distribution for a shaft of the cross section given in Fig. 9(*d*). There are several for a simple rectangle of all degrees of side ratio, including the extreme case used in the skew barrel arch and the case of the skewed rib used by the authors.

At the time that the skew arch theory[9] was first published (1924), there was some criticism of it because of the length of the equations involved and because the results were not in accord with the then popular idea of the action of a skew arch. Since the publication of this paper, a number of papers have been written on the subject and many models have been tested until now (1941) it has been definitely substantiated that the theory is sound. The writer conducted a series of tests and published the results in 1930.[10] Later he tested[11] a multiple skew arch designed by an extension of this same theory by the method developed by the late George E. Beggs, M. Am. Soc. C. E. These tests proved the validity of the theory and also the extension of the theory to the case in which there is more than one arch ring. In 1926 the Society's Special Committee on Concrete and Reinforced Concrete Arches took issue[12] with the findings of the first paper.[9] Professor Beggs was instructed to conduct a series of experiments subjecting four skew arches of varying widths to all possible conditions of vertical load. After more than a hundred loadings were investigated, the Committee came to the conclusion that the theory as published is correct and leads to correct results.[13]

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[7] "Tests of Reinforced Concrete Buildings Under Load," by Arthur N. Talbot and Willis A. Slater, *Bulletin No. 64*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., January, 1913.

[8] Prof., Civ. Eng., College of the City of New York, New York, N. Y.

[9] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[10] "Crown Stresses in a Skew Arch," by J. Charles Rathbun, *loc. cit.*, Vol. 94 (1930), p. 135.

At present the theory is recognized as correct and is being used in design throughout the United States. The writer has not heard of any engineer, who has designed by it, indicating that it is incorrect or leads to incorrect results; or who feels that, when used understandingly, it is too long and involved. The writer, therefore, takes issue with the second sentence and the second paragraph of the "Synopsis" on the ground that quantitative, computable evidence was available, and that no test loads were applied that fell outside the special case in which the authors' assumptions were valid.

It will probably be of interest to compare the elastic theory of the skew arch with the authors' series of tests and their method of design to encourage a better understanding of the skew arch and its analysis. The problem is essentially one of space analysis. If the reaction at a section across the crown or at an abutment is considered, six components of this reaction must be evaluated—forces in the direction of each of the assumed axes and moments about the line of action of these forces. This makes the arch ring statically indeterminate with six redundants.

Six equations of elastic deformation are required to evaluate these unknown moments and forces. If the deformations due to ring shortening and shear, both horizontal and radial, are considered, these equations become quite lengthy. Therefore, the work required to make a complete analysis is prohibitive, and the practicing engineer leaves out the terms that have little effect on the final result. In the case of the symmetrical arch, even when unsymmetrically loaded, these six equations split into two groups—one of four, and the other of two, equations. In the case of certain types of symmetrical loading (those used by the authors), the one group of two equations gives zero results—that is, the vertical shear and the moment about the vertical axis at the crown are zero.

Although it has little bearing on the present problem, consider the case in which the skew is zero. These six equations split still further into two equations of one unknown each and two sets of equations of two unknowns each. If the loading is symmetrical about the center line of the upper roadway (the common case in arch analysis), three equations have zero results and are not considered by designers. The other three consist of two of two unknowns and one of one unknown. This is the case considered in textbooks on arches. If, still further, the load is symmetrical about the crown, one equation (for vertical shear) disappears. The two remaining equations can be reduced to one by placing hinges at the abutments. This is the case used by the authors in testing the central rib. The very narrow skewed arch acts the same as a right arch with a span of the skewed length.

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[11] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *Transactions* Am. Soc. C. E., Vol. 98 (1933), pp. 26–31.

[12] *Proceedings*, Am. Soc. C. E. (Society Affairs), March, 1926, p. 142.

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Returning to the six equations of the skewed arch, assume a system of coordinates with the origin at the crown. The crown section is taken parallel to the abutment and radial (vertical for symmetrical arches). The x-axis is taken perpendicular to this section, the y-axis is vertical (radial), and the z-axis is perpendicular to these two, and, therefore, is horizontal and in the crown section. The general solution requires the finding of the forces in the direction of these coordinates and the moments about them and requires six equations.

If the arch is symmetrical about the two vertical planes through the y-axis, the one through the z-axis and the other through the center line of the upper roadway (the usual case), and, in addition, if the loads are also symmetrical about these planes, the crown force in the direction of the vertical axis and the moment about this axis both become zero. At the same time two equations disappear, and four equations and four unknowns remain. Next make the rather questionable assumption that the thrust at the crown and the horizontal shear at the crown are in the same ratio to each other as the tangent of the angle of skew—in this case, 35° (the complement of the 55° given by the authors). Also assume that the moments about these axes, the z-axis and the x-axis, are in this same ratio. The problem is now reduced to two unknowns and two equations, and the force line follows the central arch rib. If the abutments are designed as hinges, this reduces the problem to one equation in one unknown (which is the case assumed by the authors), and the formula is as given by Eq. 1.

The two assumptions involving the ratios of the crown forces and moments are fulfilled only under certain conditions, the most important being that the arch and loads must be placed symmetrically about the two vertical planes through the crown indicated previously. The test loads all fulfilled these conditions. In addition, the shortening of the arch rib, and its deformation due to shear, must be neglected. Another necessary condition is that the abutments do not rotate. Unfortunately, the rotation of these abutments was not measured, but from the results obtained the rotation must have been small and in a direction that tended to correct the other effects. Another assumption involved in the loading is that the earth-fill pressure is parallel to the direction of the upper roadway. This pressure is questionable in its nature. The usual assumption is that the pressure is perpendicular to the abutment face and therefore contributes to skew action.

It is to be noted that the readings given are all on the central rib where any skew effect would be a minimum. The readings on the outside girder are not given in the paper, but the authors indicate that some skew effect exists. The slab joining the several ribs evidently has sufficient rigidity to produce this effect. The torsional stresses in the slabs due to the complicated crown section are difficult to evaluate or observe; those due to horizontal shear are not so difficult. Observations on the slabs might show interesting results. Of course, these stresses are not dangerous because, if they are beyond the allowable limit, adjustment will take place since the slab is not a vital part of the arch system. If they had been made with shear joints or expansion joints (as was done on a skew arch bridge in Tacoma, Wash., with disastrous results),[14] the structure would still have acted as anticipated, but probably with no skew effect.

The writer feels that the objects of making the tests (see "Extensometer Tests: Object") have not all been obtained. For example (Object (1)), the true stress conditions at the questionable places have not been measured. The central arch rib will follow the approximate theory; but will the slab or the outer ribs follow this theory under skew loading? Failure to measure the abutment rotation prevents a check by the theory of the skew arch ring. Unless the authors refer to their "elastic theory," a comparison of true and theoretical stresses has not been made except to a very limited degree. It is to be noted that no effects of skew have been given (Object (2)). This effect is the important and dangerous condition in a skew arch. The elastic action of the system has been studied to a very limited degree only (Object (3)). The data secured (Object (4)) is valuable and adds one more test to the verification of the theory of the skew arch. That no observations were made on the rotation of the hinges detracts very much from its value, however.

The idea of cutting a bar and noting its change of length in the unstrained condition requires considerable caution in its use. It assumes that the bar has been placed in the structure in an unstrained condition, which assumption is far from the truth. For example, assume a 1-in. bar which, due to normal handling, is slightly bent with a radius of curvature of 1,200 ft. When laid in the forms, it is forced into line and wired, but it has a bending stress of 1,000 lb per sq in. If measurements are taken on one side of this bar only, after which the bar is released, there may be an error of 1,000 lb per sq in. in the steel measurement and a corresponding error in the concrete. This error cannot be detected by observing the bar and noting if it straightens up when released because, assuming the foregoing values in a 10-in. gage length, the deflection from a straight line is only $\frac{1}{600}$ in., which is too small for field measurements. If the radius were taken as 120 ft and the deflection as $\frac{1}{60}$ in., the condition would not be unusual and the error could be 10,000 lb per sq in. for the steel. Therefore, it becomes necessary to take measurements on opposite sides of the bar. This involves a great deal of concrete chipping, unless the gage is constructed for this type of observation. The authors should clarify this point. The cutting and resplicing of the stressed bars cause a redistribution of stress among the several bars of the bridge that might be objectionable to the owners but do not affect the validity of the observation. This "cut-bar" method has been suggested several times in the writer's experience but was rejected for the aforementioned reasons.

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[14] *Engineering News-Record*, February 22, 1923.

An alternate method (which has not been used to the writer's knowledge) is to take a small bar (the diameter being governed by its slenderness ratio when acting as a column), suspend it on two supports so adjusted that the effect of bending moment is zero over the gage lengths, and embed this bar in the concrete at the point to be measured. The gage points are placed on the bar before embedment and are well greased. The distance between them is measured before and after embedment. This initially unstressed bar then can be measured for strain at any time, and finally it can be taken out and the gage length checked. The bar must be long enough to insure sufficient embedment on either side of the gage points. Even in this case gage points on opposite sides of the bar are desirable.

The authors are incorrect in their remark that few measurements have been taken on structures in service where the extensometer has been used. The writer can recall several without taking the trouble to make a bibliography. Although the point is of little value, since the question has been raised it might be of interest to see such a bibliography on the subject. It would include the field observations and extensometer measurements by Clyde T. Morris and also by S. C. Hollister, Members, Am. Soc. C. E. The Progress Report of the Committee on Concrete and Reinforced Concrete Arches, dated January 20, 1926, contains more than ten references to field measurement projects.

The bibliography of this paper appears to be one on the use of the strain gage, supplemented by a few standard works on structural analysis and a few references on impact. The important phase of the paper, the stresses in a skew arch, is not touched in this bibliography, although several papers have been written on this subject between the dates given by the author (1917 to 1941), and these papers include strain measurements on existing structures. The title of the list should be less inclusive or a number of papers on the skew arch should be included, preferably the latter because literature on this subject apparently needs to be brought more to the attention of the profession.

The writer wishes to comment on the authors' conclusions as follows:

(4) The low magnitude of temperature stresses in the structure is probably due to the fact that the abutments moved horizontally with the expansion and contraction of the arch due to temperature changes, thus tending to annul the stresses.

(6) That the arch ring follows the approximate method of skew arch analysis has been proved for the central girder only.

(7) The reasons why the basic design assumptions have not been verified have been explained at length.

(9) That the arch follows the authors' "elastic theory" has not been shown except for the central girder. They neglect the stresses due to skew which have not been touched upon in the paper. This is a very dangerous and incorrect conclusion.

(10) and (11) The conclusion that there is lateral distribution of load is correct. It is this lateral distribution through the slab that makes the skew arch analysis a problem. The complex torsional condition resulting from the skew of the structure produces the dangerous stresses, if any. The remark

that "The objectives of this investigation did not embrace a study of this phase in the facia and outer frames" should be emphasized to the extent that, with one reading of the paper in order to secure data on the skew arch analysis, one will then realize that this paper is primarily one of testing and that the authors are not putting forth the idea that the approximate method of skew arch design applied to the two-hinged skew arch has proved satisfactory. It is a dangerous method and could have been unsatisfactory had the hinges acted as designed.

(15) These tests definitely do not warrant the conclusion that the approximate theory (the authors' elastic theory) is correct for hinged skew arches; nor do the tests prove that the theory is incorrect.

(17) The conclusion that the central girder functions as a simple arch is approximately correct, theoretically, if the abutments do not rotate. The other girders will also function in this manner under certain definite conditions of loading.

(18) The data from which this conclusion was drawn are of value, and it is hoped that they will be published in the closure.

(20) In the "cut-bar" technique the authors should show how they avoided stresses due to bending the bar. If they measured both sides of the bar to secure the average strain in the center, this might well be mentioned as a warning to those who use the method in the future.

The writer is very much impressed with this paper from several angles. The method in which the obtuse abutments carried most of the load (which antedates the modern theory) has not been used, and seems at last to have been abandoned in skew arch analysis. The more simple method of using the skew arch and treating it as a right arch is nearer the truth if the hinges do not exist or act.

Although this paper by title and synopsis is one on stress measurement, it actually is one on checking the method of analysis of an arch with a 35° skew. The writer feels strongly that the work performed by the Society's Committee on Concrete and Reinforced Concrete Arches, and the several papers published by the Society, should have been considered more carefully before the measurements were taken, and that the literature on skew arches should have been given a place in the bibliography. If this had been done, the stresses in the structure as a skew arch might have been given more weight in the paper.

In closing, the writer wishes to urge that designers of the skew concrete arches investigate carefully and read the literature on the stresses due to skew. A statically indeterminate structure with six redundants is comparatively safe; but should not the stresses induced by loads be analyzed carefully so that a lack of understanding of the problem may not outweigh the factor of safety? It is hoped that this paper will revive an interest in the skew arch problem and that this structure will obtain the popularity it deserves.

THEODORE BELZNER,[15] **AFFILIATE AM. SOC. C. E.**—The statement made by the authors that few measurements of stresses on structures in service had been taken until about 1917 is incorrect. Many strain-gage measurements in actual construction or maintenance work on bridges prior to that period had been made. The writer firmly believes that a complete bibliography on this subject should be brought substantially up to date.

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[15] Insp. of Steel and Bridge Insp. in Chg., Brooklyn Bridge, Dept. of Public Works, City of New York, Brooklyn, N. Y.

One case in particular, in a bridge of large span, is the Williamsburg Bridge across the East River in New York, N. Y., not given in the Appendix. Prior to 1917, stresses were measured on the Williamsburg Bridge during the summer of 1911; they were measured under train movements; also, during 1913 and 1914, when a comprehensive series of extensometer investigations were made in connection with the strengthening of the end spans of this bridge. This work was done[16] by the Department of Bridges (later the Department of Plant and Structures, and finally Department of Public Works), City of New York, in cooperation with the National Bureau of Standards, under the auspices of the late James E. Howard, engineer physicist, on the important truss members at the main towers and legs of the Brooklyn intermediate towers.

The precision attained with the Howard extensometer used in this test proved to be an invaluable aid, and its merits were illustrated in certain vital operations, especially with the various stages of wedging, and in the transferring of stresses from the old to the new diagonal members of the end trusses.

The writer agrees with the authors that field stress measurements "require a high order of instrumental and technical skill in order to be of value." The skill required to obtain reliable results in strain-gage measurements is not in the possession of all. In other words, the most important objective to be considered in taking stress-strain measurements (other factors being equal) is to obtain consistent results; and, in order to secure such results (which are absolutely essential), persistent patience and painstaking thoroughness, combined with good judgment on the part of the observer, are required, until he experiences little or no difficulty. It is upon the observer that the value of such measurements will depend.

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[16] Discussion by Isidore Delson of "Stress Measurements on the Hell Gate Arch Bridge," by D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. LXXXII, December, 1918, pp. 1087–1089.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2165

SALTS IN IRRIGATION WATER

BY RAYMOND A. HILL,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. C. S. SCOFIELD, HERMAN STABLER, M. R. LEWIS, LOUIS J. ALEXANDER, W. D. COLLINS, JOHN H. BLISS, AND RAYMOND A. HILL.

SYNOPSIS

Those concerned with domestic and industrial water supplies were the first to appreciate the importance of the mineral salts invariably present in natural stream and ground waters. Irrigation engineers later began to realize that water in its pure form is never found in nature, and that natural water may be properly termed a mineral, of which H_2O is the principal constituent.

Unfortunately, there has been a tendency for irrigation engineers to disregard the character of the other constituents of natural water, limiting themselves largely to consideration of the total amount of dissolved solids. The interpretation of chemical analyses of irrigation water generally has been left to the agriculturalist. In an effort to bridge this gap between the engineer and the agriculturalist, the writer developed a geochemical chart. This he has found most useful for the classification of irrigation waters and for the solution of problems having to do with mixtures of water and with changes in quality resulting from the use of water for irrigation.

GEOCHEMICAL CLASSIFICATIONS

In 1911 Chase Palmer presented[2] a method for geochemical classification of natural waters. In his description he stated:

> "Nearly all terrestrial waters have two general properties, salinity and alkalinity, on whose relative proportions their fundamental characters depend. Salinity is caused by salts that are not hydrolyzed; alkalinity is attributed to free alkaline bases produced by the hydrolytic action of water on solutions of bicarbonates and on solutions of salts of other weak acids.
>
> "All the positive radicals, including hydrogen, may participate in producing salinity; but of the negative radicals only those of the actively strong acids can perform a similar function. The principal strong acids in natural waters are represented by the sulphates, chlorides, and nitrates."

NOTE.—Published in June, 1941, *Proceedings*.

[1] Cons. Engr. (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

[2] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911.

In accordance with the prevalence of the reacting values of the groups of positive radicals in the system, five special properties are possible, according to Mr. Palmer—namely, (1) primary salinity (alkali salinity); (2) primary alkalinity (permanent alkalinity); (3) secondary salinity (permanent hardness); (4) secondary alkalinity (temporary alkalinity); and (5) tertiary salinity (acidity). He made the following statement with reference to these five special properties:

> "The character of natural waters with reference to the lithology of the region from which they are derived, to their solvent action on minerals with which they may come in contact, to sedimentary deposits that they are likely to form, to their effect on industrial processes, and to their chemical action in general can best be portrayed by a statement of as many of the five special properties * * * as may be found, expressed in percentages of their totality."

In any system of geochemical classification it is necessary that the reacting values be used—that is, the equivalent weights of hydrogen, usually expressed as milligram equivalents of hydrogen per liter. Substantially, 1 mg of hydrogen per liter is 1 ppm of hydrogen; consequently, "milligrams equivalent per liter" can be termed "equivalents per million," abbreviated to "epm." The equivalent weights of the more common ions are as follows:

Calcium (Ca), 20.04	Carbonate (CO_3), 30.00
Magnesium (Mg), 12.16	Bicarbonate (H CO_3), 61.01
Sodium (Na), 23.00	Sulfate (SO_4), 48.03
Potassium (K), 39.10	Chloride (Cl), 35.46
Nitrate (NO_3), 62.02	

Mr. Palmer computed the percentage reacting values of each ion from the ratio of the equivalent weight of that ion to the equivalent weight of all of the ions. However, because half of the ions are positive and half are negative, it is now more usual to express the percentage reacting value of any positive ion as the ratio of the equivalent weight of that ion to the total equivalent weight of all cations. Likewise, it is customary to compute the equivalent weight of any negative ion, such as the sulfate radical, as the ratio of the equivalent weight of SO_4 to the total equivalent weight of all anions. Regardless of the manner of computing the individual percentage reacting values, the geochemical groups are reported as percentages which themselves total 100%.

Tertiary salinity, as classified by Mr. Palmer, is rarely found in irrigation waters. The magnitude of the other four geochemical classifications suggested by him can be computed by grouping the ions normally present in natural waters in the following manner:

Primary Salinity.—Maximum grouping of Na, K, SO_4, Cl, and NO_3 ions—that is, the greatest possible amount of sodium sulfate, sodium chloride, sodium nitrate, potassium sulfate, potassium chloride, and potassium nitrate that could be in the solution.

Primary Alkalinity.—Minimum grouping of Na, K, H CO_3, and CO_3 ions—that is, the smallest possible amount of sodium bicarbonate and carbonate, and of potassium bicarbonate and carbonate, which could be in the solution.

Secondary Salinity.—Minimum grouping of Ca, Mg, SO_4, Cl, and NO_3 ions—that is, the smallest possible amount of calcium sulfate, calcium chloride, calcium nitrate, magnesium sulfate, magnesium chloride, and magnesium nitrate which could be in the solution.

Secondary Alkalinity.—Maximum grouping of Ca, Mg, $H\,CO_3$, and CO_3 ions—that is, the greatest possible amount of calcium bicarbonate and carbonate and of magnesium bicarbonate and carbonate that could be in the solution.

This system of geochemical classification was particularly adapted to the study of waters used for domestic and industrial purposes. For the interpretation of analyses of water used in agriculture it was felt desirable to group the sulfates with the carbonates rather than with the chlorides and nitrates. This resulted in the following four geochemical groups of salts that can be used to classify all but definitely acid waters:

Z_1.—Maximum grouping of Na, K, Cl, and NO_3 ions—that is, the greatest possible amount of sodium chloride, sodium nitrate, potassium chloride, and potassium nitrate that could be in the solution.

Z_2.—Minimum grouping of Na, K, SO_4, $H\,CO_3$, and CO_3 ions—that is, the smallest possible amount of sodium sulfate, sodium bicarbonate and carbonate, potassium sulfate, and potassium bicarbonate and carbonate that could be in the solution.

Z_3.—Minimum grouping of Ca, Mg, Cl, and NO_3 ions—that is, the smallest possible amount of calcium chloride, calcium nitrate, magnesium chloride, and magnesium nitrate that could be in the solution.

Z_4.—Maximum grouping of Ca, Mg, SO_4, $H\,CO_3$, and CO_3 ions—that is, the greatest possible amount of calcium sulfate, calcium bicarbonate and carbonate, magnesium sulfate, and magnesium bicarbonate and carbonate that could be in the solution.

The designations Z_1, Z_2, Z_3, and Z_4 were used rather than names to avoid confusion with the classification suggested by Mr. Palmer. Group Z_1 is composed largely of sodium chloride; hence it is termed the Common Salt Group. The second group includes those salts commonly called "white alkali" and "black alkali," and this might be called the Alkali Group. Group Z_3 is sometimes referred to as the Bittern Group. Group Z_4 is termed the Hardness Group because it is made up of those salts which most frequently produce hardness in water.

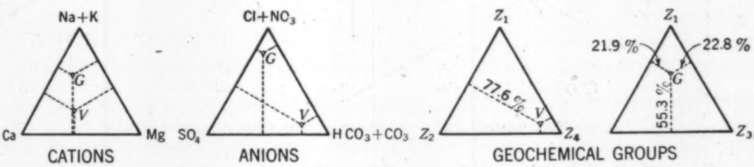
Development of Geochemical Chart

From the nature of the geochemical groups, as defined, there never can be more than three in any sample of water. Of these, Group Z_1 is almost always present, as is Group Z_4, but it is not possible to have salts of both Group Z_2 and Group Z_3 in the solution. Usually, the equivalent weight of sodium and potassium exceeds the amount of chlorides and nitrates. In such event, all of the latter are assigned to Group Z_1; should the amount of chlorides exceed the amount of the alkali bases, all of the sodium and potassium would be included in Group Z_1; in either case, nothing remains to form the fourth group.

The sum of the percentage values of the three groups present being equal to unity, the geochemical character of any water can be represented by a point within an equilateral triangle located so that the perpendiculars to that point are proportional to these percentages. A similar method of plotting can be used to show the relative proportion of the principal ions found in natural water, because there are usually only three cations present in substantial quantities, and likewise the sulfate, bicarbonate, and chloride ions usually constitute almost all of the anions. However, because of the limitation on the number of variables that can be plotted in this manner, it is necessary to combine the percentages of potassium and sodium, and to lump carbonates with bicarbonates, and nitrates with chlorides.

Typical plottings of two waters, the analyses of these waters, and the computation of the geochemical groups are shown in Table 1. The points marked "*V*" represent a sample of the Verde River below Bartlett Dam in Arizona. This water is generally characteristic of natural streams. The points marked "*G*" are for return flow in the Gila River in Arizona immediately above its confluence with the Salt River. The method of plotting is illustrated by the diagram showing the proportions of the various geochemical groups found in the Gila River water. If all of the salts were sodium chloride, the point would have fallen at the Z_1 vertex. Actually, 55.3% of the salts were of the Z_1 Group, and the plotted position is therefore 55.3% of the distance from the base opposite the Z_1 vertex toward that vertex of the equilateral triangle. Likewise, the plotted position is 22.8% of the distance from the base opposite the Z_4 vertex toward that vertex, and 21.9% toward the Z_3 vertex from the

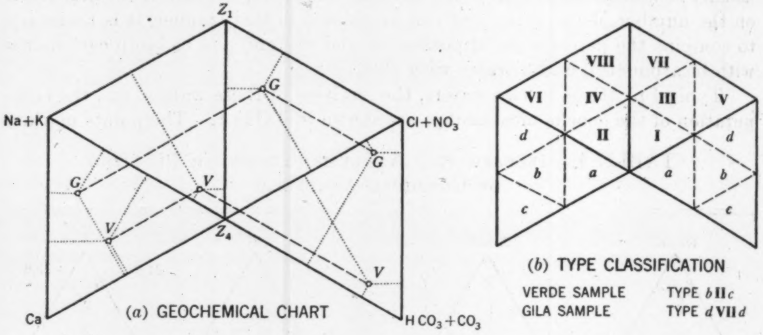
TABLE 1.—ILLUSTRATIVE ANALYSIS, TRILINEAR PLOTTING OF CHEMICAL ANALYSES



Line	Unit	Cations			Σ	Anions			Groups			
		Ca	Mg	Na		H CO$_3$	SO$_4$	Cl	Z_1	Z_2	Z_3	Z_4
Points *V*, Verde River Below Bartlett Dam, Arizona												
1	epm	2.33	1.87	1.20	5.40	3.90	0.99	0.51	0.51	0.69	0	4.20
2	%	43.1	34.5	22.4	100.0	72.2	18.3	9.5	9.5	12.9	0	77.6
Points *G*, Gila River Above Salt River, Arizona												
3	epm	15.4	8.9	30.1	54.4	4.0	8.4	42.0	30.1	0	11.9	12.4
4	%	28.3	16.4	55.3	100.0	7.4	15.4	77.2	55.3	0	21.9	22.8

opposite base. The positions of the points V and G on the cation triangle and on the anion triangle were determined in the same manner by plotting the percentage reacting values of the respective ions.

Although the chemical character of almost any natural water could be illustrated thus by the use of four equilateral triangles, the same data can be shown more effectively on a chart that combines these four triangles in the manner shown in Fig. 1.



(*a*) GEOCHEMICAL CHART

(*b*) TYPE CLASSIFICATION

VERDE SAMPLE TYPE *b*II*c*
GILA SAMPLE TYPE *d*VII*d*

FIG. 1.—TRILINEAR PLOTTING OF ANALYSES ON GEOCHEMICAL CHART

Each of the triangles used for plotting the geochemical groups (Fig. 1(*a*)) has a Z_1 vertex and a Z_4 vertex; in other words, the $Z_1 - Z_4$ base is common to both. Consequently, by superimposing these vertexes a diamond is formed, the bisector of which is the common base. The lower sides of the diamond can now be treated as oblique coordinates, with the origin at the Z_4 vertex, reading in percentage of chloride to the right and percentage of sodium to the left.

By definition, a point plotted at the Z_4 vertex would represent a water containing neither sodium nor chloride, whereas a point plotted along the side of the diamond connecting the Z_2 vertex and the Z_1 vertex would represent a water in which all salts were those of sodium and potassium. Similarly, a point on the line connecting the Z_3 with the Z_1 vertex of the diamond would indicate that all the salts were chlorides or nitrates. A point plotted on the cation triangle at the sodium vertex represents a water in which all of the positive ions are those of sodium and potassium, and a point along the opposite base would be for a water in which there was no sodium or potassium. Therefore, by bringing the cation triangle into contact with the diamond so that the sodium vertex of the cation triangle is coincident with the Z_2 vertex of the diamond and the magnesium vertex of the cation triangle is coincident with the Z_4 vertex of the diamond, a common scale of sodium percentages can be used. In like manner, the anion triangle can be placed beside the diamond with the chloride and nitrate vertex coincident with the Z_3 vertex of the diamond and with the sulfate vertex coincident with the Z_4 vertex of the diamond. When this is done, a common scale of chlorides plus nitrates can be used for the anion triangle and for the geochemical groups.

In Fig. 1 are shown the same points as those in Table 1 for samples of water from the Verde River and the Gila River in Arizona. It will be noted that the points in the cation triangle and those in the anion triangle can be projected parallel to the coordinate axes to intersections that coincide with the points plotted in the diamond. In fact, in the actual use of the geochemical chart it is customary to plot the points representing the water in the cation triangle and in the anion triangle from the percentage reacting values of the ions, and then to plot the percentage of sodium plus potassium against the percentage of chloride plus nitrate, using the sides of the diamond as oblique coordinates, only checking the position of the point by computation of the Z_2 or Z_3 perpendicular, as the case may be.

Types of Water

Waters of different chemical characteristics can be classified readily from their plotted positions on the geochemical chart. When lines are drawn parallel to each base at 50% of the distance toward the opposite vertex, each of the four fundamental triangles is divided into four secondary triangles. These have been numbered arbitrarily in the manner shown in Fig. 1(*b*), following a pattern that can be remembered easily. The sample of water from the Verde River is characteristic of many natural streams and would usually be referred to as Type II water. Were more precise description desirable, it would be termed a Type *b*II*c* water. The Roman numeral II indicates a water in which most of the salts fall in Group Z_4 and with some salts in Group Z_2 but none in Group Z_3. The prefix "*b*" indicates a water in which no one of the positive ions exceeds 50% of the total. The suffix "*c*" defines a water in which more than 50% of the anions are bicarbonates or carbonates. The sample from the Gila River is a Type VII water; for consistency, the prefix "*d*" and the suffix "*d*" could be added, although by the nature of the diagram any water of Type VII contains more than 50% sodium and more than 50% chloride. This water contains salts of Group Z_3, but ont of Group Z_2, and might well be called "irrigation sewage."

Eight primary types are thus available for cataloging irrigation waters, which can be broken down into thirty-two subtypes by use of letters indicating the preponderance of certain positive and negative ions. No attempt will be made herein to list these thirty-two different types of water in the order of their desirability for use in irrigation agriculture. Suffice it to state that, in the case of waters of equal concentration, those in Type II are preferable, and that those of Type VI should be avoided. It is hoped that some discusser will proceed further and describe the effect of different kinds of water on soils and on plant growth.

Mixtures of Different Waters

It can be demonstrated that the plotted position of a mixture of two waters must lie on a straight line connecting the plotted positions of each of the waters comprising the mixture. It can be demonstrated further that the distance of the plotted position of the mixture from either of the two waters making up the mixture depends directly upon the ratio between the total quantity of salts contributed by one water and that contributed by the other. When the respective products of the concentrations of salts in equivalents of hydrogen and the quantities of water are treated as forces normal to the plane of the paper, the position and magnitude of the mixture correspond to the resultant of the forces. Such products of concentration and quantity can be computed conveniently as equivalent tons of hydrogen, or tons-equivalent, abbreviated to "T_e." When the quantity of water is given in acre-feet, the quantity of any ion in tons-equivalent is equal to the quantity times the concentration in milligram equivalents of hydrogen per liter divided by 735, or algebraically:

$$T_e = \frac{C\,Q}{735} \qquad \text{(1)}$$

in which C is concentration of salts in equivalents per million; and Q is quantity of water in acre-feet.

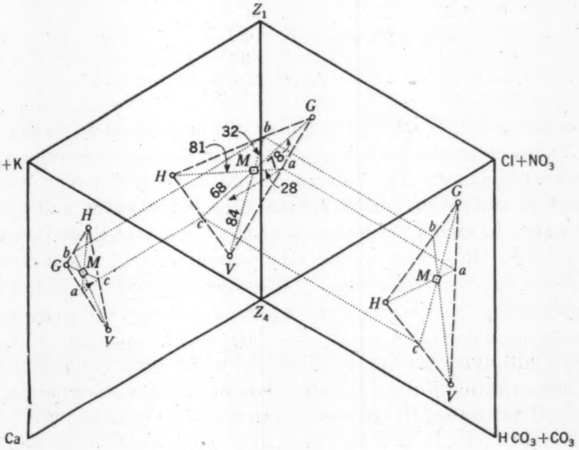
When it is known that one water is in fact a true mixture of two or even three different waters, the amount of each contribution to the mixture can be computed analytically by use of simultaneous equations. However, when it is only suspected that one water is a mixture of several different waters, or when the amount and character of one of the contributing supplies is unknown, an analytical solution becomes impracticable.

The first step in the solution of such a problem is to determine whether or not the presumed mixture is in fact a mixture. This can be readily ascertained graphically by use of the geochemical chart. A typical solution for a true mixture of three waters is shown in Table 2; in this it is assumed that the amount of water from each source is unknown.

For purposes of illustration, the sample of Verde River water (*V*) and the sample of Gila River water (*G*) described in connection with Table 1 have been used. The third water (*H*) contributing to the presumed mixture is somewhat similar to ground waters found in the same region; in fact, the mixture (*M*) is quite representative of water being used for irrigation in the Gila River Valley west of Phoenix, Ariz. The known data are shown in Table 2 and are limited to the amount of water in the mixture and to analyses of samples of the mixture, of Verde River water, of Gila River water, and of the third water. It is to be noted that the total amount of salt in each water, expressed in equivalents per million and in tons-equivalent, is the sum of the positive ions, or the sum of the negative ions, but is not the sum of both.

In each of the three figures in Table 2 the point *M* falls within the triangle formed by connecting points *V*, *G*, and *H*. This indicates, but does not prove, that the water represented by points *M* is a mixture of the other three waters. If the presumed mixture *M* is in fact a mixture of all of the three, it can be considered as being made up of any one of the three and a mixture of the other two. Consequently, if a line is drawn from point *G*, for example, through the point *M* to its intersection with the line *HV* at point *c*, the latter point represents a mixture of water *V* and water *H* in the same proportion as these two occur in the mixture of all three waters. The projection of point *c* in the cation triangle, therefore, must interest the projection of point *c* in the anion triangle at point *c* within the geochemical diamond. The same is necessarily true for points *a* and *b*, determined by drawing lines from the plotted positions of water *H* and water *V* through the plotted positions of the mixture.

TABLE 2.—ANALYSIS OF A MIXTURE OF THREE WATERS;[a] KNOWN DATA AND COMPUTED VALUES



Line	Unit	Cations			Σ	Anions		
		Ca	Mg	Na		HCO_3	SO_4	Cl
		Points *V*, Verde River (1,800 Acre-Ft)						
1	epm	2.33	1.87	1.20	5.40	3.90	0.99	0.51
2	*T.*	(5.70)	(4.57)	(2.94)	(13.21)	(9.54)	(2.42)	(1.25)
3	%	43.1	34.5	22.4	100.0	72.2	18.3	9.5
		Points *G*, Gila River (300 Acre-Ft)						
4	epm	15.4	8.9	30.1	54.4	4.0	8.4	42.0
5	*T.*	(6.28)	(3.62)	(12.28)	(22.18)	(1.63)	(3.42)	(17.13)
6	%	28.3	16.4	55.3	100.0	7.4	15.4	77.2
		Points *H*, Hypothetical Water (900 Acre-Ft)						
7	epm	1.00	2.60	6.40	10.00	2.80	4.60	2.60
8	*T.*	(1.22)	(3.18)	(7.83)	(12.23)	(3.42)	(5.63)	(3.18)
9	%	10.0	26.0	64.0	100.0	28.0	46.0	26.0
		Points *M*, Mixture (3,000 Acre-Ft)						
10	epm	3.24	2.79	5.66	11.69	3.58	2.82	5.29
11	*T.*	13.20	11.37	23.05	47.62	14.59	11.47	21.56
12	%	27.7	23.8	48.5	100.0	30.6	24.1	45.3

[a] Computed values shown in parentheses.

When it has been determined that the presumed mixture is in fact a true mixture, the salt, in tons-equivalent, contributed by each of the three waters comprising the mixture can be computed, with an accuracy commensurate with that of the analyses themselves, by measuring, at some convenient scale, the distance from point M to each of the aforementioned points, and by treating these distances as lever arms of forces normal to the plane of the paper. For example, if moments are taken about the line HG;

$$\begin{aligned}(84 + 32)\ V &= 32\ M\\ 116\ V &= 32 \times 47.62\\ V &= 13.21\end{aligned}$$

In the foregoing, 47.62 is the number of tons-equivalent in the mixture, so that 13.21 is the number of tons-equivalent in the Verde River water contributing to that mixture. The difference (that is, 34.41 T_e) represents the tons-equivalent of salts contributed by the Gila River water and by the hypothetical water, in such a proportion that the mixture of these two alone would plot at point b. In similar manner, the tons-equivalent of salt contributed by each of these waters is determined. Knowing the total amount of salt in each water making up the mixture, and the concentration of salt in samples of each, the number of acre-feet of water from each source can then be computed directly by multiplying the tons-equivalent by 735 and by dividing the product by the concentration in equivalents per million. For example, in the case of the Verde River water, the concentration was 5.40 epm and the total amount of salts was 13.21 T_e; hence, the Verde River contributed 1,800 acre-ft of water to the mixture. From the other data shown in Table 2, it was found that the Gila River furnished 300 acre-ft, and that the remaining 900 acre-ft were from the hypothetical well.

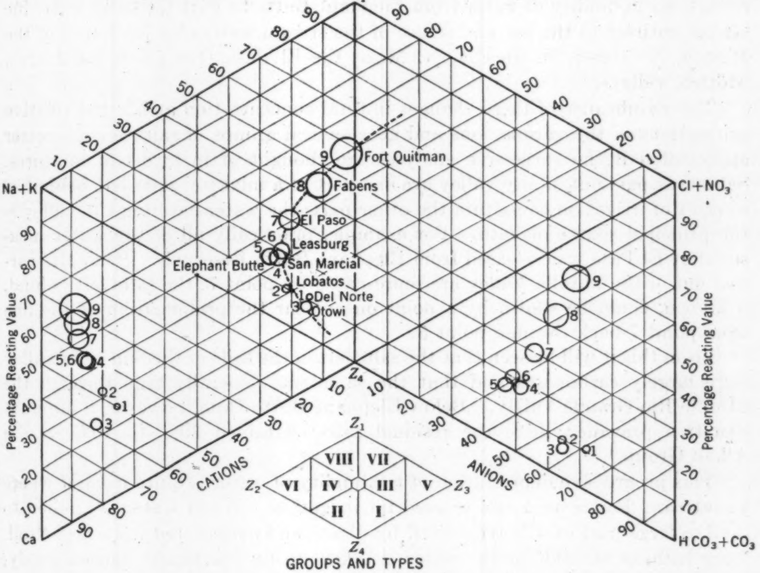
CHANGES IN QUALITY

The foregoing has been descriptive of the manner in which the geochemical chart was developed and of the fundamental principles involved in its use. One of the many applications to which the chart can be put is that shown in Table 3. On the graph have been plotted characteristic analyses of samples taken from the Rio Grande at various stations along its course from the mountains in Colorado to Fort Quitman, Tex. The data are from the 1936 report of the Rio Grande Joint Investigation. The chart itself is a reduction of a standard commercial form originally printed with 2% graduations at a scale of about 1 in. equals 20%. The weighted average concentration of the ions in equivalents per million is also given in Table 3.

As an aid in visualization of the changes that occur in the quality of the waters of the Rio Grande along its course, the analyses have been indicated by circles whose areas are proportional to the concentration of salts, ranging from the small circle representing samples from the Rio Grande at Del Norte, Colo., to the large circle corresponding to the highly concentrated waters found in the Rio Grande at Fort Quitman.

It is to be noted from the plotting of these data that the water coming out of the mountains is of Type *cIIc*—that is, a water in which calcium bicarbonate predominates. Below Del Norte lies the San Luis Valley in Colorado within which there are several hundred thousand acres of irrigated land. The change in quality resulting from irrigation use is reflected by the shift from point 1 to point 2, the latter being for the Lobatos station near the Colorado-New Mexico boundary. Below this station there are other tributary streams, particularly Rio Chama, draining the high mountains in northern New Mexico. Some of these waters obviously contain more of the Z_4 Group of salts than does the water of the Rio Grande at Del Norte, because the analysis for Otowi, N. Mex. (point 3), plots between the Z_4 vertex and point 2 and closer to that vertex than the Del Norte water.

TABLE 3.—GEOCHEMICAL CLASSIFICATION OF THE WATER OF THE RIO GRANDE ABOVE FORT QUITMAN, TEX.



No.	Location	Ca	Mg	Na + K	Total anions	CO_3 + HCO_3	SO_4	Cl + NO_3
1	Del Norte, Colo.	0.56	0.27	0.30	1.14	0.70	0.32	0.12
2	Lobatos, Colo.	1.39	0.58	0.91	2.83	1.54	1.00	0.29
3	Otowi, N. Mex.	2.19	0.69	0.90	3.71	2.12	1.28	0.31
4	San Marcial, N. Mex.	3.76	1.27	3.62	8.66	2.96	4.11	1.59
5	Elephant Butte, N. Mex.	3.75	1.28	3.88	8.96	2.66	4.76	1.54
6	Leasburg, N. Mex.	4.05	1.35	4.20	9.63	2.82	4.88	1.93
7	El Paso, Tex.	4.95	1.70	6.98	13.65	3.55	6.07	4.03
8	Fabens, Tex.	6.90	2.75	12.40	22.12	4.10	8.26	9.76
9	Fort Quitman, Tex.	9.54	3.75	19.78	33.20	4.01	10.17	19.02

Below Otowi, which is northwest of Santa Fe, N. Mex., there is the Middle Rio Grande Valley which extends along the river to San Marcial, N. Mex., near the head of Elephant Butte Reservoir. The marked reduction in quality that occurs between Otowi and San Marcial is perhaps due most to the influence of tributary streams, such as Rio Puerco, draining the plateaus of western New Mexico. The water released from Elephant Butte (point 5) differs somewhat from the inflow at San Marcial because the supply drawn out of the reservoir in 1936 included water that came into the reservoir in the preceding year. The reductions in quality of water from Elephant Butte to Fort Quitman were due almost entirely to the use and re-use of the stream waters for irrigation of the Rincon, N. Mex.; the Mesilla, N. Mex.; the El Paso, Tex.; and the Juarez, Mexico, valleys.

The significance of these changes in total concentration and in the relative proportions of the various ions and geochemical groups of salts will be better appreciated if the successive analyses are thought of in terms of mixtures. Salts that pass out of any valley are one part of a mixture; salts left behind as a result of irrigation constitute the other part; the water entering the valley is the principal source of both. For example, practically all of the water measured at El Paso was released from Elephant Butte Reservoir. The salts carried out of the Mesilla Valley are represented by point 7; the salts left behind, if known, could be shown by a point on or near the prolongation of the line from point 7 back through point 5.

From this it will be seen that the salts left in the soils of the San Luis Valley were largely carbonates and that the same was generally true through the Middle Rio Grande Valley. Below Elephant Butte Dam, however, a different situation prevailed; here the residual salts contained 30% to 40% of the Alkali Group.

This progressive depreciation of the quality of water is quite natural where the stream flow is used and re-used for irrigation. When water is applied to land, a large part of it is transpired by plants and evaporated from moist soil. Very little of the salt in the water is taken up by the plants; consequently, there is a much greater concentration of salts in the soil solution than in the irrigation supply. Since the carbonates of calcium and magnesium are sparingly soluble, they are precipitated first. The irrigation percolate, which finds its way back into the river either through drains or by natural seepage, thus contains a higher proportion of sulfates, chlorides, and sodium than the waters that were diverted for irrigation. Progressively downstream, H_2O is lost to the atmosphere by evaporation and the less soluble salts are left behind by precipitation, until only irrigation sewage remains.

Somewhat similar variations in quality exist along the Colorado River from its headwaters in the Rocky Mountains to the Gulf of California, but the degree of change is smaller, due to the fact that the Colorado River is less developed relatively than the Rio Grande above Fort Quitman. In spite of this, however, the supply released from Lake Mead contains 30% more salt per acre-foot than water released at Elephant Butte; and, although the sodium percentage is not quite as high, the proportion of chlorides and sulfates is greater in the water discharged from Lake Mead than it is in the Rio Grande below Elephant Butte.

Valley Salt Balance

In addition to presenting a picture of what happens along the course of a river whose waters are used and re-used in the irrigation of land, the geochemical chart can be used to determine whether any salt is being accumulated in the lands irrigated.

Whenever the total amount of salt removed from an area exceeds the salt load of the incoming supply, it would appear that a favorable salt balance exists; however, this is not necessarily true. In many river valleys the ground waters are much more saline than the surface stream waters, as, for example, along the Rio Grande below El Paso. Water seeping from irrigation canals and percolating from fields tends to displace previous accumulations of ground water and to cause lateral movement toward any drains. Consequently, the drainage outflow is usually more saline than the contribution to the ground water.

Wherever this situation exists, the total quantity of salt in the effluent from a valley is likely to be greater than that in the influent to the same area, and this condition will hold until all the saline ground waters have been displaced. Therefore, it is necessary to do more than compute the difference between the salt load of the irrigation water and that of the drainage water.

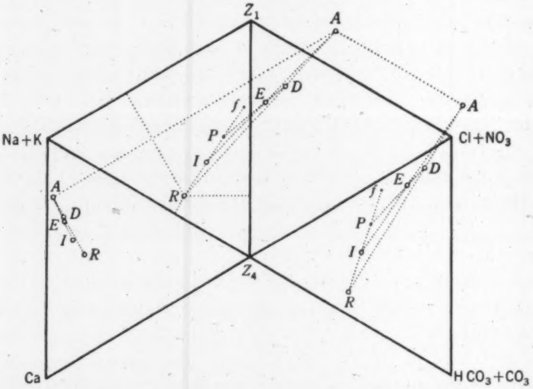
Table 4 shows the solution of such a problem for a 15,000-acre unit of the El Paso Valley. In the year 1934, which has been taken for illustration, 69,000 acre-ft of water, carrying 78,000 tons of dissolved solids of a type similar to that represented by point 7 in the graph insert of Table 3, were delivered into that area. The drainage effluent amounted to 27,000 acre-ft with a salt load of 140,000 tons. There was thus an apparent favorable salt balance amounting to 62,000 tons.

From a number of separate analyses made during that year, the total amount of each ion in the influent to that area, and in the effluent from it, were computed. These data are shown in Table 4. All other values given in the table are computed. The apparent gain (that is, the amount by which the salts in the effluent exceed the salts in the influent) is merely the difference between these quantities. The net gain in positive ions amounted to 244 T_e of calcium, 29 T_e of magnesium, and 806 T_e of sodium, making a total of 1,079 T_e removed from the area with the drainage water in excess of the amount brought in with the irrigation water.

The simple difference between the amount of chloride brought in and the amount removed shows an apparent gain of 1,261 T_e, which is 182 T_e greater than the net amount of cations removed. This excess is offset by the apparent accumulation of 122 T_e of HCO_3 and 60 T_e of SO_4

In any quantity of water there must be the same equivalent weight of positive ions as there is of negative ions. It follows, therefore, that, with the 182 T_e of chlorides in excess of the total of the positive ions, there were 182 T_e of cations in the effluent that were not brought into the area during the same year. Furthermore, with these 182 T_e of cations displaced from some source within the area, there must have been another 182 T_e of anions. In these anomalies lies the solution of the problem of how much of the influent salt was left in this part of El Paso Valley and how much of the salt already accumulated was removed from it.

TABLE 4.—VALLEY SALT BALANCE; KNOWN DATA AND COMPUTED VALUES[a]



Line	Unit	Cations			Σ	Anions		
		Ca	Mg	Na		HCO_3	SO_4	Cl
INFLUENT SALT, *I*								
1	T_e	465	166	664	1,295	332	576	387
2	%	35.9	12.8	51.3	100.0	25.6	44.5	29.9
EFFLUENT SALT, *E*								
3	T_e	709	195	1,470	2,374	210	516	1,648
4	%	29.9	8.2	61.9	100.0	8.8	21.8	69.4
APPARENT GAIN, *A*								
5	T_e	244	29	806	1,079	−122	−60	1,261
6	%	22.6	2.7	74.7	100.0	−11.3	−5.6	116.9
DISPLACED SALT, *D*								
7	T_e	458	130	1,036	1,624	92	216	1,316
8	%	28.2	8.0	63.8	100.0	5.7	13.3	81.0
RESIDUAL SALT, *R*								
9	T_e	214	101	230	545	214	276	55
10	%	39.3	18.5	42.2	100.0	39.3	50.7	10.0

[a] All values for displaced and residual salts are computed.

The necessity of considering more than just the difference between the amount of salt brought in and that removed becomes more apparent when the data are plotted on the geochemical chart. Point I, Table 4, represents the influent, point E the effluent, and point A the apparent gain. Point A, however, falls outside of the anion triangle, and likewise outside of the geochemical diamond. This could not be the case if the excess amount of salt in the effluent resulted only from the removal of previous accumulations, because no water can contain less than zero of any ion. The apparent gain must, in itself, be the difference between the total salts displaced from the area and the amount of influent salts left behind in the soil or in the ground water.

The approximate magnitude and character of the displaced salts and the residual salts can be computed from the following relationships:

$$I = R + P \qquad (2a)$$

$$E = P + D \qquad (2b)$$

$$E - I = A \text{ or } E = I + A \qquad (2c)$$

$$P + D - R - P = A \qquad (2d)$$

and

$$D - R = A \text{ or } D = R + A \qquad (2e)$$

in which: I is the amount of influent salts; E is the amount of effluent salts; P is the amount of the influent salts that passes through the area within the period of time under consideration; R is the amount of salts left behind in the soil or in the ground water; and D is the amount of salts displaced from the soil or from the ground water.

From Eqs. 2, E thus can be considered a mixture of I and A; likewise, D can be treated as a mixture of R and A. Consequently, the points I, E, and A must lie on a straight line, and the points R, D, and A must also lie on a straight line. Furthermore, the positions of the points along these lines must be such that there will be equilibrium of forces when the equivalent total weight of each is treated as a force normal to the paper.

If moments are taken about the base opposite the vertex corresponding to any ion or geochemical group, the percentage of that ion or group can be used as the lever arm of the force. In Eqs. 3 such percentages are designated by the letter C with appropriate subscripts:

$$C_i \times I + C_a \times A = C_e \times E \qquad (3a)$$

and

$$C_r \times R + C_a \times A = C_d \times D \qquad (3b)$$

There are four unknowns in Eqs. 3—namely, C_r, R, C_d, and D. Eq. 3(b) and the equation of mass (that is, Eq. 2(e)) are the only simultaneous equations available for determining these unknowns. Consequently, an infinite number of solutions of the problem are possible. The position of points R and D on the geochemical chart are circumscribed by other factors, however, so that the magnitude of these values can vary only within limits.

The smallest possible amount of residual salt was 182 T_e; if this were the correct value, point D would be at the chloride vertex of the anion triangle, indicating the displacement of no other ion from the soil or from the ground water; it also would require that the salts left behind include no chlorides. Neither presumption is warranted. There was probably no percolate from some irrigated fields, so that a part of the residual was substantially of the same character as the influent salt. All of the canal seepage could not have reached the effluent drains within the period of time under consideration; this residual was also of the same character as the influent. In other parts of the area where there was free drainage, probably no chlorides were left behind. It is reasonable, therefore, to assume that the residual salts contained substantially 10% of chlorides. Under this assumption, point *R* must lie on a line parallel to the base opposite the chloride vertex of the anion triangle and 10% of the distance from that base toward the chloride vertex.

The residual being one element of a mixture which is represented by the influent, the other element of the mixture (that is, point *P*) must lie on the prolongation of the line *RI*. Point *P* is in itself a mixture of canal water and field percolate. The canal water is the same as the influent; therefore, the position representing the field percolate must lie on the prolongation of the line *IP* at some such point as *f*, shown in Table 4. Any field percolate will be low in carbonates but presumably will contain some carbonates.

Point *D*, Table 4, is indicative of the character of the salts displaced from the soil and the ground water. These are naturally high in chlorides, and it is reasonable to assume ordinarily that only about 30% of the salts are sulfates and carbonates. In the solution of the particular problem used for illustration, certain analyses of well waters were available which indicated that the displaced salts contained more than 80% chlorides.

The location of points *R* and *D* in Table 4 were selected with consideration to these factors; the magnitude of the quantities were computed from Eqs. 2 and 3. The resulting values are shown in Table 4. It will be noted that the apparent gain of 1,079 T_e of salts during 1934 was probably brought about by the displacement of 1,624 T_e of salts from the soil and the ground water and the accumulation of 545 T_e of the salts in the irrigation supply. Other reasonable solutions of this problem are possible, but the magnitude of the residual cannot be changed to any large extent without making extreme assumptions. There is ample reason to assume that the true residual was between 450 T_e and 650 T_e and that the large net gain in chlorides resulted from the displacement of saline ground water.

The character of salts left behind as a result of irrigation in this part of the El Paso Valley can be computed from the chart. The perpendiculars to point *R* on the geochemical diamond are proportional to the magnitude of the various groups. These magnitudes are as follows:

Group	Tons-equivalent
Z_1	54.5
Z_2	175.5
Z_4	315.0

Substantially all of Group Z_1 was sodium chloride, so that the total amount of this group left behind can be computed by multiplying 54.5 T_e by the sum of the combining weights of sodium and chloride (namely, 23.0 + 35.5), giving roughly 3,200 tons. In Group Z_2 there was evidently no sodium bicarbonate because there was ample calcium present to combine with the carbonates. Accordingly, it was found that 12,500 tons of sodium sulfate were left in the area as a result of irrigation. Computation of the amount of the different salts included in Group Z_4 involves the assumption that certain combinations will be formed. These hypothetical combinations are quite uncertain, but to complete the picture it can be estimated that 6,100 tons of magnesium sulfate and 10,700 tons of calcium carbonate were included in the residual.

Approximately 32,500 tons of various salts out of the 78,000 tons brought in by the irrigation water in 1934 were thus left behind as a result of the irrigation of less than 15,000 acres. From this it follows that two thirds of the salt load of the drainage water was derived from the saline waters underlying the area, and that the salt balance within the root zone was probably adverse.

In addition to this accumulation of mineral salts in the soils of the area, it appears that there was some accumulation of sodium as a result of base exchange. The displaced salts contained 17.2% of the Z_3 Group, which is made up primarily of the chlorides of calcium and magnesium. The amount of magnesium displaced was only 8% of the cations so that about one half of the Z_3 Group of displaced salts was calcium chloride. It is unlikely that calcium chloride would be present unless there had been base exchange, because of the character of the influent water. The probability that calcium was displaced from the soil colloids by sodium is also indicated by the proximity of point *D* on the cation triangle to point *E*.

The assumption that a favorable salt balance is established whenever the total of the effluent salts exceeds the total of the influent salts may thus be erroneous. It is important to ascertain in fact whether a favorable salt balance is being maintained within the root zone of the irrigated fields, because any accumulation of salt in the upper soil horizons, no matter how slow, must eventually render irrigation agriculture unprofitable.

Necessity for Further Study

This paper has been presented for two purposes: First, to offer to the engineering profession a convenient tool for the interpretation of chemical analyses of stream and ground waters; and second, to stimulate irrigation engineers to consider quality of water to be as important as quantity.

The data presented were selected primarily in illustration of the geochemical chart developed by the writer and of the methods used in its application to irrigation problems. Much more study of these problems by irrigation engineers and agriculturalists is needed, to the end that successful irrigation agriculture may be permanent rather than temporary.

DISCUSSION

C. S. SCOFIELD,[3] ESQ.—Mr. Hill, in this paper, undertakes to extend and improve a system of water classification proposed by Chase Palmer[2] before 1911. The writer would be less than frank if he did not state at once that there exists among hydrologists a difference of opinion as to whether Mr. Palmer's classification should be retained in use even if revised. One school holds that it would be better to forget it. The writer does not feel called upon at this time to support either side of that question.

It seems both obvious and uncontrovertable that the methods currently in use for reporting on the quality of water are unsatisfactory. There are almost as many methods of reporting water analyses as there are water analysis laboratories. Under present conditions it requires the services of an expert hydrologist and a computing machine to reduce the findings of two or more laboratories to a common basis so that comparisons between them are possible.

It was not until June 25, 1941, that definite steps were taken to remedy this regrettable condition of affairs. On that date, Committee D-19 of the American Society for Testing Materials adopted the report of its subcommittee on the analyses of industrial waters. The fundamental basis adopted by Mr. Hill is in harmony with the method recommended by that Committee.

However, it is evident that Mr. Hill's objectives lie beyond the relatively simple one of obtaining uniformity of method in reporting water analyses. He has two major objectives: (1) To illustrate by means of diagrams the salient characteristics of two or more waters; and (2) to use these diagrams as an aid in solving certain perplexing hydrological problems along streams, such as finding the sources of salt contamination or finding the places in which salt is deposited.

The writer approves and supports his efforts in the direction of his first objective. These are illustrated in Fig. 1 and Tables 1 and 3. These diagrams show effectively the important differences between the waters of two streams in the Salt River Valley and the progressive changes in concentration and in composition that occur in the Rio Grande between Del Norte and Fort Quitman. No doubt, the essential facts in these two situations would be apparent to an expert hydrologist who examined the tabulated analytical data, but these facts become apparent to a much larger number of persons when they are illustrated as they are in these diagrams.

In respect to the second objective—that of solving problems of the source or the destination of water-borne salinity—the writer finds himself less positive, and in fact somewhat perplexed. The problems involved are of great importance, and any serious attempt to solve them is to be commended; but the writer must admit his own inability to evaluate this method of attack.

It may be appropriate to discuss certain phases of the subject of this paper that Mr. Hill has not emphasized—namely, the nature of the effects of salt constituents on the soil and on plants. One important constituent that is not mentioned in the paper is boron. This element is important because it is one of a small number of elements that are absolutely essential for plant growth. It is important also because in some situations it occurs in irrigation waters and ultimately in the soil solution in such high concentrations as to injure crop plants or even to prevent growth. In other situations (not only in irrigated lands but elsewhere), the quantity of available boron is so small that it is necessary to supply it artificially in order to obtain normal crop growth. Because of these facts, any general discussion of the salts of irrigation water should include boron, and any classification of such waters should take it into account.

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[3] Principal Agriculturist, U. S. Bureau of Plant Industry, Washington, D. C.

The importance of sodium as a constituent of irrigation water is fully recognized in Mr. Hill's classification. The writer will merely point out some of the reasons for its importance. So far as is known, sodium is not one of the elements essential to plant growth. However, it is almost always present in irrigation water and in the soil solution; and whenever salinity concentrations are high, sodium is almost always the dominant basic or cation constituent. Plants absorb very little sodium from the soil solution even when its concentration in that solution is high. It is not known to produce injurious effects within the plant, probably because most plants do not absorb it in large quantities. Its presence in high concentrations in the soil solution is directly injurious to plants chiefly because of osmotic effects of its salts, which retard the absorption of water by plant roots.

Sodium is chiefly injurious because of its effect on the physical condition of the soil. The soil contains minerals and organic compounds that participate in reactions of base exchange. Reactions of this type are widely known and extensively utilized in many industries, particularly in zeolitic water softeners. In the soil, high concentrations of sodium in the solution tend to replace calcium and magnesium from the mineral and organic complexes with the result that the physical properties of the soil are impaired. The soil becomes gelatinous and relatively impermeable to the movement of water.

The rate and extent of the replacement of calcium and magnesium by sodium depend upon the relative concentrations of these constituents in the soil solution. Thus the injurious effects of sodium occur only when its concentration is higher than that of the calcium and magnesium. This is the reason for emphasizing the relative concentration of sodium rather than its absolute concentration. By way of summary it may be stated that, so far as is known, sodium serves no useful purpose in irrigation water or in the soil solution. It becomes definitely harmful to the soil and thus indirectly to plants when its concentration becomes much higher than the sum of the calcium and magnesium.

It seems appropriate now to discuss briefly three other basic or cation constituents of irrigation water and the soil solution. These are calcium, magnesium, and potassium. They are all elements essential to plant growth. Of the three, potassium is the one most largely absorbed by plants. In fact, plants absorb it so avidly that it is seldom found in high concentrations even in very saline soil solutions or in drainage waters. Potassium deficiency is common, but potassium toxicity is very rare. Calcium and magnesium are often grouped together. They have some characteristics in common; for example, together they make up the hardness that is troublesome in waters intended for domestic and industrial uses. They differ in one important particular—namely, calcium combines with the anion, sulfate, to form a salt of low solubility (0.2%), whereas the corresponding salt of magnesium is one of the highest in solubility. This means that, whereas calcium seldom occurs in toxic concentrations even in very salty solutions, magnesium may do so, and may thus contribute to the high osmotic values in the soil solution that retard the absorption of water by plants.

The writer hesitates to take the space to discuss in detail the salt constituents of the anion group—namely, the carbonate-bicarbonate complex, the sulfate, chloride, and nitrate. There is much to be found in literature about the harmfulness of the carbonate-bicarbonate constituent, the so-called "black alkali." The injurious potentialities of this constituent are overrated. Most of the trouble allegedly due to black alkali is caused by sodium; and sodium is no less harmful when it occurs with sulfate, chloride, or nitrate than when it occurs with carbonate or bicarbonate. Furthermore, both carbonate and bicarbonate form salts of low solubility with calcium and may thus be made innocuous at small expense.

Of the other three anions, sulfate and nitrate at least are both essential to plant growth. In this respect there is some doubt about chloride. Nitrate is so largely absorbed by plants that it seldom occurs naturally in toxic concentrations. In artificial culture solutions it has been shown to be slightly more toxic than equivalent concentrations of chloride. Both of these constituents are more toxic than equivalent concentrations of sulfate.

In these remarks the writer has discussed four cation constituents and five anion constituents that occur in the salts of irrigation water. He has emphasized the constituents, rather than the salts they may form, because in water solutions, and in respect to their effects either on the soil or on the plants, they operate as constituents rather than as salts. Some of them operate independently, whereas the effects of others are modified by their associates. The method of reporting and illustrating the characteristics of irrigation water as proposed by Mr. Hill is unquestionably of value in the direction of simplifying this complex problem. There can be no doubt that its complexity has been one of the reasons why its economic implications have not been more widely understood. However, one must not overlook the dangers of oversimplification. With this in mind the writer wishes to indorse particularly Mr. Hill's concluding sentence in which he states that much more study of these problems by irrigation engineers and agriculturists is needed.

HERMAN STABLER,[4] M. AM. SOC. C. E.—Too few engineers are sufficiently interested in quality of water to delve into the mysteries of water chemistry, and too few chemists are sufficiently interested in the engineering applications of the water analyses they make to consider the impact of quality of water on industrial and agricultural development. The author has done a service to the engineering profession by invading that twilight zone between engineering and chemistry and bringing back and presenting to engineers a classification of waters in terms of their chemical constitution and an ingenious method of expressing chemical quality and classification in graphical form.

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[4] Chf , Conservation Branch, U S. Geological Survey, Interior Dept., Washington, D. C.

In general, the author follows the Palmer method of considering waters as a balanced system of chemical values. Palmer[5] advocated the statement of a water analysis in four parts:

(1) The properties of the solution in percentage proportions (in which he grouped radicals to indicate "properties," of which there would be but three to any water; properties so determined fixed the "class" of the water);

(2) The percentage reacting values of groups of radicals accompanied by a statement of concentration values;

(3) The "character formula"—the percentage reacting values of the individual radicals—together with a statement of concentration values; and

(4) The base analyses.

Palmer's designation of the "class" of a water, or of the "properties" that determine the class, provides a generalized chemical picture of the water that is scientifically correct and convenient for some purposes but is inadequate for industrial or agricultural evaluation because it recognizes no difference in effect of the individual radicals grouped. The author departs slightly from the Palmer grouping of radicals and states (see heading "Geochemical Classifications"): "For the interpretation of analyses of water used in agriculture it was felt desirable to group the sulfates with the carbonates rather than with the chlorides and nitrates." He thus violates the chemical logic of the Palmer grouping without seeming to gain anything in application to irrigation engineering. The writer wonders why.

Palmer's character formula gives a detailed chemical picture of a water portraying the particular place it occupies in an infinite series of "classes." Many individuals of this infinite series are grouped in the few major classes based on "properties." Together with concentration value, the character formula can be applied to the solution of many industrial and irrigation problems. The author follows the principle of Palmer's character formula with the exception that his percentage reacting values (or equivalents per million) add up to 200 instead of 100. Here again the writer sees no gain. Calculating the percentages of "anions" and "cations" separately is desirable, but it is as easy to make the sum of each 50, thus preserving the concept of 100% for the whole, as to double the values as the author has done.

The author's diagrammatic representation of a water by a single point, or by designation (for example, as a Type II or Type *b*II*c* water), carries with it a degree of significance but leaves out of the picture rather important detail. Designation of Type *b*II*c* for the Verde River water, for example, means (following the author's definitions) that this is "a water in which no one of the positive ions exceeds 50% of the total" but "in which more than 50% of the anions are bicarbonates or carbonates," in which most of the salts fall in a group of Ca, Mg, SO_4, $H\ CO_3$, and CO_3 ions, some in a group of Na, K, SO_4, $H\ CO_3$, and CO_3 ions, and none in a group of Ca, Mg, Cl, and NO_3 ions (see heading "Types of Water"). All of this and more would be disclosed in Palmer's character formula which, for the same water, would be 21.6 Ca, 17.2 Mg, 11.2 Na, 36.1 $H\ CO_3$, 9.2 SO_4, 4.7 Cl, simulating the formula for any other mineral.

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[5] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911, p. 14.

Diagrams showing characteristics of waters and adapted to comparison of several analyses have been developed by a number of people. The writer has been accustomed to use the form shown in Fig. 2, in which comparison is made of the nine waters in Table 3. This diagram shows, in parallel columns (instead of in two separate triangles as shown by the author), the factors entering into the Palmer character formula, and, in width of column (instead of area of circles as shown by the author), the concentration factor which, together with the character formula, makes up the complete detailed picture of a water analysis. The writer finds it easier to diagnose the character of a water and to compare the several characteristics of a group of waters from Fig. 2 than from the author's "geochemical chart" (Fig. 1), but recognizes that this may be largely if not wholly due to habit and his greater familiarity with the one than with the other. Both schemes accomplish essentially the same purpose, and choice between them resolves itself largely into a matter of personal preference. Fig. 2 shows at a glance the variations from upper to lower reaches of the river, which are:

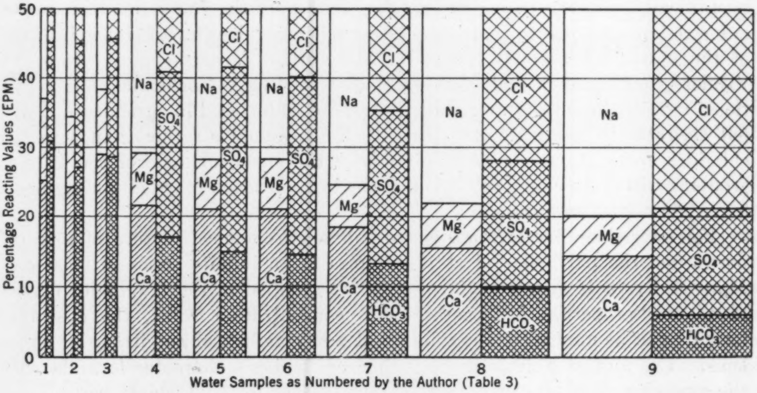

FIG. 2.—GRAPHICAL REPRESENTATION OF "CHARACTER" FORMULA (PALMER) AND CONCENTRATION VALUE FOR WATERS OF THE RIO GRANDE (WIDTH OF COLUMNS PROPORTIONAL TO CONCENTRATION, COL. NO. 1 BEING DOUBLE SCALE)

(1) Progressive increase in concentration until at Fort Quitman concentration is nearly thirty times as great as at Del Norte;

(2) Progressive decrease in the percentage of Ca from Del Norte to Fort Quitman interrupted by a maximum at Otowi;

(3) Progressive decrease in the percentage of Mg;

(4) Progressive increase in the percentage of Na + K interrupted by a minimum at Otowi;

(5) Progressive decrease in the percentage of CO_3 + H CO_3 interrupted by a sub-maximum at Otowi;

(6) Progressive increase in the percentage of SO_4 from Del Norte to Elephant Butte (except for slight decrease at Otowi), followed by progressive decrease downstream; and

(7) Progressive decrease of the percentage of Cl from Del Norte to Otowi, followed by a rather sudden increase to San Marcial and Elephant Butte and then by progressive increase downstream.

These variations can likewise be visualized from the author's anion and cation triangles in the geochemical chart in Table 3 or from a table of radicals by reacting values and concentration values (Palmer's third item in the statement of an analysis). The changes in "class" shown in the author's diamond in the geochemical chart are progressive increase (except for Otowi) in percentage hardness, progressive decrease (except for Otowi) in percentage saltness, and general progressive decrease in percentage alkali to Elephant Butte, followed by progressive increase downstream. Do these add anything of importance in evaluating an irrigation water? The writer thinks not. Had the author adhered to the Palmer grouping for properties there would be: A general progressive decrease of secondary alkalinity (beneficial or inert for irrigation); progressive increase of secondary salinity (beneficial for irrigation) to Leasburg, followed by decrease at El Paso and increase thereafter; and progressive increase (except at Otowi) in primary salinity (harmful for irrigation). The irrigation picture, however, would not be very well defined. In evaluating waters for industrial use or for irrigation the writer prefers to use formulas adapted especially to the particular problem in hand rather than to trust to any general geochemical classification.

The author does not attempt to describe the effect of different kinds of water on soil and on plant growth but expresses the hope that some discusser will do so. The writer has not been so cautious. In 1910[6] he prepared an article on "The Relative Value of Irrigating Waters" that was republished[7] one year later with modifications as part of a more comprehensive treatment of industrial interpretation of water analyses.

Irrigation waters were discussed, formulas were given for computing the "alkali coefficient" (an indicator of value for irrigation purposes), and a classification of irrigation waters was set forth. The treatment there accorded the subject of irrigation waters, though far from adequate, has proved very useful in certain phases of water evaluation by the writer. The data then available were meager, and more accurate formulas doubtless could be devised on the basis of the wealth of information and experience now available. This classification of irrigation waters includes consideration of only the common constituents reported in water analyses. Any adequate treatment of the subject would have to take into consideration such substances as boron which, although present in irrigation waters in small amounts, may exert a potent influence on plant life.

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[6] *Engineering News*, Vol. 64, No. 2, July 14, 1910, p. 57.

[7] *Water Supply Paper No. 274*, U. S. Geological Survey, Washington, D. C., 1911 (including on p. 165 a chapter on "The Industrial Application of Water Analyses")

For what it is worth the writer presents in Fig. 3, by the method outlined in *Water Supply Paper No. 274*, on semilogarithmic scale, the alkali coefficient, class, and class description of the nine waters of the Rio Grande whose analyses are supplied by the author. The waters at and above Otowi are shown to be of very high class for irrigation purposes; those at San Marcial, Elephant Butte, and Leasburg are shown as "good" but little better than "fair"; the water at El Paso is shown as "fair"; and those below El Paso as "poor." How nearly do these classes and the relative magnitude of the alkali coefficients on which they are based reflect the experience of the water users along the Rio Grande?

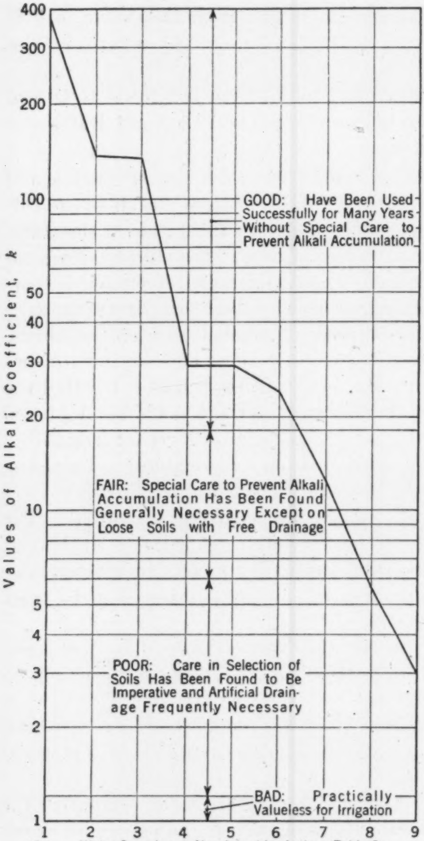

FIG. 3.—CLASSIFICATION AND RELATIVE VALUE FOR IRRIGATION OF WATERS OF THE RIO GRANDE

In 1935 Carl S. Scofield[8] discussed the many "variables that must be considered in determining the relationship between the quality of irrigation water on the one side and of a profitable and enduring system of irrigation agriculture on the other." With a warning that the class limits or standards given are intended for use in a particular area and should not be taken as applicable everywhere, he presents the values in Table 5 as an example of the permissible limits, adopted for a definite region, of classes of irrigation water with respect to certain of its characteristics.

Applying the standards of Table 5 to the nine Rio Grande waters whose analyses are submitted by the author (a procedure that Scofield would probably regard as incautious), the series of classifications in Table 6 is derived and compared with the writer's alkali coefficient and classification.

By either system the first two waters stand out as of high class; the third is equally good by the writer's classification but slightly poorer by Scofield's; the fourth rates about the same under the two classifications; the fifth and sixth, almost identical with the fourth according to the writer's classification, are slightly poorer by Scofield's; and the two classifications give about the same result on the eighth and ninth waters.

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[8] "The Salinity of Irrigation Water," by Carl S. Scofield, The Smithsonian Rept. for 1935, pp. 275–287.

TABLE 5.—PERMISSIBLE LIMITS OF CLASSES OF IRRIGATION WATER

Class	Grade	CONCENTRATION, TOTAL DISSOLVED SOLIDS[a]		Percentage of sodium[b]	BORON, PARTS PER MILLION; CROPS GROUP			CONCENTRATION IN MILLIGRAM EQUIVALENTS	
		Conductance $K \times 10^5$ at 25°C	Parts per million		A	B	C	Chlorides (Cl)	Sulfates (SO_4)
1	Excellent	<25	<175	<20	<0.33	<0.67	<1.0	<4	<4
2	Good	25–75	175–525	20–40	0.33–0.67	0.67–1.33	1.0–2.0	4–7	4–7
3	Permissible	75–200	525–1,400	40–60	0.67–1.00	1.33–2.00	2.0–3.0	7–12	7–12
4	Doubtful	200–300	1,400–2,100	60–80	1.00–1.25	2.00–2.50	3.0–3.75	12–20	12–20
5	Unsuitable	>300	>2,100	>80	>1.25	>2.50	>3.75	>20	>20

[a] The concentration of dissolved salts in water may be measured by either of two methods—that of electrical conductance and that of evaporating the water and weighing the residue. [b] This percentage represents the proportion of sodium to the total cations and is computed from the data of analysis, reported as milligram equivalents, by dividing the sum of the values for sodium and potassium by the sum of the values for all the cations.

In general, the two classifications should give similar results except with primary alkaline waters (a class not represented in the nine waters of the Rio Grande) for which the writer's classification would be substantially more severe. This is explained by a more lenient evaluation of carbonates by Scofield in the light of the experience of later years. Computation of the writer's alkali coefficients was developed under the older viewpoint and presupposes Na_2CO_3 twice as undesirable as Na Cl and ten times as undesirable as Na_2SO_4.

TABLE 6.—COMPARISON OF CLASSIFICATIONS

Water No.	SCOFIELD CLASSIFICATION				"Alkali coefficient"	Class
	Concentration	% of Na	Chlorides	Sulfates		
1	Excellent	Good	Excellent	Excellent	356	Good
2	Excellent	Good	Excellent	Excellent	139	Good
3	Good	Good	Excellent	Excellent	135	Good
4	Good	Permissible	Excellent	Good	29	Good
5	Permissible	Permissible	Excellent	Good	29	Good
6	Permissible	Permissible	Excellent	Good	24	Good
7	Permissible	Permissible	Good	Good	12.5	Fair
8	Permissible	Permissible	Permissible	Permissible	5.6	Poor
9	Doubtful	Permissible	Doubtful	Permissible	3.0	Poor

M. R. LEWIS,[9] M. AM. SOC. C. E.—The importance, not only of the total quantity of dissolved salts, but also of the character of the salts is shown well by Mr. Hill. There is need for more general knowledge and consideration of both factors in irrigation practice.

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[9] Senior Agri. Engr., SCS, U. S. Dept. of Agriculture, and Oregon State College, Corvallis, Ore.

In the Pacific Northwest there appears to be a close correlation between the pH-value of soils and soil solutions with the productivity and ease of reclamation of alkali lands. Herein and in common usage the term "alkali" includes both saline (white alkali) and alkaline (black alkali) soils. For this reason the carbonate and bicarbonate content appears to be of particular importance in agricultural practice. To the extent that this is true, it would be better to retain Mr. Palmer's method[2] of grouping the SO_4 ion with the Cl, rather than with the $H\ CO_3$ and CO_3 ions. Apparently, the alternative grouping is necessary in the development of the author's geochemical chart.

Mr. Hill's conclusion that some salts from the irrigation supply are retained in the valley and that a large part of the salts carried out with the drainage water come from the original ground water, and also his determination of the approximate character of both, seem well founded. His opinion, that the residual salts are held in the soil and not in the ground water and that the general result of irrigation in the area is adverse, seems open to argument. C. S. Scofield[10] has shown that ground water held near the surface of irrigated soils does not move rapidly to the drainage channels. Is it not probable that a large part of the "residual" salts are held in the ground water below irrigated fields rather than in the soil of the root zone? Since these residual salts are of less harmful quality than either the influent salts or the displaced salts, it would appear that the net result of the changes may well be beneficial rather than harmful.

The evidence of base exchange activity in the soil is another matter. Truly more study is needed.

Louis J. Alexander,[11] Assoc. M. Am. Soc. C. E.—Mr. Hill is to be commended for his paper. He has offered a solution to one of the most perplexing of all geochemical problems; this is, the accurate determination of the source or sources of the resultant mixture of water. Heretofore such solutions were dependent upon the method of trial and error.

Another, and perhaps more important, development in the paper is the method of forecasting what may happen to the soil complex because of the use of water of varying character. If this forecast is applied to one of the local (Southern California) sources, solution of a very controversial problem might be found.

The paper uses the words "character," "type," and "quality" synonymously. Misuse of these terms is common. The writer, however, believes that they should not be so used. "Character" is the "relationship of the ions contained in the solution," one to another and to the whole, and "type" is applicable as a subclassification under "character," whereas "quality" is the "relationship of the concentration of dissolved minerals of other impurities."

The writer has been studying the character, type, and quality of well waters pumped from the Los Angeles Coastal Plain for a number of years.[12]

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[10] "The Movement of Water in Irrigated Soils," by C. S. Scofield, *Journal of Agricultural Research*, Vol. 27, 1924, pp. 617–694.

[11] Designing Engr., Southern California Water Co., Los Angeles, Calif.

[12] "Character, Quality and Treatment of Well Water Supplies of the Los Angeles Coastal Plain," by Louis J. Alexander, presented at the meeting of the Sanitary Engineering Division of the Society, July 24, 1941, San Diego, Calif.

In Southern California consideration of domestic waters must include agricultural and industrial applications. The writer proposes four character types: Two types of carbonate water, a sulfate type, and a chloride type. These four "characters" are defined as follows:

1. Carbonate Waters.—The weak acids exceed the strong acids; alkalies exceed strong acids (no non-carbonate hardness): (CO_3 + H CO_3) exceed (Cl + SO_4); (Na + K) exceed (Cl + SO_4).

2. Carbonate Waters.—The weak acids exceed the strong acids; the earths exceed the weak acids (some non-carbonate hardness): (CO_3 + H CO_3) exceed (Cl + SO_4); and (Ca + Mg) exceed (CO_3 + H CO_3).

3. Sulfate Waters.—Strong acids exceed the weak acids and the sulfates exceed chlorides; the earths exceed the weak acids: (Cl + SO_4) exceed (CO_3 + H CO_3); (SO_4) exceeds (Cl); and (Ca + Mg) exceed (CO_3 + H CO_3).

4. Chloride Waters.—Strong acids exceed the weak acids and chloride exceeds sulfate; the earths exceed the weak acids: (Cl + SO_4) exceed (CO_3 + H CO_3); (Cl) exceeds (SO_4); and (Ca + Mg) exceed (CO_3 + H CO_3).

This is a much broader classification than the one proposed by Mr. Hill, but it is believed, for ordinary uses, more satisfactory. This classification is based upon the relative concentrations of the acid groupings rather than upon assumed combinations of various ions and radicals. The reason for this selection is that, in the consideration of the treatment of water, these relationships become important. This is especially true of the waters in sulfate and chloride groups.

A method of study and classification developed by O. A. Stone, M. Am. Soc. C. E., similar to that by Mr. Hill has been used by the writer for some years in the study of localized problems of some complication, but both methods have been found to be somewhat laborious in the study of broad classifications for general work.

Mr. Hill suggests the need for further study. The writer believes that such study should also be enlarged to include waters used for domestic and industrial as well as irrigational uses.

W. D. COLLINS,[13] ESQ. (by letter).—The new and original method of treating the subject of salts in irrigation waters that is so clearly advanced by the author will undoubtedly prove a useful tool in the hands of those who like to deal with physical and chemical phenomena by means of graphs and equations.

Graphic Representation of Analyses.—Persons not in the habit of using triangular coordinates may be discouraged by the apparent complication of the system proposed by Mr. Hill. The system of graphic representation of water analyses used mainly by the U. S. Geological Survey[14] is based on analyses in terms of equivalents per million, as is the author's system. Figs. 4 to 9 will afford a basis for judging Mr. Hill's method. It seems to some observers that the relative concentrations of the different ions in a water and the chemical characteristics of different waters are a little more easily seen in Figs. 6 and 8. The properties Z_1, Z_2, Z_3, and Z_4 of Mr. Hill's system can be read from the

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[13] Chf., Div. of Quality of Water, U. S. Geological Survey, Washington, D. C.

[14] "Graphic Representation of Water Analyses," by W. D. Collins, *Industrial and Engineering Chemistry*, Vol. 15, 1923, p. 394; also "Notes on Practical Water Analysis," by W. D. Collins, *Water Supply Paper No. 596-H*, U. S. Geological Survey, 1928.

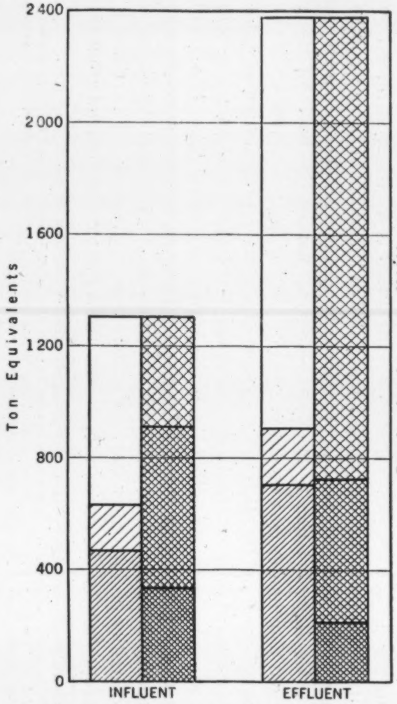

FIG. 4.—INFLUENT AND EFFLUENT WATERS FROM AN IRRIGATED AREA, IN TON EQUIVALENTS

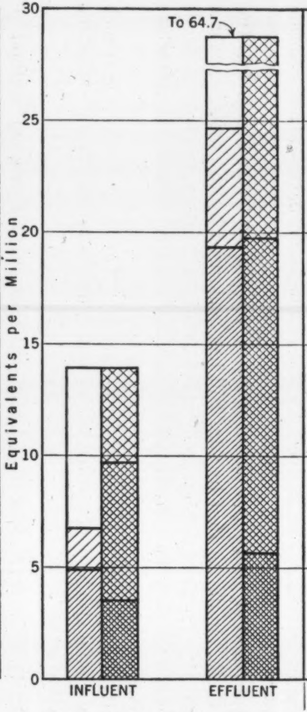

FIG. 5.—INFLUENT AND EFFLUENT WATERS FROM AN IRRIGATED AREA, IN EQUIVALENTS PER MILLION

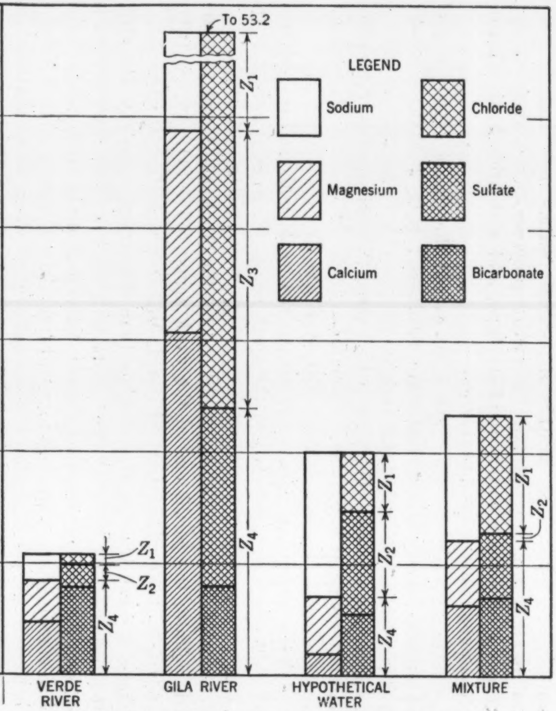

FIG. 6.—RIVER WATERS AND A MIXTURE OF RIVER WATERS, IN EQUIVALENTS PER MILLION

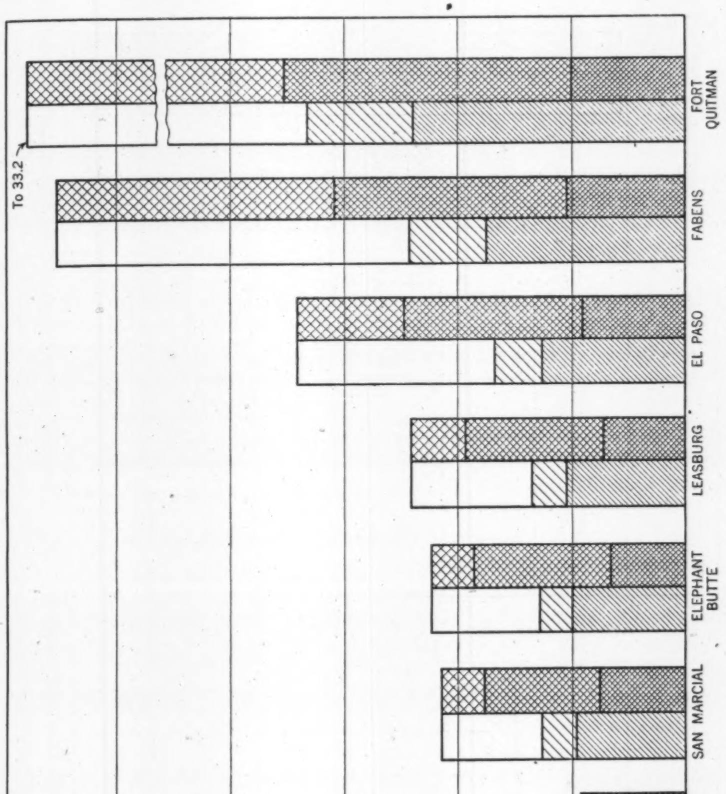

FIG. 7.—WATER FROM THE RIO GRANDE AT DIFFERENT POINTS, IN EQUIVALENTS PER MILLION

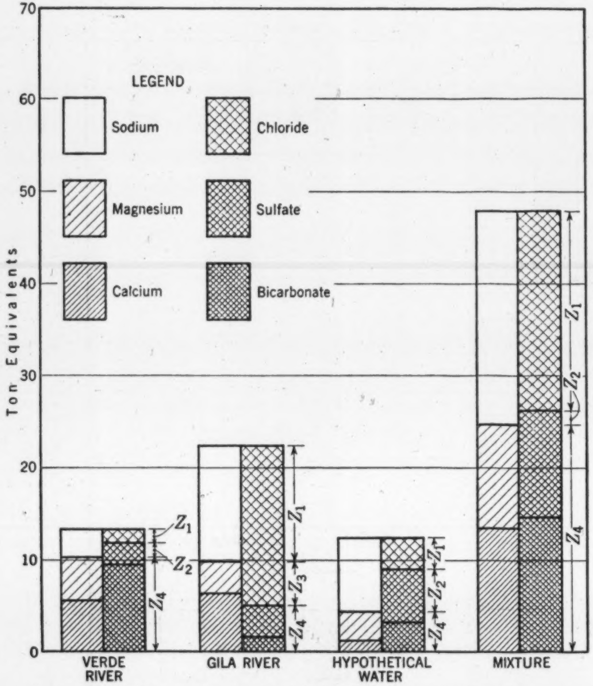

FIG. 8.—RIVER WATERS AND A MIXTURE OF RIVER WATERS, IN TON EQUIVALENTS

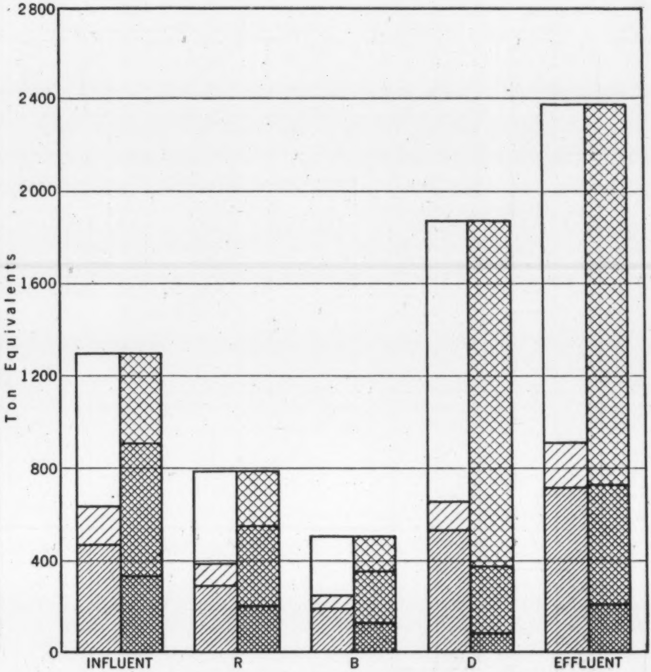

FIG. 9.—SALTS IN IRRIGATION WATERS, IN TON EQUIVALENTS

Survey diagrams. In equivalents per million the Z_1 term is the total quantity of sodium when this is less than the chloride, or the total quantity of chloride if the chloride is less than the sodium. The Z_4 property is the total quantity of bicarbonate and sulfate if this is less than the total of the calcium and magnesium, or the total of the calcium and magnesium if this is the smaller. The remainder of the Survey diagram between the Z_1 and the Z_2 parts is either Z_2 or Z_3 according to whether the chloride is less than, or greater than, the sodium. These measurements give the different properties in equivalents per million, which can be converted promptly to percentages by dividing by the total height of the diagram. The Z_1, Z_2, Z_3, and Z_4 properties are indicated in Figs. 6 and 8 which show the analyses of Fig. 1 and Tables 1 and 2.

It is customary to plot Geological Survey analyses in terms of equivalents per million, with the height indicating the concentration. When it is desired to emphasize the percentage composition this can be done by plotting all of the analyses to the same height and representing the concentration of total salts by the width of the diagram.[15]

Mixtures of Different Waters.—The fact that water *M* is a mixture of the other three (which is shown so clearly by the graphs in Table 2) would not be apparent from the analytical results tabulated in equivalents per million or shown in Geological Survey graphs. Granted that the one water is a mixture of the other three, the average person could probably compute the relative quantities of the component waters more quickly by the use of any three of the equations that may be written from the analytical results in the table.

Changes in Quality.—The analyses shown in Table 3 when plotted according to the Geological Survey system (Fig. 7) seem likely to give to the casual observer a clearer picture of the changes in character of the water along the river. Like Table 3, the Survey plot shows an increasing concentration of soluble salts from Del Norte to Fort Quitman. The water at Del Norte, of course, is excellent for all uses. Its total quantity of dissolved mineral matter is not large and the percentage of sodium is small. Increasing quantities of all the soluble salts appear at Lobatos and Otowi without much change in the chemical character of the waters, although the increase is more in calcium and magnesium and bicarbonate than in the other ingredients. From Otowi to San Marcial the water changes in chemical character. As far as Otowi the bicarbonate is the predominating anion and the calcium and magnesium together make up a very large percentage of the cations. To this point the water of the river is a typical calcium-bicarbonate water. The increase in bicarbonate from Otowi to San Marcial is insignificant. The largest relative increase is in sodium and sulfate although the chloride and calcium increase appreciably. The general character of the water remains practically unchanged from San Marcial through Leasburg. At El Paso, Fabens, and Fort Quitman, the sodium, sulfate, and chloride have increased greatly and the calcium has increased moderately, whereas the bicarbonate remains at about the same level.

The changes noted are almost universally characteristic of waters in irrigated areas. When water such as that at Otowi is evaporated to dryness and the residue is treated with distilled water, the calcium carbonate goes into solution slowly and in small quantities, whereas the sodium, magnesium, and calcium chlorides and sulfates go into solution rapidly. The calcium sulfate is not so soluble as the other salts and appears to have more of a tendency to be left behind in the land. These considerations appear more easily evident to the average observer from the Geological Survey graphs although they are all shown in Table 3.

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[15] "The Significance of Geologic Conditions in Naval Petroleum Reserve No. 3, Wyo.," by W. T. Thom, Jr., and E. M. Spieker, *Professional Paper No. 163*, U. S. Geological Survey, Washington, D. C. (see section on the waters of the Salt Creek-Teapot Dome uplift, by Herman Stabler).

Valley Salt Balance.—In order to consider the valley salt balance shown in Table 4, the analyses of influent and effluent waters have been plotted according to the Geological Survey system in two units (Figs. 4 and 5). They are shown first in ton equivalents, as in Table 4. To obtain a better picture of the actual composition of the waters the two analyses are also plotted in equivalents per million. A glance at the diagrams in ton equivalents (Fig. 4) might suggest that the outgoing water was lower in bicarbonate than the incoming water. The diagrams in equivalents per million (Fig. 5) show that the effluent water is more concentrated than the influent water in every constituent. The fact that the effluent shows fewer tons of bicarbonate and sulfate than the influent is due in part to the fact that the volume of the effluent is only 27,000 acre-ft to 69,000 acre-ft for the influent.

The discussion by Mr. Hill centering around the imaginary water called "*A*" is ingenious and interesting and leads to practical conclusions that seem wholly sound. It would appear that substantially the same conclusions might be reached by direct consideration of tangible quantities without resort to an imaginary water.

From the table of analyses or from the Survey diagrams (Figs. 4 and 5) it can be seen readily that the effluent salts (or water) from the plot under consideration have a different character from the influent salts and water. The difference is not one of simple concentration as by evaporation of water, nor could it be brought about by the normal phenomena common in irrigated regions of precipitation of calcium carbonate or calcium sulfate. Besides the relative increase of sodium as compared to calcium, and of chloride as compared to bicarbonate, the diagrams show clearly, in equivalents per million, a shift of the sodium-magnesium boundary in relation to the chloride-sulfate boundary. In terms of hypothetical combinations the influent water carries sodium sulfate and no chloride of magnesium or calcium and the effluent water carries no sodium sulfate, but does have some magnesium chloride. In Mr. Hill's terms the diagrams of influent water show the Z_2 property and the effluent diagrams

TABLE 7.—ANALYSES OF INFLUENT AND EFFLUENT WATER FOR AN IRRIGATED VALLEY OVER A PERIOD OF ONE YEAR; WITH CERTAIN COMPUTATIONS FROM THE ANALYSES (RESULTS IN TON EQUIVALENTS)

Ion	I	R	D	E' $(=I+D)$	E $(=E'-R)$	G $D-R$ $E-I$
Ca...	465	122	366	831	709	244
Mg...	166	60	89	255	195	29
Na...	664	0	806	1,470	1,470	806
Sum..	1,295	182	1,261	2,556	2,374	1,079
HCO_3	332	122	0	332	210	−122
SO_4..	576	60	0	576	516	− 60
Cl....	387	0	1,261	1,648	1,648	1,261
Sum..	1,295	182	1,261	2,556	2,374	1,079

show Z_3. The diagrams do not show any imaginary water or any loose unattached or unbalanced ions such as can be made to appear from the analytical results in Table 4.

It is recognized that an actual irrigated plot of 15,000 acres does not act like a glass vessel or like a bed of pure silica sand. The action with respect to applied water varies from spot to spot in the area. The analyses I and E in Table 4 give only an over-all picture of the relation of incoming and outgoing water and salts at a given time or over a given period. Assuming that the analyses are truly representative of actual conditions it appears that the influent water brought into the area more bicarbonate and sulfate than the effluent water removed, and that the effluent water took away more chloride than was brought in.

The simplest over-all solution of the problem would be to assume that some of the incoming calcium or magnesium bicarbonate and sulfate remained in the soil and that the outgoing water removed some calcium or magnesium chloride from the area. This can be set up as in Table 7 which shows, in ton equivalents:

I, the constituents of the influent water;
R, the tons of calcium bicarbonate and magnesium sulfate left in the area, out of the amount brought in by the influent water;
D, the tons of different constituents displaced from the store of soluble salts in the area at the beginning of the year;
$E' = I + D$, the sum of the influent salts and the salts displaced from storage in the ground (this might be called the over-all total of salts moved into and in the area);
$E = E' - R$, the difference between the tons of salts moved into and in the area and the tons of influent salts left in the ground; and
$G = D - R = E - I$, the apparent over-all removal of salts from the area.

TABLE 8.—RESIDUAL (R) AND DISPLACED (D) SALTS, AND APPARENT GAIN (G) IN AN IRRIGATED AREA DURING A PERIOD OF ONE YEAR

(Results in Ton Equivalents)

Ion	ASSUMPTION 1[a]			ASSUMPTION 2[b]			ASSUMPTION 3[c]			ASSUMPTION 4[d]		
	R	*D*	*G*	*R*	*D*	*G*	*R*	*D*	*G*	*R*	*D*	*G*
Ca	122	366	244	214	458	244	283	527	244	465	709	244
Mg	60	89	29	101	130	29	101	130	29	166	195	29
Na	0	806	806	230	1,036	806	404	1,210	806	664	1,470	806
Sum	182	1,261	1,079	545	1,624	1,079	788	1,867	1,079	1,295	2,374	1,079
HCO_3	122	0	−122	214	92	−122	202	80	−122	332	210	−122
SO_4	60	0	−60	276	216	−60	350	290	−60	576	516	−60
Cl	0	1,261	1,261	55	1,316	1,261	236	1,497	1,261	387	1,648	1,261
Sum	182	1,261	1,079	545	1,624	1,079	788	1,867	1,079	1,295	2,374	1,079

[a] No residual salts except the minimum bicarbonate and sulfate and no extra displaced salts except the equivalent calcium and magnesium chlorides. [b] The solution by the use of the imaginary water and diagrams of Mr. Hill's paper. [c] The salts in 27,000 acre-ft of influent water pass on through; the salts in 42,000 acre-ft of the influent remain in the area. [d] None of the influent salts leave the area; all the effluent salts come from the stored salts.

It is obvious that any number of solutions can be made by adding equal quantities to corresponding items in Cols. R and D. Quantity E will remain the same. The top limit will occur when *R* is the same as *I*. That will mean that none of the influent salt for the year was carried out of the area and the effluent salt was all from the store of soluble salts in the area.

Table 8 shows values of *R* and *D* for different assumptions. Fig. 9 shows, in ton equivalents, the influent salts in 69,000 acre-ft of water, the residual salts *R* from 42,000 acre-ft of the influent water, the salts carried through the area by 27,000 acre-ft of the influent water *B*, the salts taken up from the store in the area *D*, and the effluent which is the sum of *B* and *D* (see Assumption 3, Table 8).

The consideration of the relations between influent and effluent waters, whether by means of Mr. Hill's charts, by means of computation from the analyses, or by the Survey diagrams, points to the possible danger of accepting a false sense of security with reference to the salinity of lands in an irrigated area merely because the over-all salt balance seems to be favorable. The results show the need for continuous study of the character of the waters in contact with the soil as well as the influent and effluent waters for individual plots if salt damage to the land is to be avoided.

JOHN H. BLISS,[16] ASSOC. M. AM. SOC. C. E.—An interesting and ingenious method of plotting and classifying irrigation waters has been advanced by Mr. Hill. His geochemical chart is an excellent tool for portraying the relative amounts and characteristics of dissolved solids and for comparing differences in water supplies. It is also an aid in solving certain problems such as valley salt balance, although, in this connection, it does not appear to be quite so successful. The difficulty is not in the method but in the inadequacy of the basic data.

Because of the number of dissolved solids and the complexity of the effect of their active constituents upon crops and soils, it is obviously difficult, in a simple graph, to tell the entire story of a water's character and suitability for irrigation purposes. The problem is a complex one and the author is to be congratulated for his effort to simplify it and to standardize the method of reporting the results of water analyses.

By combining the strong sulfate ion with the weak carbonates and bicarbonates, rather than the chlorides and nitrates, Mr. Hill has departed somewhat from the Palmer method[2] of classification. By so doing he has tended to overlook the geologic significance of the Palmer classification in order to portray better the irrigation significance of natural waters.

The writer is somewhat familiar with the Rio Grande, one of the illustrative examples used by the author, and is particularly interested in his discussion of valley salt balance (upper El Paso Valley). Arriving at an indeterminate mathematical solution, Mr. Hill has solved the problem by reasonable deduction and has obtained an approximate answer to the true salt debits and credits in the area considered.

It is obvious that a permanent agriculture depends upon the maintenance of a salt balance in the root zone of cropped areas—that is, that as much of the undesirable dissolved constituents are removed from the root zone as are carried to it in the irrigation supply. The author has demonstrated that a balance between influent and effluent salts in an irrigated valley does not necessarily mean a favorable balance to the growing crops. The writer agrees, but rather questions the inference that, because unfavorable residual salts are being left behind somewhere in the soils of an area, the entire valley is affected adversely. In the illustrative case it would seem that many if not most of the residual salts were left in those areas of relatively impervious soils whose useful life as irrigated lands are automatically limited by their "tightness." In the remaining more porous soils of the valley, any residual accumulation may be removed easily from time to time by flushing the soil with excess quantities of water.

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[16] Engr., State Engr's Office, Santa Fe, N. Mex.

Classifications of water according to their suitability for irrigation purposes have been advanced for certain limited areas. Realizing the pitfalls that surround any attempt at broad classification of waters, the writer feels that there is much work to be done along this line. C. S. Scofield[8] has presented a table of classes of irrigation water, which (he warns) are adopted for a definite irrigation region. The table sets up five classes of water from "excellent" to "unsuitable" and gives permissible limits of concentration of those constituents likely to be injurious to growing crops. For example, Class 5, or "unsuitable" irrigation water, is described in part as carrying more than 2,100 ppm of dissolved solids, more than 80% (of total positive ions) sodium, and more than 20 milligram equivalents each of sulfates and chlorides.

The following examples may be of interest and are included herein because the writer believes that they demonstrate the need for additional information on the effects of irrigation waters upon soils and crops.

The Pecos River in New Mexico and Texas carries one of the highest concentrations of dissolved solids of any western stream used for irrigation. For the three years 1938–1940, the water supply for the Carlsbad (N. Mex.) project, as measured on the Pecos River near the Artesia (N. Mex.) gaging station by the U. S. Geological Survey,[17] averaged 3,120 ppm of total dissolved solids, 37% sodium, and 28.8 and 18.5 milligram equivalents, respectively, of sulfates and chlorides. Water of this type and approximate average concentration has been applied on the project lands for almost fifty years with little apparent adverse effect except in those areas where drainage is difficult.

Texas projects, now combined in a single district under the Red Bluff reservoir, receive a much more unsuitable water supply than the Carlsbad project upstream. The quality of the district's irrigation supply as measured by the U. S. Geological Survey at the Orla (Tex.) gaging station below the reservoir, for the 1938–1940 period, averaged 4,520 ppm of total dissolved solids, 44% sodium, and 33.2% each of sulfates and chlorides. On the Pecos River at Red Bluff (N. Mex.) station above the reservoir, the sodium content for the same period averaged about 52%, a more representative value than that at Orla because the Orla data are affected by the holdover storage of the relatively good waters caught in the reservoir during the flood of June, 1937. Water of about the Red Bluff type and concentration has been used on Texas lands for many years. Considerable areas of heavy soils, particularly those in the flood channel along the river, have been "alkalied" and abandoned. However, large areas where natural or artificial drainage has been effective have been farmed successfully for many years with little apparent adverse effect.

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[17] "Quality of Water," by C. S. Howard, W. F. White, Jr., and W. W. Hastings, U. S. Geological Survey—a section of the report, "Pecos River Joint Investigation," National Resources Planning Board, 1941.

TABLE 9.—CLASSIFICATION OF PECOS RIVER WATERS

Station	Palmer Classification				Hill classification (type)
	Class	Primary salinity	Secondary salinity	Secondary alkalinity	
Artesia...	3	37%	58%	5%	*b*I*a*
Red Bluff	3	52%	45%	3%	*d*VII*d*

The classification of these two waters according to the methods of Messrs. Palmer and Hill is shown in Table 9. It will be noted that the water at Red Bluff falls in Mr. Hill's Type VII, which he aptly calls "irrigation sewage."

Both waters, except as to sodium percentage, fall in Mr. Scofield's Class 5 or "unsuitable" irrigation water (assuming the Scofield classification to be applicable to this area). A favorable predominance of the earth bases over sodium and potassium exists in the Artesia water, but this advantage is lost at Red Bluff where sodium and potassium constitute more than 50% of the positive ions. There is insufficient evidence at this time to show what effect "base exchange" may have made upon the soils in either area.

How long successful irrigation agriculture can endure, particularly in the Red Bluff district, under the continued use of such highly concentrated, inferior quality water remains to be seen. It would appear, however, that even poor quality saline water may be used more or less successfully for irrigation purposes for considerable periods of time if the excess salts are flushed away from time to time and thus prevented from accumulating in the root zone of growing crops. There is a definite need for additional study of this complex problem.

RAYMOND A. HILL,[18] M. AM. SOC. C. E.—It is significant that most of the discussions of this paper contain an endorsement of the writer's conclusion regarding the necessity for much more study of the problem of salinity by irrigation engineers and others concerned with the use of water, even if the variance in views presented in these discussions did not in itself make this apparent. The writer regrets that so much of the discussion was directed toward the tool offered the engineering profession and so little toward its application in solving the problems of the engineer and the agriculturalist.

Geochemical Classification.—It must be recognized that the system of classification suggested by Chase Palmer more than thirty years ago has had very limited application and has remained largely a matter of mystery to engineers.[2] The writer realizes that his modification of that system, although perhaps better adapted for use in the interpretation of analyses of water used in agriculture, will enjoy no better fate. Neither is it believed that the classifications suggested by W. F. Langelier and H. F. Ludwig, Jun. Am. Soc. C. E., in their discussion published elsewhere[19] will come into more general usage.

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[18] Cons. Engr. (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

[19] "Graphical Methods for Indicating the Mineral Character of Natural Waters," by W. F. Langelier and H. F. Ludwig, *Journal*, A.W.W.A., March, 1942, p. 335.

All three systems of classification depend upon the relative concentration of various combinations of ions which are not easy to remember and which have little significance to the engineer. Evidently Mr. Scofield was correct when he stated that there exists among hydrologists a difference of opinion as to whether Palmer's classification should be retained in use even if revised.

It is important, however, to develop some system of designation of waters of different character. Any such system must be based upon the predominance of one or another of the various ions; otherwise, the designation will be without significance. On the other hand, the system must be simple and easily remembered by the engineer as well as the chemist; otherwise the terminology can never be used generally. It has since occurred to the writer that the general purpose of any geochemical classification could best be accomplished by avoiding any grouping of ions and by merely defining the predominant cation and the predominant anion, if any, by the capital letters *A*, *B*, *C*, and *D* for the positive ions, and the lower-case letters *a*, *b*, *c*, and *d* for the negative ions, as follows:

A—Any water in which calcium constitutes more than one half of the cations;

B—Any water in which magnesium constitutes more than one half of the cations;

C—Any water in which the sodium and potassium ions constitute more than one half of the cations;

D—Any water in which *A*, *B*, or *C* does not constitute more than 50% of the total cations;

a—Any water in which the carbonate and bicarbonate ions constitute more than one half of the anions;

b—Any water in which the sulfate ions constitute more than one half of the total anions;

c—Any water in which the chloride and nitrate ions constitute more than one half of the anions; and

d—Any water in which *a*, *b*, or *c* does not constitute more than 50% of the total anions.

There are, of course, sixteen possible two-letter combinations, which should be sufficient for descriptive purposes; for example, any water containing a predominance of calcium carbonate or bicarbonate would be designated as Type *Aa*; water from Salt River in Arizona would be designated as Type *Cc*, indicating that most of the dissolved solids were sodium chloride; and the sample of Verde River water, for which the analysis is given in Table 1, would be of Type *Da*.

Bar Diagrams.—Mr. Collins and Mr. Stabler both indicate their preference for bar diagrams as a means of showing graphically the character and concentration of salts in any water. Mr. Collins gives several illustrations of the use of such diagrams where the concentration is indicated by the length of the bar. Mr. Stabler, on the other hand, shows the concentration by the width of the bar and maintains the same over-all length in each case. The use of a graph having bars of different heights to show the relative concentration of the principal ions and the relative total concentration of different waters is particularly advantageous when the subject matter is to be considered by some lay body. For example, the differences in the quality and character of water available from various sources to communities northeast of Los Angeles, Calif., are obvious from Fig. 10, but no group of laymen could be expected to interpret the corresponding plotted points on a geochemical chart, regardless of its form.

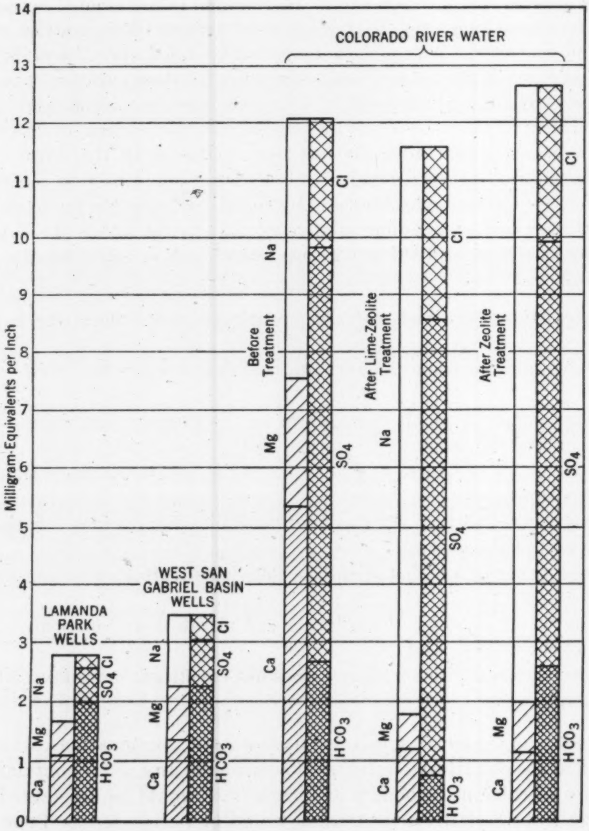

Fig. 10.—Comparative Quality of Water from Typical Recent Analyses

In general, it may be said that the bar diagram is well adapted to the presentation of the results of some study, but it is believed that the point representation possible on a geochemical chart is better adapted to the study itself.

Mixtures of Waters.—The writer accepts the statement of Mr. Collins that, if one water is actually a mixture of three others, the average person probably could compute the relative quantities of the component waters more quickly by the use of any three of the equations that may be written from the analytical results in the table. This was the case with the data presented in Table 2, which represented a true mixture for purposes of illustration. However, a true mixture rarely is found in nature, and even if one water is known to be a mixture of two or more waters from different sources, there will usually be sufficient inaccuracy in the sampling or in the chemical analyses of the samples to prevent a direct and precise analytical solution.

The practical limitations involved in any problem of mixtures are illustrated by the condition that prevails at Granite Reef Dam, the point of diversion into the Arizona Canal and the South Canal, both of which serve the Salt River Project of Arizona. The Verde River enters the Salt River from the north about 3 miles east of Granite Reef, and the supply diverted is composed almost entirely of water released from reservoirs on the two streams. Between the confluence of the streams and the point of diversion there appears to be full opportunity for the two waters to mix thoroughly, yet it is obvious from the data in Table 10 that this is not true.

TABLE 10.—CHEMICAL ANALYSES OF VERDE RIVER AND SALT RIVER WATERS; COMPOSITE SAMPLES FOR NOVEMBER, 1940

(Units: Equivalents per Million, epm)

Points	River	Location	CATIONS			Σ	ANIONS		
			Ca	Mg	Na+K		H CO_3	SO_4	Cl+NO_3
V	Verde River	Below Bartlett Dam	2.41	2.00	1.24	5.65	4.21	0.91	0.53
S	Salt River	Below Stewart Mountain Dam	2.72	1.35	10.27	14.34	2.60	1.22	10.52
M_A	Arizona Canal	At Granite Reef Dam	2.32	1.75	3.29	7.36	3.50	0.94	2.92
M_S	South Canal	At Granite Reef Dam	2.46	1.77	4.45	8.68	3.51	1.04	4.13

When these data are plotted on the geochemical chart (Fig. 11), it is evident that the water diverted into each canal is almost a true mixture of the supply from the Verde River and from the Salt River, but that the proportion of Verde River water taken into the Arizona Canal is greater than the proportion taken into the South Canal. The relative amounts of Verde River water taken into each of the canals can be computed graphically, as indicated in Fig. 11, or by the use of simultaneous equations, assuming a true mixture in each case. No precise solution is possible under the circumstances, but evidently about 77% of the water diverted at the north end of Granite Reef Dam originated in the Verde River, as compared with about 65% of the water diverted from the other side of the river.

The divergence between the plotted positions of the samples from the Arizona Canal and from the South Canal, and the divergence of both from the position of a true mixture, are illustrative of the fact that the character of water in a river cannot be judged from samples taken at one point or at one time.

Alternate Form of Geochemical Chart.—The data given in Table 10 are also shown in Fig. 12, which is in the form of the geochemical chart required to conform with Palmer's classification. The same methods of graphical analysis, which are applicable to the geochemical chart shown in Fig. 1, can be used on this alternate form, and the choice between them depends largely upon the individual and the problem at hand.

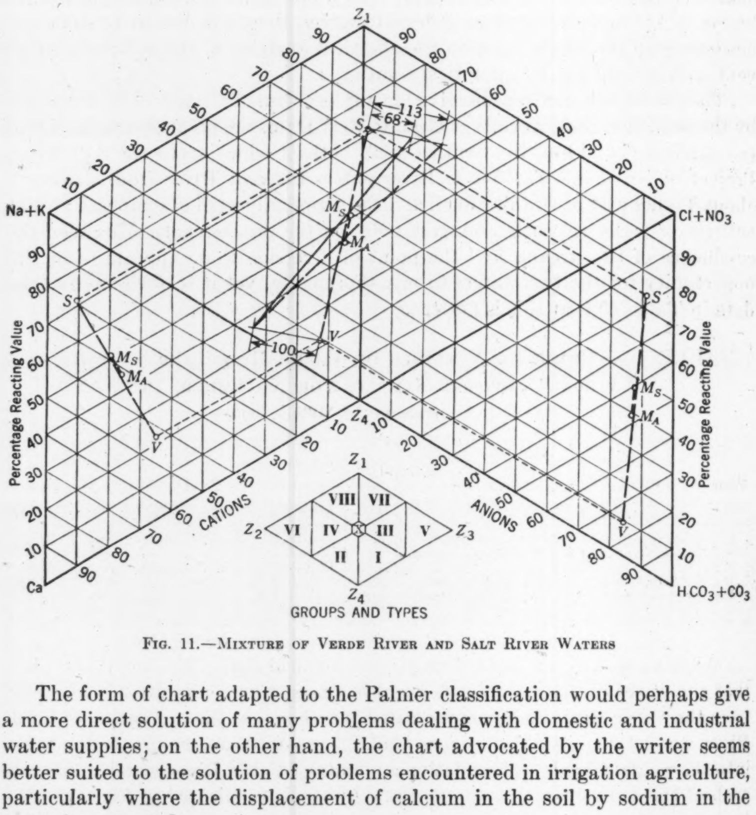

FIG. 11.—MIXTURE OF VERDE RIVER AND SALT RIVER WATERS

The form of chart adapted to the Palmer classification would perhaps give a more direct solution of many problems dealing with domestic and industrial water supplies; on the other hand, the chart advocated by the writer seems better suited to the solution of problems encountered in irrigation agriculture, particularly where the displacement of calcium in the soil by sodium in the water is suspected.

Valley Salt Balance.—Mr Bliss properly pointed out that the solution of the valley salt balance problem gives an average result, and that, although worse conditions will occur locally, the situation may be satisfactory generally. However, if the salt balance is adverse for the area as a whole, the danger of salt accumulation is greater than if the opposite is true.

Mr. Collins referred to the imaginary water called "*A*" in the writer's opening discussion of this matter. The value of so considering the difference between the quantities of salt in the influent water and the quantities of salt in the effluent water is largely a matter of convenience, but where the point *A* falls outside of the figure, it is immediately apparent that a simple difference between the influent quantities and the effluent quantities cannot be used. Although it is true that substantially the same conclusion (as stated by Mr. Collins) might be reached by direct consideration of tangible quantities, it is believed that a more rational solution of the complex problem can be had by use of a geochemical diagram upon which can be shown all of the analyses required for the solution.

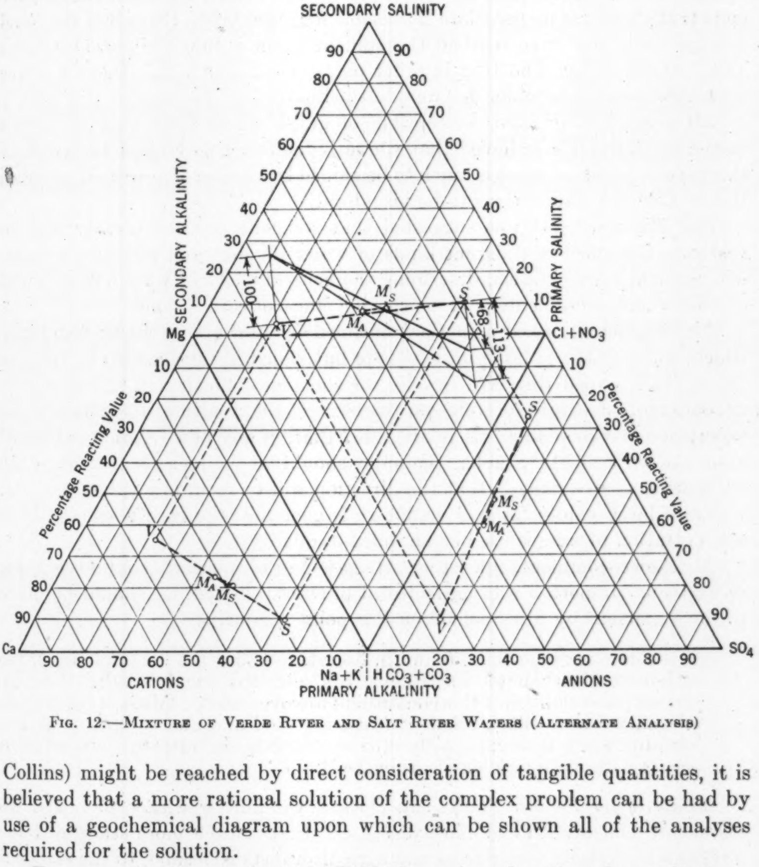
SECONDARY SALINITY
90
80
70
60
50
40
30
20
10
SECONDARY ALKALINITY
PRIMARY SALINITY
Mg
Cl+NO3
Percentage Reacting Value
Percentage Reacting Value
100
68
113
M_S
M_A
V
S
Ca
SO4
90 80 70 60 50 40 30 20 10
10 20 30 40 50 60 70 80 90
Na+K
HCO3+CO3
CATIONS
PRIMARY ALKALINITY
ANIONS

FIG. 12.—MIXTURE OF VERDE RIVER AND SALT RIVER WATERS (ALTERNATE ANALYSIS)

For example, Assumption 1 in Table 8 requires that there be no displacement from the soil or from the ground water of any chloride ion, and at the same time that the salts left behind should include no chloride. This is contrary to the statement of Mr. Collins, to wit: "It is recognized that an actual irrigated plot of 15,000 acres does not act like a glass vessel or like a bed of pure silica sand." Assumption 3 in Table 8 likewise depends upon an improbable premise, which could be true only if 27,000 acre-ft of influent water passed through the canal system without being diverted, and all of the remainder of the water was consumed, leaving the salts in the soil. Assumption

4 in Table 8 is merely an extension of Assumption 3 and clearly could not be the case in fact.

Messrs. Langelier and Ludwig state[19] their agreement with Mr. Collins but, in doing so, fail to realize that in the solution of the problem given in Table 4 the writer attempted to weight the various factors involved, including the facts that there was no percolate from some irrigated fields, that all of the canal seepage could not have reached the effluent drain within the period of time under consideration, and that in other parts of the area where there was free drainage probably no chlorides were left behind.

Although any problem susceptible of graphical analysis may be solved mathematically, it is believed that where judgment is so large a factor, as in this type of problem, there is much to be gained by the use of graphical methods that enable the assumptions to be visualized.

Salt Tolerance.—It was suggested that someone proceed further and investigate the effect of different kinds of water on soils and on plant growth. Mr. Scofield briefly did so, describing the general effect of waters of different character and, to some extent, the specific effect of certain ions.

Mr. Stabler reviewed a method outlined by him about thirty years ago[6] which, as he states, is far from adequate but which has proved useful in some cases. It is hoped that Mr. Stabler, or someone following in his steps, will develop more accurate formulas on the basis of the wealth of information and experience now available. It is suggested that, in any new formula patterned upon the work of Mr. Stabler, the index should be the reciprocal of his alkali coefficient, as modified, so that the direction will be from low values for good water to large values for bad water, as is generally the case when the total concentration of salts is used as an index of quality.

Mr. Lewis comments upon the carbonate and bicarbonate content as being of particular importance in agricultural practice. Reference should be made to the statement by Mr. Scofield on this point—namely,

> "There is much to be found in literature about the harmfulness of the carbonate-bicarbonate constituent, the so-called 'black alkali.' The injurious potentialities of this constituent are overrated. Most of the trouble allegedly due to black alkali is caused by sodium, and sodium is no less harmful when it occurs with sulfate, chloride, or nitrate than when it occurs with carbonate or bicarbonate."

Necessity for Further Study.—In closing, the author wishes to repeat his statement, concurred in by most discussers, that "Much more study of these problems by irrigation engineers and agriculturalists is needed, to the end that successful irrigation agriculture may be permanent rather than temporary."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2166

A DIRECT METHOD OF FLOOD ROUTING

By C. O. Wisler,[1] M. Am. Soc. C. E., and E. F. Brater,[2]
Jun. Am. Soc. C. E.

With Discussion by Messrs. Ray K. Linsley, Jr., Harold C. Hickman, Robert B. Horonjeff and Herbert G. Crowle, L. K. Sherman, Alfred L. Brosio, W. B. Langbein, Fred E. Tatum, H. C. Woster, and C. O. Wisler and E. F. Brater.

Synopsis

A method of flood routing is described herein, the successful use of which depends only upon the availability of dependable stream-flow records during a typical flood at various points on the main stream or on the tributaries whose flow is to be routed downstream. No cross sections of stream channel or velocities of flow are required. Nor are discharge records on all of the tributaries needed. A hydrograph of inflow from the unmeasured area is directly computed. This flow and that at each of the upstream stations is then routed downstream.

These routed flows show the extent to which each of the upper tributaries contribute to the flood peak at each downstream point. A check on the accuracy of the results is provided by adding the routed flows and comparing the resulting hydrograph with the actual records.

The entire procedure is based upon the storage equation and upon the principle that, for all high stages, there is a straight-line relationship between the volume of storage contained in any reach of river channel and the sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach. Except perhaps for unusual channel conditions, this relationship holds true.

Introduction

In large drainage basins, floods usually wreak their greatest havoc at some downstream point. At any instant the total flow at this point comprises waters contributed by many different tributaries. The percentage of the total flow occurring at any instant that is contributed by any single tributary is constantly changing throughout the flood period. Perhaps the maximum flow from no tributary reaches this point at the same time that the total combined flow

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Note.—Published in June, 1941, *Proceedings*.

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becomes the maximum. The flood waters from some of the tributaries may have passed this point long before the peak stage occurs, whereas the waters from some of the more remote tributaries do not arrive until after that stage has passed.

In all studies in which it is proposed to secure improved conditions by reducing the flood peak through the use of reservoirs, by reforestation, or by means other than local protective measures, such as levees and flood walls, it is essential to know to what extent each of the several tributaries contributes to the resulting flood peak. Without this information no intelligent planning of flood control is possible. With a hope of reducing the downstream flood peak, vast expenditures might be made on a certain tributary, the net result of which would actually be negligible or even damaging instead of beneficial.

The writers' interest in this problem was aroused by the Potomac River flood-control studies of the U. S. Department of Agriculture. These studies are being made as an inter-bureau endeavor by the Forest Service, Soil Conservation Service, and Bureau of Agricultural Economics under the chairmanship of the Appalachian Forest Experiment Station at Asheville, N. C. In connection with that work this method of flood routing was developed and successfully applied to the Shenandoah River, a tributary of the Potomac, and later to the Monongahela River in West Virginia.

It is believed that the clearest presentation of this method of flood routing can be made by means of an illustrative example. Because of the excellent discharge records that are available and because of a complication resulting from a subsequent rain that fell during the recession period of the hydrograph, the March, 1936, flood on the Monongahela has been selected for a detailed presentation.

The Monongahela River rises in the north central part of West Virginia, and flows in a general northerly direction through Morgantown, W. Va., which is a few miles south of the Pennsylvania boundary (see Fig. 1). The drainage area above Morgantown is 2,670 sq miles. The principal tributary is the Tygart River which empties into the Monongahela approximately thirty miles upstream from Morgantown. The drainage area above Fetterman, W. Va. (later a part of Grafton, W. Va.), which is about twenty miles above the mouth of the Tygart, is 1,340 sq miles. Discharge records taken at 2-hr intervals at Morgantown and Fetterman are published in U. S. Geological Survey *Water Supply Paper No. 800.*

The first operation described in the paper is the derivation of the hydrograph of inflow into the intervening area between Fetterman and Morgantown. This inflow and the measured discharge at Fetterman are then routed separately to Morgantown. The procedure for determining the effect at Morgantown of flood-control measures on the Tygart is discussed, and a method of determining corresponding effects at other points on the stream is suggested. Finally, the results of two more applications of the method are briefly presented.

DETERMINATION OF UNMEASURED INFLOW

The first step in the solution of this problem lies in the determination of the hydrograph of channel inflow from the intervening area. The intervening

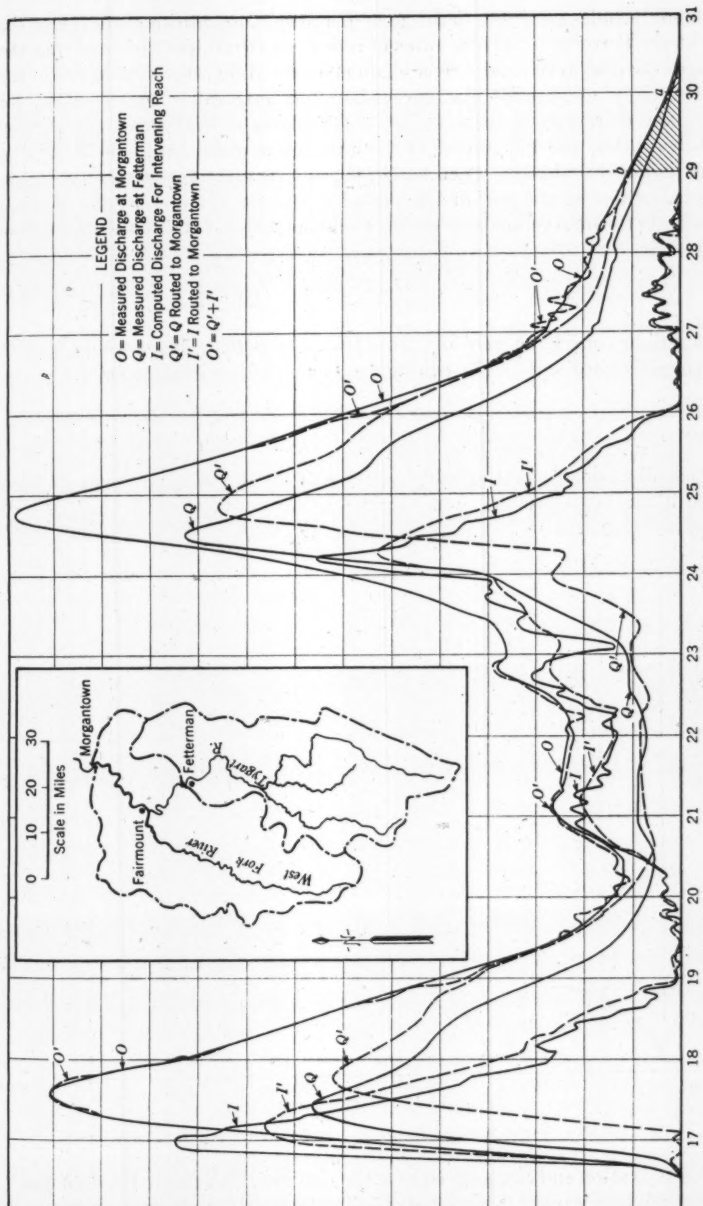

FIG. 1.—HYDROGRAPHS OF SURFACE RUNOFF, WATERSHED OF THE MONONGAHELA RIVER

area here includes all of the drainage area above Morgantown exclusive of the area above Fetterman. These rates of inflow are determined by applying the storage equation to the entire river channel above Morgantown exclusive of the channel above Fetterman for successive 2-hr intervals. In so doing, let Q_0, O_0, and S_0 represent, respectively, the discharge at Fetterman, the outflow at Morgantown, and the storage in the intervening reach, all at the beginning of the period considered. Then let Q_1, O_1, and S_1 represent, respectively, the same quantities at the end of the period. Also let I represent the average inflow from the intervening area during the same period. The resulting storage equation is

$$S_0 + 2\left(\frac{Q_0 + Q_1}{2}\right) + 2\,I - 2\left(\frac{O_0 + O_1}{2}\right) = S_1 \ldots\ldots\ldots\ldots (1)$$

If the periods considered were any other than 2-hr periods, Eq. 1 would have to be changed accordingly. The equation may be written in the form

$$I = \frac{S_1 + O_0 + O_1 - (S_0 + Q_0 + Q_1)}{2} \ldots\ldots\ldots\ldots\ldots (2)$$

in which all quantities on the right-hand side of the equation are known except S_1 and it can be determined from a curve of relation between S and $(Q + O)$ shown in Fig. 2.

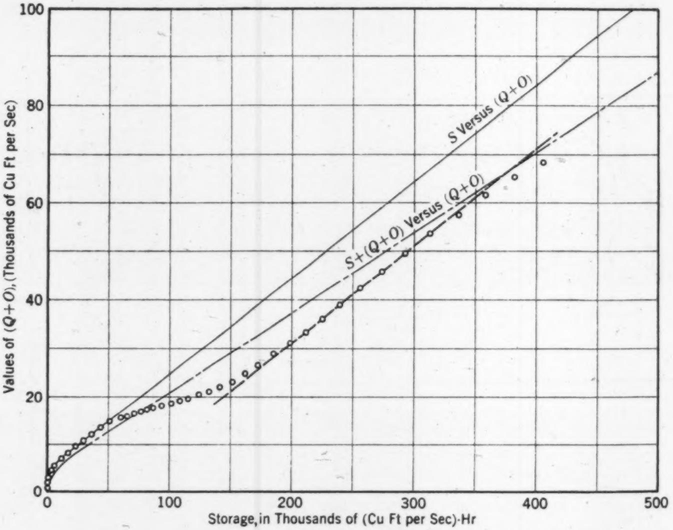

Fig. 2.—Relation Between $(Q + O)$ and the Storage in the Intervening Reach

The derivation and characteristics of the curve of relationship between S and $(Q + O)$ will be discussed in some detail. The storage in the intervening reach is obtained by making use of a principle that was apparently first utilized by R. E.

Horton,[3] M. Am. Soc. C. E.—namely, that at any time after the point of contraflexure occurs on the recession side of a hydrograph of surface runoff, the total volume of channel storage is equal to the total area beneath the hydrograph subsequent to that time. For instance, in Fig. 1, the total volume of channel storage above Morgantown at noon on the 30th is equal to the area of the shaded part beneath the hydrograph marked *O* and to the right of *a*. Also the total channel storage at noon on the 29th is represented by the shaded area to the right of *b*, etc. In a similar manner, the total channel storage occurring at any instant in the Tygart River above Fetterman can be determined from the hydrograph *Q*, Fig. 1. By deducting the channel storage above Fetterman occurring at any instant from the simultaneous storage above Morgantown, the volume of channel storage contained in the intervening reach is obtained.

For any given reach of river channel, there is a definite relationship between the volume of channel storage contained therein at any instant and the sum of the simultaneous discharge rates at the two ends of the reach. Such a relationship exists because *Q* is proportional to the river stage at the upper end of the reach, *O* is proportional to the stage at the lower end, and therefore $(Q + O)$ is proportional to the average stage which in turn depends upon the volume stored. When these two quantities are plotted on ordinary cross-section paper, the graph is a straight line, except for the lower stages. This result is most fortunate because if this relationship did not plot as a straight line, it would be extremely difficult to extend the curve into the higher stages in such a manner as to eliminate the effects of overland flow. It could probably be done only by trial and error and would be a long and laborious process.

In the foregoing, the fact was stated that Mr. Horton's method of determining the volume of channel storage from the area beneath the hydrograph is applicable only back to the point of contraflexure. It is believed that overland flow ceases at or near this point. This concept is substantiated by the experience of the writers that, in every application made to date (1941), this straight-line relationship holds true only for values derived to the right of the point of contraflexure. If the derivation is extended much beyond that point, the storage values all fall to the right of the straight line, indicating that some overland flow is being included as channel storage. As an illustration, it may be noted that in Fig. 2 the points fall to the right of the straight line for values of $(Q + O)$ greater than 58,000 cu ft per sec. It may be seen from Fig. 1 that $(Q + O)$ is equal to 58,000 at noon on March 26, which is approximately at the point of contraflexure of the curves of *Q* and *O*.

To obtain the hydrographs of surface runoff at Morgantown and at Fetterman (see graphs *O* and *Q*, Fig. 1) it was necessary to deduct the ground-water flows from the total discharges. This was done by drawing, on the hydrographs of total flow, straight lines connecting the points at which surface runoff appeared to start and end. Although this procedure is perhaps not accurate, it is sufficiently correct for this purpose.

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[3] "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935, p. 35.

In Fig. 2 the points indicated by small circles were obtained by plotting the value of $(Q + O)$ at any instant as shown in Fig. 1 against the area between the curves Q and O subsequent to that instant. For example, at 2:00 p.m. on the 29th, $O = 4{,}800$, $Q = 3{,}540$, and $S = 18{,}840$ (cu ft per sec)-hr, the latter value representing the area between the graphs O and Q (Fig. 1) after 2:00 p.m. on the 29th. At noon on the 29th, $O = 5{,}200$, $Q = 3{,}720$, and therefore

$$S = 18{,}840 + 2\left(\frac{5{,}200 - 3{,}720 + 4{,}800 - 3{,}540}{2}\right) = 21{,}580 \text{ (cu ft per sec)-hr.}$$

In this same manner the other points indicated by circles in Fig. 2 were obtained.

It should be observed that for values of $(Q + O)$ between 8,000 and 15,000, and also between 30,000 and 55,000, these points fall on two straight lines and furthermore that these lines are parallel. Now the value of $(Q + O)$ was 30,000 at 4:00 a.m. on the 27th and 15,000 at 6:00 p.m. on the 28th. During the intervening period channel inflow was taking place as a result of a subsequent rain that fell during the recession period. This channel inflow from the intervening area amounts to about 65,000 (cu ft per sec)-hr and is represented by the area under the hydrograph I during this period. Its net effect is to move the S versus $(Q + O)$-curve horizontally to the right a distance representing 65,000 (cu ft per sec)-hr. Had it not been for this subsequent channel inflow, the curve of relation between S and $(Q + O)$ would have extended beyond the value of $(Q + O) = 15{,}000$ cu ft per sec as shown by the solid line in Fig. 2. This solid line, therefore, is taken as the curve of relation between S and $(Q + O)$. Since both Q and O are known, it is possible to determine the values of S corresponding to any value of $(Q + O)$ and thus solve equation 2 for I. The graph of I (Fig. 1) was obtained in this manner.

A check on the accuracy of these computations can be made by deducting the total discharge at Fetterman from the total outflow at Morgantown and comparing this quantity with the total computed inflow from the intervening area. In this case the total outflow at Morgantown is equal to 3,894,200 (cu ft per sec)-hr, and the total discharge at Fetterman is 2,326,900 (cu ft per sec)-hr. The difference between these two quantities is 1,567,300 (cu ft per sec)-hr as compared with a total computed inflow of 1,566,900 (cu ft per sec)-hr.

Such a remarkably close agreement should not be expected in all cases. Nevertheless, the maximum error experienced by the writers to date has amounted to about 6%. In such cases all computed inflows from the intervening area may be corrected by a constant percentage in order to make them equal to the difference between Q and O. If the error should be considered to be excessive, however, it might be better to reconstruct the S versus $(Q + O)$-curve by trial until the error virtually disappears. This latter procedure would have to be followed if the stations were separated by such a great distance that this curve would perhaps deviate from a straight line.

ROUTING *Q* AND *I* THROUGH TO MORGANTOWN

The next step in the solution of this problem is the determination of the hydrographs at Morgantown resulting from (1) the foregoing discharge rates at Fetterman, and (2) the inflows from the intervening area. In other words, when do the peak flows from these two sources reach Morgantown? What are their magnitudes? How are the total flows distributed at that point? In answering these questions, the intervening channel is considered to be a storage reservoir through which the flow from each source is routed separately and independently of the other. A check upon the accuracy of the results is then obtained by combining these two graphs of routed flows at Morgantown and comparing the resulting hydrograph with the actual recorded hydrograph at that point.

First, route the Fetterman discharge. For this purpose, the inflow from the intervening area, I, will be ignored and the storage formula (Eq. 1) may be written as follows:

$$S_0 + Q_0 + Q_1 - O_0 = S_1 + O_1 \qquad (3)$$

All of the quantities on the left-hand side are known, thus giving the sum of S_1 and O_1 but not their individual values. These may be obtained by adding Q_1 to each side of the equation, thus giving Eq. 4,

$$S_0 + Q_0 + 2\,Q_1 - O_0 = S_1 + (Q_1 + O_1) \qquad (4)$$

which can be solved by means of a curve of relation between $(Q + O)$ and $S + (Q + O)$, as shown in Fig. 2. Since Q is known, O can be obtained, and finally S. These values of S_1, Q_1, and O_1 become the values of S_0, Q_0, and O_0 for the next period and the computations become continuous.

It should be observed that O here takes on a significance quite different from that which it possessed previously. Heretofore it represented the total recorded outflow at Morgantown, whereas it now represents the computed outflow at Morgantown resulting from the flow Q at Fetterman. In Fig. 1 this quantity is designated as Q' and the same symbol would be used here were it not for the confusion that would result from the use of Fig. 2.

In routing I through to Morgantown, since Q is now ignored completely, Eq. 1 may be written

$$S_0 + 2\,I - O_0 = S_1 + O_1 \qquad (5)$$

In this equation O again takes on a new significance, now representing the computed outflow at Morgantown resulting from the inflow from the intervening area. In the solution of Eq. 5 all the quantities on the left-hand side are known, and therefore $S_1 + O_1$ is known. Referring now to Fig. 2, since Q is ignored, the dash-dot curve becomes one of O versus $(S + O)$, and from it O can be determined, and finally S. These values of O_1 and S_1 then become the values of O_0 and S_0 for the next period and the procedure is repeated. These outflows at Morgantown resulting from the inflows from the intervening area are shown graphically in Fig. 1 by the dotted line marked I'.

By combining the graphs I' and Q', Fig. 1, the outflow graph, O', at Morgantown is obtained. The agreement of this graph with the actual hydrograph O is quite remarkable, especially throughout the higher stages during which the greatest need exists for accurate and dependable results.

Reduction of Flood Peak at Morgantown

After the flood of March, 1936, a dam and storage reservoir were constructed on the Tygart River near Fetterman. The effectiveness with which the storage thus provided may be expected to reduce the flood peak at Morgantown or at any other downstream point depends upon the rules that are formulated governing the operation of that storage. The reservoir outflows resulting from any such proposed set of rules can be computed and this revised Q can then be routed through and the reduction benefits determined at points downstream.

On the other hand, if it is proposed to reduce the flood flows at the upper station by terracing, by change in land use, or by some other means, then a revised hydrograph of Q at that point must necessarily be based on judgment.

After the revised Fetterman hydrograph has been routed through to Morgantown, this revised graph of Q' can be combined with I' to give a revised graph of O'. Making use of the stage-discharge relationship curve this revised O' hydrograph can be converted into a stage graph which, upon comparison with the original records, will show the amount of reduction in stage at Morgantown resulting from the flood control improvements made above Fetterman. A similar procedure can be followed for determining the amount of stage reduction that can be effected at Morgantown from any contemplated improvements in the intervening area.

Reduction of Peak at Other Points

One is not always so fortunate as to have discharge records available at all damage centers where flood reduction benefits are to be appraised. So frequently are such records missing that it seems desirable to summarize, briefly, a procedure that may be followed in such cases. Suppose, for instance, that the damage center is at Fairmont, West Virginia, which is just below the junction of the Tygart and the Monongahela. The reduction at this point may be found as follows:

(1) Determine the inflow from the intervening area between Fetterman and Morgantown and route this flow and the Q at Fetterman through to Morgantown in the manner already described.

(2) Measure the intervening area above Fairmont and find the percentage that this area is of the total intervening area above Morgantown. Then apply this percentage to the various inflows already computed (shown graphically as I, Fig. 1) to obtain a hydrograph of I for the intervening area above Fairmont.

(3) Construct a stage-discharge relationship curve for Fairmont based upon any discharge measurements that may be obtainable during normal and low stages and upon high water-marks for floods whose discharges are known at Fetterman and at Morgantown. If the approximate stage at Fairmont corresponding to zero discharge can be determined, a satisfactory stage-discharge curve should be obtained by plotting the foregoing data on logarithmic paper.

(4) A curve must now be derived showing the relation between channel storage in the intervening reach above Fairmont and the sum of the discharges at Fetterman and at Fairmont. For any value of $(Q + O)$ the total channel

storage above Fairmont can be closely approximated by prorating the total storage above Morgantown in the ratio of the drainage area above Fairmont to that above Morgantown. For the ordinates of this curve Q remains the same as before, but O must also be reduced in the ratio of these two drainage areas. An auxiliary curve showing the relation between $(Q + O)$ and $S + (Q + O)$ should now be plotted. With the aid of this curve and the storage equation, the outflows at Fairmont can be found in a manner previously described.

From this point, the procedure for the determination of the effect of a storage reservoir or other proposed improvement, either above Fetterman or in the intervening area, is the same as has already been described.

Applications to Shenandoah River

In order to test this method of flood routing for different types of stream pattern and shapes of drainage basin, two other routings were made, both on the Shenandoah River in Virginia. A comparison of the watersheds in Figs. 1 and 3, together with Table 1, shows these differences of conditions quite clearly.

For these Shenandoah tests a flood was used that occurred in March, 1936, at the same time as the preceding one used on the Monongahela. The discharge records are also obtained from *Water Supply Paper No. 800.*

In determing the inflow occurring at any instant from the intervening area above Lynnwood the sum of the channel storages existing simultaneously above the stations on North River at Burketown, Middle River near Grottoes, and South River at Harriston was deducted from the total channel storage above Lynnwood. This storage was plotted against the sum of the outflow at Lynnwood and the discharges at the upper three stations. These three discharages were then routed simultaneously through to Lynnwood although it is believed that the same result would have been obtained had each been routed separately and then combined later. The Lynnwood discharge, and also that from the intervening area below Lynnwood, were then routed to Front Royal.

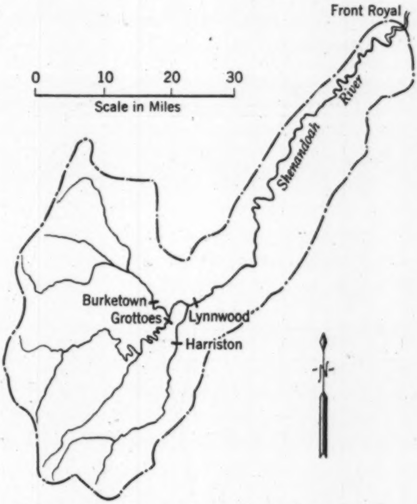

Fig. 3.—Watershed of the Shenandoah River, Above Front Royal, Va.

The results of these two routings are shown in Fig. 4. At Lynnwood the sum of the routed flows agrees almost perfectly with the actual records. At Front Royal, however, the agreement is not as good, although it can hardly be classed as poor.

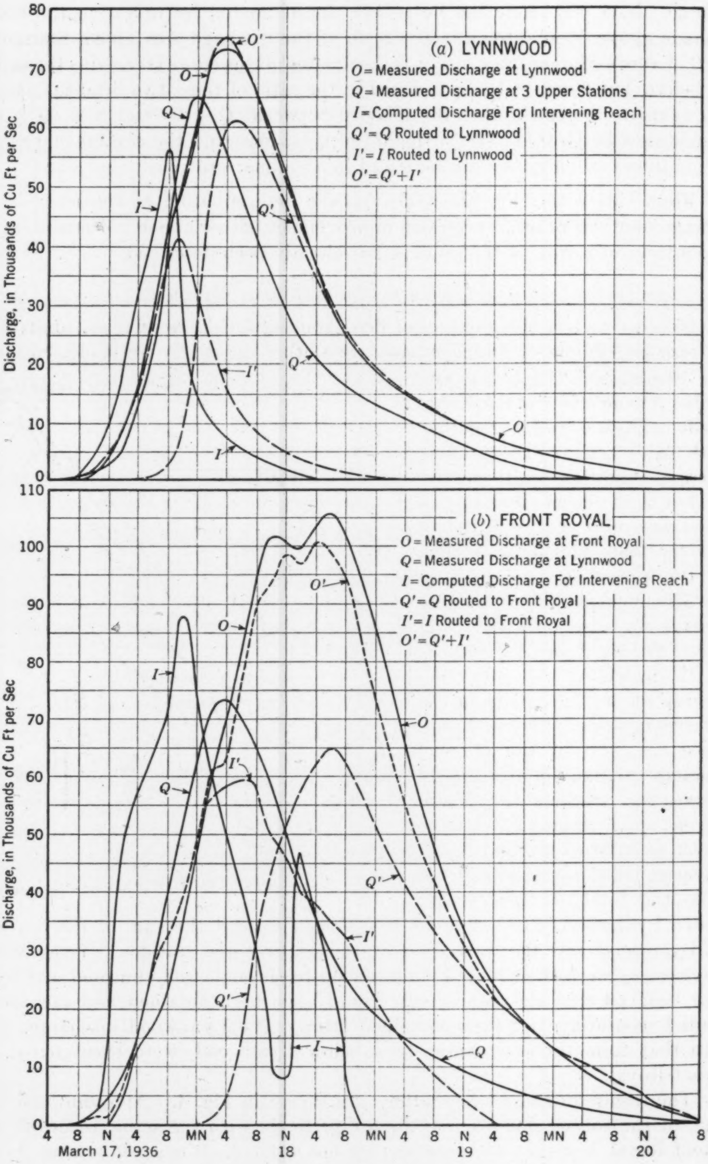

FIG. 4.—HYDROGRAPHS OF SURFACE RUNOFF

TABLE 1.—WATERSHED CHARACTERISTICS

Lower station	Area, in sq miles	Upper station	Area, in sq miles	INTERVENING:	
				Distance, in miles	Area, in sq miles
Morgantown, W. Va.	2,670	Fetterman, W. Va.	1,340	50	1,330[a]
Lynnwood, Va.	1,076	Burketown, Va. Grottoes, Va. Harriston, Va.	381 360 222	15 10 10	113[b]
Front Royal, Va.	1,638	Lynnwood, Va.	1,076	70	562[c]

[a] Long and nearly parallel with upper basin. [b] Wide, short, and entirely downstream from upper basins. [c] Long, narrow, and entirely downstream from upper basin.

The probable reason for the failure to obtain better results at Front Royal may be found in the following statement:[4]

> "Stage-discharge relation.—(At Front Royal) Defined by current-meter measurements below 25,000 second-feet; extended to peak stage by velocity-area study near control section, slope-area determination, and comparison of peak discharge and total run-off of flood with other records on the same or neighboring streams. Discharge may have been affected by backwater from North Fork of Shenandoah River from about 8 p.m. Mar. 17 to about 2 p.m. Mar. 18. Discharge at 8 a.m. Mar. 18 may have been about 75,000 second-feet instead of 99,900 second-feet. Volume of run-off during the flood rise would be reduced by a small percentage."

Because of the backwater to which reference is made in the foregoing notes, and also because of the fact that it was necessary to extend the discharge curve from 25,000 cu ft per sec to 113,000 cu ft per sec, it is easily conceivable that the entire disagreement between the computed graph O' and the hydrograph O, Fig. 4(*b*), may be due to faulty discharge records at Front Royal.

CONCLUSION

It is believed that, except possibly under unusual channel conditions, the curve of relationship between channel storage in the intervening reach and the sum of the discharges at the two ends of the reach is a straight line for the normal and high stages of the river. This straight line together with a supplementary curve expressing the relationship between $(Q + O)$ and $S + (Q + O)$ permits the direct determination of: (*a*) A hydrograph of channel inflow from the intervening area; and (*b*) the routed flows from the upper area and from the intervening area as they appear at the lower station. A positive check on the accuracy of both of these determinations can be made quickly and easily.

Furthermore, it is the belief of the writers that the accuracy of the results that can be obtained by this method of flood routing depends almost entirely upon the availability of: (*a*) Good stage-discharge relationship curves at various points on the stream; and (*b*) an actual flood record. With these data at hand, either the known flood or any hypothetical flood can then be routed downstream, and the benefits that may be expected from any proposed system of storage reservoirs or other plan of flood control can be determined definitely, and the benefits can be weighed against the cost.

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[4] *Water Supply Paper No. 800*, U. S. Geological Survey, p. 114.

DISCUSSION

Ray K. Linsley, Jr.,[5] **Jun. Am. Soc. C. E.**—In all engineering work the procedure adopted for any particular work is determined by the desired accuracy, available working data, purpose of the work, and other considerations. The routing method presented by the authors requires only a small amount of data and therefore may be extremely useful if it meets the other requirements of a particular problem. Some further consideration of the proposed method seems necessary before it can be evaluated properly.

The familiar storage equation, which serves as the basis for nearly all flood routing methods, is an exact formula. There are no simplifying assumptions in the statement "inflow minus outflow equals change in storage." It is not surprising, therefore, that the authors report close agreement between the computed volume of local inflow from the intervening area and the difference in volume of flow at the upstream and downstream stations. Working as they have from zero storage at the beginning of a flood to zero storage at the end of a flood, it would be impossible to have any error in volume other than an arithmetic one.

It is likewise to be expected that the computed hydrograph at the downstream station, obtained by combining the computed local inflow and the observed upstream flow, should check quite closely with the observed hydrograph at the lower station. The volume of flow, of course, should check exactly. Any difference in shape between the computed and observed hydrographs results from the procedure followed. In computing the local inflow, the reach storage was determined by using the sum of the observed inflow and outflow as an index of storage. In the recombination of the hydrographs, the inflow at the upper end and the local inflow are routed separately, using storage values dependent on the particular flow quantities. The process followed is that of dividing the total inflow into a reach into two parts and then adding these parts to get the whole. The error shown in Fig. 4 cannot be explained by a possible error in the Front Royal rating. The changes a flood wave undergoes are dependent on the total storage occupied within a reach. It does not seem rigorous, therefore, to divide the inflow arbitrarily into two parts and route these parts through separate and lesser storage volumes. Because the storage curves used are straight lines, this procedure can be used without introducing serious error.

A mathematically correct solution of the storage equation guarantees a true volumetric answer. The major problem in routing, therefore, is that of obtaining the correct distribution of the stream flow throughout the hydrograph. This requires accurate values of the storage changes during each routing period. True storage volumes are not necessary as long as the changes in storage can be determined correctly. Reviewing the proposed method in the light of known facts about stream flow, it is apparent that certain simplifying assumptions have been made which must of necessity affect the accuracy of the method to a greater or less degree, depending on the characteristics of the particular reach to which it is applied.

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Assuming that there is no change in the channel characteristics of a reach during a flood, the storage within the reach is fixed by the water-surface profile. If this profile is a straight line, the stages at the two ends of the reach offer the most accurate measure of the storage within the reach. The authors have used the sum of the discharges at the end points of the reach as their index of storage volumes. This presumes a single-valued relation between stage and discharge, which is not strictly rigorous, although the error is not great for headwater stations such as were used in the examples. The actual water-surface profile in a reach is rarely a straight line during the passage of a flood wave. This fact probably introduces the greatest error in routing. In effect, the authors have used the mean discharge in the reach as their index of storage. Therefore, they are unable to distinguish between the rise and fall of the flood wave; yet it is well known that the shape of the water-surface profile, and therefore the storage volumes, may differ considerably between the rising and falling phases for a given mean discharge. The use of the channel recession to determine storage volumes permits defining the storage curve for falling stages only. The use of stages or discharges at the end points of a reach as separate values by means of a family of curves[6] more effectively defines the slope in a reach than does the mean stage or discharge. Weighting Q and O to give a weighted value of $Q + O$ for the storage index[7] provides a very practical way of adjusting for the effect of rise and fall.

The procedure proposed has not been put to a test effectively by the methods outlined in the paper. The routing of a flood through a reach with large amounts of ungaged local inflow permits only an estimate of the reasonableness of the computed local inflow hydrograph. Test routings of special cases offer an opportunity for a check on the accuracy of a routing method. One such case might be a release from a reservoir without concurrent rainfall. In this case the local inflow will be low and nearly constant, thus permitting a good comparison between observed and computed hydrographs at a downstream station. Similarly, if a river reach within which a large part of the tributary flow is gaged can be found, the local inflow hydrograph can be computed from the upstream and downstream stations only, and this computed hydrograph can then be compared with the total gaged tributary flow adjusted for the ungaged inflow.

The authors suggest that, by prorating the storage volumes and the local inflow on a basis of drainage area, hydrographs for intermediate points of a reach can be computed by routing. Channel storage is a function of channel characteristics and need not show any relation to drainage area. Local inflow volume is controlled by rainfall distrubution and the runoff characteristics of the area under consideration. The shape of the local inflow hydrograph is a function of rainfall distribution with respect to both time and area and to the shape of the contributing area. It would seem advisable to avoid any arbitrary adjustment of storage and flow purely on the basis of drainage area. Contour maps and channel cross sections can be of considerable assistance in the prorating of storage, and the synthetic unit graph[8] can be of much help with the tributary flow.

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[6] "Flood Routing," by Edward J. Rutter, Quintin B. Graves, and Franklin F. Snyder, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 275.

[7] Discussion by Ralph W. Powell, *ibid.*, p. 298.

HAROLD C. HICKMAN,[9] ASSOC. M. AM. SOC. C. E.—Flood routing based on accurate and substantial information is at best an intricate and laborious problem. The method described in the paper is particularly interesting in that no cross sections of stream channels, no velocities of flow, and no discharge records on the tributaries are needed. The entire procedure is based upon the storage equation and upon the principle that, for all high stages, there is a straight-line relationship between the volume of storage contained in any reach of the river channel and the sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach. To continue further, it is assumed that at any point downstream the total flow at any instant is comprised of water contributed by many different tributaries, and that the percentage of the total flow occurring at any instant, that is contributed by any single tributary, is constantly changing throughout the flood period.

In analyzing the procedure for the determination of unmeasured inflow, it has been assumed that for any given reach of river channel there is a definite relationship between the volume of channel storage contained therein at any instant and the sum of the simultaneous discharge rates at the two ends of the reach. There is no doubt that this relationship exists, because the discharge is proportional to the river stage at the upper end of the reach, the outflow is proportional to the stage at the lower end, and discharge plus outflow is proportional to the mean stage which, in turn, is dependent on the volume stored in the reach. The relationship between these two quantities varies as a straight line, but, in obtaining the storage in the reach by measuring the area beneath the hydrograph, only a small section of the hydrograph can be used. In other words, this relationship holds good only after the stream has obtained "equilibrium" and is not influenced by overland flow. The entire procedure depends upon the relationship between $(Q + O)$ and the storage in the intervening reach. There is some doubt as to whether a solution should be based on a relationship of this kind.

Assuming for the present that the theory is rational, the next step is to route Q and I through the system to Morgantown. For this purpose, the inflow from the intervening area, I, was ignored, and the storage formula, Eq. 1, was written as in Eq. 3. The quantities on the left-hand side of Eq. 3 are known, giving a quantity which is the summation of $S_1 + O_1$. If Q_1 is added to each side, Eq. 4 results, which was solved by means of the curve in Fig. 2.

The relationship was determined by introducing I into Eq. 1 and therefore cannot be used as a means of solving Eq. 4. In other words, the manner in which the flood peak moves downstream, if the intervening area were not there, is decidedly different from the manner in which the peak will move downstream under the influence of the discharge of the intervening area. On the other hand, if the routing is accomplished by moving the flood peak, as influenced by the discharge of the intervening area, downstream, the problem resolves itself into the simple procedure of segregating the flood peak at Morgantown into its two component parts, depending on the stage-discharge relationship at Fetterman.

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[8] "Synthetic Unit Graphs," by F. F. Snyder, *Transactions*, Am. Geophysical Union, 1938, p. 447.

[9] Asst. Engr., Flood Control Section, U. S. Engr. Dept., Detroit, Mich.

In comparing the foregoing method of flood routing with other methods, certain facts must be taken into consideration. In the former method, only stream-flow records during a typical flood at a few points on the main stream, or on the tributaries whose flow is to be routed downstream, are necessary. It is universally accepted that when a known volume of water falls on the surface of a drainage area, the rate of resulting river flow at the lower end of the drainage area (point *P*) is influenced by many complex factors. Some of these, such as the consistency and moisture control of the soil, determine how much water will become surface runoff and enter the stream in time to contribute to the flood. Topography, valley storage, and channel storage are all factors in determining the shape of the hydrograph at *P*. After surface runoff has ceased, valley storage governs the flow. When dealing with small homogeneous areas, the evaluation of these factors is needless because the unit hydrograph method evaluates their combined effect as a single entity. With larger and more complex areas there is some question regarding accuracy.

Flood routing divides itself into two distinct problems and combinations of the two: (1) Flood synthesis, which is the determination of the magnitude and time of concentration of flood peaks on the various tributaries; and (2) flood routing, which is the determination of the manner in which these peaks travel and combine themselves with other peaks from other tributaries. Flood routing can be defined further as the procedure by which the hydrograph at any point on a stream is determined from a known hydrograph at some point upstream. As explained, the term "routing" is commonly used to include not only the modification of the flood wave caused by progress downstream, but the further modification caused by any additional flow that enters the river from tributaries or local runoff during this progress.

Factors Affecting Travel.—If a flood hydrograph is divided into elements of equal time, it can be said that certain factors affect the travel of the various elements and alter the shape of the flood wave as it travels downstream.

At the beginning of the flood hydrograph, the first few elements represent a part of the uniform flow of the stream, and they tend to maintain a condition of uniform flow. The elements following, which pass during the rise, have greater slopes and hydraulic radii than the normal river, and, since $V = C\sqrt{RS}$, their velocities are greater than the normal. As the stages rise, part of the discharge is temporarily lost in valley storage. As the flow at the lower end of the reach decreases, the valley storage in the reach will flow out and add to the discharge.

Valley storage in a reach at any time is equal to the inflow minus the outflow, and maximum storage occurs at the point of intersection of the two flow hydrographs. The point of intersection also represents time elapsed, and prior to this time the inflow exceeds the outflow and storage increases; after this, the outflow exceeds the inflow and storage is reduced. The curves intersect at the maximum point on the outflow hydrograph and the areas between the two curves are each equal to the increase of valley storage in the reach. Several factors affect the shape of the flood wave. They may be listed as: Rate of rise, height of rise, slope of channel, stages downstream, downstream channel sections, length of reach, length of crest of hydrograph, and rate of fall.

(1) Rate of Rise.—A rapid rate of rise causes high velocities in the first stages of the flood, which in turn result in the rapid dissipation of these first elements in valley storage, and the consequent retardation of the peak of the flood wave.

(2) Height of Rise.—The higher the stage, the greater the valley storage will be, which in turn causes a flattening of the peak. This causes a greater coefficient of roughness, which reduces velocities.

(3) Slope of Channel.—Greater slopes of the channel will allow more rapid runoff, causing lower stages and less valley storage.

(4) Stages Downstream.—With stages low downstream, a large part of the flood wave goes into valley storage. Increases in downstream stages may cause spreading to the alluvial plain, and thereafter the loss in valley storage will again become large. The propagation and intensity of the peak will be lessened. As the downstream stages are falling at a rate equal to the rate of increase in the rising flood wave, the latter will pass without loss or change due to valley storage, and the stream will tend to maintain a uniform discharge.

(5) Downstream Channel Sections.—If channel sections increase proportionately downstream, loss of valley storage will be greater than if they remain uniform, and the height and rate of propagation of the peak will be correspondingly reduced. If channel sections decrease, the reverse is true.

(6) Length of Reach.—The longer the reach the greater will be the valley storage and the more the peak will be reduced and retarded. These effects vary as a fractional power of the length of reach. Mathematically, in a uniform channel of indefinite length, the rise would be propagated downstream indefinitely, and, since the rate of peak reduction is a finite quantity at the beginning and can never become zero, the curve of successive peaks is concave upward and not a straight line.

(7) Length of Crest of Hydrograph.—The longer the crest, the less is the peak height affected by valley storage.

(8) Rate of Fall.—The more rapid the rate of fall, the more rapidly valley storage will flow out.

The effects that factors (1) to (8) have on a flood wave as it proceeds downstream are individually obvious. In order to route a flood, it is necessary to evaluate the combined effect of all factors. This presents a problem so complex that it defies exact solution by any known method.

A number of empirical methods have been used satisfactorily, but in all of them the stream has been divided into reaches. Since it is desirable to know the stage-discharge relationship at the downstream limit of each reach, the division is generally influenced by the location of gaging stations. Since the shape of the flood wave is constantly being warped by the flow from tributary streams, the accuracy of the computations is inversely proportionate to the length of the reaches. Reaches should be of such length that the simultaneous rates of rise or fall at their extremities are approximately equal and are of the same sense (the latter condition being impossible while the peak is passing through the reach). In the Allegheny and Monongahela rivers, reaches of from 10.6 to 80.7 miles have been used successfully.

All methods of flood routing are based on the solution of the basic equation:

$$I = O + \Delta S \qquad (6)$$

in which, for a given period of time T: I = total inflow; O = total outflow; and ΔS = change in storage. The time T should be no longer than the time of travel of the flood wave through the reach. Various methods, taking into consideration the aforementioned complex factors, have been developed and are generally accepted as giving good results.

Method of Inflow—Storage-Discharge Curves.—In general, the method consists of dividing the stream into reaches and determining the natural valley storage in each reach corresponding to the various discharges at the lower end of the reach. Assuming a straight line function for change of inflow and outflow during a time interval t, Eq. 6 may be written—

$$t\left(\frac{i_1 + i_2}{2}\right) = t\left(\frac{d_1 + d_2}{2}\right) + \Delta S \qquad (7)$$

in which: i_1 = inflow, in cubic feet per second, at initial instant of period; i_2 = inflow in cubic feet per second, at final instant of period; d_1 = discharge, in cubic feet per second, at initial instant of period; d_2 = discharge, in cubic feet per second, at final instant of period; ΔS = increase in storage, in cubic feet per second, during period; and t = length of period, in seconds.

In applying the method, consideration must be given to the flow that enters the reach from tributaries while the flood wave is passing along the reach. For a given value of discharge there will be several different values of valley storage, depending on whether the inflow enters all at one point, or at two points, or at a greater number. If it enters from several different locations, there will be different values, depending on the relative proportions entering at these different locations. Strictly, there are an infinite number of such variations. For practical purposes, however, on any specific reach there may be distinguished a limited number of "types" of floods, depending on which tributary or combination is contributing most of the flow. Separate computations must be made for each different type.

The decision as to how many such general types of floods will be distinguished, of course, must be made by a skilled engineer from a study of the specific reach in question. Among other things, it will depend on the location of the reach. In headwater areas a large percentage of the inflow usually enters from tributaries, whereas lower down the river the flow in the main stream is usually the principal component of the total inflow into the reach.

The storage and discharge curves are obtained by plotting storage and discharge, respectively, against gage height. Daily inflow at the head of the reach and rated tributaries entering the reach are tabulated. From plots of the storm, the rainfall over the area draining directly into the reach can be determined. The local inflow is computed from these data by the distribution-graph method. The daily discharge, gaged at the lower end of the reach, is recorded. The total outflow from any particular flood must equal the total inflow. Ordinarily, they do not check exactly, since there is a greater chance for error in computing the inflow.

With these data, the accumulated storage in the reach can be computed. A time interval of one day is used, and it is assumed that 1 cu ft per sec for one day equals 2 acre-ft (a very close approximation). Eq. 7 can then be written—

$$(i_1 + i_2) - (d_1 + d_2) = \Delta S \qquad (8)$$

in which: i_1 and i_2 are numerically equal to the daily values of inflow in cubic feet per second, but are acre-feet in this equation; d_1 and d_2 are numerically equal to the daily values of outflow in cubic feet per second, but are acre-feet in this equation, and ΔS = increase in storage for the first day, in acre-feet. The curves are then plotted.

This method is used by the Mississippi River Commission to predict flood stages and the time of their occurrence at various points in the main river, resulting from known or predicted conditions of flow in the Mississippi and Ohio rivers at Thebes and Metropolis, Ill., and in the major tributaries downstream.

It is extremely difficult to evaluate all the varied factors that influence flow in a river such as the Mississippi. The computation of the data, although simple in principle, is very difficult in fact. In certain instances, assumptions regarding computations, such as those for evaluating the effect of local and tributary inflow, must be made.

Every such prediction, of course, is based on the assumption that no rains of major proportions will fall after the date on which the prediction is made. If such rains occur, new predictions must be made. For this reason, predictions should be published with caution, since the layman does not always understand their inherent limitations.

Steinberg Method.—A method was developed by I. H. Steinberg, junior engineer, U. S. Engineer Office, St. Paul, Minn., in connection with the 9-ft channel project on the upper Mississippi River.[10]

A group of storage-discharge curves are used for a certain reach of a certain river. They have been obtained by observations of past floods on the reach, and are the same as the inflow storage-discharge curve except that in this case storage is plotted against discharge. There are five different curves, corresponding to five different conditions of backwater resulting from the inflow of a large tributary just below the reach, the volume of that tributary's flow being the parameter of each curve. (This is analogous to the development of several sets of curves in the inflow storage-discharge method, one for each "type" of flood.)

Starting with the fundamental Eq. 7, writing $\Delta S = S_2 - S_1$, and transposing:

$$\frac{t}{2}(i_1 + i_2 - d_1) + S_1 = \frac{t}{2} d_2 + S_2 \qquad (9)$$

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[10] Civil Engineering, July, 1938, p. 476.

All the knowns are on the left-hand side of Eq. 9, and the two unknowns, d_2 and S_2, are on the right-hand side. Call the left-hand member of the equation the "storage factor," and represent it by M; and let $t = 1$. Then: $M = \frac{d_2}{2} + S_2$ which, transposed, equals:

$$S_2 = M - \frac{d_2}{2} \ldots\ldots (10)$$

If Eq. 10 is plotted, S_2 against d_2, it is a straight line with a slope of -2 and an intercept of the S-axis of M. (Had t been given some other value than 1, the slope, of course, would have been different.) For different values of "M," a family of parallel straight lines may be plotted. Given the foregoing data, and certain basic data, flood routing may be accomplished by simple computations and by interpolation on the set of curves.

Other Methods.—Other methods of flood routing have been used successfully and the Muskingum method is one of them. It is based on the assumption that the ratio of valley storage within a reach to a weighted flow factor determined from both inflow and outflow is constant, and is dependent upon the physical shape of the valley within the reach. This is not theoretically correct, but is a close approximation within a limited range of flood stages.

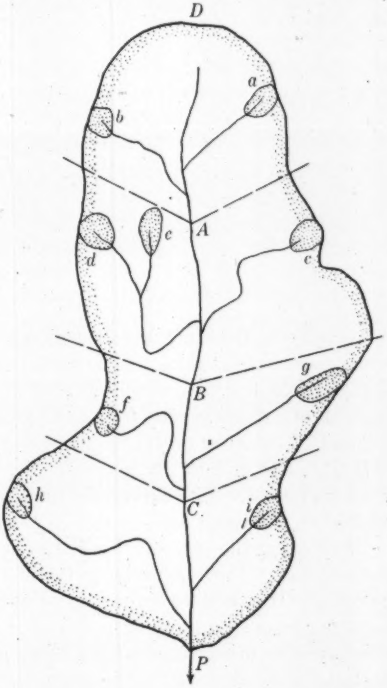

FIG. 5

There is another method that is a combination of the unit hydrograph with flood routing. Where the drainage area is too large to be dealt with by a single unit hydrograph, flood stages are forecast directly from rainfall data. For rivers such as the Connecticut, where the flood wave travels downstream very rapidly, predictions based on rainfall are usually the only ones that can be made far enough in advance to be of much use.

Assume the area PD, Fig. 5, to be drained by the system of streams shown. The method of forecasting stages at P is as follows:

At points A, B, C, and P, unit hydrographs were constructed. These are the hydrographs resulting from 1 in. of runoff over the successive divisions of the total drainage area—that is, the hydrograph at A is for the runoff from area AD, that at B is for the runoff from area BA alone, that at C is for the runoff from area CB alone, etc. The river was considered to be in flood, and these hydrographs were routed separately to *P*. Curves *A′*, *B′*, *C′*, and *P′*, Fig. 6, are the resulting hydrographs at *P*. They show the magnitudes of the elementary flows at *P* resulting from the unit hydrographs at the other points, and also the time lag between the appearance flow at point *A*, *B*, or *C* and of the resultant flow at *P*. The rain is assumed to have started at zero time.

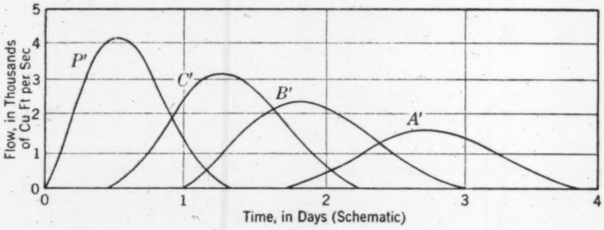

FIG. 6

Gaging stations are installed at points *a*, *b*, *c*, *d*, etc. The small stippled areas are the index areas for the subdivisions. The runoff factors for the index areas are applied to the rainfall on the larger areas in order to determine the number of inches of runoff from these areas. The ordinates of the curves *A′*, *B′*, *C′*, and *P′*, which show the resulting flow at *P* from a runoff of 1 in. over areas *AD*, *BA*, *CB*, and *PC*, respectively, are multiplied by the runoffs in inches as thus computed. The curves are then compounded to obtain the hydrograph at *P* that will result from the given storm.

In routing the individual unit hydrographs at *P*, the river was assumed to be in flood. The operation is therefore accurate only when this condition prevails. It would be erroneous to assume, for example, that the flow at *P* resulting from a 2-in. runoff from area *AD* alone could be found by multiplying the ordinates of curve *A′* by 2. Since no rain fell on areas *PC*, *CB*, and *BA*, the stages of the stream in those areas would be normal when the flood waters from upstream arrived, and much of the flood flow would be taken up in valley storage. The peak at *P* would therefore occur later than curve *A′* indicated, and would be lower than the value obtained by multiplying the maximum ordinate of *A′* by 2.

Conclusions.—It can be noted from the foregoing that flood routing with all of its complex factors cannot be considered in the light of the information from one or two flood hydrographs. When seven or eight factors, each having considerable individual effect on the size and shape of the flood hydrograph, join together in different combinations, depending upon the assimilation of predetermined conditions, the resulting effect on the flood hydrograph may be unpredictable unless all of the information that can possibly be collected is analyzed and given consideration in a manner suitable to its importance to the problem.

It is believed that the method described by Messrs. Wisler and Brater is unique and should be tested under varying conditions. It may be that this method eliminates most of the tedious work connected with the routing of floods in a large and intricate drainage system. However, the most serious objection to the method is that it is based on the assumption that routing within a basin may depend on what has happened during one flood. Can it be said that a major storm, and the resulting flood, will proceed according to the same plan every time? Unless it is true that all of the factors affect all floods on a particular stream in the same manner and to the same degree during every storm, there is no reason why each flood will not be different from its predecessor even though the storm producing it begins under identical conditions. In other words, every storm should be studied separately, so that each factor affecting the flood can be isolated and compared if possible.

Any method, in which the contributing factors can be used in different combinations to determine the resulting effect, appears to be more rational than a method that proposes to assume that all floods proceed according to the same plan under all combinations of meteorological, channel, and basin conditions.

ROBERT B. HORONJEFF,[11] AND HERBERT G. CROWLE,[12] JUNIORS, AM. SOC. C. E.—The flood-routing technique presented in this paper may prove to be of considerable value, provided the full significance of the various operations is understood by those who use the method.

The writers are in complete agreement with the basic equations presented, but they believe that the authors have not given sufficient explanation of the following features:

(1) The expression "storage in the intervening reach" includes the storage in all channels tributary to the reach as well as the storage in the main channel. This fact must be kept in mind in any attempt to visualize the proportionality of "storage in the intervening reach" and the "sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach."

(2) Referring to Fig. 1, the agreement of the composite of the routed hydrographs with the gaged hydrograph O at the lower end of the reach is merely a check on the arithmetical operations performed in deriving the hydrograph marked I, and in routing I and Q through the reach with the straight-line relation between S and $(Q + O)$ shown in Fig. 2. The agreement is in no sense a check on the accuracy of the gaging at the lower station, or the correctness of the lines in Fig. 2.

As proof of the foregoing statement, the writers present the set of hydrographs for the Pajaro River, California, in Fig. 7, corresponding to Fig. 1, and the inflow-outflow-storage lines in Fig. 8, corresponding to Fig. 2. A characteristic flood is routed from Sargent to Chittenden, Calif., discharge records being available at both of these places. The hydrographs marked I, I', and Q' were obtained by the methods outlined in the paper, using the "S versus $(Q + O)$" and "$S + (Q + O)$ versus $(Q + O)$" curves derived from the recession curves of hydrographs Q and O. The sum of the hydrographs I' and Q' was so close to the hydrograph O as to be indistinguishable. In Fig. 8, a second "S versus $(Q + O)$" curve was then drawn at random, with the corresponding "$S + (Q + O)$ versus $(Q + O)$" curve, and the hydrographs I'', I''', and Q''' derived with these new lines. The curve I'' represents the fictitious hydrograph of inflow from the intervening reach, I''' that hydrograph routed through the reach, and Q''' the fictitious routed hydrograph of Q. The sum of I''' and Q''' agrees exactly with the original gaged hydrograph O, proving that the agreement depends only upon the use of a straight line—any straight line—for the routing operation. This is perhaps the most important criticism of the paper—namely, that the agreement of the composite of the routed hydrographs with the gaged hydrograph does not prove the accuracy of the results in a physical sense.

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[11] Associate Engr., U. S. Engrs., War Dept., San Francisco, Calif.

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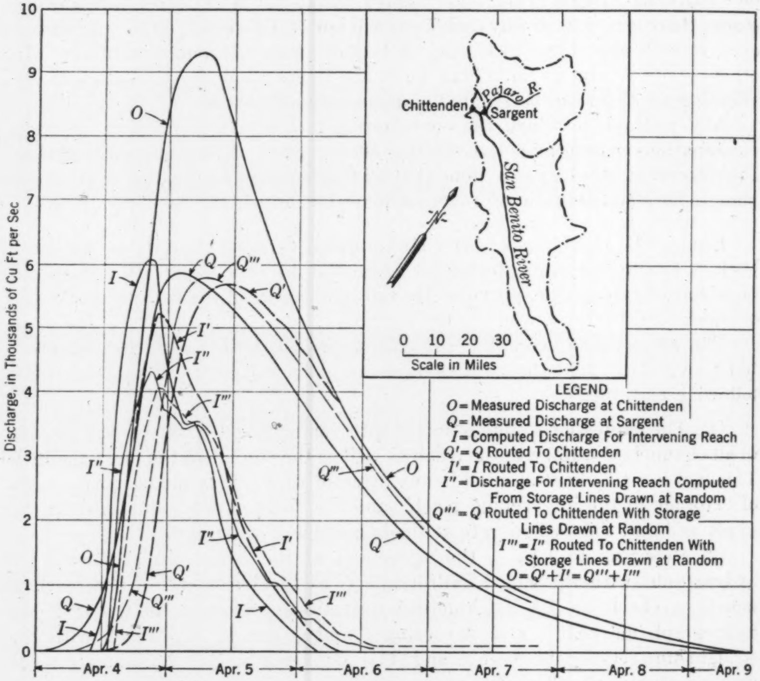

FIG. 7.—HYDROGRAPHS OF SURFACE RUNOFF, WATERSHED OF THE PAJARO RIVER, CALIFORNIA

(3) The authors did not indicate whether the storage curve derived from one flood would plot in very nearly the same position as those obtained from other floods. The writers found, in the case of the Pajaro River, that "S versus $(Q + O)$" curves, which were derived from different floods, plotted in different positions and with appreciably different slopes, indicating that there is no one curve that is characteristic of a particular reach during all floods.

(4) Base flow may be included in the routing operation, thus eliminating the necessity of adding this value to obtain the complete routed hydrograph at the lower end of the reach.

(5) Finally, the writers wish to emphasize that the authors so far have presented no evidence to prove that their method of routing gives the correct reduction in peak discharge to be expected from a reservoir constructed on a

tributary. In fact, it appears to the writers that the nature of the method precludes the possibility of a check upon the physical results. Any errors in gaging or in deriving the "S versus $(Q + O)$" curve are automatically included in the derivation of the hydrograph of computed discharge I for the intervening reach.

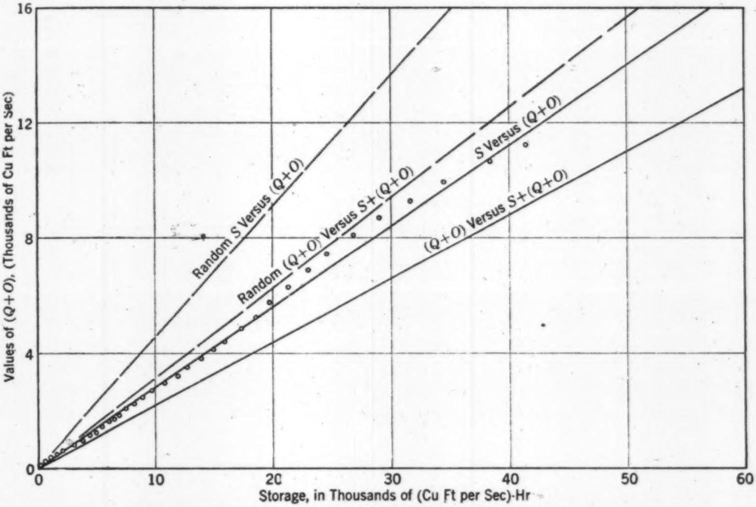

FIG. 8.—RELATION BETWEEN $(Q + O)$ AND THE STORAGE IN THE INTERVENING REACH, PAJARO RIVER, CALIFORNIA

L. K. SHERMAN,[13] M. AM. SOC. C. E.—The writer has tested the authors' flood routing method by utilizing an example published in 1940.[14] The gaged stations, reported by the authors, at Morgantown and Fetterman correspond with the two points at the ends of the lower (6-ft base) reach of the main channel, shown in Fig. 9.[15] The authors' hydrograph O corresponds with "(b) Heavy Rain," in Fig. 10. The hydrographs, corresponding to the authors' Q and routed Q_1, are plotted as curves d and e in Fig. 10.

The relation of cubic feet per second at the outlet station, to the valley storage, was determined from recession curves of the hydrographs O, Q, and also from the small hydrograph in Fig. 10 marked "(a) Light Rain." The points were platted on logarithmic paper similar to the diagram given by Robert Horton,[3] M. Am. Soc. C. E. All of the points below the contraflexure on the recession curves of the hydrographs fell on a straight line, except low values of cubic feet per second. In this example, all surface runoff ceased at the time 8,400 sec, as shown by the hydrograph of overland flow,[16] Fig. 11, for one unit of area. This time checks with the points of contraflexure. The total overland flow from the "heavy rain" on one unit strip of 1,100 ft is 71 cu ft. The total volumes of runoff, for checking purposes, are: Hydrograph Q or Q_1 is 44,000 × 71 cu ft. The volume I for the reach is 16,000 × 71, and the volume of hydrograph O is 60,000 × 71.

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[13] Cons. Engr., Chicago, Ill.

[14] "The Hydraulics of Surface Runoff," by L. K. Sherman, *Civil Engineering*, March, 1940, p. 165.

[15] *Ibid.*, Fig. 1.

[16] *Ibid.*, p. 165, Fig. 3.

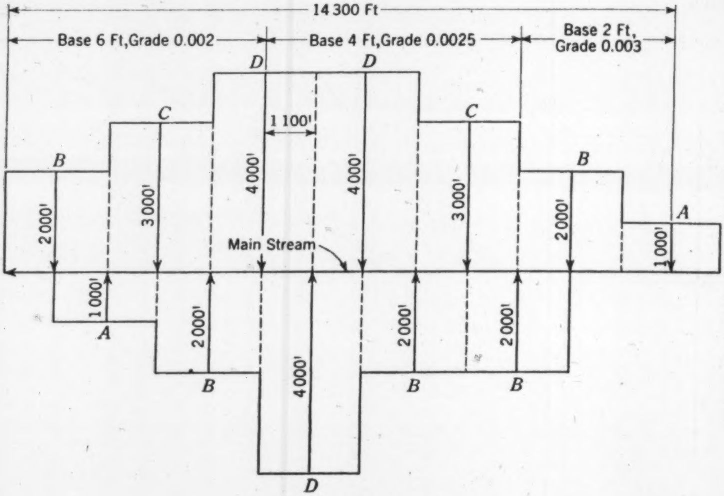
14 300 Ft
Base 6 Ft,Grade 0.002
Base 4 Ft,Grade 0.0025
Base 2 Ft, Grade 0.003
D
D
C
1 100'
C
B
B
2 000'
3 000'
4 000'
4 000'
3 000'
2 000'
A
1 000'
Main Stream
1 000'
A
2 000'
2 000'
2 000'
B
4 000'
B
B
D

Fig. 9.—The Drainage Area

The writer platted a diagram of $(Q + O)$ versus storage, with time units of 250 sec, corresponding with Fig. 2. The fifteen points all fell on, or close to, a straight line through the two points $(Q + O) = 150$ cu ft per sec versus $S = 392$ cu ft per sec (250 sec) and $(Q + O) = 1,353$ cu ft per sec versus $S = 3,260$ cu ft per sec (250 sec). The line $(Q + O)$ versus $S + (Q + O)$ was also platted.

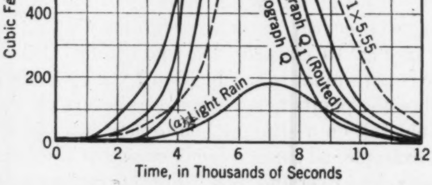

Fig. 10.—Main Channel Hydrographs for Both Light Rain and Heavy Rain

The writer's routed hydrograph Q_1 was derived with this diagram and with Eq. 4. The tails of his recession curves Q_1 and O did not coincide as do the similar curves of the authors. The writer knows of no reason why they should coincide. Even with equal durations of rainfall, the recession curve of the hydrograph with the lesser peak and volume will fall inside the larger. As an extreme case, note the two hydrographs in Fig. 10.

Conclusions.—(1) The authors have presented a reliable method that will be useful in many problems of flood routing.

(2) In selecting recession curves for the relation between discharge rate and storage, avoid, if possible, hydrographs affected by rainfall or surface runoff during the recession period.

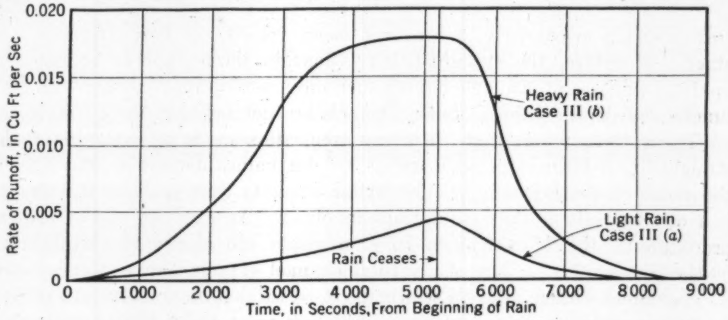

FIG. 11.—OVERLAND FLOW FROM A 1-FT STRIP 1,100 FT LONG, WITH DETENTION STORAGE

(3) The straight-line relation for $(Q + O)$ versus storage exists for a longer period than the similar line for discharge and storage.

A supported hydrograph (that is, one receiving increments of water as it moves downstream) may be routed by Seddon's wave velocity if the channel areas are known. The routed hydrograph will check with the storage equation at any time. A hydrograph routed by stream velocity will not check with the storage equation. The total volumes will agree, but the forms of the hydrographs will not agree.

An attempt was made by the writer[14] to compare the known volume of the reach with the volume derived by the difference of total hydrograph volumes, but it was not too successful.

It would appear that, on natural streams, more reliable values of storage could be derived by the authors' method than by cross-section surveys.

ALFRED L. BROSIO,[17] JUN. AM. SOC. C. E.—The method of flood routing that the authors have presented involves assumptions that need further explanation. The comments which this writer wishes to present will be separated into two parts—I. Discussion of the assumption of a straight-line flow-storage relation; and II. Analysis of the assumption: Storage = function (inflow + outflow).

I. Assumption of Straight-Line Flow-Storage Relation.—In the "Conclusion," the authors have stated their belief that "except possibly under unusual channel conditions, the curve of relationship between channel storage in the intervening reach and the sum of the discharges at the two ends of the reach is a straight line for the normal and high stages of the river." Although such a relationship is sometimes found to exist in natural channels, it is not believed to represent the "usual" channel conditions. This can be shown as follows: Let S = volume of channel storage in second-foot-hours; Q_u = uniform rate of flow, through entire reach, in cubic feet per second. A straight-line relationship between S and (inflow + outflow) implies that $\frac{S}{Q_u}$ = constant; but a constant value of $\frac{S}{Q_u}$, which is in units of time, represents a constant time of travel, and, consequently, a constant mean velocity of flow irrespective of stage. Therefore, the straight-line relationship should not be assumed to apply in the case of channels having such cross section that mean velocity is a function of depth of flow. Such channels are not believed to be "unusual."

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[17] Engr., Missouri Val. Bridge & Iron Co., Evansville, Ind.

The authors report that "in every application made to date (1941), this straight-line relationship holds true * * * for values derived to the right of the point of contraflexure." The writer suggests that perhaps this finding is a direct result of the assumptions involved. An analysis of the authors' procedure in the Monongahela River example will clarify the implications of the last sentence. Let: S_m = total channel storage above Morgantown; S_f = channel storage above Fetterman; S_a = total storage in the channels of the "intervening area"; S = storage in the Fetterman-to-Morgantown channel; S_i = storage in the channels of the "intervening area," exclusive of S (the intervening area comprises 50% of the total area above Morgantown, and many miles of channel are included therein in addition to the Fetterman-to-Morgantown channel); O = Morgantown flow; Q = Fetterman flow; and I = flow from the "intervening area."

The example is somewhat complicated because of rain that fell from March 26 to 28, during the period of recession which the authors chose for their study. However, the writer does not contest the statement of the authors to the effect that, had not this additional rain fallen, values of $(S_m - S_f)$ computed from the respective hydrographs subsequent to the point of contraflexure would have plotted against $(Q + O)$ as a straight line; but $S_m - S_f = S_a$; and $S_a = S + S_i$. Therefore, by adopting a curve conforming with plotted points of $(S_m - S_f)$ versus $(Q + O)$, to represent S versus $(Q + O)$, the authors in effect assumed S_i, and consequently I, to equal zero. Going on to the next step in the authors' procedure, the hydrograph of I was developed, using the adopted flow-storage relation, and found to equal zero during the periods in which it had been so assumed by implication; but it is obvious that had any other assumption been implied with regard to the value of I, the results would have confirmed such assumption quite as well. For example, the writer has made a test in which a curve of S versus $(Q + O)$ was adopted conforming with the authors' curve up to $(Q + O)$ = 10,000 cu ft per sec and having a "concave-upward" shape thereafter. The assumption implied by such a curve is that S_i did not deplete itself until $(Q + O)$ receded to a value of 10,000 cu ft per sec, shortly before noon on March 29. The hydrograph of I, developed as required by the authors' equations and using the concave curve, confirms the implied assumption of the writer, although it is understood that such assumption is probably no better than the one implied by the authors. The detailed calculations and results of this test are not presented herewith, but a similar test may be made in a short time by any one desiring to verify the foregoing statements.

It appears, therefore, that the method used by the authors to compute channel storage may be somewhat unreliable. Unfortunately, the writer is not at all familiar with the streams studied by the authors, nor with the quantity and quality of meteorological and hydrological data available. However, assuming the existence of a very minimum of such data, the writer would recommend the following general procedure to obtain the volume of storage in a channel such as the one from Fetterman to Morgantown:

1. Choose for analysis that flood, for which both Fetterman and Morgantown flows are known, which offers the best combination of the following: (*a*) Precipitation data in, or near, the "intervening area"; and (*b*) a minimum percentage of total runoff above Morgantown originating from the "intervening area."

2. Using the method devised by Otto H. Meyer,[18] Assoc. M. Am. Soc. C. E., for determining hydrographs for areas lacking in runoff records, the hydrograph of flow from the intervening area during the flood chosen for analysis may be estimated as to shape, total volume being known from the difference "Morgantown minus Fetterman." Should the basic data used in conjunction with Mr. Meyer's method be very poor, the choice of a flood having the quality of Item 1(*b*) will be of definite advantage.

3. Storage in the channel Fetterman-to-Morgantown may then be computed at any instant, knowing total inflow, as well as outflow, during any interval of time. This channel storage may then be used in further studies, to determine its relation to other variables. Since Mr. Meyer's method makes available not only the hydrograph *I* subsequent to the point of contraflexure, but the entire flood hydrograph, larger values of channel storage may be computed than from a method limited to analysis of recessions.

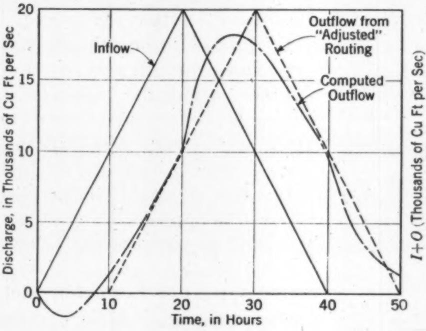

FIG. 12

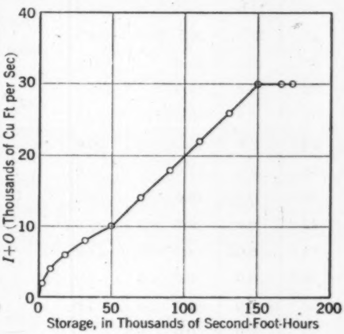

FIG. 13

Of course, it is not to be understood that the method outlined herein to compute channel storage is the only acceptable one. With more basic data the selection of methods becomes correspondingly greater. It is possible that had the authors determined channel storage in the various reaches by a more reliable method, the correlations of S versus (inflow + outflow) might not have yielded straight lines.

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[18] "Analysis of Run-Off Characteristics," by Otto H. Meyer, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 83–100.

II. Storage as a Function of (Inflow + Outflow).—The statement (see heading "Determination of Unmeasured Inflow") that "there is a definite relationship between the volume of channel storage * * * at any instant and the sum of the simultaneous discharge rates at the two ends of the reach" sounds very logical at first thought. However, it is an assumption, the weaknesses of which should be fully understood.

Let the hydrograph represented by the full line of Fig. 12 be the total flow into a reach of channel, entering entirely at the upper end of the reach. Also, assume that the relationship between storage and uniform flow is a straight line, having a value of inverse slope equal to 10 $\frac{\text{second-foot-hours}}{\text{second-foot}}$, or 10 hr. If the inflow is routed through the reach according to the method presented by the authors, using 2-hr periods, the outflow hydrograph indicated in Fig. 12 by the dash-dot symbol is the result. There are three apparent inconsistencies in that hydrograph: (1) Outflow is negative during the first 8 hr; (2) rate of rise of outflow suddenly increases at Hour 20, when inflow begins to fall; and (3) rate of fall of outflow suddenly increases at Hour 40, when it would be expected to do the opposite.

The writer does not believe that a detailed explanation is required, relating to the foregoing inconsistencies. It is obvious that the assumption storage = function (inflow + outflow) assumes a uniformly varying flow between the two ends of the reach and that such a condition does not exist during the three periods of discrepancy. The same types of inconsistencies occurred in the examples presented by the

TABLE 2.—ADJUSTED ROUTING COMPUTATIONS FOR THE TRIANGULAR-SHAPED INFLOW HYDROGRAPH OF FIG. 12

Hour	I	O	S	Eq. 11	$I_1 + O_1$
0	0.0	0.0	0.0		
				4.0	
2	2.0	~~−1.3~~ 0.0	2.0		0.7
				12.0	
4	4.0	~~−2.0~~ 0.0	8.0		2.0
				24.0	
6	6.0	~~−2.0~~ 0.0	18.0		4.0
				40.0	
8	8.0	~~−1.3~~ 0.0	32.0		6.7
				60.0	
10	10.0	0.0	50.0		10.0
				84.0	
12	12.0	2.0	70.0		14.0
				108.0	
14	14.0	4.0	90.0		18.0
				132.0	
16	16.0	6.0	110.0		22.0
				156.0	
18	18.0	8.0	130.0		26.0
				180.0	
20	20.0	10.0	150.0		30.0
				196.0	
22	18.0	~~14.7~~ 12.0	166.0		32.7
				204.0	
24	16.0	~~18.0~~ 14.0	174.0		34.0
				204.0	
26	14.0	~~20.0~~ 16.0	174.0		34.0
				196.0	
28	12.0	~~20.7~~ 18.0	166.0		32.7
				180.0	
30	10.0	20.0	150.0		30.0
				156.0	
32	8.0	18.0	130.0		26.0
				132.0	
34	6.0	16.0	110.0		22.0
				108.0	
36	4.0	14.0	90.0		18.0
				84.0	
38	2.0	12.0	70.0		14.0
				60.0	
40	0.0	10.0	50.0		10.0
				40.0	
42	0.0	~~6.7~~ 8.0	32.0		6.7
				24.0	
44	0.0	~~4.0~~ 6.0	18.0		4.0
				12.0	
46	0.0	~~2.0~~ 4.0	8.0		2.0
				4.0	
48	0.0	~~0.7~~ 2.0	2.0		0.7
				0.0	
50	0.0	0.0			0.0

authors. Some good illustrations may be found in the routing of Q into Q', in Fig. 1. The writer believes that the authors must also have obtained negative values of flow, not shown in the hydrographs of Figs. 1 and 4.

It is possible to make certain "running adjustments," during the process of routing the triangular hydrograph of Fig. 12, to eliminate the effect of the discrepancies already mentioned. Table 2 presents the calculations by means of which this is accomplished. The general procedure consists of routing by the authors' method except that, for each impossible value of outflow which is obtained, the nearest possible value is substituted. The following nomenclature is used: I = total flow into reach, entering entirely at upper end, 1,000 cu ft per sec; O = outflow in 1,000 cu ft per sec; and S = channel storage in 1,000 sec-ft-hr. In Table 2, the fifth column contains progressive values of

$$S_0 + I_0 + 2\,I_1 - O_0 = S_1 + I_1 + O_1 \qquad (11)$$

which corresponds to Eq. 4 of the paper. Assuming that S = function $(I + O)$, the uniform-flow-versus-storage relation which has been assumed for the reach gives

$$S = 5\,(I + O) \qquad (12)$$

It follows that the relation required for obtaining the sixth column of Table 2 by the authors' method is

$$I + O = \tfrac{1}{6}\,(S + I + O) \qquad (13)$$

When the values of storage in Table 2 are plotted against simultaneous values of $(I + O)$, the result shown in Fig. 13 is obtained. It seems as if the effect of the writer's "adjustments" may have been in the direction of "correcting" the relation between storage and $(I + O)$ for the fact that the variation of flow between inlet and outlet is concave upward from Hour 0 to Hour 10, and from Hour 40 to Hour 50, while being concave downward from Hour 20 to Hour 30. The resulting hydrograph of outflow is shown in Fig. 12 by the short dashes. It is seen to be a perfect image of the inflow hydrograph, with a time lag of 10 hr.

It is generally agreed that if, at all sections of a reach, discharge were entirely dependent on stage, the outflow and inflow hydrographs would be identical in shape. Although this condition is sometimes approached in natural river reaches, it should not be assumed to represent the general case. Since the assumption that storage is a function of average flow in the reach obviously implies such a condition, a method of flood-routing based on that assumption is not adaptable to general application.

Several routing methods used in present practice take care of the individualities of any given reach by means of factors embodied in the procedure. These factors, however, are generally unreliable unless they are evaluated from analysis of at least one flood, for which hydrographs of total inflow and outflow are either known or obtainable by estimation. The writer believes, however, that a practical method of flood routing, the required factors in which can be determined directly from the measurable physical properties of the reach, can be developed.

In the meantime, it is suggested that a more satisfactory solution of the authors' problems could perhaps be effected by the following procedure:

1. Using the procedure outlined in part I of this discussion, obtain the hydrograph of channel storage, for the flood chosen for study.
2. Plotting values of outflow against simultaneous values of storage should give a loop showing a larger volume of storage for a given discharge during rising stages than during falling stages. By plotting values of storage against later values of outflow, a number of trials will indicate what time interval between storage and outflow effects the best closure of the loop into a curve.
3. The inflow hydrograph could then be routed according to a method presented by Otto H. Meyer,[19] using a "storage lag" as nearly as possible equal to the time interval which effected best loop closure. The result should be a more or less satisfactory outflow hydrograph, depending on the reliability of the data used in previous steps. Incidentally, the process of obtaining the storage-outflow relation and storage lag is described in more detail by Mr. Meyer in the publication cited.[19]
4. Using the same routing procedure, outflow may then be computed for any recorded or assumed total inflow, or by reversing the routing procedure, total inflow may be computed for any recorded or assumed outflow.

W. B. LANGBEIN,[20] ASSOC. M. AM. SOC. C. E.—A reading of this paper seems to give the impression that the objectives of flood routing as set forth in the first paragraph are not being realized at present. In their presentation the authors seem to overlook the fact that floods are being routed daily and expeditiously by empirical methods as described in the following references which are evidently as simple as those they proposed:

(1) "Flood of Ohio and Mississippi Rivers, January–February, 1937," by N. C. Grover, M. Am. Soc. C. E., *Water-Supply Paper No. 838*, U. S. Geological Survey, 1938.

(2) "Natural Stream-Channel Storage," by R. E. Horton, M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1936, pp. 406–415.

(3) "Definitions and Classification of Flood Waves," by R. E. Horton, *Bulletin No. 25*, Permanent International Assoc. of Navigation Congresses, Brussels, Belgium, 1938.

(4) "A Method for Correcting River Discharge for a Changing Stage," by B. E. Jones, M. Am. Soc. C. E., *Water-Supply Paper No. 375*, U. S. Geological Survey, 1916, pp. 117–130.

(5) "Some Channel-Storage and Unit-Hydrograph Studies," by W. B. Langbein, *Transactions*, Am. Geophysical Union, 1940, pp. 620–627.

(6) "The Unit Hydrograph and Flood Routing," by G. T. McCarthy (unpublished manuscript presented at conference of North Atlantic Div., Corps of Engrs., U. S. Army, June 24, 1938).

(7) "Hurricane Floods of September 1938," by C. G. Paulsen, M. Am. Soc. C. E., and others, *Water-Supply Paper No. 867*, U. S. Geological Survey, 1940.

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[19] "Simplified Flood Routing," by Otto H. Meyer, *Civil Engineering*, May, 1941, pp. 306–307.

[20] Associate Hydr. Engr., U. S. Geological Survey, Washington, D. C.

(8) "Flood Routing," by E. J. Rutter and Q. B. Graves, Assoc. Members, Am. Soc. C. E., and F. F. Snyder, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 104 (1939), pp. 275–313.

(9) "River Hydraulics," by the late J. A. Seddon, M. Am. Soc. C. E., *ibid.*, Vol. XLIII, June, 1900, pp. 179–243.

(10) "Engineering Construction: Flood Control," the Engineer School, U. S. Army, Fort Belvoir, Va., 1940, pp. 127–177.

(11) "Synthetic Unit-Graphs," by F. F. Snyder, *Transactions*, Am. Geophysical Union, 1938, pp. 447–454.

(12) "The Hydraulics of Flood Movements in Rivers," by H. A. Thomas, M. Am. Soc. C. E., *Engineering Bulletin*, Carnegie Inst. of Technology, 1934.

(13) "Storage and the Unit Hydrograph," by C. O. Clark, Jun. Am. Soc. C. E., U. S. Engr. Office, Norfolk, Va., 1940 (unpublished).

The authors undertake to compute a hydrograph of inflow from the unmeasured area and refer (paragraphs preceding "Routing Q and I Through to Morgantown") to a comparison between two methods of computing the volume of inflow. It appears to the writer that the two methods are one; any difference should be ascribed to arithmetic discrepancies.

For example, in method 1, $\sum$ outflow at Morgantown $-$ $\sum$ flow at Fetterman = total intervening inflow, whereas in method 2,

$$I = \frac{O_2 + O_1}{2} - \frac{Q_2 + Q_1}{2} + \frac{S_1 - S_2}{\Delta t} \dots\dots\dots\dots (14)$$

in which Δt is time intervening between events indicated by subscripts 1 and 2.

From the beginning of the hydrograph to its end, storage fluctuates from zero back to zero; hence,

$$\sum I = \sum \frac{O_2 + O_1}{2} - \sum \frac{Q_2 + Q_1}{2} \dots\dots\dots\dots (15)$$

as before. Hydrographs of channel inflow from intervening, partial, and total basin areas were computed by the U. S. Geological Survey by Eqs. 14 and 15 as a part of its analysis of the Ohio River flood of January, 1937 (1).[21]

The authors state (see "Synopsis") that

> "The entire procedure is based upon the storage equation and upon the principal that, for all high stages, there is a straight-line relationship between the volume of storage contained in any reach of river channel and the sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach. Except perhaps for unusual channel conditions, this relationship holds true."

This statement is not generally supported by studies of others. G. T. McCarthy, Assoc. M. Am. Soc. C. E. (6), showed that, in the general case, unless proper weights were assigned to inflow and outflow, the discharge-storage relation was a loop. The special case of the authors, of equal weights to outflow and inflow, has been shown by the writer (5) and others (13) to be comparable to simple translation.

The straight-line relationship between weighted discharge and storage is approximated by many reaches. It is a convenience but not a necessary corollary to general flood-routing technique, and is not a proof of the authors' general statement that overland flow ceases and channel storage-drainage begins at the time of the point of contraflexure on the recession side of the hydrograph. It may be true for a minor tributary but certainly does not hold true for larger basins. For example, in Fig. 1 the point of contraflexure occurs later on the Morgantown hydrograph than on the one at Fetterman; evidently, this is simple hydraulic lag. The time of contraflexure was even later at Charleroi, Pa., and, as one proceeds downstream, even the peak may be considered as drainage from upstream areas from which overland flow had long since terminated.

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[21] Numerals in parentheses, thus: (1), refer to the foregoing references to current literature on "Channel Storage and Flood Routing."

The writer has used the method of computing storage relations from recession curves (5). From his experience he would be very much surprised if points prior to the point of contraflexure did not plot to the right of the curve as in Fig. 2, and he cannot accept this as proof that this is the point of termination of overland flow.

The method of verification adopted by the authors (applying empirical formulas directly to the material used to derive them) may be deficient to the extent that the data used in the derivation of the technique may have sampled only a limited number of experiences. A later flood representing an entirely different combination of events may prove such empirical methods to be unreliable. Present flood-routing technique has many shortcomings, and there is a great need for the application of some of the more general laws of fluid motion to the problem to replace present empiricisms. It seems probable that the authors did not take full advantage of the considerable amount of published material relating to the technique of flood routing.

FRED E. TATUM,[22] JUN. AM. SOC. C. E.—The method advanced by the authors for obtaining their storage curves saves considerable time. The curve thus obtained represents the true storage for that particular storm rather than the average curve as computed by other methods.

The true recession curve is an exponential curve[23,24] and the recession factor can be designated by "K." The recession factor "K," of both the upper and lower gaging stations, must be equal for the S versus $(Q + O)$-curve to be a straight line. In the case of unequal recession factors, S versus $(Q + O)$ will plot as a curved line.

Since the recession curves are exponential and equal, the S versus $(Q + O)$-curve should be a straight line starting at the origin and drawn parallel to the curve used by the authors. The curved line obtained at lower stages by the authors is apparently due to the fact that the point at which surface runoff appeared to end was sooner than the actual ending time.

This routing method gives good results when the two gages are reasonably close together, but if the reaches are too long for the period of time required for routing, the results cannot be depended upon. One of the requirements in routing is that the reach should be of such length that the time of travel through the reach is about the same as the length of the period used in routing.

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[22] Junior Engr., U. S. Engrs., Rock Island, Ill.

[23] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

[24] "The Mathematical Synthesis of the Flood Hydrograph," by R. T. Zoch, *Transactions*, Am. Geophysical Union, 1939, p. 207.

It seems that the authors went to considerable trouble in computing the local inflow and then routing it down to the gage, when it is readily obtainable by an easier method. The hydrograph of local inflow, of necessity, would be at the lower gaging station, and therefore would not need to be routed. The local inflow hydrograph can then be obtained by subtracting the routed flows of the upper gaging station from the flows of the lower gaging station. The difference will be the hydrograph for the local inflow.

H. C. WOSTER,[25] ASSOC. M. AM. SOC. C. E.—The method of flood routing developed by the authors is felt to be a distinct contribution to the methodology of delineating flood and damage source areas. The method is particularly adapted to use in the flood-control preliminary examinations and surveys of the Department of Agriculture, in the prosecution of which it is usually impracticable and economically impossible to obtain the channel cross sections and other hydraulic data requisite to the more exact methods of flood routing.

The authors' method has been used during and since its development in the flood-control preliminary examinations conducted by the Appalachian Forest Experiment Station, at Asheville, N. C., and, since May, 1941, under the supervision of the Regional Office, Bureau of Agricultural Economics, at Atlanta, Ga. The method was used first as a means of defining the principal sources of flow contributing to downstream damages, but in studies made since May, 1941, routings made by this method have been used as a basis for allocating actual flood damages at critical downstream points to the respective tributary areas. The allocation of damages to source areas in turn becomes one of the bases for determining the critical areas on which a land-use improvement program can be expected to contribute to flood reduction. The flow routing is used also for approximating the contribution of each area to the anticipated reduction.

This discussion is intended to present some of the difficulties encountered in the application of the method, in the hope that they may be considered by the authors in any further development and refinement of the method. It is regretted that solutions of the difficulties cannot be offered in this discussion.

The authors state that their selection of gaging stations for illustration was based partly on the excellent discharge records available. It is evident from the presentation that very good results have been obtained with these records. In the studies made by the writer it has been necessary to use the available discharge records for the watersheds studied, many of which are less reliable than those used by the authors because of lack of definition of stage-discharge relationships for high stages or because of complications due to backwater, slight regulation, or other causes. Gaging station records affected by pronounced stream regulation have not been used. The results obtained by using the regular run of discharge records are less fortunate than those encountered by the authors. It has been found necessary in most cases to adjust the routed hydrographs of Q and I after completion of the procedure because when they are combined they do not check, even closely, the actual hydrograph. In some cases the adjustments have been rather large, necessitating the exercise of personal judgment and hence detracting from the usefulness of the routing method as an exact procedure.

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[25] Takoma Park, Md.

In most cases where adjustment of the routed hydrographs has been made, it has been found necessary to reduce the peak of the routed hydrograph of Q and move it ahead in point of time. This appears to indicate that the relationship of $(Q + O)$ to channel storage does not yield sufficiently large storage values. This has been found to be particularly true in cases where the distance between gaging stations is long. One reason for this may be that the relationship between $(Q + O)$ and channel storage was developed from the recession side of the hydrograph and does not take account of the fact that for a given stage the channel storage is greater on the rising than on the recession side of the hydrograph. Consequently, the application of the authors' channel storage relationship reduces the peak discharge in transit between stations less than actually occurs. Also, the actual time of transit of the flood wave is greater than that obtained by this method.

It has been found that, in computing outflow from the inflow hydrograph of Q, negative values of outflow are often obtained for a number of periods at the beginning of the outflow hydrograph. Since the total volume of Q', including the negative values, equals the original volume of Q, then, if the negative values of Q' are called zero, the total outflow becomes greater than the original volume. This is believed to be due to errors in the slope of the storage curve near its lower end and doubtless could be corrected by adjusting the slope of the storage curve by trial and error. It has been found to be quicker and more convenient, however, to adjust the hydrograph of Q' directly to the original volume.

C. O. WISLER,[26] M. AM. SOC. C. E., AND E. F. BRATER,[27] JUN. AM. SOC. C. E.—The interest shown in this paper was very gratifying to the writers. Many of the discussions showed the results of long hours of study and preparation. To all those who contributed constructive criticisms, the writers wish to express their very sincere appreciation.

Several pertinent questions were raised that are of fundamental importance and vitally affect the validity of this method of flood routing. In seeking the answers to these questions, one must keep in mind that no exact method of flood routing has ever been devised. All methods are based on certain assumptions. Naturally, the accuracy of the results depends upon the extent to which the assumptions are in error. Any theoretical proof that a method is wrong is utterly meaningless unless it can be shown that the resulting error is of a magnitude so great as to render the results of little or no value. The writers recognize the assumptions involved in their flood-routing procedure and have attempted to evaluate the resulting errors so as to determine whether or not their magnitude is so great as to seriously impair the value of the procedure.

One of the assumptions upon which this method of flood routing is based is that the relationship between $(Q + O)$ and S is substantially the same during rising and falling stages. In many cases, especially in the upper reaches of a stream, this is true, but there are instances where this assumption may not seem justifiable. The error in such cases is due to the fact that a given amount of channel storage corresponds to a larger discharge when it occurs on the rising side of a graph than when it occurs on the falling side. In such cases the curve relating $(Q + O)$ to S becomes a loop for which, if $(Q + O)$ is plotted on the vertical axis, the upper part applies to the rising side of the graph and the lower part to the falling side. Each routing requires its own particular loop, the location of which must be fixed step by step as the routing proceeds. A method for accomplishing this is illustrated by an example. For this purpose the reach of the Ohio River from Pittsburgh, Pa., to Wheeling, W. Va., was selected because it is known that for any given stage at Wheeling there is an unusually great variation in discharge between the rising and falling stages. In order to simplify the example it is assumed that no correction is required at the upper station.

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[26] Prof., Hydr. Eng., Univ. of Michigan, Ann Arbor, Mich.

[27] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

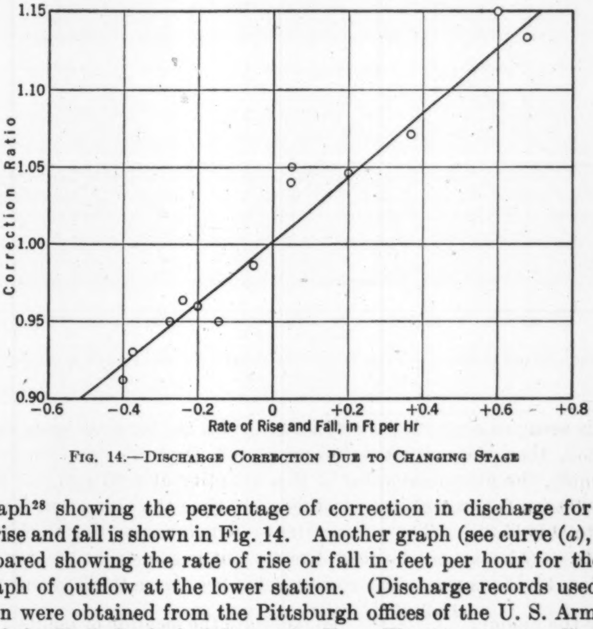

FIG. 14.—DISCHARGE CORRECTION DUE TO CHANGING STAGE

A graph[28] showing the percentage of correction in discharge for various rates of rise and fall is shown in Fig. 14. Another graph (see curve (*a*), Fig. 15) was prepared showing the rate of rise or fall in feet per hour for the actual hydrograph of outflow at the lower station. (Discharge records used in this discussion were obtained from the Pittsburgh offices of the U. S. Army Engineers and the U. S. Geological Survey.) From Fig. 14, corresponding correction factors were read and plotted as curve (*b*), Fig. 15. From this graph, the correction factor at any time during the course of the flood may be determined.

The curve relating $(Q + O)$ to S is obtained as in the original example except that values of O must be corrected before being plotted. For example, at noon of March 21, O is 111,000 cu ft per sec, Q is 17,000 cu ft per sec, and the storage is 1,778,000 (cu ft per sec)-hr. The rate of fall at this time was such that a correction factor of 8.0% must be applied to O. Since 8% of 111,000 is 9,000, the foregoing value of storage is plotted against 111,000 + 17,000 + 9,000, or 137,000 (cu ft per sec)-hr. When such values are plotted, a straight line is obtained that shows the relationship between S and $(Q + O)$ for stable river conditions. The auxiliary curve relating $(Q + O)$ to $S + 3\,(Q + O)$ is also plotted (see Fig. 16).

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[28] Data obtained from "River Discharge," by J. C. Hoyt and N. C. Grover, 3d Ed., John Wiley & Sons, Inc., New York, N. Y., 1914.

The routing is then determined by successive solutions of the following equation:

$$S_0 + 3\,Q_0 + 6\,Q_1 - 3\,O_0 = S_1 + 3\,(Q_1 + O_1) \qquad (16)$$

Eq. 16 corresponds to Eq. 4, the difference in coefficients being due to the fact that 6-hr intervals are used here instead of 2-hr intervals.

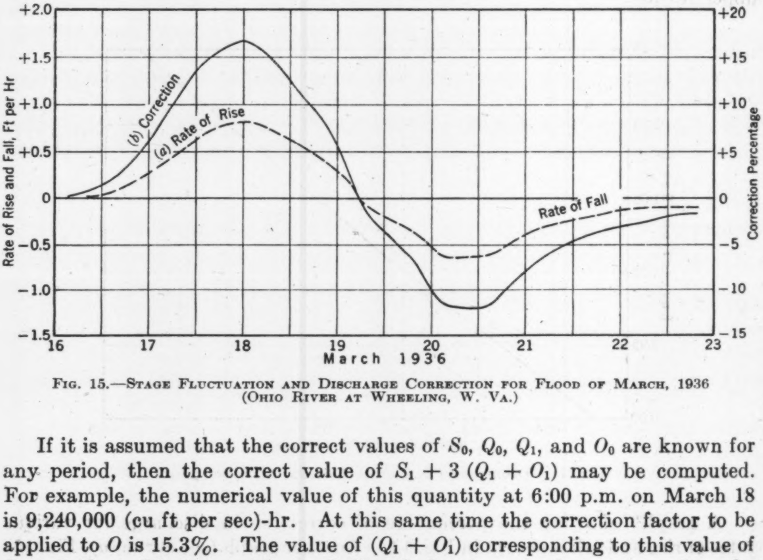

FIG. 15.—STAGE FLUCTUATION AND DISCHARGE CORRECTION FOR FLOOD OF MARCH, 1936 (OHIO RIVER AT WHEELING, W. VA.)

If it is assumed that the correct values of S_0, Q_0, Q_1, and O_0 are known for any period, then the correct value of $S_1 + 3\,(Q_1 + O_1)$ may be computed. For example, the numerical value of this quantity at 6:00 p.m. on March 18 is 9,240,000 (cu ft per sec)-hr. At this same time the correction factor to be applied to O is 15.3%. The value of $(Q_1 + O_1)$ corresponding to this value of $S_1 + 3\,(Q_1 + O_1)$ may not be read directly but is somewhat greater than the value given by the curve. The correct value lies above the S versus $(Q + O)$-curve by the amount $\frac{15.3}{100} \times O$. To facilitate the location of points at various distances above or below the S versus $(Q + O)$-curve, a series of straight parallel lines at intervals of 10,000 cu ft per sec above and below the line is drawn. Corresponding parallel lines are drawn for the auxiliary curve.

If, as a first trial, O is assumed to be about 200,000 cu ft per sec, the correction will be + 30,000 cu ft per sec. A point on the + 30,000 correction line of the auxiliary curve is located that has an abscissa of 9,240,000 (cu ft per sec)-hr. The corresponding value of $(Q + O)$ is seen to be 602,000 cu ft

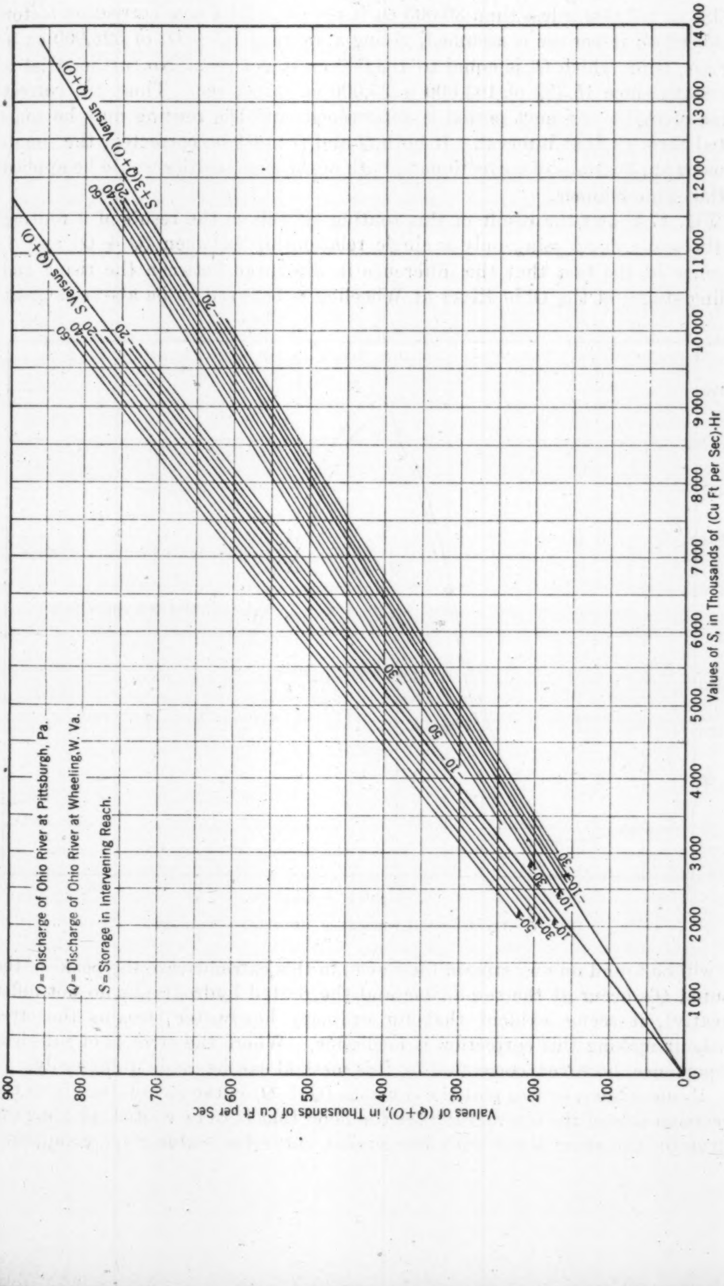

FIG. 16.—RELATION BETWEEN STORAGE AND DISCHARGE FOR STABLE RIVER CONDITIONS

per sec. Since Q is 435,000 cu ft per sec, O_1 is 167,000 cu ft per sec. However, 15.3% of 167,000 is less than 30,000 cu ft per sec, and a new correction factor of 25,000 cu ft per sec is assumed, giving a value of $(Q + O)$ of 598,000 cu ft per sec, from which O_1 is equal to 163,000 cu ft per sec. No further trial is necessary since 15.3% of 163,000 is 25,000 cu ft per sec. Thus, the correct value of O_0 for the next period is determined, and the routing may be computed through that interval. If both Q and O must be corrected, the simultaneous applications of corrections to both of these quantities would be applied in this same manner.

Fig. 17 shows the result of this routing as well as the result of a routing of the same flood using only a single relationship between $(Q + O)$ and S. In view of the fact that the difference in discharge between the rising and falling stages of the Ohio River at Wheeling is believed to be about as great as will be found on any stream (and even in this extreme case the peaks of the routed Q's occur at the same time and the routed hydrographs do not differ greatly), it seems evident that for ordinary headwater streams the error without making this correction is negligible. Where the error is of sufficient importance, however, correction by this method can be made quite easily.

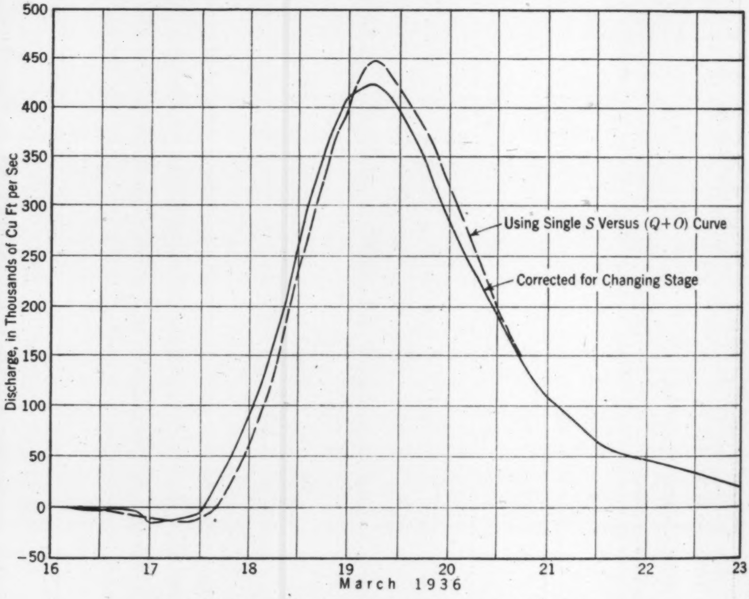

FIG. 17.—PITTSBURGH HYDROGRAPH ROUTED TO WHEELING

If one wishes, he can plot the S versus $(Q + O)$-curve as obtained from the recession side of the hydrograph for the flood that is to be routed, plotting the curve on the same sheet with the similar curve for stable river conditions.

Then after the routing has progressed until the time when the corrected values of $Q + O$ fall on this curve, it can be used for the remainder of the computations without further corrections.

In several of the discussions the question was raised as to whether or not the relationship between $(Q + O)$ and S is necessarily the same for all floods on any given basin. Some variation in this curve might be expected for floods produced by different rainfall patterns, as suggested by Messrs. Tatum and Hickman. As an illustration of the extent of this difference, four storms on the drainage area above Sewickley, Pa., were chosen for study. In these examples O is the discharge of the Ohio River at Sewickley, and Q is the sum of the discharges at Greensboro, Pa., on the Monongahela River, at Connellsville, Pa., on the Youghiogheny River, and at Parkers Landing, Pa., on the Allegheny River. Table 3 shows the discharge at each station and also that contributed by the intervening area, as well as the percentage that each is of the total Sewickley discharge. It may be observed from the table that storms of considerably different magnitude are represented and that the percentage of the total runoff contributed from each source varies over a wide range. For instance, the contribution from the intervening area ranges from 22.4% to 55.5% of the total runoff.

TABLE 3.—COMPARISON OF YIELDS FROM DIFFERENT TRIBUTARY AREAS DURING FOUR STORMS

Date	Discharge at Sewickley[a]	CONNELLSVILLE		GREENSBORO		PARKERS LANDING		INTERVENING AREA	
		Discharge[a]	%[b]	Discharge[a]	%[b]	Discharge[a]	%[b]	Discharge[a]	%[b]
March 18, 1936	21,384	2,280	10.6	4,908	22.9	2,328	10.9	11,868	55.5
October 30 1937	8,784	1,416	16.1	4,824	54.9	576	6.6	1,968	22.4
January 27, 1937	12,240	368	3.0	2,400	19.6	4,056	33.1	5,410	44.2
February 5 1939	5,856	1,008	17.2	2,712	46.2	420	7.2	1,716	29.3

[a] Discharge volumes are in 1,000 (cu ft per sec)-hr. [b] Percentage of total.

Storage-discharge curves derived from each of these storms are shown in Fig. 18. Although these curves are not identical, neither do they differ greatly. The most divergent are those for the floods of February 5, 1939, and January 27, 1937. The flood of March 18, 1936, was routed to Sewickley, using each of these two curves. The resulting routed graphs are shown in Fig. 19. It will be observed that the timing of the peak is about the same for both. The peak discharge differs by 7,000 cu ft per sec, which is about 2.3% of the total peak discharge. In this case it would seem to make little practical difference as to which of the four curves is selected for use in routing. However, even the difference between these two routed graphs does not represent an error that is inherent in the method. It represents such an error only when an S versus $(Q + O)$-curve that has been derived from the hydrograph of a flood which was produced by one particular storm pattern is used for routing a flood that resulted from a storm of a different pattern. Ordinarily, such use is unnecessary. Unless an assumed or hypothetical flood is to be routed, an S versus $(Q + O)$-curve can be derived readily and used for each different flood, and no such error will result. However, if a hypothetical flood is to be routed, its routing will depend upon the storm pattern producing it. If the pattern corresponds to any of the S versus $(Q + O)$-curves of the recorded floods of the past, then that curve can be used properly. When it is not known what storm pattern caused the hypothetical flood, the best procedure would be to route the flood by two or more of the most divergent S versus $(Q + O)$-curves obtained from past records. This procedure would show the

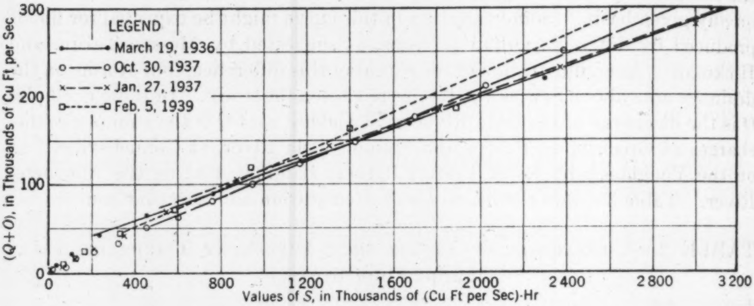

FIG. 18.—STORAGE-DISCHARGE RELATIONS DERIVED FROM FOUR FLOODS

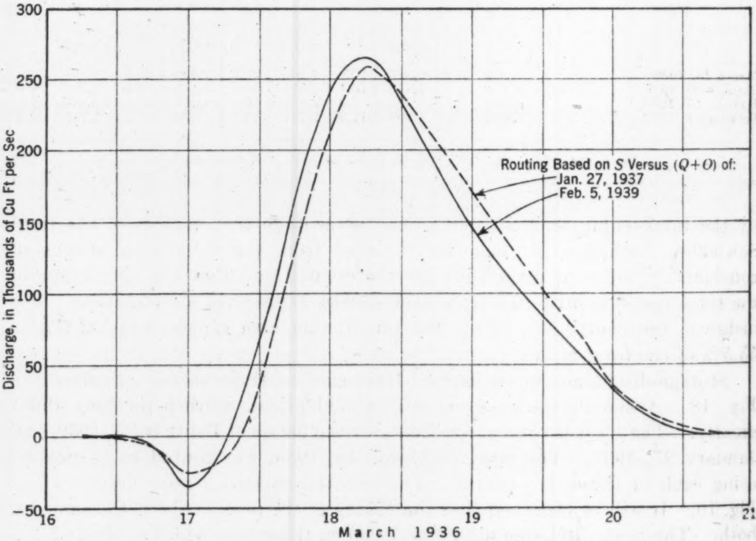

FIG. 19.—FLOOD OF MARCH, 1936, ROUTED TO DASHSHIELDS, USING S VERSUS $(Q + O)$-CURVES DERIVED FROM TWO DIFFERENT FLOODS

approximate range of results that might be expected because of different storm patterns.

The writers agree with the comment by Mr. Linsley that the check on the computed inflow, obtained by deducting the total observed upstream flow from the total observed outflow at the lower station, is only an arithmetic check. It is difficult, however, to reconcile the subsequent statement of Mr. Linsley, and also that of Messrs. Horonjeff and Crowle ("* * * the agreement of the composite of the routed hydrographs with the gaged hydrograph O at the lower end of the reach is merely a check on the arithmetical operations" and "is in no sense a check on the accuracy of the gaging at the lower station, or the correctness of the lines in Fig. 2"), with the statement of Mr. Woster that "It has been found necessary in most cases to adjust the routed hydrographs of Q and I after completion of the procedure because when they are combined they do not check, even closely, the actual hydrograph." This difference in opinion is of minor importance, however, for it involves only the question as to whether the agreement of the combined routed hydrographs with the gaged hydrograph O constitutes a bona fide check on the accuracy of the routing procedure or whether it is only a check on the arithmetic accuracy—the question of the fundamental correctness of the method is not at issue. For the sake of argument let it be granted that by the use of the two S versus $(Q + O)$-curves obtained by Messrs. Horonjeff and Crowle, and shown in Fig. 8, two different sets of routed Q-curves and I-curves will be obtained which, when combined, will coincide exactly with the measured hydrograph of O. This fact proves nothing with respect to the correctness of the method for the simple reason that the random S versus $(Q + O)$-curve does not represent the relation between S and $(Q + O)$ for the Pajaro River, whereas the other one does.

Inasmuch as the dependability of the storage equation is unquestioned, the accuracy of the Q-curve in Fig. 7 can depend only upon the following:

(*a*) The accuracy of the computations and of the discharge records at Sargent and at Chittenden;

(*b*) The correctness of the separation of the base flow from the surface runoff;

(*c*) The existence of the same relation between S and $(Q + O)$ during the rising stage as during the receding stage; and

(*d*) The use of an S versus $(Q + O)$-curve that has been derived from the flood hydrograph which is to be routed; otherwise a small and relatively unimportant error may result.

The accuracy of all methods of routing depends upon these factors. The error resulting from an incorrect separation (item (*b*)) is usually negligible and never of any great importance. This separation is necessary only for obtaining the S versus $(Q + O)$-curve. Base flow can be included in the routing operation as explained by Messrs. Horonjeff and Crowle. On streams for which the relation in item (*c*) differs excessively, the error from this source can be eliminated through the use of a loop for the expression of the relation between S and $(Q + O)$, as previously explained.

The writers are deeply indebted to Mr. Sherman for his interesting and clever suggestion for testing the validity of this method of flood routing. Mr.

Sherman's example provided an excellent check of the quantitative values of storage in the intervening reach.

Several of the observations of Mr. Brosio are decidedly at variance with the views of the writers. The significance which Mr. Brosio assigns to the quantity $\frac{S}{Q_u}$ is open to serious question. In the first place, "Q_u = uniform rate of flow, through entire reach," is a purely fictitious quantity and cannot exist in any flood-routing problem. If Mr. Brosio means to imply that, because $\frac{S}{(Q + O)}$ = a constant, this quantity "represents a constant time of travel, and, consequently, a constant mean velocity of flow irrespective of stage," it seems to the writers that this is quite a broad assumption. In this expression either Q or O can be zero without S becoming zero. If O is zero, then the expression gives the time required to provide the storage S at the average rate Q; if Q is zero, then the quantity represents the time required to deplete the storage at the average rate O; but when Q and O both have finite values, the physical significance of $\frac{S}{(Q + O)}$ is not so apparent.

It should be remembered in this connection that S, as used in the paper, represents the storage in all of the channels above Morgantown, exclusive of those above Fetterman. That this is true is apparent from the fact that it was derived by deducting from the total channel storage above Morgantown the channel storage above Fetterman. Mr. Brosio has taken the liberty of defining S as the storage in the Fetterman-to-Morgantown channel. If his definition were correct, then, of course, Mr. Brosio's conclusion that the writers had assumed that I is equal to zero would be justified.

Under the heading "II. Storage as a Function of (Inflow + Outflow)," Mr. Brosio assumes an inflow curve and then computes an outflow curve from it. He then finds certain inconsistencies in the computed outflow curve and concludes that there is something wrong with the method. The assumed inflow curve is just as glaringly inconsistent as the outflow curve. No stream ever produced such a hydrograph, but if it had it would be physically impossible for the resulting outflow hydrograph to be like the "adjusted" outflow graph shown by the short-dash line in Fig. 12. The base of the routed graph as it proceeds downstream must become broader and the crest lower. To deny this statement is to deny that velocity varies with the hydraulic radius and slope.

The statement by Mr. Hickman that "The time T should be no longer than the time of travel of the flood wave through the reach" is correct. In fact, there are reasons for believing that this time interval should be equal to the time of travel of the flood wave through the reach. However, several different routings of the same flood, in which different time intervals were used, produced nearly identical results, so it is not believed that this rule must be adhered to rigidly. Mr. Hickman suggests that

> "* * * the manner in which the flood peak moves downstream, if the intervening area were not there, is decidedly different from the manner in which the peak will move downstream under the influence of the discharge of the intervening area."

This criticism would be valid if the S versus $(Q + O)$-curve were not a straight line; but inasmuch as it is a straight line the routed flows are dependent only upon the changes in storage and not upon the actual volumes of storage existing at the different times.

Mr. Hickman further states that

> "* * * the most serious objection to the method is that it is based on the assumption that routing within a basin may depend on what has happened during one flood. Can it be said that a major storm, and the resulting flood, will proceed according to the same plan every time?"

The answer to this question is most assuredly "No!" However, the writers did not intend this paper to provide a method of determining runoff from rainfall. It is true that the hydrograph that results from any given storm depends upon the storm pattern, the condition of the ground, and a great many other factors, as Mr. Hickman states, and will therefore not proceed according to the same plan every time. It is beyond the province of flood routing, however, to start with rainfall. The basic data for this problem are flood hydrographs, either real or assumed; the determination of those hydrographs is an entirely separate and distinct problem. The problem of flood routing, as understood by the writers, is to take a flood hydrograph at an upstream station and determine when and how that hydrograph appears when it reaches a downstream station —just that and nothing more.

The most satisfactory use of the method depends upon the existence at the upper and lower stations of at least one, and preferably several, records of major floods, for it is only through records of actual performance that one is able to learn of the channel characteristics and their influence upon the flood hydrograph. The hydrographs resulting from any particular storm, and the corresponding curve of relation between S and $(Q + O)$, constitute a veritable encyclopedia of information pertaining to the characteristics of the storm pattern, drainage basin, and stream channel. To collect, in the field, all the data and to make all the measurements and computations that would be needed to derive the S versus $(Q + O)$-curve would involve a tremendous amount of work.

The actual significance of I as determined by the solution of Eq. 2 is interesting. Does this hydrograph I represent the actual rate at which overland flow from the intervening area reaches the stream channel and, if so, why is it that, when this hydrograph I is routed down to the lower station, the same hydrograph I' is obtained, regardless of whether this inflow comes from tributaries in the upper or lower reaches? The answer to this seeming inconsistency is to be found in the fact that the computed values of I represent the inflows into an imaginary reservoir located just upstream from the lower station. That this is true is indicated by the manner in which these values of I are derived. As suggested by Mr. Tatum, the hydrograph of I' could have been derived by deducting the values of Q' directly from the corresponding values of O. It may be observed that the graph I intersects I' at the crest of the latter, whereas the graph Q' reaches its peak considerably after its intersection with the graph of Q. This delayed peak in Q' is caused by the lag interval required for Q to reach the lower station, whereas there is actually no such lag in I since, from the nature of its derivation, I represents the rate of inflow in the imaginary reservoir located just above the lower station.

Several correspondents have suggested to the writers that a convincing test of the accuracy of the method could be made by routing a flood to which there was no contribution from the intervening area. Although no record could be found of a flood that exactly met that specification, the records of several floods to which the contribution from the intervening area amounted to only 15% or 20% of the total flow were found and routed. In one of these instances the total flow from the intervening area had passed the lower station before the peak occurred, and in another case very nearly all had passed ahead of the crest. In every case the routed peak occurred at the same time as the observed peak, and the graphs were nearly identical.

Perhaps others interested in this problem may have access to flood records for which there was no inflow from the intervening area. Records of floods occurring on two or more tributaries would be especially interesting and valuable, such as the North, Middle, and South rivers (tributaries to the Shenandoah River and shown in Fig. 3) with no inflow from the intervening area above Lynnwood. If one should route the flow from each of these tributaries by the use of its own S versus $(Q + O)$-curve, and then combine these routed graphs and obtain a graph that checks the recorded graph at the lower station, then surely this would constitute a bona fide check of the method. Conversely, a failure in agreement would provide similar evidence that the method is faulty, provided only (in both cases) that the discharge records are accurate.

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Paper No. 2167

PIPE-LINE FLOW OF SOLIDS IN SUSPENSION

A SYMPOSIUM

PAGE

Effect on the Characteristics of Centrifugal Pumps.
By LEIGH C. FAIRBANK, JR., ESQ. 1564

Mechanics of Flow, with Noncolloidal, Inert Solids.
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WITH DISCUSSION BY MESSRS. ARTHUR L. COLLINS, H. A. EINSTEIN, H. E. BABBITT AND D. H. CALDWELL, AND WARREN E. WILSON.

NOTE.—Published in October, 1941, *Proceedings*.

EFFECT ON THE CHARACTERISTICS OF CENTRIFUGAL PUMPS

BY LEIGH C. FAIRBANK, JR.,[1] ESQ.

SYNOPSIS

The centrifugal pump, by merit of its principle and construction, offers one of the best means for pumping liquids containing relatively high concentrations of suspended material, as is evidenced by its wide use in dredging and mining operations and various industrial processes. However, the assumption has often been made that the total head developed by a centrifugal pump at a given discharge is independent of the concentration of foreign material carried by the liquid being pumped. Tests have proved that this assumption is erroneous, but no methods have been developed to predict this change in head. The design and choice of a pump to meet the conditions of any particular job are based upon the past performance of other pumps, operating under similar conditions, and quite frequently on the clear-water characteristics of a standard pump; this latter method, however, amounts to nothing more than a very rough estimate.

The effect of suspended material upon the head-capacity characteristics is of particular importance in the dredging field since the performance of a pipe-line dredge is dependent on the characteristics of the various units of the dredge, the prime mover, the pump, the pipe line, and the cutter head. Since the type of material handled and the head developed may vary over a wide range, it is necessary to know the effect of various materials upon the individual units if an accurate prediction or estimate of the dredge performance is to be made and the economic operating conditions are to be determined.

Past investigation of this subject is very limited, including only brief reports by the Mississippi River Commission (1)[2]; by the Japanese investigators, E. Mikumo, T. Nishihara, and M. Takahara (2); by W. B. Gregory, M. Am. Soc. C. E. (3); and by M. P. O'Brien, M. Am. Soc. C. E., and R. G. Folsom (4). These investigations established the fact that pump characteristics are different for solid-water mixtures than for water alone, but since none of these reports include a complete description of the pumps used or of the material in suspension it is impossible to make any quantitative analysis of the observed change in characteristics.

The purpose of this investigation was to obtain the characteristics of a centrifugal pump of known dimensions while pumping in suspension material of various known sizes and concentrations, and to make a quantitative analysis of the change in characteristics due to the material in suspension.

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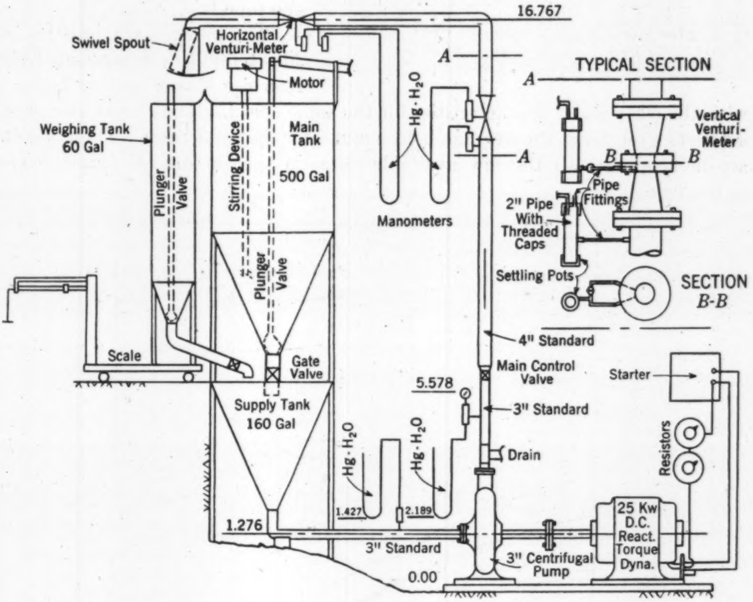
[1] Capt., Corps of Engrs., U. S. Army.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (Appendix II).

EXPERIMENTAL TESTS

The system used in conducting the experimental tests was of a recirculating type as shown in Fig. 1. The pump impeller was 12 in. in diameter, with an inlet diameter of $5\frac{1}{4}$ in. and a width of $1\frac{3}{4}$ in. The shape of the impeller vanes closely approximated the form of a logarithmic spiral of the equation



16.767
Swivel Spout
Horizontal Venturi-Meter
Motor
A
A
A
TYPICAL SECTION
Hg·H$_2$O
Weighing Tank 60 Gal
Main Tank
500 Gal
Stirring Device
Plunger Valve
Plunger Valve
Vertical Venturi-Meter
B
B
Pipe Fittings
2" Pipe With Threaded Caps
Manometers
Settling Pots
SECTION B-B
4" Standard
Scale
Gate Valve
Main Control Valve
Starter
5.578
3" Standard
Supply Tank 160 Gal
Hg·H$_2$O
Hg·H$_2$O
Drain
Resistors
1.427
2.189
25 Kw D.C. React. Torque Dyna.
1.276
3" Standard
3" Centrifugal Pump
0.00

FIG. 1

$$r = e^{a\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which the average value of a is 0.440, r is the radius, and θ is the central angle (see Appendix I).

The suspended material used in the tests included two grades of sand and one grade of oil-well drilling mud. Specific gravity and size distribution were determined for each material in accordance with American Society for Testing Materials (A. S. T. M.) standards. Average fall velocities for the two sands were determined by weighting observed fall velocities for fractions of each sand obtained from mechanical analysis (see Table 1).

Clear-water runs were made before and after pumping each different material in suspension, for the purpose of determining the effect of wear on the pump characteristics. A complete series of runs was made at pump speeds of 1,000 and 1,200 rpm. The concentration of the suspended material was varied from 10% to 25% by net volume. Generally, the rates of discharge covered a range from 0.3 cu ft per sec to 1.0 cu ft per sec, although at the higher concentrations this lower limit could not be reached without clogging the pipe line.

TABLE 1.—RELATIVE FALL VELOCITIES

Material	Specific gravity	Median diameter, in mm	Fall velocity, in ft per sec
Monterey No. 4 sand.....	2.655	0.800	0.384
Crushed Del Monte sand..	2.630	0.034	0.0012
Mojave rotary mud.......	2.726	<0.001	....

The pump characteristics for Monterey No. 4 sand at 1,000 rpm are shown in Fig. 2, along with the clear-water characteristics for the same speed. The clear-water characteristics represent the average before and after the sand runs. These results are in keeping with the few available data on suction dredge pumps taken in the field.

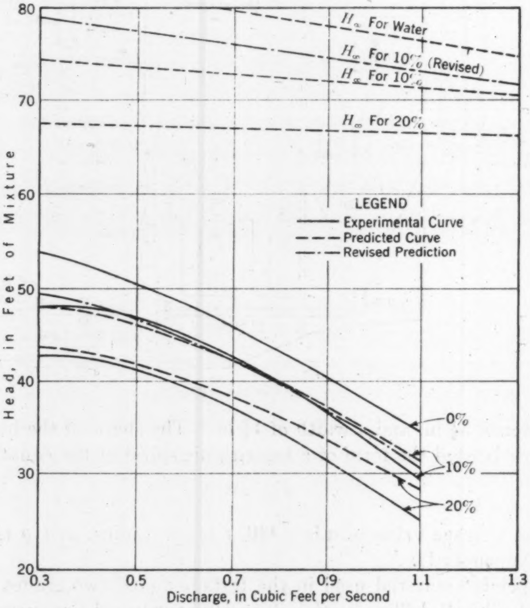


FIG. 2.—COMPARISON OF EXPERIMENTAL AND PREDICTED CHARACTERISTICS WITH MONTEREY NO. 4 SAND PUMPED AT 1,000 RPM

The head-capacity characteristics, expressed in feet of mixture, and the efficiency characteristics for the crushed Del Monte sand for all concentrations were found to coincide with the average clear-water characteristics. The ratio of power input to specific gravity of mixture also coincided with the power input characteristic for clear water. The pump characteristics for the Mojave rotary mud at 1,000 rpm are shown in Fig. 3.

The usual affinity relations for rotational speeds ($Q \propto N$, $H \propto N^2$, and $HP \propto N^3$) were applied to the solid-water characteristics at 1,000 rpm, and results compared favorably with experimental characteristics at 1,200 rpm.

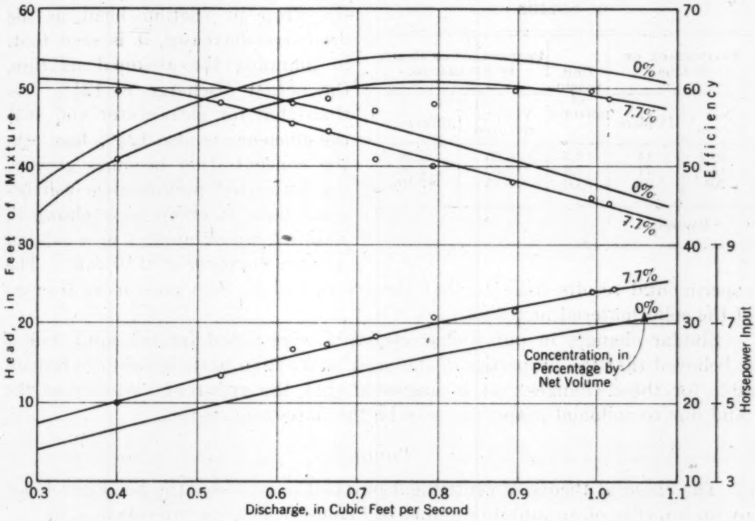


FIG. 3.—CHARACTERISTICS OF A 3-IN. DREDGE PUMP WITH MOJAVE ROTARY MUD, AT 1,000 RPM

By applying these results to a practical problem some indication is afforded as to the effect of the errors in the assumptions commonly made in the field in estimating dredge performance. If a dredge pump is assumed of the same specific speed as the pump tested herein, the characteristics of the two pumps will be proportional. Assume a dredge pump of the following description:

Description	Dimension
Dimensions of Discharge Pipe, in Ft:	
Diameter	2
Length	1,950
Discharge velocity, in ft per sec	16.3
Pump speed, in rpm	212
Impeller diameter, in ft	6.7
Static lift, in ft	30
Total head, in ft	86

The foregoing conditions correspond to the conditions of maximum efficiency, both for the test pump at 1,000 rpm and the dredge pump at 212 rpm. Assume now that the dredge is pumping a mixture consisting of 15% by net volume of

Monterey No. 4 sand. (This is equivalent to a unit weight of mixture of 77.8 lb per cu ft, the upper limit reached in dredging operations in the field.) In ordinary practice the rate of discharge and efficiency of the dredge pump would be estimated as the same as for clear water. However, referring to Fig. 2, and taking into account the drop in friction head as the discharge decreases, it is seen that, in pumping the assumed mixture, the actual discharge is 12½% less than that for clear water and that the efficiency is also 12½% less. As the concentration becomes greater, the estimated performance will become more in error, as is shown in Table 2, which applies to a sand of median diameter of 0.80 mm. The experimental results indicate that the errors will become greater as the size of the solid material increases.

TABLE 2.—ESTIMATED PERFORMANCE WITH VARYING PERCENTAGES OF SOLIDS

Percentage of Solids		Unit weight of mixture	Percentage of Error in Estimated:	
Net volume	In place		Yardage removed	Efficiency
10	14	72.5	6.25	9.38
15	21	77.8	12.50	12.50
20[a]	28	83.0	18.75	17.19

[a] Excessive.

Similar changes in pump characteristics were noted for the mud, but it is believed that the properties of the mud, other than particle size, are responsible for these changes. It is suggested that the apparent viscosity of the mud due to colloidal properties may be the important factor.

THEORY

The classical theory of centrifugal pumps (5) expresses the head developed by an impeller of an infinite number of blades, neglecting pre-rotation, as

$$H_\infty = \frac{U_2 v_{u2}}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which H_∞ is the head, in feet of fluid, U_2 is peripheral velocity of impeller, and v_{u2} is the tangential component of the absolute velocity at exit from the impeller. Eq. 2 gives the head in feet of fluid being pumped. To derive an expression for the actual head developed by a pump, it is necessary to include certain losses occurring both within the impeller and within the pump casing. Experiments indicate, however, that these losses are similar for both sand-water mixtures and clear water.

Referring to Eq. 2, it is seen that under conditions of constant speed the only manner in which the head developed could become less is that in which the value of v_{u2} also decreases. Since experiments indicate that for a sand-water mixture the head developed is less than that for water alone, at the same discharge, then the loss must be due to a decrease in the value of v_{u2} for the solid-water mixture, or, in other words, the solid particle must leave the impeller with a greater relative velocity than the water particles. This is shown graphically in Fig. 4.

From the geometry of the velocity diagrams at the exit from the impeller, in which $V_{f(2)}$ is the radial component of relative velocity with respect to the impeller blade at the exit, and β is the blade angle:

$$v_{u2} = U_2 - V_{f(2)} \cot \beta \qquad (3)$$

Assume an impeller of an infinite number of blades and add to the assumptions made in the classical theory the assumption that the solid particles also travel a relative path that follows the contour of the vanes. Then an expression

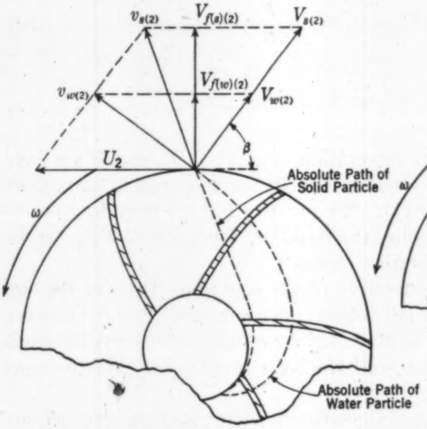


FIG. 4.—VELOCITY DIAGRAMS OF SOLID AND WATER PARTICLES AT EXIT FROM IMPELLER

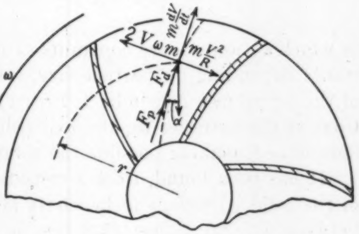


FIG. 5.—DIAGRAM OF FORCES ACTING ON SOLID PARTICLES PASSING THROUGH IMPELLER

can be obtained for the head developed by the impeller by adding the expressions for power imparted to the water and to the solid particles (which are determined from the time rate of change of moment of momentum), equating this sum to the expression for total power (which equals the product of the discharge per unit time, the unit weight of the mixture, and the head developed in feet of mixture), and then solving for the value of this head. Performing the foregoing operations, the expression for the theoretical head is obtained, expressed in feet of mixture,

$$H_\infty = \frac{S_s\, p_s\, U_2\, (U_2 - V_{f(s)(2)} \cot \beta) + (1 - p_s)\, U_2\, (U_2 - V_{f(w)(2)} \cot \beta)}{g\, S_a} \qquad (4)$$

In Eq. 4, subscript s refers to solid particles, w to water particles, 2 to the exit from the impeller, a to the average of solid-water mixture, S is specific gravity, and p is concentration of solids in percentage by volume.

The solution of Eq. 4 requires the value of $V_{f(w)(2)}$, which can be determined by the continuity relationship on the basis that the impeller is completely filled with water, and the value of $V_{f(s)(2)}$, which is not so readily determined.

Referring to Fig. 5, which shows the forces acting on a solid particle in a fluid at any position within the impeller, it is seen that there are six forces acting. If the forces acting tangent to the impeller vane are equated:

$$m \frac{dV_s}{dt} = m\, \omega^2\, r \cos \alpha - F_d - F_p \qquad (5)$$

in which m is the mass of a solid particle and ω is the angular velocity of the impeller. By assuming that F_d is a viscous force, that F_p is the force due to the pressure gradient in the fluid surrounding the particle, and that the impeller vane approximates a logarithmic spiral, and by taking an average value of V_w as it appears in the expressions for F_d and F_p, it is possible to obtain the following equations describing the motion of the solid particle:

$$r = C_1 e^{Xt} + C_2 e^{Yt} - \frac{F}{B} \qquad (6a)$$

and

$$V_s = \frac{\sqrt{1 + a^2}}{a} (C_1 X e^{Xt} + C_2 Y e^{Yt}) \qquad (6b)$$

in which C_1 and C_2 are constants of integration, and X, Y, F, and B are constants depending on particle size, settling velocity, and operating conditions of the pump (see Appendix I, Eqs. 11). By substituting the boundary conditions at the entrance to, and exit from, the impeller, the value of $V_{s(2)}$ can be determined, making possible the solution of Eq. 4.

It has been found, from a consideration of the equations, that, as the size of the solid particle, and hence its fall velocity, become smaller, the difference between V_s and V_w becomes less, so that, for very small solid particles (such as crushed Del Monte sand) in suspension, the head developed will be the same as for water alone.

The predictions of the operating characteristics while pumping a suspension may be obtained by expressing the difference between H_∞ as computed by Eq. 4 and H_∞ for clear water as a percentage loss, and applying this loss to the actual clear-water characteristics of the pump.

A comparison between predicted and test characteristics for Monterey No. 4 sand at 1,000 rpm is given in Fig. 6. For the rating point of the pump the predicted characteristic is correct for a concentration of 10% by net volume and is within 5% of the test value for a concentration of 20%. This same agreement was found to hold true at 1,200 rpm. A sample computation for this prediction is given in Appendix I.

The errors introduced through the assumptions made in this discussion are well appreciated. For the most part they are the same as those made in the classical theory, which are well known to be in error and for which it is possible to make corrections. Probably the least valid of the assumptions, at least when the suspended material has a high settling velocity, is that as to the nature of the resistance force between the solid and water particles. It is a well-known fact that Stokes' law is valid only in the range of Reynolds' number less than 0.50, and, since this value is probably greatly exceeded in most cases, it would be more nearly correct to use Rittenger's or Allen's law (6). However, the application of such a law involves the solution of a complicated differential equation. For the pump and suspended material used in the experimental tests, values of H_∞ were computed by a trial-and-error method, using Rittenger's law. These values were about 7% higher than those determined by the less accurate solution. However, if the difference between H_∞ for water and H_∞ for the suspension, computed by trial and error, is applied directly as a loss to the actual clear-water characteristics, the predicted characteristics for the suspension agree within 4% with the experimental characteristics. This agreement is shown in Fig. 6. In view of the increased difficulty and expenditure of time in making the trial-and-error solution, the method first described is the more practical, especially since both methods are subject to several approximations.

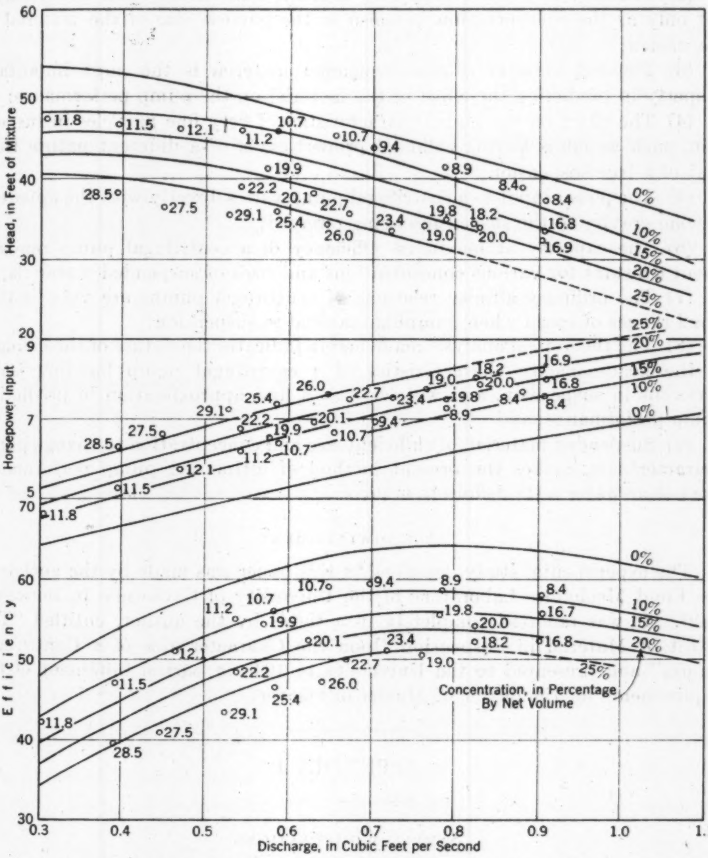


FIG. 6.—CHARACTERISTICS OF A 3-IN. DREDGE PUMP WITH MONTEREY NO. 4 SAND, AT 1,000 RPM

CONCLUSIONS

The following conclusions are based on the observations made in this investigation:

(1) At a given capacity the head developed, in feet of mixture, by a centrifugal pump handling material in suspension, is less than that developed for water alone;

(2) The drop in the head-capacity characteristics at constant speed varies not only as the concentration but also as the particle size of the material in suspension;

(3) The fall velocity of the suspended material is the most important property in predicting the effect of the material on the pump performance;

(4) The effect on the pump characteristics of very fine particles in suspension, such as muds having colloidal properties, is of a different nature than that of a true suspension;

(5) The power input to a centrifugal pump varies directly with the apparent specific gravity of the suspension being pumped;

(6) The capacity at maximum efficiency of a centrifugal pump remains about constant for various concentrations and sizes of suspended materials;

(7) The ordinary affinity relations of centrifugal pumps are valid within small ranges of speed when pumping material in suspension;

(8) The theoretical analysis made herein indicates the nature of the changes in the head-capacity characteristics of a centrifugal pump handling solid materials in suspension, and will serve as a first approximation in predicting pump performance; and

(9) Suspended material, of sufficient size and concentration to change pump characteristics, makes the present method of estimating pump performance from clear-water tests definitely in error.

ACKNOWLEDGMENT

The experimental study described in this paper was made by the author in the Fluid Mechanics Laboratory of the University of California, in Berkeley, Calif. It was reported completely in a thesis by the author, entitled "The Effect of Material in Suspension Upon the Characteristics of a Centrifugal Pump," and presented to the University in 1940 in partial fulfilment of the requirements for the degree of Master of Science.

APPENDIX I

SAMPLE COMPUTATIONS

In order to make a sample computation it will be necessary to review briefly the steps in deriving Eq. 5 so as to determine the constants X, Y, F, and B. The term F_p which appears in Eq. 5 will be neglected since, for the particular material in suspension, this term affected the solution of that formula only 0.5%. From the assumption that F_d is a viscous force, Stokes' law states

$$F_d = K\,(V_s - V_w) \qquad (7)$$

From the assumption that the impeller vane is a logarithmic spiral: $r = e^{a\theta}$

(Eq. 1); cos α = a constant; and

$$V_s = \frac{dr}{dt}\frac{\sqrt{1 + a^2}}{a} \qquad (8)$$

By taking an average value of V_w as it appears in Stokes' law, Eq. 5 may be written:

$$\frac{d^2r}{dt^2} = \frac{a\,\omega^2\,r \cos\alpha}{\sqrt{1 + a^2}} - \frac{K}{m}\frac{dr}{dt} + \frac{a\,K\,V_{(w)\,\text{average}}}{m\,\sqrt{1 + a^2}} \qquad (9)$$

Let:

$$A = \frac{K}{m} \qquad (10a)$$

$$B = -\frac{\omega^2\,a \cos\alpha}{\sqrt{1 + a^2}} \qquad (10b)$$

and

$$F = -\frac{a\,K\,V_{(w)\,\text{average}}}{m\,\sqrt{1 + a^2}} \qquad (10c)$$

Substituting Eqs. 10 in Eq. 9:

$$r = C_1\,e^{Xt} + C_2\,e^{Yt} - \frac{F}{B} \qquad (6a)$$

and

$$V_s = \frac{\sqrt{1 + a^2}}{a}\,(C_1\,X\,e^{Xt} + C_2\,Y\,e^{Yt}) \qquad (6b)$$

in which (to simplify typography):

$$X = \tfrac{1}{2}\,(-A + \sqrt{A^2 - 4B}) \qquad (11a)$$

and

$$Y = \tfrac{1}{2}\,(-A - \sqrt{A^2 - 4B}) \qquad (11b)$$

The physical properties of the pump are:

a = 0.44 (the constant in equation of a logarithmic spiral);
r_1 = 0.167 ft (inlet radius of impeller);
r_2 = 0.500 ft (exit radius of impeller);
b_1 = 0.183 ft (width of impeller passage at inlet);
b_2 = 0.146 ft (width of impeller passage at exit);
A_1 = 0.192 sq ft (area of impeller passage at inlet);
A_2 = 0.459 sq ft (area of impeller passage at exit);
α = 66° 16′ (from the equation of the blade; the acute angle between the tangent to the blade and the radius of the impeller);
β = 23° 44′ (from the equation of the blade; blade angle at 1,000 rpm);
ω = 104.72 radians per sec (angular velocity of impeller); and
U_2 = 52.36 ft per sec (peripheral velocity of impeller).

The physical properties of Monterey No. 4 sand are: Median diameter, D = 0.800 mm; fall velocity, v = 0.384 ft per sec; and specific gravity, S_s

= 2.655. From Stokes' law—

$$K\,v = \frac{\pi\,D^3}{6}\,(S_s - 1)\,w \qquad (12)$$

Also, the mass of a solid particle is

$$m = \frac{\pi\,D^3\,S_s\,w}{6\,g} \qquad (13)$$

and $\frac{K}{m} = 52.271$.

Assume a pump speed of 1,000 rpm, a discharge of 1 cu ft per sec, and a concentration of sand of 10% by net volume. Substituting in Eqs. 10 and 11: $A = \frac{K}{m} = 52.271$; $B = -1{,}776.603$; $F = -174.188$; $\frac{F}{B} = 0.098$; $X = 23.460$; and $Y = -75.731$.

When $r = r_1 = 0.167$ ft, $t = 0$ and $V_{s(1)} = \frac{Q}{A_1}\sin\beta = 12.956$ ft per sec. Substituting in Eq. 6*a*:

$$0.167 = C_1 + C_2 - 0.098 \qquad (14a)$$

Substituting in Eq. 6*b*:

$$5.221 = 23.460\,C_1 - 75.731\,C_2 \qquad (14b)$$

Solving for C_1 and C_2: $C_1 = 0.255$ and $C_2 = 0.010$.

When $r = r_2 = 0.500$ ft, substitute in Eq. 6*a*: $0.500 = 0.255\,e^{23.460t} + 0.010\,e^{-75.731t} - 0.098$; and $t_2 = 0.0362$ sec. Substituting in Eq. 6*b*: $V_{s(2)} = 34.536$ ft per sec. Furthermore, $V_{f(s)(2)} = V_{s(2)}\sin\beta = 13.928$ ft per sec; and $V_{f(w)(2)} = \frac{Q_w}{A_2} = 1.961$ ft per sec. Substituting in Eq. 4: $H_\infty = 67.809$ ft, for Monterey No. 4 sand, at 1,000 rpm, 1 cu ft per sec, and 10% concentration. Repeating the foregoing steps for the same conditions but for a discharge of 0.3 cu ft per sec, $H_\infty = 73.195$ ft.

It is to be noted that the foregoing results vary with those shown in Fig. 6 but give a much better agreement between the predicted and actual characteristics.

APPENDIX II

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(3) **"Pumping Clay Slurry through a Four-Inch Pipe,"** by W. B. Gregory *Mechanical Engineering*, Vol. 49, 1927, pp. 609–616.

(4) **"The Transportation of Sand in Pipe Lines,"** by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Vol. 3, No. 7, 1937, pp. 343–384.

(5) **"Centrifugal Pumps,"** by R. L. Daugherty, McGraw-Hill Book Co., Inc., New York, N. Y., 1915.

(6) **"The Laws of Motion of Particles in a Fluid,"** by R. G. Lunnon, *Transactions*, Institution of Mining Engrs., Vol. LXXVII, 1929, pp. 65–74.

(7) **"A Study of Some Phases of Chemical Control in Clay Suspensions,"** by A. D. Garrison and K. C. Ten Brink, *Petroleum Technology*, Am. Inst. of Mining and Metallurgical Engrs., Vol. 2, No. 4, 1939.

MECHANICS OF FLOW, WITH NONCOLLOIDAL, INERT SOLIDS

By Warren E. Wilson,[3] Assoc. M. Am. Soc. C. E.

Synopsis

The investigation of the laws governing the pipe-line flow of liquids containing solids in suspension has received the attention of a number of investigators in several fields. The presentation and analysis of the data in most cases have been of such a nature as to provide essentially an immediate answer to questions concerning the efficiency of the pumping process. Little progress has been made toward the unification of the data by means of a comprehensive theoretical background.

In the field of the transportation of solids by streams in open channels notable work has been done by a number of men, prominent among whom are Morrough P. O'Brien and E. W. Lane, Members, Am. Soc. C. E., and J. E. Christiansen, A. A. Kalinske, and Hunter Rouse, Assoc. Members, Am. Soc. C. E., to provide a clearer understanding of the mechanics of the turbulence process in maintaining material in suspension. However, a direct application of their approach to the problem of the transportation of solids in pipes does not seem feasible at the present time.

The purpose of this study is to provide an elementary theory on the basis of which certain features of the available experimental data on the transportation of solids in pipes may be interpreted. The theory is neither complete nor directly concerned with the details of the mechanics of the turbulence process in maintaining solids in suspension, but rather represents an analysis of the mechanics of the flow on the basis of energy relationships; and it results in an explanation of several important and previously noted but unsatisfactorily explained features of the flow.

Experimental data from several sources are presented and analyzed on the basis of the background provided by the theory. Due to lack of complete data in many old studies of flow of the type under consideration it is impossible to make use of the results of much of the experimental work.

Theory

The analysis will be limited to the consideration of the transportation of noncolloidal, inert solids for which the average velocity of settling relative to the transporting liquid is known. Complicating factors such as chemical reaction between solids and liquid and the formation of semi-solid plugs, as noted by some observers, will be assumed negligible or nonexistent.

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Consider a liquid flowing in a horizontal pipe, as illustrated in Fig. 7, toward the right at the constant average velocity V. Particles such as P are in suspension. If there were no turbulence the particle P would settle at the velocity V_s and in the period of time T would move downstream the distance $L = V\,T$ and vertically downward the distance $V_s\,T$. The flow being steady the particles will maintain their average positions in the vertical relative to the pipe wall. Now, since the particles individually do settle and are raised up to new positions by the turbulence of the liquid, maintaining the same average position, it is clear that the liquid must do work on the particles definitely related to the decrease in potential energy due to settling. It is not to be expected that the work done is exactly equal to this change in potential energy since the motion of the particles in settling relative to the liquid is accompanied by a loss in mechanical energy of the particles due to the viscosity of the liquid. Similarly additional energy will be required in raising the particles due to the viscous resistance requiring that more work be done than is necessary to raise the particles vertically a given distance.

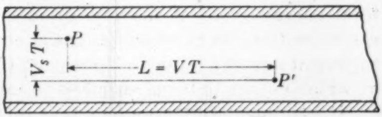

FIG. 7

In order to express this relationship between work done by the liquid on the particles in maintaining the suspension, and the decrease in potential energy during settling, one may write:[3a]

$$K_p\,A\,L\,V_s\,T\,w_e = w\,h_1\,A\,L \qquad (15)$$

in which: A is the cross sectional area of the flow section; p is the weight of solids per unit weight of solid-liquid mixture; w is the unit weight of the mixture; w_e is the effective weight of the particles suspended in the liquid; K_p is a coefficient associated with the efficiency of the process; and h_1 is the decrease in the elevation of the energy grade line in the length L due to the maintenance of the solids in suspension. Now recalling that

$$L = V\,T \qquad (16)$$

Eq. 15 may be simplified into the form,

$$\frac{h_1}{L} = \frac{K\,p\,V_s}{V} \qquad (17)$$

which is a very simple expression for the slope of the energy gradient due to the maintenance of the solids in suspension.

The Weisbach-Darcy equation may be used to determine the slope of the energy gradient due to the resistance of a homogeneous liquid flowing in a given pipe, and it will be assumed that this equation will apply equally well to a non-homogeneous liquid-solid mixture. Therefore,

$$\frac{h_2}{L} = \frac{f\,V^2}{2\,g\,D} \qquad (18)$$

[3a] Note corrections to Eq. 15 given in the closing discussion on page 1593.

in which: f is the friction factor; D is the pipe diameter; $\frac{h_2}{L}$ is the slope of the energy gradient due to the flow of the liquid mixture expressed in feet of mixture per foot of pipe; and g is the acceleration due to gravity.

An expression for total energy gradient follows immediately from the addition of Eqs. 17 and 18—

$$\frac{H}{L} = \frac{f\,V^2}{2\,g\,D} + \frac{K\,p\,V_s}{V} \qquad (19)$$

Eq. 19 expresses the fundamental principle of this study, the concept that the total energy gradient may be divided into two parts—the first, that which expresses the rate at which potential energy is being dissipated to maintain the turbulent flow of a liquid in a pipe, and the second, that which expresses the rate at which energy is being supplied to maintain the solids in suspension.

It is immediately apparent that for a given pipe, percentage of solids, and particle size and shape which are characterized by V_s, the total energy gradient approaches that for a clear liquid as V increases, since the second term of Eq. 19 decreases as V increases and the first term is the energy gradient for a clear liquid expressed in terms of head of the mixture per unit length of pipe. This phenomenon has been repeatedly observed by investigators in this field.

It is evident also that for a given solid-liquid mixture and size of pipe there will be a velocity that gives a lesser energy gradient than any other velocity. The approximate magnitude of this velocity may be determined by assuming p, V_s, f, K, and D constant, then differentiating the expression for $\frac{H}{L}$ with respect to V and setting the first derivative equal to zero. The resulting criterion for the critical value of the velocity is,

$$V^3_c = \frac{K\,p\,V_s\,g\,D}{f} \qquad (20)$$

in which V_c is the critical velocity, giving minimum energy gradient for the given combination of pipe and mixture.

Thus far it has been assumed that all the solids remain in suspension. However, it is apparent and a matter of common observation that under certain circumstances the solids will settle out, and eventually clog the pipe. The criterion for settling out of the particles or the picking-up of particles from the bed has been discussed by Messrs. Lane and Kalinske.[4] These writers demonstrate that, if the quantity $\frac{V'_s}{\sqrt{\frac{\tau_0}{\rho}}}$ (in which V'_s is the settling velocity of the solids, τ_0 is the bottom shearing stress per unit area, and ρ is the unit density of the liquid) is equal to or greater than unity little or none of the material with settling velocity V'_s will be placed in suspension.

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[4] "Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, 1939, p. 637.

Assuming that the shearing stress τ_0 is equal for all parts of the wetted perimeter it follows that, for an element of the liquid, of length L and diameter D_l, the equation of equilibrium in the direction of flow is:

$$(p_1 - p_2)\frac{\pi D^2_l}{4} - \tau_0 \pi D_l L = 0 \ldots\ldots\ldots\ldots\ldots \tag{21}$$

in which p_1 and p_2 are the unit pressures at the upstream and downstream ends of the element, respectively. If H is the decrease in pressure head in the length L, it follows that

$$\frac{\tau_0}{\rho} = \frac{H g D_l}{4 L} \ldots\ldots\ldots\ldots\ldots\ldots \tag{22a}$$

or upon substitution of the hydraulic radius for 0.25 D_l:

$$\frac{\tau_0}{\rho} = \frac{H g R}{L} \ldots\ldots\ldots\ldots\ldots\ldots \tag{22b}$$

which will give for the quantity $\sqrt{\frac{\tau_0}{\rho}}$ the expression,

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{H g R}{L}} \ldots\ldots\ldots\ldots\ldots\ldots \tag{23}$$

Thus, the condition for the settlement of solids becomes,

$$\frac{V'_s}{\sqrt{\frac{H g R}{L}}} = 1 \ldots\ldots\ldots\ldots\ldots\ldots \tag{24}$$

The assumption of uniform shear stress around the wetted perimeter is not likely to be fulfilled and there will be settling out of solids at values of the ratio of Eq. 24 corresponding to velocities somewhat different from those indicated by a value of unity of the ratio.

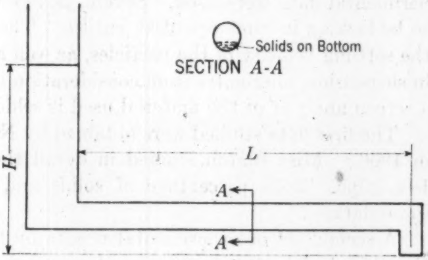

FIG. 8

Although some data are available concerning the minimum velocities necessary to prevent clogging the auxiliary data are incomplete and afford no basis for checking the validity of Eq. 24.

A means of determining the suitability of Eq. 24 as a criterion for deposition of solids was made possible by the observations on the flow in the tailings disposal line of the Climax Molybdenum plant at Climax, Colo. The important characteristics of the situation that exists in this pipe line are depicted in Fig. 8. Since the flow in this line represents a type entirely different from that in pipe which is commonly supplied by a pump, it merits consideration under this section, setting forth the elements of the mechanics of the flow of solids suspended in liquids in pipe lines.

Fig. 8 illustrates a horizontal pipe fed from a vertical stand-pipe open to the atmosphere, and discharging from a right-angled bend, vertically downward. The elevation of the free surface in the stand-pipe is determined by the loss of head in the pipe line. This loss, in turn, is dependent upon all the factors indicated by Eq. 19. It is apparent that if the discharge, percentage of solids, or type or size of particles changes the head loss will change. An automatic adjustment of the head takes place in the case of any of these changes and the flow eventually becomes steady. It is apparent that if, with a given discharge, the velocity is so low that solid particles are deposited on the bottom, this deposition will continue until the cross section has become obstructed to such an extent that the velocity is high enough to maintain the suspension, and the pipe will not clog since the head will automatically adjust itself to the new steady state. Such is found to be the mode of operation of this pipe line. Obviously a pipe supplied from a pump that maintains a constant head could not operate in this manner since the automatic head adjustment would not be possible.

Thus far, a horizontal pipe has been assumed. If the slope of the pipe is very large the component of the settling velocity perpendicular to the pipe wall must be considered as the quantity V_s rather than the full vertical velocity with which the solids settle relative to the liquid. In the case of small slopes this factor will be of minor importance but will doubtless be a factor of considerable importance if the slope is very large. If the pipe is vertical, obviously there is no component of the settling velocity perpendicular to the pipe wall; hence, in this case, one would expect no increase in the energy gradient due to the maintenance of the suspension.

DESCRIPTION OF EXPERIMENTAL DATA

To investigate the applicability of the foregoing theory, three sets of experimental data were used. Several others were investigated but were found to be lacking in some essential feature. The fact that it is necessary to know the settling velocity of the particles, as well as the percentage of solid material in suspension, eliminates from consideration much data that are available since a screen analysis of the material used is seldom available.

The first data studied were obtained by Nora Stanton Blatch and published in 1906.[5] Miss Blatch studied in detail the flow of sand-water mixtures in 1-in. pipe. Both percentage of solids and screen analysis are available in these data.

A second set of experimental results by G. W. Howard, Jun. Am. Soc. C. E., published in 1939,[6] contains data on the flow of sand-water and gravel-water mixtures in 4-in. pipe. These data are complete with screen analyses.

A third set of data (see Table 3) was supplied by the Climax (Colo.) Molybdenum Company and is complete in practically every detail. This material describes the flow of tailings in an 18-in. wood-stave pipe line so constructed and connected as to give a unique flow of the mixture differing in several important respects from the flow in the conventional pipe line setup.

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[5] *Transactions*, Am. Soc. C. E., Vol. LVII, December, 1906, pp. 400–408.

[6] *Loc. cit.*, Vol. 104 (1939), pp. 1334–1348; also, *loc. cit.*, Vol. 106 (1941), pp. 135–157.

The essential data are introduced in the following sections when necessary for the discussion. The settling velocity (Table 3) in all cases was estimated on the basis of the grain size only and no allowance was made for shape of particles; hence, the data may well be in error to some extent. However, this fact will not materially detract from the value of the conclusions drawn.

Analysis of Data

Fig. 9 is a plot of the experimental data assembled by Miss Blatch. The head loss in a 1-in. pipe per unit length, expressed in terms of the head of the mixture, is plotted as the ordinate and the average velocity of the mixture in the pipe is plotted as the abscissa. Each curve represents the data for the flow with a given average settling velocity of particles in the mixture and a given percentage of solids.

Sands designated as Nos. 1 and 2, respectively, were used in the work done by Miss Blatch. All of sand No. 1 passed the 20-mesh and was retained on the 40-mesh sieve. It is estimated that the average settling velocity of this material in water was 0.213 ft per sec. Sand No. 2 all passed the 60-mesh and was retained on the 100-mesh

TABLE 3.—Flow Data, Tailing Line, Climax Molybdenum Company, Climax, Colo.

Run No.	Flow (cu ft per min)	Solids		% plus 100 mesh	Hydraulic gradient, *S*	Velocity[a] (ft per sec)	Open cross section, %
		%	Dry tons in 24 hr				
(*a*) Series 1 (Length, 1,900 Ft; Slope, 0.003; Wood-Stave Pipe, Inside Diameter, 18 In.; Fed by Penstock 3 Ft by 4 Ft [Inside], 25 Ft High)							
1	412	34.3	7,890	20.0	0.0069	5.1	77[c]
2	402	39.4	9,480	25.6	0.0074	5.15	76[c]
3	406	41.4	10,340	31.3	0.0076	5.1	76[c]
4	404	44.0	11,080	34.8	0.0078	5.1	75[c]
5	409	49.4	12,920	40.8	0.0089	5.1	76[c]
6	405	51.1	13,740	43.4	0.0092	5.15	74[c]
7	402[b]	55.8	15,600	47.8	....	....	....
8	206	34.3	3,945	20.0	0.0063	4.35	45[c]
9	201	39.4	4,740	25.6	0.0070	4.35	44[c]
10	203	41.4	5,170	31.3	0.00765	4.25	45[c]
11	202	44.0	5,540	34.8	0.0083	4.33	44[c]
12	205	49.4	6,460	40.8	0.0090	4.30	45[c]
13	202	51.1	6,870	43.4	0.0100	4.35	44[c]
(*b*) Series 2; East Line, Station 4400 to Station 6174 (Length 1,774 Ft; Slope 0.00384; Penstock at Station 4400, of Circular Tile, Inside Diameter 30 In.)							
1	426	36.1	8,250	21.7	0.0080	5.9	68
2	402	40.8	9,120	25.6	0.0089	5.9	64
3	415	43.1	10,140	29.8	0.0083	5.65	69
4	399	44.7	10,350	33.6	0.0087	5.9	63
5	408	51.1	12,600	40.0	0.0090	5.7	67
6	405	53.1	13,290	43.4	0.0102	5.9	64
7	412	58.3	15,600	47.8	0.0100	6.6	59
8	213	36.1	4,125	21.7	0.0067	4.6	44
9	201	40.8	4,560	25.6	0.0076	4.6	42
10	208	43.1	5,070	29.8	0.0083	4.4	44
11	200	44.7	5,175	33.6	0.0092	4.1	40
12	202	51.1	6,300	40.0	0.0092	4.4	42
13	202	53.1	6,695	43.4	0.0112	4.6	42
(*c*) Series 2; East Line, Station 4400 to Station 5911 (Length, 1,511 Ft; Slope, 0.00384)							
14	260	37.8	5,400	22.9	0.0084	4.9	50
15	240	42.9	5,820	26.4	0.0077	5.0	47
16	294	42.7	6,720	31.0	0.0106	5.1	55
17	297	47.9	8,460	36.9	0.0098	4.8	59
18	297	51.6	9,330	39.5	Lost	4.8	58
19	267	57.2	9,510	46.7	0.0123	4.8	52
(*d*) Series 2; West Line, Station 5800 to Station 7700 (Length, 1,900 Ft; Slope 0.003; Penstock, 3 Ft by 4 Ft Wood Crib)							
20	412	34.9	7,560	20.0	0.0060	5.1	77
21	406	42.8	10,020	31.3	0.0077	5.1	76
22	404	46.0	10,860	34.8	0.0079	5.1	75
23	425	47.2	11,820	34.1	0.0077	5.3	76
24	409	51.2	12,540	40.8	0.0091	5.1	76
25	430	49.2	12,060	38.3	0.0085	4.9	83
26	207	43.1	5,070	29.8	0.0087	4.2	46
27	200	44.7	5,175	33.6	0.0092	4.1	46
28	212	47.2	5,910	34.1	0.0074	4.4	46
29	215	49.2	6,030	38.3	0.0088	4.7	43
30	202	51.1	6,300	40.0	0.0094	4.4	44

[a] Velocity determined by adding salt to the intake of the penstock, analyzing the pipe discharge for chlorine and noting the time for salt to appear at the discharge end.

[b] The tailing on this grind and density surged in the penstock; evidence of a tendency to become choked.

[c] Average free area; the part of the pipe area required to carry the given volume at a determined velocity.

sieve. It is estimated that the average settling velocity of this material in water was 0.0655 ft per sec.

In Fig. 9 it is easily seen that the important characteristics of the flow of the sand-water mixtures are in essential agreement with the predictions of the foregoing theory. Each curve, representing individually a given mixture, shows the characteristic minimum energy gradient at a certain velocity, with the exception of the curve for $V_s = 0.21$ ft per sec and $p = 0.12$ which does not clearly show such a minimum. Although the curves do not all become asymptotic to the curve for clear water, and in some cases drop below the clear-water curve, the general tendency is as predicted and the explanation for the dropping below the clear-water curve is doubtless that the presence of the solids modifies the characteristics of the flow to such an extent that the energy gradient for the mixture is no longer comparable to that for clear water. The presence of the solids has been thought to act as a damping influence on the turbulent fluctuations of the velocity thus decreasing the energy gradient.

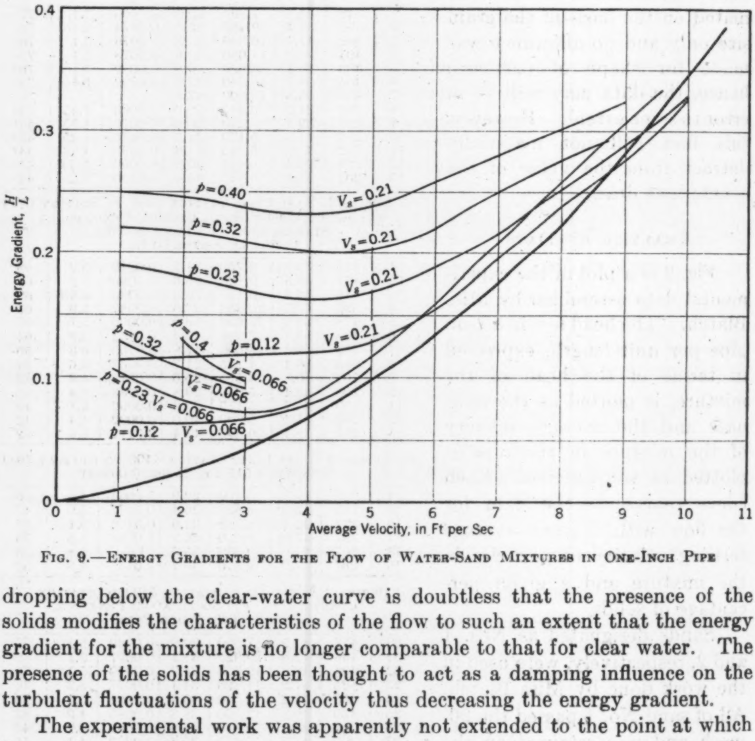

FIG. 9.—ENERGY GRADIENTS FOR THE FLOW OF WATER-SAND MIXTURES IN ONE-INCH PIPE

The experimental work was apparently not extended to the point at which it would be possible to determine the clogging velocities for the various mixtures. Therefore, it is impossible to learn anything concerning the applicability of the criterion for settling of the solids.

It is important to note that, in general, the head loss is a function of velocity at low velocities, and as had been observed in some experimental work independent of velocity over only small ranges of velocity. The variation of the head loss with velocity in the low velocity range is small in some cases.

Fig. 10 is similar to that of Fig. 9 and depicts the data obtained by Mr. Howard, who used a 4-in. pipe and a sand for which he presented a screen analysis. On the basis of the screen analysis it is estimated that the average settling velocity of the sand in water is 0.16 ft per sec.

The same general characteristics which were indicated by Miss Blatch's data are discernible in Mr. Howard's work, but the fact that the data cover a relatively narrow velocity range precludes the exact determination of the velocity for minimum head loss. The clogging velocities, experimentally determined by Mr. Howard, are indicated on the diagram by the short vertical lines so marked.

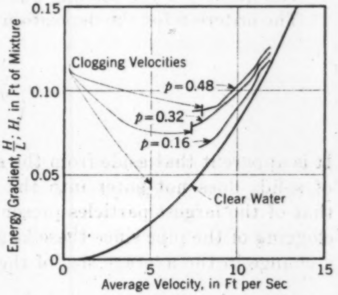

FIG. 10.—ENERGY GRADIENTS FOR THE FLOW OF WATER-SAND MIXTURES IN FOUR-INCH PIPE (AVERAGE SETTLING VELOCITY, V_s = 0.16 FT PER SEC)

Fig. 11 shows a graphical representation of the data observed at Climax. The slope of the energy gradient is plotted as the ordinate and the quantity $\frac{p\,V_s}{V}$ as abscissa. The points are grouped according to approximately equal rates of discharge since the data were gathered by maintaining nearly constant rates of discharge with varying solids content in each series of runs. The data cover so narrow a range of velocities that a plotting of the type shown in Figs. 9 and 10 is not possible.

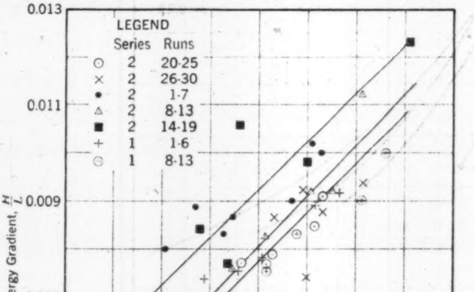

FIG. 11.—ENERGY GRADIENTS FOR THE FLOW OF TAILINGS IN 18-IN. WOOD-STAVE PIPE (f COMPUTED FOR A GIVEN DISCHARGE AND APPARENT $\frac{H}{L}$ FOR ZERO SOLIDS CONTENT)

In each series of runs both the percentage and size of solids were varied. Corresponding changes in the slope of the energy grade line took place. These data are summarized in Table 3. It is interesting to note that for a given discharge and with varying percentages and sizes of solids, the average velocity in the cross section remains nearly constant. In other words, the variation in the quantity and average size of the solids had a minor effect upon the amount of deposition of material. This is in accordance with the conditions for the settling out of the solids.

The criterion for the deposition of solids from the suspension is expressed as

$$\frac{V_s}{\left(\frac{g\,r\,H}{L}\right)^{0.5}} = 1 \qquad (25)$$

It is apparent that aside from the effect on the energy gradient, the percentage of solids does not enter into the consideration. The settling velocity V_s is that of the largest particles present in sufficient quantity to be a factor in the clogging of the pipe since these large solids would settle first. Evidently, then, a change in the average size of the particles need not greatly affect this maximum settling velocity. The fact was observed that the open area of the cross section and average velocity changed little for a given discharge when the solids content was varied over a wide range. This phenomenon is explained satisfactorily on the basis of a slight change in the maximum settling velocity and an increase in the energy gradient sufficient in amount to compensate for the increase that did take place. The increase in the energy gradient is due to the increase in the value of the quantity $\frac{p\,V_s}{V}$ upon which, as may be seen in Eq. 19, the energy gradient is directly dependent.

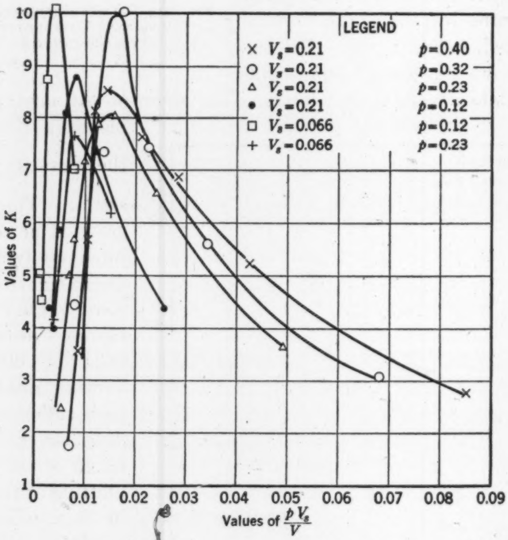

FIG. 12.—COEFFICIENT K AS A FUNCTION OF THE QUANTITY $\frac{p\,V_s}{V}$

The plotting of Fig. 11 indicates a value of the coefficient K of very nearly unity in most cases. The solid lines drawn through the groups of points have slopes which, in most cases, are approximately equal to 1. A slope of $S = 1$ corresponds to $K = 1$. The indicated values of f, the coefficient of Darcy's equation for pipe flow, were computed on the basis of the energy gradient for zero solids content as determined by the extension of the straight lines to the line, $\frac{p\,V_s}{V} = 0$.

The fact that the value of K in this situation is approximately unity is of interest in connection with the plotting shown in Fig. 12. In this curve the value of K, computed from the data obtained by Miss Blatch, is plotted as a function of the quantity $\frac{p\,V_s}{V}$. The most notable feature of this plotting is the apparent existence of a maximum value of K at a value of V which gives a minimum of the energy gradient for a given mixture in the pipe. Also, it is apparent that as the velocity decreases $\left(\frac{p\,V_s}{V}\text{ increases}\right)$, the value of K decreases and may well approach 1.

Fig. 13 contains parts of data supplied by Miss Blatch, and parts by Mr. Howard, plotted in the manner used in Fig. 11 for the data obtained at Climax. It is clear that in the ranges of variables covered in these plottings K is not unity, whereas it did have a value of practically unity for the data obtained at Climax.

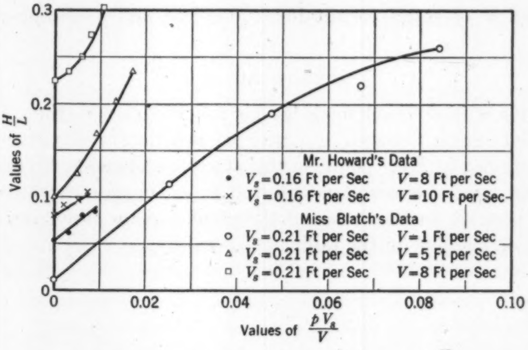

FIG. 13.—ENERGY GRADIENT AS A FUNCTION OF $\frac{p\,V_s}{V}$

Although adequate data are not available to make a thorough investigation, a superficial examination of the data already cited and some other less complete and reliable data on flow in dredge pipe lines quoted by Miss Blatch[7] indicates that the value of K may decrease as the diameter of the pipe increases.

It is to be expected that the following dimensionless quantities would form a very minimum of parameters with which to describe the coefficient K:

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[7] *Transactions*, Am. Soc. C. E., Vol. LVII, December, 1906, pp. 402 and 406.

Reynolds number, **R**; $\frac{V_s}{V}$; the ratio, $\frac{\text{mean diameter of particles}}{\text{diameter of pipe}}$; and percentage of solids (p). Obviously, extensive information is necessary to determine experimentally the laws of the variation of K.

SUMMARY

The fundamental assumption of this study is that the energy, expended in maintaining the flow of a suspension of solids in a liquid, may be expressed as the sum of the energy required to maintain the flow of a homogeneous liquid of the same density, and the energy required to maintain the solids in suspension, through the mechanism of the fluid turbulence overcoming their tendency to settle.

The equations developed on the basis of this assumption lead to conclusions concerning the general features of the flow which are in good accord with the observed facts.

The unique flow observed in the tailings disposal line at Climax is adequately explained in a qualitative manner and certain features of the flow are analyzed quantitatively with good agreement between theory and observation.

In all cases in which complete data are available, and the variation of the coefficient K is not a factor, good agreement is found between theory and observation.

Additional experimental work in which all variables are carefully controlled and observed is essential to obtain a complete understanding of this type of flow.

ACKNOWLEDGMENT

The writer wishes to acknowledge his indebtedness to the officials of the Climax Molybdenum Company for their cooperation in permitting the use of the data, obtained at their plant, in this study. Particular thanks are due to C. J. Abrams, general superintendent, and E. J. Duggan, mill superintendent, for making possible an inspection of the plant and for supplying the detailed information on the flow in the tailings disposal line.

DISCUSSION

Arthur L. Collins,[8] Assoc. M. Am. Soc. C. E.—A system of measurement has been developed by the writer that is an aid in the solution of some of the important problems encountered in dredge operation. Several measuring units make up the system and are described as follows:

Velocity and Load Meter.—A section of the discharge pipe line can be made to serve as a meter. This requires a horizontal section of discharge pipe, 35 to 50 ft long, to which is attached a chart-recording, differential gage. The drop in pressure due to the resistance to flow generally will be about 20 in. of water pressure, or 0.75 lb per sq in.

As the fluid from the pump passes this section, three stages of flow resistance are recorded: First is the clear-water velocity; and second is the increase in the pressure drop due to the heavier fluid, which may be as great as 50% when the dredge is digging. The third stage may be considered a phenomenon. The pressure drop may increase suddenly as much as 100%, which indicates that there has been some marked change in the distribution of the solids in the fluid.

The records from the so-called load chart obtained from the meter section are supplemented by a gadget that is attached to the end of the pipe line. The object here is to determine how constant the pipe-line velocity is in value. This can be accomplished by the use of a revolving shaft to which are attached steel spokes with vanes on the ends. The vanes dip into the stream flow and provide a displacement record of velocity. The principal purpose of this gage is to obtain relative values. Thus, over a 60-min period the load-meter record, when synchronized with the end of the pipe record, will indicate how the pump responds to the load, in so far as maintaining a continuous velocity is concerned.

In dredge-pump operation it is desirable to use a pump that will maintain a constant pipe-line velocity regardless of the load. If the pump cannot maintain a fairly constant velocity, but varies several per cent and has a tendency to slow down under load, it cannot maintain its maximum output continuously.

Tests on small pumps pumping sand under laboratory conditions have led investigators to make broad statements regarding the operating characteristics of a dredge pump. Conventional velocity diagrams, which are purely assumptions, are used to account for variations in pump performance when pumping solids.

Other conditions that have their effect on pump characteristics should be mentioned. The fluid enters the eye of the pump runner at a high velocity and must make a right-angle turn to follow the passages in the pump runner. An additional loss of energy is concealed here that is probably more effective in altering the pump characteristics than the explanation given by the velocity diagram (see Fig. 4).

Slippage in a dredge pump means to the practical dredge operator a spinning of the runner that results in a decreased discharge. The peripheral speed of the pump runner is generally so high that the fluid leaves the runner nearly in a tangential direction. Conditions apparently arise in which the fluid revolves in the case and will not respond to the load pickup. This is another factor that must be taken into consideration in analyzing dredge pumps.

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[8] Cons. Engr., Berkeley, Calif.

Thus, it must be realized that dredge pumps vary greatly in design. So many conditions enter into the final characteristics of the pump that field tests must be made on each dredge to determine how it can be operated at its best efficiency.

Another gage that is very important to this system of measurement has not been mentioned. This is a depth gage which is attached to the cutter and records on a chart the travel of the cutter. This chart, when synchronized with the load chart, indicates the load that is constantly being picked up by the suction pipe. Each cut or ledge can be rated in terms of load percentage. Thus, the difference in pickup between the port and starboard swings, as well as the shallow and deep cuts, is definitely known.

The following data were obtained from a load chart. It is for the duration of one dredge setup or step:

Cut No.	Time	Load by weight
1	11:30 to 11:37 a.m.	Load is 46.5% starboard swing
2	11:37 to 11:45 a.m.	Load is 21% port swing
3	11:45 to 11:52 a.m.	Load is 25.5% starboard swing
4	11:52 to 12:00 Noon	Load is 9% port swing
5	12:00 to 12:07 p.m.	Load is 23% starboard swing
6	12:07 to 12:17 p.m.	Load is 14% port swing
7–8	12:17 to 12:33 p.m.	Load is in the 3d stage

This overload occurs at the finish of the setup. It is to be noted that the peak pickup amounted to 46.5%, whereas the average is only 23%. Obviously, it should be possible to improve the cutter conditions, the design of the pump, and the method of operating the dredge, and thus increase the output materially.

The chief improvement in dredge operations will not be accomplished by work done in the laboratory on small pumps but by making the large dredge unit a laboratory in itself and recording the various operations.

Tests to determine pipe-flow coefficients fail to take into consideration one important factor that undoubtedly helps to maintain a high percentage of solids in suspension. A large number of discharge lines are made with 15.5-ft sections of pipe with slip joints. The tapered section is 1 in. smaller in diameter than the pipe. Thus in a 24-in. pipe the fluid increases its velocity by 8% at each joint. This has a nozzle effect that diffuses the solids. Even with this restricted area, the pipe coefficient may be as high as 130, using the Hazen and Williams tables.

On the other hand, a smaller pipe (say 15 in.) with long sections and rubber sleeves at the joints may have a coefficient of only 100. Excess friction should react directly upon the dredge output. If by using all the power available it is possible to maintain only 14 ft per sec, it should be possible to increase the output 25% by decreasing the frictional resistance and boosting the velocity to 17.5 ft per sec. In other words, the output will vary as the velocity. There is scarcely a dredge in operation that cannot be improved by applying better engineering technique.

H. A. Einstein,[9] Assoc. M. Am. Soc. C. E.—The problem of determining the pump and flow characteristics of a mixture of water and granular solids is of great practical importance. Only the abundance of difficulties involved may explain why so little research has been done and so few papers published in this special field. The numerous complications may also be the reason why the reports of earlier investigators failed to give full descriptions of all technique and apparatus. Both authors stress this fact but omit some information from their own reports. The writer misses especially a short description of the methods of making measurements.

In the pump test, it would be interesting and important to know if volumetric or venturi measurements were used for the discharge and how the two methods compare. For sufficiently fine particles, the venturi meter should give the correct discharge, if the pressure difference is measured by the height of a column of the mixture. For coarser particles, a relative movement of the particles against the water is to be expected in the venturi meter as well as in the pump. The solid particles will probably lag when the acceleration in the venturi meter occurs. Therefore, the water will be accelerated more than the total mixture as an average, and a slightly high manometer reading may be expected. Mr. Fairbank may be able to give some information about this point in his final discussion.

Another important measuring detail pertains to the lines between the taps in venturi, pipe, or pump, and the manometers or piezometer tubes. Part of these lines will be filled with a water-sand mixture, part with clear water. Are the points known at which this change occurs in each line? What correction for any differences in their level has been made? These questions are especially important for the measurement of the "hydraulic gradient *S*" in Table 3 of Professor Wilson's paper.

There is a simple device that can be used for measuring discharge for almost all flows through pipes. This device is the measuring elbow. Any elbow following a sufficient length of straight pipe can be used. The dynamic pressure in certain parts of the elbow seems to be extremely stable and therefore can be used for the measurement. The writer found that the point *b* (see Fig. 14), where the axis of the preceding straight pipe intersects the wall of the elbow, is very reliable. The difference in pressure between point *b* and a reference point *a* in front of the elbow is proportional to the velocity head $\frac{v^2}{2g}$ for wide ranges of velocities. For most normal elbows this coefficient seems to be close to unity. This principle of measuring can be used in almost every line of any size without additional energy loss, pro-

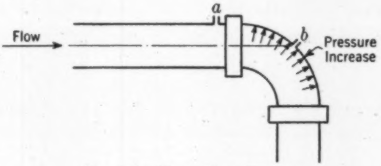

Fig. 14.—Measuring Elbow

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[9] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

vided there is an elbow in the line. Here, too, a correction may be necessary for coarse sand in suspension.

Eq. 19 is the backbone of Professor Wilson's paper and, as such, perhaps is not sufficiently explained. It seems to be one of those hydraulic rules that simply add energy losses without regard to their interdependence. Its first term represents the amount of mechanical energy dissipated in a flow without the solids. The second term is the amount of turbulence energy dissipated by lifting the solid particles. The two terms, therefore, originate in two altogether different phases of the problem. The same energy may appear in the first term as mechanical energy being transformed into turbulence, and again in the second term, when this same turbulence is dissipated in lifting particles. The measurements themselves seem to indicate this tendency. The scatter of the points of Fig. 11 is considerable, but, still, some groups of points (series 1, runs 1–6; series 1, runs 8–13; and series 2, runs 1–7) unmistakably follow a slope of less than 45°. According to Eq. 19, this can be explained only as an efficiency above 100%, which is impossible. The writer's own experiments, therefore, seem to indicate that Eq. 19 does not explain the problem, even if it does seem to describe it satisfactorily.

Professor Wilson is definitely correct when he calls the criterion for settling of the sediment on the bottom of the pipe "a bed-load problem." It is definitely to be solved as a problem of the stability of this deposit. Unfortunately, present knowledge about bed load does not include any flows that could be compared with the flow in one of these pipes. Therefore, caution must be used when data from rivers are applied to pipes. However, if all the necessary information were available, these and similar pipe measurements could very well give exceedingly important information pertinent to certain bed-load problems. However, at least one series of measurements with 100% open cross section should be included, and, as stated previously, the measurements themselves should be described in more detail.

Both papers of this Symposium will prove to be very helpful to the engineer who must design apparatus for pumping solids in suspension. They provide him with some necessary information in a convenient and sufficiently accurate form.

H. E. Babbitt,[10] M. Am. Soc. C. E., and D. H. Caldwell,[11] Esq.—The contribution of Professor Wilson to the knowledge of the flow of solids in suspension is of value in providing information applicable to the turbulent flow of settleable solids suspended in water flowing in a circular pipe. The analysis is applicable to the condition of turbulent flow in which the relation between the settling velocity (V_s) of the suspended particles and the transporting velocity (V) of flow is appreciable in terms of the head loss $\left(\frac{H}{L}\right)$. Professor Wilson clearly states that his analysis is limited to the consideration of the transportation of noncolloidal, inert solids for which the average velocity of settling relative to the transporting liquid is known. It is apparent that the analysis is limited to mixtures in which the concentration of suspended solids is insufficient to allow mutual contact between the particles in such a manner that stress may be transmitted through the mixture. It is to be noted that the analysis does not supply information concerning the velocity required to maintain the particles in suspension; nor, from the information given, would it be possible to determine the head losses due to friction for the flow of any given mixture of solids and water.

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[10] Prof., San. Eng., Univ. of Illinois, Urbana, Ill.

[11] Instructor, Civ. Eng., Univ. of Illinois, Urbana, Ill.

Information given by the writers[12] in 1939 and 1940 supplements that given by Professor Wilson. Laminar flow and turbulent flow of sludges in circular pipes were discussed, and a method was given for computing head losses due to the flow, in a circular pipe, of mixtures of solids and dispersion fluids through which stress may be transmitted. The writers' analysis covers the transportation of solids through circular conduits under conditions in which the settling velocity, V_s, of the suspended particles is negligible with respect to the transporting velocity.

If there is sufficient concentration of suspended particles to permit the transfer of stress, the mixture is said to have a yield value. This is defined as the shearing stress (τ_y) of the mixture, at which flow is impending. High concentrations of suspended particles commonly have a yield value. The existence of a yield value causes the mixture flowing in a pipe to exhibit a pseudoviscosity, higher than the viscosity of the liquid in which the particles are suspended, but decreasing as the velocity in laminar flow increases. A result is that the mixture moves in laminar flow in much the same manner as a very viscous liquid except that, in the case of the mixture, there is a moving plug of material in the center of the pipe and within which there is no movement between adjacent particles. The writers have shown that the velocity, V_c, at which so-called "plug flow," plastic flow, or laminar flow breaks into turbulent flow, is given by the expression

$$V_c = \frac{32\,\eta + 32\sqrt{\eta^2 + \dfrac{f\,\tau_y\,\rho\,D^2\,g}{0.96}}}{f\,\rho\,D} \quad \ldots\ldots\ldots\ldots\ldots (26)$$

in which η = coefficient of rigidity of mixture, in pounds per foot-second; f = friction factor in the expression

$$H = f\,\frac{L}{D}\,\frac{V^2}{2\,g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

τ_y = shearing stress at the yield point of a plastic material, called yield value, in pounds per square foot; ρ = density of flowing substance, in pounds per cubic foot; and H = difference in static head between two points in a pipe, measured or expressed in feet of the flowing mixture. Hence, when V is less than V_c, Professor Wilson's analysis cannot be expected to apply.

An analysis of the experimental data reported by the writers, particularly for a Tennessee ball clay slurry, shows that the values of V_c lie between 2.0 and 15 ft per sec. The rate of settling of this material was of the order of 2 to 3 in. per day. Hence, at the velocity of 2 ft per sec (the lowest at which turbulence occurred in the writers' tests), the influence of the term $\frac{K\,p\,V_s}{V}$, shown in Eq. 19, on the friction loss, was negligible when compared with the term $\frac{f\,V^2}{2\,g\,D}$.

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[12] "Laminar Flow of Sludges in Pipes, with Special Reference to Sewage Sludge," by H. E. Babbitt and D. H. Caldwell, *Bulletin No. 319*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1939; also "Turbulent Flow of Sludges in Pipes," by the same authors, *Bulletin No. 323*, *ibid.*, 1940.

When the velocity of flow is greater than V_c, the friction loss due to the flow of a slurry, through a circular pipe, will be the same as the friction loss for the flow, through a circular pipe, of a liquid with the same viscosity as the dispersion medium of the mixture. Hence, in the case of suspensions of solid particles in water, the common hydraulic formulas can be used. This implies that for such a suspension the conditions of flow in a circular pipe can be expressed as

$$\frac{H}{L} = \frac{f\,V^2}{2\,g\,D} \qquad (28)$$

This agrees with Professor Wilson's conclusion. It should be emphasized again that Eq. 28 is applicable only to turbulent flow at a velocity high enough to maintain the particles in suspension.

Importance should be attached to the limiting concentration of suspended matter above which a yield value appears. It is only by differentiating the regions of solids concentration above and below the existence of a yield point that confusion may be avoided. When the concentration of suspended particles is sufficiently low to show little or no yield value in the mixture, when the term $\frac{K\,p\,V_s}{V}$ (Eq. 19) becomes appreciable with respect to the term $\frac{f\,V^2}{2\,g\,D}$, when the flow remains turbulent, and when particles remain in suspension, Professor Wilson's analysis will prove of most value.

Warren E. Wilson,[13] Assoc. M. Am. Soc. C. E.—The comments of those who have discussed this paper have been most helpful. An effort will be made herein to clarify certain details.

Mr. Einstein describes the difficulties inherent in measuring head losses in pipe lines carrying solids in suspension, and notes that the method of measuring the hydraulic gradient S in Table 3 was not described. These measurements were made by the staff of the Climax Molybdenum Company and it is the writer's understanding that the hydraulic gradient was determined by observing the difference in elevation between the surface of the liquid-solid mixture in the stand-pipe and the outfall of the pipe. It is apparent that this method embodies the measurement of the length of a column of liquid-solid mixture which would have an average density equal to that of the flowing mixture.

As was stated by Mr. Einstein, Eq. 19 is the underlying feature of the discussion. To clarify and elaborate upon the concept involved, and to correct an error, the equation might be rewritten in the form:

$$\frac{H}{L} = \frac{f\,V^2}{2\,g\,D} + \frac{w_s - w}{w_s}\left[\frac{C_1\,p\,V_s}{V} + \frac{p\,V_s}{V}\right] \qquad (29)$$

in which the terms have the following significance:

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[13] Prof. and Head, Dept. of Mechanics, Colorado School of Mines, Golden, Colo.

The term $\frac{f V^2}{2 g D}$ represents the energy per pound of fluid per unit length of pipe that would be dissipated in the turbulent flow of a homogeneous fluid. The term $\frac{C_1 p V_s}{V}$ represents the energy per pound of mixture per unit length of pipe, in excess of, or less than, that which would be dissipated in turbulent action by the homogeneous fluid due to the presence of the solids in suspension. The term $\frac{p V_s}{V}$ represents the energy per pound of fluid per unit length of pipe dissipated by the settling of the solids. The factor $\frac{w_s - w}{w}$ (in which w_s is the weight per unit volume of the solid material and w is the weight per unit volume of the mixture) is necessary to take into consideration the fact that the change in potential energy of the solids is a function of the effective weight of the immersed solids and not of the unit weight w_s alone. This consideration requires that Eq. 15 be written as follows:

$$\frac{w}{w_s} K\, p\, A\, L\, V_s\, T\, (w_s - w) = w\, h_1\, A\, L$$

and all equations derived therefrom should be revised accordingly. However to simplify the situation all that is necessary is to consider that the original equations are to be used only in discussing the flow of sand of specific gravity 2.65. The factor K may then be considered to include the factor $\frac{w_s - w}{w_s}$.

Now Eq. 19 is equivalent to Eq. 29 if one writes

$$K = (1 + C_1) \qquad (30)$$

The value of K therefore depends upon the value of C_1 and may be either greater or less than unity, depending upon the sign and magnitude of C_1.

Negative values of C_1 apparently exist in pipe flow as may be deduced from Fig. 9 in which the energy gradient for flows with solids dips below that for flow without solids, implying a negative value of C_1 greater in magnitude than unity. Similar evidence exists in certain data on open channel flow investigated by the writer,[14] in which flows of solid-water mixtures with gradients less than the expected gradient for pure water are found.

That such a condition is possible may be shown analytically in a qualitative manner by considering the elementary equation for shear stress in a liquid-solid mixture based on the momentum exchange concept, thus

$$\tau = \epsilon \frac{d}{dy} (\rho_m u) = \epsilon \rho_m \frac{du}{dy} + \epsilon u \frac{d\rho_m}{dy} \qquad (31)$$

in which ϵ is the so-called "austausch" coefficient, ρ_m is the local density of the mixture, u is the local velocity in the direction of mean flow, and τ is the shear stress.

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[14] "Transportation of Debris by Running Water," by G. K. Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, 1914.

The quantity $\frac{d\rho_m}{dy}$ is negative since the density of the mixture necessarily decreases as the distance y from the channel bottom decreases. Hence the shear stress τ may be either less or greater than that for clear water under similar circumstances, and the resulting distribution of velocity may give either a lesser or greater flow than for clear water under like conditions.

Professor Babbitt and Mr. Caldwell clearly define the limits of the applicability of the writer's analysis and their equation for the critical velocity defining "plug flow."

It is true that the present analysis does not yield a result which will predict, directly, either the minimum velocity for transporting solids or the head loss to be expected under given conditions. It is hoped that further treatment of the problem, both experimentally and by theoretical analysis, may yield useful results in predicting these two vital factors.

Elaboration of this elementary attack on the problem has yielded some interesting results in the problem of open channel flow of solids in suspension but requires further work to yield more than semi-empirical results.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2168

DESIGN AND CONSTRUCTION OF SAN GABRIEL DAM NO. 1

By Paul Baumann,[1] M. Am. Soc. C. E.

With Discussion by Messrs. E. Soucek, William P. Creager, Joseph Jacobs, Jacob Feld, Walter L. Huber, Federico Barona, G. H. Hickox, and Paul Baumann.

Synopsis

Dam sites distinctly favorable for masonry structures, particularly of the massive type, are rapidly being exhausted the world over. Hence, engineers are faced, more and more, with the problem of design and construction of safe and economical dams for dam sites far from ideal, relative to topography, geology, and geography. The latter is particularly important with regard to earthquake belts, such as are well established, and to the latitude and elevation at which the dam is to be located.

In the design and construction of the San Gabriel Dam No. 1 in California, it was necessary to consider all of these features. The solution of the problems thereby encountered, and particularly the one connected with stability and watertightness during major earthquakes, is believed to be novel. For the dam proper it expresses itself in the combination of compacted earth with loose rock—and particularly with compacted rock (that is, with artificial stone). The manufacturing methods of the latter and its physical qualities established by tests prior to, and during, construction are believed to be of special interest. Hence this paper treats this subject in particular detail. It also deals with features of special interests, incorporated in appurtenant works.

Introduction

The 4,000 sq miles of surface area of Los Angeles County, California, may be divided into four principal sub-areas from a standpoint of flood control: Mountain area (catchment, erosion) with 1,500 sq miles; foothill area (debris cones) with 500 sq miles; valley area (inundation) with 1,700 sq miles; and desert (arid) occupying the remainder.

Note.—Published in September, 1941, *Proceedings*.

[1] Senior Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

The San Gabriel Mountains form the principal range and the San Gabriel River is its principal drain. Its drainage area includes 220 sq miles above the mouth of the canyon with a maximum range in elevation of nearly 10,000 ft.

Particularly interesting is the nearly straight alinement of the two main forks, east and west, which approach each other in opposite directions and the confluence of which is a distinct landmark known as "The Fork."

San Gabriel Dam No. 1 is located approximately 2¼ miles downstream from the Fork. It is the principal one of an originally proposed system of three dams, two of which (Dam No. 2 and Dam No. 3) were to control the West and East Forks, respectively. Although Dam No. 2 on the West Fork was completed in the spring of 1935, Dam No. 3 on the East Fork was abandoned for reasons of economy.

The Original Design

In order to comprehend, fully, the revised or final design of Dam No. 1 as constructed, it is necessary to refer back to the original design that was prepared by the Los Angeles County Flood Control District prior to 1933 when the original contract was let.

The original design called for a rock-fill dam chiefly consisting of large, hard, and strong rock with voids of corresponding size. This rock was to be placed dry—that is, without the aid of sluicing water. The dam was to be provided with a laminated, shotcrete water slab with a substantial layer of packed rock directly beneath.

The rock was to be produced by approved methods in what is known as Quarry 10 which is located approximately one-half mile downstream of the dam site (described subsequently in connection with Fig. 28). It is a cut in a ridge with its floor at El. 1390 and an average slope of 1 to 1. It consists chiefly of a granitic diorite, sheared and crushed due to faulting and crossed by numerous intrusions of varying magnitude and composition. The quantity, quality, and size of the quarried rock was specified for the various parts of the dam.

In the spring of 1933 the work was started under contract. After excavation of the stream-bed material down to bedrock and the construction of a section of the cutoff wall along the base of the upstream contact, the placing of the rock fill in the excavated stream bed was started.

For the purpose of producing the rock to be placed in the fill, the specifications provided for the elimination of fines and spalls by means of a screen, commonly known as a "grizzly" in western construction parlance. This "grizzly" had clear slot openings of 6 in. The material passing through these openings was wasted, and the material passing over was acceptable for the fill, provided it satisfied the requirements regarding weight, strength, and soundness.

However, as the quarrying of rock progressed, it became quite evident that rock, satisfying the specifications, could not be produced economically in sufficient quantity in that for each acceptable cubic yard approximately three had to be wasted. Under the contract the waste had to be paid for at 40¢ per cu yd or 53⅓% of the acceptable rock and the waste had been estimated at less than one third of the latter. It was obvious, therefore, that completion of the dam inside the estimated cost was out of the question. Consequently, construction was suspended in October, 1934, at which time some 450,000 cu yd out of an estimated total of 5,562,000 had been placed in the dam and studies for a revised design were commenced.

The original design had been approved by consulting engineers and geologists of national repute.

The Revised Design

The Dam Proper.—The knowledge regarding relative quantities and characteristics of the rock, gained during quarry operations, led to the conclusion that the design of a feasible dam would have to include the material originally classed as waste—that is, principally the material passing through the "grizzly" openings. Hence, in principle it was necessary to adapt the design to existing conditions, which conditions, in turn, called for a change in the design from a true or homogeneous rock-fill type with a small, unavoidable percentage of fines to a mixed or heterogeneous rock-fill type with a large and, in fact, predominant percentage of fines. Hence, it called for a change to a type of fill whose physical characteristics (particularly in relation to stability and permeability) were of a radically different nature from the original type. This recognition was given further emphasis through the indispensable requirements regarding earthquake resistance of major structures in California.

The inclusion of a large percentage of fines in the fill naturally called for the flattening of the upstream and downstream slopes relative to the original design. Clearly, this requirement was primarily governed by the behavior of such a fill when saturated, particularly when under the influence of lateral mass forces in addition to gravitational forces during earthquakes. The open-type rock fill, as originally designed, was not subject to saturation above the stream bed and hence was not affected by this important phenomenon.

The flattening of the slopes as well as experience gained during the construction of San Gabriel Dam No. 2 (which was of substantially identical design as to type as the original of Dam No. 1) called for a change in the watertight membrane. This change was encouraged by the discovery of a large deposit of clayey sand or loam of unusual uniformity approximately 1¾ miles upstream from the dam site. It was an old homestead, known as Persinger Flat, situated inside the reservoir area which the Los Angeles County Flood Control District had acquired. The character of this material suggested its use as a relatively thin, impervious layer or zone whose plasticity (and with it, flexibility) would be assured so long as it was protected against drying out. The intimate contact of this zone with the existing part of the cutoff wall, as well as with the ultimate extension of the cutoff wall, was naturally a pertinent feature. To enhance the sealing quality of this zone a 6-in. shotcrete blanket was provided along its contact area with bedrock. This was to prevent the intrusion of water along paths shorter than the full normal thickness of the zone. A watertight connection was provided between this blanket and the cutoff wall.

These studies in connection with the revised design were accompanied by extensive tests on the available rock material particularly for the purpose of establishing suitable methods of compaction or petrifaction of the waste material which, as previously outlined, formed the bulk of the quarried rock. The purposes of these tests were to establish:

(1) The maximum size of rock which, if embedded in finer material, could be rolled with standard sheepsfoot rollers;

(2) The necessary roller weight to produce adequate compaction;

(3) The necessary number of roller passes;

(4) The most favorable proportion between fines, defined as particles $\frac{1}{4}$-in. or less in size, and rock, defined as particles greater than $\frac{1}{4}$ in. in size;

(5) The breakdown of the rock;

(6) The moisture content of the fines which would lead to adequate compaction with least work;

(7) The permeability of the compacted fines;

(8) The plastic and the elastic volume changes of the compacted fines;

(9) The penetration pressure of plasticity needles for the compacted fines; and

(10) The shearing strength of the compacted fines and its relation to plasticity.

Previously, tests had been conducted on loose rock and further tests were now made on mixes of loose rock and fines for the purpose of establishing: (*a*) Settlement under load for dry and wet material; (*b*) breakdown under load; (*c*) change in unit weight due to settlement; and (*d*) permeability.

Figs. 1 and 2 show the revised design in plan and section. The cross section of the dam is divided into six zones as follows:

Zone 1, consisting of quarry-run material varying from very fine material next to Zone 2 to relatively coarse material at the upstream slope. This zone may be considered largely a protective zone.

Zone 2, consisting of a clayey sand compacted to a dry weight of at least 115 lb per cu ft.

Zone 3, which may be considered the backbone of the dam, or the core, consisting of quarry-run material having passed 6-in. by 9-in. "grizzly" openings, compacted by rolling and tamping to a dry weight of the fines (particles smaller than $\frac{1}{4}$ in. in size) of at least 120 lb per cu ft.

No gradation of material between the upstream and the downstream slope was proposed for either Zone 2 or 3.

Zone 4, consisting of loose quarry-run material with the finest material next to the downstream slope of Zone 3 and gradually becoming coarser and more porous toward the downstream end of Zone 4.

Zone 5, consisting of relatively large rock forming a free draining mass and a protection against erosion to the zone below.

Zone 6, consisting chiefly of rock originally placed in the stream-bed excavation and moved under the revised design into its new position. This rock is relatively hard.

A statical analysis of equilibrium was made of a theoretical maximum vertical dam section of unit width under the influence of simultaneous full hydrostatic water pressure and lateral mass forces due to earthquake. The latter are based on the acceleration of one tenth that due to gravitation—that is, on 0.1 g of the mass $\frac{V}{g}$ of the dam, as well as of the so-called body of inertia of water next to the dam.[2] The resultant force H of all horizontal loads was

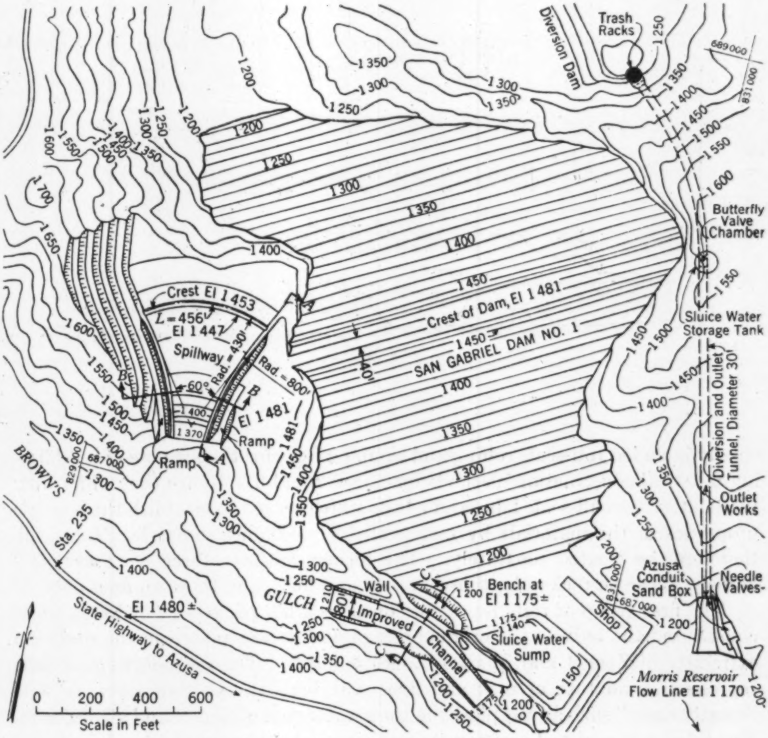
Diversion Dam
Trash Racks
689 000
831 000
Butterfly Valve Chamber
Sluice Water Storage Tank
Diversion and Outlet Tunnel, Diameter 30'
Outlet Works
Azusa Conduit Sand Box
Needle Valves
Shop
831 000
687 000
Morris Reservoir Flow Line El 1 170
Crest of Dam, El 1 481
SAN GABRIEL DAM NO. 1
40'
Crest El 1 453
L=456'
El 1 447
Spillway
Rad.=430'
Rad.=800'
60°
El 1 481
Ramp
Ramp
1 400
1 370
A
A
B
B
C
C
BROWN'S
829 000
687 000
GULCH
Sta. 235
State Highway to Azusa
El 1 480±
Wall
180'
Improved Channel
Bench at El 1 175±
El 1 200
Sluice Water Sump
1 175
1 140
1 150
1 200
1 250
1 300
1 350
1 400
1 450
1 500
1 550
1 600
1 650
1 700
-N-
0 200 400 600
Scale in Feet

FIG. 1.—PLAN VIEW, SAN GABRIEL DAM NO. 1

assumed to be resisted along a horizontal plane at El. 1120, which coincides approximately with the bedrock level. Due to the protective rather than constructive nature of Zone 1 and the plastic condition of Zone 2 as previously described, no resistance was credited to Zone 1 and Zone 2. Hence, because of their mass and acceleration these zones acted entirely as a load.

The analysis shows a necessary average shear coefficient $\frac{H}{V}$ of 0.37. Test results outlined in the course of this paper will show this value to be well within the actual shearing strength of the critical material—namely, the saturated fines in Zone 3.

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[2] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

The safety against sliding due to shear is further enhanced by the configuration of the dam site (see Fig. 1).

THE ROCK AND SOIL TESTS

With few exceptions the physical tests on materials for the dam were conducted in the field, either in the laboratory or in the open. The latter principally included full-scale rolling and tamping tests for the purpose of establishing practical construction methods for Zones 2 and 3. Hence they differ from the relatively small-scale laboratory tests which served to establish the intrinsic properties of the materials by means of soil mechanics research. They will, therefore, be treated separately. The former fall into three basic groups—namely, those on rock only, those on rock and fines, and those on fines only.

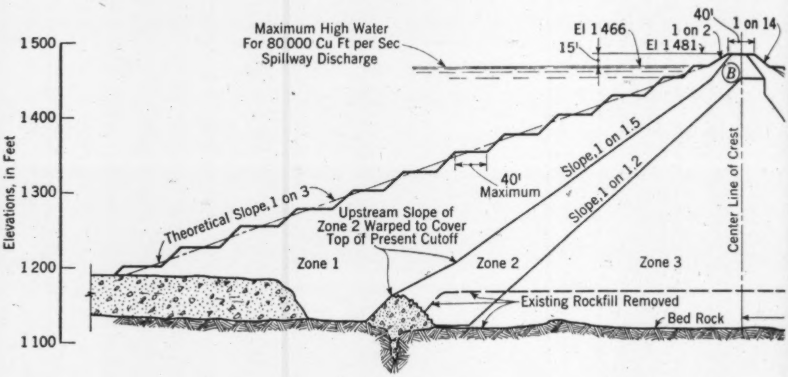

1 500
1 400
1 300
1 200
1 100
Elevations, in Feet
Maximum High Water
For 80 000 Cu Ft per Sec
Spillway Discharge
El 1 466
15'
El 1 481
40'
1 on 2
1 on 14
B
Slope, 1 on 1.5
Slope, 1 on 1.2
40'
Maximum
Theoretical Slope, 1 on 3
Upstream Slope of
Zone 2 Warped to Cover
Top of Present Cutoff
Zone 1
Zone 2
Zone 3
Center Line of Crest
Existing Rockfill Removed
Bed Rock

FIG. 2.—COMPOSITE MAXIMUM

The first group of tests represents open rock fill, placed dry or wet, such as constituted the bulk of the original design and as was placed in the upstream extremity of Zone 1 and in all of Zones 5 and 6. The second group of tests represents a mixture of rock and fines, varying between "mostly rock" and "mostly fines," such as was placed in the greater part of Zone 1 and all of Zone 4. The third group of tests represents fine material such as was placed in Zone 2 and such as forms the governing part of Zone 3.

Whereas the first two groups principally served to establish relations between load and settlement as well as permeability of the loose material, the third group served to establish all of the pertinent properties of compacted sand and soil.

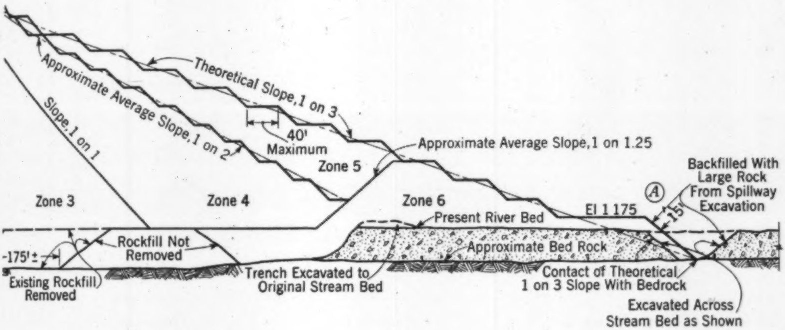
The discussion of all of the tests conducted in each group would require far more than the space available. Therefore, it will be limited to those, the results of which are believed to be typical.

First Group of Tests.—Tests to establish the settlement under load of representative rock samples were conducted in the field laboratory by means of cylinders. Two sets of three cylinders, each 24 in. and 30 in. in diameter and 24 in. and 30 in. high, respectively, were used. Pressures as great as 325 and

200 lb per sq in., respectively, could be applied by means of hydraulic jacks. Apparatus for the purpose of sluicing the rock was provided overhead. Samples of variable gradation of igneous rock were used.

Fig. 3 shows the test result of typical rock samples from Quarry 10. In Tables 1 and 2 test data are shown in detail.

The influence of sluicing, such as that due to a torrential rain, is evidenced in the sudden change in the settlement trend in Cylinders 2 and 3 upon the application of water. Cylinder 1 contained a sample from the Devil's Canyon Quarry which supplied the rock for Dam No. 2. It is a granitic gneiss of excellent quality. The corresponding results are shown in Table 1 for the purpose of comparison. They are omitted from Fig. 3.


Theoretical Slope, 1 on 3
Approximate Average Slope, 1 on 2
Slope, 1 on 1
40'
Maximum
Approximate Average Slope, 1 on 1.25
Zone 5
Backfilled With
Large Rock
From Spillway
Excavation
Ⓐ
15'
Zone 3
Zone 4
Zone 6
El 1 175
Present River Bed
Rockfill Not
Removed
Approximate Bed Rock
-175' ±
Existing Rockfill
Removed
Trench Excavated to
Original Stream Bed
Contact of Theoretical
1 on 3 Slope With Bedrock
Excavated Across
Stream Bed as Shown

SECTION, SAN GABRIEL DAM NO. 1

These tests combined with the experience gained during the construction of San Gabriel Dam No. 2 prompted the provision, in the revised specifications, for the application of approximately 2 cu yd of water to 1 cu yd of loose rock at the time of placing.

The breakdown due to the settlement under load (Table 2) is reflected in the screen analysis made at the end of the test as compared to the original one and particularly in the respective fineness moduli.

Second Group of Tests.—Samples used in these tests included fines varying between 20% and 60% by weight. Three 24-in. by 24-in. cylinders were used. The following tests are typical for this group:

A mixture of 80% rock and 20% fines was placed in Cylinder 1, 60% rock and 40% fines in Cylinder 2, and 40% rock and 60% fines in Cylinder 3. Uniform initial dry weight (placing density) was striven for. The cylinders were equipped with porous disks at the top and bottom of the sample to permit the passage of percolating water. A small initial vertical load was applied to balance the pressure due to water percolating in an upward direction until the rate of percolation had become constant.

The load was then increased in steps of 25 lb per sq in. until an ultimate load of 325 lb per sq in. was reached. Before the application of each load increment, movements due to settlement had diminished to negligible values. The total settlement and the rate of percolation were observed for each load

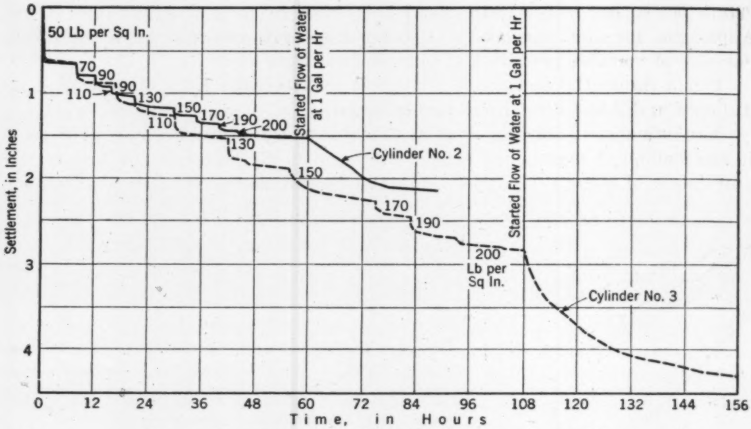


FIG. 3.—ROCK SETTLEMENT TESTS; SAMPLES FROM QUARRY 10

TABLE 1.—CONSOLIDATION, SAMPLES FROM QUARRY 10

Load (lb per sq in.)	Cylinder 1			Cylinder 2			Cylinder 3		
	Hours at the given load	Height of rock (in.)	Total settlement (%)	Hours at the given load	Height of rock (in.)	Total settlement (%)	Hours at the given load	Height of rock (in.)	Total settlement (%)
0	0	29.000	0	0	29.000	0	0	29.000	0
50	7	28.160	2.90	8	28.344	2.26	8	28.343	2.27
70	6	27.960	3.59	4	28.203	2.75	9	28.107	3.08
90	16	27.632	4.72	4	28.074	3.19	13	27.744	4.33
110	4	27.472	5.27	5	27.960	3.59	12	27.470	5.28
130	5	27.261	6.00	9	27.822	4.06	14	27.127	6.46
150	5	27.075	6.63	5	27.720	4.41	19	26.739	7.80
170	8	26.858	7.39	5	27.633	4.71	8	26.553	8.44
190	6	26.704	7.92	4[a]	27.550	5.00	20[a]	26.208	9.63
200	5[a]	26.612	8.24	16	27.479	5.25	5	26.174	9.75
200	27	25.077	13.53	29	26.864	7.37	49	24.710	14.80
Total	89	....	....	89	....	....	157	....	....

[a] Water added at the end of this period.

TABLE 2.—PERCENTAGE GRADATION, SAMPLES FROM QUARRY 10

Description	Cylinder 1		Cylinder 2		Cylinder 3	
	Before loading	After loading	Before loading	After loading	Before loading	After loading
Particle Sizes (In.):						
1½ to 3	28.5%	24.3%	27.6%	25.4%	100.0%	66.3%
¾ to 1½	28.5%	24.5%	20.5%	19.6%	0	18.9%
No. 8 to ¾	40.0%	40.4%	29.5%	31.0%	0	11.5%
Passing No. 8	3.0%	10.8%	22.4%	24.0%	0	3.3%
Volume (cu ft)	12.180	10.53	12.174	11.28	12.174	10.38
Unit weight (lb per cu ft; dry)	106.7	123.4	121.4	131.1	89.1	104.5
Fineness moduli	7.35	6.85	6.35	6.20	9.00	8.29
Average rate of flow through rock	1.15 gal per hr		1.01 gal per hr		1.0 gal per hr	

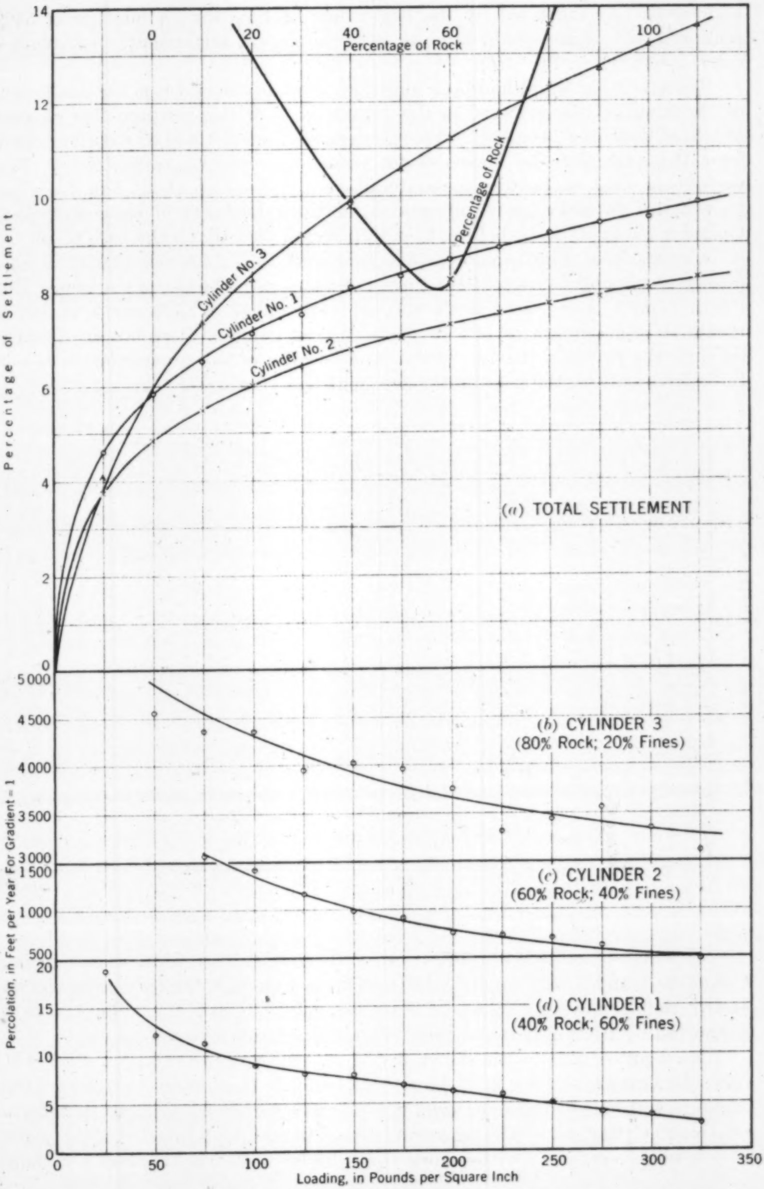


FIG. 4.—TEST RESULTS, SECOND GROUP OF TESTS ((b), (c), AND (d)) ARE FOR PERCOLATION

increment. As indicated by the heavy line in Fig. 4(*a*), a mixture of 55% rock and 45% fines would tend to show minimum settlement. Percolation rates are shown in Fig. 4(*b*).

Thus far no attempt had been made to ascertain the effect on the settlement of "arching" of the material in the cylinders. For this purpose the bottoms of two of the three 24-in. by 24-in. cylinders (Cylinders 1 and 3) were separated from the base plate by means of set screws and wedges, respectively. The perforated disks supporting the material were blocked up about 6 in. from the base plates to make ample allowance for the settlement of these cylinders. Cylinder 2 was set up as before; that is, it rested directly on the base plate.

A mixture of approximately 65% rock and 35% fines was placed as uniformly as possible in each of the three cylinders with the aid of sluicing. The top disks were then placed and a vertical load applied in 25 lb per sq in. increments to a maximum of 325 lb per sq in. as before. Percolation of water through the samples was not practicable because of the separation of two of the cylinders from the base and consequent lack of a water seal.

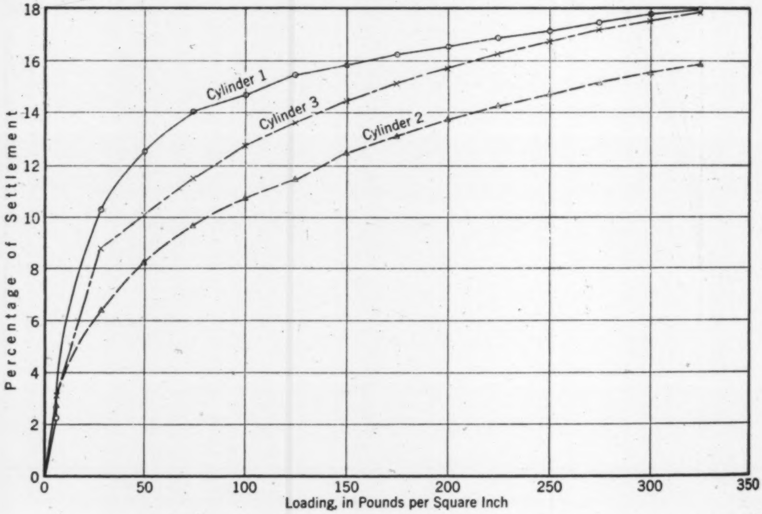


FIG. 5.—ARCHING EFFECTS, SECOND GROUP OF TESTS

Immediately after the application of the initial load of 25 lb per sq in., Cylinders 1 and 3 were completely freed from base support by raising the set screws and removing the wedges, respectively. Hence the cylinders were supported by friction between their inner surface and material only.

The result of this test is shown in Fig. 5. It shows that with cylinders of these dimensions arching is an important factor. It is expressed through the excess in settlement of the material in Cylinders 3, and particularly Cylinder 1 relative to Cylinder 2. The set screws served to raise Cylinder 1 slightly before each load increment. Furthermore, the cylinder walls were tapped. Arching to the walls was thereby not only reduced but substantially eliminated. Fineness moduli resulting from these tests are given in Table 3.

TABLE 3.—FINENESS MODULI, SAMPLES TESTED FOR ARCHING EFFECT

Description	CYLINDER 1		CYLINDER 2		CYLINDER 3	
	Before	After	Before	After	Before	After
Percentage of fines	35.3	43.1	35.3	43.0	35.9	39.9
Fineness moduli	6.01	5.61	6.01	5.68	5.96	5.82

Before proceeding with the third group of tests (which is concerned with the fines only), the field tests will be discussed.

The Rolling Tests.—The average quarry material, passing either over or through the 6-in. by 9-in. "grizzly" openings showed a typical gradation based on dry screening, as shown in Table 4.

TABLE 4.—TYPICAL GRADATION OF QUARRY MATERIAL, BASED ON DRY SCREENING

Description	LARGE PARTICLES[a] (29% > 6 IN. BY 9 IN.)		SMALLER PARTICLES[b] (71% < 6 IN. BY 9 IN.)				
			43%		50%		7%
	From	To	From	To	From	To	
Size range, in millimeters	150	500	6	150	0.10	6	<0.10
Classification	Rock		Rock		Fines		Fines

[a] Passing over. [b] Passing through.

Hence the material, subject to compaction in Zone 3, contained an average of about 43% rock and 57% fines. Obviously the stability and permeability of such a fill are governed by the density of the fines. The latter, in turn, is governed by the breakdown of the rock due to compaction, because unless breakdown occurs, pressure is transmitted from rock to rock through point contact and relatively little pressure is transmitted by fines in the interstices.

Test fills of quarry materials, having passed through "grizzlies" with openings varying from 3 in. by 9 in. to 6 in. by 9 in. in the clear were placed in 9-in. to 12-in. layers and compacted by means of a single-drum sheepsfoot roller. The drum was 58 in. long and 42 in. in diameter. It had eighty teeth and feet, each 8 in. long, and a bearing area for the feet of approximately 8 sq in. The weight of the roller was varied between 4.75 and 12.1 tons by filling the drum with suitable materials. A total of twenty-four test fills were rolled, each about 80 ft long, 12 ft wide, and 3 ft high. The test fills were divided into four sections, *A*, *B*, *C*, and *D*, roughly 12 ft by 20 ft. Six layers were spread, between 9 in. and 12 in. thick, and compacted by means of from ten to forty roller passes. The moisture content was varied between the four sections in intervals of approximately 2%—namely 7%, 9%, 11% and 13% more or less. Immediately after completion of each fill, twelve samples were taken (three from each section) to ascertain the degree of compaction. For this purpose a shallow pit about 4 ft by 4 ft in size was first dug to remove the top layer and its floor leveled off. A metal ring 12 in. in diameter was then set and plaster of Paris placed around it. A hole or pocket was then dug within the ring by suitable hand tools and the excavated materials placed in a can, sealed, and weighed. The volume of the hole was determined by filling it with fine, dry beach sand of known unit weight. The unit weight, inclusive of moisture, of the compacted fill was thereby established.

Next, the rock was separated from the fines by means of a No. 4 screen, spread out on burlap, dried, and weighed. Its apparent specific gravity and volume were then determined through immersion. Subtracting the rock volume from the total volume gave the volume of fines. After determining the average moisture content of the latter, the dry density of the fines resulted. The dry density served as the criterion of compaction as it proved to be the governing factor in determining the intrinsic properties of the fill.

For one and the same roller weight and number of passes, the dry density of the fines was principally governed by: The relative hardness, the percentage of rock, and the moisture content. The respective typical gradation before and after rolling is shown in Fig. 6. The breakdown due to rolling is evident.

Based on the rolling tests and particularly on the screen analyses before and after rolling, a relation between roller weight and breakdown of the average quarry rock was obtained. This is shown in Fig. 7. It is based on equal percentages of fines; that is, 40% ± before rolling. Guided by this result, the District proceeded with the design and construction of five double-drum roller units as shown in Fig. 8 and one single-drum unit. The dead weight of each double-drum unit was 14.50 tons and averaged 24.75 tons, or 12.375 tons per drum when filled with saturated sand.

TABLE 5.—TEST LOADS IN TONS TO DETERMINE ECONOMICAL SIZE OF ROLLERS

Load	ROLLER UNIT[a]			
	D_1	D_2	D_3	S_1
Unloaded	14.50	14.50	14.50	7.00
Water loaded	19.00	19.25	17.25	9.25
Water and sand loaded	25.50	24.25	24.50	11.00

[a] D = double drum and S = single drum.

Studies were made to ascertain the most economical combination of rollers that would satisfy the requirements regarding the compaction of the material in Zone 3. They were accompanied by tests. Three double-drum units and one single-drum unit were used unloaded, partly loaded and fully loaded, with weights in each case as given in Table 5.

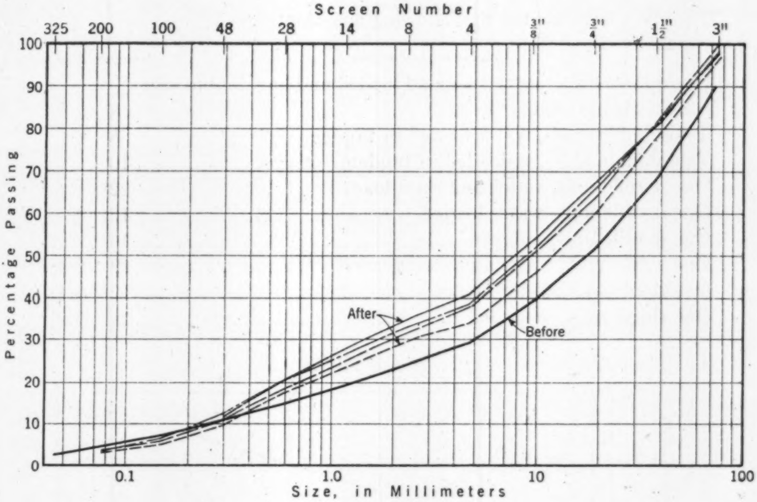


FIG. 6.—GRADATION OF ROCK FROM QUARRY 10, BEFORE AND AFTER ROLLING

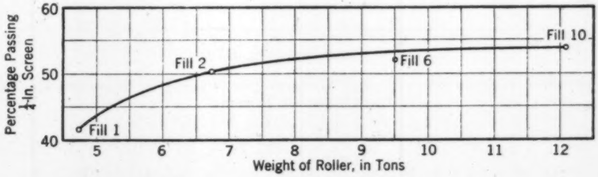


FIG. 7.—BREAKDOWN DUE TO ROLLING

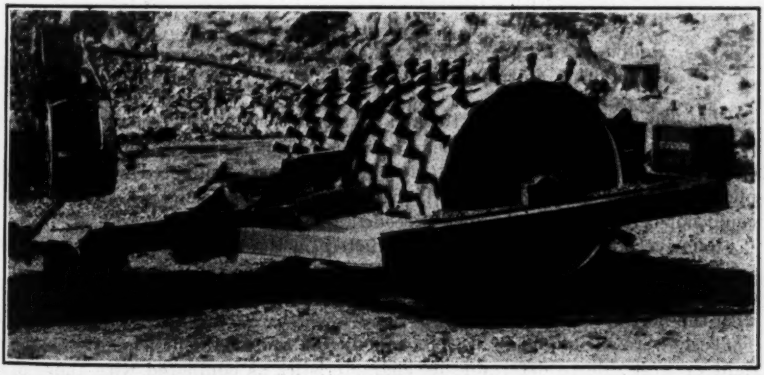


FIG. 8.—TWO-DRUM SHEEPSFOOT ROLLER

The rollers were used in the following combinations:

Combination of rollers	Number of passes	
One double unit, water and sand loaded and one double unit unloaded in tandem	8	
Two double units, water loaded in tandem	8	
Two double units, unloaded in tandem	10	
One double unit, water and sand loaded	12	
One double unit, water loaded	12	
One double unit, unloaded	16	
One single unit, water and sand loaded	12	(24)
One single unit, water loaded	12	(24)
One single unit, unloaded	16	(32)

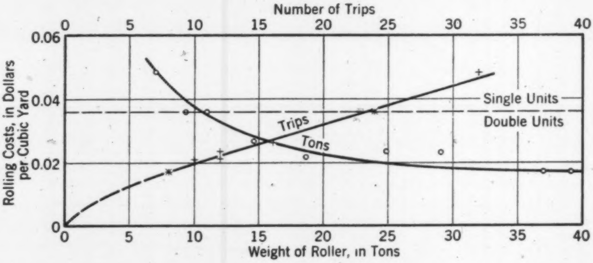


FIG. 9.—RELATION BETWEEN ROLLER WEIGHTS AND NUMBER OF TRIPS FOR CERTAIN UNIT COSTS OF ROLLING

The result is shown in Fig. 9. The field cost, without overhead, in dollars per cubic yard of rolled materials, is based on the following charges:

Equipment	Rate, in dollars per hour
One tractor with bulldozer	3.75
Tractor operator	0.8625
Double-drum unit roller	0.50
Single-drum unit roller	0.30

The Tamping Tests.—Next to the abutments a strip, 2 ft to 3 ft wide, had to be compacted by means of hand tampers because the rollers could not operate there. For this purpose, tamping tests were conducted with electric, pneumatic, and gasoline tools. It was found that satisfactory compaction could not be obtained if the rock exceeded about 40%.

Relatively few field tests were conducted on the material in Zone 2. Its typical characteristics are reflected in Figs. 10, 11, and 12, showing wet and dry screen analyses and compaction data. This material compared favorably with the best of the soil on record.

Tests were also made on samples taken from the abutments, mainly on the right or west side to determine its characteristics as compared with those of the materials in Zone 2 and Zone 3. It was found that even the softest of rock encountered after stripping showed dry densities and corresponding stability and permeability which were at least as satisfactory as those pertaining to the fines in Zone 3.

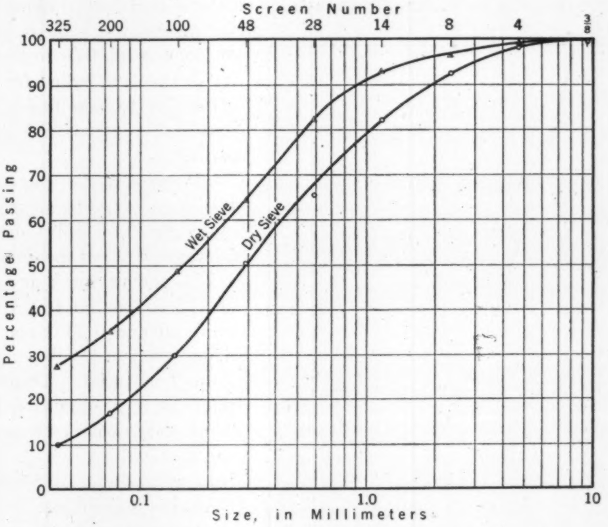


FIG. 10.—SIEVE ANALYSIS OF COMPOSITE SAMPLE, ZONE 2

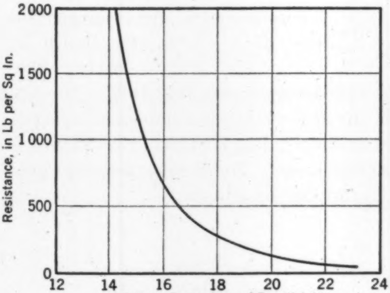


FIG. 11.—PLASTICITY-NEEDLE PENETRATION RESISTANCE; ZONE 2

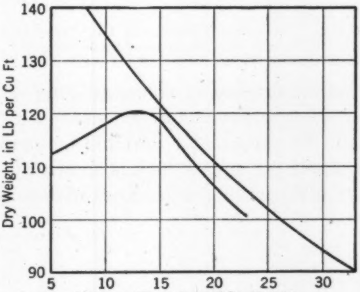


FIG. 12.—DRY WEIGHT; ZONE 2

Samples from the natural stream bed taken at various depths and distances downstream from the proposed toe of Zone 3 showed satisfactory stability and permeability under surcharge. The latter had to be such as to freely drain the flow due to percolation through the dam on the assumption of a full reservoir for an indefinite period of time.

The Third Group of Tests

All of these tests were conducted in the laboratory. They served to determine the intrinsic properties of the fines in Zone 2, and particularly in Zone 3, and hence were the most significant tests, from the standpoint of experimental soil mechanics. The importance of Zone 3 relative to the stability of the dam called for special attention to the fines of that material, particularly when saturated.

Permeability.—Percolation tests, conducted by means of 8-in. cylinders (fines only) showed an average rate of percolation of 1.5 ft per yr for a dry density of 125 lb per cu ft for Zone 3 material and 0.50 ft per yr for a dry density of 116 lb per cu ft for Zone 2 material. These values refer to a gradient of 1 on 1. They were obtained by reducing the test values, which resulted from a gradient of 80 on 1, in the same proportion.

Percolation tests conducted on composite and mixed materials of Zone 3 in the 24-in. cylinders substantially confirmed the results. They are shown in Fig. 13. They clearly reflect the importance of adequate compaction of the fines (see Fig. 13).

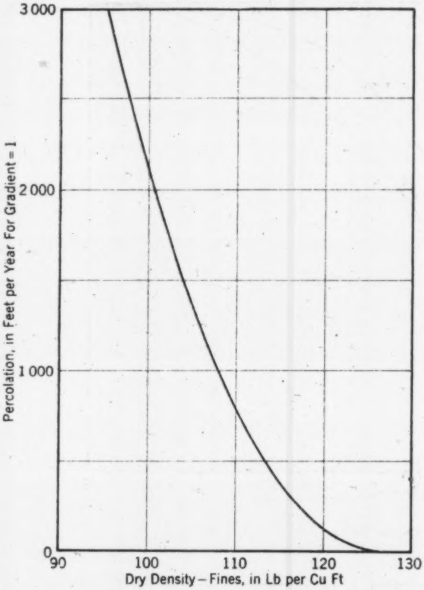


Fig. 13.—Relation Between Dry Density of Fines and Rate of Percolation

Compaction and Breakdown.—Fig. 14 shows the relation between compaction and breakdown for variable moisture contents. Isopenetration resistance in pounds per square inch is shown by broken lines. The results were obtained by compacting typical fines with a 5½ lb rammer as follows:

Curve (Fig. 14)	Number of drops	Force
A	5	4-in. drops
B	15	4-in. drops
C	15	Medium tamps
D	10	Hard tamps
E	25	Hard tamps

Shearing Strength.—The tests to establish the shearing strength of the saturated fines in Zone 3 led to results that are believed to be most significant so far as the intrinsic properties of this material are concerned. They were based on the fundamental concept that the shearing strength is the only criterion of safety of a saturated, granular mass in a dam fill in so far as structural stability is concerned, because, on the one hand, such a granular mass has no tensile strength and, on the other hand, it cannot fail due to compression as long as its lateral movement is restricted—that is, as long as it is confined. This material placed in a cylinder, for example, could be subjected to any pressure without the danger of failure, as long as the cylinder successfully resisted lateral pressure. Failure of the cylinder, however, would elimi-

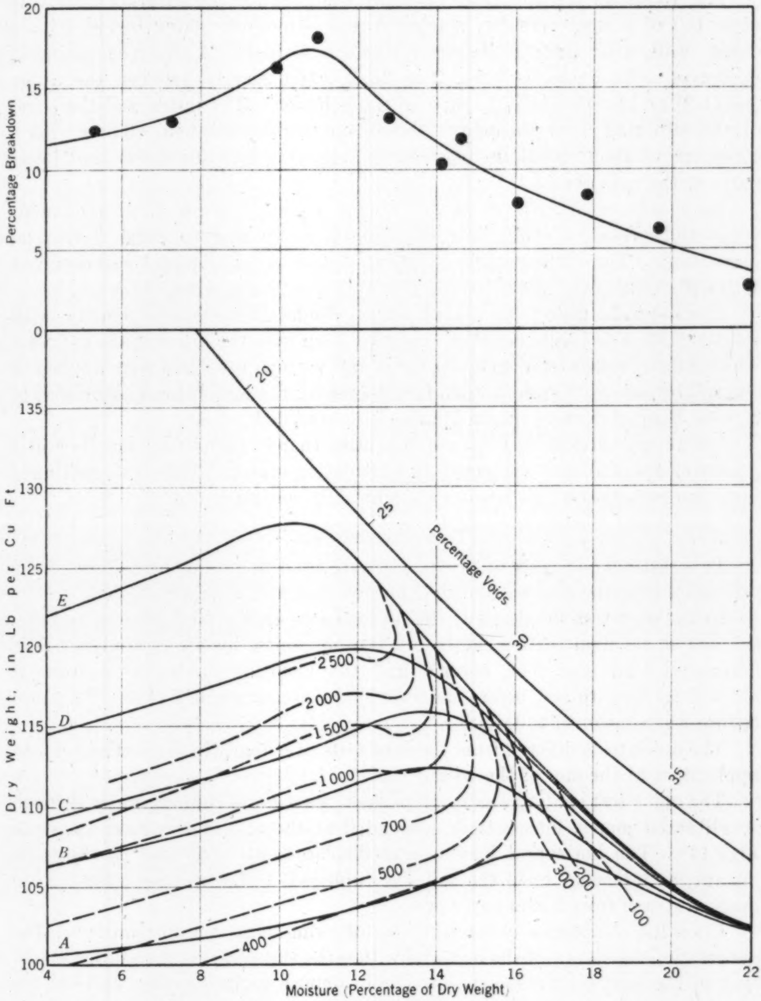


Fig. 14.—Compaction, Breakdown, and Penetration Resistance

nate the outer resistance to lateral pressure and would permit particles to move laterally and relative to each other unless restricted by internal friction; that is, by shearing strength.

Fig. 15 shows the apparatus specially designed for the shearing tests. It consisted of a steel cylinder, machined and chromium plated inside, with a 0.5-in. wall, 4 in. internal diameter, and 11 in. high. A slip ring (point 1) reinforced with a heavy band, 2 in. high, fitted snugly between the upper (point 2) and lower (point 3) parts of the cylinder. The inside and the edges of the slip ring were chromium plated and highly polished. There was a clearance of about 0.001 in. between the slip ring and the upper and lower parts of the cylinder.

By means of a counterbalanced (point 4) lever (point 5) arrangement, horizontal pressures could be exerted on the slip ring to cause horizontal movement. The ratio between vertical action and horizontal reaction was 1 : 14.40.

The soil to be tested was placed in the cylinder at the desired density, with the slip ring at its initial position—namely, flush with the inside of the cylinder. The material rested on a perforated seat and vertical pressures were applied to the soil by means of an hydraulic jack (point 6), the load being transmitted to the soil through a stem (point 7) and perforated disk.

Before any vertical load was applied (other than enough to balance the water pressure) the soil was subjected to percolating water. This was continued until the percolation rate became substantially constant.

Test Procedure

Two distinct methods of procedure were used in producing failure of the soil under shearing stress—namely, the slow test and the quick test.

In the slow test the desired vertical load was applied and allowed to bring the soil to its final settlement before the horizontal load was applied. The horizontal load was then applied and the resulting horizontal movement allowed to come to rest before the next load increment was added. This procedure was continued to failure.

The quick tests differed from the slow tests principally in the method of load application at the end of the lever.

The quick test loading was accomplished by means of running water through a calibrated meter, into a tank suspended at the end of the lever (point 5, Fig. 15). The horizontal loading and displacement were read at the same instant, at intervals, until the soil was ruptured. The time required for the quick test was from 2 min to 10 min.

Upon the completion of the test, the following data were obtained: (*a*) The penetration resistance of the material within the slip ring; (*b*) the moisture content of the soil; (*c*) the weight of soil, to determine its density, and (*d*) the specific gravity of the material, to determine the relative saturation of the soil under test conditions.

Soil Characteristics

Gradation.—Enough of typical Zone 3 material was secured to run all of the tests before they got under way in order that there would be no appreciable variation in the material to influence the results.

The soil used in the tests could be classed as silty sand. When wet the soil was more plastic than elastic. Periodic gradation analyses were made on the soil to check its variation as the series of tests progressed. The fineness modulus ranged from 1.81 to 2.07. The mean effective size (size for 10% passing) averaged 0.080 mm, with the uniformity coefficient (ratio of size at 60% passing to size at 10% passing) being about 6.5. The foregoing values

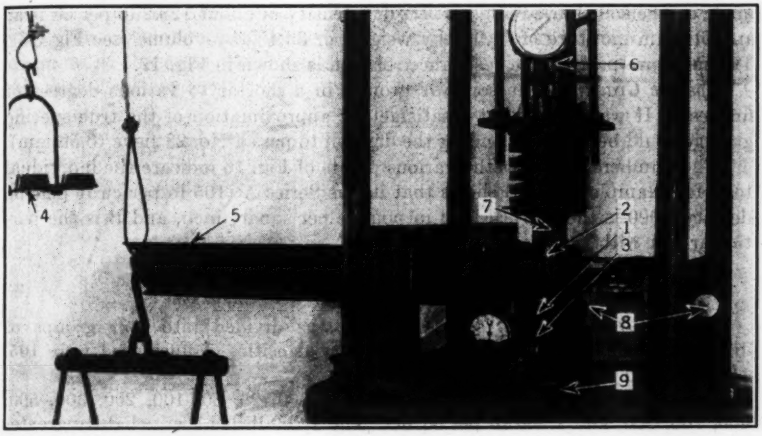


FIG. 15.—SHEAR TEST APPARATUS

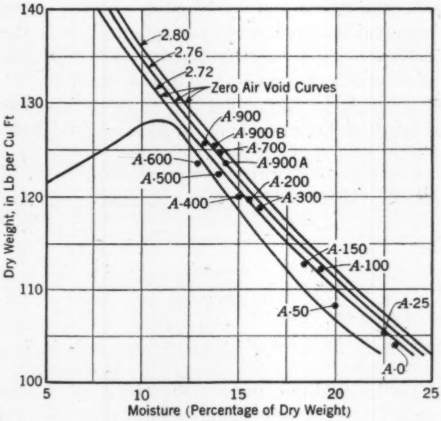


FIG. 16.—DRY WEIGHT; ZONE 3

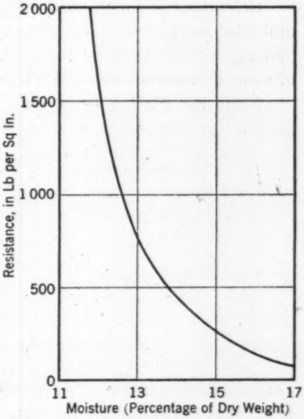


FIG. 17.—PLASTICITY-NEEDLE PENETRATION RESISTANCE; ZONE 3

are for the material screened dry. When the material was screened wet, the average fineness modulus dropped to 1.45 with a corresponding drop in the effective size and increase in the uniformity coefficient.

Compaction and Penetration Resistance.—An average of six compaction tests made on the soil showed a maximum dry density of about 128.2 lb per cu ft at an optimum moisture of 11.0% by weight, or 30.4% by volume (see Fig. 16). The moisture-penetration resistance relation is shown in Fig. 17.

Specific Gravity.—The soil was ground in a mortar to various degrees of fineness. It was found that a satisfactory approximation of the true specific gravity could be had by grinding the dry soil to pass a No. 28 sieve (0.59 mm).

The numbers shown at the various points of Fig. 16 indicate the individual test; for example, A-900B shows that it was Series A (105 lb per cu ft placing density), 900 is the vertical load in pounds per square inch, and B is the particular test with this vertical load.

TEST FEATURES

Placing Density.—The series of tests was divided into four groups of different placing densities. The placing dry densities of fines used were 105, 110, 115, and 120 lb per cu ft.

Vertical Load.—The vertical loadings were 0, 25, 50, 100, 200, 300, 500, 600, 700, 900, and 1,200 lb per sq in. The 1,200-lb load was slightly greater than that recommended by the hydraulic jack manufacturer. Consequently, all tests at 1,200 lb per sq in. vertical load were dropped from consideration.

Rebound.—At the completion of each individual test the dry density of the material within the slip ring at removal was corrected for rebound from the density at the time of incipient failure. This was the density used in determining the degree of saturation at the time of incipient failure.

Incipient Failure.—It was quite simple to determine the point of incipient failure for the quick tests. The relation between time of loading and horizontal displacement was plotted on logarithmic paper (Fig. 18). It is seen that there appear to be two distinct trends, one being plastic displacement with respect to time and horizontal loading and the other being beyond stability. Therefore, the point of intersection can be called the time of incipient failure.

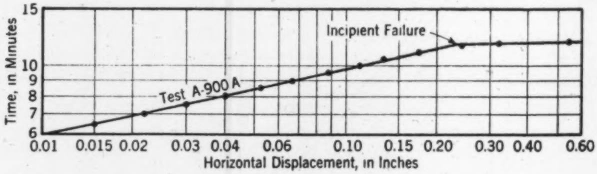


FIG. 18.—LOCATION OF POINTS OF INCIPIENT FAILURE FOR QUICK TESTS

It was not always possible to determine the point of incipient failure in this manner for the slow tests. However, failure of the material generally resulted shortly after the horizontal displacement had passed 0.2 in. At this point a small increment of horizontal load caused failure. The relation between horizontal and vertical pressures at that instant was used in determining the shear coefficient.

Horizontal Load.—The shearing stress was determined by dividing the horizontal load, applied by the lever, by the shear area, after the latter had been corrected for reduction due to the horizontal displacement. No correction was made in the compressive stress due to the vertical load because of the horizontal displacement.

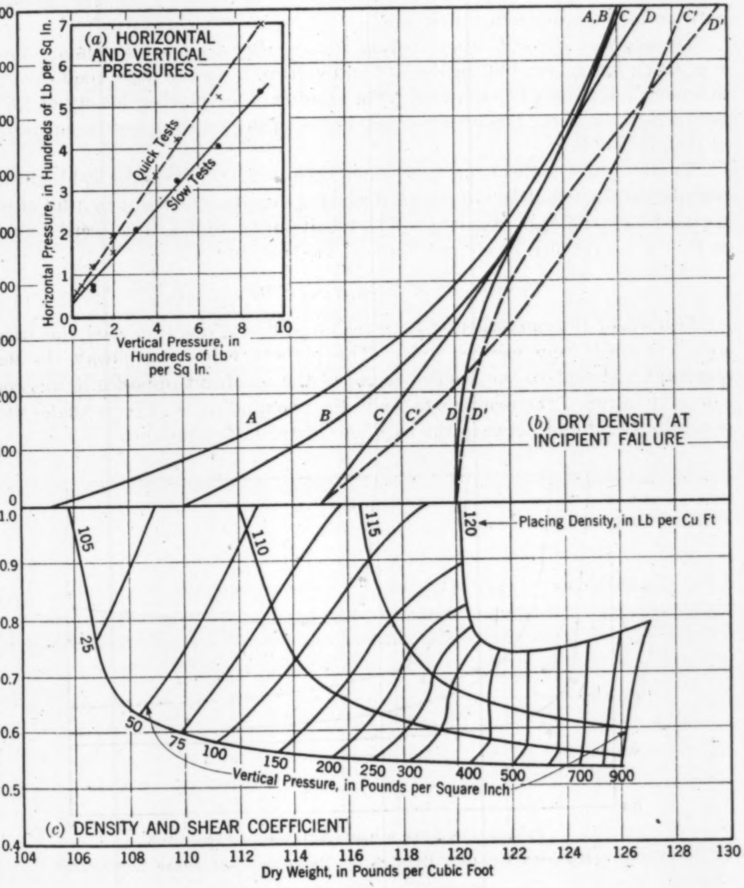


FIG. 19.—TEST RESULTS; ZONE 3

RESULTS

Shear Coefficient.—Fig. 19(*a*) shows the relation between the vertical pressure and the horizontal pressure (in thousands of pounds per square inch) required to reach incipient failure for the series having a placing density of 115 lb per cu ft. This relation is typical of the four series conducted. The shear coefficient is the ratio of horizontal pressure to vertical pressure.

Consolidation.—Fig. 19(*b*) shows the density attained by the soil at incipient failure under the various vertical pressures. There was no appreciable difference between the slow and quick tests for the two lower placing densities ($A = A'$, $B = B'$). However, there was a noticeable increase in density for the quick tests over the slow tests for the two higher placing densities ($C' > C$, $D' > D$). Obviously the influence of the placing density decreases with an increase in vertical pressure.

Density and Shear Coefficient from Placing Density-Load Relation.—From Fig. 19(*c*), for a given placing density and vertical load, the shear coefficient for incipient failure may be obtained. For example, for a placing density of 117 lb per cu ft and a vertical pressure of 300 lb per sq in. the shear coefficient is 0.70 at incipient failure.

These results confirm the conclusion already drawn from the field and the percolation tests that 120 lb per cu ft placing density will result in a fill of low permeability and high stability. The results of the quick tests were omitted as they substantially agree with those of the slow tests.

PENETRATION RESISTANCE RELATIONS

Features of Determinations.—Four readings, that is one from each quadrant, were obtained from each sample. The highest value was always in that quadrant adjacent to the applied force; in the quadrant opposite it was considerably lower. The readings taken in the two quadrants at right angles were generally equal and between the high and low values mentioned.

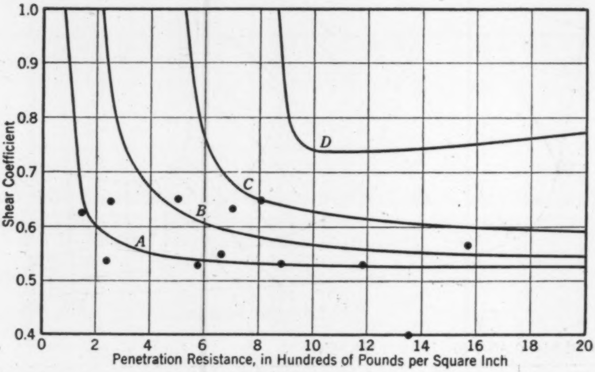


FIG. 20.—RELATION BETWEEN PENETRATION RESISTANCE AND SHEAR COEFFICIENT

Shearing Strength Relation.—A relation between shearing strength and penetration resistance may be derived. The result is shown in Fig. 20. Although it is purely theoretical it shows a characteristic similar to the dry-weight versus shear-coefficient relation curves.

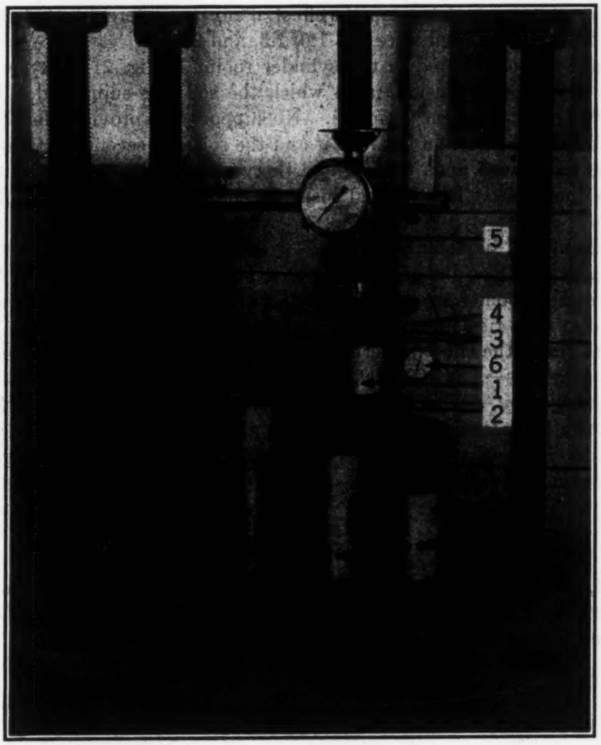


FIG. 21.—LATERAL PRESSURE TEST

The heavy dots represent actual penetration needle readings for 105 lb per cu ft placing density only. They show too wide a spread to check the theoretical relation. Hence, it would appear a hopeless task to predict the shearing strength and with it the stability of similar materials based on the penetration resistance.

LATERAL PRESSURE

The shearing strength may also be determined if the lateral pressure produced by vertical loading is known. With this in mind the following series of tests was conducted:

Method.—The determination of the lateral pressure was accomplished by measuring the change in diameter of a thin cylindrical shell, produced by various vertical loads applied to the soil. By knowing the change in diameter and the modulus of elasticity of the shell material, the value of the lateral pressure can be determined.

Apparatus.—The apparatus (Fig. 21) consisted of four brass cylinders, 1/16 in. thick, 4 in. internal diameter, and of various heights. The object of the different heights was to determine the effect, if any, for different height-diameter of soil ratios. Those used were 0.5, 1.0, 1.5, and 2.0.

The soil was compacted in the cylinder (point 1, Fig. 21). A perforated disk was fitted loosely inside, upon which the soil was supported. This, in turn, rested upon the stem (point 2). No support was provided for the brass cylinder other than friction between it and the soil. A second perforated disk (point 3), was placed upon the soil and the vertical load transmitted through the stem (point 4). The vertical load was produced by an hydraulic jack (point 5).

The change in diameter of the brass cylinder was measured to the nearest 0.0001 in. by an Ames dial gage (point 6). The cylinder wall was divided into several rings of points spaced at regular intervals vertically. Six diameters were used at each ring in obtaining an average change in diameter.

To determine the effect of vertical loads, the test was performed under five different pressures—200, 400, 600, 800, and 1,000 lb per sq in., respectively.

Shear Coefficient.—With the change in diameter and the modulus of elasticity being known the lateral pressure created by the various vertical loads could be calculated. For active thrust with the surface horizontal and for frictionless walls, Coulomb's equation

$$\frac{E}{P} = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (1)$$

was used, in which E is the lateral pressure, P is the vertical pressure in pounds per square inch, and ϕ is the angle of internal friction. The tangent of ϕ would then be equal to the shear coefficient. For example, since

$$\frac{E}{P} = \tan^2\left(45 - \frac{\phi}{2}\right) \qquad (2)$$

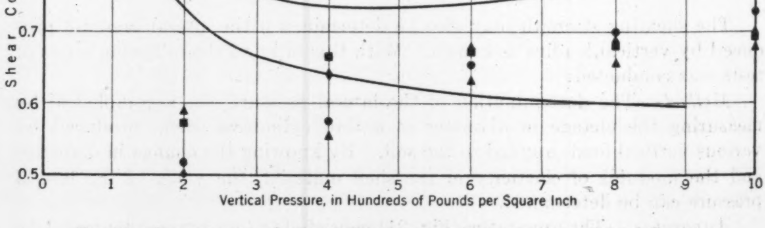


FIG. 22.—COMPARISON OF SHEARING STRENGTHS AS DETERMINED BY DIRECT SHEAR AND LATERAL PRESSURE

it follows that

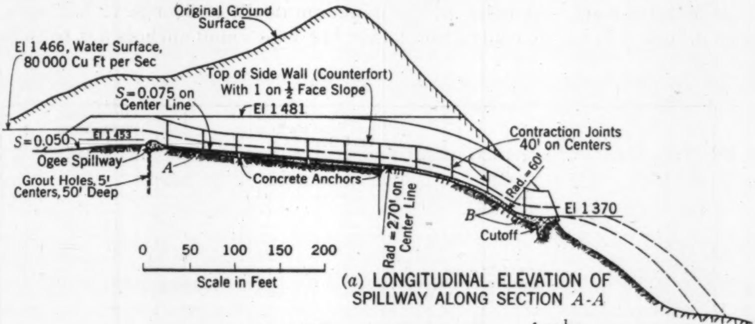
$$2 \tan^{-1}\left(\frac{E}{P}\right)^{0.5} = 90 - \phi \text{ .... } (3)$$

The angle of pressure transmission is

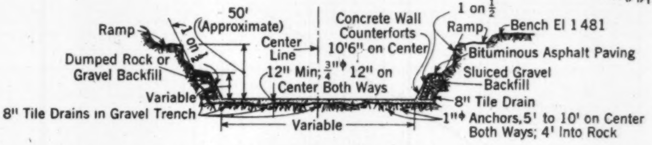
$$\theta = \tan^{-1}\left(\frac{E}{P}\right)^{0.5} \text{ .... } (4)$$

Then $\phi = 90 - 2\,\theta$; and $\tan\phi$ = coefficient of shear.

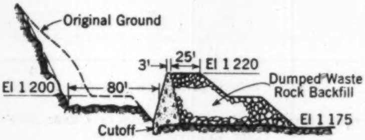
The comparison between the coefficient of shear as determined by the lateral pressure tests and by direct shear is shown in Fig. 22.


Original Ground Surface
El 1 466, Water Surface, 80 000 Cu Ft per Sec
Top of Side Wall (Counterfort) With 1 on ½ Face Slope
S = 0.075 on Center Line
El 1 481
S = 0.050
El 1 453
Ogee Spillway
A
Grout Holes, 5' Centers, 50' Deep
Concrete Anchors
Rad = 270' on Center Line
Contraction Joints 40' on Centers
Rad. = 60'
B
El 1 370
Cutoff
0 50 100 150 200
Scale in Feet

(a) LONGITUDINAL ELEVATION OF SPILLWAY ALONG SECTION A-A


50' (Approximate)
Ramp
1 on ½
Dumped Rock or Gravel Backfill
Variable
8" Tile Drains in Gravel Trench
Center Line
Concrete Wall Counterforts 10'6" on Center
12" Min; ¾"ϕ 12" on Center Both Ways
Variable
1 on ½
Ramp
Bench El 1 481
Bituminous Asphalt Paving
Sluiced Gravel Backfill
8" Tile Drain
1"ϕ Anchors, 5' to 10' on Center Both Ways; 4' Into Rock
0 40 80 120 160
Scale in Feet

(b) SECTION B-B OF SPILLWAY


Original Ground
3'
25'
El 1 220
El 1 200
80'
Dumped Waste Rock Backfill
El 1 175
Cutoff
0 40 80 120 160
Scale in Feet

(c) SECTION C-C OF WASTE CHANNEL

FIG. 23.—SECTIONS, SAN GABRIEL DAM NO. 1 (SEE FIG. 1)

THE SPILLWAY

As shown in the plan view of the project (Fig. 1), the spillway cuts through the ridge forming the right or west abutment of the dam. The original contract had no provision for this work. The revised or supplemental contract, however, included the provision for the rough excavation of the spillway cut as an auxiliary quarry, known as Unit 2 of Quarry 10. This rough excavation was estimated at 1,200,000 cu yd above El. 1400 and the contractor agreed to quarry this rock and place it in Zone 1 at the regular unit cost of 75¢ per cu yd for rock fill. Without this saving in cost of excavation the project would not have been feasible.

Fig. 23 shows three sections indicated in the general plan view (Fig. 1). At point *A*, Fig. 23(*a*), the concrete floor is 18 in. thick with $\frac{3}{4}$-in. round rods, 12 in. on centers both ways. This floor is laid on a minimum of 8 in. of gravel, with tile underdrains. Contraction joints are approximately 40 ft apart, with copper water seals. At point *B*, the minimum depth of floor is 12 in., being laid on rock. It has no contraction joints, but 1-in. round anchors 5 ft to 10 ft on centers.

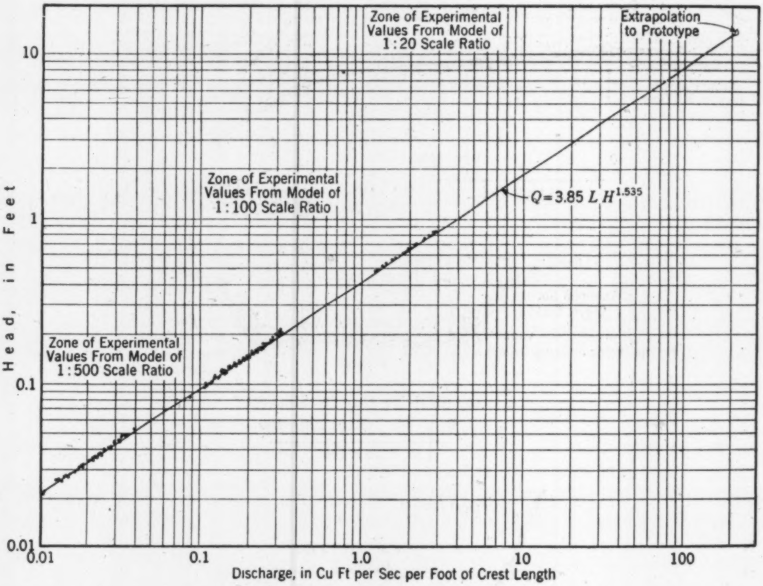


FIG. 24.—RELATION BETWEEN HEAD ON SPILLWAY CREST AND DISCHARGE PER UNIT LENGTH OF CREST

Hydraulic tests were conducted on spillway models. They served to determine:

(1) The discharge coefficient of the overflow section;

(2) The best shape and slope of the spillway channel from a standpoint of both hydraulics and economy; and

(3) The discharge conditions in Brown's Gulch.

Fig. 24 shows the result of the coefficient tests which were conducted on models of three different scales—namely, 1 : 20, 1 : 100, and 1 : 500. It shows that the discharge for all models and probably also for the prototype can be expressed by the equation

$$Q = 3.85\,L\,H^{1.535} \qquad (5)$$

Hence the weir coefficient $\mu = 0.74 =$ constant because $\frac{1}{1.535}\sqrt{2\,g}\,\mu = 3.85$.

This parabola satisfies only those tests that were not affected by backwater. Adhering to the conventional head exponent of 1.50 the same tests are satisfied if the weir coefficient is varied between 0.61 and 0.80. This exponent, however, strictly applies to free discharge only—that is, for atmospheric pressure above the upper and below the lower nappe. This was not the case here and very seldom is with ogee sections in general. Hence, there seems to be no reason why a correction of the pressure head exponent should not be more logical than the variation of the weir coefficient.

Based on these model tests it is believed safe to assume that at least 290,000 cu ft per sec would be discharged through the spillway before the dam were overtopped. This happens to be about three times the peak flow into the reservoir during the flood of March 2, 1938, and more than six times the peak flow on record prior to that date.

The heterogeneous character of the rock, in place, called for a spillway channel with side walls independent of the rock cut. Hence counterfort walls with a 1 on 0.5 face batter and backfilled with gravel against the rock were provided as shown in Fig. 23(*a*) and 23(*b*).

For the same reason the floor slab was placed on a gravel mat in the upper (that is, relatively flat) part of the spillway channel. Stops, in the form of shallow concrete anchors on circular lines, were provided for the stability of the slab as shown in Fig. 23(*a*). A system of drains was provided under the slab and the backfill behind the side walls.

Fig. 23(*c*) shows a section through the waste channel.

THE OUTLET WORKS

The outlet works were incorporated in the diversion tunnel which directed the stream flow through the east or left abutment during construction of the dam. Fig. 25 shows the general arrangement. The cross section of the dam and its position relative to the outlet works are indicated in Fig. 25. The valve chamber at Station 9, Fig. 25, contains four units: Two 123-in. butterfly valves; one 96-in. butterfly valve; and one 51-in. butterfly valve.

At Station 21 there are: One 51-in. butterfly valve; two 51-in. by 39-in. needle valves; one 39-in. by 75-in. energy absorber; one 24-in. by 60-in. energy absorber; one 30-in. by 24-in. needle valve; one 30-in. butterfly valve; one 12-in. by 9-in. needle valve; one 102-in. by 90-in. needle valve; and two 129-in. by 117-in. needle valves. Section *A-A*, in Fig. 26, shows the diversion or outlet tunnel to be circular in cross section and 30 ft in the clear. The capacity of the outlet works was based on the regulation of a so-called 50-yr capital flood, with a peak of 118,000 cu ft per sec to a maximum outflow of 50,000 cu ft per sec.

The necessary outlet capacity was thereby found to be 14,000 cu ft per sec with the water surface at the spillway lip.

A cylindrical trash-rack tower, circular in cross section, was provided at the intake end. Its principal design features are evident from Fig. 27. The four heavy columns, diametrically opposite, serve to make the structure earthquake resistant. This feature will become increasingly important with the anticipated future raising of the tower in keeping with the siltation of the reservoir. In this connection the design provided the blocking off by means of solid plain concrete of one or more lower tiers of trash-rack panels and the adding on top at least an equal number of tiers or stories of new trash-rack panels.

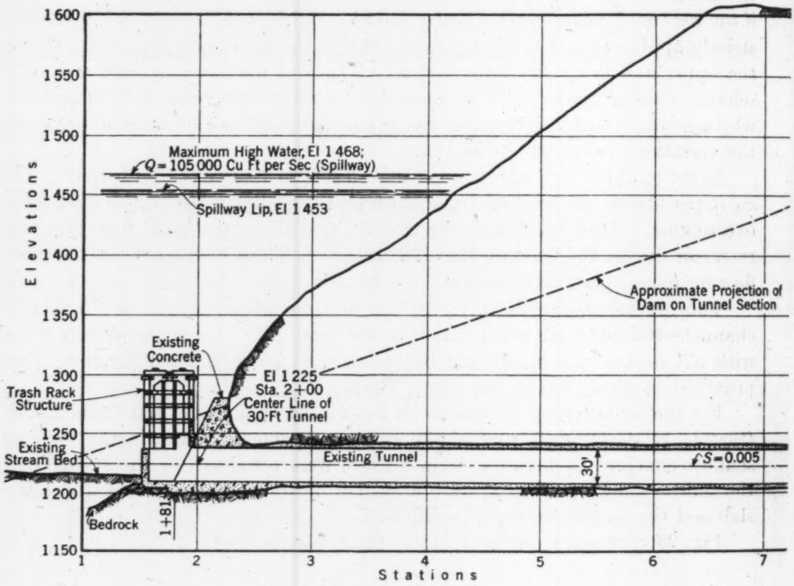
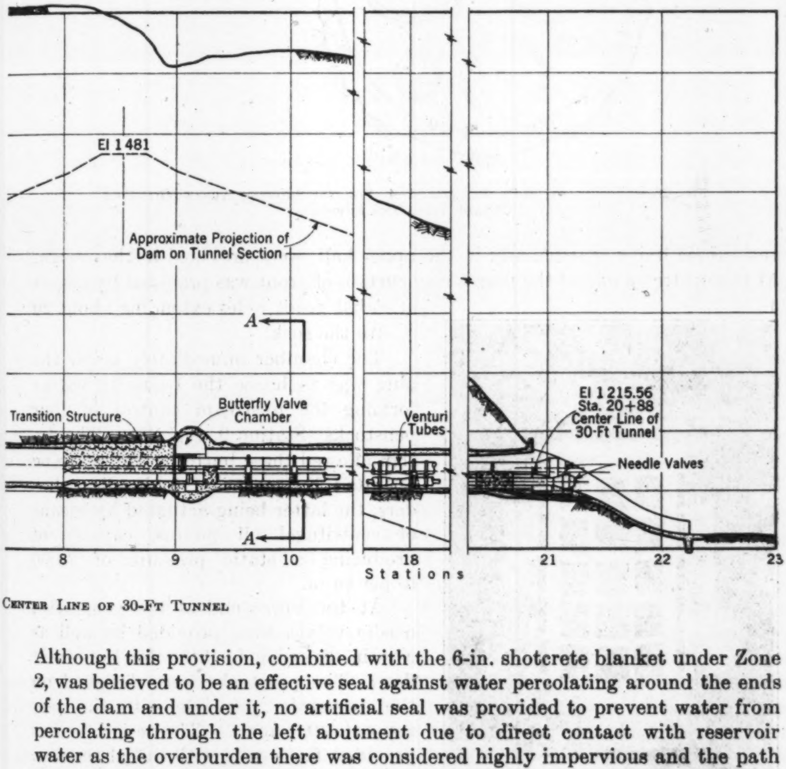

1 600
1 550
1 500
1 450
1 400
1 350
1 300
1 250
1 200
1 150
Elevations
Maximum High Water, El 1 468; Q = 105 000 Cu Ft per Sec (Spillway)
Spillway Lip, El 1 453
Approximate Projection of Dam on Tunnel Section
Existing Concrete
El 1 225
Sta. 2+00
Center Line of 30-Ft Tunnel
Trash Rack Structure
Existing Stream Bed
Existing Tunnel
30'
S=0.005
Bedrock
1+81
1
2
3
4
5
6
7
Stations

FIG. 25.—LONGITUDINAL SECTION ON

The gross area of trash racks initially provided was 5,920 sq ft or a net area of 4,950 sq ft after deduction is made for the bars. The velocity through the racks for the maximum discharge is 3.0 ft per sec.

Although the solid cylinder shell of the tower, including its base slab, was designed for pressures due to silt and water up to the spillway crest, the metal bars and ring girders supporting them were designed for pressure due to the water surface 50 ft above the top of the tower.

Approximately in line with the axis of the dam (Fig. 25) a concrete plug 100 ft long was provided with a chamber roughly ellipsoidal in shape immediately downstream from it. It was naturally desirable to place the plug as far downstream as possible in order to shorten the lengths of the penstocks. Obviously the feature governing the location was the allowable unbalanced internal pressure in the tunnel above the plug. Due to its seamy character, the rock was expected to carry some seepage water unless special provisions were made to stop it. As previously described, the cutoff wall and grout curtain along the upstream contact of Zone 2 were used to prevent this seepage.


El 1481
Approximate Projection of Dam on Tunnel Section
A
A
Transition Structure
Butterfly Valve Chamber
Venturi Tubes
El 1215.56
Sta. 20+88
Center Line of 30-Ft Tunnel
Needle Valves
8
9
10
18
21
22
23
Stations

Center Line of 30-Ft Tunnel

Although this provision, combined with the 6-in. shotcrete blanket under Zone 2, was believed to be an effective seal against water percolating around the ends of the dam and under it, no artificial seal was provided to prevent water from percolating through the left abutment due to direct contact with reservoir water as the overburden there was considered highly impervious and the path of percolation relatively long. Hence a conservative assumption had to be made as to the probable effective gradient of water percolating through the left abutment more or less parallel to the tunnel. This led to the maximum allowable excess internal pressure based on the strength of the existing tunnel lining.

The anchorage of the plug was provided by means of saw-tooth recesses in the existing tunnel lining. It was designed to safely resist full reservoir pressure combined with the maximum pull due to thermal contraction of the penstocks. Facilities for the grouting of the contact zone between the plug and tunnel lining, particularly in the upper half, were provided in the design. At the upstream end of the plug a ring curtain of grout was provided by means of radial grout holes extending about 20 ft into the rock.

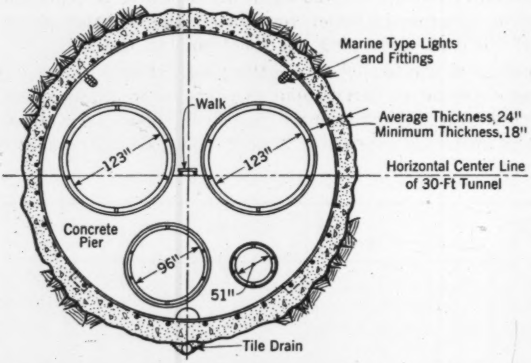


Fig. 26.—Section of 30-Ft Tunnel (*A–A*, Fig. 25) Showing Arrangement of Outlet Pipes and Supports

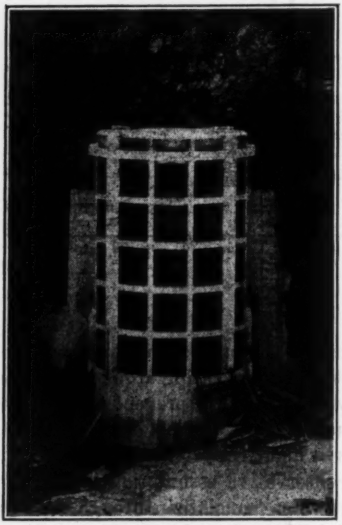


Fig. 27.—Trash-Rack Tower

The chamber immediately below the plug was to house the butterfly valves forming the upstream control of four penstocks (Station 9, Fig. 25). The design allowed these butterfly valves to be operated by means of hydraulic cylinders, the latter being actuated by means of centrifugal oil pumps, capable of producing a static pressure of 3,000 lb per sq in.

At the lower end of the penstocks, needle valves were provided as well as certain diversion facilities for low-water flow. The penstocks were designed on the ring-girder principle with the pier supports 40 ft apart. The design further provided fixed ends with initial tension.

The penstocks were to be fabricated—that is, machine welded in 40-ft lengths in the shop, with provision for field-riveted joints and welded seams. An internal enamel coating approximately $\frac{1}{8}$ in. thick was to be applied by a centrifugal process. The joints were to be treated by hand application of the enamel in the field.

The hydraulic gradients were computed upon the following assumptions:

Entrance losses—

Trash rack (through the bars)	$0.5 \frac{V^2}{2g}$
Outlet tunnel (in the tunnel)	$0.065 \frac{V^2}{2g}$
Plug (in the plug)	$0.10 \frac{V^2}{2g}$

Friction losses (Manning's *n*)—

In the tunnel	0.013
In the plug	0.011
In the pipe	0.011

Valve losses—

Butterfly valves (in the pipe)	$0.32 \frac{V^2}{2g}$
Needle valves (at the outlet and including velocity head)	$1.562 \frac{V^2}{2g}$
Venturi tube losses (in the pipe)	$0.02 \frac{V^2}{2g}$

Approximately 300 ft upstream from the needle valves (Station 18, Fig. 25) venturi tubes were provided in all of the penstocks for the purpose of measuring the flow. Remote controlled flow-indicators and recorders were provided at the needle valves and in the control house, respectively.

CONSTRUCTION OF THE REVISED PROJECT

On August 12, 1935, the revised design was approved by the California state engineer, and on August 13 construction work was resumed.

Rock-fill material underlying Zones 2 and 3 was excavated and moved to Zone 6 as indicated in Fig. 2. Simultaneously the contractor started work on a large "grizzly" plant in Quarry 10, Unit 1 (original quarry) with a peak capacity of screened materials of 1,600 cu yd per hr. The plant arrangement was perhaps one of the boldest ever used. It is shown in Fig. 28. At the toe of the existing, nearly 600 ft high quarry face, a concrete tunnel was built to protect the "grizzly" mechanism and the loading trucks from falling rock. The excavated quarry material was dumped from the edge of the cut into a chute, blasted out of the quarry face, where it partly slid and partly fell down into a receiving crater, formed by a loose rock rim immediately above the "grizzly."

With the "grizzly" in motion the material would "crawl" along the slanting "grizzly" bars, pieces greater than 6 in. by 9 in. moving over the bars and into a waiting 24-yd truck and pieces smaller than 6 in. by 9 in. including fines falling through the openings into hoppers below the floor of the tunnel. The hoppers fed this material on to a belt conveyer which lifted it into two storage bins of about 400 cu yd capacity each, shown in the foreground of Fig. 28. From these bins it was loaded into 24-yd trucks and hauled to the dam.

Immediately after completion of excavation of the old rock fill under Zones 2 and 3, the placing of compacted materials was begun. Specially designed sheepsfoot rollers were used, each two-drum unit (as previously stated) weighing about 25 tons. The minimum bearing pressure of each foot was 375 lb per sq in.

FIG. 28.—VIEW OF QUARRY 10

The cutoff wall across the canyon bottom had already been completed prior to the shut-down. Its extension, first on the east side and afterward on the west side, was now begun. The depth of the cutoff was estimated at 50 ft at the base and 25 ft at the top of the dam. It was excavated to sound rock either as a continuous trench or a series of shafts.

Special excavation was made where faults crossed the cutoff wall. According to specifications the cutoff wall had to be extended along these faults to a depth equal to one half the static head measured at the intersection of the fault with the top of the cutoff wall.

Grout holes varying from 1 in. to 8 in. in size and 100 to 150 ft in depth were drilled along the cutoff in two rows, 5 ft apart. The holes were spaced a maximum 10 ft on centers and staggered. The smaller holes, those 2 in. in size or smaller, were drilled with diamond core drills whereas the 8-in. holes were drilled by blast hole drills. The former were pressure grouted in the usual way and the latter served to apply the circulation method of grouting.

The principal excavation of the abutments was done in the conventional manner by benching from the top down and the rock surface in contact with the rock fill was stripped by means of a dragline which worked from the fill level to a height somewhat in excess of the next lift. The rock fill was placed in 25-ft lifts.

Special attention was given to the stripping of the rock surface under the shotcrete blanket, and a reasonably careful stripping job was done along the contact with Zone 3. Fig. 29 shows stripped areas and grouting operations.

To produce the water necessary for the sluicing of the loose rock fill and the sprinkling of the compacted rock and earth fill two 400-hp turbine pumps were installed in a sump below and one 150-hp turbine pump above the dam.

Monitors were used for the sluicing of the loose rock in Zones 1, 4, 5, and 6 and sprinkling trucks were used to supply the necessary moisture for the compaction of Zones 2 and 3.

The placing density of the fines in Zones 2 and 3 was checked by means of tests throughout the job. A total of 2,250 tests were taken in Zone 2 and 5,221 in Zone 3. This amounts to one test for every 500 and 620 cu yd, respectively, of materials placed. The average placing density was 117.3 and 122.20 lb per cu ft for Zones 2 and 3, respectively. Hence the actual average placing density was more than 2 lb greater than the specified one. The average unit dry weight of Zone 3 material, including rock and fines, was 144 lb per cu ft, when placed. It was probably raised to an average of 150 lb or more due to self-compaction. It assumed the properties of good concrete or artificial stone. The average moisture content was 5% of the dry weight of rock and fines combined. The 1¾-yd-dragline bucket was unable to "bite" into this mass until it had first been loosened with suitable tools.

FIG. 29.—GROUTING AND DRILLING OPERATIONS

An unusual opportunity offered itself to measure the actual consolidation when the fill reached a height of some 70 ft above bedrock. At this level an abandoned power tunnel 4 ft by 6 ft in size, "day-lighted" near the axis of the dam, and had to be sealed by means of a 20-ft concrete plug. As the tunnel afforded access to the back of the plug it was possible to observe the settlement of the Zone 3 fill by means of manometers. These manometers consisted of two calibrated vertical glass tubes ½ in. (inside diameter) connected to ½-in. copper tubes. These copper tubes were installed inside of 2-in. standard pipe and extended some 75 ft and 150 ft, respectively, into the fill. The copper tubes and 2-in. pipes were bent up at the fill end, the former being open and the latter capped. They were placed on an initial slope of 1%, the fill end being higher than the tunnel end.

By filling the copper tubes with water from the tunnel end it was possible to establish the elevation of the open end of the copper tubes by means of the manometer tubes.

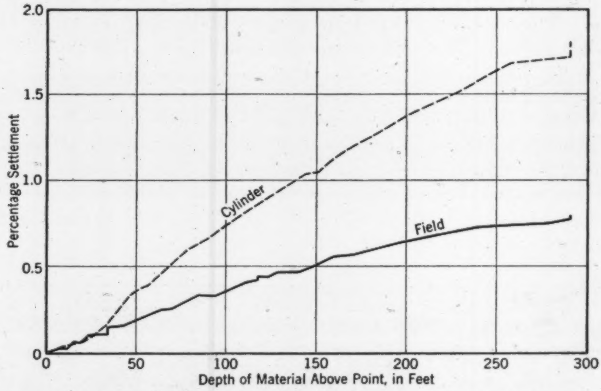


Fig. 30.—Settlement, Zone 3, Showing Relation Between Field and Laboratory Tests

Simultaneously Zone 3 material was placed in one of the 24-in. cylinders in the laboratory at the same average density as had been established for Zone 3 up to that level. The cylinder was freed from the base to minimize arching, as previously outlined. Vertical pressures corresponding to those due to Zone 3 fill above the tubes were brought to bear on the material in the cylinder, and the settlements were recorded. The results are shown in Fig. 30.

Overfill and Settlement (Revised Project)

Overfill to allow for residual settlement was provided as follows: None at the toe of the upstream slope, with a linear increase to maximum 2 ft at the crest; and none at the toe of the downstream slope, with a linear increase to maximum 3 ft vertically above the toe of Zone 3 and a linear decrease to maximum 2 ft at the crest.

Maximum vertical settlements to July, 1941, were as follows: 0.40 ft at the crest and 1.57 ft vertically above the toe of Zone 3. Under nearly full water load, the dam showed a maximum downstream deflection of 0.11 ft at the crest. With the emptying of the reservoir the crest returned substantially to its initial position.

The rock was quarried in 50-ft lifts by means of blast holes. Approximately 0.6 lb of black powder was used per cubic yard of solid rock. Including waste due to the original design as well as abutment excavation, a total of about 10,000,000 cu yd of solid rock was blasted.

The average production for two eight-hour shifts (6 days per week) was 26,000 cu yd of rock per day, placed in the dam. The maximum daily production was 42,000 cu yd and maximum monthly 975,000 cy yd. An average of 750 men were employed. Fig. 31 shows the construction of the dam completed and the spillway construction in progress.

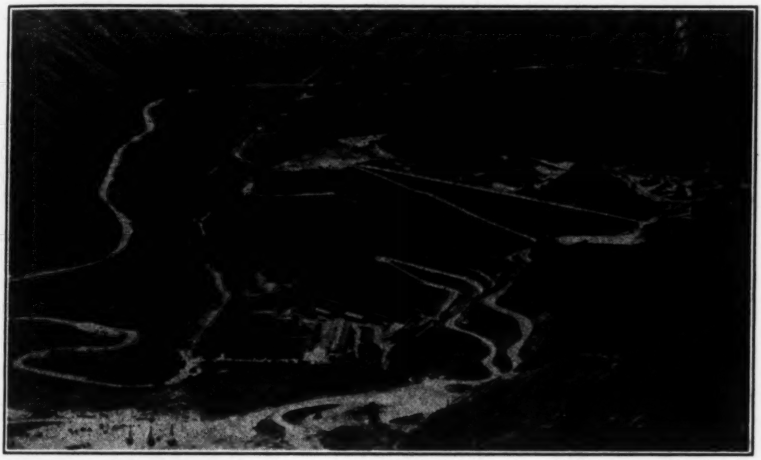


FIG. 31.—SAN GABRIEL DAM NO. 1, COMPLETED; SPILLWAY UNDER CONSTRUCTION

PERCOLATION THROUGH THE DAM (REVISED PROJECT)

Based on the actual placing densities of material in Zones 2 and 3 and the corresponding permeability conditions, a study was made on the percolation through the dam with the water surface at the spillway lip for an indefinite period.

The results are shown in the following analysis, which is based on Darcy's law of laminary flow through soil:

$$\Delta Q = K\, i\, \Delta A = K \frac{\Delta h}{\Delta L} \Delta D \qquad (6)$$

in which: ΔQ = flow in cubic feet per year per longitudinal foot of dam between two adjacent flow lines; i = hydraulic gradient for any path of flow; K = coefficient of permeability expressed as velocity in feet per year for unit area of flow and a hydraulic gradient of one; Δh = head or pressure interval in feet; ΔL = path interval in feet; and ΔA = area interval per unit length of dam in square feet = $\Delta L \times 1$. The flow net consists of flow lines and equipotential lines. The latter are perhaps better defined as equipiezometric lines. They are normal to each other and for equal head intervals form a system of squares or quasi squares as shown in Fig. 32. The flow lines are spaced such as to have equal flow ΔQ through any area interval ΔA between two flow lines. Hence, because $\Delta D = \Delta L$, Eq. 6 becomes

$$\Delta Q = K\,\Delta h \qquad (7)$$

The coefficients of permeability K_1 and K_2 for Zones 2 and 3 were found from samples of test holes to be 0.35 and 1.05 ft per yr, respectively.

Continuity of flow through Zones 2 and 3 requires that

$$\Delta Q = K_1\,\Delta h_1 = K_2\,\Delta h_2 \qquad (8)$$

or

$$\frac{K_1}{K_2} = \frac{\Delta h_2}{\Delta h_1} = \frac{0.35}{1.05} = \frac{1}{3} \qquad (9)$$

The flow net was constructed by trial and error. The flow through the dam was determined as follows:

$$Q = n_1\,\Delta Q = n_1\,K\,\Delta h = \frac{n_1\,K\,h}{n_2} \qquad (10)$$

in which: n_1 = number of flow paths and n_2 = number of head intervals. Hence, for Zone 2,

$$Q = \frac{K_1\,h\,15}{16} \qquad (11a)$$

and, for Zone 3,

$$Q = \frac{K_2\,h\,5}{16} \qquad (11b)$$

Let $h = 303$ ft; $K_1 = 0.35$; and $K_2 = 1.05$. By Eqs. 11, therefore, $Q = 99.50$ cu ft per yr, and

$$V_{\max} = K_2\,\frac{\Delta h}{\Delta L_{\min}} \times \frac{1}{(\%\ \text{voids})} \qquad (12)$$

That is, $V_{\max} = 1.05 \times \frac{303}{16 \times 20 \times 0.16} = 6.20$ ft per yr.

THE SPILLWAY

The spillway was constructed under separate contract. It was begun in June, 1937, and was completed in May, 1938. The spillway channel proper (that is, the concrete ogee section, walls and floor) was completed two weeks before the flood of March 2, 1938, at which time the spillway carried 57,000 cu ft per sec. Fig. 33 shows typical features of the spillway construction.

THE OUTLET WORKS

The installation of valves and penstocks was done under separate contract. The 51-in. butterfly valve and penstock were installed first and served to control the low-water flow of the river. For this purpose the piers were built in sections. The 96-in. butterfly valve and penstock were installed next. This latter work was in progress at the time of the flood. Hence the outlet works

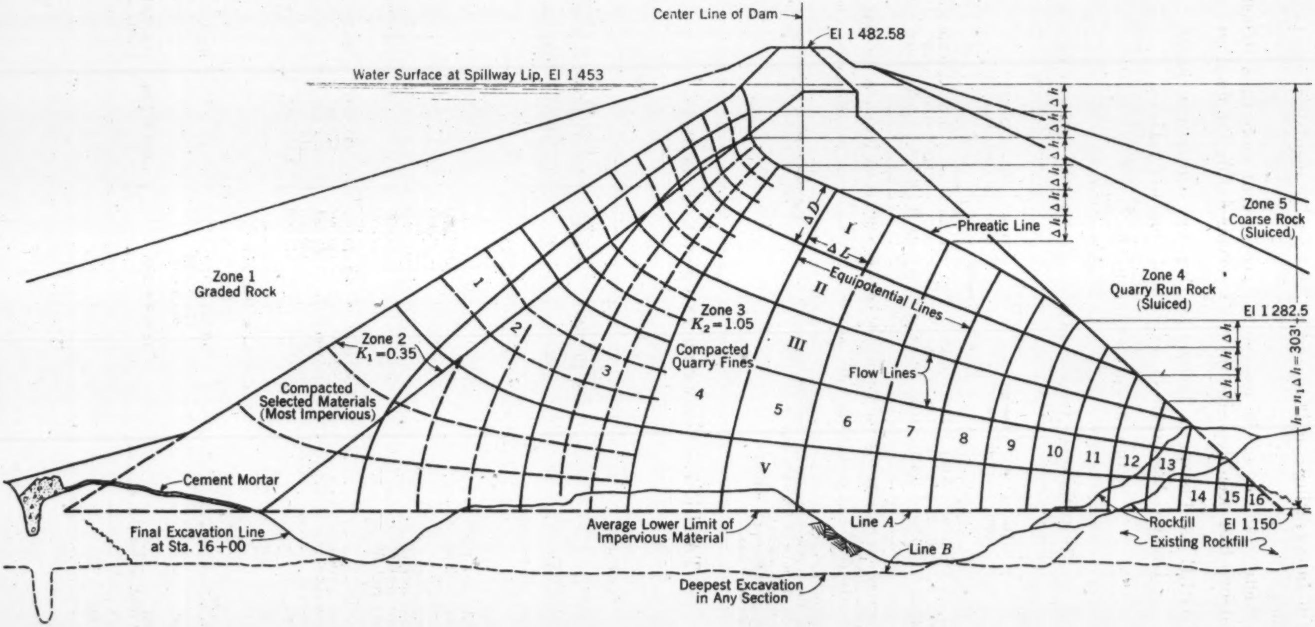

Center Line of Dam
El 1 482.58
Water Surface at Spillway Lip, El 1 453
Δh
Zone 5
Coarse Rock
(Sluiced)
ΔD
I
ΔL
Phreatic Line
Zone 1
Graded Rock
II
Equipotential Lines
Zone 4
Quarry Run Rock
(Sluiced)
El 1 282.5
$h = n_1 \Delta h = 303'$
Zone 2
$K_1 = 0.35$
1
2
Zone 3
$K_2 = 1.05$
III
Compacted
Quarry Fines
3
Flow Lines
Compacted
Selected Materials
(Most Impervious)
4
5
6
7
8
9
10
11
12
13
Cement Mortar
V
14
15
16
Final Excavation Line
at Sta. 16+00
Average Lower Limit of
Impervious Material
Line A
Rockfill
El 1 150
Existing Rockfill
Line B
Deepest Excavation
in Any Section

FIG. 32.—TYPICAL FLOW NET DIAGRAM FOR AN AVERAGE MAXIMUM SECTION

were not in an operating condition and no appreciable regulation was possible at that time.

A complete re-survey of the reservoir indicated that more than 10,000,000 cu yd of debris had been deposited in the reservoir during the flood and prac-

TABLE 6.—PRESSURE LOSSES IN OUTLETS, COMPUTED FROM DESIGN ASSUMPTIONS AND FROM OBSERVED DATA

$\left(\text{All Values to Be Multiplied by } \frac{V^2}{2g}\right)$

Outlet	TOTAL ENERGY[a]		TOTAL LINE LOSSES[b]			VENTURI[c]		NEEDLE VALVE LOSSES[d]		
	De-sign	Ob-served	Design losses[e]	Observed		De-sign	Ob-served	Design losses[f]	Observed	
				n	Losses				c	Losses
51 in.	0.796	0.661	3.82	0.0095	2.87	0.020	....	0.562	0.738	0.555
123 in. (east)	0.535	0.410	1.187	0.0087	0.744	0.020	0.062	0.562	0.680	1.147
123 in. (west)	0.530	0.377	1.193	0.0087	0.746	0.020	0.240[g]	0.562	0.690	0.916

[a] Total energy loss from the reservoir through the butterfly valve in the tunnel, on the basis of the velocity head in the pipe.

[b] Total line losses between the butterfly valve and the needle valve on the basis of the velocity head in the pipe.

[c] Net loss in the venturi tube on the basis of the velocity head in the pipe.

[d] Loss in the needle valve on the basis of the velocity head in the valve outlet (valve 100% open), based on the gross area of the valve outlet.

[e] Manning's $n = 0.011$.

[f] Discharge coefficient $c = 0.80$.

[g] Includes bend loss.

TABLE 7.—ESTIMATED AND FINAL QUANTITIES

Description (1)	ESTIMATED		FINAL	
	Prior to Aug. 13, 1935 (2)	Revised design (3)	Prior to Aug. 13, 1935 (4)	Revised design (5)
(a) QUANTITIES, IN CUBIC YARDS				
Rock Fill and Earth Fill:				
Zone 1, loose rock fill		2,685,000		2,575,349
Zone 2, earth fill		1,042,000		1,144,220
Zone 3, compacted rock fill		3,273,000		3,211,415
Zone 4, loose rock fill		1,905,000		1,885,563
Zone 5, loose rock fill		1,068,000		1,019,170
Zone 6, loose rock fill		722,000		753,714
Totals	5,562,000[a]	10,675,000	453,659[a]	10,599,431
Stream-bed excavation	1,292,000	110,597	873,403	104,127
Abutment excavation	525,000	559,881	283,119	478,667
Quarry strippings	900,000	150,000	2,201,028	460
Quarry waste	1,800,000	75,000	1,411,892	none
(b) COSTS, IN DOLLARS				
Dam proper	10,355,910.00			10,391,834.63
Spillway	478,303.46			376,393.30
Outlet works	804,000.00			1,209,586.33
Sub-total	11,638,213.46		5,023,317.39	11,977,814.26
Grand total (Col. 5 plus Col. 4)				17,001,131.65

[a] Rock fill.

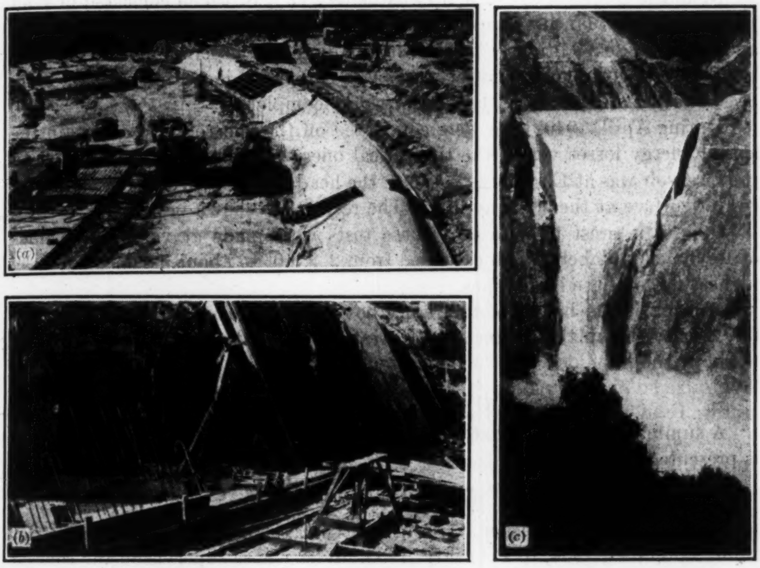


Fig. 33.—Typical Features of the Spillway Construction
(a) General View of the Upper Part; (b) Wall Section Showing Steel and Forms; (c) Spillway in Action

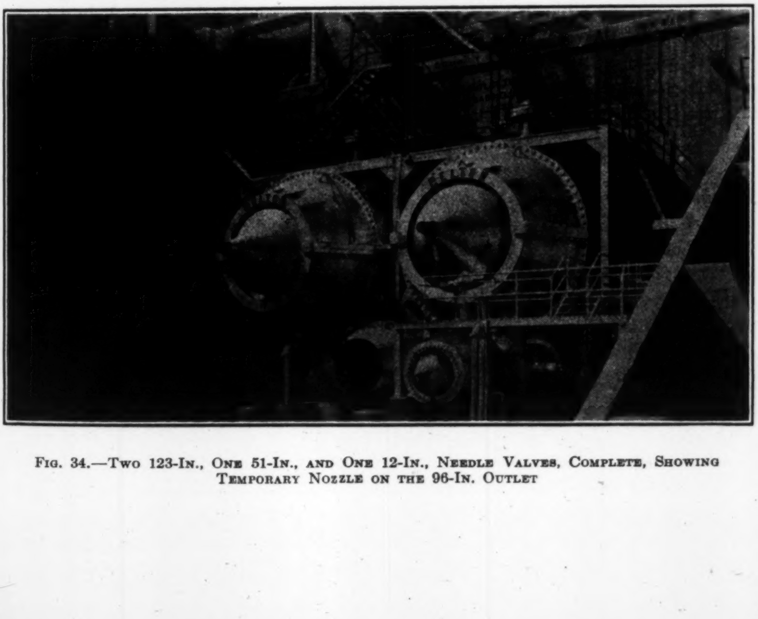


Fig. 34.—Two 123-In., One 51-In., and One 12-In., Needle Valves, Complete, Showing Temporary Nozzle on the 96-In. Outlet

tically in the course of one day, approximately 75% of which consisted of sand and silt. This quantity happens to be about equal to the total yardage of rock and earth fill "expeditiously" placed in the dam over a period of two years!

The installation of the outlet works was completed in October, 1938. Fig. 34 shows the installation of the outlet works completed.

During April, 1940, tests were conducted on the penstocks to ascertain the actual energy losses versus the theoretical ones. The initial water surface in the reservoir was at El. 1360±. Hence the head was ±60% of the design head (water surface at the spillway lip). The results are shown in Table 6.

It is of interest to note that these tests were made at very high values of Reynolds' number, which extended from 2×10^6 to about 3.8×10^7 for the highest flows in the large pipes.

The maximum height of the dam above its foundation is 376 ft. Therefore, it may claim the distinction of being the highest dam of its type in the world at this time.

QUANTITIES AND COSTS

A summary of the estimated and final quantities and their handling costs, is presented in Table 7.

ACKNOWLEDGMENTS

Credit is due S. M. Fisher, M. Am. Soc. C. E., chief engineer of the District from September, 1934, to February, 1935, under whose supervision the revised design of the dam was prepared in its essentials.

C. H. Howell, M. Am. Soc. C. E., was chief engineer of the District from February, 1935, to October, 1938. By his untiring efforts, he succeeded in the elimination of seemingly unsurmountable obstacles to make the completion of the project possible in accordance with the revised design, which had been supplemented and perfected under his supervision.

The writer is indebted to H. E. Hedger, M. Am. Soc. C. E., chief engineer of the Los Angeles County Flood Control District, for valuable advice and for making District records available for the preparation of this paper.

DISCUSSION

E. Soucek,[3] **Jun. Am. Soc. C. E.**—This discussion is confined to Fig. 24 and one of the author's comments in connection with it. The ranges of heads plotted for each model and the corresponding prototype ranges (in feet) are approximately as follows:

Model	Model heads	Prototype heads
1 : 20	0.50 to 0.85	10 to 17
1 : 100	0.10 to 0.20	10 to 20
1 : 500	0.02 to 0.05	10 to 25

It may be that there was no necessity for discharges corresponding to heads lower than about 10 ft. If this was the case, the tests served their purpose but they do not justify the author's statement (see heading "Spillway") "that the discharge for all models and probably also for the prototype" can be expressed by Eq. 5. This inference should be qualified as to the range of head within which the formula is to be applied. The conformity between the models is excellent in the range of heads compared. An extension of the comparison to lower heads would be equally valuable and interesting.

William P. Creager,[4] **M. Am. Soc. C. E.**—For his able description of the tests, design, and construction of this notable dam, the author is to be congratulated. The consulting engineers on this work (L. F. Harza, M. Am. Soc. C. E., and the writer) and the chief engineer (Mr. Howell) advocated a somewhat smaller section for this dam, the finally adopted section being a compromise between their ideas and those of the state engineer's consultants. This point is indicated in view of the steeper slopes adopted for subsequent designs of dams of similar type.

The writer cannot agree with Mr. Baumann's method of stepping up to the prototype the results of tests of the three models as shown in Fig. 24; nor can he agree with the statement (see "The Spillway") that the discharge for all models and probably also for the prototype can be expressed by Eq. 5, having an exponent of 1.535.

In the three models and the prototype, consider, respectively, homologous heads of 0.03, 0.15, 0.75, and 15.00 ft for the scale ratios of 1 : 500, 1 : 100, 1 : 20, and 1 : 1 (prototype). The test head of 0.03 ft in the 1 : 500 model, of course, is entirely too small for consideration on account of the relatively great influence of skin friction. The test head of 0.75 ft in the 1 : 20 model should closely simulate the action of the prototype since, for that size, the influence of skin friction, with a properly constructed model, is negligible.

Beginning with a head of 0.75 ft in the 1 : 20 model and a flow Q of 2.475 cu ft per sec, and assuming that the true exponent is 1.5:

$$Q = 3.81\, H^{1.5} \qquad \text{.......................(13)}$$

If this parabola were plotted in Fig. 24 it would give $Q = 221.0$ for the homologous head of 15.0 ft in the 1 : 1 prototype instead of 246.0 determined by the author.

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[3] Iowa City, Iowa.

[4] Cons. Engr., Buffalo, N. Y.

Similarly, the corresponding homologous discharge for the homologous head of 0.03 ft, in the 1 : 500 model, is $Q = 0.0198$ instead of the measured discharge of $Q = 0.0177$. The lower measured discharge is accounted for by the relatively greater effect of skin friction.

JOSEPH JACOBS,[5] M. AM. SOC. C. E.—This is a splendid paper, presenting in commendable detail the procedures and technique of design and construction, and many of the results obtained in the actual building of a very important and somewhat unusual earth and rock-fill dam structure. It is desired to refer briefly to a few of the items discussed in the paper.

(1) *The Original Design.*—It is not clear why entirely adequate tests of quarry composition, as to the proportions of fine and coarse materials that would probably be yielded, were not made in advance of the preparation of plans and specifications. Presumably, more extensive tests than apparently were made would have avoided, or at least materially lessened, the train of unfortunate events (not fully discussed in the paper) that resulted from the lack of correct foreknowledge as to quarry composition. A dam structure of magnitude always justifies thorough and extensive advance exploration for required construction materials, foundation conditions, etc.

(2) It seems also, to the writer, that the specifications imposed too severe an exclusion of the fines, particularly in view of the fact that the rock was to be placed dry. The inclusion in the rock-fill section of a dam of a fair proportion of such fines, for example, as were eliminated by the original specifications, makes for a denser and more stable structure (even though the friction coefficient may be somewhat decreased) and one that is satisfactory as to permeability. In the present case, as was disclosed soon after construction was begun, the disparity in the proportions of coarse and fine material of Quarry 10 was too great to warrant construction, economically, of a rock-fill dam with borrow from that source.

(3) *The Revised Design.*—The assumption, or rather the crediting, of no sliding resistance to Zones 1 and 2 in determining dam stability was in the direction of increasing the factor of safety against sliding failure, but that assumption may be regarded as too severe, for those zones definitely do have appreciable sliding resistance, particularly Zone 1 with its dominant rock content and with a shear coefficient probably not less than 0.75. An alternate, and perhaps a more economical procedure, would be to assign their nearest probable value to all zone resistances, rather than to ignore some of them as was done for Zones 1 and 2, and then apply a conservative over-all safety factor to the entire cross section of the dam, or to such portion of it as was involved in the plane or curved surface of least shear resistance. It is hoped that the author, in his closure, will indicate the location of the critical line of shear weakness and just how he obtained his necessary average shear coefficient of 0.37. For a horizontal section at the base of the dam, the shear coefficient would seem to be less than 0.37 for a safety factor of one, although that value must, of course, be appreciably exceeded for assured safety.

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[5] Cons. Engr., Seattle, Wash. (Mr. Jacobs died on March 16, 1942.)

(4) It is difficult to obtain entirely dependable cohesion and friction factors by laboratory tests and both caution and good judgment are required in making, interpreting, and applying such tests to full-size embankment conditions. This is particularly so because laboratory tests are made on practically homogeneous soil samples, whereas embankments are rarely, if ever, entirely homogeneous. Laboratory tests, however, are indispensable aids in connection with earth-dam design and stability analyses, but their inherent limitations must be recognized and conservative safety factors must always be used in applying the results of such tests to actual design and construction.

(5) The author's several graphs of laboratory tests showing the relationship, under varying conditions, of shear coefficients, vertical pressures, penetration resistances, densities, etc., are all of great interest. Among other things they indicate the following:

> (*a*) That density increases with vertical pressure but at a very much slower rate, and that under high vertical pressure the final densities are practically equal despite appreciable differences in the initial densities at which the samples were tested (Fig. 19(*b*));
>
> (*b*) That, except for sample *D* for the higher pressures, there is a progressive decrease of the shear coefficient with increase of vertical pressure, the initial density remaining constant; and a progressive increase of the shear coefficient with increase of initial density, the vertical load remaining constant (Figs. 19(*c*) and 22); and
>
> (*c*) That, except for sample *D* for the higher pressures, there is a progressive decrease of the shear coefficient with increase of penetration resistance, the initial density remaining constant; and a progressive increase of the shear coefficient with increase of initial density, the penetration resistance remaining constant (Fig. 20).

(6) Each of the indications cited, when considered alone, seems to be logical and, apparently, is in general accord with observations elsewhere; but a comparison of the shear coefficients for varying vertical pressures (Fig. 19(*c*)) with those for varying penetration resistances (Fig. 20), for identical arguments in pounds per square inch, shows differences that are not so readily accounted for. These differences vary irregularly and range from zero to 70% or more, the higher shear values being those pertaining to penetration resistances. To be sure, vertical pressure and penetration resistance, as now determined, are not identical, and, although the latter is intended primarily as a quick field index of density and moisture content, yet it would seem, if it were accurate, that it also should be, in so far as shear values are concerned, a fair index of pressure equivalence even though its technique does differ from that of the normal pressure test.

(7) The preceding statement is not intended to imply that the laboratory tests were in any sense below par (apparently, they were all carefully made) but merely to indicate that such tests are not free of difficulties and limitations that may mar results, and also to emphasize that there is yet much that is elusive, vague, and unknown in the field of soils mechanics, particularly in the relationship of shear, internal friction, and cohesion. It is suspected that

part of the cited difference in shear values is due to the fact that the present usual method of determining penetration resistance is not very accurate. Too much of the personal equation enters into that determination, involving, as it does, both the time rate and the intensity of man-power load application. Some new device should be developed that will largely eliminate the personal equation and permit more accurate correlation to density and supporting power of the soil. Such a device might take the form of a small fixed weight falling through a fixed height on a penetration needle of known diameter, and with a gage or scale calibration correlated to fairly large-size pressure tests on the same soil as that of the penetration needle tests. A series of such tests should afford a fairly dependable correlated calibration for manufacture of the apparatus.

(8) It is also possible that the generally accepted formula for shear may not be, and probably is not, entirely correct. It usually takes the form

$$F_t = C + f P \quad \text{(14)}$$

in which F_t = total shear resistance; C = total cohesion resistance, or total resistance at zero loading, the cohesion per square foot being regarded as a constant for a given soil; f = coefficient of internal friction, regarded, usually, as a constant for a given soil; and P = total imposed vertical load.

It is the writer's concept that both total friction and total cohesion increase with density and with vertical loading; that the coefficients of friction and cohesion generally decrease with increase of vertical loading; and, as previously stated, that soil densities increase with vertical loading at a greatly retarded and progressively decelerating rate. On the basis of these concepts the shear formula might take the basic form,

$$F_t = C + c P + f P \quad \text{(15)}$$

in which the additional factor, c, is the coefficient of cohesion. The thought is that C is a base constant for a given soil and for a specific standard dry density, say 100 lb per cu ft, and that c and f are not constants but are variable functions of P—probably exponential functions and not identical for c and f. Only extensive laboratory tests correlated to proper mathematical analysis can determine what these functions are and what the equation in its final form should be.

(9) *Roller Tests, Zone 3.*—Fig. 9 seems to indicate that the lower compaction costs resulted from the use of the heavier roller units with relatively few passes rather than from the lighter units with a larger number of passes. Apparently, the lowest unit cost, particularly if considered on the basis of cost per cubic yard, or cost per square foot per ton-pass of roller, was for eight passes of the tandem combination, consisting of one double unit, loaded with sand and water, and one double unit, unloaded. The indicated initial unit cost, however, is not necessarily the final single criterion as to the best roller combination to be adopted. The final dry densities desired, and those actually attained under the different tests, must also be considered, and no doubt were.

(10) It would be informative, and would add appreciably to the completeness of the record as presented in the paper, if the following data were added to

the tabulation immediately preceding Fig. 9, and for each of the roller combinations listed: The average cost of compaction per cubic yard, the average dry density attained at optimum moisture and what that optimum was, and the average travel speed in miles per hour. It would also be of interest if the author, in his closure, would state just what roller combination, and just what number of passes were actually used, in the construction of Zone 3.

(11) *Densities.*—The densities attained in San Gabriel Dam are a good illustration of what can be accomplished with any reasonably good earth material under the present efficient methods of constant check of the properties of the materials as placed, and unremittent compaction and moisture control during the entire period of embankment construction. The degree of density obtained is largely a question of economics—what one feels warranted in expending for synthetic proportioning and mixing of the constituent materials, and, in compaction effort, to secure the density desired. The Zone 3 dry density of 144 lb per cu ft for material in place, including rock and fines, is perhaps 10% to 15% in excess of that which, in earlier years, was attained or attainable with the equipment then available, and then regarded as acceptably good. There are many intact old embankments whose dry densities are less than 120 lb per cu ft. With its moisture content of 5%, the wet density of the Zone 3 material in place was about 151 lb per cu ft, a density, as the author indicates, comparable to that of good concrete.

(12) Other recent examples of high densities attained in earth embankments are Green Mountain Dam on the Blue River in Colorado, and Deer Creek Dam on the Provo River in Utah, completed in 1941, both being structures of the U. S. Bureau of Reclamation. The former, for its impervious zone, attained an average dry density of about 141 lb per cu ft for the material in place and an average wet density at optimum moisture (±9%) of about 153 lb per cu ft, with departures from about 5% below to 2% above these values. For Deer Creek Dam, the average wet field density of the impervious section was also about 153 lb per cu ft with an optimum moisture somewhat greater than that for Green Mountain Dam. These densities, in the main, were secured with twelve passes of duplex and triplex sheepsfoot roller units, ranging in weight from 1 ton per lin ft of roller, unloaded, to nearly 2 tons per lin ft fully sand and water loaded.

(13) *Seepage.*—The author states that the laboratory tests of Zone 3 material indicate a permeability coefficient of 1.05 (1.05 cu ft per sq ft per yr for unity gradient), a maximum velocity of 6.2 ft per yr (also for unity gradient), and a seepage loss of 99.5 cu ft per yr per lin ft of dam at its maximum section. This means, with reservoir water surface at spillway crest elevation, a total seepage for the structure of less than 0.01 cu ft per sec—an entirely negligible amount. If the data are available, a comparison of the seepage loss as indicated by laboratory test with that of the actual structure in operation at full reservoir stage would be of interest. The writer suspects that the latter is very much the greater.

(14) The fact is that for any acceptably good dam material, properly placed and compacted by modern methods, as already referred to in a previous para-

graph, seepage, as indicated by laboratory test, is almost always negligibly small, and is almost always greatly exceeded by the actual seepage of the finished structure. This by no means implies that the laboratory work is poorly done, or that the dam is not well and carefully constructed, or that the actual seepages exceed safe permissibility although, in some cases, they may be of that character. Usually, the reverse of these conditions prevails.

(15) Seepage through the actual structure is not uniformly distributed; nor is it of uniform intensity, as through an entirely homogeneous mass of uniform density and porosity, such as is practically the case for laboratory tests. Usually, it manifests itself in distinctive areas or sections of the dam where, possibly, a minor shear plane may have developed from slight differential settlement, or where conditions as to compaction or composition of the material itself may have been somewhat below normal. Also, if adequate cutoffs have not been provided, the major seepage may be under or around the ends of the dam. Whatever the cause or causes, however, it may be anticipated that actual seepage will exceed laboratory seepage. It should be appreciated then, that, although laboratory tests are highly informative, indeed indispensable, and are valuable indexes of the comparative impermeabilities of the different borrow-pit materials from which selections must be made for dam construction, they should not be regarded as safe measures of the absolute seepage that will obtain in the finished structure.

Jacob Feld,[6] M. Am. Soc. C. E.—A wealth of data on granular materials and rock fills is contained in this paper. Results of field experience and laboratory tests are correlated with each other and also with actual results in the construction work. The paper merits intensive study not only by the practical engineer who directs the performance of work but even more by the laboratory technicians, because the author clearly describes the type of information needed in the design and construction of rock-fill dams.

In works of such magnitude, the honest and commendable example set by this project, of revising the design after the field organization has had sufficient experience with the materials involved to permit true evaluation of the assumptions upon which the design was based, should be seriously followed. The mathematical development of the design can be beyond any criticism, and the design can be approved "by consulting engineers and geologists of national repute"; yet all of it is based on assumptions of what the materials will be and how they will act. It has been the writer's feeling that no amount of laboratory tests on soils, whether natural or (as in this project) artificial, can give a complete quantitative answer. The best that can be expected is a determination of the limits within which the properties of the soil may range. Such limits may be far apart, and designs of the structure for each limit may be even further apart. Averaging the laboratory results and using such data for an "average" design is not honest, economical, or even safe. When the possible range is large enough, why cannot alternate designs be prepared, or, for a project of this magnitude, several designs, to be adjusted as the true character and action of materials encountered are learned?

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[6] Cons. Engr., New York, N. Y.

Economical moving of rock by blasting as in excavation work is not always economical rock production in quarry work. As a matter of fact, most economical excavation work is often dependent upon the size of the rock produced, where mucking equipment is limited by space requirements. In the excavation of a rock tunnel portion of the Sixth Avenue Subway in New York, N. Y.,[7] a slower acting dynamite was used to prevent a seamy schist rock from breaking out in slabs too large for the mucking machine. In describing the quarry operation, Mr. Baumann states that only one fourth of the rock was of proper size. There is a description of the type of quarrying and the kind of explosive used, and the writer would like to ask whether tests were made with other explosives and different drilling methods. Certainly, the quarrying of 10,000,000 cu yd of rock deserved a little study.

The tests of consolidation and percolation for different mixtures of fine and coarse materials are of great value. From the data as summarized in Fig. 4 and Table 2, two conclusions can be reached:

(1) The mixture of maximum density does not provide maximum resistance to seepage, because for similar materials the shape of the channels formed by the voids, and not the total percentage of voids, is the chief factor of percolation; and

(2) The mixture of maximum initial density shows the minimum change in density under load, chiefly because the more intimate contact of grains in the mixture of maximum density reduces local overstresses and thereby reduces edge and point crushing within the mass.

The author's attempt to evaluate the shear coefficient by measuring the total earth pressure within a small cylinder is subject to several objections. The validity of the formula for enclosed shallow bins is one objection; another is the variations in total pressure with volume expansion of the retaining cylinder. The direct shear test seems a preferable method for evaluating the shearing coefficient.

The complete coverage of the subject of compaction under various moisture contents, as well as with different equipment, is a valuable contribution. It is noteworthy that saturation practically eliminates breakdown of the grains, whereas the maximum effect of compaction equipment can be obtained when the moisture is between 10% and 15% of the dry weight. The explanation is probably in the tensile strength provided by the moisture films in the granular mass.

That even field tests will not provide accurate estimates of consolidation settlement is shown by Fig. 30, which reports an observation of the actual consolidation of about half the value expected from cylinder tests. One explanation of the discrepancy is in the difference of time during which the same unit loads were applied, as well as the lateral restraint found in the actual dam fill as contrasted with the artificial cylinder restraint with less frictional resistance along the boundary of the test sample inside the cylinder. The action of the completed dam in deflecting under full water load with perfect rebound shows that the structure is substantially elastic in nature; no better test of the excellence of design and performance could be expected.

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[7] "Tunnel Construction, Sixth Avenue Subway, New York, N. Y.," by Jacob Feld, *Proceedings*, Am. Soc. C. E., April, 1941, p. 617.

WALTER L. HUBER,[8] **M. AM. SOC. C. E.**—As stated by the author, a revised design for San Gabriel Dam No. 1 was approved by the state engineer of California on August 12, 1935. This approval was secured only after many conferences among the state's Board of Consulting Engineers for this dam (F. C. Herrmann, M. Am. Soc. C. E., the writer, and the late Charles D. Marx, Past-President and Hon. M. Am. Soc. C. E.), the chief engineer of the Los Angeles Flood Control District, and the consulting engineers of the District. The section of the dam, although considered conservative, was the minimum which, under all of the local conditions, was satisfactory to the state's consultants.

The capacity of spillway required by the state's consultants was 80,000 cu ft per sec with 15-ft freeboard. Had certain information since secured (partly the record of subsequent floods) been available, it is probable that a larger discharge capacity would have been required. However, the requirement of freeboard that was fixed as a condition is quite conservative, and by encroaching upon this freeboard a discharge that is ample is indicated—this without jeopardy. The model tests referred to by the author lead him to conclude "that at least 290,000 cu ft per sec would be discharged through the spillway before the dam were overtopped." As he states, this is "about three times the peak flow into the reservoir during the flood of March 2, 1938, and more than six times the peak flow on record prior to that date" (see heading "The Spillway").

FEDERICO BARONA,[9] **ESQ.**—The information given in the paper by Mr. Baumann is interesting and constitutes a valuable contribution to the subject of the intelligent use of the materials available, in the construction of rock-fill and earth-fill dams.

The use of gravel or rock as coarse aggregate, bound by a sand-clay mortar, is undoubtedly an efficient manner of increasing the fill density and internal friction. If the coarse aggregate is added properly so that a certain limit is not exceeded, and so that sufficient compaction is assured, the composite fill can be made as watertight as the mixture of sand and clay mortar by itself. "El Palmito" Durango, in Mexico, is an earth-fill dam 280 ft high. In its construction, about 40% gravel was used as an addition to the sandy-clay loams available.

The addition of gravel has been economical since the depth of the layers of sand-clay in the borrow pits is limited by layers of gravel. By using the gravel, the borrow pits close to the site can be excavated to greater depths. Furthermore, the density of the fill has been increased.

The experience obtained in "El Palmito" Dam shows that the gravel-sand-clay mixture can be compacted more satisfactorily by means of smooth rollers. The gravel pieces act like the sheepsfoot elements of a roller to transfer the pressure from the smooth roller to the lower layers. For this kind of material, smooth rollers are more efficient than sheepsfoot rollers, because the latter tend to dislodge the pieces of gravel already embedded in the sand-clay mortar.

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[8] (Huber & Knapik), Cons. Engrs., San Francisco, Calif.

[9] Testing Engr., Mexican Commission of Irrigation, Mexico, D.F., Mexico.

The trend in the construction of compacted fill dams must be toward the formation of soil concretes, with improvements in technique, grading, water content, etc., until they are as well known as cement concretes. Of course, if coarse aggregate is used in compacted fills, the friction to overcome is greater and so the compacting equipment must be much heavier.

Soil concretes have not been studied much, because generally, in the soil laboratory, only small specimens are tested, and only the fine portion (sand-clay mortar) can be included. San Gabriel Dam has the distinction of being the first project providing for the testing of larger specimens, with rocks included.

It is of interest to notice that in the beginning of Portland cement investigations, tests were made only on small specimens of neat cement or mortar. Later the technique of testing the entire mass of concrete was developed until 36-in. by 72-in. cylinders were tested.

In the case of soil concretes it would be worth while to determine permeability, settlement, compaction, and triaxial compression strength in specimens (24 in. diameter or more) sufficiently large to include gravel or rock such as is used in the fill.

Tests, in place, for permeability, settlement, compaction, etc., must be given serious consideration. They are especially important in the case of soil concretes as a means of approaching a greater knowledge of the true conditions that occur within the finished fill.

Experimental sections of fill compaction, as observed in San Gabriel Dam, are of great practical importance. In the Mexican Commission of Irrigation it is general practice to make these experimental sections whenever the construction of a new dam is begun. Thus it has been possible to determine many factors of technical and economic importance, such as: Thickness of layers to compact, number of trips for rollers, type and weight of equipment, etc. The measurement of settlement in embankments is also a general practice in the Mexican Commission of Irrigation.

Permeability tests, in place, have been made by the Commission to some extent, but for such an important soil characteristic the cooperation of all those engaged in the construction of earth dams is desirable to establish standard testing methods.

G. H. HICKOX,[10] M. AM. SOC. C. E.—The part of Mr. Baumann's paper that describes the tests on the spillway model, particularly the tests on models at three different scales, is especially interesting. The results of these tests lead to Eq. 5. This equation can be developed if it is assumed that the velocity of flow through a horizontal element of area over the crest is

$$V = \sqrt{2\,g\,H^n} \qquad \qquad (16)$$

in which n is equal to 1.07.

If Eq. 5 with a constant coefficient is valid for the prototype as well as for the model, it leads to the curious relationship that the ratio of prototype discharge to model discharge varies as the 2.535 power of the length ratio instead of the 2.5 power predicted from considerations of dynamic similarity.

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[10] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

It has been supposed generally that discharge over a prototype spillway would be greater than that predicted from a model because of the relative reduction of friction loss in the prototype, and that the change in discharge would be represented by a change in the coefficient. The points of Fig. 24 seem to be equally well represented by three lines having a slope of 1.5 and different coefficients. It is unfortunate that data on the various models do not overlap.

It would be interesting to know if advantage was taken of the discharge illustrated in Fig. 33(*c*) to obtain a prototype verification of the curve of Fig. 16. Such verification of model tests is rare and is needed acutely.

Paul Baumann,[11] M. Am. Soc. C. E.—In the process of condensing the original text, it was necessary to make certain deletions. Among them was a statement pertaining to the spillway. It appears that this caused certain difficulties on the part of some of the discussers in comprehending the significance of the range within which the model tests were conducted. The statement set forth the state engineer's basic requirement—namely, that the spillway should be designed to carry a discharge of 80,000 cu ft per sec with a residual, minimum freeboard of 10 ft. The chief engineer (Mr. Howell) and his consultants (W. P. Creager and L. F. Harza, Members, Am. Soc. C. E.) raised the residual freeboard to 15 ft.

A design study based on an assumed discharge coefficient over the ogee section indicated a necessary water surface elevation of 1466—that is, 13 ft above the spillway crest to produce this discharge. The primary purpose of the hydraulic model tests was to ascertain whether the design discharge value would be realized.

It is hoped that this will explain to Mr. Soucek why the tests were not taken to heads of less than 10 ft. The writer agrees with Mr. Soucek that an extension of the tests to lower heads would have been desirable to check, more completely, the conformity between the models. Unfortunately, the necessary time to do so was not available.

The District's studies for the San Gabriel watershed indicate a maximum probable spillway flow of approximately 200,000 cu ft per sec, which is well within the capacity of the spillway. The writer therefore does not agree with Mr. Huber that a larger spillway capacity than the one provided would have been called for, even if a flood such as the one of March 2, 1938, had been on record at the time the revised design was submitted to the state engineer for approval. Had Mr. Huber referred to the diversion tunnel instead of the spillway, the writer would have agreed readily.

Mr. Hickox correctly sets forth the basic equation for the velocity of elemental flow normal to a vertical, cylindrical surface through the crest. However, his statement that three lines having a slope of 1 on 1.5 and different coefficients would represent the points in Fig. 24 equally well is not correct if, as outlined in the paper, such points are neglected which represent conditions affected by backwater. Unfortunately, discharge measurements below the spillway were not made during the March flood of 1938, as there were no facilities for such a purpose. The writer fully agrees with Mr. Hickox that such flow measurements would have been very valuable indeed.

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[11] Senior Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

For the interpretation of model tests operating under the influence of gravity only, it is generally acknowledged that the discharge scale varies as the 2.5 power of the length scale. For models operating under purely viscous influences in which gravity plays a relatively small part, it may be shown from Manning's equation that the discharge scale varies as the 2.67 power of the length scale.[12]

In models influenced by both gravity and viscosity, as practically all hydraulic models are, it may be reasoned that the discharge-scale value will be somewhere between the 2.5 and 2.67 power of the length scale, the exact location depending upon the relative preponderance of the gravity or viscosity influence. The primary purpose of testing models at three different scale ratios was to determine the value of this exponent. In model tests made subsequently by the District, and in the model tests reported by the late F. Eisner,[13] results at five different scale ratios could be correlated by the use of the 2.52 power instead of the conventional 2.50. All these experiments deal with overflow weirs having some kind of approach channel. The writer is not yet prepared to specify the power to be used for other models in which both gravity and viscosity enter. However, the points for models of different scale ratios of arbitrary structures are not necessarily satisfied by one and the same equation. It should also be kept in mind that each prototype may be considered a model of a larger prototype.

Mr. Creager offers a substitute for the parabola shown in Fig. 24 in the form of a parabola of the equation $Q = 3.81\ H^{1.5}$. He correctly shows that if these two parabolas have a common point in the 1 : 20 model range—namely, $H = 0.75$ ft and $Q = 2.475$ cu ft per sec—the discrepancy between his parabola and the one shown in Fig. 24 is of the order of 10% for the corresponding points of the 1 : 500 model, as well as for the prototype. For any other point in the 1 : 20 model range, presumably Mr. Creager would compute the coefficient in a similar way and extrapolate to prototype dimensions by the equation he suggests. This is tantamount to accepting the 2.5 power of the length scale as adequately representing the discharge scale factor, and disregarding the results of the other two model scale tests. To the writer, this does not seem justified in the light of the evidence presented.

The revised design of the dam proper, including field and laboratory tests on materials for the rolled rock core section (Zone 3), was prepared in its essentials during a period that preceded Mr. Creager's connection with the District. The success achieved in these tests was due in great part to the assistance and initiative of R. R. Proctor, M. Am. Soc. C. E., and N. M. Imbertson, Assoc. M. Am. Soc. C. E. Subsequently, the revised design of the dam proper, including many additional large and small size tests, was perfected in collaboration with the state engineer and his consultants, and the spillway, outlet works, and appurtenances were designed. In the light of these circumstances, Mr. Creager's reference to a compromise is not clear to the writer. Prior to the completion of the revised design an alternate design was prepared which did not include the rolled rock core section. Instead, similar "grizzlied" quarry material was proposed to be placed by sluicing. Tests were conducted in this connection. They may serve to confirm some of the statements made by Messrs. Jacobs, Feld, and Barona. The material was placed in 25-ft lifts and was thoroughly sluiced at the time of placing. Shafts were then sunk and the material tested at 5-ft intervals in elevation between top and bottom. In spite of the greater care in placing than could be reasonably expected as an average during construction, the dry density of the fines averaged 89 lb per cu ft, and the dry weight of rock and fines averaged 122 lb per cu ft. In comparing these dry densities with those obtained by rolling (122.2 lb for the fines and 144 lb for rock and fines in Zone 3 at the time of placing and on the surface), it is obvious that the two masses are distinctly different in character from the standpoint of stability under saturation, particularly under the influence of earthquake.

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[12] "Hydraulic Laboratory Results and Their Verification in Nature," by Herbert D. Vogel, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 597.

[13] "Überfallversuche in verschiedener Modellgrösse," by F. Eisner, *Preussische Versuchsanstalt für Wasserbau und Schiffbau*, Berlin, 1933.

In his interesting and constructive discussion, Mr. Feld raises a question regarding tests conducted in the quarry to ascertain the most suitable quarry methods. The writer was not connected with the District prior to October, 1934. However, records indicate that such tests were conducted by the contractor. According to the specifications, his quarry methods were subject to the approval of the chief engineer. No suitable method was found to produce the rock acceptable under the original specifications in economical quantities.

Mr. Feld also draws conclusions from the cylinder tests, which are supported by the data presented. He further stresses the significance of a feature that is perhaps not generally recognized. It is the breakdown of the grain structure under load. This characteristic for the material in question is particularly brought out in Fig. 14. The breakdown characteristics are extremely important relative to the compaction of a mixture of coarse and fine materials secured from a quarry. Therefore, it would not be a wise undertaking to apply the same design to a dam at a location where this type of rock does not exist. The breakdown effect may become somewhat paradoxical, in that it is at least conceivable, if not practicable, to produce expansion of this type of material in a cylinder with increasing load rather than consolidation. This phenomenon naturally would be in line with the swelling of broken rock as compared to solid rock. Tests on nonsaturated samples resulted at least in relatively large reductions in the void ratio, $e = \frac{n}{1 - n}$, of the fines for relatively small differential settlements. On the other hand, it is obvious that, if such a mixture were unrestricted and were caused to break down into fines of quasi uniform grain size, the volume occupied by such a mass would be larger than the volume occupied by the mixture of course and fine material. A vertical shake-up due to earthquake would tend to do this if the compacted mixture were not restricted by overlying, loose material of adequate weight, such as was provided. A critical void ratio could establish itself as a result of both loosening and breakdown of the grain structure without this provision. Mr. Feld evidently was under the impression that the increment of load in the cylinder which produced the settlement curve, as shown in Fig. 30, was applied at shorter intervals than the increment of load due to the dam fill itself. This was not the case, in that the load increment on the cylinder was actually timed with the load increment on the section of the dam for which the settlement was observed. The discrepancy between the two curves is believed to be due principally to two factors—namely, greater average density (self-compaction) and greater arching effect in the fill.

The writer is duly impressed with the late Mr. Jacobs'[14] exhaustive and valuable contribution. It is hoped that the following supplemental statements will answer his query regarding the analysis of the dam section for earthquake resistance. The dam was divided into two nearly symmetrical parts, relative to its axis. The reservoir surface was assumed at El. 1466 (H.W.S.). Active forces due to static and dynamic water and earth pressures were assumed to be resisted by the downstream prism. The plane of minimum resistance was determined by the *E*-line method.[15] This plane was found to be sloping upward at an angle of 5° 30′ for an average angle of internal friction of 35°. A factor of safety of two resulted, or a minimum average shear coefficient of 0.37. The writer agrees with Mr. Jacobs that the assumptions were conservative. He also agrees "* * * that there is yet much that is elusive, vague, and unknown in the field of soil mechanics" (quoting from Mr. Jacobs' discussion) and it is in part for these reasons that conservatism in respective designs is considered advisable. Mr. Jacobs suggests an interesting and logical modification of the conventional shear formula. The writer shares his concept regarding a decrease of the angle of internal friction with increasing, vertical pressure within normal ranges, so far as damp sand is concerned. This was previously[16] shown by him, based on passive pressure tests.

The final average cost of spreading, sprinkling, rolling, and tamping the material in Zone 3 was 16¢ per cu yd. The roller combination that proved most economical during construction was one double drum, loaded with sand and water to a weight of approximately 24 tons. Five of these units were used. An average of four forward and four backward, or a total of eight, passes for each strip of compacted material was necessary. Mr. Jacobs' difficulties in accounting for the penetration resistance and shear coefficient relation as shown in Fig. 20 are perhaps due to overlooking the statement that the curves are "purely theoretical." They were derived from two independent, experimental relations—vertical pressure versus shear coefficient, and penetration resistance versus vertical pressure. There does not seem to be a definite relation between penetration resistance and shearing strength, possibly due, in part, to the "personal equation" as suggested by Mr. Jacobs. Since the completion of the dam, the reservoir has been full, or nearly so, twice, and in the intervals the dam has been under considerable water pressure for several months at a time. Nevertheless, to date (June, 1942) there has been no measurable seepage through the dam.

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[14] Mr. Jacobs died on March 16, 1942.

[15] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 719.

[16] *Ibid.*, p. 791.

In comparing the San Gabriel Dam No. 1 with a similar dam in Mexico, Mr. Barona reveals some features of interest. Among these is his reference to the use of smooth rollers for the compaction of gravel-sand-clay mixtures. Such a mixture is quite different in character from the one used in Zone 3. It is primarily an earth fill with a dissimilar material (gravel) embedded in it. Definite contrasts always remain between the two. In contradistinction, the material used in Zone 3 "fuses" into a quasi homogeneous mass. The contrasts between rock and fines disappear and with them the surface friction phenomena. The latter material could not be adequately compacted with a smooth roller. The writer would have welcomed data on embankment settlement of Mexican dams for comparison with Fig. 30. Mr. Barona's reference

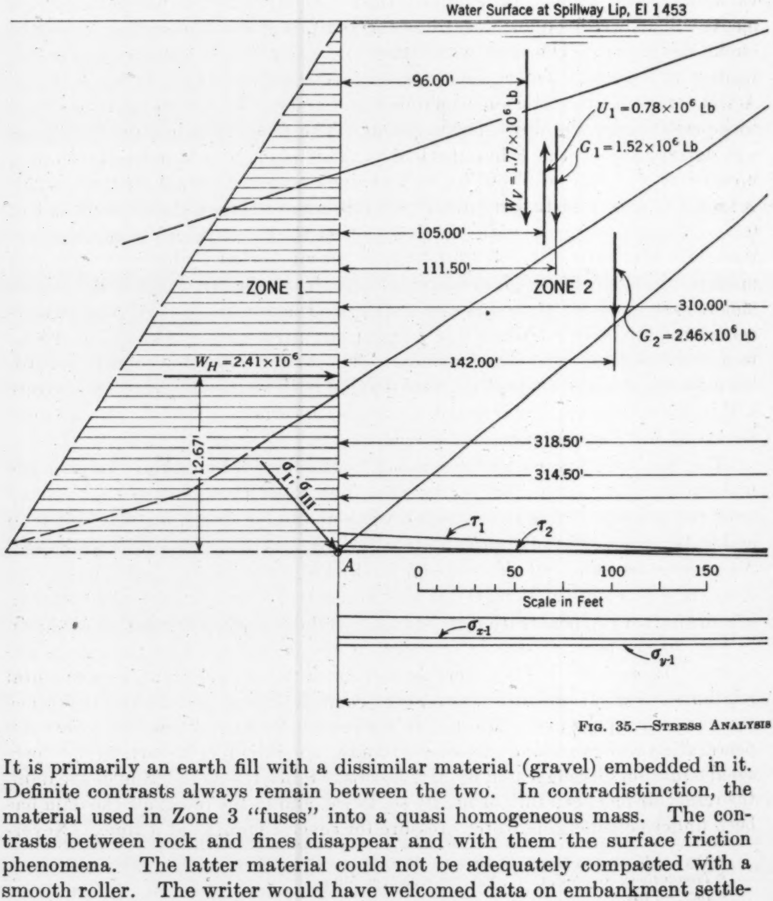
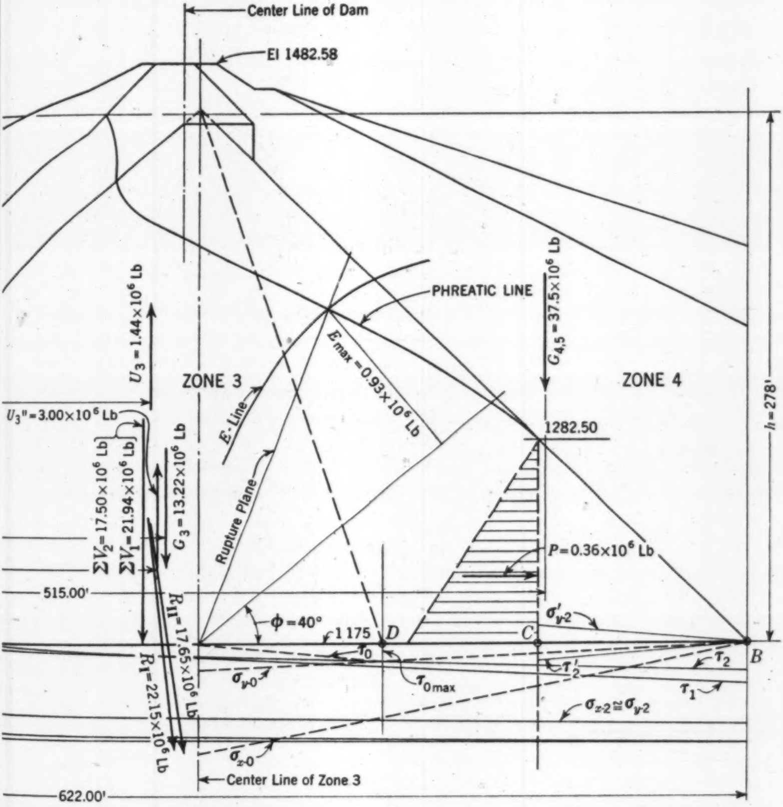

Water Surface at Spillway Lip, El 1453
96.00'
$W_V = 1.77\times10^6$ Lb
$U_1 = 0.78\times10^6$ Lb
$G_1 = 1.52\times10^6$ Lb
105.00'
111.50'
ZONE 1
ZONE 2
310.00'
$G_2 = 2.46\times10^6$ Lb
$W_H = 2.41\times10^6$
142.00'
318.50'
314.50'
12.67'
σ_I, σ_{III}
τ_1
τ_2
A
0
50
100
150
Scale in Feet
$\sigma_{x\text{-}1}$
$\sigma_{y\text{-}1}$

FIG. 35.—STRESS ANALYSIS

to soil concrete merits attention. However, much research is required before knowledge of soil concrete equals that of compaction.

The question of stresses and their distribution in dam embankments has been much debated. The writer is satisfied that present knowledge of soil properties is inadequate to justify refined, mathematical analyses, based on stress functions such as are applicable to monoliths for the usual type of embankment. On the other hand, relatively simple and accurate stress analyses can be made if it is reasonable to treat an embankment section as a monolith, the criterion being whether or not the stress condition so obtained is possible within the strength of the material and the boundary conditions of the section. Based on the known properties and the elastic behavior of the dam under maximum and minimum load before, during, and after, the March flood of 1938, it is believed that Zone 3 acts as a monolith by virtue of its density and


Center Line of Dam
El 1482.58
PHREATIC LINE
$G_{4,5} = 37.5\times10^6$ Lb
$U_3 = 1.44\times10^6$ Lb
ZONE 3
ZONE 4
$E_{max} = 0.93\times10^6$ Lb
E-Line
Rupture Plane
$h = 278'$
1282.50
$U_3'' = 3.00\times10^6$ Lb
$\Sigma V_2 = 17.50\times10^6$ Lb
$\Sigma V_1 = 21.94\times10^6$ Lb
$G_3 = 13.22\times10^6$ Lb
$P = 0.36\times10^6$ Lb
515.00'
$R_{II} = 17.65\times10^6$ Lb
$\phi = 40°$
1 175
D
C
B
$\sigma'_{y\text{-}2}$
τ_0
$\tau_{0\,max}$
τ'_2
τ_2
τ_1
$\sigma_{y\text{-}0}$
$\sigma_{x\text{-}2} \cong \sigma_{y\text{-}2}$
$R_I = 22.15\times10^6$ Lb
$\sigma_{x\text{-}0}$
Center Line of Zone 3
622.00'

(COMPLETED IN FIG. 36)

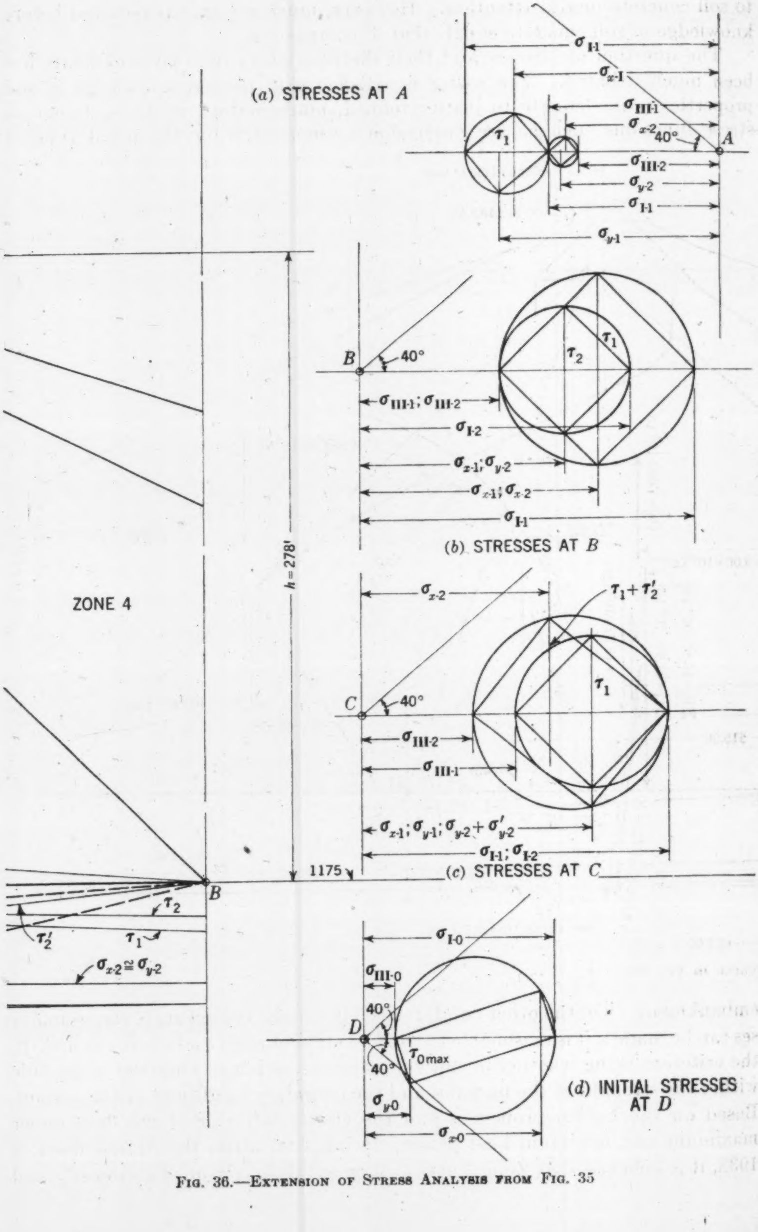

(a) STRESSES AT A
$\sigma_{I\text{-}1}$
$\sigma_{x\text{-}1}$
$\sigma_{III\text{-}1}$
$\sigma_{x\text{-}2}$
τ_1
40°
A
$\sigma_{III\text{-}2}$
$\sigma_{y\text{-}2}$
$\sigma_{I\text{-}1}$
$\sigma_{y\text{-}1}$
B
40°
τ_1
τ_2
$\sigma_{III\text{-}1}; \sigma_{III\text{-}2}$
$\sigma_{I\text{-}2}$
$\sigma_{x\text{-}1}; \sigma_{y\text{-}2}$
$\sigma_{x\text{-}1}; \sigma_{x\text{-}2}$
$\sigma_{I\text{-}1}$
(b) STRESSES AT B
ZONE 4
h = 278'
$\sigma_{x\text{-}2}$
$\tau_1 + \tau_2'$
C
40°
τ_1
$\sigma_{III\text{-}2}$
$\sigma_{III\text{-}1}$
$\sigma_{x\text{-}1}; \sigma_{y\text{-}1}; \sigma_{y\text{-}2} + \sigma_{y\text{-}2}'$
$\sigma_{I\text{-}1}; \sigma_{I\text{-}2}$
1175
(c) STRESSES AT C
B
τ_2
τ_2'
τ_1
$\sigma_{x\text{-}2} \cong \sigma_{y\text{-}2}$
$\sigma_{I\text{-}0}$
$\sigma_{III\text{-}0}$
40°
D
$\tau_{0\,max}$
40°
σ_{y0}
$\sigma_{x\text{-}0}$
(d) INITIAL STRESSES AT D

FIG. 36.—EXTENSION OF STRESS ANALYSIS FROM FIG. 35

rigidity, which are much greater than those of the loose rock and of the prestressed (compressed) condition due to surcharge. The analysis is shown in Figs. 35 and 36 for:

(*a*) Water penetration to the upstream face of Zone 2 only;
(*b*) Water penetration (percolation) through the section as in Fig. 32; and
(*c*) Zone 3 unloaded (initial condition).

Referring to cases (*a*), (*b*), and (*c*), the stresses are marked with the indexes 1, 2, and 0, respectively. Normal σ_x stresses were determined in the usual manner for the first two cases, and the σ_y, τ, and principal stresses at the upstream face (point *A*) with the aid of Mohr's circle.[17] In a triangular section under triangular load the stress distribution is linear. Hence, the shearing stress may be determined at the downstream face (point *B*) to satisfy the requirement that the sum of all horizontal forces must be zero. Principal stresses may then be determined for any point along the base. The third principal stress (σ_{III}) at point *B* must, as a boundary condition, be within the active pressure of the surcharge.

In case (*b*) with percolation through the dam, the stress distribution is affected by buoyancy and water pressure within the section. Point *C* merits attention because of increased shear. An angle of internal friction, $\phi = 40°$, is believed conservatively representative for Zone 3 material. The analysis shows that this stress condition is possible, in that there is no tension in any direction and shearing stresses are well within the shearing strength.

In case (*c*) the rupture plane and point of maximum shear, *D*, were determined by the *E*-line method.[18] In this case the normal σ_x stresses are proportional to the height of the section, and the shearing stress at *D* exceeds the shearing strength. The fact that sections with much steeper slopes of this material actually stood up under complete saturation and even wave action without the slightest sign of weakening would tend to make the assumed angle of internal friction of 40° appear decidedly conservative.

In conclusion, the writer wishes to express his appreciation to the discussers for their valuable additions to the paper. As the project engineer, he naturally had close contact with the office and field personnel engaged in the work. He regrets having been unable to do better justice to their outstanding work. It will always be remembered with respect and gratitude.

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[17] "Soil Mechanics," by D. P. Krynine, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1941, p. 94.

[18] *Ibid.*, p. 272.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2169

SALVAGE OF SEWAGE

FINAL REPORT OF THE JOINT COMMITTEE OF THE SANITARY ENGINEERING DIVISION AND THE IRRIGATION DIVISION ON THE SALVAGE OF SEWAGE

General Statement and Summary

Consideration is limited in this report to the direct salvage of products from sewage—water, fertilizer, gas, grease, and such other materials as may be separated from sewage in the treatment plant or elsewhere and used. It does not include consideration of such salvage as may occur after the sewage leaves the collecting system or disposal works and, mingling with the waters of streams, lakes or underground strata, is again put to useful service. Factors limiting the study are arbitrary and were adopted by a majority vote of Committee and Subcommittee members after an extended period of investigation had indicated that any wider view would involve consideration of many topics which might better be subjects for separate reports.

Public Health Considerations

The utilization of products salvaged from domestic sewage should be with a full realization of the public health significance of such use. The potential health hazards incident to the utilization of the salvaged products will, to a varying degree, modify their usage.

Standards for the use of sewage and its by-products promulgated by health authorities afford the best criteria in this field, and in general imply that:

(*a*) Raw sewage, or its untreated solids content, or the soil which it has recently irrigated shall not come in contact with foodstuff designated for human consumption; nor shall live stock graze upon pasture irrigated therewith;

(*b*) Forage crops which are to be harvested and cured may be irrigated with the untreated effluent from adequate subsidence tanks;

(*c*) For use in the cultivation of human foodstuff, particularly that to be eaten raw, the water reclaimed from sewage must be well oxidized and thoroughly sterilized at all times; and (*d*) Sewage solids to be used as fertilizer must be digested and dried, or if undigested may be kiln-dried at temperatures which will destroy all inimical pathogenic organisms.

SEWAGE

Sewage is a heterogeneous combination of organic and inorganic, suspended, colloidal, and dissolved solids dispersed in large quantities of water. Principal among items of potential value to be salvaged therefrom are water, fertilizer, gas, and grease. It contains many other constituents but usually in such small quantities, or so cheaply obtained from other sources, as to render their salvage from sewage uneconomical.

Sewage characteristics vary widely in different localities. The effect of water abundance, customs of the citizens, industrial development and whether separate or combined sewers are in use, are reflected in the general per capita contribution to the sewers, which will vary from 30 to 200 gal per capita daily (G.C.D.). Solids contribution will vary from 0.1 to 0.3 lb per C.D.

Water.—In the more arid West, where irrigation is practiced extensively, and where reclamation of water from sewage assumes a greater importance than elsewhere, the average daily flow in separate sewers is usually less than 100 G.C.D. and will average about 65. Among others, a notable example to the contrary is the City of North Yakima, Wash. (1),[1] where, because of the rise of ground water during the irrigation season, the flow increases from 100 G.C.D. in the winter to 530 G.C.D. in the summer. The quantity of water recoverable from sewage is substantially equal to the entire sewage flow.

Fertilizer.—A survey in 1924 (2) indicated throughout the United States an average solids contribution to the sewers of 0.24 lb per C.D. Wide variation from this average will depend upon whether or not the area served is industrial in character, as well as the character of the industries and their periods of operation. Certain types of industries (notably packing houses, breweries, distilleries, and the like) will contribute a reasonably high uniform load throughout the year; whereas others—such as wineries, fruit-canning establishments, and vegetable processing plants—produce large volumes of solids for comparatively short periods and no solids for the remaining months.

The grade of fertilizer obtained from sewage solids will depend upon the preliminary treatment afforded the sewage and processes involved in sludge treatment. Dried activated sludge is highest in nitrogen, ranging from 3% to 6%, and generally averaging more than 5%. During the process of digestion, sewage solids will lose about the same percentage of nitrogen as of organic material (3*a*), and sludge digested to a 50% reduction in organic material will range from 1.5% to 3% nitrogen, depending upon its original composition, and will average about 2%. Phosphoric acid is found in sludges in concentrations from 0.3% to 4% and averages about 1.5%. Potash, present in dried activated sludge in concentrations usually less than 1% (4) (16*b*), is absent from digested sludge. Dried activated sludge will range from 60% to 70% organic material. Sludge is considered well digested when the organic material is about 50% consumed (5), although this figure may be exceeded at times; therefore digested sludge will range from 45% to 55% organic material. Digested sludge which is being prepared for fertilizer will seldom run less than 50% organic.

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[1] Numerals in parentheses, thus (1), refer to corresponding numbers in the Bibliography.

Nitrogen lost from the sludge during the digestion period is to be found in the form of ammonium bicarbonate in solution in the sludge liquor (3*b*), whence it may be recovered as ammonia or ammonium sulfate by distillation or by treatment with zeolite; or it may be recovered as magnesium ammonium phosphate by evaporation with salt works bitterns and mono-calcium phosphate (6). Its recovery is not deemed economical at this time, although production of the phosphate salt holds fair possibilities.

Gas.—Methane gas, a product of the anaerobic digestion of sewage solids is of value as a fuel in internal combustion engines and heat generation (7). Per thousand cubic feet of gas, composed of two thirds methane and one third carbon dioxide, about 55 lb of organic sewage solids will have been consumed in the digestion tank. The current prices of gas and fertilizer in the area under consideration will be factors in determining whether the solids are more valuable in the form of fertilizer or gas, and thus will help to decide the most appropriate method of sludge disposal. In general, the sludge will have more potential value as fertilizer than as gas, except in localities where artificial gas of low calorific value is used exclusively.

Grease.—The grease content of sludge from domestic wastes varies widely. Grease recovery from combined domestic and industrial wastes, or from their sludges, is not deemed an economical venture in the United States at this time because of the abundant supply of cheap greases from other sources. Extracted, the grease from sewage is usually found to contain considerable inorganic oil from which it must be separated before it becomes of much value as a commercial product (8) (9) (10). Direct recovery of grease from wool-scouring and packing-house wastes offers more advantages than its recovery from sewage.

Other Constituents.—Recovery of organic acids from supernatant liquor during acid fermentation of sewage sludges is a possibility (11), although with the abundance of organic acids cheaply produced commercially it cannot be considered economically feasible.

There is some possibility of re-use of ash following incineration of sewage sludge. Added to the raw sewage, its lime content may aid coagulation of solids. If it is high in iron, that metal may be leeched out with sulfuric acid to be re-used as a coagulant. The use of ash for filling low-lying land may assume importance in certain areas.

PUBLIC HEALTH CONSIDERATIONS

> "An important factor in epidemiology is the longevity of pathogenic bacteria in nature, for upon such longevity rests the ability of various agents such as foods, to disseminate communicable diseases. Chief among such agents may be fresh vegetables which are eaten raw" (12).

Recognition of this has caused health officers almost without exception to view suspiciously the use of sewage or sewage sludge in crop culture, and notwithstanding that the evidence upon which to base hygienic misgivings is contradictory, official prejudice persists (13). Left to the mandates of health officers, it is unlikely that the use of water reclaimed from sewage, unless clear, well oxidized, stable, and sterile, would be permitted for the irrigation of vegetables consumed raw, or that sewage effluents of any character would be permitted to be so utilized as to threaten contamination of domestic water supplies. Their attitude toward the use of sewage sludge would be more tolerant as a rule, permitting heat-dried activated, and sun-dried or heat-dried digested, sludge to be used on garden vegetables other than those to be eaten raw, but excluding from any consideration in crop culture the use of raw sewage solids (13).

There is ample evidence to indicate that pathogenic bacteria to be found in nearly all domestic wastes will persist for days in sewage-irrigated soil (12), as well as upon the outer skins and leaves of vegetation grown therein; and under such a wide variety of conditions as to appear to render inevitable their transmission to those working in the soil or handling and consuming the produce therefrom. Despite the fact that evidence tracing epidemics to such sources is tenuous and inconclusive in many of the instances which have been the subject of epidemiological investigation, Fred W. Tanner concluded, in 1935 (12),

> "Good epidemiological evidence seems to exist in the literature to the effect that fresh vegetables may and have caused communicable diseases and that the sanitary condition of the soil from which they are harvested is important."

He then cited twelve examples which indicate the possibility of communication in which three seem to offer conclusive evidence, tracing epidemics to sewage-contaminated green vegetables. Mr. Tanner states further:

> "The review of literature indicates quite clearly that fresh vegetables which are raised on polluted soil may transmit pathogenic bacteria. The increase of typhoid fever every year in Paris during July to October was attributed to fresh vegetables harvested from sewer farms."

The Paris report states (14) that during the summer at the time the paper was written in 1923, the incidence of typhoid fever in Paris, France, increased markedly from July to October due to the fact that many Parisians summered in the environs of Paris and consumed polluted water, and that the increase of disease among those citizens who had not left Paris could undoubtedly be traced to carriers; and possibly to the use of sewage-polluted green vegetables which investigation disclosed were being raised in large quantities upon the sewer farms of Paris. The article stated that farms were established to be used for pasturage, but proximity to the city afforded a great incentive to raise vegetables, including among others, lettuce, dandelion greens, endive, onions, parsley, and radishes, which were the subject of epidemiological investigation in attempting to account for the high incidence of fever among citizens. Commenting upon this and other situations, in letters to the Committee, E. A. Reinke, Assoc. M. Am. Soc. C. E., senior sanitary engineer of the California State Board of Health, stated in part: "It appears from the public health aspect that we have far more suspicion than fact to guide us. Epidemiological studies are rarely absolutely conclusive." According to Dr. E. N. Chapman (15):

> "It has been shown
>
> (1) That gross pollution with sewage of some of our streams used for irrigation of vegetables exists in Colorado.
>
> (2) That this contamination of irrigation water offers an ideal method for the spread of filth diseases—typhoid fever, amoebic dysentery and infectious diarrhea—since these conditions are transmitted by human excreta.
>
> (3) That there is an epidemic each year of diarrhea by the time the bulk of vegetables raised with contaminated water comes upon our markets."

Despite the fact that Dr. Chapman's evidence is about as conclusive as any regarding the spread of communicable diseases by sewage-irrigated vegetables, inspection indicates that in Denver, Colo., where a large portion of the vegetables irrigated with sewage-polluted water from the South Platte River are used, the average annual death rate throughout the growing season, from typhoid fever and from diarrhea and enteritis, is equal to or less than the average for the United States; which may be construed as evidence to the effect that the increase in the incidence of these diseases in Colorado may not be the result of transmission by green vegetables shipped to the city, but rather from drinking sewage-contaminated water in the rural area.

Referring to the extensive Berlin sewage farms in Germany, H. J. N. Hodgson states in part (16*a*): "Bacteriological tests are made at intervals on the milk, cheese, vegetables, etc., produced at the farm, with no positive evidence of the presence of bacteria indicative of contamination by sewage," and in an excellent review of current knowledge upon the hygienic aspects of the use of sewage sludge, the Committee on Sewage Disposal with the American Public Health Association states in part (17*a*): "Obviously the index of the danger of using wet sludge or air-dried sludge for the growth of crops must lie in the evidence of experiments to determine the life of pathogenic organisms contained therein and of experience where such material is used." Thus the potential danger is coupled with experience to indicate the menace in the unguarded and unwise use of these sewage products.

Investigation indicates that there is much evidence that bacteria do live for many days and under a wide variety of conditions; but experience discloses that it has been thought unnecessary to investigate more than comparatively few instances in which reasonably positive evidence can trace epidemics to the use of sewage or sewage sludge in crop culture (13). From a canvass of state sanitary engineers in 1937 it was learned that in sixteen states they advised against the use of sewage sludge on garden crops or did not permit it (17*a*). Evidence of eight national authorities pointed to the fact that no harm should result if the sludge is properly processed and used with discretion. The conclusions of Langdon Pearse, M. Am. Soc. C. E., upon this phase of his investigation were as follows:

> "From the hygienic standpoint, heat dried activated sludge and heat dried digested sludge appear safe for any reasonable use in agriculture or

> horticulture. Digested sludge, air dried, appears safe for such purposes if used like manure and plowed in when preparing for a crop; and if care is taken not to apply such sludge thereafter on root crops or low-lying leafy vegetables which are eaten uncooked. Thorough digestion and air drying, as well as storage of the air dried sludge afford a sufficient protection. The Committee knows of no case of sickness traceable to the use of digested sludge or activated sludge" (17*a*).
>
> "Fresh sludge should be regarded as only one degree removed from night soil and treated as such, being used only on forage crops and then applied and plowed in promptly thereafter" (17*b*).

In 1934 the directors of public health in each of the forty-eight states and the District of Columbia were requested for information relative to the extent of sewage use for irrigation in each of the states (13), the influence of such use on the public health, and the standards which should be prescribed and enforced if water and fertilizer reclaimed from sewage are to be used in the production of foodstuffs (*a*) to be eaten raw; (*b*) to be cooked; (*c*) to be used as fodder crops, or on public grounds. The questionnaire also sounded out the attitude of state health officers toward the use of reclaimed sewage products. Thirty-nine states and the District of Columbia replied. Of these, nine admitted having had experience with sewage irrigation and sewage irrigated crops; four were able to trace illness of an epidemic order to such use, whereas one other stated that cattle, pastured on a raw sewage irrigated field, developed poorly and were infected with cysticercus bovis.

The celebrated instance of the development of teania saginata from eating beef infested with cysticercus bovis which had pastured on the raw-sewage irrigated farm of Melbourne, Australia, is further substantiation of the general danger (18). In this instance, the cattle, which at one time were sold for beef for human consumption, greatly reducing the cost of operating the sewage farm, were forced out of the market and could thenceforth be used only for purposes other than human consumption. In the nine states where sewage irrigation was or had been practiced, the attitude was one of tolerance for the act and rigid regulation in the production of the effluent as well as the nature of the crops for which used (13). Only one of the nine was agreeable to the use of treatment plant effluent on vegetables to be eaten raw; however, in this instance, the standard prescribed was such as to render the effluent the substantial equivalent of potable water. Thirty states reported that there was no sewage irrigation within the borders. A number of these declined to express any opinion about the measure, and seventeen would not tolerate its use. Many found the idea of sewage irrigation repugnant.

More than half of the state officers without experience in the use of sewage sludge would prohibit its use in any form on vegetables or fruits to be eaten raw (13). A number indicated repugnance to the idea, without expressing any definite knowledge of the hazards. Opinion was about equally divided for and against the use of sewage sludge on vegetables to be eaten cooked. The majority expressed a willingness to use it on fodder crops and as a fertilizer for lawns and parks. Wells A. Hutchins states (19*a*):

> "Notwithstanding the fact that raw sewage was being applied to crops in some localities in 1937, the practice has little support among public

> health authorities. It is generally considered hazardous, and therefore is either definitely prohibited or at least discouraged by the state officials."

At a given time sewage may, or may not, contain pathogenic organisms. "In case of illness, particularly in epidemic form, among the inhabitants contributing to the flow, the sewage may carry large quantities of such organisms not otherwise disposed of by direction of the public health authorities" (19*a*). The ever-present possibility of such occurrence is considered by health authorities to render untreated sewage a potential source of infection. It is appreciated that raw sewage has been used for irrigation for many years, in many places, both in the United States and abroad, and that evidence of ill results is generally absent or not conclusive. Nevertheless, the hazard exists, and its perpetuation is not in the public interest (19*a*).

For the purpose of regulating the use of sewage water and its solids for irrigation, the states in which irrigation is practiced have adopted legislation which grants executive authority to public health officers (19*b*). Health officers in western states are not all endowed with the same degree of authority in coping with the problem. California has defined its position most clearly and has described sewage fertilizers and effluents to be used in irrigation under a wide variety of conditions (5). In the remaining western states regulations vary from the implications of the Montana law that the sale of all vegetables grown on farms irrigated with human sewage is prohibited (19*b*), to that of Oregon, where no formal regulations have been issued, and of Texas, where the Department of Health acts in an advisory capacity only. National legislation respecting the use of sewage irrigation is not at all comprehensive. Regulations promulgated by the Secretary of the Treasury prohibit the movement in interstate commerce of articles known to have been exposed to infection by certain disease organisms (19*c*), and, upon occasion, the United States Public Health Service has cooperated with state and local authorities to prohibit shipment of vegetables known to have been irrigated with sewage or with water containing large quantities of raw sewage.

There seems little doubt that the presence of pathogenic bacteria will persist in raw sewage, or ground contaminated therewith, for many days (12), and that under the circumstances there is danger of the spread of contamination through the use of vegetables grown under such conditions, particularly if eaten raw. It has been reasonably well determined, however, that water of such purity and freedom from bacterial contamination may be reclaimed from sewage as to be fairly safe for use in irrigation; and it is also demonstrable that sewage sludge undergoing digestion for a period of ten days or more is quite free of pathogenic bacteria causing intestinal diseases, although the cyst of the amoeba has been discovered in well-digested sludge (20).

As in the activated sludge process, oxidation, followed by disinfection, should produce an effluent which is the bacteriological equivalent or superior of many irrigation supplies, although no sterilization process is current today which is 100% effective against the dysentery amoeba (21) (22). In his investigation of sewage irrigation as practiced in the western United States, Hutchins states in part:

> "From a sanitary standpoint it is theoretically true that any kind of irrigation water that does not meet the recognized bacterial standards for domestic water is a potential source of danger. As a matter of fact, however, in no instances in the present study, have epidemics been traced definitely to irrigation with sewage in which the Bacillus coli content has been substantially reduced, even though the standard of drinking water was not attained; and in only a few cases have individual cases of illness been suspected of coming from such sources" (19*d*).

Thus it would appear that, except for use upon vegetables or fruits to be eaten raw, an effluent from about the treatment afforded by subsidence, trickling filters, and secondary subsidence effecting about 85% reduction in bio-chemical oxygen demand (B.O.D.) and suspended solids, followed by chlorination, should satisfy health requirements, and that the use of reclaimed water for irrigation of growing crops—as well as of processed sewage solids as fertilizer—may not be ruled out on strictly public health grounds.

THE RECLAMATION AND USE OF WATER FROM SEWAGE

For What Purposes Used.—In some cases, water reclaimed from sewage is adaptable for industrial and irrigational usage. There are no known instances in the civilized world where a direct reclamation of water from sewage is used to supplement a domestic supply, although there are many underground strata which receive some measure of repletion from the drainage of cesspools and from rivers carrying the discharge of sewage treatment plants, and otherwise. It is also a fact that many grossly sewage-polluted streams are used as raw water supplies of cities in the United States and other countries, this report is confined to direct reclamation or re-use of products from sewage; further discussion will therefore relate to the use of reclaimed sewage water for industrial and irrigation purposes.

Chemical Limits of Good Irrigation Waters.—The usual limiting constituents in irrigation water are sodium, carbonate, chloride, sulfate, and boron (23) (24) (25) (26). It is generally conceded as desirable to have the ratio of sodium to sodium-plus-magnesium-plus-calcium less than 0.6, although many crops are irrigated where ratios are much higher. It is also desirable that water for irrigation use have a sulfate content less than 200 ppm, and a chloride content less than 150 ppm, although there are waters used for irrigation purposes in western states with sulfate contents in excess of 550 ppm (27), and chloride contents of more than 700 ppm. Another general requirement for satisfactory irrigation water is that the total salinity shall be less than 1,000 ppm. In contradiction to this, waters are used containing 3,000 ppm (27).

The effect of boron in irrigation water has been given particular attention in California. Boron is found to be essential to the development and growth of nearly all plants, but becomes toxic to most plant life if present in concentrations greater than a few parts per million. Critical concentrations of boron in irrigation water used for lemons, walnuts, avocados, and some other tropical crops is from 0.5 ppm to 1.0 ppm, whereas the tolerance of many other crops is much higher (26).

In the analysis of water reclaimed from sewage, particular attention must be paid to the salt concentrations, which, without exception, will be greater than in the original water supply, although not necessarily greater than those tolerated by, or of benefit to, growing crops. From a number of sewage treatment plants in the West, including some from which sewage effluents are used for irrigation, it is found that boron varies from the tolerance applicable to citrus and other types of tropical trees to concentrations much higher (26). Fruit-packing establishments, where borax is used in washing fruit, are at times responsible for excessive concentrations of boron in sewage. Other sources may be found in the widely divergent applications of various forms of borates in industry and domestic use.

In his report on the "Analysis of Ten Sewage Treatment Plants in California," Hutchins states (19*e*) in part:

> "Chlorides were well on the safe side. As to percent of sodium, the result from one city showed half the samples contained 60% or more and most of the remainder above 50%. The critical ratio is considered to lie between 50 and 60%."

Effect of Harmful Salt Concentrations in Irrigation Water.—Irrigation experts are agreed that some of the irrigated lands in western United States have suffered from excessive concentrations of toxic salts in the soil, rendering the ground less friable and productive than when irrigation was first practiced in the area. This results from the base exchange qualities of the soil itself and over concentrations of sodium in the water tending toward the exchange of sodium for calcium and magnesium with resultant high concentrations of the dense sodium salts in the soil. In addition, indications are pretty strong that where water pumped from under the land being irrigated tends again to reach the same source of supply, carrying with it its additional burden of salts, the point of toxicity is reached more rapidly.

If the practice of irrigation with water, deemed unsafe because of salt concentrations (excepting boron), is continued in western United States, there appears to be little, if any, reason for excluding the water content of sewage for such purpose. If it is not to be used, the reason must ordinarily be found otherwise than in its concentration of salts or its unbalanced sodium radical.

Beneficial Salts in Sewage.—Sewage water contains certain valuable plant foods in solution, notably nitrogen, boron, phosphoric acid, and potash, in proportions which may render it more valuable for crop culture than other irrigation supplies. A million gallons of average effluent from plain subsidence may add as much fertility to the soil as two tons of ordinary barn manure, provided the water is used wisely and not as in ordinary farm procedure (29). The value of the salts contained in sewage water has more often than not been lost because of the heavy applications of water to soil required for disposal during the growing season as well as its continued abundant application when no crops are being raised (30). The net result of such procedure is to carry most of the dissolved salts through to the subsurface drainage, and the soil may actually suffer impairment. It is essential that the farm be operated primarily for crop production and not for sewage disposal if the benefits to be derived from the dissolved salts are to be secured.

Pathogenic Implications.—Water used for irrigation should be reasonably free from disease bacteria, and the best of bacteriological standards should be enforced in the use of sewage effluents where irrigation water may be cross-connected with other supplies or mixed with an otherwise reasonably pure irrigation supply from which domestic water is at times drawn by irrigators and their help. Hutchins states in part (19*d*):

> "As a matter of fact, however, in no instances in the present study have epidemics been traced definitely to irrigation with sewage in which the Bacillus coli content has been substantially reduced even though the standard of drinking water was not attained; and in only a few cases have individual cases of illness been suspected of coming from such sources."

This statement is limited not only to the irrigation of fodder crops and of vegetables to be eaten cooked, but also to irrigation with effluents from subsidence tanks of vegetables and fruit to be consumed raw.

From the standpoint of existing facts, it would appear that bacteriological standards for irrigation waters to be used without restriction might be drawn without reference to those of drinking water, and that sewage effluents from about the quality of treatment imposed to reduce the biochemical oxygen demand and suspended solids from 85%, followed by chlorination to a residual, should be adequate although esthetic considerations might reject this view. It should also be clear, that sewage effluent need not be ruled out of the irrigation scheme because of its bacterial content when the latter has been substantially reduced from that of raw sewage.

Incentives for Irrigation with Sewage.—Irrigation for crop culture is practiced extensively in the eleven western states of the United States (30). Three or four large irrigated tracts extend easterly into neighboring states, but for the purpose of an analysis of the utilization of sewage water for agriculture, the statistical data gleaned from an examination of the eleven western states and the arid, less populous, southwestern strip of Texas, serve to illustrate the picture clearly enough. Eastern and Central United States depend upon rainfall; irrigation is not practiced.

The area under discussion is peopled with approximately twelve and one-half million individuals, of whom more than six million reside either in tidewater metropolitan areas along the Pacific Coast, where sewage flows are discharged directly into the sea, or in areas where ample rainfall precludes the need for irrigation (30) (31). Of the remaining six and one-half million, about four million reside in cities which are sewered and in the environs of which irrigation is practiced. Therefore, in the collected sewage of these states there exists an available supply of irrigation water, in the form of sewage, approximating 300 mgd which, assuming a maximum month's demand equal to 30% of a total yearly demand of 3 acre-ft per acre, might, without storage, be utilized to irrigate about thirty thousand acres of land. In passing, it may be mentioned that the irrigated area in western United States, now under cultivation or in immediate prospect of cultivation, is nearly twenty-three million acres.

Of this 300 mgd a considerable proportion is, in fact, now being utilized as irrigation water after mingling with stream water into which it is discharged for disposal. Some of it, however, is wasted into streams from which it is not again taken for re-use and is, therefore, lost for irrigation purposes. Sewage from the City of Denver is a material contribution to the South Platte River, the entire summer flow of which, including the sewage of Denver, is at times diverted for irrigation use. To the contrary, it cannot be said that, to date (1942), sewage reaching the Columbia River through its tributaries, the Snake and Spokane rivers, has been a contribution to irrigation water. Probably 40 mgd are lost by being wasted into streams from which the water is not recovered for irrigation; of the remaining 260 mgd, about 100 mgd reach streams or underground waters from which they are again taken for re-use in irrigation.

Thus, eliminating:

(*a*) Seaboard cities;

(*b*) Those cities which are located in areas not using irrigation water;

(*c*) Cities from which the sewage is carried away by streams not utilized for irrigation waters; and

(*d*) Cities, the sewage of which is reclaimed from streams and the like and re-used—

there remains for potential irrigation use about 160 mgd of sewage which are sufficient, without storage, to provide annual irrigation water for about sixteen thousand acres of land.

Not including those areas which derive irrigation water from streams, the waters of which contain sewage or sewage plant effluent, there are about ten thousand acres of western land, four fifths municipally owned and two fifths municipally operated, which are irrigated with sewage or the effluent from municipal sewage treatment plants. Scattered about among the seven hundred million acres of arid land west of the one-hundredth meridian, there are, therefore, some six thousand acres of land to which the practice of sewage water irrigation might be economically extended if sufficient incentive were present. Of the seaboard metropolitan areas, only those of California are located in, or close by, extensive irrigated farm sections, and from the California cities the sewage of Los Angeles looms largest in offering a possibility of exploitation for irrigation use.

The incentives needed to make direct irrigation with sewage either popular or much of a success appear to be lacking. There is no great need of additional irrigated lands in western United States or for the fertilizing qualities of the solids in suspension or solution in sewage. Neither of these needs or desires exist to the extent that objections and restrictions to the use of sewage in crop culture are readily surmounted. The burden of proof that irrigation with sewage is not objectionable from the standpoint of health hazards, violation of esthetic standards and general cleanliness is placed on the sewage irrigation practice by health authorities and by those who are expected to consume the sewage irrigated crops. The net result of this is that with but few minor exceptions, sewerage works in the United States have not been planned to take account of crop culture with plant effluent as a source of revenue.

A fact which mitigates against the reclamation of water from sewage and its application in oxidized and disinfected form directly to crops is that, generally, irrigation water is not relatively expensive where sewage flows exist or are abundant. This follows logically from the fact that cities in the western irrigated land were established, almost without exception, where irrigation was easiest and consequently where water rights and rates are today based upon early appropriation of inexpensive irrigation supplies. In its flow from the city, sewage travels downhill, usually toward water that is even less expensive. Uphill and away from the place of concentration, cost of water may be considerably higher, particularly so if it is pumped or if it has been brought to the land through the construction of costly hillside ditches or as a result of expensive water storage. If the sewage is partly treated and transported to such an area, the cost involved in treatment, transportation, and segregation from other water supplies which it might be expected to contaminate, constitutes an item that almost invariably makes this water more expensive than the water applied to neighboring farms.

The reclamation of sewage water and its use in agriculture probably has a better chance of success in the metropolitan area of Los Angeles, Calif., than anywhere else in the country. Los Angeles County is arid, notwithstanding which it is the richest county, agriculturally, in the United States. Prices paid for irrigation water to raise the valuable citrus and other crops would be prohibitive elsewhere. Although potentially productive arid lands lie fairly close to the municipality, reclamation of sewage water, and its application to the soil, does not appear a safe economical venture.

In the metropolitan area of Los Angeles, there are two small activated sludge plants and one trickling filter plant producing effluents which, under existing rules, may be used for limited crop culture. A group of four cities operates an activated sludge plant close to the city limits of Los Angeles, from which the effluent, amounting to about 8 mgd, is discharged into a near-by wash to be diverted later for irrigation use or to sink into the water-bearing sands below. With but these and one or two other minor exceptions, the sewage from the entire metropolitan population—more than 2,500,000—flows to the sea.

Many schemes have been proposed for using the fresh-water component of this 125 mgd of sewage in agriculture and industry; however, all fail to receive public support because they cannot be made to indicate a return commensurate with the proposed expenditure, to say nothing of the many complications attendant upon exchanges of water, separate systems of distribution, legality of water diversion, governmental and financial structures, and general public disapproval of wholesale sewage water irrigation of farm produce.

Barring the fact that the case for Los Angeles is complicated by the complex structure of its political and social life, it is the counterpart of sewage irrigation attempts elsewhere in the West. Lacking the elements of good business, it is not probable that sewage irrigation will become a very important consideration except as a means for sewage disposal.

Public Attitude.—In any use of sewage water for irrigation, the public attitude is a factor with which to reckon. Quoting Hutchins (19*f*):

> "Aesthetic considerations are important in that they may, and sometimes do, affect the market for produce from sewage irrigated farms. The

> direct applications of polluted water to vegetables which may be eaten uncooked is not only objected to by sanitary authorities, but is obviously repugnant from an aesthetic standpoint. In case of vegetables always eaten cooked, regardless of whether the danger of infection is minimized or entirely removed by the cooking, aesthetic objections to the bringing of such foodstuffs into the kitchen remain on the part of many people. The same considerations apply to low-growing small fruits, such as strawberries, which remain close to or in contact with the ground. If many consumers in a given area have such objections, the marketability of vegetables grown commercially may be impaired and the result may extend to many food products from the same general locality."

It is doubtful that edible products of sewage irrigation could long be sold unless prepared for eating before being brought into private kitchens, and there is considerable doubt even then that there would be a ready market for such foodstuff if its origin were known. Were the source concealed and the crop culture carefully supervised and controlled by health officers, it is altogether likely that no nuisance or menace to public health would ensue—from which it develops that action taken to disclose the origin of the irrigation waters would be a potent factor in the sale and use of such produce even though it might not be readily ruled out on the score of public health menace.

Sewage Water Rights.—Sewage water rights are at times important factors in the use of sewage for irrigation (19*g*) (32) (33). Laws regarding the rights to re-use water which has served the purpose for which originally appropriated are not alike in all states. Under judicial ruling it is possible in some states to re-use the sewage if such re-use is had before the sewage enters a natural stream, whereas in others the right of the original appropriator extends to ownership of the sewage even after it has mingled with other waters. In still others, ownership ceases once the diverted water has become sewage and the latter may be taken by another appropriator whose right to diversion may not be disturbed.

Three cases which serve to illustrate are found in Colorado, Wyoming, and Washington. In Colorado it has been held by the State Supreme Court that a city does not own its treated sewage; that once it becomes spent water—having served the use for which it was originally diverted or appropriated—it becomes public property. The City of Cheyenne, Wyo., was permitted to dispose of its sewage to an irrigator by contract, but lost ownership if the sewage was permitted to mingle with stream water in the natural water course from which originally diverted. In correspondence with the Committee, S. A. Greeley, M. Am. Soc. C. E., states that Walla Walla, Wash., was permitted to change the character of effluent from its treatment process but was compelled to locate its plant so as not to disturb prescriptive rights established by an appropriator who diverted the sewage for irrigation use.

Summarizing, it may be stated, with respect to the reclamation of water from sewage for irrigation purposes, that its use is probably not greatly restricted by salt content other than if it is heavily impregnated with boron, but rather because of the possibility of transmission of disease unless adequate treatment is provided. Incentives for irrigation with sewage are almost totally lacking in western United States because of the high relative cost of reclamation of sewage water and the relatively insignificant acreage which may be reclaimed.

Furthermore, the attitude of the public toward sewage irrigation, particularly with respect to the esthetic considerations involved, tends to exclude sewage irrigated crops from competition with those raised with fresh water. These are the potent factors that will probably limit the use of reclaimed sewage water for irrigation in the United States for many years.

USE OF RECLAIMED SEWAGE WATER IN INDUSTRY

The quality of water to be used in industry depends in each instance upon the service the water has to perform. At times water is used simply as a mechanical means to transport substances or material from one part of a manufacturing process to another. In others, it may be used for boiler purposes; while again, it may enter directly into the manufactured product. Cooling and condensing offer a wide use for water in industry.

It is difficult to imagine any circumstance where water entering directly into a manufactured produce would be reclaimed from sewage. In general, such waters would be used for canning purposes, brewery waters and soft drink bottling, motion picture industries, and the like. It is likewise extremely unlikely that reclaimed sewage water could be used without extensive treatment and assurances in the meat products packing industry or in the transportation of fruit and vegetables from one portion of a plant to another. Furthermore, as a means of preventing the spread of water-borne disease through infection, it is unlikely that anything in a manufacturing process, which would need to be handled wet by those engaged upon the job of manufacturing, would be transported by sewage water through flumes. These would include such products as sugar beets, oranges, sugar cane, and allied materials—in fact, almost anything except waste product from sawmills, cabinet manufacturers, and the like. It seems a little incredible that sewage, or water reclaimed from sewage, could find any place in industry except possibly as boiler feed or cooling and condensing waters. The Committee, after due consideration, has concluded that only the latter uses are to be considered in this report.

Water for Boiler Feed Purposes.—In boiler feed water it is desirable to have water practically devoid of calcium, magnesium, carbon dioxide, and free oxygen; sodium chloride should not be present in concentration in excess of 500 ppm so as to avoid priming troubles, and in water with a high ratio of sodium to calcium, caustic embrittlement is caused by soda bicarbonate breaking up by heating into caustic soda and carbon dioxide. The American Society of Mechanical Engineers has placed the ratio of sulfate, computed as sodium sulfate, to alkalinity, as sodium carbonate, as one, for pressures up to 150 lb per sq in.; two, for pressures up to 250 lb per sq in.; and three for pressures greater than 250 lb per sq in. (34).

Unusual circumstances generally dictate whether or not reclaimed sewage water should be used for boiler feed. Perhaps the most notable example of such use in the United States is at the Grand Canyon of the Colorado in Arizona (35), where reclaimed sewage water is used for this purpose in the Santa Fe (N. Mex.) machine shops, and otherwise. In this instance, reclaimed sewage water is substituted for fresh water which would have to be pumped vertically 4,000 ft from springs near the floor of the Grand Canyon to the Canyon rim where the shops are situated, or transported from Flagstaff, Ariz., to the Grand Canyon in tank cars. The process of reclamation employed is activated sludge treatment in a plant which is of ample proportions and from which an effluent of sparkling clarity, low B.O.D., and suspended solids, is produced at a relatively high operation cost. However, it is much less than the cost of water from the low-level springs or from Flagstaff. The sewage treatment plant is not operated as a commercial venture, but by the park authorities on behalf of the Santa Fe Northwestern Railway Company. In addition to this industrial purpose, the water is used to sprinkle lawns and for the flushing of toilets. Naturally, a dual system of water piping is employed.

It is not unusual to find sewage water being used for cooling purposes. For many years the circulating water coils from an internal combustion gas engine operated at the Joint Disposal Plant of the Los Angeles County Sanitation Districts (California) were immersed in the effluent wet well of an activated sludge plant, and operated with perfect satisfaction. A recent example (36) is the contract for purchase of 40 mgd of sewage effluent from the Baltimore, Md., sewage disposal plant by the Bethlehem Steel Company plant at Sparrows Point, Md., at about $1.64 per million gal. The effluent is that from the city's activated sludge process. It would otherwise be wasted to the river. Obviously, as a commercial venture in industrial water reclamation it would be a financial failure.

No small amount of difficulty is involved in algal and slime growths in cooling and condensing water. The use of disinfectants, such as chlorine, in circulating systems, is quite common and current, and it appears obvious that where reclaimed sewage water is to be used in such a manner these difficulties will be multiplied because of the chemical consistence of sewage water. According to William O'Connell (letter to Committee) this factor should be taken into account in evaluating the comparative merits of reclaimed sewage water and other waters for this purpose.

The Committee concludes that the use of reclaimed sewage water for boiler feed purposes is perfectly logical where such water is better than any other adequate supply available in the area, or is less expensive than a supply of equal character, and that the salvage of water for this purpose is not to be excluded on any grounds that it implies in nuisance or menace to health, but rather that it must depend for its success upon comparative features in the area and industry in which it is to be applied. It is significant that the use of sewage water in industry is so rare as to command notice in the editorial column of national engineering journals when it occurs.

THE RECLAMATION AND USE OF FERTILIZER FROM SEWAGE

The production of sludge at a sewage treatment plant is unavoidable. The most economical and acceptable manner of disposal determines the type and degree of conditioning available. If the disposal includes its distribution or sale as fertilizer, such procedure will probably be invoked.

Barring health hazards, the acceptability of sewage fertilizer produced in a given area:

(*a*) May be evaluated by comparison with the type and characteristics of commercial fertilizers prepared for soils and crops within the economical distribution distance from the plant where the sewage fertilizer is produced;

(*b*) Its value may be reckoned in terms of fertilizer filler—a material which is utilized in the preparation of commercial fertilizers in much the same manner as peat, tankage, and other organic materials low in plant essentials bolstered up by the addition of the elemental plant requirements, and

(*c*) It has been said to contain "plus" values (37) which are not readily susceptible to measurement by laboratory yardsticks, the determination of which depends upon observation of influence upon plant culture following application of the sludge.

Elements Essential to Plant Growth.—The elements essential for plant growth which may be added in the form of plant fertilizer are nitrogen, phosphorus, and potassium. These three are drawn upon so heavily that soil which is being constantly cropped is depleted quite rapidly in the absence of careful soil culture and conditioning. The elements hydrogen, oxygen, and carbon, essential to all plant growth, are secured from air and water, are not subject to depletion, and need not be added to the soil in fertilizer form. There are, in addition, many elements needed in small amounts for a wide variety of plants and crops (38), and which are usually found in sufficient quantity in the average soil, although at times they may be added either with the fertilizer or otherwise. They include tin, calcium, magnesium, sulfur, iron, manganese, zinc, boron, aluminum, arsenic, cobalt, chromium, radium, uranium, silicon, strontium, and lead.

Nitrogen is required by all plants—particularly those in which extensive leaf development is required (17*a*) (39). It is thus of particular value in fertilizing grass, lettuce, spinach, celery and the like, because it stimulates the growth of leaf and stem. Phosphorus is essential for some phases of plant growth. It accelerates ripening, develops root growth, and is said to increase resistance to disease. Potassium is an important factor in vigorous growth. It increases resistance to disease but delays ripening and is needed in the formation of chlorophyl.

Almost without exception the commercial fertilizer applied to the soil is blended to provide the essential elements—nitrogen, phosphorus and potassium—in a ratio which will provide maximum benefit to the soil and the crop to be grown. The variables in this ratio are endless. They are designated to indicate numerically the percentages of nitrogen, phosphorus, and potassium, in that order:

> "No general rule can be given for fertilizing soils since conditions are so variable and soils differ so widely in their characteristics. The only sure way of determining the fertilizer needs of crops of any particular soil is by actual field tests" (40).

Untreated Sewage Solids.—It has long been recognized by American health authorities (17*b*) and others engaged in the practice of sewage treatment and disposal, that fresh domestic sewage sludge is, to all intents and purposes, substantially the same as night-soil and, as such, is unfit for use in fertilizing soils which may, shortly thereafter, be cropped with foodstuffs, particularly those which are to be eaten raw. Untreated domestic sewage solids have at times been disposed of by broad irrigation in the culture of crops used for pasturing beef cattle, sheep, horses, and the like, as well as for raising harvested forage crops. There are, however, some classic examples of the transmission of tapeworm through its life cycle by this means, although such transmission may be thwarted if the meat is fully cooked (13) (18).

It is the universal rule that raw sewage shall not be used in the production of foodstuffs to be consumed by humans in the United States although it cannot be said that the rule is always observed (19*a*). Health authorities in the United States are also quite universally opposed to the use of sewage products in any form being applied to vegetables and fruits which grow in or close to the ground and are to be eaten uncooked.

The longevity of pathogenic bacteria in an environment created by the spread or distribution of raw sewage or its wet, untreated solids content, upon a field or ploughed into the soil, is material (12). Bacterial life will persist for long periods of time therein, and consequently will constitute a potential menace to the health of those who will later come in contact with the soil or consume foodstuff raised thereon. Although a careful review of the literature fails to disclose many instances in America in which illness may be traced directly to contact with raw sewage solids under these conditions, it must be remembered that the use of such solids in crop culture is greatly restricted by prohibition and is practiced only infrequently, generally through ignorance of the law or in violation of the current concepts of health and decency, both factors which will limit the occasions where such contact is possible. It is, therefore, improbable that there are, or will be, many instances in which direct contact with raw sewage may account for illness in endemic or epidemic form. It is probably safe to predict that even in the absence of restrictive health laws, the use of raw sewage solids for crop culture or the fertilizing of fields, which are later to be cultivated and used for raising garden truck and the like, will be quite universally avoided because of public health knowledge and the violation of the esthetic standards which are incidental to present public sanitation concepts in a nation where the pressure of hunger is not greatly felt.

It is unfortunately true, however, that where the presence of famine, starvation, or hunger is experienced, as has long been the case in certain parts of the Orient and is now the case in some European countries, prohibition to the practice of irrigating table foods with raw sewage breaks down, and in an effort to conserve to the utmost the resources of these lands, urban sewage is applied to suburban farms in an effort to make the greatest use of its fertilizer value in the production of foodstuffs (30).

In 1936 the trend in sewage utilization in Germany as a land conditioner and crop fertilizer was described as follows (16*c*):

> "In Germany, sewage farms have always been popular. Now, mainly due to the present economic situation, there use is being extended at a number of places, notably, at Berlin and Leipzig. It is a significant fact, however, that some years ago before the economic position became so acute, plans were prepared for the replacement of the Berlin sewage farms

> by seven modern sewage treatment works, two of which were built. A similar situation existed at Leipzig. Summing up the situation, it appears fair to state that irrigation, as a means of treating raw or settled sewage, is largely being superseded by more modern methods of sewage treatment."

In 1938, the situation in Germany, France, Spain, and Italy was expressed in the following language (30):

> "In contrast to this report [that from the United States] the reports from the European countries, France, Spain, Italy and Germany, show a more positive attitude towards the agricultural utilization of sewage. The reason should be sought in the fact that in the highly populated European countries the increase of agricultural production has quite a different significance than in the United States, but also here, it can be observed that the experience of numerous irrigation fields in the European countries does not confirm the hygienic misgivings felt in the United States.
>
> "A review of the reports of the European countries shows that intensive study of the question of modern agricultural utilization of sewage is at present practically confined to Germany" (30*a*).

In France,

> "He sees economic possibilities of such utilization in the cultivation of vegetables, but he also draws attention to the frequent refusals of the public to buy vegetables grown on sewage land" (30*b*).
>
> "In Spain the conditions for practical utilization of sewage are especially favorable * * *. It cannot be justifiable to destroy the values of sewage which are given in the form of plant nutrients * * * by treatment in clarifying plants * * *. The aim is * * * in Spain to carry out comprehensively the agricultural utilization of sewage" (30*c*).
>
> "In Italy, the increase of agricultural production is a vital problem, and that in the Italian policy of self-sufficiency the agricultural utilization of sewage must also be considered * * *. The complete elimination of all kinds of hygienic objections to this system of sewage utilization has been established by long time observations" (30*d*).

In Germany,

> "It is intended to develop this modern system of utilization of sewage as fully as possible * * * to achieve a partial reenrichment of underground water supplies * * * to insure and increase agricultural production in Germany * * *. Whereas [heretofore] we find municipal plants for the disposal of sewage, we now find that sewage disposal is regarded as a matter of soil improvement with the principle that irrigation must be adapted to farming requirements and not vice versa * * *. Vegetables are excluded to the greatest possible extent in the utilization of the irrigated areas. The Reich authorities * * * recognize the importance of agricultural sewage utilization for the security and increase of farm production and have therefore promoted such measures on the greatest scale. It has been decreed that sewage clarification plants may only be established where the utilization of sewage is impossible. The systems practiced are on grasslands with favorable topographic conditions, trickling irrigation, otherwise here, as elsewhere, artificial raining. On arable land the principle is always artificial raining of sewage with semi-stationary equipment * * *. It can already be recognized that modern sewage utilization is an excellent means of production, suited to ensure and increase agricultural production * * *. There has been no evidence whatever that sewage utilization has any detrimental effect for the public."

There has been no public nuisance.

> "The reason for this fact is the use of fresh sewage, the avoidance of processes of decomposition and the activity of micro-organisms on the irrigated areas which instantaneously split the low quantities of organic matter that are discharged at a time * * *. With regard to the economic aspects of such systems, the investment and operation expenses of another otherwise necessary effective clarification of sewage must be deducted" (30*a*).

No information regarding the further advances of sewage utilization in the European countries is available.

From the foregoing comments, it appears that the estimate of danger in the use of untreated sewage solids as fertilizer in European countries is modified by the country's needs; and where the pressure of economic necessity is being felt as strongly as today, authorities are willing to chance the danger of contamination in the interest of more food production. This economic pressure combined with the public health hazards inherent in the use of raw sewage solids tends to modify food habits and to eliminate the consumption of uncooked vegetables. It appears that under carefully controlled conditions fresh sewage solids may be utilized as fertilizer without great menace to public health particularly in a country where economic conditions have probably greatly restricted the use of green vegetables in an effort to produce the more stable nutritious cooked varieties needed in its present national economy; and although it is altogether likely that such use will be barred as a potential menace in a country where the pressure for food products is no greater than in the United States at this time, the situation as it exists elsewhere, and probably will continue to exist for some time, insists upon the admission that the use of raw sewage solids for fertilizer may not be ruled out solely because of the health implications involved, particularly in the culture of cooked foodstuffs.

Limiting Considerations.—Since it is decidedly unlikely that untreated night-soil or settled solids from domestic sewage will be utilized in the United States as table-food fertilizer, and because the health restrictions upon use of air-dried digested, or heat-dried activated, sludge, as well as heat-dried or composted screenings, are not unfavorable, admitting these products to the channels of commercial fertilizer trade and lending them a definite value (17*a*), further discussion relating to the use of sewage solids for fertilizer will be limited to the processed varieties.

Activated sludge which has been dried at high temperature is said to be free from bacteria inimical to human life. Thorough digestion of sewage solids coupled with air-drying is generally deemed sufficient to destroy all pathogenic bacteria contained in the sludge mass (17*c*) (20) (41) although some evidence points to the contrary, and it probably follows that sewage screenings—heat-dried or composted—are fit for use as fertilizer or filler for substantially any type of crop culture.

The quantity of solids obtained from sewage by plain subsidence will vary from 0.1 to 0.3 lb per capita per day. A survey in 1924 indicated that throughout the United States there was an average solids contribution to the sewers of 0.24 lb per C.D. (2). In untreated sewage solids, the product of subsidence of sewage from a separate system, the average of 0.24 lb per C.D. will contain approximately 3% nitrogen (39) (17*a*) (16*a*) (3*b*) (42); 2.5% P_2O_5; and 0.7 K_2O and 70% organic material. It will be removed from the tanks at about 95% moisture content. The actual quantity per million gallons will vary between quite wide limits but may be expected to average about one ton.

From sewage treated by the activated sludge process the sludge will contain the following approximate quantities of fertilizer essentials (39) (17*a*) (16*a*) (3*b*): 5% nitrogen; 3.5% P_2O_5; and 0.7 K_2O. This material will be removed from tanks at 96% to 98% moisture content. When the solids, which are the product of plain subsidence—termed primary solids—or those from secondary treatment, usually activated sludge, or a mixture of the two, are digested, the nitrogen loss is approximately equal to the loss in organic material (3*b*). Thus, if digestion is considered complete at a 50% reduction in organic solids, the digested sludge may be expected to contain from 1.5% to 3% nitrogen; 1½% P_2O_5 and little, if any, K_2O (16*b*). Heat-dried undigested activated sludge will retain substantially all its fertilizing constituents, little, if any, of which will be lost in the process of drying.

At the New York (N. Y.) Jamaica plant it is reported that screenings are centrifuged to 70% moisture, mixed with 2% chopped straw and 2% gypsum, and spread in beds 18 in. deep where they compost for 30 days (42). The resulting mass is inodorous, contains approximately 2.6% nitrogen and 1.21% P_2O_5. Its ash content is said to be less than 20%.

Fine screenings at the Hyperion Plant of Los Angeles are heat-dried to zero moisture and sold as fertilizer (43). On a dry-weight basis they are reported to contain from 1.5% to 2% nitrogen.

PROPORTION OF ANNUAL NATIONAL REQUIREMENTS

Fertilizer.—The total suspended matter currently received annually at sewage works in the United States is about one and three-quarter million tons of dry solids (estimated from U. S. P. H. records of per capita discharge and the present population tributary to plants). Under ordinary conditions this might be deemed to produce about 575,000 tons of dried digested solids, which, at 2% nitrogen, would yield 11,500 tons per year of the latter. Were all of the sewage treated by the activated sludge process and the resulting sludge dried without digestion, it would yield some 87,500 tons of nitrogen annually. Actually, the total annual production of dried activated sludge in the United States is less than 100,000 tons (17) (44) (45) (46). This is equal to 6,000 tons of nitrogen, which is about 2% of the total annual amount of nitrogen currently sold in mixed commercial fertilizers.

Filler.—The current annual consumption of mixed fertilizers of all kinds is nearly 9,000,000 tons—about two thirds of which is in the form of commercial mixtures (39). Of this, nearly 15% or 900,000 tons, is said to be filler; an amount which is more than twice the annual national production of dried sewage sludge of all kinds.

From the foregoing quantitative analyses, it may be assumed that the production of sewage sludge fertilizer need not be restricted because of present overproduction of either fertilizer or fillers. Current production at sewage treatment works is but a small fraction of requirements; even potential production does not present a very impressive figure.

The "plus" values contained in sewage sludges which do not appear to be readily measurable in the laboratory have been described in the following manner (37):

> "The fertility values together with the fertilizer values in sewage sludges may be summarized in general as follows: In addition to relatively small amounts of the principal fertilizer ingredients, sewage sludge contains 'minor' chemical elements and growth-promoting substances essential and necessary for proper plant growth. It prevents depletion of humus and is capable of increasing the fertility of the soil. It improves soil structure in general and increases the water holding capacity of sandy soil in particular. The value of sludge is not the same for all soils; neither is it the same for all crops. Except for their growth observations there is no adequate measuring stick for revealing the over-all values of sewage sludges as soil improvers and growth promoters—in short, their fertility values as contrasted to their limited worth as fertilizers based on the chemist's analysis alone."

Those who use sewage sludges extensively for fertilizing certain types of crops insist that the results obtained cannot be attributable entirely to the proportions of nitrogen, phosphorus, and potash contained therein (37*a*) (37*b*). Sludge, particularly digested sludge, promotes the growth of ground worms; and greatly increases friability of heavy soils and the water holding qualities of sandy soils. The introduction of large numbers of bacteria and Protozoa results in an increased rate of decomposition. Microorganisms such as molds, etc., appear to make inorganic fertilizers more readily available, and the almost total absence of weed seed conserves the fertilizer for the use intended. It appears that sludge does have values greater than laboratory analyses will indicate—they are tangible and real and have a marked effect upon the sale price of the material where the beneficial "plus" values have been demonstrated. It is otherwise impossible to account for the difference between the selling price of sludge and barnyard manure as citrus tree fertilizers—the former commanding about three times the price of the latter per unit of nitrogen. The extra values are not always evident; they appear in certain soils and crops. In their absence the fertilizer value must be determined solely upon analysis and in this case the digested products suffer greatly by comparison with most of the commercial brands.

Relation between Sludge Disposal and Fertilizer Production

Following the separation of sludge from sewage, there appears to be but one acceptable method of final disposal which is not a step toward the production of sewage fertilizer if the latter is desired—namely, barging to sea, as exemplified by operations at the Wards Island Treatment Plant, New York City, the Joint Meeting of Municipalities in Essex and Union Counties, New Jersey, and elsewhere. Any other method currently practiced involves one or more moves in the direction of fertilizer production—not particularly with that objective in mind, but because they are inherent in sludge disposal processes.

Manufacture and sale of heat-dried activated sludge fertilizer involves:

(*a*) Separation of sludge from water by subsidence (a necessary corollary of any type of sludge disposal);
(*b*) Conditioning sludge with chemicals;
(*c*) Filtration;
(*d*) Heat drying; and
(*e*) Grinding, sacking, warehousing, sales, and distribution.

The cost of barging activated sludge from the Wards Island plant to sea approximates $4.00 per ton of dry solids (47), and this cost is significant because it probably represents, in whole or in part, the anticipated loss per ton in the production of fertilizer at the sewage treatment plant. Doubtless all methods of sludge disposal, including the manufacture of fertilizer, were investigated by the New York Authority as a preliminary to the adoption of the method now employed. At least one is justified in this conclusion. In view of this circumstance, the $4.00 figure becomes important. At Pasadena, Calif., there is no opportunity to dispose of activated sludge from the Tri-Cities Disposal Plant by barging to sea, but the cities which are parties to this sewage disposal works have recently signified their desire to dispose of wet activated sludge from the Tri-Cities Disposal Plant through lines of another authority to which the wet sludge may be rendered tributary at small cost, and have expressed willingness to pay approximately $5.00 per ton of dried solids for such disposal. This verification of the New York experience helps to establish the economics of this operation if the wet activated sludge can be disposed of without further processing than that indicated by Item (*a*) in the foregoing tabulation—namely, the separation of sludge from water by subsidence.

Experience in Pasadena is of value in determining other factors. At that plant, nearly two decades of experience in fertilizer manufacture indicates that the cost of chemicals and filtration is about one half of the total cost of manufacturing the fertilizer, so that if it is not intended to digest the activated sludge and other sewage solids but to dispose of them by a process of partial dehydration (Items (*b*) and (*c*)) of the undigested solids followed by some method leading to their final disposal, that portion of the cost of manufacture (Items (*b*) and (*c*)) of fertilizer may be deducted and the remaining processes of drying, grinding, sacking, warehousing, and selling will usually cost less than the revenues derived from the sale of the resulting sludge as fertilizer. In fact, based upon such a method of computation, the fertilizer plant may show a considerable profit.

In correspondence with the committee during 1941, Longdon Pearse called attention to the fact that the Chicago Sanitary District installed equipment at its West Side Plant for the incineration of activated sewage sludge. As a preliminary to burning, the sludge is conditioned and vacuum filtered. This comprises operations (*a*), (*b*), and (*c*) of the foregoing tabulation. By operating the incineration equipment to de-water the sludge, the City is producing a ton of dried activated sludge fertilizer by the use of 0.6 ton of coal. Substantially the only change in the originally adopted process for sludge disposal in this instance is the substitution of drying for incineration—using the same equipment. The features of distribution and selling are added, but the revenues which accrue from the sale of fertilizer effect an economy to the Sanitary District. The case for Chicago is influenced by the fact that steps (*a*), (*b*), and (*c*) and, in part (*d*), were deemed essential in sludge disposal and less expensive or troublesome than other methods. To show a return the fertilizer must only sell for more than Item (*e*) plus 0.6 ton of coal.

It has been proved many times that activated sludge fertilizer is not worth its cost of production if each of the items included ((*a*) to (*c*)) are charged to it. Only when certain of the steps necessary in its making are incidental to the selected method of disposal can it be made to show other than a loss.

Discounting the "plus" values in sewage fertilizers, they may usually be counted upon to command a selling price commensurate with their beneficial organic and mineral content and as a rule dried activated sludge is worth two or three times its digested counterpart. Value at the point of use must not be confused with value at the plant site. The fact that from two to three tons of digested sludge must be transported from plant to point of use to obtain the plant food value of one ton of digested sludge is a potent factor and greatly reduces the radius of the use area of digested sludge except as a filler.

Manufacture and sale of dried digested sludge fertilizer involves:

(*a*) Separation of sludge from water by subsidence (a necessary corollary of any type of sludge disposal);

(*b*) Digestion;

(*c*) Transfer to air drying beds; or (alternate) conditioning and filtration, with possible composting or drying;

(*d*) Removal from the beds, maceration, storage, sales, and distribution; or (alternate) direct application to lawns, parks, orchards or crop lands; including sales and distribution.

Each of these items is involved in the disposal process indicated by sludge digestion; therefore the preparation of the resulting dried mass as a low-grade fertilizer or filler is a comparatively simple, inexpensive procedure. (In some instances where the dried sewage sludge fertilizer has an especially good reputation, the dried residue from the beds is ground, sacked, and sold as a finely divided organic fertilizer.)

An impressive number of sewage disposal plants in the United States and Great Britain dispose of digested sludge to those who use it as a fertilizer and soil conditioner (16*d*). Classifying typical procedure, one finds that:

(*a*) The greatest number give the sludge to those who are energetic enough to haul it away, in most instances removing it from the beds at public cost, thus realizing the cost of hauling from the site as the sole return;

(*b*) At a second general group of plants, favorably situated with respect to soil and crops which respond to digested sludge fertilizer, sludge removed from the drying beds is sold for prices ranging from 50¢ to $2.00 per dry ton at the plant site, transported and applied to the soil by the purchaser; and

(*c*) A few plants go to considerable lengths to prepare the sludge in attractive form and packages. This apparently yields dividends—especially in the smaller plants where prices for the prepared product range from $3.00 to $10.00 per ton, although it is not always clear whether the selling price includes transportation to place of use.

In class (*b*) (where the larger plants successfully disposing of digested sludge as a fertilizer generally fall), it is probable that the revenues defray from 20% to 50% of the sludge digestion costs. Small plants operating in the manner described in class (*c*) have at times paid the entire cost of sludge disposal and it has been stated by experienced operators of such works that no large-sized sludge digestion plant should be without the necessary equipment and facilities to convert dried digested sludge into well-prepared fertilizer attractively packaged.

Many cities and other authorities use the entire sludge production from plants under their jurisdiction on parks and other public grounds (48). A notable example of this is Golden Gate Park in San Francisco, Calif., where the entire output of the Golden Gate activated sludge and the Richmond-Sunset sedimentation plants is used exclusively in the park area.

The financial outcome of a venture into the manufacture and sale of sewage sludge fertilizer is a function of so many variables that the decision must be made for each plant individually, or at least for each general area. Choice may rest with:

(*a*) The most acceptable means for disposal;
(*b*) The size of the plant, type of treatment, and daily quantity of sludge;
(*c*) Soil analysis within a reasonable transporting distance;
(*d*) Crops grown within a reasonable transporting distance; as well as numerous others not so general in application.

Certainly one of the strongest deterents to such manufacture and sale is the inherent lack of economy in the operation unless many or all of the steps involved in production are included in the otherwise selected process of sludge disposal and charged to plant operations and not to fertilizer production.

Except where sludge can be disposed of without any further processing than concentration, the most potent reason for its use is the fact that it exists in abundance and must be disposed of, and that any return in labor or money which will offset a portion of the cost of disposal is profitable to the producing plant. Therefore, its continued use will be urged. It is a product of necessity and its use makes a virtue thereof.

During the process of biological digestion of sludge the nitrogen which the sludge loses goes entirely to form ammonium bicarbonate in the sludge liquor (3). For instance, if an untreated sludge containing 5% of solids is digested to approximately 50% reduction in organic content, the sludge liquor should contain about 1,000 ppm of ammonium nitrogen, and the quantity of such liquor that can be decanted from the sludge will be at least half of the original volume of the sludge. If the solids removal from the sewage approximates 250 ppm, the fresh sludge liquor will amount to 3,000 gal, more or less, from 1 million gal of sewage and this will contain 25 lb of ammonium nitrogen or 30 lb of ammonia.

There is no known plant at which nitrogen is recovered from sludge liquor; nevertheless, the fact that it exists in abundance therein leads to the following discussion of four general methods which suggest themselves for its recovery (49):

A calculation based upon average values indicates that a well-designed still with suitable heat exchanges could recover the ammonia, utilizing less than one fourth of the sludge gas produced in digestion as fuel, since it would be necessary to vaporize only a small part of the liquor. In addition, about 3.85 parts of Ca O will be required to recover one part of ammonia. The value of ammonium nitrogen weighed against the capital cost involved in the construction of the still, its operation, cost of the lime, and the value of the sludge gas, will decide whether or not such a method of recovery can be made profitable. The picture is not very promising. Its best chance of success will exist in a plant which uses lime to aid in the filtration of digested sludge where the lime constitutes a necessary expense to filtration and is not a charge against ammonium nitrogen production. The second method is to mix the sludge liquor with sulfuric acid and crystallize ammonium sulfate by solar evaporation. Approximately one ton of sulfuric acid is required to produce one ton of ammonium sulfate, the latter having a value somewhat less than twice the former. The economy of this operation will depend upon annual evaporation statistics, cost of, evaporation beds, and capacity to operate without odor nuisance. It will probably have its best chance of success in the arid West or Southwest.

A third procedure might be the use of zeolite. The Guggenheim process for sewage treatment proposes recovery of ammonia from sewage by this method. The net result of the use of zeolite is to concentrate the ammonia from its dilute solution into a fairly concentrated solution of ammonium chloride, mixed with sodium and calcium chlorides. It is then necessary to add lime and distill off the ammonia; but the lime requirement in this instance is less than one half that for direct distillation. It appears doubtful that the advantages of concentrating the ammonia are sufficient to justify the expense of zeolite treatment.

A fourth possibility is by precipitating magnesium ammonium phosphate. For this purpose salt works bitterns may be used to furnish the magnesium and mono-calcium phosphate. The resulting product would be a white granular material which would not cake or deteriorate in storage. It would furnish both ammonia and phosphate in a form readily available for plants, and yet its solubility is low enough so that it is not readily washed out of the soil. Production would be expensive, but the somewhat unique value of the product is such that investigation may eventually prove this process to be of some value where the necessary constituents are available at moderate cost.

Gas Reclamation and Utilization

A valuable end product produced in the anaerobic digestion of sewage solids is sludge gas. Typical analysis of this gas indicates that it has the following chemical properties:

Methane (CH_4)	55 to 75%	(Average 65%)
Carbon dioxide (CO_2)	25 to 40%	(Average 35%)
Nitrogen	Zero to 3%	
Hydrogen sulfide (H_2S)	Trace to 3%	
Illuminants	Trace	

There is doubt about the presence of nitrogen in undiluted sludge gas (3), it being argued that any nitrogen released in the decomposition process will go into solution in the supernatant liquor. This consideration is not of especial importance, however, as the principal constituents with which one is concerned when determining the value of the sludge gas, as indicated by its ability to produce heat and do work, is methane; the principal dilutent carbon dioxide being an inert gas, the presence of which merely reduces the calorific value per unit of total volume of gas. The presence of excessive quantities of H_2S may render the gas so corrosive as to indicate treatment for the removal thereof (16*e*), slightly increasing the cost of production (see Manufacturers Specifications).

In anaerobic sewage sludge digestion some of the volatile substances are rapidly decomposed. Among these are the fatty substances, greases, fats, and soaps (50) (51). These substances in turn furnish the best quality of gas (62% to 75% CH_4). Proteins decompose into a similar grade of gas but in lesser amounts per pound digested. Comparatively small amounts of cellulose and cellulosic material digest in the usual period allowed for sludge digestion at a sewage treatment works; although the methane content of the resulting gas is considerably lower (averaging 45% to 50%).

The weight of the gas produced in each instance is greater than the weight of the material decomposed (51), the increase corresponding to the quantity of water which has entered into the reaction. The weight of the gas produced is approximately 1.2 times the weight of organic material digested. The weight of a standard cubic foot of gas of 65% CH_4 will average 0.064 lb.; thus per pound of organic material consumed in the digestion tanks, approximately 18.5 cu ft of gas, with an approximate calorific value of 650 Btu per cubic foot, is available. Specific analysis of the solids to be digested will, of course, modify this generalization.

Under controlled digestion, the average rate of gas production at a sewage treatment works will approximate 1 cu ft per capita per day which, equated to horsepower, indicates that about 1 hp-hr may be reckoned for each 17 persons tributary to the sewage treatment works. Thus a city of 50,000 population will have a potential sludge gas power generating capacity of 2,900 hp-hr per day.

Regarding sulfides, it is suggested by manufacturers of internal combustion gas engines that any sulfides in the gas are harmful, but that within reasonable limits their effects can be controlled by care in operation. If the gas that is to be used contains 2% sulfides by weight, it is frequently specified that the owner of the gas engine must provide for the maintenance and replacement of parts which are subject to corrosive attack (16*b*). Cast iron is not readily susceptible to corrosion and generally 0.25% of sulfides will not seriously affect steel and copper elements in the engine, although care must be taken to scavenge the engine with air as soon as it is shut down. Conversely, if the vapors that remain in the engine when it is shut off are allowed to cool and the acid condensate is allowed to precipitate in the cylinder walls and elsewhere, even 0.25% of sulfides will destroy or seriously impair the valve and exhaust assemblies, and to some extent the cylinder walls and rings; 0.1% of sulfides does not appear to have a harmful effect upon the engine if the lubricating oil is kept reasonably fresh.

Within the past few years engine manufacturers have developed what is known as the vapor phase system of cooling engines. In this system the circulating water ranges from 210° to 220° F constantly and enough of it is circulated so that the temperature rise through the jackets is only 5° to 6° F. The water coming from the engine is led to a flash tank where pressure is released to the atmosphere, resulting in the formation of some steam which later is condensed in a radiator cooler and pumped back into the main stream of water. The main stream does not undergo any cooling process before it is returned to the engine; thus the engine operates constantly at a high temperature. This type of cooling greatly decreases the corrosion ordinarily caused by sulfur and organic acids formed through the combustion of the fuel. It permits the use of gas with a higher sulfur content than it has heretofore been possible to use.

Sludge gas for power generation was first utilized in the United States at the sewage treatment plant at Charlotte, N. C., in 1928 (52). This, in turn, was the first American counterpart of the original sewage gas engine installed at a leper colony in Bombay, India, in 1907. In the early 20's a number of plants were installed in England and Germany and, in the United States, since 1928, the practice of utilizing sludge gas had progressed to the place where an incomplete tabulation prepared in 1941 indicates (53) that 115 sewage treatment plants, 180 sludge gas engines operate 79 generators, 49 blowers, and 40 pumps. The engines range in size from 3 to 1,440 hp, with most of them less than 500-hp capacity.

The impressive numbers of installations noted in the 1941 tabulation indicate that sludge gas is an acceptable fuel for internal combustion gas engines. Its value may be reckoned in terms of substitute fuel or other source of power; the value of the destroyed organic sewage solids as fertilizer and the cost to produce and capture the gas.

Dried activated sewage sludge fertilizer has usually been found to cost more to prepare and market than is returned from its sale provided the complete processing is included in its cost of preparation (53*b*). The over-all loss to the treatment plant in the production and disposal of 1 ton approximates $4.00. To determine the value of sludge gas in terms of undigested sewage solids utilized as fertilizer, it is necessary to determine the net cost of digestion, sludge disposal, and gas collection. The following approximation is made in terms of 1941 costs.

The cost to digest, dry, and collect the gas from 1 ton of sewage solids may be summarized about as shown in Table 1. This analysis assumes a 50% reduction in volatile solids in a raw sludge mass averaging 70% volatile solids originally. It further assumes that the 50% reduction in volatile solids, amounting to 700 lb per ton, will produce about 12,000 cu ft of gas (a little less than 18 cu ft per lb of volatile solids destroyed) having a calorific value of 650 Btu per cu ft.

Between the limits of a profit of $3.10 per ton of raw solids removed from the sewage and the loss of $3.43 are almost as many different possibilities as there are sewage treatment plants, and there are, of course, plants where unusual conditions will result in a profit or loss lying without the extremes noted. Most, however, will fall within. Actually, circumstances seldom combine to show returns fully equal to costs; nor is such a demonstration necessary to indicate that the values remaining in the sludge residue or derived from the process of digestion are equal to or greater than those inherent in dried activated sludge or other undigested sewage solids—fertilizer. Therefore, it may be stated that the destruction of volatile sewage solids by digestion will yield a gas, the value of which universally exceeds the value of the solids themselves as a processed fertilizer.

TABLE 1.—COST OF DIGESTING, DRYING, AND COLLECTING GAS FROM ONE TON OF SEWAGE SOLIDS

No.	Description	COST, IN DOLLARS PER TON[a]	
		From:	To:
1	Controlled separate sludge digestion	1.75	
	Air Drying		
2	On underdrained beds	1.60	
3	In shallow lagoons	0.15	
4	On drying beds ready for final disposal	1.80	3.35
5	Selling price of dried digested sludge	−0.65[b]	+1.30[c]
6	Net cost, removed from drying beds	0.50	4.00
7	12,000 cu ft of gas at 4.75¢ to 30¢ per thousand[d]	0.57	3.60
8	Sludge digestion, including recovery of all gas values	−3.10	+3.43

[a] Entire cost, in dollars per ton of (dry) raw solids.
[b] Equivalent of $1.00 cost per ton of dried digested sludge for removing from beds.
[c] Equivalent of $2.00 revenue per ton of dried digested sludge on drying bed.
[d] Equivalent of 8¢ to 50¢ per 1,000 cu ft for 1,100 Btu of gas.

It has been demonstrated repeatedly that the cost of generating electrical energy using a sludge gas engine as the prime mover costs about 0.6¢ per kw-hr unless conditions differ materially from those commonly encountered (54). Costs have been variously reported at from 0.4¢ to 1.0¢ per kw-hr. Commercial power purchased in large quantities can be bought, at times, for about 0.6¢ per kw-hr, but it is seldom possible for a plant, required to serve a city of 100,000 or 200,000 population, to buy electrical energy for such a price or less. More often the cost will approximate 1.0¢ per kw-hr unless special concessions are made by municipally owned public utilities or the like. Commercial power at 0.6¢ per kw-hr indicates that it is being purchased in relatively large quantities, and if available at that price, one must not look to power generation with sludge gas as an economy measure except under the most favorable circumstances. However, if the over-all cost of power generation with sludge gas equals, or is less than, the cost of commercial power, the value of the sludge gas may be reckoned in terms of a substitute fuel which would serve the same purpose.

Natural gas with a calorific value of about 1,100 Btu wholesales for from 8¢ per 1,000 cu ft in the South-Central oil producing states, to 50¢ in or near the New England States area. This is equivalent to from 4.75¢ to 30¢ per 1,000 cu ft for 650 Btu of gas and establishes those limits of value for sludge gas. If no natural gas is available, as is reported in some of the Northeastern areas, artificial gas of about the same heat value as sludge gas costs from $1.00 to $2.00 per 1,000 cu ft, which obviously eliminates it as a substitute fuel. In no instance is the wholesale price of natural 1,100 Btu gas reported as less than 8¢, and only under the most favorable circumstances does it sell for less than 15¢, which is the price for large blocks purchased in Southern California, typical of an area where natural gas is cheap and plentiful.

Sludge gas is frequently utilized in heating the digesting sludge mass, control of temperature being one of the important factors in accelerating sewage sludge decomposition and consequently gas production. Therefore, the gas assumes a value for this purpose. Fortunately the generation of power from an internal combustion engine may be a by-product of digestion tank heating, although the reverse view is usually assumed (55) (58) (59) (60). Of a possible capture of 80% of the energy in the gas, 26% alone goes to power, and the remaining over-all 54% may be captured from the exhaust gases and the jacket water and utilized in sludge heating; that is, "the heat balance for temperature control remains about the same whether the gas is used to produce power or burned directly under a boiler."

The economy of sludge heating for uniform temperature control is unquestioned where the plant size is sufficient to warrant continuous supervision. However, since either controlled or uncontrolled temperatures in the digesting mass eventually produce the same results, the value of temperature control is expressed only in reduced cost of sludge digestion structures and an increased and more uniform flow of gas and is not to be judged by comparison with fertilizer value of destroyed solids.

Quality of sludge gas is dependent upon quality of solids and hydrogen-ion balance in the digesting mass. Rate of production varies with the:

(1) Temperature;
(2) Quantity, character, and percentage settleable of solids;
(3) Rate of flow of sewage through the plant with all of its implications;
(4) Sludge bulking; and
(5) Filter unloading and the like, all factors which cause wide variations and render difficult the economic utilization of 100% of the gas produced.

Without supplemental fuel, or extensive gas storage facilities, or a combination of both, much of the gas produced will be wasted or at least will not be available as a uniform, continuous source of power.

Oil burning Diesel engines which will also operate on gas, render Diesel oil a substitute fuel (56). Butane, or rock gas, natural and artificial gas, as well as gasoline, may be used as substitute fuels with success and with but minor adjustments to the carburation of internal combustion gas engines (57).

In the determination of the economics of the use of sludge gas as internal combustion engine fuel each plant will present a different combination of circumstances and factors which must all be equated to determine (*a*) if all of the sludge gas is to be utilized, following which (*b*) the cost of gas storage must be equated to the cost of a substitute fuel which latter may be used to iron out deficiencies in sludge gas production. Average approximate equivalents are as follows:

1,000 cu ft of sludge gas equals:
1,000 cu ft of artificial gas;
590 cu ft of natural 1,100 Btu gas;
6.4 gal of butane;
5.2 gal of gasoline;
4.6 gal of Diesel oil.

The increasing popularity of sludge digestion with its corollary of power generation is testimony to the economy of this process of solids disposal (53). For the present at least, the by-product of the process with the most consistent value is sludge gas, with digested sludge finding increasing favor as a low-grade fertilizer or filler and as an excellent conditioner for public grounds where it is otherwise not commercially valued. Since there are no health implications involved in the use of sludge gas as fuel, it may depend for its salvage value upon comparison with the purchase price, availability, and adaptability of equivalent fuel substitutes.

Reclamation of Grease from Sewage

The term "grease" as it relates to sewage is usually descriptive of all ether or chloroform soluble materials (depending upon which is used as the solvent) contained therein and may include fats and their acids, insoluble soaps, inorganic oils and (again depending upon the solvent used), some waxes and resins (9) (61) (62). Grease is present in varying amounts in all sewage of domestic origin. It is present in organic form in industrial wastes from plants processing vegetable and animal products. Likewise, it appears in inorganic form in wastes from plants and shops manufacturing, maintaining or operating machinery, refining oils and manufacturing bituminous materials, gas, and the like.

Quantitatively and qualitatively the grease content of sewage will vary widely. In the purely domestic sewage treated at the Tri-Cities activated sludge plant in Pasadena (63) (tributary population of 140,000), grease (ethyl-ether extract) averages about 0.011 lb per capita per day, the equivalent of approximately 15 ppm in the sewage. In this sewage it is almost wholly organic, saponifiable fat, slightly mixed with oils and greases from garages, automobile washing plants, and the like. In the sewage of a municipality where there are reasonably extensive manufacturing enterprises, the grease content will vary from 15 ppm to as many as 100 ppm and the quality of the grease will be found to be influenced greatly by the character of the industrial establishments served (9). An example is cited of 489 ppm in a sewage flow of 145 mgd at Gloversville, N. Y., where industrial wastes from tanneries are present in large quantities (9). No general or average quantity may be stated, the influence of industry being proportional to the number and character of industries served and the flow therefrom relative to the domestic flow. About 15 ppm may be stated as the minimum value and from that the grease content of sewage and industrial wastes range upward through combined sewage and domestic wastes as illustrated at Gloversville, to a grease content of 2,000 ppm to 5,000 ppm for fish wastes, meat scraps rendering plants (62*a*); 150 ppm to 15,000 ppm for wool scouring, and the extraordinary 70,000 ppm for rice-bran processing plants manufacturing Vitamin "B" concentrates. There are as many variations in the quality and quantity of grease in sewage as there are combinations of sanitary sewage and industrial wastes in the sewers.

In every sewage treatment works which includes subsidence as one of its processing features, a portion of the grease in the sewage will be removed by settling or flotation and it is from these concentrates that there is afforded the possibility of grease reclamation. Methods are provided which will assist in the flotation of grease and increase the over-all removal from sewage. Current knowledge on this score has been summarized briefly as follows (9*a*): "a considerable proportion of the ether-soluble matter in sewage can be removed by sedimentation and plain skimming. Moreover, the available data show, in general, that a somewhat larger proportion can be removed if the sewage is aerated prior to sedimentation and skimming. Experiments indicate, furthermore, that aero-chlorination prior to sedimentation and skimming accomplishes a still larger removal of grease."

In 1938 a plant sale experiment (64) indicated the percentage removal of grease from the sewage without aeration or chlorination to be 38; with aeration alone, 50; and with aero-chlorination, 80. These results were achieved with 0.11 cu ft of air per gallon of sewage and 2 ppm chlorine. Data indicating the effectiveness of these methods are limited, and insufficient data are available to predict the effect of given quantities of air or air and chlorine upon sewage of known characteristics.

Grease recovery from sludge has been much discussed in sanitary engineering literature and it is rather conclusively indicated that, unless alternative sources of cheap grease are eliminated, there is little prospect of profitable grease recovery from sludge except under the most unusual conditions as, for instance, the presence of large proportions of wool scourings, packing-house wastes, or the like.

Grease recovery from sewage skimmings has better prospects. Analyses of sewage skimmings indicate 30% to 50% grease and may average 25 to 50 lb per million gallons of sewage. Grease is recovered by acidifying the skimmings, heating with steam, and pressing. The resulting product contains saponifiable matter mixed with inorganic grease from which it must be separated to be of much value. The combined product is dark in color and unsalable in a market where other greases may be cheaply procured.

The sewage works at Bradford, England, is the outstanding example of a grease reclamation plant (65) (16*f*). Industrial wool washing plants tributary to the Bradford works process one fifth of the wool of the entire world and four fifths of that grown or imported into England. Weekly the sewage works produces from 500 to 600 tons of grease from wool scourings. The Bradford works has presented the following financial statement:

> "In the year 1903 it is recorded that 166 tons of grease were recovered and the first sales realized $1,110.00 [the committee computation is based on: £1 equals $5.00]. In five years the tonnage had grown to 1,606 with

> prices ranging at about $40.00 per ton naked, and sales amounting to $57,500. In 1918 the tonnage became 4,524; with the peak price of $325.00 per ton naked, and an income from sales of $633,000.00. In 1925 the yearly tonnage had risen to 6,271, selling price about $45.00 per ton naked, the income $482,000.00 and the stock at a very low point." [For the] financial year ending March 31, 1940, grease sales will exceed $400,000.00, the month of February constituting a record with a sales value of $80,000.00, although the full effect on world trade had not been fully felt at that time. It is predicted that during the year 1941 several times the return of $400,000.00 is quite within the bounds of possibility. Twenty-thousand tons of sulfuric acid is used annually in the production of about 28,000 tons of grease."

Without the stimulus given to the sale of native grease by the current war, the works at Bradford would probably be severely pressed to show economy in grease recovery by itself. The plant report states that at the outbreak of present hostilities "tens of thousands" of tons of grease were in storage at the plant site (65). There is nothing so far in the experience at Bradford from which to conclude that grease reclamation from sewage is a profitable venture, except under the most unusual and favorable circumstances.

For each thousand pounds of grease added to digesting sludge, approximately 760 lb of grease will be destroyed in one month of controlled digestion (61), yielding about 13,250 cu ft of gas (76% CH_4). At from 4.75¢ to 30¢ per thousand this gas will have a value of $0.65 to $4.00 if it is entirely utilized in power or heat production. Grease is of doubtful value as a fertilizer. It is, in fact, considered injurious to many soils. Its value as fuel in a process incorporating incineration of sewage solids roughly approximates 30,000,000 Btu per ton. These factors may be considered in more specific form in judging the merits of an individual venture in grease reclamation from sewage. With respect to grease recovery at the Bradford works, it is stated (65):

> "The presence of the grease would greatly have enhanced the value of such gas, but such a procedure would have resulted in heavy losses to the corporation as the grease values would have been affected without any compensating advantage."

Until the industrial waste containing grease leaves the establishment and enters the sewer, mingling with the other sewage flowing therein, its reclamation is not deemed to be implied in the title of this report and therefore such wool scouring, tannery and meat-packing wastes and the like, as are treated at the source for grease recovery, are considered as outside the scope of this investigation.

The sewage treated in the Bradford works contains approximately 890 ppm of grease, 60 ppm of which are lost in the effluent (16*f*). Sludge which is finally pressed contains 40% grease; whereas the dried residue after grease has been removed contains 16%. The 40% grease content of the sludge prior to pressing compares favorably with the grease content found in settling tank scum, and were there sufficient quantity of the latter collected in the ordinary sewage treatment works; and further, were the grease almost wholly of organic origin, it is possible that the results at Bradford could be duplicated as and when desirable at other sewage treatment plants.

Unfortunately this is seldom the case. Almost without exception the quantity of grease obtained as scum is insufficient to warrant the construction of reclamation works, and its quality is so impaired by admixture of mineral oils that the cost of recovery exceeds the revenue likely to be produced, except under the most unusual circumstances.

No health hazards seem to be involved in the use of grease reclaimed from sewage (16*g*). The processes of acidification and heating, as well as those involved in the further manufacture of grease-bearing products, all contrive to render the reclaimed grease free from harmful bacteria. The reclamation of grease apparently must "rest its case" with its value as a commercial product in competition (particularly in the western hemisphere) with the many other sources of cheap grease. It is not apparent at this time that the reclamation of grease from sewage is a fruitful source of revenue or is needful in national economy.

Respectfully submitted,

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ADDRESS AT THE ANNUAL CONVENTION, MINNEAPOLIS, MINNESOTA, JULY 22, 1942

By E. B. Black,[1] President, Am. Soc. C. E.

America today is joined by other nations in an effort to prevent the world-wide destruction of civilization, and our responsibility as American citizens puts us in the position where it is necessary for us to pledge and to deliver to that cause whatever we have of engineering ability, of constructive effort. Our debt to America for the privileges we enjoy as Americans, and for the preservation of those privileges for the future, is one we can never fully repay.

Such a pledge presents problems, but our engineering forebears met and solved problems of relatively equal importance and complexity. There are times when we can best fit ourselves to meet the demands of the present and of the future if we

> "Turn back the clock of time 100 years
> And for an hour or two show once again
> The hope and courage of those dauntless men
> Who laid our firm foundation."

I trust that we shall never forget that the present and the future of our country, and of the American Society of Civil Engineers, depend on what you and I do with the heritage bequeathed to us as engineers and Americans by those dauntless men who laid our firm foundation through the 320 years of our country's existence, and the 90 years of our Society's activity.

The pressure of our obligations to National Defense prevents our proceeding, at this time, along the course we normally would follow. War conditions change rapidly from day to day, and from day to day we must change our plans, and our rate of speed if necessary, to accomplish the results our country requires. Occasionally, however, we must take time out to take stock of our assets and liabilities, to plan both for the present and the future, so that at the end of this war we will be prepared to secure a just peace, a world peace planned to control war.

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[1] Cons. Engr. (Black & Veatch), Kansas City, Mo.

You are familiar with the oft-repeated statement by Maj. Gen. Eugene Reybold, M. Am. Soc. C. E., that "This is an engineers' war,"[2] that there "can be no engineering as usual" until after this war is over. The General's statement does not mean that engineers had anything to do with starting the war, but that engineers will have to stop the war; that engineers will have to provide the ideas, develop the plans, and perform the engineering work necessary to prepare for and win the war. The problems to be met and solved are here now. Because of their number and complexity, and the number of engineers required for their solution, it is easy to see why there "can be no engineering as usual" until this war is over.

We know, of course, that engineers cannot do all of the thinking and all of the work necessary to prepare for and win the war, but, at the same time, no one familiar with the work required of engineers at this time can do other than agree that never before has the engineer played so important a part in the preparation for and the direction of the activities necessary because of the war. Today engineers are in demand; and, each day that the war continues, our responsibility increases and there are changes in the problems that we will be required to meet and solve in order to win and to secure a peace worthy of our effort.

It is my hope that, when this war is ended, this Society will be prepared to direct the continuation of constructive engineering work necessary for our country to continue as nearly as possible on a "business as usual" basis. Such a result will tax our ability to perform effectively as citizens as well as engineers. Under no circumstances can we hope for such success unless we make the necessary plans before we require action.

With few exceptions the engineering profession is made up of men who live by the sweat of their brows; men trained to plan and think along definite lines; men whose interest in, and application to, the problems of their profession prevents their spending much time in community, state, or national affairs, at least until those affairs require unscrambling; men to whom citizenship activities that require following established political practices in order to get things done mean little. The June issue of *Headlines*[3] brought to us the comment of one of our well-known columnists about the need and the lack of engineers in the governing bodies of the United States, in these words:

> "This war requires sound leaders to be familiar with science, engineering, technological planning and management.
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> * * *
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> "Too bad half the members of Congress aren't engineers, so we would have intellectual honesty and precision of thought."

Whether or not members of the profession need go to the extreme suggested is a problem, but there can be no doubt about our obligation to prepare now to fight effectively for a peace to control war; that should be our first postwar activity.

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[2] "This Is an Engineers' War," by Eugene Reybold, *Civil Engineering*, June, 1942, p. 293.

[3] *Headlines* (Society House Organ devoted to the Promotion of Local Section Publicity), June 15, 1942.

Our obligation to our country and to our profession requires us to carry on without interruption the work of the Society so that we may also carry on the National Defense work required of us as a Society and as individuals.

In November, 1852, almost 90 years ago, the American Society of Civil Engineers was founded. Its record has been one of effective support of our government. As years have passed, and as new conditions have produced problems requiring changes in operation, no effort has been spared to keep the Society in pace with the constantly increasing requirements of the profession. Perhaps the most effective change has been to bring the Society to the members, through the Local Sections, and to students (those future members of the Society), through the Student Chapters.

The assets of the Society consist mainly of men who have had the vision to spread the gospel of the importance of citizenship and of sound, ethical engineering practice among its members and among those who come in contact with the profession through the usual channels of everyday life. A sound financial policy, a bank account, and the ownership of real property are also assets, and the Society has all of these things.

No professional organization can operate effectively without the unstinted effort of its governing body and its members, and without doubt the Society has long had the benefit of such effort. It is directed by an elected Board of twenty-six men (officers who devote from four to eight weeks each year to Society affairs), and by the executive organization at Society Headquarters operating full time under the direction of a full-time Secretary.

The Board of Direction plans and directs operations of the Society through eight committees of the Board—the Executive, Honorary Membership, Districts and Zones, Professional Conduct, Publications, Membership Qualifications, Division Activities, and Society Relations committees.

Seven Committees serve in an advisory capacity to the Board on the Alfred Noble Joint Prize, the Columbia University Scholarship, the Freeman Fund, the J. Waldo Smith Hydraulic Fellowship, the Retirement System, Securities, and Society Property.

Three administrative committees, each with one Director of the Society as a contact member, are in charge of Local Sections, Juniors, and Student Chapters.

Six Professional Committees, each of which also has a Director of the Society as a contact member, have in hand Engineering Education, Registration of Engineers, Fees, Salaries, Professional Objectives, and Employment Conditions.

A Committee on Technical Procedure and the National Committee on Civilian Protection in War Time complete the list of committees other than those in the Technical Divisions.

The Society supports twelve technical divisions operating under small executive committees, dividing into eleven sections all design, construction, and operating problems with which civil engineers are concerned. The twelfth, the Engineering Economics Division, combines the financial problems of the other eleven divisions and cooperates with those divisions through liaison members of those divisions.

The Society has joint representation with other national engineering societies on the administrative bodies of eleven organizations, such as United Engineering Trustees, Inc., Engineering Foundation, National Research Council, American Standards Association, and the Engineers Council for Professional Development. It also has thirty-four representatives on sectional committees of the American Standards Association and on seven technical committees on Engineering History, Lumber Standards, Concrete and Reinforced Concrete Specifications, Paving Brick, Federal Map Making Efforts, Safe Highways, and Yield Point of Structural Steel.

The Technical Division committees total more than 100 and bring the total number of operating committees of the Society to more than 125. All of the committees of the Society are necessary and of importance for the planning and guidance of Society activities. Time prevents detailing some of the interesting activities of these committees, but there are five on whose proper and continuous functioning the welfare of our Society depends, and to which our attention should be especially directed: (1) The Publications Committee, (2) Membership Qualifications, (3) Local Sections, (4) Juniors, and (5) Student Chapters.

The Publications Committee has charge of all Society publications; in the year 1941 it released a total of twenty-eight issues: *Proceedings* (10), *Civil Engineering* (12), *Transactions* (1), *Manuals* (4), and *Yearbook* (1), totaling about 493,000 separate numbers or volumes, containing 102,337,000 pages. It is through these publications that the news of the Society is made available to the membership and the record of its technical operations is made permanent.

Outside the medium of Society publications, the members of the Society are kept in contact with Society operations through the Local Sections, and the guidance of these Sections is through the Local Sections Committee, an Administrative Committee of the Board.

The sixty-four Local Sections of the Society now cover the United States, the Republic of Panama, Hawaii, and Puerto Rico, and include the 16,532 members of the Society resident in the United States, Hawaii, the Republic of Panama, and Puerto Rico.

Prior to the formation of Local Sections, groups of members of the Society used to gather from time to time to read papers and otherwise promote an understanding of engineering problems. These groups operated under such rules and regulations as seemed necessary. The forerunner of the Society Local Section in our host city of Minneapolis, Minn., was organized 59 years ago to entertain the June, 1883, meeting of the Society in Minneapolis, and to discuss the forming of a local Engineers Club.[4] Both of these objects were effectively accomplished.

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Minneapolis, April 30th, 1883.

Dear Sir:

You are respectfully invited to meet the Engineers of this city in the Board of Trade Room, in the City Hall, on Friday, May 4th, at 7:30 p.m. to consider the question of appointing a committee to join with those of the Board of Trade and Chamber of Commerce, in making preparations for the entertainment of the American Society of Civil Engineers in June.

The subject of forming a local Engineers Club will also be discussed.

Signed, Andrew Rinker,
City Engineer.
Wm. A. Pike,
E. T. Abbott.

All local sections now operate under constitutions approved by the Board of Direction of the Society, and Society funds are allocated to each Local Section. Each Local Section constitution defines the objects of the organization as "the advancement of engineering knowledge and practice; the cultivation of friendly relations with all engineers; the maintenance of high professional standards; and co-operation with other engineering societies, with a view of promoting the general welfare of the American Society of Civil Engineers and the engineering profession."

The first Local Section to operate under a constitution approved by the Society was organized in 1905 in San Francisco, Calif., and today that section ranks fourth in size, with 841 members. Organization of Local Sections was a slow process. The second section, Colorado, was organized in 1909; the third, Georgia, in 1912; but in 1913 six sections were formed. In 1914 five more were formed, and at the end of 1919 a total of twenty-four sections had been organized, which have a present-day membership of 51.5% of the total of 16,532 members.

Twenty more sections were organized from 1920 to 1923, which have a present-day membership of 31.2% of the total membership, and the remaining twenty sections were formed, one or more each year, between 1923 and 1940, with the exception of the years 1926 and 1935, in which no sections were formed. Additional sections, or subsections may be formed for the purpose of decreasing the area covered, and to bring section meetings closer to individual groups of members. Texas is an outstanding example of successful operation of a one-state section, with a number of subsections, covering 8.8% of the area of the United States.

Without doubt the Local Sections have been giving members closer contact with the Society, and the Society first-hand contact with the members' problems. Both of these results are necessary if the Society is to continue to serve the membership as the membership has the right to expect.

The future of the engineering profession depends on the engineering students and the young engineers of today; and the future of the young engineer, and to some extent that of the engineering student, depends on guidance by the professional engineers. The Society puts the direction of Juniors and of Student Chapters each under the direction of an Administrative Committee.

The Society's work with engineers in training and the engineering student starts with centralized points of contact, such as our 121 Student Chapters, with 5,303 members, in the forty-eight states and the District of Columbia. Fourteen of these states have one chapter each; twenty-three states and the District of Columbia have two each; and the number ranges upward from three per state to the ten chapters in Pennsylvania and eleven in New York.

The first ten of these chapters were organized in 1920, and seventy-two were in operation before the end of 1926. From one to five chapters were added each year from 1926 to 1941, except in 1934 when no chapter was formed.

Each Student Chapter has a faculty adviser, a contact member, and, where possible, a junior contact member. All three of these men are members of the Society, and the contact and junior contact members are men in the active practice of the profession.

The rate at which civil engineers are graduated affects the Society's effort to serve them, to see that its facilities keep pace with the demand. Of the 126 colleges reporting to the Society for the Promotion of Engineering Education, 103 have adopted a plan to speed up engineering courses without shortening them by making the college year 48 weeks in every 12 months.

At the start of the 1941–1942 school year, in October, 1941, 149 undergraduate engineering schools in Canada and the United States reported a total of 10,838 civil engineering students matriculated. In the spring of 1941, 1,430 were granted degrees. The school year of 1940–1941 had a total enrollment of 108,091 engineers in all branches of engineering in the 129 accredited schools in the United States, and a total of 12,709 were graduated.

Student Chapters are receiving effective support from the schools in which they are located. In the first six and one-half months of this year, 797 applications were received for admission to the Society as Juniors. Not all of these applicants were graduate engineers, and 70 of the 768 applications analyzed had an average of 5.7 years professional experience. Fifty-nine of this group of 70 were engineering graduates. The remainder of the group analyzed, 698, were graduates from 111 engineering schools. Applications have been received this year from 100% of the 1942 civil engineering graduates from seven engineering schools.

Contact with a young engineer after he enters the active practice of the profession is something which can be maintained only as it has been in times past—through local and state sections of the societies and professional organizations, and through the contacts we as individuals can make with these young engineers. On February 2 of this year there were 4,858 Juniors in the Society, indicating that these new men in the profession see the advantage of such contacts. When 27% of our membership is made up of Juniors, men under 35 years of age, there should be little fear but that the Society will be kept in step with the present-day requirements of the profession.

Members should be interested in the fact that, of approximately 800 Juniors elected in 1941, 135 failed to qualify by paying the initiation fee and their first year's dues. Inquiry develops that many of them have been called to service in the Army and Navy, and the uncertainty of their future has prevented them from investing in the necessary membership fees.

Statistics for 1942 show that the membership of the Society is continuing on the upgrade. The year 1941 showed a net growth of 776 additions to the membership. On July 8, 1942, the net growth was 764 over and above the same date in 1941, and the indications are that by the end of the year our membership, which now totals 17,877, will reach a total of about 18,200. This year it appears probable that a smaller percentage of the membership will be dropped for non-payment of dues on September 1 than were dropped last year. The ratio, not the number of members involved, is about 100 this year compared with 125 in 1941.

An analysis of applications for admission to membership in the Society develops numerous interesting facts, a number of which are indicated in Table 1, which contains data from 1,241 of the applications received in the first six and one-half months of this year. These data are offered for the purpose of emphasizing the fact that one of the Society's principal obligations to the profession is to continue its active support of civil engineering students and Junior engineers, through Student Chapters, Local Sections, and personal contact.

TABLE 1.—DATA FROM APPLICATIONS FOR ADMISSION TO THE AMERICAN SOCIETY OF CIVIL ENGINEERS IN FIRST HALF OF 1942

Description	APPLICATIONS FOR:				
	Member	Transfer from Assoc. Member to Member	Assoc. Member	Transfer from Junior to Assoc. Member	Junior
Number of applications analyzed	116	76	178	103	768
Average age	45	44	36	31	23
Maximum age	73	66	66	33	32
Minimum age	34	35	27	27	20
Graduates, percent of applicants	74	80	69	93	99[a]
Average experience in years	23	23	13	11	....[b]
Average years in grade before transfer		11		11	....

[a] The calculated percent of 768 applications for Junior grade is 98.6. [b] Seventy of the 768 applicants average 27 years of age, and have had an average of 5.7 years of professional experience; 84.3% are graduate engineers; 698 of the number average 23 years of age; and all of them are engineering graduates.

It is difficult to forecast what the future has in store for us as members of the American Society of Civil Engineers and as Americans, but at this time it seems safe to state definitely that in 1942 the Society will show increases in membership and in financial stability, and that it will continue to make contributions of time and effective service to the profession and to America during this war period, when such service counts. There is a possibility that service of this sort is the reason for the Society's continued growth during the 90 years of its successful existence.

MEMOIRS OF DECEASED MEMBERS

LINCOLN BUSH, Past-President and Hon. M. Am. Soc. C. E.[1]

DIED DECEMBER 11, 1940

Lincoln Bush was born of Vermont and Ohio parentage on December 14, 1860, in a log house on a pioneer farm in Palos Township, Cook County, Illinois, some twenty miles from the young, ambitious little city of Chicago. His father's name was Lewis Bush; his mother's, Mary (Ritchey) Bush.

Because of his birth a few weeks after the momentous election of November, 1860, that stirred the United States to its very foundation, and because of his own heritage and environment, it was fitting that he should be named Abraham Lincoln Bush, and should later be influenced by the traditions of that name. Afterward he adopted the designation A. Lincoln Bush, and eventually this became simply Lincoln Bush, a name by which he was known throughout his later life. In his modest way he used to explain that he hardly felt he could live up to the grandeur of the fuller name.

Expecting to become a teacher, Lincoln Bush studied in the Cook County Normal School (the teachers' training school for the City of Chicago), receiving the degree of Master of Arts in 1881. After three years spent in teaching in the public schools, he studied four years in the University of Illinois, at Urbana. He attained high scholarship standing and was graduated in the class of 1888 with the degree of Bachelor of Science in Civil Engineering. Sixteen years afterward the University made recognition of his engineering achievements by conferring upon him the honorary degree of Doctor of Engineering.

Mr. Bush's early experience was on railroad and structural engineering in the West. Several years were spent on railroad location, construction, and maintenance on the Wyoming Division of the Union Pacific Railroad Company and with the Pacific Short Line Company in Wyoming and Utah. After a year in the bridge office of Morison and Corthell in Chicago, he served five years as chief draftsman and office engineer for the Pittsburgh Bridge Company in its Chicago office, followed by three years as engineer on the design of masonry for the bridges of the Chicago Sanitary District and as assistant bridge engineer and acting division engineer for the Chicago and North Western Railway Company. In the latter position he had valuable experience in moving the Kinzie Street drawbridge in Chicago and the Kinnikinnic drawbridge in Milwaukee, Wis. In these various ways he served an apprenticeship, grew in stature, and developed his powers to take larger responsibilities.

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[1] Memoir prepared by a Committee consisting of C. A. Mead, M. Am. Soc. C. E. (Chairman), C. W. Hudson, M. Am. Soc. C. E., and H. S. Crocker and the late A. N. Talbot, Past-Presidents and Hon. Members, Am. Soc. C. E.

Then in January, 1900, came the opportunity to join the headquarters staff of the Delaware, Lackawanna and Western Railroad Company in Hoboken, N. J., as bridge engineer. This railroad was just entering what proved to be an era of tremendous betterments—with an extensive program of new structures and line improvements; heavier, stronger, and safer steel bridges; shortened and straightened alinements and greatly reduced grades; elimination of highway grade crossings in its suburban territory; and improved station, shop, and terminal facilities. Altogether this was a most promising prospect for an ambitious man. In less than a year Mr. Bush was principal assistant engineer as well as bridge engineer. In two years more he became chief engineer of the Lackawanna Road. In his nine years of service with this railroad, many improvements were completed and many others planned.

With a view of entering a larger field, in January, 1909, Mr. Bush established an office in New York, N. Y., as consulting engineer. Thus he was able to give expert engineering service on arbitrations, grade eliminations, foundations, bridges, and other large projects. Also, as a member of large construction contracting companies and carrying the heavy responsibilities as chief engineer of these organizations, Mr. Bush became a conspicuous figure in the construction of notable engineering structures.

He was vice-president and chief engineer of F. M. Talbot Company, Contractors, 1910 to 1917; president and chief engineer of Talbot Construction Company, 1911 to 1913; vice-president, chief engineer, and treasurer of Flickwir and Bush, Inc., 1912 to 1916; and president of Bush, Roberts and Schaefer Company, 1920 to 1925.

The engineering feats of Lincoln Bush are characterized by boldness as well as caution, by ingenuity and research, and by detailed planning and extracts from experience. At times he undertook daring procedures unprotected by precedent, and again, he resorted to plodding computations based on prior practice and experience. Any one who studies his work will be stirred by the appearance of novelty, by the daring and magnitude of a feat; he will marvel at the caution and care, the foresight and multitudinous provisions against failure, that were considered in advance.

The element of novelty in design and construction characterized his work. The Bush train shed, in use by many railroads, is an example. He originated and patented this construction. His many and varied engineering structures and constructions, in their nature and variety, would fill a large catalog. An idea of some of his work may be given by referring to an unusual project successfully completed a few months after he became chief engineer of the Lackawanna Road.

This feat, which drew the headlines of New York newspapers, was the removal of a 1,000-ton bridge truss to a new position against the odds of time and tide and weather. This four-track, double-deck swing bridge had been constructed the year previous for provisional use over the Passaic River, at Newark, N. J., preliminary to track-elevation construction. Permission from the United States War Department to obstruct the navigable waterway for the duration of one tide, twelve hours, was obtained for Sunday, December 20, 1903. Barges carrying immense sand jacks were placed under the truss, so that the incoming tide and removal of water ballast lifted the bridge from its support. Thus it was towed upstream to the new location for which it was designed. Reversing the action of the sand jacks and the water ballast, the truss was successfully lowered into its position on the now pivot pier, the barges were removed, and all traffic was resumed within the time limit before the turn of the tide would have lifted the burden again.

Sand jacks had previously been used for minor purposes. Mr. Bush, however, had to create his mode of procedure, learn the characteristics and eccentricities of sand jacks, and make his own precedents. Months were spent with experimental jacks and their operation. Provisions for pumps and boilers and duplications for emergencies were made. The variabilities of the tide and the currents had been given careful study; and it was well that this had been done, for that day the heavy rain, the tide, the wind, the current of the river, and the weather conditions formed as unfavorable a combination for such an undertaking as would probably ever occur. The next day, Monday, 200 passenger trains and 40,000 commuting passengers sped over that bridge as though it had always been there.

Mr. Bush's many achievements in engineering construction of large scale are well exemplified by the part he took in the building of the mammoth Tunkhannock Viaduct for the Lackawanna Road. This double-track viaduct is 240 ft high above the bed of Tunkhannock Creek north of Scranton, Pa. It consists of ten 180-ft arches of concrete masonry and two of 100 ft. The conception, design, and construction of the viaduct were under the direction of George J. Ray, M. Am. Soc. C. E., at that time chief engineer of the Delaware, Lackawanna and Western Railroad Company. Mr. Bush, as vice-president and chief engineer of Flickwir and Bush, Inc., Contractors, had the responsibility for the methods, equipment, and engineering procedure adopted and used in the entire construction of the viaduct and for the constructional progress of the work.

Many older members of the Society will remember that this structure, then well along toward completion, was the main objective of a most enjoyable all-day inspection trip, by special train on the Lackawanna Railroad, on the occasion of the Annual Meeting of the Society in January, 1914. Towers and cableway, transportation of materials, steel truss arch centers and their transfer, the conveying of concrete—these and all other details and ensemble were in Mr. Bush's hands. The speed, economy, certainty, and quality attained in the construction of one of the great masonry bridge structures built up to that time redounds to the constructive talents of Lincoln Bush.

The service he contributed to his country during the first World War is worthy of emphasis. With his long experience in construction and in the management of men, and with his knowledge of both rapid procedure and permanent construction, he gave loyally, unselfishly, and unreservedly of his professional knowledge, administrative skill, and driving energy. As an Officer in Charge of the Engineering Branch, under Gen. R. C. Marshall, Jr., M. Am. Soc. C. E., chief of the Construction Division of the United States Army, he was in responsible charge of the design and plans for practical operation for the seven terminal ports at Boston, Mass., Brooklyn, N. Y., Port Newark, N. J., Philadelphia, Pa., Norfolk, Va., Charleston, S. C., and New Orleans, La.; for fourteen interior warehouses located at industrial centers; for three arsenals; and for innumerable miscellaneous projects at other points, altogether totaling in cost $234,000,000. He entered the service in October, 1917, was commissioned Major in January, 1918, Lieutenant-Colonel in March, 1918, and Colonel in August, 1918. He assisted in harmonizing the work done by employees within the Division and with other branches of the service and also assisted in building up personnel of the Engineering and Construction divisions largely from his former associates and employees on engineering work.

His superior officer, in an official communication, reported that the services of Colonel Bush were regarded as invaluable to the success attained by the Engineering Division; that he had displayed most unusual qualities of engineering genius, sound judgment, and quick decision, and deserved the highest commendation; and that he was indefatigable in his application to duty, sound in his judgment, unremitting in his zeal, and successful in the achievement of a task of almost unprecedented magnitude.

Colonel Bush did his "bit" in helping win the war. When it was over and questions were raised regarding the conduct of the engineering construction, he was just as ardent in defending what he thought was right. In his private relationship and before congressional investigators he sprang loyally to the protection of the accused engineers from civilian life. Thereafter he maintained a great, although restrained, pride in his accomplishments during the war. Of all the titles and honors he received, he preferred to use that of "Colonel."

It is said that it is the duty of every professional man to contribute to his profession in a concrete way. This was true of Colonel Bush in his acceptance of office in the Society. He was Director for three years, 1912, 1913, and 1914; Treasurer for two years, 1915 and 1916; Vice-President for two years, 1924 and 1925; and President for one year, 1928.

In addition, after the untimely accident and subsequent death of Secretary John H. Dunlap on July 29, 1924, Colonel Bush gave daily counsel and advice in the administration of the affairs of the Society—over a period of a number of months—until a new Secretary assumed office the following January. He was never designated as Secretary or as Acting Secretary, but he nevertheless performed many functions of the office. From this standpoint, it may be said that he was the only member in Society history who had had personal experience in all the main offices of the organization.

Colonel Bush treated his election to office not as a mere honor, an empty recognition, but rather as an opportunity for service, a call to duty. In every instance he gave lavishly of his time and energy. One case stands out as a special evidence of his great loyalty to the Society. When Secretary Dunlap was fatally injured, in 1924, in the service of the Society, he took personal charge of Mr. Dunlap's interests. He assumed the responsibility of safeguarding Mr. Dunlap's estate, saving the family thousands of dollars. Colonel Bush's accomplishment was acknowledged by the Board of Direction at its meeting on October 22, of that year, when it adopted a formal vote of thanks for the great sacrifice the Colonel had made in stepping into the breach and rendering this valuable service.

From 1928 to 1932 Colonel Bush was consulting engineer for the City of Elizabeth, N. J. During this period the major project for the separation of grade crossings between the city streets and the tracks of the Central Railroad Company of New Jersey was being developed.

Colonel Bush was an outstanding member of the Society. He was also a member of the American Railway Engineering Association, the Western Society of Engineers, and the Engineers' Club of New York. He was the author of several articles that have appeared in the leading technical journals.

On financial matters his assistance on behalf of the Society was repeatedly sought. Among others, he served on the Board of Direction's Committee having responsibility for the 57th Street property. Here again, his financial insight and his thoroughness were of untold value to the Society. After he had convinced himself of the possibilities of this Society investment, he was adamant in his support of the course of action that to him was the only proper one. As a result, the Board approved of making a long-time lease of this property, a solution of great and lasting benefit to the organization. This case is only one instance, although perhaps the greatest, illustrating the many ways in which the Society has profited by Colonel Bush's financial intuition.

On October 17, 1890, he was married to Alma R. Greene, of Colfax, Ill., who, with their two sons, Cedric Lincoln and Denzil Sidney, survives him.

Of all Colonel Bush's noble traits, that of character stands out preeminent. He was as rugged and as dependable as the old log cabin home where he first saw the light of day. To his many intimate friends he was known to excel in forthright honesty and fearlessness. He was loyal to them and never hesitated to support them when they were attacked. He was persistent, painstaking, logical, and thorough. When he finally reached a conclusion, his judgment was similarly stable and complete. He would not be the first of a group to form a final opinion; but once formed, no man could hold a more rational one or defend it more reasonably. In this characteristic of dogged determination for what was right lay one key to his great success as an engineer and to his invaluable attributes as a man.

Colonel Bush was elected a Member of the American Society of Civil Engineers on October 4, 1905, and an Honorary Member on October 3, 1932.

DONALD HUBBARD SAWYER, Past-President, Am. Soc. C. E.[1]

DIED JUNE 21, 1941

The memory of Donald Hubbard Sawyer lives after him and is perpetuated in his lasting accomplishments, which, in retrospect, afford the truest measure of his stature. The lives of those who have known and loved him, and been associated with him, have been fuller and richer because of his inspiring influence. To countless of those who may not have been so favored, the record of his high professional ideals and noteworthy achievements will be a stimulant to higher aspirations and nobler efforts. Since June, 1941, he has rested in Arlington Memorial Cemetery beneath the canopy of the open sky, unmindful of sunshine or storm, drought or flood, peace or war. Amid the hectic turmoil of international strife, the knowledge that he has found eternal serenity carries with it the solace of a benediction.

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[1] Memoir prepared by W. E. Reynolds, *Chairman*, and Daniel C. Walser and John C. Hoyt, Members, Am. Soc. C. E.

He belonged to the ninth generation of Sawyers in the United States. His paternal forbears came to America from England and settled in the New England states. The three generations immediately preceding him were born in Chester, Vt. His grandfather, having the pioneer spirit, moved to the West, and his father, George Silas Sawyer, settled at Mount Pulaski, Logan County, Ill., almost in the center of that great state, where he met and was married to Phoebe Content Hubbard. Donald Hubbard Sawyer was born there on August 26, 1878.

When he was six years of age, his parents moved to Oak Park, Ill., where Don attended the public schools and was graduated from high school in 1897. Already attracted to the profession of engineering, he spent a year with John W. Alvord, Hon. M. Am. Soc. C. E.—even at that time an eminent engineer—and then entered the College of Engineering at the University of Illinois, in Urbana, from which he received the degree of Bachelor of Science in 1902.

Whether Don possessed the divine gift of prophecy, in his attraction to the profession of engineering in his early life, no one will ever know, but, looking backward over the years, it is now plain that the golden age of engineering dawned at about the time that he was born. In the span of sixty-two years, between his birth and his untimely death, engineering made greater progress and offered greater opportunity for scientific genius than had ever occurred before in any comparable period of time. Engineering achievement went far toward revolutionizing our way of living and profoundly affected the course of civilization itself. Don and his fellows in the engineering profession contributed substantially and importantly to the social and economic progress that has relieved life of much of its drudgery, broadened the field of opportunity for all, and raised the standards of living to levels far above the wildest dreams of even the "wise men" living at the time that he was born.

After his graduation from the University of Illinois, Don began, step by step, to lay the foundation of a notable engineering career in which he was destined to achieve distinction and exert a powerful influence. His first two years after graduation from college were spent with the late Alexander Potter,[2] M. Am. Soc. C. E., of New York, N. Y., on projects in New Jersey and Johnson City, Tenn., and as city engineer at Paris, Ill. For the following six years he was with the Illinois Traction System and advanced to the position of chief engineer in charge of interurban construction.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1652.

Already well on the way to eminence in his chosen profession, Don Sawyer, in 1910, sought wider fields of opportunity in the Pacific Northwest. As a partner in the firm of Sawyer Brothers, he turned his talents to irrigation and power projects and general municipal engineering. This association continued until the outbreak of the first World War.

After the entry of the United States into the first World War, and because of the urgent need of the Army for skilled engineers, Don Sawyer answered the call of his country and was commissioned a Major in the Construction Division of the Quartermaster Corps. He was in charge of the construction of Camp Grant, at Rockford, Ill., and built a nitrate plant at Cincinnati, Ohio, and another army cantonment—Camp Bragg, at Fayetteville, N. C. His success in these undertakings is attested by his promotion to Lieutenant Colonel and his retirement after the war with the rank of Colonel in the reserves.

When he returned to civilian life, he became associated with James Stewart and Company, of New York City. Two years later he was appointed secretary of the Associated General Contractors of America. After filling this office for seven years, he was made secretary of the Heating Board of Trade of New York. In 1931 the United States government again sought his engineering services, and from that time until his death he served successively as director of the Federal Employment Stabilization Board, president of the United States Housing Corporation, and chief of the Real Estate Section, Public Buildings Administration. When, to combat the depression, the Public Works Administration was created to launch a great program of public construction to set the wheels of industry moving again, he was designated by the President to act as temporary administrator of public works. After serving as vice-president of the Washington Building Congress, Colonel Sawyer was elected president of that organization but his death prevented his induction into office.

Other honors, too, were his. They are duly recorded in *Who's Who in America* for 1940–1941 and *Who's Who in Engineering* for 1941. There is no need to repeat them here, for great as were his engineering achievements, generous as were his sacrifices in the interest of his profession (particularly among students and junior engineers), and richly deserved as were the honors heaped upon him, it is upon Don Sawyer the man, rather than upon Don Sawyer the engineer, that memory lingers. It is upon the charm of his personality, his sincere affection for his friends, and his unswerving loyalty to them and to all who were associated with him that our thoughts dwell today.

The distinguished public service that Don Sawyer rendered is now a matter of recorded history, but he did something more important than that. He added to the sum of human happiness and joy. He was eyes for the blind, strength for the weak, and a shield for the defenseless. He embodied all the refinements of a brilliant mind, a chivalrous soul, and a heart as big as humanity itself.

He was a gifted public speaker, with an analytical mind, a ready wit, and a fine sense of humor. As such, hundreds outside the circle of his close friends came to know and admire him. It was in the intimacy of his home, however, that he was at his best. He and his wife, Elizabeth Osborn Merriam, to whom he was married on September 25, 1929, were the kind of friends before whom it was possible to think aloud. The real ornaments of his home were the friends who frequented it, for Don Sawyer loved people and this affection was universally reciprocated.

His professional engineering activities were conducted in a thorough, efficient manner, in which he showed the high administrative and organizing ability for which he was well known and highly respected. His sound judgment on engineering, financial, and other questions was fully recognized and sought for by those connected with his various activities. His tact and firmness in handling difficult questions were appreciated by all who came in contact with him.

To have known Don Sawyer, to have been favored with his friendship, to have been entrusted with his confidence, are things of a value that can be neither fully measured nor set down in cold type. Few men had more real friends than he and few men were accorded more generous affection by those who knew him best.

He is survived by his wife and two brothers, George L. Sawyer, M. Am. Soc. C. E., and Fred Sawyer.

Colonel Sawyer was elected an Associate Member of the American Society of Civil Engineers on April 6, 1909, and a Member on November 25, 1919. He was a Director from 1926 to 1928 and Vice-President during 1935 and 1936. The Society honored him by choosing him as its President in 1939, one of the youngest men ever to hold that high office.

ALEX DOW, Hon. M. Am. Soc. C. E.[1]

Died March 22, 1942

Alex Dow was born in Glasgow, Scotland, on April 12, 1862, the son of a Highland family whose members for generations had been skilled iron workers. His father was an expert accountant but had served an apprenticeship at the forge.

Young Alex attended the school of a Scots Presbyterian parish for six years, closing, at the age of less than twelve years, the only formal education he ever received. The rest of his education, his knowledge of the world's great literature, of science, economics, commerce, and his thorough and intimate knowledge of engineering, he gained himself.

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[1] Memoir prepared by Alfred C. Marshall, M. Am. Soc. C. E., James W. Parker, Vice-Pres. and Chf. Engr., Detroit Edison Co., Detroit, Mich., Harry A. Snow, Controller, Detroit Edison Co., J. H. Walker, Engr. Asst. to the General Manager, Detroit Edison Co., and Henry E. Riggs, Past-Pres. Hon. M. Am. Soc. C. E.

As a boy his ambition was to become a marine engineer, but circumstances forced him to go to work at the age of eleven years, first as a messenger boy, later as junior clerk and stenograher in a railroad office. At the age of seventeen he was employed by the Cunard Steamship Line in Liverpool, England, in a clerical capacity which often caused him to be afloat. On such trips he spent all his spare time in the steamship engine rooms.

In November, 1882, in his twenty-first year, he came to the United States and was first employed by the Baltimore and Ohio Railroad Company. Then he became a supervisor of local line and instrument maintenance for the Baltimore and Ohio Telegraph Company and began his study of electricity. In October, 1888, he was employed by the Brush Electric Company of Cleveland, Ohio, organized by Charles F. Brush. His first assignment for that company was as installing electrician of the Chicago, Ill., office.

In 1889 he became district engineer of the Brush Electric Company at Chicago, and was responsible for the design and construction in 1892 of the lighting of the South Park system of Chicago, a job which involved the placing of thirty miles of underground cable.

Detroit, Mich., was then (1893) a city of 260,000 inhabitants. The Public Lighting Commission had been organized and was about to install a municipally owned electric plant. Mr. Dow was employed in 1893 as municipal engineer at a salary of $3,000 per year. He designed the plant and placed it in commission at an expenditure of $600,000.

In 1896, being convinced that private ownership is more efficient than public ownership, and having had enough of municipal employment, he resigned his position with the Public Lighting Commission to become Vice-President and General Manager of The Edison Illuminating Company of Detroit. When The Detroit Edison Company was formed in 1903, Mr. Dow was appointed to the corresponding position in the new organization with the same title. In 1912 he became President and held that position until 1940 when he retired and became Chairman of the Executive Committee. In this capacity he was actively engaged as adviser and counselor in the affairs of the company until his death.

During these years of service with The Detroit Edison Company, Mr. Dow was called twice to serve the City of Detroit. From 1916 to 1921 he served as engineer member of the Detroit Board of Water Commissioners, and again, from 1925 to 1932, he was President of the Board. During these twelve years of service on the Water Board, all of them years of immense expansion of plant and construction of major improvements, Mr. Dow not only protected the department from politics but devoted much time to study of the engineering problems and to developing and establishing the sound business procedures which have made this one of the outstanding water works properties of the nation.

In 1932 he was appointed civilian chief of the Detroit Ordnance District, War Department, a post whose duties he was particularly fitted to discharge both because of his technical knowledge and his keen interest in ordnance manufacture. Traveling privately he inspected arsenals in the United States, Canada, England, and France to learn first-hand the exacting requirements of shell manufacture. Returning with considerable knowledge of the subject, he initiated the development by the Ex-Cell-O Corporation of Detroit, of the special purpose shell-turning machine now in use in considerable numbers in the United States and Canada. That first machine was built by private subscription and presented to the United States government by way of demonstration. It was but one of his many contributions toward improved methods both in production and in contract procedure. He held the position until his death.

The preceding paragraphs give the chronology of Mr. Dow's professional career, but they fall far short of telling the complete story. He was a great engineer, a great executive, a great public servant, and above all, a great man and citizen who can never be replaced in the public life of the State of Michigan or in the hearts of those who knew him.

During the more than forty-five years of his connection with the Detroit Edison properties he never ceased to keep the closest contact with engineering problems or to be in a very real sense the active head of the engineering activities. Space does not permit a listing of the major problems. Their magnitude and importance are indicated by the statement that when Mr. Dow first joined the company in 1896 its total fixed capital was $389,000, and at the end of 1941 it was $341,652,000. The total utility earnings of the company were $149,000 in 1896 and lighting rates were four times greater than they were in 1941 when the revenues were $73,575,000. During these forty-five years the number of customers increased from 2,500 to 743,086 and the total output from 2,700,000 kw-hr to 4,319 million kw-hr. These figures give some idea of the magnitude of the problems of design, construction, and administration for the correct solution of which he was responsible. They also indicate clearly that the destiny of the property, which depended upon the accuracy of his decisions, was safe in his hands.

In 1896 there were half a dozen little electric light companies in Detroit in cut-throat competition for business. The Edison Illuminating Company was one of them. Mr. Dow decided that he would not give special discounts to gain or hold business; that all customers should pay the same rates for the same service; that the service should be as good as he could make it; that rates should be as low as possible. The wisdom of that decision is not only proved by the fact that his company survived and absorbed all of its competitors, but is abundantly proved by the figures of growth just cited.

The outstanding fact found in any study of Alex Dow's career and of his writings is that he kept ahead of the industry through wise decisions, some of them made years before their general acceptance.

In 1896 he wrote "A Frequency of 60 cycles is best for central station use." Certain other utilities adopted 25 cycles and are cursed with it, as it would cost millions of dollars to change over now.

In 1898 he inaugurated the differential rate for electric service, under which the liberal use of electricity was encouraged by a low follow-on rate after a relatively high charge for the first few kilowatt-hours. In 1902 Mr. Dow recommended to his directors that a new alternating current turbine-driven generating station be built with an ultimate capacity three times the total capacity of all the engine-driven generators then in service in the city. Turbines were then new and untried; today reciprocating engines are obsolete. When Delray Power House No. 2 was built, Mr. Dow decided to use boilers which were nearly five times the size of those in Power House No. 1, and they were much larger than any boilers in use anywhere. Today large boilers are in general use, but Mr. Dow was the courageous pioneer who first blazed the way.

In 1898 Mr. Dow was probably the first man in the United States to propose that electric utilities operate without competition under commission regulation. He wrote, "Two electric lighting plants in the same territory is an economic error * * * competition is to the detriment of the public * * * in the absence of competition it is necessary that prices be approved by a competent authority." Nine years later the Michigan Railroad Commission was given jurisdiction over electric rates.

In 1908 he insisted that no dividend be paid to stockholders unless an equal sum were set aside to start a reserve against the depreciation of plant. This was recognition of the need of reserves for the replacement of property years before the practice became general. The annual reports of the Bell Telephone Company discussed the subject at length in 1911 and later years. The Detroit Edison policy of reserves has not been founded on formula but has been wise and conservative. Mr. Dow expressed it in a letter in 1932 as follows:

> "My disagreement with you does not mean that I accept the foolishness of the doctrinaires who persist in teaching that each year of our Lord is a person-at-law having rights and sanctions as against every other year in respect to accruals for depreciation, whether the same be proven by inspection or be estimated by some scholastic formula. I insist on providing for accruals which I know must inevitably accrue. But I propose to follow the rule which was good business when Joseph was Grain Controller for old Rameses—namely to make the good years contribute to reserves or to betterments which will maintain value, and to let the lean years travel into the past without marking them as defaulters."

In 1912 Mr. Dow laid the foundation for the "Original Cost" property records of The Detroit Edison Company. Twenty-five years later the Federal Power Commission officially adopted "Original Cost" and ordered all utility companies coming under its jurisdiction to revise their records.

In 1915 he wound up the "Holding Company" phase of The Detroit Edison Company's existence, being of the opinion that the simplest possible corporate structure has advantages outweighing any possible benefits of indirect ownership. Since then the company has conducted its business directly instead of through subsidiary distributing companies. Not until twenty years later did Congress enact the death sentence for holding companies.

In 1915 the new tungsten lamp promised light for less than half of the electricity required by the old carbon and gem lamps. Some managers, fearing loss of revenue, opposed its adoption. Mr. Dow, taking the ground that opposition to an innovation that promised to be for the public good was un-ethical and futile, offered to and did replace burned-out lamps of the old type with new tungsten lamps. The old types are now completely obsolete.

In 1932, when times were bad and work was scarce, Mr. Dow adopted the forty-hour week a full year before the federal statute was enacted.

During his whole administration decision after decision was made, important decisions fundamental to the welfare of his company. Enough cases have been cited to indicate that they covered not only engineering but finance, public relations, rates, labor and economic problems, from one or two to twenty years ahead of public acceptance. There were undoubtedly some mistakes, but Mr. Dow was right far more than half the time.

Mr. Dow's whole philosophy of public utility management was so admirably expressed in a letter written in 1914 to a woman stockholder that we quote from it as follows:

> Your line of reasoning (as I follow it) leads you inevitably to your stated conclusion as to dividends and wages; that whatever is added to the one must be taken from the other. This conclusion is wrong * * *. The big mistake is the assumption that a manager, or operator, or manufacturer has a duty to stockholders or has accepted a trust from them which debars him from doing his duty to his own employees and to humanity as a whole.
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> * * *
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> "I have refused consistently to make long time contracts or agreements fixing rates and, while I have reduced rates, I have been able to increase the payment to labor per *unit of time* because I was simultaneously reducing the cost of labor *per unit of production.* Whereas, if my industry had already been highly developed and systematized when I took charge of it, so that no economies of organization or machinery were possible, I should have been unable thus to increase the day's pay of the men, unless I had been permitted to raise the selling price of my product to consumers. That I would have been able to do so is inconceivable. Even though I might have had legal right and governmental consent, any material rise in price would have made my product unmarketable. In all these years I have held that the duty lay upon me to protect not only the interest of my stockholders but those of my employees and to so manage my relations with the public which purchases my product that while the public was well served and got relatively low prices, it nevertheless paid enough to let me distribute a fair return to stockholders AND a satisfactory wage to employees. To do this with fair dealing between the three parties—Capital, Labor, and Public Service—has not only been my first duty but my whole duty. Because all the other duties merge into this."

His successor, and close associate, A. C. Marshall, M. Am. Soc. C. E., writes of him:

> "Mr. Dow was a great man in so many ways that it is impossible to know at this time what particular facet of his brilliant mind will be remembered the longest, or what particular trait of his lovable nature will be treasured the most—because right now the sense of personal loss is overwhelming. * * *
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> "As time goes on, we will think of other characteristics of his greatness. We will think of his knowledge of history—his grasp of ancient as well as modern philosophy and religions—of his knowledge of and love for the King James version of the Bible, and of his high standing in the

field of pure science—physics and astronomy. All of this in addition to his recognized position as an authority in economics, finance and religion.

"He was, in a true sense, a gentleman and a scholar. To know him and to work with him was a liberal education."

H. A. Snow, Controller of The Detroit Edison Company, and his long associate, in closing a summary of Mr. Dow's career, writes:

> "How was it accomplished? By luck? No—the percentage of accuracy was far too high. By intuition? Perhaps—he claimed, smiling, some acquaintance with the pixies. By high intelligence? Certainly. His was undoubtedly one of the best minds in the Nation. By character? Yes! His motto was live and let live. He had no great use for personal wealth—money was something to be spent, freely, joyously. By hard work? Of course. The forty-hour week was for his men—not for him. His appetite for information was insatiable. * * * His mastery of the English language was unequalled, and his writing had a clarity and brevity and charm that could only come from the mind of a man such as he.
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> "He dealt on somewhat better than even terms with specialists in a dozen lines of human thought—he outfigured engineers and outtalked lawyers; he expounded theories to professors and theology to parsons; he fearlessly raided predatory politicians and he out-bargained financiers. Now it is better than even money that he is showing Charon how to row.
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> "This imposing record of accomplishment, with its undeniable proof of intelligence and ability may account for the respect but it does not account for the affection with which he is remembered by the ten thousand men and women who make up his staff. After years and years of being praised and reprimanded and bulldozed by him, a deep feeling of personal loss comes to all of us at his death."

Mr. Dow was an Honorary Member and Past-President of the American Society of Mechanical Engineers; Fellow and Honorary Member of the American Institute of Electrical Engineers; Member of the Institution of Electrical Engineers (Great Britain); Member of the Institution of Mechanical Engineers (Great Britain). He was one of the founders of the old Detroit Engineering Society, and was an Honorary Charter Member of the Engineering Society of Detroit; member and past chairman of the Rates Committee National Electric Light Association; and past-president of the Association of Edison Illuminating Companies. He was awarded the Edison Medal by the American Institution of Electrical Engineers in 1936.

Since 1936 Mr. Dow has been President of the Trustees, Rackham Engineering Foundation.

In 1911 the University of Michigan, at Ann Arbor, conferred on him the Honorary degree of Master of Engineering, and in 1924 that of Doctor of Engineering. He was given the degree of Doctor of Science by the University of Detroit in 1936; and the degree of Doctor of Arts by Wayne University (Detroit) in 1938.

Alex Dow was married in 1889 to Vivia Kinnersley of Baltimore, Md., who survives, together with their four children, Lothrop Dow, Col. Douglas Dow, Dorothy (Mrs. James W. Parker), and Margaret (Mrs. Lawrence H. Connell).

After a full life that had brought him more than the usual satisfactions for which men live, he left this world as he would have wished, working actively until Friday, March 20 when he went to the hospital. He passed away in his sleep on the following Sunday morning.

Mr. Dow was elected a Member of the American Society of Civil Engineers on December 5, 1906, and an Honorary Member on October 11, 1936.

THOMAS ULVAN TAYLOR, Hon. M. Am. Soc. C. E.[1]

DIED MAY 28, 1941

On the Texas frontier on January 2, 1858, a red-headed son was born to John Henry and Louisa (Allison) Taylor. This boy was christened Thomas Ulvan Taylor. His father died when he was about ten years old, and he had to help his mother make the living for the family. His mother was a wonderful woman, for she instilled into young Tom a burning desire to get an education. In spite of adversity, and poverty, he succeeded in doing this and as a result he has left his influence on the engineering profession in Texas and the Southwest.

The first school he attended was held under an oak tree. Indians were very common in that section of the state at that time, and the boys attending school carried rifles for protection; during school hours these rifles were stacked against the tree.

Railroads were just being developed in Texas. The Houston and Texas Central Railroad was being promoted north from Houston. As a young boy, Tom would accompany the freighters that went to the depot—the end of the railroad—Hempstead, Navasota, Hearne—to get supplies and haul them inland. Also, he made trips to Jefferson, Tex., which was the terminus of navigation at that time. Steamboats would come up from the Mississippi River, the Red River, and through Caddo Lake to discharge their cargoes at Jefferson. The freighters would carry this merchandise to the inland towns and forts.

When he was twelve years old he went to Cleburne where he obtained work in a brickyard "trotting brick." He supplemented his income by getting up early in the morning and carrying water from a spring to the saloons for five cents per bucket. He enjoyed telling of the arrival of the stage coach; this occurred about 4 o'clock each morning. The driver of the coach was provided with a long tin horn, and as he approached the town he would blow several long, loud blasts on this horn to announce his arrival.

When he was about fourteen he went to live with an uncle in Fannin County. Here, he worked on the farm and did other chores. He made friends with an older man, who coached him in his school work—in effect he had a private tutor. In 1879, the Sam Houston Normal, in Huntsville, Tex., opened its doors. The first students were selected by competitive examination and T. U. Taylor won first place in a group of eight from his senatorial district; he was admitted to college. His thirst for knowledge was increased; after graduation from college, when his uncle offered to lend him the money to buy a farm and a pair of mules, young Taylor borrowed the money, but he went to the University of Virginia, at Charlottesville, Va. He was graduated from this institution in 1883, after two years, with a degree of Civil Engineer. Here, he came in direct contact with famous characters of the Civil War days. He was so thoroughly saturated with the "Honor System," as it existed in the University of Virginia at that time, that he believed in it to the very last. He made such an excellent record at Virginia that he was offered a place teaching physics and mathematics at the Miller Institute, Crozet, Va., where he stayed for five years.

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[1] Memoir prepared by John A. Focht, M. Am. Soc. C. E.

An opening developed at The University of Texas, in Austin, for a teacher of Mathematics, and he could not resist the call to return to his native state. He joined the faculty of The University of Texas in 1888, and he continued there until his death. There were thirteen on the faculty and about two hundred and fifty students when he came to the University. In 1895, the Department of Civil Engineering was created and he was made Dean. This position he held for forty-one years; in 1936 he was retired as Dean Emeritus.

He was deeply interested in the welfare of his boys. Not only did he keep up with the boys' work in school, but he also knew about their financial conditions and their love affairs. He had a wonderful memory and recalled names and incidents easily. Many a man has walked into his office after an absence of twenty, twenty-five, or thirty years; Dean Taylor would look up from his work, push back his glasses, and call the man by his nickname, throw in his middle name for good measure, and then start talking about school-boy pranks and affairs in which this individual had been involved. He had an obsession to help the poor boy who was struggling financially to make his way through school. At first he would lend to the students from his own pocket, but as time went on he established Loan Funds which took care of the financial needs of the students. First came the Engineers Loan Fund with a Board of Trustees; to borrow money from this fund the student must have a "C" average. The Dean realized that there were worthy boys who did not have the "C" average, so he established the Big Brothers Fund, and he administered this Fund without consulting any one. He was always quick to say that during the entire time he had dealt with students he had never lost one penny—the loans had always been repaid.

Some four years ago Dean Taylor provided funds for a Foundation; cash awards are made to some six or seven senior engineers each year from the income of these funds. No applications are made for these awards, but the Board of Trustees makes inquiry as to worthy students to receive these awards. Dean Taylor definitely specified that they be given to seniors who, during the fall semester of their senior year, earned part of their college expenses by the sweat of their brow—such jobs as waiting on tables, washing dishes, carrying a paper route, etc., were acceptable, but white-collar jobs such as grading papers, office work, etc., were not to be considered.

One night, in 1905 or 1906, some of his students came to his house and reported that one of their classmates had withdrawn from the University, and was leaving on the night train for his home in East Texas. Dean Taylor went to the depot, and talked with the boy; his money was exhausted, and he was returning to the farm. The next morning this boy registered in the University—the Dean had taken care of financial troubles—and he completed his course with credit. This man is one of the prominent hydraulic engineers in the United States today.

In 1920, a young Russian refugee reported to Dean Taylor's office and inquired about going to school. After a number of questions Dean Taylor agreed to permit him to enter the engineering department. The boy admitted he had no money but his answer to how he would make his living was—"I can washa the dish." Dean Taylor helped him get a job, and kept close watch over him. In due time he was graduated with honors, having been elected to Tau Beta Pi. He became a prominent figure in oil refining. He became a naturalized citizen. Years elapsed. A delegation came from Russia to study oil refining and obtain technical help for some of their problems. He was finally persuaded to return to Russia to help solve some of their difficulties. Truth is stranger than fiction for he was sent as a consultant back to his home town, from which he had run away a few years before seeking opportunity in the United States. Just two or three years ago an organization in Texas offered a prize to high school students for the best paper on "Why I Am Glad I Am an American." The daughter of this Russian refugee whom Dean Taylor had befriended won first place in this state-wide contest.

He helped others than engineering students. One day the writer went to Dean Taylor's office immediately after lunch, and he found the Dean seated at his desk and a boy standing in front. The Dean asked the boy, "Will this be enough?" and the boy answered, "Yes Sir." The Dean filled out a check and handed it to him. After the boy had left, the Dean turned and said, "That boy is a student in the College of Arts; his funds were completely exhausted and it would be a week or ten days until his check came; he had no money to buy his meals; he did not feel close enough to any of his professors to ask for help, so he came to me." No one will ever know how many boys Dean Taylor helped in one way or another; he never let his left hand know what his right hand did.

Dean Taylor was a great favorite with the students; every one of the students who attended the University since 1888 knew him. This is illustrated by the fact that in the early stages of his last illness he went to a clinic in Temple, Tex., for examination. A young doctor came in the room and said, "Hello, Dean Taylor; you may not know me but I know you. I was a premed student at The University in 1930." Every year he was asked to speak at one of the principal football rallies. In 1923 when one of the Campus characters—a dog—died, an enormous parade was formed which marched around the campus to the front of the Main Building. From the front steps Dean Taylor delivered the funeral oration of this dog, which had been the pet of the entire student body for eight or ten years. Several have said that this was one of the best speeches Dean Taylor ever made.

Early in his teaching career, Dean Taylor was affectionately christened "The Old Man" by his students. More of his students referred to him by this title than any other. From the beginning Professor Taylor developed the habit of marking a perfect paper with a check mark rather than the customary 100. This check mark accumulated a flourishing tail. One day after the Professor had returned some papers, the students gathered in the hall to compare grades. One of the boys exclaimed, "The Old Man gave me a Ram's Horn." A tradition was born, and the name stuck. In 1913 when the Dean had been at the University twenty-five years—the first person to have that honor—he was given a banquet, and was christened by his former students as "The Grand Old Man." The Dean presented to every man present, and every man who had been a student under him during that twenty-five years, a brass watch fob with the inscription "The Old Man's Engineers, 1888–1913," and the embellishment of a Ram's Horn. At that time he said he was giving all of his students, and former students, a Ram's Horn.

In 1897, he was appointed hydrographer for Texas by the U. S. Geological Survey. At that time practically nothing was known about the discharge of streams in Texas. He established a number of gaging stations and arranged for observers to record the elevation of the water at least daily. At intervals, Dean Taylor went to these stations and made discharge measurements, from which rating curves were constructed. He was so interested in this work that when the government funds were discontinued he carried on the work for several years at his personal expense.

During this time he became interested in the silting of Lake McDonald, the first lake constructed on the Colorado River near Austin; he made periodic measurements on this and Lake Austin, and published several bulletins on the silting problem. He was an outstanding authority on the silting of reservoirs in the Southwest.

When he was more than seventy five, he organized a party of ten consisting of professors and graduate students and went to Lake Worth to make one of his periodic silting surveys of that lake. The party obtained a cabin at the lake, and the next morning he had the group out in the boats before sunup; the Old Man spent the entire day in a boat working as hard as, or harder than, any one else. After this survey was completed, he insisted that the party return home that night—more than two hundred miles. The next morning he was at his office as usual—seven o'clock.

Dean Taylor was an early riser; he was up and had his breakfast by daylight. His regular scheduled office hour was seven to eight in the morning. He always requested that his classes be at eight o'clock in the morning. When boys would miss a quiz or make an extremely low grade and ask for a makeup, he usually granted the request, but he would set the time for six o'clock the following morning.

In the summer of 1892, Professor Taylor was assistant engineer in the office of the late J. A. L. Waddell, Hon. M. Am. Soc. C. E., on the design of Inter-State Draw Bridge, and the Halstead Street Lift Bridge. In the winter of 1893, he assisted Mr. Waddell in the competitive design for a bridge over the Danube River at Budapest, Hungary.

Professor Taylor was called in a consulting capacity by a number of the Texas cities to assist in the solution of their water problems. Some of these cities were Austin, Abilene, Dallas, Fort Worth, and Wichita Falls.

Dean Taylor was a member of the Texas Academy of Science from its organization until his death. He was also an active member of the Texas State Teachers' Association, the Society for the Promotion of Engineering Education, the American Mathematical Society, and the American Association for the Advancement of Science. He was a member of Phi Beta Kappa, Tau Beta Pi, Chi Epsilon, Pi Tau Sigma, Pi Epsilon, and Eta Kappa Nu.

He was the author of a number of books including "Earthwork and Prismoidal Formulas," "Backbone of Perspective," and "Surveyors Handbook." He was the author of a number of Water-Supply Papers and Bulletins based on data gathered while working for the U. S. Geological Survey. In his later life he was interested in preserving the early history of Texas, and wrote "The Chisholm Trail and Other Routes," "Jesse Chisholm," "Fifty Years on Forty Acres," and innumerable articles on the life and happenings of early Texas pioneers.

He was married in 1888 to Maria Moon in Albemarle County, Virginia. He is survived by his widow, and one daughter, Julia (Mrs. D. C. Strother). A son, Dr. Summerfield M. Taylor, died in 1940.

One of the reasons for Professor Taylor working for Mr. Waddell in 1892 was to gain acquaintance with engineers who would give their names as references on his application for membership in the American Society of Civil Engineers. It is said there were not five members in Texas at that time. He was elected an Affiliate (then Associate) on January 31, 1893; an Associate Member on October 2, 1895; a Member on October 2, 1901; and an Honorary Member on October 6, 1939. He became a Life Member in 1928.

FRANK ELWIN WEYMOUTH, Hon. M. Am. Soc. C. E.[1]

Died July 22, 1941

Upon the Empire-building water conservation and development projects that have transformed vast wastelands of Western America into productive and populous areas since the turn of the twentieth century no mind or hand has made a deeper imprint than Frank Elwin Weymouth's. From the day in 1902 when the federal government launched its first reclamation project until his death on July 22, 1941, he ranked among the foremost of those who planned and directed the successful completion of the far-flung system of dams, canals, and conduits which now conserve and distribute for the service of Man the life-giving water of a score of great rivers in that boundless territory which old geography textbooks once marked out as "The Great American Desert."

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[1] Memoir prepared by Julian Hinds, M. Am. Soc. C. E.

One of Mr. Weymouth's earliest engineering assignments as a young man fresh from the halls of his University was work in the planning of the Massachusetts Metropolitan Water District. In this task he gave his talents to America's first great publicly owned domestic water-supply system, designed to serve through a common effort the needs of numerous cities. Later, and for many years, he concentrated his remarkable engineering and organizing abilities on the government's gigantic irrigation and reclamation program. At the close of his career he had completed his greatest accomplishment—the planning and building of the Colorado River Aqueduct through which an abundant and unfailing domestic water supply has been made available to Los Angeles, Calif., and twelve other enterprising cities which comprise The Metropolitan Water District of Southern California.

Frank E. Weymouth was born in Medford, Me., on June 2, 1874, the son of Andrew Jackson and Charlotte Prudence (Powers) Weymouth. After finishing high school at near-by Fort Fairfield, Me., he took a civil engineering course at the University of Maine, at Orono, Me. He was graduated with the degree of Bachelor of Science in Civil Engineering in 1896, and subsequently honored by this institution with the degree of Doctor of Engineering in 1934. His first job after graduation was with the City of Malden, Mass., and later with The Metropolitan Water Board in Massachusetts, on water works construction, the field in which his greatest distinction was to be attained in later life. From this work he went to the City of Winnepeg as assistant city engineer, holding this office until June, 1899, when he joined the engineering organization of the Isthmian Canal Commission of the United States government and was engaged until 1901 in making surveys and studies in connection with the proposed Nicaraguan Canal.

In 1902, when the U. S. Reclamation Service was created, Mr. Weymouth joined its engineering organization and served with this bureau in various capacities and with ever-increasing responsibilities for the following twenty-two years. During his early years with the Service, he was in charge of surveys and investigations for irrigation projects in Montana and North Dakota. In 1908 he became supervising engineer in charge of the Idaho District, and in this capacity directed the design and construction of irrigation projects in the Snake River region, an engineering work affecting more than 400,000 acres of land. Among the important structures built under his supervision during this period were the storage dam on the Snake River at Jackson Lake, Wyoming, and the Arrowrock Dam, on the Boise River in Idaho. This latter dam, 349 ft high, was the tallest in the world at the time of its construction.

From 1916 to 1920 Mr. Weymouth served as chief of construction in charge of all work undertaken by the Reclamation Service in the Western States, and in 1920 he was made chief engineer of the Bureau, a post which he held until his resignation in October, 1924. During this period the Reclamation Service engineering organization, under Mr. Weymouth's leadership, was one of the pioneers in the development of new methods of dam design and construction, which culminated in the successful construction by the Bureau some years later of the Boulder Canyon Dam to a maximum height of 720 ft, more than twice that of Arrowrock, which stood unchallenged for so many years after its completion in 1915 as the "world's highest." Indeed, one of Mr. Weymouth's outstanding achievements as chief engineer of the Reclamation Service was his work, with the late Arthur P. Davis,[3] Past-President, Am. Soc. C. E., and at that time director of the U. S. Reclamation Service, on surveys and the preparation of plans and estimates for the Boulder Dam project. The "Weymouth report" furnished the basis of facts and figures upon which the final decision as to the feasibility of this project rested.

Upon leaving the Reclamation Service, Mr. Weymouth became president of the engineering firm of Brock and Weymouth, of Philadelphia, Pa. In 1926 he severed this connection to take the position of chief engineer of the J. G. White Engineering Corporation in Mexico, in charge of all engineering work being handled by that organization for the Republic of Mexico. In this capacity he worked with the National Irrigation Commission of Mexico and was in active charge of all the irrigation and reclamation projects that were carried forward during the administration of President Plutarco Elías Calles. This work involved extensive investigation of irrigation possibilities throughout the republic and the supervision of the planning and actual construction of a number of important projects. Notable structures were the Don Martin Dam near Nuevo Laredo and the Calles Dam in the State of Aguascalientes. There were several other dams and many miles of canals and laterals, with a variety of incidental structures. This work was completed with notable success and dispatch.

In 1929 Mr. Weymouth was retained by the City of Los Angeles as chief engineer of Water Works, and placed in charge of the Colorado River Aqueduct studies then being conducted by that city. Later in the same year he was appointed chief engineer of The Metropolitan Water District of Southern California, which had been formed to take over this project. In 1931 he was appointed to the position of general manager and chief engineer of the District, and became the District's chief administrative officer, as well as its chief engineer.

The Metropolitan Water District of Southern California is composed of thirteen cities located in what is known as the South Coastal Basin of California. This basin contains 45% of the wealth and population of the State of California and only 1% of the water resources.

The region is arid, and upon his arrival, Mr. Weymouth found that physical development had gone far beyond the safe annual yield of existing water resources, this condition having been made possible by seriously overdrawing the waters accumulated during prehistoric times in the deep gravel beds underlying the basin. The community was facing the alternative of bringing in a new supply from an outside source or of serious retrogression. The possibility of importations from near-by sources had been exhausted. The only possible origin of a new supply was the Colorado River, which lay 300 miles away and at the time lacked storage facilities required to conserve its variable flow for domestic diversion. The problem of tapping this river for the benefit of Southern California was committed to Mr. Weymouth.

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1582.

Planning and building the Colorado River Aqueduct involved tremendous problems at every step of its development. The first task was the selection of the best route across a vast extent of desert, the geological and topographical details of which were practically unknown. Simultaneously, it was necessary to prepare preliminary estimates and to convince the voters of the District of the need for and the practicability of the project. That these tasks were well accomplished was evidenced on September 29, 1931, when a proposed bond issue of $220,000,000 was carried by a five-to-one majority.

Actual construction was started late in 1932 and the aqueduct was completed and delivering water to District cities on June 18, 1941, a month before Mr. Weymouth's death. The present development involves 242 miles of main aqueduct and 150 miles of laterals, or wholesale delivery lines—a total of 392 miles. Of this length, 92 miles is 16-ft-diameter tunnel and 16 miles is 10-ft tunnel. There are four important dams, five high-head pumping plants, 237 miles of high-voltage transmission lines bringing Boulder Dam power to the pumping plants, and all of the many miscellaneous structures that go with a project of this magnitude.

On December 3, 1900, Mr. Weymouth was married to Mary Maude Lane, who died on January 2, 1937. On November 10, 1938, he was married to Barbara Turner, who survives him.

For almost forty years Mr. Weymouth was engaged in responsible engineering work on large government projects. Early in his career he demonstrated the ability to deal effectively and cordially with members of Congress and ranking government department heads, and yet there was nothing of the politician in his character or his methods. He possessed a deep and abiding aversion to the "handshaking" and "back-scratching" technique of certain types of professional politicians, and steadfastly refused to permit such operators to interfere with or influence his public duties in any particular. Nevertheless, his unquestioned ability and his unshakable honesty won for him the support of responsible government heads, where, in many instances, the political maneuvers of others had failed.

He was a man with no personal hobbies outside the orbit of his professional work. Reading appeared to be his only relaxation. Biographical and historical works made up a large part of his non-technical reading. He carried his professional interests and his work with him into his home, where he spent long hours of almost every day in the review in minute detail of the reports of his assistants, and in the analysis of current problems.

One of the secrets of his outstanding success as an executive and an organizer was his uncanny ability to pick the right man for a particular task. To his subordinates he delegated ample authority to do the job, and at the same time he held each man strictly accountable for the success or failure of his work. As every successful executive must, he generally passed on to his aides the manifold details of an assignment, and yet these details never escaped him. Frequently he appeared to concentrate upon what, to others, appeared to be a minor phase of a problem under consideration. In such instances, it developed that the "minor phase" was the crux of the entire problem, a realization which sometimes dawned slowly and painfully in the minds of his assistants.

To the preparation of reports and written recommendations for specific action by governing bodies, he gave careful and meticulous attention. Such documents, he insisted, must be clear, accurate in all details, solidly buttressed with definite findings and incontrovertible facts. Once, twice, or a half dozen times any such report might come "sailing" back to the desk of a subordinate bearing the marginal suggestions and constructive criticisms of the Chief. It had to be right, not only in its general outlines but in every phrase and every single word; and not until it was right would it pass the barrier of Mr. Weymouth's desk.

Few men have given more of their waking hours to the daily tasks before them than Frank E. Weymouth. Behind him he has left an enduring record, in steel and concrete, of his accomplishments in the service of his fellow men. It has been the lot of few men to leave a greater record.

Mr. Weymouth was elected an Associate Member of the American Society of Civil Engineers on September 4, 1901; a Member on February 5, 1907; and an Honorary Member on October 10, 1938.

CHARLES MELVIN ADAMS, M. Am. Soc. C. E.[1]

DIED AUGUST 31, 1941

The passing of Charles M. Adams at the ripe age of 81 years removed a widely known and respected engineer from civil engineering activities in northwestern Washington.

Mr. Adams was born at Prattsburg, Steuben County, N. Y., on June 11, 1860. He attended public schools in Prattsburg and was graduated from the Franklin Academy, in Prattsburg, in 1878. Following graduation he taught in rural schools. Later, he taught mathematics for three years at the Franklin Academy, during which time he took up the study of civil engineering.

Mr. Adams' first engineering experiences were with the Western New York and Pennsylvania Railway Company and later with the Lackawanna and Pittsburg Railroad Company in New York State, and the Chicago, Burlington and Quincy Railroad Company in Illinois. From 1885 to 1888 Mr. Adams was division engineer with the Chicago, Rock Island and Pacific Railway Company in Kansas. He later held the same position with the New York, Pennsylvania and Ohio Railway (Nickel Plate) and the Toledo and Ohio Central Railway companies in Ohio, and the Lehigh Valley Railroad Company in New York.

He went West late in 1890, arriving at Whatcom, Wash. (later named Bellingham), on Thanksgiving Day. He was engaged as engineer on the

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[1] Memoir prepared by C. N. Reitze, M. Am. Soc. C. E., and H. M. Hadley and M. S. Campbell, Assoc. Members, Am. Soc. C. E.

Bellingham Bay and British Columbia Railroad (Chicago, Milwaukee, St. Paul and Pacific Railroad) until completion of that line in 1891.

On three different occasions, 1892–1895, 1900–1901, and 1918–1924, he served Bellingham as city engineer, having charge of design and supervision of construction of paving, sewers, bridges, water works, etc. He also served Whatcom County as county engineer from 1895 to 1897, 1901 to 1905, and 1913 to 1917, having charge of the design and construction of the county's highway system, including bridges, river protection, and drainage works. Some of the earliest concrete pavement used in the State of Washington was laid by Mr. Adams during his last term as Whatcom County engineer.

From 1898 to 1899, Mr. Adams was construction engineer in charge of an extension of the Canadian Pacific Railway through the mountainous Kettle River district.

From 1927 to 1929, Mr. Adams served as municipal engineer for the government of the Dominican Republic, in direct charge of the installation of various projects totaling $3,000,000 and including the water supply system, storm and sanitary sewer system, and street paving of the capital city, Cuidad Trujillo. While in the Dominican Republic, he served as consultant for other projects of a similar nature. The West Indian climate, however, did not prove well suited to Mr. Adams, and following his return to Bellingham, his earlier good health deserted him. For the last eleven years of his life he was an invalid, confined quite closely at home.

Mr. Adams was a member of the Presbyterian Church, of the Prattsburg Lodge No. 583 F. and A. M., of New York, and of the Bellingham Elks Club. In May, 1888, he was married to Ida Belle Middleton of Prattsburg, who survives him, together with a son, John M. Adams, Assoc. M. Am. Soc. C. E.

Mr. Adams was elected a Member of the American Society of Civil Engineers on August 27, 1928.

LAYSON ENSLOW ATKINS, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 17, 1940

Layson Enslow Atkins, only child of Abraham L. and Louise (Cummings) Atkins, was born in Hurleton, Butte County, Calif., on October 12, 1892. He was of Scotch and English descent.

Soon after graduation from high school at Berkeley, Calif., he was appointed to the United States Military Academy, at West Point, N. Y., from which he was graduated in 1915. He was then commissioned a Second Lieutenant, Corps of Engineers, and was enrolled as a student officer at Washington Barracks, District of Columbia, from September, 1915, to September, 1916, and again from January to April, 1917. His military education was further supplemented by studies at the Army Industrial College at Washington, D. C., in August, 1934, from which he was graduated in June, 1935. In August, 1935, he entered the Army War College and was graduated from it in June, 1936.

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[1] Memoir prepared by B. C. Dunn, Assoc. M. Am. Soc. C. E.

His foreign military service commenced in 1916 with an assignment to Company B, 2d Engineers, during the Punitive Expedition into Mexico. From May, 1917, to March, 1918, he was Captain and Adjutant in the 14th Engineers, Railway, and was engaged in organizing and training troops in the operation and maintenance of light railways in France. From March to August, 1918, he was in charge of training of engineer candidates for commissions in the American Expeditionary Forces, and was senior instructor in the Bridge Section of the Army Engineer School at Langres, France.

On returning to the United States, his military duties were continued as Commanding Officer of the 9th Engineers at El Paso, Tex., from October, 1918, to September, 1919; and as Adjutant, 5th Engineers, at Fort Belvoir, Virginia, in 1920. In a later tour of duty with troops, from July, 1932, to February, 1934, he served as Commanding Officer of the 2d Battalion of the 6th Engineers at Fort Lewis, Wash.

From June, 1936, to July, 1939, Lieutenant-Colonel Atkins was Chief of the Supply Section, in the Military Division of the Office of the Chief of Engineers, in charge of procuring and furnishing engineering equipment to all components of the United States Army.

His ability as an instructor in military science was early recognized by his superiors and, as Major Atkins, he was associate professor of Military Science and Tactics at the University of Illinois, in Urbana, Ill., from August, 1920, to August, 1924, and served as instructor at summer camps at Fort Humphreys, Va., and at Camp Custer, Mich., during the same period.

During his career as an engineer officer, Lieutenant-Colonel Atkins fulfilled several assignments on civil works that are performed by the Corps of Engineers. It is somewhat difficult to record the professional engineering accomplishments of an individual engineer officer. In a sense he directs teamwork, and his personal contribution to engineering achievements, barring exceptional cases, is submerged in the mass effort of the team. To state that a man was assistant district engineer, or district engineer, does not mean much, but in order to convey an idea of the contribution of the officers and engineers engaged upon these works to the engineering achievements of the age in which we live, the following outline thereof is presented:[2]

Since 1820, the Engineer Department of the U. S. Army has been continuously engaged in the performance of various works on the navigable waters of the country and their tributaries. Its civil works in the field are effected through forty-seven districts grouped into eleven divisions. Each district and division is in charge of an officer of the Corps of Engineers assigned as district or division engineer. The Corps of Engineers is the federal agency responsible for planning, constructing, and, with some exceptions, maintaining and operating improvements for navigation and flood control on rivers and harbors of the United States, Alaska, Hawaii, Puerto Rico, and the Virgin Islands. Such improvements include breakwaters, canals, locks, dams, power houses, and accessory works. The department is also responsible for regulating the use of navigable waters and surveying the Great Lakes. Since its establishment, its officers have also been employed in municipal engineering and management at the Nation's capital. For several years, prior to 1932, roads, bridges, and trails were constructed in Alaska by the Alaska Road Commission, which was then directed by officers of the Corps of Engineers. In the performance of these civil works, engineer officers are afforded greater opportunity for contact with other professional engineers, and with the general public, than officers engaged in military duties.

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[2] See "The River and Harbor Functions of the Corps of Engineers," by Thomas M. Robins, Brigadier General, U. S. Army, published in *The Military Engineer*, September–October, 1940, p. 325.

Lieutenant-Colonel Atkins' assignments in civic works constructed by the Corps of Engineers included two years, September, 1924, to August, 1926, as assistant to the district engineer of the Louisville, Ky., District; four years, September, 1926, to May, 1930, in Washington, D. C., as assistant to the engineer commissioner of the District of Columbia, where he was in charge of the municipal architect's office; two years, June, 1930, to July, 1932, at Juneau, Alaska, as engineer officer of the Alaska Road Commission and assistant to the district engineer on river and harbor projects; and finally, from August, 1939, he was district engineer at Seattle, Wash., until relieved on July 23, 1940, by reason of illness.

The Seattle Engineer District embraces most of the State of Washington, northern Idaho, western Montana, and the Territory of Alaska. During Lieutenant-Colonel Atkins' incumbency as district engineer, there were nineteen river and harbor projects and ten flood-control projects in active operation in the district. Among the latter was the White River project, then in its early state of construction, the principal feature of which is the Mud Mountain Dam. This dam is 425 ft above the lowest bedrock in the river—the highest rock-fill dam ever constructed. The original design contemplated an earth-fill dam, but change to the rock-fill type was made during his term as district engineer.

The career of Lieutenant-Colonel Atkins as an officer in the various grades from Second Lieutenant to Lieutenant-Colonel exemplified notably the principles of probity, honor, devotion to duty, and other characteristic ideals of the Corps of Engineers. A summary giving dates of his various appointments in the military establishment is deemed appropriate:

Layson E. Atkins was a cadet at the U. S. Military Academy from June 14, 1911, until June 12, 1915, when he was graduated and commissioned as Second Lieutenant in the Corps of Engineers. He was promoted to First Lieutenant on July 1, 1916, and to Captain on May 15, 1917. He was given a temporary commission as Major on February 18, 1918, which expired April 26, 1919, returning him to the grade of Captain. He was again promoted to Major on July 1, 1920. In accordance with Acts of Congress approved June 30, 1922, and September 14, 1922, relating to Army reorganization, he was discharged as Major and appointed Captain on November 4, 1922. He was permanently promoted to Major on January 22, 1924, and to Lieutenant-Colonel on May 1, 1936.[3]

He possessed a keen sense of humor and a kindly regard for his fellow men. He was therefore popular in military circles and well known and respected in those parts of the United States in which he served. He was well acquainted in Alaska, having traveled extensively there. His early passing from life cut short a career of much promise.

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[3] See Official Army Register, January 1, 1940.

Lieutenant-Colonel Atkins was District Commander of the American Legion in Washington, D. C., in 1929–1930, and a member of the Forty and Eight, Torch Club, and Army-Navy Country Club in that city. He was also a member of the Society of American Military Engineers and a Thirty-Second Degree Mason.

On May 10, 1919, he was married to Helen H. Clark in Washington, D. C. He is survived by his widow, his son, Douglas C., and his mother.

Lieutenant-Colonel Atkins was elected a Member of the American Society of Civil Engineers on January 15, 1940.

WILLIAM JAMES BACKES, M. Am. Soc. C. E.[1]

DIED JANUARY 20, 1942

William James Backes was born at Hartford, Conn., on April 29, 1879, the son of Frederick W. and Marie Anna (Schwarz) Backes. He died suddenly in Boston, Mass., on January 20, 1942.

Mr. Backes was educated in the public schools of Hartford, the Cheshire Academy at Cheshire, Conn., and Sheffield Scientific School of Yale University at New Haven, Conn. He was graduated from the latter institution with the degree of Bachelor of Philosophy in 1899. In 1920 he received from the Pennsylvania Military College at Chester, Pa., the honorary degree of Civil Engineer.

After graduation, Mr. Backes went to Mexico in July, 1899, as a leveler on railroad location. Later he was employed as an assistant superintendent of erection. He returned to the United States in December, 1900. From January, 1901, to July, 1903, he was employed as an assistant on the Engineering Corps, and later as a transitman, of the Pennsylvania Railroad Company's Lines West. In July, 1903, he entered the employ of the New York, New Haven and Hartford Railroad Company as a transitman in the Maintenance of Way Department. He resigned in April, 1905, to enter the employ of the New York, Pennsylvania and Long Island Railroad Company as an assistant engineer in charge of all field work for driving tunnels and sinking the central shaft on the Bergen Hill Section of the Pennsylvania tunnels into New York, N. Y. In May, 1906, Mr. Backes was appointed chief engineer of the Central New England Railway Company.

This latter appointment enabled Mr. Backes to display that unusual energy, initiative, foresight, enthusiasm, judgment, and ability that characterized his professional life. Some of the major projects completed under his direction were: The construction at Maybrook, N. Y., of the first gravity switching yard in New England, together with the erection of a modern engine terminal and coal handling and icing station; the double tracking and relocating, with heavy grade revisions, of forty miles of line; the replacing of timber bridges with concrete for permanence and heavier loads; the establishing of new and improved standards of maintenance; and the installation of fifteen miles of what at that time was a novel system of automatic signals for single-track operation, being one of the original installations of its kind.

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[1] Memoir prepared by Walter D. Warren, *Retired* Div. Engr., New York, New Haven & Hartford R. R. Co., Newton, Mass.

Mr. Backes was promoted to engineer of maintenance of way of the New York, New Haven and Hartford Railroad Company in May, 1913, in charge of maintenance and minor construction. He gave his especial attention to improving the property, to raising the standards of maintenance, to increasing the use of labor-saving and time-saving tools and machines, to increasing the interest, and to encouraging the young engineer to advance to higher and more responsible positions in the department. The energy and enthusiasm to improve the department and its members, as displayed by Mr. Backes, reflected not only to his credit as an achievement, but created an increased efficiency in operation and an esprit de corps that did not previously exist. He was again promoted in November, 1923, to assistant general manager in charge of Maintenance, Electrical, Telegraph, and Signal departments. He resigned in June, 1925, to become associated with the Lundie Engineering Company.

In February, 1926, Mr. Backes returned to railroad service as engineer of maintenance of way for the Boston and Maine Railroad Company. In April, 1927, he was appointed chief engineer, and in May, 1933, his jurisdiction was extended by appointment as chief engineer of the Maine Central Railroad and the Portland Terminal companies. At the time Mr. Backes entered the employ of the Boston and Maine Railroad Company, the management had decided on an extensive program of improved standards of maintenance and the construction of several major improvements. In conducting this program, Mr. Backes displayed his unusual energy and efficiency in the rehabilitation of the railroad and established recognition of credit because of his accomplishments. Some of the projects were: Extensive main-track renewals with heavier rail and the installation of stone ballast; replacing with heavier structures, or strengthening, some 700 bridges; increasing the main-line clearances, involving clearances through the Hoosac Tunnel in Massachusetts; construction of a modern retarder-equipped classification yard at the Boston (Mass.) freight terminal; installation of important interlocking plants, extensive centralized traffic-control installations, remote control and color-light installations, and other extensive signal improvements, some of a pioneer nature; replacing the timber trestle area over the Charles River with heavier and more permanent construction, including the erection of four, two-track bascule bridges at the Boston passenger terminal; and the razing of the old facilities and the construction of the new Boston North Station area, including passenger station with arena above, and connecting hotel and terminal buildings. Mr. Backes' acknowledged achievement in these projects was reflected in his organization and direction of the repair work and early restoration to service of the many lines damaged by the disastrous New England floods of 1927–1936. He was appointed consulting engineering in April, 1936, and held this position until his death on January 20, 1942.

On September 25, 1901, he was married to Elsie Marie Goerz. He is survived by Mrs. Backes; two sons, Edward William Backes, Assoc. M. Am. Soc. C. E., and Frederick William Backes; two daughters, Helen M. (Mrs. Wilbur E. N. Manosh) and Marie Gertrude (Mrs. Duncan E. Littlefair); and two sisters, Mrs. Karl Peiler and Mrs. Helen Stearns.

Mr. Backes was active in the local affairs of the community in which he resided. He was a Mason, maintaining that affiliation until his death. He was a member of the Episcopal Church. He was also a member of the Yale Club of New York City and Boston; Albemarle Country Club at Newton, Mass.; New England Traffic Club; Boston Traffic Golf Club; New York Railroad Club; the New England Railroad Club, which he served as president from 1919 to 1920; and the Yale Engineering Association. In 1908 he became a member of the American Railway Engineering Association and for a number of years served on the Committee on Rail.

He was intensely interested in the railroad world, and gave loyal service to the company with which he was connected. He devoted much time to, and took an active part and interest in, all problems relating to increasing the efficiency, the economy, and the capacity of the existing railroad system. He was distinguished and recognized for his outstanding achievements in railroad maintenance and construction work, and for his ability to pioneer and progress through the force of his energy, ability, and enthusiasm. He was highly regarded by all who knew him, and all recognized his unusual personal integrity, sterling honesty, and fairness, which were so well shown by his record and career.

Mr. Backes was elected a Junior of the American Society of Civil Engineers on May 31, 1904; an Associate Member on July 9, 1906; and a Member on October 1, 1912.

CHARLES JOSEPH BENNETT, M. Am. Soc. C. E.[1]

DIED MARCH 24, 1941

Charles Joseph Bennett was born in Frome, Somerset County, England, on February 9, 1878, the son of William T. and Lydia (Perkins) Bennett.

When he was three years old, his parents came to the United States, and he received his early education in the grammar and high schools of Amsterdam, N. Y. Here he first came in contact with engineering by working as rodman for the city engineer of Amsterdam while still in high school. He continued this work through his college course and for some months thereafter. In 1901 he was graduated from Union College, at Schenectady, N. Y., with the degree of Bachelor of Engineering.

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[1] Memoir prepared by Robert J. Ross, M. Am. Soc. C. E.

During the early part of 1902 he entered the service of the Metropolitan Street Railway Company in New York, N. Y., serving as an assistant in layout and inspection of construction. He remained on this work until October, 1903, when he went with the New York Central Railroad Company as assistant engineer in elimination of grade crossings at Schenectady.

In March, 1905, he went to Connecticut and became successively transitman and assistant engineer on construction of the freight terminal in Hartford for the New York, New Haven and Hartford Railroad Company. After completion of this work he was appointed division engineer of the same railroad's midland division, which position he retained until September, 1909, when he resigned to become assistant engineer of the Saybrook and Lyme Bridge Commission of Connecticut.

On March 28, 1910, he was appointed deputy superintendent of streets of the City of Hartford, under the late Frederick L. Ford, M. Am. Soc. C. E. He was appointed superintendent on February 10, 1911, and remained until his appointment as Connecticut state highway commissioner on February 26, 1913. During the next ten years Mr. Bennett continued as highway commissioner and under his direction many hundreds of miles of Connecticut highways were constructed. Leaving the Highway Department in 1923, he became general manager of the Edward Balf Company and, later, director of production for the Fuller Brush Company, both of Hartford.

In 1928 he entered private practice as a consultant, and the work of the firm of Bennett and Terry included not only a diversity in type of projects but projects of considerable importance and magnitude. In recent years he had served as consultant for the Hartford Hospital, Trinity College (in Hartford), the Groton-New London Bridge Commission, the Connecticut Highway Department, and other public and private agencies.

At the time of his death Mr. Bennett was acting as executive secretary to the Hartford Flood Commission, a post which he filled most ably, and it is unfortunate that his span of life could not have been extended so that he might see the consummation of this project, to which he had devoted so much time and energy.

Among his other engineering accomplishments and affiliations were membership in the Connecticut Society of Civil Engineers (of which he was a past-president and honorary member), the American Road Builders Association (of which he was a past director), and the American Association of Highway Officials, which he once served as president. He conducted special surveys of the Michigan state highways in 1924 for the Detroit Automobile Club and an investigation of the Street Department of New Rochelle, N. Y., in 1921 for the Chamber of Commerce of that city. He also made a special report on the Merritt Parkway in Connecticut to Governor Wilbur Cross. He acted as manager of the Hartford Community Chest work relief program and at various times lectured on engineering at the University of Michigan, in Ann Arbor; at Yale University, in New Haven, Conn.; and at Columbia University, in New York City.

Two educational institutions conferred honorary degrees on Mr. Bennett. Yale University honored him with the degree of Master of Arts in 1922 and in 1926 his alma mater, Union College, conferred the degree of Master of Science.

Mr. Bennett had a strong sense of duty to the community and served on numerous committees and civic bodies, including the Building Commission of the City of Hartford, the Park Board of the City of Hartford of which he was president for a term, the Regional Planning Commission of the (Hartford) Metropolitan District, and the Board of Trustees of the Connecticut State College (later University of Connecticut) in Storrs. He was appointed by the Governor as delegate to the 1931 National Conference on Home Building and Home Ownership. He was particularly active in work for the colored people in the Hartford tenement district and served as president of the Board of Directors of the North End Federated Clubs, an organization sponsoring work of all kinds among the tenement dwellers. A "fresh-air" camp maintained near Hartford was named "Camp Bennett" in his honor.

Mr. Bennett was a member of St. John's Episcopal Church, in Hartford, and Beta Theta Pi Fraternity. He was a past-president of both the Hartford Rotary Club and the University Club of Hartford.

On August 22, 1905, he was married to Marguerite Balch of Galway, N. Y. He is survived by his wife; two sons, Charles W. and Gordon; and three daughters, Elizabeth J. (Mrs. George A. Wardinski), Alison M. (Mrs. Kingsbury Dyke), and Patricia H. Bennett.

It is seldom that a man is found who can offer such able and diversified assistance to his fellow men and who at the same time is willing to devote such boundless energy to civic and professional activities. Such a man, however, was Charlie Bennett, for whom no eulogy is necessary. His works and deeds speak for themselves. It is particularly fitting that the affection and respect of his fellow men could be exemplified by the beautiful and appropriate funeral services, held in the chapel of Trinity College, in the design and erection of which he rendered able professional assistance. Every station in the community's life was represented. All were there because of a profound sense of loss. The inspiration to be derived by his fellow men from the life of such a citizen, however, can never be lost.

When Yale in 1922 conferred the honorary degree on Mr. Bennett the citation concluded with the following words:

> "Governor Baldwin appointed him State Highway Commissioner which office, fortunately for the State, he still holds. The annual expenditure of public money under his direction is between eight and ten millions. He has built admirable roads under difficult conditions, and is giving the State full value for money expended. He is skillful, shrewd, and honest, and can neither be tempted nor deceived. He is a scientific expert with a Puritan Conscience."

Mr. Bennett was elected a Member of the American Society of Civil Engineers on February 4, 1914.

WILLIAM ALLEN BLAKEMORE, M. Am. Soc. C. E.[1]

DIED NOVEMBER 8, 1941

William Allen Blakemore, the son of Thomas Mark and Jane (Allen) Blakemore, was born on March 2, 1889, at Prairie Grove, Ark. Both the Blakemore and Allen families were planters, descended from English ancestors, who came to the South in Colonial days, and both families contributed soldiers to all of the American wars. His grandfather, John W. Allen, whose slaves built the typical southern colonial home in which Mr. Blakemore was born, came from Virginia to Texas as a young man to fight with Gen. Sam Houston in the Texas War of Independence.

Although the families were all from plantations they believed in education There were college graduates in every generation, many of whom were doctors. Young Allen Blakemore received his preparatory education in the public schools of Prairie Grove. After graduating from the high school, he entered the University of Arkansas, at Fayetteville, where he received the degree of Bachelor of Science in Civil Engineering in 1911.

During the first summer after receiving his degree, Mr. Blakemore, as instrumentman, helped to survey and build the Missouri, Oklahoma and Gulf Railroad from Muskogee, Okla., to Denison, Tex., and then worked in the Maintenance of Way Department after the construction was completed. The following summer, he went to Gary, Ind., as draftsman for the American Bridge Company, where he stayed until the spring of 1913. He then went to Ontario as instrumentman on the construction of the Canadian Pacific Railway. While on this work, a friend wrote him that the United Fruit Company had formed the Trujillo Railroad Company in Honduras, Central America, and engineers were needed to survey a line across the mountains, so he signed up for a year and took charge of locating a part of this difficult line through a wild country.

After a year in the tropics, Mr. Blakemore, in 1915, entered the employ of the Gulf Oil Corporation (then known as Gulf Production Company), in Beaumont, Tex., with which company he spent the remainder of his career with the exception of two years in the armed services of the United States Army during the First World War. His duties consisted in general oil field surveying, retracing old land lines and marking the boundary lines of company leases, staking locations for oil wells to be drilled, locating and constructing pipe lines leading from the oil fields to market, and constructing pump stations, tanks, and other facilities necessary in the development of new oil fields.

In May, 1917, one month after war was declared on Germany, the government called for its first volunteers, and Mr. Blakemore answered the call. He joined the 12th Engineers at Kansas City, Mo., enlisting as a private, and sailed for France a few weeks later. He was promoted to Second Lieutenant while in France and served until the Armistice in railroad construction work immediately back of lines through France, Belgium, and Alsace-Lorraine. Lieutenant Blakemore received his discharge in May, 1919, and returned to the Gulf Oil Corporation in Houston.

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[1] Memoir prepared by John H. Bringhurst, M. Am. Soc. C. E.

In December, 1919, "Blake," as he was called by all of his friends, was made assistant chief engineer under Frank L. Buford and assumed responsible charge of all surveys for the company. Being a tireless worker himself, he insisted that his men work long hours. Hardships in the field, waist or shoulder deep in oil and mud, meant nothing to him, and he expected his men to follow where he led. The oil field work on the Gulf Coast in Texas, during the "Booms," demanded ingenuity, stamina, and perservance in the surveyor, all of which Mr. Blakemore possessed in a marked degree, and the difficulties of terrain were met and overcome just as Lieutenant Blakemore met and solved his railroad problems under shell fire in France.

In 1931, at the death of Mr. Buford, Mr. Blakemore was made chief engineer, and his direct supervision of work in the field had to give way to his work of more executive nature. Boundary controversies involving enormous sums were frequent. So-called "vacancies" were being claimed and filed in almost every proven oil field. He devoted much of his time to such problems and his skill in directing and interpreting the work of his land survey parties prevented much litigation and won most of the cases that reached the courts.

A friend who was closely associated with him says—

> "I worked with him on a tedious and difficult job in one field where we wallowed daily in the oil covered marsh and slush pits for several weeks in preparing evidence on which a suit involving several million dollars depended. It was settled favorably out of court. I have 'tailed' a surveyor's chain, with 'Blake' on the head end for miles down a county road, with my tongue hanging out in the August heat while the two regular chainmen rested and drove the car. I have worked with him over the scorched prairie grass in another oil field, our lunches tied to our belts but our empty canteens preventing us from eating all day. In over thirty-five years of engineering experience I have worked with no one who had his capacity for hardships and dogged determination to get a job done."

On January 20, 1920, Mr. Blakemore was married at Siloam Sp[illegible], Ark., to Mamie John Brockman, to whom he was engaged before going to France. His first child, Nancy Allen, was born in 1922 and the other, John William, was born in 1930.

His marriage, his devotion to his wife and children, complemented a routine of hard work to which he had given his entire time unstintingly. His innate reserve, his tranquil and unruffled demeanor, which at times concealed from others occasional strong emotions locked within him under perfect self-control, were shed in his home life and, though always gentle and quiet, joy, happiness, and pride in his family radiated from his countenance.

He enjoyed attending all kinds of athletic contests and especially those of his own college. While watching a football game between the University of

Arkansas team and that of Rice Institute, in Houston, with his daughter, a student in the Rice Institute cheering section and his wife and son helping him cheer for Arkansas, Mr. Blakemore suffered a heart attack and was dead upon arrival at the hospital.

Mr. Blakemore was elected a Member of the American Society of Civil Engineers on October 29, 1934.

EDWARD RICHARD CARY, M. Am. Soc. C. E.[1]

Died July 17, 1941

Edward Richard Cary was born in Troy, N. Y., on December 19, 1865, the son of Tallmann and Elizabeth (Lounsbury) Cary. He received his elementary education in the public schools of Troy, and at the Troy High School. Entering Rensselaer Polytechnic Institute, in Troy, in September, 1884, he was graduated with the degree of Civil Engineer in 1888.

In the fall of that year he became a member of the Faculty of Rensselaer Polytechnic Institute as assistant and instructor in the Department of Surveying. Likewise, from 1888 to 1904, he served as registrar of the Institute. In September, 1902, he was appointed assistant professor in the Department of Surveying, and in 1904 he became professor of geodesy, railroad engineering, and highways, in which capacity he served the Institute until his retirement in 1936.

Professor Cary began private practice immediately upon graduation, engaging mainly in surveying and municipal engineering. From 1896 to 1906 he was a member of Cary and Roemer, Civil Engineers. He served as city engineer for Troy from 1900 to 1902, and again from 1906 to 1908. At different periods he was engaged as engineer by near-by municipalities.

He was married on June 21, 1892, to Mary Lyman, who, with a daughter, Helen (Mrs. F. W. Cappelmann), survives him. He is also survived by a sister, Mrs. G. K. Gormly.

Professor Cary was the author of "Slide Rule Solutions of Railroad Problems"; "Geodetic Surveying"; and "Elements of Diagrams." He was a member of the American Railway Association, Sigma Xi, and Tau Beta Pi.

Generations of Rensselaer men will long remember him as a teacher and adviser. Members of the Rensselaer faculty will remember him as a faithful and ever helpful colleague. His professional associates will remember him as a distinguished engineer. All who ever knew him will long remember him as a friend.

Professor Cary was elected a Member of the American Society of Civil Engineers on April 2, 1902.

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[1] Memoir prepared by H. O. Sharp, M. Am. Soc. C. E., and H. H. Nugent, Prof., Dept. of English, Rensselaer Polytechnic Inst., Troy, N. Y.

FRANK LYNTON CHASE, M. Am. Soc. C. E.[1]

Died April 1, 1941

Frank Lynton Chase, retired vice-president of the Lone Star Gas Company and a pioneer of the natural gas industry, died April 1, 1941, and was buried in the Green Lawn Cemetery at Columbus, Ohio. He died while en route to his home in Dallas, Tex., after a vacation in Florida, during which he suffered a fall.

Mr. Chase was born in Bucyrus, Ohio, on May 23, 1865, the son of Francis and Jenny (Weatherbe) Chase. His father was from a Vermont family which produced many men of note. He was an engineer by profession and at the time of Mr. Chase's birth was chief engineer of the Pittsburgh, Fort Wayne and Chicago Railway Company. He died in Mr. Chase's early childhood. Frank Chase attended Washington and Lee University at Lexington, Va., for two years, and after working several years he entered Massachusetts Institute of Technology, in Boston, Mass., graduating in three years in the class of 1890.

Following his graduation he was for a year a transitman on location for the Union Pacific Railroad Company in the State of Washington. On July 1, 1891, he entered the engineering department of the Cleveland, Cincinnati, Chicago and St. Louis Railway at Indianapolis, Ind., under George W. Kittredge, Hon. M. Am. Soc. C. E. Mr. Kittredge was chief engineer of this road which was newly formed by the consolidation of a number of other lines. After about two years he entered the service of the Baltimore and Ohio Railroad Company and was engaged in grade revision and bridge designing. In 1900 he went to the New York Central and Hudson River Railroad as bridge engineer under William J. Wilgus, Hon. M. Am. Soc. C. E., chief engineer. He had charge of the design and construction of nearly all the bridges on the system—work which was made necessary by the heavier equipment then coming into use. In 1903 he was elected president of the Jamestown, Chautauqua and Lake Erie Railway and the Jamestown and Chautauqua Navigation Company. While in this position he conceived the idea of the practicability of a gasoline-driven railway passenger coach. His experiments were conducted during the development period of the gasoline engine and he secured patents covering his ideas.

His experience in the natural gas business, which became his lifework, began a few years later when he was placed in executive charge of eight companies that served Dayton and Springfield, Ohio, and the towns in the Miami Valley of Ohio. In this capacity he supervised both pipe-line and distribution company activities.

Mr. Chase became associated with the Lone Star Gas Company in 1917 at a time when this company was struggling to obtain sources of supply sufficient to permit its continued operation. His outstanding success in the solution of this problem was largely responsible for the present status of the company. In 1925 he organized and became the president of the Community Natural Gas Company, a Lone Star Gas Company subsidiary. This company was created for the purpose of extending domestic and industrial gas service to a large number of small cities and towns in Texas and Oklahoma located adjacent to the main trunks of the pipe-line system.

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[1] Memoir prepared by Edward C. Connor, M. Am. Soc. C. E., and Elmer F. Schmidt, Vice-Pres., Lone Star Gas Corp., Dallas, Tex.

Aside from his immediate duties Mr. Chase rendered valuable service to the gas industry as a whole, having served as a director of the former Natural Gas Association of America, as chairman of the Natural Gas Section of the American Gas Association, and as a member of the management committee of the same organization. For many years he was active in the work of every association in the Southwest dealing with natural gas and oil, and he maintained a keen interest in all matters pertaining to engineering advances and conservation measures.

In 1935 Mr. Chase was appointed to the newly formed Federal Commission, designated as the advisory board of the United States Bureau of Mines, which functions as an advisory body to the chief of the bureau and counsels with him on matters of bureau policy.

After twenty-two years of service as an official of the Lone Star Gas Company, Mr. Chase retired in 1939, but he retained offices in the company building and remained active in company affairs in an advisory capacity until his death. As late as May, 1940, he was honored as the "grand old man" of the natural gasoline division of the American Petroleum Exposition at Tulsa, Okla., an award that is an annual feature designed to honor men who have contributed to the progress of the petroleum industry.

Despite the fact that heavy executive responsibilities demanded most of his attention during the period of his association with the natural gas industry, Frank Chase was always an engineer in all that the word implies. Throughout his administrative career he kept the tools of his profession keen and bright—so keen and bright, in fact, that he frequently astonished his younger associates by his unerring approach to the correct solution of complicated technical problems. Above all, however, he was a striking example of what constitutes an educated Christian gentleman. His inquiring mind found interest in the realms of pure science. Lobachevski and Riemann, Michelson and Einstein were his familiars, and he was intrigued by the elusive problems growing out of a consideration of time and space. However, his normal predilection for things scientific did not dull his intelligent appreciation of the fine arts. He was a keen student of philosophy and economics and was an omnivorous reader of the literature of these subjects. However, in the midst of his crowded life he found time to serve as a member of the vestry of his cathedral parish.

He was a man of fine stature, and in his undergraduate days at Washington and Lee was a varsity tackle. His imposing person, coupled with his dominant intellect, made him an outstanding personality in any group. Once he had rationalized his conclusions on any debatable subject he would support these conclusions with vigor; nevertheless, he was tolerant of the opinions of others even if they conflicted with his own. He enjoyed nothing more than a clash of ideas with those whose knowledge of a subject forced him to rally his mental reserves to the attack.

To those who were privileged to know him well, several of his traits of character and accomplishments stand out above the high level of his professional attainments. His keen sense of fairness dominated his dealings with others. He never negotiated a contract without keeping foremost in his mind the question of whether or not its provisions were equitable with respect to the best interests of the other contracting party. This trait found concrete expression in the gas purchase contracts that he negotiated with the natural gas producers in Texas and Oklahoma. To him the current market price of gas was secondary to the question of whether or not gas could be produced profitably at the price agreed upon.

His interest in the advancement of his younger associates extended far beyond the normal relationship between an executive and his subordinates. He delegated responsibilities freely and exacted results with firmness, but he always maintained a sympathetic and fatherly interest in the progress of every individual responsible to him. As a result of this policy, practically all of the operating executives of the Lone Star Gas Company and Community Natural Gas Company, as well as the department heads of the operating sections of these companies, owe their advancement to his personal interest in their respective careers.

Waste and improper utilization, which for so long were the outstanding characteristics of natural gas operations in the Southwest, were abhorrent to him. Almost single handed he led the crusade of conservation against the entrenched forces of self-interest that ultimately resulted in the enactment and enforcement of rational conservation laws that have materially extended the life of natural gas service to the domestic consumers of the Southwest and have made it feasible to extend natural gas service to the large population centers of the Middle West.

He had the profound conviction that a public utility operating in a given territory was obligated to render a complete service to all of that territory without giving primary consideration to the limitations imposed by the prospects for profit that might be involved in any given extension of service. As a result of this conviction, the Community Natural Gas Company was organized and through this agency natural gas service has been provided to more than three hundred small cities and towns variously located in an area of approximately 12,000 sq miles—a program unparalleled in the history of the natural gas industry. Above all, he had an abiding faith in the destiny of Texas, the state of his adoption. Few of its citizens have contributed more to the fulfilment of this destiny than has Frank Chase—engineer.

On November 12, 1895, he was married to Sarah Winchell Forbes, who, with two daughters, Sarah Elizabeth (Mrs. Ralph W. Lee, Jr.) and Priscilla Chase Bergin, survives him.

Mr. Chase was elected an Associate Member of the American Society of Civil Engineers on November 4, 1896, and a Member on November 1, 1899.

EDUARDO JUSTO CHIBÁS, M. Am. Soc. C. E.[1]

DIED AUGUST 22, 1941

Eduardo Justo Chibás was born in Santiago de Cuba, on February 28, 1869, the son of Eduardo Chibás and Mercedes (Guerra) Chibás. He received his early education in his native city, obtaining the degree of Bachelor of Arts from the Santiago de Cuba Provincial Institute.

Cuba was at that time under the Spanish régime; there was no engineering school in the country, and the Santiago de Cuba Province (later Oriente Province) was practically isolated from Habana or the remainder of the island, except by boat. Like many other sons of well-to-do families, young Eduardo Chibás was sent abroad for a professional career. He went to the United States and attended Rensselaer Polytechnic Institute, at Troy, N. Y., where he was graduated with the degree of Civil Engineer in June, 1889. He distinguished himself as a conscientious student and became a member of Sigma Xi, honorary scientific fraternity, whose emblem he carried in his watch chain throughout his life.

After graduation he went back to Cuba and worked for a sugar estate of the Oriente Province. The internal unrest of the country, preliminary to its last and successful struggle for independence, already was felt, and there were few or no prospects for engineering activities. He went back to the United States and served as assistant engineer for the Duquesne Traction Company and for the Pittsburg, Allegheny and Manchester Electric Railways, at Pittsburgh, Pa.

Mr. Chibás was very much interested in mining, probably on account of the abundance of ore deposits in his native province of Cuba, and he examined and reported on deposits of manganese, iron, and gold, and also on other developments, in Cuba, Panama, Colombia, and Venezuela for several American and English concerns. He was in actual charge of developing some of these mines, built railroads through mountainous country, and made extensive surveys.

Then in 1898 came the Spanish-American War, and he had the opportunity to contribute in an active way to the liberation of his country from the secular Spanish dominion. He joined the United States Fifth Army Corps as assistant civil engineer under the late Brig. Gen. William Ludlow,[2] M. Am. Soc. C. E., chief engineer of the Army, and later under Col. George McC. Derby, M. Am. Soc. C. E. He had charge of reconnaissance around the San Juan and Caney forts and the City of Santiago, locating trenches and positions of the different regiments before and after the capture of those forts, until the surrender of the City of Santiago. The official maps of the Santiago campaign bear his signature with those of the officers of the U. S. Engineer Corps. It is said that in connection with his reconnoitering duties he made use of a military balloon, an unusual feature at that time in the western hemisphere, and also, that while engaged in his engineering work he took the first prisoners of the war.

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[1] Memoir prepared by Francisco J. Gastón, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1700.

Back in civil life and enjoying the invigorating air of his country's political freedom, he devoted himself to the development of the natural resources of his native Province. In January, 1899, he was appointed chief engineer and general manager of the Guantánamo Railroad Company. While with this company, he organized the Guantánamo Ice Company, of which he became president, and he held that position until 1924.

In 1902, at the time of the inauguration of the Republic of Cuba, he moved to Santiago where he acted as a promoter and practiced as a consulting engineer and contractor. Enjoying great personal prestige, because of his honesty, engineering capacity, and unusual business insight, he organized different companies and built a large number of engineering works.

He prepared plans for an electric light and power plant for Santiago and organized the Santiago Electric Company, which built the plant. Later, he made plans for the electric tramways of the city, obtained the necessary concessions, and reorganized the company under the name of the "Santiago Electric Light and Traction Company," which built and inaugurated the system in 1908. He was with this company as a member of its Board of Directors and as consulting engineer until 1921. These and other enterprises, such as the Vista Alegre Development, changed the physical aspect of his native city, Santiago de Cuba, from a sleepy colonial town to a thriving, prosperous community.

In 1906 Mr. Chibás organized the Guantánamo Electric Company, becoming its president, and built the electric plant. Later, in 1916, he reorganized the company and built a hydroelectric plant at the Guaso River, 14 miles from the city, with a head greater than 600 ft and about 1,000 hp. He served as president of this company until 1924.

In January, 1911, at the time of a water famine in Santiago de Cuba that threatened to stop all industrial activities in the city, he suggested that tubular wells be driven to an underground reservoir in San Juan Valley. Once this was authorized, he proceeded with the work and in two weeks obtained water in sufficient quantity to relieve the situation. He then built an emergency system of water supply for the city. When there is a shortage of water from other sources, the San Juan wells can still be counted on for their share of good underground water.

From 1912 to 1920 he was vice-president of the Santiago Brewing Company, of New York, N. Y., and Santiago de Cuba, originators of the "Hatuey" beer, later owned by the Bacardí Company. He was also vice-president of the Compañía Azucarera Oriente (from 1914 to 1921), which developed a new sugar state, later known as "Altagracia," in Oriente Province.

In 1922 Mr. Chibás changed his residence to Habana and, after he had disposed of most of his property and business interests in Oriente Province, he found himself in a very sound economical position. He invested in real estate and built the "Chibás Building," one of the first large apartment houses in the City of Habana.

Although not giving up engineering, and studying especially the water supply problems of Santiago de Cuba and Habana and devoting much of his time to the Cuban Society of Engineers, he contributed largely and generously to civic activities. He was a member and at one time the president of the Rotary Club of Habana, a member and president of the League Against Cancer, a member and president of the Real Estate Center of Habana, and a member and president of the National Association of Charities. His coöperation was solicited by numerous clubs and societies, and he belonged to the Habana Yacht Club, the Country Club, the Automobile Club, Pro-Arte Musical, etc. He was a member of the Institute of Mining and Metallurgical Engineers, and one of the founders and first president of the local section of the American Water Works Association, which is said to be the only branch of the Association outside of the United States. He joined the Sociedad Cubana de Ingenieros when this was founded in 1908, and acted as its delegate in Santiago for many years. He was elected president of the Society for the term 1930–1932 and was reelected for 1932–1934.

His idea of fulfilling civic duties led him into politics; although refusing to join any of the conventional political parties, he was staunch in the big national issues. During the second term of President Gerardo Machado, Mr. Chibás' elder son, Eduardo, was actively engaged in oppositional activities, which at that time could not be other than of a revolutionary character. Government agents denounced and involved Mr. Chibás, the father, and he was imprisoned and court-martialed under most absurd charges, from which, of course, he was in due time wholly exonerated. Mr. Chibás was at that time president of the Cuban Society of Engineers, and many of its members felt it a duty to attend the sessions, some of them as witnesses for the defendant. To show how engineers stood by Mr. Chibás, it may be mentioned that he was reelected president of the Society while he was in prison. During all the long and cruel process, Mr. Chibás maintained a perfect dignity. It was easily observed that most of the members of the military board, although demanding the highest penalty for the accused, were impressed by his fine character. One of the sergeants attending the sessions and acting as a stenographer was no other than Fulgencio Batista, later President of the Republic.

After the revolution that put an end to the Machado Government, Mr. Chibás was appointed Secretary of Public Works by President Carlos Manuel de Céspedes during his short term. Later, he was repeatedly asked to occupy the same or another important government position, but he sternly refused. He served, however, as president or member of many official or private commissions of a technical, economic, or social character.

Mr. Chibás was the president of the local section of the Rensselaer Polytechnic Institute Alumni Association. In 1939 he was elected president of the national association, which was considered a great honor, because never before had a foreigner and nonresident been elected president of any of the alumni associations of the great universities of the United States. This coincided with the fiftieth anniversary of his professional life, and it was the occasion for a homage to Mr. Chibás from his fellow engineers of the Cuban Society of Engineers. A bronze plaque was then presented to him as a token of appreciation.

In February of the same year, the Rensselaer Polytechnic Institute Alumni Association, under the presidency of Mr. Chibás, celebrated its annual reunion in Habana. William Otis Hotchkiss, M. Am. Soc. C. E., president of the Institute, and a host of professors and distinguished engineers assisted; the entire gathering was pronounced a great success. At the reception for the visitors at the Cuban Society of Engineers, the president of that society referred to the special honor bestowed on Mr. Chibás by the Rensselaer men, and this token of affection rendered to a Cuban, he said, "went deeply into our hearts"; and he added, addressing the visitors:

> "Chibás is now your President, but he is none the less our Chibás, a member of this Society who was elected and reelected its President, one whom as an engineer and a public citizen, we all love and respect. In this way a sort of spiritual relationship has been established between the Rensselaer Polytechnic Institute and the Cuban Society of Engineers."

A good example, it might be added, of the "Good Neighbor" policy.

Rensselaer Polytechnic Institute conferred the degree of "Doctor of Engineering," *Honoris Causa,* on Mr. Chibás in 1940. He is mentioned in "Who's Who in Engineering" for 1941.

He published a number of pamphlets and reports on many of his engineering activities and on economical subjects. He was a member of the commissions that reported on the water supply problems of Santiago de Cuba and on the 1932 earthquake in the same city. These latter reports were published in *Revista de la Sociedad Cubana de Ingenieros.*

Mr. Chibás, although of an energetic nature, was above all a kind man. He was always willing to help, and received with a smile and listened to all. It can be stated truly that he made personal friends of all who had occasion to meet him. His outstanding qualities were personal honesty and kindness, engineering and business ability, and civic spirit.

On November 11, 1906, he was married to Gloria Ribas, at Camagüey, Cuba. They had two sons, Eduardo and Raul. Mrs. Chibás died, and on February 1, 1933, he was married to Conchita Ribas Agramonte, of Washington, D. C., who together with the two sons by his first marriage, survives him. His brother, Luis F. Chibás, a mining engineer of good standing, also survives him.

Mr. Chibás was elected a Junior of the American Society of Civil Engineers on March 31, 1891; an Associate Member on May 6, 1896; and a Member on January 3, 1900. He had been enjoying life membership since 1931.

WILLIAM TURLEY COBURN, M. Am. Soc. C. E.[1]

Died January 24, 1941

William Turley Coburn was born in Washington, D. C., on May 30, 1885, the son of Jennie and Turley Coburn of French Huguenot and Irish ancestry.

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[1] Memoir prepared by Lester C. Hammond, M. Am. Soc. C. E.

His engineering training was obtained at the University of Maryland, at College Park, and at George Washington University, at Washington, D. C.

His early work to 1913 was with the office of the engineer of bridges in Washington, D. C. From 1913 to September, 1918, when he joined the United States Army, he was connected with several important engineering projects. He worked during these years with Parsons, Klapp, Brinckerhoff and Douglas, of New York, N. Y., as cost engineer on the Salmon River (N. Y.) power development and inspector of dredging for the Cape Cod Canal in Massachusetts; with Jarrett and Chambers Company, Inc., of New York City as office manager of the Bear Lake power development in Idaho; and with the Foundation Company of New York City as engineer assistant to the superintendent and office manager on the power plant at Logan, W. Va., and on the construction of several industrial plants in New York, Pennsylvania, Virginia, and Ohio.

His engineering career was interrupted by his enlistment in the Army in the fall of 1918. He was appointed to the Quartermaster Corps and ordered to France, in October, 1918, as casual officer, to rebuild roads at the front. On his way there the Armistice was signed and he was returned and assigned as executive officer at Pontanezen Barracks, Brest, France, under Gen. Smedley Butler, U. S. Marine Corps. He remained on the construction work of enlarging the camp, which was used for the debarkation of troops to the United States after the Armistice, until he became ill with Spanish influenza the following spring. He was returned in the hospital ship *Leviathan* on April 2, 1919, and was sent to the Walter Reed Hospital. He was discharged from the service on April 27, 1919. He continued his Army affiliations in the Reserve Officers Training Corps and was appointed First Lieutenant. During his services in Detroit, Mich., with the 310th Engineers, he became a Captain. Further promotion would have been given him for his conscientious and valuable service had not his engineering work in many parts of the country prevented more active duty in the Reserve Corps.

His positions after his discharge from the Army included auditor and accountant on engineering reports and income tax with the Internal Revenue Bureau, at Washington, D. C., and six years as assistant manager with Parklap Inc., of Detroit, on construction of large reinforced warehouses and refrigeration plants in Detroit; Albany, N. Y.; Buffalo, N. Y.; Toronto, Ontario, Canada; and Montreal, Quebec, Canada.

His later work as utility officer of the Veterans Administration continued until his death. On April 1, 1931, he was appointed to the Veterans Administration Facility at Lincoln, Nebr.; from there he was sent to Helena, Mont.; and on July 13, 1939, he was promoted to grade Professional 5 and transferred to Danville, Ill. The Veterans Administration operates ninety-one field stations, which include hospital facilities. The utility officer is in charge, under the manager, of maintenance of physical plant and equipment and the operation of mechanical and non-medical facilities, including power plant, laundry, farm, motor vehicles, police and fire protection, and building equipment. It naturally follows that utility officers must possess professional and administrative ability to a large degree. L. H. Tripp, director of construction of the Veterans Administration, pays this tribute to Captain Coburn:

> "Because of Captain Coburn's very high qualifications, he was sent to Danville which is the second largest mental hospital under the control of the Veterans Administration, having a capacity of approximately 1900 beds. Captain Coburn's service there was highly successful and several practices which he initiated have become standard throughout the Field service. Due to his qualities of cooperativeness, courtesy and sympathy, he was highly successful in the utilization of patient labor, and constructed several fine service buildings such as shops and laundry very largely through the use of patient labor. Captain Coburn enjoyed the unqualified esteem, respect and affection of every one who came in contact with him."

Dr. George A. Rowland, the manager of the Danville Facility, stated in his last official communication referring to Captain Coburn:

> "Captain Coburn's death came as a shock to me. I thought a great deal of him, and soon learned to depend on his judgment. He was a gentleman in every respect and a fine and efficient Utility Officer. We will all miss him greatly."

Captain Coburn enjoyed people and had an unusual talent for friendly association with them in business and social life. He was a member of Sigma Alpha Epsilon fraternity, a Thirty-Second Degree Mason, and a Shriner. One of his keen interests during his Army career was his membership in the Sojourners.

He was married in Baltimore, Md., on August 9, 1918, to Blanche E. Wilmot, who survives him.

The funeral services were held in the Veterans Administration Chapel at Danville and at Fort Meyer Chapel, Virginia. Interment was at Arlington National Cemetery in Virginia.

Captain Coburn was elected an Associate Member of the American Society of Civil Engineers on May 28, 1923, and a Member on January 13, 1941.

EMMETT FILMORE COLLINS, M. Am. Soc. C. E.[1]

DIED MAY 8, 1942

Emmett Filmore Collins, the son of Richard M. and Frances M. (Lybrook) Collins, was born at North Judson, Starke County, Ind., on January 5, 1873. He received his education at Purdue University, in Lafayette, Ind., and was graduated with the degree of Bachelor of Science in Civil Engineering in 1894.

His professional career began immediately after graduation, and from 1894 to 1898 he held various positions as rodman, instrumentman, and draftsman. Beginning with his work as chief engineer of the Escanaba and Lake Superior Railroad Company in June, 1898, he thereafter held responsible positions in the location and construction of important railways, which led him into various sections of the United States and Canada. These engagements covered work on many important railway systems, such as the Chicago and North Western; Chicago, Milwaukee and Puget Sound; Wabash; Kansas, Oklahoma and Gulf; and the St. Louis-San Francisco.

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[1] Memoir prepared by Frank G. Jonah, Hon. M. Am. Soc. C. E.

Mr. Collins was resident engineer in Oklahoma from 1901 to 1902 on construction of lines which later formed part of the "Frisco System." He held the same position in Texas from 1904 to 1905 on what was then known as The St. Louis, Brownsville and Mexico Railway. After that he had some engagements in West Virginia, and in 1910 was engaged in making surveys and estimates for various railway projects in the Southwest. His work on these various railways was at a time when they were engaged in extensive developments in new territory in the Southwest and Northwest, and this gave him a very diversified and practical experience.

He returned to the "Frisco" in August, 1912, and was with that company continuously until 1920. During this time he acted as principal assistant engineer, valuation engineer, and chairman of the Valuation Committee on Federal Valuations. He retired from the "Frisco" in 1920 to devote himself to his private business, but during his retirement he had many engagements as consulting engineer on appraisals and valuations.

Mr. Collins' work was always characterized by accuracy and good judgment. He was a gentleman of very likable personality and made many lasting friendships.

He was married in St. Louis, Mo., on June 4, 1919, to Genevieve A. Purcell, who, with one daughter, Jean Elizabeth, survives him.

Mr. Collins was elected a Member of the American Society of Civil Engineers on December 3, 1913.

CHARLES STRATTON DAVIS, M. Am. Soc. C. E.[1]

DIED JANUARY 2, 1942

Born near Oxford, N. Y., on November 16, 1866, Charles Stratton Davis was the son of John Perkins Davis and Almeda (Buckley) Davis. He attended the local district school and Oxford Academy, from which he was graduated in 1885. He then entered the College of Civil Engineering of Cornell University at Ithaca, N. Y., from which he was graduated in 1889 with the degree of Civil Engineer. He was vice-president of his class during his sophomore year and was a charter member of Phi Gamma Delta fraternity.

In 1889 he was employed by the Massillon Bridge Company at Massillon, Ohio, and in 1893 became chief engineer. He was then transferred to the headquarters of the company at Toledo, Ohio, where he had charge of designing, estimating, and erecting, both in connection with the construction of new bridges and the rebuilding of old bridges. Under his direction the Massillon Bridge Company repaired and reinforced the Eads' Bridge over the

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[1] Memoir prepared by Daniel E. Davis, M. Am. Soc. C. E.

Mississippi River at St. Louis, Mo. Today this bridge is carrying the heaviest locomotives in use.

In 1907 Mr. Davis entered private practice in Toledo. During this time he prepared stress sheets and working drawings for standard plate girder bridges for several railroads in Brazil and Mexico. He also examined, repaired, and built bridges for the Chicago and Eastern Illinois Railroad Company, the Cincinnati, Hamilton, and Dayton Railroad Company, the Pennsylvania Railroad Company, and for the City of Toledo.

In 1914 he was appointed senior structural engineer in charge of the Bridge Branch, Southern District, Interstate Commerce Commission, with headquarters at Chattanooga, Tenn. He continued in this work until 1919, except for an interval of two years when he served as structural engineer with the Pennsylvania-Detroit Railroad Company.

In 1919 Mr. Davis entered private practice in Pittsburgh, Pa. His first work was the engineering and construction of the Washington Crossing Bridge, a concrete arch bridge over the Allegheny River at Pittsburgh. Among his clients were the City of Pittsburgh, the Pittsburgh and West Virginia Railway Company, the Pittsburgh Railways Company, and the Carnegie Institute in Pittsburgh.

Mr. Davis leaves his wife, Grace Gilbert Davis; a daughter, Charlotte Marie, of Hampton Hall, Pittsburgh; and a sister, Mrs. Sackett H. Mead, of Oxford, N. Y.

Mr. Davis held a commission as Lieutenant Colonel, Engineer Reserve. He was a member of the Engineers Society of Western Pennsylvania, American Society of Military Engineers, and American Railway Engineering Association.

Mr. Davis was elected a Member of the American Society of Civil Engineers on February 1, 1905, and became a Life Member in 1937. He took an active part in Society matters, and a committee of which he was chairman presented a report[2] on sand and aggregate specifications which became a model.

CHARLES YOUNG DIXON, M. Am. Soc. C. E.[1]

Died February 18, 1938

Charles Young Dixon was born on a farm located about three miles west of Niles, Mich., on May 20, 1864. He was the son of James Dixon and Louisa (Young) Dixon, and the third son of five children.

Mr. Dixon attended the public schools at Niles, and among his schoolmates were Ring Lardner, the humorist; and John F. and Horace E. Dodge, later of the firm of Dodge Brothers. For three years he and Horace Dodge were seat mates. He was graduated from the Niles High School in 1882, and from the Ann Arbor (Mich.) High School in 1883. From 1883 to 1887 he pursued the prescribed course in Civil Engineering at the University of Michigan, at Ann Arbor, and in 1887 was graduated from the Literary Department with the degree of Bachelor of Science in Civil Engineering. The Engineering Department had not yet been established.

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[2] "Specifications for Concrete with Local Aggregates"—Engrs. Soc. Western Pa.—adopted May 3, 1927.

[1] Memoir prepared by George H. Fenkell, Hon. M. Am. Soc. C. E., and Maj.-Gen. (*Retired*) E. M. Markham, M. Am. Soc. C. E.

During the period from 1888 to 1891 Mr. Dixon was engaged on railway and land surveys. In 1892 he entered the employ of the federal government, in the Corps of Engineers, as an inspector. He was promoted to assistant engineer the following year, classified by Executive Order in 1896, promoted to engineer in 1926, continued in service by the authority of the President in 1934, and again in 1935, and retired in 1936. From 1892 to 1895 he was employed continuously on the improvement of the channel through the St. Marys River, in Michigan, and thereafter he was primarily engaged with improvements of the channels between Lake Huron and Lake Erie, and was in direct charge of the construction of the Livingston Channel.

It may be noticed that Mr. Dixon's duties, during the uncommonly extensive period of his public service, were related all but exclusively to the federal navigational improvements at the western end of Lake Erie, the Detroit River, Lake St. Clair, the St. Clair River, Lake Huron, the St. Marys River, and its enlargements into Lake Superior. As a consequence of Mr. Dixon's peculiar engineering capabilities, and his utmost devotion to his tasks, he acquired invaluable knowledge in public interest, which might appropriately be assessed at very high levels, as to national worth.

It is a matter of gratification, combined with affectionate recollections of intimate, official, and personal associations, to make special mention of the most amiable qualities of one who served the Corps of Engineers, U. S. Army, with such loyalty, efficiency, and fidelity, throughout his service in all engineering grades from inspector to the highest position in the Corps—that of principal assistant engineer.

In 1894 Mr. Dixon was married to Mary McIntosh. Mrs. Dixon passed away in 1922. Their children, Donald and Louise, and two granddaughters, survive him. In 1926 he was married to Ella Taylor, who survived him. A sister, Mrs. Hattie Post, and two brothers, Oscar and Wallace, also survive.

Mr. Dixon was known to his acquaintances as somewhat reserved and retiring, but thoroughly kind and considerate. His family and home came first. He was a Deacon in the Congregational Church, and a member of the Masonic fraternity, having been active in lodge affairs in both Ontario, Canada, and Detroit, and a long-time member of the Detroit (Mich.) Commandery No. 1, Knights Templar.

The ending of Mr. Dixon's life, and the completion of his remarkable services to the Corps of Engineers, give proper occasion to memorialize him in this simple and brief fashion, as an outstanding public servant, and a real contributor to the national welfare. Upon his retirement in 1936, his many friends tendered him a testimonial dinner at the Detroit Yacht Club.

Mr. Dixon served as President of the Detroit Engineering Society in 1918 and as President of the Detroit (later Michigan) Section of the Society in 1922–1923.

Mr. Dixon was elected a Member of the American Society of Civil Engineers on November 4, 1903.

WALTER JULES DOUGLAS, M. Am. Soc. C. E.[1]

DIED JULY 2, 1941

Walter Jules Douglas, son of August and Mary Virginia (Crawford) Douglas, was born on October 26, 1872, at Baltimore, Md.

August Douglas, although of English and Scottish descent, was born in Luxembourg. He came to America from France in 1852 and established an importing business in Baltimore. He was married there to Mary Virginia Crawford, a native of Baltimore, whose parents had come to America from Belfast, Ireland. The family moved from Baltimore to Washington, D. C., in 1876.

Walter J. Douglas received his early education in the public schools of Washington and was graduated from the scientific department of the high school. He then attended the Bethlehem Preparatory School for Engineers, in Bethlehem, Pa., from which he was graduated in 1890. Later, he served for ten years as trustee of this school. Immediately after graduation he entered Lehigh University, in Bethlehem, and in 1894 was graduated with the degree of Bachelor of Science in Civil Engineering. These years at Lehigh brought him in close touch with the late Prof. Mansfield Merriman,[2] M. Am. Soc. C. E., resulting in a lifetime friendship and a later professional association.

After graduating from Lehigh, Mr. Douglas started his engineering career as a draftsman with the Coxe Iron Manufacturing Company at Drifton, Pa., which was engaged in the manufacture of engines, pumps, and miscellaneous equipment for coal mines. His work also included the study of coal-breaker construction and the testing of anthracite coals.

In 1895 Mr. Douglas entered the employ of the Lehigh Valley Coal Company, and remained with it until 1897 in the capacity of transitman and assistant engineer. His duties included taking charge of Hazleton Mine No. 1, and its adjacent coal strippings, and the handwork strippings of the Jaensville mines. He participated in all classes of engineering work usual in coal mining, such as the construction of trestles, boilerhouses, mine railways, hoists, shafts, slopes, and tunnels.

During part of the year 1898, he was associated with F. C. Dinney and Company as engineer-in-charge at the Winterpock mine, Chesterfield coal basin, in Virginia, where he made surveys and plans for the development of the mine and of 18 miles of railroad.

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[1] Memoir prepared by H. Alden Foster and Eugene E. Halmos, Members, Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 1658.

His activities as a mining engineer ended in 1898, when he was appointed assistant engineer of bridges in the office of the engineer commissioner of the District of Columbia, where he remained about eleven years. In 1901 he was appointed engineer of bridges in the same office, and until he resigned from the position in 1909 he was in charge of the design and supervision of all bridges constructed in the District of Columbia under the general direction of the U. S. Engineer Corps. His duties also included bridge and highway repair and construction, blasting work, preparation of regulations for the storage and use of explosives, and the design of a public bathing beach and floating bath.

At this time, the commissioners of the District of Columbia were planning an extensive bridge-construction program. As preparation for this work, Mr. Douglas obtained a leave of absence during the summer of 1904, and spent about four months in Europe, traveling through France, Italy, Germany, Belgium, and Holland. In each of these countries he studied the notable bridges and conferred with the leading engineers and bridge builders. This experience was of great value in the bridge-design program in which he engaged immediately after his return to Washington.

Of the several bridges constructed in Washington during the period when Mr. Douglas was engineer of bridges, the largest and best known is the Connecticut Avenue Bridge across Rock Creek (later called the "Taft Bridge"), which was completed in 1906. At the time of its construction, it was the largest concrete bridge in the United States, comprising five semicircular concrete arches of 150-ft span, with two concrete approaches each containing an 82-ft arch span. A distinctive feature of this bridge was the use of precast concrete blocks and monolithic unreinforced concrete for construction of the arches. The original design for the bridge was prepared by the late George S. Morison,[3] Past-President, Am. Soc. C. E., having been submitted in a prize competition in 1897. The preparation of the final plans, and the construction of the bridge, however, were under the direct supervision of Mr. Douglas.

Among other bridges which he designed and the construction of which he supervised during this period were the Anacostia Bridge, the Piney Branch concrete bridge, and, in Rock Creek Park, a three-hinged concrete arch and four small concrete bridges distinguished for their rustic charm. The Anacostia Bridge, which crosses the eastern branch of the Potomac River, consists of six three hinged steel arch spans, and a double-leaf bascule span. The Piney Creek Bridge was of particular interest at the time of its construction because it comprised a 125-ft unreinforced concrete arch of parabolic shape, the first arch of this form constructed in America. Mr. Douglas also designed at this time the Edmonson Avenue Bridge and the Wyman Park Bridge, both in Baltimore.

As a result of the prominence he attained through his work in Washington, Mr. Douglas received an invitation to serve as consultant to Barclay Parsons and Klapp, Consulting Engineers of New York, N. Y., on the design of the

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LIV, June, 1905, p. 513.

Almendares Bridge at Habana, Cuba. This bridge, containing four unreinforced concrete arches (one of 190-ft span and three of 102-ft span), was supported on pile foundations. The unbalanced horizontal thrusts on the intermediate piers were taken care of by horizontal struts connecting the pile caps below the river bottom. The long-span arch was constructed with three hinges, which were grouted in after completion of the structure. A paper describing the Almendares Bridge,[4] written jointly by Mr. Douglas and Eugene Klapp, was awarded the Thomas Fitch Rowland prize in 1912.

In 1909 Mr. Douglas left Washington to become associated with the late William Barclay Parsons,[5] Hon. M. Am. Soc. C. E., Mr. Klapp and Henry M. Brinckerhoff, M. Am. Soc. C. E., operating under the firm name of Barclay Parsons and Klapp, with offices in New York City, and took charge of all the structural design work of the firm. Among the earlier projects in the design of which he participated were three reinforced-concrete piers for the harbor of Habana and two hydroelectric developments in New York State—the Mohawk Development near Ephratah and the Salmon River Development near Altmar. After completion of the first of these projects he served as President of the Mohawk Hydro-Electric Corporation.

In 1911 he designed and supervised the construction of a railroad car manufacturing plant for the National Steel Car Company at Hamilton, Ontario, Canada, and in 1928 he redesigned and rebuilt this plant, providing additional facilities for the fabrication of automobile chassis and bodies. In 1918 he constructed a similar plant for the Magor Car Company at Passaic, N. J.

From 1915 to 1917 he served as deputy chief engineer of the Cape Cod Canal Company, of which Mr. Parsons was chief engineer. Mr. Douglas took an active part in solving construction problems that developed after the canal was placed in service.

During the early months of the first World War, Mr. Douglas participated in the organization of three plants for the manufacture of ferro-silicon, an essential material for armament manufacture. These plants were located at Niagara Falls, Plattsburg, and Fort Plain, N. Y. In June, 1918, he went to Panama as engineer of maintenance of the Panama Canal, relieving the late Maj. (later General) Jay J. Morrow,[6] M. Am. Soc. C. E., of the Engineer Corps, who was assigned to active duty in France. Mr. Douglas remained in Panama until May, 1919, and during a part of this period, in the absence of the Governor of the Canal Zone, he became Acting Governor, ex officio. During this period he was also vice-president of the Panama Steamship Line and the Panama Railroad Company.

On his return from Panama in 1919, Mr. Douglas renewed his association with Mr. Parsons, becoming a member of the firm of Parsons, Klapp, Brinckerhoff and Douglas. Shortly afterward, the firm was engaged to prepare the detail drawings and to supervise construction of the Sherman Island Dam and Power House, for which the late H. de B. Parsons,[7] M. Am. Soc. C. E., was consulting engineer. The Sherman Island Development, located on the Hudson River above Glens Falls, N. Y., is unique in that it includes a multiple-arch reinforced-concrete dam, 80 ft high (which is founded for the greater part of its length on a sand stratum), and a 50,000-hp power house, also founded on sand.

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[4] *Transactions*, Am. Soc. C. E., Vol. LXXIV, December, 1911, p. 216.

[5] For memoir, *ibid.*, Vol. 98 (1933), p. 1485.

[6] For memoir, *ibid.*, Vol. 103 (1938), p. 1846.

[7] For memoir, *ibid.*, Vol. 101 (1936), p. 1617.

Between 1925 and 1930, the firm was active in various construction enterprises in Florida. Mr. Douglas participated in the design of many of these projects, including Bayborough Harbor, the Recreation Pier at St. Petersburg, a yacht basin for the Vinoy Park Hotel, and the Tampa Union Terminal.

Between 1925 and 1932, Mr. Douglas served as consulting engineer to the Arlington Memorial Bridge Commission, in connection with the design and construction of the Arlington Memorial Bridge over the Potomac River at Washington, D. C. This monumental structure consists of six reinforced-concrete arches, with maximum spans of 180 ft, and a steel double-leaf bascule bridge of 184-ft span. The project also includes a water gate and river wall along the left bank of the river, and elaborate retaining walls and other monumental structures adjacent to the entrance of the Arlington Cemetery. The design of these structures involved an appreciation of permanence exceeding that normally assigned to engineering structures.

Between 1929 and 1931, Mr. Douglas participated in the design and supervision of construction of the water power plant on the Sacandaga River at Conklingville, N. Y.; the reconstruction and extension of the Spier Falls Plant on the Hudson River; and in the design and construction of the Detroit-Windsor Vehicular Tunnel between Michigan and Ontario, Canada. During the period 1930-1932, as consulting engineer, he supervised the design and construction of a double-lane vehicular tunnel and the design of a pedestrian tunnel, both under the Scheldt River at Antwerp, Belgium. In recognition of his services on this project, he was made a Commander of the Order of the Crown of Belgium.

In 1936 Mr. Douglas served as chairman of the Board of Consultants appointed by the President of the United States to prepare an economic survey for the Atlantic-Gulf Ship Canal across Florida. About this time he also served on a Board of Consultants for the U. S. Engineers in connection with the proposed Passamaquoddy Tidal Power Project in Maine.

Between 1936 and 1938, Mr. Douglas served on the Board of Consultants for foundations and structures at the New York World's Fair. From 1936 to 1940 he was a member of the Board of Consultants to the U. S. Engineers for the design and construction of the Santa Ana, San Gabriel, and Los Angeles (Calif.) flood control projects. During this period he advised the Work Projects Administration (WPA) in New York City on foundation problems at La Guardia Airport, and on other engineering projects in the New York City area. He was also consultant to the commissioner of works of the Borough of Manhattan in connection with the East River Drive.

Among various projects in the design and construction of which Mr. Douglas participated between 1934 and 1940 were the railroad lift bridge over the Cape Cod Canal, the Jamestown Bridge over Narragansett Bay in Rhode

Island, and the Pennsylvania Avenue Bridge over the Anacostia River at Washington, D. C.

Following the 1929 depression, with the subsequent decline of private engineering practice and the increased activities of federal and quasi-federal agencies in the engineering field, Mr. Douglas realized that it would be desirable to have a geographically dispersed practice upon which economic conditions in the United States would have a minimum effect. He accordingly spent several months making investigations and studies of conditions in foreign countries. As a result of these investigations, his firm decided to open an office in Venezuela. This office was opened in Caracas in May, 1938, and was subsequently active on projects for flood control, port development, irrigation, bridges, water supply, and sewage disposal.

While he was engineer of bridges of the District of Columbia, Mr. Douglas already was recognized as one of the most distinguished designers of masonry bridges in America. In consequence, he was honored in 1909 by Professor Merriman, who asked him to be one of the associate editors of the first edition of the *American Civil Engineers' Pocket Book,* published in 1911. Mr. Douglas prepared the section of this handbook on "Masonry and Timber Structures." For the fifth edition (1930), under the editorship of Thomas H. Wiggin and the late Thaddeus Merriman, Members, Am. Soc. C. E., he prepared three sections—"Foundations and Earthwork," "Timber Structures," and "Masonry Structures."

In addition to the aforementioned paper on the Almendares Bridge, he participated in the discussion of various papers published by the Society. He also wrote numerous articles for the technical press. In 1910 he wrote a manual on reinforced concrete which, in that field, was quite similar to present structural steel handbooks.

Although Mr. Douglas never held office in the Society, he frequently assisted in the work of the Committee on Publications. He was also secretary for several years of the Committee on Bearing Value of Soils for Foundations.

Mr. Douglas had a remarkable personality. He was of less than medium stature, slightly built, modest in manners, and quietly spoken. Nevertheless, both in social and professional intercourse, he was soon recognized by people with whom he came in contact as a great man and an engineer of outstanding ability.

His extensive travels, his wide acquaintance with people in all grades of society, his unfailing humor, and his love of literature and the arts made Mr. Douglas an exceptionally versatile conversationalist. There were few topics on which he could not contribute valuable and pertinent comments. He was much interested in the modern trend of social philosophy and liked to discuss current events, both domestic and international, in the light of his broad knowledge of their historical background.

Professionally, Mr. Douglas had an excellent theoretical foundation, which he acquired in college and subsequently through constant study of the progress of engineering science. He was particularly interested in the development of structural mechanics, and in the chemical and physical composition and behavior of building materials and soils.

He had exceptional ability to apply to the design of engineering structures the knowledge of practical considerations derived from his wide experience in actual construction. He knew thoroughly the costs of all types of work, the operating possibilities of building machinery, and the methods best adapted to a particular construction problem.

From the beginning of his active engineering work, Mr. Douglas kept in close touch with his office associates and the men in the field. He was thoroughly human, took extensive interest in the welfare of all persons under his control, and thus was able to build up a following of loyal men who were ready to join him in the various organizations he set up for conducting the diversified work of the firm. His close association with the workers on the job also gave him a keen appreciation of human nature that was outstanding throughout his career.

Walter J. Douglas was gifted with a keen business sense, which was chiefly responsible for the successes his firm achieved during the difficult postwar and depression periods. On his return from service in Panama, recognizing that the changing times required a closer cooperation between engineers and contractors than was customary before the war, he organized the Parklap Construction Corporation and Parklap Incorporated, the former to undertake heavy construction work, the latter for building construction. These companies were entities independent of the firm, and were operated under Mr. Douglas as president. They were highly successful; in the brief period of less than ten years, between 1920 and 1930, they completed more than fifty million dollars worth of construction.

The Parklap companies constructed Sherman Island Dam and Power House; Feeder Dam Power House; railway and harbor terminals at Detroit, Toronto (Ontario, Canada), Montreal (Quebec, Canada), Buffalo (N. Y.), and Albany (N. Y); wharves and bulkheads at Detroit, Buffalo, and Habana; and certain sections of the Detroit-Windsor Vehicular Tunnel. A subsidiary organization in Florida constructed the Tampa Union Terminal, the St. Petersburg News Building, the St. Petersburg Recreation Pier, and various wharves and bulkheads at St. Petersburg, Belleair, and Fort Pierce.

Mr. Douglas was a member of the Phi Delta Theta fraternity. He was also a member of the Institution of Civil Engineers of England; the Society of American Military Engineers; the Plainfield Country Club of Plainfield, N. J.; the Army and Navy Club of New York City; the Engineers' Club of New York; the Union Club of Washington, D. C.; the Army and Navy Club of Washington, D. C.; the American Society for Testing Materials; the American Railway Bridge and Building Association; and the Washington Society of Civil Engineers.

From 1919 until his death, his home was in Plainfield, where he was a member of the Grace Episcopal Church. He served on the Board of Appeals of the city Building Department and for several years was chairman of the local committee on Unemployment Relief. He died in New York City on July 2, 1941, and is buried in Hillside Cemetery, Plainfield.

He was married to Elizabeth Appleton Spalding on April 17, 1906. He is survived by his widow; four children—Walter Spalding Douglas (Jun. Am.

Soc. C. E.), Mary Virginia Crawford Douglas (Mrs. Philetus Holt Riley), Elizabeth Appleton Douglas, and Caroline Hewetson Douglas; and two grandchildren.

Mr. Douglas was elected a Member of the American Society of Civil Engineers on April 3, 1907.

ERNEST WERNER EICKELBERG, M. Am. Soc. C. E.[1]

Died May 20, 1941

Ernest Werner Eickelberg was born in Haskins, Ohio, on March 25, 1890, the son of the Rev. Hugo and Selma (Kaiser) Eickelberg.

In spite of the meager salary which the father received in the ministry, the family of five sons and five daughters was well managed and well clothed, living their lives with pride and personal happiness, thanks to the mother's diligence and versatility and the father's ingenuity, as well as to the willing sacrifices of both parents.

Ernest was the fourth child. He attended grammar schools in Buffalo, N. Y., and Baltimore, Md., finishing his last five years of public school in the latter city, where he was graduated with honors from the Baltimore Polytechnic Institute in 1910. During his high school days he worked after school and on Saturdays at a grocery and also worked in the circulation department of the *Baltimore News.* He had an inherent aptitude for mathematics. In his early youth he displayed a natural teaching ability by assisting his brothers and sisters in their school work; and later during his Polytechnic days he was frequently sought as tutor by some of his classmates. The genial disposition and patience so well remembered by all his friends of later years were notable also during these early school days.

Ernest Eickelberg entered Cornell University, at Ithaca, N. Y., in the fall of 1910, and in three years obtained the degree of Civil Engineer. His cheerful smile and willingness to do more than his part of any work attracted all who met him.

During his days at Cornell, he found time for basketball and track, which he greatly enjoyed. He usually attended church services at least once every Sunday.

While at Cornell he met O. M. Leland, M. Am. Soc. C. E. (formerly a field engineer in the U. S. Coast and Geodetic Survey and later Dean of Administration, Institute of Technology, at the University of Minnesota, in Minneapolis), and through this association he became interested in geodetic field work. In his senior year he sought connection with that service and was notified to report a few days before his own graduation exercises. His interest in this work was so intense that his roommate and close friend, Harry Barnard, was able only with great difficulty to persuade him to remain for the commencement exercises, even though his sister, Martha, who had just been married, was coming to the graduation exercises on her honeymoon.

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[1] Memoir prepared by Paul A. Smith, M. Am. Soc. C. E.

Ernest Eickelberg joined the Coast and Geodetic Survey on June 20, 1913, and during the remainder of his life he had opportunity to demonstrate his unusual versatility in every phase of the Survey's work. Among his first assignments were wire drag surveys and launch hydrography surveys in Maine, Massachusetts, and Florida, and in Clarence Strait, Alaska. For the latter he was chief of party. During the winter of 1915–1916, he conducted second-order and third-order coastal triangulation operations in the Patapsco River region and in Baltimore, Md. In the summer of 1916 he went again to Alaska on triangulation, topography, and wire drag work. The following winter, 1916–1917, was spent in triangulation operations around San Francisco Bay, California; and in the summer of 1917 he again returned to Alaska on first-order triangulation and reconnaissance work.

In September, 1917, he was commissioned First Lieutenant in the U. S. Coast Artillery Corps, assigned to the mine planter, *Graham,* and given command in April, 1918. He was promoted to the rank of Captain in September, 1918. During this tour of duty he had charge of the laying and testing of mines and controlling cables.

On June 6, 1919, he returned to geodetic work as chief of party, and until January, 1921, was engaged in gravity observations, testing new gravity equipment, and on first-order triangulation and base measurements in California, Oregon, and Idaho. After spending a few months on office computations, he returned to field work in April, 1921, in the vicinity of Puget Sound, Washington. During the winter of 1921–1922 and the following year he accomplished the first-order triangulation connections between British Columbia, Canada, and the United States.

From March, 1922, to February, 1923, he was in charge of the computing office of the Coast and Geodetic Survey at Manila, Philippine Islands, where he superintended the adjustment of the triangulation nets of the Philippine Islands. From February, 1923, to May, 1924, he was assigned to the ship *Pathfinder* as executive officer. He took an opportunity to return to the United States by way of Europe, and his knowledge of some of the European languages, coupled with his sense of humor and genial character, made his trip around the world one that provided him with many interesting memories which he frequently related to the great enjoyment of his friends.

Returning to the United States in August, 1924, he spent the winter in charge of the District of Columbia-Virginia boundary survey. In February, 1925, he reported to the Survey Ship *Lydonia* as executive officer, where he remained on coastal surveys of the Atlantic Coast of the United States until January 18, 1926. This vessel was engaged in experimental work in radio acoustic ranging and echo sounding, and "Eick" (as he was by then universally known in the Survey) soon showed his ability in this work. He was accordingly sent to the Survey ship *Surveyor* in the fall of 1926, and later to the *Discoverer* in the Hawaiian Islands, to install echo sounding apparatus and instruct field engineers in its use. Eick was trained in this work by Dr.

Herbert Grove Dorsey, popularly known throughout the Survey as "Doc." As a mark of distinction for his outstanding ability, Eick was promptly dubbed "Doc" also by his colleagues in the Survey, a title which stuck with him for many years.

In January, 1928, he returned to the *Lydonia* and worked on Atlantic coastal surveys until June, 1929. During this assignment he was married to Mary Louise Stover in Wilmington, N. C., on April 10, 1928. He assumed command of the Survey ship *Explorer* in June, 1929, operating on topographic, hydrographic, and geodetic surveys in Southeastern Alaska.

Eick returned from this assignment in August, 1931, to the position of assistant chief, Division of Terrestrial Magnetism and Seismology of the Survey in Washington, D. C. Here he was active in the development of new methods and procedures, as a valued assistant to Capt. N. H. Heck, M. Am. Soc. C. E.

In August, 1938, he returned to the Pacific Coast in command of the Survey ship *Guide,* and conducted difficult and important surveys in the Bering Sea in the vicinity of Unimak Pass, Alaska. During this assignment he was stricken in the summer of 1940 with the first indication of the disease that eventually caused his death. He was confined to the hospital in Dutch Harbor, Aleutian Islands, Alaska, for several months following a serious operation; upon apparent recovery he returned to Oakland, Calif., and subsequently to Washington, D. C., where he had received orders to become assistant chief of the Division of Coastal Surveys. The last months of his life were spent at the Marine Hospital in Baltimore. Here his genial smile never failed to greet his many friends from Baltimore and Washington, D. C., who visited him, in spite of the pain he endured without complaint until his death.

Eick was slight and wiry in build, but his character raised him above many of his colleagues in those modest personal qualities that make for outstanding leadership. On mountain trails, surf landings on dangerous coasts, and in any of the hardships frequently encountered in the field work in which he spent almost twenty years of his life, he always led the way with more than his share of the undesirable chores and burdens.

He was always a willing participant in any of the playful badinage commonly indulged in by members of survey parties in remote places. On one occasion he cheerfully permitted himself to be hauled by a line through glacier streams by several of his fellow engineers when they discovered that he was too light to ford the numerous swift and icy streams flowing into the sea from the Fairweather Range south of Lituya Bay, Alaska, and along which coast his party was running a control survey.

During his service with the Survey, he rose to the rank of Lieutenant-Commander, and at the time of his death was one of the most popular senior officers in the service. He was active in sports, especially golf and bowling, and usually held office as secretary or treasurer of the sports association in spite of his heavy administrative duties.

Commander Eickelberg always became an active participant in the life of the community where he resided with his family. He was especially well known in Arlington, Va., where he lived during his last duty in Washington, D. C., and where funeral services were held in St. Mary's Episcopal Church. Burial was in Arlington National Cemetery with military honors. He is survived by his wife; a daughter, Jean Stover; a son, Ernest Werner; and all of his brothers and sisters, who are Hugh, Erwin H., Arthur P., Dr. Walter A., Helen I., Martha (Mrs. E. O. Youngen), Selma (Mrs. William C. Miles), Clara (Mrs. Charles E. Lyon), and Edna R. (Mrs. H. Middleton Heer).

He was a member of Dawson Lodge No. 16, F. and A. M., in Washington, D. C., the Philosophical Society of Washington, Society of American Military Engineers, Washington Academy of Sciences, American Geophysical Union, American Society of Photogrammetry, Society for Research on Meteorites, Sojourners, and Heroes of '76.

The signature "E. W. Eickelberg" is probably found on as wide a variety of survey observations and records as that of any engineer in the service. His resourcefulness in devising improvements to surveying apparatus and instruments was notable. Some of these improvements are described in various technical reports and papers under his name in the archives and publications of the Coast and Geodetic Survey. His meticulous concern for thoroughness, accuracy, and a job well done was outstanding in the organization whose work he enjoyed so well, and from whose personnel he won the highest tribute a man may have—the confidence of his superiors, the esteem of his associates, and the respect and loyalty of his subordinates.

Commander Eickelberg was elected an Associate Member of the American Society of Civil Engineers on April 25, 1921, and a Member on October 14, 1935.

NATHAN RANDALL ELLIS, M. Am. Soc. C. E.[1]

DIED FEBRUARY 28, 1941

Nathan Randall Ellis was born in San Francisco, Calif., on January 5, 1876, the son of Nathan Rodney Ellis and Katherine (Quigley) Ellis. He attended the historic Lincoln Grammar School and Boy's High School in San Francisco, developing a remarkable aptitude for mathematics. Unable to continue his educational course through college, he remained a diligent and conscientious student throughout his life.

Upon graduation from high school in 1894, he immediately found employment in the construction of a water power plant on Mokelumne River. The first successful utilization of water power for the generation of electricity for commercial use in California had been effected only two years previously, and in 1894 there were four additional plants under construction in the state, and a large number in various stages of study and projection. This Blue Lakes plant on the Mokelumne River, when completed in 1897, was the fourth California installation of this kind to be placed in operation, and the first to supply service from transmission lines operated at the then unprecedented potential of 17,000 volts. These pioneer installations were of primary importance in demonstrating the extent and complexity of the physical, technical, economic, industrial, and financial problems confronting the development of California's vast water resources.

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[1] Memoir prepared by Paul Leo Beck, M. Am. Soc. C. E., and James Tillman Ryan, Assoc. M. Am. Soc. C. E.

To the young high school graduate, these problems presented both a challenge and an opportunity. He responded to this by entering upon a course of self-directed study in engineering with characteristic energy and thoroughness, using all the available textbooks and current literature, but, to a more important degree, the personal instruction and example of engineers to whom he reported in the practical work of design and supervision on the job.

The Blue Lakes Water Company realized, even before the completion of its first plant, that the power service it proposed to supply required far more capacity than it could develop at that site or could finance from its available resources. It was, therefore, reorganized as The Standard Electric Company of California, which was incorporated on November 27, 1897. With Mr. Ellis employed in successive positions of increased responsibility, the company began the studies that resulted in the construction of the Electra plant, at a point 4 miles upstream from the Blue Lakes plant, and an extensive system of canals and reservoirs to supply water for its operation. Actual construction was begun in July, 1900, with Mr. Ellis assigned to the supervision of all work except the power-house equipment and penstocks. Thus, he was already well advanced in his chosen field at the age of twenty four.

His work from 1894 to 1900 was, therefore, both an apprenticeship and the equivalent of a college course in engineering, his diploma for the latter being his appointment as superintendent. In this capacity he designed and constructed two diversion dams in the main channel of Mokelumne River, five storage and two forebay reservoirs, and a canal 22 miles long with a flow capacity of 125 cu ft per sec. He also superintended the reconstruction and enlargement of a parallel canal 21 miles long with a flow capacity of 75 cu ft per sec. This work was completed and the plant placed in operation on May 6, 1902.

The excellence of its design and construction is attested by the fact that all of it has been in continuous operation since, except the diversion and intake structures, which were by-passed by the Tiger Creek power plant, completed in 1931. Probably one of the most significant things that Mr. Ellis ever did was to turn over to the Pacific Gas and Electric Company a complete report of construction methods and progress and an analysis of the cost of this project.

In the latter part of 1904 he was appointed manager of the Humboldt Construction Company, the work involving electric railway construction and the installation of a steam power plant for the local railway system in Eureka, Calif. In the early part of 1906 he became chief engineer and assistant general manager of the Shattuck-Edinger Construction Company, which was engaged in the construction of the Northern Electric Railway, the Western

Pacific Railroad between Oroville and Sacramento, Calif., and the Southern Pacific Railroad from Santa Cruz to Vicente, Calif.

During 1907 he was employed as the engineer for the contractor on the construction of the Market Street sewer system in San Francisco. This was a cost-plus contract and, as the total cost greatly exceeded the preliminary estimates, the contractor was charged with gross waste and inefficiency, culminating in an investigation by the Grand Jury. After a thorough investigation, the Grand Jury not only cleared the contractor but in addition commended Mr. Ellis for the efficient conduct of the work. This commendation was due in part to the fact that he maintained and was able to present a complete record of construction methods and costs.

In 1908 Mr. Ellis was appointed engineer in charge of construction for the Crocker Estate Company in San Francisco on the subdivision of the Crocker Estate. This is now largely that part of Daly City, Calif., which lies east of Mission Road.

In the latter part of 1908 he became manager of the Desmond Construction Company in Los Angeles, Calif. In this capacity he supervised the construction of the Inter-California Railway in Mexico and the Arizona-Colorado Railroad in Arizona, and a reconstruction of the Gila Valley, Globe and Northern Railroad in Arizona. In addition to this railroad work, Mr. Ellis directed the Desmond Company's subsistence and commissary contracts on the Los Angeles Aqueduct, operating some fifty camps and dining rooms. On the cessation of work on the aqueduct in 1910, he became office engineer for the Los Angeles Highway Commission, which was engaged in the construction of highways costing some three and a half million dollars. In 1911 he became a partner in the contract for the construction of the Interurban Electric Railways from Van Nuys to Canoga Park in the vicinity of Los Angeles.

His first contact with valuation work was made in November, 1911, as an assistant engineer for the J. G. White Company of San Francisco on an appraisal of the properties of the Spring Valley Water Company. In July, 1912, he was appointed consulting engineer for the Finance Committee of the Board of Supervisors of the City and County of San Francisco to study the functions of the various departments of the Board of Public Works. In the street repair department he made exhaustive studies of paving methods and costs, resulting in the reorganization of that department. He also made similar studies of the operations of the sewer department, the street cleaning department, and the department of buildings.

On the completion of this work in August, 1913, Mr. Ellis was appointed valuation engineer for the city attorney's office of San Francisco. From 1913 to 1932 he supervised the following valuations: Spring Valley Water Company, appraised twice; the gas and electric properties of the Pacific Gas and Electric Company in San Francisco; all of the properties of the Pacific Gas and Electric Company's gas and electric system in California, as well as all of the properties of the Great Western Power Company, later merged with the Pacific Gas and Electric Company; and properties of the Pacific Telephone and Telegraph Company in San Francisco and the East Bay Cities in

1913 and 1914 and again in 1927. He was also in charge of the preparation of evidence presented by the city attorney in all utility rate cases before the Federal Court and the Railroad Commission of the State of California.

Mr. Ellis' activities were not confined to the rate work and valuation work of the city attorney. He was frequently called in as a special advising engineer to the Board of Supervisors on the operation of the city's municipally owned utilities, programs of construction on the Hetch Hetchy Project, and in proceedings looking to the acquisition of the water and electric properties in San Francisco. From 1913 until 1932, he was at various periods employed as consulting engineer on valuation and rate matters by the cities of San Jose, Marysville, and Fresno (all in California), and by the Sierra Railway Company of California and the McCloud River Railroad Company in proceedings before the Interstate Commerce Commission.

With the ratification of a new charter by the electorate of the City of San Francisco in 1932, Mr. Ellis' title was changed to chief valuation and rate engineer. From 1932 until his death, he continued to supervise the preparation of all data presented by the city attorney before the Railroad Commission of the State of California in cases involving gas, electric, and telephone rates and street railway fares. He also advised and reviewed the reports of the Public Utilities Commission of the City and County of San Francisco on the cost of acquiring the electric properties of the Pacific Gas and Electric Company, and the financial results to be expected under municipal operation. The last important report prepared by Mr. Ellis was on the financial results to be derived from the leasing of the electric distribution properties of the Pacific Gas and Electric Company in San Francisco.

Randall Ellis faced the problems and issues of life with resolute courage and discharged his duties with conscientious integrity. To him there was but one standard of right, but one measure of excellence, and to these he exacted even more rigid conformity from himself than he expected from others. With these principles and with a positive and aggressive mentality, a man whose pathway constantly led through fields of controversy must needs beget opposition. He was a participant in a number of the most important and bitterly contested rate and condemnation cases in American court history, in which he was on successive occasions in opposition to many of our most famous technical experts and subjected to cross-examination by some of the most adroit practitioners of the American Bar. The significant result of all of these cases was the very great extent to which the final decision in each conformed to Mr. Ellis' analysis and conclusions.

In a condemnation case involving the most valuable property ever subject to such proceedings in America, he arranged for a series of conferences with engineers representing the various interests involved. These resulted in the presentation of an appraisal that was accepted by all and adopted by the Commission which decided it. In numerous proceedings involving the rates of utilities, with hundreds of millions of invested capital and hundreds of thousands of customers, it was not infrequent that he was able to bring all parties concerned into agreement as to the pertinent facts, leaving only questions of policy for argument and decision.

These controversies usually extended into months and several of them into years of intensive study and patient analysis. The ultimate agreement was reached only after Mr. Ellis had established the soundness of his own logic, and had virtually shamed prejudice from its bias, entrenched interest from its selfishness, and political objective from its subterfuges.

The esteem in which Mr. Ellis was held was probably best expressed by Resolution No. 1658 (Series 1939) of the Board of Supervisors of the City and County of San Francisco, as follows:

> "WHEREAS, Almighty God has summoned to eternal rest, N. Randall Ellis, utility rate expert for the City and County of San Francisco; and
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> "WHEREAS, Mr. Ellis, for twenty-five years an employee of the City and County, participated in all important rate litigations involving San Francisco, and immediately prior to his death was engaged in a study of electric power distribution facilities of the Pacific Gas and Electric Company, in connection with the proposed City lease thereof; and
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> "WHEREAS, the affability of Mr. Ellis was, to his many friends and colleagues, of equal distinction with his worth as a man of erudition, expert in his chosen field, and his passing will be a distinct loss to San Francisco and his death deeply mourned by his many friends in private and public life;
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> "Now, therefore, be it resolved, that this Board notes with keenest regret the passing of N. Randall Ellis, and when it adjourns this day it does so out of profound respect to his memory; and the Clerk is hereby directed to forward to his bereaved widow and family a copy of this Resolution as an expression of the Board's heartfelt sympathy and condolence."

Mr. Ellis was married to Clara Cook at Mokelumne Hill, Calaveras County, Calif., on December 31, 1901. They became the parents of three daughters, Eleanor (Mrs. Donald Mazer), Edith, and Katherine Elizabeth (Mrs. Donald Fleming), who, with Mrs. Ellis, survive him.

He was a Registered Professional Civil Engineer of the State of California, and a member of the American Water Works Association and Commonwealth Club of San Francisco.

Mr. Ellis was elected a Member of the American Society of Civil Engineers on June 23, 1916.

JULIAN CLARENCE FEILD, M. Am. Soc. C. E.[1]

DIED DECEMBER 22, 1940

Julian Clarence Feild was born in Denison, Tex., on September 9, 1881, the son of Julian Clarence and Susan Elizabeth (McClain) Feild. His father was a physician in the rapidly opening Southwest and a leader of community activities, having chosen to make Denison his home shortly after the townsite was established in 1872 by the builders of the Missouri, Kansas and Texas Railway. His mother was of a pioneer family that went to Texas many years previously and had settled at Whitesboro. His boyhood was spent in the frontier town, where he attended the public schools and Harshaw's Literary Academy.

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[1] Memoir prepared by A. M. Brenneke, Assoc. M. Am. Soc. C. E.

His higher education began with one year at the University of Texas, in Austin, after which he transferred to Cumberland University at Lebanon, Tenn., where he was graduated in June, 1902, with the degree of Bachelor of Science. During vacations he gained valuable experience in survey parties with various railroads expanding throughout Texas and Oklahoma.

After graduation it was natural for him to continue in railroad location and construction, which he did until May, 1907, first as assistant engineer for the Shreveport and Red River Valley Railroad Company; then as draftsman and instrumentman with the St. Louis-San Francisco Railway and the Missouri Pacific Railroad companies; and finally as division engineer of construction on the Springfield Division and as division engineer on the Colorado Division of the Missouri Pacific Railroad Company.

In May, 1907, he decided to open his own office in Denison for general engineering practice. During the next ten years he served four years as construction engineer and one year as city engineer for Denison; two years as county engineer, Grayson County; as consulting engineer for Grayson, Hunt, Robertson, and Milam counties on $2,000,000 worth of road construction; and as consulting engineer for the Southern Surety Co., Lion Bonding and Surety Company, and Fidelity and Bonding Company, advising on contract bonds and adjusting losses. As consulting engineer for the Red River Bridge Company, he designed and built a highway and interurban bridge over the Red River, near Denison, in 1914. It was during this period that rapid strides were being made in road construction, in which move Julian Feild was an active leader both as an organizer of the Texas Good Roads Association and as consultant for the Oklahoma, Texas and Gulf Highway Association and the Jefferson Highway Association.

In 1916, he became associated with the Healy Construction Company in the construction of streets and roads. While connected with this company he built many miles of improved roads around Pine Bluff, Grady, Hope, and Prescott, Ark. His ability as an organizer and his keen foresight led him to continue road contracting work, from 1920 until about 1931 under the firm name of Julian C. Feild and Company, and his was a pioneer firm in concrete road construction in the Southwest. Julian C. Feild and Company built concrete roads during the decade following the first World War in many counties in Texas, doing a volume of business often in excess of $1,000,000 annually. In the early 30's contracting became less interesting to him, and he fulfilled an earlier desire to enter the construction material business by building a sand and gravel plant just north of Denison, which plant he operated until his death.

Julian C. Feild had a keen interest in, and sense of individual responsibility toward, all civic activities and took an active part in community development. He was President of the Denison Chamber of Commerce for two years, 1927 and 1928, a director of the State National Bank, President of the Denison Hotel Company, and a director of the Denison Building and Loan Association. He was a lifelong member of St. Luke's Episcopal Church, in Denison, and served as Senior Warden and as a Vestryman for many years. He was a Thirty-Second Degree Mason, a member of the Denison Rod and Gun Club, Denison Rotary Club, and Beta Theta Pi fraternity.

He was married on June 10, 1903, at Lebanon, to Mary Lea Sanders, whom he met and won while a student at Cumberland University. One daughter, Kathryn Elizabeth Feild, was born of this union. He is survived by his widow and his daughter.

Mr. Feild was elected an Associate Member of the American Society of Civil Engineers on January 17, 1916, and a Member on November 27, 1917. He was active in the organization of the Texas Section of the Society in 1913, being one of those present when such an organization was proposed, and he assisted in the organization activities.

JAMES EASTON FERGUSON, M. Am. Soc. C. E.[1]

Died March 3, 1941

James Easton Ferguson, who died on March 3, 1941, at his residence in Cleveland, Ohio, at the age of sixty five, had been manager of the Fabricating Division of The Austin Company, Engineers and Builders, for a period of twenty-one years, during which he played an important rôle in the development of efficient methods for shop welding of structural steel.

A native of Marshall, Mich., where he was born on April 21, 1875, the son of Thomas E. and Margaret (Porter) Ferguson, Mr. Ferguson attended public school and high school at Toledo, Ohio, and in 1894 entered Cornell University, at Ithaca, N. Y. Prior to entering college he had spent one year as assistant to the manager of the Cincinnati, Hamilton and Dayton Railway Elevator Company at Toledo, handling general work and assisting in the supervision of construction of a grain elevator. He continued this work during the succeeding summers. Upon completion of his studies in civil engineering at Cornell in January, 1898, Mr. Ferguson obtained his first regular employment with the Toledo Bridge Company (later a part of the American Bridge Company). As draftsman and estimator he was engaged in the detailing and checking of shop drawings from January, 1898, until October, 1900. He spent the following six months there designing and estimating highway bridges and mill buildings. From March, 1901, to March, 1902, he was chief draftsman of the Macdonald Engineering Company in Chicago, Ill., in charge of structural design and drawing room, on work which included design and detailing steel for several fireproof, steel, grain elevators representing an aggregate investment of about

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[1] Memoir prepared by W. G. Paton, M. Am. Soc. C. E., and Louis H. Burr, Mgr., Fabricating Div., The Austin Co., Cleveland, Ohio.

$1,500,000. During this period he also supervised shop and field work on these projects.

In March, 1902, he went to Detroit, Mich., as assistant engineer for the American Bridge Company in that area, and was in charge of the drawing-room squad, with direction over the detailing for various classes of bridges and buildings. Mr. Ferguson continued in this capacity until June, 1906, when he joined the Toledo-Massillon Bridge Company at Toledo, serving as chief draftsman and assistant plant engineer until May, 1908. He was in complete charge of detail design and drawing-room work for railroad and highway bridges and buildings for fabrication at the Toledo and Massillon plants. From May, 1908, to January, 1909, as contracting engineer, he made designs and estimates for all classes of bridge and building work and handled some contracting. Following some erection work during this period, he had charge of the construction of a $200,000 highway and street railway bridge over the Monongahela River between Webster and Donora, Pa., where the channel span was 515 ft long. In this connection, he designed the falsework, travelers, and erection schemes. From 1909 until 1917, when he left the company, Mr. Ferguson functioned first as plant and construction engineer in full charge of shop operation, construction work, drafting department, and steel purchases, and later as operating manager. In the latter capacity he was directly responsible for the fabrication and erection of the Cherry Street Bridge[2] at Toledo and several large bridge projects in the Chicago area, all of which were bascule span bridges.

Leaving the Toledo organization in 1917, Mr. Ferguson assumed the responsibilities of general management at the American Steel Company of Cuba at Habana, which at the time operated a structural steel plant, railroad car plant, and a bolt and rivet works. In this capacity he was closely connected with the expansion of Cuba's industrial plant facilities, particularly for the sugar industry, for which the American Steel Company produced a large number of new freight cars. He left Cuba at the end of 1918, returning to Toledo, where he established his own consulting engineering service and collaborated in the design of additional bridges and industrial structures in the Middle West.

He joined The Austin Company, Engineers and Builders, as manager of its Manufacturing Division in Cleveland on April 1, 1920, and in that capacity, for the succeeding twenty-one years, was largely responsible for the development of the efficient shop practices that have distinguished this plant. Under his management, the plant was expanded repeatedly and grew from a capacity of between 300 and 400 tons per month in 1920 to a capacity of approximately 1,800 tons per month in 1941.

The standard cross sections developed by Austin engineers during the previous decade, and which figured so largely in construction of defense plants for the first World War, were produced in large quantities and at record speeds during the factory building boom of the middle 1920's. It was largely as a result of Mr. Ferguson's organizing ability and his understanding of production problems that his plant, one of the smallest in Ohio, was able to produce more tonnage than other shops twice its size. During one month in 1925, for instance, the plant fabricated 1,031 tons of structural steel in a shop area of less than 7,500 sq ft.

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[2] "The Cherry Street Bridge, Toledo, Ohio," by Clement E. Chase, *Transactions*, Am. Soc. C. E., Vol. LXXX, December, 1916, p. 744.

With the introduction of electric welding, the Austin shop became a center of research and experimental work in this field. It was under Mr. Ferguson's direction that the shop developed test procedures and set up equipment for the efficient fabrication of structural elements by welding. Structural members for a 50-ton crane runway at the Pittsburgh Transformer Company, later a part of the Allis-Chalmers Company, constituted the first commercial tonnage handled at this shop. In 1929 the shop furnished steel for a four-story and basement store and office building, which The Austin Company built as an investment and to gain practical welding experience before it undertook the design and construction of welded plants on a commercial basis. This building was erected entirely without the aid of rivets and temporary bolts. After the difficulties encountered in erecting the 117 tons of steel for this structure, Austin engineers recognized the importance of adequate temporary field connections and Mr. Ferguson played a major part in working out these problems.

With one corner equipped with special grids which expedited fabrication, the shop in 1930 promptly turned to jig welding of trusses for a long series of industrial plants. Although many of these conform to standard Austin cross sections, a variety of special designs demonstrated the widespread adaptability of welding.

Most significant of all the jobs undertaken at the Austin shop, under Mr. Ferguson's direction, was the fabrication of 2,600 tons of structural steel for the Electro-Motive Corporation's Diesel Locomotive Works at La Grange, Ill., in 1935. Designed for production of Diesel locomotives weighing up to 200 tons, this plant called for roof trusses of 104-ft span and built-up columns 65 ft high, weighing 6 and 7 tons, respectively, in its main erection aisle, where one 200-ton electric traveling crane and two cranes of 30-ton and 40-ton capacity were to operate. The extent of shop welding on this project is indicated by the fact that only 17% of the welding electrodes (totaling 7½ tons) were used in the field.[3]

When the Austin research organization during the early depression years initiated studies into ways and means of providing more clean-cut, sturdy factory cross sections, experiments were executed under Mr. Ferguson's supervision at the fabricating shop. Late in 1936 the shop turned out columns and roof bents for one bay of an all-welded, rigid-frame, saw-tooth structure in which "tree-form" curves supplanted the angles normally encountered at roof and column connections. Several months later the shop produced its first commercial tonnage, applying this cross section for the International Silver Company. This was followed in July, 1937, by the fabrication of 1,327 tons for the Lincoln Electric Company's addition, where four 40-ft aisles, ranging from 240 to 442 ft in length, involved the use of this cross section, and where a lofty, all-welded, 80-ft "cathedral" aisle, 39 ft to the peak, was also provided.

Steel for a series of other welded, rigid-frame, saw-tooth plants was fabricated under Mr. Ferguson's direction. These were followed by fabrication of welded rigid-frame portal trusses, which Austin utilized to accommodate conveyers in otherwise useless sub-ceiling areas immediately under factory roofs. This was accomplished by eliminating diagonal truss members and was tested at the fabricating shop, where the cross section was applied experimentally in a building addition. This application of the portal truss principle was used in a fertilizer plant built for the International Agricultural Corporation at Chicago Heights, Ill., in 1938.

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[3] "Welding in the Design and Construction of Industrial Plants," by Albert S. Low, *The Welding Journal*, Am. Welding Soc., May, 1940, p. 329.

A total of more than 60,000 tons of steel was welded at the Cleveland shops under Mr. Ferguson's management and included the framework for several hundred buildings, among which are the American Rolling Mill Company's research laboratory at Middletown, Ohio; a bicarbonate of soda plant at Syracuse, N. Y.; the controlled-conditions factories of Simonds Saw and Steel Company, at Fitchburg, Mass.; and the Allison Division of General Motors in Indianapolis, Ind.

Essentially a "family man," Mr. Ferguson's chief interest was always in his wife, the former Bessie Norris of Toledo, to whom he was married in 1900, and his four children, Edmund Easton, Thomas Warner, Norris Porter, and Elizabeth (Mrs. R. E. Daus). An occasional golfer in earlier years, his primary source of active recreation was motoring, and he was known to have driven as many as 725 miles, just for the love of it, on a Sunday.

He was an occasional contributor to technical publications, and served on arbitration boards and as an expert witness in court cases involving structural practices. He was a member of the American Welding Society and was a registered professional engineer for civil and structural work in the State of Ohio.

Mr. Ferguson was elected an Associate Member of the American Society of Civil Engineers on June 1, 1904, and a Member on January 3, 1911. He became a Life Member on January 1, 1939.

FRED EUGENE FOSS, M. Am. Soc. C. E.[1]

DIED JANUARY 18, 1942

The friends and associates of Prof. Fred Eugene Foss were deeply grieved at his passing on January 18, 1942, at Bethesda, Md.

He was born at Wales, Me., on June 11, 1862, the son of Uriah and Abby (Furlong) Foss. A practical viewpoint inherited from his New England parentage was evidenced in his splendid sense of proportion and his well-balanced judgment throughout his active professional life.

He was graduated from Bates College at Lewiston, Me., in 1883 with the degree of Bachelor of Arts, and in 1886 he received the degree of Master of Arts from the same institution. Following his graduation from Bates, he enrolled in the course in civil engineering at the Massachusetts Institute of

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[1] Memoir prepared by R. C. Brumfield and Elton D. Walker, Members, Am. Soc. C. E.

Technology in Boston, Mass., from which he was graduated in 1886 with the degree of Bachelor of Science. Later, Bates College conferred upon him the honorary degree of Doctor of Science.

From 1886 to 1889 he was employed by the Minnesota and North Western Railroad Company (later the Chicago, St. Paul and Kansas City Railway Company, and, still later, the Chicago, Great Western Railroad Company) in the various grades of transitman, resident engineer, and assistant engineer of maintenance of way. From 1889 to 1891 he was employed as resident engineer by the Chicago Union Transfer Railway Company.

In 1891 Mr. Foss returned to the Massachusetts Institute of Technology as instructor in civil engineering to arrange the material for, and to teach, a new course in railway and highway engineering. In 1893, after two years at the Institute, he went to The Pennsylvania State College at State College, Pa., as head of the Department of Civil Engineering. Here he installed the first hydraulics laboratory at the college, and had an important part in the development of the testing materials laboratory. In addition to his teaching work he was engaged in engineering practice that included work as borough engineer for State College from December, 1900, to June, 1906.

In 1907 Professor Foss resigned his position at The Pennsylvania State College to accept an appointment as head of the Department of Civil Engineering at Carnegie Institute of Technology in Pittsburgh, Pa. He remained at Carnegie Institute until 1909, when he went to New York, N. Y., as head of the Department of Civil Engineering at The Cooper Union. He held this position until his retirement as Professor Emeritus in 1938.

During his period of service as head of the Department of Civil Engineering at The Cooper Union, the Department expanded several fold in both personnel and in its services to the students of the institution. Particularly notable among the accomplishments of Professor Foss during this period was his work in planning and installing the Materials Testing and Hydraulics Laboratories. These laboratories are unique in the completeness and compactness of their equipment, and in the opportunities for experimental work which they offer to the student body. Professor Foss served as Secretary to the Faculty from 1914 to 1938, and exerted a great influence on that body through his gentlemanly qualities and sound judgment.

While he was at The Cooper Union, he also served for eighteen years as engineering examiner for the Municipal Civil Service Commission of New York City.

Professor Foss's long teaching service was interrupted by two leaves of absence, each for one year. During his first leave he took the position of principal assistant superintendent of construction for the J. G. White Engineering Corporation on the construction of the Raleigh and Pamlico Sound Railway from 1906 to 1907. During his other leave he was director of the Physical Testing Laboratories, Ordnance Department, United States Army, in the first World War.

Undoubtedly, one of the greatest of Professor Foss's many quiet achievements during his long engineering career was his spiritual influence on the many students with whom he came in contact. No class left him without the impress of his fine engineering knowledge and the splendid example of gentlemanly qualities which he embodied. Such achievements are difficult to measure, but they are very important.

Professor Foss did committee work for the National Research Council during the first World War. He was a Director of the Railroad Cooperative Building and Loan Association. He was a charter member of the Society for the Promotion of Engineering Education and a life member of the American Railway Engineering Association. He held membership in the American Society for Testing Materials, American Road Builders' Association, Sons of the American Revolution, Phi Beta Kappa fraternity, Phi Kappa Phi fraternity, The Seigniory Club of Quebec, and Lucerne-in-Maine Country Club. He was a member of the Board of Governors of the latter club.

He was married three times: In 1889 to Mittie Hanscom at Oak Park, Ill.; in 1908 to Louise Rust at Cleveland, Ohio; and in 1939, in New York City, to Jane Emerson Chadbourn, who survives him.

Professor Foss was elected a Member of the American Society of Civil Engineers on April 1, 1903, and became a Life Member in January, 1933.

HENRY NEWTON FRANCIS, M. Am. Soc. C. E.[1]

Died February 28, 1942

Henry Newton Francis, the eldest son of Blinn and Lucy (Hart) Francis was born in West Hartford, Conn., on November 6, 1853. He was the second eldest of a family of nine children. His father was a building mover. Both parents were Connecticut Yankees.

Mr. Francis was educated in the elementary schools of West Hartford and spent part of two years in the Hartford, Conn., High School. In the spring of 1871 he went to Cranston, R. I., where he was employed by the City of Providence, R. I., as a junior engineer under the late Samuel M. Gray,[2] M. Am. Soc. C. E., on the new water supply then being built. He continued on this work until its completion, about 1877, when he became connected with the city engineer's office in Providence as an assistant engineer in charge of various street, sewer, bridge, and harbor improvement projects, as well as further extensions of the water supply system.

In 1893 Mr. Francis became superintendent of the Narragansett Improvement Company of Providence, which was engaged in laying asphalt paving in Providence, Newport, R. I., Woonsocket, R. I., New Bedford, Mass., Taunton, Mass., and other near-by localities. In 1901 Mr. Francis was for a short time a contractor on his own account, and then became employed as engineer for the late Frederick E. Shaw, a prominent New England contractor. In

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[1] Memoir prepared by Paul H. Francis, Washington, D. C., and Carl Francis, Audubon, N. J.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1679.

1913 Mr. Francis was again employed under Mr. Gray and was engaged in preliminary property surveys for acquisition of property necessary for the new Scituate Reservoir, which was a major extension of the Providence water supply system. He was one of the two engineers who were connected with the first water supply system for the City of Providence which was planned to be supplemented later by a second system. He was the only one to live to see the second system completed. He resigned and retired in 1923. His last work was the construction of the small Horseshoe Dam at North Scituate, R. I., a part of the Scituate Reservoir project.

Mr. Francis was active in politics and was a member of the School Committee, and a member and president of the City Council of Cranston for two terms. He was also a Representative for two terms from the third ward of the City of Cranston in the General Assembly of the State of Rhode Island.

He was active in fraternal organizations and was an Odd Fellow, a Red Man, a Past Sachem of Wamsutta tribe, a Past Great Sachem of the District of Rhode Island, a Thirty-Second Degree Mason, a Past Master of Nestell Lodge, a Past High Priest of Providence Royal Arch Chapter, and a member of numerous other Masonic organizations.

Mr. Francis was one of the earliest advocates of the "good roads" movement and was instrumental in starting and building many miles of water-bound macadam in his own town of Cranston about 1890. It is also interesting to note that Mr. Francis saw the economic advantages of good roads to transportation as well as to real-estate values. Probably he was one of the first engineers to sense the fact that hard-surfaced roads need a solid concrete foundation, and he put this idea into practice in the actual construction of a number of miles of streets in Providence, which stood up well under heavy travel. Easy grades, long-radius curves, and good drainage of new roads were advocated and put into use by Mr. Francis many years before it was generally realized how important they were to become in modern road construction.

He was one of the first bicycle enthusiasts, and a charter member of the League of American Wheelmen. He was also a member of the Touring Club de France and of the Cyclists Touring Club of England. He made two bicycle trips to Europe accompanied by his eldest son, Carl, and toured England, France, Italy, and Switzerland. He was also one of the early automobile owners in Rhode Island, starting in 1903 with the well-known "one-lung" Cadillac. He drove his car to Florida for the winter and returned north in the spring each year until well past eighty.

He was married to Emily Jane Pettis of Putnam, Conn., in 1878. There were three children by this marriage, two of whom, Carl and Paul Hart, survive him. Two sisters also survive him. His first wife died in 1914 and in 1917 he was married to Louise Lissette Hilkeny who died in 1939.

Mr. Francis was elected a Junior of the American Society of Civil Engineers on March 1, 1876, and a Member on November 7, 1888. At the time of his death he was the oldest member, in years of membership, of the Society.

MANFORD W. FURR, M. Am. Soc. C. E.[1]

DIED NOVEMBER 16, 1941

Manford W. Furr was born on a farm near Veedersburg, Ind., on August 24, 1888, the son of Sylvan and Eva (Wilson) Furr. After completing the high school course at Veedersburg, he entered Purdue University at Lafayette, Ind., from which he was graduated with the degree of Bachelor of Science in Civil Engineering in 1913. He was awarded the degree of Civil Engineer by Purdue in 1925 and the degree of Master of Science by Kansas State College, at Manhattan, in 1926.

After subordinate positions of a few months each with the American Tie and Creosote Company and the Lafayette and North-Western Traction Company, he became an instructor in civil engineering at the State University of Iowa, at Iowa City, in September, 1915. In September, 1917, Mr. Furr went to Manhattan as an instructor in civil engineering at the Kansas State College. He was advanced rapidly and attained a full professorship, in charge of instruction in highway engineering, in 1927.

Manhattan was his residence during the remainder of his life, and Mr. Furr's fine character and technical ability were soon recognized in the community. He accepted many assignments to service in its civil and social activities, and was an active member of the Christian Church, serving, more than once, on building committees and for several years as treasurer and deacon. He was a member of the Manhattan Country Club and of the Chamber of Commerce, in both of which organizations he filled frequent committee appointments. For several years he was a member of the Manhattan Planning Commission.

Mr. Furr was an enthusiastic member of the Society, and of its Kansas Section. He was for several years the faculty sponsor of the Kansas State Student Chapter, and on numerous occasions took student representatives to the regional meetings. Nearly every summer, when not prevented by other work, he attended the Annual Convention of the Society and the annual meeting of the Society for the Promotion of Engineering Education (S. P. E. E.), of which society he was also a member. At different periods he was chosen as chairman of the Kansas Section of the Society, and of the Kansas-Nebraska Section and the Civil Engineering Section of the S. P. E. E. He was a member of the American Road Builders Association, the Kansas Engineering Society, and the honorary fraternities of Phi Kappa Phi and Sigma Tau.

Although primarily a teacher, Mr. Furr never missed a chance to improve his contact with outside affairs through summer work with railroads, consulting firms, or state departments. His first interest was in highway engineering. He was closely associated with the entire development of the Federal Aid System in Kansas, and he assisted with the preparation of plans for a number of the earlier projects.

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[1] Memoir prepared by L. E. Conrad, F. F. Frazier, and C. H. Scholer, Members, Am. Soc. C. E.

On September 12, 1915, he was married to Lela Osborn in Lafayette. He is survived by his widow; his mother; a sister, Mary E. Furr; and a brother, Omer Furr.

Mr. Furr was elected an Associate Member of the American Society of Civil Engineers on January 15, 1923, and a Member on August 17, 1936.

JOSEPH GOODMAN, M. Am. Soc. C. E.[1]

Died May 9, 1941

Born in Sáros-Pâtak, Hungary, on January 14, 1876, Joseph Goodman, at the age of ten, was brought to the United States by his parents, Leopold and Amelia (Feldmesser) Goodman. At that time he became a resident of New York, N. Y., and remained so until his death.

He attended New York City public schools and later the College of the City of New York, from which he was graduated with the degree of Bachelor of Arts in 1894. He then entered the School of Mines of Columbia University, in New York City, from which he was graduated with the degree of Civil Engineer in June, 1898. His work in college was outstanding and, after graduation, he served for several months as assistant instructor in surveying at Columbia University's Summer School, near Litchfield, Conn.

In October, 1898, he entered the employ of the City of New York as a topographical draftsman and computer in the Department of Public Improvements. He served in this position until July, 1900. After a short period of private employment as structural draftsman and detailer, he re-entered the employ of the city in January, 1901, as transitman in the Bureau of Water Supply of the Borough of Brooklyn, being promoted to assistant engineer in March, 1904.

At that time, the maintenance, operation, and extension of the water supply system in the Borough of Brooklyn were under a separate chief engineer, there being a chief engineer with headquarters in Manhattan who functioned in the other four boroughs of the city. The water system of the Borough of Brooklyn was unique, in that the supply was derived entirely from ground-water sources through a system of wells and infiltration galleries located along the south shore of Long Island. This was probably the largest development of its kind in the country. Mr. Goodman's early work in the Bureau was largely in connection with the maintenance and extension of such supplies, he having been in charge of the preparation of plans and specifications and of the construction of new wells and filters. He was responsible also for the planning and construction of a system of high-pressure fire service mains installed in Brooklyn in 1909, and of the three driven well developments and pumping plants within that borough that directly supplied its distribution system. His work at this period included the construction of pumping plants on Long Island, which were required to supply additional water for the Borough of Brooklyn; the operation of these pumping plants (approximately thirty in number); and the maintenance and operation of the high-pressure fire service stations.

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[1] Memoir prepared by Patrick Quilty, M. Am. Soc. C. E., in collaboration with William Flannery, Chf. Engr., Dept. of Water Supply, Gas and Electricity, City of New York, New York, N. Y., and Tobias Hochlerner, M. Am. Soc. C. E.

In 1910, the position of chief engineer, Borough of Brooklyn, was abolished, and the engineering activities of the entire department were placed under a single chief engineer. In this reorganization, the Division of Investigation, Design and Statistics of the Bureau of Water Supply, an adjunct to the office of the chief engineer, was established with Mr. Goodman in charge.

The functions of the division were sufficiently broad in scope to cover the entire field of water supply engineering. They included the preparation of plans, specifications, contracts, etc., for all water-main extensions and improvements to the distribution system in all boroughs, and for well plants, purification works, buildings, and miscellaneous structures, as well as the installation of mechanical equipment; also investigations of all kinds pertaining to the maintenance, operation, and extension of the system. These also included studies of the adequacy of plants of the several private water companies then operating within city limits, many of which were subsequently acquired by the city, there being only two remaining companies operating in 1941. Mr. Goodman had charge of, and personally performed, a large amount of work in connection with the appraisal of these plants for their acquisition by the City of New York. He also had an important part in determining the rates to be paid these companies for fire protection in the form of hydrant rental, and developed special methods of procedure for the solution of the problems involved.

The work of his division also included the compilation of statistical data relating to rainfall and runoff in the watersheds, the consumption of water, etc., such data being the basis for determining the most advantageous and economical operation of the combined systems of the city's supply and for estimates of the anticipated needs of water in the immediate future.

Following a vacancy that occurred on February 10, 1933, Mr. Goodman was appointed acting deputy chief engineer and, on February 1, 1934, deputy chief engineer of the Bureau of Water Supply. On June 1, 1934, he was appointed acting chief engineer and on May 26, 1936, chief engineer of the Bureau, taking complete charge of the maintenance, operation, and extension of the water supply system. Thus, after thirty-five years of service, he had risen from the lowest ranks to the highest technical position in the Department, but it was not long before he was called to even higher administrative duties. On September 17, 1936, Mayor Fiorello H. LaGuardia designated Mr. Goodman as acting commissioner and, on October 27, 1936, he was appointed by the Mayor as commissioner of the Department of Water Supply, Gas and Electricity.

The ceremony at which he was sworn in to his new office by the Mayor on that date, at the City Hall, was witnessed by a large gathering of distinguished engineers, public officials, and Department employees. The Mayor then stated that the appointment of Mr. Goodman was one which was out of the ordinary, being in conformity with the new city government policy of placing at the head of certain departments Civil Service career men who had long been connected with these departments and had acquired specialized knowledge to fill the positions, such appointments being without regard to political standing or affiliations of the appointees. He noted that Mr. Goodman had devoted his life to city service in the Water Department, thus making him an ideal or "No. 1 career man," and he was frequently so referred to thereafter.

The appointment was singularly opportune and fortunate for the city because of the water supply situation then prevailing. It was one of those critical periods of impending water shortage that have occurred at intervals during the city's growth, the demand on the system having about reached the limit of the dependable supply of the existing sources, and a project to provide an additional supply had just been initiated. This project, involving the construction of the Delaware Aqueduct and the development of the watersheds of the upper Rondout Creek, the Neversink River, and the East Branch of the Delaware River, is now (1941) well under way, but the delivery of the first installment from these new sources is not expected until 1945. There was, therefore, particular need for an engineer having Mr. Goodman's long experience in and knowledge of the water system to head the Department. He was admirably fitted to take measures to carry on and maintain an adequate supply for the city until the new sources became available.

Despite onerous duties in the administration of the two other bureaus of the Department, Mr. Goodman devoted much time to the engineering activities of the Bureau of Water Supply and the operation of the system. An active program was undertaken for the installation of new facilities to pump Croton water within the city and for improvements in the Long Island watershed, so as to enable these sources to be utilized to the fullest possible extent. Commissioner Goodman also initiated an application to the State Water Power and Control Commission for permission to drive new wells to secure an additional supply of 50 mgd in Nassau County, such supply to be developed, if and as it might be needed, before 1945.

The water supply situation was rendered still more serious by the severe drought that began in May, 1939, and continued into March, 1940, being relieved then by heavy rains. During this dry period, the impounding reservoirs, which were full in April, 1939, were steadily depleted so that their contents were only about 40% of their capacity in March, 1940. In this situation, Commissioner Goodman was in active charge of a publicity campaign for the reduction of consumption and the conservation of the supply, this being carried on in the press, over the radio, in the schools, and by notices placed in the subways, hotels, etc. The use of water for flushing and sprinkling streets was temporarily prohibited by the Commissioner.

The results of this campaign of conservation and the measures taken to reduce the demand on the system were very gratifying, the consumption of water in the early months of 1940 being reduced by about as much as 120 mgd, as compared with the same months of 1939. The effects were evident throughout 1940 during which the consumption, within the municipal system, averaged 862 mgd, as against 938 mgd in 1939.

In addition to his many other duties, Commissioner Goodman also was occupied with problems relating to the defense situation, having been appointed a member of Mayor LaGuardia's City Defense Council. His first concern in this matter was to take steps to provide for guarding the system and to make arrangements, in cooperation with the Board of Water Supply, for a force to patrol the watersheds and aqueducts. He was instrumental in having the Governor of the State of New York declare an emergency, as a result of which some 20,000 permits hitherto issued for fishing and boating on the reservoirs were rescinded. A comparatively small number of such permits were re-issued under new regulations, which restricted the areas for these activities.

In the administration of the Bureau of Gas and Electricity, Commissioner Goodman also effected substantial savings in the cost of the lighting of streets and public buildings, these services being provided under contracts with private utilities.

Commissioner Goodman, although of slight physique, was an indefatigable worker. He underwent a severe operation in June, 1940, and retired on July 8 of that year in the belief that his usefulness would be impaired to such an extent that he could not continue his many duties as commissioner. However, he was reappointed by the Mayor and, as soon as his health improved, he returned to service, continuing active almost until his death.

His long service was always marked by intense loyalty and devotion to the interests of the city, and he endeavored to imbue and inspire his subordinates and associates with the same objectives. He was held in high esteem and affection by all of the personnel and never failed to attend any gathering, whether social or professional, of Department employees so as to obtain more intimate contact with them and become personally acquainted with as many of the 3,000 employees as possible. Commissioner Goodman's name will be added to those of the many distinguished engineers and administrators who have been engaged on the construction and operation of New York City's vast water supply system since its inception more than one hundred years ago.

In addition to his membership in the Society, he was also for many years a member of the American Water Works Association and the Municipal Engineers of the City of New York.

On June 28, 1916, he was married to Dorothy Goldberger of Perth Amboy, N. J., who survives him. Their only son, Lawrence Eugene Goodman, Jun. Am. Soc. C. E., is following his father's profession, having graduated from the Columbia School of Engineering with the degree of Civil Engineer in 1939. Mr. Goodman is also survived by a sister, Mrs. Rose Glucksman; and two brothers, Louis Goodman, Assoc. M. Am. Soc. C. E., and George Goodman.

Mr. Goodman was elected an Associate Member of the American Society of Civil Engineers on June 6, 1906, and a Member on October 10, 1916. He became a Life Member in 1941.

CHARLES SAMUEL GREEN, M. Am. Soc. C. E.[1]

Died October 23, 1941

Charles Samuel Green was born in Toomsboro, Ga., on July 4, 1873, the son of Samuel Gartensteig and Esther (Green) Gartensteig, later Weiss. In 1918 he changed his name to Green, adopting his mother's maiden name.

As a young child, he moved to New York, N. Y., with his family. He was a graduate of the New York City public schools, the College of the City of New York (which he entered at the age of fourteen), and Columbia University, School of Mines, in New York City. In 1891 he received the degree of Bachelor of Science from the College of the City of New York and in 1895 the degree of Civil Engineer from Columbia University.

Upon his graduation from the latter university in 1895, he was appointed assistant engineer of the Long Island Railroad Company, Montauk Division, and put in charge of field party on location and construction. From 1895 to 1896 he was assistant in the office of the late William Barclay Parsons,[2] Hon. M. Am. Soc. C. E., on design of the first rapid transit subway for New York City.

In 1896 Mr. Green became assistant engineer, Bureau of Sewers, Borough of The Bronx, New York City, in charge of construction of sewers, including the contract for the Broadway sewer (approximate cost, $1,000,000). He held this position until 1902, when he became assistant engineer, Bureau of Highways, Borough of The Bronx, in charge of numerous contracts for grading and paving streets and of design and construction of bridges and viaducts of steel, masonry, and reinforced concrete. From 1910 to 1918 he was engineer of design, Office of the President of the Borough of The Bronx, in charge of design and preparation of plans and contracts for all improvements under the jurisdiction of the President of the Borough, including paving, bridges, viaducts, walls, culverts, etc. (approximate cost, $3,000,000 during 1911). He was also at various times construction engineer and designer of foundations and steel frames for buildings.

In 1918, during the World War, he was commissioned a Captain in the Engineer Corps of the United States Army. He served overseas in France in 1918 and 1919 as acting Major in command of a labor battalion of engineers.

Mr. Green was a member of the Harmonie and City Athletic Clubs of New York for several years. He was a charter member of Fairview Country Club, serving as president, governor, and for many years chairman of the Green Committee. In the latter capacity he laid out and built three different golf courses.

At the time of his death he was in private practice as a consulting engineer, with an office at 119 West 57th Street, New York City. He was an expert at drawing difficult specifications and contracts and was a master in the use of correct, terse English. His letters and contracts were models of comprehensive brevity.

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[1] Memoir prepared by Charles Griffith Moses, New York, N. Y.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1485.

During the last few years of his life he devoted much time to the instruction in English, and Americanization, of European war refugees, chiefly professional men. He not only taught a considerable number of German, Austrian, and Italian engineers colloquial English and American practices, but was remarkably successful in instilling in these foreign, professional contemporaries, many of whom were past middle age, the fine ideals of democracy and constitutional government so dear to his heart.

Mr. Green was never married. All his life he was deeply interested in religious, ethical, charitable, and educational matters and cheerfully shared with others his wealth of wisdom, good judgment, and kindliness. His keen sense of humor and his remarkable fund of general information made him outstanding and popular with his host of friends. He was an ardent fisherman and golfer.

Mr. Green was elected a Junior of the American Society of Civil Engineers on October 6, 1896; an Associate Member on May 6, 1903; and a Member on April 30, 1912.

JOHN LYLE HARRINGTON, M. Am. Soc. C. E.[1]

DIED MAY 20, 1942

John Lyle Harrington was born on December 7, 1868, at Lawrence, Kans., the son of Robert Charles and Angeline Virginia (Henry) Harrington. During most of his youth, the family lived on their homestead near De Sota, about 15 miles east of Lawrence. His early schooling was very irregular, as was most often the case with the boys on Kansas farms in those days. He entered the University of Kansas, at Lawrence (by passing entrance examinations), with a total of only about four years attendance at grade and secondary schools. He was graduated from the University in 1895 with the three degrees of Bachelor of Arts, Bachelor of Science, and Civil Engineer. He found time, also, for a large amount of extra reading and for at least the normal participation in student affairs and the social life of the University. He was a member of Sigma Nu fraternity.

His intense desire for knowledge is evidenced by the record of his experience in the twelve years from 1895 to 1907, and he often said of those years that, when he had lost the interest of learning something new in one job, he began to look for another. During two summer vacations, and from June, 1895, to March, 1896, he was in the employ of the late J. A. L. Waddell, Hon. M. Am. Soc. C. E., consulting engineer, at Kansas City, Mo. He then held various positions, as follows: March to August, 1896, in the drafting room of the Elmira Bridge Company at Elmira, N. Y.; August, 1896, to July, 1897, with the Pencoyd Iron Works at Philadelphia, Pa., in charge of work in the drafting room; July, 1897, to January, 1898, with the Keystone Bridge Works of the Carnegie Steel Company at Pittsburgh, Pa., in charge of preparation of general and detail plans for numerous bridges, including various bridges and viaducts for the Union Railroad approach to the Pittsburgh and Lake Erie Railroad bridge over the Ohio River; January to September, 1898, with the Cambria Steel Company at Johnstown, Pa., first as assistant superintendent, Structural Department, designing the structural shops, then in charge of engineering, and later in charge of the shop and structural material yards; September, 1898, to March, 1899, with the Bucyrus Company at South Milwaukee, Wis., as assistant chief engineer and assistant superintendent; March to December, 1899, with the Northwestern Elevated Railroad Company in Chicago, Ill., as assistant to the chief engineer, in charge of preparation of shop plans and of inspection of fabrication of steelwork at the shops of the Elmira Bridge Company and Union Bridge Company; December, 1899, to November, 1900, with the Berlin Iron Bridge Company at East Berlin, Conn., as designer; November, 1900, to November, 1901, with the Baltimore and Ohio Railroad Company at Baltimore, Md., as assistant engineer of bridges and buildings; November, 1901, to January, 1905, with the C. W. Hunt Company in New York, N. Y., first in charge of preparation of detail drawings of structural work and of general contract plans for bidding purposes, and later as executive engineer in charge of estimating, contracting, etc.; and January, 1905, to January, 1907, with the Locomotive and Machine Company at Montreal, Quebec, Canada (a subsidiary of the American Locomotive Company), as chief engineer and general manager, having charge of the building and then the operation of its plant.

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[1] Memoir prepared by Frank M. Cortelyou, M. Am. Soc. C. E., in collaboration with R. P. Woods, E. E. Howard, and O. W. J. Anschuetz, Members, Am. Soc. C. E.

During these twelve years from 1895 to 1907 Mr. Harrington unquestionably always had in mind the definite goal of the independent practice of his profession as a consulting engineer. He chose such a course in preference to accepting an offer of an annual salary of $25,000 to become a vice-president of one of the large railway systems of the United States. Thus began thirty-five years of practice as consulting engineer—from 1907 to 1914 as a partner in Waddell and Harrington, from 1914 to 1928 as senior partner of the firm of Harrington, Howard and Ash, and after 1928 as senior partner of the firm of Harrington and Cortelyou, all specializing in the field of bridge engineering. Some 250 projects were handled by these firms, the services performed being generally the complete engineering work from inception to completion, although at times the services covered only the design of and detail plans for the structure and in rarer cases only consultation work. Their work covered all sections of the United States, numerous structures in Canada, and some in foreign countries, as shown by the following principal projects:

Bridges over the Mississippi River at St. Paul, Minn. (Chicago Great Western Railroad), Keithsburg, Ill., Alton, Ill., Louisiana, Mo., Cape Girardeau, Mo., Vicksburg, Miss., and Baton Rouge, La.; bridges over the Missouri River at Fort Bellefontaine, Mo., Booneville, Mo., Waverly, Mo., Kansas City, Mo., Rulo, Nebr., Blair, Nebr., Yankton, S. Dak., and Mondak, Mont.; bridge over the Ohio River at Paducah, Ky.; seven bridges over the Chesapeake and Delaware Canal in Delaware and Maryland; seventeen bridges over the Welland Ship Canal in Canada; nineteen bridges and viaducts for the Canadian Northern Pacific Railway in British Columbia; three bridges in Vancouver, B. C.; four bridges in Tacoma, Wash.; two bridges in Spokane, Wash.; the Columbia River Bridge between Portland, Ore., and Vancouver, Wash.; four bridges over the Willamette River in Oregon, including the double-deck, vertical lift bridge at Portland; two vertical lift bridges and one bascule bridge over the Sacramento River, the Dumbarton Highway Bridge over San Francisco Bay near Palo Alto, and the concrete arch bridge over Arroyo Seco at Pasadena, all in California; two bridges over the Tombigbee River and one over the Tennessee River, in Alabama, and also the Mobile Bay Bridge at Mobile, Ala.; bridges over the Arkansas River at Van Buren, Morrilton, Little Rock, and Mozart, Ark.; two bridges over the St. Johns River at Jacksonville, Fla., and the Lee Street Viaduct at Jacksonville; nine bridges over the Miami River, all of the bascule type, at Miami, Fla.; the Spring Street Viaduct at Atlanta, Ga.; the vertical lift bridges of the Pennsylvania Railroad over the Chicago and Calumet rivers and that of the Lake Shore and Michigan Southern Railway (later the New York Central System) over the Calumet River at Chicago; Market Street Viaduct at Shreveport, La.; the Louisiana and Arkansas Railway Bridge over the Atchafalaya River (extension on west end) at Simmesport, La.; the vertical lift bridge over the Duluth Ship Channel at Duluth, Minn.; fifteen bridges for Kansas City and Jackson County, Missouri, of which the most important are the 12th Street and 23d Street viaducts; bridges over Great Egg Inlet at Ocean City, over the Cooper River at Camden, and over the Hackensack River at Hackensack, all in New Jersey; the West Third Street Viaduct in Jamestown, N. Y.; the Northhill Viaduct at Akron, Detroit Superior Viaduct at Cleveland, Sandusky Bay Bridge at Sandusky, and eight bridges over the Miami, Mad, and Wolf rivers at Dayton, all in Ohio; the concrete arch bridge over the Colorado River at Congress Avenue, Austin, Tex.; the vertical lift bridge over the Southern Branch of the Elizabeth River at Norfolk, the Wasena Viaduct at Roanoke, and the Ninth Street Bridge at Richmond, all in Virginia; the Piscataqua River Bridge at Portsmouth, N. H.; the Massachusetts Avenue and K Street bridges over Rock Creek and Rock Creek and Potomac Parkway in Washington, D. C.; the Waikato River Bridge at Hamilton, New Zealand; and the Don River Bridge at Rostoff, Russia.

His most outstanding contribution to bridge engineering was in the development of the vertical lift type of movable span. He brought to his partnership with Mr. Waddell a considerable experience in mechanical engineering work, principally gained during his four years with the C. W. Hunt Company. This experience, applied to the operating problems of the vertical lift type of span, resulted in numerous patents for the firm's designs and the construction of twenty-seven vertical lift spans. Two of these are worthy of special mention because of their variation from the usual type. The double-deck railway and highway bridge over the Missouri River at Kansas City has the upper (highway) deck carried by a through, fixed-truss span and the lower deck (double-track railway), over 400 ft long, suspended on vertical hangers and capable of being raised and lowered, the hangers telescoping into the vertical members of the through fixed trusses. The double-deck highway and railway bridge over the Willamette River at Portland has a similar lower deck, but the entire span can also be raised and lowered, the operation of the lower deck relative to the upper being independent of the operation of the whole span. In all, seventy-four vertical lift bridges have been built by Mr. Harrington's firms.

In August, 1932, Mr. Harrington was called to Washington, D. C., by President Herbert Hoover, Hon. M. Am. Soc. C. E., to become a member of the Engineers Advisory Board of the Reconstruction Finance Corporation (RFC) when that agency was formed to aid in combating the 1929–1933 depression. This board dealt with only one phase of the Corporation's activities—the financing of new construction projects. Mr. Harrington had a most prominent part in the organization of the Board's activities and in its work, and he remained with the RFC as chief engineer after the authority to make such loans was transferred to the Public Works Administration. Essentially, his entire time from August, 1932, to January, 1934, was devoted to this service, which he accepted without hesitation as a patriotic duty at considerable financial loss and, as it developed, at material cost to his health. This Board dealt with nearly 700 applications for loans on self-liquidating projects for amounts ranging from a few thousand to over 100 million dollars. Loans were made on approximately one third of the applications and all but one proved profitable, enabling the RFC to dispose later of the securities taken.

Mr. Harrington had a wide range of interests outside of his own immediate business affairs. He was greatly interested in engineering education and gave liberally of his time in addressing engineering students. He was particularly active in the American Society of Mechanical Engineers (A. S. M. E.), serving as vice-president from 1920 to 1922 and as president in 1923. He was a member of the American Engineering Council as representative of the A. S. M. E. from 1926 to 1932. For many years he was on the Board of Trustees of Robert College at Istanbul, Turkey, resigning only a few months before his death. He was a member of the American Railway Engineering Association, the American Society for Testing Materials, the Institution of Civil Engineers (England), The Engineering Institute of Canada, the American Association for the Advancement of Science, and the Newcomen Society. He was a member of Sigma Xi, Tau Beta Pi (president in 1917), and Theta Tau. He received from McGill University, in Montreal, the degree of Bachelor of Science in 1906 and that of Master of Science in 1908. In 1930 he received the honorary degree of Doctor of Engineering from Case School of Applied Science in Cleveland, Ohio.

In Kansas City, which was his home continuously after 1907, he was a member of the University Club, Mission Hills Country Club, and the Engineers Club (its first president in 1911). He was a life member of the German

Hospital Association (later renamed Research Hospital). He was from 1932 to 1941 a member of the Cosmos Club in Washington, D. C.

On June 21, 1899, Mr. Harrington was married to Daisy June Orton of White Cloud, Kans., also a graduate of the University of Kansas. He is survived by her and their only son, Thomas Orton Harrington.

Mr. Harrington had a most forceful character and remarkable energy and ability. His success was not a result of chance but of his own qualities. He had a keen and active mind and the power of memory such as few possess. He was a fluent speaker and writer, but wrote very little for publication. Essentially all the addresses he made were given extemporaneously. He was ever a voluminous reader, and there was no limit to the variety of subject in which he took interest and sought information. The structures which he built are monuments to his memory, but these do not measure his accomplishments through the influence he exerted on the lives of others. Many could say as one has said that "My close personal contacts with him during those five years have always been considered invaluable. Mr. Harrington was one of the most remarkable and capable men I have ever known, and I deeply appreciated his status as one of the outstanding men of our time, both in engineering and without."

Mr. Harrington was elected a Junior of the American Society of Civil Engineers on August 31, 1897; an Associate Member on October 4, 1899; and a Member on December 1, 1903.

BURR POWELL HARRISON, M. Am. Soc. C. E.[1]

Died January 21, 1942

Burr Powell Harrison was born of a family long prominent in Virginia history. His forebears included pre-Revolutionary ancestors who seemed prophetic of life interest in engineering. For instance, the earliest known exploration of the area of Loudoun County, Virginia, where Mr. Harrison lived and died, was made in 1699 by Burr Harrison of Chipawamsic, a lineal ancestor, under the direction of Col. Francis Nicholson, Governor of Virginia. In 1787 Col. Leven Powell, another ancestor, who had raised and fought with the 16th Regiment of Virginia Continentals under Gen. George Washington, laid out the town of Middleburg on the old road running to Ashley's Gap. In addition, there have been distinguished contributors to American colonial and national life, including a member of the Continental Congress, an ambassador to the Court of St. James, Senators, Congressmen, and other men of prominence.

Born on December 7, 1875, Burr Powell Harrison was the son of Capt. Powell Harrison, a prominent lawyer of Leesburg, Va., who had served as a Captain of Cavalry under the Confederate Cavalry leader, Gen. "Jeb" Stuart.

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[1] Memoir prepared by J. D. Fauntleroy, M. Am. Soc. C. E.

His mother was Janet Fauntleroy, daughter of Capt. Charles Fauntleroy, who during the "War Between the States" commanded the Confederate cruiser *Shenandoah.*

In his early youth Mr. Harrison attended public and private schools in Leesburg. In 1893 he entered Virginia Military Institute at Lexington, Va., as a third classman. He was graduated with the degree of Bachelor of Science in Civil Engineering in 1896 and in 1914 he was awarded the degree of Civil Engineer. For two years he taught in the public schools of Fairfax County, Virginia. He then sought employment as a civil engineer. At that time work of such nature was scarce in Virginia, so he accepted employment as an instrumentman for the Nicaraguan Canal Commission. He returned to the United States in 1901 and was employed by the Baltimore and Ohio Railroad Company as a transitman. The next year he was made resident engineer, in charge of the construction of 7½ miles of the Quemahoning Branch of the Baltimore and Ohio Railroad in Somerset County, Pennsylvania. Upon completion of this work he was promoted to assistant engineer of the Baltimore and Ohio Railroad Company, in charge of locating parties. He left the employ of this company in 1904 and for a while was employed as engineer for the Filtration Company of America, on the construction of a filtration plant at Washington, D. C. From Washington, D. C., he went to Kansas City, Mo., where he was employed in 1904 and 1905, working up plans for the terminals and the new depot for the United Railroads.

During the years 1905 to 1907 he lived in New York, N. Y., where he was employed by N. R. Chapman and Company and by Halle and Steiglits, Brokers. From 1907 to 1912 he was employed by the U. S. Engineers, first as inspector and then as junior engineer in charge of dredging operations on the Potomac River and of deepening the channel leading to Baltimore, Md.

In 1912 he entered the employ of the Maryland State Roads Commission. One of his duties here was to organize the State Maintenance System, one of the first road maintenance systems in the United States. He remained with the Maryland Roads Commission and received several promotions, becoming first assistant engineer to the chief in 1918. He left the Roads Commission in 1918 and became harbor engineer of Baltimore City. Subsequently, he practiced as a consulting engineer in Baltimore City.

In 1922 he was appointed engineer of location and plans to the State Highway Commission of Virginia. He located the new state highway roads under the "pay-as-you-go" régime inaugurated by Gov. Harry Byrd. He was always a hard worker and never allowed anything to interfere with his duties. His great interest was to eliminate unnecessary curves and bends on highways to make them safe for the traveling public. He also tried to reduce grades, and was one of the first highway engineers to pay close attention to the elevation of curves. Once, when taking part in a discussion among a gathering of engineers, he was asked what, in his opinion, was the best type of road surface, and he replied: "The most important feature on any highway is its location. Any type you may select will wear out, but the location is permanent." Visiting engineers often praise the beautiful locations of the state highways of Virginia. They stand as a living testimonial to his skill and wisdom.

Mr. Harrison was nicknamed "Long Tangent" at the Virginia Highway Department. The chiefs of fifteen survey parties presented him with a gold Hamilton watch and gold chain, in token of their appreciation and affection. Inscribed on the inside of the watch was: "B. P. Harrison, Locating Engineer, a token of appreciation from the Locating Parties, 1927."

He left the State Highway Department in August, 1930, and entered the U. S. Bureau of Public Roads, where he had charge of the economic work in the states comprising the 10th Federal Aid District. This necessitated constant intercourse with state and county officials to find out the revenues received by various states, counties, and subdivisions, the sources from which these funds were derived, and the purposes for which they were expended and the results obtained. He was generally well liked by all with whom he came in contact. His services in the Bureau were so satisfactory that he received several promotions, the last only a short time before his death.

Mr. Harrison was proud of his birth and lineage and was always dignified and courteous in his bearings. He was a devoted husband and father. It has been commented on by several engineers who have traveled with him over Virginia that, no matter how far he had driven that day and notwithstanding how hard he had worked, he always found time at night to call his wife by long distance telephone and to inquire about her and the family.

He was a fine conversationalist, and for his friends it was always a pleasure to hear him tell about his varied experiences and how he had solved various complex problems that had arisen in his work. He was a Christian and a member of St. James Episcopal Church in Leesburg. He never used profane language.

In 1910 he was married to Louise Goldsborough, who came from a family long prominent on the eastern shore of Maryland. Her father, Dr. Brice Worthington Goldsborough, was a very prominent and much beloved surgeon and physician. Mr. Harrison is survived by his widow; a son, Burr Powell Harrison, Jr., a Captain in the U. S. Army; three daughters, Louise Dulin (Mrs. Francis Landey Patton), Nannie (Mrs. James S. Cox), and Lalla Fauntleroy Harrison; five grandchildren, Anna Harrison Cox, James S. Cox, Jr., Brice Goldsborough Cox, Louise Goldsborough Patton, and Francis Landey Patton IV; a brother, Charles Fauntleroy Harrison; and three sisters, Mrs. E. B. White, Mrs. Henry Lynn, and Mrs. Walter H. Taylor.

Mr. Harrison was elected a Member of the American Society of Civil Engineers on October 3, 1911.

GEORGE WILLIAM HAWLEY, M. Am. Soc. C. E.[1]

DIED MARCH 17, 1942

The happy smile and gracious, cheerful greeting known to his fellow workers in the office of the state engineer for more than twelve years was erased and stilled forever at 6:00 p.m. on March 17, 1942. The end came less than twenty-four hours after George Hawley had gone to the hospital for a check-up of a heart ailment of some years' standing.

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[1] Memoir prepared by Edward Hyatt and W. A. Perkins, Members, Am. Soc. C. E., and Emerson Dolliver, Assoc. M. Am. Soc. C. E.

George William Hawley, the son of James and Margaret Hawley, and the eldest of three children, was born at Portland, Ore., on November 19, 1889. He acquired his elementary and preparatory education in the schools of Portland and entered Stanford University (California) in the fall of 1909. He received the degree of Bachelor of Science in Civil Engineering in 1913 and the degree of Civil Engineer in 1916. Because of his scientific achievements, he was elected a member of Sigma Xi.

After his graduation, his entire life was spent in the pursuit of his chosen profession. His first engagement was with the Oregon Electric Company, which he left in 1914 to become construction engineer with the South San Joaquin Irrigation District. His principal work for the District was as engineer in charge of the construction of Woodward Dam, a large semi-hydraulic fill. He severed this connection in 1917 to enter the employ of the East Bay Water Company, where he rose to the position of chief construction engineer. Two major projects were completed under his régime—the later stages of San Pablo Dam and the construction of Upper San Leandro Dam, both large hydraulic-fill dams. These were in addition to the regular construction program of a large domestic water system. He remained in this position until the company was sold to the East Bay Municipal Utility District in 1928, when he became assistant director of operations for that organization. From 1925 until the latter part of 1929, he also maintained a private practice as consulting engineer on hydraulic projects.

In September, 1929, following an enactment by the Legislature placing all dams in the State of California under the jurisdiction of the state engineer, Mr. Hawley was appointed deputy state engineer in direct charge of construction, maintenance, and repair of more than 800 dams in the state. Through later amendment, more than 200 small-sized dams were removed from the list, and at the end of his régime the number under his supervision was 639. The duties of the office called for travel to all parts of the state and contact with all the hydraulic engineers and operators of all types of water projects in which dams were involved. Under his able direction, the Dam Department of the State Division of Water Resources became a highly efficient organization that had the full confidence of the general public and such federal agencies as the U. S. Forest Service and the Federal Power Commission, with which it had frequent and close contact.

On March 7, 1916, Mr. Hawley was married to Sara Nancy Williams of Portland, Ore., who survives him. Also surviving are a brother, Henry James Hawley, and a sister, Mary E. Hawley. Mr. and Mrs. Hawley were most welcome attendants at most of the engineering gatherings held on the Pacific Coast for many years.

One of George Hawley's outstanding characteristics was his kindly and gentlemanly demeanor, whether in the presence of acquaintances, superiors, associates, or subordinates. His example of loyalty to his superiors, fidelity to his job, and keen-mindedness and alertness to protect the interests of the state could well be emulated by all.

Because of his wide technical interest he held membership in the American Concrete Institute (serving as a member of the Committee on Properties of Mass Concrete), the American Geophysical Union, and the American Water Works Association. He served as a member of several committees of the Society, the more important of which were the Special Committee on Irrigation Hydraulics, and the Subcommittee on Structural Design of Embankments and Foundations of the Committee on Earth Dams and Embankments.

Mr. Hawley was elected a Junior of the American Society of Civil Engineers on September 2, 1914; an Associate Member on May 13, 1918; and a Member on January 14, 1929.

EDWARD ADOLPH HERMANN, M. Am. Soc. C. E.[1]

Died June 12, 1941

Edward Adolph Hermann was born at Hermannsburg, Washington County, Ark., on December 28, 1856. He was the son of Charles F. Hermann and Caroline (Wilhelmi) Hermann, who had come to the United States from Germany. With a relative, they founded the town of Hermannsburg, where they maintained a store and a flour mill.

During the Civil War the country was terrorized by guerrillas who robbed and murdered adherents to both sides impartially. When a Union column came through this district, the Hermann family moved to St. Louis, Mo., which was Edward A. Hermann's home most of his life.

Mr. Hermann attended the public schools in St. Louis, and in 1879 was graduated in Civil Engineering from Rensselaer Polytechnic Institute at Troy, N. Y. His early career was as follows: Draftsman in the office of M. E. C. Darly, mechanical engineer, at St. Louis, from July, 1879, to October, 1879; assistant engineer, Grand Tower Mining, Manufacturing and Transportation Company at Grand Tower, Ill., from October, 1879, to January, 1880; draftsman in Bridges Department of city engineer's office of St. Louis from January, 1880, to May, 1880; draftsman with Atlantic Refining Company of Philadelphia, Pa., from May, 1880, to July, 1880; assistant engineer, Lake Erie and Western Railroad Company, at LaFayette, Ind., from July, 1880, to October, 1881; assistant engineer, Pennsylvania Railroad Company (Pittsburgh, Fort Wayne and Chicago Railway Company and branches), from October, 1881, to April, 1882, in charge of improvements for third and fourth main tracks between Allegheny, Pa., and Dixmont, Pa., a distance of ten miles; and April, 1882, to January, 1887, assistant engineer, Cincinnati, Indianapolis, St. Louis and Chicago Railway Company (later part of Cleveland, Cincinnati, Chicago and St. Louis Railway Company [Big Four]). While with the latter company, Mr. Hermann was in charge of construction of changes in alinement of main track at several places, extension of second main track, building of several short branch lines, erection of wooden and iron bridges, and construction of piers, abutments, yards, and depots. About this time he wrote "Steam Shovels and Steam Shovel Work,"[2] which was the first book written on this subject.

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[1] Memoir prepared by C. E. Hermann, Pres., Reliance Whiting Co., Alton, Ill.

In 1899 he resigned from his position as assistant chief engineer of the Big Four Railroad Company and became sewer commissioner and member of the old Board of Improvements in St. Louis. He took an active part in the work of the Board and served on many of its committees assigned to the practical working out of plans. He was particularly active in compelling the street railways to use power brakes on their cars, and was an expert on this subject, due to his long experience on steam railroads.

After serving his term of four years as sewer commissioner, he declined reappointment, and in 1904 bought a small quarry at Alton, Ill., which he incorporated under the name of Reliance Quarry and Construction Company. This plant grew considerably under his management. The quarry contains a large quantity of high calcium rock suitable for the manufacture of pulverized limestone for domestic whiting. In 1919 a pulverizing plant was added, and the company was reincorporated as the Reliance Whiting Company. This company has continued in operation ever since and was Mr. Hermann's chief interest. He was active in its management, and continued in good health until his sudden death from a cerebral hemorrhage on June 12, 1941.

Mr. Hermann was liked and respected by the entire business community in Alton, and had a great many friends. His chief interests outside of his business were his family, his books, and his activities in the engineering profession generally.

He was married to Florence Pitzman at St. Louis on April 30, 1896, and is survived by his widow and a son, Charles Edward Hermann, who succeeded him as president of the Reliance Whiting Company.

Mr. Hermann was elected a Member of the American Society of Civil Engineers on April 6, 1887.

BENJAMIN ALEXANDER HODGDON, M. Am. Soc. C. E.[1]

DIED APRIL 6, 1941

Benjamin Alexander Hodgdon was born at Tremont, Mount Desert Island, Me., on September 6, 1875, the son of Benjamin E. and Mary (Beverly) Hodgdon. He was a descendant of Nicholas Hodgdon who came from Herfordshire, England, and settled in Hingham, Mass., in 1635. His great-grandfather, Joseph Hodgdon, enlisted in the Continental Army from Scarboro, Me., and at the close of the Revolutionary War took up a grant of land on

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[2] "Steam Shovels and Steam Shovel Work," by Edward Adolph Hermann, Engineering News Publishing Co., New York, N. Y., 1894.

[1] Memoir prepared by Alfred M. Wyman and John H. Myers, Members, Am. Soc. C. E.

Mount Desert Island, where his descendants continue to live. His father, following the traditions of the family, was owner and captain of one of the fleet of New England schooners that have now disappeared from the sea.

When Benjamin Hodgdon was still a young boy, his father was forced to retire from active service at sea because of ill health, and the family moved to Somerville, Mass. Mr. Hodgdon was educated in the public schools of Somerville and Boston, Mass., and was graduated from the Massachusetts Nautical Training School, at Boston, in 1897 after a cruise to England and the Mediterranean Sea, during which he held the rank of cadet captain.

His first engineering engagements consisted of various assignments in the vicinity of Boston. They included surveys for parts of the original Metropolitan Park System, construction of the Boston Elevated Railway, and work as inspector and transitman at the Boston Navy Yard. These assignments were interspersed with sea trips as deck officer on steamships sailing from Boston to South America, the West Indies, and New York, N. Y. He held a commission as Lieutenant (junior grade) in the Massachusetts Naval Reserve.

In September, 1904, Mr. Hodgdon joined the engineering force of the New York City Rapid Transit Railroad Commission as assistant engineer. He was employed continuously from that time until his retirement in 1938 on subway construction with that Commission and its successors. As section engineer he was in immediate charge of several of the more difficult parts of the subway construction work, now forgotten, since, unlike many of the more spectacular structures of engineers, they function out of view beneath the highways of a great city.

Between 1904 and 1907 Mr. Hodgdon was serving his apprenticeship in subway construction. Between May of the latter year and November, 1909, he was chief of party on the construction of part of the so-called Center Street Loop, which connects the Williamsburgh and Manhattan bridges over the East River with the Whitehall Street Tunnel beneath it. After an assignment to office work, he was connected, between February, 1911, and July, 1929, with another part of this loop as chief of party and as section engineer. During the period from July, 1912, to May, 1917, he was section engineer in charge of Section 2A of Route 5 and Section 2 of Route 20. These were the official names of the subways along Broadway from Walker to Howard streets and on Canal Street between Broadway and the Bowery. The latter section included a terminal station connected by passageways with the subway along Broadway. Owing to the high elevation of ground water, this was a difficult piece of construction. It was necessary to underpin several buildings and the existing subway on Lafayette Street.

Following this work, Mr. Hodgdon was in charge of the building of Section 5 of Routes 4 and 36, which was the subway under 59th and 60th streets from Seventh to Second avenues. From January, 1919, until May, 1925, he was assigned to the Track Division and during this period was in charge of laying the tracks in the 180th Street and the Jerome Avenue yards, and also superintended the laying of the tracks in the 60th Street Tunnel under the East River.

In May, 1925, he returned to construction work and until February, 1928, was in charge of the work of building a section of the four-track subway in a deep rock excavation along Central Park West from 79th to 89th streets. This work was officially known as Section 1A of Route 78 and was part of the Independent System. From February, 1928, to December, 1930, Mr. Hodgdon was in charge of construction of another section of the Independent System. It was officially known as Section 5 of Route 101 and extended along Greenwich and Sixth avenues from Bank Street to West 3d Street. The work included the underpinning of buildings and the column foundations of the Sixth Avenue elevated railroad. The material encountered in the excavation was water bearing and unstable. In the latter part of 1930 and the early part of 1931, Mr. Hodgdon took charge of the construction of a passageway leading from the subway on Eighth Avenue through 41st Street to the subway on Seventh Avenue. Following this he assisted in the preparation of data for the defense of legal actions that arose out of certain of the subway contracts. On this latter work he was engaged until his retirement in 1938.

After retirement he busied himself about his home, in which he took great pride. Both before and after retirement he often went in summer to his native isle of Mt. Desert, where he could enjoy his hobby of sailing, of which he never tired.

Always conscious of his responsibility, Benjamin Hodgdon was conservative in his judgments, hard working, and ever faithful to his high ideals. He supervised for the City of New York the construction of underground railroads amounting in value to more than $20,000,000.

Mr. Hodgdon was married in June, 1906, to Margaret A. Magee of Newburgh, N. Y. He is survived by his widow and two daughters, Sally Beverly (Mrs. Richard G. Dubraska), and Frances Wilson (Mrs. Paul A. Danielson, Jr.).

As a boy, his summer vacations were often spent at sea with his father and uncle and, in keeping with his nautical background, his principal pastime was sailing. He was known to his friends as the "Skipper," a title that he acquired on week-end yachting trips "down the harbor" with his associates in Boston. His assistants on his work used the same title in addressing him but always with the utmost respect.

His genial disposition, frankness, and old-fashioned honesty, coupled with a keen sense of humor and a homely philosophy, endeared him to his friends, who included almost every one with whom he came in contact.

He was a member of the Ridgeview Congregational Church of White Plains, N. Y.

Mr. Hodgdon was elected an Associate Member of the American Society of Civil Engineers on March 2, 1909, and a Member on January 14, 1919.

HOWARD CECIL LEE HUMPHREYS, M. Am. Soc. C. E.[1]

DIED JULY 18, 1941

Howard Cecil Lee Humphreys (known as Cecil Lee Howard Humphreys) was born at Wembley, Middlesex, England, on September 7, 1893. His parents were Henry Howard Humphreys and Alice (Page Tazer) Humphreys. He was educated at Burford Grammar School and Westminster School, London, England, which he left in 1911 to join his father who was then conducting a consulting civil engineering practice in Westminster, London. Mr. Humphreys' period of pupilage lasted from 1911 to 1914, during which time he was engaged upon the construction of a reservoir for the Letchworth Garden City Company in Hertfordshire and on surveys and the preparation of drawings for a number of water and drainage projects for which his father was the engineer.

At the outbreak of the World War in 1914, Mr. Humphreys was among the foremost of those who volunteered for military service and, within a few days, his life had changed completely from that of an engineer engaged upon the preparation of schemes for the betterment of living conditions to that of a soldier in the Honourable Artillery Company. After a few weeks of training he was sent to France where he served for some months; he returned to England on leave and became acutely ill with trench fever. On his recovery he was granted a commission in the 17th London Regiment, and he again saw service in France and later in Salonika (Greece) and Palestine. He was in France when the war ended.

In 1919 he was demobilized and rejoined his father's practice which was then beginning to recover from the stagnation of the war period. The struggle was a difficult one, for Britain's economic situation was bad, prices were high, and the cry for curtailment of expenditure was insistent. Contractors who had become accustomed to the free and easy percentage-on-cost war contracts were reluctant to give firm-priced bids, and it was for a time almost impossible to let contracts. Gradually conditions improved, but some years elapsed before they began to approach normal. Much faith and perseverance were required in this difficult period, but Mr. Humphreys never gave up hope and, in the end, he was rewarded. In 1925 he was made a partner, and public works by then were being undertaken freely.

In 1930 Mr. Humphreys' father died, and he became senior partner of the firm of Howard Humphreys and Sons. The severe economic crisis of 1930–1933 soon afterward descended upon Britain, and public works were curtailed or stopped in so far as possible. A dreary period of partial inaction followed, during which the engineering profession suffered the greatest possible hardships. Once again, however, conditions became better, and from 1933 to 1939 public works were undertaken in ever-increasing volume with the result that, by the time of the outbreak of war in 1939, Mr. Humphreys' firm had a very large volume of work in hand. This work comprised projects for roads, bridges, water supply, and drainage in England and for water supply and drainage in the British colonies. A particularly interesting English scheme was the improvement of the London-Holyhead (Wales) road at Penmaenmawr and at Penyclip, described in a paper written by Mr. Humphreys and delivered at the Institution of Civil Engineers on April 6, 1937.[2] A colonial scheme of equal interest was that of the main drainage of Georgetown, British Guiana, described in a paper delivered at the Institution of Civil Engineers on April 4, 1933.[3]

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[1] Memoir prepared by G. Howard Humphreys, London, England.

Mr. Humphreys had maintained his connection with the Territorial Army from 1919 onward, and soon after the outbreak of war in 1939 he found himself again in France; by this time he held the rank of Lieutenant-Colonel. He was in the fighting in Belgium, was evacuated from Dunkirk, France, and received the Military Order of the British Empire for "distinguished services in the field." On his return to England he was appointed Chief Signal Officer to the 3d Corps. Late in 1940 he was offered and accepted the post of Director of Works at the Ministry of Works and Buildings, an appointment that gave the greatest satisfaction to all those who knew him. He retired from the Army with the rank of Colonel. His health unfortunately began to fail and at Whitsun, 1941, it was patent that he was dangerously ill.

Colonel Humphreys was a member of the Council of the Institution of Civil Engineers, the Committee of the Association of Consulting Engineers, the Institution of Mechanical Engineers, the Institution of Water Engineers, the Société des Ingénieurs Civils de France, and the Westminster City Council. He was a Freemason and belonged to the Bath and Union clubs.

During his wide and varied experience Colonel Humphreys frequently was called upon to give expert evidence for which he had a special aptitude. He also functioned as an arbitrator on many occasions, and acquired a reputation for his fearless findings and for the manner in which he refused to allow extraneous matters to be brought into dispute so as to cloud the issue.

His personality was one of genial charm coupled with a great sense of humor and an exceptionally generous outlook on life. He enjoyed living and was never more happy than when every moment was fully occupied. His mind worked at high speed, and he generally outstripped any one who was collaborating with him on a problem. He was an ardent advocate of the strictest adherence both to the letter and to the spirit of professional etiquette, and his early death has removed an influence for the best from the British engineering profession. With a view to dispersing misunderstandings as to the professional status of an engineer, Colonel Humphreys, in conjunction with his brother, wrote a book entitled *The Training of a Civil Engineer* in 1932.

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[2] "The Reconstruction of the Chester-Holyhead Road near Penmaenmawr, North Wales," *Journal,* Inst. C. E., Vol. 6, 1936–1937.

[3] "The Main Drainage of Georgetown, British Guiana," *Proceedings,* Inst. C. E., Vol. 236, May, 1934.

In 1917 he was married to Ailsa Yoxall, the elder daughter of the late Sir James Yoxall, M.P. He is survived by his widow; a daughter, Barbara; and a brother, Guy Howard Humphreys.

Colonel Humphreys was elected a Member of the American Society of Civil Engineers on February 23, 1932.

MURRAY LEE HUTTON, M. Am. Soc. C. E.[1]

Died February 18, 1941

Murray Lee Hutton was born in Washington County, Iowa, on February 11, 1886, the son of Robert and Elizabeth (Duncan) Hutton. He received his early education in the public schools of the county, and was graduated from the Columbus Junction High School in 1904. He was a student in the Civil Engineering Department of Iowa State College, at Ames, for three years and would have been graduated with the Class of 1910 had he completed his college course. He left school at the end of his Junior year, however, to engage in the practice of engineering.

As an engineering student, and later as a practicing engineer, Mr. Hutton's work was always of the intensive, result-producing variety. He had a rare capacity for concentrating on and finishing the task at hand, whether it was a pleasant or distasteful one. Meticulously honest in his thinking as well as in his every act, his reports and recommendations came to be accepted by those who knew him as representing the most complete and accurate information currently available on the subject under consideration.

His ability as a county engineer was early recognized by the Iowa State Highway Commission, with the result that he was offered a position with that organization. As assistant engineer in the Administration Department, he handled most of the details incident to the federal-aid construction program, as well as a wide variety of miscellaneous assignments.

One of the miscellaneous assignments was that of dealing with railroad companies with regard to grade separation and elimination projects. Since at that time there were no special funds for this class of work, such as have since been made available, and since there are no state laws in Iowa that specify a division of costs on projects of this kind, each project was subject to specific negotiations and agreement with the affected railroad. Mr. Hutton made extensive studies and did much research work to determine a fair and equitable division of the cost of each project as between the public and the railroads. As a result of his efforts along this line, agreements were negotiated with railroad companies that resulted in the separation or elimination of many dangerous railway grade crossings.

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[1] Memoir prepared by W. H. Root, Assoc. M. Am. Soc. C. E., and C. Coykendall, M. Am. Soc. C. E.

Another one of the miscellaneous assignments handled by Mr. Hutton while he was assistant administration engineer for the State Highway Commission was the supervision of much of the engineering work that the Commission did for the State Board of Conservation. Through his association with this work he developed a keen interest in the state's conservation program. As a result of contacts made in this work, in 1929 he was offered, and accepted, the position of chief engineer of the Iowa State Board of Conservation. In 1935 he became director of State Conservation and held this position until his death.

Mr. Hutton will be long and gratefully remembered by the people of Iowa for his work in connection with the state's conservation program. To this work he devoted the energy and ability that had always characterized his efforts in his professional work. He familiarized himself with the conservation programs being conducted throughout the United States, and from the knowledge thus gained he formulated the program best suited, in his judgment, to the needs of Iowa. He was a vigorous champion of sound, prudent, forward-looking public administration. He visualized for Iowa a system of state parks so located as to be of easy access to all of the people of the state, and so developed as to afford wholesome and diversified recreation for all—streams and lakes freed from contamination and pollution and abundantly stocked with fish; and a game conservation and propagation program that would again make open hunting seasons possible. Definite and encouraging progress is being made toward all of these goals.

Mr. Hutton was an associate member of the Iowa Academy of Science, and a member of the Association of Midwest Fish and Game Commissioners. He served as president of the latter in 1940. Among his writings were: "Proposed Work in Louisa County for the Coming Year" (*The Iowa Engineer,* 14: 111–11, 1913); "Engineering Phases of Conservation," Associated State Engineering Societies (*Bulletin 8,* No. 4: 40–7, October, 1933); "Upstream Reservoirs" (*American Wildlife,* May–June, 1937). He also made a statement for the Association of Midwest Fish and Game Commissioners to the Joint Congressional Committee in Forestry at its hearing in Madison, Wis., on December 18 and 19, 1939, before the Select Commission on Conservation of Wildlife resources (House of Representatives 76th Congress, 3d Session, Pursuant H. Session 64, 1940: 73–81).

That Mr. Hutton's program and leadership in the conservation field met with public approval is best shown by the fact that he was retained in office through changing state administrations. Numerous changes were made in the personnel of the State Board of Conservation, but Mr. Hutton continued as the executive officer of the organization. To know him was to have high respect for his ability and complete confidence in his integrity and sincerity. His untimely death is a personal loss to his many friends, a public loss to the conservation program in Iowa and the Nation, and a professional loss to the civil engineering fraternity.

On April 25, 1913, he was married to Tecla Johnson in Pocatello, Idaho. He is survived by his widow and a son, Donald W. Hutton.

Mr. Hutton was elected an Associate Member of the American Society of Civil Engineers on June 23, 1916, and a Member on April 17, 1923.

FRANK CALEB KELTON, M. Am. Soc. C. E.[1]

DIED FEBRUARY 25, 1942

Frank Caleb Kelton was born in Montgomery, Vt., on November 25, 1881. He was the son of Judge Otis N. Kelton and Harriet (Clapp) Kelton.

Having a desire to participate in the development of the West, after two years at the University of Vermont, at Burlington, and at the instigation of G. E. P. Smith, M. Am. Soc. C. E., he went to Tucson, Ariz., in 1901, where he entered the University of Arizona from which he was graduated in June, 1904, with the degree of Bachelor of Science. He later attended the University of California, in Berkeley, where he received the degree of Master of Science in June, 1916.

Mr. Kelton engaged in private survey work in Tucson in 1904 and 1905 and thereafter was with the engineering staffs of the Gila Valley, Globe, and Northern Railroad Company, in Arizona, as draftsman and computer, and the Ferrocarril de Cananea Rio Yaqui and Pacifico companies, in Sonora, Mexico, on railroad construction. In January, 1907, he became associated with the Agricultural Experiment Station of the University of Arizona where he held the position of assistant irrigation engineer. Later he was transferred to the teaching college and held the positions of assistant and associate professor of civil engineering, until May, 1919. In June, 1919, he became head of the Civil Engineering Department of the University of Arizona as professor, a position he held at the time of his death.

He was joint author with O. E. Meinzer of a paper on the "Geology and Water Resources of Sulphur Spring Valley, Arizona," published as *Water-Supply Paper No. 320* of the U. S. Geological Survey.

On June 9, 1906, Professor Kelton was married to Eleanor H. Harrison, who, with their three sons, Kenneth Otis, Frank C., Jr., and Raymond Harrison, survives him.

The Society listed his nature of business as "instruction," and he loved it. His students are scattered all over the world. His interest in youth and its development was unwavering. He was faculty adviser for the University of Arizona Student Chapter of the Society from its organization (1926) until his death. The success of this chapter was due to his unstinted efforts and guidance, and poor health in the last few years failed to hamper his work with this group. He was president of the Arizona Section in 1932, which, in extending condolences to his wife, adopted the following resolution as an expression of the Section's feeling regarding him:

> "WHEREAS, a member of long standing has been lost to the Arizona Section of the American Society of Civil Engineers, and
>
> "WHEREAS, it is fitting and proper that his record should be reviewed and recorded,

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[1] Memoir prepared by John H. Gardiner, M. Am. Soc. C. E.

"THEREFORE, be it resolved by the Arizona Section, assembled at Tucson, Arizona, this second day of May, 1942, that the following record be spread upon the minutes and that copies be sent to members of his family and to the Headquarters of the Society.

"Frank Caleb Kelton joined the American Society of Civil Engineers as an Associate Member in 1912 and was made a full Member in 1926. He was President of the Section in 1932. In 1926 he organized the University of Arizona Student Chapter and was its advisor until the time of his death.

"He was a faithful and valued member of the Society and of the Section. He considered Society affairs of first importance and discharged the duties imposed by it with meticulous care. The affairs of the Student Chapter were always uppermost in his mind and heart. He made the Student Chapter one of the best in the Country, and invested its meetings with importance and conducted them with dignity. He served always with energy, great attention to detail and complete thoroughness. His performance will be hard to duplicate.

"He was a man of great courage and strength, unswerved by winds favorable or adverse. He carried on without fear, completely uninfluenced by considerations which sought to turn him from the path he thought to be right. His example will be long remembered. The Society has suffered an irreplaceable loss."

Professor Kelton was a man of determined character and one who loved his home and his family. He felt keenly his obligation to his fellow men, and his career enlarged and enriched all his human relations.

He was a member of Phi Kappa Phi honorary fraternity, Kappa Sigma, and the Society for the Promotion of Engineering Education.

Professor Kelton was elected an Associate Member of the American Society of Civil Engineers on February 6, 1912, and a Member on October 1, 1926.

CHARLES HANFORD KENDALL, M. Am. Soc. C. E.[1]

DIED DECEMBER 10, 1941

Charles Hanford Kendall was born on May 27, 1872, in Rushford, Allegany County, N. Y., the son of Charles Bemus Kendall and Antoinette (Smith) Kendall. He attended the public schools of Rushford between 1886 and 1890. He entered Cornell University, at Ithaca, N. Y., in 1891 and was graduated in 1895 with the degree of Civil Engineer. In 1896 he received the degree of Master of Science from the University of Minnesota, in Minneapolis, which he attended from 1895 to 1897.

During 1897 and part of 1898, he was employed by the Central Railroad Company of New Jersey on valuation engineering work. In 1898 and 1899, he was employed as locating engineer on the Guayaquil and Quito Railroad in Ecuador, South America. This work was in the wild country of the Andes

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[1] Memoir prepared by J. D. Fauntleroy and G. G. Wickline, Members, Am. Soc. C. E.

Mountains, and transportation for supplies and equipment was by pack mule. From 1899 to 1901, he was employed as maintenance engineer with the Pittsburg, Shawmut and Northern Railroad Company in Pennsylvania.

From 1901 to 1907 Mr. Kendall was with the U. S. War Department in the Philippine Islands—first, as provincial supervisor of public work of Batangas Province for two years; second, as railroad engineer for two years; and for four years he was assistant director of public works. He remained in this province about two years and then was promoted to assistant engineer to James W. Beardsley, M. Am. Soc. C. E., consulting engineer for the insular government, and was placed in charge of "Surveys for the Friar Lands." Much of this work was in the region of the fierce-fighting Moro natives. During this period he acquired a large collection of native bolo knives and spears. His work was mostly on the island of Luzon, but he also worked on other islands of the Philippine group, including Mindanao, where he was located in the City of Davao. He received much credit for the manner in which this work was completed.

He then received an offer from the Manila and Dagupan Railway Company to take charge of extensive surveys that the company was making, and thereupon left the government service. After several years of service with the railroad company, he returned to the United States because of the serious illness of his parents. While in the Philippines, he was married to Elizabeth Mullen in Manila in 1906.

During the period from 1909 to 1914, Mr. Kendall engaged in private engineering practice at Buffalo, N. Y., and Washington, D. C., in various capacities. In April, 1914, he was employed by the U. S. Public Roads, in which capacity he served until 1919. During this period he had the title of U. S. Senior Highway Engineer and served on location and construction of forestry roads and national parks, and also in connection with certain duties in the Washington Office pertaining to forestry roads. Some of his duties carried him into the states of Idaho and Utah. He worked under the supervision of T. Warren Allen, M. Am. Soc. C. E., formerly chief of the Division of Control of the U. S. Bureau of Public Roads and earlier chief of the Division of Forest and Park Roads.

Mr. Kendall's next assignment came in 1919; he was transferred to Fort Worth, Tex., and served for a period of eight months in District 6 of the Public Roads Administration. In 1919 he was employed by the State Highway Department of Texas as assistant state highway engineer, which position he held for three years. From July, 1922, to May, 1927, he was county engineer in Fort Bend County, Texas, and was stationed at Sugar Land, Tex., at which place he had supervision over an extensive highway improvement program on U. S. Highway 59, known at that time as State Highway No. 3. During this period of employment, Mr. Kendall had charge of work between the Harris County line and the City of Richmond, Tex. This work included grading, structures, paving, and the building of a large bridge across the Brazos River for which he supervised the engineering work. The highway designed by Mr. Kendall had a typical cross section with side slopes of 1 on 6, which many engineers at that time considered extremely flat and a considerable waste of money. The value of this added safety factor to the road was apparent after its completion. This served as a forerunner for the flat slopes for the highway shoulders in the State of Texas, which no doubt has been a factor in the saving of lives and in the reduction in the number of accidents. This practice has since been followed by quite a number of states in the United States. Mr. Kendall generally is regarded as being the originator of the plan for building flat slopes on the highways of Texas.

In May, 1927, Mr. Kendall's work was completed in Fort Bend County and he was employed as resident engineer for the State Highway Department in Brooks County, where he remained for a period of one year to supervise the engineering work of a concrete highway. This highway is the main route out of the Rio Grande Valley for the shipment of winter vegetables and fruits to northern points.

From 1928 to October, 1930, Mr. Kendall served as resident and county engineer in Anderson County, Texas, with headquarters at Palestine, where he had supervision of the building of considerable portions of new highways for the State Highway Department. In October, 1930, he became resident engineer for the State Highway Department in Tyler County, Texas, with headquarters at Woodville. He held this position until June, 1938. For a short period of time he served on special assignments with the Maintenance Division of the State Highway Department, with various miscellaneous projects over the State of Texas. In May, 1939, Mr. Kendall began residence at Pecos, Reeves County, Tex., where he served with the District Highway Engineering Office, first as field engineer on location, and later as a special office engineer.

In the performance of his duty on December 9, 1941, Mr. Kendall received injuries that resulted in his death. He was returning from an engineering assignment with another engineer when the car overturned because of a tire blowout. He is survived by one daughter, Elizabeth Kendall. His wife passed away in Pecos in 1941. His son, Horace Kendall, died on January 1, 1924, at the age of twelve, in Sugar Land.

Mr. Kendall was noted for his energy and friendly attitude toward all people. He had a great host of friends to whom he was very generous. He spent much of his time and effort in helping to provide comfort and pleasure for his friends and others. He possessed an unusually tender heart and a sympathetic understanding of his fellow men, and his passing is grieved by his many friends.

Because of the fact that he had seen engineering service in practically all parts of the United States, in South America, and in the Philippines, he had a great number of very interesting experiences. He had a collection of various art objects and souvenirs, instruments of warfare, and fishing equipment that he acquired in various sections of the world. He had thirteen large trunks of souvenirs and books packed away in a storage house in San Antonio, Tex. Mr. Kendall was a great lover of books, and he had a large library covering many subjects. He was also fond of hunting and fishing. One of his happiest periods of relaxation was in fishing. Had he lived until May 27, 1942, he would have been seventy years old. He had been notified of his change to the life membership grade in the Society as of January 1, 1942.

Mr. Kendall was elected a Member of the American Society of Civil Engineers on November 6, 1907.

WILSON SHERMAN KINNEAR, M. Am. Soc. C. E.[1]

DIED AUGUST 8, 1941

Wilson Sherman Kinnear was born at Circleville, Ohio, on May 25, 1864. He was the son of Richard and Mary Hall (Crew) Kinnear. During the years that he was in the public schools he often accompanied his father on survey parties and thus early acquired a liking for, and the ambition to become a member of, the engineering profession. He matriculated at the University of Kansas, at Lawrence, Kans., as a member of the class of 1884, and remained in college two years, leaving school at the end of his junior year to enter railway service. At the commencement of the class of 1907 he was granted the degree of Civil Engineer by the University.

In 1883 and 1884 he served as axman, rodman, draftsman, leveler, and transitman with the Atchison, Topeka and Santa Fe Railroad Company; from the autumn of 1884 to the autumn of 1885 he was assistant engineer of the Kansas City, Clinton and Springfield Railway Company; in 1885 and through the year 1886 he was division engineer for the Gulf, Colorado and Santa Fe Railway Company. With this background of early surveying experience prior to his college work, followed by three and a half years of experience in railway location and construction during the most active period of railway building the nation ever had, he went to Los Angeles, Calif., and during 1887 and 1888 was engaged in hydraulic engineering work. Late in 1888 he was employed by the North and South American Construction Company, of Santiago, Chile, first as office engineer, then assistant chief engineer, and after a few weeks as acting chief engineer in charge of the construction of railways, which constituted a very large part of the program of internal improvements which had been inaugurated by President José Manuel Balmaceda.

By the autumn of 1889 the opposition to President Balmaceda had become so powerful and the coming Revolution was so evident that Mr. Kinnear left Chile "while the going was still good" and returned to the United States. In February, 1890, he was employed by the Michigan Central Railroad Company as supervising engineer in charge of construction of second track on the Canada Southern Railroad between Buffalo, N. Y., and Windsor, Ont., Canada.

Mr. Kinnear served the Michigan Central Railroad Company for twenty years, during which time his advancement was rapid. He held the positions of assistant engineer on maintenance of way and construction; supervising engineer on the construction of the Toronto, Hamilton and Buffalo Railroad for the Michigan Central; principal assistant engineer; assistant superintendent; assistant general superintendent; chief engineer; and assistant general manager.

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[1] Memoir prepared by Henry E. Riggs, Past-President and Hon. M. Am. Soc. C. E.

During the period 1904 to 1910 the Michigan Central was engaged in the design and construction of the Detroit River Tunnel. Previous to the completion of the tunnel all trains had been ferried across the river at Detroit, Mich., at great expense and with much inconvenience and delay.

In 1904 Mr. Kinnear was named a member of a committee of which William J. Wilgus, Hon. M. Am. Soc. C. E., then Vice-President of the New York Central and Hudson River Railroad, and the late E. A. Handy, M. Am. Soc. C. E., then chief engineer of the Lake Shore and Michigan Southern Railway Company, were the other members. This committee had the responsibility of reporting on the feasibility of building a tunnel beneath the Detroit River. Following its favorable report and the decision to proceed with the work, an advisory board was appointed composed of Mr. Wilgus as chairman, Mr. Kinnear, and the late Howard Adams Carson,[2] Hon. M. Am. Soc. C. E. Mr. Kinnear was appointed assistant general manager of the Michigan Central and chief engineer of the Detroit River Tunnel Company, and the late Benjamin Douglas,[3] M. Am. Soc. C. E., was appointed tunnel engineer.

The underwater conditions in the Detroit River, owing to the extreme depth of rock, the light covering over the tunnel, and the poor character of the clay through which the tunnel must be built, were such as to make the use of the usual shield-driven tunnel hazardous and highly undesirable. Four methods of construction were considered and the advisory board finally decided upon a method conceived and presented by Mr. Wilgus and supported by Messrs. Kinnear and Carson. This trench and tremie method was novel and untried, and was at the time severely criticized by prominent engineers; but it was adopted.

The job was completed under this engineering organization: The trench was dug across the bed of the river; the steel tunnel shell or core was built about thirty miles north of the tunnel site, floated to place, sunk, connected up and anchored; the concrete was placed using the tremie or bucket deposit method; and, finally, the shore sections and the river tunnel were connected—all without any serious difficulty or delay. Mr. Kinnear was chief engineer of the work from start to finish.

The record of the conception and completion of this remarkable achievement is filed in the Engineering Societies Library[4] among the professional papers of Colonel Wilgus and is splendidly summarized in the paper by Mr. Kinnear, "The Detroit River Tunnel."[5] This paper resulted in the award to Mr. Kinnear in 1912 of the Norman Medal.

In connection with the building of the tunnel Mr. Kinnear had charge of the design and construction of the electrified terminals of the Michigan Central Railroad in Detroit and Windsor, and of the construction of the electrified section of the main line through the tunnel connecting the two terminal yards.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1386.
[3] For memoir, *ibid.*, Vol. LXXV, December, 1912, p. 1151.
[4] 29 West 39th St., New York, N. Y.
[5] *Transactions*, Am. Soc. C. E., Vol. LXXIV, December, 1911, pp. 288–374.

In 1910, upon completion of the tunnel, Mr. Kinnear was offered, and he accepted, the presidency of the Kansas City Terminal Railway Company, which had just begun the construction of the new Union Station. Under his supervision the plans were perfected for the complete reconstruction of the Belt Line railway, the Union Station, and all adjacent improvements; contracts were let; and the entire major project was nearing completion before he severed his connection with it. As one of the engineers who was on this work expressed it, "Mr. Kinnear and the men he brought with him from the Detroit River Tunnel were the whole thing."

On May 1, 1912, Mr. Kinnear resigned and went to New York, N. Y., as president of the United States Realty and Improvement Company, serving until January 1, 1918. He organized the U. S. Realty and Improvement Construction Company, which built many major projects in New York, among them being the Harlem River four-tube subaqueous tunnel of the Lexington Avenue Subway; the Hell Gate Bridge foundations, including all the foundations from Bronx Kills Bridge to the Long Island Viaduct piers at Stemler Street; Section Two of the Broadway Subway; and two sections of the Seventh Avenue Subway.

On January 1, 1918, he organized the firm of W. S. Kinnear and Company, Consulting Engineers, and continued in private practice until 1936 when he retired and moved to Detroit to make his home with his daughter. During these sixteen years his practice was of a broadly diversified nature and included many engagements on major work of great importance.

Mr. Kinnear was a member of Beta Theta Pi fraternity and of Sigma Xi honorary scientific fraternity. He was a member of the Detroit Club, the Detroit Golf Club, and the Union League Club of New York City, and the Sleepy Hollow Country Club, of Tarrytown, N. Y. He was a member of the American Railway Engineering Association, and a member and Past-President of the American Institute of Consulting Engineers.

Mr. Kinnear was married in 1887 to Carrie M. Nichols of Springfield, Ohio, who died in September, 1932. Surviving are a daughter, Carmen (Mrs. Elwood Johnson), and a son, Lawrence Wilson Kinnear.

Wilson Sherman Kinnear was an able engineer and an excellent executive. The magnitude of the work that he did and its diverse nature easily entitles him to rating as one of the Master Builders of his generation. He was a man of fine personality who made a host of friends both in his profession and in the social circles in which he moved.

Mr. J. C. Mock, retired Signal-Electrical Engineer of the Michigan Central, who was intimately associated with Mr. Kinnear on the Michigan Central System and the Tunnel, gives the following estimate of his personality:

> "His six feet four inches in height, carried with modest dignity, distinguished him in any company. His humanity and kind consideration for those associated with him professionally made them his friends, insured him unquestioned loyalty, and largely accounts for his excellent record as an organizer and coordinator."

Mr. Kinnear was elected a Member of the American Society of Civil Engineers on July 9, 1906.

STUART KELSEY KNOX, M. Am. Soc. C. E.[1]

Died June 28, 1941

Stuart Kelsey Knox, the son of Judge David M. and Maria Louise (Kelsey) Knox, was born in Three Rivers, Mich., on May 8, 1879. He attended the University of Michigan at Ann Arbor, Mich., and was graduated with the degree of Bachelor of Science in Civil Engineering in 1903. While obtaining his technical education he gained practical experience with the Oldsmobile Company of Detroit, Mich.; the city engineer of Sault Ste. Marie, Mich.; International Transit Company, Sault Ste. Marie, Ontario, Canada; and the late Clarence G. Wrentmore, M. Am. Soc. C. E., consulting engineer of Ann Arbor.

Mr. Knox's first engagements after graduation were with the United States Engineers' office on pier and revetment construction at Muskegon, Mich., and with consulting engineers on irrigation surveys in Texas. From July, 1904, to August, 1907, he was employed by the Thompson-Starrett Company, Inc., of New York, N. Y., on construction of such buildings as the Continental Trust Building in Baltimore, Md.; the Kenesaw Apartment Hotel and Union Station in Washington, D. C.; and the United States Express Building in New York City. He was superintendent of construction for the Foundation Company of New York City on work in Washington, D. C., from September, 1907, to December, 1907.

In January, 1908, Mr. Knox joined the staff of the late Nicholas S. Hill, Jr.,[2] M. Am. Soc. C. E., specializing in sanitary engineering work. He served as principal staff member until 1916, and then as an associate member of the firm until 1926, when he was admitted to a partnership that terminated in June, 1934. In this consulting engineering work he was identified with notable enterprises in the United States, Canada, Mexico, and Cuba.

During World War I Mr. Knox was chief designing engineer for all utilities at Camp Merritt, New Jersey, and engineer in charge of design and construction for the United States Housing Bureau in connection with its developments near Norfolk and Portsmouth, Va.

In January, 1935, Mr. Knox established a professional business as hydraulic and sanitary engineer that endured to the day of his death. Chief among the undertakings were water works rehabilitations at Binghamton, N. Y., and Montclair, N. J.

Mr. Knox's technical and professional affiliations included the American Society of Mechanical Engineers, American Public Health Association, American Water Works Association, and New York State Sewage Works Association. He was a contributor to the technical press.

Possessed of a keen, analytical, versatile mind, Mr. Knox was a man of great force of character and the utmost honesty, was exceptionally conscientious in his work, and was respected by his associates and his friends for his ability, integrity, and fundamentally sound principles of conduct.

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[1] Memoir prepared by Samuel F. Newkirk, Jr., M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1808.

On June 8, 1914, he was married to E. Isabel Lane of Brooklyn, N. Y., at Wrentham, Mass. He is survived by a son, Stuart Kelsey Knox, Jr., and a brother, Edward M. Knox.

Mr. Knox was elected a Member of the American Society of Civil Engineers on April 2, 1913.

EDWIN WEED KRAMER, M. Am. Soc. C. E.[1]

DIED OCTOBER 31, 1941

Edwin Weed Kramer, the son of William Paul Kramer and Jean (Mobley) Kramer, was born at Louisville, Ky., on March 4, 1877. He obtained his technical education at Cornell University, in Ithaca, N. Y., and was graduated in 1905 with the degree of Civil Engineer.

Following his graduation from Cornell University, Mr. Kramer worked on the construction of the New York State Barge Canal, on dry docks for the U. S. War Department, and on other projects. Early in his career he chose the government service and served in that service continuously until his death. He entered the employment of the U. S. Forest Service in September, 1907, as construction engineer, at first employed in the Denver, Colo., office and later in Missoula, Mont., where he was designated district engineer, effective January 1, 1910. This district embraced Montana, northern Idaho, eastern Washington, and included in addition the National Forests in Minnesota, Wisconsin, and northern Michigan. The National Forests of the region in those early days consisted literally of primeval wilderness areas, practically without roads or means of communication. Transportation was effected generally by means of crude pack trails, and Mr. Kramer's initial work concerned the design and construction of bridges and other structures of the Forest Service improvement program. In 1913 he was promoted to hydraulic engineer and his activities broadened to include the field of water power administration. This embraced the issuance of permits covering hydroelectric developments utilizing land in the National Forests. In connection with this activity Mr. Kramer conducted notable investigations of the utilization of the Missouri River for power and other purposes.

The law creating the Federal Power Commission was passed in 1920 for the purpose of encouraging the development of water power resources under federal control. This law, combined with the spectacular increase of demand for electrical energy in California, resulted in an extensive program of hydroelectric development in that state. Since most of the power sites were in the National Forests, the administrative activities relating to the issuance and supervision of licenses were greatly increased. Mr. Kramer was called from Montana to serve as special assistant to the district engineer of the California District of the Forest Service, who was also serving as representative of the Federal Power Commission for the state. Throughout the decade of the

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[1] Memoir prepared by Frank E. Bonner, M. Am. Soc. C. E.

1920's, when most of the great hydroelectric projects of California were being constructed, Mr. Kramer occupied a prominent position in the government administration phase of the program. During this period his technical work relating to the design of concrete arch dams was particularly noteworthy.

Mr. Kramer was advanced to the position of district engineer of the California District on July 1, 1929, and, in addition to directing all the Forest Service engineering activities of the state, he served as representative of the Federal Power Commission. The latter agency established a regional headquarters office at San Francisco, Calif., on July 1, 1936, to supervise the field activities of the seven western states, and Mr. Kramer was chosen to serve as regional director. He held this position until his death in 1941.

In connection with water power development, Mr. Kramer was always a staunch defender of the public interest, and in the course of his official duties he constantly endeavored to enforce the fullest scope of federal authority. His many years in the government service gave him a broad knowledge of the power and irrigation developments throughout the West, and as a result he was called upon frequently to serve as expert witness in litigation involving water rights and water uses.

Mr. Kramer was co-author of the reports published by the Federal Power Commission on "Uses of the Stanislaus River"[2] and "Uses of the American River."[2] He was author of "Construction of Fish Dams for Water Storage in California,"[3] and many articles in the technical press. He was also the author of "Curves for Determining Stresses in Arch Dams by the Cain Formula," which, although never published in their entirety, gained wide use among engineers designing arch dams. He was a member of the Sons of the American Revolution, and was a Spanish-American war veteran.

On June 22, 1905, he was married to Ruth Edwards. He is survived by his widow; by two sons, Edwin Weed Kramer, Jr., Assoc. M. Am. Soc. C. E., and Paul Kramer; by three daughters, Ruth (Mrs. Joel M. Popper), Jean Mobley (Mrs. Samuel H. Berry), and Margaret (Mrs. David Mealiffe); and by seven grandchildren.

Mr. Kramer was elected a Member of the American Society of Civil Engineers on December 14, 1925.

DWIGHT ARNOLD LANE, M. Am. Soc. C. E.[1]

DIED JULY 1, 1941

Dwight Arnold Lane was born at Boise, Idaho, on August 24, 1896, the son of Frank and Frankie (Arnold) Lane. He received his early education at Gardena, Calif., and later attended the University of Southern California, at Los Angeles, Calif.

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[2] Federal Power Commission, 1927.

[3] U. S. Forest Service, 1935.

[1] Memoir prepared by Milton Anderson, Assoc. M. Am. Soc. C. E.

He entered the service of the Department of Water and Power, Bureau of Water Works and Supply, of the City of Los Angeles, in November, 1916, as chainman. He continued as such until he enlisted in the 26th Engineers, United States Army, on November 9, 1917. He served with this organization, as a part of the American Expeditionary Forces in France, in charge of a field party at Mesves, Bulcy Base Hospital, on water and sewer lines, buildings, railway lines, property surveys, etc. He also worked on the installation of a water system during the Meuse-Argonne drive and was with the Army of Occupation in Germany.

Upon his return to the United States and his discharge from the Army, Mr. Lane reentered the service of the Department of Water and Power, Bureau of Water Works and Supply, of Los Angeles, in April, 1919, as transitman in topographic surveys and water works construction. Later, he received appointments as draftsman, junior civil engineer, and finally assistant civil engineer.

It was during the time he was junior civil engineer that Mr. Lane rendered his important work of ground-water reconnaissance surveys and reports for the location and development of ground-water supplies for the City of Los Angeles. His supervision and general charge of irrigation studies and estimates of water demand, together with hydrographic and hydrologic information, formed the basis for the design, construction, and operation of water-spreading works. The recapture of water so spread, by his previously located water wells, was recognized as a water-conservation measure. It was also during this time that he became recognized as an authority on water rights. Under his direction were prepared engineering data and exhibits used in litigation in which he served as an expert witness.

His promotion to assistant civil engineer placed him in charge of Hydrologic and Engineering Investigations for the Los Angeles area, which included engineering research and collaboration with consulting engineers and governmental agencies, both state and federal, where his specialized knowledge was used to its best advantage. He continued in this capacity until his death.

On February 7, 1920, he was married to Gladys Show. He is survived by his widow; two sons, Dwight Arnold, Jr., and Kay Armand; and his father.

Mr. Lane was a member of Unity Lodge No. 368, Free and Accepted Masons, in Glendale, Calif.; Southern California Horticulture Institute; American Water Works Association; American Geophysical Union; and Aqueduct Post No. 342, American Legion, Department of California, of which he was a Past-Commander. For two years he was secretary of the Los Angeles Section of the Society.

Among his published papers were "Transforming Surface Water into Underground Storage,"[2] "The Loss of Wells in Construction and Operation,"[3] "Increasing Storage by Water Spreading,"[4] and "Artificial Storing of Groundwater by Spreading."[5]

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[2] *Water Works Enginering*, September 2, 1936.

[3] *Journal*, A. W. W. A., October, 1932.

[4] *Ibid.*, April, 1934.

[5] *Ibid.*, September, 1936.

He had been in failing health for several years prior to his death; however, he was able to attend to his work until late in June, 1941, and died rather suddenly on July 1, 1941.

Dwight Arnold Lane was a quiet, unassuming gentleman. He was kind and sympathetic, interested in people whose pathways were beset with troubles and difficulties, always public-spirited, and ever ready to lend his aid and coöperate to the full in undertakings in which he could be of assistance. His passing marks the end of a life of service. He is missed and mourned by his host of friends.

Mr. Lane was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924, and a Member on December 9, 1935.

SAMUEL HILL LEA, M. Am. Soc. C. E.[1]

Died March 28, 1941

Samuel Hill Lea was born at Marion, Perry County, Ala., on October 22, 1861, the son of Sumter and Susan I. (Hill) Lea. His maternal grandfather was a Colonel in the Mexican War, and his father served as Colonel in the Confederate Army.

Mr. Lea received his elementary school education in the public schools of his native city. At the age of twenty he was graduated from the Virginia Military Institute at Lexington, Va.

Soon after graduation, Mr. Lea entered the railroad engineering field in the South and served in various positions from levelman to locating engineer and resident engineer with several railroads. The latter included the Birmingham and Tennessee, the Sheffield and Birmingham, the Kansas City, Memphis and Birmingham, and the Louisville, St. Louis and Texas railroad companies.

During his early experience, Mr. Lea also engaged in a general engineering practice at Salem, Ala., in 1885 and 1886, at Fort Payne, Ala., from 1889 to 1891, and at Birmingham, Ala., from 1901 to 1904.

After several years of a wide and diversified experience on levee construction and maintenance, and on sewer projects and canal operation, he took charge of the construction of buildings, sewer system, and water supply at Fort Meade, S. Dak.

Mr. Lea's outstanding services at Fort Meade for the U. S. War Department brought him to the attention of the Governor of South Dakota who was seeking a suitable engineer to appoint to the newly created position of state engineer. In March, 1906, Mr. Lea became state engineer, a responsible public post which he filled with distinction for a term of seven years. He did pioneer work in the preparation of a State Irrigation Code; in the supervision of appropriations of water from non-navigable streams for irrigation, water power, and other beneficial uses; and in the direction of the construction of small dams across "dry draws" and valleys to conserve storm water and to prevent soil erosion. At the direction of the Governor, Mr. Lea supervised the building of the new State Capitol at Pierre. He planned the Capitol Park and lake and was responsible for the installation of a modern heating and air-conditioning system. The Hon. Harry L. Gandy, former congressman from South Dakota, wrote: "The South Dakota Capitol will stand as a monument to Mr. Lea long after he is dead."

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[1] Memoir prepared by A. B. McDaniel, M. Am. Soc. C. E.

Returning to his native Southland in 1913, he served as city engineer of Charlotte, N. C., for four years. During this period he extended the paved streets into new sections and initiated important municipal improvements.

The highway field claimed Mr. Lea's professional talents in the later years of his life. He designed, laid out, and supervised the construction of highways in Mineral and Berkeley counties, West Virginia, from 1919 to 1923. During the following decade he was designing and estimating engineer with the Pennsylvania Department of Highways at Harrisburg.

In November, 1933, he was appointed engineer examiner for the Public Works Administration at Washington, D. C. On March 1, 1935, Mr. Lea left the federal service and retired to his home in Harrisburg. He passed out of this life on March 28, 1941, after an illness of several months.

Mr. Lea was married in New York, N. Y., to Bella B. Hackett of Easton, Pa., in 1901. After her death, he was married to Amy S. Le Van on November 26, 1930. He is survived by his widow.

A Mason, a devoted attendant of the Episcopal Church, and a faithful civic worker, Mr. Lea lived a life filled with interest in, and service to, his fellow men. Generous, kindly, and genial, he was well liked by his fellow workers. A man of high ideals, honesty, and integrity, he was impatient of careless and dishonest work and frank in his criticism of bureaucratic methods and political chicanery. As a facile writer, he wrote a treatise on "Hydrographic Surveying," issued a report on "Irrigation in South Dakota" while State Engineer, and contributed many articles to technical journals.

Mr. Lea was elected a Member of the American Society of Civil Engineers on November 3, 1897.

FRED HERBERT MARSH, M. Am. Soc. C. E.[1]

Died June 24, 1941

Fred Herbert Marsh was born on October 4, 1880, at Humboldt, Iowa, the son of Herbert W. and Ella J. (Harvey) Marsh. He received his early education in the public schools and was graduated from Iowa State College, at Ames, in 1904 with the degree of Bachelor of Science in Civil Engineering.

From 1904 to 1910 he was employed by various steel companies in Iowa and Colorado as draftsman and estimator, and on construction of all types of railroad structures and coal-mining equipment.

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[1] Memoir prepared by A. B. Reeves, M. Am. Soc. C. E.

In June, 1910, he entered the employ of the Des Moines (Iowa) Bridge and Iron Works, which two years later became the Pittsburgh-Des Moines Steel Company. Here he was advanced through various positions to sales manager of the Des Moines office. In 1923 he resigned the latter position to become associated with the Paxton and Vierling Iron Works at Omaha, Nebr., as sales manager and member of the Board of Directors. In 1931 the management of this firm changed hands and Mr. Marsh retired from active connection with the company.

The next year was spent as estimator and designer for the Patterson Steel Company at Tulsa, Okla. Another year was spent on various jobs in connection with structural steel work.

In 1933 Mr. Marsh entered the employ of the U. S. Government with the War Department at Fort Lewis, Wash., on construction of improvements. Later he was transferred to the Bonneville Dam where, for more than three years, he was in charge of structural steel construction and inspection on the installation of the navigation lock gates.

Upon completion of the Bonneville job, he was transferred to the Pittsburgh, Pa., office, where he was employed on structural steel design. In July, 1938, he was transferred to the Denver, Colo., office of the Bureau of Reclamation, where he was engaged on structural steel design until his death.

Fred Marsh was a man who was helpful, cooperative, and considerate in his relations with others, and he had the good will and respect of all who knew him. Although he took very little part in political or social life, he was, for many years, active in local engineering groups and commercial organizations. He enjoyed hunting and fishing, and was interested in all outdoor activities.

On September 3, 1921, he was married to Eloise Winhafer. He is survived by his widow; three sons, Richard A., Edward H., and Frederick E.; and by his aged mother.

Mr. Marsh was elected a Member of the American Society on March 12, 1923.

CHARLES MEADS, M. Am. Soc. C. E.[1]

Died April 24, 1942

Charles Meads was born in Portsmouth, Va., on January 11, 1870, the son of Charles Meads and Anna (Herbert) Meads. He attended the schools of Portsmouth and Washington, D. C., and then entered the Corcoran Scientific School of the old Columbian University, at Washington, D. C., where he studied for two years.

Upon leaving school, he was employed as an architectural and structural draftsman. In 1894 he entered the employ of Richardson and Burgess, General Contractors of Washington, as an assistant superintendent of construction on contracts for power houses, car barns, and other structures for the Eckington Street Railway and the Columbia Railroad Company.

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[1] Memoir prepared by Hugh J. Donnelly, Charles Meads & Co., New York, N. Y.

In 1896 Mr. Meads left Richardson and Burgess to work for R. H. Hood, M. Am. Soc. C. E., as an estimator and superintendent of construction on various public works contracts, including buildings in the Washington Navy Yard, car sheds for the Washington and Alexandria Railroad Company near Alexandria, Va., and car sheds for the Metropolitan Railroad Company of Washington. In 1897 Mr. Hood moved his main office to New York, N. Y., and Mr. Meads remained in Washington in charge of the Washington office. In 1898, upon the closing of the Washington office, Mr. Meads moved to New York, where the R. H. Hood Company had been incorporated. As an officer in this company, he was associated for several years with Mr. Hood and the late Daniel A. Garber, who subsequently formed the North Eastern Construction Company and who was also the organizer and first president of the General Contractors Association in New York. The R. H. Hood Company engaged in the construction of many large municipal and government projects in New York City from 1898 to 1904, including the Main Immigration and Hospital Buildings on Ellis Island in New York Harbor, the steel floor and concrete foot-walk construction of the Williamsburg Suspension Bridge, and the construction of four lift bridges over the Gowanus Canal in Brooklyn, N. Y.

Mr. Meads organized and became president of Charles Meads and Company in 1904, and of the Meads Engineering Company, Inc., in 1931. These companies have been engaged in engineering and contracting work in New York City continuously since their organization. Under his direction as President, they have completed a long list of municipal and government projects. Among these are the Lighthouse of Lloyd Harbor, Long Island, N. Y.; the first contract for the Central Building of the Brooklyn Public Library; the Municipal Ferry Terminal at the foot of 39th Street, Brooklyn; the steel viaduct and upper level approaches of the Bronx Terminal Market; the reconstruction of the old Elevated Station on the Interboro Rapid Transit (I.R.T.) at the World's Fair Grounds in Flushing, changing this from a small local station to a large express station, and the extension of the Independent Subway into the World's Fair Grounds, including the Terminal Station. Perhaps the most notable work done by his companies was the construction between 1928 and 1940 of more than eighty subway stations of the Independent Subway System in New York City, as well as many elevated railroad stations, and the very difficult work of lengthening existing subway stations.

During World War I, Mr. Meads' company was engaged almost exclusively on projects for the U. S. Navy, including several large warehouses in the Brooklyn Navy Yard, service buildings and utilities at the Rockaway Beach Naval Air Station, the Naval Air Stations at Chatham, Mass., and Hampton Roads, Va., and the construction of the U. S. Marines Flying Field at Quantico, Va.

Mr. Meads was active in the affairs of these companies up to the time of his death. He brought to the field of public works contracting a high sense of honor and obligation to the public good. He always believed that a successful business in public works contracting could be run with only the highest standards of business and professional ethics. He followed these beliefs consistently with success throughout his entire business career of fifty years.

Mr. Meads was a member of the Brooklyn Engineers' Club, an associate of the Municipal Engineers of New York City, and a member of the American Military Engineers.

He was married in Washington, D. C., on October 24, 1894, to Jean Estelle Reeves, who, with one daughter, Estelle (Mrs. Charles Crane) survives him. His home was in Mountain Lakes, N. J.

Mr. Meads was elected a Member of the American Society of Civil Engineers on September 6, 1910.

THADDEUS MERRIMAN, M. Am. Soc. C. E.[1]

Died September 26, 1939

Thaddeus Merriman, eminent civil engineer and life-long student of science, died prematurely in the full tide of his mental activity and richness of engineering experience. He left, only partly finished, extremely valuable original researches in the nature and manufacture of Portland cement, but the results of his studies were and are already in wide use.

Mr. Merriman was born in New Haven, Conn., on April 6, 1876. His father was Prof. Mansfield Merriman, well known to all civil engineers by his treatises on hydraulics and other engineering subjects and his *Civil Engineers' Handbook*. He was especially well known to the many students who had the benefit of his personal teaching at Lehigh University, Bethlehem, Pa. Professor Merriman was of New England stock, his ancestor, Capt. Nathaniel Merriman, having come to America in 1632. Thaddeus Merriman's mother was Wanda (Kubale) Merriman. She was born in Germany where Professor Merriman met her after his graduation from Yale University, at New Haven. She was a warm-hearted, sensitive, generous woman, and her son Thaddeus much resembled her in these and other ways.

On January 12, 1904, Thaddeus Merriman was married to Margaret Mather, a Vassar graduate, of New Haven, Conn. It happened that her father and Mr. Merriman's father were classmates at Yale. She and their two children survive him. Their only son, Mansfield, is associate geologist for the U. S. Bureau of Reclamation. The only daughter, Margaret Mather Merriman, Ph.D., is married to W. George Parks, who is (1941) Professor of Chemistry at Rhode Island State College, at Kingston, R. I. The scientific bent remains strong in the family.

Thaddeus Merriman was graduated from Lehigh University in 1897 with the degree of Civil Engineer, receiving the degree of Doctor of Engineering in 1930. He began his engineering experience even before entering college by summer work as rodman, in 1891, at the age of fifteen. Two years later he spent another summer as draftsman at the Bethlehem Iron Company, South Bethlehem, Pa. The following three summers were likewise spent in getting practical engineering experience, each year in a different way.

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[1] Memoir prepared by Thomas H. Wiggin, M. Am. Soc. C. E., and Mansfield Merriman, Bureau of Reclamation, Denver, Colo.

The year and a half following graduation were spent in a geological survey of western Pennsylvania and then on water works construction in New Britain, Conn. From December, 1897, to July, 1900, Mr. Merriman was engaged in increasingly important positions as rodman, transitman, draftsman, and computer, and finally assistant engineer and chief of party on studies and surveys for a canal across Nicaragua, Central America, to connect the Atlantic and Pacific oceans. The early part of this work was in Washington, D. C., but the last year was spent in the field in Nicaragua with the U. S. Isthmian Canal Commission. This experience in the wilds was greatly to his taste and, to those who knew him in later life, its influence was apparent in the acuteness of his engineering field sense and his appreciation of the romance of pioneer engineering.

In the fall of 1900 and the spring of 1901, Mr. Merriman was designer with the New York Continental Jewell Filtration Company, a newly formed amalgamation of three active filter companies which had received the contract for equipping the first of the modern, masonry, rapid sand filter plants. This pioneer plant of 32 mgd capacity, designed by the late George W. Fuller[2] and William B. Fuller,[3] Members, Am. Soc. C. E., and based on the historic filter experiments of George W. Fuller at Louisville, Ky., attracted Mr. Merriman, who participated during the period of its development.

From March, 1901, to April, 1902, Mr. Merriman returned for a time to the steel business as inspector, in the United States, of bridge and other material used in the construction of the Guayaquil and Quito Railroad Company of London, England.

In April, 1902, Mr. Merriman began his long career in water supply design and construction, serving for two years as assistant engineer and one year as division engineer in charge on the construction of the important masonry dam at Boonton, N. J., built by the Jersey City Water Supply Company. During this latter period Mr. Merriman also did work for the group of water companies affiliated with the East Jersey Water Company and the parent Society of Useful Manufactures, a society founded by Alexander Hamilton. This group of companies has been served by such other celebrated hydraulic engineers as the late Clemens Herschel,[4] Past-President and Hon. M. Am. Soc. C. E., inventor of the venturi meter (which should have borne his name), the late J. Waldo Smith,[5] Hon. M. Am. Soc. C. E., one time chief engineer of the Board of Water Supply under whom the Catskill water system was built and who preceded Mr. Merriman in that position, and the late Allan W. Cuddeback,[6] M. Am. Soc. C. E.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1653.

[3] For memoir, *ibid.*, Vol. LXXXVII (1924), p. 1337.

[4] For memoir, *ibid.*, Vol. 95 (1931), p. 1419.

[5] For memoir, *ibid.*, Vol. 101 (1936), p. 1502.

[6] For memoir, *ibid.*, Vol. 105 (1940), p. 1815.

The Boonton dam is believed to be the first Cyclopean masonry dam. This masonry, invented or at least first used in dams by William B. Fuller, Mr. Merriman's chief on the Jersey City water supply work, consisted of large stones up to about 2 cu yd in volume embedded in concrete instead of being set in mortar. The exterior faces of the Boonton dam were of squared stone masonry. Since the large dams subsequently built for the New York water supplies were also of Cyclopean masonry, this first experience of Mr. Merriman in the construction of a large dam is of note.

In this dam also were placed electric thermometers, perhaps for the first time in such a dam, and the observations of the high setting temperatures of masses of Portland cement concrete colored Mr. Merriman's subsequent work on large dams and on the technique of cement manufacture and use.

In August, 1905, Mr. Merriman was one of the nucleus of engineers assembled by J. Waldo Smith for his first report as chief engineer of the Board of Water Supply on the Catskill water system for the City of New York. This first report had the benefit of the 1903 report of the Burr, Hering, Freeman Commission on "Additional Water Supply for the City of New York," but it was made with incredible celerity—namely, in August and September of 1905. Mr. Merriman had an important part in it; his estimates of dam and aqueduct cost, made quickly and with apparent roughness, yet with a nice regard for the precision warranted by the available data, furnished much of the basis of the estimates of cost of about $162,000,000 for this great undertaking. These estimates subsequently were found close to the actual cost of the work.

Following the acceptance of the Catskill plan and the beginning of its construction, Mr. Merriman went to the Reservoir Department of the Board of Water Supply engaged in the construction of the huge Ashokan (N. Y.) Reservoir of 120 billion gallons capacity. His work in that department between November, 1905, and April, 1907, was as assistant engineer and then division engineer on surveys and studies for the main dam and the dikes.

In 1907 Mr. Merriman became assistant to J. Waldo Smith at the main office in New York, N. Y., his title being, at first, division engineer and then department engineer. This work continued through the most active period of design and construction of the Catskill water supply system. Between August, 1914, and February, 1918, he was department engineer in charge of Headquarters Department. This section was charged with preparation of designs, specifications and reports, the inspection of material, and keeping records, personnel, etc. The most active work during this period was the completion and testing of the Catskill aqueduct and City tunnel and the design of the Gilboa dam and Shandaken tunnel for impounding the waters of Schoharie Creek, bringing them through the mountains to Esopus Creek, whence they flow into Ashokan reservoir and become available for supplying the city through the Catskill aqueduct.

From February, 1918, to June, 1922, Mr. Merriman was deputy chief engineer of the Board of Water Supply of the City of New York, and in June, 1927, he succeeded J. Waldo Smith as chief engineer. He retired on November 29, 1933, but continued an active interest in the work while engaged in private practice. In September, 1936, he was appointed consulting engineer to the Board, a position which he held until his death.

Mr. Merriman's work as deputy chief engineer and chief engineer of the Board of Water Supply saw the completion of the Schoharie dam and Shandaken tunnel, also the surveys, geological investigations, and the preparation of preliminary plans for the additional supply of water to New York City from the Rondout watershed, the Neversink, and the East Branch of the Delaware. The original plans contemplated an additional supply of 760 million gallons but, except for the 100 mgd from Rondout Creek, the water to be taken was interstate water flowing into the Delaware River and affecting the interests of the states of New Jersey and Pennsylvania. These states took their case to the courts and an important legal battle resulted which was finally settled in 1931 by the U. S. Supreme Court. This decision awarded the City of New York 440 mgd of the 660 mgd asked for from Delaware River sources, but gave the city the right to apply for more when it was needed.

In this case, Mr. Merriman took a leading part both as witness and as collaborator with the Corporation Counsel of the City of New York, Arthur J. W. Hilly. Mr. Merriman wrote the engineering and much of the legal parts of the brief and was credited, not only by his own board but by Corporation Counsel Hilly, as the major contributor to the successful outcome.

Among the interesting interludes during his employment on the water supply of New York City was a period of about five months in the fall and winter of 1919–1920, on leave from the city. This time was spent in company with a former colleague, William E. Swift, M. Am. Soc. C. E., on behalf of the firm of Ford, Bacon and Davis, Engineers, making a field investigation and report on a ground-water supply for the City of Athens, Greece.

During at least twenty-five years of his employment with the Board of Water Supply of New York City, Mr. Merriman pursued original studies into the nature, behavior, and manufacture of Portland cement. The laboratory of the Board of Water Supply had begun these studies as early as 1907, but in the later years the studies were in great part guided by Mr. Merriman. He was stimulated in these studies by the experience in the New York water supply and elsewhere as to the destruction by solution, frost, and weather of concrete structures. His studies led to new specifications for the manufacture of sulfate resistant Portland cement, which were adopted in 1933 for the Fort Peck Dam in Montana. His further studies resulted in the specification adopted by the Board of Water Supply of the City of New York for the Delaware water supply project, and soon thereafter on other important hydraulic works. The Metropolitan District Water Supply Commission of Massachusetts used this cement in its great reinforced concrete pipe, the Department of Power and Water of the City of Los Angeles now (1941) uses it in the lining of its steel pipe, and private concerns have used it for pipe linings.

Mr. Merriman's specifications provide for both laboratory and manufacturing control. Experiment and experience demonstrated to him the necessity for control of the burning zone temperature, as indicated by recording pyrometers, the time of burning, as determined by the rate of kiln rotation, the protection of the clinker from exposure to the weather or other access of moisture and a limitation on the age of the clinker before grinding. They also provided for the limitation of alkalinity and water soluble alkalis, for the fixing of a minimum percentage of silica, of a maximum lime-silica ratio, of a maximum molecular ratio of lime to the sum of silica, alumina, and ferric-oxide (Colony's ratio), of maximum percentages of alumina and a maximum alumina-iron ratio. A sugar solubility test was also devised as a laboratory check to indicate the thoroughness of burning. This, together with other manufacturing and chemical controls, he believed essential to the production of a consistently high-grade cement that could be depended upon to make durable concrete. The sugar solubility test has been included in the 1938 and later cement specifications drawn by the Metropolitan Water District of Southern California and has been used by private companies in purchasing low solubility cement for water pipe linings.

Those who have critically used cement, made in accordance with these specifications, have generally remarked on the greatly diminished production of laitance, the diminished tendency to cracking, and the greater resistance to destruction by saturation and frost action. It is too soon for a final appraisal of Mr. Merriman's work on cement, but there is no room for doubt as to the value of his analysis of the imperfections of cement, his emphasis on the importance of finding remedies, and his insistance on care of the clinker in manufacture. He infused life into a too dormant field of chemical and engineering inquiry and it is regrettable that he could not have lived to carry his researches further and lend his great influence to keeping this important study alive until it was further along toward final and unquestioned solution.

During the period from his retirement in 1933 from the chief engineership of the Board of Water Supply of the City of New York until his death, Mr. Merriman was called to act as consulting engineer on a number of large hydraulic projects. He was chairman of the Board of Review of the Metropolitan Water District of Southern California in its project for supplying water from the Colorado River. He was also consultant on the Fort Peck project in Montana, on flood control projects in Los Angeles and Orange counties in California, and consultant to the Tennessee Valley Authority on Norris and other large dams, to the Water Power and Resources Board of the Commonwealth of Pennsylvania and to the Metropolitan District Water Supply Commission of the Commonwealth of Massachusetts.

To his father's and his own school, Lehigh University, he contributed of his knowledge and experience by lecturing on hydraulics and water supply. Mr. Merriman was a life-long student of pure science apart from his study of applied science in engineering. In 1933 he submitted to the American Association for the Advancement of Science a mathematical paper entitled "Gravitation in Terms of a Substantive Æther." Few of his friends know of the sustained interest and thought given to the theories concerning gravity, inertia, mass, and energy. He left a partially completed book entitled

"Reality," on which he had been working for more than thirty years, and dedicated, "To my wife, whose help and patience through many years finally made it possible for me to visualize a little of the reality of the universe in which we live." The material is being assembled and compiled for publication by his son. Another complete paper on "A Kinetic Theory of Hydraulic Flow" was likewise never published.

Apart from his necessary engineering reports and briefs, Mr. Merriman was not a prolific writer. His brief on the Delaware project for the U. S. Supreme Court has been mentioned, also his monographs on Gravitation, on Reality, and on Hydraulic Flow. In recent years he wrote much on Portland cement, mostly in the organization paper of the Board of Water Supply called "Delaware Water Supply News." His views on Portland cement are also given in a paper presented before the Boston Society of Civil Engineers on October 19, 1938.[7] The intense activity of his professional life together with his insistence on "understanding" was responsible for the small number of his published articles. One of his favorite sayings was to the effect that finding an obscure detail and assigning it a name did not constitute a discovery.

He enjoyed nothing more than to be with his family. His relaxation was invariably found out of doors as a woodsman, fisherman, canoeist, and archer. He was no stranger to the mountains and streams from Maine to California, yet the near-by woods held equal attraction. In the field he was always the geologist and student of land forms, instilling his interest in all about him. He often sought and found evidences of much greater floods than any experienced within recent times, and believed in making spillways for dams as large as the physical conditions would permit. Later, his activity in engineering left him too little time to follow the leisure that gave him so much contentment.

Endowed with a most sociable personality, he enjoyed the companionship of a few or large group of friends and associates. He knew the popular songs and was fond of leading a group in a serious rendering of "Nellie Gray."

He lived a many sided life, bringing to each subject the illumination of a scientifically trained, keen, logical, and resourceful mind and, withal, giving a surprisingly large amount of time to the obligations of friendship.

Mr. Merriman belonged to the following societies, appropriate to his many interests: American Association for the Advancement of Science, The Franklin Institute, American Society for Testing Materials, American Concrete Institute, New England Water Works Association (Honorary Member), American Water Works Association, Merchants Association of New York, Century Association of New York, Arkwright Club, Engineers Club, Engineering Foundation (Chairman of Research Committee), and Tau Beta Pi and Sigma Xi fraternities.

Mr. Merriman was elected a Junior of the American Society of Civil Engineers on April 4, 1899, an Associate Member on April 1, 1908, and a Member on December 6, 1910.

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[7] "Portland Cement," by Thaddeus Merriman, *Journal*, Boston Soc. of Civ. Engrs., Vol. XXVI, January, 1939, p. 1.

CHARLES HAMILTON MITCHELL, M. Am. Soc. C. E.[1]

DIED AUGUST 26, 1941

After twenty-two years of service as Dean of the Faculty of Applied Science and Engineering, of the University of Toronto, in Toronto, Ont., Canada, Brig.-Gen. Charles Hamilton Mitchell passed away on August 26, 1941, at the age of sixty-nine years. Of unusually robust physique, he had been highly active and interested in many pursuits until the summer of 1940 when his health began to fail.

Born in Petrolia, Ont., Canada, on February 18, 1872, the son of the Rev. George A. and Agnes A. (Beckett) Mitchell, Charles Hamilton Mitchell was a graduate in civil engineering of the University of Toronto in 1892, receiving the professional degree of Civil Engineer in 1898. Upon graduation he became assistant city engineer of Niagara Falls, N. Y. From 1894 to 1901 he served as city engineer of Niagara Falls, Ont., at the same time maintaining a private practice in hydraulic engineering, which brought him a wide reputation as a specialist in this field and led to his appointment, in 1901, as principal assistant engineer of the Ontario Power Company. He occupied this position until 1906 when he went to Europe to investigate hydroelectric power practice there. On returning to Canada he practiced as a consulting engineer in Toronto, specializing in hydraulic and hydroelectric work, involving the design and construction of plants in the Maritimes, Ontario, and the Canadian West.

During 1909 and 1910 he was consulting engineer for the Ontario Power Company on its extension. From 1911 to 1914, and again from 1919 to 1920, he was consulting engineer of the Water Power Branch of the Federal Department of the Interior. Very frequently he was engaged as an arbitrator or expert witness in technical and legal cases.

Long before the outbreak of the first World War, military activities had claimed much of his attention. On the outbreak of that conflict he was a Major in the Corps of Guides and District Intelligence Officer on the General Staff of Military District No. 2, with headquarters at Toronto. For five years he served in France, Belgium, and Italy on the General Staff. In 1914 and 1915 he was a Lieutenant-Colonel in the First Canadian Division under Lord Byng; in 1915 and 1916 he was Chief of Intelligence for Lord Plumer, Commander of the Imperial Second Army; and in 1917 and 1918 he held the same post under Lord Cavan, at general headquarters of the British Army in Italy. In 1919 he was promoted to the rank of Brigadier-General and attached to the Intelligence Staff of the British War Office in London. Four governments honored him with decorations, and he was mentioned seven times in dispatches.

During the present war he gave valuable service as a civilian to the military authorities in connection with important matters of intelligence, and only his age prevented him from taking a more active part.

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[1] Memoir prepared by C. R. Young, M. Am. Soc. C. E.

Upon returning from the first Great War in 1919, he was appointed Dean of the Faculty of Applied Science and Engineering of the University of Toronto, a post that he filled until his retirement on June 30, 1941. In this position he was able to maintain active and enthusiastic contact with young Canadian engineers in training, a task which he found greatly to his liking. Following his retirement he was made Dean Emeritus of the Faculty, an honor which he did not live long to enjoy.

In 1920 General Mitchell was appointed a member of a Royal Commission to report on radial railways in Ontario, and in 1924 he became a Canadian member of the International Joint Board of Engineers to study and report on the navigation and power problems of the St. Lawrence River.

An honor in which he took great pleasure came to him in 1929 when he was elected President of the Engineering Institute of Canada. He had been elected the first President of the Ontario Association of Professional Engineers in 1922. He was a member of the Board of Governors of the University of Toronto from 1913 to 1919, and President of the Toronto Board of Trade in 1927 and again in 1933.

General Mitchell occupied a highly active and influential position in the public life of Canada. The wide expressions of appreciation of his genial personality, and of his contribution to engineering and to engineering education, that have been voiced indicate general acknowledgment of the value of his services to his country.

In 1901 he was married to Myra E. Stanton in St. Catherines, Ont. Mrs. Mitchell survives him.

General Mitchell was elected an Associate Member of the American Society of Civil Engineers on June 4, 1902, and a Member on January 5, 1904.

ALFRED MITTON MOSSCROP, M. Am. Soc. C. E.[1]

DIED MARCH 27, 1941

Alfred Mitton Mosscrop was born in Brooklyn, N. Y., on January 23, 1865, the son of Thomas and Emma (Mitton) Mosscrop. He attended the public schools of Brooklyn and studied at the Brooklyn Polytechnic Institute for one year. He then entered Cornell University, at Ithaca, N. Y., from which he was graduated in June, 1885.

After his graduation from Cornell he was assistant office engineer with the Missouri Pacific Railroad Company in Kansas in 1886. He returned to Cornell University as instructor in the College of Civil Engineering from November, 1886, to July, 1887. In 1887 he began work as an assistant in the office of the Rochester Bridge and Iron Works at Rochester, N. Y., and in 1889 was made engineer of the company in charge of the city office, covering the estimating and contracting departments. For part of the years 1892 and

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[1] Memoir prepared by Edwin A. Fisher, Hon. M. Am. Soc. C. E.

1893 he was chainman and topographer for the State of New York in the Adirondack Survey.

From 1900 to March, 1902, he was engineer and manager of the Rochester Works, then owned by the American Bridge Company. He was vice-president and manager of the Baltimore Bridge Company at Baltimore, Md., from April, 1902, to July, 1903.

In August, 1903, he became manager of the Bridge Department, Dorman Long and Company, Ltd., of Middlesbrough, England. From April, 1905, to June, 1909, he was general manager of the same company in the Rolling Mills Open Hearth Plant and Bridge Department. He was elected a director of the company in 1907. From July, 1909, to May, 1914, he was a director of the Dorman Long and Company, Ltd., with residence in the United States, where he looked after company interests. During this period he resided in Rochester. From July, 1909, to July, 1914, he was active as a consulting engineer, dealing mainly with reorganization problems for foreign bankers.

At the outbreak of the World War in 1914, he became active in Red Cross activities, giving up all other work. From March, 1918, to February, 1919, he was production manager of the Rochester District Ordnance Department, U. S. Army. From February, 1919, to about one week before his death on March 27, 1941, he was semi-retired, devoting time to the study of economic problems and giving opinions concerning them from time to time.

His chief fields of work during his active life were bridges and the operating end of rolling mills and open-hearth practice. His most important accomplishments were the reorganization, from an operating standpoint, of the Rochester plant of the American Bridge Company; the Baltimore Bridge Company; bridge shops; and the rolling mills and open-hearth plant of Dorman Long and Company, Ltd.

Mr. Mosscrop's professional standard was of the highest and best. In his business relations he had the complete confidence of all his associates. He enjoyed the respect and confidence of the community in which he lived, as a gentleman and a scholar.

He was a charter member of the Rochester Engineering Society in 1897, and of the Rochester Section of the American Society of Civil Engineers in 1923. He was also a member of the Iron and Steel Institute of England; the Military Order of the Loyal Legion in England; the Engineers Club of New York City; the Genesee Valley Club; Country Club of Rochester; and the honorary fraternity of Sigma Xi.

He is survived by his wife, the former Linnie Gambee of West Fayette, N. Y., to whom he was married on September 19, 1888. A son, Roger Gambee Mosscrop, and a daughter, Alfreda Mosscrop, also survive him.

Mr. Mosscrop was elected a Junior of the American Society of Civil Engineers on May 4, 1887; an Associate Member on May 3, 1893; and a Member on October 4, 1899.

EARLE DAWSON PARKER, M. Am. Soc. C. E.[1]

DIED JANUARY 9, 1942

Earle Dawson Parker was born on November 27, 1888, at Paris, N. Y., the son of the late Frank Parker and Julia (Morse) Parker.

After completing his course in the Sidney High School, at Sidney, N. Y., in June, 1907, he entered Rensselaer Polytechnic Institute at Troy, N. Y., and was graduated in 1911 with the degree of Bachelor of Science in Civil Engineering. At intervals during his college course Mr. Parker worked in survey parties for the New York State Highway Department, and after graduation was employed for about six months by the Public Service Commission, First District, New York, N. Y.

In June, 1912, he entered the employ of the New York State Highway Commission, and continued work for this Commission until April, 1922. He was promoted in regular steps to responsible charge of various types of highway construction and location.

In April, 1922, he entered the organization of the Lane Construction Corporation of Meriden, Conn., one of the larger contracting companies doing street and highway construction in the North Atlantic States. His advancement was consistent, and he rose to the position of assistant district engineer. In December, 1926, he became district engineer and was in charge of estimating cost, and had general supervision, of construction of highways, streets, and bridges, and structures incidental thereto, in New York and Connecticut. This included the development and assignment of construction organization, layout of plant setup for production and manipulation of all types of material, and selection and operation of appropriate transportation and performance facilities. His keen analysis of cost was particularly successful, covering an average estimate for successful bidding of more than a million dollars per year.

Contracts completed under Mr. Parker's direction were the grade-crossing eliminations in New York State at Newburgh, Chester, Cornwall, and Monroe, and dual reinforced-concrete pavement in Connecticut at East Hartford and South Windsor.

Mr. Parker's death was caused by an automobile accident on January 5, 1942, while driving from his home at Meriden to supervise construction work in New York State. He died on January 9 in the Sharon, Conn., Hospital.

He was a member of the First Methodist Church of Meriden, the Ancient Free and Accepted Masons, and the Connecticut Society of Civil Engineers.

On September 24, 1914, he was married to Grace Beagle, who survives him. One brother, Cecil M. Parker, also survives him.

Mr. Parker was elected an Associate Member of the American Society of Civil Engineers on July 9, 1923, and a Member on January 13, 1930.

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[1] Memoir prepared by George E. Smith, M. Am. Soc. C. E.

CHARLES HOWARD PAUL, M. Am. Soc. C. E.[1]

DIED OCTOBER 6, 1941

Charles Howard Paul was born on the rock-bound New England coast at Rockport, Mass., on March 10, 1875. His father, Howard H. Paul, was a paint manufacturer and son of a ship carpenter. His mother, Lucy (Dousett) Paul, was a direct descendant of Richard Tarr, one of two shipwrecked mariners who founded Rockport about 1690. The stability and ruggedness of these ancestors and of the surroundings in which they lived seem to have been distinctly reflected in the character of Mr. Paul.

While his boyhood was not devoid of a desirable amount of play and enjoyment of the maritime surroundings, he learned how to work at an early age. He caught and sold fish to earn spending money and also operated a fleet of dories for fishermen and sightseers. The summers were spent at the Paul's camp on Cape Ann opposite Thacher Island where the favorite sports were swimming, fishing, yachting, etc. Encouraged by his father, he had a live interest in athletics and became proficient in such sports as football, baseball, tennis, and track events. He began playing basketball when it was first introduced. He was brought up in a New England Puritan atmosphere where one went to church and Sunday school, where sports and boisterous fun were taboo on Sundays, and when boys did not argue with their parents.

Charles Paul's education began in the grade schools of Rockport where he continued through the first year of high school. He finished high school at Malden, Mass., where the family moved in 1889. In 1892 he entered Massachusetts Institute of Technology, at Cambridge, Mass., taking up the study of civil engineering. He was an honorary member of Tau Beta Pi fraternity. At the end of his third year he was compelled to drop his college work because of his father's death. At this time it became necessary for him to support the family consisting of his mother, grandmother, and two younger brothers, two and four years old.

His first professional job was that of pipe inspector on the construction of the Rockport water works during a summer vacation. After leaving school he was employed for a short time by the Sewer Department of Boston, Mass., and then by the Massachusetts State Board of Health on surveys of the Neponset River basin. This work kept him busy for about twelve hours per day and consisted largely of carrying a stadia rod through the Neponset meadows and swamps. This was a good job for a young engineer in those days and paid $52.00 per month.

In 1896 he transferred to the Boston Metropolitan Water District, which had recently been organized. Here he worked on surveys and designs for pipe lines, reservoirs, and pumping stations. When construction started he was made inspector of pile foundations, laying of large mains, and other construction work. On this work he became acquainted with a number of outstanding engineers. These associations he appreciated more and more in later years. According to Mr. Paul's statement they were men who insisted on building safe and efficient structures and getting full value for every dollar spent.

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[1] Memoir prepared by a Committee consisting of Walker R. Young, H. S. Morse, C. S. Bennett, and C. H. Eiffert, Members, Am. Soc. C. E.

In 1900 Mr. Paul accepted a position with the Bureau of Filtration in Philadelphia, Pa., on plans and specifications. During construction he was assigned to the Belmont plant which included a sedimentation basin, sand filters, and a reservoir. He was placed in charge of this work and on completion became responsible for its operation. The work of this bureau marked a great forward step in water supply engineering. At the time of its beginning typhoid epidemics were quite common in large cities, such as Philadelphia, Pa. With the completion of the work done at this time they became a matter of history in that city. It was here that Mr. Paul made his first acquaintance with dam and reservoir design and construction, a branch of engineering in which he became an eminent authority in later years. The following is a quotation from a letter written to him by the chief engineer when he left Philadelphia: "I have always felt that any work entrusted to your care would be executed in the same manner as though I were personally on the ground to supervise it."

In October, 1904, Mr. Paul entered the employ of the newly organized U. S. Bureau of Reclamation, the purpose of which was to design and build irrigation works in the arid West. His first assignment was on the Lower Yellowstone project near Glendive, Mont.

No doubt most of Charles Paul's friends and acquaintances would class him as quite conservative in every way. However, his start with the Metropolitan Water Board when it was first organized, his move to Philadelphia at the beginning of the work of the Bureau of Filtration, and then his change to the Reclamation Service at the time of its inception, indicate that he possessed in a marked degree the spirit of pioneering and the ability to adapt himself to new surroundings and circumstances.

While on this work Mr. Paul met Camilla M. Wheeler of Northville, Mich., who was teaching in one of the rural schools of Montana. This proved an important event in his life. Some of the trips to see her were made on horseback over a distance of 60 miles. They were married on June 19, 1907, at Northville. A few days later they returned to the Montana camp where a small cottage had been prepared for them. For 1½ years following, Mrs. Paul was one of two women residents at the camp. During this time the temperature dropped as low as 48° below zero and sometimes remained between 20° and 30° below for a week or more.

The next few years with the Reclamation Service brought a variety of experience on irrigation work, ranging from surveys through design and construction to maintenance and management of completed projects.

During the winter of 1910–1911 Mr. Paul was placed in direct charge, as construction engineer, of the design and construction of the Arrowrock Dam on the Boise River in Idaho. This was another job of pioneering as the Arrowrock Dam was to be higher than any dam built to that time. Its height, as originally planned and built, was 348.5 ft. It is of the curved gravity type, 1,100 ft long, and contains 600,000 cu yd of concrete. Excavation to bedrock reached a depth of 90 ft below the river bed. In 1938 this dam was raised to a height of 353.5 ft. On this work Mr. Paul was employed as consultant.

Because of possible contingencies that might arise in connection with abutments and foundations due to the unusual height of the dam, this work was done by force account rather than by contract. The organization was highly successful and the dam was completed one year ahead of schedule at a cost about 20% below the official cost estimate. It was a notable accomplishment. Mr. Paul, together with members of his organization, was commended highly by officials of the Reclamation Service and by the engineering profession.

In the fall of 1915 Mr. Paul joined the staff of The Miami Conservancy District, a political subdivision of the State of Ohio organized for flood control in the Miami Valley. Here again he joined in an undertaking which was of a pioneering nature. The method of flood control by a combination of retarding basins and channel improvements was radically different from anything previously attempted.

This project was ready for construction in 1917 when the world was in a turmoil. Because of war conditions and uncertainties it was impossible to secure reasonable bids from contractors for quantities such as were involved here. It was decided, therefore, to develop a construction organization to do the work by force account. The experience gained by Mr. Paul at Arrowrock was of great value in perfecting this organization. He soon became assistant chief engineer and in 1921 was made chief engineer of this organization. This work, which was estimated to cost more than $30,000,000, was completed below the cost estimate and ahead of schedule.

In 1924 Mr. Paul resigned as chief engineer of the Miami Conservancy District, but was retained as consulting engineer until his death. Before he left the District several floods occurred in the Miami Valley which were of sufficient magnitude to demonstrate the efficiency of the flood control works. Everything functioned as planned and there was great public interest particularly in the action of the dams.

At this time Mr. Paul established a consulting office in Dayton, Ohio, which he continued until his death.

In 1924 he managed successfully the International Air Races which were held in Dayton under the sponsorship of a large group of citizens, with the cooperation of officers of the U. S. Army Air Corps at Wright Field.

The following year he was made Managing Director of the Dayton Industrial Association. The purpose of this organization was to make a study of the local industrial situation, to act as a clearing house for local industrial problems, and to furnish authentic information to outside industries seeking a location. Mr. Paul managed this successful organization for about six years. He resigned in 1931 because of the demands on his time in other fields. From this time on he devoted practically his entire time to consulting work on large projects. The list of these is so extensive that only the principal ones will be mentioned here.

In 1933 he was appointed by the Secretary of the Interior as a member of the committee charged with the task of exploring the water problems of the

Mississippi Valley. This committee published a report in 1934 which was in popular form and was well received. From 1933 to 1940 Mr. Paul and his associates on this work passed on more than 3,000 proposed projects. Of course many of these had little or no merit but it was nevertheless necessary to give them consideration. This work later broadened into general consulting service for the Public Works Administration (PWA) and for the National Resources Board and led to the renewal of many old friendships and the making of new ones.

In 1933 he was appointed as one of the consulting engineers for the newly organized Tennessee Valley Authority (TVA). On this work he was connected with the design and construction of all the large dams built by the Authority. Most of these are outstanding in some particulars. Here also he renewed associations with many old friends, a number of whom had been his former employees and assistants.

In 1934 he was appointed as chairman of the consulting board for the Grand Coulee Dam and continued on this work until his death. This is the largest concrete dam in the world and probably the largest concrete structure of any kind ever built. During the years following this appointment he acted as consultant on many other dams of the U. S. Reclamation Bureau and served as chairman of the consulting board on a number of these projects. Among them were the Central Valley project of California, Marshall Ford Dam in Texas, Seminoe Dam in Wyoming, Bartlett Dam in Arizona, Deer Creek Dam in Utah, Green Mountain Dam in Colorado, and the inspection of many other dams and dam sites. Altogether he had an important part in the design, construction, or inspection of more than 200 dams, both earth and concrete. This work was located in 34 of the States. He had an unusually broad experience in the investigation, study, and treatment of dam foundations.

Mr. Paul always took an active part in civic and community affairs. At the age of 22 he was elected a member of the City Council of Malden. He was active in the Young Men's Christian Association (Y. M. C. A.) whenever he was so located that this was possible. At Dayton he was a member of the board of trustees and the advisory council of that institution for many years. In 1933–1934 he served as president. He served as a director of the Dayton Community Chest from 1927 to 1939. He was a member of the Chamber of Commerce for many years and also served as a director. In 1924 he became a member of the Dayton City Plan Board; and in 1925, at the request of a number of prominent citizens, he became a candidate and was elected to the City Commission, which is a non-partisan legislative and advisory group at the head of the Dayton Commission-Manager government. He served in this capacity for 4 years, during which several major projects were initiated in Dayton. During his service with the Dayton Industrial Association, which has been previously mentioned, he selected the site and directed the acquisition of the land for the Dayton Municipal Airport. In the years 1924–1926 he was a member of the building committee of Westminster Presbyterian Church and later of a similar committee for the new Y. M. C. A. building. Each of these structures was valued at about $1,000,000. He was a member of the Rotary Club of Dayton until his work took him away from Dayton too often.

One of Mr. Paul's outstanding characteristics was his ability to make friends, not only in the engineering profession but also among those with whom he associated in his daily community life. This applied to young men in minor positions as well as to persons of higher rank. Many of his assistants and subordinates, and others not directly connected with his work, were guided by the friendly counsel and advice which he was always ready and willing to give.

He was the author and co-author of many reports, articles, and papers published in engineering magazines and periodicals. In addition, he wrote a number of chapters for technical books.

Mr. Paul was one of the early presidents of the Engineers Club of Dayton, and of the Technology Club of Dayton. He was one of the organizers and the first president of the Dayton Section of the American Society of Civil Engineers. In 1922 he was awarded the Norman Medal of the Society for his paper entitled "Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District."[2] From 1924 to 1928 he served as a Director of the Society. During that time he served on a number of important committees including those on Flood Protection Data, Reorganization of Government Departments, Public Relations, Technical Activities and Publications, Aims and Activities, Professional Conduct, Technical Procedure, and Technical Expansion. He also contributed several technical papers and numerous discussions to the publications of the Society and was awarded Life Membership in 1940.

Mr. Paul is survived by his widow, and two brothers, Edwin L. and Elliot H. Paul.

Mr. Paul was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905, and a Member on September 1, 1908.

ROBERT JUSTICE PAULETTE, M. Am. Soc. C. E.[1]

Died August 28, 1941

Robert Justice Paulette was born at St. Joseph, Mo., on June 2, 1886. He was the son of Charles T. and Alma (Wilson) Paulette. He attended the public schools of St. Joseph and was graduated from the University of Missouri, at Columbia, in 1912 with the degree of Bachelor of Science in Mining Engineering. He received the degree of Civil Engineer from the same University in 1932.

Mr. Paulette was early attracted to engineering and spent his summer vacations, while attending high school, with surveying parties in and around St. Joseph. During his vacations, while in the University, he worked at various times for the St. Joseph-Southern Railway Company, the City of St. Joseph, and The Home Telephone Company.

He spent the two years following his graduation with the Anaconda Copper Mining Company at Great Falls, Mont. This engagement was followed by two years with the U. S. Reclamation Service on the Sun River Project at Fort Shaw, Mont.

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[2] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1181.

[1] Memoir prepared by Lloyd B. Smith, M. Am. Soc. C. E.

In 1915 Mr. Paulette became construction engineer for Black and Veatch, Consulting Engineers, in Kansas City, Mo. Then in 1917 he entered the armed forces of the United States as First Lieutenant of the 315th Engineers and two years later returned from overseas as Captain with the 513th Engineers.

In 1925, after serving the City of Emporia, Kans., for five years as city engineer and superintendent of utilities, he became director of public works for the City of Salina, Kans.

In 1931 he went into private consulting engineering practice and a year later organized the Paulette and Wilson Engineering Company. He remained active in this organization until his death. For two years during this time, 1933 to 1935, however, he retired from the company to act in the capacity of state engineer, Federal Emergency Administration of Public Works.

The engineering work of Mr. Paulette extended into a wide variety of fields. He handled the many heavy responsibilities that fell to him with engineering exactness and efficiency, with fairness to all, and with unusual satisfaction to his clients and employees.

He was active in the passage of the Engineer's Registration Law in 1931. He was appointed a member of the first Registration Board of Professional Engineers and was elected secretary of the Board, which office he held until his death. This indicates the high regard in which he was held, not only by the engineers of the state but by the general public, since the appointing authorities changed political parties during the time of his incumbency.

Mr. Paulette took an active part in the various engineering organizations. He was past-president of the Kansas Water and Sewage Works Association and a member of the Kansas Engineering Society, the Topeka Engineers Club, and the American Water Works Association.

He was married to Jessie A. Griebie at Farmington, Minn., on August 24, 1917. He is survived by his widow and three children, Grace Alma (Mrs. D. V. McClure); Robert Griebie Paulette, Jun. Am. Soc. C. E., Second Lieutenant, U. S. Air Force; and Charles Harvey Paulette.

Mr. Paulette was elected a Member of the American Society of Civil Engineers on November 15, 1926.

RAYMOND ALBERT PEASE, M. Am. Soc. C. E.[1]

Died April 19, 1941

Raymond Albert Pease was born at Kelso, N. Dak., on June 19, 1888, the son of Albert Edward Pease and Jennie (Files) Pease, pioneers in North Dakota. He received his elementary education at Kelso, attended Central High

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[1] Memoir prepared by Z. E. Sevison, M. Am. Soc. C. E., and Ray Robinson, Maintenance Engr., North Dakota State Highway Dept., Bismarck, N. Dak.

School in Minneapolis, Minn., and was graduated in 1908. He then entered the University of Minnesota, at Minneapolis, receiving the degree of Bachelor of Science in Civil Engineering in 1912 and a degree in Civil Engineering in 1913. While a student there, he became a member of Psi Chapter of Theta Xi, engineering fraternity.

Mr. Pease began his professional work in September, 1913, as instrumentman and draftsman with B. J. Reynolds, at Redwood Falls, Minn. From June, 1914, until November, 1915, he held the position of assistant city engineer, City of Chisholm, Minn. From June, 1916, until September, 1917, he served as deputy county surveyor, Ward County, North Dakota. At that time he entered the United States Army, serving as a Private, Corporal, and then Sergeant in the 313th Engineers, and as 2d Lieutenant in the 29th Field Artillery.

Following his service in the U. S. Army, Mr. Pease resumed his position as deputy county surveyor of Ward County in April, 1919, and served in this capacity until May, 1921. In July, 1921, he became a resident engineer for the State Highway Department. He held this position until September, 1925, when he became division engineer of the Minot Division of the North Dakota State Highway Department. He continued in this position until June, 1933.

Mr. Pease was camp superintendent of Civilian Conservation Corps (CCC) camps and was in charge of parties on survey and design of water conservation dams, serving under A. D. McKinnnon, coordinator of the Federal Soil Conservation Service, at Bismarck, N. Dak., from June, 1933, to December, 1934. From April, 1935, until June, 1935, Mr. Pease was assistant civil engineer with the Bureau of Biological Survey. He then became resident engineer inspector for the Public Works Administration (PWA). He resigned from this position in February, 1940, when his health failed.

Besides his membership in the Society, Mr. Pease was a member of the Society of American Military Engineers. He was licensed in the State of South Dakota as a professional engineer and land surveyor. Mr. Pease was active in the community life of his home city of Minot, N. Dak., being a member of the American Legion, Veterans of Foreign Wars, and the Lions Club.

He was ever alert to promote the interests of good roads and water conservation in Minot and in the whole northwestern North Dakota area. He was of unassuming nature and not many knew of his assistance to young men, particularly those interested in engineering, in completing their college education when their parents were unable to give financial assistance. As a tribute to his memory, immediately following his death, friends in Minot collected a considerable sum of money as a contribution to the Children's Home in Fargo, N. Dak. It was from this home that Mr. and Mrs. Pease adopted their infant son, Theodore, in 1930.

On May 7, 1922, Mr. Pease was married to Laurel Vivian Alpstag, who, with their son, survives him.

Mr. Pease was elected a Member of the American Society of Civil Engineers on August 17, 1931.

JOHN JOSEPH PHELAN, M. Am. Soc. C. E.[1]

Died February 11, 1941

John Joseph Phelan was born in Lowell, Mass., on April 28, 1879, the third of four children born to Arthur Quin and Mary Ann (Molloy) Phelan. His father was a physician, having received his degree in medicine at Harvard University, Cambridge, Mass., after his earlier studies at Dublin College, Ireland. His mother and sister were graduate practicing pharmacists.

As a youth Mr. Phelan received his primary and high-school education in Lowell. Even at an early age he showed a deep interest in military training and ranked as a first lieutenant in his high-school battalion. His histrionic flare found expression in amateur school and college theatricals as minstrel end man, as comedian in musical comedy and light opera performances, as drummer in the college band, and with the glee club at college as bass and soloist, as well as in the church choir. This was a direct inheritance from his mother's brother, Terrence Molloy, a great tenor singer and church organist, who was cast in a stage play with the well-known Raymond Hitchcock.

He was graduated from the Lowell High School in 1898 and employed by "The Proprietors of the Locks and Canals" on the Merrimack River, at Lowell, as hydraulic engineer on water and mill maintenance until 1903. He attended Tufts College, at Medford, Mass., from which he received the degree of Bachelor of Science in Civil Engineering in June, 1907. During two summers of his college years (1905 and 1906), he was employed by the New England Telephone and Telegraph Company as assistant to the division chief inspector in Lowell, assigning work to forty-five inspectors and instrument-men, while engaged in making payrolls, writing checks, and accounting for stock supplies.

For the next three years (until 1910) Mr. Phelan was engaged in triangulation, surveying, and chart making of the waters of Chesapeake Bay and tributaries with the United States Coast and Geodetic Survey in cooperation with the Maryland Shell Fish Commission on oyster bar survey. This work included the navigation and thorough knowledge of these waters, as well as the maintenance of steam, gasoline, and kerosene oil boats.

For the next year until May, 1911, he was with the United States Geological Survey as hydraulic engineer in the Water Resources Branch on stream gaging and surveys in the Adirondack Mountains in New York State and office work in Washington, D. C.

His next engagement (May, 1911, to December, 1917) was with the International (Canadian) Boundary Commission. The last four of these years he was chief of party, establishing and marking the United States-Canadian boundary between Minnesota and Ontario, working in topography, astronomy, triangulation, leveling, and disbursements and subsistence supply for a topographic and leveling party of thirty men and a monumenting and referencing party of ten men.

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[1] Memoir prepared by Lynn Perry, M. Am. Soc. C. E.

When the United States entered the war in April, 1917, Mr. Phelan applied for a commission, and on June 19, 1917, was commissioned Captain in the U. S. Army Engineers, but was not released from his work with the Boundary Commission until December. On December 14, 1917, he was assigned by the War Department to military mapping in Virginia and Georgia and later as an instructor in Astronomy, Geodesy, Topography, Hydrography, Photography, and Map Reproduction in the Regular Army Post-Graduate Engineer School, Camp Humphreys (later Fort Belvoir), Virginia.

Being discharged from the Army on October 31, 1919, Captain Phelan was employed until July, 1920, as civil engineer and principal civilian assistant to the Constructing Quartermaster and Utility Officer on Construction and Operation of hospital buildings. He was placed in charge of design and construction of buildings, highways, utilities, and power plant at Walter Reed, United. States Army General Hospital, Washington, D. C. He was commissioned Major, Engineer Reserves, U. S. Army, on May 1, 1920.

Major Phelan was employed from August, 1920, to September, 1922, by the Federal Board for Vocational Education and its successors, the United States Veterans' Bureau and United States Veterans' Administration, in New York, N. Y., Newark, N. J., and Hartford, Conn., as vocational adviser in the rehabilitation of veterans and the preparation of appeals, and as Sub-District Manager.

He received his commission as Lieutenant Colonel, Engineer Reserve, U. S. Army, on December 1, 1923.

From September 1, 1922, to January, 1924, he was on the Board of Registration for the U. S. Veterans' Bureau, and U. S. Veterans' Administration, and from then until August, 1933, was civil engineer, Construction Division, with headquarters in Washington, D. C. On this assignment he was engaged in hospital design, construction and inspection, surveys, and maintenance of Veterans' Hospitals. Colonel Phelan also completed special assignments from the office of the Director and Administrator and for the Federal Board of Hospitalization on new hospital projects, inspections and investigations as well as selection, acquisition, title searches, and condemnation of Veterans' Hospital and Home sites. During these years Colonel Phelan's work carried him into most of the forty-eight states, and he made a host of friends wherever he went.

Because of close contact with legal matters and proceedings which his work entailed, he attended evening classes whenever he had the opportunity at the Columbus University Law School in Washington, D. C., to increase his efficiency in his work. As a result Colonel Phelan received the degrees of Bachelor of Laws in May, 1928, and Master of Laws and Master of Patent Law in June, 1930, from the Columbus University. He was admitted to the Indiana State Bar in March, 1930, and to the Supreme Court of Appeals of West Virginia as a Visiting Attorney in April of the same year.

From August to December, 1933, he was employed as Regional Utility Officer, in charge of water supply, sewage disposal, and the utilities and service employees of the United States Veterans' Hospital at Lyons, N. J.

Colonel Phelan was then appointed regional engineer in charge of engineering and inspection of field work, of relief and construction, for New England, Florida, and the Great Lakes states, also work in Washington, D. C., with the United States Civil Works Administration and Federal Emergency Relief Administration, in which capacity he served until July, 1934, when he was made inspection engineer of construction, and of the construction and space surveys of federal buildings, with the Public Works Branch of the Procurement Division, Treasury Department.

In July, 1936, he accepted the position of Utility Officer at the United States Veterans' Administration Facility, Mount Alto Hospital, in Washington, D. C., in charge of the utilities, maintenance, construction, and service employees of this hospital.

Though he had received a "certificate of capacity" for the rank of Colonel, Corps of Engineers Reserves, July 19, 1927, he was not appointed until March 12, 1936. Colonel Phelan was then assigned commanding officer of the 376th Engineers Regiment. He was very active in army reserve work (including the monthly problems) and looked forward to his yearly training periods at Fort Belvoir where he was in full command of all reserve officers in camp during his last detail there in August and September, 1939.

In April, 1938, Colonel Phelan was transferred to the United States Veterans' Administration Hospital at Wichita, Kans., in the capacity of utility officer, in charge of hospital maintenance and operations. Previously he had been instrumental in determining the site of this hospital, and in its construction. He had many friends in Wichita, made in former years. He was well and favorably known to the entire staff at the hospital, medical and service. They were grieved at his death, after a brief illness, on February 11, 1941.

Colonel Phelan was a member of the American Society of Mechanical Engineers; the Society of American Military Engineers (Charter Member); the Washington Society of Engineers; the Reserve Officers Association; the Military Order of the World War; the Fort Stevens (Washington, D. C.) and the Thomas Hopkins (Wichita) Posts of the American Legion; the Tufts Club of Washington, D. C.; the Tau Chapter of Alpha Kappa Pi fraternity; and, formerly, the Boston Society of Civil Engineers.

He was a member of the Sacred Heart Church in Washington, D. C., and a Fourth Degree Knight of the Washington Assembly of the Knights of Columbus.

Backed by years of travel and an almost inexhaustible fund of rich and interesting experiences, Colonel Phelan was a delightful conversationalist and entertaining raconteur. His outstanding characteristics were his ready wit and contagious good humor. He was a true and loyal person, with a keen sense of justice and charity. He was thorough, accurate, trustworthy, and dependable. His standards of morals, honor, and service were high. He practiced them on a high level and insisted on the same from those over whom he had jurisdiction. Many younger men who have served under his direction are indebted to him for their training in the simple old-fashioned virtues along with the accuracy and efficiency that are too frequently considered sufficient. Colonel Phelan was a student of men and of nature. He knew human nature and the motives that inspire men to "carry on" and to see the job through to the end. The years that he spent on the Maryland Oyster Bar Survey and on the Canadian Boundary Survey provided the opportunity for a thorough acquaintance with quiet Nature which he loved so much. From this he developed a fund of knowledge which was of technical value and which furnished a basis for more sublime speculation. He was not a salesman nor advertiser in any sense. Publicity had no place in his philosophy of life. He was an engineer and a leader of men.

On November 24, 1914, he was married to Emily Wherry Matter. He is survived by his widow, his son, John Joseph, Jr., and his grandson, Joseph Michael.

Colonel Phelan was elected a Member of the American Society of Civil Engineers on November 9, 1920.

WILLIAM HALE PHILLIPS, M. Am. Soc. C. E.[1]

DIED JUNE 22, 1940

William Hale Phillips was the youngest of seven children of John Phillips and Sarah (Rhoads) Phillips. He was a native Californian, born in the San Joaquin Valley Town of Lemoore, on October 26, 1877. He attended the local schools and worked on the family farm.

At a very early age he lost both of his parents. Although provided for by his grandparents, he soon found that as far as his future was concerned he would have to think out his own plans. He never cared for farming, but a liking for mathematics, which was encouraged by his high school principal, suggested that he follow engineering as a lifework. Perhaps he realized that he needed an active outdoor vocation to keep him in health, for as a youngster he had had considerable sickness. Perhaps it was to show his people that in spite of a serious physical disability he could make good. William Phillips' right hand was missing—severed at the wrist. At all events, in the face of his elders' protests and discouragement, he held to his conviction that he could become an engineer.

In 1900 he entered the University of California, at Berkeley. After three years he went to the University of Colorado, at Boulder, where he finished his undergraduate work. It was necessary for him to find employment throughout his college years to supplement his limited resources. Later he returned to the University of California where he received the degree of Master of Science in Civil Engineering in 1905.

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[1] Memoir prepared by John R. Cahill and H. G. Sharp, Members, Am. Soc. C. E.

As he had planned years before, most of his professional activity was in the field. His first work was with the Realty Syndicate in Oakland, Calif., on surveying, subdivision, roads, and streets. In a few years he was off to the Northwest where in Spokane, Wash., he worked with the city engineer. Later he opened an office of his own.

In Spokane he met and was married to Jane Palmer of Chicago, Ill., in 1913. They had two children, Sarah and Frank, who, with Mrs. Phillips, survive him.

For several years William Phillips followed railroad work under the late James J. (Jim) Hill,[2] F. Am. Soc. C. E., of the Great Northern Railway Company. Then he went to British Columbia, Canada, on surveys for a water project. The first World War interrupted this work so he moved to Chicago and engaged in building construction for one of the steel companies; then he went to Philadelphia, Pa., on similar work for the U. S. Navy. After the Armistice in 1918 he returned to California and opened an office in San Francisco. Later he entered the office of the state architect at Sacramento to superintend the construction of schools and hospitals. Among these were the Teachers College at Chico, The State Hospital at Ukiah, and others.

During the 1920's and 1930's he worked for contractors, engineers, and architects. For several years he was with Weeks and Day, Architects, and later with Cahill Brothers, General Contractors. He also worked for the late A. J. Cleary, M. Am. Soc. C. E., and associates on investigation and study of water resources for the San Francisco Bay area.

In 1937 he entered the employ of the San Francisco Bay Exposition where he remained during the construction period of the Fair. He left in 1939 to accept a position with the Work Projects Administration which was helping to finance the construction of school buildings. This work took him to Sonora, then to Los Angeles, and later to Redding, Calif. He was active until his death in June, 1940.

William Phillips loved the outdoors and chose his work accordingly. He was also a home-loving man and spent what time he could with his family, working in his home or in his garden, for one of his hobbies was flowers. He was always interested in boys, and in his later years he gave much time to boys' welfare work, actively serving the local Boy Scout Troup as committeeman. His passing is a real loss to the profession and to the community.

Mr. Phillips was elected a Junior of the American Society of Civil Engineers on November 1, 1904; an Associate Member on May 4, 1909; and a Member on May 15, 1917.

JOHN HORTON POPE, M. Am. Soc. C. E.[1]

DIED JULY 26, 1941

John Horton Pope was born at Pomeroy, Ohio, on December 16, 1866, the son of the late Maj. Gen. John Pope and Clara Pomeroy (Horton) Pope.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. LXXXI, December, 1917, p. 1829.

[1] Memoir prepared by F. H. Pope, Brigadier General, U. S. Army (*Retired*), Washington, D. C.

After attending schools at Fort Leavenworth, Kans., where his father was stationed, he entered Washington University, St. Louis, Mo., from which he was graduated in 1888 with the degree of Civil Engineer. For some months thereafter he was engaged in railroad surveying in Kansas.

From 1889 to 1894 he was employed by the American Bridge Company on the new Merchants Bridge at St. Louis, which was completed in the latter year. During the next three years, he was engaged on several bridge constructions, including service at Phoenixville, Pa.

In 1897 he was appointed bridge engineer of the Missouri, Kansas and Texas Railway Company, with station in Parsons, Kans., where he remained until 1902. Then he went to Manila, Philippine Islands, to do construction work on sea-wall and harbor improvements.

In 1906 Mr. Pope entered the employ of the Pearson-Farquhar Syndicate, which had large and diversified interests in Central and South America. His activities took him to Guatemala, Costa Rica, and Nicaragua. In 1908 he made a survey for the construction of the Madeira-Marmoré Railway, from the headwaters of the Amazon in Brazil to the west coast of South America. On this trip he was stricken with black-water fever, from which he never fully recovered.

He served in Bolivia with the Pearson-Farquhar Syndicate until 1913, when he was sent to Montevideo, Uruguay, to be the South American engineering representative of the Syndicate. The effect of the World War forced the Syndicate into liquidation, but he remained in Uruguay as its representative until its affairs were finally settled in 1920.

He then returned to the United States and retired from the active practice of his profession. He resided in Philadelphia, Pa., from that time until his death.

In 1897 he was married to Hope Cox, daughter of the late Jacob D. Cox, who was a Major General during the Civil War, later Governor of Ohio, and Secretary of the Interior in the cabinet of President Ulysses S. Grant. She predeceased her husband.

Surviving Mr. Pope are his sister, Lucretia Pope, and two brothers, Horton Pope, and Brig. Gen. F. H. Pope, U. S. Army (*Retired*).

Mr. Pope was elected a Member of the American Society of Civil Engineers on December 7, 1904. He became a Life Member in 1937.

JOSEPH HYDE PRATT, M. Am. Soc. C. E.[1]

DIED JUNE 2, 1942

Joseph Hyde Pratt was born in Hartford, Conn., on February 3, 1870, the son of James Church and Jennie A. (Peck) Pratt. He was educated at the Sheffield Scientific School of Yale University in New Haven, Conn., from which he received the degrees of Bachelor of Philosophy and Doctor of Philosophy in 1893 and 1896, respectively. He also received an honorary degree of Master of Arts in 1923. He was instructor in mineralogy at Yale University from 1895 to 1897, and at Harvard University, in Cambridge, Mass., during the summer of 1895.

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[1] Memoir prepared by T. F. Hickerson, M. Am. Soc. C. E.

In 1892 Mr. Pratt went to North Carolina on his vacation to collect minerals with State Geologist Joseph A. Holmes. In 1897 he returned as a permanent citizen of North Carolina, and made his home at Chapel Hill, where he lived for forty-five years. His was an active and useful life in which he won distinction as a geologist, engineer, soldier, and all-round public servant. He lectured on mineralogy at the University of North Carolina, at Chapel Hill, from 1898 to 1901, was state mineralogist from 1897 to 1906, state geologist and director of the North Carolina Geological and Economic Survey from 1906 to 1924, and professor of economic geology at the University of North Carolina from 1904 to 1925.

Professor Pratt aided in the discovery of several Carolina minerals—pirssonite, wellsite, rhodolite, and mitchellite. He also helped to organize Western North Carolina, Inc., the Made-in-Carolina Association, and a number of civic organizations.

Interested in good roads, he was one of the pioneer advocates of better roads in North Carolina and the remainder of the United States. For ten years or more prior to the establishment of the North Carolina Highway Commission, of which he was the first secretary, he conducted an extensive educational program throughout the state, consisting of lectures and informative articles on methods of road building and the advantages of better roads. He served for years as secretary of the North Carolina Good Roads Association, conducted road institutes, and sponsored field demonstrations of road location, construction, and maintenance. He was a prominent member of the Southern Appalachian Good Roads Association and the American Road Builders Association, and was a director of the American Association for Highway Improvement.

Many other activities allied to the development of North Carolina claimed his attention. He was at one time the president of the American Peat Society; a member of the National Association of Shell Fish Commissioners, the National Drainage Association, the North Carolina Drainage Association, the Forestry Congress (South), the American Chemical Society, the American Association for the Advancement of Science, the American Institute of Mechanical and Metallurgical Engineers, the North Carolina Academy of Science, and the Fisheries Society; chairman, executive committee of the Appalachian Press Conference; past secretary, North Carolina Fisheries Association; and director of the American Forestry Association, the National Parks Association, and the Military Engineers Society.

At the St. Louis (Mo.) Exposition in 1904, he was a member of the international board of awards. In 1907 he was chief of the Department of Mines and Metallurgy at the Jamestown (Va.) Exposition. From 1904 to 1905 he was expert, Twelfth United States Census on asbestos, etc., for the U. S. Coal Testing Plant at St. Louis. He contributed many articles to scientific magazines, both domestic and foreign, on mineralogical, geological, and chemical subjects. Among his later publications were articles entitled "Topographical Mapping" and "American Meridians."

When America entered World War I, Joseph Hyde Pratt was commissioned Major in the 105th Regiment of Engineers of the North Carolina National Guard in the 30th ("Old Hickory") Division. He was promoted to Lieutenant Colonel in November, 1917; went to France in the summer of 1918; served in Flanders and Belgium, and in the Somme offensive as commander of the regiment; and was commissioned Colonel in October, 1918. It was the 30th Division which made the first big break in the Hindenburg line, and Colonel Pratt received the Distinguished Service Medal in recognition of his services in that and other operations. His War Diary, consisting of a series of daily letters to his wife and son during the period May, 1918, to May, 1919, was published in book form by Colonel Pratt in collaboration with the North Carolina Historical Commission.[2]

After the war he returned home with impaired health, which caused his retirement to the hospital for about three years. He was unable to take up again all the former connections with the North Carolina Geological Survey, and so he decided to resign this position in 1924. Later, with improved health, he served as regional engineer of the Civil Works Authority, the Federal Emergency Relief Administration, and the Resettlement Administration; still later he was contact engineer for the Work Projects Administration. For a time he was senior engineer consultant for the U. S. Geological Survey.

As one of North Carolina's most prominent civic leaders, Colonel Pratt was president of the North Carolina Symphony Orchestra Society, president of the North Carolina Society for the Preservation of Antiquities, chairman of the Chapel Hill chapter of the Red Cross, and chairman of the Orange County Welfare Committee. Declared Chapel Hill's "most valuable citizen" in 1940, he was presented with the annual award for his work in local welfare groups and for establishment of the negro community center.

He was a member of the Cosmos Club (Washington, D. C.); Washington Philatelic Society; Sigma Xi; Alpha Tau Omega; and the Order of Gimghouls (Carolina). He was a member of the Episcopal Church. One of his hobbies was old stamps, of which he had a very interesting collection. Chapel Hill will long hold Colonel Pratt in grateful and affectionate remembrance.

He was married to Mary Dicus Bayley of Springfield, Ohio, in 1899, and they had one son, Joseph Hyde Pratt, Jr., a physician at the Mayo Clinic in Rochester, Minn. After the death of Mrs. Pratt, in 1929, Colonel Pratt was married in 1930 to Harriet White Peters of Baltimore, Md., and Washington, D. C. He is survived by his widow; one son; two sisters; two stepsons, James Girvin Peters and Horace White Peters; and several nieces and nephews.

Colonel Pratt was elected a Member of the American Society of Civil Engineers on June 6, 1921.

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[2] "War Diary," by Joseph Hyde Pratt (Edwards and Broughton, publishers), Raleigh, N. C.

GEORGE WILLIAM REAR, M. Am. Soc. C. E.[1]

DIED FEBRUARY 10, 1942

George William Rear was born in Uxbridge, Ontario, Canada, on June 13, 1873. His parents were John and Mandilla (Hilborn) Rear. He received his education at Campbellford Collegiate Institute, Northumberland County, Ontario, Canada, and was graduated in 1891. He spent his vacations of 1889 and 1890 (a total of nine months) in the engineering department of the Midland Railway Company of Canada as an engineering apprentice. After his graduation from college, he was employed for four years by the Midland Railway Company of Canada as rodman, chainman, and transitman, and also worked in the office on line changes, grade revision, and bridgework.

From 1895 to 1901 Mr. Rear was with the Grand Trunk Railway Company in Canada, after it had absorbed the Midland Railway Company. He acted as draftsman, checker, inspector, foreman, and accountant, and worked on various other engineering assignments. His last work for the Grand Trunk Railway Company was as chief clerk in the office of the superintendent. While with the Canadian railways, Mr. Rear was engaged in work on surveys and construction of the bridges over the Trent, Otonabee, Moira, and other rivers, including the work of rebuilding the Victoria Bridge at Montreal, Quebec. He was also engaged on the Don improvement at Montreal and many grade improvements, including those at Trenton, Ontario, St. Annes, Quebec, and Vaudrieul, Quebec.

Mr. Rear's services with the Southern Pacific Company in the United States began on July 3, 1901, as foreman of bridge erection, and he served in this capacity until March, 1902, when he became assistant general bridge inspector. From August, 1905, until his death he was engineer of bridges (part of this time he had the title of general bridge inspector).

A complete list of the various structures that Mr. Rear designed and constructed for the Southern Pacific Lines during his long service would be too lengthy to include herein. A few of the more important ones are the Suisun Bay Bridge, California; Sacramento River Bridge at Sacramento, Calif.; and bridges over the American River, California, Colorado River at Yuma, Ariz., Wilmington Harbor, California, Los Angeles River, California, and the Willamette River and other waterways in the State of Oregon.

Mr. Rear had many friends. A fine, honest, upstanding man, he was unusually sincere in his friendships, and all who knew and worked with him loved him. His honesty, loyalty, self-sacrifice, belief in fair play, tolerance, and keen sense of humor were outstanding characteristics. His generosity and willingness to help a friend or associate in need, and his kindliness toward others, were among his finest traits. He took great pride in his profession and work.

On April 8, 1903, he was married to Madge Virginia Jackman at Santa Rosa, Calif. He is survived by his widow; a son, George W. Rear, Jr.; a daughter, Madge D. (Mrs. Robert R. Root); and a granddaughter, Geraldine M. Root. He was a wonderful husband and father.

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[1] Memoir prepared by W. H. Kirkbride, M. Am. Soc. C. E.

Mr. Rear was very active in fraternal work, and was a life member of Royal Victoria Lodge No. 57, F. and A. M., of Montreal; a member of Park Boulevard Lodge No. 568, F. and A. M., of Oakland, Calif.; a Knight Commander, Court of Honor, Oakland Scottish Rite Bodies; Past Venerable Master, Knights Kadosh, Oakland Consistory No. 2; Senior Warden, Oakland Lodge of Perfection No. 2; and a member of Aahmes Temple, Ancient and Accepted Order, Nobles of the Mystic Shrine. He was one of the organizing members of the Pacific Railway Club of San Francisco, Calif., in 1917, and was president of the Club in 1918 and on its Board of Governors from 1918 to 1922.

Mr. Rear was a member of the American Railway Engineering Association, a member and past president of the American Railway Bridge and Building Association, and a member of the Engineers' Club of San Francisco.

He passed away in San Francisco on February 10, 1942, and the funeral services were held on February 13 at the Scottish Rite Temple, in Oakland, and were conducted by Gethsemane Chapter No. 2, Knights Rose Croix, and the Rev. J. A. B. Fry, D.D.

Mr. Rear was elected a Member of the American Society of Civil Engineers on October 15, 1923.

CHARLES RUSS RICHARDS, M. Am. Soc. C. E.[1]

DIED APRIL 17, 1941

Charles Russ Richards was born in Clarks Hill, Ind., on March 23, 1871, the son of Charles and Sarah Elizabeth (Watt) Richards. His mother was of the family that produced the famous Scottish inventor and engineer, James Watt, so it was but natural that young Richards turned to engineering as his profession.

Mr. Richards was graduated from Purdue University, at Lafayette, Ind., in 1890 with the degree of Bachelor of Mechanical Engineering. In 1891, he received the degree of Mechanical Engineer from Purdue, and in 1895 the degree of Master of Mechanical Engineering from Cornell University, in Ithaca, N. Y., doing his work *in absentia*. In his undergraduate days he was a college mate of George Ade and, together with that later famous humorist and others, he promoted Purdue's early football teams. Too small to play, he was an enthusiastic supporter and aided in the management of the team.

In 1891 he began his career as a teacher and engineer by becoming an instructor of mechanical engineering at Colorado State College of Agriculture and Mechanical Arts, in Fort Collins, Colo., where he designed the college foundry. He went to the University of Nebraska, at Lincoln, in 1892 and remained there until 1911. He spent the first two years as adjunct professor of manual training and during these two years designed and erected the college shops. During the next four years he was professor of practical mechanics. In 1898 he became professor of mechanical engineering. Nebraska's Mechanical Engineering Laboratory was designed and erected by Professor Richards. From 1907 to 1909 he served as associate dean of the Industrial College and had charge of all engineering. From 1909 to 1911 he was dean of the College of Engineering.

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[1] Memoir prepared by Walter R. Okeson, M. Am. Soc. C. E.

During this period of nineteen years Professor Richards was active in field sports, being an ardent hunter and fisherman. Later his life became too sedentary, and his scholarly and administrative duties pushed into the background all forms of active exercise. Doubtless this immolation of outdoor recreation on the altar of duty had much to do with his subsequent breakdown while still in his early sixties.

In 1911 Professor Richards was called to the University of Illinois, at Urbana, as professor of mechanical engineering. He reorganized the system of shop instruction and designed and supervised the reconstruction of the Mechanical Engineering Laboratory. His great administrative talent was recognized, and from 1913 to 1915 he was acting dean of the College of Engineering and acting director of the Engineering Experiment Station. From 1917 to 1922 he was dean of the College of Engineering and director of the Engineering Experiment Station, and was in general charge of the original organization of the School of Military Aeronautics of the University of Illinois. When the United States entered World War I, he added to his other duties by serving on the National Committee of Coal Conservation and as consulting engineer of the U. S. Bureau of Mines.

In the fall of 1922, Dean Richards went to Lehigh University, in Bethlehem, Pa., as president. The impression he made on his fellow faculty members at the University of Illinois is evidenced by the engrossed resolution adopted and signed by his colleagues in the College of Engineering of that University. It reads as follows:

> "As you leave the University of Illinois, we wish to express the pleasure we have had in our association with you and our regret that it must be interrupted.
>
> "In our dealings with you as Dean of the College of Engineering, we have prized especially the sympathetic co-operation, the frankness and the impartiality upon which we have all come to rely. We have felt ourselves, as a faculty, safely and adequately and vigorously represented both within and without the University. We shall always remember the cordiality and sincerity that have marked our personal as well as our official relations.
>
> "We assure you of our appreciation of the important contribution you have made to the progress of the College of Engineering through the sound judgment and vision with which you have administered its affairs.
>
> "Our regret in having you go from us is tempered by our pride in your election to the presidency of Lehigh University. We heartily wish you the highest measure of success in that position, and the fullest opportunities for service and accomplishment in the years to come."

At Lehigh he began the work which will stand as a monument to one of the great educators and administrators of his day and generation. In thirteen short years he made over this institution. Under his inspiring leadership the alumni raised millions for endowment and millions more for plant, and the faculty grew steadily in teaching strength and in productive scholarship and research.

In his first year at Lehigh he made a survey of the University's needs and laid out a constructive program designed to produce a unique and outstanding educational institution. He had no patience with shirkers or those who lacked vision. Faculty members who had no desire to add to the sum of human knowledge but preferred merely to follow routine teaching methods had short shrift. Alumni who felt that winning football teams were more important than educational standards were horrified when, taking the "bull by the horns" and supported by his trustees, he abolished athletic scholarships and took the lead among eastern colleges in putting athletics at Lehigh on a strictly amateur basis. To the surprise of those who thought he was destroying alumni support and believed he would shrink student enrollment, the exact opposite happened. The enrollment of the University doubled, and the endowment and plant trebled through alumni contributions.

President Richards was an excellent business man as well as an outstanding educator. During his administration Lehigh University showed an operating surplus each year. Although the salaries of faculty members had been increased during his early years as president to a scale comparable with other first-class institutions, there was no cut in them during the depression years. By rigid economy in other directions he kept the University "out of the red."

Seldom does an institution receive such devoted service as President Richards gave to Lehigh. Every waking hour was given to thought and action that resulted in a complete reorganization of the work in the three colleges which make up the University. Administrative machinery and methods were expanded and improved. To list the results he produced in material welfare and educational improvement would take many pages. Suffice it to say that few college presidents have equaled his accomplishments.

In the summer of 1934 he was stricken with a heart attack while on a vacation in Minnesota. He was able to return to his home on the campus but never improved sufficiently to "get back into harness." In the spring of 1935 he definitely decided to resign although this move was vigorously opposed by the trustees. As always, seeing what he conceived to be his duty, he was unmoved by their protests and insisted that his resignation be accepted. No body of men ever parted from a leader with more regret than did the trustees of Lehigh University when they finally and reluctantly accepted his resignation.

It was typical of President Richards that no sooner was his resignation accepted than he began to plan to move away from Bethlehem. He loved the hills and valleys of eastern Pennsylvania, but he felt that his presence in any close proximity to the University would be unfair to his successor and might hamper that successor's administration. With him, Lehigh's welfare always came ahead of any personal pleasure or comfort. He moved to Minneapolis, Minn., to be near his daughter, Lenore, and there continued his fight to regain his health, steadily obeying—like the good soldier that he was—every mandate of his physician, even though it was torture for his active mind to be compelled to lie dormant instead of playing its usual part in the affairs of education.

President Richards was elected President-Emeritus of Lehigh University on the date of his retirement. During his presidency he had been a consulting engineer member of the Philadelphia Ordnance District and a member of the Board of Visitors of the U. S. Naval Academy. He was an active member and Fellow of the American Society of Mechanical Engineers, having been on its Council from 1918 to 1921, and a Fellow in the American Association for the Advancement of Science. He was a member of the Western Society of Engineers; the Sigma Xi honorary research society; Tau Beta Pi and Sigma Tau honorary engineering societies; Pi Tau Sigma, honorary mechanical engineering fraternity; the Sigma Chi fraternity; Omicron Delta Kappa; and Scabbard and Blade.

In 1920 he received the degree of Doctor of Engineering from the University of Nebraska. He received the honorary degree of Doctor of Laws from the following colleges: Lafayette College, at Easton, Pa., in 1922; Purdue University in 1932; University of Pennsylvania, at Philadelphia, Pa., in 1935; and Lehigh University in 1936.

On November 26, 1891, he was married to Alida Russell Beardsley at Lafayette, Ind. Of this union were born two children, a daughter, Lenore, and a son, Robert Watt. Mrs. Richards and the two children survive him.

After seven years of suffering President Richards passed away on Thursday, April 17, 1941. His body was brought to Bethlehem for interment after services in the chapel of the University he so deeply loved and so greatly served.

President Richards was elected a Member of the American Society of Civil Engineers on November 26, 1923.

HENRY CHANDLER ROBBINS, M. Am. Soc. C. E.[1]

Died May 6, 1941

It is granted to some men that they leave behind them one or more striking monuments to their skill and knowledge. To others, the opportunity to perform any single work which is outstanding does not come; but they do leave a mark in the world by doing everything that comes to their hand with care and thoroughness, no matter how unimportant it may seem. Henry Chandler Robbins was of the second group, for although there are many fine buildings through New England, in the planning and design of which he had an important part, there are none in which imposing appearance or ornamental treatment is nearly as important to the value of the building as the knowledge and care that entered into every detail.

During the earlier years of his association with the late Edward D. Densmore and Gifford LeClear, much of his work was along the lines of structural engineering, and at that time he made the structural design for one of the first large reinforced concrete buildings in the immediate vicinity of Boston, Mass., an undertaking that demanded courage as well as knowledge, for at that time the first Joint Committee had not made its report[2] and the trade still spoke of concrete buildings as "mud houses." With the passing of the years, and especially after the deaths of his partners, he was forced to give up active participation in the engineering part of his work and delegate that to others. His interest in that branch of architecture, however, never lessened and he was always ready to give an attentive ear to any proposed method of construction that gave promise of contributing to safe and economical buildings.

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[1] Memoir prepared by Francis S. Wells, M. Am. Soc. C. E.

Henry Robbins was born in Westbrook, Me., on September 12, 1876, the son of Frederick E. C. and Elizabeth A. (Rogers) Robbins.

In 1895 he entered the employ of F. H. Fassett, Architect, of Portland, Me., remaining there until 1897 when he entered the Harvard University School of Architecture, at Cambridge, Mass., as a special student. From 1898 to 1905 he was employed by Warren, Smith and Biscoe, Architects, of Boston; and from 1905 to 1907 he was an architect in the Boston office of the American Telephone and Telegraph Company. In 1907 he made his connection with the firm of Densmore and LeClear, an association that was to prove his real life work. Prior to this time, the firm had been engaged in strictly mechanical engineering work, and was just beginning to broaden out into general architecture, the objective being to build an organization that would cover all parts of the planning of buildings, including the work they had previously done. From 1907 to 1914 Mr. Robbins was an employee; he became a silent partner in 1914; and in 1923 the firm name was changed to Densmore, LeClear and Robbins.

This was the era of prosperity for architects and, during his association with Messrs. Densmore and LeClear, many buildings were planned in which they could, and did, take genuine pride. Among them are the Paine Furniture Company Building, the Salada Tea Company Building, and the Park Square Building, all in Boston. At the time of its erection, the Park Square Building was the fifth largest office building in the world from the point of view of rentable area. Although there was no permanent agreement between the firm and the New England Telephone and Telegraph Company, Mr. Robbins and his associates did so much work for that company that many believed them to be their official architects. Of the scores of Exchange Buildings designed for that company, those at Worcester, Mass., and in Bowdoin Square, Boston, are outstanding.

He served the American Institute of Architects as a member of the Committee on Architectural Services for the two last years of his life and was Treasurer of the Boston Society of Architects in 1938 and 1939.

Mr. Robbins was elected a Member of the American Society of Civil Engineers on June 18, 1918.

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[2] Final Report of the Special Committee on Concrete and Reinforced Concrete, *Transactions*, Am. Soc. C. E., Vol. LXXXI, December, 1917, p. 1101.

CHARLES VICTOR SEASTONE, M. Am. Soc. C. E.[1]

Died September 26, 1940

Charles Victor Seastone was born in New Boston, Ill., on April 18, 1872, the son of John August Seastone and Eva Sophia (Hjalmar) Seastone. His parents were born in Sweden, but came to the United States independently in their youth and were married in this country. His father was a manufacturer, and this early association probably influenced the son to enter the engineering profession. His early education was obtained in the public schools of New Boston.

Mr. Seastone entered the College of Civil Engineering of the University of Illinois, at Urbana, in the fall of 1889. He continued his University work until 1893 when he contracted typhoid fever. This so incapacitated him that he was obliged to discontinue his college work for a year. He reentered the University in the fall of 1894 and was graduated with the degree of Bachelor of Science in Civil Engineering in June, 1895. He early won the marked regard and respect of his classmates—regard for him personally for his intrinsic worth, and respect for his mental keenness. He was entirely free from the infirmity of self-assurance that bright young men sometimes exhibit. An able student with a high scholastic record, he had a keen analytical mind and an uncanny facility in rapid mathematical computations. It is related that, during a discussion in class as to the merits of the slide rule and the computing machine in shortening the time required for mathematical computations, young Seastone was chosen to compete with others operating these devices. On this occasion he was able to furnish the correct answers to the same problems before they were solved by the operators of the slide rule or the calculating machine.

Mr. Seastone was a classmate, and for some time a roommate, of the late Milo Smith Ketchum,[2] Hon. M. Am. Soc. C. E. During their senior year these two undertook a thesis entitled: "An Investigation of the Flow of Water through Siphons." This was a unique and outstanding piece of experimental work. They designed the test layout and erected the apparatus without assistance. Such matters as vacuum, flow, accumulation of air, length of time before cessation of flow, etc., were included in the test program under heads of from 2 to 8 ft and with lifts up to 27½ ft. The conduct of the test was very commendable.

In his early life Mr. Seastone was noted for his somewhat taciturn manner, a habit which remained with him even in the later years of his life, except among personal friends or in his professional work, where he was noted for the free, frank, and candid expression of his opinions. His early teachers at the University recall that Mr. Seastone disclosed in a marked degree what well may be called a "professional" spirit, and he was habitually a good stride ahead of his classmates.

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[1] Memoir prepared by Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1487.

While a student, Mr. Seastone engaged in practical engineering work during most of his summer vacations: In 1890 as an assistant surveyor in Mercer County, Illinois; in 1891 as engineer in charge of highway and levee location and construction; in 1892 as engineer in charge of levee construction; and in 1893–1894 as engineer on location of twenty units of county roads, and as assistant surveyor.

For two years after his graduation Mr. Seastone was employed by the Mississippi River Commission as assistant hydrographer in charge of instrument work. In 1897 he accepted the invitation of Prof. A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., to return to the University of Illinois as his personal assistant in charge of laboratory work in theoretical and applied mechanics (including hydraulics), where his services were reported as unusually satisfactory. He was considered an excellent teacher. He served in this position for three years, and in 1900 he went to Purdue University, at Lafayette, Ind., as associate professor of sanitary engineering under Prof. William D. Pence, M. Am. Soc. C. E., whose acquaintance he had previously made at the University of Illinois. Here he made an excellent record as a teacher and practical engineer. Upon his resignation in 1907 he gave up teaching as a career.

Professor Seastone did considerable practical work during his vacation periods. He designed the West Lafayette (Ind.) sewer system and was in charge of the design and construction of the roadway from Lafayette to the Soldiers Home and also of the relocation of the Lafayette Street Railway branch to the Soldiers Home.

In 1898 he had served as assistant engineer to the writer on the reconstruction of the Rock Island (Ill.) reservoir and slow sand filter system; and in the summer of 1900 he had served as resident engineer on the construction of a bridge over the Rock River at Rockford, Ill., also for the writer. This contact later led to many years of professional association in consulting work.

In the fall of 1906 Professor Seastone was offered the position of principal assistant engineer to the writer in his consulting office at Madison, Wis. He at first declined the appointment as he had previously agreed to continue his teaching at Purdue University for the year 1906–1907. President W. E. Stone of that institution, hearing of the opportunity, offered to release Professor Seastone from his position. He finally accepted, and began work in January, 1907. He served in this position until January 1, 1913, when he became a partner. This partnership lasted until January 1, 1933, when the business was turned over to three associates of the firm under the name of Mead, Ward and Hunt. Mr. Seastone continued as a consultant until disabled by sickness about a year before his death.

The scope and character of Mr. Seastone's experience in the course of this partnership is best indicated by brief descriptions of typical projects handled during this period. At the time that he entered the Madison office, plans for the construction of a hydroelectric plant at Kilbourn (later Wisconsin Dells), Wisconsin, on the Wisconsin River, were under way. This plant was one of the earliest of this type in the Middle West, and was built to transmit electric power at 66,000 volts to Milwaukee and Madison, Wis. The dam was built between the high sandstone bluffs of the "Dells" of the Wisconsin River where the section of the river was sufficient only for flood passage; therefore the river had to be carried over half of the dam, which was first constructed some 4 ft above tailwater, until the other half of the dam was built. Gates 8 ft in height were needed to reduce the headwater elevation during floods. Because of the flood height the gates had to be lowered into the dam at high flood and, as the floods subsided, had to be raised to maintain maximum headwater. As the entire river section was needed for flood flow, the headworks, forebay, and power house had to be constructed on a side of the river involving the removal of about 120,000 cu yd of sandstone rock.

The modern development of the hydraulic turbine had only just begun at this time, and the choice was necessarily confined to the horizontal type, with six turbines on a common shaft being necessary to develop 2,700 hp, direct-connected to a generator of 1,670-kva capacity. To develop the maximum head of 20 ft, the turbine had to be set so low that at the time of floods the generators were below the level of the tailwater; and yet the upper turbine gates were so high that umbrella covering was necessary over the gates to prevent breaks in the vacuum. The generators were set in a watertight concrete pit, and the walls were extended well above flood-water level. These conditions required considerable ingenuity in design and were so well met that, in the thirty years which have elapsed since the plant began operation, few desirable changes have developed except the use of the more powerful turbines now available. This plant, costing about $2,500,000, has been successful both from an engineering and an economic standpoint.

In 1908, the Madison office under Mr. Seastone began plans for a hydroelectric plant at High Falls on the Peshtigo River in Wisconsin. This was a 6,000-kw plant under an 82-ft head and carried power to Green Bay, Wis., about 60 miles distant. It was a very different type of plant from the Kilbourn plant and, under the higher head, was constructed for about half of the unit cost of the low-head plant at Kilbourn.

In the summer of 1911 work was begun on a hydroelectric plant on the Wisconsin River just above Prairie du Sac. The river bed at this point is of sand, 100 ft or more in depth, and the entire structure, including both power house (350 ft long) and dam (about 1,000 ft long), was built on a pile foundation on this sand deposit. Excessive underflow was cut off by 50-ft steel sheeting above the power house and dam, and by 30-ft steel sheeting below these structures. Some erosion took place in the sand deposit downstream from the dam, and in 1935 the apron was extended about 55 ft, which added greatly to the safety and stability of the structure. The plant has been in use for twenty-seven years under heads of 37 to 38 ft and is apparently even more stable and safe than when first constructed.

These plants were followed by the design and construction of hydroelectric plants at Alpena, Mich., on Thunder Bay River; at Twin Falls on the Menominee River near Iron Mountain, Mich.; on the Brule River near Florence, Mich.; and on the Pine River in Wisconsin, southwest of Iron

Mountain (this was one of the earlier automatic plants and was operated from the main station at Twin Falls). Hydroelectric plants were also designed and constructed on the Peshtigo River at Johnson Falls, Caldron Falls, and Sandstone Rapids; and dams and plants were improved at Potato Rapids and at Peshtigo, Wis., on the same stream. Numerous other plants were also designed and constructed or revised and improved. A number of steam plants were built or rebuilt. An auxiliary steam plant was designed and constructed at Iron River, Mich., for the Peninsular Power Company. The steam plant of the Madison Gas and Electric Company was redesigned and reconstructed at Madison, first with a 5,000-kw unit and later with 10,000-kw and 15,000-kw units. The steam plant of the Mississippi Valley Public Service Company at Winona, Minn., was reconstructed and enlarged.

During Mr. Seastone's active period, water works plants were built, revised, or reconstructed at a number of places under his immediate supervision. Among these plants may be mentioned a new pumping station, water supply system, and storage reservoir systems at the original site in Madison. Operations had to be maintained while the new work was being done at the old site. A new artesian well and air-lift system, with new pumping plant and reservoir, were constructed at Rockford (Ill.). An electric pumping system was designed and constructed at Dubuque, Iowa, to replace a steam plant, and the water supply was also increased. A new pumping plant was designed and constructed at Decatur, Ill.; and a storage dam and improved filters were built for the Danville (Ill.) Water Company. A water works plant was designed and built at Prairie du Sac, and improvements were designed for Two Rivers, Wis.; Chisholm, Minn.; and West Allis, Wis.

Many reports were furnished on water power and water supply developments in various sections of the United States. One of the largest projects on which a report was rendered was the possible development of hydroelectric power on the Missouri River for the State of South Dakota at some five different localities within that state.

One of the most interesting, and perhaps most difficult, pieces of construction work designed and constructed under the general supervision of Mr. Seastone was a dam foundation on the Elwha River on the Olympic Peninsula in the State of Washington. This dam was a concrete structure built to create a 100-ft head in a narrow canyon on the river and was designed and constructed under an engineer who was evidently unfamiliar with hydraulic work. The dam had a pea gravel and sand foundation and was provided with an inadequate cutoff. When the head was raised and the plant was started, the river broke under the dam and washed out the foundation to rock at a depth of about 100 ft below the base of the concrete, leaving the dam suspended in the air. The plant had already cost 50% more than was warranted by the work done. The company was therefore almost exhausted financially, and the cost of diverting the river through the canyon walls, cofferdamming, and building a concrete foundation to the overhanging dam seemed to entail an expense perhaps equal to the expenditure already made.

Under these conditions such an expenditure was impossible. As an economic solution, it was decided to blast into the canyon and into the flowing river the rock cliffs on each side from a point about 350 ft below the dam up to the structure itself, after which the rock immediately above the dam was blown *en masse* into the stream to fill the gap and stop the stream flow. Before the placing of the rock fill was started, the stream had refilled with sand and gravel to a depth of about 50 ft. On this, immediately below the dam, a block of concrete weighing about 2,000 tons was built in a rectangle of steel sheeting to furnish stability to the rock fill. The space under the dam was filled, so far as practicable; and above the dam facines and mattresses of Douglas fir were used to catch and hold the fine material to make the rock fill above the dam watertight.

In the fall of 1913 some 40,000 cu yd of rock were blown into the stream just above the dam without injury to the structure. Although stoppage of flow was not immediately attained, this was soon accomplished by filling the depressions in the rock fill and by adding gravel and finer material. Although there is still a small flow through the gravel under this mass, the length of flow is so considerable that the velocity of seepage is reduced to such a degree that it leaves the toe of the rock fill without cutting velocity. This simple construction accomplished its object at a very low cost. The dam has been in use for twenty-eight years since its reconstruction, and the cost was less than one tenth of the estimated cost of a concrete substructure. Thus the financial life of the original company was saved and an otherwise useless plant was made serviceable.

Another interesting piece of work done under Mr. Seastone's supervision was the construction of a foundation under a completed power plant built on the soft bottom lands along Lake Michigan. This plant was designed and constructed by engineers not familiar with foundation work and was built on what is known as a "floating foundation." Unfortunately, one unit consisting of boilers and a 5,000-kw turbogenerator was first installed on one side of the building intended ultimately for two such units. Such a foundation always must be expected to settle somewhat, and for success such settlement must be anticipated and the foundation uniformly loaded. In this particular case, however, before the first installation was complete, the building settled unevenly, tipping toward the loaded side. The Madison office, under Mr. Seastone, was called in to underpin this completed building. The first substantial stratum that would carry the load was 40 ft below the ground level. After much study it was decided to support the structure by pairs of piles placed at about 25-ft intervals around the building. In each case the outer pile (40 ft long) was driven just outside the wall of the building to a firm foundation, while the twin pile inside of the building had to be sunk in 10-ft lengths joined together by steel sleeve joints. After these twin piles were driven, they were suitably capped and connected by heavy I-beams that supported a pair of lighter I-beams placed longitudinally in the wall of the building. After all the piles and I-beams had been placed, pairs of steel wedges were driven between the two sets of I-beams and concrete was placed around the beams. The settling stopped with the tightening of the wedges.

On February 17, 1930, the Madison Metropolitan Sewerage District was organized, and Mr. Seastone was one of the three original commissioners appointed by the County Court. The Madison Metropolitan Sewerage District, which included the City of Madison, four contiguous villages, and part of five adjacent townships, was formed to furnish sewerage services to a population in excess of 85,000. Mr. Seastone was elected President of the Board.

The work of the District was directly in the line of Mr. Seastone's study and experience. He was of great assistance in formulating and directing the policies of the District from the time of its organization. He thoroughly studied and gave close attention to every problem that arose. During his term of office the District expanded its interceptor system and greatly enlarged its sewage treatment facilities to make its treatment works one of the most efficient in the United States.

Mr. Seastone required practical solution of the problems of the District and was insistent that every dollar of public money spent under his direction return its full value in accomplishment. The people in the District, as is common in work performed for the public, cannot realize the debt they owe to the untiring interest and careful studies and supervision of Charles V. Seastone.

Considerable ingenuity is necessary to meet all of the contingencies of the diverse work in an engineering office, and it is seldom possible, with any degree of exactness, to place the credit for suggestions that bring success. If failures result, however, the responsibility is always with those in responsible charge. There is no doubt, however, that Mr. Seastone should be credited with his full share of the successful completion of the manifold problems and projects undertaken by his firm during the years of his connection with it. Honest and honorable in all of his thoughts and actions, and with high professional ability, he studied every project that came to his attention with an unbiased judgment and from every point of view. He was uniformly kind and considerate of his associates and assistants, and he utilized with the fullest possible credit all sound suggestions that came to him from any source.

On June 25, 1900, Mr. Seastone was married to Susan Sarah Bouton at Anna, Ill. Mrs. Seastone was the daughter of Harvey Cady Bouton, owner and publisher of the weekly, *Farmer and Fruitgrower*. Mr. Seastone is survived by his wife and two sons, John Bouton Seastone and Dr. Charles Victor Seastone, Jr.

He was a member of the Engineering Society of Wisconsin and the Technical Club of Madison. He was also a member for many years of the Madison Club and the Madison Rotary Club.

Mr. Seastone was elected an Associate Member of the American Society of Civil Engineers on December 5, 1906, and a Member on September 12, 1916.

VOLNEY RICHARD SEAWELL, M. Am. Soc. C. E.[1]

Died September 19, 1941

Volney Richard Seawell was born in Ukiah, Calif., on February 17, 1884, the son of John Henry and Lucy Frances (Watson) Seawell. John Henry

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[1] Memoir prepared by De Witt L. Reaburn, M. Am. Soc. C. E.

Seawell was born in Lafayette County, Missouri, in 1847. He crossed the plains with his parents in 1853 and settled in Napa County, California. He was graduated from Pacific Methodist College, at Santa Rosa, Calif., studied law, and began practice in 1887. He was very successful. For many years he was a member of the California State Assembly and Senate. Lucy Frances Seawell was born in Sacramento, Calif., in 1852, and was a graduate of the first class of Mills College, at Oakland, Calif. Watsonville, Calif., was named for her family. Justice Emmet Seawell, a brother of John Henry Seawell, was a member of the California State Supreme Court for many years, and sat for some time as Chief Justice.

Volney Seawell's early education was acquired in the public schools of Ukiah. After high school, he educated himself by private study and extension courses in subjects pertaining to his work. In his school days, he was interested in athletics. He played football, held Pacific Coast records in the pole vault and high jumps, and was an enthusiastic "fan" for all sports.

The Seawell family were inclined to the law; but "Barney" (his nickname) was more interested in material things that were governed by the laws of nature. He evinced an early interest in electrical engineering, and began his long construction career in 1906, as electrician, with the Nevada Power and Mining Company, at Goldfield, Nev.

During the period 1907 to 1923, he successfully followed this field of work in the capacities of chief electrician, general foreman, and superintendent of construction on various projects in California, Montana, and British Columbia, Canada. His major assignment was with the Southern California Edison Company, at Big Creek, Calif., where he served, from 1921 through 1923, as superintendent of construction on that company's $19,000,000 hydroelectric project. In addition to the power houses, forebays, and penstocks, this project included 6 miles of 21-ft tunnel and a constant radius arch dam across the San Joaquin River, with tunnel intake, spillway, and gate-house structures.

In 1923 and 1924 he had charge of gravel testing and bridge foundation testing for the Exchequer Dam project of the Merced (Calif.) Irrigation District. Following this work, from 1924 to 1928, he was with the City of Los Angeles, Calif., as superintendent on the construction of the Mulholland Highway, Beverly Boulevard, and Cahuenga Pass projects, which involved heavy earth and rock excavation, paving, and structures.

In 1928 he was superintendent of construction for the H. W. Rohl Company, in Los Angeles, on additions to the Riverside (Calif.) Water System, which consisted of a reservoir and reinforced-concrete pipe lines. Following completion of this work, he was with the Alphonso E. Bell Corporation, at Bell Air, West Los Angeles, Calif., on research work for construction of a cement plant.

From 1929 to 1934 he was construction superintendent for the Macco Construction Company, of Clearwater, Calif., in charge of the branch office at Ellwood, Calif., 13 miles north of Santa Barbara, on steel piers, concrete foundations, etc., for marine oil wells (total cost, $2,000,000).

In 1934 Mr. Seawell became building inspector for the Los Angeles Board of Education, in charge of demolition of several large school buildings. He retained this position until 1935, when he became project engineer for the Work Projects Administration (WPA), at Los Angeles. He was soon made chief material inspector, and then zone engineer and manager of the Project Control Section.

In 1938 and 1939 he was with the Public Works Administration (PWA), at Los Angeles, as resident engineer inspector on school-building construction, but in the summer of 1939 returned to the WPA, at Los Angeles, as director, Division of Operations, in charge of all engineering, planning, and construction activities in Southern California, on which about 35,000 men were employed.

Director Seawell had an affable and agreeable disposition. He was kind and considerate and enjoyed the affection and respect of all who were associated with him. His cheery greetings and genial companionship will be missed by a host of friends and fellow engineers, who will continue to reflect, directly and indirectly, the ability, energy, and pioneering spirit of the man they knew and admired. The respect of his associates was evidenced by the large memorial wreath that was placed in the entrance of the WPA building in Los Angeles for a week following his death.

On February 1, 1911, he was married to his schoolmate, Ethel Elledge, who was his constant companion and partner through the years that followed. He is survived by his widow and his mother.

Mr. Seawell was elected a Member of the American Society of Civil Engineers on February 13, 1940.

ROBERT LOUIS SHAPE, M. Am. Soc. C. E.[1]

DIED JUNE 16, 1941

Bob Shape, as he was familiarly known to his many friends, was born in Milwaukee, Wis., on January 4, 1872, of German ancestry. His parents were Gustave Hermann Shape and Gertrude (Laubenheimer) Shape. He was the sixth of a family of nine children, and a good share of home duties and chores fell to his lot. In addition, however, he found time to study German, take swimming and violin lessons, and attend a gymnasium twice a week under a strict German instructor. At the age of twelve he played a violin solo in a concert hall where German families met on Sunday afternoons for beer, cakes, and coffee.

When he was thirteen his father took the family to Switzerland to live. There his study of the German language was continued and French was begun. He afterward became proficient in both languages. Because of the previous training in Milwaukee, he excelled in the gym work there and developed a fine physique, attaining his six-foot height at an early age. Mountain climbing and expeditions on foot to near-by points of interest, in which the entire family joined, afforded an instructive and enjoyable pastime as well as a physical benefit.

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[1] Memoir prepared by Isaac Harby, M. Am. Soc. C. E.

In 1887, it was decided that Bob would study architecture, and his father entered him in the University of Winterthur in Canton of Zurich, Switzerland; but fate ruled otherwise, for soon after this the father died and the family moved back to Milwaukee. Here he re-entered the public school where he was definitely handicapped through a deficiency in English grammar. This handicap was made up the following year, however, so that he was able to enter the only high school in Milwaukee at that time. He enrolled in the three-year science course. He also continued his musical education in the Milwaukee Conservatory of Music, played in the orchestra of the Congregational Church, and at seventeen became the leader at a salary of $5 per week. Summers were now being spent on the farm of an uncle where the harvest season afforded beneficial physical exercise from sunup to sundown.

Having graduated from high school with honors, the second highest in his class, and from the Conservatory of Music with a gold medal for progress and proficiency, he entered Cornell University, at Ithaca, N. Y., in the fall of 1891 for the course in architecture. He joined the Beta Theta Pi fraternity, was elected to the Aleph Samach Society and to Sphinx Head, and at graduation to Zeta Epsilon.

His six-foot athletic body envisioned an oarsman, so he tried out for the crew and made it in his freshman year. He rowed in the varsity through his sophomore, junior, and senior years under the coaching of the famous Charles Edward Courtney. In his senior year he captained the crew and went with it to England when it rowed at Henley-on-Thames. His last-used oar is still a cherished trophy at Cornell.

Mr. Shape's professional career began in New York, N. Y. After graduation from Cornell in 1895 with first honors, he went to work as a draftsman in the office of the important architectural firm of George B. Post and Son. Through his continued interest in rowing he joined the Atlanta Boat Club and participated in some races in which it was a contestant.

When the war with Spain began in 1898, Mr. Shape enlisted with the 8th U. S. Infantry for a period not to exceed three years and was in active service in Cuba. One day, while he was at headquarters near Sabony, there suddenly was heard a heavy cannonading and all hands rushed to a point of vantage. Here was witnessed the naval battle of Santiago de Cuba, and those at headquarters on that day were the only persons in the army to see that victory. Private Shape's enlistment lasted thirteen months—five in active service and eight off duty on the sick list.

After traveling in the West for three months to recuperate, he returned to his old job in New York City. His duties now consisted of structural steel designing and superintending the construction of the new Stock Exchange and the College of the City of New York. Other engagements followed, including designing with Clinton and Russell, architects, and with the Cleveland (Ohio) Electric Illuminating Company.

In 1905 the architectural firm of Shape and Bready (afterward Shape, Bready and Peterkin) began to function. Its work included the design of the following buildings: (1) The Candler Building on West 42d Street; (2) a number of Edison phonograph shops; (3) a cooperative apartment at 50 West 67th Street; and (4) a cooperative garage on 67th Street, the first to obtain a permit under the New York City Zoning Law.

From 1918 to 1926 Mr. Shape was associated in the work of the following organizations:

(*a*) Todd Drydock and Shipbuilding Corporation, Tacoma, Wash.—production manager.

(*b*) J. J. Coughlan Shipbuilding Company, Vancouver, B. C., Canada—works manager.

(*c*) William J. Taylor Company, contractors, New York City—in charge of construction of Canadian Pacific Building.

(*d*) Starrett and Van Vleck, architects, New York City—in charge of construction of Equitable Life Assurance Society Building.

(*e*) Clinton and Russell, architects, New York City—designing.

(*f*) Charles M. Hart, architect, New York City—engineer and construction superintendent.

Mr. Shape subsequently became a member of the firm of Hart and Shape, which has to its credit some apartment houses, schools, hotels, an armory in Brooklyn, N. Y., and a hospital in Rockville Center, Long Island, N. Y. The last five years of his life were spent in the service of the federal government.

In 1935 the Resettlement Administration began its activities in the construction of suburban projects popularly known as "Greenbelt Towns." Mr. Shape was appointed principal architect in connection with the planning of some five hundred single-family homes at Greenhills near Cincinnati, Ohio. After this work was completed, he became senior engineer for the Public Works Administration with work on housing projects at various locations throughout the United States.

In December, 1940, he received an appointment from the Public Buildings Administration as senior construction engineer with instructions to report for duty on December 12, but on that day he was not well and requested a postponement of the duty date. He entered the Veterans' Hospital in New York City for examination and treatment, and it was found that he was suffering from an insidious disease for which there was no known cure. After a few months, death overtook the career of one who had achieved excellence in his chosen field and who had served his country well.

Throughout his life he maintained an interest in music, and, being well trained, was a skilful violinist. His unassuming attitude and genial manner to all engendered lasting friendships. In addition to membership in the Society, he was a member of the Architectural League, the Cornell Club, and the New York Athletic Club. His home was at Ridgefield, Conn.

He is survived by his wife, the former Elizabeth Powell, to whom he was married on July 24, 1911. A son, Norman Lee Shape; a daughter, Elizabeth

G. (Mrs. Denis Mulligan); and two stepdaughters, Margaret Stoddart (Mrs. Theodore Cuyler Speers) and Marion Stoddart (Mrs. Harry P. Warren), also survive him.

Mr. Shape was elected a Member of the American Society of Civil Engineers on November 14, 1927.

HENRY GARNETT SHIRLEY, M. Am. Soc. C. E.[1]

Died July 16, 1941

Henry Garnett Shirley was born at "Locust Grove," Summit Point, W. Va., on October 28, 1874, the son of Robert Vinson and Julia (Baylor) Shirley. He received his preparatory education in the public schools of Jefferson County, West Virginia, and at the Charles Town (W. Va.) Male Academy. He entered the Virginia Military Institute at Lexington, Va., in the fall of 1892 and was graduated from the engineering course in 1896. He was prominent in many phases of barracks life at Virginia Military Institute, a member of the football team, and first lieutenant of Company "A," corps of cadets. Upon graduation he accepted the position as Commandant and Professor of Military Science at Horner Military School, in Oxford, N. C.

At the outbreak of the war with Spain in 1898, Mr. Shirley enlisted and served with the U. S. Volunteers. At the close of that war he began his engineering work on the New York Central and Hudson River Railroad. Later he worked for the West Virginia Central and the Baltimore and Ohio railroad companies; for a time he also was connected with the engineering department of the District of Columbia. After five years of experience in engineering—which were required for a degree from the Virginia Military Institute—he was awarded the degree of Civil Engineer by that institute.

While working for the Baltimore and Ohio Railroad Company, Mr. Shirley was with the corps of engineers that retraced the location of the South Pennsylvania Railroad, which was then being surveyed by the Baltimore and Ohio Railroad Company for a possible low grade line over the Allegheny Mountains. This location is now a part of the recently completed superhighway from Carlisle to Pittsburgh, Pa.

Mr. Shirley's principal engineering work was in the development of modern highways. He first began highway work as roads engineer in Baltimore County, Maryland, in April, 1904, following which he was appointed to the office of chief engineer of the Maryland State Roads Commission. While in this position Mr. Shirley was one of the organizers of the American Association of State Highway Officials in 1914 and was the first president of that organization. He served as president of the Association for two years, having been the only president to serve for that length of time. At the time of his retirement as president he was presented with a large silver cup by his associates. He continued as a member of the Executive Committee of the Association and was a member of the Special Committee on Highway Transport that cooperated with the National Automobile Chamber of Commerce. In 1918 Mr. Shirley was made executive secretary of the Federal Highway Council, a body which advised the federal and state governments and colleges and universities in the matter of highway research conducted by various agencies. It also gave advice as to the problems that should be investigated.

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[1] Memoir prepared by W. R. Glidden and E. M. Hastings, Members, Am. Soc. C. E.

At the outbreak of the first World War Mr. Shirley was made a member of the Highway Transport Committee, Council of National Defense, and gave his untiring energy to the end that the roads of the United States might be in shape so that necessary materials and munitions of war could be trucked over them to the camps and ports.

In 1920 Mr. Shirley was again made engineer of Baltimore County. At this time the suburbs of the City of Baltimore were undertaking a huge program of street and road building and the construction of various public utilities. He remained in this office until 1922 when he was called to Virginia to be the chairman of the State Highway Commission. From that date until his death Mr. Shirley headed the Virginia State Department of Highways, first as chairman of the Highway Commission and later as commissioner, Department of Highways. He served under five governors, and his work in this department is known throughout the United States. Under his administration the highway system was expanded so that today the State of Virginia occupies a most enviable position in modern highway development.

In 1925 Mr. Shirley was elected to the Executive Committee of the American Road Builders Association and the following year was elected president of that Association. As president he appointed delegates to the International Road Congress at Milan, Italy, and was himself subsequently appointed as a delegate to various International Road Congresses by federal authority.

Mr. Shirley was awarded the honorary degree of Doctor of Science by the University of Maryland, in Baltimore, and a similar honorary degree by Hampden-Sydney College, in Hampden-Sydney, Va. He was one of the organizers of the Virginia Section of the Society and served a term as president of the Section. He was always deeply interested in the welfare of engineers and the furtherance of the engineering profession. He maintained a loyal and active contact with his alma mater, the Virginia Military Institute, was constantly concerned as to its welfare, and assisted in progressive efforts to further its interests. He was a man of rare intellect and had a peculiar personal charm that gave him great ability as an executive; this, coupled with the skill of a master of engineering, made him an international figure in the highway world. These same traits placed him, in the minds of his friends and close associates, in a position even higher than that of his national reputation, so that it would be difficult to find a department more completely coordinated and cooperative than the Virginia State Department of Highways. Mr. Shirley possessed a disarming frankness in his approach and discussions with men that brought out the best in all those with whom he came in contact; he was frank and open in his dealings and handled many difficult situations in his administration of the Virginia State Department of Highways so successfully that he was looked upon as one of the outstanding officers of the state government.

On November 29, 1904, Mr. Shirley was married to Alice Robertson Graham, of Oxford. She died on October 22, 1916. There were five children by this marriage, Robert Vinson, Lucy Graham (Mrs. Leon Otis), Augustus Graham, Julia Baylor (Mrs. Fred Patterson), and Alice Graham, all of whom survive him. On September 17, 1930, he was married to Sara Anne Berkeley of Richmond, Va., who survives him. He is also survived by a brother, George; three sisters, Mary, Ruth, and Julia; and a grandson, Henry G. Shirley 2d.

Mr. Shirley was elected a Member of the American Society of Civil Engineers on June 30, 1911.

BURR HENRY SIMPSON, M. Am. Soc. C. E.[1]

DIED AUGUST 10, 1941

Burr Henry Simpson was born on January 6, 1885, at Buckhannon (Upshur County), W. Va., the son of Allen A. and Elizabeth (Lorentz) Simpson. He had one sister who preceded Mr. Simpson in death. He was a descendant of the Burr family, of whom Aaron Burr is the best known.

Mr. Simpson was educated in the public schools at Buckhannon, and continued his studies with a scientific course at West Virginia Wesleyan College, in Buckhannon. Later he studied civil engineering at Ohio State University, in Columbus, Ohio. His collegiate training extended from 1900 to 1903.

He began his practical experience in engineering work in September, 1904, as a chainman, rodman, and instrumentman with the Coal and Coke Railway Company in West Virginia, where he remained until July, 1905. From April, 1906, to October, 1907, he served as instrumentman with the Chesapeake and Ohio Railway Company on double-track construction between Richmond and Newport News, Va.; as instrumentman and chief of party on the Lewiston and Southeastern Electric Railway, in Idaho; as chief of survey party for the Inland Power and Electric Company, in the State of Washington, on reconnaissance, preliminary, and partial location (53 miles); and as assistant engineer for the Oregon Railroad and Navigation Company in the states of Washington and Oregon on branch-line construction, drainage, tunnels, and stations.

From October, 1907, to November, 1909, he was engaged in private practice at Spokane, Wash., on location, design, and construction of a 9-mile mountain railroad spur; surveys for water-power development and irrigation projects; underground and surface work for copper mining companies; and as United States Deputy Mineral Surveyor.

In April, 1911, Mr. Simpson became a draftsman and computer with the Frog and Switch Department of the Pennsylvania Steel Company, at Steelton, Pa., on special track work, and then with the Track Department of The Falk Company, in Milwaukee, Wis. From July, 1912, to January, 1913, he was a civil engineer for the Gogebic and Iron Counties Railway and Light Company, in Ironwood, Mich., on location and construction of 7 miles of electric railway, including a filtration plant. Following this work he was a computer and assistant engineer for the J. G. White Company, in New York, N. Y., on an appraisal survey of the Atlantic Coast Line Railroad from September, 1913, to May, 1914.

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[1] Memoir prepared by Howard A. Levering, M. Am. Soc. C. E.

Mr. Simpson entered the highway engineering field in May, 1914, with the Ohio State Highway Department, in Columbus. He was acting resident engineer on construction of the national road between Hendrysburg and New Concord, Ohio, and spent one year as deputy engineer in Knox County, in charge of surveys and construction for the State Highway Department of Ohio. Between April, 1917, and August, 1918, he was field engineer, and from March, 1919, to January, 1921, he was field engineer and assistant district engineer for the Portland Cement Association, in Pittsburgh, Pa., on promotion work on concrete roads.

During the period from August, 1918, to March, 1919, he was with the U. S. Ordnance Department, in Cleveland, Ohio, first as production assistant, visiting shell plants (until November, 1918), and then as special assistant to the Claims Board, being referee in the settlement of claims.

Mr. Simpson was sales engineer for the Kentucky Rock Asphalt Company, in Louisville, Ky., from January, 1921, to November, 1926. He was city engineer for his home city of Buckhannon from November, 1926, to January, 1929, in charge of design and construction of streets, sewers, etc. Mr. Simpson supervised the construction of the Pittsburgh-Butler (Pa.) Airport in 1929.

From 1930 to August, 1933, Mr. Simpson served again as city engineer of Buckhannon. He then became district engineer of the fifth district, at Keyser, W. Va., for the State Road Commission. Later he was made project engineer for the Commission with headquarters at Charleston, W. Va., in which position he served until he was appointed state road commissioner by Governor H. G. Kump in August, 1935. He was reappointed to the same office by Governor Homer A. Holt in March, 1937, and served in this capacity until his resignation in March, 1941.

Following his resignation as state road commissioner, Mr. Simpson accepted the position of president and general manager for the West Virginia Sand and Gravel Company, and of the Standard Brick and Supply Company, both of Charleston. These positions he held until about three weeks prior to his death, when he resigned from them because of ill health.

Mr. Simpson was a Mason and belonged to the Shrine. He was a member of the West Virginia Society of Professional Engineers, and was a former chairman of the State Aeronautics Board. He also was a former member of the State Publicity Commission.

His varied and far-flung experience in the field of engineering gave him a bright outlook on life and a broad vision as to the value of engineering principles and accomplishments in behalf of the public and humanity. Good engineering was paramount in his code of life.

He was a highly efficient and indefatigable public servant. His striking personality and genuine sincerity won for him a legion of friends. He placed the public welfare ahead of his own aggrandizement. His retiring disposition and humble modesty were true tests of his real ability and untrammeled honesty. Mr. Simpson would make no promises unless he knew that they could be fulfilled, which is an admirable and outstanding tribute to any man. His love for his fellow men and his devotion to his friends were ever manifest in his firm and enthusiastic handclasp.

The career of Mr. Simpson, which came to an abrupt close at the end of more than fifty-six years, was climaxed by his service to the people of his native State of West Virginia as its state road commissioner. The greatest monument that could be erected to his memory, aside from the sterling quality of his character and friendship, was the great advancement that was made in improved highways of West Virginia during his tenure of office. His time, thought, and energy were completely devoted to a well-rounded highway system. Better roads throughout the entire state for all of the people was his dream and almost his realization. The vast network of farm-to-market roads was planned and consummated during his commissionership, and was a rare tribute to Burr Henry Simpson, the man, engineer, and friend.

He died on August 10, 1941, and interment was at the Heavener Cemetery in Buckhannon on August 12. He is survived by his mother.

Mr. Simpson was elected an Associate Member of the American Society of Civil Engineers on December 6, 1915, and a Member on October 1, 1928.

CHARLES HENRY SMITH, M. Am. Soc. C. E.[1]

Died December 20, 1940

Charles Henry Smith was born at Winthrop, Mass., on June 29, 1868, the son of Alexander and Mary Ann (Mears) Smith.

He began his engineering work in 1890 as rodman with the Metropolitan Sewerage Commission, at Boston, Mass., and continued with that organization through the grades of instrumentman, inspector, draftsman, and foreman of construction until 1896. His next engagement was with the Boston Transit Commission, first as inspector on contract work, then as engineer in charge of construction (night) on two sections of open-cut, heavy brick and concrete masonry; tunneling; street paving; and relaying sewer, gas, and water pipes. In 1898 he built a dredge and superintended dredging in Back Bay Fens for Perkins, White and Company, in Boston.

Mr. Smith went to Honolulu, Territory of Hawaii, in 1898, where he served as designing engineer for the Department of Public Works for more than eight years, designing and constructing reservoirs, sewers, etc. For nearly a year he was engineer and general superintendent with E. J. Lord, general contractor, in Honolulu.

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[1] Memoir prepared by O. E. Stanley, M. Am. Soc. C. E.

Returning to "The States" in 1908, Mr. Smith spent the next two years in private practice in Seattle, Wash., and Portland, Ore. In 1910 he began his long service in the Bureau of Sewers, City of Portland, which lasted until his death, with the exception of about five years. During these five years he was associated with Barr and Cunningham, Consulting Engineers, in Portland, as project engineer on the Malheur Drainage District; as resident engineer on other work outside of Portland; and with the Warm Springs Irrigation District at Vale, Ore., for nearly a year, as construction engineer on the Warm Springs Dam.

In his work for the City of Portland, Mr. Smith began as chief inspector of sewer construction, and then was promoted to sewer engineer in charge of sewer design and construction. Among his more important studies were preliminary plans and estimates for the Park Rose and Altamead District and for the Portland Sewage Disposal System. The latter included the necessary intercepting sewers. This was the work upon which he was engaged at the time of his final illness.

In his early manhood, Mr. Smith was interested in building and sailing small yachts and continued to enjoy this sport while living in Honolulu. He later derived pleasure from collecting and refinishing antique furniture, but during his later years his whole interest appeared to be occupied by his profession.

Mr. Smith was a registered professional engineer in Oregon and was a life member of the Professional Engineers of Oregon. He was a member of the Order of the Mystic Shrine, first as a member of Aleppo Temple in Boston, and later as one of the sixteen charter members of Aloha Temple in Honolulu.

On June 7, 1900, he was married to Margaret McCorriston in Honolulu. Mrs. Smith and two daughters, Theresa (Mrs. Gail Miller), and Frances E. (Mrs. Hollister M. Stolte), survive him. He is also survived by three brothers, Chester, Percy, and Lewis.

Mr. Smith was elected a Member of the American Society of Civil Engineers on August 17, 1936.

CARLETON WILLIAM STURTEVANT, M. Am. Soc. C. E.[1]

Died September 16, 1941

From pioneer home in Missouri to construction on the Panama Canal in one's own lifetime seldom falls to the lot of an engineer—that is what makes the life of Carleton William Sturtevant stand out. Advance from assistant on a Mississippi River Survey party to Colonel of the 15th Engineers, Civilian Member of the Mississippi River Flood Control Board, and Consulting Engineer of the Fort Peck Dam, show the advance that can be made by an engineer in the U. S. Government service.

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[1] Memoir prepared by L. F. Bellinger, M. Am. Soc. C. E.

Aside from government work, Mr. Sturtevant spent one year in railroad work as transitman and resident engineer. For 7½ years (1902 to 1909), he was in dredge work for the Atlantic Gulf and Pacific Company at Vicksburg, Miss., Mobile, Ala., Tampa, Fla., and Fort Ann, N. Y., made examinations of Cape Cod Canal, and the harbor of Lynn, Mass. He built the first two dredges for the Panama Canal Commission.

He was with the James Stewart Company for four years and built 93 miles of New York State Barge Canal from Montezuma Swamp to Oneida Lake and Fulton. For three years he was with the New York State Dredging Company as general superintendent on six barge canal contracts, and on a 5,000,000-cu-yd fill at Utica, N. Y., for the New York Central Railroad Company.

After six months with James Stewart again, on Hudson River work, he was ordered to active duty as Major, Engineers Reserve Corps. From May 25, 1917, to July 6 of that year, he was Major of the 15th Engineers under the then Colonel Edgar Jadwin,[2] M. Am. Soc. C. E., under whom he had served in Cuba as a Captain in the 3d U. S. Volunteer Engineers. He was made Lieutenant Colonel on July 6, and sailed on the *Baltic* for Liverpool, England, on July 9, arriving there on July 19. The Regiment's 32 cars of railroad tools filled 100 English cars, and all were transferred from Southampton to Le Havre and to Vierzon. Only the 16th Infantry and the 18th Infantry of the regular Army preceded them to France, having gone directly there. The English said they were the first armed regiment of foreign troops to be in England since William the Conqueror arrived from France, in September, 1066, nearly 851 years before. The Normans defeated the English on October 14, 1066, at the Battle of Hastings.

The work in France included selecting and building a harbor, a railroad, and storage yard at Gievres. He moved headquarters from Vierzon to Liffol-le-Grand, and to Solmary yard. He helped build the railroad from Verdun to Conflans that was destroyed by the Germans just before the Armistice, and took the first trainload of food in 3½ years into Sedan.

Colonel Jadwin was promoted December 17, 1917, to Brigadier General and the work mentioned previously was nearly all done under the command of Lieutenant Colonel Sturtevant, who was commissioned Colonel on November 13, 1918, two days after the Armistice.

His regiment moved to Bordeaux, France, on January 21, 1919, and then to Poullic to await a ship to the United States. On April 13, the regiment sailed on the *Santa Clara,* arrived in New York, N. Y., on April 27, 1919, and was mustered out after two years of service.

Colonel Sturtevant was the civilian member of the Jadwin Board of three, for Mississippi River Flood Control. Duties on this Board occupied his time from May 16, 1928, to January 12, 1929, on which date the Board's work was completed.

A Citation signed by Gen. John J. Pershing was awarded to Colonel Sturtevant on April 19, 1919, "For exceptionally meritorious and conspicuous services at Gievres, France." In addition, on June 7, 1919, General Pershing wrote him a personal letter commending him as follows: "Your services both as Lieutenant Colonel and Colonel of the 15th Engineers on the important construction projects at Gievres and at Liffol-le-Grand have been of the highest character."

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1498.

The U. S. Engineers ordered Mr. Sturtevant in 1891 to build a small scale detail of a "caving bank revetment mattress" which was sent to the World's Fair in Paris, France. In 1900 the French Republic awarded him a diploma and a silver medal for this model. The medal was awarded in silver. It was delivered in bronze, because of lack of funds.

On the more personal side of Carleton William Sturtevant's record we have:

Carleton William Sturtevant was born on January 25, 1864, in Ruggles Township, Ashland County, Ohio, and died at Atlanta, Ga. He had four sisters. He was the son of Samuel Carter Sturtevant of Ashland County, Ohio. (On his father's side, seven ancestors came to the United States on the *Mayflower*. The name is of Norman-French origin.) His mother was the former Adelaide Elizabeth Taylor of Cedarville, N. Y. (She was a fourth cousin of her husband; hence they had about the same ancestry—Norman French.) With such a heritage, is it any wonder that Colonel Sturtevant had so distinguished a career?

He was married three times—first, on May 10, 1887, to Mary Jane Atcheson, who died on May 16, 1892. His second wife was Eva May Peterson, to whom he was married on February 16, 1893. She died on July 31, 1899. She bore him his two children. On September 3, 1902, he was married to Rosalie Lightcap Denio, who survives him.

As an interesting personal note, attention is invited to what may be called the family domiciles of three Life Members of the Society. Mention was made of Cedarville (about eight miles south of Mohawk, N. Y.) as being the home of the mother of the Colonel of the 15th Engineers. The Colonel of the 17th Engineers, William Greene Atwood, M. Am. Soc. C. E., had a family domicile at Middleville, N. Y. (about eight miles north of Mohawk). A Captain of the 3d U. S. Volunteer Engineers was born in Mohawk (the writer of this memoir).

On certain occasions much too widely separated, these three dined together and thoroughly enjoyed reminiscences of Mexico, Alaska, Cuba, Europe, and the Orient. Speaking for myself, there prevailed a warmth of feeling, a lasting friendship, and an eager anticipation of joy at the hope of an early repetition of the reunion of these three.

Colonel Sturtevant's father left a diary covering the period from July 4, 1859, to July 5, 1869, which abounds in wonderfully sound patriotic paragraphs of a political and economic nature. It is given in full in the Colonel's family genealogy, from which the foregoing personal data are taken.

The Colonel was a graduate of the University of Missouri, at Columbia, in the Class of 1884. In 1929 his Alma Mater conferred on him the honorary degree of Doctor of Laws.

At the time admission to the Society was through election by the membership, and a certain number of adverse votes precluded admission, he was elected a Member on October 2, 1901. He was thereby senior to the writer of this memoir, who was elected a Member on November 6, 1901. Some good engineers were repeatedly blackballed by contractors' friends who were already in the Society.

Captain Sturtevant's services in the Spanish American War with the 3d U. S. Volunteer Engineers deserve a separate paragraph. The data are taken from the history of the regiment written by Maj. John L. Van Ornum, M. Am. Soc. C. E. Our Col. D. D. Gaillard was a "regular" Captain in the Corps of Engineers. One of our Majors, Edgar Jadwin, was a "regular" First Lieutenant, Corps of Engineers. When he was retired, he became Lieutenant General.

The law creating the Volunteer Engineers regiments was the Act of May 11, 1898. Colonel Gaillard accepted his commission on June 16; most of the regimental officers were commissioned on July 13. The first company was mustered in, July 25, two thirds of the companies by August 4, and the last company on August 20. From St. Louis, Mo., the regiment moved to Lexington, Ky., on September 20; then to Macon, Ga., on November 13. A mutinous colored regiment, the 6th Virginia, was placed "in arrest" by the regiment on November 19. We were reviewed by President McKinley on December 19. Our 3d Battalion with Captain Sturtevant was in Matanzas, Cuba, on December 23, and raised the American Flag there, on January 1, 1899, when the Spanish flag was hauled down. The headquarters Battalion reached Cienfuegos, Cuba, on February 10, 1899, "received" General Gomez and his "army" of about fifty, on February 13, and "received" our Secretary of War, Russell A. Alger, on March 30, in Cienfuegos.

Captain Sturtevant was commanding Officer of the 3d Battalion in Matanzas, from March 1 to March 15, 1899. In this period he had occasion to discipline an enlisted man, so he "preferred charges" as a company commander, tried the charges as Summary Court Officer, then executed the sentence as Battalion Commander. This is some kind of a record, I believe.

The second Battalion sailed from Cienfuegos on April 13; 1st and 3d Battalions sailed from Havana, Cuba, on April 15; all left Savannah, Ga., on April 23, and all were mustered out on May 17, 1899, at Atlanta. Every year in various parts of the United States the surviving members of the "Old Third" have celebrated the May 17th muster-out of the regiment. What other regiment can boast that it has never missed a single anniversary meeting in forty-two years?

The national organization known as "Third U. S. Volunteer Engineers" (War with Spain) has kept alive the sentiments, the friendships, and the patriotic loyalties of its members through all these years. The National President for 1941 was Carleton William Sturtevant; he died in harness. His last duty on the Board of Consulting Engineers for Fort Peck Dam was in July, 1941, when he was seventy-seven and one-half years old.

Colonel Sturtevant was six feet tall. He was strong in mind, strong in body, strong in character. He was indeed a man!

A brief outline of his work is given in *Who's Who in Engineering,* page 1274, 1931 edition. His application for membership in the American Society of Civil Engineers in 1901 is in the files of that Society. Details of his professional engagements are in his "Professional Record" received by the Society on April 16, 1941.

Colonel Sturtevant was elected a Member of the American Society of Civil Engineers on October 2, 1901. He became a Life Member in 1935.

FRANK STONE TAINTER, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 25, 1941

Frank Stone Tainter, son of Thomas Phillips and Josephine (Brooks) Tainter, was born on December 16, 1862, at Morristown, N. J. He was educated at Morris Academy, in Morristown, and took special engineering courses at Princeton University, in Princeton, N. J.

In 1883, before reaching the age of twenty-one, Frank S. Tainter began his engineering career by joining the staff of the Delaware, Lackawanna and Western Railroad Company on location and maintenance work. Railroad construction was at its peak at that time, and in the following decade Mr. Tainter did work on many other lines, including the Delaware and Hudson, Boston and Maine, Southern, and the Texas and Pacific railroads. Between 1888 and 1891 he was in the employ of the Metropolitan Telephone and Telegraph Company as construction engineer and superintendent of pay stations. In the course of these engagements he gained considerable experience not only in accurate surveying, but also in the design and construction of heavy masonry work (such as retaining walls, bridge foundations, and building constructions) and of water supply, water treatment, and sewage disposal plants. This experience he put to excellent use in later years.

In 1896, Mr. Tainter established his own consulting engineering office at Bernardsville, N. J., which he maintained for the forty-five years preceding his death. During this period, between 1902 and 1910, he was a member of the engineering firm of E. A. Stevens and Company, with offices at Hoboken, N. J.; and, between 1920 and 1941, he also maintained an office in New York, N. Y., as an associate of Parsons, Klapp, Brinckerhoff and Douglas, Consulting Engineers.

In 1917, when the United States entered the first World War, Mr. Tainter applied for a commission in the service and was appointed by President Woodrow Wilson as a Major in the Corps of Engineers, U. S. Army; he retired with the rank of Lieutenant-Colonel in 1920. During his service he designed and was in charge of construction of the Aberdeen Proving Ground in Maryland.

In the course of his engineering practice, which extended over more than half a century, Colonel Tainter designed and supervised the construction of a large number of engineering works. He was chiefly active in surveying, water supply and sewage disposal projects, bridges, and private and public property developments. Only a few of his many works in these several lines will be mentioned.

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[1] Memoir prepared by Eugene E. Halmos, M. Am. Soc. C. E.

Surveys.—Map of 130,000-acre game preserve in Beaufort and Hampton counties, South Carolina; map of Oyster Bay Township, Long Island, N. Y.; and map of Southampton, Long Island, N. Y.

Water Supplies.—Pottersville, N. J.; Castleton on Hudson, N. Y.; Kingsport, Tenn.; Appalachia, Va.; and Norwood, Ohio.

Most of the water supply projects included dams of various types and heights, some of which reached national recognition, such as the Carnegie Dam at Princeton and the dam at Kingsport.

Sewage Systems.—Aberdeen Proving Ground; Clearwater, Fla.; Kingsport, Tenn.; and Collingswood, N. J.

Bridges.—Carnegie Bridge at Washington Crossing, Princeton; highway bridge, Peapack, N. J.; causeway at Blairsden, Peapack; and railroad bridge, Dansville, N. Y.

Property Developments.—Estate of C. Ledyard Blair, Peapack; estate of Marshall Field, Lloyds Neck, Long Island, N. Y.; Peachtree Heights Park, Atlanta, Ga.; and Somerset Hills Country Club, New Jersey.

Colonel Tainter was one of the last of the generation of engineers, affectionately termed "old-timers," whose status in the profession paralleled that of the family physician. The first part of his career came in a period when the profession was not crowded as it is now, and when the engineer with general practice could not afford to specialize. He splendidly fulfilled the duties of a general practitioner.

Colonel Tainter, although a man of slight build, had tremendous energy, was uncommonly active, and was a tireless worker. He personally conducted all work that he undertook, delegating only details to subordinates. Although this method of managing his affairs kept him extremely busy, he was always willing to devote his time and money in the interest of the community. Thus, in 1911, on the request of the then Governor Woodrow Wilson of New Jersey, he went to England to obtain firsthand information on methods of construction, surfacing, and maintenance of British roads. His report was published in the *Year Book* of the State of New Jersey in 1912. For periods of one to four years he held the following positions: Acting chief consulting engineer and acting engineer of the Highway Department of the State of New Jersey; acting county engineer for Morris and Somerset counties, New Jersey; engineer commissioner and acting chairman of New Jersey State Board of Health; engineer member and vice-chairman of Tri-State Commission for the abatement of contamination of waters surrounding the New York metropolitan area; and consulting engineer for the Roosevelt Memorial Association.

In addition to his professional work, Colonel Tainter took great interest in the social life of his community. He was well known, liked, and respected in the entire State of New Jersey and particularly among his neighbors at Far Hills, where he had a beautiful home, built and developed by himself, and of which he was justly proud. His hobby was horseback riding and he participated in fox hunting until a few years before his death.

In 1892, Colonel Tainter was married to Susan Bayard, who survives him.

He was a member of the following organizations: American Institute of Mining and Metallurgical Engineers, American Association for the Advancement of Science, Society of American Military Engineers, American Water Works Association, Institution of Civil Engineers, London, Royal Society of Arts, and Sons of the American Revolution. He was also a member of the Engineers' Club, Down Town Association, Morristown Club, and Essex Fox Hounds.

Colonel Tainter was elected an Associate Member of the American Society of Civil Engineers on January 3, 1900, and a Member on March 31, 1908. He became a life Member in 1933.

BENJAMIN FRANKLIN TAYLOR, M. Am. Soc. C. E.[1]

Died April 18, 1942

Benjamin Franklin Taylor was born at Proctor, Vt., on November 9, 1881, the son of Benjamin Franklin Taylor and Jessie (Smith) Taylor. His preliminary education was obtained in the grade school and high school at Proctor, and he was graduated from the University of Vermont, at Burlington, with the degree of Bachelor of Science in Civil Engineering in 1907.

Immediately after graduation, Mr. Taylor accepted a position as rodman with the New York, Ontario and Western Railway Company. In 1908 he became a rodman with the New York State Highway Department where he was employed until 1917. In the latter year, he entered the employ of the Construction Division of the War Department, Quartermaster Corps, at Washington, D. C., as highway engineer.

On April 4, 1919, Mr. Taylor was employed by the Public Roads Administration as highway engineer at Washington, D. C. Shortly after his entrance into the service, he was transferred to Division 5 with headquarters at Omaha, Nebr. He was promoted to senior highway engineer in 1928, which position he held until his death.

During 1939 Mr. Taylor took a leave of absence for a year due to nervous prostration. When he returned to duty, he appeared to be in good health. He had been residing at an apartment hotel in Kansas City, Mo., and on or about April 3 he was awakened by flames and smoke in his room. He undoubtedly received a severe shock. He was taken to the Crestview Sanitarium, in Kansas City, where he died on April 18, 1942, at 9:30 p.m. His body was taken to Proctor for interment.

He leaves a wife, Mrs. Reland S. Taylor, to whom he was married on June 3, 1935; a son by a former marriage, Brooks S. Taylor; a sister, Mildred E. Taylor; and two brothers, C. M. Taylor and Richard H. Taylor.

Mr. Taylor was elected a Member of the American Society of Civil Engineers on March 10, 1930.

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[1] Memoir prepared by E. L. Brown, M. Am. Soc. C. E.

ROBERT ANDREW THOMPSON, M. Am. Soc. C. E.[1]

DIED MAY 30, 1941

Robert Andrew Thompson was born at New Waverly, Walker County, Tex., on July 11, 1869, the son of Col. Andrew Jackson and Mary (Gillespie) Thompson. The family lost its holdings after the War Between the States and, although Colonel Thompson's negroes settled around him and worked for him, he was too old to make a new start. Before he was twelve years old, young Thompson was managing his father's plantation and marketing such products as honey, and especially prepared mutton, for which products the plantation had been noted.

Mr. Thompson's early education was obtained by night study under his mother's direction, after days spent in fields and in stock feeding and breeding. He read every book he could secure within horseback distance of the plantation home. At nineteen he was admitted to the College of Engineering of the University of Texas, at Austin, Tex., from which he received the degrees of Bachelor of Science in Civil Engineering in 1892, Master of Arts in 1893, and Civil Engineer in 1900. He made a very distinguished scholastic record, earning membership in the Phi Beta Kappa and Tau Beta Pi fraternities. During the undergraduate period he worked each summer and in his spare time during the school years for the Texas Geological Survey.

In 1893 Mr. Thompson worked for the Pencoyd (Pa.) Bridge Company, resigning to become instructor in civil engineering at the University of Texas during the 1893–1894 session. He also served in this capacity during the 1897–1898 session. In the interim he worked for the Texas Railroad Commission, constructed a sewer system for Palestine, Tex., and worked for the Kansas City Southern Railway Company. He served as chief engineer for the Texas Railroad Commission from 1898 to 1908. He became chief engineer of the Wichita Falls and Northwestern Railway Company in January, 1909, and served as such until September, 1911.

Mr. Thompson was chief engineer of the California Railroad Commission from October, 1911, to May, 1913, resigning to accept membership on the Engineering Board, Bureau of Valuation of the United States rail lines, Interstate Commerce Commission, in charge of the Pacific District, San Francisco, Calif. This extremely important work lasted until October, 1921. Returning to Texas, he became chief engineer of the Wichita County Water Improvement District (Wichita Falls, Tex.) in charge of two large dams and a 40,000-acre irrigation system costing $4,500,000 which was completed in 1924. From 1924 to 1927 Mr. Thompson was co-chief engineer (with the late J. C. Nagle,[2] M. Am. Soc. C. E.) in charge of location, design, and construction of the Garza Reservoir Dam for the City of Dallas (Tex.) water supply, costing $5,000,000. In 1925 and 1926 he was a member of the firm of Thompson and Butler, Consulting Engineers, in San Francisco.

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[1] Memoir prepared by Gibb Gilchrist, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1878.

In 1927 the Texas Highway Department, being in need of reorganization, chose Mr. Thompson for the job and he earned the gratitude of a people by his courage and organizing ability in putting the department in order. Here his influence will be felt for many years. The work was strenuous and confining and when the organization had been perfected he returned to Dallas and resumed his consulting practice. He served as consultant to the Dallas Chamber of Commerce from 1928 to 1933 and from 1934 to 1938, particularly in connection with highway transport in the Dallas trade territory. During 1933 and 1934 Mr. Thompson served as state engineer for the Public Works Administration for Texas. He made a long-range plan for highways for Tarrant County, Texas. He served as chief engineer of the Wichita Falls and Southern Railway Company and as consulting engineer for the Brown County Water Improvement District. He made reports on power development for the Wichita County Water Improvement District and on a water supply for the City of Abilene, Tex. Appointed by the Governor in 1938, he served as engineer member of the Texas State Board of Health. There were many other lesser assignments.

Robert A. Thompson was married on December 21, 1897, to Evelyn Dickson of Cleburne, Tex. Seven children were born of this union—Frances, Robert Andrew, Jr. (Assoc. M. Am. Soc. C. E.), Elizabeth, Eleanor, Evelyn, Frederick, and Lucy, all of whom, with the pioneer wife and mother, survive. The writer knows that much of Mr. Thompson's success was due this wife who stood with him through every trial and whose every thought was for the welfare of her husband and children.

Mr. Thompson made some of his finest friends while living in Austin from 1900 to 1908. Frequently he played chess, bridge, and whist with four men who were to become famous within a decade, three as members of a wartime president's cabinet and one as his chief adviser. These men were Edward M. House, T. Watt Gregory, Albert S. Burleson, and David F. Houston.

Robert A. Thompson was endowed with a brilliant mind, which he constantly improved by study and hard work. He had an unquenchable and uncompromising honesty that was refreshing and inspired confidence. In passing, he has left a heritage of professional conduct and performance that is an inspiration to young engineers who have known and admired him. The engineering profession in Texas is better by reason of Robert A. Thompson's professional career and personal life.

The following editorial from the "Dallas Morning News" is appropriate:

> "A man of few words and many deeds, Robert A. Thompson, who died at Fort Worth Friday, typified the best in the modern engineering profession. From the time he was graduated from the University of Texas with high honors until the close of his professional career, his life was one of achievement, though his unassuming nature kept from him the public recognition that many another man would have gotten for a far less constructive career. A native Texan, he served his state faithfully, first as engineer to the Railroad Commission and later as State Highway Engineer under the administration of Governor Dan Moody. It was Highway Engineer Thompson, with the sponsorship of Governor Moody, who laid the basis for the present efficient department and its brilliant

record of accomplishment. Dallas owes Robert A. Thompson a lasting debt of gratitude."

Mr. Thompson was elected a Junior of the American Society of Civil Engineers on April 6, 1897; an Associate Member on October 4, 1899; and a Member on January 3, 1911. He became a Life Member on January 1, 1934.

CURTIS McDONALD TOWNSEND, M. Am. Soc. C. E.[1]

DIED MAY 26, 1941

Curtis McDonald Townsend, the son of Martin L. and Alice Sophia (McDonald) Townsend, was born in Brooklyn, N. Y., on March 22, 1856. He was graduated from the College of the City of New York in 1875, with the degree of Bachelor of Arts, at nineteen years of age, having completed a five-year course. He was undecided whether to study law or medicine but, taking the advice of a friend, he entered a competitive examination and received an appointment to the United States Military Academy at West Point, N. Y., on July 1, 1875. He was graduated in June, 1879, and was commissioned Second Lieutenant in the Corps of Engineers.

His first assignment as an officer was at Willets Point, New York, where he served until June, 1882, when he was promoted to First Lieutenant. He then became assistant engineer at Baltimore, Md., and Petersburg, Va., becoming assistant to the engineer commissioner in the District of Columbia on June 15, 1886. He returned to Willets Point in May, 1887, as Quartermaster of the Engineer Battalion there, in charge of the construction of buildings and the introduction of sewers and a water supply. In June, 1890, having been promoted to Captain, he was assigned to duty in Washington, D. C., in connection with the construction of the Washington Aqueduct.[2]

Captain Townsend was placed in charge of the Third Mississippi River District, at Memphis, Tenn., in July, 1896. In May, 1898, he was transferred to Grand Rapids, Mich., and placed in charge of construction there. While at Memphis he fell critically ill from typhoid fever, and he overheard the doctor tell his father, "We'll carry him out in the morning." This aroused his fighting spirit, and he said, "Not if I can help it." He recovered, but his body bore marks of the struggle throughout his life. With characteristic reticence, however, he said nothing. After he left Memphis one of his chief assistants paid this tribute: "Captain Townsend was a fighter."

After a brief tour of duty at Rock Island, Ill., he was assigned to the 3d Engineer Battalion in the Philippine Islands, where he served on the staff of the chief engineer, Philippine Division. During this time he was promoted to Major. While in the Philippines, he supervised the construction of military roads and the building of the Manila breakwater, and started wharfs for the City and Quartermaster Department; he also supervised the construction of harbors and wharfs in other parts of the islands.

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[1] Memoir prepared by Col. James W. Skelly, M. Am. Soc. C. E., in collaboration with Col. John A. Laird, Cons. Engr., St. Louis, Mo., Lt. Col. Frank G. Jonah, Hon. M. Am. Soc. C. E., Lt. Col. Frederick W. Green, M. Am. Soc. C. E., and Maj. John J. Barada, Secy., Twelfth Engrs. Assn., St. Louis, Mo.

[2] *Who's Who in Engineering*, 1937, p. 1392.

Major Townsend returned to Washington, D. C., in May, 1906, as a member of the Board of Engineers for Rivers and Harbors. He went to Cleveland, Ohio, in February, 1908, and later to Detroit, Mich., meanwhile having been promoted to Colonel. In July, 1912, he became division engineer of the Western Division, at St. Louis, Mo. He also served as president of the Mississippi River Commission and as a member of the Experimental Towboat Board, formed to develop, through experiments, a suitable towboat for use on inland waterways.

Shortly before the outbreak of World War I, while stationed at St. Louis, he was called into consultation with the chief of engineers, concerning the organization of engineer regiments to construct and operate railways in France, in response to an urgent request from Marshal J. J. C. Joffre, who was then touring the United States. Colonel Townsend recommended that these regiments should have two battalions, one organized for construction, the other as a pioneer unit. Later this was modified, the pioneer unit being replaced by a railway operation battalion. Colonel Townsend was assigned the task of organizing one of these regiments in St. Louis, which was first called the Second, and later the Twelfth, Engineers. The officers were taken mainly from the Officers Reserve Corps, and the enlisted men from the railroads entering St. Louis from the south and southwest.

The Twelfth Engineers regiment sailed from New York City on July 28, 1917, and was assigned to the British on the Picardy front. It reached there on August 21, 1917, being among the first 30,000 American troops to reach France. The regiment was immediately placed in charge of the operation and maintenance of light railways near the Hindenburg line, and in due time built quite a number of extensions.

One day while Colonel Townsend was driving near the front, shells began dropping near-by. The chauffeur turned to him and said, "What shall we do Colonel, they are getting pretty close." Colonel Townsend replied, "Better keep on going, this is not a safe place."

Colonel Townsend was transferred to Paris, France, on February 12, 1918, to assume the duties of engineer representative on the General Purchasing Board for the American Expeditionary Forces. He left on the Regiment the impress of his own high standards of justice, democracy, steadfastness, and sound judgment. In October, 1918, he returned to the United States. For his World War services he was awarded the French decoration of Officer de le Legion d'Honneur.

On his return to the United States, he served for a time in the office of the chief of engineers. In December, 1918, he was again named division engineer of the Western Division, at St. Louis, and president of the Mississippi River Commission. He served in these capacities until February, 1920, when at his own request he was retired from active duty.

After retirement from active service, he lived for a time in Brooklyn. During this time he wrote a textbook for students on improvement of rivers and harbors, in the course of which he acquired a working knowledge of the Italian language in order to read references on the subject. Later he moved to Washington, D. C., where he continued to live until shortly before his death.

He was recalled to active service in the Corps of Engineers for two assignments. From December, 1923, until June, 1926, he served with the Rivers and Harbors Commission in New Orleans, La., and from July, 1926, until October, 1926, he served as Gulf district engineer in New Orleans, in charge of creating a 36-ft waterway in the Southwest Pass of the Mississippi River.

In the spring of 1927, the Belgian Ambassador to America requested Colonel Townsend to go to Brussels, Belgium, as a member of the International Board on Improvement of the Mouth of the Congo River. He did not go on to Africa for a personal inspection, but advised the Belgian Government as to what further data and surveys were necessary. When this information was assembled, in conference with Belgian and French engineers, a satisfactory report was formulated.

Colonel Townsend was a member of the Western Society of Engineers, the Engineers Club of St. Louis, and the Society of American Military Engineers. He was president of the latter society in 1934. He contributed numerous articles to engineering and technical journals, relating to commerce and improvement of navigable rivers. He had the ability to grasp a subject quickly; yet his considerations were deliberate, and his conclusions were reached only after a careful "estimate of the situation." Although respectful of the opinions of others, he would permit no motive of convenience or expediency to swerve him from what he believed to be his duty. He exemplified the highest type of honor and integrity.

As a boy, one of his hobbies was collecting stamps, which he continued throughout his life. He also paid considerable attention to geology, particularly of the Manhattan and New Jersey regions, from which he made quite a collection of stones. He was a scientific and skilful chess player.

He was by no means devoid of a sense of humor. When he was a Captain at Rock Island, two of his assistants were field officers in the National Guard of Illinois. One day, as he related to one of the writers, a messenger passed the word through the office, "Have the Major tell the Colonel the Captain wants to see him."

One of the field officers of the Twelfth Engineers paid this tribute: "He was a fearless soldier, a courteous gentleman, a staunch friend and an honorable man."

Colonel Townsend was never married. Since he had no family, he devoted his time to his profession and to his friends. In stature he was tall and erect, and of distinguished appearance.

He died on May 26, 1941, and was buried in Arlington National Cemetery.

Colonel Townsend was elected a Member of the American Society of Civil Engineers on November 1, 1893.

ROBERT WILLIAM WADDELL, M. Am. Soc. C. E.[1]

DIED APRIL 19, 1940

Robert William Waddell was born in Port Hope, Ontario, Canada, on October 31, 1859, the son of Robert N. and Angeline E. (Jones) Waddell. At the age of twenty two he was graduated from McGill University, in Montreal, Quebec, Canada, with the degree of Bachelor of Applied Science.

The early years of Mr. Waddell's professional life afforded him a wide and varied experience in engineering. His first interest was in railroad engineering, in the service of the Denver and New Orleans and the Denver, Texas and Fort Worth railroad companies. He next engaged in private practice from 1885 to 1887. In 1889 and 1890 he supervised the erection of the Castlewood Dam, in Colorado. After completing some government contracts, as deputy land surveyor, Mr. Waddell went to Kansas City, Mo., and during the years 1895, 1896, and 1897 he again turned his talents to railroad construction.

In 1898 Mr. Waddell served as engineer and superintendent of construction on the Hotel Baltimore in Kansas City. It seems almost symbolic that the structure that he built so well should continue, for the identical span of years as its builder, to serve the community so well and be such an outstanding landmark in the great transition period between 1900 and 1940.

In 1899 Mr. Waddell accepted a position as chief engineer of railroads for a mining company in Indian Territory, later Oklahoma, but returned to Kansas City the following year to become city engineer. During the period from 1900 to the date of his death, April 19, 1940, Mr. Waddell served the city as its city engineer for twenty-two years. Twice, during that period, he left the city service to engage in contracting. During the last few years of his life, he did considerable personal inspection of the many projects that he had envisioned and had seen materialize. Mr. Waddell's era saw the reconstruction of his community in the changes necessitated by the introduction of the automobile, and its attendant traffic problems, and also the general modernization of all utilities that occurs when a small community develops into a large metropolitan center. The personal interest which Mr. Waddell had in his work, and the outdoor life made possible by his personal inspections, undoubtedly contributed much to the prolongation of his life during that period when his health was failing slowly.

In addition to his membership in the Society, Mr. Waddell was an honored member of the Engineers' Club of Kansas City, the American Road Builders Association, the American Society for Municipal Improvements, and the National Conference of City Planning. His interest in civic affairs was as great as that in his profession and he also held membership in the Knife and Fork Club of Kansas City, The Safety Council, The Citizen's League, and the City Club of Kansas City. During his many years of service in Kansas City, Mr. Waddell was known and loved by engineers and contractors and, for at least a generation, he was to them "Uncle Bob."

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[1] **Memoir prepared by Arthur C. Everham, M. Am. Soc. C. E.**

As a pastime, Mr. Waddell was an ardent philatelist, and he had an unusually fine collection of stamps, some of considerable value. He said that he developed his interest in stamp collecting by cleaning out the wastebaskets in his father's office. He specialized in Canadian and United States stamps and owned one ninety-cent United States stamp which was so rare that he was unable to find any one who could determine its value.

Mr. Waddell was an Episcopalian and was a member of St. George's Church, which he served for many years as Vestryman. He was most generous with his charities, and many a younger man can remember his readiness to help out in times of financial distress.

His death at the age of eighty concluded a long and useful life during which he established a standard in municipal service that always will be a goal for those who follow him. He left a family consisting of Sarah E. Waddell, to whom he was married on November 24, 1897, and two sons, Gerald Needham and Robert Douglas Waddell. Many tributes have been paid Robert W. Waddell, but none was greater than that expressed by the head of one family who said he would indeed be very happy if he could raise his two boys to be as fine men as Mr. Waddell had raised his sons to be.

Mr. Waddell was elected a Member of the American Society of Civil Engineers on September 4, 1901.

EDWARD WEGMANN, M. Am. Soc. C. E.[1]

Died January 3, 1935

Edward Wegmann was born in Rio de Janeiro, Brazil, on November 27, 1850. His ancestors had settled in the City and Canton of Zürich, Switzerland, where they became a very distinguished family. The records show that one, A. Wegmann, moved from the City of Tagelschwangen, Switzerland, to the City of Zürich in 1469. Lewis Edward Wegmann, the father of Edward Wegmann, was born in Steckborn, Switzerland, on March 14, 1814, and came to New York, N. Y., when he was seventeen years old. There he soon obtained a position with the firm of De Rham, Iselin and Moore. When he was twenty-three years old he accepted an offer to become a partner of Mr. Daniker of Zürich, who was in the export and import business in the City of Rio de Janeiro. Lewis Wegmann went to that city in 1837, and remained there for about fifteen years. During an epidemic of yellow fever, he and his wife (the former Mary W. Sand) both contracted the disease, so they returned to New York City and Mr. Wegmann became a partner of the well-known firm of A. C. Rossire and Company. During the panic of 1857 this firm suffered heavy losses, which caused him to sever his connection with it and go to Galveston, Tex., where he went into business under the name of Wegmann and Parizot, shortly before the outbreak of the Civil War. Before leaving for Texas, however, he took his family to Zürich. He left his wife and four children there until 1866, when he took them back to New York City. He again engaged in business in that city until his death in September, 1895. His wife passed away in April, 1896.

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[1] Memoir prepared by T. Kennard Thomson, M. Am. Soc. C. E.

Their son, Edward Wegmann, born during the family's stay in Brazil, received his first education in a private school in Brooklyn, N. Y., from his sixth to his ninth year. From his ninth to his sixteenth year, he attended the public schools of Zürich. On returning to America he spent two years in the Polytechnic Institute of Brooklyn, and then became a clerk in his father's business for a short time. Not finding the work congenial, he decided to become a civil engineer. He was graduated from New York University, in New York City, in June, 1871, with the degree of Civil Engineer.

During the next twelve years he was engaged on the construction of railroads. He started as axman and was then chainman on the New York, West Shore, and Chicago Railway during August, September, and October, 1871. From January to December, 1872, he was a rodman on the New Haven, Middletown and Willimantic Railway. He acted as assistant engineer on the same railroad from January to December, 1873. For the greater part of 1874, Mr. Wegmann was working in the Wyandotte (Mich.) Rolling Mill, and in the Machine Shop, learning the manufacture of iron (there was no steel in those days).

From March, 1875, to August, 1877, Mr. Wegmann, in partnership with the late Robert W. Creuzbaur,[3] M. Am. Soc. C. E., a mechanical engineer, developed a new steam engine, and some other inventions, first at their own shop, and then, during 1876, at the Danforth Locomotive Works in Paterson, N. J. From November, 1877, to June, 1878, Mr. Wegmann was engineer for the Keystone Bridge Company (a Carnegie subsidiary), in charge of building foundations and superstructures of the Sixth Avenue Elevated Railway in New York City. From June, 1878, to August, 1879, he was assistant engineer on the Ninth, Second, and Third Avenue Elevated railways, all in New York City.

He was on the engineering staff of the New York and New England Railway Company from January to May, 1880, when he became assistant engineer on the location of the Ohio River and Western Railway. From August, 1880, to January, 1884, Mr. Wegmann was resident engineer on the construction of The New York, West Shore and Buffalo Railroad, in charge of 14 miles of work near Haverstraw, N. Y., and then 16 miles in Genesee County, New York.

In 1884 Mr. Wegmann became assistant engineer on the New Croton Aqueduct (New York), engaged in the office of the chief engineer. The first duty to which he was assigned was to make studies for the design of the proposed Quaker Bridge Dam, which was to be built across the Croton River. This structure was to have a height of 297 ft above the lowest foundation. This was to be a monumental structure, more than 100 ft higher than any masonry dam built up to that time.

After a thorough study of the subject, material for which was found mainly in the professional memoirs of French Engineers as published in the

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[3] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1160.

Annales des Ponts et Chaussées, he arrived at a very simple formula for designing masonry dams. It was used for the New Croton Dam (originally called the Quaker Bridge Dam) and for some other very high dams in America.

In 1888 the first edition of Mr. Wegmann's monumental work on *The Design and Construction of Dams,* including treatments of masonry, earth, rock-fill, timber and steel structures, and principal types of movable dams, was published by John Wiley and Sons, Inc. The eighth edition of this great book (740 pages, 9 by 12 inches, with many illustrations, and, in addition, 111 plates) appeared in 1927. It gives a full explanation of the simple analytical method he devised for the New Croton Dam and a description of the principal masonry dams, completed before 1888, in Spain, France, and other foreign countries. He was working on the ninth edition before he died.

In 1885 Mr. Wegmann was made division engineer of the Manhattan Section of the New Croton Aqueduct, which included the tunnel under the Harlem River and the section from 179th Street to 135th Street, the terminal gate-house at this street, and eight pipe lines 4 ft in diameter that terminated at the Central Park Reservoir.

Following this assignment, in 1892 Mr. Wegmann was transferred to the Croton watershed and placed in charge of the Croton River Division of the New Croton Aqueduct. This included the construction of the Muscoot Dam (about nine miles upstream from the New Croton Dam) and all the changes of highways and railroads made necessary by the construction of the Croton Reservoir. This work naturally caused Mr. Wegmann to take a great interest in the early history of the New York water works, so he wrote another monumental book on *The Water-Supply of the City of New York,* covering the years 1658 to 1895. The book was published in 1896 by John Wiley and Sons, Inc.

Before this book appeared, Mr. Wegmann was engaged by the Committee of the Legislature in charge of the exhibit of the State of New York at the Columbian Exposition held in Chicago, Ill., in 1892, to prepare for this Exposition a "Description of the Waterworks of the City of New York," illustrated by plans and photographs. As a result of this exhibit the Aqueduct Commissioners of the City of New York (in whose service Mr. Wegmann remained while preparing the exhibit) received a diploma from the Commissioners of the Columbian Exposition. Mr. Wegmann also was awarded a diploma of honorable mention for his work.

After twenty years spent in the field in charge of this important construction for the Aqueduct Commissioners, he was transferred from the Croton Division to the headquarters in New York City. He was then instructed to write the final report of the works completed under the direction of these Commissioners, from 1895 to 1907, when the New Croton Reservoir was completed. This report, which is complete with practical and technical details, was published by the Commissioners in 1907, in which year they appointed Mr. Wegmann their consulting engineer. Three years later he was promoted to the position of chief engineer for the Commission; he was the last person to hold this title.

In the same year, however, as the projects constructed under their supervision were about completed, the Commission was abolished by Act of Legislature and the employees were transferred to the Department of Water Supply, Gas and Electricity of the City of New York. Mr. Wegmann then was appointed consulting engineer for the Bureau of Water Supply of this Department, and held the position for four years, when he resigned to open an office as a consulting engineer. He had many clients in various parts of the world, advising them especially on dams and water works.

Another of his books was *Conveyance and Distribution of Water for Water Supply; Aqueducts, Pipe-Lines and Distribution Systems,* published by D. Van Nostrand Company in 1918. In 1890 Mr. Wegmann and C. J. Bates published a pamphlet on "A System of Rapid Transit for the City of New York." This was a comprehensive plan for a construction three decks high, consisting of one subway with two elevated structures above it, each having two railroad tracks, or six in all. They proposed one line between Third and Fourth avenues, and another line between Ninth Avenue and the North River.

In 1893 Mr. Wegmann published a *Catalogue of the Illustrations of the Water-Supply of the City of New York.* He contributed many articles to the press and to various societies. For the American Society of Civil Engineers alone, he took part in writing or discussing some twenty-five papers, on dams, railroads, elevated railroads, pneumatic plants, water mains, and surveying in tunnels.

From July, 1918, to January, 1920, Mr. Wegmann was retained by The New York and New Jersey Harbor Commission (later The Port of New York Authority), making estimates of the cost of constructing subways, elevated railroads, and tunnels for freight service. For this Commission he wrote a report on "The Water Supply of the Metropolitan District of New York," including part of New Jersey. It contained estimates of existing available resources of water supply and probable consumption until 1975.

For the last time Mr. Wegmann reentered the service of the City of New York in 1920. He served until 1925, when he retired with a pension for life.

In 1922 Mr. Wegmann was honored by The International Congress of Engineers, which was held that year in Rio de Janeiro, the city of his birth. The Commission of the four American societies at the Congress asked him to write a paper on "The Design of Masonry Dams." The Commission also sent to The Engineering Society of Rio de Janeiro a copy of Mr. Wegmann's book on dams. As a result, Mr. Wegmann was unanimously elected an Honorary Member of The Engineering Society of Rio de Janeiro.

He was also a member of The New England Water Works Association, The American Water Works Association, and The American Society of Engineering Contractors.

Mr. Wegmann and Albert Nelson Aeryns developed a new formula, based on about 1,000 tests, to determine the flow of water in new and old cast-iron pipes. They also contracted with The Research Association of Cast Iron Pipe and Foundry Companies to prepare tables for the flow of water in clean and old cast-iron pipes that varied in diameter from 4 in. to 60 in.

Mr. Wegmann lectured at Harvard University (Cambridge, Mass.), Yale University (New Haven, Conn.), and other leading universities, and at many other gatherings of professional men. His books have been translated into several foreign languages. He spoke fluently four languages: English, French, Italian, and German. Probably the reason he did not speak Portuguese was because he left Brazil when he was about one year old.

The *American Swiss Gazette,* published in New York City, printed in 1931 a number of biographical sketches of outstanding men of Swiss birth or ancestry, including a very fine one of Edward Wegmann. The publishers were justifiably proud of his great record. (The writer wishes to express his thanks for having had the opportunity to make many extracts from that fine paper for this memoir.)

He spent the last years of his life in Yonkers, N. Y., and in 1927 wrote a comprehensive letter to *The Yonkers Herald* indorsing the late Mayor Thomas F. Larkin's efforts to obtain more water from New York City for Yonkers. He made some good suggestions on the subject.

He became a Mason on April 26, 1910, and became a charter member of Fernbrook Lodge No. 898, F. and A. M., at Yonkers, in June, 1912. He was a regular attendant at the meetings of that Lodge until a short time before he died, and was given a Masonic funeral. He was also a good Episcopalian and attended St. John's Church in Yonkers. Mr. Wegmann also was an excellent musician and at the age of nine received an unusually fine violin, which he played well all his life.

On May 6, 1901, he was married to Charlotte H. Drummond. Their son, Charles Edward Wegmann, was born on February 11, 1905, and became a successful businessman, importing and exporting rubber. A daughter, Cecile D. Wegmann, born in September, 1907, is a fine concert pianist. Mr. Wegmann must have been a devoted son and brother, and he certainly was a devoted and proud husband and father. Furthermore, he was a true citizen and a faithful friend.

Edward Wegmann was the most outstanding authority of his time on the design and construction of dams and water works. In addition, he had an unusually broad experience in many other branches of engineering, as is evidenced by the printed works he left behind him and the memory of those who knew him.

Mr. Wegmann was elected a Member of the American Society of Civil Engineers on March 7, 1888, and cherished that membership as long as he lived.

LESLIE CLIFFORD WHITTEMORE, M. Am. Soc. C. E.[1]

DIED JANUARY 13, 1941

Leslie Clifford Whittemore was born at Milford, Mass., on April 22, 1885. His father, the late George H. Whittemore, lived in Milford, and was both a jeweler and an electrical contractor. His mother, Minna L. (Mann) Whittemore, is the daughter of Capt. Elbridge Mann, who was a shoe manufacturer of Milford and one-time representative in the Massachusetts State Legislature.

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[1] Memoir prepared by Langdon Pearse and Norval E. Anderson, Members, Am. Soc. C. E.

Mr. Whittemore's early life was spent in Milford, where he attended the public schools. In June, 1907, he was graduated from the Massachusetts Institute of Technology, in Boston, Mass., with the degree of Bachelor of Science in Sanitary Engineering.

His first engineering experience was with the Massachusetts State Highway Commission, on surveys for state roads, during the summer of 1906. After graduation from the Massachusetts Institute of Technology, he was employed as a draftsman in the building department of the American Brass Company, at Waterbury, Conn., for about six months. His first sanitary engineering position was that of assistant engineer for the Massachusetts State Board of Health, from December, 1907, to January, 1909, on examinations and reports for water supply, sewage treatment, and stream pollution. From January to May, 1909, he was assistant chemist at the Washington, D. C., Filtration Plant on chemical, bacteriological, and microscopic analyses of water for control of plant operation.

In May, 1909, Mr. Whittemore joined the Board of Water Supply, in New York, N. Y., as assistant engineer in the Northern Aqueduct Department, where he served until July, 1910, as assistant to the department engineer. From July, 1910, to May, 1911, he had charge of construction at Shaft No. 5 of the Moodna Siphon, a deep-rock tunnel 14 ft in diameter. Thereafter he served as principal office assistant to the department engineer until October, 1912.

For twenty-nine years Mr. Whittemore was associated with The Sanitary District of Chicago, Ill., beginning October 15, 1912, at a time when the Sanitary District was investigating methods for treating the rapidly increasing volume of domestic sewage and industrial wastes that was overloading the canal dilution system. For several years he was concerned with the operation of the various sewage testing stations on industrial wastes, reports on the tests, and the study of preliminary plans for treating industrial wastes, as well as plans for intercepting sewers and studies for the treatment of sewage for the entire Sanitary District.

When actual work began, in 1919, Mr. Whittemore, as principal assistant to the sanitary engineer, was active in the design and preparation of plans and specifications for the Calumet and Des Plaines River sewage treatment works. He gradually took over the responsibility for this part of the work until, in 1925, he was placed in charge of the separate division of Sewage Treatment Plant Design. In 1930 he was appointed engineer of design, with general supervision and coordination of the various design divisions of the Sanitary District. He continued in this capacity until his death.

Mr. Whittemore was active throughout his connection with The Sanitary District of Chicago in the development of its sewage treatment program, from the early experiments through the studies and reports on a general scheme; studies and experimental work on treatment processes; preparation of data for litigation and for the review of consulting engineers; and the design, preparation of plans and specifications, and construction of all the sewage treatment plants. Under his supervision, as engineer of design, were prepared the plans for the four major plants, Calumet, North Side, West Side, and Southwest. Before he died the initial units of the Southwest Sewage Treatment Plant were put into operation, the largest sewage treatment plant in the world.

As an engineer, Mr. Whittemore was known for his painstaking attention to detail and thorough analysis of basic data and the alternatives available, as well as for the soundness of his conclusions. His decisions and judgment were universally respected by his associates. He applied himself unsparingly in time and effort to the problems at hand, sometimes to the detriment of his health. Always considerate of his associates and subordinates, he was not impatient with those of lesser ability. Although the author of a number of technical articles and papers presented to the Society, he wrote little, considering his knowledge and authority, because of his modesty and constant application to his work. He was a member of the Society's Committee on Activated Sludge Practice.

Mr. Whittemore was a member of the Central States Sewage Works Association. In addition to his professional work, he was interested in athletics. He was a great reader, especially of history and travel, and a lover of good music. He was a member of the Congregational Church.

In 1936 Mr. Whittemore suffered a severe heart attack, from which he soon recovered and was able to resume his duties. However, there were succeeding attacks. In the late summer of 1940, his condition became such that he was confined to his home, in Chicago, where he died on January 13, 1941.

He was married on June 4, 1913, in Poughkeepsie, N. Y., to Lucile Marion Taber of that city. He is survived by his widow; his only son, Robert; his mother; and his sister, Mildred E. Whittemore.

Mr. Whittemore was elected a Junior of the American Society of Civil Engineers on May 3, 1910; an Associate Member on June 24, 1914; and a Member on July 11, 1921.

JOSEPH WHITE WILLIAMS, M. Am. Soc. C. E.[1]

DIED FEBRUARY 3, 1941

Joseph White Williams was born in Milan, Erie County, Ohio, on October 20, 1879, the son of Daniel Newton Williams and Sophie (White) Williams. His early life was spent in and about Milan, where he was graduated from the Milan High School in 1897.

In 1898 he enlisted as a private in the 52d Iowa Volunteer Infantry in the Spanish-American War. On the cessation of hostilities, after receiving his discharge, Mr. Williams held various minor positions on railroad location and construction until 1903. From 1903 to 1906 he served in the capacity of transitman and assistant engineer on location and construction for the Atchison, Topeka and Santa Fe Railway Company in New Mexico and California. In 1907 he was appointed assistant chief engineer of the San Francisco and Northwestern Railway Company with headquarters in San Francisco, Calif. Later, when the Northwestern Pacific Railroad Company was organized, Mr. Williams continued in this capacity on all the vicissitudes of the Eel River Canyon construction in California. His brilliant handling of this particularly difficult piece of construction work earned for him the promotion to chief engineer of the Northwestern Pacific Railroad Company in July, 1914.

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[1] Memoir prepared by H. M. Smitten, M. Am. Soc. C. E.

Mr. Williams was selected for and accepted a commission as Major in the 18th Engineers, United States Army, in May, 1917. He arrived at American Lake, Wash., on June 19, 1917. After a period of about a month, during which time the regiment was thoroughly organized and placed on a war basis, it left American Lake for Hoboken, N. J. On August 23, 1917, the regiment sailed for France, one of the first of the United States armed forces bound for foreign parts. On arrival at Le Havre, France, on August 29, 1917, work was quickly begun on the railroad yards at San Sulpice and Bassens and later on the Bassens docks. Major Williams played a prominent part in the construction of these projects and later, when Col. J. B. Cavanaugh was called away, was promoted to Lieutenant Colonel of the regiment.

Colonel Williams returned to the United States in July, 1919, and resumed his position as chief engineer of the Northwestern Pacific Railroad Company. He retained this position until August, 1921, when he was called to the Western Pacific Railroad Company as its chief engineer. The latter position he held until his death in February, 1941.

Colonel Williams held a commission in the Engineer Reserve, U. S. Army, from 1920 until his death. He was a charter member of the Society of American Military Engineers, and was president and director of the San Francisco Post. He was also president of the San Francisco unit of the Association of the Army of the United States and of the Army and Navy Club of San Francisco.

In 1902 he was married to Eliza Hunt who, with a son, Joseph H., and a daughter, Mary (Mrs. J. F. Loughery), survives him.

Colonel Williams was elected a Member of the American Society of Civil Engineers on January 13, 1930.

RONALD BLAIR WILLS, M. Am. Soc. C. E.[1]

DIED OCTOBER 31, 1941

Ronald Blair Wills was born on September 8, 1896, in Calusa, Calif., the son of James and Katherine (Blair) Wills. The family moved to Evansville,

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[1] Memoir prepared by F. W. Epps, M. Am. Soc. C. E., and Ross C. Keeling and W. S. McDaniel, Assoc. Members, Am. Soc. C. E.

Ind., in October, 1896. Mr. Wills attended grade school and high school at Evansville, and after graduation from high school entered the Missouri School of Mines and Metallurgy at Rolla, Mo., enrolling in the course in civil engineering. He was graduated from this school in 1920 with the degree of Bachelor of Science in Engineering.

At the end of his junior year, in 1919, he was employed for the summer as draftsman with the Southwestern Engineering Company at Hutchinson, Kans. After graduation in 1920, he went back to Hutchinson and was employed by Reno County, Kansas, as draftsman from May to August, 1920. He worked on highway plans for federal-aid paving projects in that county.

In August, 1920, he moved to Garden City, Kans., and from that date to April, 1922, was employed by Finney County, Kansas, as office engineer and field engineer on federal-aid grading and paving projects throughout that county. In April, 1922, he was given the position of resident engineer on the various highway projects under construction in that county, continuing in this position until March, 1923. At that time he moved to Wellington, Kans., and was given the position of field engineer and office engineer for Sumner County, Kansas. This involved the designing of highway projects in that county and the field work connected with their construction.

In April, 1924, he was given the position of resident engineer on highway paving projects in Jefferson County, Kansas. During this period he resided at Oskaloosa, Kans. The highway paving consisted of concrete, brick, and bituminous macadam.

The period from 1920 to 1925 marked the start of highway construction in the State of Kansas in the various counties in which he worked, and the projects on which he was employed covered many of the first federal-aid highway projects constructed in Kansas. They marked the beginning of highway construction in this state. The highway laws of Kansas were such that federal-aid highway projects were initiated by the counties, the counties making their own plans and letting the contracts. The work was done under the supervision of what was then the Bureau of Public Roads and a skeleton organization of the State Highway Commission of Kansas. In May, 1925, the State of Kansas was divided into six highway divisions, and, because of Mr. Wills' wide experience throughout the beginning of highway construction in Kansas, he was sent to Hutchinson as division engineer in the Fifth Highway Division, with supervision over highway work in eighteen counties. He continued in this capacity until April, 1934, having an outstanding record in this position. During the time he was division engineer with the Kansas Highway Commission, the state, through a constitutional amendment, took over complete control of all the state highways, thereby relieving the counties of their connection with highway construction and maintenance on the state highway system. Mr. Wills took an active part in solving the problems of the organization of the State Highway Commission during this period of change from the county system of control of highways to full control by the state, and his organizing ability had much to do with the final organization of the State Highway Commission of Kansas.

In April, 1934, Mr. Wills was transferred from his position as division engineer to the headquarters office at Topeka, Kans., and was given the position of assistant maintenance engineer. This was during the early period of bituminous road construction in Kansas, and he was brought into headquarters because of his original work in the construction and maintenance of this type of road, the first project of this type having been constructed under his supervision while he was division engineer.

In August, 1934, he was given the position of engineer of construction and served in that capacity until January, 1936. At this time he was given the position of engineer of highway planning and organized this department, continuing in that capacity until May, 1937, at which time he became engineer of design.

Mr. Wills was appointed state highway engineer in April, 1939, and continued in that capacity until his death. Because of his wide experience in the field and in the various headquarters departments of the State Highway Commission, he was especially well qualified for the position of state highway engineer. His professional career was an example of devotion to public work. He took an active part in the meetings of the American Association of State Highway Officials, the Mississippi Valley Conference of State Highway Departments, and local state highway meetings. During the time he served as state highway engineer, much was accomplished by the Department in the establishment of better public relations, due principally to his character and reputation as an engineer and to his personality and outstanding ability to make friends.

He was a member of the Kansas Engineering Society and the Masonic Lodge, and was a trustee of the Westminster Presbyterian Church of Topeka.

Mr. Wills was married on May 25, 1923, to Kathryn Brown, who, with two daughters, Sara Frances and Shirley Ann, and his mother, survives him.

Mr. Wills was elected an Associate Member of the American Society of Civil Engineers on April 23, 1928, and a Member on December 13, 1937.

WALTER SCOTT WINN, M. Am. Soc. C. E.[1]

Died May 31, 1941

Walter Scott Winn was born near Winchester, Ky., on May 15, 1869, the son of Joshua Nicholas and Ellen (Poston) Winn. His early education was received in schools at Winchester and at Kentucky University, at Lexington, Ky., where he was a student in the civil engineering course for two years. Immediately after departing from the university, he was associated for two years (until June, 1889) with Thompson, Hall, and Bosler, Consulting Engineers, at Chattanooga, Tenn. During this time he developed his engineering education by constant study, reading, and extensive and varied experience. This was further amplified by continuation of his studies at Lehigh University, at Bethlehem, Pa., which he attended from 1889 to 1891.

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[1] Memoir prepared by Lee G. Warren, M. Am. Soc. C. E.

In May, 1891, he joined the civilian staff of the U. S. Engineer Corps, with which he identified himself, except for brief periods, until July, 1918. Starting as a levelman on the Tennessee River, he was employed by the Chattanooga (Tenn.) Engineer District under the late Col. George W. Goethals,[2] M. Am. Soc. C. E., who became famous as the builder of the Panama Canal. He rose to assistant engineer in 1894. He continued in that capacity until 1909. In these various assignments he was in charge of dredging operations and open channel improvements (1894 to 1898), and construction of Colbert Shoals Lock on the Tennessee River (1898 to 1901). From 1901 to 1909, his work covered the operation and maintenance of Muscle Shoals Canal, and design, construction, and repair of construction equipment and machinery for improvement of the Tennessee and Cumberland rivers.

In an intermission from December, 1909, to September, 1911, Mr. Winn was general manager of the North Alabama Construction Company, engaged upon construction of irrigation, drainage, and lock and dam projects. From the latter date, following his first period of private practice, and ending February, 1912, he was employed by the Alabama Power Company as chief of surveys on investigation of power sites, which concluded in May, 1913. From 1920 to 1921 he was engaged in private engineering practice in Chattanooga.

In May, 1913, he returned to the Engineer Corps, as principal assistant engineer for the Tennessee River, continuing in that assignment until 1917. He had charge of construction on the river and of surveys, investigations, and preliminary plans for combined scheme for navigation improvement and power development at Muscle Shoals, Ala. From the middle of 1917 to July 31, 1918, he held the appointment of district engineer for the Chattanooga and Nashville U. S. Engineer District. In 1918, he was appointed disbursing officer and assistant engineer, Florence District, U. S. Engineer Corps, on the construction of Wilson Dam, in Alabama.

He was engaged in almost every phase of river work, but particularly in the extensive operations of open channel improvement by dikes and dredging. One of the outstanding accomplishments of Mr. Winn was during the period of 1915 and 1916 when, as assistant to the U. S. Engineer, he prepared the report of the Tennessee River between Browns Island, Ohio, and Florence, Ala., which included the projects of Dams No. 1 and No. 2 (Wilson Dam) and Dam No. 3 (Wheeler Dam). This is House Document No. 1262 and the basic survey and report for the improvement which has since been completed in the Muscle Shoals section.

In August, 1918, he became associated for a year with the Air Nitrate Corporation, on what was probably one of his most important jobs, as works engineer and camp supervisor at U. S. Nitrate Plant No. 2, at Sheffield, Ala. More than 20,000 employees were under his supervision.

In July, 1919, he became general manager of the Southland Exploration Company, mining and milling zinc in eastern Tennessee. He continued in that position until July, 1921, when he was appointed assistant to the chief engineer of the Alabama Power Company, carrying out that assignment until April, 1929. Here he supervised all inspection of concrete on the construction of Mitchell Dam, in Alabama, and other large projects.

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[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1813.

He again became associated with the U. S. Engineer Corps of the War Department in September, 1929, continuing in that association until March, 1940. He was assigned, in succession, to the Cincinnati (Ohio) and Huntington (W. Va.) Districts. He had charge of various studies and investigations on the Ohio River and several tributaries. The official assignment was supervising the "Studies and Reports Section of the Cincinnati Division."

Walter Scott Winn was always called the "Major" by his friends and close associates. He was a Major in the Engineer Reserves, but the title was really coupled with great respect and feeling of endearment by those who knew him well. He was always affable. His personality was attractive and his bearing dignified. Major Winn was deeply considerate toward his junior associates, being ever ready to assist them in their problems. He had a splendid character with the mark of the "old school." Major Winn's experience was particularly wide and long in river improvements. He had several assignments which were very intensive; for example, his work at Mitchell Dam. The details of his experience records describe the scope of his work.

Major Winn was married to Lena McDermott on September 11, 1894. He is survived by a daughter, Martha (Mrs. C. E. Parker), and two sons, Herbert P. Winn, Assoc. M. Am. Soc. C. E., and Lt.-Col. W. S. Winn, Jr.

Major Winn was elected an Associate Member of the American Society of Civil Engineers on October 6, 1897, and a Member on October 1, 1926.

ALBERT JOSEPH WISE, M. Am. Soc. C. E.[1]

DIED FEBRUARY 3, 1942

Albert Joseph Wise was born at Suffield, Ohio, on December 18, 1871, the son of Bernard Wise and Rosina (Knapp) Wise. He was educated in the public schools of Suffield, attended St. Josephs Preparatory School at Ada, Ohio, and was graduated in 1898 from Ohio State University, at Columbus, with a degree of Civil Engineer. After graduation he was employed as roadmaster on the Baltimore and Ohio Railroad.

In 1902 he came to Texas on a vacation trip, and liked the state so well that he went back north only to resign. He returned to Texas that same year, settling in Houston, where he remained until his death.

After several jobs, including one with the Dayton Goose Creek Railway Company, he formed a partnership in 1909 with the late Joseph Milton Howe,[2]

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[1] Memoir prepared by Charles R. Haile, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 1569.

M. Am. Soc. C. E. The firm of Howe and Wise had a prominent place in the building of Houston and the great Southwest.

They served Harris County as county engineers from 1911 to 1914 and from 1917 to 1929, and again as consultants on bond-issue work in 1935 and 1936. The excellent network of roads and bridges in Harris County stand today as a monument to this firm.

An excerpt from an editorial of a Houston newspaper expresses the high regard in which Mr. Wise was held in his community.

> "In 1909 he formed a partnership with the late J. Milton Howe who died in 1937. The partnership was so successful that most Houstonians immediately think of the firm when either name is mentioned.
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> "The firm served Harris County as County Engineers for many years. Their work was so noteworthy, particularly their highway construction methods, that they became nationally famous. Two new methods of road construction developed in Harris County roads by the partners were adopted as standard practice all over the country.
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> "Besides their outstanding engineering achievements, the partners were respected for their professional integrity and high standard of ethics. During their many years of service as County Engineers, they successfully resisted all forms of political pressure and maintained a notable reputation for honor and skill."

This high standard of ethics and his pride in the engineering profession is well expressed in an interview he gave, published in the *Houston Magazine:*

> "To analyze and search for the truth is the function of the engineer. In the measure of his mental equipment and training for analysis of the natural resources and conditions in relationship to the present and future material needs of his fellow man, will he be able to set down the true facts for the guidance of others. Essentially, the engineer is a pathfinder in human progress."

It can be said truly that A. J. Wise has left his mark in his community, both in the engineering works he planned, designed, and constructed and in the inspiring memory of his christian character, high ideals, and friendly and wise counsel, which lies deep in the hearts of his friends, colleagues, and the young engineers whom he guided.

A. J. Wise always took an active part in the civic and social life of his community and is well known and highly regarded as a citizen.

He was a charter member of the Houston Engineers Club, serving as one of its first directors and later as President. He was a member of the Knights of Columbus, Holy Name Society of Holy Rosary Church, and, until his health forced his resignation, a member of Houston Rotary.

Mr. Wise was married on September 30, 1914, to May Cushman, who survives him.

Mr. Wise was elected a Member of the American Society of Civil Engineers on September 3, 1912. He became a Life Member in 1942.

GEORGE PILLSBURY WOOD, M. Am. Soc. C. E.[1]

DIED OCTOBER 15, 1940

George Pillsbury Wood was born in Lebanon, N. H., on March 2, 1866. His father, James Wood, and his mother, Rebecca (Pillsbury) Wood, both natives of New Hampshire, were representatives of that energetic, clear-thinking, and public-spirited stock that forged New England into a political and industrial power far beyond its natural advantages.

The common school, the town meeting, the unfailing industry, and the all-round mechanical skill required to wrest a living and an education from a New England hill farm found their reflection in a notable engineering mind and in a man whose character won the love and respect of all his associates.

Beyond his common school education, Mr. Wood had only part of one year in attendance at Kimball Union Academy at Meriden, N. H., before entering the New Hampshire College of Agriculture and Mechanic Arts, then a department of Dartmouth College at Hanover, N. H. He was graduated first in his class with the degree of Bachelor of Arts in 1886. In 1888 he entered the Thayer School of Engineering of Dartmouth College for a two-year graduate course, which he did not finish. He left college at the end of the first year to undertake the practical work of life. Suffice it to record that in 1903 Dartmouth College conferred upon Mr. Wood the degree of Civil Engineer for distinguished services already performed, and in doing so honored itself in recognizing the worth of the engineer and the man.

Mr. Wood entered the drafting room of the King Bridge Company at Cleveland, Ohio, in the spring of 1889 and three months later took service with the late Gustav Lindenthal,[2] Hon. M. Am. Soc. C. E., as rodman on a survey contemplating the development of the then Maryland Central Railroad into a coal line from the anthracite coal fields of Pennsylvania to the City of Baltimore, Md. The funds for this project failed in the latter part of the year 1889, and in November of that year Mr. Wood went to Roanoke, Va., where he found immediate employment with William M. Dunlap, city engineer, under whose direction he served for four years.

His first assignment was the topographical survey and construction of a real estate project at Craig City, Va. Although this undertaking was in advance of the requirements of the region, the experience served to impress upon Mr. Wood the needs of comprehensive municipal planning.

Returning to Roanoke in 1891, still with Mr. Dunlap, Mr. Wood was called upon to prepare plans for a survey of the city that were unique in two particulars. First, Mr. Wood prepared an ordinance for the city council which not only provided for the survey and its expense, but also gave authority to the results of the survey in the location of streets and property lines, together with means of adjusting claims which might arise later. This ordinance was adopted as submitted, and the procedure has been justified by its results. Second, Mr. Wood prepared the plans for a precise triangulation survey, which gave a base for accurate traverse surveys upon which were hung not only the street and property limits, but also the topographical survey for the locations of city extensions with their required drainage systems. All positions of this survey were expressed in rectangular coordinates based on a central meridian established by careful observations on Polaris, with triangulation and traverse points practically located and permanently fixed.

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[1] Memoir prepared by George P. Bard, M. Am. Soc. C. E., in collaboration with Robert A. MacGregor, M. Am. Soc. C. E., and C. H. Rice, Research Engr. (*Retired*), New York Central R. R. Co., Hillsdale, N. Y.

[2] For Memoir, see *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1790.

This was the work of an engineer two years out of college who, with few precedents, developed plans that should be standard practice and put the entire project into effect. A description of this survey with its enabling ordinances was sent, in 1891, to the *Engineering News* and was received with such appreciation by the late George H. Frost,[3] Assoc. M. Am. Soc. C. E., then editor, that an advance article was printed and mailed to five hundred city engineers in the United States as a procedure worthy of thorough study by the profession. Before this survey was complete, Mr. Wood read a paper before the Association of Engineers of Virginia, of which he was a charter member, describing the methods and results of the triangulation and traverse surveys. The paper is still good reading for any engineer planning similar work.

The foundation of the Roanoke survey was practically completed in the early part of 1893 when Mr. Wood went to Baltimore where a similar survey was being contemplated. Here he submitted such a comprehensive plan for the undertaking, backed up by the experience and results attained at Roanoke, that he was at once placed in charge of the triangulation, precise traverse work, and property surveys by the chief engineer of The Topographical Survey of Baltimore, Col. H. T. Douglas. After eighteen months Mr. Wood was placed in charge of the entire field work, including street extensions, plans for annexed areas, and map publication. During this time Colonel Douglas submitted to the late Samuel M. Gray,[4] M. Am. Soc. C. E., of Providence, R. I., a notable engineer of his time, some of the maps produced from this survey and received word from Mr. Gray that they were the finest that he had ever seen.

Aside from the precision, thoroughness, and scope of this survey, there was a unique feature that has a distinct bearing upon municipal and state surveys now being undertaken—namely, that it was then thought desirable to tie in the city triangulation to a Coast and Geodetic Secondary, and this was properly done. However, the traverse surveys made with accurately compared tapes within minute limits of error demonstrated that somewhere an error existed, and it became necessary to measure two base lines which were found to be accurate with a minute probable error comparable to the most accurate United States Geodetic Survey work. The Coast Survey Secondary Triangulation to which the Baltimore survey was tied was readjusted accordingly. There was therefore in this survey, in addition to precise triangulation and traverse work, a definite tie-in with the fundamental government triangulation.

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[3] For memoir, see *Transactions*, Vol. LXXXI, December, 1917, p. 1827.

[4] For memoir, *ibid.*, Vol. LXXXV (1922), p. 1679.

In succeeding years the survey maps, notes, and records have been of continuing value not only in adjusting property boundaries, but in providing an accurate basis for municipal additions and unexcelled maps for drainage and sanitary planning. The entire system was and is expressed by rectangular coordinates centering on the head of the statue on the Washington Monument, center of Baltimore City, and after fifty years the survey is the accepted Baltimore standard.

Work on the Baltimore survey was discontinued in April, 1897, when Mr. Wood entered the service of the District of Columbia in the development of street and sewer additions. In July of the following year he was employed by the Bureau of Yards and Docks of the United States Navy Department as draftsman, where he was rapidly advanced to the position of chief draftsman, serving under the late Admiral Mordecai T. Endicott,[5] Past-President, Am. Soc. C. E., and the late Admiral H. H. Rousseau,[6] M. Am. Soc. C. E., with a distinction that is verified by a correspondence that attests his sound judgment and practical capacity.

In 1906 the late J. Waldo Smith,[7] Hon. M. Am. Soc. C. E., was preparing plans for the first Catskill Aqueduct. Since this project offered a wider scope for his engineering career, Mr. Wood resigned from his position with the Bureau of Yards and Docks and entered the service of the Board of Water Supply of New York, N. Y., remaining with that organization to the completion of the Aqueduct in 1917.

His first assignment was an investigation of the possible use of the Peekskill Creek watershed as a temporary source of water for New York City, and this report still stands as an example of thorough investigation followed by logical and practical deductions. When construction of the Aqueduct began the following year, Mr. Wood was assigned to the Peekskill Division as assistant engineer. Later he was advanced to senior section engineer, and in 1911 he was made division engineer of the Peekskill Division, where he remained to the completion of the enterprise. During this time the contractor on this Division failed, and the sureties for completion then divided the work into four contracts, entailing upon the engineer not only the technical attention required for a large and difficult piece of construction, but also the use of mature judgment and diplomatic handling of the various contracting interests. All of the work was completed successfully. The work on this Division included not only the conventional cut-and-cover construction, but a number of tunnels and steel siphons with the attendant details for handling water transportation.

The Aqueduct finished, Mr. Wood resigned from the service of the Board of Water Supply and for the twelve succeeding years devoted himself to private business, to the development of patents applying to concrete construction, and to agriculture. At the same time he took an active part in the town government of Peekskill, N. Y., where he lived until his death. Good citizenship ranked with good engineering in Mr. Wood's estimation, and his associates still bear witness to this trait of his character.

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[5] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1051.

[6] For memoir, *ibid.*, Vol. 95 (1931), p. 1597.

[7] For memoir, *ibid.*, Vol. 101 (1936), p. 1502.

In the year 1929 the town of Peekskill found that its water system needed improvement and employed Mr. Wood to rebuild the pumping plant and extend the distribution facilities. At the expiration of a year he was appointed superintendent of the Peekskill Water Board, serving until his death. During this time he extended the storage and elaborated the filter and treatment systems and so conducted the management of the water property that it became more than self-supporting and is now (1942) rapidly liquidating the obligations incurred by construction and maintenance. At the same time the work of maintenance was so planned that during the "depression" it was made to employ many that otherwise would have been dependent on charity.

Mr. Wood's last work was the extension of the Wickopee watershed and the design and building of the second Wickopee Dam, completed in the summer of 1940. The extension of this water supply was Mr. Wood's own conception and the dam built on his own design now properly bears his name, a memorial from the municipality which he served so faithfully.

In September, 1940, Mr. Wood prepared a paper[8] descriptive of the Peekskill water system, especially related to the Wickopee watershed and the Wickopee Dam. He submitted copies to the Water Board at its regular meeting on the night of October 14, 1940, and on presentation of this last record of his work he was stricken with a cerebral lesion. He was taken home by his associates and passed to his reward on the following day.

Mr. Wood's library bears evidence of unremitting study during his entire career, the record of his work shows sound judgment in the application of his reading and research, his management of every duty assigned to him was admirable, and his character was above reproach. He was considerate in all his dealings. To some he seemed reticent but he had a courage of his convictions that enforced quality and fairness in all he undertook. His seeming diffidence was never due to any lack of confidence in his own conclusions. He wished to look at all matters from all sides; then his decisions stood.

Such is the record of a wise and understanding engineer, an upright public servant, a public-spirited citizen, and an unfailing friend. The following is the characterization of one who was associated with him in his early enterprises: "Personally, I consider George P. Wood as one of the best men I have ever known. He was utterly incapable of any meanness. I am proud to have had his friendship for fifty years."

Mr. Wood was married to Mary Ridout, of Roanoke, on April 12, 1894, and is survived by Mrs. Wood and by two sons, Maj. James Wood, U. S. Marine Corps, and Winchester Wood. A daughter, Mary, died on October 10, 1913.

He was a charter member of the Association of Engineers of Virginia, the University Club of Washington, D. C., and the University Club of Peekskill; and a member of the Thayer Society of Dartmouth College.

Mr. Wood was elected a Member of the American Society of Civil Engineers on June 6, 1911.

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[8] *Water Works Engineering*, September, 1940.

KENNETH CRAWFORD WRIGHT, M. Am. Soc. C. E.[1]

Died December 21, 1940

Kenneth Crawford Wright was born in Brigham, Utah, on November 11, 1891, the son of Cornelius and Frances (Crawford) Wright, highly respected citizens of northern Utah.

He received his early education in the public schools of his home city, and attended the University of Utah, in Salt Lake City, receiving the degree of Bachelor of Science in Civil Engineering on June 2, 1915. On June 9, 1931, he received the degree of Civil Engineer from the same university.

Mr. Wright was essentially a highway engineer, as nearly all his professional work until March, 1937, was in that field. For the Utah State Road Commission he served as bridge designer, road agent in charge of maintenance, resident engineer on construction on numerous projects, district engineer, assistant chief engineer from April, 1928, finally becoming chief engineer for the Commission on March 1, 1933. He continued in this capacity until March, 1937, when he resigned. He then served as manager of the industrial division of the John Lang Company until his death.

The John Lang Company for a long time has been a leader in its field. It was begun many years ago as a one-man welding enterprise and rapidly expanded to include manufacturing and selling of a very complete line of heavy equipment, the greater part of which is used in heavy engineering construction. This industrial division dealt chiefly with tractors, bulldozers, power shovels, motor graders and carriers, and since Mr. Wright's work of a great many years formed an excellent background for the selection and use of such equipment it was natural that John Lang should engage him for this important post of manager. He filled the position for nearly four years, with credit to himself and satisfaction to his company.

In March, 1935, the Utah legislature enacted the Professional Engineers and Land Surveyors law, patterned closely after the Society's Model Law for the Registration of Professional Engineers and Land Surveyors. Mr. Wright took an active part in advocating this and securing its passage, and in fitting recognition of that he was given license No. 1. He was chairman of the board of seven engineering examiners, provided for in the law, from the time it was formed until his death, and he rendered a real service to the profession in that capacity.

He was a man of real character; he was public spirited, and bore the respect of his fellow engineers, his associates, and citizens generally. His decisions were reached after careful consideration of all pertinent facts; and when so determined he was tenacious of his opinions, but did not hesitate to modify them should that be justified.

Mr. Wright was president of the Utah Section of the Society in 1936 and Secretary of the Western Association of State Highway Officials in 1937.

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[1] Memoir prepared by R. K. Brown and H. S. Kerr, Members, Am. Soc. C. E.

He was married to Grace McMaster on September 11, 1916. They had four sons—exceptionally fine boys. The two eldest were attending the University of Utah at the time that their father died. Shortly after Mr. Wright's passing, their second son was killed by a snowslide while skiing—another sad bereavement for Mrs. Wright and her three remaining sons.

Mr. Wright was elected an Associate Member of the American Society of Civil Engineers on June 16, 1924, and a Member on January 14, 1929.

CARL LINCOLN BANNISTER, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 14, 1940

Carl Lincoln Bannister was born in Bergen, N. Y., on November 23, 1865, the son of Luther and Mary (Kelsey) Bannister. During his early childhood the family moved to Le Roy, N. Y., and Mr. Bannister received his primary education at the neighborhood district school. Later, he attended Le Roy Academy from which he was graduated. He then returned as teacher of the district school he had once attended. During his spare time he worked for some of the neighboring farmers until the fall of 1891, when he entered Union College, at Schenectady, N. Y. He was graduated from Union College in 1895 with the degree of Bachelor of Engineering. After his graduation, he spent the balance of that year in Schenectady, as rodman on city water works construction and as draftsman with the General Electric Company.

On January 13, 1896, he received an appointment as rodman in the New York State Department of State Engineer and Surveyor. This period of service with the state covered about two years, the last year being under the title of leveler.

From September 18, 1898, to July, 1899, Mr. Bannister was employed as draftsman and leveler with the U. S. Deep Waterway Commission. In August, 1899, he entered the employ of the U. S. Engineering Corps, where he remained until June, 1904, when he returned to the New York State Department of State Engineer and Surveyor as assistant engineer. His active work terminated upon his retirement from state service on July 31, 1932. Following his retirement Mr. Bannister resided in Bergen. His death occurred in St. Petersburg, Fla., where he had been spending his winters.

Except for a few months of miscellaneous engineering and short periods of highway surveys and construction, Mr. Bannister's work was centered on waterways. In 1897, he had charge of contracts covering 8 miles of construction for canal improvement for New York State. His work with the Deep Waterway Commission included stadia surveys in the studies of a water supply for a possible ship-canal connection between the Great Lakes and the Atlantic seaboard. Work with the U. S. Corps of Engineers was principally on river and harbor improvement for which he made surveys and inspected construction. His assignments included work on concrete and timber pier construction and dredging at Erie, Pa.; stone breakwater construction at Buffalo, N. Y.; surveys of harbors on Lake Ontario; and river and harbor improvements at Savannah, Ga.

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[1] Memoir prepared by Leroy C. Hulburd, Assistant Commissioner of Canals of New York State, Albany, N. Y.

In 1904 the New York State Engineer and Surveyor was building an organization for the improvement of the State Canal System, which improvement had been authorized in the previous year. This was one of the major waterway projects undertaken to that time, and many engineers were attracted to the work. Mr. Bannister's ability was recognized from his previous work with the State Engineer and Surveyor, and he was immediately assigned to make preliminary surveys and estimates and to prepare plans for the project. One of his assignments was a survey and report on a proposed extension of the project to include the Chemung (N. Y.) Canal. Although Mr. Bannister conducted surveys and supervised some of the construction on the Improved Canal System, his work was confined principally to office supervision over the preparation of plans and estimates. When that work was brought to a close, his assignment was transferred to the Division of Highways. His employment in that division continued until he retired.

The integrity and loyalty of Mr. Bannister were recognized by all of his associates. His interest in the successful completion of any work under his supervision overshadowed any personal considerations or desires, and the faithful administration of each duty assigned to him contributed to efficient, honest public work.

Mr. Bannister never married. He is survived by two sisters, Cordelia Grace (Mrs. Almer C. Seward), and Mary Charlotte (Mrs. Shelton E. Crittenden).

Mr. Bannister was elected an Associate Member of the American Society of Civil Engineers on May 3, 1905.

EDWARD MORRIS BASSETT, Assoc. M. Am. Soc. C. E.[1]

Died August 2, 1941

Edward Morris Bassett was born in Salem, N. J., on March 13, 1883, the son of Edward Hicks Bassett and Emma (Hall) Bassett. He prepared for college at the public high school in Salem and the Swarthmore Preparatory School, entering Swarthmore College, at Swarthmore, Pa., in the fall of 1900. He left college after his sophomore year to spend the year 1902–1903 with the United Gas Improvement Company as sheet clerk in the Department of Records, where his work consisted largely of surveying and recording changes in the street mains. In March, 1903, he went to Gary, W. Va., as rodman for the United States Coal and Coke Company, working on topographical surveys, layout, and inspection of the construction of coke ovens.

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[1] Memoir prepared by Scott B. Lilly, M. Am. Soc. C. E.

In the fall of 1903 he returned to Swarthmore College, receiving his degree of Bachelor of Science in Engineering with the class of 1905. He showed his versatility by becoming editor of the *Phoenix*, the college newspaper, at that time a monthly; leader of the mandolin club; and member of the football team, a most unusual combination for an engineer. He received the degree of Civil Engineer from Swarthmore College in 1911.

His first job after graduation was with the Pennsylvania Railroad Company. He stayed with it for only one year, during which time he worked upon changes in line in connection with the Union Station then being built in Washington, D. C. He resigned from the Pennsylvania Railroad Company in 1906 to take a position with Ransome and Smith Company, General Contractors, in New York, N. Y., and began his career in building construction. This company had many contracts for concrete buildings in New York State and Connecticut.

He returned to Philadelphia, Pa., in 1907 to become engineer of construction with Doyle and Doak, later Doyle and Company. He remained with this company until 1916, during which time it built the Bulletin Building, the Curtis Building, the Penn Mutual Building, the Hotel Vendig, and many other important structures in downtown Philadelphia.

From 1916 to 1918 he was with Warren, Moore and Company, again on building construction, the most notable contracts being with the United States Government for the foundry building at League Island, Pa., the projectile plant at Charleston, W. Va., and structures in the Washington (D. C.) Navy Yard.

From 1918 until his death he was vice-president and general superintendent of Wark and Company. With this company he continued to build notable buildings in Philadelphia and elsewhere. The following list is chosen to show the different types of construction in which Wark and Company engaged, all of the buildings being in Philadelphia: Babies Hospital, 7th and Delancey streets; Dispensary Building, Presbyterian Hospital; 18-story Samuel Gustine Thompson Annex, Jefferson Hospital; Western Union office, 11th and Locust streets; office buildings at 1600 Walnut Street, 1608–14 Walnut Street, and 1916–22 Walnut Street; and Architects Building, 17th and Sansom streets. Outside of Philadelphia the company built large manufacturing buildings for E. G. Budd and Company and the Viscose Company, and many large structures for the United States Government. Readers who are familiar with Philadelphia will realize what a large part Mr. Bassett had in changing the sky line of that city.

During the time that Mr. Bassett was general superintendent, Wark and Company used plywood for concrete formwork in the construction of the United States Marine Corps Building, 15th and Washington Avenue, Philadelphia. It is believed that this is the first time that it was used for this purpose in the Philadelphia area. Wark and Company were also pioneers in using adjustable shores for reinforced-concrete building construction. These were used for the first time in the Philadelphia area in connection with the Gulf Oil Company office building in 1924.

Mr. Bassett's success was due not only to his engineering knowledge but also in a marked degree to his ability to encourage men to work together. From his early days with Doyle and Doak, when he was charged with seeing that each subcontractor maintained his schedule, until the time of his death, when he had under his supervision construction worth millions of dollars, Mr. Bassett succeeded in getting a close coordination among all the firms and individuals involved. This resulted in buildings being built quickly and well.

This ability to lead men is shown throughout Mr. Bassett's career. In college he was a member of Phi Kappa Psi fraternity. In later life he was elected treasurer of the national Phi Kappa Psi in 1926, again in 1928, and later was president of this fraternity. He was a member of Swarthmore Borough Council and later Burgess. He was president of the Swarthmore Building Association, and for many years was a member of the Board of Managers of Swarthmore College. All of these items testify to the regard and confidence of his associates. The pleasure that Mr. Bassett derived from association with his fellowmen is shown by the fact that he was a member of the Union League of Philadelphia, the Penn Athletic Club, and the Rolling Green Golf Club.

Mr. Bassett is survived by his widow, Ellie Simons Bassett, and his children: Kathryn, Elizabeth, and E. Morris, Jr.

Mr. Bassett was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916.

ADAM CARL BEYER, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 30, 1941

Adam Carl Beyer was born in St. Paul, Minn., on June 20, 1901, the son of Adam Carl Beyer and Mina (Butler) Beyer. His father died before he was born. At an early age he moved to California with his mother and completed his education at the University of California, in Berkeley, from which he was graduated in 1925 with the degree of Bachelor of Science in Civil Engineering. He was an outstanding student leader in college, was elected to many honor societies (including Phi Beta Kappa, Tau Beta Pi, and Sigma Xi), and was president of the student body.

Following his graduation, he accepted several professional engagements with the California Highway Commission, the Southern Pacific Railroad Company for the City of San Rafael, Calif., and the Pelton Water Wheel Company. In February, 1929, he entered the employ of the Wallace and Tiernan Company in San Francisco, Calif., as sanitary engineer and district manager. He remained in the employ of this company until his death. He was a specialist in the field of sanitary engineering equipment, designing and constructing many important installations throughout the West.

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[1] Memoir prepared by Gerald E. Arnold, M. Am. Soc. C. E.

Mr. Beyer was a member of numerous technical and professional societies and served as an officer in most of them. He was the representative of the San Francisco Section of the Society on the San Francisco Engineering Council. He was a director of the San Francisco Junior Chamber of Commerce.

In May, 1932, he was married to Mary Katherine Barry. He is survived by his widow and by two children—a daughter, Mary Ellen, and a son, Barry Michael.

His was an outstanding life of service and fellowship to his friends and associates. He was known by many, and all who knew him loved him.

Mr. Beyer was elected a Junior of the American Society of Civil Engineers on December 14, 1925, and an Associate Member on February 24, 1931.

JOHN CANTLEY, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1941

John Cantley was born in Nicetown, Philadelphia, Pa., on December 9, 1874, the son of Andrew and Mary Ann (Beatty) Cantley.

As a boy he attended the public schools of Philadelphia in the vicinity of East Falls, later enrolling in the mechanical engineering course in the evening school of Temple University, in Philadelphia.

In December, 1891, at the age of sixteen, he entered the service of the Pencoyd Iron Works (later merged with the American Bridge Company) as a structural draftsman. While in the employ of the Pencoyd Iron Works, he furthered his engineering education by diligently attending a special training course which the company offered to the ambitious young men in its employ.

By 1903, Mr. Cantley had gained sufficient recognition to be placed in charge of a squad of engineers and draftsmen engaged in the detailing of numerous buildings, such as the New York, N. Y., and Philadelphia stores of John Wanamaker, and the Hudson Terminal and Pennsylvania Railroad Station in New York.

In 1901, Mr. Cantley, having been granted a short leave of absence, assisted Sax and Abbott, Consulting Engineers of Philadelphia, in the preparation of plans and details for the Pennsylvania Arcade and Union League Buildings in Philadelphia. Again in 1905, on a short leave of absence, he assisted the American Bridge Company office in the design in the Sonneborn Building, Baltimore, Md.—a structure of steel and concrete.

In 1909, Mr. Cantley became associated with Steel and White Company, Iron Works, in full charge of all engineering work on many well-known department stores, churches, and hospitals in the vicinity of Philadelphia.

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[1] Memoir prepared by Samuel Bernhang, Assoc. M. Am. Soc. C. E.

In April, 1913, Mr. Cantley entered the service of the City of Philadelphia as an engineer in the Bureau of Building Inspection. His work included the examination of plans for all types of buildings and structures.

Mr. Cantley was known throughout Philadelphia as the outstanding designer of structural steel and as an expert on theater design, being of great assistance to numerous architects with advice and suggestions.

On October 6, 1895, he was married to Mary Jane Clayton. He is survived by his widow and three children, Walter John, Herbert Watson, and Mrs. J. Franklin Wooler. He was a devoted husband and father and to his many friends and admirers his sincerity and kindness were a constant source of encouragement.

Deeply interested in his profession, he was always ready with advice and encouragement to help the young architect and engineer along the path of the profession.

Mr. Cantley was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916.

HARTLEY ROBERT CHURCH, Assoc. M. Am. Soc. C. E.[1]

DIED APRIL 21, 1941

Hartley Robert Church was born in Remington, Ind., on November 13, 1876, the son of Osman W. and Emroy Jane (Lyon) Church. He was graduated from Earlham College at Richmond, Ind., in 1896 with the degree of Bachelor of Science.

From 1901 to 1905 he served with the Western Division of the Atchison, Topeka and Santa Fe Railway Company in various capacities from chainman to assistant engineer.

For the next seventeen years he was connected with various private engineering and construction firms engaged in the design and construction of sewage systems and sewage disposal works, irrigation systems, city paving, etc. During this period he was connected with the following organizations in California: Stephen E. Kieffer, M. Am. Soc. C. E., Berkeley; Haviland and Tibbetts, San Francisco; Church and Beugler, Oakland; the late Philip Schuyler,[2] M. Am. Soc. C. E., Oakland; City of Berkeley; Interstate Commerce Commission; Fair Oaks Irrigation District; Southern Pacific Railroad Company; and the Cosumnes Irrigation District.

He entered the service of the California Division of Highways in February, 1922, and remained in this service until his death, advancing from the junior engineer grade to that of associate highway engineer with the San Luis Obispo District Office.

Mr. Church was a member of King David Lodge No. 236, F. and A. M., of San Luis Obispo. He was extremely active in the church, and was an ardent worker in the Boy Scouts and other worthy local civic activities. His lovable character endeared him to all with whom he came in contact.

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[1] Memoir prepared by T. E. Stanton, M. Am. Soc. C. E.

[2] For memoir, see *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1547.

On November 20, 1902, he was married to Katharine Parker, who, with two sons, Osman and Robert, survives him.

Mr. Church was elected an Associate Member of the American Society of Civil Engineers on June 6, 1911.

JAMES SINGLETON CLAY, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 2, 1942

James Singleton Clay was born in Germantown, Philadelphia, Pa., on April 10, 1889, the son of George and Mary (Musgrove) Clay. He attended the University of Pennsylvania in West Philadelphia and was graduated in 1912 with the degree of Bachelor of Science in Civil Engineering. In 1917 he received the degree of Civil Engineer.

After graduation Mr. Clay was employed as bridge designer with the Department of Public Works of the City of Philadelphia, where he remained from June, 1912, to March, 1917.

On March 16, 1917, he entered the service of the federal government as highway bridge engineer for the Bureau of Public Roads (later the Public Roads Administration) in Washington, D. C. He served two assignments as bridge engineer in two of the District offices as follows: From September, 1920, to December, 1927, at Montgomery, Ala., and from December, 1927, to April, 1930, at Troy, N. Y. From April, 1930, to the time of his death, he served as senior highway bridge engineer in the Washington, D. C., office as first assistant to the principal bridge engineer.

Mr. Clay was a man of unusual thoroughness and was a lover of nature, music, and literature. He had a keen insight into engineering problems and was an accurate analyst and logician. His associates found him to be a man of retiring disposition—quiet, unassuming, and independent in thought and action. He was never given to boasting or self-aggrandizement and, although he naturally shouldered responsibility, he continually won the respect and friendship of his associates.

Mr. Clay was never married. He devoted his life to his family. He is survived by his sister, Mary.

He was an associate member of the Society of American Military Engineers.

Mr. Clay was elected an Associate Member of the American Society of Civil Engineers on May 12, 1919.

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[1] Memoir prepared by Nathan Wilson Morgan, M. Am. Soc. C. E.

ROBERT HARE POWEL CORLEY, Assoc. M. Am. Soc. C. E.[1]

Died December 8, 1941

Robert Hare Powel Corley, the son of Peter A. and Lilly (Lees) Corley, was born on October 19, 1885, at Saxton, Bedford County, Pa. The family moved to Philadelphia, Pa., establishing a residence in Roxborough. Mr. Corley was educated in the Philadelphia public schools and was graduated from Central High School in June, 1904, with the degree of Bachelor of Science.

He began his engineering work with the Pencoyd Iron Works Division of the American Bridge Company, where he completed a term of apprenticeship in structural design on May 31, 1907. The following spring he was transferred to the American Bridge Company plant at Ambridge, Pa. In 1909 he accepted a position as assistant engineer with the Milliken Company in Staten Island, N. Y., which position he held until the failure of the firm in 1910.

From 1911 until 1918 he held responsible engineering positions with the Hay Foundry Company and the Interboro Rapid Transit Company of New York, N. Y., and the Midvale Steel Company of Philadelphia. While with the Interboro Rapid Transit Company his entire time was occupied on design for reinforcing the various elevated structures operated and controlled by this company. His special assignment was the design of the added overhead structure for third-track express operation during morning and evening peak loads.

In 1920 Mr. Corley accepted the position of structural designer with the Philadelphia Rapid Transit Company and remained in the employ of this organization and its successor, the Philadelphia Transportation Company, until his death on December 8, 1941. During the twenty-two years that he was connected with the Philadelphia Transportation System, his engineering ability, resourcefulness, and untiring energy were recognized by his superiors. Several times he was advanced to positions of higher authority, culminating in that of engineer of designs. In this position he was responsible for all structural and building design and construction for the entire system.

Quiet and unassuming in manner, and highly conscientious, Mr. Corley possessed a sense of honor and understanding that will long be remembered by his friends and associates. The men who worked for and with him loved him and swore by him. He was absolutely reliable in everything he said and did, true and devoted to his family, his friends, his masonic lodge, and his church, as well as to the engineering profession and the Society of which he was so proud. He was an active member of St. Timothy's Episcopal Church in Roxborough, and a member of the Masonic Fraternity, Lafayette Lodge No. 27, located in Rahway, N. J.

In October, 1910, Mr. Corley was married to Elizabeth Bottomley of Roxborough. He is survived by his widow; by four children, Robert, Jr., Ruth,

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[1] Memoir prepared by L. F. Parlette, M. Am. Soc. C. E.

Edith, and Helen; by four brothers, James, Thomas, Paul, and William; and by two sisters, Mary Braddock and Margaret Hertica.

Mr. Corley was elected an Associate Member of the American Society of Civil Engineers on July 6, 1936.

GEORGE WARREN CUTTING, JR., Assoc. M. Am. Soc. C. E.[1]

Died February 4, 1942

George Warren Cutting, Jr., the son of George Warren Cutting and Josephine (Brown) Cutting, was born in Weston, Mass., on October 12, 1877, of New England Puritan stock. From the Weston public schools he went to the Massachusetts Institute of Technology, at Boston, Mass., where he was a member of the Class of 1900. His engineering career began in his summer vacations when he acted as rodman for the city engineer of Newton, Mass. After three summers and one winter in the latter position, he served for a year as rodman and acting transitman with the Massachusetts Highway Commission.

Between 1900 and 1905, the Metropolitan Water Works was extending the water supply of the City of Boston by building the Wachusett Reservoir at Clinton, Mass., and the Weston Aqueduct, with a smaller aeration reservoir in Weston. At first Mr. Cutting was recorder on construction at Clinton. Later, at Weston, he was advanced from rodman to engineer and for two years was in charge of the Division office work.

After the completion of this Metropolitan system, Mr. Cutting took a position with the National Board of Fire Underwriters, Committee of Twenty, in New York, N. Y., for whom he examined and reported upon water-works systems and fire departments of various cities, mostly in the South. He returned to Massachusetts and spent a year with the Massachusetts State Board of Health as assistant on sanitary and hydrographic investigations.

In 1906 he opened a private office in Waltham, Mass., where he engaged in general engineering practice, surveys, road and masonry construction, drainage, grading, abolition of grade crossings, etc. In 1910 he received an appointment in the Quartermaster Department of the United States Army, and for a year he was engineer with the Constructing Quartermaster at Fort Mills, Corregidor Island, in the Philippines. This work was particularly interesting. He went to Corregidor by way of the Pacific Ocean, and returned to the United States via Asia, Africa, and Europe.

Upon his return, he opened a private office in Boston, instead of in Waltham. This he maintained from 1911 to 1941. During the first World War, he served as engineer for Stone and Webster on the housing project connected with the Atlantic Ship Building Corporation at Portsmouth, N. H. When the houses were completed, he went into the Engineering Department of the Atlantic Ship Building Corporation, where he remained until 1920. When he returned to

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[1] Memoir prepared by Helen E. Cutting, Weston, Mass.

Boston, he did a great deal of local work in Weston, Wayland, and Lincoln, Mass. For two years he was with the Metropolitan Planning Board, at the time when it was laying out the Northern Artery and the Gallivan Boulevard.

Throughout his life, Mr. Cutting was interested in military affairs. His early training was at Massachusetts Institute of Technology. For some years he was in the Massachusetts State Guard, a member of the First Squadron Cavalry, and a National Lancer. Later he was in the 94th Division, 319th Engineers, U. S. Army. His commission as Captain was in Washington, D. C., awaiting signature when the Armistice was signed in November, 1918, but after the war he remained a member of the Engineer Reserves. When a physical condition prevented his remaining with the Engineers, he was transferred to the Quartermaster Department and commissioned a Major.

The Cutting family in Weston always gave devoted service in public office. Mr. Cutting's father was Town Clerk of Weston for fifty-two years, and his brother, Alfred L. Cutting, was a member of the Board of Selectmen for twenty-five years; so it was natural enough for George Cutting to take on the duties of Treasurer and Collector, an office which he held for nearly twenty-one years.

With deep devotion to his church and to the principles of a true Christian, he brought to his work and to his relationships with the townfolk such faithfulness, such keen interest, and such kindliness that he won the love and respect of all.

In 1912 he was married to Helen Ellms Warren. He is survived by his widow, by one daughter, Mary, and by two sisters, Sarah L. (Mrs. Arthur B. Nims), and Eleanor M.

Mr. Cutting was elected an Associate Member of the American Society of Civil Engineers on May 3, 1910.

HARRY JOHNSON DEUTSCHBEIN, Assoc. M. Am. Soc. C. E.[1]

Died February 8, 1942

Harry Johnson Deutschbein was born in Canaan, N. Y., on August 13, 1880, the son of Joseph E. Deutschbein and Eliza (Johnson) Deutschbein. His father came to the United States, at the age of eighteen, from Southern Germany soon after the revolutionary disturbances of 1848. His mother was descended from a family that had been settled in and near Canaan since the time of the American War of Independence.

After their marriage, his parents settled in Albany, N. Y., where young Deutschbein spent his early life. He received his education from the public schools there until his graduation from the Albany High School in 1898. Although a few months under the age limit, he enlisted in the 1st New York Volunteers, then being recruited for service in the Spanish-American War.

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[1] Memoir prepared by Francis S. Crowell, M. Am. Soc. C. E.

After several months of training, he accompanied his regiment to the Pacific Coast, where it was embarked for duty in the Philippine Islands. However, the termination of the war caused the detention of the expedition in Hawaii, where the soldiers were encamped until early in 1899, when they were returned to the United States and the regiment was disbanded.

In the autumn of 1899 Mr. Deutschbein enrolled in the freshman class of Rensselaer Polytechnic Institute at Troy, N. Y. He was graduated four years later with the degree of Civil Engineer. During the intervening summers he held several positions as transitman and inspector on survey and construction work in and near Albany. After graduation he went to work in the office of the chief engineer of the Duluth and Iron Range Railroad Company in Minnesota, where his duties were of a general nature, involving projects of survey, maintenance, and new construction. After a year, however, he returned to the East to enter the employ of The Foundation Company, with which he was later to be so intimately associated, but after a short period of work in New York, N. Y., he was induced to return to Albany, where in October, 1904, he was appointed deputy city engineer. In this position, which he held for the succeeding three or four years, he directed the program of modernization of the paving and sewerage of the growing city. In 1908 he was transferred from the Bureau of Engineering to the Bureau of Water Supply as superintendent of water works. His duties in the latter position included the construction and later the operation of the new slow-sand filters, as well as the reconstruction of the pumping and distribution facilities throughout the city and its environs. This work resulted in the practical obliteration of typhoid fever in Albany.

In 1912 Mr. Deutschbein returned to The Foundation Company, this time to continue with it for almost twenty years. After serving as construction engineer and superintendent on several large projects on the Ohio River and the New York Barge Canal, he was recalled to the head office of the company in New York City as district manager. Two years later, in 1917, he became vice-president and general manager. From this time onward he was in direct charge of all the company's work in the United States, Mexico, and South America. After 1926, when he was advanced to president, through affiliated companies, he had charge of all the company's work in Europe as well. It was during these years that the company was built up to the dominating position which it occupied in its field during the 1920's and early 1930's, especially in the construction of deep piers and caissons by the pneumatic method.

In 1929 Mr. Deutschbein withdrew from The Foundation Company to form the company which bears his name. He served as its president until his death. Under his direction this company has completed a large number of important construction projects in the field of deep foundations and heavy engineering work, and has been engaged in construction work of magnitude for the United States Government.

Mr. Deutschbein was a member of the American Concrete Institute; the Chi Phi fraternity; the Sigma Xi honorary society; and the Engineers', Downtown Athletic, and Rockefeller Center Luncheon clubs of New York City. He served on the Joint Committee on Masonry Dams of the Society, and on the Sectional Committee of the American Standards Association on Building Code Requirements for Excavation and Foundations. For the latter Committee he was chairman of the Subcommittee on Caissons and Cylinders.

Mr. Deutschbein was known to a large circle of clients, associates, and friends as an engineer of more than usual ability. He was widely recognized as an authority on all questions appertaining to construction work and methods, especially those in the field of deep and difficult foundations involving subterranean and subaqueous problems. His advice on these questions was eagerly sought and widely adopted. His untiring industry, a direct and forceful personality, a highly trained and logical power of reasoning, and a lovable personal charm explain his success and the friendships that he formed wherever he went. He was an intensely active man to whom death came too soon. His departure leaves a gap which is keenly felt by many.

In 1905 Mr. Deutschbein was married to Frances Ethel Dresser, who, with their two children, Harry D. and Ethel H. (Mrs. F. Ashton Smith), survives him.

Mr. Deutschbein was elected a Junior of the American Society of Civil Engineers on December 6, 1904, and an Associate Member on January 7, 1913.

SAMUEL DOUGLASS DODGE, Assoc. M. Am. Soc. C. E.[1]

Died December 11, 1941

Samuel Douglass Dodge was born on November 29, 1870, in Grinnell, Iowa, and died at his home in Suffern, N. Y., on December 11, 1941. He was the son of Joshua G. and Mary F. (Herrick) Dodge, a descendant of an old New England family.[2]

Samuel Dodge possessed a scholarly mind. He was graduated from the Massachusetts Institute of Technology at Boston, Mass., with the degree of Bachelor of Science in Civil Engineering in 1893. His high standing in mathematics, applied mechanics, etc., resulted in an invitation to return to the Institute after graduation as an instructor in the theory of structures. He accepted, and remained at the Institute for about one year.

For the next year and a half he was instrumentman with the Water Board of Winchester, Mass. From 1895 to 1905 he was in the employ of the Metropolitan Water Board of Boston on the location and construction of pipe lines, reservoirs, masonry water towers, subaqueous tunnels, etc.

The ten years from 1906 to 1916 were spent with the Board of Water Supply of New York, N. Y. As section engineer he located several miles of the Catskill Aqueduct just west of the Hudson River and was in charge of construction of three deep shafts and about three miles of tunnel on the Moodna siphon of the Aqueduct. At the latter end of this period he was engaged on statistical and literary work in connection with a history of the work of the Board of Water Supply of New York City.

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[1] Memoir prepared by William E. Swift, M. Am. Soc. C. E.

[2] *Who's Who in Engineering*, 1925, p. 568.

During World War I he was supervising and production engineer in the Ordnance Department of the United States Army. First he supervised the construction of a water supply system for an extensive smokeless powder plant at Nitro, W. Va. Later, he was connected with the production of picric acid in the New York Ordnance District. Mr. Dodge became interested in the Richmond Screw Anchor Company and after the war accepted the position of treasurer of this company. He resigned about five years before his death.

Samuel Dodge was a gentleman in the truest sense. He was interested in music, art, and drama, and was well informed on topics of the day. He maintained a live interest in civic affairs. In 1917 he purchased a large apple orchard near Suffern. He built a comfortable house among the apple trees in 1918, and this was his permanent residence until his death. Because of his unselfishness, absolute straightforwardness, and keen sense of humor, his home life was ideal. He was to be depended upon as a friend.

On June 12, 1913, he was married to Margaret Stone of Cornwall, N. Y., who, with two sons, David and Eric Douglass, survives him.

Mr. Dodge was elected an Associate Member of the American Society of Civil Engineers on November 6, 1907.

WALTER THOMAS EVANS, Assoc. M. Am. Soc. C. E.[1]

DIED AUGUST 9, 1941

Walter Thomas Evans was born at Wales, Wis., on December 2, 1887, the son of David Richard Evans and Sarah (Price) Evans. His grammar school and high school days were spent in Racine, Wis. He entered Illinois Wesleyan University, at Bloomington, Ill., in 1907, and at the end of his freshman year transferred to the University of Illinois, at Urbana, where he pursued a course in railroad engineering and administration for two years.

In June, 1910, he entered the employ of the Chicago and Alton Railroad Company in its engineering department. He served as rodman and instrumentman from June, 1910, to January, 1914. From January, 1914, to January, 1916, he served as assistant engineer on maintenance and construction at Bloomington. He served as chief draftsman from January, 1916, to 1923, with headquarters in Chicago, Ill., and during this time he had charge of mapping the entire Chicago and Alton Railroad system. He was in charge of making valuation maps for the entire road and of compiling Valuation Order No. 7, which consisted of looking up and listing all the title instruments for each individual parcel of land belonging to, or used by, the railroad. He also acted as pilot engineer for the Interstate Commerce Commission parties on federal valuation. He assisted in the design of yards and terminals.

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[1] Memoir prepared by Alfred Jones, Assoc. M. Am. Soc. C. E., and Gifford G. Todd, Chf. Valuation Engr., Office of County Surv. and County Engr., County of Los Angeles, Los Angeles, Calif.

In the latter part of 1923 Mr. Evans severed his connection with the Chicago and Alton Railroad Company and moved to California. From January, 1924, until the spring of 1927 he served as secretary and plant superintendent of the California Crushed Fruit Corporation at Los Angeles, Calif. In this connection he equipped the plant and had charge of the installation of the machinery. This concern shipped their products to all parts of the United States and foreign countries.

In the spring of 1927 he entered the employ of the Los Angeles County surveyor's office, still retaining a financial interest in the Crushed Fruit Corporation. Soon after entering the county surveyor's office, he was selected for valuation work in connection with highways and other governmental matters. He served as a valuation engineer until his death.

Mr. Evans was a member of the Phi Kappa Sigma Fraternity, a registered civil engineer of the State of California, a charter member of the American Right of Way Association, and a member of Crescenta Valley Lodge No. 652, F. and A. M. in Montrose, Calif.

At the time of his death he was residing in Glendale, Calif. He is survived by his widow, Jessie J. Evans, to whom he was married on May 14, 1913, and a daughter, Gloria Irene Evans.

A conscientious and kindly individual, and an inspiring worker, he will long be remembered, as a real friend, by those who had the pleasure of working with him.

Mr. Evans was elected an Associate Member of the American Society of Civil Engineers on March 5, 1928.

WARREN GLENN FOWLER, Assoc. M. Am. Soc. C. E.[1]

Died February 14, 1941

Warren Glenn Fowler was born on December 15, 1895, in Vandalia, Mo., the son of James W. and Mary Susan Fowler. He was educated at the University of Missouri at Columbia, Mo., and was graduated in 1919 with the degree of Bachelor of Science in Engineering. In 1919–1920 he took postgraduate work at the University of Missouri. As research assistant at the Engineering Experiment Station, he collaborated in the preparation of a bulletin, "Study Relating to the Water Resources of Missouri." He was granted the degree of Civil Engineer in June, 1920, at the conclusion of this work. He was awarded the "Gregory Senior Scholarship" in 1919 and was a member of Tau Beta Pi.

In the summer of 1918 he was engaged on drainage work in the St. Francis River lowlands in southeastern Missouri, and in the summer of 1919 served as a transitman, maintenance of way, with the St. Louis-San Francisco Railway Company in Springfield, Mo.

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[1] Memoir prepared by Charles A. Haskins, M. Am. Soc. C. E.

For three years after the completion of his school work he was structural and mechanical designer with Heyl and Patterson, Inc., of Pittsburgh, Pa., engaged in design and estimating on conveying machinery, coal and coke plants, etc. In May, 1923, Mr. Fowler was employed by the Bureau of Bridges, Missouri State Highway Commission, as special designer. During this period he was engaged on the design of three bridges across the Missouri River. In May, 1924, he was promoted to chief draftsman and from then until August, 1926, was in charge of the drafting room on bridge plans and standards for more than four hundred bridges, including fourteen concrete arches and a large steel cantilever.

In August, 1926, he was appointed bridge engineer, Board of Park Commissioners, at St. Joseph, Mo., and was in charge of the preliminary study, design, and erection of ten bridges for the extensive boulevard and parkway project in that city. From 1927 to the time of his death Mr. Fowler was office engineer for the firm of Black and Veatch, Consulting Engineers, of Kansas City, Mo., and was in responsible charge of a large number of engineering projects of great variety.

Mr. Fowler took an active interest in the affairs of the Society, and he served the Kansas City Section faithfully and well. He was elected secretary-treasurer in 1940 and vice-president in 1941, which office he held at the time of his death. His friendly but quiet personality and his efficiency in handling the affairs of the Local Section endeared him to all with whom he came in contact.

In 1924 he was married to Dorothy Brittingham of Jefferson City, Mo., who survives him.

Mr. Fowler was elected a Junior of the American Society of Civil Engineers on December 4, 1922, and an Associate Member on March 14, 1927.

RAY STUART GATENS, Assoc. M. Am. Soc. C. E.[1]

Died April 10, 1941

To Patrick Gatens, of Donegal County, Ireland, and Hanna (Cunningham) Gatens, his wife, a native of Quebec, Canada, was born Ray Stuart Gatens on April 29, 1886, at Memphis, Tenn. Ray Gatens' father was a pioneer cotton merchant, an active and honorary member of the Memphis Cotton Exchange, and held a seat on the New York Cotton Exchange. Ray was educated at Christian Brothers College in Memphis and at Notre Dame and Purdue universities, in Notre Dame and Lafayette, Ind., respectively.

Returning to Memphis in June, 1904, he began a career of varied and interesting engineering activities as an inspector on levee construction with

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[1] Memoir prepared by J. C. Carpenter, M. Am. Soc. C. E.

Le Vasseur and De Beughem, Consulting Engineers, 2d Mississippi River District, U. S. Engineers. During his service he was assigned the supervision of an important part of the work that seemed too difficult for his experience. He asked his father's advice, and Patrick Gatens was concerned for fear that the responsibility might result in a failure that would discourage his son. Mr. Gatens, Sr., consulted an old friend who had had construction experience and his advice was "Let the boy decide for himself. Of course he can handle it, and, if he's 'stuck' I'm here to see him through." The boy decided to take the risk, handled it successfully, and was promoted to resident engineer.

On completion of his first assignment Mr. Gatens was employed for a year in Chicago, Ill., as a city inspector on heavy tunnel construction. For the next six years, 1907 to 1913, he was contractors' engineer for the Parker-Washington Company of St. Louis, Mo., in executive charge of construction; plant layout; and financing and construction design of paving, sewers, water works, and tunnels in Chicago, St. Louis, Kansas City (Mo.), and Oklahoma, involving several million dollars' worth of work.

During 1914 Mr. Gatens engaged in general engineering and surveying at Greenville, Miss., in private practice. As general superintendent and construction engineer for the Continental Public Works Company, of New York, N. Y., in 1915 and 1916, Mr. Gatens was directly or indirectly in charge of a large volume of construction of street paving, highways, sewers, and sewage disposal.

Volunteering for service in the United States Army at the beginning of its participation in the first World War, he was commissioned a First Lieutenant on February 26, 1918, and assigned to the 42d Engineers at American University in Washington, D. C. On completion of the training course he sailed for France on April 11, 1918, on the *President Lincoln.* Immediately on his arrival, April 24, he was placed in command of a company of the 20th Engineers and served throughout the remainder of the war. He was promoted to Captain on May 6, 1919. He returned to the United States on the *Zeelanda* and was honorably discharged at Camp Pike, in Arkansas, on June 21, 1919.

From August, 1920, to October, 1921, Mr. Gatens was general superintendent of construction for the Liberty Contracting Company of New York City on the installation of 18 miles of 72-in. steel pipe line as a part of the water supply for Jersey City, N. J. During the next three years he served as general superintendent for the William P. McDonald Construction Company of New York City in general charge of estimating and construction of street paving, roads, bridges, and sewers in New Jersey, New York, and Florida.

Mr. Gatens was city engineer and city manager of Winter Haven, Fla., in executive charge of all municipal departments from December, 1924, to June, 1928. He was seriously ill and obliged to suspend all professional activities from June, 1928, to the end of 1929.

On his return to practice he again joined the William P. McDonald Construction Company as assistant to the general superintendent and in charge of cost control on projects in New York State and in Creek de Prairie, Quebec, covering construction of street paving, roads, and sewers.

Mr. Gatens was again forced to retire from active practice after a year with the William P. McDonald Company and returned to Florida. He conducted a private engineering practice in Winter Haven and Lakeland in 1932 and 1933.

On October 24, 1933, he was appointed an associate highway engineer in the United States Bureau of Public Roads. His first assignment was on a special force account project in Texas, where he demonstrated a broad-visioned ability beyond the stipulated requirements. He was then placed in charge of cost studies and landscaping projects for the district which includes Arkansas, Louisiana, Oklahoma, and Texas. In August, 1935, he was appointed maintenance engineer for the district, in which position he served until his death. His tact and analytical ability, reinforced with his wide experience in important engineering projects and coupled with a pleasing personality, marked by good humor, served to establish such satisfactory relations with the maintenance organizations throughout the district as undoubtedly resulted in marked improvement in maintenance practice in this area.

During the last two years of his life Mr. Gatens was forced to suspend his field operations several times due to severe illness. He always returned to the field with dogged determination, even when he had not fully recovered. In spite of these handicaps he maintained a cheerful and good-natured mien and few of his business associates realized that he was not in the best of health. He had a magnetic personality and was admired and respected by his many friends.

On February 1, 1913, he was married to Fannie Pringle Cherry, a descendant of the old families of Cherry and Pringle of Charleston, S. C., and a granddaughter of a South Carolina governor. In addition to his widow he is survived by three sisters, Mrs. C. K. H. Byrne and Ann and Agnes Gatens; and two brothers, Thomas J. and James T. Gatens.

Mr. Gatens' college fraternity was Kappa Alpha, and he was a member of the Winter Haven post of the American Legion.

Mr. Gatens was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917.

PHILIP HOLDEN GLOVER, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 30, 1941

The death of Philip Holden Glover brought to a close an unusually interesting and diversified engineering career. Although in poor health for several years, Mr. Glover performed his duties as city engineer of Bangor, Me., cheerfully and efficiently, resigning only a few weeks before his death.

Philip Holden Glover was born in Portland, Me., on February 23, 1883, the son of Capt. Russell Glover and Elizabeth (Nash) Glover. (He was a direct descendant of Dr. Phineas Holdon, who was a surgeon attending the wounded of the battle of Bunker Hill. At the time of Mr. Glover's death, he was in possession of the medicine case used by Dr. Holdon during that battle.) He received his early education in the schools of that city, and then entered the University of Maine, at Orono. He was graduated with high honors in 1906, receiving the degree of Bachelor of Science in Civil Engineering. While in college he became a member of the honorary engineering fraternity, Tau Beta Pi, and the honorary scholastic fraternity, Phi Kappa Phi.

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[1] Memoir prepared by Henry L. Doten, M. Am. Soc. C. E.

After graduation, he spent six years in the U. S. Reclamation Service, working on the Garden City, Minidoka, Blackfeet, Flathead, Shoshone, and Sun River projects in Montana and Wyoming and advancing to the positions of resident engineer on construction and location engineer on trunk drainage ditches, and canals.

In 1912 Mr. Glover returned to the East and entered the University of Maine as a graduate student. The degree of Master of Science in Civil Engineering was conferred upon him in 1913.

During the next few years, he held engineering positions with several different companies, notably the Bangor and Aroostook Railroad Company, the International Paper Company, and the Barahona Sugar Company in Santo Domingo, Cuba. On the latter project he was the engineer in charge of the construction of a large sugar plantation.

In 1921 he returned to his native state and accepted the position of resident engineer on concrete bridge construction for the Bridge Division, Maine State Highway Commission. He served in this capacity until the winter of 1923.

During the next six years he held engineering positions with Murrie and Company, New York, N. Y.; McCrossin and Company, New York; and Structural Gypson Company, Linden, N. J. He was also office engineer on the Buggs Island Project of the Electric Bond and Share Company at Clarksville, Va.

In August, 1929, he reentered the employ of the Bridge Division, Maine State Highway Commission, and during the next two and one-half years was resident engineer on several important and difficult bridge projects.

In February, 1932, he was invited to accept a position with the City of Bangor, which had just adopted the city manager form of government. He served the city as assistant engineer for ten months and was then appointed to the position of city engineer, serving in this capacity until April 5, 1941, when he resigned because of failing health. While in Bangor, he had a summer home in the Village of Harrington and, upon his retirement, adopted this town as his permanent residence. It was his hope to indulge in his lifelong hobbies of hunting, fishing, and gardening, but fate decreed otherwise and Mr. Glover succumbed to a heart attack on May 30, 1941.

Although an unusually brilliant engineer, it may well be said that his outstanding characteristics were devotion to duty, loyalty to his superiors, and friendliness and helpfulness to his associates and subordinates. There are many engineers who will remember the kindly acts and the professional advice and assistance that helped them along the difficult pathways of engineering experience. He was always keenly interested in younger engineers and, prior to 1936, served several years as contact member with the Student Chapter of the Society at the University of Maine. While residing in Bangor, he frequently attended the chapter meetings at the University, often contributing to the program.

Mr. Glover was a member of the Masonic Order, Beta Theta Pi fraternity, and a life member of the National Geographic Society. He was also a patror of the Smithsonian Institution.

He is survived by his widow, Zena Talmage Glover, to whom he was married in 1926; a brother, Russell; and a stepson, Thomas Nickerson.

Mr. Glover was elected an Associate Member of the American Society of Civil Engineers on February 6, 1912.

ARTHUR GREEN, Assoc. M. Am. Soc. C. E.[1]

Died April 6, 1942

Arthur Green was born in Edwina, Cocke County, Tenn., on March 25, 1890, the son of Joseph D. Green and Melissia (Burke) Green. His early education was received in the public schools of Newport, Tenn., and Maryville Polytechnic School in Maryville, Tenn.

In September, 1915, he secured employment as rodman with the Aluminum Company of America in Alcoa, Tenn., and subsequently was promoted to chief instrumentman. He remained with this company until May, 1918, when he entered the United States Army. He was in the Army until February, 1919, and then returned to the Aluminum Company of America at Alcoa as instrumentman on water lines, reservoir construction, street locations, grades, and sanitary and storm sewers.

In January, 1921, he was employed at Hopewell, Va., as material estimator for the Tubize Corporation and remained there until August, 1922, at which time he resigned to accept a position as instrumentman and assistant to the city engineer of Petersburg, Va. He was in charge of the Mapping Section of the Engineering Division, and also directed the installation of water and sewer systems throughout the city. In August, 1923, he resigned from his position at Petersburg to accept employment as resident engineer with the Ambler Engineering Company of Richmond, Va., and was sent to Bristol, Tenn., to supervise and direct the surveys, planning, and installation of a filter plant for the Bristol City Water Works. In January, 1924, he was appointed city engineer of Bristol, and held this position until June, 1931. During this period he planned and directed a large street improvement program and water and sewer extension, and installed the first garbage collection system for the City of Bristol.

His good record as city engineer was accepted by the citizens of Bristol, in that he was chosen as their commissioner of streets and public improvements in June, 1931, and served until June, 1937. During these six years as city commissioner his service to the community was outstanding, as indicated by city progress. He designed and supervised construction of the Bristol City Municipal Stadium during the early years of the relief program under the Emergency Relief Administration and the Work Projects Administration. This stadium was an outstanding achievement in stone masonry of medieval architecture, and will remain as a monument to him through the years for his foresight and vision in fulfilling the needs for a recreational center for the entire city of Bristol and the surrounding territory. During his term of office he also initiated the first Civil Works Authority project in the United States in the design and construction of the Beaver Creek Channel change, thereby relieving the business section of Bristol from damage caused by flash floods, which for years had been a menace to the city.

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[1] Memoir prepared by Robert L. Morrison, Assoc. M. Am. Soc. C. E.

In June, 1937, he established in Bristol an office for the practice of engineering and until his death on April 6, 1942, he continued to serve his community by making engineering plans and surveys for the construction of projects that will benefit mankind for many years to come.

On December 22, 1920, Arthur Green was married to Mildred Manges, of Maryville, who survives him. They had no children. He is also survived by four brothers, W. Bartley Green, John Green, Isaac Green, and George W. Green; and by two sisters, Charity (Mrs. L. B. Allen), and Flossie (Mrs. R. D. Bland).

He was a member of the Board of Deacons of the First Presbyterian Church of Bristol, and was deeply interested in his church and community affairs. He was an active member of the Kiwanis Club, the Elks Club, and the American Legion.

Time has not lessened the difficulty of consoling relatives and friends who have lost a companion and counselor, and Arthur Green's relatives and many friends can take comfort in the worthy life he lived and the assistance he gave to his fellow men.

Mr. Green was elected an Associate Member of the American Society of Civil Engineers on June 10, 1929.

CLAUDE CLEMENT HOCKLEY, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 4, 1941

Claude C. Hockley lived intensely. He was a man of strong convictions and exceptional energy. He was not an egotist, but he believed in his ability to accomplish. He was never satisfied to drift along but was always looking for more attractive fields of endeavor. When confronted with engineering or executive problems, he was not content until he had studied all phases of the subject and had arrived at a solution which, to him, appeared to be the only logical one. Early in his professional career he became an executive in responsible charge of important projects. One of his outstanding traits was the ability to win the loyalty of those under his supervision. He was exacting in his requirements, and did not spare his men or himself. The determination to give all his energies to each problem was largely responsible for his untimely death.

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[1] Memoir prepared by M. E. Reed, M. Am. Soc. C. E.

Claude Clement Hockley was born in Watsontown, Pa., on July 5, 1878, the son of Jacob and Elizabeth Hockley. He attended the public schools of Pittsburgh, Pa., and was graduated from Ohio Northern University, at Ada, Ohio, in 1904 with the degree in Civil Engineering.

His professional life can be segregated into four periods. The first one was the formative period. It includes his college life and the railroad location and construction period before and after college. Like many engineers of the 1890's, he turned to railroad work as a means of earning a livelihood until a more acceptable type of engineering was available.

From 1899 to 1901 Mr. Hockley was a chainman, leveler, and transitman for the Carnegie Steel Company at Rankin, Pa. In 1902 he worked for the Greene County Railroad Company. The following year he became chief of party for the Wabash Terminal Railroad Company, where he remained until 1905, when he became locating engineer for the Western Maryland Railway Company. He remained with the latter company for a year, and in 1906 accepted a position as engineer for the Davis Coke and Coal Company at Fairmont, W. Va.

The second period of Mr. Hockley's career dates from 1906 when he gave up railroad work to become resident engineer for the Shawinigan Water and Power Company at Shawinigan Falls, Quebec, Canada. His duties included the installation of a 10,500-hp turbine and generator, together with head gates, penstock, and transformers. He also acted as borough engineer for Shawinigan Falls.

In 1908 Mr. Hockley was appointed chief engineer of the Laurentide Paper Company at Grand Mère, Quebec, Canada, in charge of engineering and construction. In 1910 he was also made superintendent of the Sulfite, Acid, and Wood Departments. His executive ability was soon demonstrated in the reorganization and extension of the several subsidiary companies. Surveys were made which resulted in the construction of the St. Maurice Power Company (175,000-hp capacity). He also acted as city engineer of Grand Mère, in charge of the water supply system, sewers, and pavements. A 4,000-hp steam plant was built under his supervision. In 1915, as chief engineer and mill manager for the Union Bag and Paper Company, he was in charge of properties both in Canada and the United States, including the St. Maurice Paper and Pulp Company, at Cap Madeleine Falls, Quebec, Canada, the bag factory and paper mills at Hudson Falls, N. Y., and a large bag storehouse at Wausau, Wis.

In addition to the foregoing projects, Mr. Hockley designed and built a water supply and filtration plant at Fenimore, N. Y.; built a 3,000-hp steam plant and a 50,000-kw turbogenerator at Fenimore; designed and rebuilt the paper bag factory at Hudson Falls; and rebuilt Mill No. 5 at Hudson Falls.

In 1917 Mr. Hockley was made chief engineer and manager of manufacture for the Thilmany Pulp and Paper Company in Kaukauna, Wis., and also for its subsidiaries, including the Paper Conversion Corporation at Niagara Falls, N. Y., and the American Lakes Paper Company at Waukegan, Ill. He was in charge of reconditioning the paper mills at Appleton and Kaukauna, Wis., of the construction of a 5,000-kw turbogenerator at Kaukauna, and of the construction of a two-machine mill at Niagara Falls for manufacturing and printing waxed waterproof paper. Mr. Hockley invented and patented several improvements in paper-mill operation.

The third period in his career dates from 1925 when he opened a consulting engineering office in Portland, Ore., specializing in power development and paper-mill design. A special study was made of the possibility of using mill refuse in pulp manufacture and also for wallboard. Studies also were made for hydroelectric power developments in the Northwestern States and Alaska. While making these studies, Mr. Hockley was retained by paper mills and other corporations in the Northwest.

The fourth period of Mr. Hockley's career began in 1933 when he was appointed regional director of Region No. 9 of the Public Works Administration, in charge of projects in Oregon, Washington, and Idaho. These projects covered a broad field of engineering and architecture with costs running into many millions of dollars.

Mr. Hockley's executive ability was soon demonstrated in the organization of the personnel that was required to complete this large program. Perhaps the most difficult problem was to keep the relations among the engineers, the contractor, and the owner harmonious and at the same time meet the requirements of the Washington officials. Mr. Hockley was able to meet the problems as they arose and eventually to win the confidence of all involved.

On October 5, 1910, he was married to Edith Moore. He is survived by his widow, a son, Claude C., Jr., and a daughter, Helene.

Mr. Hockley was elected an Associate Member of the American Society of Civil Engineers on April 2, 1912.

WILLIAM LOUIS KUEHNLE, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 8, 1942

William Louis Kuehnle was born in New York, N. Y., on March 1, 1881, the son of George W. and Bertha L (Pratt) Kuehnle. He attended Rutgers College at New Brunswick, N. J., and received the degree of Bachelor of Science in 1902. In 1911 he received the degree of Civil Engineer.

His first engineering experience was gained during the summer of 1901, when he was employed as an instrumentman on hydrographic surveys by the Corps of Engineers in the New York District Office. After graduation he returned to the Corps of Engineers, this time as assistant to the chief inspector on dredging and timber bulkhead construction at Philadelphia, Pa. From this time until the end of his life, Mr. Kuehnle served with the Corps of Engineers, except for a short period spent in private practice and with an industrial concern. From 1903 until 1923, he was stationed at the U. S. Engineer Office, First District, New York City, where he served in various capacities, with steadily increasing responsibility. During this time he laid the groundwork for his broad experience in both construction operations and design. His assignments included surveying, inspection, construction, design of various types of navigation projects, dredging, and other aspects of the work done by the Corps. He also organized the cost-accounting system used in this District Office.

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[1] Memoir prepared by Norman H. Chaffee, Chf. Administrative Asst., U. S. Engr. Office, Tulsa, Okla.

In 1923 Mr. Kuehnle severed his connection with the Corps of Engineers to enter private practice at Long Beach, N. Y. His engagements during this period were in many fields, including town planning, design and construction of municipal improvements (such as sewers and streets), the design and construction of building foundations, beach erosion, and structures. Late in 1928 he was engaged by the Dry Ice Corporation of America as chief engineer on the design, construction, and operation of plants for the manufacture of dry ice or solid carbon dioxide.

In 1931 he again entered private practice for a short period but was prevailed upon to rejoin the Corps of Engineers in the Pittsburgh, Pa., District Office, where he remained until 1936. In the Pittsburgh Office he was in charge of all contract and hired labor construction and inspection and was acting principal engineer in the absence of the principal engineer. In 1936 Mr. Kuehnle was transferred, first to Galveston, and then to Mineral Wells, Tex., as acting principal engineer on design and investigations for the Possum Kingdom Dam on the Brazos River. In 1937 he was again transferred, this time to the Binghamton, N. Y., District, where he was in charge of the design of earth-fill dams and levees and flood-wall projects for five localities. Mr. Kuehnle transferred to the Little Rock, Ark., District in 1938 to take charge of the Operations Division in that office. In this position he was responsible for all contract inspection and hired labor operations on the structures being built under the supervision of that office. He also was in charge of safety, land acquisition, and labor relations. In July, 1939, a new District Office was formed at Tulsa, Okla., where Mr. Kuehnle was stationed at the time of his death. Mr. Kuehnle went to the Tulsa District Office as principal civilian assistant in charge of the Engineering Division. He was responsible for all phases of investigations and engineering studies in the design and construction of major flood-control dams and reservoirs. Upon the transfer of Air Corps construction to the supervision of the Corps of Engineers, he assumed charge of the design and construction of all such projects assigned to the Tulsa District.

Mr. Kuehnle was an able engineer. He had good judgment and common sense, and was honest and extremely conscientious. He took a keen interest in all phases of the work being performed under his supervision, and due to his large and varied experience was able to give helpful criticism. He possessed an ability to express himself forcefully and aptly, with a sense of humor that aided in keeping a balanced view of the many problems presented to his attention. His death, following an automobile accident on February 7, 1942, near Watonga, Okla., leaves a vacant place which cannot soon be filled. Men of his ability cannot well be spared in this time of need.

He was married in 1906 to Luella Mae Van Dusen, who survives him. A brother and two sisters also survive him.

Mr. Kuehnle was elected an Associate Member of the American Society of Civil Engineers on June 6, 1911.

ENGELBERT CONRAD LAWRENCE, Assoc. M. Am. Soc. C. E.[1]

Died October 25, 1941

Engelbert Conrad Lawrence was born in Baltimore, Md., on June 11, 1875, the son of Joseph and Isabelle (Scott) Lawrence. He was educated at St. Michaels Parochial School in Baltimore; took business courses at St. Clements College in Saratoga Springs, N. Y., and St. Marys Seminary at North East, Pa.; studied engineering in the International Correspondence School; and studied architecture at the Maryland Institute of Art and Design.

In 1892, when only 17 years old, he began his technical career as an estimator with one of the largest and best-known contracting firms in Baltimore. Before he left this firm, after about four years of service, he had had charge of the construction of some buildings.

From that firm he went to a large bolt and steel works in Baltimore for three years and was in charge of its Estimating and Drafting Department. Following this work he became an estimator and detailer of structural steel for buildings with the Baltimore Bridge Company and then with Purdy and Henderson of New York, N. Y. He was with each of these companies for about a year, and then was engaged by an important Baltimore building contractor to take charge of the Estimating Department and act as superintendent of construction. In his five and one-half years of service for this company he designed and constructed buildings costing more than a half million dollars. He left that firm to go to the Engineering Department of the Baltimore and Ohio Railroad Company as a designer of bridges and foundations. In July, 1907, after only about six months of service there, the Harbor Board of Baltimore selected him as engineer on the construction of a $5,000,000 pier and improvement project for that city's harbor. After about eight months he was promoted to the position of principal assistant engineer in charge of this work. About four years later, when the work was practically finished, he went with a large general contracting firm in Newport News, Va., as an estimator and engineer.

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[1] Memoir prepared by Alfred M. Quick, M. Am. Soc. C. E., and Charles W. Wisner, Jr., Asst. Director, Claim Dept., U. S. Fidelity and Guaranty Co., Baltimore. Md.

After about a year's service with this firm, he was appointed by the United States Fidelity and Guaranty Company as an engineer and adjuster in its Claim Department, and there he began the final link in his chain of forty-two years of continuous technical service. His period of service in this company exceeded by two years the total of all of the periods of his previous service. This work required all his knowledge and skill as an engineer plus the practical and working knowledge of the rights and duties of a surety in a bond conditioned for the faithful performance of a construction contract. It involved the duty of protecting the rights of the obligee, or owner, seeing that he received all that his contract called for, and at the same time conserving the interests of the principal—that is, the contractor.

This line of work took Mr. Lawrence into every section of the United States. In almost every state there are roads, schoolhouses, dams, water works, sewerage systems, post offices, memorial structures, and many other constructions of various types, which were built either under his supervision or under his general direction. Space is not available to enumerate even the most outstanding of these, but to mention a very few, there are: The Soldiers and Sailors Memorial Auditorium, Chattanooga, Tenn.; high school building, Waynesburg, Pa.; Seattle (Wash.) water works; sewerage system, Westchester County, New York; Concourse Apartments, New York City; Post Office, Brooklyn, N. Y.; the Wills Creek Dam, Zanesville, Ohio; and pier, bulkhead, and ramps, Naval Air Station, Jacksonville, Fla.

This work brought Mr. Lawrence into contact not only with other engineers, architects, and contractors, but also with various governing bodies and commissions of the school district, the county, the state, and federal authorities; with the lawyers representing all these; and with bankers and financial representatives, often in consultation, not infrequently in contest—a work which was broadening but which required fundamental ability and acumen. Mr. Lawrence measured up to these requirements.

In his later years Mr. Lawrence was frequently engaged as a consulting engineer. On many occasions his services as supervisor of construction work were sought, but he always refused because of possible conflict with his company duties.

He died on October 25, 1941, a character of sincerity and reliability, a hard worker, clear-headed, and able to concentrate, with a gift and developed ability of sound reasoning which brought sound conclusions. He was a man of diligence, great energy, and vitality.

To the home there has been the irreparable loss of husband and father; to the company there has been the loss of a most able representative in one of the most important phases of its business; to many, the loss of a dependable friend; to his profession the loss of an able and experienced engineer, one who strove to maintain and maintained the high intent of his profession and his Society.

Mr. Lawrence was married to Anne Heyer of Baltimore on October 13, 1896. She died on June 23, 1915, leaving five children: Three daughters, Isabelle (Mrs. William A. Fingles, Jr.); Genevieve (Mrs. Charles I. Jamieson); and Anne (Mrs. Edward J. Donnelly); and two sons, Edgar C. Lawrence, and John Heyer Lawrence, Assoc. M. Am. Soc. C. E. Also surviving

Mr. Lawrence are his second wife, Rachel Mills of Baltimore, to whom he was married on September 28, 1916, and nine grandchildren.

Mr. Lawrence was elected an Associate Member of the American Society of Civil Engineers on November 8, 1909.

THOMAS JOSEPH LEAHY, Assoc. M. Am. Soc. C. E.[1]

Died August 8, 1940

Thomas Joseph Leahy was born in Central City, Colo., on September 3, 1879, the son of Marcus Leahy and Hanna (Foehy) Leahy, a pioneer mining family of Central City. He received his education in the public schools of Central City and was graduated from the Central City High School.

His engineering career started in March, 1903, with employment as chainman on a surveying party for the Denver, Northwestern and Pacific Railroad Company—that famous 50-mile section of line between Denver, Colo., and the Continental Divide, having twenty-nine tunnels, then under construction. Mr. Leahy rose rapidly from chainman to instrumentman on construction, having charge of work. He remained with the railroad until March, 1909.

After several short engagements on irrigation and railroad survey work, and construction on the Juniper Coal Company property, he became resident engineer on survey and construction of water conduits, and other work for the Denver Union Water Company. With the sale of the water plant to the City of Denver, on November 1, 1918, he was retained by the Denver Board of Water Commissioners as construction engineer until June, 1926. During this period of nearly sixteen years, Mr. Leahy had charge of the surveys and construction of six large, steel, concrete and wood stave conduit lines, forming an important part of Denver's water supply system. He also had charge of the construction of a diverting dam, and other important structures.

In 1927 and 1928, Mr. Leahy was resident engineer on the construction of 200 miles of railroad for the Fort Worth and Denver South Plains Railway Company in western Texas. From 1929 to 1939 he was employed on many short engagements, mostly in or near Denver, involving the construction of stockyards, hospital buildings, sewers, a swimming pool, South Platte River improvement, and other work, being employed by the Denver Union Stock Yards, the Public Works Administration, and the Work Projects Administration.

Mr. Leahy was one of the first members of the Colorado Society of Engineers. He was a member of the Roman Catholic Church.

In 1910 he was married to Agnes Flynn. He is survived by his widow; two sons, Thomas C. and Marcus C.; one daughter, Florence Leahy; one brother, Richard; and two sisters, Margaret (Mrs. Frank Jones), and Mary (Mrs. John Hughes). He was highly respected by all who knew him.

Mr. Leahy was elected an Associate Member of the American Society of Civil Engineers on October 1, 1913.

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[1] Memoir prepared by Dwight D. Gross, M. Am. Soc. C. E., and Arthur A. Matthews, Esq.

JAMES BELL LEIGH, Assoc. M. Am. Soc. C. E.[1]

Died February 16, 1942

James Bell Leigh was born in Columbus, Miss., on April 21, 1893, the son of William F. and Julia (Riddick) Leigh.

He received his grammar and high school education in Columbus. In 1910 he was graduated from Franklin Academy (Columbus) as valedictorian of his class. Then he was employed for a year as clerk in a drug store, Weaver and Harrington, in Columbus, and one year as clerk in the Yarborough Hotel, Raleigh, N. C.

He entered Vanderbilt University, Nashville, Tenn., in 1913, and was graduated in 1917 with the degree of Bachelor of Science in Engineering. Here he completed both the civil and mechanical engineering courses and was graduated cum laude, in spite of the fact that he was doing outside work to help defray the expenses of his college education. In his senior year he taught chemistry classes in the University.

Franklin Academy received affiliation with Vanderbilt University and entrance examinations were waived on account of the high record of James Bell Leigh on his entrance examinations and his work at Vanderbilt, by letter written from Vanderbilt University to Franklin Academy.

From June 1 to September 30, 1917, he was employed as engineer for the Suwanee Iron Company, superintending the reconstruction of two blast furnaces and the housing and sanitary facilities for the workmen at Grand Rivers, Ky. From September 30 to November 30, 1917, he was assistant engineer for the Hillman Land and Iron Company, supervising the surveying of land, establishing permanent corners, locating prospect holes, and surveying and staking out roads on the extensive holdings of this company in Kentucky with headquarters at Kuttawa, Ky.

From November 30, 1917, to April 4, 1918, he was employed as Arkansas representative, with headquarters at Jonesboro, Ark., for Klyce and Kackley, Engineers of Nashville, supervising the surveying, designing, inspecting and testing materials, and the construction of highways, municipal improvements, and drainage canals. He was in charge of work amounting to six million dollars at one time.

During World War I, from April 4, 1918, to July 21, 1919, he was in the U. S. Army Engineer Corps, 43d Engineers, which was afterward combined with the 20th Engineers, at St. Julien-Clensy, France. He was acting master engineer, repairing roads, etc., and on special detached service, serving in active duty in France for more than a year during this period.

From July 21, 1919, to June 1, 1923, he was again engineer in Arkansas for Klyce and Kackley with the same duties as before. From June 1, 1923, to August 1, 1925, he was chief construction engineer and principal assistant to chief engineer in charge of all field work and construction for Mississippi

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[1] Memoir prepared by William F. Leigh, Pottsville, Pa.

County Road Improvement District No. 1, building 58 miles of concrete and bituminous highway, and paving, storm sewers, curbs and gutters through six towns, From August 19, 1925, to September 1, 1925, he was engineer for Magehee, Ark., acting as consultant in designing. He supervised the construction of a sewage system and reinforced concrete sewage disposal plant, consisting of approximately eleven miles of 8 in. T.C. gravity sewer, one mile of 15 in. T.C. pressure sewer, one ejector pit with a 50-gal ejector, one pit with two 300-gal ejectors, one reinforced concrete double-chamber Imhoff tank, filter beds, etc.

From September 1, 1926, to June 1, 1931, he was employed as resident engineer, assistant district engineer, and district construction engineer for the Arkansas Highway Department, first in District No. 1 comprising nine counties and then in District No. 2 comprising eleven counties. He was in charge of all the construction work of an extensive construction program of the Highway Department in these districts. This included different types of highways, such as gravel, bituminous, concrete, etc., and such bridges and causeways, etc., as the Harrihan Viaduct, which is the 2¾-mile earth and concrete approach on the Arkansas side to the Harrihan Bridge at Memphis, Tenn.; the White River Bridges with the 4-mile approach at Clarendon, Ark., a reinforced concrete and steel high-arch bridge; the bridges and approaches at DuValle Bluff, Ark., on the U. S. Highway No. 70; and many others.

From December 1, 1931, to April 30, 1933, he was employed as contractor's engineer for the Merit Concrete Corporation, and was in charge of the engineering work on reinforced concrete arches, concrete deep foundation walls, etc., on ten new buildings for the New York Hospital at Wingdale, N. Y., and five new buildings for the New York State Hospital at Creedmore, Long Island, also on part of the reinforced concrete work for the New York Central Railroad overhead approach to the Parcel Post Building in New York, N. Y.

From April 30, 1933, to March 1, 1934, he was employed as professional superintendent in charge of CWA work at Langley Field, Virginia. This included remodeling work on wind tunnels, moving and building shops, etc.

He was employed from March 1, 1934, to January 1, 1935, as contractor's engineer in charge of the engineering work for the Washington Concrete Corporation of New York City on concrete fireproofing, cinder arches, reinforced concrete arches, concrete foundations, etc., on Knickerbocker Village, and the Italian Building, Radio City.

From February 16, 1936, to September 29, 1936, as project head, he was in charge of the Schuylkill River Survey for the Work Projects Administration (WPA) in Schuylkill County, Pennsylvania, in cooperation with the United States Engineer Department. From September 29, 1936, to March 13, 1938, he was employed as project head and project engineer for the WPA, in charge of a complete survey map for the City of Pottsville, Pa. From March 13, 1938, to the time of his last illness he was employed as assistant engineer, Classification 1, with Pennsylvania State Office of WPA, Special Consultant on Highway and Air Port Construction.

With his brilliant mind, superior ability, and conscientious application to all duties and responsibilities, he gave much to the world; but those who mourn his passing think most of his modesty, his unselfishness, his clever original wit, his nobility of character, and the quiet unassuming way in which he did for others. He bore misfortunes unflinchingly and alone, never asking others to share them; and what evil and wrong he discovered in others, he kept absolutely to himself.

It seemed especially fitting, because of his unswerving patriotic spirit, that he should be given a funeral with full military honors which his Forty and Eight and Legionnaires Comrades requested.

He was a Mason, a Shriner, an Elk, and a member of the American Legion and the Forty and Eight.

Mr. Leigh was elected an Associate Member of the American Society of Civil Engineers on January 15, 1923.

ROY EZEKIEL NELSON, Assoc. M. Am. Soc. C. E.[1]

Died January 7, 1941

On August 29, 1889, Roy Ezekiel Nelson was born at Halleck, W. Va. He was the tenth of eleven children of David and Melinda (Galliher) Nelson. Mr. Nelson's early life was spent on the farm where he developed his love of the outdoors and sports.

In 1913 Mr. Nelson was graduated from Fairmont State Normal College, in Fairmont, W. Va. Previous to his graduation, and interspersed with his studies, he taught in the public schools for a period amounting to about four years. Until interrupted by the first World War, Roy Nelson continued to pursue his education at West Virginia University, in Morgantown.

When the United States declared war, he was one of the first three in Monongalia County to volunteer for service. At first he was rejected because of a weak hip that had developed since childhood, but upon continued persistence he was admitted to the service of his country on September 5, 1917, and sent to Camp Lee, Virginia, where he was assigned to duty in the Field Artillery, and subsequently was graded as corporal.

On May 26, 1918, Corporal Nelson sailed for Europe. Upon arriving there he was trained at the artillery school in Saumur, France. When the war was over, Corporal Nelson was honorably discharged from the 314th Division of the Field Artillery and thereupon commissioned a Second Lieutenant of Field Artillery on March 23, 1919.

Once again Mr. Nelson continued his pursuit of education, and in 1921 received the degree of Bachelor of Science in Civil Engineering from West Virginia University. In February of the same year he joined the faculty of his alma mater as instructor of surveying.

The year of 1925 proved to be an outstanding one in Mr. Nelson's career, for in this year he received his Master's Degree in Civil Engineering from the

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[1] Memoir prepared by Richard T. Feller, West Virginia Univ. Student Chapter, Am. Soc. C. E., Morgantown, W. Va.

University, and also a Master's Degree in Highway Engineering from Iowa State College, in Ames. On gaining these honors he was promoted to assistant professor of railway and highway engineering, in which position he served until his sudden death.

Professor Nelson always occupied his summers very profitably, such as in 1930 when he served with the U. S. Engineers on dam and flood-control surveys. During the summers of 1935 to 1940 he was manager and instructor of the University's surveying camp.

In September, 1938, Professor Nelson was married to Dorotha Armstrong, a friend of many years standing, by whom he is survived. He is also survived by two sisters, Iva and Bessie (Mrs. James W. Boyers); and two brothers, George H. and Edward M. Nelson.

Professor Nelson took a great interest in the Masonic Order. He earned his Thirty-Second Degree and was a past master of Morgantown Union Lodge No. 4, A. F. and A. M. He was also a member of the American Society of State Highway Officials.

In the twenty years that Professor Nelson served the University as a teacher and adviser, he made contacts and associations that resulted in lasting friendships with many men now in the engineering profession who mourn his departure. His close association with members of the teaching profession repeatedly demonstrated his sterling qualities of friendship, honesty, and integrity. Although handicapped by a physical infirmity that caused him great suffering in the later years of his service, he always maintained a disposition to keep going under circumstances that would cause many men to falter and fail.

Professor Nelson was elected an Associate Member of the American Society of Civil Engineers on September 18, 1939.

HARLAN ROWE ROBBINS, Assoc. Am. Soc. C. E.[1]

DIED MAY 22, 1941

Harlan Rowe Robbins was born in Owosso, Mich., on August 15, 1893, the son of William V. and Lillian (Sargent) Robbins. In 1916 he was graduated from the School of Mines of the University of Utah, in Salt Lake City, with the degree of Bachelor of Science in General Engineering.

His father is associated with the mining industry, and Mr. Robbins' early work was in that field. For two years after leaving college he was manager and foreman of the Metallurgical Laboratory of the General Engineering Company in Salt Lake City. During the first World War he served in the Signal Corps of the United States Army and was given a special Signal Corps Officers' training course at Yale University, in New Haven, Conn. After the war he returned to Utah, and in 1919 served as a shift boss for the Western Utah Copper Company, followed by the position of foreman on structural contracting work. In the early part of 1920 he worked as a shift boss for the Vipont Silver Mining Company in Idaho.

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[1] Memoir prepared by a Committee of the Sacramento Section consisting of A. G. Mott, M. Am. Soc. C. E., Chairman, Walter E. Stoddard, Assoc. M. Am. Soc. C. E., and Thomas B. Waddell, M. Am. Soc. C. E.

In June, 1920, he moved to California and was appointed an assistant engineer with the California Railroad Commission. The Commission has under its jurisdiction the regulation of public utilities in the State of California, including water, telephone, telegraph, electric, gas, railroad, and other privately owned public utilities. Its duties include fixing rates and granting reparations for excess charges, the determination of the need for utility service, the granting of certificates of public convenience and necessity where such need exists, the control of utility financing, the authorization for the disposal of utility property, and the fixing of just compensation for property taken under condemnation. During the fourteen years that Mr. Robbins was with the Commission, he prepared appraisals of lands, leaseholds, right of ways, easements, and franchises to assist the Commission in the foregoing duties and in other duties when appraisals were directly or indirectly involved.

In 1933 California adopted a major change in its tax system, which required the appraisal for tax purposes, for the first time in twenty-five years, of all properties of public utilities. For this purpose the Valuation Division of the State Board of Equalization was created, and Mr. Robbins was chosen as one of the most qualified land appraisers in California to take charge of the land valuation work of the Division for this major task. This work required the appraisal each year of many classes of lands widely situated throughout the State of California and having an aggregate value in excess of $200,000,000. In this work he won for himself a reputation of outstanding ability.

At the time that he took up his duties with the Valuation Division, Mr. Robbins moved to Sacramento, Calif. He resided there until the time of his death, which occurred very suddenly while he was away on a business trip in Los Angeles, Calif.

Mr. Robbins had various outside interests and hobbies, among which were rifle marksmanship, amateur photography, and astronomy. In following the latter subject, he developed an interest in lenses and telescope manufacture, and contributed in no small measure to the development of amateur interest in astronomy in his community because of his ability in grinding and mounting lenses.

In 1917 he was married to Ethelyn Slater in Salt Lake City. He is survived by his widow and parents.

Mr. Robbins was elected an Associate Member of the American Society of Civil Engineers on July 25, 1932.

EARL MILTON SPENCER, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 7, 1941

Earl Milton Spencer was born at Topeka, Kans., on February 25, 1889, and died on January 7, 1941. He was the son of Francis Milton Spencer and

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[1] Memoir prepared by F. W. Epps, M. Am. Soc. C. E.

Catharine (Miller) Spencer. His early education was obtained in the public schools of Topeka, after which he attended Washburn College, in Topeka, for two years, where he took the engineering course.

From January, 1910, to November, 1911, he was superintendent and engineer for his father, who was a general contractor. From 1911 until the time of his death he was owner of the firm of F. M. Spencer and Son, General Contractors. During this time Mr. Spencer was in charge of the following large contracts:

1924—Capital Building and Loan office building at Topeka
1924—Parsons High School Building at Parsons, Kans.
1926—Central National Bank and Trust Company at Topeka
1926—Ursuline Academy at Paola, Kans.
1927—Mutual Ice and Cold Storage Company at Topeka
1930—Hospital at Topeka for Security Benefit Association
1930—Clear Water Reservoir at Topeka
1930—Residence for Mrs. Frank P. MacLennan at Topeka
1931—National Bank of Topeka
1934—Depot for Atchison, Topeka and Santa Fe Railway Company at Oklahoma City, Okla.
1938—Residence for Alfred M. Landon, at Topeka
1938—Municipal Auditorium and offices at Topeka
1939—Residence for Mrs. Dan L. Meisinger at Topeka
1939—Manufacturing Building for John Morrell and Company at Topeka

All of these contracts were executed faithfully and honestly.

On April 17, 1912, Mr. Spencer was married to Zella Chapin. Two children were born to this union, Mary Catharine and Francis Milton. His wife and the two children survive him.

Mr. Spencer was elected an Associate Member of the American Society of Civil Engineers on January 16, 1922.

JOHN JOSEPH SWEENEY, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 6, 1942

John Joseph Sweeney, the son of Patrick M. Sweeney and Mary (Morrow) Sweeney, was born in Eckley, Pa., on February 28, 1889. He attended the public school and his college preparatory work was taken at the Mining and Mechanical Institute of Freeland, Pa., where he was graduated with honors in June, 1907.

After graduation, from June, 1907, to September, 1908, and the subsequent summers of 1909, 1910, and 1911, he worked as a rodman, chainman, transitman, and chief of party for the G. B. Markle Coal Company of Jeddo, Pa.

In September, 1908, Mr. Sweeney entered Villanova College at Villanova, Pa., and was graduated in 1912 with the degree of Bachelor of Science in Civil Engineering. He received the degree of Civil Engineer from Villanova in

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[1] Memoir prepared by Wilmer Z. Kline, M. Am. Soc. C. E.

June, 1918. While at college Mr. Sweeney was active in the Phi Kappa Pi Society and was a member of the track team.

After graduation he obtained a position with the New York Central and Hudson River Railroad Company as a rodman, inspector, and field engineer on its property subdivision work on grade-crossing elimination, remaining there until November, 1913. He then entered the employ of Villanova College as instructor in the Civil Engineering Department. In 1914 he became an associate professor in the Civil Engineering Department, and in 1917 he was appointed professor of highway engineering. He taught at the College in the latter capacity until June, 1930.

While teaching, Mr. Sweeney went into the practical field of engineering each summer and gained much engineering experience. In the summer of 1914 he traveled in Europe and was there when the first World War started.

From June to September, 1915, he worked as an assistant engineer for the Lehigh Valley Coal Company on the valuation of coal lands. He spent his vacations in 1916, 1917, 1918, and 1919 as superintendent for H. P. Friend, Inc., Engineers, of Norristown, Pa., on general building construction, railroad construction, estimating and street paving.

Mr. Sweeney spent the summer of 1920 in travel. During the vacation periods of 1921 and 1922 he taught summer school at Villanova College. In the summer of 1923 he acted as inspector of building materials, steel, stone, concrete, and concrete highways for the Highway Department of Monmouth County, New Jersey. During vacations in 1924, 1925, and 1926, he worked on land development and subdivision work, laying out the Merion Golf Manor tract for Over and Tingley at Oakmont, Pa. In the summer of 1927 Mr. Sweeney was resident engineer on the stadium construction at Villanova College. During the following two summers he was resident engineer for Carl R. Camp, Inc., on roads and bridge construction.

In June, 1930, after much deliberation, Mr. Sweeney gave up teaching to accept membership in the firm of Carl R. Camp, Inc., Engineers and Contractors, with whom he had been associated the two previous summers. During this association, as resident engineer, he actively supervised the construction of the Sewerage Disposal Plant for the City of Burlington, N. J., and the following bridges in Delaware County, Pennsylvania: The Fourth Street Bridge in Chester, Pa.; Waterville Bridge No. 9, Delaware County, between Nether Providence and Chester; Crozierville Bridge over West Branch of Chester Creek, Ashton Township, Delaware County; Johnsons Mill Bridge No. 11, West Branch of Chester Creek in Ashton, Pa.; and the State Bridge 22d Street in Chester, Pa.

In 1932 their contracts were all finished and a major depression was in progress; to employ his time to advantage Mr. Sweeney attended the Graduate School of the University of Pennsylvania, at West Philadelphia, from September, 1932, to June, 1933, majoring in mathematics and physics. In March, 1934, he obtained a position as general superintendent on Pennypack Park, Philadelphia, Pa., on road construction. He gave up this work in June, 1934, to become resident engineer on the construction of the State Bridge 22d Street between Ridley Township and the City of Chester.

In September, 1935, Mr. Sweeney went to the Navy Yard in Philadelphia on construction of roads, etc. In July, 1940, he was appointed associate civil engineer, a post which he held until his death.

When the construction of five large buildings in the Navy Yard was undertaken by the Public Works Administration, Mr. Sweeney was placed in charge in the field of construction of all of the buildings. This work was handled successfully and expeditiously. He was subsequently placed in charge of the specification group of the Design Section and for several years until his death he was responsible for the preparation of specifications for a large amount of construction work that was performed by bid contracts. Mr. Sweeney's part in getting various projects under contract was handled carefully and with a minimum amount of difficulties. The specifications prepared by his group were found to describe the work accurately and without conflict.

After leaving the teaching field for engineering work, Mr. Sweeney kept in close touch with the student activities at Villanova College and did much to further student participation in the Student Chapter of the Society. He served on the Student Chapter Board for many years. As a member of the Reception Committee, Meetings Committee, and the Villanova representative for Student Chapters, all of the Philadelphia Section of the Society, he was a tireless worker in the interests of the Section.

In June, 1916, Mr. Sweeney was married to Florence Marie O'Rourke of Philadelphia, who survives him.

Mr. Sweeney was elected an Associate Member of the American Society of Civil Engineers on January 16, 1922.

JAMES SYKES, Assoc. M. Am. Soc. C. E.[1]

Died August 31, 1941

James Sykes was born at Mossley, Cheshire, England, on May 18, 1884, the son of Hezekiah and Eleanor (Hollingworth) Sykes. At the age of twenty two, he was graduated from the Municipal School of Technology, in Manchester, England, and came to the United States shortly thereafter.

His first work in engineering after reaching the shores of America was that of inspector on the Sixth Avenue Extension, Hudson and Manhattan Railroad, in New York, N. Y., from April, 1907, to June, 1908. While with this company he had the experience of construction in concrete, waterproofing, excavation, timbering, and other phases covering the general scope of civil engineering as practiced on such contracts. His training in actual construction work at this time proved of great value in later years when he assumed definite responsibilities in the field of railroad engineering.

Early in the summer of 1908, he began work with the Oklahoma Strenolith Paving Company, at Oklahoma City, in the design and construction of paving machinery. This assignment led to his appointment, in October, 1909, as assistant city engineer. This work included the design and construction of sewers, a disposal plant, and other municipal works.

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[1] Memoir prepared by Ralph L. Parshall, Assoc. M. Am. Soc. C. E.

James Sykes' ambitions and interest brought him back into the field of railroad engineering when he took up his work with the Kansas City Terminal Railway Company, at Kansas City, Mo., in January, 1912. His activities here centered on the design of classification yards and track material, together with the many other details related to the job.

To broaden his field of experience, he began work as assistant engineer with Burns and McDonald, in Kansas City, in January, 1915, and was supervising engineer on the Montrose, Kans., and Orchard City, Kans., water works system. While with this firm of engineers, he made valuation studies on public utilities and was also engaged in other important lines of engineering.

After completing his work at Kansas City, he went to Denver, Colo., in July, 1916, as engineer for the Great Western Sugar Company. Because of his experience in general railroad work, he was assigned to the engineering staff of the Great Western Railway Company, a subsidiary of the Sugar Company. His work with this company covered valuation studies, which brought him national recognition. He was given the responsibility of maintenance of the railroad, operating in northern Colorado, where the line connected with several sugar factories in the transportation of sugar beets, livestock, and other agricultural commodities. His ability in the field of railroad engineering was soon recognized and he was later made chief engineer of this important transportation line. He held this position at the time of his death.

James Sykes possessed a pleasing personality; he was ambitious—a characteristic necessary for the progressive engineer—a most excellent companion, and a true friend. His home library, of many volumes, covered a wide range of subjects that were indicative of his broad knowledge of engineering and scientific matters. His training and experience in his chosen profession made him a keen observer, a leader, and one capable of assuming responsibilities under adverse conditions. He was a treasurer of valuable old records, a correlator of the past and present general world conditions, and a traveler in his own right.

He was a lover of the great outdoors, finding both interest and pleasure in his contact with Nature. At home, he was a contented man, and prided himself on his talent for cooking and sewing. He was also a musician and painter of landscapes. His untimely death, due to a heart ailment, was a distinct shock to his many friends and close acquaintances, who suffer in the loss of a true friend and a thorough gentleman.

He was married to Emmaline Leachman, at Louisville, Ky., on April 17, 1910. There were no children. His widow survives him.

He was associated with a number of professional organizations and was a leader in local civic activities. He was a member of the Episcopal Church and of the Stayley Masonic Lodge of Stalybridge, England.

Mr. Sykes was elected an Associate Member of the American Society of Civil Engineers on January 14, 1918.

HANSON ZIMRI WILSON, Assoc. M. Am. Soc. C. E.[1]

DIED DECEMBER 27, 1940

Hanson Zimri Wilson was born on April 22, 1874, near "Brick Meeting House" on the site granted by William Penn to his followers in Nottingham, Pa. The post office, at that time called "B. M. House," was later moved to the nearest railway station at Sylmar, Md. Mr. Wilson's parents were the late William and Mary (Taylor) Wilson. His maternal grandfather, Zimri Taylor, was married to Julia Brown, whose family had been among the first settlers of Penn's Nottingham Lots in 1701. His paternal grandparents, Samuel Dixon and Mary (Pierson) Wilson, founded the homestead in Maryland, which was also property belonging to the Nottingham Lots previous to the survey of the Mason and Dixon Line in 1765.

Hanson Wilson attended Swarthmore College at Swarthmore, Pa., and in 1896 was graduated with first honor, with the degree of Bachelor of Science. Early in the following year he started on surveying work for the Erie Railroad Company at Port Jervis, N. Y. He also continued his studies in civil engineering, and prepared a thesis entitled "Railway Track Maintenance." He was granted the degree of Civil Engineer by the managers of Swarthmore College in 1899.

In 1899 Mr. Wilson became assistant engineer in charge of the Engineering Corps, Maintenance of Way Department, for the Erie Railroad Company, and had responsible assignments in Port Jervis and in Jersey City, N. J. Some of these duties were: The designing of engineering structures, re-alining and putting spiral ends on curves, preparing plans and estimates for the cost of track extensions, yard design, signaling for interlocking, new water stations, and preparing plans and laying out masonry for new bridges.

From 1901 to 1905, as supervisor for the Erie Railroad Company at Bradford, Pa., he was in charge of engineering and track foremen. He also acted as engineer and prepared plans for, and directed the building of, spur tracks, one with a 30-ft span bridge. Similar engagements until 1910 completed a continuous service of nearly fourteen years with the Erie Railroad Company.

At this time Hanson Wilson entered the employ of the Standard Oil Company of New York as engineer in its Construction Department. In 1912 he became associated with the Standard Oil Company of New Jersey. He was one of the original members of the Civil Engineering Department when it was organized in New York, N. Y. Later, with head office at Newark, N. J., he was construction engineer in charge of the design and building of domestic trade stations in the State of New Jersey. During this engagement there were favorable comments from other divisions of the company on the efficient way in which the engineering work was conducted. Mr. Wilson was one of the first to employ photography as a means of recording the progress of construction work in the field. On his retirement in 1932 he was presented with a service pin in recognition of his twenty years of service.

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[1] Memoir prepared by C. Victor Wilson, Licensed Professional Engr., State of New York, Nassau County, Dept. of Public Works, Mineola, N. Y.

In 1921 he built a colonial dwelling in the Midwood Manor section of Brooklyn, N. Y., which became a very attractive residential district. Here he resided for nearly twenty years, making annual vacation visits to his relatives in Maryland. There were four brothers and one sister. Three of his brothers survive him. Although Hanson remained a bachelor, he was always interested in the younger generation of his relatives and friends, several of whom he assisted in obtaining an education. Others were encouraged by his timely advice when getting started on their careers in civil engineering.

Mr. Wilson was a member of the Men's Club of the Religious Society of Friends in Brooklyn, which he attended regularly for many years; he was also a member of its Friendly Hand Society. Likewise, he served on committees of the Yearly Meeting of Friends. He was fond of taking long walks with a Friend and of taking snapshots with his favorite camera, some of which he made available to the Society.

Different mathematical subjects, including astronomy, were of special interest to him. In 1935 and 1936 he attended classes in astronomy at Columbia University in New York City. At this time he became a member of the Amateur Astronomers Association, with headquarters at the American Museum of Natural History in New York City. On January 1, 1925, he took photographs of the total eclipse of the sun from the top of Bear Mountain, New York. Enlarged copies of these photographs, with details of the observation, were contributed to the Amateur Astronomers Association. Also, in 1934 he contributed negatives and photographic prints of star trails to the collection of the American Museum of Natural History. For these and other activities he became in 1936 a member of the Columbia University Camera Club.

Hanson Wilson was highly respected by his associates in professional life and elsewhere. His reliability and uprightness of character won the esteem of many friends.

Mr. Wilson was elected an Associate Member of the American Society of Civil Engineers on June 30, 1911.

JOHN SOULE BUTLER, JR., Jun. Am. Soc. C. E.[1]

DIED DECEMBER 18, 1940

John Soule Butler, Jr., will be but a name to the many who read these pages; but to those who shared the privileges of intimate friendship with John the name brings to life an unusually gifted young man well founded in sincerity, integrity, and intellect.

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[1] Memoir prepared by Almon H. Fuller and Frank Kerekes, Members, Am. Soc. C. E.

The son of the late Lt.-Col. John Soule Butler, M. Am. Soc. C. E., and Abbie Holton (Throop) Butler, John was born on March 18, 1914, in Nashville, Tenn. Colonel Butler, a graduate of Vanderbilt University in Nashville, had been with the Corps of Engineers in civilian capacity for twenty-three years when he was commissioned a Major. One of the many outstanding achievements of Colonel Butler was a report upon the Columbia River Basin in the State of Washington, including the possibilities of the Grand Coulee Dam.[3]

As a son of an army engineer, John benefited from travel and broad contacts, as exemplified by his elementary school attendance at the Wilson Dam County School near Florence, Ala.; at the Balboa Grade School, Panama Canal Zone; and at the Reid Grade School, Seattle, Wash. He gave early indication of his brilliance by completing "With Honors" in two and one-half years the regular four-year high school course in Seattle. At Iowa State College, in Ames, where he enrolled in 1931, he soon established himself as an exceptional student who was more interested in high achievement than in grades. Upon graduation in June, 1936, from the Department of Civil Engineering with the degree of Bachelor of Science, his scholastic average placed him at the top of the list of civil engineering graduates. He was elected to the honor societies of Tau Beta Pi and Phi Kappa Phi.

The summer after John's freshman year he worked for the General Land Office on surveys in northern Minnesota. He left college during the junior year to work again with the General Land Office, returning a year later to finish his college course. Immediately following graduation he was employed by the Chicago, Milwaukee and St. Paul Railroad Company but had to relinquish his position in less than a year because of ill health. After two years, when his health had improved sufficiently, he began work for the Building Department of the City of Denver, Colo. In September, 1940, he left Denver to accept an appointment in the National Advisory Committee on Aeronautics in Virginia, where he was at the time of his death.

A real student by nature, he read and studied almost constantly. During the last year of his life he took graduate courses in mathematics, leading toward a Master's Degree, at the University of Colorado in Boulder. Barred by physical disability from participation in all forms of athletics except swimming, his recreational interests lay largely in chess and contract bridge. His college roommate for three years writes:

> "As a man, John was loyal and dependable. He did not have a wide circle of friends, but was a very sincere, close friend of those who knew and understood him. He had the highest respect for those who earned their own way through college. He loved the simple way of living. Whatever he undertook, work or play, he put into it the best that was in him. He played chess and bridge with the champions. He was an excellent swimmer and, although he was not on the team, he acted gratuitously as the tutor for many varsity swimmers who needed this type of coaching more than training in swimming. He was a profound student of mathematics. He had the ability to set up facts which were

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[3] *Civil Engineering,* September, 1931, p. 1075; see also "Columbia River and Minor Tributaries," H. R. Doc. No. 103, 73d Cong., 1st Session, Vol. 2.

necessary for the solution of a problem. He accepted no man's word, theory or solution until he had worked out the proof by his own reasoning."

Those who really knew him, whether classmate, faculty, or employer, recognized a keenness of intellect which few possess. The tragedy of an early death has deprived society of the benefits of a very unusual constructive mind. The loss is lessened, however, by the inspiration and the guidance which John had given his associates for more than a decade and by the fact that his influence will be reflected in the lives and the accomplishments of others.

On July 27, 1935, he was married to Mary Evelyn Smith who attended Mundelein College in Chicago, Ill. A daughter, Jill Butler, was born on June 2, 1936. Besides his widow and daughter, John is survived by his mother; by two brothers, Phares Throop Butler and Francis Stringfield Butler; and by a sister, Clyde Butler (Mrs. Stanley DeVore).

Mr. Butler was elected a Junior of the American Society of Civil Engineers on November 9, 1936.

MAURICE LEONARD SHULTZ, Jun. Am. Soc. C. E.[1]

Died April 29, 1941

Maurice Leonard Shultz was born in Atlantic City, N. J., on April 17, 1913, the son of John and Fannie (Sacks) Shultz. He received his primary education in Atlantic City and Pleasantville, N. J., and then followed in the footsteps of his brother Ralph in becoming a civil engineer. He received his degree in Civil Engineering from the Drexel Institute of Technology, Philadelphia, Pa., in June, 1936. While at Drexel he was active in the engineering profession, being employed by the United States Coast and Geodetic Survey. He was also a member of the Reserve Officers Training Corps.

After graduation, Mr. Shultz became a civilian employee of the United States Engineer Department where he devoted most of his time to the engineering technique of flood control. He left the flood control field to engage in the work of rebuilding army camps. Soon after the call to the colors (February 9, 1941) he entered active service as a Second Lieutenant in the regular Army Engineer Corps. After a brief stay at Fort Dix, New Jersey, he was transferred to Fort Buchanan, Puerto Rico, as a First Lieutenant in the Engineer Corps. His duties were particularly concerned with the building of strategic roads and bridges on this vital island. He was in the process of redesigning and reconstructing bridges built 200 years ago by the Spanish, so that they would be wide enough and capable of supporting military loads, when he was stricken with lobar pneumonia. After a short illness Lieutenant Shultz passed away on April 29, 1941. His sudden death came as a tremendous shock to his many friends in the social and business world.

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[1] Memoir prepared by I. M. Levitt, Asst. Director Fels Planetarium, The Franklin Inst., Philadelphia, Pa., and Edward Semel, Philadelphia, Pa.

The outstanding qualities of Lieutenant Shultz were his character, personality, and fellowship. These features endeared him to all who considered him a friend. His hobby, photography, became his servant in his work; he made extensive photographic surveys of all operations in which he was engaged.

He is survived by his mother, his brother, Capt. Ralph Shultz, and three sisters, Esther (Mrs. L. W. Potler), Dora (Mrs. M. E. Baker), and Tilma.

Lt. Maurice Leonard Shultz's early departure from this earth cut short a promising career, for, with his character, vision, and keen intellect, much was expected of him by those who knew him well.

Lieutenant Shultz was elected a Junior of the American Society of Civil Engineers on December 14, 1936.

HENRY JOHN WENDEROTH, Jun. Am. Soc. C. E.[1]

DIED JULY 2, 1941

Henry John Wenderoth was born at Salem, Ore., on November 19, 1914, the son of Harry J. and Grace (Keizer) Wenderoth.

His schooling was begun in the public schools of Salem, and he attended high school at North Bend, Ore. He entered Oregon State College at Corvallis, Ore., when he was seventeen, and received the degree of Bachelor of Science in June, 1937. In college he was elected to the Sigma Tau National Society. He was also a member of the Alpha Chi Rho social fraternity.

As a graduating student Mr. Wenderoth was awarded the Portland, Ore., Section prize for his paper entitled "Forecasting Rainfall by Mean Seasonal Distribution."[2] The prize was the entrance fee into the Society and dues for the first year.

Mr. Wenderoth's first experience with the Topographic Branch of the U. S. Geological Survey was in 1934 when he worked as a rodman during the summer vacation. He worked also during the next season; then, after his graduation in 1937, he took up the work again, first as recorder and then as instrumentman, until July 1, 1939, when he received his Civil Service appointment as junior topographic engineer. He had mapping or leveling assignments in the states of Oregon, Washington, California, Arizona, and Nevada. He was very much interested in photogrammetry and was engaged in this comparatively new field of engineering at the time of his last illness. His death occurred at Salem on July 2, 1941.

In the brief period allotted to him, Mr. Wenderoth had already demonstrated a standard of industry and performance that left no doubt of his ability to excel in his chosen field of work. He was well liked by his associates and had those outstanding traits of character that are essential to success in any field of endeavor.

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[1] Memoir prepared by George S. Druhot, Assoc. M. Am. Soc. C. E.

[2] *Civil Enginering*, November, 1937, p. 770.

On March 24, 1939, Mr. Wenderoth was married at Del Monte, Calif., to Bettyrae Ella McGahan, who survives him. He is also survived by his parents and a grandmother, Mrs. Mary Wenderoth.

Mr. Wenderoth was elected a Junior of the American Society of Civil Engineers on November 22, 1937.

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OF THE

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